1 Number and algebra

Teaching support and guidance

Concepts

- Patterns
- Equivalence
- Generalization

Outcomes

Students will understand the relationships between different quantities and patterns in number, finance, and sequences and series.

Conceptual Understandings

- Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.
- Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.
- The binomial theorem is a generalization that provides an efficient method for expanding binomial expressions.

Inquiry Questions

- Factual: What equivalences exist in mathematics?
- Conceptual: How does Pascal's triangle help us expand brackets?
- Debatable: How does a pattern become a rule?

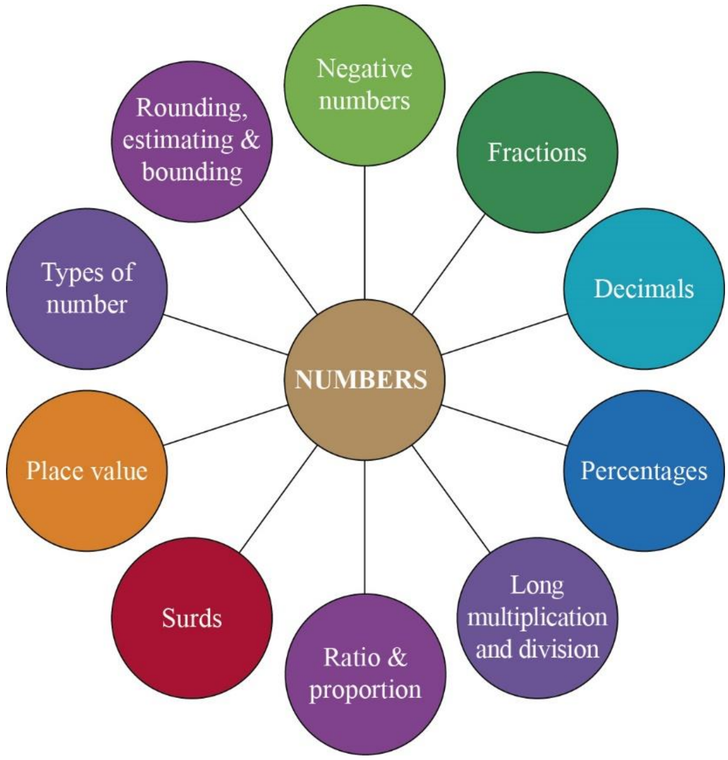
Factual: What equivalences exist in mathematics?

Concept: Equivalence

Standard Level

Discussion: Introduction to equivalence (S1.1)

When starting the unit, it is a good idea to gauge students' understanding of equivalences. Have them list as many as they can think of within mathematics. It may be useful for students to work in small groups to produce mind maps.



How do we calculate 10% of a number? We multiply by 0.1. This is a simple relationship that allows us to use multipliers to find percentage amounts.

If we want to increase a number by 10%, we multiply by 1.1. This exemplifies how basic relationships in mathematics enable us to simplify a process.

PowerPoint: The Fibonacci numbers and the golden ratio (S1.1)

Patterns and relationships can appear in many different ways. What is obvious to one person may not be clear to another. This PowerPoint introduces the Fibonacci numbers and the golden ratio and includes links to the following videos.

- *The magic of Fibonacci numbers* – a TED talk by mathematician Arthur Benjamin: www.youtube.com/watch?v=SjSHVDfXHQ4
- *Places You Won't Believe The Golden Ratio & Fibonacci Sequence Appears* – a look at some of the many natural occurrences and practical applications of Fibonacci numbers and the golden ratio: www.youtube.com/watch?time_continue=8&v=RqqErDSLtwE

The PowerPoint addresses the TOK question: Is all knowledge concerned with the identification and use of patterns?

Links: Patterns in Pascal's triangle (S1.1)

As an extra activity, which will benefit them in work on future topics such as Binomial expansions or Combinations, students can explore the patterns within Pascal's triangle.

There are some interesting examples on the website:

- www.mathsisfun.com/pascals-triangle.html

Activity: Large numbers (S1.1)

This task is designed as a consolidation of students' knowledge of scientific notation. They will have seen this notation before but may not have used the applications of it. Questions 9 to 15 will challenge students to apply their knowledge and develop problem-solving skills.

Higher Level

Activity: Introduction to absolute value (H1.10)

Potentially a homework task, or a research task for students working in a flipped classroom context, this task covers the basics of absolute value and links the idea of absolute value being equivalent to the distance from zero.

Conceptual: How does Pascal's triangle help us expand brackets?

Concept: Patterns

Standard Level

Activity: Pascal's triangle and binomial expansions (S1.9)

An investigation activity that can be done by students working individually or in groups. It highlights the link between the patterns in Pascal's triangle and the expansion of brackets. It is heavily scaffolded because this is not an easy concept for students to grasp. You may want to cover Question 8 on the board in front of the class or invite students to volunteer their discoveries.

Debatable: How does a pattern become a rule?

Concept: Generalization

Higher Level

PowerPoint: Proof by induction (H1.15)

The link to ‘ways of knowing’ in TOK can be discussed before teaching proof by induction. Start with the link in the PowerPoint to the regions problem:

- www.murderousmaths.co.uk/books/regions.htm

Students can attempt the task themselves or can follow the website discussion. It is an excellent and clear example of why proof by induction is necessary. It is important to highlight this during the discussion.

Proof by induction is a tricky topic, mainly because students struggle to see the relevance of it. The video link in the PowerPoint is a straightforward example that proves a nice introduction to the methods:

- www.youtube.com/watch?v=WHVOLY9mrJ4