5 Calculus

Teaching support and guidance

Concepts

- Patterns
- Approximation
- Systems
- Change

Outcomes

Students will understand how we use calculus to represent the rate of change between two variables and describe the accumulation of limiting areas.

Conceptual Understandings

- Limits describe the output of a function as the input approaches a certain value and can represent convergence and divergence.
- Understand the links between the derivative and the rate of change and interpret the meaning of this in context.
- Areas under curves can be approximated by the sum of the areas of rectangles, which may be calculated even more accurately by using integration.

Inquiry questions

- Factual: Can a ‘difference’ be equal to zero?
- Conceptual: How close are the links between mathematical models and physical reality?
- Debatable: When does science become mathematics?
- Conceptual: Can we reconcile mathematics with our intuition?

Factual: Can a 'difference' be equal to zero?

Conceptual: How close are the links between mathematical models and physical reality?

Concepts: Patterns, Approximation, Change

Standard Level and Higher Level

PowerPoint: Differentiation from first principles (S5.1, H5.12)

Differentiation from first principles is not listed as a specific topic in the SL course; however, there are many benefits in showing students the reasoning behind limits and their use in differentiation from first principles. The PowerPoint scaffolds the process, to enable SL students to comprehend the idea of limits while linking to the inquiry questions.

PowerPoint: Familiar applications of volume of revolution (S5.11, H5.17)

While 'volume of revolution' is not listed as a topic in the SL course, there should be no problem in introducing the idea that an area rotated about the x-axis forms a three-dimensional shape. This may offer students ideas for exploration pieces around the topic of integration.

Students can use this link to an area calculator to check any solutions that they have attempted:

- www.symbolab.com/solver/area-under-curve-calculator

It is important also to note that students are able to use their GDCs to calculate areas.

Debatable: When does science become mathematics?

Concept: Systems

Standard Level

PowerPoint: Kinematics (S5.9)

This PowerPoint explores ideas behind kinematics and simplifies the logic in the calculations. The three-step process is a useful visual for students when they are attempting kinematics questions.

For example, suppose they are given the function for velocity. Using the visual aid, when they integrate the function, they will have a function for displacement. If they differentiate the function, they will have a function for acceleration.

Conceptual: Can we reconcile mathematics with our intuition?

Concepts: Systems, Approximation

Higher Level

PowerPoint: Gabriel's horn

TOK: An infinite area sweeps out a finite volume. Can this be reconciled with our intuition? What does this tell us about mathematical knowledge?

This PowerPoint discusses and addresses the TOK question related to infinite areas and finite volumes. It leads students through the mathematics involved and relates it to a famous mathematical concept called 'Gabriel's horn'. There is potential for an exploration piece based on the surface area of a volume of revolution within the PowerPoint.

For the graphing, students may find it useful to refer to the GeoGebra applet at:

- www.geogebra.org/m/zBRtUVfR

Links: Indeterminate form and L'Hôpital's rule

The HL material in this unit is very content heavy. The links below address indeterminate form and l'Hôpital's rule. The websites and videos provide good discussion points for the concepts of systems and approximation when studying these topics.

- YouTube video – Using L'Hôpital's Rule
  www.youtube.com/watch?v=W5HYPtriZ7M
- Indeterminate Forms at brilliant.org
  https://brilliant.org/wiki/indeterminate-forms
- L'Hôpital's Rule at Math24
  www.math24.net/lhopitals-rule
- L'Hospital's Rule and Indeterminate Forms at Paul's Online Notes
  http://tutorial.math.lamar.edu/Classes/CalcI/LHospitalsRule.aspx