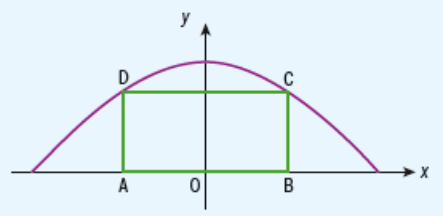
Chapter 13 / **Example 9**

Optimization with derivatives

Derivatives are useful in optimization problems, such as the maximizing or minimizing profits, costs, areas, volumes or distances.

A rectangle is inscribed under the curve $f(x)=2\cos\left(\frac{1}{2}x\right)$, for $-\pi \le x \le \pi$. Points *A* and *B* lie on the *x*-axis, and points *C* and *D* lie on the curve, as shown in the diagram.



The coordinates of *B* are (*x*, 0) and the coordinates of *C* are $\left(x, 2\cos\left(\frac{1}{2}x\right)\right)$.

a Write expressions for the lengths *AB* and *BC* in terms of *x*.

b Write an expression for the area of the rectangle, *A*(*x*), in terms of *x*.

c Find *A*'(*x*).

d Use your answer from part **c** to find the value of x for which the area is a maximum.

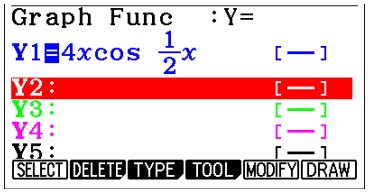
e Use your GDC to plot a graph of *y* = *A*(*x*) and verify your answer from part **d**. Find the maximum area of the rectangle.

$AB = 2x$ and $BC = 2\cos\left(\frac{1}{2}x\right)$

$A(x) = 4x\cos\left(\frac{1}{2}x\right)$

Press MENU 5 GRAPH to display the equation entry screen.

Type $4x\cos\left(\frac{1}{2}x\right)$ and press EXE to enter the equation as Y1.

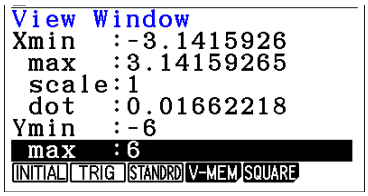


Press SHIFT F3 V-WIN.

Set the axes to show $-\pi \le x \le \pi$ with a scale of 1 and $-6 \le y \le 6$ with a scale of 1

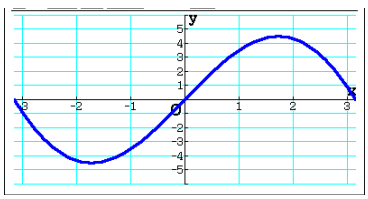
You can leave the other items as they are.

Press EXIT when you have finished.



Press F6 DRAW to display the graph screen

The GDC displays the function $A(x) = 4x\cos\left(\frac{1}{2}x\right)$.



Chapter 13 / **Example 9**

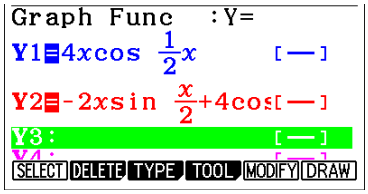
Optimization with derivatives

$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$

To graph the derivative function press [EXIT]

In Y2 type $-2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$

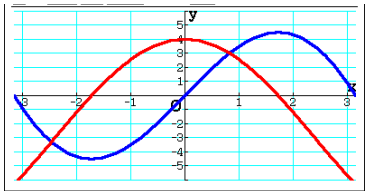
Press [EXE].



Press [F6] DRAW to display the graph screen

The GDC displays $A(x) = 4x\cos\left(\frac{1}{2}x\right)$ as *f*1(x) and

$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$ as *f*2(x).



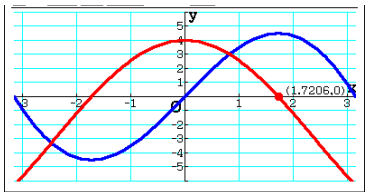
To find the zeros press [F5] G-SOLVE and then press [F1] ROOT.

Use ▼ the select the graph Y2 and press [EXE] to show the coordinates.

The GDC shows the zero. Press ▶ to move the cursor to the zero that corresponds to the maximum point.

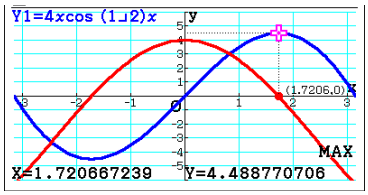
Press [EXIT] to leave G-Solv mode and [F6] DRAW to display the graph screen again.

$A'(x) = 0$ when x = 1.72.



press [F5] G-SOLVE and then press [F2] MAX

Use ▼ the select the graph Y1 and press [EXE] to find the maximum. Press [EXE] to display the coordinates.



Press [EXIT] to leave G-Solv mode and [F6] DRAW to display the graph screen again.

The GDC displays the vertex and verifies that the maximum occurs when $x = 1.72$. The maximum area is 4.49.