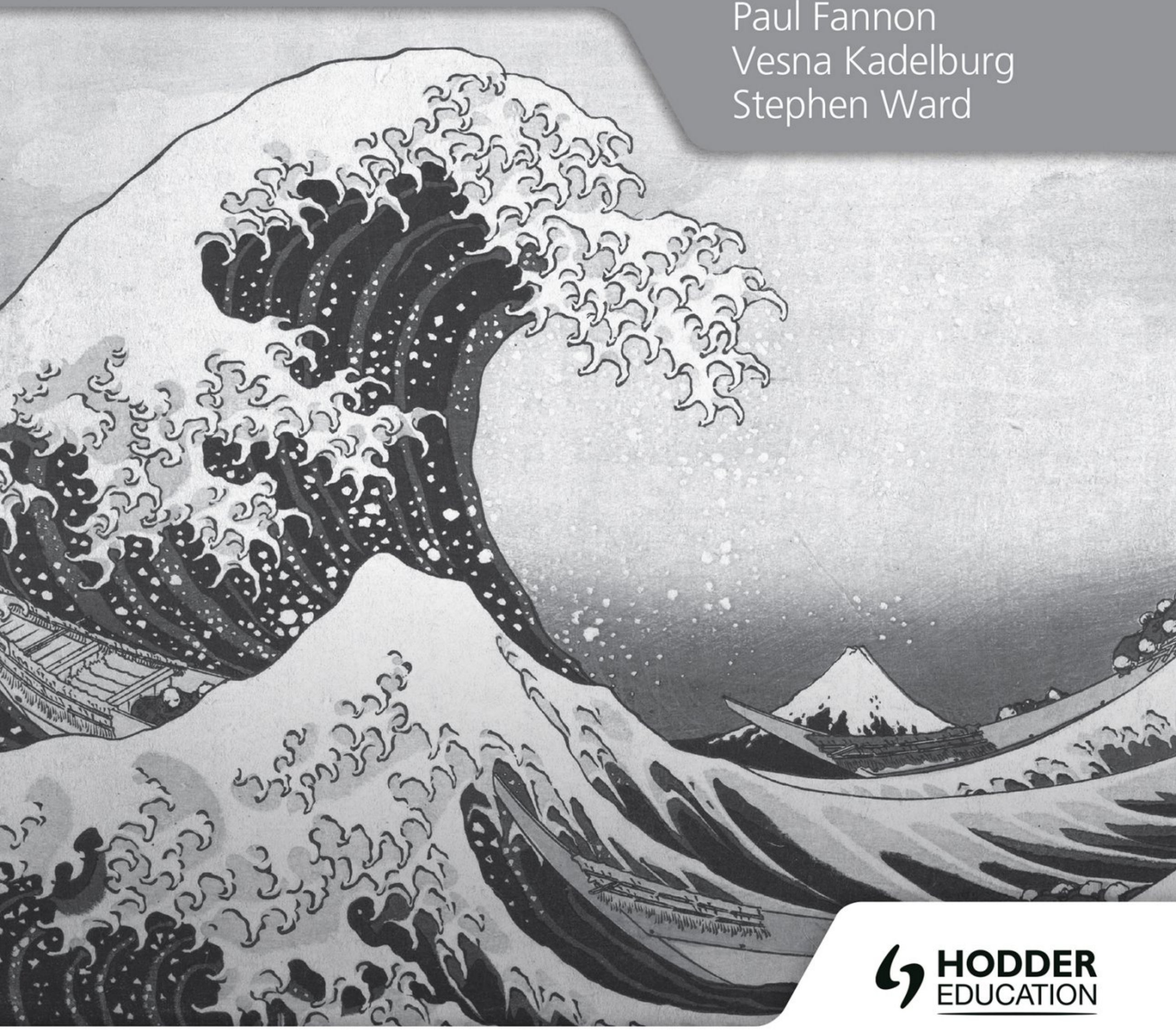
FOR THE
IB DIPLOMA
PROGRAMME

Mathematics
APPLICATIONS AND INTERPRETATION SL
EXAM PRACTICE WORKBOOK

Paul Fannon
Vesna Kadelburg
Stephen Ward



HODDER EDUCATION

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Introduction

Revising for exams can sometimes be overwhelming. This book is designed to help you structure your revision and provide opportunities to practise sitting exam-style papers. Revision should be a cycle of going through your notes and textbooks, practising exam-style questions, reviewing your strengths and weaknesses, and returning to your notes and textbooks.

There are five skills that are needed for exam success:

- knowledge of all the topics required in the syllabus
- facility with basic algebra and arithmetic
- familiarity with your calculator
- the ability to make links and solve problems
- calmness under test conditions.

You need to be aware of your own strengths and weaknesses in each of these areas. This revision guide is designed to help with each of them.

How to use this book

The book comprises four sections that are designed to help you master the five skills listed above.

Calculator checklist

This lists all the tools provided by your GDC (graphical display calculator) that you need to be familiar with. Different calculators might have slightly different input methods, so it is best to use your own calculator manual (these can be found online) to find out the exact syntax yours uses.

Syllabus revision

This section goes through the syllabus line by line to make sure you have covered every part thoroughly. Each skill required in the syllabus is exemplified by a question. You can either start by going over the syllabus content, or by doing the questions. These questions illustrate the basic skills you need; however, they are not all designed to be exam-style questions as they are designed to check syllabus understanding rather than problem-solving. The answers to these questions can be found online at www.hoddereducation.co.uk/ib-extras. Once you are happy, tick the 'revised' box. If you need more details, there are references to the section in the accompanying Hodder Education *Mathematics for the IB Diploma: applications and interpretation SL* textbook corresponding to each syllabus item.

Questions with calculator icons are designed specifically to test calculator or non-calculator skills. Those without an icon could be done either with or without a calculator.

Paper plan

This table provides an overview of the entire syllabus that maps the practice papers in this book and in the *Mathematics for the IB Diploma: applications and interpretation SL* textbook to the different topics and also serves as a revision checklist. You should use the mastery section to tick off and make sure that you have covered each topic. When you have revised the topic, you can tick the second column. Then try doing some questions – either from the textbook or the practice papers – and tick the final column once you feel confident with the topic.

The practice paper section shows the corresponding topic for each question in the textbook practice papers and the sets of practice papers in this book. You can use this to check the type of questions that you might get on each topic.

Practice papers and mark schemes

The best way to practise for exams is to do exams. These papers are designed to mimic the style of real IB papers. The questions often combine more than one syllabus topic and can require you to make novel links. As in the real exam papers, there is space for you to write in your calculations and answers to questions in Section A; for Section B, you will need to use a separate notebook.

Understanding mark schemes

Once you have tried some of the practice papers in this book, it is a very good idea to mark your own (and also mark other people's) to see what makes things easy or difficult to award marks.

There are three different types of marks awarded in exams:

M These are method marks. They are awarded for a clear and obvious attempt to use the correct method. There is a certain amount of subjective opinion needed to award these. For example, if you are asked to find the length of the hypotenuse, h, of a right-angled triangle with shorter sides 5 and 7, which of the following would be awarded a method mark?

I $h = 5 + 7 = 12$

II $h = \sin(5) + \cos(7) = 1.08$

III $h = \sqrt{5 + 7} = 3.46$

IV $h = 5^2 + 7^2 = 74$

V $h = \sqrt{7^2 - 5^2} = 4.90$

VI $h = \sqrt{5^2 + 7^2} = 5 + 7 = 12$

Most examiners would agree that the first three examples are not good enough to award a method mark. In case VI, even though there is subsequent incorrect working and the wrong answer, a method mark would still be awarded. Cases IV and V are on the boundary of what might be acceptable and would probably require discussion among the examiners to find a clear boundary, but it is likely that both answers would be awarded a method mark. However, an answer of 74 or 4.90 by itself would not be awarded any marks because, even though we might have suspicions about where these numbers have come from, it has not been clearly communicated.

Sometimes method marks have brackets around them, for example, **(M1)**. In this case they do not have to be explicitly seen and can be inferred from the correct answer.

Remember that sometimes the question requires a particular method (for example, find the maximum value of the function by differentiating) or it might require you to explicitly use the previous working (generally indicated by using the word 'hence'). If you use a different method in either of these instances, even if it works, you will not normally gain any credit.

For Paper 2, many questions will be answered primarily by using a calculator. However, you can still get some method marks for communicating what you are doing. Remember to write down any numbers that you put into your calculator that are not given in the question (for example, midpoints of grouped data). If you are using a graph to solve an equation, then draw a quick sketch.

A These are accuracy marks. They are for obtaining the correct answer. If there is a previous method mark without a bracket around it then these marks can only be awarded if the previous method mark was awarded (this tends to happen in situations where examiners think the correct answer can be guessed so they need to see supporting evidence, or when the question was a 'show that' or 'proof' question, where the communication rather than just the final answer is assessed).

Often lines are denoted M1A1 – this means one method mark for a correct attempt and one accuracy mark for doing it correctly.

The accuracy mark is withheld if the value or expression is wrong; however, it can also be withheld if the answer is written in calculator notation (for example, $1.8E9$ rather than 1.8×10^9) or is given to the wrong accuracy – remember that all final answers should be given either exactly or written to three significant figures unless the question says otherwise. It is usually a good idea to write down intermediate working to more significant figures to ensure that the final answer is correct to at least three significant figures (and ideally store the answer to the full accuracy your calculator can hold using the calculator memory store).

Accuracy marks are also awarded when sketching graphs. It is important to choose an appropriate window to show all the important features of the graph and to label any relevant features (for example, axis intercepts, turning points, asymptotes).

Unless a particular form is required, most equivalent forms are accepted – for example, $x^2 + x$ or $x(x + 1)$ would normally both be fine. However, there is also an expectation that you understand the general requirements of the course. For example, if the question asked you to find the area under the curve $y = x^2$ between 0 and 1, the answer $\int_0^1 x^2 \mathrm{d}x$, while *technically* equivalent to the correct answer, is not sufficiently simplified – the acceptable answer would be $\frac{1}{3}$ or 0.333.

R These are marks awarded for clear reasoning – often in response to a question asking for explanation or justification. They might also be used when choosing particular solutions from equations (for example, saying that the solution of a quadratic that is negative cannot be a probability).

You may also see an **AG** notation in the mark schemes. This is when the answer is given in the question and it is just a reminder to the examiner that the correct answer does not mean that people have reasoned properly and to be particularly watchful for flawed arguments that just happen upon the right answer.

Sometimes later parts of the question use an answer from a previous part. If you got the earlier part of the question wrong, the examiner will try to award 'follow through' marks by checking whether your method works for your prior incorrect answer. However, follow through marks may only be awarded if you have clearly communicated how you are using your previous answer, if you have not fundamentally changed the nature of the question (for example, solving a quadratic equation turned into solving a linear equation) and if your answer is not implausible (for example, a negative probability).

Revision tips

- Do not leave all your revision until the last minute. The IB is a two-year course with many later topics building on previous topics. One psychological study suggested that you need to 'learn' something seven times for it to be really fixed in your mind. Try to use each class test, mock exam or new topic as an opportunity to revise previous work.
- Revision should be an active rather than a passive process – often you can read through notes for hours and gain very little new knowledge. Try to do some questions first, then read through your notes and textbooks once you get stuck. Your reading will be far more focused if you are trying to find the solution to a particular difficulty.
- Try varied memorization strategies until you find one that works for you – copying out pages of notes does not work for most people. Strategies that do work for some people include using colour to prioritize key facts, using mind maps and making up silly songs to memorize techniques. Psychologists have found a strong link between memory and smell, so you could try using a particular perfume or deodorant while revising, then using the same one in the final exam!
- Work positively with others – some group revision can be a great way of improving your understanding as you can bounce ideas off each other, try to explain a concept to someone who is struggling or design exam-style questions for your friends to do. However, do be careful – avoid feeling bad by comparing yourself to people who seem to be good at everything and do not be tempted to feel good about yourself by making others feel bad – neither scenario is productive.
- Practise checking your answers. This is a vital skill that you will not suddenly be able to do in the final exam if you never do it in your revision. Think about good ways to check answers; for example, with and without a calculator, working backwards and sanity checking that the answer is plausible.
- Become an expert at using your exam calculator. You cannot start working on this skill too early, as each calculator has its own quirks that you need to get used to. Make sure you are using it in the mode required for the exam and know what happens when the memory is cleared and it is reset ahead of the exam; for example, does it default to radians or degrees mode?
- Become familiar with the exam formula booklet. It has lots of useful information, but only if you are used to using it – make sure you know what all the symbols mean and where everything is, well ahead of your final exam. Formulae that are included in the formula booklet are indicated in the syllabus content sections of this book by this icon .
- Make sure some of your revision is under timed conditions. During the exam, the time flashes by for some people whereas others have to pace themselves or they run out of steam towards the end of an exam.
- Do not get downhearted if you are getting lots of things wrong, especially at the beginning of the revision process. This is absolutely normal – in fact, you learn a lot more from the things that you get wrong than the things you get right!

- Weirdly, too much revision can actually be counterproductive. You will have your own personal concentration span beyond which there is no point revising without a small break. Check that your revision plan is achievable, and schedule in plenty of relaxation time.
- Try to get into stable sleeping and eating patterns in the run-up to the exam. If you are getting up each day at noon and never having caffeine, then a 9am exam with lots of coffee is unlikely to go well!
- Unless you know that you only have a very short-term memory, it is unlikely that the night before an exam is the best time to revise. Going for a run, doing some yoga or reading a good book and having a good night's sleep is likely to be worth far more marks than last minute panic revision.
- If you choose to do any revision between Paper 1 and Paper 2, do use the syllabus checklist to see if there are any major topics not covered in the first paper and focus your revision on those.

Exam tips

- Use the reading time wisely. Every mathematics exam starts off with five minutes of reading time in which you are not allowed to write. This time is vital – make sure you read as much of the paper as you can and mentally start making a plan.
- The examiners design the difficulty of the questions to be in increasing order in the short questions, and in increasing order within and between each long question; however, their judgment of difficulty is not always going to align with yours, so do not assume that you should do the questions in order. Many people try all the short questions first, spend too long on the last, often tricky, short question and then either panic or run out of time on the long questions. Sometimes the first long question is the easiest question on the paper, so consider doing that first. There is no substitute for potentially gaining lots of marks early on to build confidence for the rest of the exam.
- Keep an eye on the time. Each mark equates to approximately one minute – so do not spend 10 minutes on a question worth only 2 marks. Sometimes you have to abandon one question and move on to the next.
- Do not get dispirited if you cannot do a question – the exam is meant to be challenging and you will not need 100% of the marks to get the grade you are aiming for. The worst thing you can do is let one bad question put you off showing your ability in other questions.
- Look at the mark schemes to understand what is given credit. Even when many method marks are implied, only putting down the final answer is a high-risk strategy! Even the best mathematicians can make mistakes entering numbers into calculators. Mathematical communication is an important skill so try to convey your reasoning clearly – this has the advantage of enabling you to score some marks even if you make a mistake and of marshalling your ideas so you are more likely to get the answer right in the first place.
- Especially in the long questions, do not assume that just because you cannot do an early part, you cannot do later parts. Even if you get an early part wrong, follow through marks may still be available in later parts if you clearly communicate the correct method, even if you are using the wrong numbers. Sometimes the later parts of questions do not need the results from earlier parts anyway. The only way that you can guarantee getting no marks for part of a question is by leaving it blank!
- In Paper 2, identify which questions are 'calculator questions'. Too many people try to do these questions using non-calculator techniques that do work, but often absorb a lot of time.
- Keeping the exam in perspective is perhaps more important than anything else. While it is of some importance, always remember that exams are artificial and imperfect measurements of ability. How much you can achieve working in silence, under timed conditions and by yourself without any resources on one particular set of questions is not what is most valued in mathematics. It should not be the only outcome from the course that matters, nor should it be how you judge yourself as a mathematician. It is only when you realize this that you will relax and have a chance of showing your true ability.
- Finally, make sure that you understand the command terms used in exams – these are listed below. In particular, 'write down' means that you should be able to do it without any major work – if you are finding that it requires lots of writing then you have missed something!

Command term	Definition
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Explain	Give a detailed account including reasons or causes.
Find	Obtain an answer showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.
Integrate	Obtain the integral of a function.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show	Give the steps in a calculation or derivation.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Calculator checklist

You should know how to:

	Skill	Got it!	Need to check
General	Change between radian and degrees mode		
	Set output to three significant figures		
	Store answers in calculator memory		
	Edit previous calculations		
Number and algebra	Input and interpret outputs in standard index form ($a \times 10^n$)		
	Use the sequence functions to find terms of an arithmetic and geometric sequence		
	Use tables to display sequences		
	Use the sum function to sum sequences		
	Use the TVM package to answer questions about compound interest and depreciation, including finding unknown interest rates and interest periods		
	Use the TVM package to answer questions about amortization and annuities		
	Evaluate logarithms to base 10 and e		
Functions	Graph equations of the form $y = f(x)$		
	Use the zoom/window functions to explore interesting features of graphs		
	Use the trace function to explore graphs, especially suggesting asymptotes		
	Find axis intercepts of graphs		
	Find the coordinates of local maxima or minima of graphs		
	Find the points of intersection of two graphs		
	Solve polynomial equations		
	Solve simultaneous linear equations		
	Solve equations using solve functions on the calculator		
Statistics and probability	Input data, including from frequency tables and grouped data		
	Find mean, median, mode, quartiles and standard deviation from data		
	Input bivariate data		
	Find Pearson's correlation coefficient of data		
	Calculate probabilities for a given binomial distribution		
	Calculate probabilities for a given normal distribution		
	Calculate boundary values from probabilities for a given normal distribution		
	Conduct chi-squared goodness of fit tests		
	Conduct chi-squared contingency table tests		
	Conduct *t* tests to compare two populations		
Calculus	Estimate the value of a limit from a table or a graph		
	Find the derivative of a function at a given point		
	Use the calculator to sketch the derivative of a function		
	Find definite integrals		
	Find areas using definite integrals		

Syllabus revision

1 Number and algebra

Syllabus content

S1.1	Standard form	
	Book Section 1B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Operations with numbers of the form $a \times 10^k$ where $1 \leqslant a < 10$.	Input and interpret numbers of this form on the calculator.	1 ☐
	Factorize to add or subtract numbers in standard form.	2 ☐
	Use the laws of exponents when multiplying or dividing numbers in standard form.	3 ☐

S1.2	Arithmetic sequences and series	
	Book Section 2A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Use of the formulae for the nth term and the sum of the first n terms of the sequence.	Find the nth term of an arithmetic sequence. Use √x $u_n = u_1 + (n-1)d$	4 ☐
	Use the formula to determine the number of terms in an arithmetic sequence.	5 ☐
	Set up simultaneous equations to find the first term and common difference.	6 ☐
	Find the sum of n terms of an arithmetic sequence. There are two formulae in the formula booklet. You should be able to use: √x $S_n = \frac{n}{2}(2u_1 + (n-1)d)$	7 ☐
	Or use: √x $S_n = \frac{n}{2}(u_1 + u_n)$	8 ☐
Use of sigma notation for sums of arithmetic sequences.	Understand how sigma notation relates to arithmetic sequences.	9 ☐
	Evaluate expressions using sigma notation.	10 ☐
Applications.	Recognize arithmetic sequences from descriptions.	11 ☐
	In particular, be aware that simple interest is a type of arithmetic sequence.	12 ☐
Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	Find the common difference as an average of the differences between terms.	13 ☐

S1.3	Geometric sequences and series	
	Book Section 2B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Use of the formulae for the nth term and the sum of the first n terms of the sequence.	Find the nth term of a geometric sequence. √x $u_n = u_1 r^{n-1}$	14 ☐
	Use the formula to determine the number of terms in a geometric sequence.	15 ☐
	Set up simultaneous equations to find the first term and common ratio.	16 ☐
	Find the sum of n terms of a geometric sequence using √x $S_n = \frac{u_1(r^n-1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, r \neq 1$	17 ☐
Use of sigma notation for sums of geometric sequences.	Understand how sigma notation relates to geometric sequences.	18 ☐
	Evaluate expressions using sigma notation.	19 ☐
Applications.	Recognize geometric sequences from descriptions.	20 ☐

S1.4	Financial applications of geometric sequences		
	Book Section 2C	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Financial applications of geometric sequences and series. • Compound interest. • Annual depreciation.		Calculate values of investments with compound interest using financial packages or √x $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ where: FV is the future value PV is the present value n is the number of years k is the number of compounding periods per year r% is the nominal annual rate of interest.	21 ☐
		Calculate interest rates required for particular outcomes.	22 ☐
		Calculate the number of periods required for a particular outcome.	23 ☐
		Calculate the values of goods suffering from depreciation.	24 ☐
		Calculate the real value of investments after inflation.	25 ☐

S1.5	Exponents and logarithms		
	Book Section 1A, 1C	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Laws of exponents with integer exponents.		Evaluate expressions involving integer exponents including using $a^m \times a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $(a^m)^n = a^{mn}$ $a^{-n} = \frac{1}{a^n}$ $(ab)^n = a^n \times b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	26 ☐
		Simplify algebraic expressions using the above rules.	27 ☐
Introduction to logarithms with base 10 and e.		Use the fact that √x $a^x = b$ is equivalent to $\log_a b = x$	28 ☐
		Know that natural logarithms, in x, are equivalent to $\log_e x$ where e = 2.718 …	29 ☐
Numerical evaluation of logarithms using technology.		Use your calculator to evaluate logarithms to base 10 and e.	30 ☐

S1.6	Approximation and estimation		
	Book Section 11A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Approximation: decimal places, significant figures.		Round to a given number of decimal places.	31 ☐
		Round to a given number of significant figures.	32 ☐
		Choose an appropriate degree of accuracy for an answer.	33 ☐
Upper and lower bounds.		Report upper and lower bounds of rounded numbers as an inequality.	34 ☐
Percentage errors.		Calculate percentage errors using √x $\epsilon = \left\lvert\frac{v_A - v_E}{v_E}\right\rvert \times 100\%$ where v_E is the exact value and v_A is the approximate value.	35 ☐
		Find the maximum percentage error caused by rounding.	36 ☐
Estimation.		Determine if an answer is reasonable.	37 ☐

S1.7	Amortization and annuities using technology		
	Book Section 11B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Amortization and annuities using technology.		Calculate the outstanding amount of loans being regularly paid off.	38 ☐
		Calculate the value of investments with regular contributions made.	39 ☐
		Calculate the amount of annuity that can be purchased.	40 ☐

S1.8	Solving equations		
	Book Section 12A, 12B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Use technology to solve systems of linear equations in up to three variables.		Recognize linear simultaneous equations and solve them using the appropriate GDC function.	41 ☐
Use technology to solve polynomial equations.		Recognize the order of a polynomial equation, understand the terms zero and root and solve polynomial equations.	42 ☐

Practice questions

1 Evaluate $(3 \times 10^{40})^2 - 5 \times 10^{80}$.

2 Evaluate $3 \times 10^{n+1} - 4 \times 10^{n}$, leaving your answer in the form $a \times 10^{k}$ where $1 \leqslant a < 10$.

3 Evaluate $(6 \times 10^{n}) \div (8 \times 10^{-n})$, leaving your answer in the form $a \times 10^{k}$ where $1 \leqslant a < 10$.

4 Find the 25th term of the following arithmetic sequence:
20, 17, 14, 11 …

5 An arithmetic sequence has first term 21 and last term 1602. If the common difference is 17, how many terms are in the sequence?

6 An arithmetic sequence has 4th term 10 and 10th term 34. Find the 20th term.

7 Find the sum of the first 30 terms of the arithmetic sequence 13, 10, 7, 4 …

8 An arithmetic sequence has $u_1 = 4$, $u_{20} = 130$. Find the sum of the first 20 terms.

9 Determine the first term and common difference of an arithmetic sequence where the sum of the first n terms is given by $S_n = \sum_{r=1}^{n} (5r + 11)$.

10 Evaluate $\sum_{1}^{100} (5r + 11)$.

11 On the first day of training for a race, Ahmed runs 500 m. On each subsequent day Ahmed runs 100 m further than the day before. How far has he run in total by the end of the 28th day?

12 Juanita invests \$300 at 2.4% simple interest. How much will be in her account after 10 years?

13 A ball is dropped and the velocity (v m s^{-1}) is measured at different times (t seconds).

t	0	0.1	0.2	0.3
v	0	1.1	1.9	2.7

It is assumed that the velocity when $t = 0$ is correct, but there is uncertainty in the remaining measurements.

a By modelling the situation as an arithmetic sequence, estimate the velocity when $t = 0.5$.

b Make one criticism of the model.

14 Find the 10th term of the geometric sequence 32, –16, 8, –4 …

15 Find the number of terms in the geometric sequence 1, 2, 4 … 4096.

16 A geometric series has third term 16 and seventh term 256. Find the possible values of the first term and the common ratio.

17 Find the sum of the first eight terms of the sequence 162, 54, 18 …

18 Determine the first term and common ratio of a geometric sequence where the sum of the first n terms is given by $S_n = \sum_{r=1}^{n} 2 \times 5^r$.

19 Evaluate $\sum_{1}^{10} 2 \times 5^r$.

20 The population of bacteria in a petri dish grows by 20% each day. There are initially 50 000 bacteria in the dish.

a Find the number of bacteria in the dish after 12 days.

b Explain why this model cannot continue indefinitely.

21 £2000 is invested in an account paying 4% nominal annual interest, compounded monthly. Find the amount in the account after 10 years, giving your answer to two decimal places.

22 Samira wants to invest £1000 in an account paying a nominal annual interest rate of i%, compounded quarterly. She wants to have £1500 in her account after 8 years. What value of i is required?

23 James invests $100 in an account paying 2.1% annual interest. How many complete years are required until he has doubled his investment?

24 A car suffers from 12% annual depreciation. If the initial value is $40 000, find the value after 4 years.

25 Clint invests \$2000 at 3.2% annual compound interest. He estimates that the annual inflation rate is 2.4%. Find the real value of his investment after 5 years, giving your answer to the nearest dollar.

26 Evaluate $(2^{-2})^{-2}$.

27 Simplify $(2x)^3$.

28 Solve $10^x = k$.

29 If $e^{2x-6} = 5$, find x in terms of natural logarithms.

30 Evaluate $\ln 10 + \log_{10} e$.

31 Round 0.0106 to three decimal places.

32 Round 105 070 to three significant figures.

33 x is measured as 500 000 to one significant figure. y is measured as 0.1235 correct to four significant figures. Calculate xy giving your answer to an appropriate level of accuracy.

34 If $x = 12.45$ to four significant figures, find the range of possible values of x.

35 An angle is 38°. A student estimates the answer as 45°. Find the percentage error in the student's work.

36 The side of a square is 7 cm to one significant figure. Find the maximum percentage error if the area is quoted as 49 cm^2.

37 Jenny calculates the probability of getting five heads when flipping a coin eight times and gets 2.5. Explain why Jenny's answer cannot be correct.

38 Jamie borrows $1000 at 3% interest, compounded annually. She pays back the loan over 5 years. How much should she pay at the end of each year? Give your answer to two decimal places.

39 Kumar invests $100 at the beginning of each year in an account that pays 2.3% interest at the end of the year. What is the value of his investment at the end of 10 years. Give your answer to two decimal places.

40 Heidi has $50 000 she wants to use to pay out an annuity over 30 years. If she invests the money in an account paying 4% annual interest, how much can she withdraw at the end of each year? Give your answer to the nearest dollar.

41 Solve:

$3x + 2y + 4z = -1$

$x + y + z = 0$

$10x + 7y + 4z = 6$

42 Find the roots of the equation $x^3 - 4x^2 + 2x + 1 = 0$.

2 Functions

Syllabus content

S2.1	Equation of a straight line		
	Book Section 4A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Different forms of the equation of a straight line.		Use the gradient-intercept form $y = mx + c$, the general form $ax + by + d = 0$ and the point-gradient form $y - y_1 = m(x - x_1)$. √x	1 ☐
		Find the equation of a line given its gradient and a point on the line.	2 ☐
		Find the equation of a line given two points on the line using the gradient formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ √x	3 ☐
Parallel lines $m_1 = m_2$.		Find the equation of a line through a given point parallel to another line.	4 ☐
Perpendicular lines $m_1 \times m_2 = -1$.		Find the equation of a line through a given point perpendicular to another line.	5 ☐

S2.2	Concept of a function		
	Book Section 3A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Function notation.		Use function notation.	6 ☐
Domain, range and graph.		Find the domain of a function.	7 ☐
		Find the range of a function.	8 ☐
Informal concept that an inverse function reverses or undoes the effect of a function.		Understand that an inverse function reverses the effect of a function.	9 ☐
Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$		Sketch the graph of the inverse function from the graph of the function.	10 ☐

S2.3	Graph of a function		
	Book Section 3B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Creating a sketch from information given or a context.		Sketch a graph from a list of features or from a given context.	11 ☐
Using technology to graph functions.		Sketch the graph of a function from a plot on the GDC.	12 ☐

S2.4	Key features of graphs		
	Book Section 3B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Determine key features of graphs.		Use your GDC to find vertices (maximum and minimum values) and lines of symmetry.	13 ☐
		Use your GDC to find vertical and horizontal asymptotes.	14 ☐
		Use your GDC to find zeros of functions or roots of equations.	15 ☐
Finding the point of intersection of two curves or lines using technology.		Use your GDC to find intersections of graphs.	16 ☐

S2.5a	Linear models		
	Book Section 13A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Linear models: $f(x) = mx + c$.		Form a linear model from given data.	17 ☐
		Form a piecewise linear model from two or more line segments.	18 ☐

S2.5b	Quadratic models	
	Book Section 13B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Quadratic models: $f(x) = ax^2 + bx + c$.	Form a quadratic model from given data.	19 ☐
Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis.	Find the axis of symmetry, vertex, zeros and y-intercept of the graph of a quadratic model. √x $f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$	20 ☐

S2.5c	Exponential models	
	Book Section 13C	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Exponential growth and decay models: $f(x) = ka^x + c$, $f(x) = ka^{-x} + c$, $f(x) = ke^{rx} + c$	Form an exponential growth/decay model from given data.	21 ☐
Equation of a horizontal asymptote.	Find the equation of the horizontal asymptote of the graph of an exponential model.	22 ☐

S2.5d	Direct/inverse variation and cubic models	
	Book Section 13D	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Direct/inverse variation: $f(x) = ax^n$.	Find a direct/inverse relationship from given data.	23 ☐
The y-axis as a vertical asymptote when $n < 0$.	Sketch the graph of a function of the form $y = \frac{a}{x^n}$.	24 ☐
Cubic models: $f(x) = ax^3 + bx^2 + cx + d$.	Form a cubic model from given data.	25 ☐

S2.5e	Sinusoidal models	
	Book Section 13E	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Sinusoidal models: $f(x) = a\sin(bx) + d$, $f(x) = a\cos(bx) + d$.	Find the amplitude, period and principal axis of a sinusoidal model of the form $f(x) = a\sin(bx) + d$ or $f(x) = a\cos(bx) + d$.	26 ☐

S2.6	Modelling skills	
	Book Section 13F	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Develop and fit the model.	Select an appropriate model based on the shape of the graph or context of the situation.	27 ☐
	Find the parameters of the chosen model from the given data.	28 ☐
	Determine a reasonable domain for a model.	29 ☐
Use the model.	Use a model to make predictions.	30 ☐
Test and reflect upon the model.	Comment on the appropriateness and reasonableness of a model.	31 ☐
	Suggest improvements to a model.	32 ☐

Practice questions

1 Find the gradient and y-intercept of the line $3x - 4y - 5 = 0$.

2 Find the equation of the line with gradient -3 passing through the point $(2, -4)$. Give your answer in the form $y = mx + c$.

3 Find the equation of the line passing through the points $(-3, -5)$ and $(9, 1)$. Give your answer in the form $ax + by + d = 0$, where a, b, d are integers.

4 Find the equation of the line through the point $(1, 4)$ parallel to the line $y = 2x - 7$.

5 Find the equation of the line through the point $(-2, 3)$ perpendicular to the line $y = -\frac{1}{4}x + 1$.

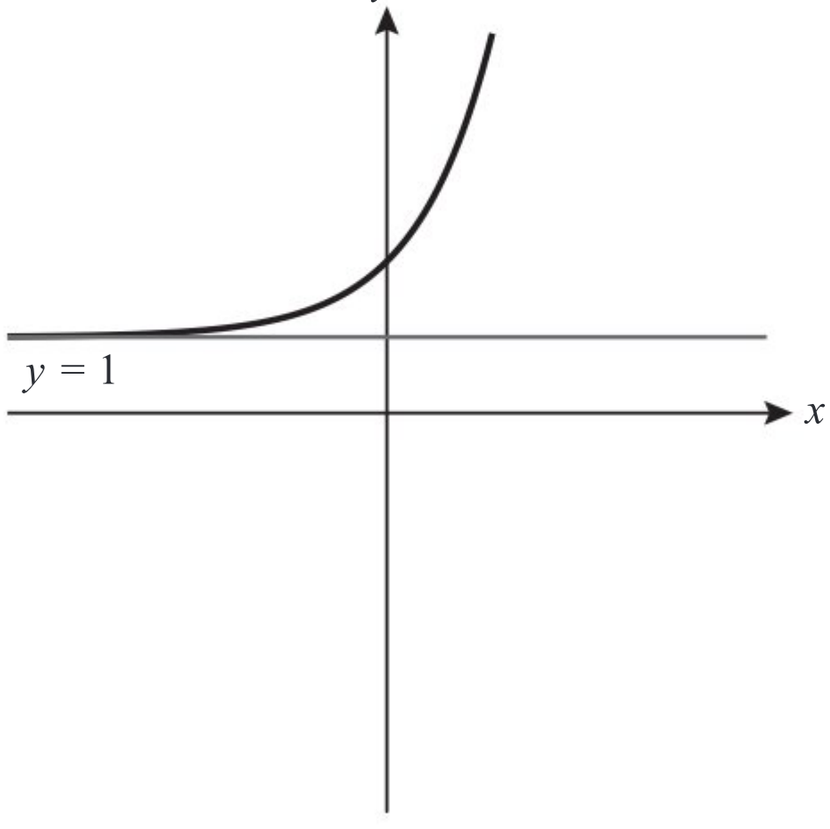
6 If $f(x) = 3x^2 - 4$, find $f(-2)$.

7 Find the largest possible domain of the function $f(x) = \ln(2x - 1)$.

8 Find the range of the function $f(x) = \sqrt{5 - x}$, $x \leqslant 1$.

9 If $f(x) = 4 - 3x$, find $f^{-1}(-8)$.

10 Sketch the inverse function of the following graph.



11 The graph of $y = f(x)$ has zeros at -1 and 3 and no vertices. It has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = -2$.
The range of f is $f(x) > -2$.
Sketch a graph with these properties.

12 Sketch the graph of $y = x^5 - x^4 + 6x^2 - 2$, labelling the y-intercept.

13 a Find the coordinates of the vertices of $y = x^4 + 4x^3 - 3x^2 - 14x - 8$.

b Given that the curve has a line of symmetry, find its equation.

14 Find the equations of all vertical and horizontal asymptotes of the function $f(x) = \frac{x^2}{x^2 + x - 6}$.

15 Find the zeros of the function $f(x) = \frac{3}{\sqrt{x}} + 2x - 6$.

16 Find the points of intersection of $y = 3^x$ and $y = 3x + 2$.

17 Find a linear model, $f(x) = mx + c$, that satisfies $f(-5) = 10$ and $f(1) = -8$.

18 $f(x) = \begin{cases} 0.5x + 11, & 0 \leqslant x \leqslant 10 \\ 2x - 4, & x > 10 \end{cases}$

Find

a f(3)

b f(11).

19 Find a quadratic model, $f(x) = ax^2 + bx + c$, that satisfies $f(1) = -3$, $f(2) = 4$ and $f(3) = 17$.

20 Find a quadratic model with the following properties:

- vertex at $x = -2$
- y-intercept at $y = 9$
- passes through (1, 19).

21 Find an exponential model of the form $f(x) = k \times 3^{-x} + c$, that satisfies $f(-2) = 38$ and $f(1) = 14$.

22 $f(x) = 8e^{-0.5x} + 3$. Sketch the graph of $y = f(x)$ labelling the axis intercept and the asymptote.

23 y is inversely proportional to the square of x. Given that $y = 3$ when $x = 2$, find the relationship between x and y.

24 Sketch the graph of $y = \frac{12}{x^2}$, stating the equations of all asymptotes.

25 Find a cubic model, $f(x) = ax^3 + bx^2 + cx + d$, that satisfies $f(-2) = 1$, $f(-1) = 7$, $f(1) = 1$ and $f(2) = 13$.

26 Find a model of the form $f(x) = a \sin(bx) + d$ with the following properties:

- amplitude 4
- period 240°
- principal axis $y = -1$.

27 The following data are collected:

x	0	1	2	3	4
y	5	2.5	1.25	0.625	0.3125

Determine which one of the following models is most appropriate for the data:

$$y = ax + b$$

$$y = ax^2 + bx + c$$

$$y = \frac{a}{x}$$

$$y = k \times 2^{rx}$$

28 Find the values of the parameters in your chosen model from question **27**.

29 A publisher predicts that the percentage market share it can gain for a newly published book (s%) is given by the function $s(x) = 3x + 4$, where x (in thousands of \$) is the amount spent on marketing. Determine a suitable domain for this model.

30 From the function in question **29**, find the predicted market share when \$18 000 is spent on marketing.

31 State two reasons why the model in question **29** is unrealistic.

32 Suggest an improved model to that given in question **29** (only the form is needed; not the precise model).

3 Geometry and trigonometry

Syllabus content

S3.1a	Distance and midpoints		
	Book Section 4B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
The distance between two points in three-dimensional space, and their midpoint.		Find the distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) using √x $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$	1 ☐
		Find the midpoint using √x $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$	2 ☐

S3.1b	Volume and surface area of 3D solids		
	Book Section 5A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Volume and area of three-dimensional solids.		Find the volume and surface area of a sphere using √x $V = \frac{4}{3}\pi r^3$ √x $A = 4\pi r^2$ where r is the radius.	3 ☐
		Find the volume and curved surface area of a right cone using √x $V = \frac{1}{3}\pi r^2 h$ √x $A = \pi r l$ where r is the radius, h is the height and l is the slant height.	4 ☐
		Find the volume and surface area of a right pyramid using √x $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height.	5 ☐
		Find the volume and surface area of combinations of solids.	6 ☐

S3.1c	Angle between intersecting lines and planes		
	Book Section 5B, 5C	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
The size of an angle between two intersecting lines or between a line and a plane.		Find the angle between two lines in two dimensions.	7 ☐
		Find the angle between a line and a plane.	8 ☐
		Find the angle between two intersecting lines in three dimensions.	9 ☐

S3.2a	Trigonometry in right-angled triangles		
	Book Section 5B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.		Find lengths and angles in right-angled triangles using the sine, cosine and tangent ratios.	10 ☐

S3.2b	Trigonometry in non-right-angled triangles		
	Book Section 5B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
The sine rule: √x $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.		Find lengths and angles in non-right-angled triangles using the sine rule.	11 ☐
The cosine rule: √x $c^2 = a^2 + b^2 - 2ab\cos C$ √x $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$.		Find lengths and angles in non-right-angled triangles using the cosine rule.	12 ☐
Area of a triangle as $\frac{1}{2}ab\sin C$.		Find the area of a triangle when you do not know the perpendicular height.	13 ☐

S3.3	Applications of trigonometry	
	Book Section 5C	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Angles of elevation and depression.	Use trigonometry in questions involving angles of elevation and depression.	14 ☐
Construction of labelled diagrams from written statements.	Construct diagrams from given information (often involving bearings) and solve using trigonometry.	15 ☐

S3.4	Arcs and sectors of circles	
	Book Section 14A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
The circle: length of an arc; area of a sector.	Find the length of an arc, √x $l = \frac{\theta}{360} \times 2\pi r$ where θ is the angle measured in degrees, r is the radius.	16 ☐
	Find the area of a sector, √x $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle measured in degrees, r is the radius.	17 ☐

S3.5	Perpendicular bisectors	
	Book Section 14B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Equations of perpendicular bisectors.	Find the equation of a perpendicular bisector given the equation of a line segment and its midpoint.	18 ☐
	Find the equation of a perpendicular bisector given two points.	19 ☐

S3.6	Voronoi diagrams	
	Book Section 14B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Voronoi diagrams: sites, vertices, edges, cells.	Identify sites, vertices, edges and cells in a Voronoi diagram.	20 ☐
Addition of a site to an existing Voronoi diagram.	Use the incremental algorithm to add a site to an existing Voronoi diagram.	21 ☐
Nearest neighbour interpolation.	Use nearest neighbour interpolation to find the value of a function at any point in a Voronoi diagram.	22 ☐
Applications of the toxic waste dump problem.	Solve the toxic waste dump problem to find the point that is as far as possible from any of the sites in a Voronoi diagram.	23 ☐

Practice questions

1 Find the distance between (2, –4, 5) and (7, 3, –1).

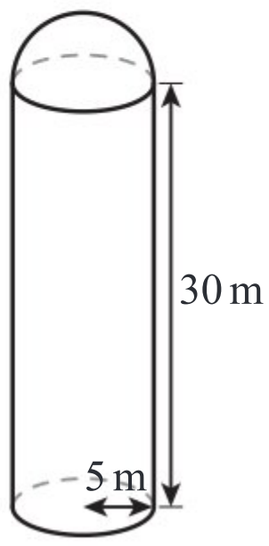
2 Find the midpoint of (1, 8, –3) and (–5, 2, 4).

3 Find, to three significant figures, the volume and surface area of a sphere of diameter 16 cm.

4 Find, to three significant figures, the volume and surface area of a cone with base radius 6 cm and height 15 cm.

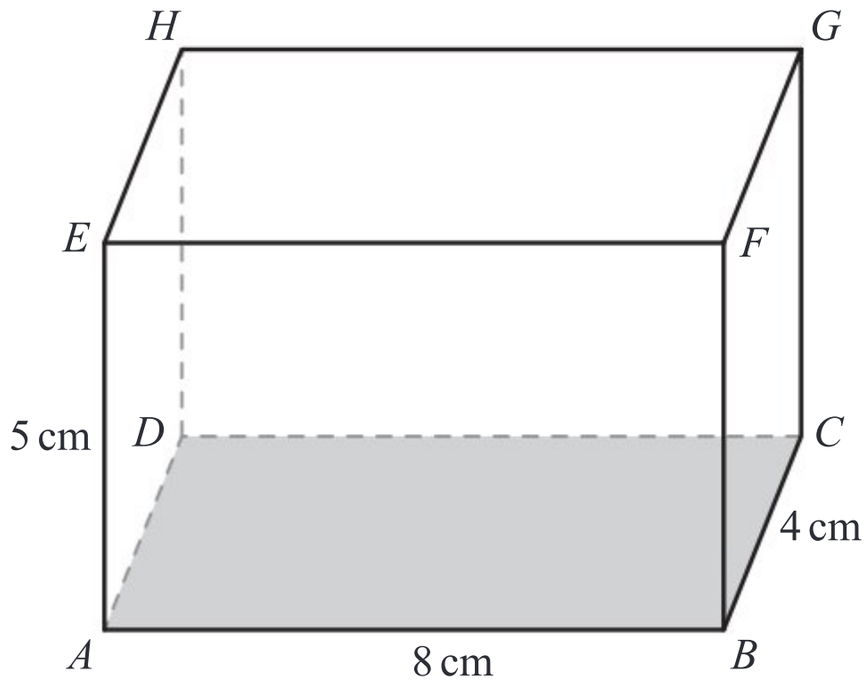
5 Find, to three significant figures, the volume and surface area of a square-based pyramid with base side 5 cm and height 9 cm.

6 A grain silo is formed of a hemisphere on top of a cylinder of radius 5 m and height 30 m as shown. Find the silo's volume.



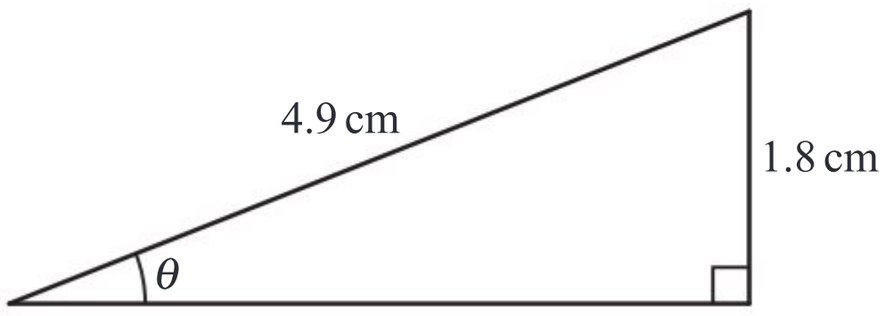
7 Find the acute angle between the lines $y = 4x - 3$ and $y = 5 - 3x$.

8 Find the angle between the line AG and the base plane $ABCD$ in the cuboid below.

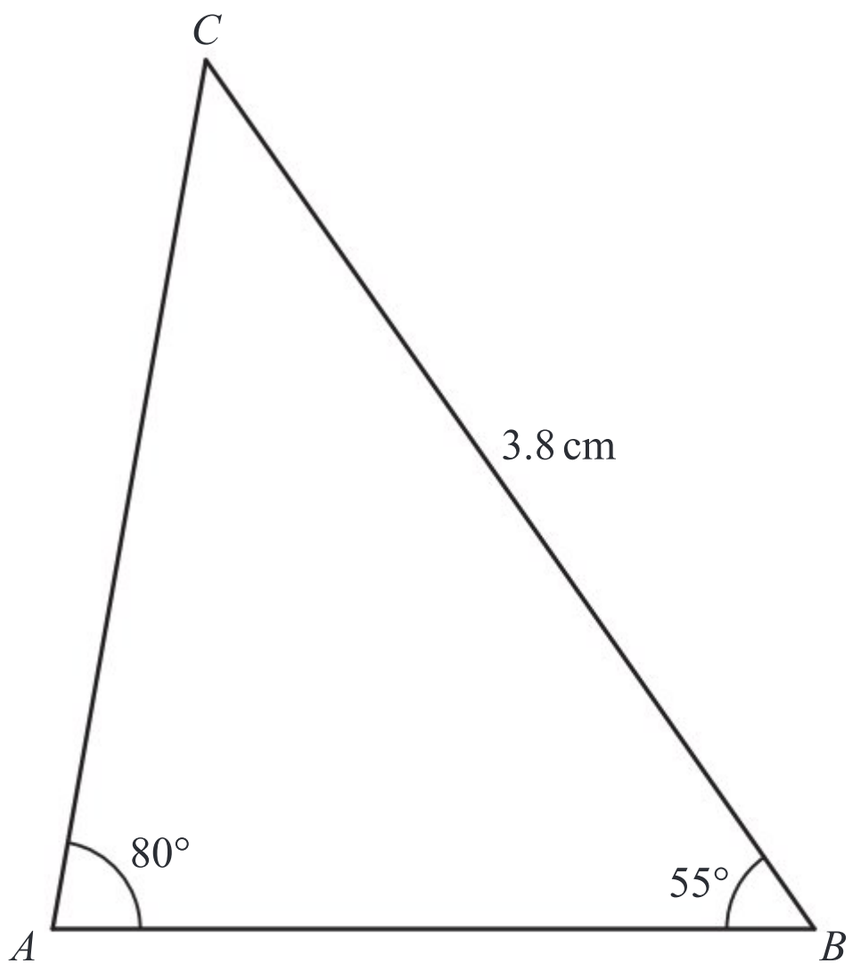


9 Find the acute angle between the diagonals AG and EC in the cuboid from question **8**.

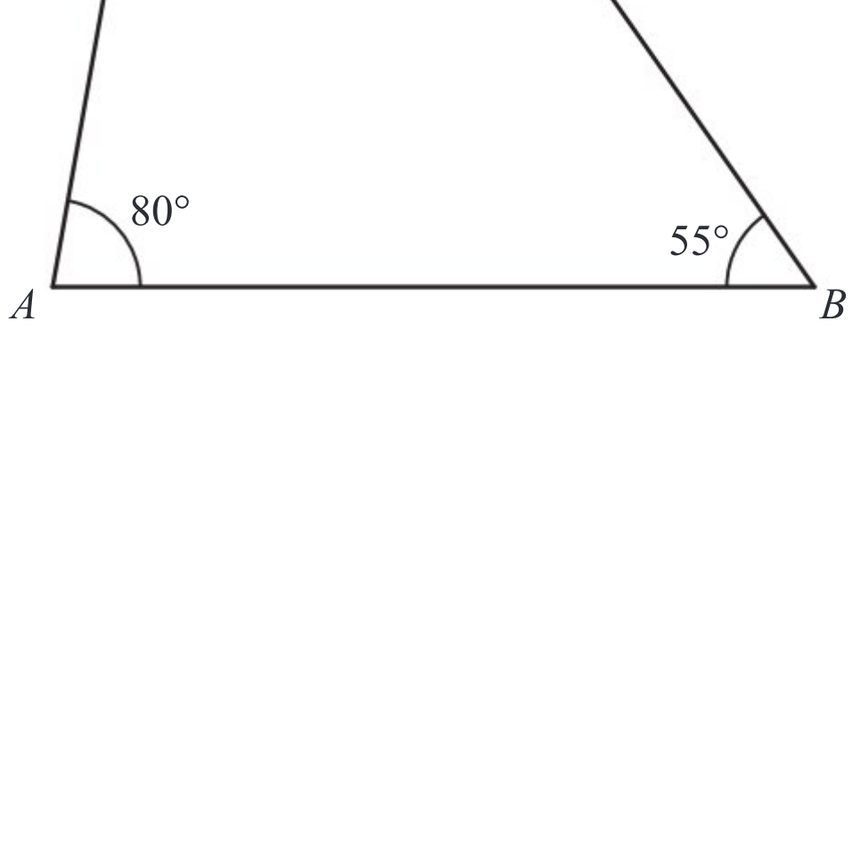
10 Find the angle θ in the following triangle.



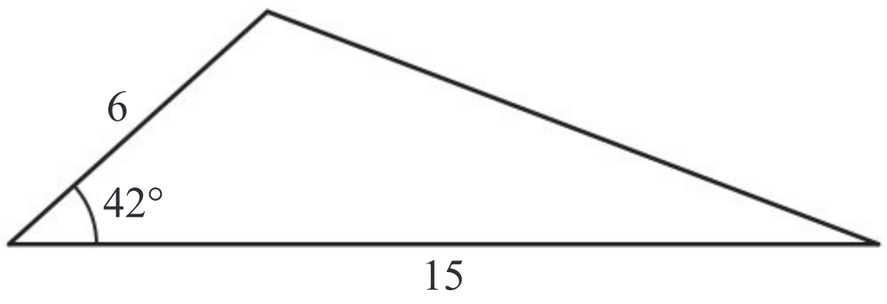
11 Find the length AC in the following triangle.



12 Find the angle C in the following triangle.



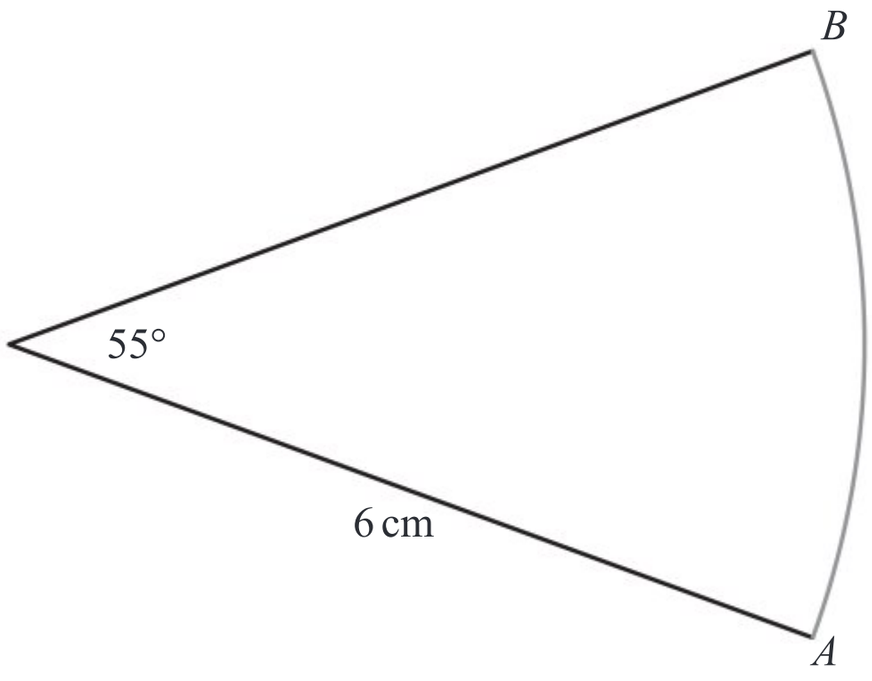
13 Find the area of the following triangle.



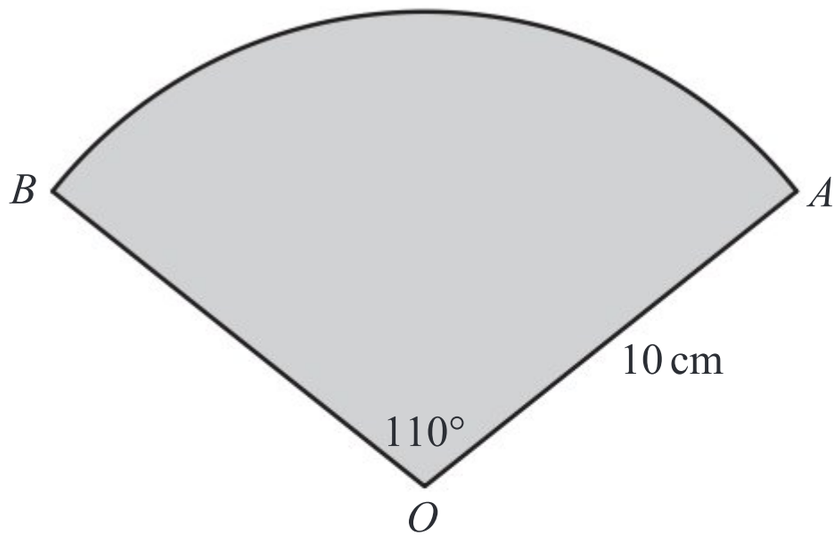
14 The angle of elevation of the top of a tree at a distance of 6.5 m is 68°.
Find the height of the tree.

15 A ship leaves port on a bearing of 030° and travels 150 km before docking. It then travels on a bearing of 110° for 80 km before docking again.
Find the distance it must now travel to return to where it started.

16 Find the length of the arc AB.



17 Find the area of the sector AOB.



18 The line segment AB has equation $2x + 3y = 5$ and midpoint (4, 7).
Find the equation of the perpendicular bisector of AB.

19 Find the equation of the perpendicular bisector of (–3, –2) and (1, 8).

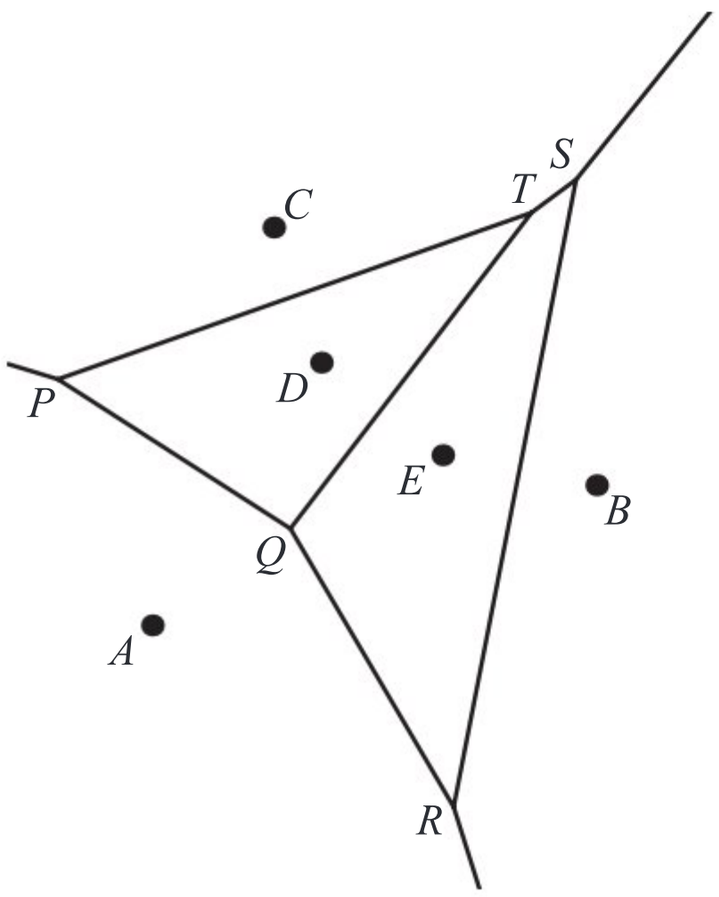
20 For the Voronoi diagram below, identify all the:

a sites

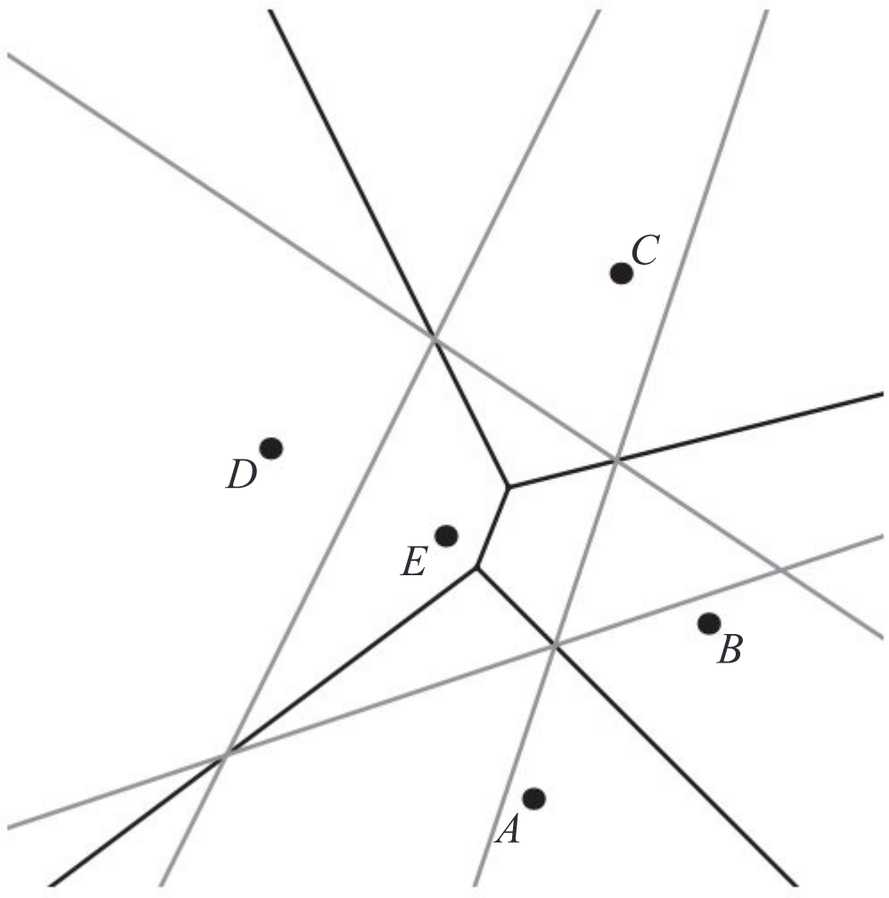
b vertices

c finite edges

d finite cells.



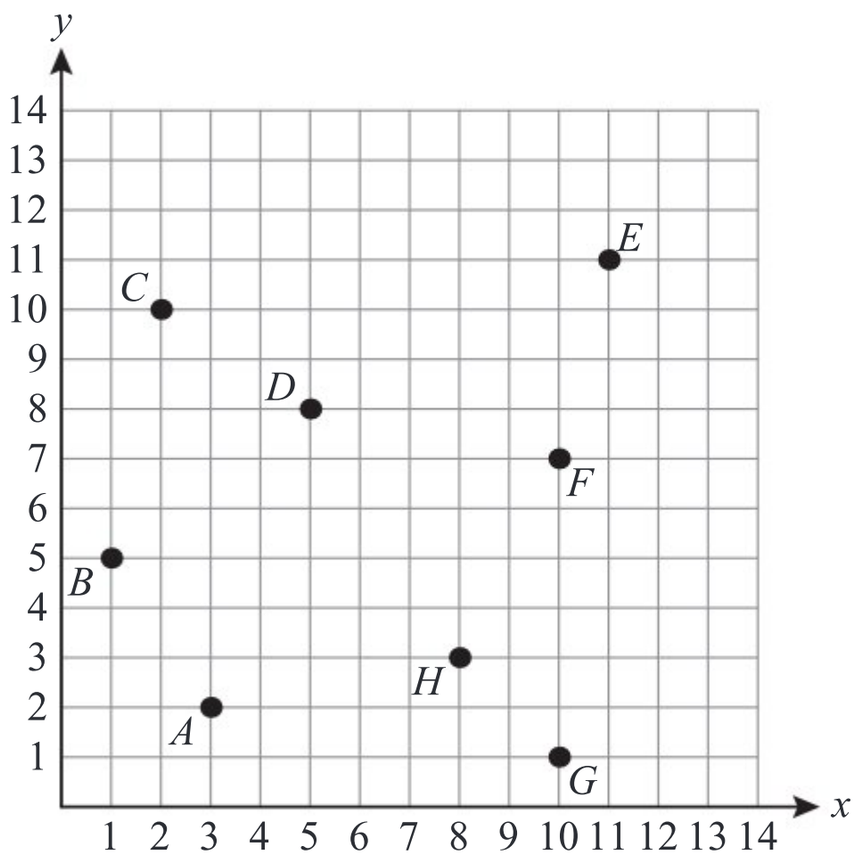
21 The Voronoi diagram for sites A, B, C and D is shown. An additional site E is added and is shown together with the perpendicular bisectors of line segments joining E to the other sites.



Sketch on the above diagram the Voronoi diagram for A, B, C, D and E.

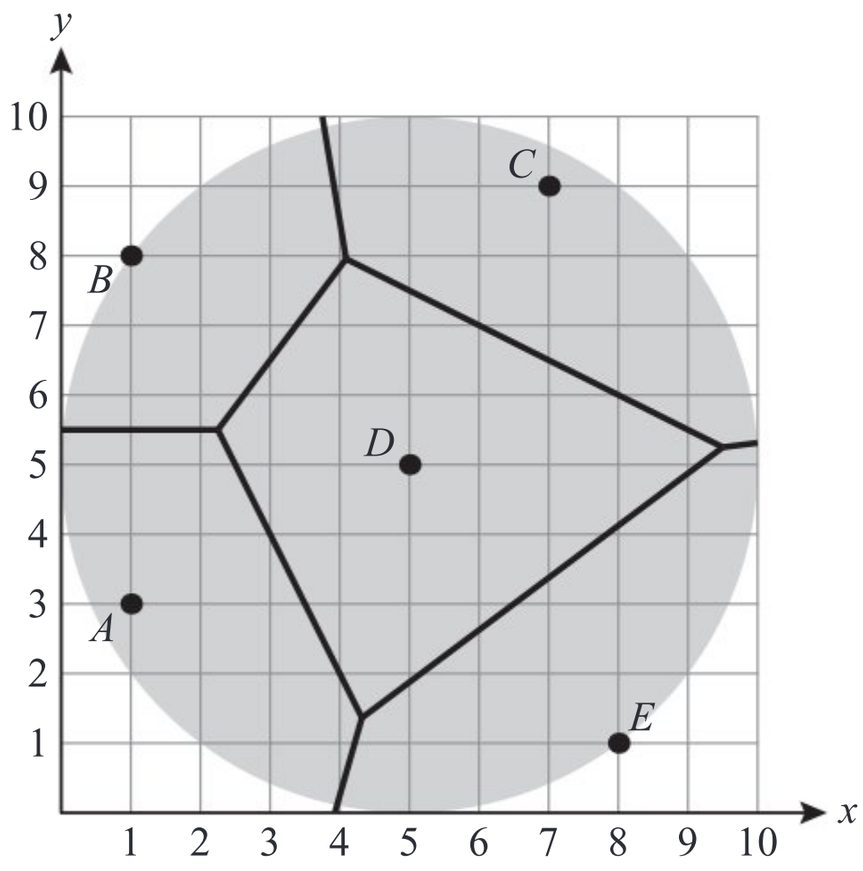
22 The function f has the following values at each site:

Site	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
f	12	10	6	9	11	15	7	13



Use nearest neighbour interpolation to estimate the value of the function at (5,4).

23 Towns are located at the points A, B, C, D and E; one unit on the graph represents 1 mile. The local authorities need to locate a toxic waste dump in this area, within 5 miles of town D but want it to be as far as possible from any of the towns.
Find the coordinates of the required location.



4 Statistics and probability

Syllabus content

S4.1	Sampling	
	Book Section 6A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Concepts of population, sample, random sample, discrete and continuous data.	Identify if data are continuous or discrete.	1 ☐
	Identify in context what the population is, what the sample is and whether it is random.	2 ☐
Reliability of data sources and bias in sampling.	Identify bias in sampling (a tendency for the sample to include more of one type of object).	3 ☐
	Identify reliability of data (strictly the consistency of their results and in a more colloquial sense, how trustworthy they are).	4 ☐
	Deal with missing data or errors in the recording of data.	5 ☐
Interpretation of outliers.	Know that an outlier is defined as more than 1.5 × IQR from the nearest quartile, and be able to suggest how to determine if an outlier should be removed from the sample.	6 ☐
Sampling techniques and their effectiveness.	Be able to identify and evaluate the following sampling techniques: • simple random • convenience • systematic • quota • stratified.	7 ☐
	Calculate the number of data items in each category of a stratified sample.	8 ☐

S4.2	Statistical diagrams	
	Book Section 6C	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Presentation of data: Frequency distributions.	Interpret frequency distribution tables.	9 ☐
Histograms.	Interpret frequency histograms.	10 ☐
Cumulative frequency graphs.	Interpret cumulative frequency graphs, including finding median, quartiles, percentiles, range and interquartile range using √x $IQR = Q_3 - Q_1$	11 ☐
Box and whisker plots.	Produce box and whisker diagrams.	12 ☐
	Interpret box and whisker diagrams, including using them to compare distributions and use their symmetry to determine if a normal distribution is plausible.	13 ☐

S4.3	Summary statistics	
	Book Section 6B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Measures of central tendency.	Calculate the mean, median and mode of data.	14 ☐
	Use the formula for the mean of data √x $\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ where √x $n = \sum_{i=1}^{k} f_i$	15 ☐
Estimation of mean from grouped data.	Use mid-interval values to estimate the mean of grouped data.	16 ☐
Modal class.	Find the modal class for grouped data using tables or histograms.	17 ☐
Measures of dispersion.	Use technology to calculate interquartile range (IQR), standard deviation and variance.	18 ☐
Effect of constant changes on the original data.	Calculate the mean and standard deviation (and other statistics) of the new data set after a constant change.	19 ☐
Quartiles of discrete data.	Use technology to obtain quartiles.	20 ☐

S4.4	Correlation and regression	
	Book Section 6D	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Linear correlation of bivariate data: Pearson's product moment correlation coefficient, r.	Calculate the correlation coefficient of bivariate data using technology, and interpret the result, including being aware that correlation does not imply causation.	21 ☐
Scatter diagrams.	Estimate the line of best fit by eye, knowing that it should pass through the mean point.	22 ☐
Equation of the regression line of y on x.	Calculate the equation of the regression line using technology.	23 ☐
Use of the equation of the regression line for prediction purposes.	Use the regression line while being aware of the dangers of extrapolation. Be aware of when a y-on-x regression line is appropriate.	24 ☐
Interpret the meaning of the parameters, a and b, in a linear regression.	Put the meaning of the parameters into context.	25 ☐
Piecewise linear models.	Create and use piecewise linear models.	26 ☐

S4.5	Definitions in probability	
	Book Section 7A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Concept of trial, outcome, equally likely outcomes, relative frequency, sample space and event.	Estimate probability from observed data.	27 ☐
The probability of an event A is √x $P(A) = \frac{n(A)}{n(U)}$.	Find theoretical probabilities by listing all possibilities.	28 ☐
The complementary events A and A'.	Link the probability of an event occurring and it not occurring √x $P(A) + P(A') = 1$.	29 ☐
Expected number of occurrences.	Calculate how many times an outcome will be observed by multiplying the number of trials and the probability.	30 ☐

S4.6	Probability techniques	
	Book Section 7B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.	Use Venn diagrams to organize information and find probabilities.	31 ☐
	Use tree diagrams to organize information and find probabilities. In tree diagrams you multiply along the branches and add between the branches.	32 ☐
	Use sample space diagrams to organize information and find probabilities.	33 ☐
	Use tables of outcomes to organize information and find probabilities.	34 ☐
Combined events.	Work with the notation $A \cap B$ meaning A and B occurring. Work with the notation $A \cup B$ meaning A or B or both occurring. Use √x $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	35 ☐
Mutually exclusive events.	Know that mutually exclusive means that the two events cannot both occur, so that $P(A \cap B) = 0$. Therefore √x $P(A \cup B) = P(A) + P(B)$.	36 ☐
Conditional probability.	Know that $P(A \mid B)$ means the probability of A given that B has happened. Use Venn diagrams, tree diagrams, sample space diagrams or tables of outcomes to find conditional probabilities.	37 ☐
Independent events.	Know that if two events, A and B, are independent (that is, do not affect each other) then √x $P(A \cap B) = P(A)P(B)$.	38 ☐

S4.7	Discrete random variables		
	Book Section 8A	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Concept of discrete random variables and their distribution.		Create probability distributions from context.	39 ☐
		Use the fact that the total probability in a probability distribution equals 1.	40 ☐
Expected value (mean) for discrete data.		Use $E(X) = \sum xP(X = x)$.	41 ☐
Applications.		Use probability distributions to answer questions in context.	42 ☐
		Know that $E(X) = 0$ indicates a fair game if X represents the gain of a player.	43 ☐

S4.8	Binomial distribution		
	Book Section 8B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Binomial distribution.		Recognize that if a situation has • a fixed number of trials • outcomes that can be classified into two, 'successes' and 'failures' • fixed probability of being in each group • independent trials then the number of successes follows a binomial distribution.	44 ☐
		Use technology to calculate binomial probabilities.	45 ☐
Mean and variance of the binomial distribution.		Use $E(X) = np$ $Var(X) = np(1 - p)$ where X is the number of successes when there are n binomial trials each with a probability p of success.	46 ☐

S4.9	Normal distribution		
	Book Section 8C	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
The normal distribution and curve; properties of the normal distribution.		Recognize that many natural situations are well modelled by a normal distribution. One way to validate this is to use the fact that about 68% of normally distributed data should fall within one standard deviation of the mean, about 95% within two standard deviations and about 99.7% within three standard deviations.	47 ☐
Diagrammatic representation.		Recognize that a normal distribution can be represented by a symmetric bell-shaped curve with area representing probability.	48 ☐
Normal probability calculations.		For a given mean and standard deviation, find the probability of a random variable falling in a given interval.	49 ☐
Inverse normal calculations.		For a given probability, find the boundary of the region it describes.	50 ☐

S4.10	Spearman's rank correlation coefficient		
	Book Section 15C	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Spearman's rank correlation coefficient, r_s.		Calculate Spearman's rank using technology.	51 ☐
		Average rank of equally ranked items.	52 ☐
Awareness of the appropriateness and limitations of Pearson's and Spearman's correlation coefficients.		Choose an appropriate correlation coefficient, justifying your choice (Pearson's when testing for linearity, Spearman's for any monotonic relationship).	53 ☐
		Understand that Spearman's is less sensitive to outliers than Pearson's.	54 ☐

S4.11	Hypothesis testing		
	Book Section 15A, 15B	Revised ☐	
Syllabus wording		*You need to be able to:*	*Question*
Null and alternative hypotheses, significance levels and p-values.		Write down appropriate hypotheses for a given situation.	55 ☐
Expected and observed frequencies.		Find expected frequencies when a given ratio is expected.	56 ☐
		Find expected frequencies for a binomial distribution.	57 ☐
		Find expected frequencies for a normal distribution.	58 ☐
χ^2 test for independence: contingency tables; degrees of freedom; critical value.		Use technology to find the p-value and the χ^2 statistic, including determining the degrees of freedom.	59 ☐
χ^2 test for goodness of fit.		Use technology to find the p-value and the χ^2 statistic, including determining the number of degrees of freedom.	60 ☐
		Use a given critical value to determine a conclusion.	61 ☐
The t-test		Use a t-test to determine if a population mean has changed from a prior belief using a given sample.	62 ☐
		Use a t-test to determine if a population mean has changed from a prior belief using summary statistics.	63 ☐
Use of the p-value to compare the means of two populations using one-tailed and two-tailed tests.		Use technology to find the p-value (using pooled two-sample t-test) and interpret the result of the test.	64 ☐
		Distinguish between one-tail and two-tail tests.	65
		Understand that a t-test is only valid if the underlying distributions are normal.	66 ☐

Practice questions

1 Determine whether each of the following variables is continuous or discrete.

a Number of people in a family.

b Time for a nucleus to decay.

c Age in complete years.

2 A doctor wants to find out whether exercise can lower the incidence of illness. He asks patients who come to his clinic to fill in a survey about their exercise habits; 20% of them agree to do this.

a Suggest a possible population that the doctor is interested in.

b Is his sample random?

3 Is the sampling in question **2** likely to be biased? Justify your answer.

4 Five independent groups of people were asked to estimate the length of an arrow that is 5 cm long. The average for the groups was 4.6 cm, 4.6 cm, 4.7 cm, 4.8 cm, 4.8 cm. Does this suggest that the results are reliable?

5 Five people were asked to record their height in metres:

A: 1.83 B: 1.45 C: 1.77 D: 5.10 E: 1.60

Suggest which data item is an error. What should be done with this item?

6 A data set has lower quartile 7 and upper quartile 11. Explain why 18 should be considered an outlier and suggest how to determine if it should be excluded from the data.

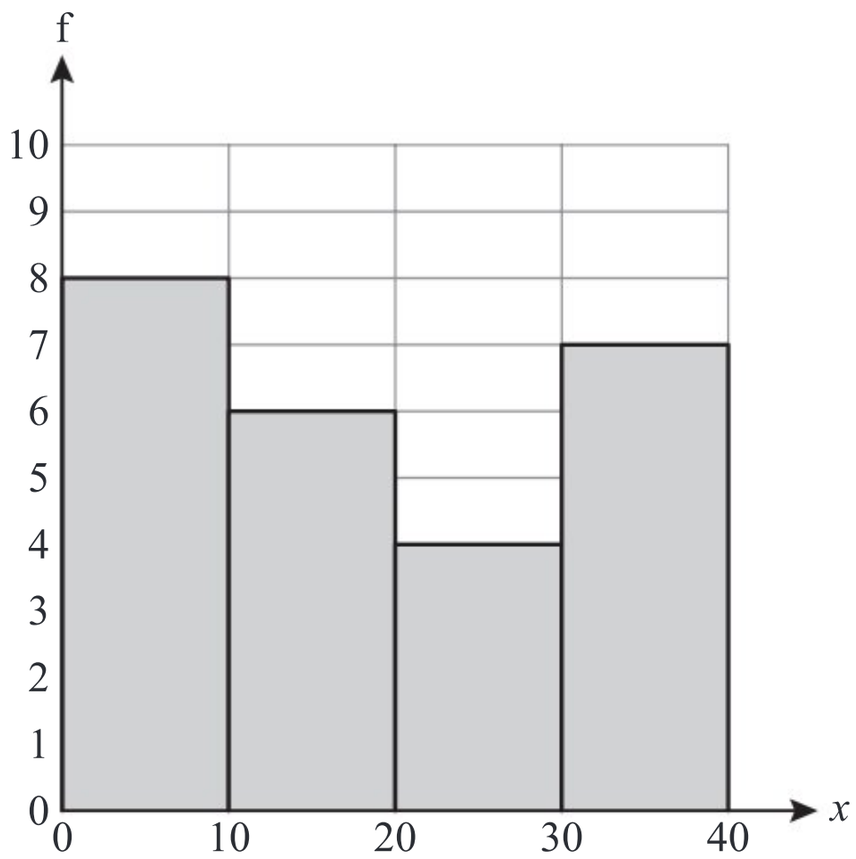
7 Write down the sampling method used by the doctor in question **2**.

8 A language school consists of students from either Italy or Spain. There are 60 from Italy and 90 from Spain. In a stratified sample of 20 students, how many should be from Italy?

9 For the following frequency table, find the proportion of data items above 20.

x	$0 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$
Frequency	15	18	12

10 For the following histogram, estimate the number of data items above 25:

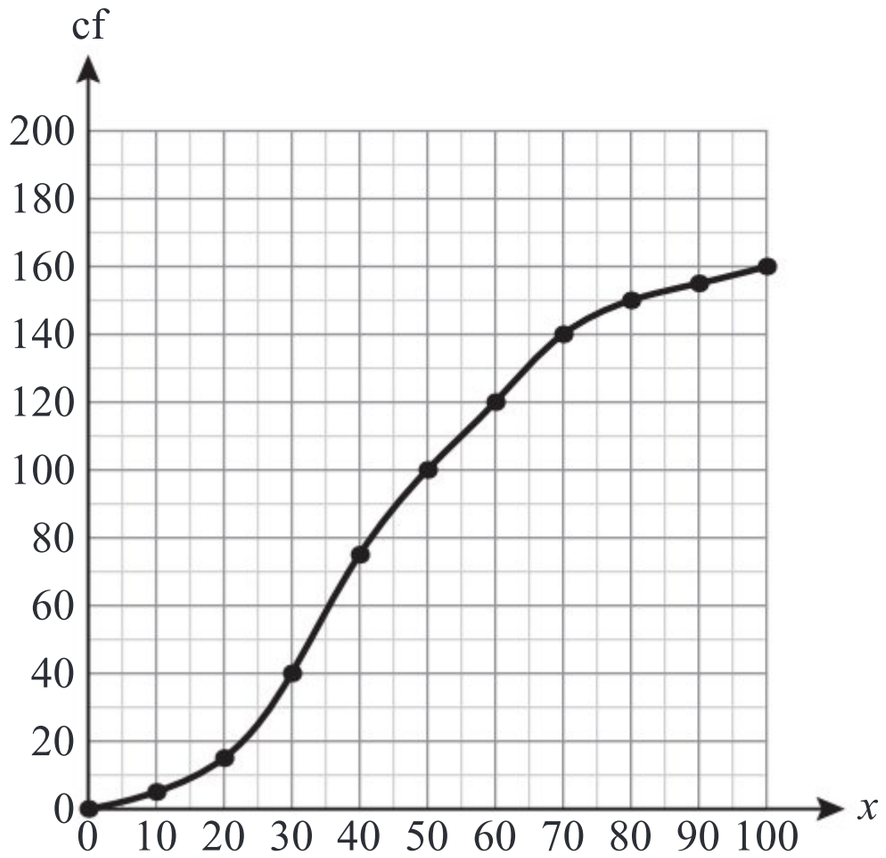


11 For the following cumulative frequency graph find

a the median

b the interquartile range

c the 90th percentile.

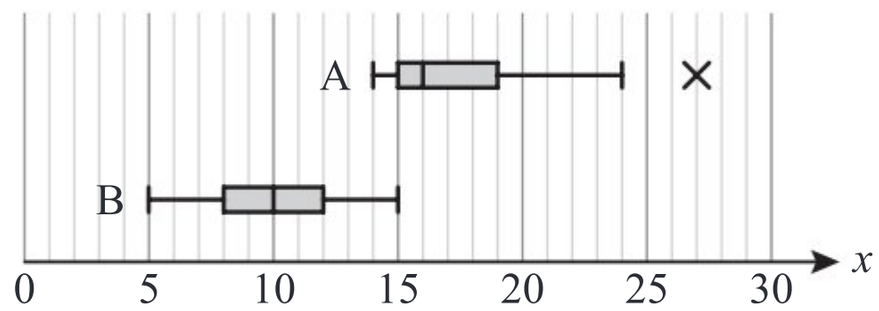


12 Sketch a box and whisker plot for the sample below:
12, 13, 15, 16, 16, 18, 18, 19, 20.

13 For the following box and whisker plots:

a Compare the two distributions.

b Determine, with justification, which of the two distributions is more likely to be a normal distribution.



14 Write down the mean, median and mode of the following data:
14, 14, 16, 17, 19, 20, 23, 25.

15 The numbers 4, 8, 2, 9 and x have a mean of 7. Find the value of x.

16 a Estimate the mean of the following grouped data.

x	$10 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 50$	$50 < x \leqslant 60$
Frequency	10	12	15	13

b Explain why it is only an estimate.

17 Find the modal class for the data below:

x	$0 < x \leqslant 5$	$5 < x \leqslant 10$	$10 < x \leqslant 15$	$15 < x \leqslant 20$
Frequency	16	12	15	18

18 For the data set 6, 7, 9, 12, 14, 18, 22, find

a the interquartile range

b the standard deviation

c the variance.

19 A set of data has mean 12 and standard deviation 10. Every item in the data set is doubled, then 4 is added on. Find the mean and standard deviation of the new data set.

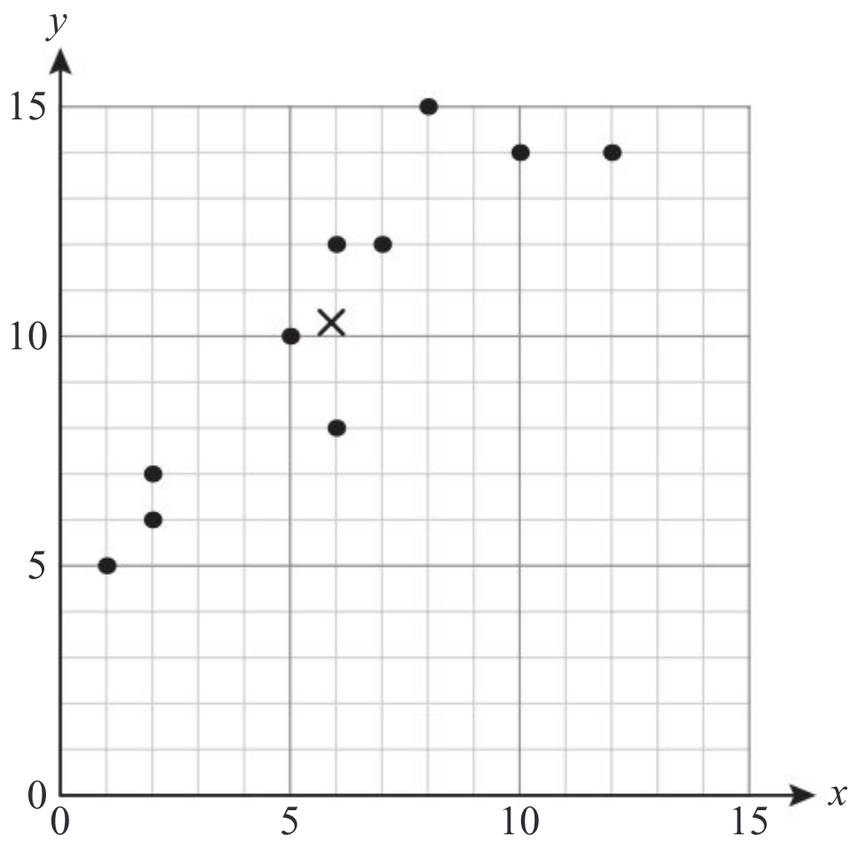
20 Find the quartiles of the following data:
17, 15, 23, 29, 15, 22, 28, 30.

21 a Calculate Pearson's product moment correlation coefficient for the following data:

x	2	4	4	7
y	1	3	6	8

b Interpret your result.

22 The following diagram shows a set of 10 data items with the mean point labelled with a cross.



a Sketch a line of best fit on the diagram.

b Hence estimate y when $x = 3$.

23 Find the y-on-x regression line for the following data, in which y is dependent on x.

x	1	2	2	5	6	6	7	8	10	12
y	5	6	7	10	12	8	12	15	14	14

24 a Use your answer to question **23** to estimate:

i y when $x = 9$

ii y when $x = 20$

iii x when $y = 10$.

b Which of the predictions made in part **a** is valid. Justify your answer.

25 A social scientist investigates how the number of text messages sent by pupils each day (y) depends on the number of hours they spend on social media each day (x). He finds the regression line $y = 6.7 + 1.4x$.

Interpret what each of the following numbers mean in context.

a 6.7

b 1.4

26 A veterinary researcher believes that the growth of a breed of snake is very different during their first 6 months compared to their next 6 months.

She collects the following data showing the length (L cm) and age (A months) of a sample of snakes.

A	1	2	4	4	7	7	10	11	12
L	4	8	15	18	30	32	34	36	34

a Create a piecewise linear model to reflect the researcher's belief.

b Use your answer to part **a** to estimate the length of a 3-month-old snake of this breed.

27 A coin is flipped 200 times and 134 heads are observed. Estimate the probability of observing a head when the coin is flipped.

28 Find the probability of rolling a prime number on a fair six-sided dice.

29 If $P(A) = 0.6$, find $P(A')$.

30 If there are 30 pupils in a class and the probability of a student being absent is 0.05, find the expected number of absent pupils.

31 In a class of 30 students, 14 study French, 18 study Spanish and 4 study both languages. Find the probability that a randomly chosen student studies neither French nor Spanish.

32 A drawer contains three white socks and five black socks. Two socks are drawn without replacement.

a Find P(2nd sock is black|1st sock is white).

b Find the probability that the socks are different colours.

33 A fair four-sided dice is thrown twice.

a What is the probability that the total score is greater than 5?

b If the total score is greater than 5, what is the probability that it is 7?

34 One hundred students were asked whether they preferred soccer or cricket. They were also asked if they prefer mathematics or art. The results are summarized in the table:

	Soccer	**Cricket**
Mathematics	40	20
Art	30	x

a Find the value of x.

b Find the probability that a randomly chosen student prefers mathematics to art.

35 If $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$, find $P(A \cup B)$.

36 Events A and B are mutually exclusive. If $P(A) = 0.4$ and $P(B) = 0.2$, find $P(A \cup B)$.

37 For the sample in question **34**, determine the probability that a randomly chosen person who prefers soccer also prefers mathematics.

38 Independent events A and B are such that $P(A) = 0.4$ and $P(B) = 0.6$. Find $P(A \cap B)$.

39 A drawer contains three white socks and four black socks. Two socks are drawn at random without replacement. Find the probability distribution of W, the number of white socks drawn.

40 The random variable X can take values 0, 1 or 2 with probability $P(X = x) = k(x + 1)$. Find the value of k.

41 For the distribution given below, find $E(X)$.

x	0.5	1	2.5
$P(X = x)$	0.5	0.4	0.1

42 The value of prizes ($\$X$) won by an individual each month in a prize draw is shown in the table.

X	0	10	2000
$\mathbf{P}(X = x)$	0.9	0.095	0.005

a Given that an individual wins a prize, find the probability that it is \$2000.

b Find the probability of winning more than the expected amount.

43 The gain, $\$X$, of a player in a game of chaºnce follows the distribution shown below.

X	-1	0	k
$\mathbf{P}(X = x)$	0.6	0.3	0.1

Find the value of k that would make the game fair.

44 A drawer contains 5 black socks and 10 red socks. Four socks are drawn at random without replacement. Explain why the number of black socks drawn does not follow a binomial distribution.

45 If X is a random variable following a binomial distribution with five trials and a probability of success of 0.4, find

a $P(X = 2)$

b $P(X \geqslant 3)$.

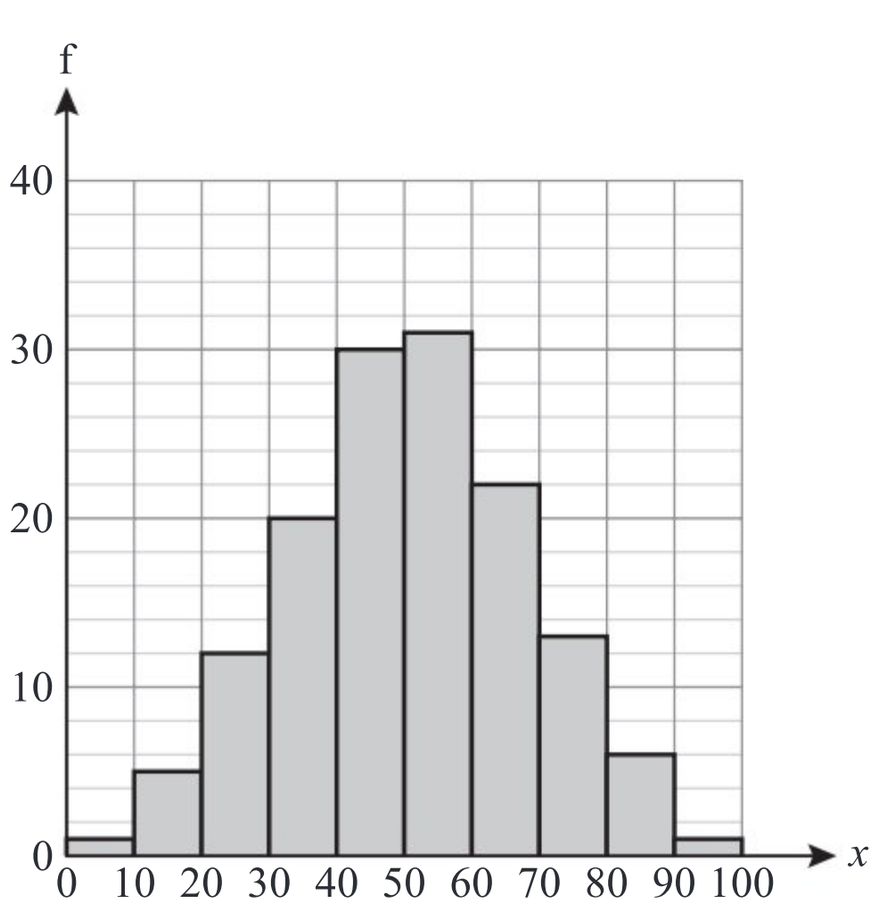
46 A biased coin has a probability of 0.6 of showing a head. It is flipped 10 times.
If this experiment is repeated many times:

a Find the expected mean number of heads.

b Find the expected standard deviation in the number of heads.

47 The time for a child to learn a new dance move is found to have a mean of 2 weeks and a standard deviation of 4 weeks. Explain why this variable is unlikely to be modelled by a normal distribution.

48 The following histogram shows the results of an experiment.



a What feature of this graph suggests a normal distribution might be a good model for the outcome of the experiment?

b Visually estimate the mean of the distribution.

49 A random normal variable has mean 12 and standard deviation 2. Find the probability that an observation is between 11 and 15.

50 A random normal variable has mean 100 and standard deviation 15. The probability of being above k is 0.7. Find the value of k.

51 Calculate the Spearman's rank correlation coefficient for the following data set:

x	1	4	3	8
y	2	–1	0	3

52 Calculate the Spearman's rank correlation coefficient for the following data set:

x	0	5	5	7
y	4	4	7	4

53 A scientist wants to test to see if as x increases, y also tends to increase. Would r or r_s be a more appropriate correlation statistic to use?

54 A scientist is concerned that her data include outliers. Should she use r or r_s to test for possible correlation?

55 James believes that the mean of a population is 0.4. Write down appropriate null and alternative hypotheses if he wants to determine if the mean has

a changed

b increased.

56 A geneticist believes that the ratio of blond to brown to black hair should be 1:4:3. He collects a sample of size 100. Find the expected values of each hair colour.

57 Thirty-five experiments are conducted. The outcome of each experiment is a number that is believed to be taken from a binomial distribution with two trials and a probability 0.4 of success. Find the expected frequencies of the experimental outcomes.

58 The following data were observed:

x	$100 \leqslant x < 120$	$120 \leqslant x < 130$	$130 \leqslant x < 140$	$140 \leqslant x < 150$
Frequency	30	35	20	5

Find the expected frequencies if the data are drawn from a normal distribution with mean 125 and standard deviation 10.

59 The following data were collected for variables X and Y:

		Variable X		
		A	B	C
Variable Y	D	12	24	16
	E	32	15	18

a Assuming that variables X and Y are independent, find the expected values.

b Conduct a hypothesis test at the 5% significance level to see if the two variables are dependent.

60 The following table shows the observed frequencies from an experiment.

Outcome	A	B	C	D
Observed frequency	10	15	12	13

Is there evidence, at the 5% significance level, that the outcomes are not all equally likely?

61 The outcome of a chi-squared goodness of fit test is 15.3. A table of critical values says that the appropriate critical value is 14.07. Is there significant evidence that the observed frequencies differ from the expected frequencies? Justify your answer.

62 Does the following sample suggest, at the 5% significance level, that the population mean is bigger than 10?
14, 9, 12, 11, 15

63 Kwami wants to test if the mean length of newts is different from the 13 cm he read in a textbook. Based on a sample of 12 newts he finds that $\bar{x} = 11.8$ and $s_{n-1} = 1.2$. Determine if there is evidence, at the 5% significance level, of a difference from the textbook value.

64 Determine whether there is evidence at the 10% significance level that the two groups below are drawn from populations with different means.

Group A	13	18	19	
Group B	8	15	20	20

65 For the following data determine at the 5% significance level, if group A is drawn from a population with a larger mean than group B.

Group A	23	16	19
Group B	11	16	14

66 State one distributional assumption required when using a *t*-test.

5 Calculus

Syllabus content

S5.1	The concepts of a limit and derivative	
	Book Section 9A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Introduction to the concept of a limit.	Estimate the value of a limit from a table.	1 ☐
	Estimate the value of a limit from a graph.	2 ☐
Derivative interpreted as gradient function and as rate of change.	Understand and use the notation for derivatives: $\frac{dy}{dx}$ and $f'(x)$.	3 ☐
	Interpret the derivative as a rate of change.	4 ☐
	Interpret the derivative as a gradient function.	5 ☐
	Estimate the gradient at a point as a limit of gradients of chords.	6 ☐

S5.2	Increasing and decreasing functions	
	Book Section 9B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$.	Identify intervals on which a function is increasing ($f'(x) > 0$) and decreasing ($f'(x) < 0$).	7 ☐
	Sketch the graph of the derivative from the graph of a function.	8 ☐
	Sketch the graph of a function from the graph of its derivative.	9 ☐

S5.3	Derivatives of polynomials	
	Book Section 9C	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
The derivative of the functions of the form $f(x) = ax^n + bx^{n-1} + \cdots$ where all exponents are integers.	Apply the rule to differentiate polynomials using √x $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$	10 ☐
	Rearrange an expression into the form $f(x) = ax^n + bx^{n-1} + \cdots$ before differentiating.	11 ☐

S5.4	Equations of tangents and normals	
	Book Section 9D	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Tangents and normals at a given point, and their equations.	Evaluate the gradient at a given point.	12 ☐
	Find the point on the curve with a given gradient.	13 ☐
	Find the equation of the tangent to the curve $y = f(x)$ at the point (x_1, y_1) using $y - y_1 = m(x - x_1)$ where $y_1 = f(x_1)$ and $m = f'(x_1)$.	14 ☐
	Find the equation of the normal to the curve using $y - y_1 = -\frac{1}{m}(x - x_1)$	15 ☐
	Solve problems involving tangents and normals.	16 ☐
	Use technology to find the gradient and the equation of the tangent at a given point.	17 ☐
	Use technology to draw the graph of the gradient function.	18 ☐

S5.5	Introduction to integration	
	Book Section 10A, 10B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \cdots$, where $n \in \mathbb{Z}$, $n \neq -1$.	Use √x $\int ax^n \, dx = \frac{a}{n+1} x^{n+1} + c$, for $n \neq -1$.	19 ☐
	Rearrange an expression into the form $f(x) = ax^n + bx^{n-1} + \cdots$ before integrating.	20 ☐
Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.	Use technology to evaluate integrals of the form √x $\int_a^b f(x) \, dx$, and interpret this as the area between the curve and the x-axis.	21 ☐
Anti-differentiation with a boundary condition to determine the constant term.	Find the expression for y in terms of x when given $\frac{dy}{dx}$ and one pair of (x, y) values.	22 ☐

S5.6	Local maximum and minimum points	
	Book Section 16A	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Values of x where the gradient of the curve is zero. Solving $f'(x) = 0$.	Use the expression for $\frac{dy}{dx}$ to find points where the gradient is zero.	23 ☐
	Use technology to sketch the graph of $f'(x)$ and solve $f'(x) = 0$.	24 ☐
Local maximum and minimum points.	Locate local maximum and minimum points on a graph.	25 ☐
	Be aware that the greatest/least value of a function may occur at an end-point of the domain.	26 ☐

S5.7	Optimization problems in context	
	Book Section 16B	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Optimization.	Find the maximum or minimum value of a function in a real-life context.	27 ☐

S5.8	The trapezoidal rule	
	Book Section 16C	Revised ☐
Syllabus wording	*You need to be able to:*	*Question*
Approximating areas using the trapezoidal rule.	Estimate area given a table of data. √x $\int_a^b y \, dx = \frac{1}{2} h ((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}))$, where $h = \frac{b-a}{n}$	28 ☐
	Estimate the area given a function.	29 ☐

Practice questions

1 In this question, x is measured in degrees. Use a table to estimate, to two decimal places, the limit of $\frac{\sin 3x}{0.2x}$ when x tends to zero.

2 Use a graph to estimate the limit of $\frac{\ln\left(\frac{x}{2}\right)}{x-2}$ when x tends to 2.

3 Given that $y = 3x^2 - 5x$ and $\frac{dy}{dx} = 6x - 5$, what is the value of the derivative of y when $x = 2$?

4 Write an equation to represent the following situation:
The area decreases with time at a rate proportional to the current area.

5 The table shows some information about a function f(x).

x	1	3	4
f(x)	4	8	5
f'(x)	–1	4	2

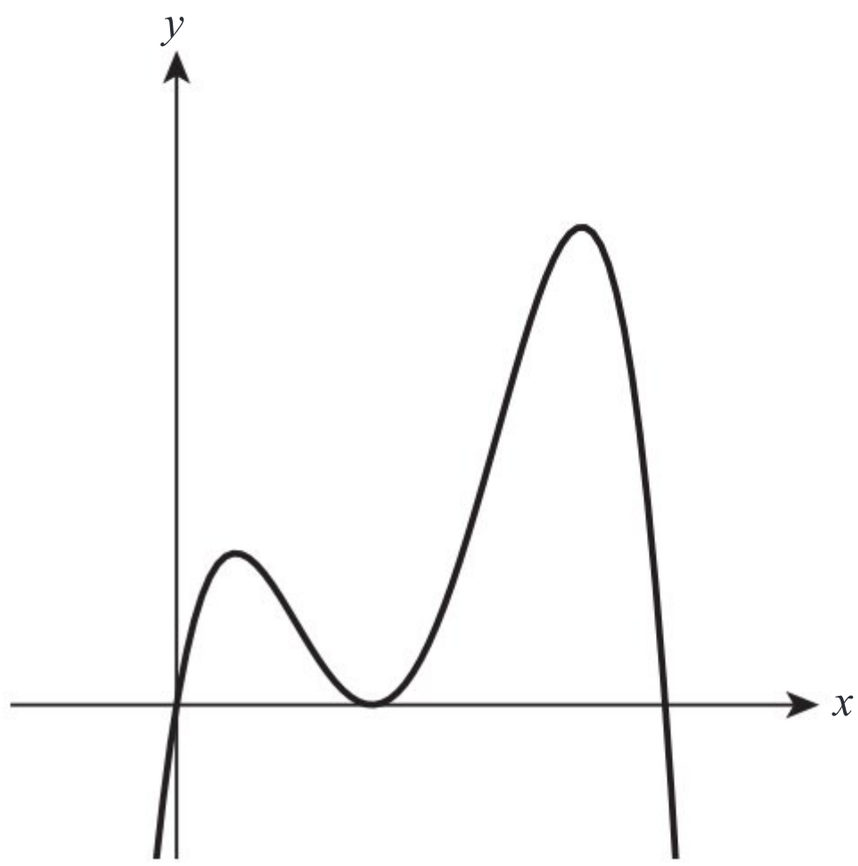
A graph has equation $y = f(x)$. Find the gradient of the graph at the point where $y = 4$.

6 Point $P(4,2)$ lies on the curve with equation $y = \sqrt{x}$. The table shows the coordinates of a variable point Q and the gradient of the chord PQ. Complete the table and use it to estimate the gradient of the curve at P.

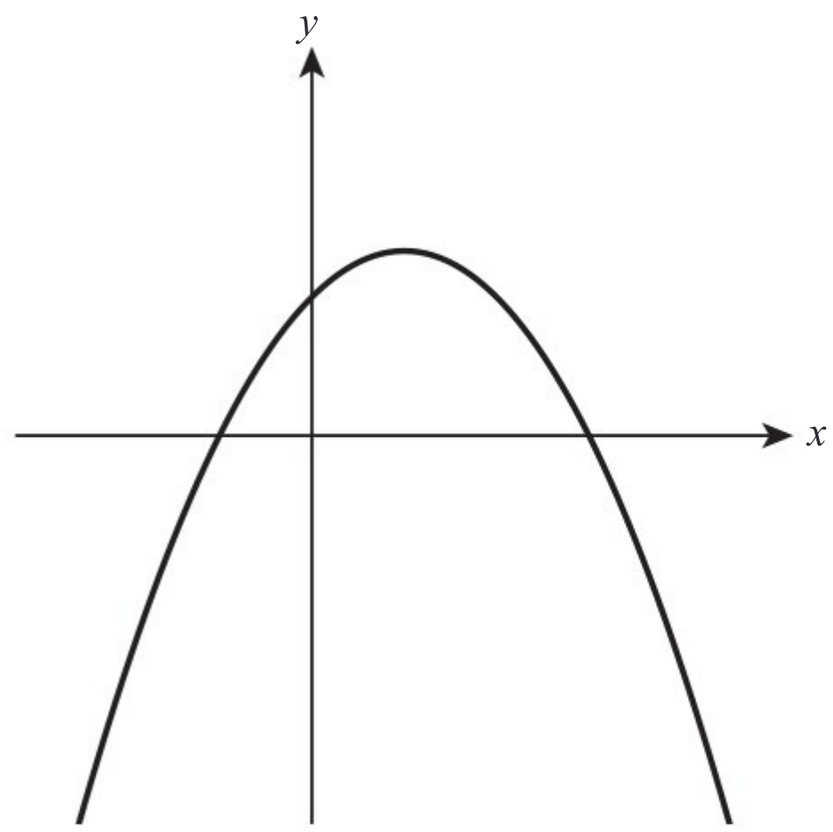
x_Q	y_Q	Δx	Δy	Gradient of PQ
5	2.236	1	0.236	0.236
4.1	2.025	0.1		
4.01				
4.001				

7 Use technology to sketch the graph of $f(x) = x^3 - 5x + 2$ and use it to find the range of values of x for which $f'(x) < 0$.

8 The graph of $y = f(x)$ is shown here. Sketch the graph of $y = f'(x)$.



9 The graph of $y = f'(x)$ is shown here. Sketch one possible graph of $y = f(x)$.



10 Differentiate $y = 4x^2 - \frac{1}{10}x^{-5} - 3x + 2$.

11 Find $f'(x)$ when

a $f(x) = 3x^2(4 - x^4)$

b $f(x) = 1 - \frac{3}{2x^4}$

c $f(x) = \frac{4x^2 - 3x + 1}{5x}$.

12 Given that $f(x) = 4x^2 - 2x^{-1}$, find $f'(x)$ and evaluate $f'(2)$.

13 For the curve with equation $y = 12x + 5x^{-1}$, find $\frac{dy}{dx}$.
Hence find the x-coordinates of the points on the curve $y = 12x + 5x^{-1}$ where the gradient equals 2.

14 A curve has equation $y = x^2 - 3$. Find the equation of the tangent to the curve at the point where $x = 4$.

15 Find the equation of the normal to the curve $y = 3x - 2x^{-1}$ at the point where $x = 2$.

16 The tangent to the curve with equation $y = x^2 - 3$ at the point (a, b) passes through $(0, -12)$. Find the possible values of a.

17 For the curve with equation $y = \dfrac{4\sqrt{x} - 3}{7x^2}$ find, correct to two decimal places,

a the gradient when $x = 3.2$

b the equation of the tangent at the point where $x = 3.2$.

18 A curve has equation $y = \dfrac{4\sqrt{x} - 3}{7x^2}$. Find the coordinates of the point on the curve where the gradient is 2.

19 Find $\int 9x^2 + 6x^{-3}\,dx$.

20 Find $\int \dfrac{x^5 - 3}{2x^2}\,dx$.

21 Find the area enclosed by the curve $y = 2x^3 - 1$, the x-axis and the lines $x = 2$ and $x = 3$.

22 Given that $\frac{dy}{dx} = 4x + 2$, and that $y = 3$ when $x = 2$, find an expression for y in terms of x.

23 A curve has equation $y = 2x^3 - ax^2 + 3$. Find, in terms of a, the x-coordinates of the points where the gradient of the curve is zero.

24 Given that $f(x) = \frac{2}{x} + \sqrt{x}$, use the graph of $y = f'(x)$ to solve the equation $f'(x) = 0$.

25 Find the coordinates of the local maximum point on the graph of $y = 2x^3 - 0.4x^2 - 0.7x + 2$.

26 Find the smallest value of the function $f(x) = 0.1x^5 - 2x^3$ in the interval $-5 \leqslant x \leqslant 5$.

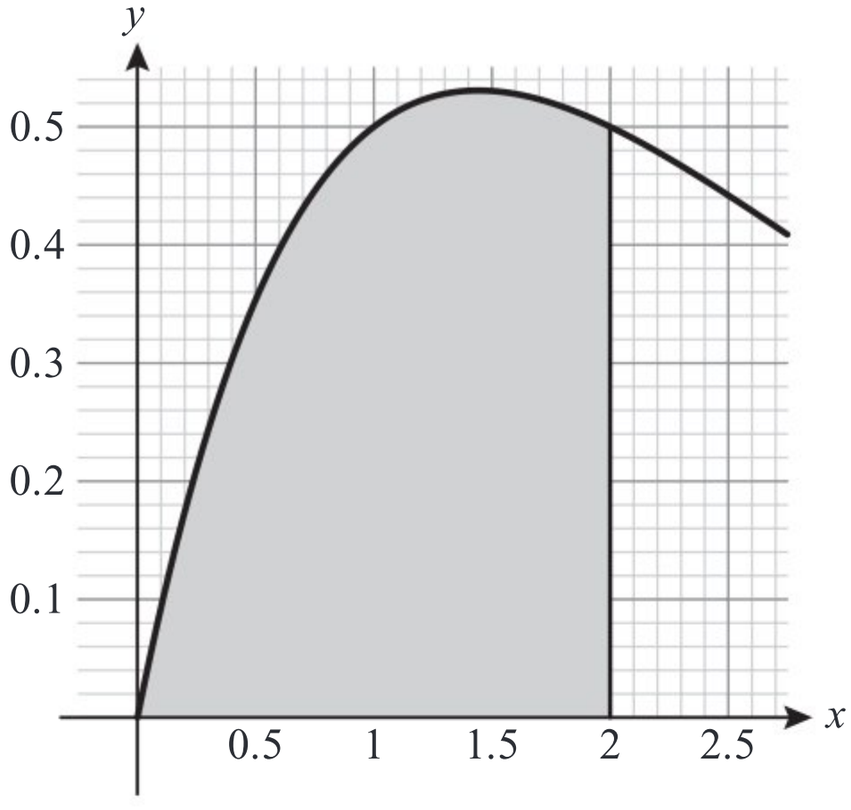
27 An open box has a square base of side x cm and height $\frac{32}{x^2}$ cm. Show that the surface area of the box is given by $S = x^2 + \frac{128}{x}$, and find the minimum possible surface area of the box.

28 Some of the values of the function f(x) are given in the table:

x	2	2.5	3	3.5	4	4.5	5
f(x)	1.6	2.1	2.3	2.2	2.0	1.5	0.8

Use all the values in the table to estimate the value of $\int_2^5 \text{f}(x)\,\text{d}x$.

29 The diagram shows a part of the graph of $y = \frac{x}{2^x}$.



Use the trapezoidal rule with five strips to estimate the shaded area.

Paper plan

	Syllabus Section	Description	Book Section	Mastery			Practice Paper Coverage							
				Covered	Revised	Confident	Book P1	Book P2	A1	A2	B1	B2	C1	C2
Number and algebra Core	S1.1	Standard form	1B						1				1a	
	S1.2	Arithmetic sequences and series	2A				3		9		3		4, 11ab	
	S1.3	Geometric sequences and series	2B								7			
	S1.4	Financial applications of geometric sequences	2C							4				
	S1.5a	Exponents with integer coefficients	1A									5b		6b
	S1.5b	Introduction to logarithms	1C						4					
Number and algebra SL	S1.6	Approximation and estimation	11A				7b	4c	1		4	3f		
	S1.7	Finance: Amortization and annuities	11B				5				8		9	
	S1.8a	Solving systems of equations	12A										7a	
	S1.8b	Solving polynomial equations	12B						6b	1d	7		7b	
Functions Core	S2.1	Equations of straight lines	4A					1	10					5cd
	S2.2	Concepts of functions	3A				2ab	3bcd	4c					
	S2.3	The graph of a function	3B				2c	4e	4b	1f	11b			
	S2.4	Key features of graphs	3B						6	1f		3d	8	
Functions SL	S2.5a	Linear models	13A					5cde	12					
	S2.5b	Quadratic models	13B				12					3c	7	
	S2.5c	Exponential models	13C				8							6
	S2.5d	Direct/inverse variation and cubic models	13D					4d		1				
	S2.5e	Sinusoidal models	13E					4a			11			
	S2.6	Modelling skills	13F				13c			3e			7c	
Geometry and trigonometry Core	S3.1a	Distances and midpoints	4B											2, 3ad
	S3.1b	Volume and surface area of 3D solids	5A				9	3a	1b, 2			5ab	1	2
	S3.1c	Angle between intersecting lines and planes	5B						2b					2
	S3.2a	Trigonometry in right-angled triangles	5B							6				2
	S3.2b	Trigonometry in non-right-angled triangles	5B						11			2cde	2	
	S3.3	Applications of trigonometry	5C				9		11	6		2b		
Geometry and trigonometry SL	S3.4	Arcs and sectors of circles	14A				7a				1		5	
	S3.5	Perpendicular bisectors	14B						13			2a		5bc
	S3.6	Voronoi diagrams	14B				11		13					5

	Syllabus Section	Description	Book Section	Mastery			Practice Paper Coverage							
				Covered	Revised	Confident	Book P1	Book P2	A1	A2	B1	B2	C1	C2
Statistics and probability Core	S4.1	Sampling	6A									1ab		
	S4.2	Statistical diagrams	6C				1						3	
	S4.3	Summary statistics	6B							3a	2		6	1a
	S4.4	Correlation and regression	6D					5		3		1cde	11c	1a–e
	S4.5	Definitions in probability	7A						3					4c
	S4.6	Probability techniques	7B				4		3	2a, 5			14	4be
	S4.7	Discrete random variables	8A					2abc		2	12			
	S4.8	Binomial distribution	8B					2def		5		4		4d
	S4.9	Normal distribution	8C							5	10b			4ab
Statistics and probability SL	S4.10	Spearman's rank correlation coefficient	15C						5					1fg
	S4.11a	χ^2-tests	15A				6		7		10			
	S4.11b	*t*-test	15B				10				6		13	
Calculus Core	S5.1a	Concept of a limit	9A								5			
	S5.1b	Interpretation of derivatives	9A							1c	9a		12	
	S5.2	Increasing and decreasing functions	9B							1e			12	
	S5.3	Derivatives of polynomials	9C							1b			10	
	S5.4	Tangents and normals	9D					6a	8b					3bc
	S5.5a	Integration as anti-differentiation	10A				13b		8a		9b			
	S5.5b	Definite integrals and areas using technology	10B						6			3e	8	
Calculus SL	S5.6	Maximum and minimum points	16A					6d			9a		10, 12	
	S5.7	Optimization problems	16B									5		
	S5.8a	Trapezoidal rule	16C									3abg		

Practice exam papers

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set A

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1 *[Maximum mark: 5]*

a The radius of the Earth is found to be 6.4×10^6 m correct to two significant figures. Find the upper bound on the possible value of the radius. *[1]*

b Hence find the upper bound on the surface area of the Earth, modelling it as a perfect sphere. *[2]*

c A textbook states that the area of the Earth is $5.10 \times 10^{14}\,m^2$. Find the percentage error if the upper bound found in part **b** had been used as an estimate. *[2]*

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2 *[Maximum mark: 6]*

A right pyramid has a square base with sides of length 4 cm. It has a height of 6 cm.

a Find the volume of the pyramid. *[2]*

b Find the acute angle between the sloping edge of the pyramid and the base. *[4]*

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3 *[Maximum mark: 6]*

A biologist investigated the characteristics of a group of fruit flies. Her results are shown below:

	Red eyes	**Black eyes**
Wings	54	156
Wingless	12	34

Use the results to estimate

a the probability of having red eyes and wings *[2]*

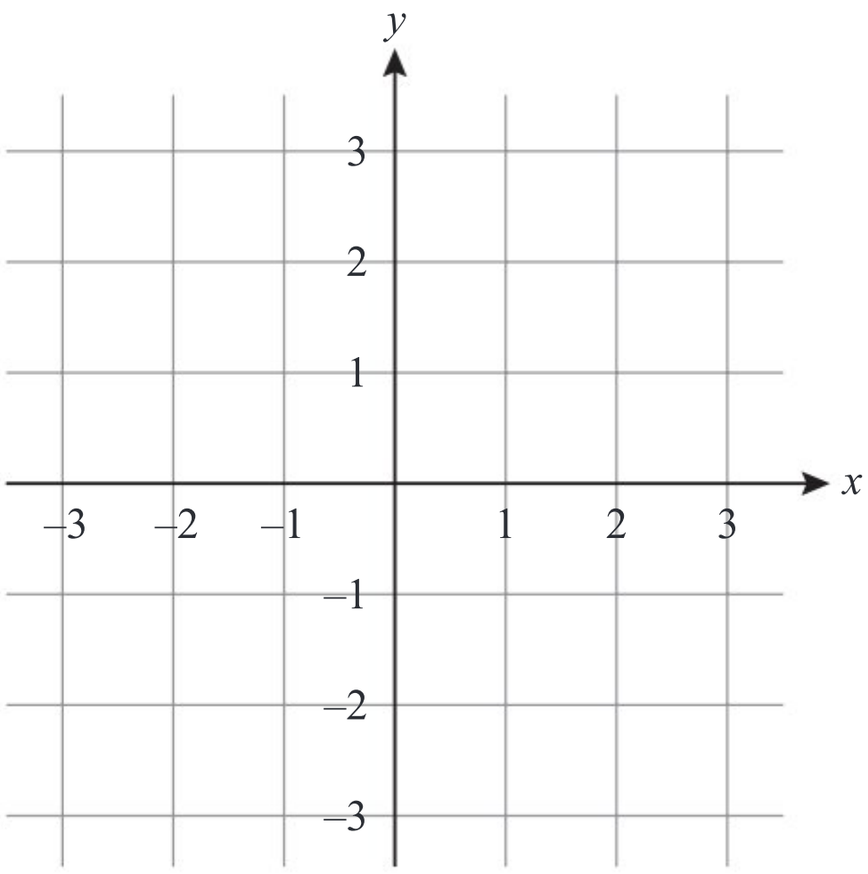
b the probability of not having red eyes *[2]*

c the probability of having red eyes if the fly is wingless. *[2]*

4 *[Maximum mark: 5]*

a Evaluate $\ln 3$. *[1]*

b On the axes below, sketch the graph of $y = \ln x$ *[2]*



c **i** The graph $y = \ln x$ is reflected in the line $y = x$. Sketch the new graph on the axes above.

ii Write down the equation of the new graph. *[2]*

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5 *[Maximum mark: 6]*

A geographer is investigating the link between the population of a city (P million) and the number of doctors found there (D hundred). She finds the following data:

P	1.2	1.9	2.1	2.5	2.5	3.4	5.6	6.8
D	3.1	3.4	3.3	4.8	4.5	4.1	7.2	7.7

a Find the Spearman's rank correlation coefficient between P and D. *[3]*

b The one-tailed critical value at 5% significance is 0.643. Does the sample provide evidence of a positive association between the population and the number of doctors? *[1]*

c The number of doctors in the largest city was found to be underestimated. How would the value of the Spearman's rank correlation coefficient be affected if the true value were used? Justify your answer. *[2]*

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6 *[Maximum mark: 6]*

a Find the maximum value of the function $f(x) = 8 + 2x^2 - x^4$. *[2]*

b Find the area enclosed by the curve $y = 8 + 2x^2 - x^4$ and the x-axis. *[4]*

7 *[Maximum mark: 6]*

A gene in cats can be expressed in three different ways, depending upon the genetic material in the cell. These are described as HH, Hh and hh. A geneticist believes that this should be found in the ratio 1:2:1. He investigates a sample of 150 cats and finds the following data:

Gene expression	HH	Hh	hh
Frequency	31	77	42

Determine, at the 5% significance level, if there is evidence that the 1:2:1 prediction is incorrect. *[6]*

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8 *[Maximum mark: 8]*

The graph of y against x has a gradient at any point equal to $3x^2 + 2$. It passes through the point (0,1).

a Find the equation of the graph. *[4]*

b Find the equation of the normal to the graph passing through (0,1) in the form $ay + bx = c$ where a, b and c are integers. *[4]*

9 *[Maximum mark: 5]*

The sum of the first n terms of an arithmetic sequence is given by $9n - 2n^2$. Find the first term and the common difference. *[5]*

10 *[Maximum mark: 8]*

For the points $A(0, 4)$, $B(2, 8)$ and $C(1, 3)$:

a Find the line perpendicular to AB through C. *[5]*

b Hence find the point on the line through A and B that is closest to C. *[3]*

11 *[Maximum mark: 6]*

A ship leaves a dock and travels 10 km on a bearing of 040°. It then travels 20 km on a bearing of 170°.

a Find the distance of the ship from the dock. *[3]*

b On what bearing should it travel to return to the dock? *[3]*

12 *[Maximum mark: 6]*

The cost of a mobile phone contract depends on the number of minutes spent talking each month.

Contract A has a fixed cost of \$8 then charged at a rate of \$1 per 20 minutes.

Contract B has a cost of \$10 for the first 100 minutes, then charged at a rate of \$2 per 10 minutes.

Joanne wants to investigate the cost, \$$C$, of spending x minutes talking each month on both contracts.

a Find a model to describe C in terms of x under contract A. *[1]*

b Find a piecewise linear model to describe C in terms of x under contract B. *[2]*

c Hence find values of x for which contract B is cheaper than contract A. *[3]*

13 *[Maximum mark: 7]*

Three mobile phone masts are located, in arbitrary units, at $A(1, 2)$, $B(3, 4)$ and $C(5, 0)$. Sanjay's phone picks up the signal from all three simultaneously. Assuming the land is flat and all three signals travel at the same speed without obstruction, find the coordinates of Sanjay's phone. *[7]*

Mathematics: applications and interpretation
Standard level
Paper 2
Practice Set A

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer **all** questions.

Do **not** write solutions on this page.

Answer all questions in an answer booklet. Please start each question on a new page.

1 *[Maximum mark: 13]*

The number of people (in thousands) subscribed to a website x weeks after it is launched is modelled by

$$f(x) = x^3 - 6x^2 + 9x + 4,\ x \geqslant 0$$

a Find the initial number of subscribers when the website launches. *[1]*

b Find $f'(x)$. *[2]*

c Interpret $f'(x)$ in context. *[1]*

d Find all solutions of $f'(x) = 0$. *[2]*

e Find the values of x for which $f(x)$ is increasing. *[2]*

f Sketch $y = f(x)$. *[3]*

g How long does it take the website to reach 10 000 subscribers? *[2]*

2 *[Maximum mark: 12]*

A fair four-sided dice is rolled twice. S is the sum of the scores.

a Copy and complete the probability distribution of S. *[2]*

s	2	3	4	5	6	7	8
$P(S = s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$			

b Find the expected value of S. *[2]*

c Given that the total is more than 4, find the probability that it is more than 6. *[3]*

Eric plays a game where he rolls a fair four-sided dice twice. If the score is four or less, he loses and pays \$1. If he scores 5 or more, he receives \$$k$.

d Find the value of k if the game is fair. *[5]*

3 *[Maximum mark: 14]*

Kwami keeps a record of his best 5000 m time (y minutes) for each week (x) in the 13 weeks after he starts training for a competition season. The results are shown here:

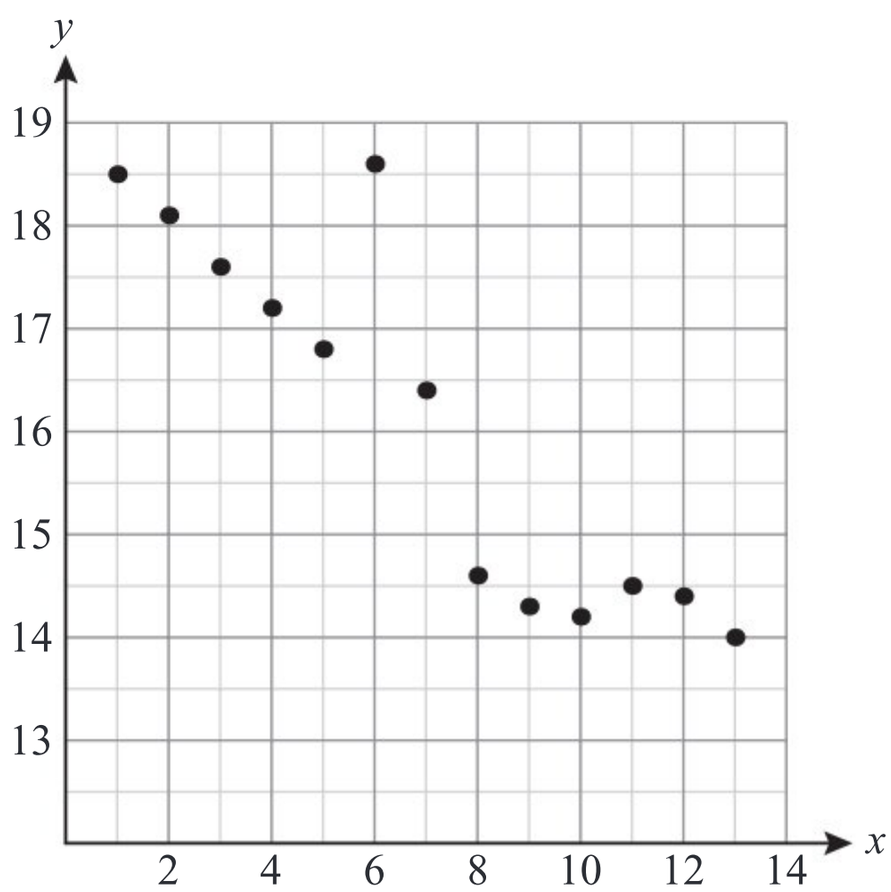
x	1	2	3	4	5	6	7	8	9	10	11	12	13
y	18.5	18.1	17.6	17.2	16.8	18.6	16.4	14.6	14.3	14.2	14.5	14.4	14

a i Find the mean of Kwami's best times each week.

ii Find the standard deviation in Kwami's best times each week.

iii Find Pearson's product moment correlation coefficient for these data. What type of correlation is suggested by this value? *[4]*

The results are illustrated in the following scatter diagram.



b i Competitions occurred every week from week n until week 12. Athletes generally have improved performance in competitions. Use the graph to suggest the value of n.

ii During one of the weeks before competitions began, Kwami was ill. Use the scatter graph to suggest which week this was. *[2]*

For the rest of this question, the result from the week where Kwami was ill should be excluded.

c i Create a piecewise linear model to predict y for a given x.

ii Compare and contrast, in context, the coefficients of x in each part of the linear model. *[5]*

d Use your model to predict the time Kwami would have achieved in the week he was ill if he had not been ill. *[2]*

e Explain why it would not be valid to use this model to predict Kwami's times in the following season. *[1]*

4 *[Maximum mark: 13]*

Almira is considering two different savings schemes. Both schemes involve an initial investment of $1000 in an account.

In scheme A, at the end of each year $50 is added to the account.

In scheme B, at the end of each year 4% compound interest is added to the account.

a How much will be in Almira's account at the end of the fifth year after investment in

i Scheme A

ii Scheme B. Give your answer correct to two decimal places. *[4]*

b What annual compound interest rate would achieve the same outcome for Almira as investing in scheme A for five complete years? *[2]*

c Almira wants to invest for n complete years. For what values of n would Almira be better off investing in scheme B? *[3]*

d Almira estimates that there is 2.5% depreciation each year. How long would Almira need to save in scheme B to use her savings to purchase something currently valued at $1400? *[4]*

5 *[Maximum mark: 15]*

The results in an intelligence test are normally distributed with a mean of 100 and a standard deviation of 30.

a Find the probability that a randomly chosen individual will have a score above 150. *[1]*

b Only 10% of people have a score above k. Find the value of k. *[2]*

To enter a high intelligence society, people need to have a score of at least 150. Five people are chosen at random to take the test.

c Find the probability that at least two of them qualify to enter the high intelligence society. *[4]*

d Find the probability that the fifth person to take the test is the second person to attain a score of at least 150. *[3]*

People with a score of more than 170 in the test are allowed to enter a merit stream within the society.

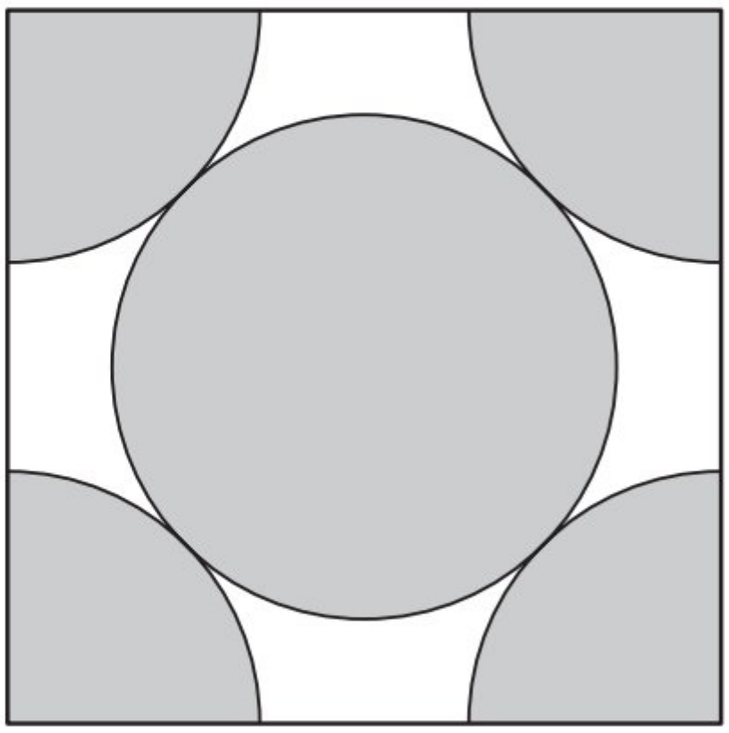
e What percentage of the society are members of the merit stream? *[4]*

f State one assumption required in your answer to part **e**. *[1]*

6 *[Maximum mark 13]*

Metal rods are modelled as perfect cylinders with radius 1 cm. They are packed into a box in two different ways.

In method 1, the repeating unit is shown below:



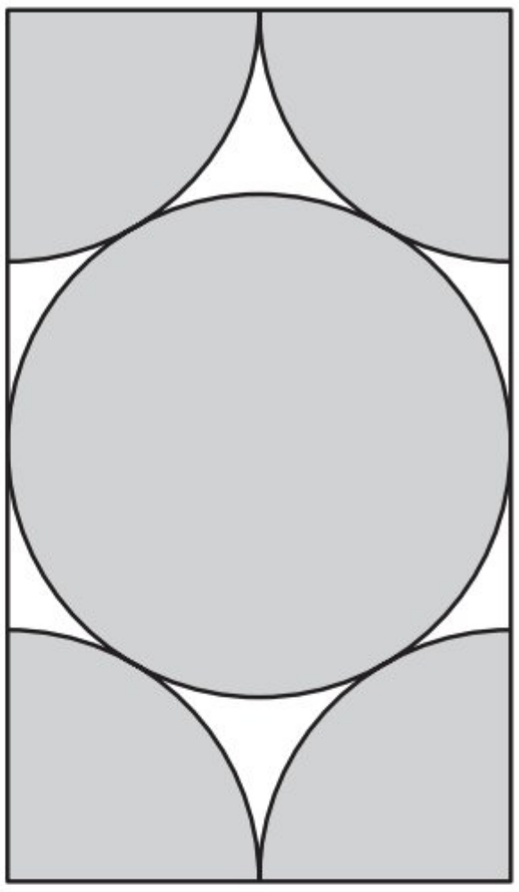
The repeating unit contains four quarter circles and one full circle.

a Explain why the diagonal of the square has length 4 cm. *[1]*

b Find the proportion of the box that is filled with metal. *[4]*

c State one assumption required in your answer to part **b**. *[1]*

In method 2, the repeating unit is shown below:



d Find the proportion of the box that contains metal in method 2. *[5]*

e Determine, with justification, whether method 1 or method 2 packs more rods into the same box. *[2]*

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set B

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
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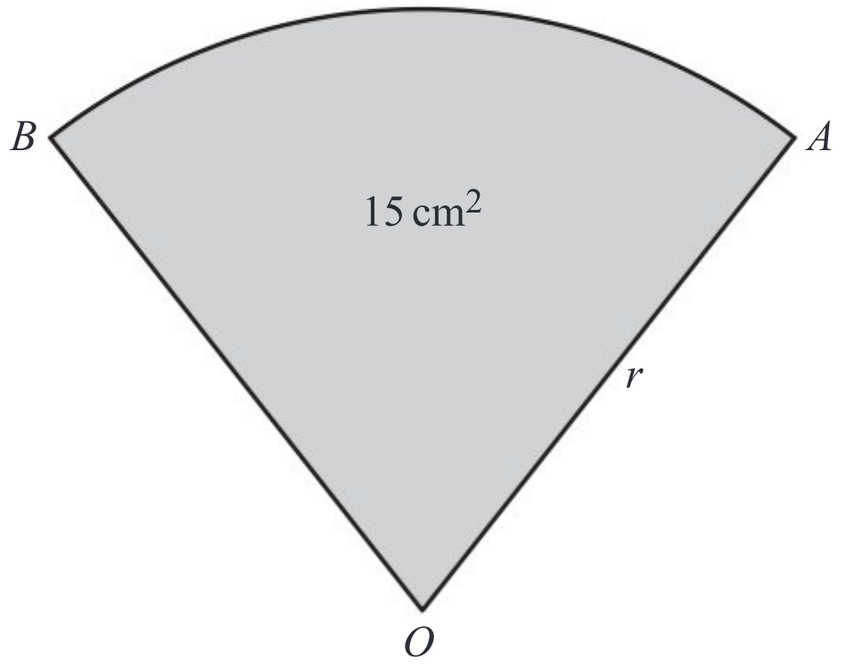
1 *[Maximum mark: 4]*



The sector OAB has area $15\,\text{cm}^2$.

The perimeter of the sector is 4 times the length of the arc AB.

Find the radius, r. *[4]*

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2 *[Maximum mark: 5]*

The scores out of 10 in a mathematics test are summarized in the table:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	1	0	1	1	2	2	3	2	1	1

a Find the median score. *[1]*

b Find the interquartile range. *[1]*

c Determine, with clear calculations, whether any of the scores are outliers. *[3]*

3 *[Maximum mark: 6]*

Laura is training for a marathon. The number of miles she runs in each training session forms an arithmetic sequence.

In her fifth session she runs 8 miles. The total distance she has run in the first eight sessions is 58 miles.

a Find the distance she ran in her first training session and the increase in distance between consecutive sessions. *[4]*

b Hence find the number of the training session in which she first runs a full marathon distance of 26 miles. *[2]*

4 *[Maximum mark: 6]*

The voltage, V, current, I, and resistance, R, in an electrical circuit are related by the formula $V = IR$.

Suzi measures the voltage to be 219 to three significant figures and the current to be 18.4 to three significant figures.

a Find

i the upper bound on the resistance

ii the lower bound on the resistance. *[4]*

b Hence state the value of the resistance to an appropriate degree of accuracy, justifying your choice. *[2]*

5 *[Maximum mark: 6]*

a Find an expression for the gradient of the chord joining the point (1, 0) to the point $(x, \ln x^2)$. *[2]*

b Use technology to find the limit of the gradient of this chord as x tends towards 1. *[2]*

c Explain the significance of your answer to part **b**. *[2]*

6 *[Maximum mark: 8]*

Sven has recently moved from Gamla Stan to Nya Stan. He suspects that the average temperature in Nya Stan is warmer than in Gamla Stan at this time of year and has gathered the following temperature readings from each town.

Gamla Stan (°C)	12.3	14.1	16.3	15.9	10.6	13.2		
Nya Stan (°C)	14.8	11.7	16.5	17.6	19.3	12.1	16.9	15.4

He uses these samples to carry out a test at the 10% level to investigate his suspicion.

a State the null and alternative hypotheses. *[2]*

b Calculate the p-value for this test. *[2]*

c State the conclusion of the test. *[2]*

d State two assumptions needed to conduct this test. *[2]*

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7 *[Maximum mark: 6]*

The fourth term of a geometric sequence is 13.5 and the sum of the first three terms is 74.

Find the first term and common ratio of the sequence. *[6]*

8 *[Maximum mark: 6]*

Erica pays £6000 each year into a pension scheme that guarantees a 2.5% per annum interest rate.

She plans to retire in 30 years.

a Find the value of her pension on retirement. *[3]*

She will use the sum saved in the scheme to buy an annuity also at a 2.5% per annum interest rate.

The annuity will pay out £750 per month for life.

b How long after retiring must Erica live before she starts receiving money that was not saved in her pension? *[3]*

9 *[Maximum mark: 8]*

A manufacturer produces x hundred items of a particular product each week and makes a profit $P(x)$ in thousands of US dollars.

He knows that the rate of change of profit with respect to the number of items produced is given by $-3x^2 + 5x + 2$.

a Find the number of items he should produce each week to maximize profit. *[3]*

He makes a profit of $2000 when producing 100 items.

b Find $P(x)$. *[5]*

10 *[Maximum mark: 9]*

A surgery manager claims that patient waiting times for pre-booked appointments at his surgery are normally distributed with a mean of 14 minutes and a variance of 36 minutes.

A sample of the waiting times for 80 patients is taken:

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Observed frequency	3	8	23	30	16

This sample is used to conduct a χ^2 goodness of fit test to investigate the manager's claim. The test is conducted at the 5% level.

a State the null and alternative hypotheses. *[2]*

b Copy and complete the following table. *[2]*

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Expected frequency	5.34				

c Find the p-value for the test. *[3]*

d State the conclusion of the test. *[2]*

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11 *[Maximum mark: 8]*

The London Eye is an observation wheel with a diameter of 120 m that rotates once every 30 minutes. The pods that carry customers are arranged around the rim of the wheel.

A particular pod starts at the lowest point of the circle 2 m above ground level. The height, h metres, of that pod at time t minutes can be modelled by the function $h = a\cos(bt) + c$.

a Find the values of a, b and c. *[5]*

b Find the length of time for which the pod is higher than 50 m above ground level in any one revolution of the wheel. *[3]*

12 *[Maximum mark: 8]*

The table below is the probability distribution for the score obtained in a game of chance:

x	0	1	2	3	4
$\mathbf{P}(X = x)$	0.1	a	b	0.2	0.15

A player is charged \$2 to play and they win, in dollars, the score they achieve in the game.

a Show that $a + b = 0.55$. *[2]*

b Given that the game is fair, find a and b. *[3]*

Zhuo plays the game twice.

c Find the probability that he makes a loss overall. *[3]*

Mathematics: applications and interpretation
Standard level
Paper 2
Practice Set B

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer **all** questions.

Do **not** write solutions on this page.

Answer all questions in an answer booklet. Please start each question on a new page.

1 *[Maximum mark: 15]*

Fola is researching the possible relationship between height and weight of staff at his school.

He initially decides to select his sample by taking an alphabetic list of all staff and selecting every 10th person on the list.

a i Name this sampling technique.

ii Explain why this does not produce a simple random sample. *[2]*

He then decides to change his sampling technique by taking a stratified sample of men and women.

He wants a sample size of 12 and knows there are 46 men and 63 women at the school.

b i Find the number of men he should include in his sample.

ii State the sampling method he then needs to employ to select the particular men and women.

iii State how cluster sampling differs from stratified sampling. *[4]*

He collects the following data:

Height/cm	153	158	161	162	164	165	167	172	175	179	184	190
Weight/kg	52.4	54.6	59.7	57.1	58.5	74.2	62.8	73.1	82.3	60.2	74.3	86.6

c Find Pearson's product moment correlation coefficient and interpret this value in context. *[2]*

d Use an appropriate regression line to estimate the weight of a person with height

i 140 cm

ii 170 cm. *[3]*

e Comment on the reliability of the predictions in parts **di** and **dii**. *[2]*

f Suggest two ways Fola could improve the reliability of any predictions made from linear regression for this population. *[2]*

2 *[Maximum mark: 18]*

A pleasure boat runs trips around the local bay.

It leaves its mooring and manoeuvres onto a straight line path that keeps it equidistant from the end of the harbour walls located at the points with coordinates (1, 8) and (5, 2).

a Find the equation of its path in the form $ax + by + c = 0$. *[4]*

As the boat passes between the harbour walls, the captain observes that the angle of elevation to the top of one of the walls is 12°. The harbour master is 50 m closer to that wall and observes that the angle of elevation is 55°.

b Find the height of the harbour wall. *[5]*

Once clear of the harbour, the boat reaches a buoy at A and from there moves on a bearing of 310° for 20 km until it reaches point B.

It then moves on a bearing of 055° for 30 km to point C.

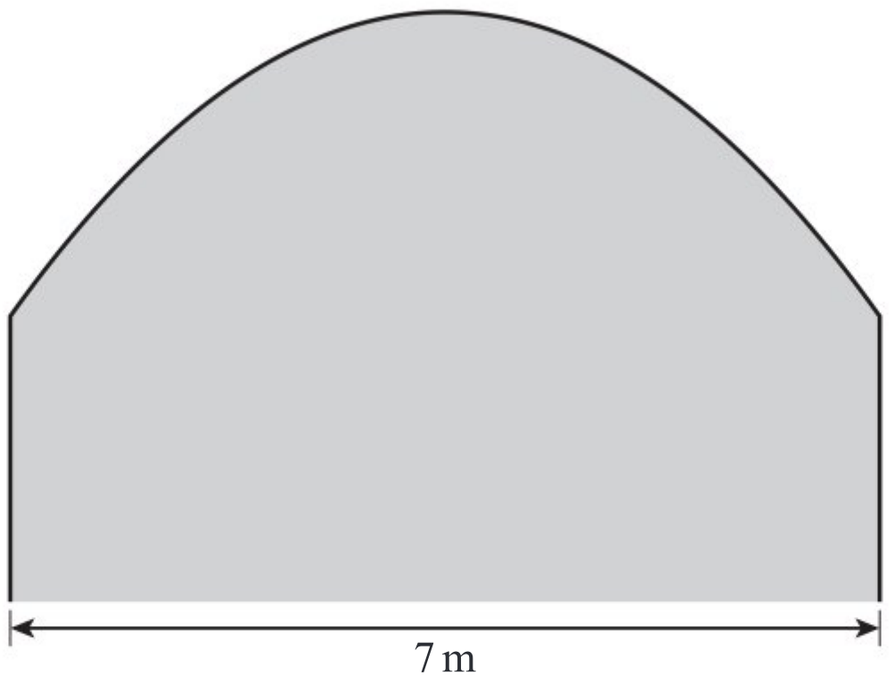
c Find the angle $A\hat{B}C$. *[2]*

d Find the shortest distance from C back to the buoy at A. *[3]*

e Find the bearing the boat must travel on to cover the shortest distance from C back to A. *[4]*

3 *[Maximum mark: 16]*

The entrance to a railway tunnel is shaped as shown below:



John measures the height, h, at various distances, x, from one side.

x/m	0	1	2	3	4	5	6	7
h/m	2.3	3.5	4.3	4.7	4.7	4.3	3.5	2.3

a Use the trapezoidal rule with 5 strips to estimate the cross-sectional area of the tunnel. *[3]*

b Explain whether your answer in part **a** is an underestimate or overestimate of the true cross-sectional area. *[2]*

In fact the curve of the entrance is a parabola, $h = ax^2 + bx + c$.

c Find a, b and c. *[4]*

d Find the maximum height of the tunnel. *[2]*

e Find the exact value of the actual cross-sectional area. *[2]*

f Find the percentage error in the estimate from part **a**. *[2]*

g How could the accuracy of the estimate in part **a** be improved? *[1]*

4 *[Maximum mark: 16]*

A telesales worker has constant probability of 0.04 of a call resulting in a sale.

a Find the probability of achieving exactly two sales in the first 10 calls made. *[2]*

b Find the probability of achieving at least two sales in the first 10 calls made. *[2]*

c **i** Find the number of calls he needs to make in a day to average two sales per day.

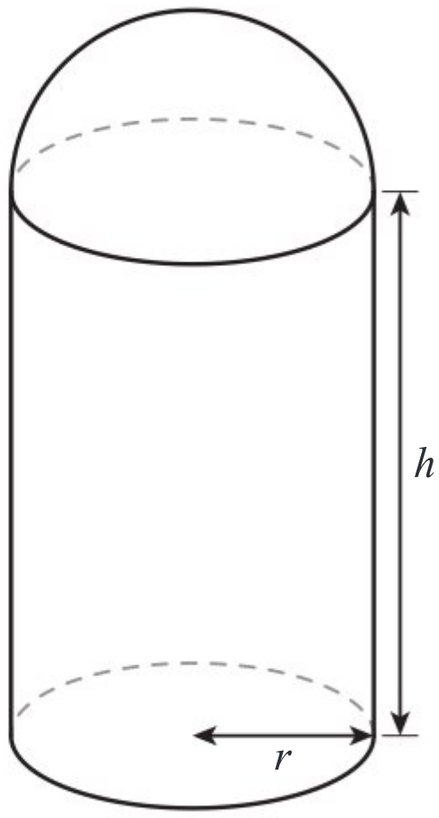
ii In this case, find the variance of the number of sales achieved. *[4]*

d In a 5-day week, find the probability that he achieves at least two sales in the first 10 calls made on more than one day. *[4]*

e Find the least number of calls he needs to make in order that the probability of making at least one sale is greater than 95%. *[4]*

5 *[Maximum mark: 15]*

A wooden salt shaker is formed from a hemisphere of radius r on top of a cylinder of height h as shown.



The volume of the salt shaker is $300\,\text{cm}^3$.

The manufacturer wants to use the least amount of wood possible in the production process.

a Show that $h = \dfrac{900 - 2\pi r^3}{3\pi r^2}$. *[4]*

b Hence find an expression for the surface area, A, of the salt shaker in the form $A = ar^b + cr^d$, where a, b, c and d are constants to be found. *[5]*

c Find

i the minimum amount of wood needed

ii the radius to achieve this minimum

iii the height to achieve this minimum. *[5]*

d State one reason why the manufacturer might not wish to use the dimensions found in parts **cii** and **ciii**. *[1]*

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set C

Candidate session number

1 hour 30 minutes

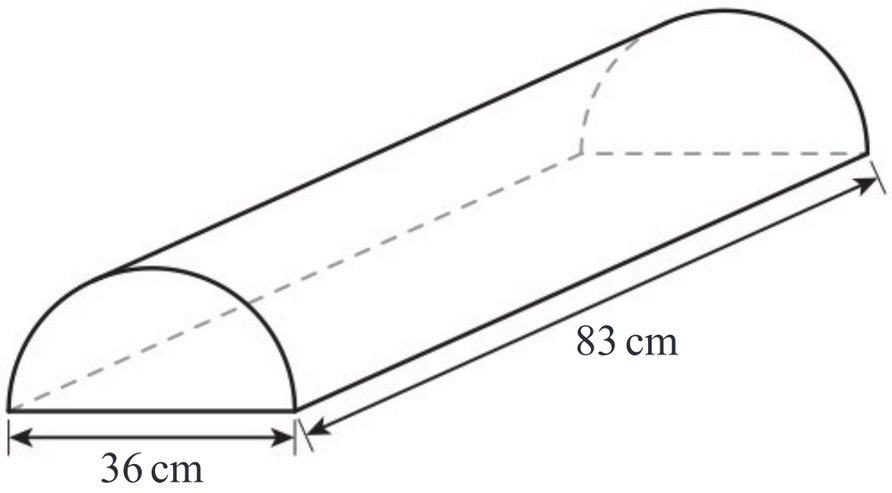
Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1 *[Maximum mark: 6]*

A metal bar is in the shape of a prism with a semicircular cross-section. The dimensions are shown in the diagram.



a Find the volume of the bar. Give your answer in cm^3, in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. *[3]*

The bar is melted down and all the metal used to make a sphere.

b Find the radius of the sphere. *[3]*

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2 *[Maximum mark: 5]*

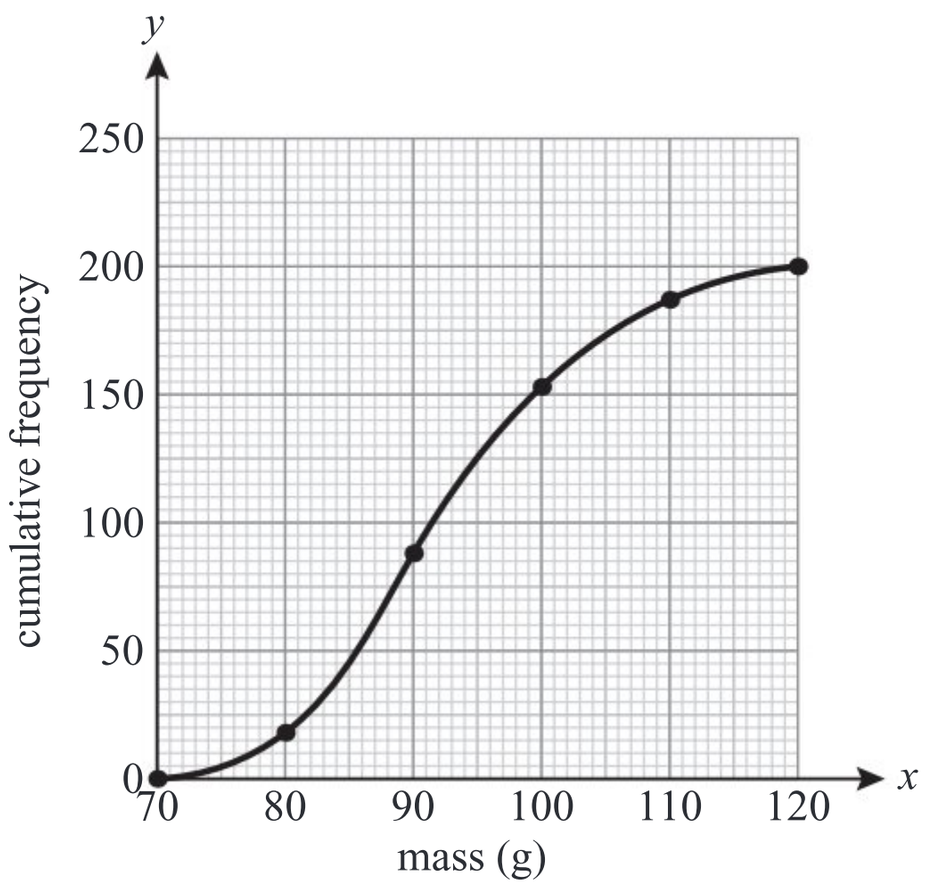
A triangle has sides $AB = 6.8\,\text{cm}$, $BC = 4.7\,\text{cm}$ and $CA = 9.1\,\text{cm}$.

a Find, in degrees, the size of the angle $A\hat{B}C$. *[3]*

b Find the area of the triangle. *[2]*

3 *[Maximum mark: 5]*

Roshni collected 200 apples from her orchard. The cumulative frequency graph below shows their mass in grams.



a Estimate how many apples weigh more than 90 g. *[2]*

b The heaviest 15% of the apples are going to be sent to the local restaurant. Estimate the least weight of an apple sent to the restaurant. *[3]*

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4 *[Maximum mark: 4]*

An arithmetic sequence has first term 7 and the sum of the first 20 terms is 640. Find

a the 20th term of the sequence *[2]*

b the 39th term of the sequence. *[2]*

5 *[Maximum mark: 5]*

A sector of a circle has radius 10 cm. The angle at the centre of the sector is $\theta°$. The area of the sector is 75 cm^2.

a Find the value of θ. *[2]*

b Find the perimeter of the sector. *[3]*

6 *[Maximum mark: 6]*

The heights of 30 flowers, measured in cm, are summarized in the table.

Height/cm	8–12	12–15	15–20	20–25	25–28
Frequency	5	6	8	7	4

a Estimate the mean and the standard deviation of the 30 flowers. *[3]*

The measurements are converted into inches, where 1 inch = 2.54 cm.

b Find the mean and variance of the heights in inches. *[3]*

7 *[Maximum mark: 7]*

Notebooks are delivered to schools in boxes of different sizes. A teacher thinks that the volume, $V\,\text{cm}^3$, of a box of height $x\,\text{cm}$ can be modelled by the equation

$V = ax^3 + bx^2 + cx$.

He measures heights and volumes of three boxes and obtains the following results:

x	8	10	15
V	1890	1690	703

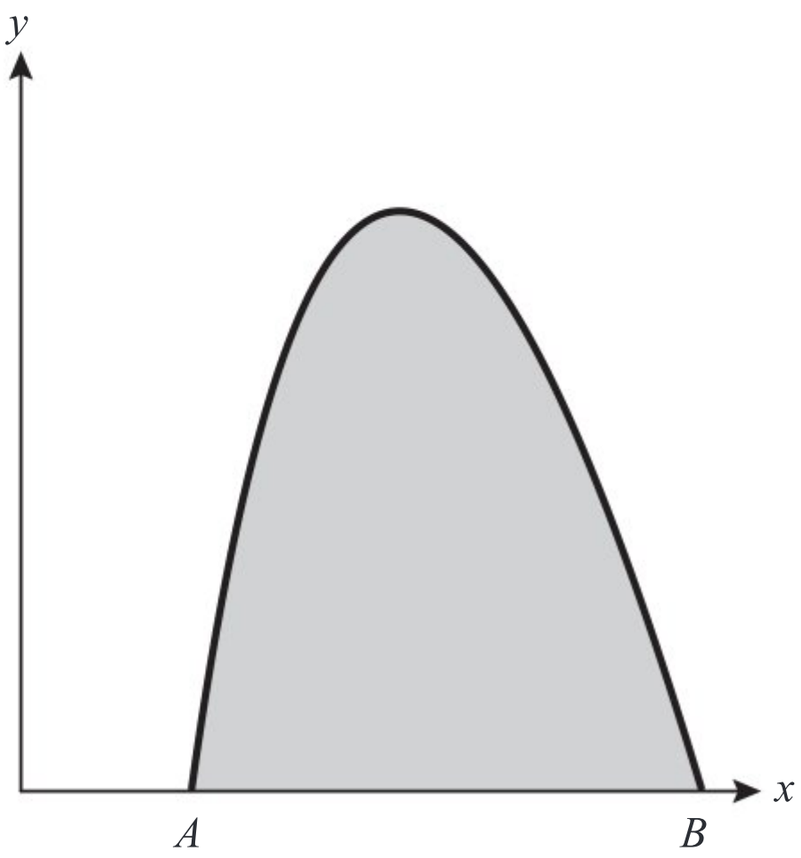
a Find the values of a, b and c. *[3]*

b According to this model, what should the height of a box with volume $1720\,\text{cm}^3$ be?
Give your answer correct to one decimal place. *[2]*

c Would this be a good model for a box of height 20 cm? Explain your answer. *[2]*

8 *[Maximum mark: 6]*

The diagram shows a cave entrance, whose outline can be modelled by the equation $y = 0.8\,(4 - x)\ln x$, where x and y are measured in metres. Points A and B are on the ground.



a Find the coordinates of A and B. *[2]*

b Find the height of the cave entrance at its highest point. *[2]*

c Find the area of the opening. *[2]*

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9 *[Maximum mark: 7]*

A small business needs to take out a loan of $50 000. A bank offers a loan with an annual interest rate of 2.4%, compounded monthly.

a If the business pays back $1000 per month, how long will it take to pay off the loan? *[2]*

b Find the monthly payment if the loan needs to be repaid in four years. *[2]*

c Compare the total amounts the business will pay for the loan in cases **a** and **b**. *[3]*

10 *[Maximum mark: 6]*

The graph of $y = 2x^3 - ax^2 + x + 2b$ has a local minimum point at $(2, -6)$.

Find the values of a and b. *[6]*

11 *[Maximum mark: 6]*

Rin has started a new martial arts club. The number of members during the first six months is shown in the table:

Month	1	2	3	4	5	6
Number of members	26	34	44	51	59	66

Rin thinks that the number of members can be modelled by a sequence that is approximately arithmetic.

a Find an average increase in the number of members over the first six months. *[2]*

b Use this model, with the first term equal to 26, to predict the number of members at the end of the year (in month 12). *[2]*

Gabor thinks that a better way to predict the number of members in future is to use a regression line.

c Use the data from the table to find the equation of the regression line and use it to predict the number of members at the end of the year. *[2]*

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12 *[Maximum mark: 5]*

The fuel consumption of a car depends on the speed at which the car is travelling. When the speed of the car is $v\,\text{km}\,\text{h}^{-1}$, the fuel consumption, in litres per 100 km, is modelled by the function F(v). The table shows the rate of change of F(v) for some values of v.

v	20	30	35	40	50
F'(v)	–0.0773	–0.0023	0.0195	0.0307	0.0217

a For which values of v in the table does the fuel consumption decrease as the speed increases? *[1]*

b Suggest a whole number speed, v, with $20 \leqslant v \leqslant 50$, at which the car should travel to minimize its fuel consumption. Justify your choice. *[2]*

c The minimum fuel consumption is 4.2 litres per 100 km. When the car is travelling at $20\,\text{km}\,\text{h}^{-1}$, the fuel consumption is 4.6 litres per 100 km. Use the information in the table to sketch the graph of F against v for $20 \leqslant v \leqslant 50$. *[2]*

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13 *[Maximum mark: 6]*

A running coach investigates whether different running shoes make a difference to athletes' running times. He times eight of his runners over the same distance, wearing two different pairs of shoes. The times, in seconds, are recorded in the table.

	Athlete							
Shoes	A	B	C	D	E	F	G	H
Pair 1	26.2	31.5	28.2	22.7	33.8	25.2	29.7	30.3
Pair 2	27.3	30.8	29.3	25.7	31.8	26.5	31.2	33.1

The coach decides to conduct a hypothesis test, using a 5% significance level, to test whether the mean times are different for the two pairs of shoes.

a State suitable hypotheses for the test. *[1]*

b Complete the table showing the difference in times for each athlete. *[2]*

Athlete	A	B	C	D	E	F	G	H
Time difference	–1.1	0.7						

c Hence carry out the test and state your conclusion. *[3]*

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14 *[Maximum mark: 6]*

Events A and B are such that:

$P(B) = \frac{1}{6}$, $P(A \cup B) = \frac{1}{5}$ and $P(A \mid B) = 4P(A)$.

Find $P(A \cap B)$. *[6]*

Mathematics: applications and interpretation
Standard level
Paper 2
Practice Set C

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
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- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
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Answer **all** questions.

Do **not** write solutions on this page.

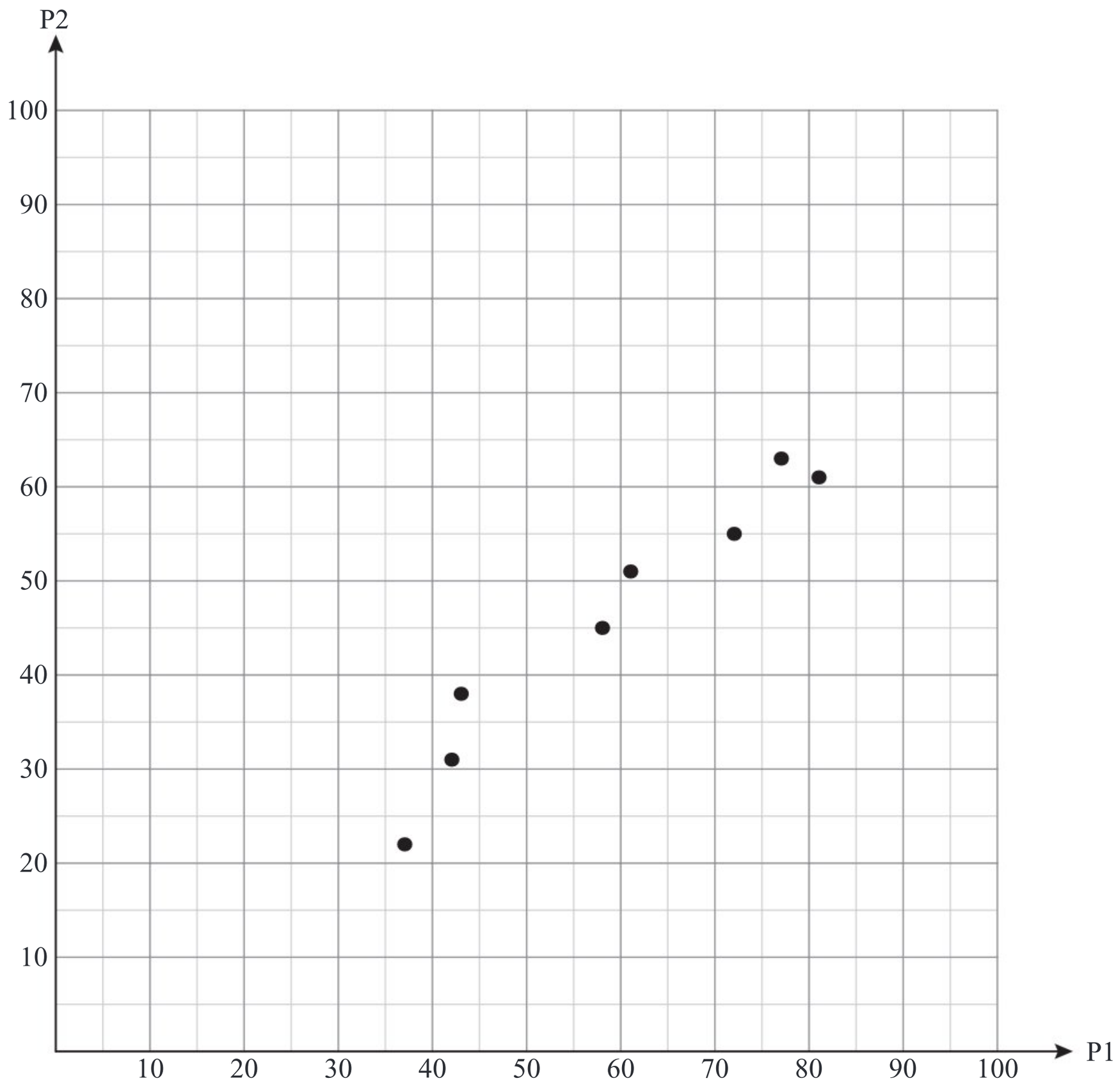
Answer all questions in an answer booklet. Please start each question on a new page.

1 *[Maximum mark: 17]*

The table shows the marks eight students obtained on two different exam papers.

	Student							
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Paper 1	37	61	81	43	42	72	58	77
Paper 2	22	51	61	38	31	55	45	63

The data are also shown on the scatter diagram.



a Find the mean mark for each paper and add the corresponding point to the scatter graph. *[2]*

b Draw a line of best fit. *[1]*

c Calculate Pearson's product moment correlation coefficient for the data. *[1]*

Two students did not take the second paper and a teacher wants to estimate what mark they would have got in it.

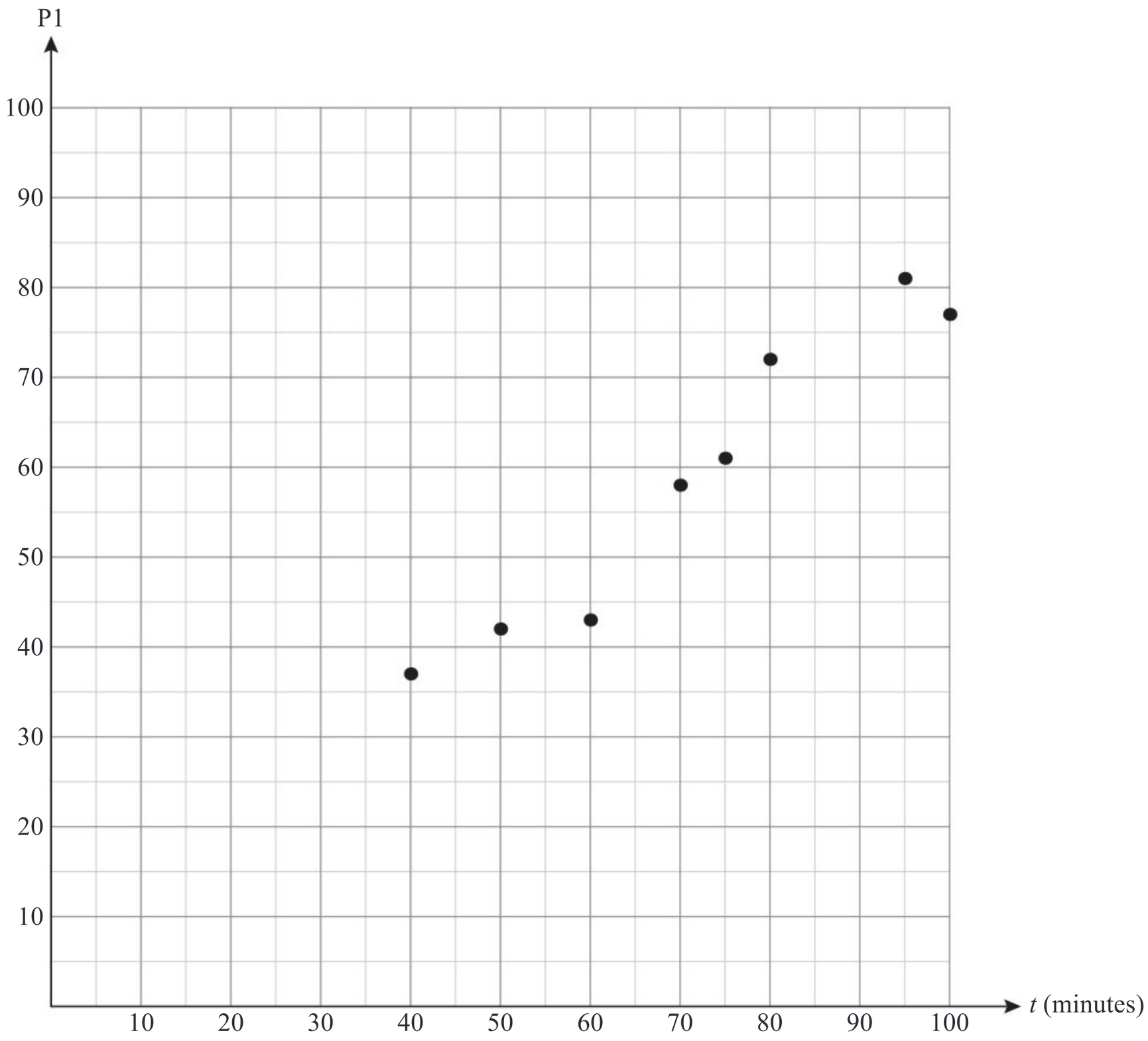
d Find the equation of the appropriate regression line that the teacher should use. *[2]*

e In Paper 1, Student J got 57 marks and Student K got 23 marks.

i Use your regression line to estimate how many marks each student would have got in Paper 2.

ii For each student, comment on the reliability of the estimate, giving reasons for your answers. *[5]*

Students A to H recorded how long they spent revising for Paper 1. The graph shows the time and the Paper 1 mark for each student. The teacher wants to determine whether there is any evidence of positive correlation between the time spent revising and the mark on Paper 1.



f By referring to the graph, explain why Pearson's product moment correlation is not an appropriate measure of correlation. *[1]*

g **i** Complete the table of ranks below.

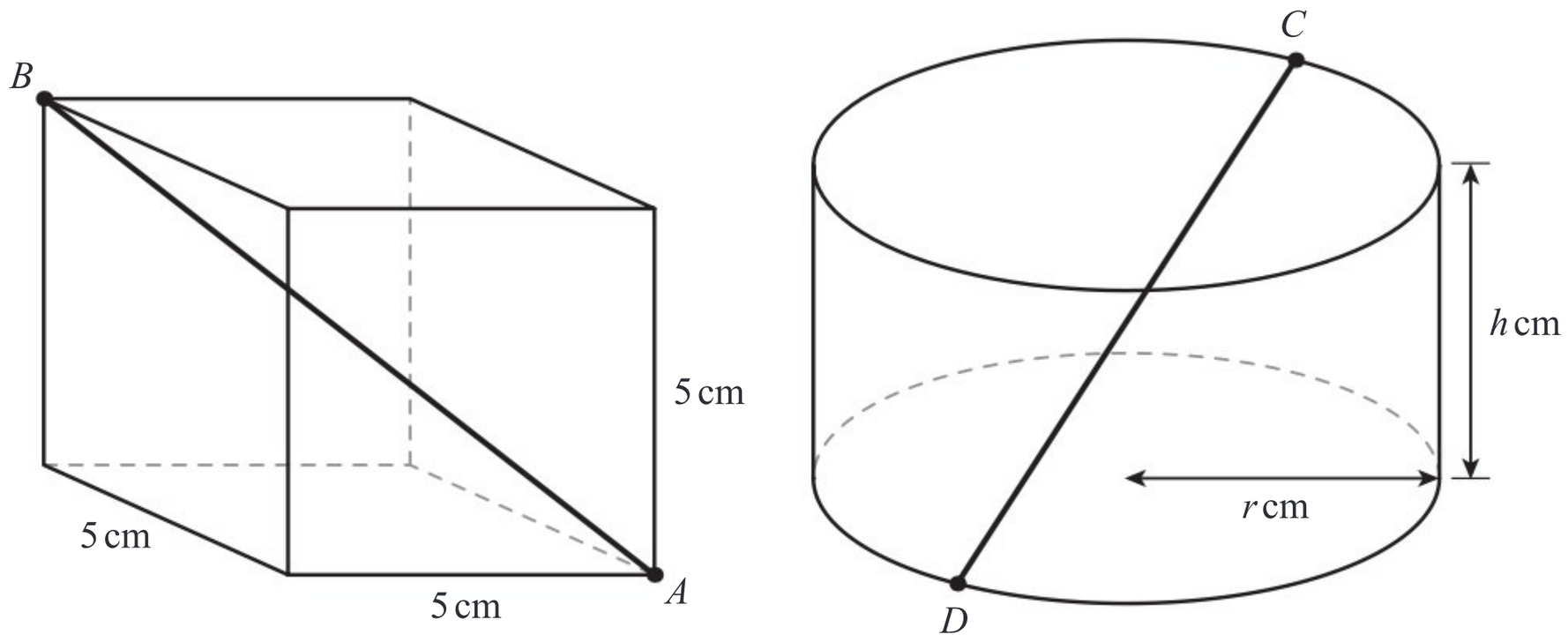
Student	A	B	C	D	E	F	G	H
Revision time rank	1	5		3			4	8
Paper 1 rank	1			3	2		4	7

ii Calculate Spearman's rank correlation coefficient.

iii The critical value of the correlation coefficient for the 5% significance level is 0.643. Stating your hypotheses and conclusion clearly test, at the 5% significance level, whether there is evidence of positive correlation between the time spent revising and the mark on Paper 1. *[5]*

2 *[Maximum mark: 14]*

The diagram shows a cube with side 5 cm and a cylinder with base radius r cm and height h cm.



a Find the length of AB. *[2]*

b Find the angle that the line AB makes with the horizontal base of the cube. *[2]*

The cylinder and the cube have the same volume.

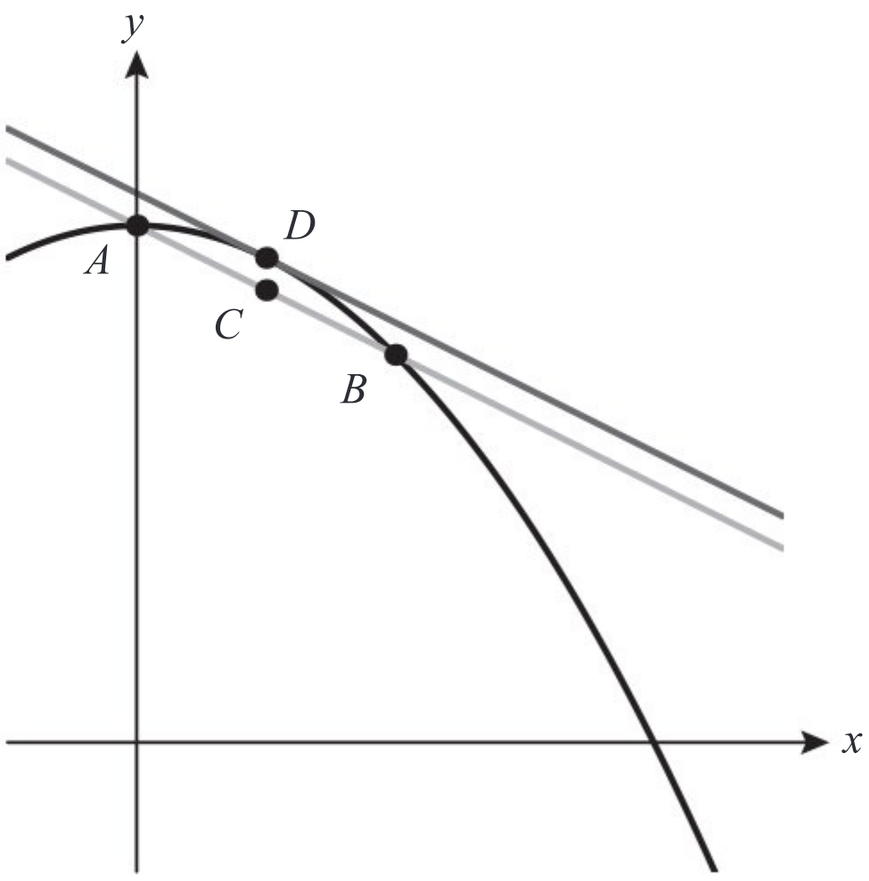
c Show that the surface area of the cylinder is given by $\frac{250}{r} + 2\pi r^2$. *[4]*

d Compare the minimum possible surface area of the cylinder to the surface area of the cube. *[3]*

e Assume the cylinder has the minimum possible surface area found in part **d**. The line CD is the longest line that can be drawn between the bottom base and the top base of the cylinder.
Find the angle that this line makes with the base of the cylinder. *[3]*

3 *[Maximum mark: 11]*

The diagram shows the curve with equation $y = 4 - x^2$. The line $y = 4 - x$ intersects the curve at the points A and B. The point C is the midpoint of AB. The line $y = k - x$ is tangent to the curve at point D.



a Find the coordinates of C. *[3]*

b Find the x-coordinate of D. *[3]*

c Find the value of k. *[3]*

d Find the distance CD. *[2]*

4 *[Maximum mark: 11]*

The times taken by children to complete a race can be modelled by a normal distribution with mean 5.56 minutes and standard deviation 2.5 minutes.

a Find the probability that a randomly selected child completes the race in less than 9.2 minutes. *[1]*

b Given that a randomly selected child completes the race in less than 9.2 minutes, find the probability that they complete the race in less than 8.3 minutes. *[2]*

Twenty randomly selected children run the race.

c Find the expected number of children who complete the race in less than 9.2 minutes. *[2]*

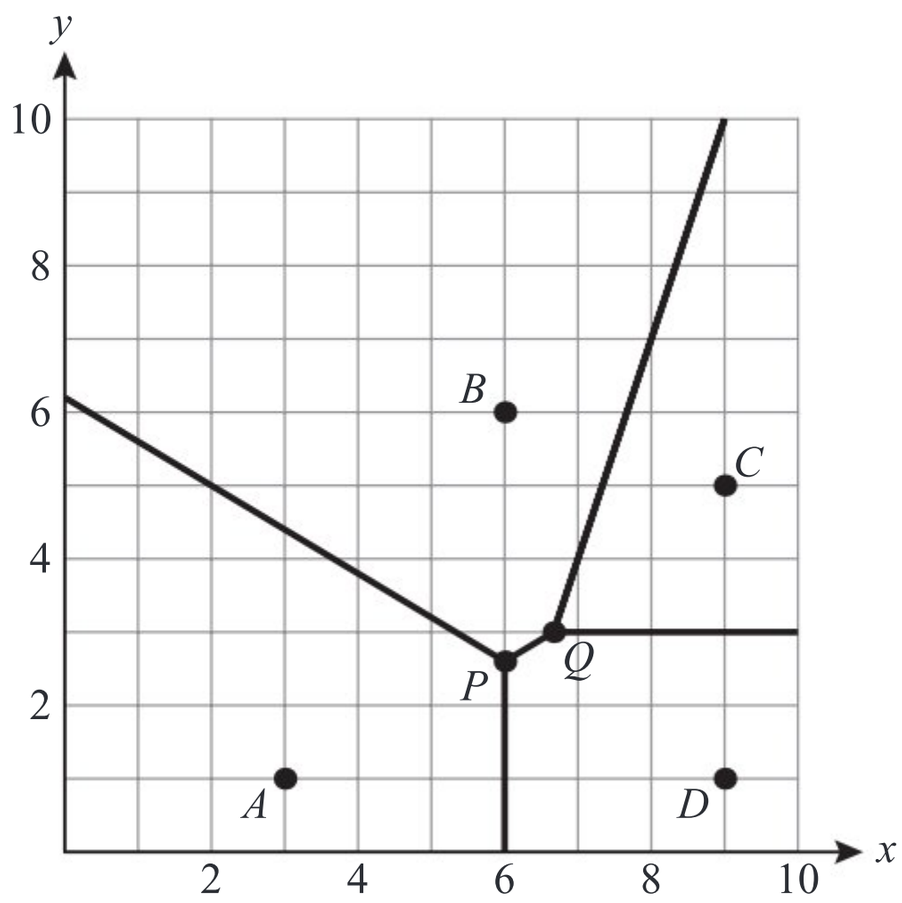
d Find the probability that at least 18 of the 20 children complete the race in less than 9.2 minutes. *[3]*

Two separate groups of 20 children run the race.

e Find the probability that in exactly one of the groups, at least 18 children complete the race in less than 9.2 minutes. *[3]*

5 *[Maximum mark: 15]*

In the Voronoi diagram below, post offices are located at sites $A(3, 1)$, $B(6, 6)$, $C(9, 5)$ and $D(9, 1)$.



a A shop is located at the point with coordinates (5, 4). The manager wants to go to the nearest post office. Which post office should she go to? *[1]*

b Write down the equations of the perpendicular bisectors of AD and CD. *[2]*

c Find the equation of the perpendicular bisector of BD, writing your answer in the form $ax + by = c$ where a, b and c are integers. *[4]*

d Find the coordinates of the vertices P and Q. *[3]*

e A new post office is to be opened at one of P or Q. Which of the two locations should be chosen if the new post office is to be as far as possible from the existing post offices? Show your method clearly. *[5]*

6 *[Maximum mark: 12]*

Newton's law of cooling states that the difference between the temperature of a cooling object and the background temperature decreases exponentially with time. This model can be represented by the equation $T = B + A \times 10^{-kt}$, where T is the temperature of the object in °C, B is the background temperature, t is the time in minutes, and A and k are constants.

A hot cake is placed in a room whose temperature can be assumed to be constant. The difference between the temperature of the cake and the room temperature halves every 3 minutes. The initial temperature of the cake is 93 °C.

a Show that the temperature of the cake after 9 minutes is given by $T = \frac{93 + 7B}{8}$ *[4]*

b Show that $10^{3k} = 2$. *[3]*

It is found that the temperature of the cake after 9 minutes is 30 °C.

c How much longer will it take for the cake to cool down to 24 °C? *[5]*

Practice Set A: Paper 1 Mark scheme

1 **a** 6.45×10^6 (m) A1

[1 mark]

b $4 \times \pi \times (6.45 \times 10^6)^2$ (M1)

$= 5.23 \times 10^{14}$ A1

[2 marks]

c $\dfrac{5.23 \times 10^{14} - 5.10 \times 10^{14}}{5.10 \times 10^{14}} \times 100$ (M1)

$= 2.51\%$ A1

[2 marks]

Total [5 marks]

2 **a** $V = \dfrac{1}{3} \times 6 \times 4^2$ (M1)

32 (cm^3) A1

[2 marks]

b Diagonal of square $= \sqrt{4^2 + 4^2}$ (= 5.66) (M1)

Length from corner to centre of square = 2.83 (A1)

Angle is $\tan^{-1}\left(\dfrac{6}{2.83}\right)$ (M1)

= 64.8° (1.13 radians) A1

[4 marks]

Total [6 marks]

3 **a** $\dfrac{54}{54 + 156 + 12 + 34}$ (M1)

$\dfrac{54}{256}\left(=\dfrac{27}{128}\right)$ A1

[2 marks]

b $\dfrac{156 + 34}{54 + 156 + 12 + 34}$ (M1)

$\dfrac{190}{256}\left(=\dfrac{95}{128}\right)$ A1

[2 marks]

c $\dfrac{12}{12 + 34}$ (M1)

$\dfrac{12}{46}\left(=\dfrac{6}{23}\right)$ A1

[2 marks]

Total [6 marks]

4 **a** 1.10 A1

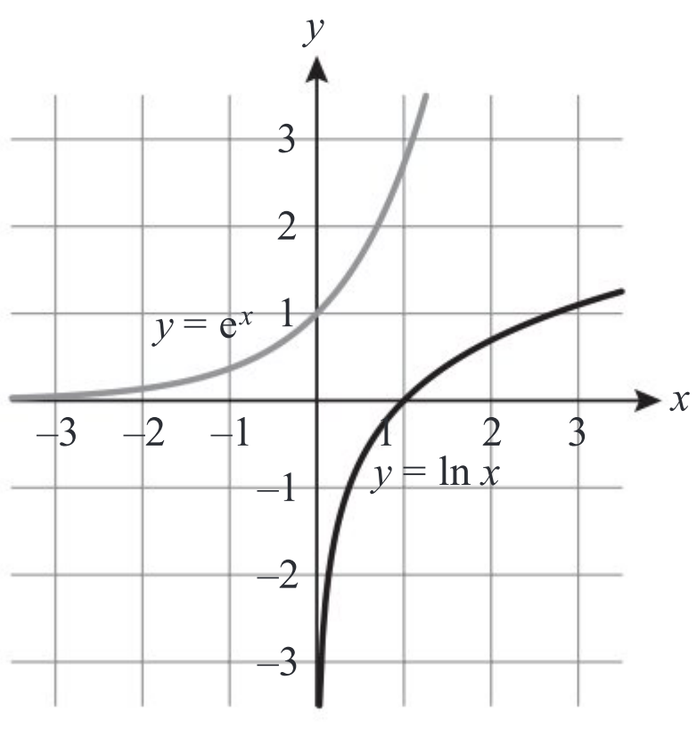
[1 mark]

b Logarithmic graph with y axis as asymptote A1

Passing through (1,0) and roughly (3,1.10) A1

[2 marks]

c **i** Exponential graph (as shown below) passing through (0, 1) A1



ii $y = e^x$ A1

[2 marks]
Total [5 marks]

5 a Ranks are

P	8	7	6	4.5	4.5	3	2	1
D	8	6	7	3	4	5	2	1

M1A1

(Or ranks could be reversed)
So from GDC, $r_s = 0.898$ A1

[3 marks]

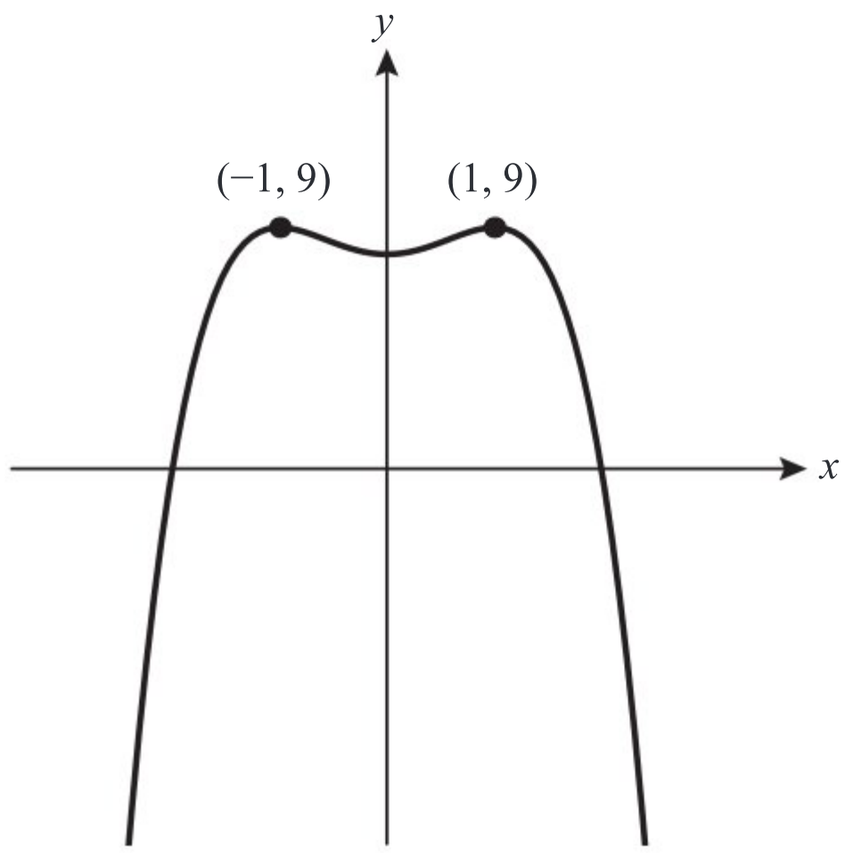
b $0.898 > 0.643$, therefore there is evidence of a positive association A1

[1 mark]

c Value would not change A1
Since an increase in the largest value of D would not change its rank R1

[2 marks]
Total [6 marks]

6 a Sketch of graph (M1)



From GDC, max value is 9 A1

[2 marks]

b Solve $8 + 2x^2 - x^4 = 0$ (M1)
$x = \pm 2$ (A1)
Area $= \int_{-2}^{2} 8 + 2x^2 - x^4 \, dx$ (M1)
$= \frac{448}{15} \approx 29.9$ A1

[4 marks]
Total [6 marks]

7 Expected frequencies are

HH	**Hh**	**hh**
37.5	75	37.5

M1A1
2 degrees of freedom A1
$\chi^2 = 1.72$ (M1)
p-value = 0.423 A1
p-value > 0.05, therefore there is no evidence that hypothesis is incorrect R1

Total [6 marks]

8 a $y = \int 3x^2 + 2 \, dx$ (M1)
$= x^3 + 2x + c$ (A1)
When $x = 0$, $y = 1$ so $y = x^3 + 2x + 1$ (M1) A1

[4 marks]

b When $x = 0$, gradient of tangent is 2 (M1)

So gradient of normal is $-\frac{1}{2}$ (A1)

$y = -\frac{1}{2}x + 1$ (M1)

$2y + x = 2$ A1

[4 marks]

Total [8 marks]

9 EITHER

$S_1 = 7$ (A1)

$S_2 = 10$ (A1)

$u_1 = 7$ A1

Note: Must be made clear that this is the first term

$u_2 = 3$ (M1)

$d = -4$ A1

[5 marks]

OR

$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$ (M1)(A1)

Comparing coefficients:

$\frac{d}{2} = -2$ and $a - \frac{d}{2} = 9$ (M1)

$d = -4$ A1

$a = 7$ A1

[5 marks]

Total [5 marks]

10 a Gradient of AB $= \frac{8-4}{2-0}$ (M1)

$= 2$ (A1)

So gradient of perpendicular line is $-\frac{1}{2}$ (A1)

So equation is $y - 3 = -\frac{1}{2}(x - 1)$ M1A1

$(y = -0.5x + 3.5)$

[5 marks]

b Equation of AB is $y = 2x + 4$ (A1)

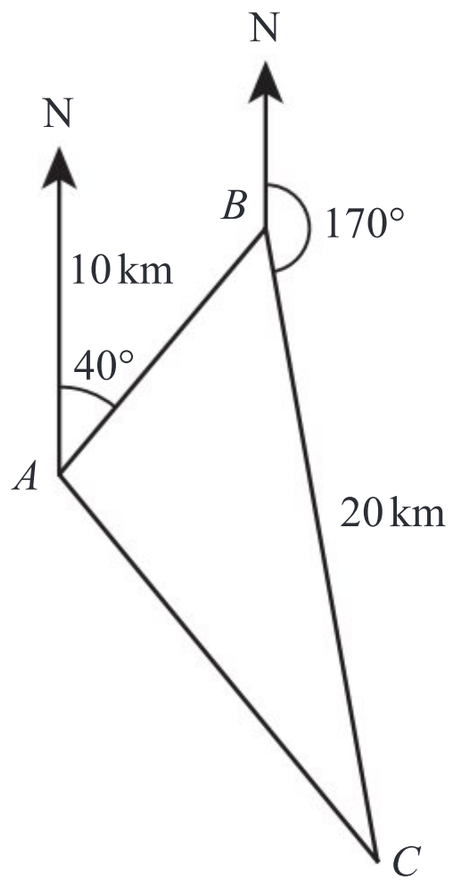
Solve simultaneously to find point of intersection (M1)

$(-0.2, 3.6)$ A1

[3 marks]

Total [8 marks]

11 a



Angle between trajectories is 50° (A1)

Using cosine rule:

$b^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 50$ M1

$(= 242.88 \ldots)$

$c = 15.6$ km A1

[3 marks]

b Using sine rule

$\frac{\sin C}{10} = \frac{\sin 50}{15.6}$ (M1)

$C = \sin^{-1}\left(\frac{10 \sin 50}{15.6}\right) = 29.4°$ A1

Note – could also be found using the cosine rule

Bearing is $360 - 10 - 29.4 = 321$ A1

[3 marks]

Total [6 marks]

12 a $C = 8 + 0.05x$ A1

[1 mark]

b $C = \begin{cases} 10 & 0 < x \leqslant 100 \\ 0.2x - 10 & x > 10 \end{cases}$ A1A1

[2 marks]

c Intersects first branch at $x = 40$ A1

Intersects second branch at $x = 120$ A1

So cheaper for $40 < x < 120$ A1

[3 marks]

Total [6 marks]

13 The midpoint of AB is (2, 3) (A1)

The gradient of AB is 1 (A1)

Therefore, the equation of the perpendicular bisector is

$y = 5 - x$ A1

Then EITHER perpendicular bisector of BC is $y = \frac{1}{2}x$

OR perpendicular bisector of AC is $y = 2x - 5$ A2

Intersecting any two perpendicular bisectors M1

$\left(\frac{10}{3}, \frac{5}{3}\right) \approx (3.33, 1.67)$ A1

Total [7 marks]

Practice Set A: Paper 2 Mark scheme

1 **a** 4000 A1

[1 mark]

b $3x^2 - 12x + 9$ (M1)A1

Note: Award M1 for at least one correct term.

[2 marks]

c Rate of change of number of subscribers A1

[1 mark]

d From GDC: $x = 3$ or $x = 1$ A1A1

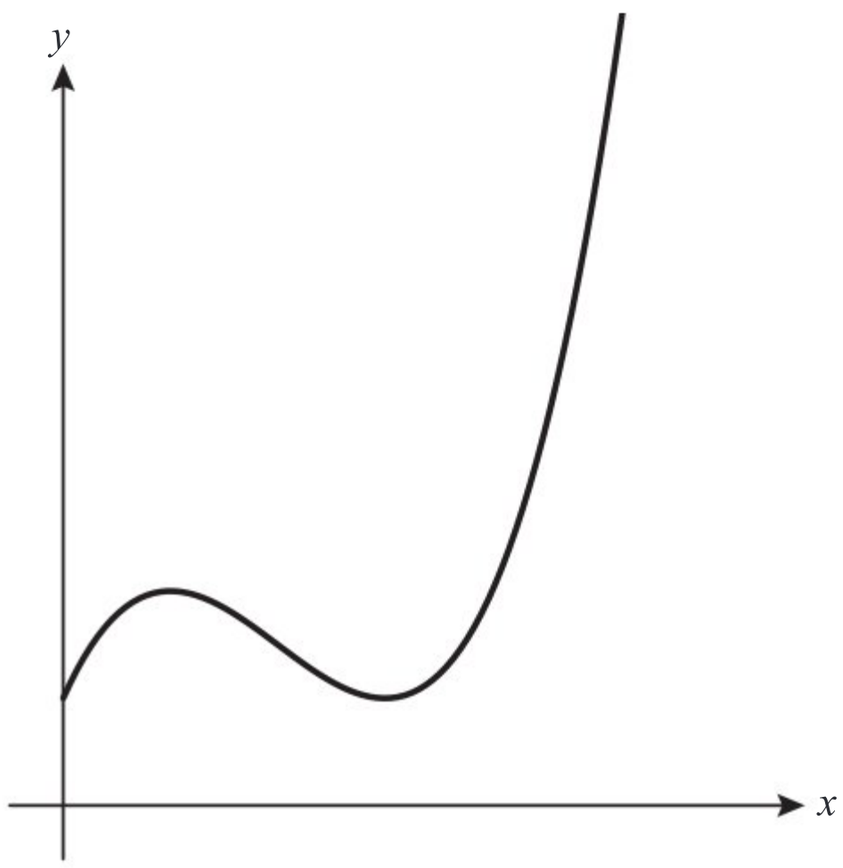
[2 marks]

e $f'(x) > 0$ (M1)

$x > 3$ or $x < 1$ A1

[2 marks]

f



Correct shape, with no negative x values A1

Intercept labelled at $y = 4$ A1

Max labelled at (1, 8), min at (3, 4) A1

[3 marks]

g Solving $x^3 - 6x^2 + 9x + 4$ graphically or using polynomial solver M1

4.20 (weeks) A1

[2 marks]

Total [13 marks]

2 **a** Using a lattice diagram or other systematic list (M1)

s	2	3	4	5	6	7	8
$P(S = s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\mathbf{\frac{3}{16}}$	$\mathbf{\frac{2}{16}}$	$\mathbf{\frac{1}{16}}$

A1

[2 marks]

b $E(S) = 2 \times \frac{1}{16} + 3 \times \frac{2}{16} \cdots$ (M1)

$= 5$ A1

[2 marks]

c $P(X > 4) = \frac{10}{16}$; $P(X > 6) = \frac{3}{16}$ A1

$P(X > 6 \mid X > 4) = \frac{3}{16} \div \frac{10}{16} = \frac{3}{10}$ M1A1

[3 marks]

d If W is the winnings then:

w	-1	k
$\mathbf{P}(W = w)$	$\frac{6}{16}$	$\frac{10}{16}$

M1A1

$\mathrm{E}(W) = -\frac{6}{16} + \frac{10k}{16}$ A1

For the game to be fair, $\mathrm{E}(W) = 0$ so M1

$\frac{10k}{16} = \frac{6}{16}$

$k = 0.6$ A1

[5 marks]

Total [12 marks]

3 **a** **i** 16.1 A1

ii 1.73 A1

iii −0.901 A1

Strong negative correlation A1

[4 marks]

b **i** 8 A1

ii 6 A1

[2 marks]

c **i** $y = \begin{cases} -0.361x + 18.8 & x < 8 \\ -0.0686x + 15.1 & x \geq 10 \end{cases}$ M1A1A1

ii Both are negative so there is a trend of improving in both parts of the season A1

The modulus of the first coefficient is larger, so there is greater improvement each week in the pre-competition training A1

[5 marks]

d $-0.361 \times 7 + 18.8 \approx 16.6$ (minutes) M1A1

[2 marks]

e This is an example of extrapolation, which is not generally valid A1

[1 mark]

Total [14 marks]

4 **a** **i** $1000 + 5 \times 50 = 1250$ (M1)A1

ii $1000 \times 1.04^5 = 1216.65$ (M1)A1

Note: May be done using TVM so no working shown.

[4 marks]

b 4.56% (M1)A1

Note: award M1 for any evidence of using TVM package, eg stating principal value of 1000 and final value of 1250.

[2 marks]

c Solving $1000 + 50n = 1000 \times 1.04^n$ (M1)

Evidence of graphical, tabular or trial and error approach (M1)

$n \geq 12$ A1

Note: do not accept non-integer values.

[3 marks]

d Effective interest rate = 1.5% (A1)

Evidence of TVM or $1400 = 1000 \times 1.015^n$ (M1)

22.599 years (A1)

So needs 23 years A1

[4 marks]

Total [13 marks]

5 **a** From GDC, 0.0478 A1

[1 mark]

b Using inverse normal distribution M1

138(.4465) A1

[2 marks]

c If X = "number of people with score ⩾ 150 out of 5"
$X \sim B(5, 0.0478)$ (M1)(A1)
$P(X \geqslant 2) = 1 - P(X \leqslant 1)$ (M1)
$= 0.0207$ A1
[4 marks]

d We need one success in the first four, then a success (M1)
If Y = "number of people with score ⩾ 150 out of 4"
$Y \sim B(4, 0.0478)$
Required probability is $P(Y = 1) \times 0.0478$ (M1)
$= 0.00789$ A1
[3 marks]

e If A is the score of a member then we require
$P(A > 170 | A > 150)$ (M1)
$= \dfrac{P(A > 170 \cap A > 150)}{P(A > 150)}$
$= \dfrac{P(A > 170)}{P(A > 150)}$ (OR use a Venn diagram) (M1)
$= \dfrac{0.0107}{0.05}$ (M1)
Note: Award M1 for evidence of using GDC to calculate any probability from a N(100, "their value") distribution, even outside of context of conditional probability.
$= 0.214$ A1
[4 marks]

f That the membership of the high intelligence society is representative of the whole population R1
[1 mark]
Total [15 marks]

6 a 1 diameter of 2 cm and 2 radii each of 1 cm R1
[1 mark]

b Total area of metal in each repeating unit $= 2 \times \pi \times 1^2 = 2\pi$ M1A1
If side of the square is x then $x^2 + x^2 = 16$ M1
So proportion of box filled is $\dfrac{2\pi}{8} = \dfrac{\pi}{4}$ A1
[4 marks]

c for example, that the extra space at the edge of the box is negligible R1
Note: Accept any reasonable criticism of the model.
[1 mark]

d Diagonal is 4 and width is 2 A1
So height is $\sqrt{4^2 - 2^2} = \sqrt{12}$ M1A1
Ratio is $\dfrac{2 \times \pi \times 1^2}{2\sqrt{12}} = \dfrac{\pi}{\sqrt{12}}$ M1A1
[5 marks]

e $\sqrt{12} < \sqrt{16} = 4$ R1
Therefore method 2 can pack more rods A1
Note: Do not award R0A1
[2 marks]
Total [13 marks]

Practice Set B: Paper 1 Mark scheme

1 $\frac{1}{2}r^2\theta = 15$ (A1)

$2r + r\theta = 4r\theta$

$\theta = \frac{2}{3}$ A1

Substituting their θ into their area equation: (M1)

$\frac{1}{2}r^2\left(\frac{2}{3}\right) = 15$

$r = \sqrt{45} = 6.71\text{ cm}$ A1

Total [4 marks]

2 **a** Median = 6.5 A1

[1 mark]

b $IQR = 8 - 5 = 3$ A1

[1 mark]

c x is an outlier if $x < Q_1 - 1.5(Q_3 - Q_1)$, so if $x < 5 - 1.5 \times 3 = 0.5$ A1

OR if $x > Q_3 + 1.5(Q_3 - Q_1)$, so if $x > 8 + 1.5 \times 3 = 12.5$ A1

No data values smaller than 0.5 or larger than 12.5 so no outliers R1

[3 marks]

Total [5 marks]

3 **a** $a + (5 - 1)\,d = 8$

$a + 4d = 8$ A1

$\frac{8}{2}(2a + (8 - 1)d) = 58$

$4a + 14d = 29$ A1

$a = 2,\ d = \frac{3}{2}$ A1A1

Note: If a and d both incorrect then award M1A0 for attempt to solve simultaneous equations.

[4 marks]

b $2 + (n - 1)\frac{3}{2} = 26$ (M1)

$3(n - 1) = 48$

$n = 17$ A1

[2 marks]

Total [6 marks]

4 **a** **i** Upper bound $= \frac{219.5}{18.35} = 11.961\,85$ M1A1

ii Lower bound $= \frac{218.5}{18.45} = 11.842\,81$ M1A1

Note: Award M1 each time for $\frac{219.5 \text{ or } 218.5}{18.35 \text{ or } 18.45}$

[4 marks]

b Agreement between upper and lower bound to two significant figures R1

so, $R = 12$ (2 s.f.) A1

[2 marks]

Total [6 marks]

5 **a** $m = \frac{\ln x^2 - 0}{x - 1}$ (M1)

$= \frac{\ln x^2}{x - 1}$ A1

[2 marks]

b Finds sequence of values of m for values of x that approach 1 M1

m tends towards 2 A1

[2 marks]

c The gradient of the function $f(x) = \ln x^2$ A1

at $x = 1$ A1

[2 marks]

Total [6 marks]

6 **a** $H_0: \mu_G = \mu_N$ A1
$H_1: \mu_G < \mu_N$ A1
[2 marks]

b 0.0986 A2
[2 marks]

c $0.0986 < 0.1$ R1
So reject H_0. There is sufficient evidence at the 10% level that Nya Stan is warmer A1
Note: Award R1 for correct comparison of their *p*-value. Must have conclusion in context for A1. Do not award R0A1.
[2 marks]

d The population temperatures are normally distributed A1
The population variances are equal A1
[2 marks]
Total [8 marks]

7 $ar^3 = 13.5\ldots$ (1) A1
$a\left(\frac{1-r^3}{1-r}\right) = 74\ldots$ (2) A1
Dividing their (1) by (2) or substituting: (M1)
$\frac{r^3(1-r)}{1-r^3} = \frac{13.5}{74}$
$74r^3 - 74r^4 = 13.5 - 13.5r^3$
$74r^4 - 87.5r^3 + 13.5 = 0$ M1
Note: Award M1 for rearranging to a quartic equation $p(r) = 0$
$r = \frac{3}{4}$ (reject $r = 1$) A1
$a = 32$ A1
Total [6 marks]

8 **a** $N = 30$
$I\% = 2.5$
$PV = 0$
$PMT = -6000$
$P/Y = C/Y = 1$ (M1)(A1)
Note: Award M1 for attempt to use financial app; A1 for all values correct.
$FV =$ £309 263 416.22 A1
[3 marks]

b $I\% = \frac{2.5}{12}$
$PV = 309\,736.06$
$PMT = 750$
$FV = 0$
$P/Y = C/Y = 1$ (M1)(A1)
Note: Award M1 for attempt to use financial app; A1 for all values correct.
$N = 263.8$
So, 264 months or 22 years A1
[3 marks]
Total [6 marks]

9 **a** Attempt to solve $-3x^2 + 5x + 2 = 0$ (M1)
$x = 2\left(\text{reject } -\frac{1}{3}\right)$ (A1)
So 200 items A1
[3 marks]

b $P(x) = \int -3x^2 + 5x + 2 \, dx$ (M1)

Note: Award M1 for attempt at integration

$= -x^3 + 2.5x^2 + 2x + c$ A1A1

Note: Award A1 for any two correct terms in x; second A1 for all correct including constant of integration

$2 = -1^3 + 2.5 \times 1^2 + 2 \times 1 + c$ M1

$c = -1.5$

So, $P(x) = -x^3 + 2.5x^2 + 2x - 1.5$ A1

[5 marks]

Total [8 marks]

10 a H_0: Waiting times follow a $N(14, 36)$ distribution A1

H_1: Waiting times do not follow a $N(14, 36)$ distribution A1

[2 marks]

b

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Expected frequency	5.34	14.85	25.10	22.01	12.69

A2

Note: Award A2 for all four correct; A1 for two or three correct; A0 otherwise.

[2 marks]

c $v = 4$ (A1)

p-value = 0.0871 A2

[3 marks]

d $0.0871 > 0.05$ R1

So do not reject H_0. There is insufficient evidence to reject the manager's claim A1

Note: Award R1 for correct comparison of their p-value. Must have conclusion in context for A1. Do not award R0A1

[2 marks]

Total [9 marks]

11 a $30 = \frac{2\pi}{b}$ so $b = \frac{\pi}{15}$ A1

When $t = 0$, $h = 2$ so $2 = a \cos 0 + c$ (M1)

$2 = a + c$

When $t = 15$, $h = 122$ so $122 = a \cos\left(\frac{\pi}{15} \times 15\right) + c$ (M1)

$122 = c - a$

Solving simultaneously, $a = -60$, $c = 62$ A1A1

[5 marks]

b $50 = -60 \cos\left(\frac{\pi}{15} t\right) + 62$ (M1)

From GDC, $t = 6.54, 23.5$ (A1)

So time above 50 m is 16.9 minutes A1

[3 marks]

Total [8 marks]

12 a $0.1 + a + b + 0.2 + 0.15 = 1$ M1A1
$a + b = 0.55$ AG
[2 marks]

b Game fair so $E(X) = 2$
$(0 \times 0.1) + a + 2b + (3 \times 0.2) + (4 \times 0.15) = 2$ (M1)
$a + 2b = 0.8$ A1
Solving simultaneously with $a + b = 0.55$,
$a = 0.3$, $b = 0.25$ A1
[3 marks]

c Will make loss if $X_1 + X_2 < 4$ (M1)
(0, 0), (1,1)
(0,1), (0,2), (0,3), (1,2) AND REVERSES
$P(X_1 + X_2 < 4) = 0.1^2 + 0.3^2$
$+ 2\ (0.1 \times 0.3 + 0.1 \times 0.25 + 0.1 \times 0.2 + 0.3 \times 0.25)$ M1
$= 0.4$ A1
[3 marks]
Total [8 marks]

Practice Set B: Paper 2 Mark scheme

1 a i Systematic sampling — A1

ii Not all samples are possible, eg adjacent people on the list cannot be chosen — A1

[2 marks]

b i $\text{Men} = \frac{46}{46 + 63} \times 12 = 5.06$ — M1

So, 5 men — A1

ii Simple random sampling — A1

iii Uses opportunity sampling rather than simple random sampling to select the participants in each group — A1

[4 marks]

c $r = 0.787$ — A1

Reasonable positive correlation between height and weight; as one increases, so does the other — A1

[2 marks]

d i $w = 0.806h - 70.0$ — (M1)

$w = 0.806 \times 140 - 70.0 = 42.8\,\text{kg}$ — A1

ii $w = 0.806 \times 170 - 70.0 = 67.0\,\text{kg}$ — A1

[3 marks]

e 140 cm is significantly outside the range of the given data so extrapolation of the relationship makes the prediction unreliable — A1

170 cm is within the range of the data and reasonable positive correlation so prediction reasonably reliable — A1

[2 marks]

f for example, take a larger sample; create separate regression lines for men and women — A1A1

[2 marks]

Total [15 marks]

2 a Midpoint of (1, 8) and (5, 2) $= \left(\frac{1+5}{2}, \frac{8+2}{2}\right) = (3,5)$ — A1

Gradient of line segment from (1, 8) to (5, 2) $= \frac{2-8}{5-1} = -\frac{3}{2}$ — A1

So gradient of perpendicular bisector is $\frac{2}{3}$ — (M1)

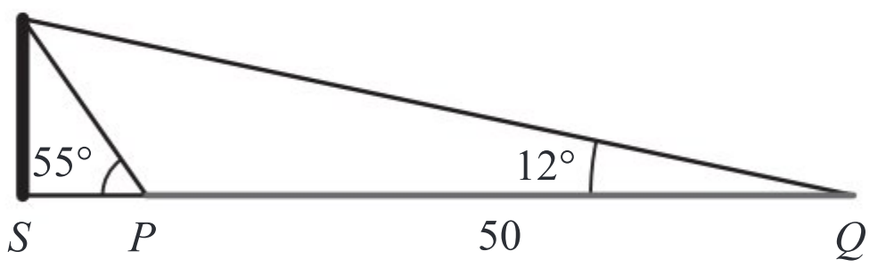
Note: Award M1 for gradient of perpendicular $= -\frac{1}{\text{their } m}$

Equation of perpendicular bisector: $y - 5 = \frac{2}{3}(x - 3)$

$2x - 3y = -9$ — A1

[4 marks]

b



$R\hat{P}Q = 125°$ so $P\hat{R}Q = 180 - 125 - 12 = 43°$ — A1

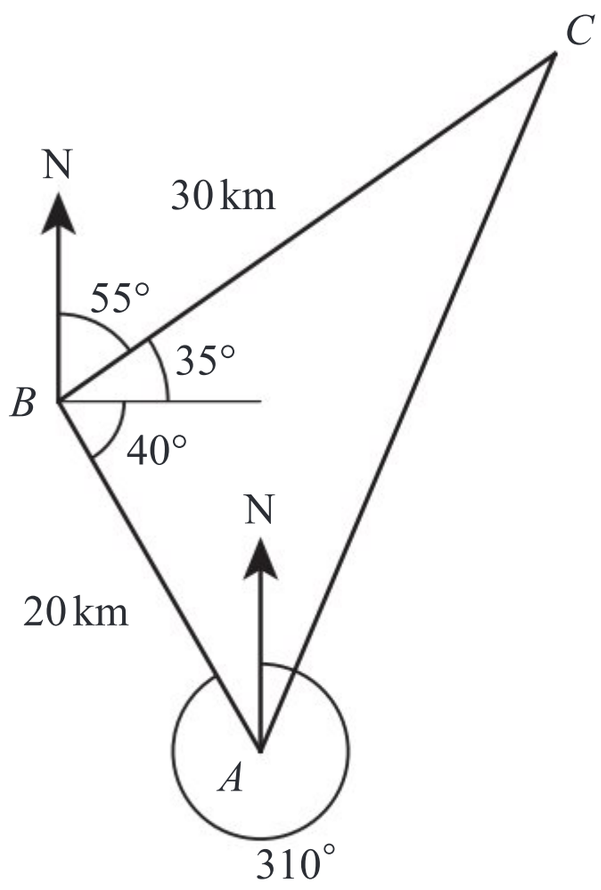
By sine rule $\frac{PR}{\sin 12} = \frac{50}{\sin 43}$ — (M1)

$PR = \frac{50 \sin 12}{\sin 43} = 15.24283\ldots$ — A1

$h = PR \sin 55 = 12.5\,\text{m}$ — (M1)A1

[5 marks]

c



$A\hat{B}C = 40 + 35$ M1
$= 75°$ A1
[2 marks]

d By cosine rule, $AC = \sqrt{20^2 + 30^2 - 2 \times 20 \times 30 \cos 75}$ (M1A1)
Note: Award M1 for attempt to use cosine rule
$= 31.5\,\text{km}$ A1
[3 marks]

e By sine rule, $\frac{\sin B\hat{C}A}{20} = \frac{\sin 75}{31.455}$ (M1)
Note: Award M1 for attempt to use sine rule
$B\hat{C}A = 37.9°$ A1
So bearing = 360 – 125 – their $B\hat{C}A$ (M1)
$= 197°$ A1
[4 marks]
Total [18 marks]

3 a $A = \frac{1}{2}[2.3 + 2.3 + 2(3.5 + 4.3 + 4.7 + 4.7 + 4.3 + 3.5)]$ M1A1
$= 27.3\,\text{m}^2$ A1
[3 marks]

b Since the curve bows out, the trapezia are all under the curve... R1
... so this gives an underestimate A1
Note: Do not award R0A1
[2 marks]

c $h = ax^2 + bx + 2.3$ A1
Substitute in any other two pairs of data: M1
$3.5 = 1^2a + 1b + 2.3$
$4.3 = 2^2a + 2b + 2.3$
Solve simultaneously to give $a = -0.2$, $b = 1.4$ A1A1
[4 marks]

d Finds max point of their quadratic from GDC (M1)
Max height is $h = 4.75\,\text{m}$ A1
[2 marks]

e $A = \int_0^7 -0.2x^2 + 1.4x + 2.3\,\text{d}x$ (M1)
$= \frac{413}{15}$ A1
[2 marks]

f $\%\text{ error} = \frac{\frac{413}{15} - 27.3}{\frac{413}{15}} \times 100$ (M1)
$= 0.847\%$ A1
[2 marks]

g Take more strips A1
[1 mark]
Total [16 marks]

4 **a** $X \sim \text{B}(10, 0.04)$ (M1)
$\text{P}(X = 2) = 0.0519$ A1
[2 marks]

b $\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1)$ (M1)
$= 0.0582$ A1
[2 marks]

c **i** $0.04n = 2$ (M1)
$n = 50$ A1
ii $\text{Var}(X) = 50 \times 0.04 \times 0.96$ (M1)
$= 1.92$ A1
[4 marks]

d $Y \sim \text{B}(5, 0.0582)$ (M1A1)
Note: Award M1 for use of binomial with $n = 5$
$\text{P}(Y > 1) = 1 - \text{P}(Y \leq 1)$ (M1)
$= 0.0301$ A1
[4 marks]

e $\text{P}(X \geq 1) > 0.95$
$1 - \text{P}(X = 0) > 0.95$ (M1)
$\text{P}(X = 0) < 0.05$
$0.96^n < 0.95$ M1
$n > 73$ A1
that is, smallest number of calls is $n = 74$ A1
[4 marks]
Total [16 marks]

5 **a** $\pi r^2 h + \frac{2}{3}\pi r^3 = 300$ M1A1
Note: Award M1 for correct volume of cylinder or hemisphere
$3\pi r^2 h + 2\pi r^3 = 900$ A1
$3\pi r^2 h = 900 - 2\pi r^3$ A1
$h = \frac{900 - 2\pi r^3}{3\pi r^2}$ AG
[4 marks]

b $A = 2\pi rh + \pi r^2 + 2\pi r^2$ (M1A1)
$= 2\pi r\left(\frac{900 - 2\pi r^3}{3\pi r^2}\right) + 3\pi r^2$ M1
$= \frac{600}{r} - \frac{4}{3}\pi r^2 + 3\pi r^2$
$= 600r^{-1} + \frac{5}{3}\pi r^2$ A1A1
Note: Award A1 for $ar^{-1} + cr^2$; second A1 for all correct
[5 marks]

c **i** Attempt to find minimum point of $y = 600x^{-1} + \frac{5}{3}\pi x^2$ from GDC or otherwise (M1)
$A = 233 \text{ cm}^2$ A1
ii $r = 3.86 \text{ cm}$ A1
iii Substituting their r into $h = \frac{900 - 2\pi r^3}{3\pi r^2}$ M1
$h = 3.86 \text{ cm}$ A1
[5 marks]

d For example, may want taller and thinner design for aesthetic reasons, or for ergonomic reasons A1
[1 mark]
Total [15 marks]

Practice Set C: Paper 1 Mark scheme

1 **a** $\frac{18^2\,\pi}{2} \times 83$ (M1)

$4.22 \times 10^4 \text{cm}^3$ A1A1

[3 marks]

b $\frac{4}{3}\pi r^3 = \text{their volume}$ M1

$r^3 = \frac{\text{volume} \times 3}{4\pi}$ (M1)

$r = 21.6\,\text{cm}$ A1

[3 marks]

Total [6 marks]

2 **a** $9.1^2 = 6.8^2 + 4.7^2 - 2(6.8)(4.7)\cos B$ (M1)

$\cos B = -0.227$ (A1)

$B = 103°$ A1

[3 marks]

b $\frac{1}{2}(6.8)(4.7)\sin(\text{their } B)$ M1

$15.6\,\text{cm}^2$ A1

[2 marks]

Total [5 marks]

3 **a** (90, 88) (M1)

$200 - 88 = 112$ A1

[2 marks]

b 15% of 200 = 30 (M1)

Line at 170 on graph crosses at (104, 170) A1

104 g A1

[3 marks]

Total [5 marks]

4 **a** Use $640 = \frac{20}{2}(7 + u_{20})$ or $640 = \frac{20}{2}(14 + 19d)$ (M1)

57 A1

[2 marks]

b $19d = 50$ or $u_{39} - u_{20} = u_{20} - u_1$ (M1)

107 A1

[2 marks]

Total [4 marks]

5 **a** $\frac{\theta}{360} \times \pi \times 10^2 = 75$ M1

$\theta = 85.9$ A1

[2 marks]

b $\frac{\text{their } \theta}{360} \times 2\pi \times 10$ M1

+20 (M1)

35 cm A1

[3 marks]

Total [5 marks]

6 **a** midpoints: 10, 13.5, 17.5, 22.5, 26.5 (M1)

mean = 17.8 A1

SD = 5.40 A1

[3 marks]

b "17.8" × 2.54 (M1)

mean = 45.3 A1

variance = 188 A1

[3 marks]

Total [6 marks]

7 **a** Attempt to find three simultaneous equations — M1

$\begin{cases} 512a + 64b + 8c = 1890 \\ 1000a + 100b + 10c = 1690 \\ 3375a + 225b + 15c = 703 \end{cases}$

All three equations correct — A1

$a = 1.31$, $b = -57.3$, $c = 610$ — A1

[3 marks]

b Attempt to solve $ax^3 + bx^2 + cx = 1720$ — M1

4.6 cm, 9.7 cm or 29.4 cm — A1

[2 marks]

c Find $V(20)$ — M1

$V = -240$; No, model predicts negative volume — A1

[2 marks]

Total [7 marks]

8 **a** $A(1, 0)$, $B(4, 0)$ — A1A1

[2 marks]

b Maximum point marked on a sketch — (M1)

1.14 m — A1

[2 marks]

c $\int_1^4 0.8x\,(4 - x)\,dx$ (condone lack of limits) — M1

$= 2.27\,m^2$ — A1

[2 marks]

Total [6 marks]

9 **a** Using TVM solver: $PV = 50\,000$, $PMT = -1000$, $I = 2.4$, P/Y = C/Y = 12

[to get $N = 52.73$] — M1

53 months (4 years and 5 months) — A1

[2 marks]

b Change N to 48 and find PMT — M1

\$1093.51 — A1

[2 marks]

c In part **a**: Amount left after 52 payments of \$1000 — M1

($FV = 731.56$)

Total paid = \$52 731.56 — A1

In part **b**: Total paid = 48×1093.51 = \$52 488.48, which is less — A1

[3 marks]

Total [7 marks]

10 $\frac{dy}{dx} = 6x^2 - 2ax + 1$ — (M1)

$24 - 4a + 1 = 0$ — M1

$a = \frac{25}{4}$ — A1

$-6 = 16 - 4\left(\text{their } \frac{25}{4}\right) + 2 + 2b$ — M1A1

$b = 0.5$ — A1

Total [6 marks]

11 **a** increases: 8, 10, 7, 8, 7 — (M1)

average = 8 — A1

[2 marks]

b $26 + 11 \times$ "8" — M1

114 — A1

[2 marks]

c $y = 8.06x + 18.5$ — M1

115 — A1

[2 marks]

Total [6 marks]

12 a 20 and 30 A1

[1 mark]

b Any one of 31, 32, 33, 34 A1
The gradient is zero somewhere between 30 and 35 R1

[2 marks]

c Minimum at (v, 4.2) where $v \in \{31, 32, 33, 34\}$ A1
Decreases from 4.6 to 4.2, then increases A1

[2 marks]
Total [5 marks]

13 a $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$ A1

[1 mark]

b

A	B	C	D	E	F	G	H
–1.1	0.7	–1.1	–3.0	2.0	–1.3	–1.5	–2.8

M1A1

[2 marks]

c $\bar{x} = -1.01$, $t = -1.72$ (evidence of using t-test) (M1)
$p = 0.130 > 0.05$ A1
Insufficient evidence that the means are different R1

[3 marks]
Total [6 marks]

14 $P(A \cap B) = P(A \mid B)P(B) \left[= \frac{2}{3}P(A)\right]$ M1

Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ M1

$\frac{1}{5} = P(A) + \frac{1}{6} - \frac{2}{3}P(A)$ A2

$P(A) = \frac{1}{10}$ (A1)

$P(A \cap B) = \frac{1}{15}$ A1

Total [6 marks]

Alternative:

Draw a Venn diagram with $\frac{1}{30}$, x, $\frac{1}{6} - x$ M1A1

$\frac{x}{1/6} = 4\left(x + \frac{1}{30}\right)$ M1A1

$x = \frac{1}{15}$ M1A1

Practice Set C: Paper 2 Mark scheme

1 **a** Means 58.9 and 45.8 A1
Point added on the diagram A1
[2 marks]

b Line of best fit through the means A1
[1 mark]

c 0.969 A1
[1 mark]

d Attempt the correct line M1
$y = 0.833x - 3.28$ A1
[2 marks]

e **i** Attempt to use the line to find y from x M1
Student J: 44 marks A1
Student K: 16 marks A1
ii Student J: reliable as strong correlation R1
Student K: not reliable, as extrapolation R1
[5 marks]

f The correlation does not seem to be linear R1
[1 mark]

g **i** Time ranks correct A1
Paper 1 ranks correct A1

Student	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Revision time rank	1	5	7	3	2	6	4	8
Paper 1 rank	1	5	8	3	2	6	4	7

ii $r_S = 0.976$ A1

iii H_0: There is no correlation between the revision time and the marks
H_1: There is a positive correlation
[both correct] A1
$0.976 > 0.643$, so there is evidence of positive correlation between the revision time and the marks A1
[5 marks]
Total [17 marks]

2 **a** $5^2 + 5^2 + 5^2$ [=75] M1
8.66 cm A1
[2 marks]

b $\sin^{-1}\left(\frac{5}{8.66}\right)$ or $\tan^{-1}\left(\frac{5}{\sqrt{50}}\right)$ M1
35.3° A1
[2 marks]

c $\pi r^2 h = 125$ M1
$h = \frac{125}{\pi r^2}$ A1
$SA = 2\pi rh + 2\pi r^2$, replace h by $\frac{125}{\pi r^2}$ M1
Simplify $2\pi r \times \frac{125}{\pi r^2}$ to $\frac{250}{r}$ A1
[4 marks]

d Graph of $y = \frac{250}{x} + 2\pi x^2$ M1
Minimum value is 138 A1
The surface area of the cylinder is smaller (138 versus 150) A1
[3 marks]

e $r = 2.71$, $h = \frac{125}{\pi r^2} = 5.42$ A1
$\tan\theta = \frac{5.42}{2 \times 2.71}$ M1
$\theta = 45.0°$ A1
[3 marks]
Total [14 marks]

3 **a** $[4 - x^2 = 4 - x]$ or use GDC (M1)
$A(0, 4)$, $B(1, 3)$ A1
(0.5, 3.5) A1
[3 marks]

b $\frac{dy}{dx} = -2x$ A1
$= -1$ M1
$x = \frac{1}{2}$ A1
[3 marks]

c $y = 4 - \left(\frac{1}{2}\right)^2 = \frac{15}{4}$ M1
$k - \frac{1}{2} = \frac{15}{4}$ (M1)
$k = \frac{17}{4}$ A1
[3 marks]

d (their y_D) – (their y_C) M1
$\frac{1}{4}$ A1
[2 marks]
Total [11 marks]

4 **a** 0.927 A1
[1 mark]

b $\frac{P(X < 8.3)}{\text{answer } \mathbf{a}}$ M1
0.931 A1
[2 marks]

c 20 × answer **a** (M1)
18.5 A1
[2 marks]

d Using B(20, answer **a**) M1
$1 - P(X \leq 17)$ M1
0.824 A1
[3 marks]

e Using answer **d** (M1)
$2 \times 0.824 \times (1 - 0.824)$ M1
0.290 A1
[3 marks]
Total [11 marks]

5 **a** B A1
[1 mark]

b $x = 6, y = 3$ A1A1
[2 marks]

c Gradient of $BD = -\frac{5}{3}$ A1
Midpoint $= \left(\frac{15}{2}, \frac{7}{2}\right)$ A1
Equation: $y - \frac{7}{2} = \frac{3}{5}\left(x - \frac{15}{2}\right)$ M1
$3x - 5y = 5$ A1
[4 marks]

d Intersect $3x - 5y = 5$ with $x = 6$ and with $y = 3$ M1
$P\left(6, \frac{13}{5}\right), Q\left(\frac{20}{3}, 3\right)$ A1A1
[3 marks]

e Attempt to find distances from P and Q to one of B or D. M1

$PB = 6 - \frac{13}{5} = 3.4$ A1

$QB = \sqrt{\left(\frac{20}{3} - 6\right)^2 + (3-6)^2} = 4.01$ A1

The post office should be built at Q A1

Because $QB > PB$ R1

[5 marks]

Total [15 marks]

6 a Using $T - B$ halves every 3 minutes

When $t = 0$: $93 - B = A$ A1

When $t = 9$: $T - B = \frac{1}{8}A$ M1

$T - B = \frac{1}{8}(93 - B)$ M1

Rearranges correctly to $T = \frac{93 + 7B}{8}$ A1AG

[4 marks]

b Using $t = 9$ and $t = 0$: $\frac{1}{8}A = A \times 10^{-9k}$ M1

$\frac{1}{8} = 10^{-9k}$ A1

$10^{9k} = 8 \Rightarrow 10^{3k} = 2$ A1AG

[3 marks]

c When $t = 9$: $30 = \frac{93 + 7B}{8}$

When $t = 0$: $93 = B + A$ M1

$A = 72, B = 21$ A1

$10^{3k} = 2$ so $k = \frac{1}{3}\log 2$ (=0.1003) A1

Attempt to solve $24 = 21 + 72 \times 10^{-kt}$ with $k = \frac{1}{3}\log 2$ M1

$t = 13.75$, so another 4.75 minutes A1

[5 marks]

Total [12 marks]

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