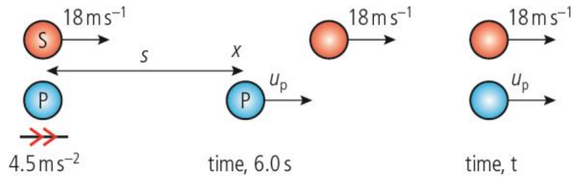
A Space, time and motion

A.1 Kinematics

Practice questions

1.



(**a**) Car S travels at constant velocity so $s_S = 18 \times t$ (1)

(**b**) Police car has constant acceleration so: $s = ut + \frac{1}{2}at^2$

$$s_P = 0 + \frac{1}{2} \times 4.5 \times 6.0^2 = 81\text{m}$$ (1)

(**c**) Using $v = u + at = 0 + 4.5 \times 6.0 = 27\text{ m s}^{-1}$ (1)

You could use $v^2 = u^2 + 2as \Rightarrow v = \sqrt{2 \times 4.5 \times 81} = 27\text{ m s}^{-1}$ but it would not be good practice to use s in case you calculated it wrongly in part (**b**). However, you would technically not lose marks if you did this (error carried forward).

(**d**) The police car travels at constant velocity from 6.0 s until the cars meet at time t. So, time at constant velocity $= (t - 6.0) \Rightarrow x = 27(t - 6.0)$ (1)

(**e**) The police car catches up (draws level) with car S when they have traveled the same distance.

Distance traveled by S in time t is $18t$

Distance traveled by P is $81 + 27(t - 6.0)$

So, when they meet $18t = 81 + 27(t - 6.0)$

$18t = 81 + 27t - 162$

$18t - 27t = 81 - 162$

$t = 9.0\text{ s}$ (2)

(*Total 6 marks*)

2. (**a**) (**i**) The height of the building is the vertical displacement: 4 m (1)

(**ii**) Considering vertical motion with upwards as positive, the variables are:
$u = 14\sin 60$; $s = 4$; $a = -9.81$

$$s = ut + \frac{1}{2}at^2$$

$$4 = 14\sin 60 \times t - \frac{1}{2} \times 9.81 \times t^2$$

Rearranging: $4.91t^2 - 12.1t + 4 = 0$

This is a quadratic equation with two solutions for t : 0.39 and 2.1 s

The first of these is during the ball's rise and the second (the correct answer) is during its fall.

So time of flight = 2.1 s (3)

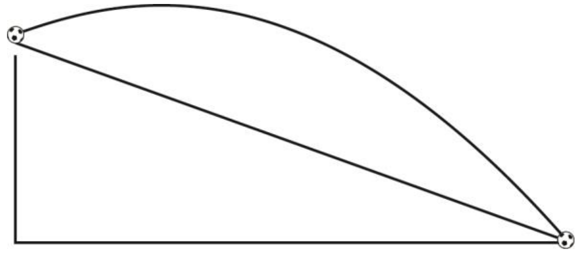
(**iii**) Considering horizontal motion, speed is constant (= $14\cos 60$) and the time is the same as the time of flight.

$s_h = 14\cos 60° \times 2.1$ = 15 m (2)

(**b**) On the way up, air resistance is in the same direction as weight. On the way down, air resistance is in the opposite direction to weight. The magnitude of acceleration is greater on the way up than down so the time to reach the highest point is smaller than the time back to the ground. (3)

(*Total 9 marks*)

3. (**a**) The ball is a projectile and so will follow a parabolic path. This should be symmetrical on either side of the maximum height.



(2)

(**b**) The boy at the top of the hill will kick the ball with a smaller magnitude of velocity than the boy at the bottom. The direction of the shot from the boy at the top will be at an angle closer to the horizontal; the angle of the shot from the boy at the bottom will be closer to the vertical. (2)

(*Total 4 marks*)

4. Instantaneous speed is the gradient of the tangent at a point and average speed is the gradient between the start and end positions. These are parallel when the tangent for instantaneous velocity is drawn at C.

Answer: C (1)

(*Total 1 mark*)

5. Take care to notice that the axes are labelled v and s. Therefore, we need to consider an equation that involves u (0 since starting at rest), a, v and s: $v^2 = u^2 + 2as$. Therefore, $v \propto \sqrt{s}$

Answer: A (1)

(*Total 1 mark*)

6. (**a**) You are being asked for the time to cover the horizontal range and so can use the horizontal component of the speed, which is constant. Listing the variables: $s = 11.9$; $u = 64\cos 7.0°$

$t = \frac{s}{u} = \frac{11.9}{64\cos 7.0°}$ = 0.19 s (2)

(**b**) Now use this time (to reach the net) in a vertical component calculation to find out the height of the ball when in line with the net. Listing the variables and taking downwards to be positive: $u = 64\sin 7.0°$; $a = 9.81$; $t = 0.19$

$s = ut + \frac{1}{2}at^2 = 64\sin 7.0° \times 0.19 + \frac{1}{2} \times 9.81 \times 0.19^2$ = 1.66 m

This is vertical distance fallen. The height of the ball is 2.8 – 1.66 = 1.1 m, which means the ball passes over the net (height of 0.91 m). (3)

(**c**) The horizontal component of velocity is unchanged: $v_h = 64\cos 7.0°$ = 63.5 m s^{-1}

The vertical component of velocity is calculated across the full height from which the ball was hit: ${v_v}^2 = {u_v}^2 + 2as \Rightarrow v_v = \sqrt{(64\sin 7.0°)^2 + 2 \times 9.81 \times 2.8}$ = 10.8 m s^{-1}

From Pythagoras: $v = \sqrt{63.5^2 + 10.8^2}$ = 64.4 m s^{-1} (2)

(*Total 7 marks*)

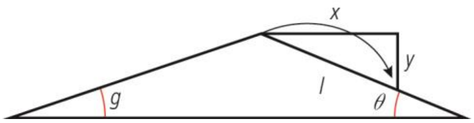
7. Listing the variables: $u = 0$; $v = 330$; $a = 9.81$

$v^2 = u^2 + 2as$

$s = \frac{330^2 - 0}{2 \times 9.81}$ = 5560 m (2)

(*Total 2 marks*)

8. Start by drawing a diagram:



(**a**) Taking the horizontal component, speed is constant.

Horizontal displacement: $x = l\cos\theta$

$$t = \frac{l\cos\theta}{u\cos\theta} = \frac{l}{u}$$ (2)

(**b**) Taking the vertical component, upwards as positive and knowing the time until the motorbike reaches P, the variables are: $u_v = u\sin\theta$; $a = -g$.

Vertical displacement: $y = -l\sin\theta$

$$s = ut + \frac{1}{2}at^2$$

$$-l\sin\theta = u\sin\theta \times \frac{l}{u} - \frac{1}{2}g\left(\frac{l}{u}\right)^2$$

Cancelling u: $-l\sin\theta = \sin\theta \times l - \frac{1}{2}g\left(\frac{l}{u}\right)^2$

Rearranging and grouping terms: $\frac{1}{2}g\left(\frac{l}{u}\right)^2 = 2l\sin\theta$

Simplifying and concluding with respect to l: $l = 4\frac{u^2\sin\theta}{g}$ (3)

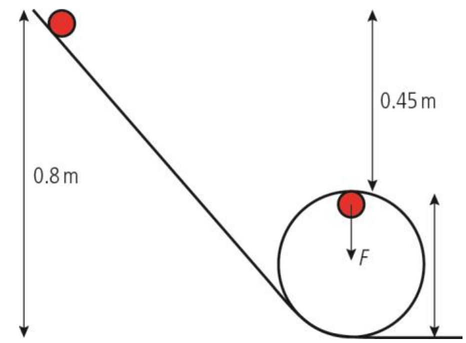
(*Total 5 marks*)

A.2 Forces and momentum

Practice questions

1. (**a**) Velocity is a vector so as the direction of the car changes, velocity must change. Acceleration is the rate of change of velocity so if velocity changes the car must accelerate. (2)

(**b**) (**i**)



(1)

(**ii**) Weight and the normal force both act downwards. Not centripetal force; this is the resultant. (2)

(**iii**) If no energy loss then loss of E_P = gain in E_K.

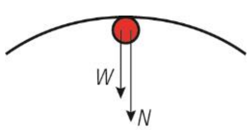
$mg\Delta h = \frac{1}{2}mv^2$

$\Rightarrow 10 \times 0.8 - 0.35) = \frac{1}{2}v^2$

$\Rightarrow v = \sqrt{9} = 3\ \text{m s}^{-1}$ (3)

(**iv**) When moving in a circle $F = \frac{mv^2}{r} \Rightarrow F = \frac{0.05 \times 3^2}{\frac{0.35}{2}}$

Now this force is caused by normal force and weight.



$\Rightarrow 2.6 = N + W$ where $W = 0.5$ N

so $2.6 = N + 0.5$

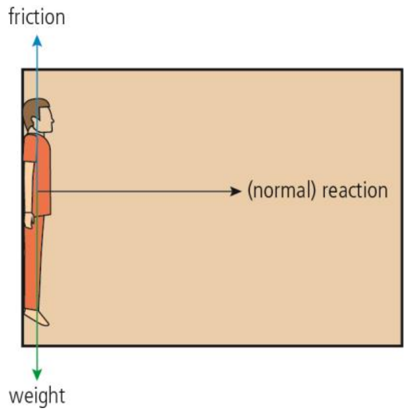
$N = 2.6 - 0.5 = 2.1$ N (4)

(*Total 12 marks*)

2. (**a**) Coefficient of friction is defined by the equation $F = \mu R$ so μ is the ratio of friction force divided by normal reaction force. (1)

(**b**) (**i**) Since the person is not moving relative to the wall, the friction would be static friction. (2)

(**ii**)



(3)

(**c**) (**i**) The minimum speed is such that friction = weight

$mg = \mu R$

So $R = \frac{mg}{\mu} = 80 \times \frac{10}{0.4} = 2000\ \text{N}$ (2)

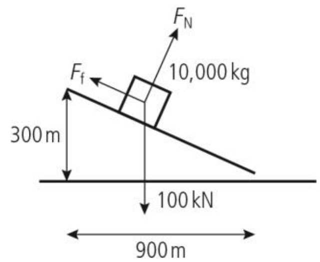
(**ii**) The body is moving in a circle so the unbalanced force = centripetal force

$R = \frac{mv^2}{r}$

$V = \sqrt{\frac{Rr}{m}} = \sqrt{\frac{2000 \times 6}{80}} = 12\ \text{m s}^{-1}$ (2)

(*Total 10 marks*)

3.



(**a**) Adding forces to the diagram and taking $g = 10\ \text{N kg}^{-1}$:

Components parallel to slope: $F_f = 100\sin\theta$

Now, $\tan\theta = \dfrac{300}{900} \Rightarrow \theta = 18.4°$

$F_f = 100\sin 18.4° = 31.6\ \text{kN}$ (3)

(**b**) On sliding, $\mu_d F_N = 100\sin 18.4°\,\text{kN}$

Components perpendicular to slope: $F_N = 100\cos 18.4°\,\text{kN}$

$\mu_d F_N \times 100\cos 18.4° = 100\sin 18.4°$

$\mu_d = \dfrac{100\sin 18.4°}{100\cos 18.4°} = \tan 18.4° \Rightarrow \mu_d = 0.33$ (3)

(**c**) (**i**) $\mu_d = 0.2$ so $F_f = 0.2 \times 100\cos 18.4°\,\text{kN}$

Resultant parallel to slope: $100\sin 18.4° - 0.2 \times 100\cos 18.4° = 12.59\ \text{kN}$

$a = \dfrac{12.59}{m} = 1.26\ \text{m s}^{-2}$ (2)

(**ii**) $v^2 = u^2 + 2as$

Now, from Pythagoras, $s = \sqrt{900^2 + 300^2} = 949\ \text{m}$

$v = \sqrt{2 \times 1.26 \times 949} = 48.9\ \text{m s}^{-1}$ (2)

(**d**) (**i**) $F_f = \mu_d F_N$, where $F_N = 100\ \text{kN}$

$F_f = 0.2 \times 100 = 20\ \text{kN}$ (1)

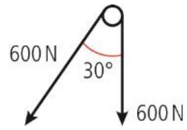
(**ii**) $v^2 = u^2 + 2as$

Now, $a = \dfrac{F_f}{m} = \dfrac{-20000}{10000} = -2\ \text{m s}^{-2}$

$s = \dfrac{0 - 48.9^2}{2 \times (-2)} = 597.8\ \text{m}$ (2)

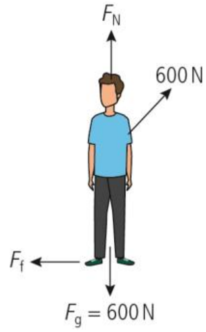
(*Total 13 marks*)

4. (**a**) (**i**) Assuming the rope is inextensible, and the pulley is smooth:



F_t = 600 N (1)

(**ii**) Drawing the forces on the standing student:



Vertical components: $F_N + 600\cos 30°$ = 520 N

$F_N = 600 - 520$ = 80 N

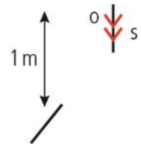
Scale reading is 8 kg (2)

(**iii**) Horizontal components: $F_f = 600\sin 30°$ = 300 N (2)

(**b**) If the rope jerks, the acceleration would be large. This means the tension would increase and, in turn, that the horizontal component of tension would increase. A greater frictional force would be needed to remain stationary. (2)

(*Total 7 marks*)

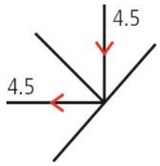
5. (**a**) (**i**)



$v^2 = u^2 + 2as = 0 + 2g \times 1 = 20$

$v = \sqrt{20}$ = 4.5 m s^{-1} (1)

(**ii**)



Horizontal component is 4.5 m s^{-1} (1)

(iii) Change in velocity perpendicular to the plate is $-4.5\cos 45° - 4.5\cos 45° =$ 6.4 m s^{-1}

Force is rate of change of momentum = mass per unit time × change in velocity

$F = 5 \times 6.4$ = 31.8 N (3)

(b) (i) The horizontal component of velocity changes, so there is an unbalanced force on the sand. According to Newton's third law, an equal and opposite force is exerted on the truck. In turn, there must be a frictional force opposing this in order for the truck to remain stationary. (3)

(ii) Change in velocity = 4.5 m s^{-1} from part (**a**) (**ii**)

Force = mass per unit time × change in velocity

$F = 5 \times 4.5$ = 22.5 N (2)

(*Total 10 marks*)

6. The maximum possible resultant force is 45 N, leading to an acceleration of 5 m s^{-2}.

The minimum possible resultant force is 9 N, leading to an acceleration of 1 m s^{-2}.

Only one of the available answers lies within this range.

Answer: C (1)

(*Total 1 mark*)

7. The change in velocity is $V - (-U) = U + V$

Change in momentum is the product of mass and change in velocity.

Answer: D (1)

(*Total 1 mark*)

8. The *resultant* force is towards the center of the circle, but this is a combination of the diagonal tension and the vertical weight.

Answer: D (1)

(*Total 1 mark*)

9. (**a**) If hovering, there is no acceleration and therefore no resultant force. (1)

(**b**) The blades exert a downward force on the air. The air exerts an equal and opposite force on the blades to form a Newton's third law pair. (2)

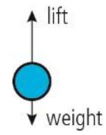
(**c**) Change of momentum of the air $= 1.7v$

Force is rate of change of momentum = $\frac{1.7v}{1}$ (in one second)

$1.7v = (0.95 + 0.45) \times 9.81$

v = 8.1 m s^{-1} (3)

(**d**) Free-body diagram on aircraft:

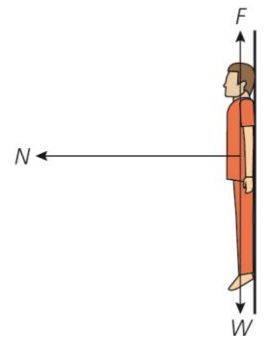


vertical force = lift force − weight

acceleration $= \frac{F}{m} = \frac{(0.95+0.45-0.95)\times 9.81}{0.95}$ = 4.6 m s^{-2} (2)

(*Total 8 marks*)

10. (**a**) Vertically, forces should be balanced. Horizontally, a resultant force is required towards the center of the circle.



(2)

(**b**) Horizontally, $F_N = mR\omega^2$

Vertically, $F_f = mg$

$F_f = \mu F_N = \mu mR\omega^2$

$\mu mR\omega^2 = mg$

$\omega = \sqrt{\frac{g}{\mu R}}$ (2)

(**c**) Using (**b**), the minimum required angular velocity $= \sqrt{\frac{9.81}{0.4\times 3.5}}$ = 2.6 rad s^{-1}

The actual angular velocity $= 2\pi f = 2\pi \times \frac{28}{60}$ = 2.9 rad s^{-1}

The actual angular velocity is greater than the minimum, so the person does not slide.
(2)

(*Total 7 marks*)

11. If acceleration were proportional only to force, then the larger weight acting on the boulder would result in a higher acceleration than that of the pebble. But acceleration has also to do with mass (its tendency to resist a change in motion). For the pebble, $a_p = \frac{mg}{m} = g$. For a boulder with 100 times the mass, $a_b = \frac{100mg}{100m} = \frac{mg}{m} = g$.

Answer: C (1)

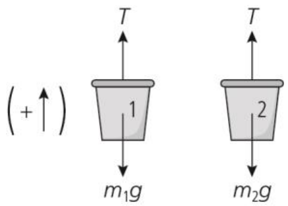
(*Total 1 mark*)

12. If the wind speed is doubled the mass of air hitting the house per second is doubled. The speed of that mass is also doubled. There is four times as much momentum, and the force exerted is proportional to the momentum per second.

Answer: D (1)

(*Total 1 mark*)

13. (**a**) Drawing the forces acting on the buckets:



On m_1, $T - m_1g = m_1a$

On m_2, $m_2g - T = m_2a$

Combining these equations to find a: $m_2g - m_1g = m_2a + m_1a$

$$a = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

Now, $v^2 = u^2 + 2as = 2as$

Substituting: $v = \sqrt{2\frac{(m_2 - m_1)g}{m_2 + m_1}h}$ (2)

(**b**) Bucket 1 also has this speed when bucket 2 hits the ground.

Using $v^2 = u^2 + 2as$, as $v = 0$ when it hits the ground, $u^2 = -2as$

To find the magnitude of the displacement, $s = \frac{2\frac{(m_2 - m_1)g}{m_2 + m_1}h}{2g} = \frac{(m_2 - m_1)}{m_2 + m_1}h$ (2)

(*Total 4 marks*)

A.3 Work, energy and power

Practice questions

1. (**a**) (**i**) Using conservation of momentum: $80 \times 10 = 180v$

$v = 4.4 \text{ m s}^{-1}$

Impulse is $\Delta mv = 100 \times 4.4 = 440 \text{ N s}$ (2)

(**ii**) $\Delta mv = 80 \times 4.4 - 80 \times 10 = -440 \text{ N s}$ (1)

(**iii**) $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 180 \times 4.4^2 = 1740 \text{ J}$ (1)

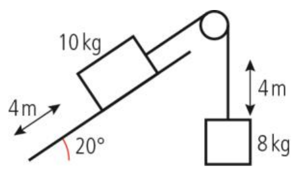
(**b**) To reach the other side, the initial kinetic energy must be equal to or greater than the change in potential energy.

$\Delta E_p = mg\Delta h = 180 \times 10 \times 1 = 1800 \text{ J}$

The kinetic energy is slightly less than this, so the competitor will not reach the far end of the wire. (2)

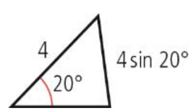
(*Total 6 marks*)

2. (**a**) (**i**)



Work done $= F\Delta x = 80 \times 4 = 320 \text{ J}$ (1)

(**ii**)



$\Delta E_p = mg\Delta h = 10 \times 10 \times 4 \sin 20° = 137 \text{ J}$ (1)

(**iii**) The masses also gain kinetic energy and, due to conservation of energy, this subtracts from the total energy input.

(**b**) Efficiency $= \frac{\text{useful work output}}{\text{total energy input}} \times 100\% = \frac{137}{400} \times 100\% = 34\%$ (2)

(*Total 4 marks*)

3. (**a**) Initial $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.25 \times 20^2$ = 50 J

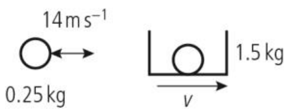
At maximum height, $E_p = mgh = 0.25 \times 10 \times 10$ = 25 J

Because of conservation of energy, initial E_k = final $E_p + E_k$

At maximum height, $E_k = 50 - 25$ = 25 J

$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 25}{0.25}}$ = 14 m s−1 (NB: This is horizontal.) (2)

(**b**)



Using conservation of momentum, $14 \times 0.25 = (0.25 + 1.25)v$

$v = \frac{14 \times 0.25}{1.5}$ = 2.3 m s^{-1} (2)

(**c**) The combined kinetic energy of the ball and bucket is: $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.5 \times 2.3^2$ = 3.97 J

Elastic potential energy, $E_p = \frac{1}{2}kx^2$ = 3.97 J

$k = \frac{2 \times 3.97}{0.1^2}$ = 794 N m^{-1} (2)

(*Total 6 marks)*

4. Taking the statements in turn:

I The gravitational potential energy is split between kinetic energy and thermal energy. As the stone is falling at constant velocity, there is zero gain in kinetic energy. The statement is true.

II Kinetic energy is constant but gravitational potential energy is decreasing. The statement is false.

III Because there is no acceleration, the work done by friction must be equal to the work done by gravity. The statement is true.

Answer: B (1)

(*Total 1 mark)*

5. The useful energy is the light energy to the lamp, which is represented by an arrow of width 2 squares. The input arrow has a width 20 squares, so efficiency is $\frac{2}{20} \times 100\% = 10\%$.

Answer: A (1)

(*Total 1 mark*)

6. Work done $= Fx = max$ $= 3.0 \times \text{area under graph}$ $= 3.0 \times \frac{1}{2}(15 + 25) \times 20$ = 1200 J

Answer: C (1)

(*Total 1 mark*)

7. $E_k = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(\Delta mv)^2}{m}$

$E_k \propto \frac{I^2}{m}$

Since I doubles and m doubles, E_k doubles overall.

Answer: C (1)

(*Total 1 mark*)

8. Since acceleration is constant, $v \propto t$

Now, $P = Fv$, and we know that the engine force F needed to combat the resistive force is a function of v. Therefore, $P \propto v^2$.

Combining the two relationships, $P \propto t^2$.

Answer: D

(1)

(*Total 1 mark)*

9. $P = Fv = 1.5 \times \frac{100}{5.0}$ = 30 kW

Answer: D (1)

(*Total 1 mark*)

10. Energy density, $\frac{E}{V} = \frac{E}{m} \times \frac{m}{V}$

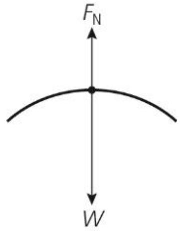
Answer: C (1)

(*Total 1 mark*)

11. (**a**) Using conservation of energy, $\frac{\frac{1}{2}mv^2}{mgh} = 0.24$

v = 11.9 m s^{-1} (2)

(**b**)



(2)

(**c**) For the skier to remain in contact with the circular slope, a centripetal force is required.

$\frac{mv^2}{r} = \frac{65 \times 12^2}{20}$ = 468 N

The centripetal force is weight minus normal reaction force. Since the weight is larger than the centripetal force, the reaction force is non-zero and therefore there is contact. (3)

(**d**) The kinetic energy lost while stopping, $E_k = \frac{1}{2}mv^2 = 0.4 \times 65 \times 8.2^2 = 2185$ J. This is equal to the work done by friction.

friction force = $\frac{E_k}{\text{distance}} = \frac{2185}{24} = 91$ N

$F_f = \mu N$ and $N = mg$

coefficient of friction = $\frac{91}{65 \times 9.81} = 0.14$ (3)

(**e**) Impulse is equal to change in momentum, $\Delta mv = 76 \times 9.6 = 730$ N s (2) (**f**) The change in momentum is the same with or without a flexible net, because the skier goes from having a speed to having zero speed. The net increases the stopping *time*.

Since $F = \frac{\Delta p}{\Delta t}$, F is smaller, which means that the body experiences less force (and harm). (2)

(*Total 14 marks)*

12. (**a**) (**i**) Using conservation of energy between the compressed spring and the top of the ramp, $E_{\text{p, elastic}} = \frac{1}{2}mv^2 + mgh$

$E_{\text{p, elastic}} = \frac{1}{2} \times 55 \times 0.90^2 + 55 \times 9.81 \times 1.2 = 669\ \text{J}$ (2)

(**ii**) Using conservation of energy between A and C (and assuming no thermal energy losses between A and B), $\frac{1}{2}mv^2 = mgh$

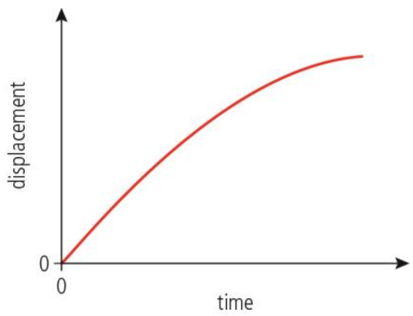
$\frac{1}{2} \times v^2 = 9.81 \times 1.2$

$v = \sqrt{2 \times 9.81 \times 1.2} = 4.9\ \text{m s}^{-1}$ (2)

(**b**) (**i**) According to Newton's first law, a body will remain at constant speed unless acted upon by a resultant force. As there is no force on the block, so it will move at a constant speed. (1)

(**ii**) According to Newton's second law, a resultant force will cause an acceleration. In this case, the resultant force is in the opposite direction to the motion, so speed decreases. (2)

(**c**) Between A and B, the gradient is constant because speed is constant. Between B and C, the gradient decreases (and displacement continues to increase) because the speed decreases.



(2)

(**d**) Force is the rate of change in momentum: $F = \frac{\Delta p}{\Delta t} = \frac{55 \times 4.9}{0.42}$

$F = 642 \approx 640\ \text{N}$ (2)

(**e**) Efficiency $= \dfrac{\text{useful energy output}}{\text{total energy input}} = \dfrac{670}{816 \times 1.5} = 0.55$ (2)

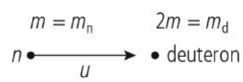
(*Total 13 marks*)

13. (**a**) Answer: C (1)

(**b**) Although equal and opposite forces act between the car and the magnet, they are connected and cancel. Alternatively, no overall work is done and therefore no kinetic energy is produced. (1)

(*Total 2 marks*)

14.



(**a**) Using conservation of momentum, $m_n u = m_n v_n + m_d v_d$

And, $2m_d = m_n$

$u = v_n + 2v_d$

Using conservation of kinetic energy (because the collision is elastic),

$\frac{1}{2} m_n u^2 = \frac{1}{2} m_n {v_n}^2 + \frac{1}{2} m_d {v_d}^2$

$u^2 = {v_n}^2 + 2{v_d}^2$

Solving these simultaneous equations by substituting for v_n,

$u^2 = (2v_d - u)^2 + 2{v_d}^2$

$u^2 = 4{v_d}^2 - 4v_d u + u^2 + 2{v_d}^2$

$6{v_d}^2 = 4v_d u$

$\dfrac{v_d}{u} = \dfrac{2}{3}$ (2)

(**b**) Initial $E_k = \frac{1}{2} m_n u^2$

Deuteron $E_k = \frac{1}{2} m_d {v_d}^2 = \frac{1}{2} \times 2m_n \times \left(\frac{2}{3}u\right)^2$

Ratio of $\dfrac{E_{k,\ \text{deuteron}}}{E_{k,\ \text{initial}}} = \dfrac{\frac{1}{2} \times 2m_n \times \left(\frac{2}{3}u\right)^2}{\frac{1}{2} m_n u^2} = \dfrac{8}{9} = 89\%$ (2)

(**c**) If $\frac{8}{9}E_k$ is given away in each collision, $\frac{1}{9}E_k$ remains.

If the number of collisions is *N*, $\left(\frac{1}{9}\right)^N \times 10 \times 10^6 = 0.01$

$\left(\frac{1}{9}\right)^N = 10^{-9}$

$9^N = 10^9$

Taking logs of both sides, $N\log 9 = 9$

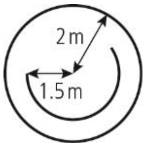
$N = 9.4$, so (rounding up) 10 collisions are needed. (2)

(*Total 6 marks*)

A.4 Rigid body mechanics

Practice questions

1. (**a**)



Total moment of inertia = moment of inertia of disc + moment of inertia of handrail

$\frac{1}{2}m_d r_d^2 + m_h r_h^2 = \frac{1}{2}100 \times 2^2 + 1.5 \times 1.5^2$ = 245 kg m^2 (3)

(**b**) (**i**) $\tau = Fr = 200 \times 1.5$ = 300 N m (1)

(**ii**) $W = Fx \times 10$ = 2000 J (1)

(**iii**) Assuming the system is frictionless, $E_k = W$ = 2000 J (1)

(**iv**) Rotational kinetic energy $= \frac{1}{2}I\omega^2$

$\omega = \sqrt{\frac{2 \times 2000}{245}}$ = 4.0 rad s^{-1} (1)

(**v**) $L = I\omega = 245 \times 4.0$ = 980 kg m^2 s^{-1} (1)

(**c**) (**i**) The child accelerates, which means there is an unbalanced force. This is supplied by the roundabout. According to Newton's third law, the child exerts an equal and opposite force on the roundabout, so the roundabout slows down. (3)

(**ii**) Using conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

Now, with the addition of the child, $I_2 = 245 + 40 \times 2^2$ = 405 kg m^2

Therefore, $\omega_2 = \frac{245 \times 4}{405}$ = 2.4 s^{-1} (2)

(**iii**) $F = mr\omega^2 = 40 \times 2 \times 2.4^2$ = 461 N (2)

(**d**) (**i**) $\tau = Fr = 20 \times 2$ = 40 N m

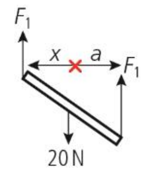
$\alpha = \frac{\tau}{I} = \frac{40}{405}$ = 0.099 rad s^{-2} (2)

(**ii**) This equation would assume that angular acceleration is constant. However, angular acceleration depends on velocity because the torque from air resistance depends on velocity.

The average angular acceleration is less than the maximum value, so using the maximum value would underestimate the answer. (2)

(*Total 19 marks*)

2. (**a**)

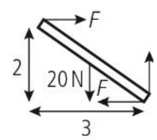


Taking moments about the bottom end:

$F_1 \times 2x = 20x$

F_1 = 10 N (2)

(**b**) (**i**)



There must be a horizontal frictional force at the bottom. (2)

(**ii**) Taking moments about the bottom end:

$$F \times 2 = 20 \times 1.5$$

F = 15 N

Using Newton's laws and noting that horizontal forces are balanced, the magnitude is 15 N. (3)

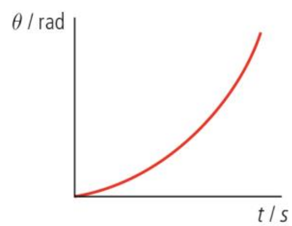
(*Total 7 marks*)

3. (**a**) Using $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$,

$\omega_f^2 = 0 + 2 \times 0.110 \times 6 \times 2\pi$

ω_f = 2.88 rad s^{-1} (2)

(**b**) Using $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$



(1)

(**c**) $\tau = I\alpha$ so $\tau = 0.110 \times 0.0216 = 2.38 \times 10^{-3}$ N m (1) (**d**) Rearranging $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$:

$$\alpha = \frac{2.9^2}{2 \times 2\pi \times 30} = -0.022 \text{ rad s}^{-2}$$

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{-2.9}{-0.0220} = 130 \text{ s}$$ (2)

(*Total 6 marks*)

4. (**a**) The person rotates anticlockwise. This is because of conservation of angular momentum; in order for the wheel to rotate clockwise, the person's angular momentum must be opposite to this. (3)

(**b**) The rotational kinetic energy has increased because the person has done work when flipping the wheel. (NB: rotational kinetic energy is a scalar quantity.) (2)

(*Total 5 marks*)

5. Using conservation of rotational and linear energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now, $I = \frac{2}{5}mr^2$

And, $\omega = \frac{v}{r}$

Substituting into the conservation equation:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2\left(\frac{v}{r}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2$$

$$10gh = 5v^2 + 2v^2$$

$$v^2 = \frac{10gh}{7}$$ (3)

(*Total 3 marks*)

6. (**a**) A torque requires a force and for there to be a distance between the force and the axis of rotation. (1)

(**b**) (**i**) Considering translational motion and components along the slope:

$Mg\sin\theta - F = Ma$

Considering rotational motion:

$F_f R = I\alpha$

Substituting $F_f = \frac{I\alpha}{R}$ gives $Mg\sin\theta = Ma + \frac{I\alpha}{R}$

Using $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$,

$$Mg\sin\theta = Ma + \frac{\frac{1}{2}MR^2\frac{a}{R}}{R}$$

$$g\sin\theta = a + \frac{1}{2}a = \frac{3}{2}a$$

$$a = \frac{2}{3}g\sin\theta$$ (4)

(**ii**) Using and rearranging $s = \frac{1}{2}at^2$ to get $t = \sqrt{\frac{2s}{a}}$

substituting to get $t = \sqrt{\frac{2 \times 1.5}{\frac{2}{3} \times 9.81 \times \sin 30°}}$ = 0.96 s (2)

(**c**) The ice slides whereas the cylinder rotates. In terms of energies, the cylinder's potential becomes both kinetic and rotational kinetic. In terms of accelerations, the cylinder's is always less than that of the ice. Through either consideration we can see that the speed of the cylinder will be less at any given point. (1)

(**d**) The hollow cylinder has a greater moment of inertia because the mass is concentrated at the full radius. The acceleration will be smaller for the same torque. (2)

(*Total 10 marks*)

7 (**a**) Answer: B (1)

(**b**) The road exerts a forward force on the tyres. This accelerates the car and produces a torque about the center of mass of the car. Whether the force is exerted on the rear or front (or all) tyres, the line of action of the force is along the roadway, which means the torque sees the front of the car rotate upward. (1)

(*Total 2 marks*)

8 (**a**) Answer: D (1)

(**b**) Linear momentum is always changing because the speed of the swing is always changing between zero and a maximum value. Angular momentum is always changing because the angular speed (which is related to the speed) of the swing is always changing.

Where does the momentum go? The string, support mechanism and earth! (1)

(*Total 2 marks*)

A.5 Galilean and special relativity

Practice questions

1 (**a**) Proper length and time are lengths and times measured by an observer in a frame of reference that is at rest relative to the events being measured. (2)

(**b**) (**i**) If Miguel sees the matches light simultaneously then the light from each strike must arrive to him at the same time. But to Carmen, Miguel is moving towards B so the light from B has traveled a shorter distance, so if the lights reach Miguel at the same time, the match A must have been struck first.

Note: This is the other way round if the events are simultaneous on the station but not on the train. (4)

(**ii**) Miguel is at rest relative to A and B so L_0 = 20 m.

Carmen is moving relative to A and B so

$$L = \frac{L_0}{\gamma} \Rightarrow \gamma = 2$$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \Rightarrow \frac{1}{\gamma^2} = 1 - \frac{u^2}{c^2}$$

$$\Rightarrow u = c\sqrt{1-\frac{1}{\gamma^2}}$$

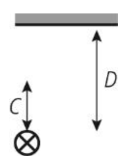
$u = 0.87c$ (2)

(**iii**) The measurements are different because they are in different frames of reference; there is no right and wrong. (2)

(*Total 10 marks*)

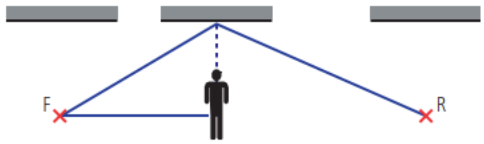
2. (**a**) An inertial frame of reference is a coordinate system covered in clocks within which Newton's laws of motion are obeyed. In other words, not accelerating. (1)

(**b**)



$$T_0 = \frac{2D}{c}\left(\frac{\text{distance}}{\text{velocity}}\right)$$ (1)

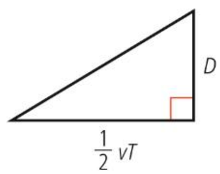
(c) (i)



(1)

(**ii**) Distance $F - R = vT$ (1)

(**iii**) From Pythagoras:



$$\text{path length } = 2 \times \sqrt{D^2 + \left(\frac{vT}{2}\right)^2}$$ (2)

(**iv**) We know that observer E sees the light travel from F to R in time T. Since the speed of light is the same for all inertial observers this must be a distance cT.

$$\text{so } cT_0 = 2\sqrt{D^2 + \left(\frac{vT}{2}\right)^2}$$

$$\text{but } D = \frac{cT_0}{2} \text{ so } cT = 2\sqrt{\left(\frac{cT_0}{2}\right)^2 + \left(\frac{vT}{2}\right)^2}$$

$$\text{squaring} \Rightarrow c^2T^2 = = 4\left(\frac{c^2T_0^{\ 2}}{4} + \frac{v^2T^2}{4}\right)$$

$$c^2T^2 = c^2T_0^2 + v^2T^2$$

$$(c^2 - v^2)T^2 = c^2T_0^2 \Rightarrow T_0^2 = \left(\frac{c^2 - v^2}{c^2}\right)T^2$$

$$T_0^{\ 2} = \left(1 - \frac{v^2}{c^2}\right)T^2$$

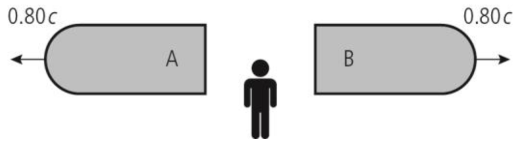
$$\Rightarrow T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$ (4)

(*Total 10 marks*)

3. (**a**) Postulate 1. The laws of physics are the same for all inertial observers.

Postulate 2. The speed of light in a vacuum is the same as measured by all inertial observers. (2)

(**b**) (**i**)

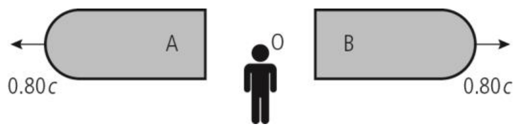


Each spacecraft moves away from the observer at 0.8c; this is not greater than c.

To find out how fast they move from each other, you would have to determine the velocity of A relative to B. (1)

(**ii**) velocity transform is $u' = \dfrac{u - v}{1 - \dfrac{uv}{c^2}}$

Measure the velocity of A from the reference frame of B.



u – velocity of A measured by O = –0.8c

v – velocity of B's reference frame relative to O = 0.8c

$$u' = \frac{0.8c + 0.8c}{\left(1 - \dfrac{-0.8c \times 0.8c}{c^2}\right)} = \frac{1.6}{1 + 0.64}$$

u' = 0.98c (3)

(*Total 6 marks*)

4 (**a**) (**i**) The ct' axis is the same as a body traveling at the same velocity as the observer.

$$\tan\theta = \frac{v}{c} = 0.5$$

$v = 0.5c$ (2)

(**ii**) $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1 - \dfrac{(0.5c)^2}{c^2}}}$ = 1.15 (2)

(**iii**) t = 4 s and x = 3 light seconds (1)

(**iv**) t' = 3 s and x' = 1 light second (1)

(**v**) $x' = \gamma(x - vt) = 1.15(3c - 0.5c \times 4)$ = 1.15 light seconds $\approx$ 1 light second

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.15\left(4 - \frac{0.5c \times 3c}{c^2}\right) = 2.9\text{ s} \approx 3\text{ s}$$ (2)

(**b**) (**i**) $v = \dfrac{1}{\text{gradient}} = 4c$ (1)

(**ii**) It would arrive before it left. (2)

(**c**) (**i**) Reading the diagonal: 4 s (1)

(**ii**) Reading the axis: 4.6 s (1)

(**iii**) 4 s because the start and end are measured by the same clock on the spaceship. (1)

(**iv**) Time intervals are longer when measured by observers moving relative to events. (2)

(*Total 16 marks*)

5. It is possible to respond to this question in terms of time or distance. Either way, the aim is to show that more muons are detected at the ground than expected without time dilation.

Earth frame transit time $= \dfrac{2000}{0.98c}$ = 6.8 μs

Earth frame dilation of proper half-life = 2.2 × 5 = 11 μs

Muon's proper transit time $= \dfrac{6.8}{5}$ = 1.4 μs

Distance muons can travel in a proper lifetime = 2.2 × 0.98c = 650 m

Earth frame lifetime distance due to time dilation = 650 × 5 = 3250 m

Muon frame distance traveled $= \dfrac{2000}{5}$ = 400 m (3)

(*Total 3 marks*)

6. If your room moves smoothly in a straight line and is not accelerating in any way, there is no way for you to know if it is moving or not. If a window was present, you would not know if it was your room moving or the things outside moving.

If the room's motion affected your mass, heart rate or size, it would affect all of these (and all clocks) in the same way. There would be nothing in the room that did not change. There would therefore be no fixed reference with which to compare. You cannot, in short, be your own personal speedometer.

Answer: E (1)

(*Total 1 mark*)

7. Bearing in mind that this person cannot be their own personal speedometer, they will experience their own mass as though it is normal, irrespective of their speed.

The reason that they cannot exceed the speed of light is because they reckon their speed by their time. They are in motion, which means their time (say, an hour) is longer to a stationary observer (say, a month). Their distance traveled (say, a kilometre) is shorter to a stationary observer (say, a centimetre). They can increase their speed as much as they want, but the resulting speed is not the sum of the additions. (2)

(*Total 2 marks*)

8. (**a**) Answer: D (1)

(**b**) As you are observing the moving spear at a location that is at rest relative to the pipe, the spear will appear shorter than the pipe. If you are traveling at the same speed as the spear, however, the pipe appears shortened. At a speed halfway between these, the spear and the pipe will have the same speed relative to you and both will be shorter the same amount. (2)

(*Total 3 marks*)

B The particulate nature of matter

B.1 Thermal energy transfers

Practice questions

1. (**a**) (**i**) Work is done against friction, which means that energy is transferred to the atoms in the bar. Temperature is linked to the average kinetic energy of the atoms. (3)

(**ii**) Using the density equation:

$$\rho = \frac{m}{V},\ \ m = \rho V = \rho \pi r^2 L = 8000 \times \pi \times 0.005^2 \times 0.1 = 0.063\ \text{kg}$$ (2)

(**iii**) Using the specific heat capacity equation for both the blade and the bar:
$Q = mc\Delta\theta = (0.030 + 0.063) \times 470 \times 6 = 261.8\ \text{J}$

Combining with the work done equation, $W = Fd$, and assuming that all heat has come from the work done by the saw, $F = \frac{W}{d} = \frac{261.8}{0.30 \times 100} = 8.7\ \text{N}$ (3)

(**b**) (**i**) Energy required to melt the half bar:

$mc\Delta\theta + mL = 0.032 \times 470 \times (1500 - 20) + 0.032 \times 270000 = 30\,900\ \text{J}$

Now, $P = \frac{W}{t}$, so $t = \frac{W}{P} = \frac{30\,900}{1000} = 31\ \text{s}$ (2)

(**ii**) If the bar does not begin to melt, because it has no means of increasing temperature, all of the electrical power must be lost: 1000 J (1)

(**c**) (**i**) Convection currents carry hot, less dense fluid upwards; if the element were at the top, only very small currents at the top could form. The boiler does not need to be filled to the top because water will leave from near the bottom. (2)

(**ii**) If no water is present then the heat loss from the element will be reduced, which means that the element will reach a higher temperature. (2)

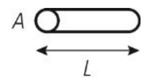
(*Total 15 marks*)

2. (**a**) $2500 + 273 = 2773\ \text{K}$ (You can assume that the '0' values in the temperature are not rounded.) (2)

(**b**) Conduction and convection require particles; radiation is the only mechanism of heat loss in a vacuum. (2)

(**c**) Using the radiation equation, $P = \sigma A T^4$:

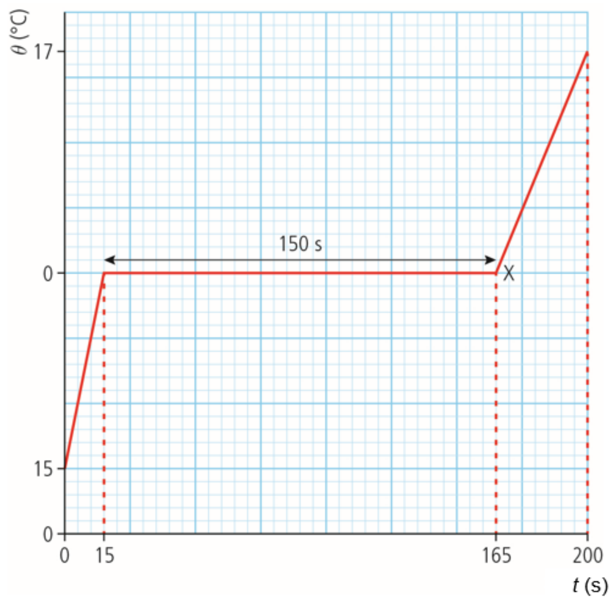
$$A = \frac{P}{\sigma T^4} = \frac{30}{5.67 \times 10^{-8} \times 2773^4} = 8.9 \times 10^{-6}\ \text{m}^2$$



$A = 2\pi rL$ (ignoring the ends), so $r = \dfrac{A}{2\pi L} = \dfrac{8.9 \times 10^{-6}}{2\pi \times 0.01} = 1.4 \times 10^{-4}\ \text{m}$ (3)

(*Total 7 marks*)

3.



(**a**) Ice melts when temperature is constant 0 °C. All melted at 165 s. (1)

(**b**) Heat goes to increase E_P not E_K so temperature remains constant. (3)

(**c**) (**i**) For last part of graph, water is heated from 0 to 15 °C in 30 s.

$Q = mc\Delta\theta \Rightarrow$ heat supplied

$= 0.25 \times 4200 \times 15 = 1.79 \times 10^4$ J

power $= \dfrac{Q}{t} = \dfrac{1.79 \times 10^4}{30}$ = 525 W ≈ 530 W (3)

(**ii**) Time to heat ice from –15 to 0 °C = 15 s

Q = power × t = 530 × 15 = 7950 J

$Q = mc\Delta\theta = 0.25 \times c \times 15$

$c = \dfrac{7950}{0.25 \times 15}$ = $2.1 \times 10^3\ \text{J kg}^{-1}\ \text{K}^{-1}$ (3)

(**iii**) Takes 150 s to melt 0.25 kg of ice

Heat given $Q = 530 \times 150 = 79\,500$ J $= mL$

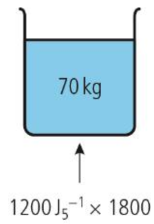
$L = \dfrac{79\,500}{0.25}$ = 3.2 × 10^5 J kg^{-1} or 320 kJ kg^{-1} (2)

(*Total 12 marks*)

4. (**a**) In this context thermal energy is the internal energy of the molecules of the runner.

This can be E_K and E_P. Increased thermal energy will increase the average E_K of the molecules which increases the temperature; in other words, the runner becomes hot. (4)

(**b**) (**i**) Energy generated = power × time = 1200 × 3600 = 2.2 × 10^6 J (2)



(**ii**) $Q = mc\Delta\theta$

$\Delta\theta = \dfrac{Q}{mc} = \dfrac{2.2 \times 10^6}{70 \times 4200}$ = 7.5 K (3)

(**c**) convection; conduction; radiation (6)

(**d**) (**i**) The molecules with greatest E_K leave the surface resulting in a decrease in average E_K and hence temperature. (3)

(**ii**) Total energy generated = 2.2 × 10^6 J

50% lost in evaporation = 1.1 × 10^6 J

This energy goes to latent heat of vaporization, $Q = mL$.

$m = \dfrac{Q}{L} = \dfrac{1.1 \times 10^6}{2.26 \times 10^6}$ = 487 g (3)

(**iii**) wind

skin temperature

humidity

air temperature

area of skin

clothing (4)

(*Total 25 marks*)

5. We know that $\lambda_{max} \propto \frac{1}{T}$ so halving the peak wavelength will double the temperature.

We also know that $P \propto T^4$. Doubling the temperature will therefore multiply the power by a factor of 16.

Answer: D (1)

(*Total 1 mark*)

6. We know that $\lambda_{max} \propto \frac{1}{T}$ so doubling the peak wavelength will halve the temperature.

We also know that $P \propto T^4$. Halving the temperature will therefore divide the power by a factor of 16.

Answer: A (1)

(*Total 1 mark*)

7. Start by finding the energy of the bicycle that is absorbed by the brake:

$$\frac{1}{2} \times \frac{1}{2} Mv^2 = \frac{1}{4} Mv^2$$

This energy is the heat added to the brake: $\frac{1}{4} Mv^2 = mc\Delta\theta$

$$\Delta\theta = \frac{Mv^2}{4mc}$$

Answer: A (1)

(*Total 1 mark*)

8. There is no atmosphere on the moon, so convection is not possible. Conduction will be possible between the rock and the section of the moon it is in contact with. Radiation will be possible in all directions.

Answer: B (1)

(*Total 1 mark*)

9. (**a**) Substitution of $L = \sigma A T^4$ into $b = \frac{L}{4\pi d^2}$:

$b = \frac{\sigma A T^4}{4\pi d^2}$ and σ and 4π are constants. (1)

(**b**) Sirius is a star and is therefore luminous. Venus is a planet; it is bright in the night sky because it reflects the Sun's light. (2)

(*Total 3 marks*)

10. Since $\rho = \frac{m}{V}$, $V = \frac{m}{\rho}$

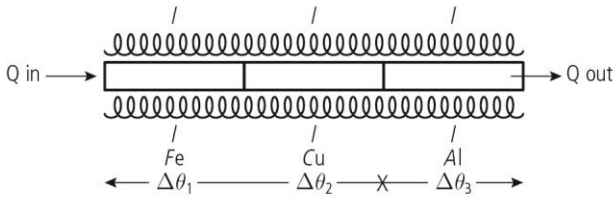
We are therefore looking for two points that share the same value when their x-coordinate is divided by their y-coordinate and which lie on the same line drawn through (0, 0).

1 and 4 have the same volume (of 20 cm^3) because a straight line through the origin can be drawn through them.

Answer: D (1)

(*Total 1 mark*)

11.



Because the rod is perfectly insulated, we know that the thermal power in each section is the same: $k_1 A \frac{\Delta\theta_1}{l} = k_2 A \frac{\Delta\theta_2}{l} = k_3 A \frac{\Delta\theta_3}{l}$

Therefore, $k_1 \Delta\theta_1 = k_2 \Delta\theta_2 = k_3 \Delta\theta_3$ (equation 1)

In addition, we know the overall temperature difference:

$\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 = 100$ (equation 2)

From this point onwards, it is all about algebra.

From equation 1: $\Delta\theta_1 = \frac{k_2}{k_1}\Delta\theta_2$ and $\Delta\theta_2 = \frac{k_3}{k_2}\Delta\theta_3$

Substituting for $\Delta\theta_2$: $\Delta\theta_1 = \frac{k_2}{k_1}\frac{k_3}{k_2}\Delta\theta_3 = \frac{k_3}{k_1}\Delta\theta_3$

Now substituting into equation 2: $\frac{k_3}{k_1}\Delta\theta_3 + \frac{k_3}{k_2}\Delta\theta_3 + \Delta\theta_3 = 100$

Factorising: $\left(\frac{k_3}{k_1} + \frac{k_3}{k_2} + 1\right)\Delta\theta_3 = 100$

Substituting the thermal conductivity values: $\left(\frac{240}{60} + \frac{240}{400} + 1\right)\Delta\theta_3 = 100$

Solving: $\Delta\theta_3 = \frac{100}{5.6} = 17.9\,°C$

$\Delta\theta_1 = 71.4\,°C$ and $\Delta\theta_2 = 10.7\,°C$

The Cu/Al junction is $17.9\,°C$ and the Fe/Cu junction is $28.6\,°C$. (5)

(*Total 5 marks*)

B.2 Greenhouse effect

Practice questions

1. (**a**) Fossil fuels are continually being made on the sea floor as organisms die; however, the rate at which they are made is much slower than the rate they are used up. (2)

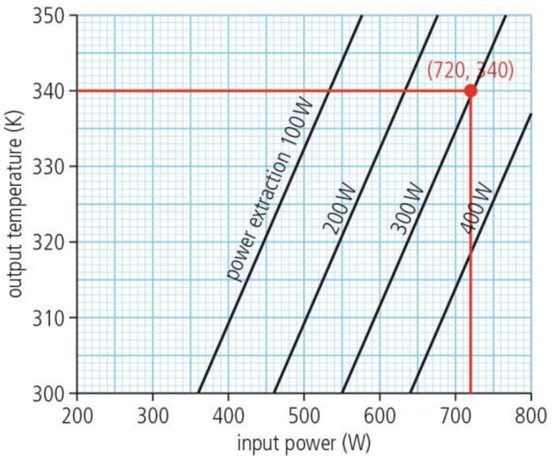
(**b**) Nuclear waste: the process of nuclear fission produces radioactive waste, which is difficult to dispose of.

Nuclear weapons: although nuclear fuel cannot be used directly to make bombs, the process of enrichment and raw materials are the same. A country with a nuclear power program could theoretically be producing weapons. (2)

(*Total 4 marks*)

2. (**a**) A solar panel is a panel containing water pipes that absorb the Sun's radiation to heat the water. This hot water can be used for showers, washing dishes, etc. A solar cell is a semiconductor device that absorbs light, converting it to electrical potential energy. (2)

(**b**) (**i**) From the graph:



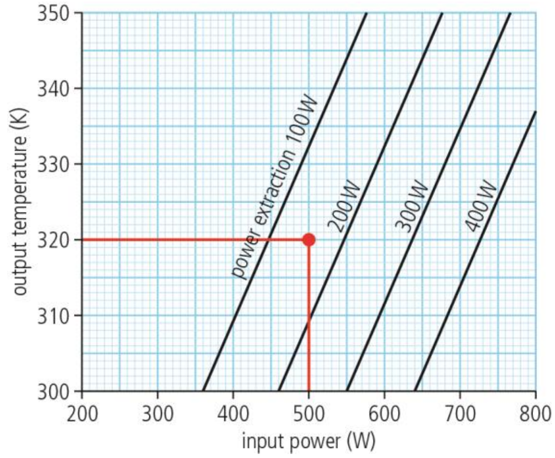
Input power = 720 W

so if intensity = 800 W m^{-2}

will need an area $= \dfrac{720}{800}$ = 0.90 m^2

You don't really need to understand what is happening; you just read the graph. (2)

(ii) The point (500, 320) is between the 100 W and 200 W lines.



Estimate power extraction ≈ 150 W

$$\text{efficiency} = \frac{\text{power out}}{\text{power in}} \times 100\% = \frac{150}{500} \times 100\% = 30\%$$ (3)

(*Total 7 marks*)

3. (**a**) Only half of the Earth is exposed to the Sun, which absorbs energy as if it were a disk of area πR^2.

So energy power absorbed = 1400 πR^2

Now calculate the power received per unit area:

$$\text{power} = \frac{1400\pi R^2}{\text{total area of the Earth}} = \frac{1400\pi R^2}{4\pi R^2} = 350\ \text{W m}^{-2}$$ (2)

(**b**) (**i**) Emissivity = power radiated by body divided by power radiated by a black body at the same temperature. From the diagram, power radiated by atmosphere = $0.7\sigma T^4$.

A black body radiates σT^4, so emissivity = 0.7. (1)

(**ii**) Power radiated per unit area = $0.7\sigma T^4 = 0.7 \times 5.67 \times 10^{-8} \times 242^4 = 136\ \text{W m}^{-2}$ (1)

(**iii**) Power in = 245 + 136 = 381 W m^{-2}

If in equilibrium, power in = power out

$\sigma T_E^4 = 381\ \text{W m}^{-2}$

$$T_E = \sqrt[4]{\frac{381}{5.67 \times 10^{-8}}} = 286\ \text{K}$$ (2)

(**c**) (**i**) Carbon dioxide molecules have a natural frequency in the infrared region of the electromagnetic spectrum. This means that infrared radiation will cause the molecule to oscillate and therefore be absorbed. The temperature of the Earth is such that the peak in the electromagnetic spectrum is in the infrared region so a lot of the power radiated from the Earth will be absorbed. (3)

(ii) The Sun has a surface temperature of about 6000 K so radiates in the visible region, which does not resonate with the CO_2 molecules. (2)

(iii) Burning fossil fuels produces CO_2. Plants absorb CO_2. (2)

(*Total 13 marks*)

4. (**a**) The power emitted per unit surface area of a black body is proportional to the fourth power of its absolute temperature.

$$\frac{P}{A} = \sigma T^4$$ (2)

(**b**) (**i**) $\frac{P}{A} = \sigma T^4 = 5.67 \times 10^{-8} \times 5800^4 = 6.4 \times 10^7$

$A = 4\pi r^2 = 4\pi \times (7 \times 10^8)^2 = 6.16 \times 10^{18}\ \text{m}^2$

So $P = 6.4 \times 10^7 \times 6.16 \times 10^{18} = 3.9 \times 10^{26}\ \text{W}$ (1)

(**ii**) At a distance of 1.5×10^{11} m, the power has spread over a larger area, so

power per unit area $= \frac{3.9 \times 10^{26}}{4\pi \times (1.5 \times 10^{11})^2} = 1400\ \text{W m}^{-2}$ (2)

(**iii**) If average power absorbed per unit area = 240 W m^{-2}, then power in = 240 × area (2)

(**iv**) power out = area × σT^4

power in = power out

area × σT^4 = 240 × area

$\sigma T^4 = 240\ \text{W m}^{-2}$

$T = 255\ \text{K}$ (2)

(**c**) The radiated radiation is in the infrared region of the spectrum so is absorbed by the CO_2 in the atmosphere. After absorption, the molecules re-radiate in all directions. A proportion of this returns to the Earth; this increases the temperature. An increase in the Earth's temperature results in more power radiated until equilibrium is maintained. (3)

(**d**) Burning fossil fuels releases CO_2 into the atmosphere. This increase in CO_2 concentration leads to more absorption of infrared radiation, enhancing the greenhouse effect, resulting in more radiation being re-radiated back to Earth. (3)

(*Total 15 marks*)

5. (**a**) (**i**) The black pattern will be a better radiator even though the temperature at the surface is no different to the white sections. (2)

(**ii**) Black surfaces have higher emissivity than white surfaces. (1)

(**iii**) Albedo is related to the reflectivity of a surface and white surfaces are better reflectors. (1)

(**b**) White roofs would reflect more radiation, which means that less would be absorbed at the Earth's surface. Because the Earth will remain at a stable temperature when the incident power is equal to the combined reflected and radiated power (and with the latter dependent on temperature), the white roofs would allow this to happen at a reduced temperature. (3)

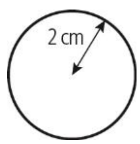
(**c**) (**i**) Greenhouse gases absorb longer wavelength infrared radiation when it is radiated from the planet's surface and re-emit it in all directions. At this planet, no infrared radiation would reach the planet's surface on entry. (2)

(**ii**) UV radiation is harmful because it damages DNA molecules. On Earth, the ozone layer acts as a UV shield, which is part of what makes life as we know it possible. (2)

(**iii**) The higher the albedo of a planet, the less radiation reaches its surface and therefore the lower its temperature needs to be to achieve equilibrium. (2)

(*Total 13 marks*)

6. (**a**) (**i**)



Calculate the radiated power, taking care to find the surface area of the sphere:

$P = \sigma e A T^4 = 5.67 \times 10^{-8} \times 0.8 \times 4\pi \times 0.02^2 \times 1000^4$ = 228 W (2)

(**ii**) Use the area of the sphere that the radiation is emitted into at a radius of 2 m:

$I = \frac{P}{A} = \frac{228}{4\pi \times 2^2}$ = 4.5 W m^{-2} (2)

(**b**) (**i**) Use the area of the sensor: $P = IA = 4.5 \times 0.5 \times 10^{-4}$ = 0.000 23 W

Rearranging $P = \frac{E}{t}$, $E = Pt = 0.00023 \times 60$ = 0.014 J (2)

(**ii**) Note that this question is not seeking any information about the sensor's efficiency; it is merely asking about absorption. As with any 'climate' model, you must make assumptions about albedo. In this case, the assumption is that no radiation is reflected. In addition, only the area given in the question has been used when calculating the energy absorbed, assuming that there was no tilt relative to the light source. (2)

(*Total 8 marks*)

7. If 300 W m^{-2} is radiated from a total of 400 W m^{-2} incident, the remainder (100 W m^{-2}) must be reflected.

Albedo is the ratio of reflected to incident radiation, which is $\frac{100}{400} = 0.25$.

Answer: A (1)

(*Total 1 mark*)

8. Carbon dioxide molecules absorb long-wavelength outgoing radiation. This radiation is then re-emitted in all directions, including towards the Earth.

Answer: B (1)

(*Total 1 mark*)

9. Intensity is power per unit area, therefore the surface areas of the planet and moon are not relevant. Using $P \propto T^4$, if the temperature of the moon is a fifth that of the planet, its intensity will be divided by a factor of 625.

Answer: C (1)

(*Total 1 mark*)

10. The solar constant, which is the intensity at the Earth's surface, is $1.36 \times 10^3\ \mathrm{W\,m^{-2}}$. We also know that $I \propto \frac{1}{r^2}$. If Earth is 1.5 times the distance of Venus from the Sun, then the intensity at Venus will be 1.5^2 times that at Earth.

Answer: D (1)

(*Total 1 mark*)

11. Since intensity is the power per unit area, $P \propto A$. Since panel 2 has only $\frac{3}{4}$ times the efficiency, it will require $\frac{4}{3}$ times the area to make up for this.

Answer: D (1)

(*Total 1 mark*)

12. The solar constant is the intensity of radiation from the Sun at the orbital radius of the Earth. This radius is an average value, and the Sun's luminosity is an average value too. On the other hand, the Earth's plane of axis is irrelevant; anything placed at the Earth's orbital radius would experience the same average solar constant.

Answer: A (1)

(Total 1 mark)

13. This question is about a specific value at a specific time at a specific location on the Earth's surface, that is, it is not about the solar constant. All of these factors affect the incident power at that location.

Answer: D (1)

(*Total 1 mark*)

B.3 Gas laws

Practice questions

1. (**a**) (i) Using $\rho = \frac{m}{V}$, $V = \frac{m}{\rho} = \frac{1}{1.21}$ = 0.83 m^3 (2)

(**ii**) Using $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ and recognising that pressure is constant:

$$V_2 = \frac{T_2V_1}{T_1} = \frac{373 \times 0.83}{293} = 1.0566 = 1.06\text{ m}^3$$

Now $\rho = \frac{m}{V} = \frac{1}{1.0566}$ = 0.95 kg m^{-3} (3)

(**iii**) In this case, the volume is for the whole balloon:

$m = \rho V = 0.95 \times 2500$ = 2380 kg (1)

(**iv**) Using the same equation but with the density of cool air:

$m = \rho V = 1.21 \times 2500$ = 3030 kg (1)

(**v**) Upthrust from the displaced cool air is 3030×10 = 30 300 N

Total weight of the hot balloon is $(2380 + 250) \times 10$ = 26 300 N

The balloon will rise because the upthrust exceeds the weight. (2)

(**b**) (i) Average $E_k = \frac{3}{2}kT = \frac{3}{2} \times 1.3 \times 10^{-23} \times 373 = 7.3 \times 10^{-21}$ J (2)

(ii) Using the ideal gas law, $pV = nRT$, and rearranging with respect to n:

$$n = \frac{pV}{RT} = \frac{100 \times 10^3 \times 2500}{8.53 \times 373} = 7.96 \times 10^4 \text{ moles}$$ (2)

(**iii**) Because the gas is ideal, the entirety of internal energy is a result of kinetic energy.

Total kinetic energy is the product of the number of molecules and the average kinetic energy. To convert from moles into number of molecules, multiply by Avogadro's number.

$E_k = 7.96 \times 10^4 \times 6.02 \times 10^{23} \times 7.3 \times 10^{-21} = 3.45 \times 10^8$ J (2)

(*Total 15 marks*)

2. (**a**) (**i**) The molecules of an ideal gas are considered to be small perfectly elastic spheres moving in random motion with no forces between them. Small and elastic is mentioned in the question so:

1. Motion is random.

2. No forces between molecules except when colliding. (2)

(**ii**) The molecules of an ideal gas have no forces between them so changing their position does not require work to be done; gas molecules therefore have no E_P; this implies that the internal energy of a gas is related to the average E_K of the molecules. If energy is added to the gas, temperature increases so temperature is related to the average E_K. (3)

(**b**) (**i**) Using $PV = nRT$

$T = 290$ K

$P = 4.8 \times 10^5$ Pa

$V = 9.2 \times 10^{-4}$ m^3

$$n = \frac{PV}{RT} = \frac{4.8 \times 10^5 \times 9.2 \times 10^{-4}}{8.3 \times 290} = 0.18 \text{ mol}$$ (2)

(**ii**) If temperature is constant, $P_1V_1 = P_2V_2$

$4.8 \times 10^5 \times 9.2 \times 10^{-4} = P_2 \times 2.3 \times 10^{-4}$

$$P_2 = \frac{9.2}{2.3} \times 4.8 \times 10^5 = 1.9 \times 10^6 \text{ Pa}$$ (2)

(**iii**) If volume is constant, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$P_1 = 19 \times 10^5$ Pa

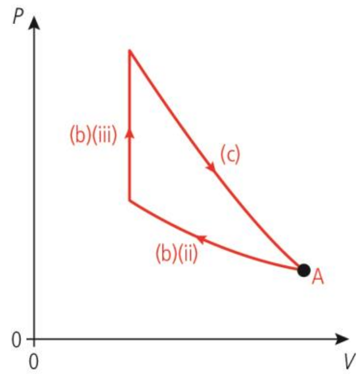
$T_1 = 290$ K

$P_2 = ?$

$T_2 = 420$ K

$$P_2 = 19 \times 10^5 \times \frac{420}{290} = 2.8 \times 10^6 \text{ Pa}$$ (1)

(c)



(3)

(*Total 13 marks*)

3. (**a**) When carrying out gas law experiments, it is important to keep control variables constant. In this experiment, pressure and volume are under investigation and so temperature should be kept constant. (1)

(**b**) Comparing the equation of the graph with $y = mx + c$, b is the gradient because $\frac{1}{H}$ is the horizontal axis. Gradients should be found using a large triangle, e.g. $\frac{2.5 - 1.0}{5.0 - 2.0} = 0.5$. The unit is Pa $(\text{m}^{-1})^{-1}$ = Pa m. (3)

(**c**) We know that volume and height are proportional because of the constant cross-sectional area. We also know that pressure and volume are inversely proportional and so $p \propto \frac{1}{H}$. The line is straight and can reasonably be extrapolated to the origin, therefore this relationship is consistent. (2)

(**d**) $n = \frac{pV}{RT} = \frac{bA}{RT}$ because $pH = b$ and so $pV = bA$

$$n = \frac{5 \times 10^4 \times 1.3 \times 10^{-3}}{8.31 \times 300} = 0.026 \approx 0.03$$ (2)

(**e**) The ideal gas laws only apply at low pressure. Now, large $\frac{1}{H}$ means very high pressures, so at small volumes (and heights) it would not. (2)

(*Total 10 marks*)

4. The molecular kinetic model about the random movement of particles and their collisions explains the ideal gas equation (including how pressure, volume and temperature are interrelated).

Answer: A (1)

(*Total 1 mark*)

5. Using $pV = nRT$, $p = \frac{nRT}{V} = \frac{2 \times 8.31 \times (127 + 273)}{0.083} = 2 \times 100 \times 400$ = 80 000 Pa

Answer: D (1)

(*Total 1 mark*)

6. The two gases are the same temperature and so we know the average kinetic energies are equal. Because the particles in X have more mass, the particles in Y must have higher speeds to compensate, since $E_k = \frac{1}{2}mv^2$.

Answer: B (1)

(*Total 1 mark*)

7. In the liquid state, we can assume that particles are separated by their diameters. In the gas state, the density is 1000 times less, which means that the distances of separation must be 10 times less (because volume is related to (distance)^3).

Answer: B (1)

(*Total 1 mark*)

8. The particles are at the same temperature and therefore the average kinetic energies are equal, irrespective of the state.

Answer: B (1)

(*Total 1 mark*)

9. In P, let $n_P = \frac{pV}{RT}$. In Q, therefore $n_Q = \frac{p \times \frac{V}{2}}{R \times 2T} = \frac{n_P}{4}$

Answer: C (1)

(*Total 1 mark*)

10. We know the pressure will increase because the initial pressure in Q is less than that in R (and because they will equalize). This reduces the possible answers to two.

The total number of particles is n, and this remains constant before and after the valve is opened: $\frac{p \times 3V}{RT} + \frac{3pV}{RT} = \frac{xp \times 4V}{RT}$

$x = \frac{3}{2}$, therefore the pressure change is $\frac{3p}{2} - p = +\frac{p}{2}$

Answer: B (1)

(*Total 1 mark*)

11. (**a**) Using $pV = nRT$, $p = \frac{3.0 \times 8.31 \times 290}{0.15}$ = 48 kPa (1)

(**b**) Using $Q = mc\Delta\theta$, $m = \frac{860}{3100 \times 23}$ = 0.012 kg (1)

(**c**) Using $\overline{E_k} = \frac{3}{2}kT$, $\overline{E_k} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 313 = 6.5 \times 10^{-21}$ J (1)

(**d**) The kinetic model is about the random movement of particles and their collisions (which, in turn, explains the ideal gas law). When temperature increases, average kinetic energy increases, which means that collisions with the walls of the container are both more frequent and more forceful. The larger the force, the larger the pressure for the same area. (3)

(*Total 6 marks*)

B.4 Thermodynamics

Practice questions

1. (**a**) As depth increases, pressure increases. As pressure increases, the volume of air decreases. Volume and pressure are inversely proportional. (2)

(**b**) The pressure at depth is equal to the sum of atmospheric pressure and the pressure difference:

$p = A + \rho gh$

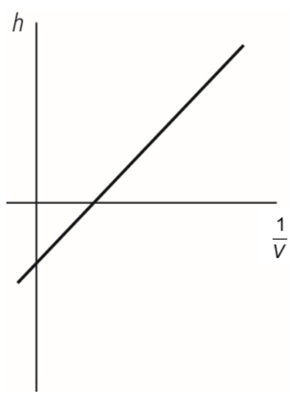
We also know that $pV = nRT$ so $V = \frac{nRT}{p} = \frac{nRT}{\rho gh + A}$

Rearranging for h:

$V\rho gh + VA = nRT$

$h = \frac{nRT}{V\rho g} - \frac{VA}{V\rho g} = \frac{nRT}{\rho gV} - \frac{A}{\rho g}$ (3)

(**c**)



Comparing this equation to $y = mx + c$, h is linearly related to $\frac{1}{V}$ with gradient $\frac{nRT}{\rho g}$ and y-intercept $-\frac{A}{\rho g}$. (2)

(**d**) The fractional uncertainty in the measurement of 50 cm^3 is $\frac{2}{50}$ = 4%, which means the uncertainty in $\frac{1}{V}$ is also 4%.

$\frac{1}{V}$ = 0.02 cm^{-3} so the uncertainty is $\pm 0.0008 \approx$ 0.001 cm^{-3} (2)

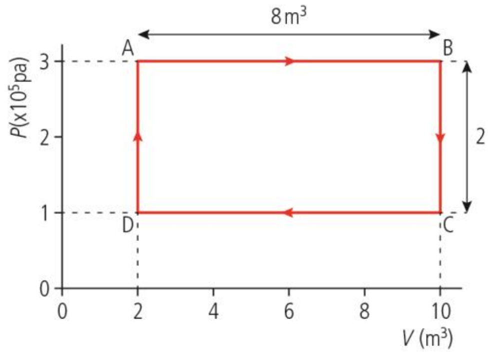
(**e**) Temperature, because this is the only other variable in the gradient equation. (1)

(**f**) The depth should be measured to the level of the water. Instead, all h values are measured too small, which means the magnitude of the intercept is too large and therefore that A is larger when calculated than when measured. (2)

(*Total 12 marks*)

2. (**a**) Work done during cycle = area inside cycle

$= 2 \times 10^5 \times 8 = 1.6 \times 10^6$ J (2)



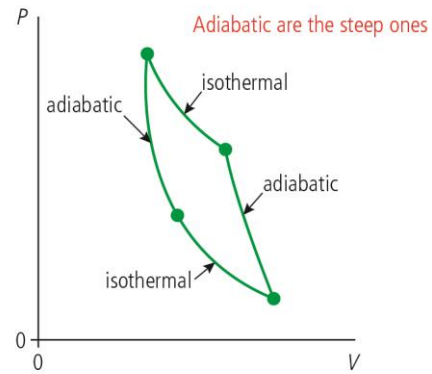
(**b**) 1.8×10^6 J thermal energy ejected

So the energy in must equal work done + energy lost

$= (1.6 + 1.8) \times 10^6 = 3.4 \times 10^6$ J

efficiency $= \dfrac{\text{work out}}{\text{energy in}} \times 100\% = \dfrac{1.6}{3.4} \times 100\% = 47\%$ (2)

(**c**)



(2)

(**d**) (**i**) Adiabatic expansion

Adiabatic compression

Isothermal expansion

Isothermal compression (2)

(**ii**) See labelled graph (2)

(*Total 10 marks*)

3. (**a**) Adiabatic transformations involve no loss or gain of heat. In this case, the expansion is fast, which means there is not enough time for heat transfer. (2)

(**b**) Using $pV = nRT$, $n = \dfrac{104\times 10^3 \times 200\times 10^{-6}}{8.31\times 300} = 0.00835$ = 0.008 moles (2)

(**c**) Using $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$:

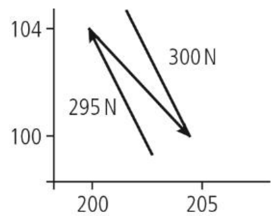
$$\frac{104\times 200}{300} = \frac{100\times 205}{T_2}$$

T_2 = 295 K

ΔT = 5 K (2)

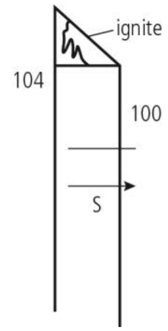
(**d**) $\Delta U = \dfrac{3}{2}nR\Delta T = \dfrac{3}{2}\times 0.00835\times 8.31\times 5$ = 0.52 J (2)

(**e**) (**i**)



(1)

(**ii**) The work done is the area under the graph.



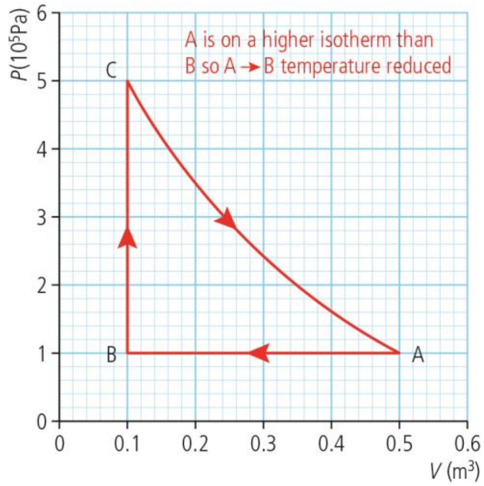
Assuming that the area is rectangular (we can ignore the change in pressure because it is considerably less than either pressure):

$5\times 10^{-6}\times 100\times 10^{3}$ = 0.5 J (3)

(**f**) The temperature change is too fast for accurate measurements. (2)

(*Total 14 marks*)

4. (**a**) (**i**) A → B the volume is getting less ⇒ gas compressed so work is done on the gas. (1)



(**ii**) Temperature of gas goes down and work is done on gas.

First law: $Q = \Delta U + W$

If ΔU and W are both negative then Q is negative ⇒ heat lost. (1)

(**b**) Work done from A → B = area under A–B = $1 \times 10^5 \times -0.4 = 4 \times 10^4$ J (2)

(**c**) Total work done = area of 'triangle' ABC = $\frac{1}{2} \times 4 \times 10^5 \times 0.4 = 8 \times 10^4$ J (2)

(**d**) Useful work done = 8×10^4 J

Thermal energy supplied = 120 kJ = 12×10^4 J

efficiency = $\frac{\text{useful work}}{\text{energy in}} \times 100\% = \frac{8}{12} \times 100\% = 67\%$ (2)

Note: (**c**) and (**d**) are overestimates since the area is a bit less than the area of the triangle.

(*Total 8 marks*)

5. Entropy always increases unless deliberate interventions are made to prevent this. The room temperature increases because the refrigerator in the room does work (and no heat can leave due to the insulation).

Answer: D (1)

(*Total 1 mark*)

6. Work done is the area under the graph. The pressure during this change has a constant value of 5×10^5 Pa and the volume increases by 3 m^3 so the work done is 15×10^5 J.

Answer: A (1)

(*Total 1 mark*)

7. (**a**) The temperatures must be converted from °C to K by adding 273 before substituting them into the equation:

$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{622}{885} = 0.297$$ (2)

(**b**) The Carnot efficiency represents a theoretical maximum. Energy losses are sure to emerge in the usual ways. (2)

(**c**) Efficiency is $\frac{\text{useful output energy}}{\text{total output energy}} = 0.71 \times 0.297$ = 0.211. Since the useful output power is 1.33 GW, the total must be 6.31 GW. The wasted power is therefore 4.98 GW and the energy wasted in an hour is 1.79×10^{13} J. (3)

(**d**) When referring to the first law, it is worth considering what impact wasted heat output has on energy conservation and the relatively constant temperatures that result in no change to internal energy.

With respect to the second law, think about the fact that entropy overall must increase and therefore how heat must enter the surroundings. (3)

(*Total 10 marks*)

8. **(a)** **Alternative 1**

Using $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, $V_2 = \frac{47.1 \times (273 + 19)}{(273 - 12)} = 52.7\ \text{m}^3$

Alternative 2

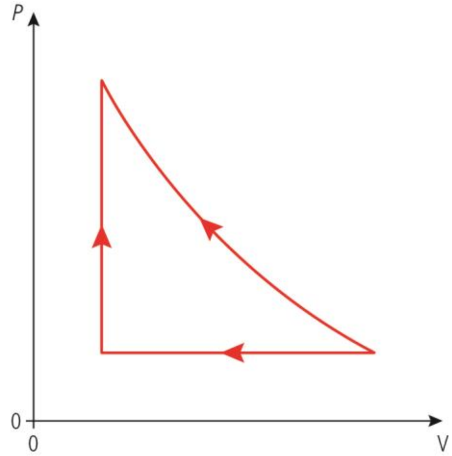
Using $PV = nRT$, $V = \frac{243 \times 8.31 \times (273 + 19)}{11.2 \times 10^3} = 52.6\ \text{m}^3$ (2)

(**b**) $W = P\Delta V = 11.2 \times 10^3 \times (52.7 - 47.1) = 62.7 \times 10^3$ J (2)

(**c**) $\Delta U = \frac{3}{2} nR\Delta T = 1.5 \times 243 \times 8.31 \times (19 - (-12)) = 9.39 \times 10^4$

Using the first law of thermodynamics:

$Q = \Delta U + W = 9.39 \times 10^4 + 6.27 \times 10^4 = 1.57 \times 10^5$ J (3) (**d**) An adiabatic compression is a curve in which pressure increases while volume increases.



Cooling at constant volume is a line in which pressure decreases. (2)

(**e**) Entropy is related to temperature when volume and number of particles are held constant, so entropy decreases with temperature. (1)

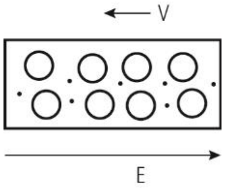
(**f**) A nature of science question, in this case related to laws as being paradigms/ways of thinking/modelling/views, as allowing testing and having applications in situations. (1)

(*Total 11 marks*)

B.5 Current and circuits

Practice questions

1. (**a**) In order for the negative charge carriers to be moving to the left, the field must be to the right:



(1)

(**b**) (**i**) Resistance is the ratio of energy changed to current flowing. In this model, when electrons collide with the lattice atoms, they lose energy. (2)

(**ii**) Heating is associated with an increase in temperature. In this model, when electrons collide with the lattice atoms, the lattice atoms gain kinetic energy. When average kinetic energy increases, temperature increases. (2)

(**iii**) When a light bulb is switched on, there is temporarily very little resistance and then the resistance rapidly increases. The more the lattice atoms vibrate due to increasing kinetic energy from collisions, the more the electrons will collide with them. This is a model for increasing resistance. (2)

(**c**) Current is defined as the rate of flow of charge. Counting the charges present (15) and selecting a time t by which all will have passed through the end of the simulated conductor:

$$I = \frac{15q}{t}$$

Now, assuming the electrons on average have a velocity v, the time taken by the right-most charge carrier to pass through the left is $\frac{L}{v}$.

Substituting into equation for current: $I = \frac{15q}{\frac{L}{v}} = \frac{15vq}{L}$ (3)

(*Total 10 marks*)

2. (**a**) (**i**) Using the equation for the area of a circle as the cross-section, and taking care to account for unit prefixes:

$$R = \frac{\rho L}{A} = \frac{1.1 \times 10^{-6} \times 0.2}{\pi \times \left(\frac{0.1 \times 10^{-3}}{2}\right)^2} = 140\,\Omega$$ (2)

(**ii**) Recognizing that the wire and the cell are in series when finding the total resistance in the circuit:

$R_T = 140 + 1$

$I = \frac{V}{R_T} = \frac{9}{141}$ = 0.06 A (2)

(**iii**) Selecting an appropriate equation for power, bearing in mind that current is the same throughout the circuit and that the resistance is for the wire only:

$P = I^2 R = 0.06^2 \times 140$ = 0.5 W (2)

(**iv**) Assuming that the wire is a black body (especially given that there is no information about emissivity), $P = \sigma A T^4$.

Rearranging for T and using the surface area of a cylinder (ignoring ends):

$T = \sqrt[4]{\frac{P}{\sigma A}} = \sqrt[4]{\frac{0.5}{5.7\times10^{-8} \times \pi \times \frac{0.1\times10^{-3}}{2} \times 0.2}}$ = 726 K (2)

(**b**) The wire is not a perfect black body. Therefore the emissivity will actually be less than 1 and the temperature will be greater. (1)

(**c**) The power dissipated in the cell's internal resistance:

$P = I^2 R = 0.06^2 \times 1 = 0.0036\text{ W}$

In, say, one minute, the energy given out will be $Pt = 0.0036 \times 60$ = 0.216 J

The cell will increase in temperature. To estimate, use the equation $Q = C\Delta T$ so

$\Delta T = \frac{Q}{C} = \frac{0.216}{10} = 0.0022\text{K}$

This increase in temperature is insufficient to damage the cell. (3)

(*Total 12 marks*)

3. We know the combined current is 6.0 A and so we require a voltage that is common to both S and T where the sum of the currents is 6.0 A. This voltage is 4 V. (It is a coincidence that the currents happen to be the same.)

Answer: A (1)

(*Total 1 mark*)

4. Using $R = \frac{\rho l}{A}$ for X, Y has resistance $\frac{\frac{\rho}{2}\times 2l}{\frac{A}{4}} = 4R$. Notice that the effect on the diameter is squared to find the effect on area.

Answer: D (1)

(*Total 1 mark*)

5. Because the emf is common to all three, we can conveniently remove any consideration of current by using $P = \frac{V^2}{R}$. Y has resistance $2R$, Z has resistance $\left(1+\frac{1}{2}\right)R$ and X has resistance $\frac{R}{2}$. This order of decreasing resistance is the same as that which increases power.

Answer: A (1)

(*Total 1 mark*)

6. P and Q lie on either side of a parallel circuit. The upper branch has resistance 4 Ω. The lower branch has resistance $4+\frac{4}{2}=6\,\Omega$. The combination of these is

$$\frac{1}{\frac{1}{4}+\frac{1}{6}} = \frac{1}{\frac{6+4}{24}} = 2.4\,\Omega.$$

Answer: B (1)

(*Total 1 mark*)

7. The voltmeter reading increases because the variable resistor holds a greater share of the overall resistance. The current decreases because the total resistance has increased.

Answer: C (1)

(*Total 1 mark*)

8. (**a**) Emf is defined as the terminal potential difference when no current flows. In this instance, a current is present and so the potential difference will be less than the emf due to 'lost volts' in the internal resistance. (2)

(**b**) Both of the routes to a solution seek to obtain the potential difference across the internal resistance, which can then be divided by the current to find the resistance itself.

Alternative 1

potential difference across cell $= 6.5\,\text{V}$

internal resistance $= \frac{6.5}{0.9} = 7.2\,\Omega$

Alternative 2

$\varepsilon = I(R+r)$ so $\varepsilon = V + Ir$

$21.0 = 14.5 + 0.9 \times r$ so $r = 7.2\,\Omega$ (3)

(**c**) The power arriving at the cell from the sunlight is:

intensity × area = 680 × 0.35 × 0.45 = 107 W.

The electrical power in the external circuit is VI = 14.5 × 0.9 = 13.1 W

Efficiency is 13.1 ÷ 107 = 0.12. (3)

(**d**) By definition, we will need to increase the proportion of energy derived from renewable sources because those that are non-renewable cannot be replaced within a human lifetime, assuming that the global population remains the same or increases. In addition, photovoltaic cells have no greenhouse gas emissions once installed. (2)

(*Total 10 marks*)

9. (**a**) This is calculated using the $\text{speed} = \frac{\text{distance}}{\text{time}}$ equation:

time taken = $\frac{2.0\times10^4}{7} = 2857 \approx 3000\,\text{s}$ (1)

(**b**) Electrical energy = qV = 43 × 10^3 × 16 = 6.88 × 10^5 J

Power = $\frac{E}{t} = \frac{6.88\times10^5}{2857} = 241 \approx 240\,\text{W}$ (2)

(**c**) Using $P = Fv$, which is appropriate both for finding the power evolved from the engine or the power given to the surroundings (because speed is constant):

$F = \frac{241}{7} = 34\ \text{N}$ (2)

(**d**) Only a small proportion of the weight acts down the slope, because the slope has such a low gradient, so $66g\sin 3°$ = 34 N (1)

(**e**) Using a total force 34 + 34 = 68 N and $P = Fv$:

$v = \frac{240}{68} = 3.5\,\text{m s}^{-1}$ (2)

(**f**) From common sense, we know that the maximum distance will decrease. The physics-related reason for this is related to opposing forces increasing and more energy being given to gravitational potential energy (which would not otherwise have been required). (2)

(**g**) The potential difference across the internal resistance is 16 − 12 = 4 V so internal resistance = $\frac{4.0}{6.5}$ = 0.62 Ω (2)

(**h**) Since there are five pairs of cells in series, $\frac{16}{5}$ = 3.2 V (1)

(**i**) We have the emf of one cell. The terminal potential difference across each cell is 2.4 V. The potential difference due to the internal resistance in each cell is therefore 3.2 − 2.4 = 0.80 V.

The current in each cell is half of the maximum because of the parallel arrangement.

$r = \frac{0.80}{3.25} = 0.25\,\Omega$ (2)

(*Total 15 marks*)

10. (**a**) Using $R = \frac{\rho l}{A}$, $l = \frac{RA}{\rho} = \frac{82 \times 8 \times 10^{-3} \times 2 \times 10^{-6}}{4.1 \times 10^{-5}} = 0.032\,\text{m}$ (1)

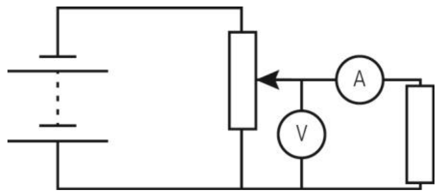
(**b**) The maximum electrical power is the power dissipated per square metre multiplied by the area = 1500 × 8 × 10^{-3} × 0.032 = 0.384 W.

current ≤ $\sqrt{\frac{\text{power}}{\text{resistance}}} = \sqrt{\frac{0.384}{82}} = 0.068\,\text{A}$ (2)

(**c**) Resistivity is a material property, which is a constant for a given material irrespective of its dimensions. (1)

(**d**) The area is larger and the length is smaller because the perpendicular face has changed. Using $R = \frac{\rho l}{A}$, we can see that the resistance will be smaller. (2)

(**e**) The carbon-film resistor will be in parallel with a variable resistor. The ammeter must be in series with the carbon-film resistor and the voltmeter must be in parallel with it.



(2)

(*Total 8 marks*)

C Wave behaviour

C.1 Simple harmonic motion

Practice questions

1. A mass on a spring has simple harmonic oscillations. The definition of simple harmonic motion is when acceleration is proportional to displacement and in the opposite direction. Therefore, we expect a straight line through the origin with negative gradient.

Answer: D (1)

(*Total 1 mark*)

2. Acceleration is represented by the gradient of a velocity-time graph, so acceleration will fstart at zero and immediately become negative. Displacement is proportional to acceleration but in the opposite direction, so it will always be a reflection of acceleration along the time axis.

Answer: A (1)

(*Total 1 mark*)

3. Velocity is represented by the gradient of a displacement-time graph, so we are looking for a point that has the highest possible magnitude of gradient (to maximize velocity) that is negative (so that velocity is to the left).

Answer: C (1)

(*Total 1 mark*)

4. Time period of a pendulum is proportional to the square root of the length so, at first glance, all would seem to have the same. However, looking more closely, the center of mass of D is highest because of its shape, and so the effective length is slightly less than the others. Mass is of no relevance for a pendulum.

Answer: D (1)

(*Total 1 mark*)

5. For a pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

For a mass on a spring: $T = 2\pi\sqrt{\frac{m}{k}}$

Nothing has changed in the move to the Moon except for gravitational field strength, which has decreased. Therefore the time period of a pendulum will increase and the time period for the mass on a spring will be unaffected.

Answer: B (1)

(*Total 1 mark*)

6. At the halfway point, $E_p = \frac{1}{2}m\omega^2\left(\frac{A}{2}\right)^2 = \frac{1}{8}m\omega^2 A^2$

$$E_k = E_T - E_p = \frac{1}{2}m\omega^2 A^2 - \frac{1}{8}m\omega^2 A^2 = \frac{3}{8}m\omega^2 A^2$$

Answer: B (1)

(*Total 1 mark*)

7. (**a**) Simple harmonic motion is defined by acceleration being proportional to the displacement from an equilibrium position and in the opposite direction to the displacement: $a \propto -x$ (2)

(**b**) Using $F = ma$ and observing the maximum acceleration on the graph:

$F = 0.28 \times 2.6$ = 0.73 N (1)

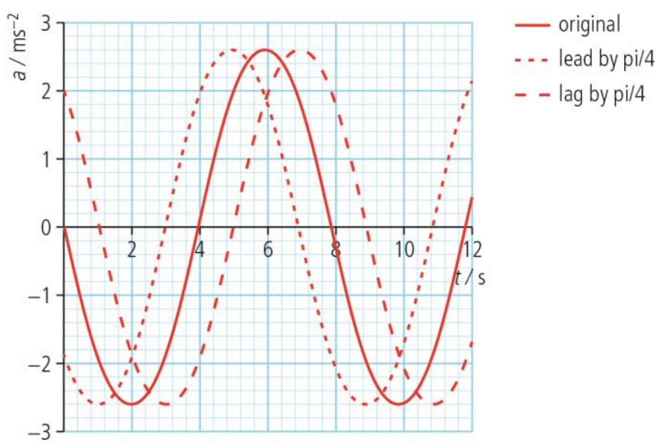
(**c**) We will use $a = -\omega^2 x$, because we already know the maximum acceleration. To find ω, we read the time period from the time axis:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.9} = 0.795$$

Therefore, $x = \frac{a}{\omega^2} = \frac{2.6}{0.795^2}$ = 4.1 m

Use two significant figures because this is what you can read from the graph. (4)

(**d**) The graph will have the same amplitude and time period; its shape should be identical to the original. The phase difference of $\frac{\pi}{4}$ means that it will be in front or behind by an eighth of a cycle. Conveniently, each cycle has a time period of approximately 8 s, so the new curve should be 1 s to the right or left.



(2)

(*Total 9 marks*)

8. (**a**) Simple harmonic motion is defined by acceleration being proportional to the displacement from an equilibrium position and in the opposite direction to the displacement: $a \propto -x$ (2)

(**b**) Since $T = 2\pi\sqrt{\frac{m}{k}}$, $\frac{m_2}{m_1} = \left(\frac{T_2}{T_1}\right)^2$

$$\frac{m_2}{0.52} = \left(\frac{0.74}{0.86}\right)^2$$

m_2 = 0.39 kg (2)

(**c**) Using $E_T = \frac{1}{2}m\omega^2 x_0^2$ with $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.74} = 8.5$:

$E_T = \frac{1}{2} \times 0.39 \times 8.5^2 \times (4.8 \times 10^{-2})^2 = 0.032$ J (3)

(**d**) When springs are in parallel, the spring constant increases. Since $T = 2\pi\sqrt{\frac{m}{k}}$, time period will decrease. Because the uncertainty in the measuring instrument is unchanged, the fractional uncertainty in time period will increase and therefore the fractional uncertainty in mass will also increase. (3)

(*Total 10 marks*)

9. (**a**) Since $F = -\rho Agx$, $ma = -\rho Agx$ and therefore $a \propto -x$. This is the defining proportionality for simple harmonic motion. (1)

(**b**) Using $ma = -\rho Agx$, $\frac{a}{x} = -\frac{\rho Ag}{m}$ and $\frac{a}{x} = -\omega^2$:

$$\frac{\rho Ag}{m} = \omega^2$$

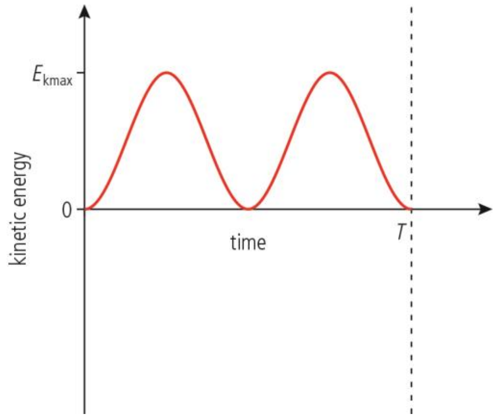
$\omega = \sqrt{\frac{1.03 \times 10^3 \times 2.29 \times 10^{-1} \times 9.81}{118}} = 4.43\,\text{rad}\,\text{s}^{-1}$ (2)

(**c**) Using $E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 118 \times 4.4^2 \times 0.250^2 = 71.4$ J

or

$E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 118 \times 4.43^2 \times 0.250^2 = 72.4$ J (2)

(**d**)



Energy is a scalar quantity, which means that it is not negative. The magnitude of velocity is maximum twice in each time period, so kinetic energy has two peaks in the same time. The question says that the body was initially pushed down, so kinetic energy will start at 0. (2)

(*Total 7 marks*)

10. (**a**) Using $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$:

The first body has $\frac{1}{2\pi}\sqrt{\frac{39.48}{0.90}}$ = 1.054 Hz

The second body has $\frac{1}{2\pi}\sqrt{\frac{39.48}{1.10}}$ = 0.953 Hz (2)

(**b**) The beat frequency is the difference in these: 1.054 − 0.953 = 0.10 Hz

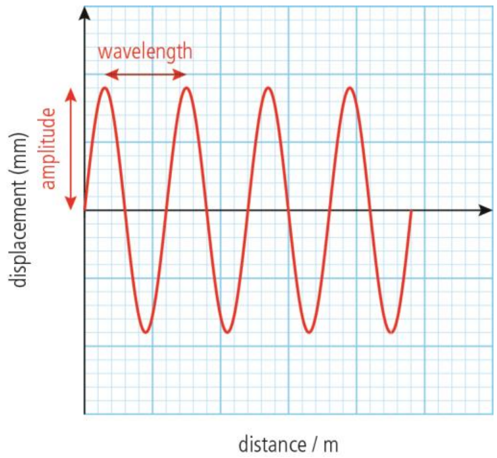
The beat period = $\frac{1}{f} = \frac{1}{10}$ = 10 s (2)

(*Total 4 marks)*

C.2 Wave model

Practice questions

1.



(**a**) Sound is a longitudinal wave. You may be confused by the graph which looks transverse but remember this is a graph not the wave. (1)

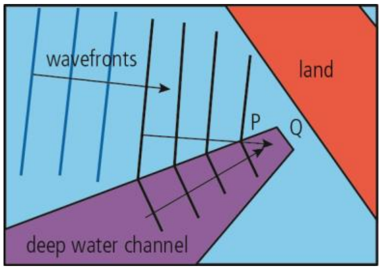
(**b**) (**i**) Wavelength = 0.5 m (1)

(**ii**) Amplitude = 0.5 mm (1)

(**iii**) Speed = frequency × wavelength = 660 × 0.5 = 330 m s^{-1} (2)

(*Total 5 marks*)

2. (**a**)



The deep water channel has waves at higher speed. Therefore, the ray will change direction away from the normal. (1)

(**b**) The waves from the shallow water and the channel superpose. They interfere constructively. (3)

(**c**) The wavelength decreases. This is because the speed of the wave decreases but the frequency is constant. (1)

(**d**) Energy is conserved, and so all must remain contained within the same, shorter, crest. (3)

(*Total 8 marks*)

3. (**a**) (**i**) Using $v = f\lambda$ with wavelength = 14 cm and f = 0.5 Hz:
$v = 0.5 \times 14$ = 7 cm s^{-1} (3)

(**ii**) Since the end pendulums are in phase and all others are within the same 2π period, consecutive pendula will have the phase difference $\frac{2\pi}{8}$ = 0.79 rad. (2)

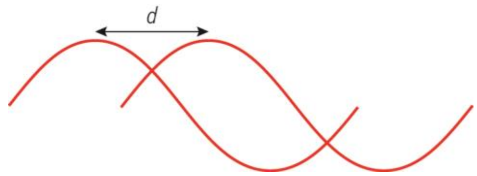
(**b**) (**i**) The wavelength remains at 14 cm because the width of the overall simulation is unchanged. (1)

(**ii**) Since $T = 2\pi\sqrt{\frac{l}{g}}$, $f \propto \frac{1}{\sqrt{l}}$ so $f_{new} = 0.5 \times \sqrt{2}$ = 0.71 Hz (1)

(**iii**) $v = 0.71 \times 14$ = 9.9 cm s^{-1} (1)

(*Total 8 marks*)

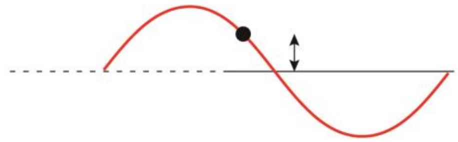
4. (**a**) (**i**) The speed of a wave is the distance traveled by the wave profile per unit time.



So if a wave progressed distance d in time t, $v = \frac{d}{t}$ (2)

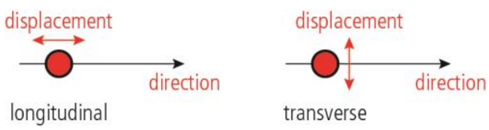
(**ii**) velocity = $\frac{\text{displacement}}{\text{time}}$ where displacement is the distance moved in a certain direction. However light spreads out in all directions. (2)

(**b**) (**i**) Displacement is how far a point on a wave is moved from its original position. For example on a water wave how far up or down a point is relative to the original flat surface of the water.



(2)

(**ii**) In a longitudinal wave, e.g. a wave in a slinky spring, the displacement is in the same direction as the wave but in a transverse wave, e.g. water, the displacement is perpendicular to the direction.



(3)

(**c**) (**i**) From gradient $v_p = \frac{1200}{125} = 9.6\text{ km s}^{-1}$ (1)

(**ii**) $v_s = \frac{1200}{206} = 5.8\text{ km s}^{-1}$ (1)

(**d**) (**i**)

P wave is fastest so gets to the detector first. (1)

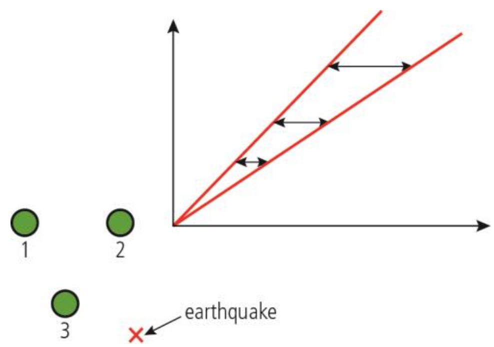
(**ii**) L_3 is closest because the signal arrives first. (1)

(**iii**) 1. First pulse arrives first.

2. Separation of pulses is shorter.

3. Amplitude of pulses is bigger. (3)

(**iv**) Measure on the graph where the horizontal distance between the lines is 68 s, 42 s and 27 s (approximately)



~ 1060 km (L_1), 650 km (L_2), 420 km (L_3)

Closest to L_3 furthest from L_1 (4)

(**v**) Shown on diagram above (1)

(*Total 21 marks*)

5. A longitudinal wave is defined by having displacement of the medium parallel to the direction of energy transfer. All waves (longitudinal or transverse) propagate parallel to the direction of energy transfer.

Answer: B (1)

(*Total 1 mark*)

6. The frequency is 10 waves per minute. Time period is $\frac{1}{f} = 0.1$ minutes.

Answer: A (1)

(*Total 1 mark*)

7. The wavelength is represented by 2π radians, which means $\lambda = 6 \times 0.05$ = 0.3 m

$v = f\lambda = 500 \times 0.3$ = 150 m s^{-1}

Answer: C (1)

(*Total 1 mark*)

8. $v = f\lambda = 1 \times 10^3 \times 0.33$ = 330 m s^{-1}

This is the speed of sound in air, and sound is a longitudinal wave.

distance = speed × time = 330 × 2 × 10^{-3} = 0.66 m

Answer: C (1)

(*Total 1 mark*)

9. Taking the points in turn:

- X and Y are in antiphase.
- The velocity at × is zero because displacement is a maximum.
- Using $v = f\lambda$, $\lambda = \frac{v}{f} = \frac{100}{25}$ = 4 m and the horizontal distance between X and Z is three-quarters of a wavelength (which is 3 m).
- The particle at Y oscillates about a fixed point; it is the wave that causes this oscillation that travels at 100 m s^{-1}.

Answer: C (1)

(*Total 1 mark*)

10. The time period is 20 × 10^{-6} s, which means the frequency is $\frac{1}{20 \times 10^{-6}}$ = 50 000 Hz.

The amplitude is the maximum displacement from the equilibrium position (i.e. half of peak to trough).

Answer: C (1)

(*Total 1 mark*)

11. One wavelength is 8 m. One time period is 10 ms.

speed = $\frac{\text{distance}}{\text{time}} = \frac{8}{10 \times 10^{-3}} = 800$ m s^{-1}

Answer: D (1)

(*Total 1 mark*)

12. Both X-rays and ultraviolet radiation are waves and therefore both can be diffracted. Both are part of the electromagnetic spectrum and therefore both have the same speed in a vacuum. X-rays do have a higher frequency (and shorter wavelength) than ultraviolet waves; they are consecutive regions on the spectrum.

Answer: B (1)

(*Total 1 mark*)

13. The time period is equal to the time taken for the wave to travel out and back, because each clap coincides with the echo from the one before.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{2L}{\frac{1}{f}} = 2Lf$$

Answer: D (1)

(*Total 1 mark*)

14. P is at a crest and about to reduce in amplitude towards zero. Q is at the equilibrium position (displacement = 0) and about to have negative displacement because it is set to fall further still.

Answer: C (1)

(*Total 1 mark*)

15. Intensity is proportional to the square of amplitude. Therefore, amplitude is proportional to the square root of intensity.

Answer: A (1)

(*Total 1 mark*)

16. (**a**) Longitudinal waves are characterized by the oscillations of particles in the medium being parallel to the direction of energy transfer. (1)

(**b**) (**i**) The two lines show displacements 0.882 ms apart. The distance traveled in this time is 0.3 m.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{0.3}{0.882 \times 10^{-3}} = 340\text{ m s}^{-1}$$ (2)

(**ii**) A wavelength is the distance of a full wave form, which (looking at either one of the lines) is 1.6 m.

Using $v = f\lambda$, $f = \frac{340}{1.6}$ = 213 Hz (2)

(**c**) (**i**) The particle appears from the graph as though it is about to go ‘down’. However, this is a longitudinal wave, and so the appropriate option is ‘left’, as this coincides with the parallel particle oscillations and wave propagation. We know this because its displacement is about to become negative. (2)

(**ii**) The particle is in the center of a rarefaction. We know this because particles to its left are already displaced to the left and particles to its right are already displaced to its right; they are spread apart. (2)

(*Total 9 marks*)

17. (**a**) (**i**) Let t_P and t_S be the times for the P and S waves, respectively, to travel D.

Then: $D = 5.50 \times 10^3 t_P$ and $D = 3.00 \times 10^3 t_S$

Subtracting these equations: $t_S - t_P = D\left(\frac{1}{3.00 \times 10^3} - \frac{1}{5.50 \times 10^3}\right)$

Since $t_S - t_P$ =917 s, D = 6052 km (2)

(**ii**) $T = \frac{D}{\text{speed}} = \frac{6052}{800}$ hours

This is 7 hours 34 minutes. (2)

(**b**) After arrival of the S wave, the time delay between the P and S waves can be determined and D can be calculated. The time before the arrival of the tsunami (i.e. the warning time) is:

$t_W = \left(\frac{D}{800} - \frac{D}{3.00 \times 60 \times 60}\right)$ hours for D measured in km

$t_W = 1.16 \times 10^{-3} D$ hours

The inhabitants therefore can be warned and have time to go to higher ground. In case of interest, seismometers can detect both wave forms. If connected to a computer, its warning can be given almost immediately after the arrival of the S wave. (2)

(*Total 6 marks*)

C.3 Wave phenomena

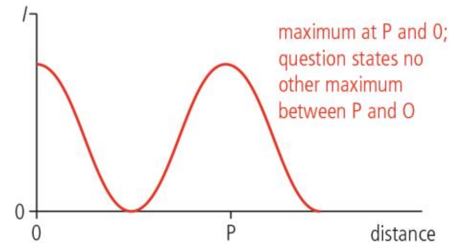
Practice questions

1 (**a**) (**i**) Two sources are said to be coherent if they have the same frequency, similar amplitude and a constant phase difference. (1)

(**ii**) To get interference the light from S_1 and S_2 must overlap; this only happens if the light is diffracted which means the slits must be narrow. (2)

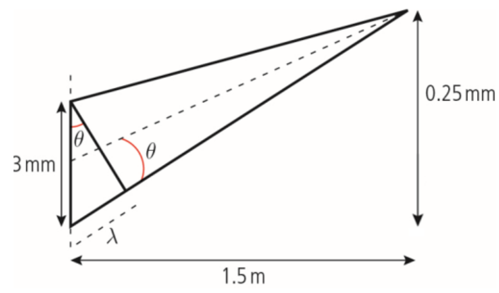
(**b**) (**i**) For maximum intensity the path difference = $n\lambda$ where n is a whole number. (1)

(**ii**)



(2)

(**c**)



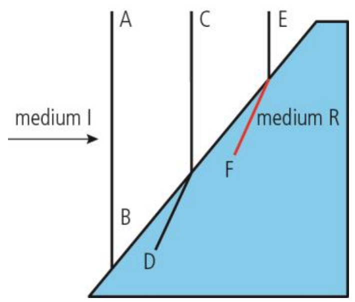
Since angle θ is

$$\theta = \frac{\lambda}{0.003} = \frac{2.5\times10^{-4}}{1.5} \Rightarrow \lambda = \frac{0.003\times2.5\times10^{-4}}{1.5} = 5\times10^{-7}\text{ m} = 500\text{ nm}$$ (2)

(*Total 8 marks*)

2. (**a**) A ray shows the direction of a wave and a wavefront is a line joining points that are in phase. A ray is perpendicular to a wavefront. (3)

(**b**) (**i**) The line should be parallel to D. (1)



(ii) We can see that the wavelength gets shorter ⇒ velocity is less in medium R.

Could also tell from the way the wave bends. (3)

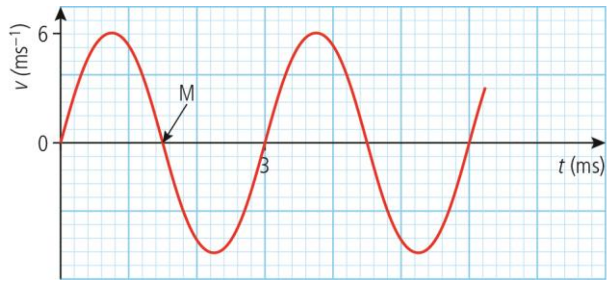
(iii) ratio = $\frac{V_I}{V_R} = \frac{3.0}{1.5}$ = 2.0 (2)

(c) **(i)** The sign of velocity changes ⇒ direction changes ⇒ body is oscillating. (2)

(ii) Time period = 3 ms

$F = \frac{1}{T} = \frac{1}{0.003}$ = 330 Hz (2)

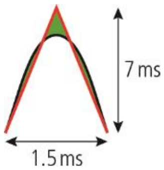
(iii) Maximum displacement is when velocity is zero – think of a pendulum; it stops at the top.



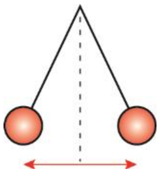
(1)

(iv) To find area either count squares or make a triangle that is a bit higher than the top.

Squares are rather small so it is easier to calculate $\frac{1}{2}$ × 7 × 0.0015 = 5.25 mm (2)



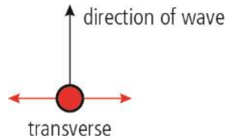
(v) Area under *v*–*t* graph is displacement. In this case it is displacement between two times when velocity is zero, i.e. 2 × amplitude.



(1)

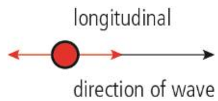
(*Total 17 marks*)

3. (**a**) (**i**)



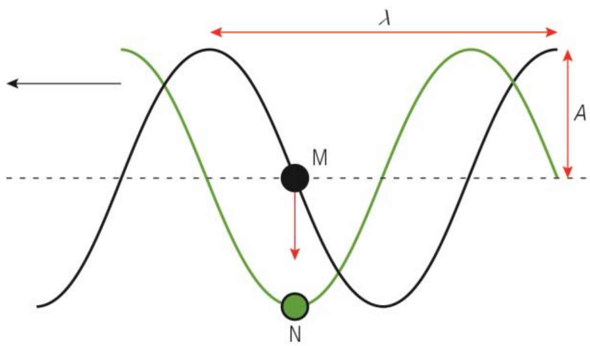
(1)

(**ii**)



(1)

(**b**) In $\frac{T}{4}$ the wave progresses $\frac{1}{4}$ cycle or $\frac{1}{4}\lambda$



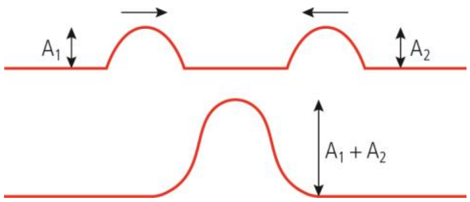
Wave moves left so trough is about to reach M. (5)

(**c**) (**i**) $\lambda = 5.0$ cm; $v = 10$ cm s^{-1}

$v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{10}{5} = 2.0$ Hz (1)

(**ii**) In $\frac{1}{4}$ wave moves $\frac{1}{4}\lambda = \frac{1}{4} \times 5.0 = 1.25$ cm (2)

(**d**) When two waves coincide the resultant displacement at any point is equal to the vector sum of the individual displacements.



If waves that have the same frequency and a constant phase overlap then, due to superposition, they will add or cancel out. This is called interference. (4)

(**e**) (**i**) If the path difference = $n\lambda$ then there will be constructive interference. n is an integer. (2)

(**ii**) Since angles are small, $\theta = \frac{S_2X}{d}$ (2)

(**iii**) Again small angles so $\phi = \frac{y_n}{D}$ (1)

(**f**) (**i**) $\theta = 2.7 \times 10^{-3}$ rad; d = 1.4 mm

$\theta = \frac{S_2X}{d}$ where $S_2X = 8\lambda$

so $\lambda = \frac{d\theta}{8} = \frac{1.4 \times 10^{-3} \times 2.7 \times 10^{-3}}{8} = 473$ nm (2)

(**ii**) From geometry:

$\varphi = \theta = 2.7 \times 10^{-3}$ rad

$D = 1.5$ m

$\phi = \frac{y_n}{D} \Rightarrow y_n = \varphi \times D = 4$ mm

spacing = $\frac{4}{8}$ = 0.5 mm (3)

(*Total 24 marks*)

4. Frequency is set from the moment the wave is produced. However, speed increases from water to air and so wavelength increases because $v = f\lambda$

Answer: C (1)

(*Total 1 mark*)

5. At an angle greater than the critical angle, total internal reflection occurs for light passing from a more dense to less dense medium.

Answer: D (1)

(*Total 1 mark*)

6. $\frac{\text{speed of light in liquid X}}{\text{speed of light in liquid Y}} = \frac{n_Y}{n_X} = \frac{\frac{n_g}{n_X}}{\frac{n_g}{n_Y}} = \frac{2}{1.5} = \frac{4}{3}$

Answer: C (1)

(*Total 1 mark*)

7. The wavefront will be blocked to an extent by the barrier, so we would expect least progress immediately to the right of the barrier. You can then ascertain the correct answer from A and D by covering the lower half of the diagram with one hand and imagining the pattern for a gap.

Answer: A (1)

(*Total 1 mark*)

8. Amplitude sums vectorially: $X_0 + X_0 = 2X_0$

Intensity is the square of the amplitude and so will be 2^2 times higher than the original.

Answer: D (1)

(*Total 1 mark*)

9. The path difference is 0.4 m, which is two full wavelengths. Therefore, the waves meet in phase and the sum of their amplitudes is double the amplitude of either one. In turn, therefore, the intensity is 2^2 times the intensity of either.

Answer: D (1)

(*Total 1 mark*)

10. When the distance from the slits to the screen is increased, the angular spread of the pattern stays the same, but its spacing on the screen increases.

Incidentally, increasing the frequency decreases the wavelength which reduces the fringe spacing. Increasing the distance between the slits also reduces the fringe spacing (as can be spotted from a glance at the equation).

Answer: C (1)

(*Total 1 mark*)

11. Using $s = \frac{\lambda D}{d} = \frac{\lambda y}{a}$, we can see that the fringe spacing will double overall.

Answer: B (1)

(*Total 1 mark*)

12. Since $\theta = \frac{\lambda}{b}$, where θ is the first minimum: $\frac{\lambda}{b} = 1 \times 10^{-2}$

Answer: A (1)

(*Total 1 mark*)

13. Statement I is correct because the fewer the lines per meter, the further apart they are.

Statement II is incorrect because θ decreases as d increases.

Statement III is correct because n increases as d increases.

Answer: B (1)

(*Total 1 mark*)

14. (**a**) $v = c\frac{\sin i}{\sin r} = \frac{3\times10^8\times\sin(33°)}{\sin(46°)}$ = 2.3 × 10^8 m s^{-1} (2)

(**b**) This question is driving towards showing that total internal reflection occurs, for which you need to compare the angle of incidence with the critical angle:

light strikes AB at an angle of 57°

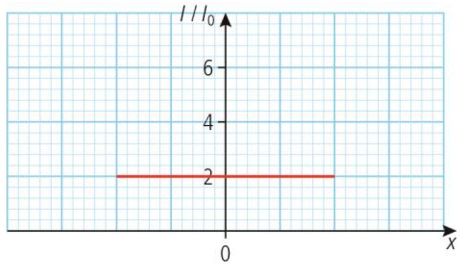
$$c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{\sin(33°)}{\sin(46°)}\right) = 49°$$ (3)

Because the angle of incidence is greater than critical angle, total internal reflection occurs.

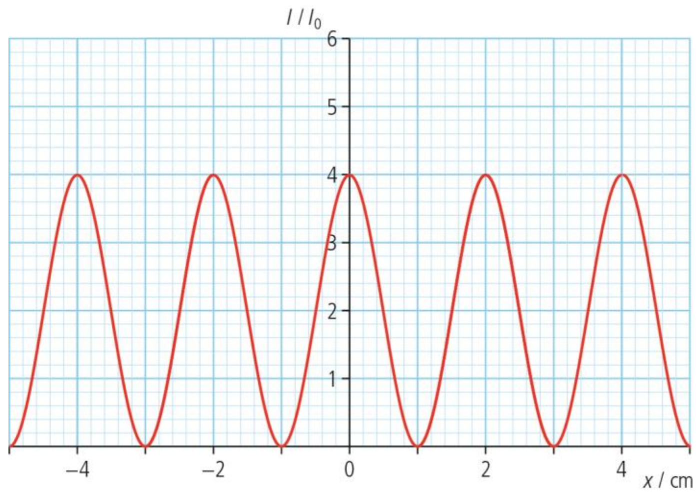
(**c**) Take care to use a protractor to measure angles to the surface where the light reflects along AB and then remember to show the light ray bending away from the normal as it leaves along the next surface. (2)

(*Total 7 marks*)

15. (**a**) There is no diffraction at either lamp because there are no slits. Therefore, the intensity has no pattern. (1)



(**b**) Now we move to the standard two slit pattern. The maximum intensity is four times the separate intensities because the amplitudes sum vectorially. The fringe spacing is calculated using $\frac{\lambda D}{d}$ to be 2.0 cm. (3)



(**c**) As the slit separation increases, the fringe spacing decreases. (1)

(*Total 5 marks*)

16. (**a**) The amplitude doubles and intensity is proportional to (amplitude)^2, so the intensity will be $22 \times 2^2 = 88\ \text{W m}^{-2}$ (3)

(**b**) $s = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{sd}{D} = \frac{0.12 \times 10^{-3} \times 7.0 \times 10^{-3}}{1.5} = 560\,\text{nm}$ (2)

(**c**) The double slit interference pattern is capped by the single slit diffraction envelope. Sometimes even double slit intensity maxima may not appear if they coincide with a single slit minimum. (1)

(**d**) The first minimum of the diffraction envelope is at $\theta = \frac{\lambda}{b} = \frac{560 \times 10^{-9}}{0.030 \times 10^{-3}}$ = 0.01867 rad.

Using trigonometry and the small angle approximation, the distance on the screen is 1.50×0.01867 = 28 mm, so intensity will be zero. (2)

(*Total 8 marks*)

17. (**a**) As the three rays drawn together form a maximum, the points an equal distance from the screen (V, Y and Z) must be in phase, which can be considered as 0 or 2π radians. (1)

(**b**) WY is two wavelengths, because we are looking at rays to the second-order diffraction maximum. XZ must therefore be two wavelengths: 4λ. (1)

(**c**) Using trigonometry and recognizing that θ is also the angle subtended at V,

$\sin\theta = \frac{\text{XZ}}{\text{VX}} = \frac{4\lambda}{2d} \Rightarrow 2\lambda = d\sin\theta$ (1)

(**d**) The equation of the line is $\sin\theta = \frac{\lambda}{d}n$ and so the gradient is $\frac{\lambda}{d} = 0.080$.

The distance between slits, $d = \frac{\lambda}{0.08} = \frac{633 \times 10^{-9}}{0.08} = 7.91 \times 10^{-6}\ \text{m}$

The number per meter is therefore $\frac{1}{7.91 \times 10^{-6}}$ = 126 000 m^{-1} = 126 mm^{-1} (4)

(**e**) (**i**) As the gradient is $\frac{\lambda}{d}$, the gradient would decrease. (1)

(**ii**) The distance to the screen has no effect on the angle at which the fringes form, but can increase their spacing if increased. (1)

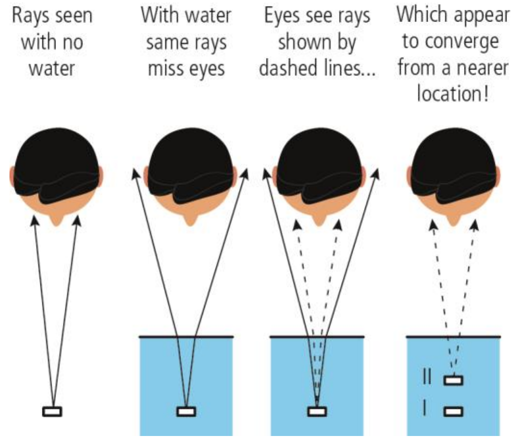
(*Total 9 marks*)

18. Although the route via B has the shortest distance, the lifeguard can travel faster while running on the beach than when swimming in the water. Therefore, route C is the fastest overall, because the time saving on the swim compensates for the increased distance on land.

Light always travels from one point to another along the path requiring least time. (2)

(*Total 2 marks*)

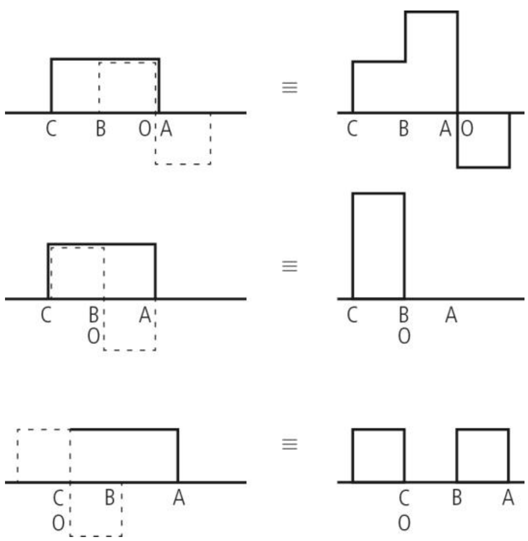
19. A human estimates distance using both eyes with the brain sensing the extent to which the eyes are crossed. The light reflecting from the coin will bend away from the normal when exiting the water, so the straight-line appearance is shorter.



Answer: A (1)

(*Total 1 mark*)

20. The resultant wave form is the sum of the two wave forms.



(4)

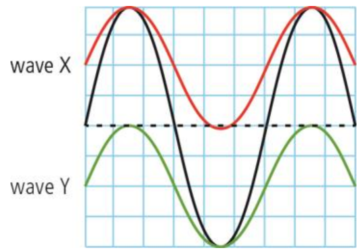
(*Total 4 marks*)

C.4 Standing waves and resonance

Practice questions

1. (**a**) Superposition is what happens when two waves coincide: the displacements of waves add vectorially to produce a resultant wave. (2)

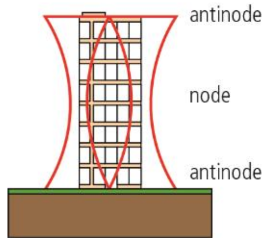
(**b**)



If X and Y are added they give the resultant. (4)

(*Total 6 marks*)

2. (**a**)



(1)

(**b**) If the standing wave is as shown then wavelength = 2 × height of building

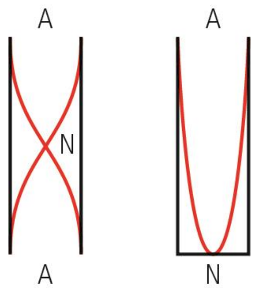
$\lambda = 2 \times 280 = 560$ m

$v = f\lambda$ so $f = \frac{v}{\lambda} = \frac{3400}{560}$ = 6.1 Hz (about 6 Hz)

So if the earthquake has a frequency of 6 Hz, the building will be forced to vibrate at its own natural frequency so will have a large amplitude vibration (resonance). (3)

(*Total 4 marks*)

3. (**a**) (**i**) (**ii**)



(4)

(**b**) (**i**) Frequency of 1st pipe = 512 Hz

Wavelength of wave = $\frac{v}{f} = \frac{325}{512}$ = 63.5 cm

pipe is $\frac{1}{2}\lambda$ so length = $\frac{63.5}{2}$ = 31.7 cm = 0.317 m (3)

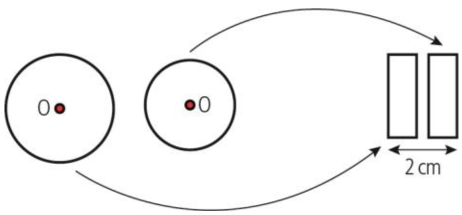
(**ii**) The length of a closed pipe is shorter than an open one of the same frequency so if the organ pipes are closed they take up less space. (2)

(*Total 9 marks*)

4. (**a**) (**i**) The motion corresponds approximately to the horizontal displacement of the joint. This is sinusoidal, which is analogous to SHM. (2)

(**ii**) Because of the angle of the connecting rod. This is reduced by the connecting rod being long (so approximately horizontal throughout). (2)

(**iii**) Because the rod is attached 1 cm from the center, the amplitude is 1 cm. (1)



(**iv**) 0.125 s is a quarter of a revolution, which means the time period is 0.5 s.

$f = \frac{1}{T} = \frac{1}{0.5}$ = 2 Hz (1)

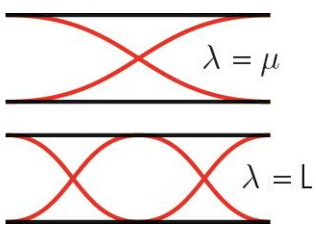
(**b**) (**i**) The distance between the centers of two rarefactions is 2 m, and so the wavelength is 2 m. (1)

(**ii**) $v = f\lambda = 2 \times 2$ = 4 m s^{-1} (1)

(**iii**) The free end means that there is no phase change when the wave reflects. (2)

(**iv**) At the point when the wave reaches the end, a standing wave has not yet been formed; only one cycle has passed.

After reflection, a standing wave will form with both ends as antinodes if the length of spring is equal to a whole number of half wavelengths.

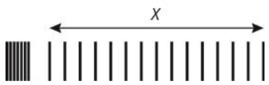


(3)

(**c**) From Hooke's law we know that $F_t = kx$

Therefore: $v = \sqrt{\frac{kx}{\mu}}$

Additionally, we can assume that $\mu = \frac{M}{x}$ because the unstretched length is negligible in comparison to the extension.



Therefore: $v = \sqrt{\frac{kx}{\frac{M}{x}}} = \sqrt{\frac{kx^2}{M}} = \sqrt{\frac{k}{M}}x$, which means that velocity and length are proportional. (3)

(**d**) The ball rattles when the acceleration of the piston is greater than *g*, which means that the ball loses contact.

Assuming SHM with maximum acceleration: $a = \omega^2 x_0$:

$\omega^2 = \frac{9.81}{0.01} = 981$

$\omega = 31.3 = 2\pi f$

$f = 5.0\text{ Hz}$ (3)

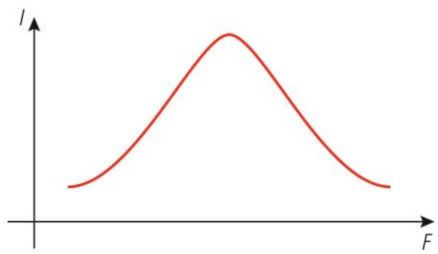
(*Total 19 marks*)

5. (**a**) If an alternating current is applied to the circuit, it will be forced to oscillate. If the frequency is the same as the first harmonic, then resonance will occur, which leads to a large current. (4)

(**b**) Using $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.00 \times 10^{-3} \times 2.00 \times 10^{-12}}} = 1.1 \times 10^7\text{ Hz}$

$f = \frac{\omega}{2\pi} = \frac{1.1 \times 10^7}{2\pi} = 1.8 \times 10^6\text{ Hz}$ (2)

(**c**) This is a resonance curve:



(2)

(*Total 8 marks*)

6. In the first harmonic there are two antinodes (one at each end) and one node (in the middle). In the second harmonic there are three antinodes (one at each end and one in the middle) and two nodes. The pattern continues; the sum is always an odd number.

Answer: A (1)

(*Total 1 mark*)

7. If it is called the third harmonic then the end conditions do not matter; it will by definition have a frequency three times the first harmonic.

Answer: C (1)

(*Total 1 mark*)

8. (**a**) Standing waves occur when two waves of equal amplitude and frequency meet in opposite directions. In most cases (including this one), they result from a reflection. (2)

(**b**) Because both ends are fixed, for the first harmonic: $\lambda = 2l = 2 \times 0.62 = 1.24\,\text{m}$

$v = f\lambda = 195 \times 1.24 = 242\,\text{m s}^{-1}$ (2)

(**c**) Point P obeys simple harmonic motion, acceleration and displacement are proportional with a negative gradient. (1)

(**d**) In the third harmonic there are three loops: $\frac{62}{3} = 21\text{cm}$ (1)

(*Total 6 marks*)

9. (**a**) Considering the molecules at both ends, which are at their amplitudes at $t = 0$, they will be at the equilibrium position after three-quarters of a time period. The centermost molecule will remain at the equilibrium position throughout. Therefore, there is no displacement at any position. (1)

(**b**) As this is a longitudinal wave and because the molecule commences to the left of its equilibrium position, it will start by moving to the right, it will then move to the left and then return to its original position (2)

(**c**) Half a wavelength is shown in the full length of the pipe, therefore:

the wavelength = 2 × 1.4 = 2.8 m

$c = f\lambda = 120 \times 2.8 = 340\,\text{m s}^{-1}$

Rearranging the equation given: $K = \rho c^2 = 1.3 \times 340^2 = 1.5 \times 10^5\,\text{kg m}^{-1}\,\text{s}^{-2}$ (4)

(*Total 7 marks*)

10. (**a**) Standing waves do not transmit energy whereas traveling waves do. Standing waves have nodes and antinodes with range of amplitudes in between, whereas a traveling wave has a constant amplitude. (2)

(**b**) A sound wave travels down tube and is reflected such that the incident and reflected waves interfere. (2)

(**c**) The water surface acts as a node. The top opening of the tube is an antinode. The drawing is of the third harmonic. (1)

(**d**) The tube is raised through a distance equal to half a wavelength, therefore $\lambda = 0.74\,\text{m}$.

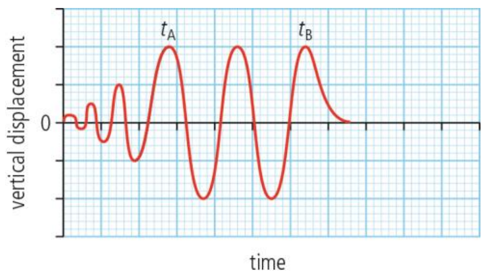
Rearranging $v = f\lambda$: $f = \frac{c}{\lambda} = \frac{320}{0.74} = 430\,\text{Hz}$ (2)

(*Total 7 marks*)

11. (**a**) (**i**) For a wave, amplitude and energy are related. When the vibrator is switched on, the energy goes into larger oscillations. (1)

(**ii**) Because the amplitude remains constant, we can assume that the energy input is equal to the energy lost due to damping. (1)

(**b**) In critical damping the system returns to zero displacement as quickly as possible without overshooting.



(2)

(*Total 4 marks*)

12. (**a**) Because each carriage is 25 m long and the speed is 10 m s^{-1}, we calculate the time period using $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{25}{10} = 2.5\,\text{s}$.

$f = \frac{1}{T} = \frac{1}{2.5} = 0.4\,\text{Hz}$ (1)

(**b**) Performing the same series of calculations shows that 30 m s^{-1} corresponds to $f = 1.2\,\text{Hz}$, which is the resonant frequency according to the graph. (2)

(**c**) The curve will be lower in amplitude everywhere with its maximum at a lower frequency. (2)

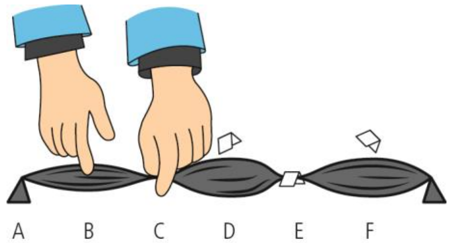
(*Total 5 marks*)

13. The larger the air space, the lower the first harmonic (just like how big organ pipes make the low notes). The air space increases as the liquid runs out.

Answer: A (1)

(*Total 1 mark*)

14. Pinching at C means a node will form at C. A node will also therefore form at E so that the loops are equal in length. B, D and F are antinodes with greatest vibration.



Answer: D (1)

(*Total 1 mark*)

15. The trick is to push the rolling ball at the right times and in the right direction. The mouse would need to match the natural rhythm of the back-and-forth of the ball.

Answer: B (1)

(*Total 1 mark*)

16. (**a**) When the pan is moving upwards with a retardation when approaching its amplitude, the mass will leave the pan if the retarding acceleration is less than *g*.

For displacement *x*, using $F = kx = ma$, the retarding acceleration is $-\frac{kx}{m}$ where *k* is the spring constant.

When $-\frac{kx}{m} < -g$, i.e. when $\frac{4\pi^2 x}{T^2} > g$ (since $T = 2\pi\sqrt{\frac{m}{k}}$) the mass leaves the pan. (3)

(**b**) As the amplitude varies, this will occur first when $\frac{4\pi^2 A}{T^2} = g$ i.e. $A = \frac{T^2 g}{4\pi^2}$

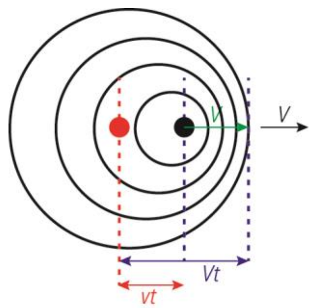
Substituting the data provided, $A = \frac{0.50^2 \times 9.81}{4\pi^2} = 0.062\text{m}$ =6.2 cm (3)

(*Total 6 marks*)

C.5 Doppler effect

Practice questions

1. (a)



(3)

(**b**) In time *t* distance moved by source = *vt*

distance moved by sound = *Vt*

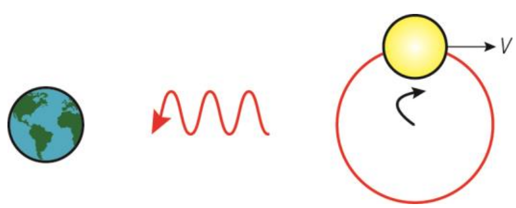
So all waves produced in time *t* are squashed into a distance *Vt* – *vt* ahead of the source, so number of complete cycles produced = f_0t

so $\lambda = \frac{Vt - vt}{f_0 t}$ but $f = \frac{V}{\lambda}$ so apparent frequency $f_1 = \frac{V}{\lambda} = \frac{V \times f_0}{V - v}$ (3)

It can be a bit confusing using *V* and *v*.

(**c**) Using the formula $\Delta\lambda = \frac{V}{c}\lambda$

$0.004 = \frac{V}{3 \times 10^8} \times 600$



$V = \frac{0.004}{600} \times 3 \times 10^8 = 2 \times 10^3\ \text{m s}^{-1} = 2\ \text{km s}^{-1}$ (3)

(*Total 9 marks*)

2. As the source of the sound is moving away, the wavelength will be perceived as increased. The wave speed is unchanged, because the observer is not moving.

Answer: B (1)

(*Total 1 mark*)

3. The intensity increases because the train is getting closer to the station and therefore its energy is spread over a smaller surface area.

The frequency decreases from frequency that would have been observed when the train was at maximum incoming speed. As the train stops the observed frequency will decrease until it matches the source frequency.

Answer: D (1)

(*Total 1 mark*)

4. In an extension from the previous question, we know that the frequency will decrease as the train approaches while decelerating.

When the train has passed the observer, the observed frequency will increase from a new minimum as it tends towards the source frequency.

If in doubt, remember that the observed frequency and the source frequency are equal when there is no relative motion.

Answer: D (1)

(*Total 1 mark*)

5. As it is the source that is moving, $f' = f\left(\frac{v}{v+u_s}\right) = f_0\left(\frac{v}{v+0.1v}\right) = \frac{1}{1.1}f_0$

It is worth performing a quick check that this yields the correct directionality of answer. In this case, we would expect for the observed frequency to be lower, because the train is moving away.

Answer: B (1)

(*Total 1 mark*)

6. As the observer is stationary, the observed speed of the sound is unchanged: *v*.

As the source is approaching, the observed wavelength will be reduced. Therefore, we have our answer!

Answer: A (1)

(*Total 1 mark*)

If you had been asked a version of this in a 'show that' format, the observed frequency for a moving source is $f' = f\left(\frac{v}{v+u_s}\right) = f\left(\frac{c}{c-\frac{c}{34}}\right) = \frac{34}{33}f$

$$\lambda' = \frac{c}{f'} = \frac{c}{\frac{34}{33}\times\frac{c}{\lambda}} = \frac{33}{34}\lambda$$

7. The observer is stationary, therefore the observed speed is unchanged. The source is moving away, therefore the observed wavelength is increased.
Answer: A (1)

(*Total 1 mark*)

8. The girl is moving away from the beach, because she is observing a higher frequency of wavefronts than that of the wavefronts themselves.

The wavelength of the sea waves is $\frac{2}{0.1} = 20$ m

If we assume momentarily that the sea is stationary, the girl's speed is $0.4 \times 20 = 8\ \text{m s}^{-1}$

Combining the girl's speed with the water speed, $8 - 2 = 6\ \text{m s}^{-1}$

Answer: B (1)

(*Total 1 mark*)

9. The observer is moving away from the gas, therefore the observed wavelengths will be increased (i.e. to the right).

As the percentage change is the same for both lines, the line that had the greatest wavelength to begin with will have the greatest shift.

Answer: C (1)

(*Total 1 mark*)

10. (**a**) There is relative motion between the approaching car and the stationary police officer, which means the observed frequency will be increased. (2)

(**b**) The microwaves are not emitted by the car but reflected off it; the frequency change doubles up: on the route of the microwaves to the car and from the car back to the police officer. (2)

(**c**) Rearranging the equation for v and using the speed of light for the microwaves:

$$v = \frac{\Delta fc}{2f} = \frac{9.5 \times 10^3 \times 3 \times 10^8}{2 \times 40 \times 10^9} = 36\ \text{m s}^{-1}$$

This exceeds the maximum speed allowed. (2)

(*Total 6 marks*)

11. (**a**) Using theory from previous waves chapters, we compare the wavelength (0.40 m) with the path difference (1.8 m): $1.8\text{m} = 4.5\lambda$

The half number of wavelengths means that the waves meet in antiphase and therefore destructive interference occurs, which means that the intensity is a minimum. (4)

(**b**) This question is about double slit interference patterns, in which maxima and minima are equally spaced with a maximum along the perpendicular between the sources and the measurement line.

Because the path difference at P is 4.5λ, there are four maxima between P and Q in the positions of whole numbers of wavelengths. (2)

(**c**) With both loudspeakers connected, the amplitudes at Q summed together in full. When B is disconnected, the amplitude of sound at Q is therefore halved. Since intensity is proportional to amplitude squared, $\frac{I_A}{I_0} = \frac{1}{4}$. (2)

(**d**) Now to the Doppler effect! In this example, the source is stationary and the observer is moving. Although the wavelength of the sound is unchanged, the observed speed of sound is less than it is at source, which means that the observed frequency is lower. (2)

(**e**) Using the equation for a moving observer:

$$f' = f\left(\frac{v - u_o}{v}\right):\ 845 = 850 \times \frac{340 - v}{340}$$

$v = 2.00\,\text{m s}^{-1}$ (2)

(*Total 12 marks*)

12. (**a**) The return to Earth is governed by the distance away, which means that the shortest distance should be selected. (2)

(**b**) The Doppler effect is governed by relative speeds, which means that the point approaching with highest velocity should be selected. (2)

(*Total 4 marks*)

13. (**a**) For a stationary source and observer, $c_s = f\lambda$

(**i**) When the observer is stationary and the source is moving away from the observer, the wavelength λ_s increases. The distance between wave crests increases by the distance the source moves during a period:

$\lambda_s = \lambda + \frac{v}{f}$, where f = 500 Hz

The wave velocity, $c_s = \lambda_s f_s$

Therefore, $\frac{c_s}{f_s} = \lambda + \frac{v}{f} = \frac{c_s}{f} + \frac{v}{f}$

$$f_s = \frac{c_s f}{c_s + v} = \frac{340 \times 500}{340 + v}$$ (3)

(**ii**) At the wall, the source is approaching, and the frequency received is obtained by replacing v by $-v$:

$$f_s = \frac{340 \times 500}{340 - v}$$

Since the observer is stationary, the frequency f_{wall} will be reflected and detected by the observer:

$$f_{reflected} = \frac{340 \times 500}{340 - v}$$ (3)

(**b**) The Doppler frequency therefore gives:

$$30 = \frac{340 \times 500}{340 - v} - \frac{340 \times 500}{340 + v} = 340 \times 500\left(\frac{1}{340 - v} - \frac{1}{340 + v}\right) = \frac{340 \times 500}{340^2 - v^2} 2v$$

$$340^2 - v^2 = \frac{340 \times 500}{30} 2v$$

$$v^2 + \frac{340 \times 500}{15} v - 340^2 = 0$$

There are two solutions to quadratic equations. As $\frac{v}{340} \ll 1$ we can neglect $\left(\frac{v}{340}\right)^2$.

$$v \approx \frac{15 \times 340}{500} = 10.2 \text{ m s}^{-1}$$ (4)

(*Total 10 marks*)

D Fields

D.1 Gravitational fields

Practice questions

1. Close to the surface of the Earth, we know that g is both the acceleration due to gravity and the gravitational field strength. In this example (using Newton's second law) $mz = ma$ so $z = a$.

Answer: A (1)

(*Total 1 mark*)

2. We know that gravitational force is equal to $G\frac{Mm}{r^2}$. Likening to $y = mx + c$, we can see that F against $\frac{1}{r^2}$ is a straight line (with gradient GMm and no intercept).

Answer: D (1)

(*Total 1 mark*)

3. Equating forces left and right: $G\frac{4Mm}{(1-x)^2} = G\frac{9Mm}{x^2}$

Simplifying and multiplying by the denominators: $4x^2 = 9(1-x)^2$

Taking square roots of everything: $2x = 3(1-x)$

$5x = 3$

Answer: C (1)

(*Total 1 mark*)

4. (**a**) A circular orbit means that the direction is constantly changing and, therefore, that the planet is always accelerating (which means a resultant force is needed). It is important to remember that a centripetal force is simply a resultant force that happens to be towards the center of a circle. (2)

(**b**) There is no force acting on a planet except the gravitational force. (1)

(**c**) Use of $g = \frac{GM}{R^2} = \frac{6.67\times10^{-11}\times8.0\times10^{24}}{(9.1\times10^{6})^2}$ = 6.4 N kg^{-1} ***OR*** m s^{-2} (2)

(*Total 5 marks*)

5. (**a**) Work done is defined as the product of a force and the displacement in the direction to the force. However, in a circle, speed is always perpendicular to the force. (1)

(**b**) ***Alternative 1***

One option is to use the fundamental equations for gravitational force/acceleration/orbital velocity. For example: $\frac{GMm}{R^2} = mR\omega^2$

Cancelling variables and defining angular speed: $GM = R^3\left(\frac{2\pi}{T}\right)^2$

Rearranging to match equation provided: $\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$

Therefore, $\frac{GM}{4\pi^2} = kM$ and $k = \frac{G}{4\pi^2}$

Alternative 2

Another option is to work backwards from the given equation $\frac{R^3}{T^2} = kM$ and reversing the steps. (3)

(**c**) Since $\frac{R^3}{T^2} = kM$, we know that $\frac{T^2M}{R^3}$ is a constant and therefore that $\frac{T_1^2M_1}{R_1^3} = \frac{T_2^2M_2}{R_2^3}$

$$M_{\text{Mars}} = \left(\frac{R_{\text{Mars}}}{R_{\text{Earth}}}\right)^3\left(\frac{T_{\text{Earth}}}{T_{Mars}}\right)^2 M_{\text{Earth}} = 6.4\times10^{23}\text{ kg}$$ (2)

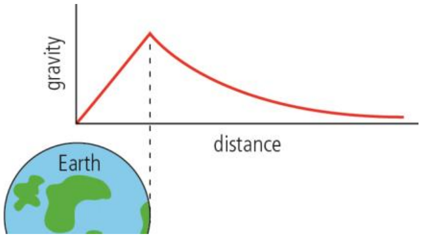
(**d**) We know that field strength and potential gradient are equal, which means that the field strengths due to the Earth and the Moon are equal and opposite at the position where the potential is maximum.

At this position, we equate the gravitational field strength of the Earth and Moon. (3)

(*Total 9 marks*)

6. (**a**) Answer: B (1)

(**b**) In fact, gravitational field strength varies linearly within planets (assuming a constant density throughout) and according to an inverse square law beyond.



(1)

(*Total 2 marks*)

7. (**a**) Answer: B (1)

(**b**) It might be helpful to visualize the Earth and Moon as being connected by a spring. The greater their separation, the stronger the force pulling them together. The difference with gravitation today is that the body with greatest mass would be at the center of the orbit and not at a focus of an ellipse. (1)

(*Total 2 marks*)

8. (**a**) Recognizing that the gravitational force is the resultant force: $mr\omega^2 = \dfrac{GM_g m}{r^2}$

Rearranging using $\omega = \dfrac{2\pi}{T}$, $T^2 = \dfrac{4\pi^2}{GM_g} r^3$ (1)

(**b**) Rearranging and substituting:

$$M_g = \frac{4\pi^2}{GT^2} r^3$$

$$= \frac{4\pi^2}{6.67 \times 10^{-11} \times \left(200 \times 10^6 \times 365 \times 24 \times 3600\right)^2} \left(30000 \times 3 \times 10^8 \times 365 \times 24 \times 3600\right)^3$$

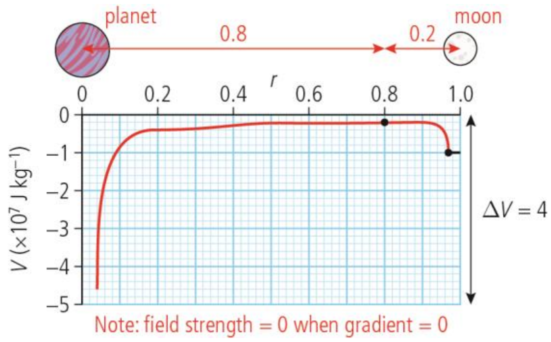
$= 3.4 \times 10^{41}$ kg (1)

(**c**) The mass of the galaxy divided by the mass of the Sun gives the number of stars like the Sun as being $\dfrac{3.4 \times 10^{41}}{1.7 \times 10^{11}} \approx 2 \times 10^{11}$ (1)

(*Total 3 marks*)

9. (**a**) Gravitational potential is the amount of work done per unit mass in taking a small test mass from infinity (a place of zero potential) to the point in question. (2)

(**b**) (**i**) If field strength = 0 then field strength of planet is equal and opposite to field strength of moon.



$\dfrac{GM_p}{r_p^2} = \dfrac{GM_m}{r_m^2} \Rightarrow \dfrac{M_p}{M_m} = \dfrac{r_p^2}{r_m^2} = \dfrac{0.8^2}{0.2^2} = 16$ (3)

(**ii**) As satellite travels from planet its $E_K \rightarrow E_P$.

To reach the moon it must have enough E_K so that it reaches the position of zero field, a distance $r = 0.8$ from the planet. From here to the moon it will be attracted by the moon's field.

Loss of E_K = gain in E_P

If final $E_K = 0$ then loss = $\frac{1}{2}mv^2$

Gain in E_P = change from planet to 0.8 from planet (from the graph)

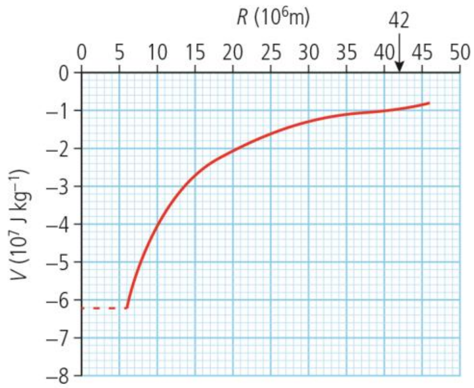
$\Delta V = 4.4 \times 10^7$

so $\Delta E_P = 4.4 \times 10^7 \times 1500 = 6.6 \times 10^{10}$ J

Original $E_K = 6.6 \times 10^{10}$ J (3)

(*Total 8 marks*)

10.



(**a**) (**i**) V at surface = -6.3×10^7 J kg^{-1} (1)

(**ii**) Height: 3.6×10^7 m = 36×10^6 m

So $R = (36 + 6) \times 10^6$ m

$R = 42 \times 10^6$ m

So from graph $V = -1.0 \times 10^7$ J kg^{-1} (2)

(**b**) As satellite leaves the Earth, $E_K \rightarrow E_P$ so if final $E_K = 0$ then original E_K = gain in E_P

From the graph $\Delta V = (6.3 - 1) \times 10^7$ J kg^{-1} = 5.3×10^7 J kg^{-1}

so $\Delta E_P = 1 \times 10^4 \times 5.3 \times 10^7 = 5.3 \times 10^{11}$ J

So minimum $E_K = 5.3 \times 10^{11}$ J (3)

(**c**) The rocket doesn't stop when it reaches the orbit; it must have enough velocity to stay in orbit.

$$\frac{mv^2}{r} = \frac{GMm}{r}$$

During the early stages, while the rocket is in the atmosphere, energy is lost due to air resistance. (2)

(*Total 8 marks*)

11. (**a**) The force is directed toward the center so is perpendicular to the direction of motion. Work done is the force × distance moved in the direction of the motion which is zero since there is no motion toward the center.

Alternatively you could argue that since the speed and distance to center are constant there is no change in either E_K or E_P therefore no exchange of energy so no work done. (1)

(**b**) (**i**) The centripetal force is provided by gravitational attraction between the masses

so $\frac{mv^2}{r} = \frac{GMm}{r}$

$v = \sqrt{\frac{GM}{r}}$ (2)

(**ii**) Total energy = $E_K + E_P$

$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$

$E_P = -\frac{GMm}{r}$

Total energy = $\frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ (2)

(**c**) The total energy is $-\frac{GMm}{2r}$ so if r is increased total energy becomes less negative i.e., bigger. If the energy has increased then work has been done. To do this the engines must be fired in the direction of motion so the rocket moves in the direction of the force. (2)

(*Total 7 marks*)

12. Gravitational potential is, in essence, work done per unit mass. The unit is therefore J kg^{-1}.

Now, J is equivalent to N m, which is equivalent to kg m s^{-2} m = kg m^2 s^{-2}.

Inserting back into the original derived unit, we have m^2 s^{-2}.

Answer: A (1)

(*Total 1 mark*)

13. In orbit, $\frac{GMm}{r^2} = \frac{m(0.5v)^2}{r}$ so $4GM = v^2 r$

Using escape speed, $\frac{1}{2}mv^2 = \frac{GMm}{r}$ so $v^2 R = 2GM$

Substituting into the orbit equation, $2v^2 R = v^2 r$

$r = 2R$ which means that the body in orbit is a full radius R above the surface.

Answer: B (1)

(*Total 1 mark*)

14. We know that potential energy is always negative and that kinetic energy is always positive. This reduces the decision to being between B and D. Of these, only B has a total energy E that could be the result of the sum of E_p and E_k.

Answer: B (1)

(*Total 1 mark*)

15. (**a**) The planet has a combination of both kinetic and gravitational potential energy.

Recalling that kinetic energy is $\frac{1}{2}mv^2$ and that $\frac{GMm}{r^2} = \frac{mv^2}{r}$ so $\frac{1}{2}mv^2 = \frac{GMm}{2r}$.

Recalling also that potential energy is negative: $E = \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r} = -\frac{GMm}{2r}$.

Finally, $V = -\frac{GM}{r}$ and so this can be substituted in. (2)

(**b**) Gravitational potential energy is associated with position. Beyond the surface of the star, mass would have to change for the potential energy to change. We need not consider kinetic energy here because total energy is expressed in (**a**) in terms of V only. (2)

(*Total 4 marks*)

16. (**a**) Escape speed is the speed needed for a body without an engine and without air resistance to *just* escape a planet's gravitational field (i.e. to just reach infinity). (1)

(**b**) This is an energy conservation task: $E_k + E_p$ at take-off = E_p at destination

Bearing in mind that $E_k = \frac{1}{2}mv_{esc}{}^2 = \frac{GMm}{R}$ at escape velocity:

$$\frac{9GMm}{16R} - \frac{GMm}{R} = -\frac{GMm}{r}$$

$$9r - 16r = -16R$$

$$r = \frac{16R}{7}$$

Subtracting the radius of the planet: $r = \frac{16R}{7} - \frac{7R}{7} = \frac{9R}{7}$ (3)

(**c**) It is important to think first of the overall energy effect (that is, it decreases) and then to consider the impact on potential energy (which also decreases) and, finally, the impact on speed (which increases, because the gravitational force increases). (3)

(*Total 7 marks*)

D.2 Electric and magnetic fields

Practice questions

1. The direction of the arrows on the field lines is away from the positive point charge, because field lines show the direction of force that would act on a positive test charge. The shape of C is correct because the field is approximately radial at the point charge and approximately uniform at the metal plate.

Answer: C (1)

(*Total 1 mark*)

2. Force and charge separation are related by an inverse square law: $F \propto \frac{1}{x^2}$

The separation is 3 times greater; the force will be 3^2 times smaller.

Answer: C (1)

(*Total 1 mark*)

3. The force on Q_2 is $\frac{kQ_1Q_2}{r^2} \approx \frac{9\times10^9 \times 2\times10^{-9} \times 2\times10^{-9}}{3^2}$ = 4 × 10^{-9} N

Irrespective of the force, the correct response must have an electric field that is equal to the force divided by the charge Q_2. In this case, $E = \frac{4\times10^{-9}}{2\times10^{-9}}$ = 2 N C^{-1}

Answer: A (1)

(*Total 1 mark*)

4. Electrical potential difference is work done per unit charge: J C^{-1}

Now, from the definition of current, C = A s so C^{-1} is A^{-1} s^{-1}

From the equations for work done and, in turn, force: J = N m = kg m s^{-2} m

The correct response is kg m^2 s^{-3} A^{-1}

Answer: C (1)

(*Total 1 mark*)

5. At constant velocity there is no resultant force. Therefore, the electric force is equal to the gravitational force: $Eq = mg$.

Answer: C (1)

(*Total 1 mark*)

6. Field lines do not really exist; fields are 3-dimensional expanses of space in which forces are experienced. Drawing field lines enables us to visualize the directions of magnetic forces and the strength of the field.

Answer: B (1)

(*Total 1 mark*)

7. (**a**) Electric field strength is force per unit charge:

$$E = \frac{k \times q}{r^2} = \frac{8.99 \times 10^9 \times 6.0 \times 10^{-3}}{0.4^2} = 3.37 \times 10^8 \, \text{N C}^{-1}$$

In 'show that' questions, remember that it is necessary to give an answer to more significant figures than those provided. (2)

(**b**) Acceleration is a result of a resultant force. To calculate the electric force:

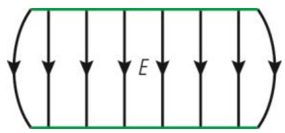
$$F = q \times E = 1.6 \times 10^{-19} \times 3.4 \times 10^8 = 5.4 \times 10^{-11} \, \text{N}$$

$$a = \frac{F}{m} = \frac{5.4 \times 10^{-11}}{9.1 \times 10^{-31}} = 5.9 \times 10^{19} \, \text{m s}^{-2}$$ (2)

(**c**) This question is testing your awareness of two things: the direction of the force (i.e. is the electron attracted to or repelled by the negative point charge) and the nature of how forces change with distance in radial fields. In this case, the electron moves away from the point charge (to the right) because it is repelled by the like charge. It accelerates because of the resultant force but the magnitude of the acceleration decreases as distance of separation increases. (3)

(*Total 7 marks*)

8. (**a**) The parallel plates have a uniform field between them. The top plate has a higher potential than the lower plate and so the field lines should be directed downwards.



(2)

(**b**) For a uniform field between two plates:

$$E = \frac{V}{d} = \frac{5.2 \times 10^3}{12 \times 10^{-3}} = 4.3 \times 10^5 \, \text{N C}^{-1}$$ (1)

(**c**) To find the magnitude of the change:

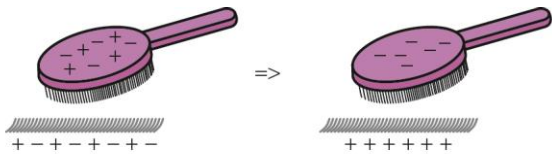
$$\Delta E_P = q\Delta V = 3.2 \times 10^{-19} \times 5.2 \times 10^3 = 1.7 \times 10^{-15} \, \text{J}$$

The charge is losing potential energy throughout; its final potential is lower than its initial potential. Therefore the change of potential energy is negative. (3)

(*Total 6 marks*)

9. (**a**) Answer: B (1)

(**b**) Using the cat as an example, the cat's fur already contained equal amounts of positive and negative charge. The stroking only serves to separate these. (1)



(*Total 2 marks*)

10. (**a**) Answer: B (1)

(**b**) The bird would, on the other hand, receive a shock if it were to touch another wire at a different voltage at the same time. Then it would have a current passing through it with $P = IV$. (1)

(*Total 2 marks*)

11. (**a**) If m is the mass of the drop and E the field strength, then according to Newton's laws:

$$En_1e = mg + 6\pi\eta a u_1$$

$$En_2e = mg + 6\pi\eta a u_2$$

We can assume that the difference in the number of charges has a negligible effect on the mass of the droplets.

Subtracting: $Ee\left(n_2 - n_1\right) = 6\pi\eta a(u_2 - u_1)$

$$n_2 - n_1 = \frac{6\pi\eta a}{Ee}(u_2 - u_1)$$

Since $E = \frac{V}{d} = \frac{5000}{1.5\times10^{-2}}$ and $u = \frac{\text{distance}}{\text{time}}$:

$$n_2 - n_1 = \frac{6\pi\times1.82\times10^{-5}\times2.76\times10^{-6}}{\left(\frac{5000}{1.5\times10^{-2}}\right)\times1.6\times10^{-19}}\left(\frac{10^{-2}}{42} - \frac{10^{-2}}{78}\right) = 1.95$$

The change in the number of electrons is a decrease of 2. (8)

(**b**) For the separate drops (this time of different masses):

$QE = m_1 g + 6\pi\eta r_1 u_1$

$QE = m_2 g + 6\pi\eta r_2 u_2$

Adding these equations: $2QE = (m_1 + m_2)g + 6\pi\eta(r_1 u_1 + r_2 u_2)$

Considering the combined drop: $2QE = m_3 g + 6\pi\eta r_3 u_3$

Now, $m_3 = m_1 + m_2$

Therefore, by inspection or subtraction: $r_3 u_3 = r_1 u_1 + r_2 u_2$

Volume is conserved, so $\frac{4}{3}\pi r_3^{\ 3} = \frac{4}{3}\pi r_1^{\ 3} + \frac{4}{3}\pi r_2^{\ 3}$

$r_3 = \sqrt[3]{r_1^{\ 3} + r_2^{\ 3}}$

Hence, $u_3 = \frac{r_1 u_1 + r_2 u_2}{\sqrt[3]{r_1^{\ 3} + r_2^{\ 3}}}$

You might like to ponder if or how this would change for two different charges. (5)

(*Total 13 marks*)

12. Although there is no resultant force at the center of the square, electric potential is a scalar and non-zero. The potential due to one point charge is $\frac{kQ}{\sqrt{2}d}$, where $\sqrt{2}d$ is the distance from the center to one of the points (calculated using Pythagoras).

Total potential is therefore $4 \times \frac{kQ}{\sqrt{2}d} = \frac{2\sqrt{2}kQ}{d}$

Answer: D (1)

(*Total 1 mark*)

13. Change in electric potential is the same as the work done per unit charge, which is same as the product of the force per unit charge (field strength) and the distance moved parallel to the force.

The distance moved parallel to the field lines is x.

Answer: A (1)

(*Total 1 mark*)

14. Take care to notice that the lines drawn here are not field lines; they are equipotentials. This diagram is analogous to gravitational field in the room in which you are situated.

When the particle moves from X to Y it experiences a resultant force. It is therefore uniformly accelerated. The work done is equal to the product of the potential difference and the charge: $400 \times 10^3 \times 2.5 \times 10^{-6} = 1\ \text{J}$

Answer: D (1)

(*Total 1 mark*)

15. Dimensionally, the product of field strength and distance is potential difference.

Had the vertical axis been force and not field strength, work done would have been calculated instead.

Answer: B (1)

(*Total 1 mark*)

16. (**a**) The question says that σ is charge per unit area. Its unit therefore is C m^{-2}.

You can find the unit of ε_0 from the data booklet: $\text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$.

$\dfrac{\text{C m}^{-2}}{\text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}} = \text{N C}^{-1}$, which is the unit of field strength. (2)

(**b**) Three forces act on the ball: weight acting vertically downwards (*mg*), electric force acting horizontally to the right (*F*) and tension in the thread acting diagonally (*T*).

Horizontally, $F = T\sin 30°$

Vertically, $T\cos 30° = \text{mg}$

Dividing these equations: $\dfrac{F}{mg} = \tan 30°$

$F = mg\tan 30° = 0.025 \times 9.8 \times \tan 30° = 0.14\,\text{N}$ (3)

(**c**) Using the definition of electric field strength: $E = \dfrac{F}{q} = \dfrac{0.14}{1.2 \times 10^{-6}} = 1.2 \times 10^{5}$

Now rearranging the equation provided:

$\sigma = 2E\varepsilon_0 = 2 \times 1.2 \times 10^{5} \times 8.85 \times 10^{-12} = 2.1 \times 10^{-6}\,\text{C m}^{-2}$ (2)

(**d**) At P, the force due to Q is equal to the force due to the ball. Therefore the field strengths are equal:

$$\frac{Q}{0.22^2} = \frac{1.2 \times 10^{-6}}{0.18^2}$$

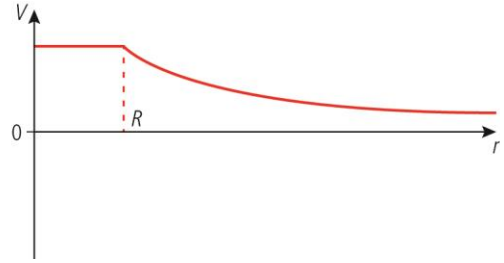
$Q = +1.8 \times 10^{-6}\,\text{C}$

Two significant figures should be used as this is the smallest number in the quantities given in this particular calculation. (2)

(*Total 9 marks*)

17. (**a**) The electric potential changes because the force and the distance moved are parallel. The potential decreases because the work done on moving a positive test charge away from a positive sphere is negative; energy is gained. (2)

(**b**) The sphere is solid and conducting, which means the field strength within the sphere's radius is zero (and therefore potential must be constant so that potential gradient is zero). Beyond the sphere's radius, an inverse proportionality exists: $V \propto \dfrac{1}{r}$



(2)

(**c**) In terms of change in potential being work done per unit charge:

$$\Delta V = \frac{W}{q} = \frac{1.7 \times 10^{-16}}{1.60 \times 10^{-19}} = 1.1 \times 10^{3}\ \text{V}$$ (1)

(**d**) In terms of change in potential being the difference between the final and initial potentials based on position:

$$\Delta V = -\frac{kQ}{r_\text{f}} - \left(-\frac{kQ}{r_\text{i}}\right)$$

$$= 8.99 \times 10^{9} \times Q \times \left(\frac{1}{5.0 \times 10^{-2}} - \frac{1}{1.0 \times 10^{-1}}\right)$$

$$= 1.1 \times 10^{3}\ \text{V}$$

$$Q = 1.2 \times 10^{-8}\ \text{C}$$ (2)

(**e**) Although there are differences between electric and gravitational fields, they also have a great deal in common. Both involve forces, field strengths, energies and potentials, and both have radial and uniform models. (1)

(*Total 8 marks*)

D.3 Motion in electromagnetic fields

Practice questions

1. By definition, field lines show the direction of force acting on a positive charge. An electron will experience a force parallel but opposite to the field lines.

No electric force is exerted horizontally.

Answer: C (1)

(*Total 1 mark*)

2. In order for the proton to experience no resultant force (i.e. so that its direction is unchanged), the electric and magnetic fields must act in opposite directions and cancel out. Any deflection that had emerged would have been a result of the proton's charge and not its mass. Therefore, because mass is not a factor, the opposite charge on the electron and the larger charge on the alpha particle also have no effect.

Answer: D (1)

(*Total 1 mark*)

3. Using Fleming's left-hand rule: the field is from left to right and the direction of conventional current is out of the page. The direction of force on the electron is therefore upward.

Answer: D (1)

(*Total 1 mark*)

4. Using Fleming's left-hand rule, the field is from right to left and the direction of conventional current is into of the page. The direction of force on the negative charges is therefore upward and they will accumulate on surface A.

Answer: A (1)

(*Total 1 mark*)

5. The two wires will tend to attract one another. The force acting on each individual wire will be perpendicular to the direction of current flow.

Answer: A (1)

(*Total 1 mark*)

6. The particle acts as though it were another current-carrying wire. It is attracted to the original wire because the currents are in the same direction.

Answer: A (1)

(*Total 1 mark*)

7. We know that as the currents are in the same direction, the wires will attract one another. We must also bear in mind Newton's third law: if wire R exerts a force on wire S then wire S exerts an equal and opposite force on wire R.

Answer: A (1)

(*Total 1 mark*)

8. (**a**) Electric field strength is defined as the force per unit charge acting on a small/test positive charge placed at a given position in the field. (2)

(**b**) The field lines show the direction of force that would act on a positive charge. Electrons are negative, and so the force would be to the left. (1)

(**c**) When comparing motion, we need to factor in both magnitudes and direction. The force acting on either an electron or proton would be the same because their charges are equal in magnitude. However, a proton has more mass, so its acceleration will be smaller. A proton would move to the right, in the direction of the field lines.

One added complexity in this diagram is the convergence of the field lines to the right, which means that the force on the proton would increase over time and therefore that the acceleration would increase over time. The opposite would be true for an electron. (4)

(*Total 7 marks*)

9. (**a**) When the force acting on a body is always perpendicular to the velocity, the motion will be a circle. Direction is always changing but speed is constant. (2)

(**b**) Equating the magnetic force with the resultant force for circular motion:

$$qvB = \frac{mv^2}{R}$$

$$R = \frac{1.67\times10^{-27}\times2.0\times10^{6}}{1.6\times10^{-19}\times0.35} = 0.060\text{ m}$$ (2)

(**c**) Time is distance divided by speed, where the distance traveled is a circumference:

$$T = \frac{2\pi R}{v} = \frac{2\pi\times0.06}{2.0\times10^{6}} = 1.9\times10^{-7}\text{ s}$$ (2)

(**d**) The force and distance traveled are always perpendicular. Therefore, no work is done, and no energy is changed. Additionally, the proton's speed is constant throughout, which means that kinetic energy is constant throughout. (2)

(*Total 8 marks*)

10. (**a**) Using Fleming's left-hand rule: the force acting is upwards (on the plane of the paper) and the direction of conventional current is to the left. The field must therefore be out of the page. (1)

(**b**) Magnetic force = Bqv = 8.5 × 1.60 × 10^{-19} × 6.8 × 10^{5} = 9.2 × 10^{-13} N (1)

(**c**) Although the electron experiences an acceleration, this is always at right angles to its velocity and therefore only its direction (and not speed) changes. (1)

(**d**) The circular path emerges from the force always being at right angles to the velocity. The magnetic force acts is the resultant, centripetal force. (2)

(*Total 5 marks*)

11. (**a**) The upper section of the wire has an associated magnetic field due to its current. Using the right-hand grip rule, the field at the position of the lower section is into the page.

Using Fleming's left-hand rule with the field into the page and the current to the left, we see that a downward force acts. Therefore the balance reading increases. (3)

(**b**) Magnetic force $= BIl$

Magnetic force per unit length $= BI = 1.3\times10^{-4}\times0.2 = 2.6\times10^{-5}\text{ N m}^{-1}$ (1)

(**c**) To calculate the speed of electrons in the wire, we need to find the size of the magnetic force overall. We start by finding the volume of the wire (a cylinder):

$$\text{volume} = \pi r^2 l = \pi\times\left(\frac{2.5\times10^{-3}}{2}\right)^2\times0.15 = 7.36\times10^{-7}\text{ m}^3$$

Then find the charge in the wire:

$$\text{charge} = 8.5\times10^{28}\times7.36\times10^{-7}\times1.6\times10^{-19} = 10\times10^{3}\text{ C.}$$

$$v = \frac{BIl}{Bq} = \frac{2.6\times10^{-5}\times0.15}{1.3\times10^{-4}\times10^{4}} = 3.0\,\mu\text{m s}^{-1}$$ (4)

(**d**) The angled upper section means that some parts of the lower section of wire will have a smaller magnetic field strength and, in turn, experience a smaller magnetic force. The reading of the balance will decrease (3)

(*Total 11 marks*)

12. (**a**) Answer: B (1)

(**b**) Currents flowing in the same direction 'pinch' together, because the magnetic fields between them are in opposite directions. (1)

(*Total 2 marks*)

D.4 Induction

Practice questions

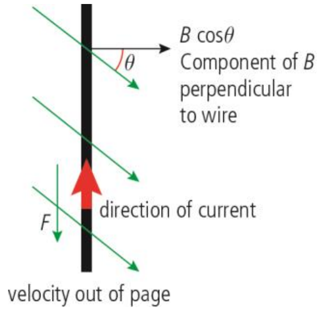
1. (**a**) Gravitational field strength is the force experienced per unit mass by a small test mass placed in the field. (2)

(**b**) From Newton's law the force on a mass m on the surface of the Earth $F = \frac{GMm}{R^2}$

Field strength $g_0 = \frac{F}{m} = \frac{GMm}{R^2 m} \Rightarrow g_0 = \frac{GM}{R^2} \Rightarrow GM = g_0R^2$ (2)

(**c**) Use Fleming's right-hand rule (1)

(**d**) $F = B\cos\theta \times V \times e$ (1)



(**e**) E = energy converted from mechanical work to electrical potential energy per unit charge.

Work done on an electron in pushing it along the wire = force × distance

$= B\cos\theta \times ev \times L$

Work done per unit charge = $\frac{\text{work done}}{e}$ = $B\cos\theta \times vL$

This is the correct answer since question says deduce from (**d**). However a more obvious solution uses Faraday's law.

E = rate of flux cut

$= B\cos\theta$ × area swept out per second

$= B\cos\theta \times vL$ since wire moves a distance v in 1 second. (3)

(**f**) For an orbiting body the gravitational force = centripetal force

so $\frac{GMm}{R^2} = \frac{mv^2}{R}$ where R is the orbit radius

so $v = \sqrt{\frac{GM}{R}}$

From the question we know that for the Earth's surface $GM = g_0R_0^2$, where R_0 is the Earth's radius.

$= 10 \times (6.4 \times 10^6)^2$

so $GM = 4.1 \times 10^{14}$ N m^2 kg^{-1}

Height = 3×10^5 m

so $R = 3 \times 10^5 + 6.4 \times 10^6 = 6.7 \times 10^6$ m

so $V = \sqrt{\dfrac{4.1\times10^{14}}{6.7\times10^{6}}} = \sqrt{6.1\times10^{7}} = 7.8 \times 10^3$ m s^{-1} (3)

(**g**) From answer to (**e**), $E = B\cos\theta \times vL$

so $E = 6.3 \times 10^{-6} \cos 20° \times 7.8 \times 10^3 \times L = 1000$ V

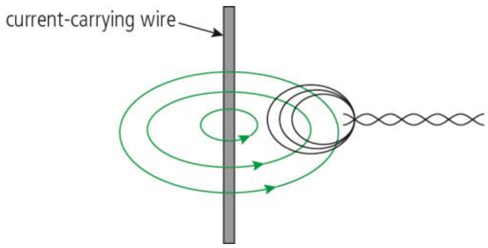
$L = \dfrac{1000}{6.3\times10^{-6}\cos 20°\times7.8\times10^{3}} = 2.2 \times 10^4$ m (2)

Error carried forward: In IB questions you generally do not get penalized for carrying an error forward so if you got part (**e**) wrong and used your answer in part (**g**), then you should get the marks for (**g**).

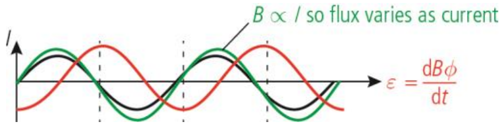
(*Total 14 marks*)

2. (**a**) (**i**) The emf induced in a conductor placed in a magnetic field is directly proportional to the rate of change of the flux it encloses. (2)

(**ii**) The loop encloses *B* field as shown. If the current changes then the enclosed flux field will change so, according to Faraday, emf will be induced. (1)



(**b**) (**i**) and (**ii**) When gradient of *B* versus *t* is maximum then ε is maximum. (3)



(**iii**) When coil is further from the wire, the *B* field will be less so flux enclosed will be smaller.

As a result $\dfrac{dN\phi}{dt}$ is less, so ε is less. (2)

(**c**) Advantage – does not need to be in contact with the wire.

Disadvantage – distance from the wire should be known. (2)

(*Total 10 marks*)

3. Using Faraday's law, emf is the rate of change of flux. It takes 2 s for the ring to go from perpendicular (maximum flux) to parallel (zero flux):

$$\text{emf} = \frac{5\times10^{-3}\times2\times10^{-4}}{2} = 5\times10^{-7}\text{ V}$$

Answer: D (1)

(*Total 1 mark*)

4. Lenz's law is a consequence of the conservation of energy; the increased kinetic energy of the charges cannot come from nowhere.

Answer: B (1)

(*Total 1 mark*)

5. The emf reaches a maximum, and emf is the rate of change of flux. The rate of change of flux is a maximum when flux is zero, and therefore the rotation needs to be about XX' (so that the plane of the coil becomes aligned with the field) and through $\frac{\pi}{2}$ (so that the timing of alignment with the field matches the maximum emf on the graph).

Answer: C (1)

(*Total 1 mark*)

6. $P = Fv = BILv = B\times\frac{V}{R}Lv = B\times\frac{BLv}{R}Lv = \frac{B^2L^2v^2}{R}$

This question could alternatively have been answered using dimensional analysis.

Answer: C (1)

(*Total 1 mark*)

7. (**a**) Flux is the product of magnetic field strength (which is constant) and perpendicular area.

The perpendicular area increases as the magnet approaches the ring. (1)

(**b**) As a result of Lenz's law, the direction of the ring's field will oppose the magnet approaching it. Therefore, a north pole is formed atop the coil, which (using the right-hand rule) means the current is counter-clockwise.

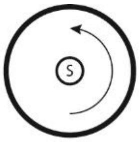


Diagram 2: view from above

(1)

(**c**) The force is upwards. Again, this is a consequence of Lenz's law, because the rate at which the magnet approaches the ring must be reduced. (2)

(*Total 4 marks*)

8. (**a**) An emf is induced when there is a rate of change of flux. The rate of change of flux is greatest when the coil is in the position shown because the cutting of the field lines is fastest. Because side AB moves upwards and CD moves downwards whilst the field direction is the same for both, a current flows in the coil. The emf is sinusoidal. (3)

(**b**) emf = $\frac{BA}{t} = Blv = 0.34 \times 8.5 \times 10^{-2} \times 2 = 0.058\,\text{V}$ (1)

(**c**) There are 80 turns on the coil and two sides so $160 \times 0.058 = 9.2\,\text{V}$ (1)

(*Total 5 marks*)

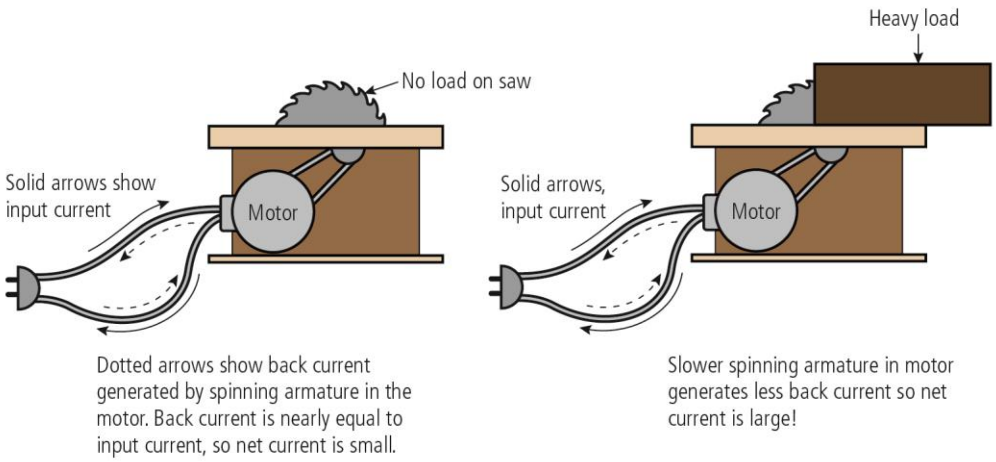
9. (**a**) Answer: A (1)

(**b**) We know that an electromagnet forms when a current-carrying conductor is wrapped around a magnetic material. This question is really about why the second statement is false: the iron is not moving, which means there is no energy to change form, and therefore a current cannot flow by the simple act of wrapping a wire around a magnet. (1)

(*Total 2 marks*)

10. (**a**) Answer: A (1)

(**b**) Every electric motor is also a generator. The generator effect leads to a 'back' emf and a 'back' current. The smaller the back current, the more you pay for your electricity. (1)



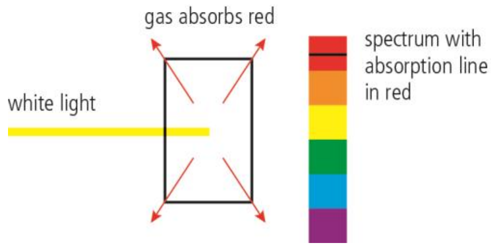
(*Total 2 marks*)

E Nuclear and quantum physics

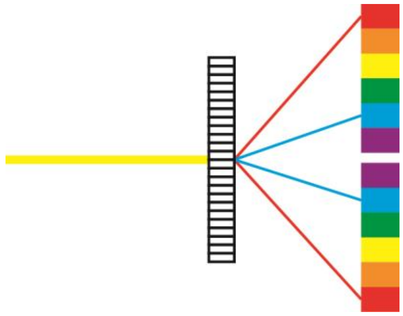
E.1 Structure of the atom

Practice questions

1. (**a**) (**i**) When white light passes through a gas photons can excite atomic electrons into higher energy levels. When this happens the photon is absorbed resulting in a dark line in the spectrum. After absorption the electron will go back to its original level, re-emitting the photon in a random direction. (2)



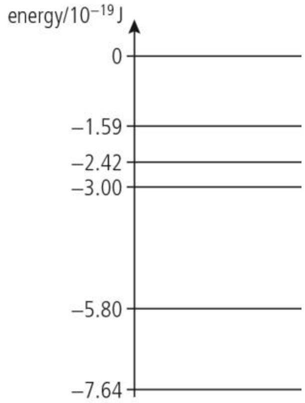
(**ii**) To produce a spectrum the light can be passed through a diffraction grating; this will produce interference maxima for each colour at a different angle. (2)



(**b**) (**i**) $E = hf$ where $f = \frac{c}{\lambda}$

so $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{588 \times 10^{-9}}$ = 3.38×10^{-19} J (2)

(ii) According to the previous calculation, absorption of a 588 nm photon will give an atomic electron 3.38×10^{-19} J of energy. This would correspond to a change from the –5.8 to the –2.42 level. 5.8 – 2.42 = 3.38



(To answer this simply try subtracting 3.38 from each level.) (3)

(c) The Bohr model assumes that electrons orbit the nucleus, like planets orbiting the Sun. To explain the line spectrum of hydrogen, the electrons can only exist in certain stable orbits defined by their angular momentum. Absorption of a photon of light causes the electrons to change to a larger orbit. (3)

(*Total 12 marks*)

2. (**a**) An electron can move from ground state to a higher energy level if

$n = 3$ ________________ –1.51 eV

$n = 2$ ________________ –3.40 eV

$n = 1$ ________________ –13.68 eV

1. It absorbs a photon.

2. It gains energy as the gas is heated. (2)

(**b**) (**i**) Photon energy = $hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{658 \times 10^{-9}}$ = 3.02×10^{-19} J

$\frac{3.02 \times 10^{-19}}{1.6 \times 10^{-19}}$ = 1.89 eV (2)

(**ii**) A transition from $n = 2 \rightarrow n = 3$ is equal to 1.89 eV so light of this wavelength will excite electrons from $n = 2 \rightarrow n = 3$ and therefore be absorbed. (3)

(*Total 7 marks*)

3. At high energies, the alpha particles become close enough to the nucleus that the strong nuclear force has an influence, which creates deviations in the electrostatic repulsion pattern.

Answer: B (1)

(*Total 1 mark*)

4. Using $R = R_0 A^{\frac{1}{3}}$, $A = \left(\frac{R}{R_0}\right)^3 = \left(\frac{6\times10^{-15}}{1.2\times10^{-15}}\right)^3 = 125$

Answer: C (1)

(*Total 1 mark*)

5. Using $R = R_0 A^{\frac{1}{3}}$, $\left(\frac{R_{\text{silver}}}{R_{\text{other}}}\right)^3 = \frac{A_{\text{silver}}}{A_{\text{other}}}$ so $(3)^3 = \frac{108}{A_{\text{other}}}$

$A_{\text{other}} = 4$, which matches the nucleon number of He

Answer: A (1)

(*Total 1 mark*)

6. Nuclear density is constant because volume is proportional to the nucleon number, and because all nucleons have approximately equal mass to one another.

Answer: A (1)

(*Total 1 mark*)

7. Emission lines can, in fact, correspond to ultraviolet and other non-visible parts of the electromagnetic spectrum.

Answer: D (1)

(*Total 1 mark*)

8. Frequency and energy are proportional, so the two smallest energy differences must be sought.

$f = \frac{E}{h}$ so the first is $\frac{6.63\times10^{-19}}{6.63\times10^{-34}} = 1.0\times10^{15}$ Hz and the second is

$\frac{13.3\times10^{-19}}{6.63\times10^{-34}} = 2.0\times10^{15}$ Hz

Answer: C (1)

(*Total 1 mark*)

9. In order of decreasing frequency (which is proportional to energy difference) and increasing wavelength, the energy differences are $\lambda_4, \lambda_1, \lambda_2, \lambda_3$.

Answer: B (1)

(*Total 1 mark*)

10. The Bohr model for hydrogen predicts all three of orbital radius, quantized energy and quantized angular momentum.

Answer: D (1)

(*Total 1 mark*)

11. (**a**) It is important to consider the relative numbers of alpha particles for each of the outcomes. Most are undeflected, which means that the atom is mostly empty space. Some are deflected, which means that the nucleus is small and dense and positively charged. (4)

(**b**) In the model of electrons orbiting the nucleus, they have fixed orbital radii. However, according to classical physics, the electrons should radiate energy and gradually spiral into the nucleus as a result of the centripetal force. (3)

(*Total 7 marks*)

12. (**a**) Since λ = 435 nm and $f = \frac{c}{\lambda}$,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4.35 \times 10^{-7}} = 4.6 \times 10^{-19}\text{ J}$$ (3)

(**b**) arrow downwards between -0.87×10^{-19} J and -5.44×10^{-19} J (1)

(**c**) The photon is emitted, therefore the transition is downwards. The energy difference is 4.6×10^{-19} J, which means that the transition must be between approximately -0.87×10^{-19} J and -5.44×10^{-19} J. (2)

(*Total 6 marks*)

13. (**a**) Recognizing that the electrical force is the resultant force, and selecting the equation for centripetal force: $\frac{2ke^2}{r^2} = \frac{mv^2}{r}$

Simplifying and rearranging: $v = \sqrt{\frac{2ke^2}{mr}}$ (1)

(**b**) The total energy is equal to the sum of kinetic and potential energy.

$$E_{\text{TOT}} = E_{\text{k}} + E_{\text{p}} = \frac{ke^2}{r} - \frac{2ke^2}{r} = -\frac{ke^2}{r}$$ (2)

(**c**) Using this equation, if E_{TOT} decreases then radius must also decrease, because total energy is negative. (2)

(**d**) Using the equation from (**a**) with $n = 3$,

$$v = \sqrt{\frac{2 \times 8.99 \times 10^{9} \times \left(1.6 \times 10^{-19}\right)^2}{9.11 \times 10^{-31} \times 9 \times 2.7 \times 10^{-11}}} = 1.44 \times 10^{6}\text{ m s}^{-1}$$ (2)

Using the de Broglie equation, $\lambda = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.44 \times 10^{6}} = 5.05 \times 10^{-10}\text{ m}$

(**e**) Since circumference is $2\pi r$, the ratio $\frac{2\pi r}{\lambda} = \frac{2\pi \times 9 \times 2.7 \times 10^{-11}}{5.1 \times 10^{-10}} = 2.99 \cong 3$ (1)

(*Total 8 marks*)

14. (**a**) According to classical orbit theory, electrons accelerate and so radiate energy, which means they ought to spiral into the nucleus. However, electrons have discrete energy levels from which no energy is radiated, and these satisfy the Bohr condition

$mvr = n\frac{h}{2\pi}$ (3)

(**b**) Recognizing that the electrical force is the resultant force, and selecting the equation for centripetal force: $\frac{ke^2}{r^2} = \frac{m_e v^2}{r}$

Simplifying and rearranging: $v = \sqrt{\frac{ke^2}{m_e r}}$ (1)

(**c**) Substituting $v = \sqrt{\frac{ke^2}{m_e r}}$ into $mvr = n\frac{h}{2\pi}$: $m_e\sqrt{\frac{ke^2}{m_e r}}r = n\frac{h}{2\pi}$

$m_e^2\frac{ke^2}{m_e r}r^2 = \frac{h^2}{4\pi^2}$

Rearranging: $r = \frac{h^2}{4\pi^2 k m_e e^2}$ (2)

(**d**) $r = \frac{\left(6.63\times10^{-34}\right)^2}{4\pi^2\times8.99\times10^9\times9.11\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$ = 5.3 × 10^{-11} m (1)

(*Total 7 marks*)

E.2 Quantum physics

Practice questions

1. (**a**) According to the wave model the energy in the wave is related to the amplitude, not the frequency. This means that the E_K of photoelectrons should be dependent on the intensity.

However, E_K is dependent on frequency, not intensity. This can be explained if we consider light to be made up of photons. Each photon

has energy = *hf*.

A photoelectron is emitted when the atom absorbs a photon so the E_K of a photoelectron is related to *f*.

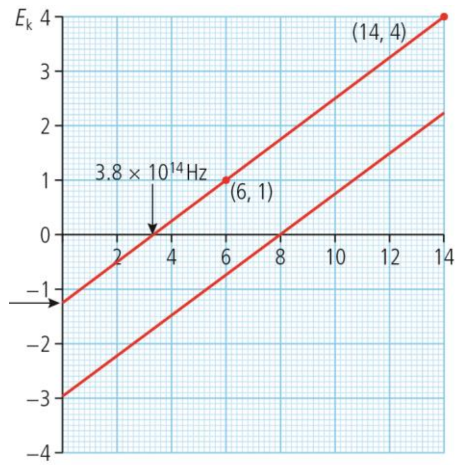
Intensity is related to the number of photons, so increased intensity increases the number of photoelectrons, not their E_K. (2)

(**b**) (**i**) From *x*-intercept, threshold frequency = 3.8×10^{14} Hz (2)

(**ii**) From gradient, Planck constant $= \frac{(4-1)\times 1.6\times 10^{-19}}{(13.5-6)\times 10^{14}}$ = 6.4×10^{-34} J s (2)

(**iii**) Work function = *y*-intercept = 1.5 eV (2)

(**c**)



$E_K = hf - \varphi$

$y = mc + c$

gradient = *h*

y-intercept = $-\varphi$ (2)

(*Total 10 marks*)

2. (**a**) The de Broglie hypothesis states that all particles have a wave associated with them. This wave gives the probability of finding the particle: its wavelength is related to the momentum of the particle by the formula $\lambda = \frac{h}{p}$ where *h* = Planck constant. (3)

(**b**) (**i**) Gain is E_K in eV = 850 eV

or, in joules, $850 \times 1.6 \times 10^{-19} = 1.4 \times 10^{-16}$ J (1)

(**ii**) $E_K = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2E_k}{m}}$ × momentum

$$p = mv = m\sqrt{\frac{2E_k}{m}} = \sqrt{2m \times E_k} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.4 \times 10^{-16}} = 1.6 \times 10^{-23} \text{ Ns}$$ (2)

This is in the data book. You ned to know what is in the data book in case you need to use a value.

(**iii**) $\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{1.6 \times 10^{-23}} = 4.1 \times 10^{-11}$ m (2)

(*Total 8 marks*)

3. Using the photon energy equation, $E = hf = \frac{hc}{\lambda}$.

Using the de Broglie wavelength equation, $\lambda = \frac{h}{mv}$ so $mv = \frac{h}{\lambda}$

Answer: A (1)

(*Total 1 mark*)

4. $E_k = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m}$

Since $E_k \propto (mv)^2$ and $mv \propto \frac{1}{\lambda}$, $\lambda \propto \frac{1}{E^{\frac{1}{2}}}$

Answer: C (1)

(*Total 1 mark*)

5. Diffraction is a behaviour exhibited by waves.

Answer: B (1)

(*Total 1 mark*)

6. The kinetic energy of the electrons released is not constant. Even though a maximum theoretical kinetic energy can be calculated based on electrons requiring exactly the (constant) work function for release, the reality is that electrons will require a range of energies more than this based on their extent of binding within the lattice.

Answer: C (1)

(*Total 1 mark*)

7. Using conservation of energy for the energy of the photon becoming the energy of the electron:

$hf = \Phi + E_{k,\ max}$

Rearranging and relating to $y = mx + c$: $E_{k,\ max} = hf - \Phi$

As h is a constant and, incidentally, the gradient of any one of these graphs, all must have the same slope.

Answer: C (1)

(*Total 1 mark*)

8. Current depends on the number of electrons.

The horizontal intercept therefore reveals the point at which no electrons can bridge the potential difference gap and, in turn, indicates the threshold frequency of the light; Y and Z have the same frequency.

Above zero current, the increased gradient for Y indicates that the quantity of electrons is higher than that for Z. Since they have the same frequency, beam Y must be more intense than beam Z.

Answer: C (1)

(*Total 1 mark*)

9. An increased intensity of light means that more photons are present. Therefore, more electrons will be emitted from the surface. Each individual photon has the same energy as before (as no change has been made to the frequency), so the kinetic energies of the electrons will remain the same.

Answer: D (1)

(*Total 1 mark*)

10. (**a**) In waves, energy transfer is a process that takes time, especially at low intensities (and therefore amplitudes). Therefore, the instantaneous emission cannot be explained by the wave nature of light. (2)

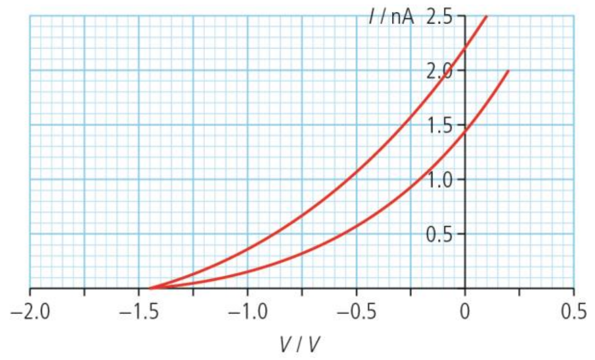
(**b**) In, the photon theory of light the electron absorbs all of the energy from the photon immediately. The number of photons (intensity) is of no relevance to the likelihood of individual electron emission or the time required. (2)

(**c**) Rearranging the conservation of energy equation: $\phi = \frac{hc}{\lambda} - E_k$

From the intercept of the graph, and knowing that the kinetic energy of the electrons is equal to the work done in stopping them by the potential difference: $E_k = 1.5\,\text{eV}$

Therefore, $\phi = \frac{1.24 \times 10^{-6}}{480 \times 10^{-9}} - 1.5 = 1.1\,\text{eV}$ (3)

(**d**)



(2)

(*Total 9 marks*)

11. (**a**) $\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.60\times10^{-19}\times4.2\times10^{8}} = 2.96\times10^{-15}$ m (1)

(**b**) This curvature is congruous with diffraction around spherical objects. Diffraction is a behavior exhibited by waves. (2)

(**c**) For single-slit diffraction, $\sin\theta_0 \approx \frac{\lambda}{d} = \frac{2.96\times10^{-15}}{4.94\times10^{-15}} = 0.599$

$\theta_0 = 37°$ (2)

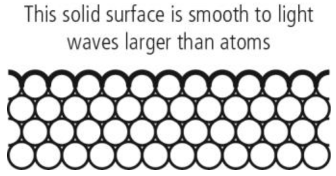
(**d**) Diffraction is optimized when the wavelength is approximately the same size as the gap or object. Using the equation provided in (**a**), we can see that a reduced energy would result in an increased de Broglie wavelength, which would be much greater than a typical nuclear radius (~10^{-15}). (2)

(**e**) Since all nucleons have approximately the same mass and diameter, and because the volume of a nucleus is proportional to $(A^{\frac{1}{3}})^3 = A$, the ratio $\frac{\text{mass}}{\text{volume}}$ is independent of *A*, hence density the same for all nuclei. (2)

(*Total 9 marks*)

12. (**a**) Answer: A (1)

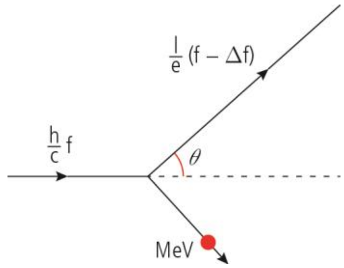
(**b**) Atoms are smaller than visible light wavelengths. This means that visible light microscopes cannot be used for imaging individual atoms.



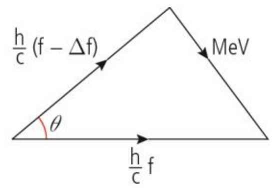
(1)

(*Total 2 marks*)

13. (a)



leading to



(3)

(**b**) Using the cosine rule:

$$m_e v = \sqrt{\left(\frac{h}{c}f\right)^2 + \left(\frac{h}{c}(f-\Delta f)\right)^2 - 2\frac{h}{c}f \times \frac{h}{c}(f-\Delta f)\cos\theta}$$

$$(m_e v)^2 = \left(\frac{h}{c}f\right)^2 + \left(\frac{h}{c}(f-\Delta f)\right)^2 - 2\left(\frac{h}{c}\right)^2 f(f-\Delta f)\cos\theta$$

(4)

(**c**) Using conservation of energy between the energy of the initial photon and the kinetic energy of the electron and the final energy of the photon:

$$hf = \frac{1}{2}m_e v^2 + h(f-\Delta f)$$ (2)

(**d**) From the energy conservation equation: $m_e v^2 = 2h\Delta f$

Substituting into the equation from (**b**):

$$2hm_e\Delta f = \left(\frac{h}{c}f\right)^2 + \left(\frac{h}{c}(f-\Delta f)\right)^2 - 2\left(\frac{h}{c}\right)^2 f(f-\Delta f)\cos\theta$$

$$2hm_e\Delta f = \left(\frac{h}{c}\right)^2 f^2 + \left(\frac{h}{c}\right)^2\left(f^2 - 2f\Delta f + \Delta f^2\right) - 2\left(\frac{h}{c}\right)^2 f^2\cos\theta + 2\left(\frac{h}{c}\right)^2 f\Delta f\cos\theta$$

Bringing constants to the left-hand side to combine like terms on the right-hand side:

$$\frac{2m_e c^2}{h}\Delta f = f^2 + f^2 - 2f\Delta f + \Delta f^2 - 2f^2\cos\theta + 2f\Delta f\cos\theta$$

$$\frac{2m_e c^2}{h}\Delta f = f^2(2-2\cos\theta) + \Delta f^2 - f\Delta f(2-2\cos\theta)$$

Dividing by 2:

$$\frac{m_e c^2}{h}\Delta f = f^2(1-1\cos\theta)+\frac{\Delta f^2}{2}-f\Delta f(1-1\cos\theta)$$

Dividing by f^2:

$$\frac{m_e c^2}{h}\frac{\Delta f}{f^2} = 1-1\cos\theta+\frac{\Delta f^2}{2f^2}-\frac{\Delta f}{f}(1-1\cos\theta)$$

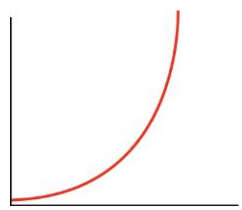
Since $\Delta f \ll f$, $\frac{\Delta f^2}{2f^2}\approx 0$ and $\frac{\Delta f}{f}\approx 0$.

Therefore, $\frac{m_e c^2}{h}\frac{\Delta f}{f^2} = 1-1\cos\theta$

Rearranging as required:

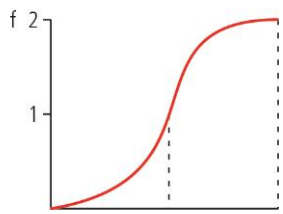
$$\Delta f = \frac{hf^2}{m_e c^2}(1-1\cos\theta)$$ (7)

(e) **(i)** $\Delta f \propto f^2$ so the curve is parabolic (and there is no intercept)



(1)

(ii) $\Delta f \propto 1-\cos\theta$ so the curve will be an upside-down cosine graph (starting at 0)



(1)

(iii) Δf has a maxiumum value of $\frac{hf^2}{m_e c^2}$ when $1-\cos\theta = 1$ and therefore when $\cos\theta = 0$ so $\theta = \pi$ (2)

E.3 Radioactive decay

Practice questions

1. **(a)** Decay is a nuclear process so rate is not affected by temperature or pressure of the sample. However the rate of decay does depend on how many nuclei are present. (2)

Property	Increase	Decrease	Stays the same
Temperature			✓
Pressure			✓
Amount	✓		

(b) **(i)** $^{226}_{88}\text{Ra} \rightarrow {}^{4}_{2}\alpha + {}^{222}_{86}\text{Rn}$

From text given it is α decay so we know A – 4, Z – 2 (2)

(ii) (loss of mass)c^2 = energy released ⇒ mass (Ra) > mass (α + Rn)

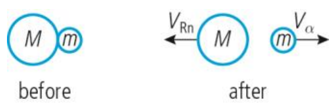
E = [mass(Ra) – mass (α + Rn)]c^2

Δm = 226.0254 – (222.0176 + 4.0026) = 0.0052u

1u ⇒ 931.5 MeV

So E = 0.0052 × 931.5 = 4.84 MeV (3)

(c) **(i)**



The momentum of the particles before the decay = 0 since the bodies are isolated; momentum is conserved so momentum after decay = 0

This means that the momentum of the nucleus is equal and opposite to the momentum of the alpha particle. In other words they move in opposite directions. (3)

(ii) $222 \times v_{\text{Rn}} = -4 \times v_{\alpha}$

$$\Rightarrow \frac{v_\alpha}{v_{\text{Rn}}} = -\frac{222}{4} = -55.5$$ (3)

(iii) $E_{\text{K}\alpha} = \frac{1}{2}mv_\alpha^2$ and $E_{\text{KRn}} = \frac{1}{2}mv_{\text{Rn}}^2$

but $m_{\text{Rn}} = \frac{222}{4} \times m_\alpha$ and $v_{\text{Rn}} = \frac{v_\alpha}{55.5}$

substituting $E_{\text{KRn}} = \frac{1}{2}\left(\frac{222}{4}\right)m_\alpha \times \left(\frac{v_\alpha}{55.5}\right)^2 = 0.018 \times \frac{1}{2}mv_\alpha^2$

so, $E_{\text{KRn}} < E_{\text{K}\alpha}$ (3)

(*Total 16 marks*)

2. (**a**) The other particle emitted in beta decay is an antineutrino.

(**b**) During beta decay the energy released (change in binding energy) is shared between the daughter, beta particle and neutrino. The daughter has a much bigger mass than the other two so doesn't receive much energy, resulting in most energy being shared between the beta particle and neutrino.

(**c**) Using the decay equation $N = N_0 e^{-\lambda t}$

where the decay constant $\lambda = \dfrac{\ln 2}{\text{half-life}} = \dfrac{\ln 2}{0.82}$ = 0.845 s^{-1}

So fraction remaining after 10 s = $\dfrac{N}{N_0}$ =e$^{-0.845 \times 10}$ = 0.000 213, which is 0.02%

3. (**a**) ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18} + \beta^+ + \nu$

The proton number has gone down by 1 but the nucleon number is constant → p^+ → $n^o + \beta^+ + \nu$ (2)

(**b**) Rock contains 1.2 × 10^{-6} g of K and 7.0 × 10^{-6} g of Ar

Originally all of this was K so the original amount of K = (7.0 + 1.2) × 10^{-6} = 8.2 × 10^{-6} g (1)

(**c**) (**i**) $\lambda = \dfrac{\ln 2}{t_{\frac{1}{2}}} = \dfrac{\ln 2}{1.3 \times 10^9}$ = 5.3 × 10^{-10} year^{-1} (2)

(**ii**) N_0 = 8.2 × 10^{-6} g

N = 1.2 × 10^{-6} g

$N = N_0 e^{-\lambda t}$

$$t = \frac{-1}{\lambda}\ln\frac{N}{N_0} = \frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$t = \frac{1}{5.3 \times 10^{-19}} \times \ln\frac{8.2}{1.2}$$ = 3.6 × 10^9 years (2)

(*Total 7 marks*)

4. There are two correct definitions for binding energy, one of which is an available response in this question: the energy released when a nucleus is formed from its constituent nucleons and the energy required to separate a nucleus into its constituent nucleons.

Answer: B (1)

(*Total 1 mark*)

5. Binding energy per nucleon is a measure of stability. The greater the binding energy per nucleon, the more energy that was released (per nucleon) during its formation and therefore the more work done needed to change the number of nucleons.

Answer: C (1)

(*Total 1 mark*)

6. In a hydrogen atom there are no strong nuclear forces present because there is only one nucleon. The electromagnetic force between a proton and an electron is far greater than the gravitational force, so it must be that responsible for the difference.

Answer: A (1)

(*Total 1 mark*)

7. α is most ionizing (because it has the greatest charge) and γ is least ionizing (because it is uncharged and therefore must be absorbed by atoms).

Answer: A (1)

(*Total 1 mark*)

8. During beta decay a neutron changes into a proton and an electron. The proton remains in the nucleus and therefore the total number of nucleons is unchanged.

Although charge is conserved overall, the charge of the nucleus increases.

Answer: A (1)

(*Total 1 mark*)

9. $^{210}_{83}\text{Bi} \xrightarrow{\beta^-} {}^{210}_{84}\text{X} + {}^{0}_{-1}\beta^-$

$^{210}_{84}\text{X} \xrightarrow{\alpha} {}^{206}_{82}\text{Y} + {}^{4}_{2}\alpha$

Answer: B (1)

(*Total 1 mark*)

10. $^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{X} + {}^{0}_{1}\beta^+ + {}^{0}_{0}\nu_e$

The sodium nucleus is required to retain the nucleons. An electron neutrino (matter) is produced in addition to the beta-plus particle (anti-matter).

Answer: A (1)

(*Total 1 mark*)

11. Gamma decay is emission of photons from the nucleus. The fact that there are discrete possible values of energy because of this suggests that the nucleus has energy levels.

Atomic energy levels are evidenced by electron and photon interactions.

Answer: B (1)

(*Total 1 mark*)

12. Nuclear energy levels do not have a continuous spectrum. Photon emission spectra are related to electron energy levels.

The existence of neutrinos was postulated because of the continuous spectrum of energies of emitted beta particles. There should only have been one, so another particle must be responsible for carrying a portion of the energy.

Answer: D (1)

(*Total 1 mark*)

13. The number of neutrons exceeds the number of protons in stable heavy nuclei. Since beta-minus decay involves a neutron changing into a proton, this will occur for isotopes above the line of stability.

Answer: D (1)

(*Total 1 mark*)

14. In radioactive decay, the rate of decay is proportional to the number of undecayed nuclei. This is what yields the exponential relationship.

Answer: A (1)

(*Total 1 mark*)

15. Since $A = \lambda N$, using the values for A and N at any one time (e.g. $t = 0$):

$$\lambda = \frac{0.7 \times 10^6}{1 \times 10^6} = 0.7\ \text{s}^{-1}$$

Answer: A (1)

(*Total 1 mark*)

16. Since $A = A_0 e^{-\lambda t}$, $\ln A = -\lambda t + \ln A_0$

$-\lambda$ is the gradient, which is -6, so $\lambda = 6$.

Answer: D (1)

(*Total 1 mark*)

17. For nuclides with long half-lives, it is not possible to measure the half-life by waiting until the activity halves. Instead, we use the equation $A = \lambda N$ combined with $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Answer: A (1)

(*Total 1 mark*)

18. A nice way that this can be done without a calculator is by considering the shape of an exponential decay curve. 4.0 s is halfway to a half-life, which means that the correct answer is greater than 5000 Bq.

To decide between the other two, notice for the appropriate shape of curve that the decay in nuclei slows with time. Therefore, the activity will be a little more than halfway on its way from 10 000 Bq to 5000 Bq.

Answer: C (1)

(*Total 1 mark*)

19. This is one definition to be memorized! It is indeed based on an atom (not a nucleus) of carbon-12, although the electrons make little difference to the number itself.

Answer: A (1)

(*Total 1 mark*)

20. $E = mc^2 = 938\text{MeV}\ c^{-2} \times c^2 = 938\ \text{MeV}$

To convert into Joules: $938 \times 10^6 \times 1.6 \times 10^{-19}$ J

Answer: D (1)

(*Total 1 mark*)

21. $E = mc^2 = 4 \times 10^{-30} \times \left(3 \times 10^8\right)^2 = 36 \times 10^{-14} = 3.6 \times 10^{-13}$ J

To convert into eV: $\dfrac{3.6 \times 10^{-13}}{1.6 \times 10^{-19}} \approx 2 \times 10^6$ eV

Answer: C (1)

(*Total 1 mark*)

22. (**a**) By adding the details to the equation for the beta-plus decay:

$^{26}_{13}\text{Al} \rightarrow {}^{X}_{Y}\text{Mg} + {}^{0}_{1}\beta^{+} + \text{Z}$, we can see what X and Y are:

$^{26}_{13}\text{Al} \rightarrow {}^{26}_{12}\text{Mg} + {}^{0}_{1}\beta^{+} + \text{Z}$

We know from the continuous spectrum of beta particle energies that a neutrino is also present: in this case a neutrino rather than an antineutrino because beta-plus is antimatter. (2)

(**b**) The total energy released in beta decay is fixed. The neutrino and the beta particle share this total energy, but in uneven amounts. (2)

(**c**) Half-life is the time take for the number of undecayed nuclei to halve and the time taken for the activity to halve – because activity is proportional to the number of undecayed nuclei. Bear in mind that weight does not change significantly, because the nuclei are decaying into nuclei of similar mass. (1)

(*Total 5 marks*)

23. (**a**) Writing out an equation: $^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + \beta^{-} + \bar{\upsilon}$

We have been provided with the binding energy per nucleon for $^{32}_{15}\text{P}$ and $^{32}_{16}\text{S}$, so we multiply these by the number of nucleons (32). The other particles have no binding energy; they are not nuclei. (2)

Energy/mass difference = 8.450 × 32 − 8.398 × 32 = 1.664 MeV

(**b**) The initial activity is 480 Bq; half this is 240 Bq. The time taken is 12 days. You can verify the half-life by halving activity again to 120 Bq, which takes an additional 12 days. (1)

(*Total 3 marks*)

24. (**a**) Starting with what we know (i.e. the alpha particle): $^{226}_{86}\text{Ra} \rightarrow \text{Rn} + ^{4}_{2}\alpha$

Now calculating what remains for the radon: $^{226}_{86}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\alpha$ (2)

(**b**) One way of approaching this is to calculate the fraction of the activity that remains, taking care to ensure consistency between the units for the decay constant and the time:

$$A = A_0 e^{-1.4\times10^{-11}\times5.8\times10^{5}} \approx 0.9999927A_0$$

Alternatively you can find the half-life using $T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = 5.0\times10^{10}$ s = 1600 years, which is much longer than the experiment. (2)

(**c**) Using what you have just learned, you can assume that activity is constant throughout.

Using $A = \lambda N_0$, we find that the number emitted per second is $5.2 \times 10^9\ \text{s}^{-1}$. Multiplying up by the number of seconds in 6 days, we find there are 2.7×10^{15} alpha particles. (3)

(**d**) Alpha particles have a low penetration power (just a few centimeters in air) because they are highly ionizing. A thin wall increases the likelihood that some alpha particles will penetrate through. (1)

(**e**) Using $pV = nRT$, $p = Nk\frac{T}{V}$

Using absolute temperature of 291 K:

$$p = 2.7 \times 10^{15} \times 1.38 \times 10^{-23} \times \frac{291}{1.3\times10^{-5}} = 0.83\ \text{Pa}$$ (3)

(*Total 11 marks*)

25. (**a**) Answer: B (1)

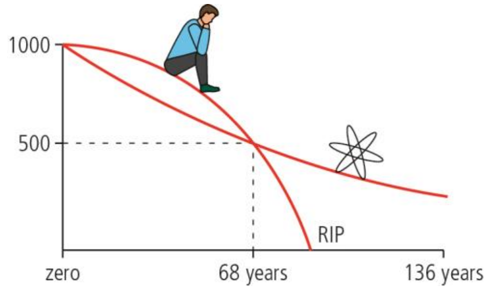
(**b**) In exponential relationships, a quantity changes by equal fractions in equal times (unlike a linear relationship). Therefore, increasing the time taken for the same fraction to decay is a much greater priority than starting with double.

There are links in this to ideas about compound interest and to virus transmission that you might like to look into in more detail. (1)

(*Total 2 marks*)

26. (**a**) Answer: C (1)

(b) Radioactive decay is a random process, which means that although we cannot know which nucleus will decay next nor when, we can calculate the fixed and constant probability of a given number of nuclei decaying in a given time window.



It theoretically takes infinite time for the entirety of a sample of radioactive nuclei to decay. In case of interest, you can however use 5 times the half-life for a rough estimate. In this case, it would take 350 years (far surpassing the oldest human lives) for the number of nuclei to approximate zero. (1)

(Total 2 marks)

27. (**a**) As we are considering just one nucleus, the probability of it not decaying is $\frac{1}{2}$.

Therefore, the probability of it decaying is: $1-\frac{1}{2}=\frac{1}{2}$ (2)

(**b**) Using a similar logic, the probability of it not decaying in the first half-life and the second half-life and the third half-life is $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$.

Therefore, the probability of it decaying is: $1-\frac{1}{8}=\frac{7}{8}$ (2)

(*Total 4 marks*)

28. (**a**) There are two volumes each year, each double the length of the one before so $2 \times 2 = 4$. (1)

(**b**) Taking the beginning of each year in turn:

1935: 1 cm

1936: 4 cm

1937: 4^2 cm

1938: 4^3 cm

1939: 4^4 cm

1940: 4^5 cm = 1024 cm (2)

(**c**) 1024 cm $\approx 1\times10^3$ cm (1) (**d**) Taking the beginning of each year in turn:

1940: 10 m

1941: 4 × 10 = 40 m

1942: 4 × 40 = 160 m

1943: 4 × 160 = 640 m (2)

(**e**) After *n* years beginning in 1941 the volume thickness will be $4^n \times 10$ m.

The speed of the front page will be $4^n \times 10 \div 6$ months.

Equating to the speed of light to find the year: $3 \times 10^8 = \dfrac{4^n \times 10}{\frac{365}{2} \times 24 \times 3600}$

$4.73 \times 10^{14} = 4^n$

Taking logs to base 10: 14.67 = *n* log 4

n = 24.4

So the year will be 1964 (4)

(*Total 10 marks*)

E.4 Fission

Practice questions

1. (**a**) (**i**) Fission is when a large nucleus splits into two smaller ones of roughly equal size.

Radioactive decay is when the nucleus emits a small particle (α, β, γ)

(4)

(**ii**) $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{90}_{38}Sr + ^{142}_{54}Xe + 4^{1}_{0}n$

Calculate how many neutrons form the change in A:

235 + 1 = 90 + 142 + 4 (2)

(**iii**) During beta decay a neutron → a proton, so the number of nucleons is unchanged but the number of protons increases by 1.

A is unchanged, Z → Z + 1 (2)

(**b**)

(**i**) Sr has E_K = 102 MeV. This is $102 \times 10^6 \times 1.6 \times 10^{-19}$ J = 1.63×10^{-11} J

mass of Sr = 90 × mass of nucleon = $90 \times 1.7 \times 10^{-27}$ kg (approximately)

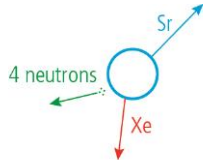
= 1.53×10^{-25} kg

$E_K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2E_k}{m}}$ = 1.46×10^7 m s^{-1}

momentum = mass × velocity = 2.2×10^{-18} Ns (4)

(**ii**) Momentum of two parts is not the same because the four neutrons will also have momentum. (2)

(**iii**)



Since we don’t know which way the neutrons go it is difficult to say which of the arrows should be biggest or their exact direction.

The way shown is with the neutrons on the side of the Xe. (2)

(**c**) (**i**) Energy released = 198 MeV = 198 × 10^6 × 1.6 × 10^{-19} = 3.17 × 10^{-11} J

25% of this = 7.9 × 10^{-12} J (2)

(**ii**) $Q = mc\Delta T$ = 0.25 × 4200 × 80 = 8.4 × 10^4 J (3)

(**iii**) The number of fissions to heat the water $= \frac{8.4 \times 10^4}{7.9 \times 10^{-12}}$ = 1.1 × 10^{16} fissions

Each nucleus has mass 3.9 × 10^{-25} kg

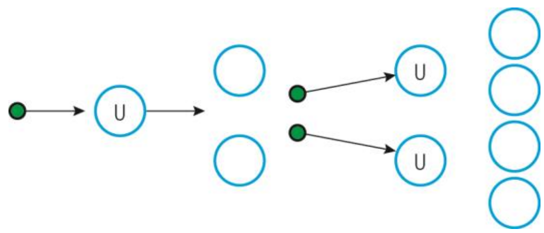
So mass required = 1.1 × 10^{16} × 3.9 × 10^{-25} = 4.1 × 10^{-9} kg (4)

(*Total 25 marks)*

2. (**a**) (**i**) This reaction is a fission reaction. (1)

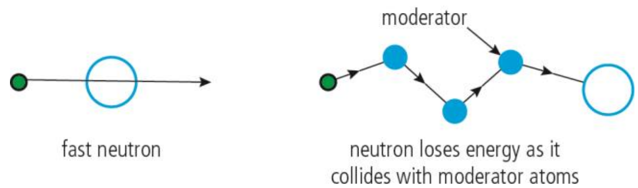
(**ii**) The energy liberated is given to the E_K of the products. Increasing the E_K of the atoms results in an increased temperature. (1)

(**b**) This is best shown in a diagram.



Neutrons from the first fission are absorbed by U nuclei, initiating further fissions. (1)

(**c**) (**i**) If the neutrons are traveling too quickly, they will pass through the U nucleus. To allow them to be absorbed they are slowed down by the moderator atoms.



(3)

(**ii**) The control rods are used to slow the reaction down. They do this by absorbing neutrons, preventing them from being absorbed by ^{235}U, leading to further fissions. (2)

(**d**) Fission of U → E_K to products. This causes the temperature to increase. The hot fuel is used to turn water into steam, which drives a turbine. The turbine turns a generator that produces electricity. (4)

(*Total 12 marks*)

3. Since energy is released we know that binding energy per nucleon is increasing during this fission reaction. We know for sure, therefore, that $B_Y > B_X$ and $B_Z > B_X$. We can make no comments on B_Y relative to B_Z except knowing that these are not the same (because the nuclei will be different).

Answer: A (1)

(*Total 1 mark*)

4. Fast-moving neutrons are not absorbed by the fuel nuclei and therefore they must be slowed down. This is the role of the moderator: to reduce the speed (and therefore kinetic energy) of the neutrons so that they are more likely to induce further fission reactions.

Answer: B (1)

(*Total 1 mark*)

5. Moderator nuclei provide a collision site for transmission of kinetic energy. Control rods absorb the neutrons altogether, making it impossible for further fissions to occur.

Answer: B (1)

(*Total 1 mark*)

6. Taking the possible answers in turn:

- Moderators slow the neutrons down.
- Fuel rods contain the fuel supply.
- Water or another substance takes the energy generated in a heat exchanger.
- Control rods regulate the number of reactions taking place in a given time.

Answer: D (1)

(*Total 1 mark*)

7. Generators are common to all electrical power stations. They convert kinetic energy (usually in a rotating turbine) into electrical energy.

The kinetic energy in a nuclear power station comes from the particles of steam that drive the turbine, which in turn came from heat exchange with the water in the reactor core.

Answer: B (1)

(*Total 1 mark*)

8. (**a**) Binding energy is defined as the energy released when a nucleus is formed from its constituent nucleons. It can also be defined as the energy required to separate a nucleus into its constituent nucleons. (1)

(**b**) Because the masses of individual atoms and the energies in individual nuclear reactions are very small, physicists might choose to select a unit that avoids the use of standard form. (1)

(**c**) $^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2{}^{1}_{0}\text{n}$

Converting the binding energy per nucleon into binding energy:

Binding energy for U-235 = 7.59 MeV × 235

Binding energy for Xe-140 = 8.29 MeV × 140

Binding energy for Sr-94 = 8.59 MeV × 94

Find the difference between the final and initial binding energies:

$140 \times 8.29 + 94 \times 8.59 - 235 \times 7.59 = 184\,\text{MeV}$ (1)

(d) Specific energy is energy per unit mass. Converting into SI units:

Specific energy = $\dfrac{180\times10^{6}\times1.60\times10^{-19}}{235\times1.66\times10^{-27}}=7.4\times10^{13}\ \text{J kg}^{-1}$ (2)

(e) Energy is power × time $=1.2\times10^{9}\times24\times3600$

Since efficiency is $\dfrac{\text{useful energy output}}{\text{total energy input}}$,

total energy input = $\dfrac{\text{useful energy output}}{\text{efficiency}}=\dfrac{1.2\times10^{9}\times24\times3600}{0.36}=2.9\times10^{14}\ \text{J}$

Since specific energy is $\dfrac{\text{energy}}{\text{mass}}$, mass $=\dfrac{\text{energy}}{\text{specific energy}}=\dfrac{2.9\times10^{14}}{7.4\times10^{13}}=3.9\,\text{kg}$ (2)

(f) The specific energy of uranium is much higher than that of a fossil fuel, and therefore much lower quantities are needed overall. This saves on transportation costs. (1)

(g) Using the equation and adding what we know for the electron: ${}^{94}_{38}\text{Sr}\rightarrow\text{X}+\bar{v}_e+{}^{0}_{-1}e$

Now complete the numbers for X: ${}^{94}_{38}\text{Sr}\rightarrow{}^{94}_{39}\text{X}+\bar{v}_e+{}^{0}_{-1}e$ (1)

(h) The initial mass of the sample is 1 kg. The time taken for this to halve to 0.5 kg is 75 seconds. This can be verified by selecting two other initial and final values. (1)

(i) This can be attempted with or without the exponential decay equation.

To use the equation, convert the half-life into the decay constant using $\dfrac{\ln 2}{T_{\frac{1}{2}}}=\dfrac{\ln 2}{75}=9.24\times10^{-3}\ \text{s}^{-1}$ and then use mass remaining

$=1.0\times e^{-9.24\times10^{-3}\times600}=3.9\times10^{-3}\ \text{kg}.$

Alternatively you might spot that 10 minutes is an integer number (8) of 75 seconds, which means that the mass will halve 8 times in the time:

mass remaining $=1.0\times\left(\dfrac{1}{2}\right)^{8}=3.9\times10^{-3}\ \text{kg}$ (2)

(*Total 12 marks*)

E.5 Fusion and stars

Practice questions

1. (**a**) The temperature of the core of the Sun is so high that all atoms would be ionized; this means that chemical reactions such as burning could not take place. Spectral analysis of the Sun shows that it is mostly H and He so the reaction taking place is fusion not fission. The amount of energy produced is also of the right order of magnitude. (2)

(**b**) (**i**) Balancing the nucleon numbers (atomic mass number) 4 + 4 + 4 = 12

Balancing proton number (atomic number) 2 + 2 + 2 = 6 (2)

(**ii**) The energy released is due to loss in mass

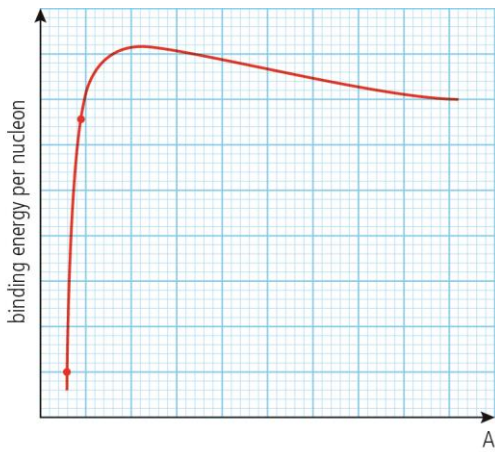
Mass defect = 3 × mass of He – mass of C

$3 \times 6.648\,325 \times 10^{-27} - 1.993\,200\,0 \times 10^{-26} = 1.2975 \times 10^{-29}$ kg

This is equivalent to mc^2 energy $= 1.2975 \times 10^{-29} \times (3 \times 10^8)^2 = 1.17 \times 10^{-12}$ J (3)

(*Total 7 marks*)

2. (**a**) Fusion is when two small nuclei join to form a larger nucleus with higher binding energy. This results in the release of energy. (3)



(**b**) Nuclear force is very short range, so to fuse, nuclei must get very close. But nuclei are positive, so repel each other.

To get them close they must move very fast. This can be achieved if the temperature is high.

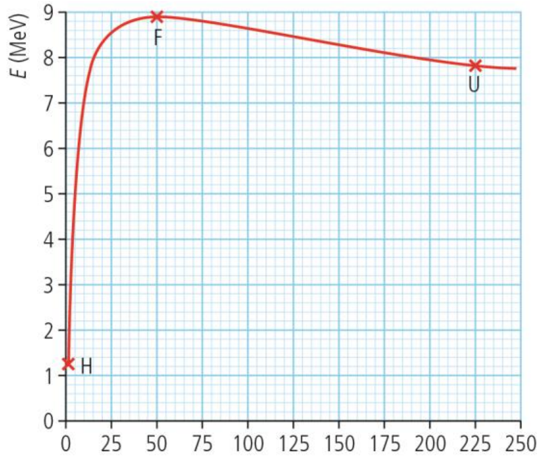
To increase the number of collisions, the density of nuclei should be high. This is achieved by increasing pressure. (5)

(*Total 8 marks*)

3. (**a**) **(i)** A nucleon is a proton or neutron; these are the particles that make up the nucleus. (1)

(**ii**) Nuclear binding energy is the energy required to pull a nucleus apart or the energy released when it is formed from its constituent nucleons. (1)

(**b**) and (**c**) (5)



(**d**) E_B = (mass of parts – mass of nucleus)c^2

If mass is in u then can convert to MeV by multiplying by 931.5 MeV

${}^{3}_{2}\text{He}$ has 2 protons and 1 neutron

E_B = [(mass neutron + 2 × mass proton) – mass He] × 931.5 MeV

= [(1.008 67 + 2 × 1.007 28) – 3.016 03] × 931.5 MeV

= 0.0072 × 931.5 = 6.7 MeV

E_B per nucleon = $\frac{6.7}{3}$ = 2.2 MeV (3)

(**e**) ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$

(**i**) This is a fusion reaction. (1)

(**ii**) When two small nuclei fuse the binding energy increases. This means energy must be released. (2)

(*Total 13 marks*)

4. Because binding energy per nucleon is a measure of stability, the peak of the curve represents the most stable nuclei. To the left of the peak, the nuclei are smaller, which means that fusion adds to the binding energy in such a way that binding energy per nucleon increases. To the right of the peak, the nuclei are larger, which means that (although binding energy itself is higher) binding energy per nucleon is lower.

Answer: C (1)

(*Total 1 mark*)

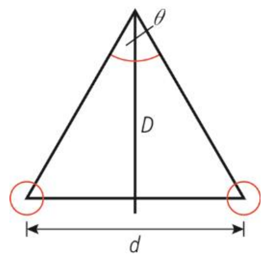
5. Nuclear reactions always increase binding energy per nucleon; this is why they occur. For binding energy per nucleon to increase, the binding energy itself must be increasing (because there are simply more nucleons present). Therefore, the mass overall must decrease in order for this energy to be released.

Answer: B (1)

(*Total 1 mark*)

6. (**a**) The star has changed position in the night sky because the Earth has completed some of its orbit around the Sun and the star is much closer to the Earth than the stars it happens to appear adjacent to. (1)

(**b**) *d* is the diameter of the Earth's orbit, so this spans the two most extreme positions of the Earth. *D* is the distance to the distant star (which is approximately the same irrespective of where on the diameter it is drawn). θ is the total subtended angle.



(1)

(**c**) Using trigonometry, $\sin\frac{\theta}{2} = \frac{d}{2D}$.

Because of the small angle approximation ($D \gg d$), $\theta = \frac{d}{D}$. (2)

(**d**) You could opt for degrees or arc-minutes for θ and metres or light-years for D or any other correct units for each individual quantity. (1)

(*Total 5 marks*)

7. (**a**) By definition, distance in parsec $= \frac{1}{\text{angle in arc-seconds}} = \frac{1}{3.64 \times 10^{-3}} = 275\text{ pc}$ (1)

(**b**) Stellar parallax relies on the star being relatively near to the Earth so that the angle subtended by the Earth's orbit is large enough to measure. (1)

(*Total 2 marks*)

8. (**a**) A nebula is a large body made of dust and gas. They are often formed at the end-point of an existing star's life. (1)

(**b**) distance in parsec $= \frac{1}{\text{angle in arc} - \text{seconds}} = \frac{1}{8.32 \times 10^{-3}} = 120\text{ pc}$

First we convert this into light-years and then we convert into metres:

distance = $120 \times 3.26 \times 3 \times 10^{8} \times 365 \times 24 \times 3600 = 3.70 \times 10^{18}\text{ m}$ (2)

(**c**) It is sometimes more convenient to work with numbers that do not require standard form or large powers of 10. (1)

(*Total 3 marks*)

9. (**a**) Luminosity is proportional to surface area and (temperature)^4. Surface area is proportional to (radius)^2:

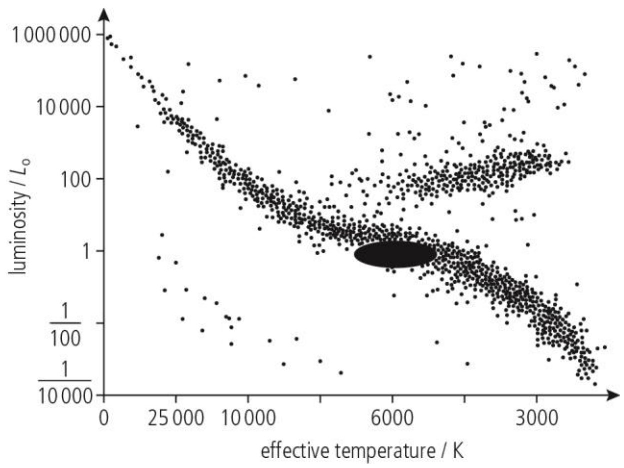
$$\frac{L_V}{L_S} = \frac{1.54\times10^{28}}{3.85\times10^{26}} = \frac{[r_V]^2}{[r_S]^2}\times\frac{9600^4}{5800^4}$$

Rearranging: $r_V = \sqrt{\frac{1.54\times10^{28}}{3.85\times10^{26}}\times\frac{5800^4}{9600^4}}r_S = 2.3\ r_S$ (3)

(**b**) We know from Wien's law that the peak wavelength in a star's blackbody spectrum is inversely proportional to the temperature: $\lambda_{max}T = 2.9\times10^{-3}$ mK (3)

(*Total 6 marks*)

10. (**a**) We know that the Sun's surface temperature is roughly 6000 K and that the Sun is in its main sequence.



(1)

(**b**) A red giant forms when hydrogen fusion in the core comes to an end. This is because the reduction in outward radiation pressure leads to the gravitational force taking over; the star contracts. When the star contracts the temperature rises, enabling the hydrogen in a layer around the core will begin to fuse. The renewed increase in radiation pressure means the Sun expands, cooling its surface (hence why it is red), and the fusion of helium means the star is more luminous. (3)

(**c**) Luminosity is proportional to surface area and (temperature)^4. Surface area is proportional to (radius)^2:

$$\frac{L_W}{L_R}=\frac{10^{-4}}{10^4}=\frac{[r_W]^2}{[r_R]^2}\times\frac{3000^4}{10\,000^4}$$

Rearranging: $\frac{r_W}{r_R}=\sqrt{\frac{10^{-4}}{10^4}\times\frac{3000^4}{10\,000^4}}=9.0\times10^{-6}$ (2)

(*Total 6 marks*)

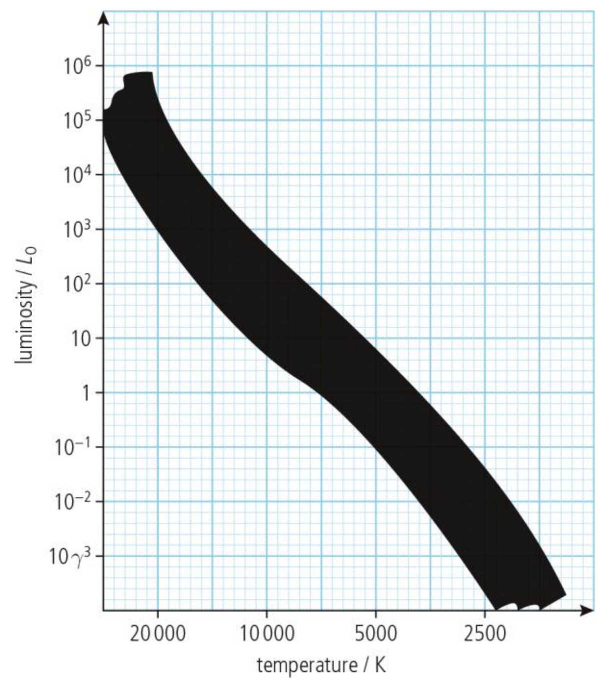
11. (**a**) Radiation pressure due to fusion is an outward force, which is balanced by the gravitational inward force. (2)

(**b**) Using $L=\sigma AT^4$, $L_{Gacrux}=5.67\times10^{-8}\times4\pi\times\left(58.5\times10^9\right)^2\times3600^4=4.1\times10^{-29}\,\text{W}$

$$\frac{L_{Gacrux}}{L_{\odot}}=\frac{4.1\times10^{29}}{3.85\times10^{26}}=1.1\times10^3$$ (3)

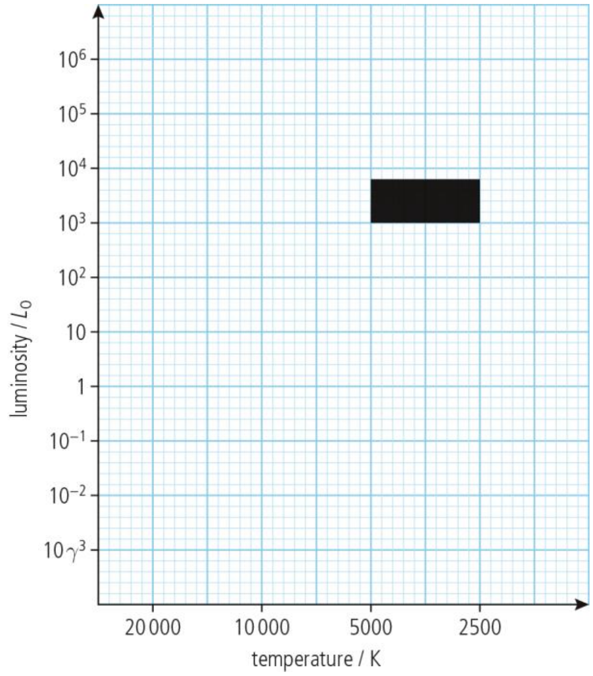
(**c**) Parallax is only effective for stars that are relatively close to the Earth because the angle subtended by the Earth's orbit needs to be large enough to measure. (1)

(**d**) The main sequence consists of stars that are luminous and hot and stars that are dim and cool (and all in between that happen to be fusing hydrogen).



(1)

(**e**) Gacrux can simply be plotted without any awareness of its type because we know its luminosity ($10^3 L_{\odot}$) and its temperature (3600 K).



(1)

(**f**) There are two routes following the main sequence. Low-mass stars become red giants, then white dwarves then (eventually) black dwarves. High-mass stars become supergiants, and then experience a supernova explosion leaving behind a neutron star or (for very high masses) a black hole. (3)

(*Total 11 marks*)

12. (**a**) By definition, main sequence stars fuse hydrogen to helium. The innermost region is hottest, and so hydrogen will already have fused into helium. The outer regions are cooler, which means that hydrogen may or may not fuse into helium in due course. (2)

(**b**) X is already very luminous. As larger nuclei are formed through fusion, its luminosity will remain mostly the same but its surface temperature will fall. (1)

(**c**) $L = A\sigma T^4 = 5.67\times10^{-8}\times4\times\left(2.0\times10^4\right)^2\times\left(10^6\right)^4 = 2.85\times10^{26}$ W (2)

(**d**) Using Wien's law, $\lambda = \dfrac{2.9\times10^{-3}}{10^6} = 2.9\times10^{-9}$ m, which is an X-ray wavelength. (2)

(*Total 7 marks*)

13. (**a**) Answer: A (1)

(**b**) Iron nuclei are the most stable in the universe and have the maximum possible binding energy per nucleon. Uranium nuclei have more binding energy in total, but less binding energy per nucleon (because they are less stable). Therefore, the formation of uranium from iron is a fusion process that requires more energy input than that which is output. It happens only in supernovae. (1)

(*Total 2 marks*)