OXFORD IB DIPLOMA PROGRAMME

EXAM PRACTICE

MATHEMATICS:
ANALYSIS AND APPROACHES

STANDARD LEVEL

COURSE COMPANION

ENHANCED ONLINE

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Exam practice: chapters 1 – 5

1 P2: Give your answers to parts **a** to **e** to the nearest dollar.

When Maria turned 18 her grandparents gave her three options of how she might receive her birthday present.

Option A: She receives \$50 each month for three years.

Option B: She receives \$1 in the first month, \$4 in the second month, \$7 in the third month, increasing by \$3 each month for three years.

Option C: \$20 in the first month and increasing by 5% each month for three years.

a If Maria chooses Option A, calculate the total value of her present. (2)

b If Maria chooses Option B, calculate

i the amount of money she will receive in the 12th month

ii the total value of her present at the end of the three-year period. (4)

c If Maria chooses Option C, calculate

i the amount of money she would receive in the 12th month;

ii the total value of her present at the end of the three-year period. (4)

d State which of options A, B or C Maria should choose to give her the greatest total value of her present. (1)

2 P2: On 1 Jan 2018, a company purchased a vehicle costing \$26 000. The company expects the vehicle to be operational for 4 years, at the end of which it can be sold for \$6 000.

a Calculate the total depreciation of the vehicle at the end of 2021. (1)

b Hence find the average annual depreciation. (2)

c Assuming that the vehicle depreciates at a constant rate during the 4-year period, sketch a graph to represent the value of the vehicle against time for $0 \le t \le 4$ years.

3 P1: Let $f(x) = a(x-b)^2 + c$. The vertex of the graph of f is at $(1,-4)$.

a State the value of b and the value of c. (2)

The graph passes through $(3,4)$.

b Find the value of a. (3)

4 P2: In an arithmetic sequence, the third term is 15 and the fifth term is 21.

a Find the common difference. (2)

b Find the first term. (2)

c Find the sum of the first 50 terms of the sequence. (3)

5 P1: Without expanding any brackets, find the derivative of the following functions.

a $f(x)=(3x^2+1)(1-2x)$ (3)

b $g(x)=(x^2+3x)^2$ (2)

c $h(x)=\dfrac{4x-1}{2-x}$ (3)

6 P2: A rock is thrown vertically upward from the surface of the moon at a velocity of v_0 m s^{-1}. After t seconds its height is given by $h=24t-0.8t^2$ metres.

a Find expressions for the velocity and acceleration of the rock after t seconds. (3)

b Hence state

i the value of v_0

ii the acceleration due to gravity on the moon. (2)

c Find the maximum height the rock reaches and state the value of t when this occurs. (3)

d State how long it takes for the rock to fall back to the moon's surface. (2)

e Find the values of t for which the rock is at half of its maximum height. (3)

7 P2: The sum of the first n terms of an arithmetic sequence is given by $S_n=2n^2+4n$.

a Write down the value of

i S_1

ii S_2. (2)

The n^{th} term of the arithmetic sequence is given by u_n.

b Find the value of u_2. (2)

c Find the common difference of the sequence. (2)

d Hence show that $u_n=4n+2$ for all $n\in\mathbb{N}$. (2)

e Find the least value of n for which $S_n>100u_n$. (4)

8 P1: Three consecutive terms of an infinite geometric sequence are $a+2$, 6 and $a+7$, where $a\in\mathbb{Z}$.

a **i** Write down two expression for the common ratio, r, in term of a.

ii Hence, show that a satisfies the equation $a^2+9a-22=0$. (4)

b **i** Find the possible values of a.

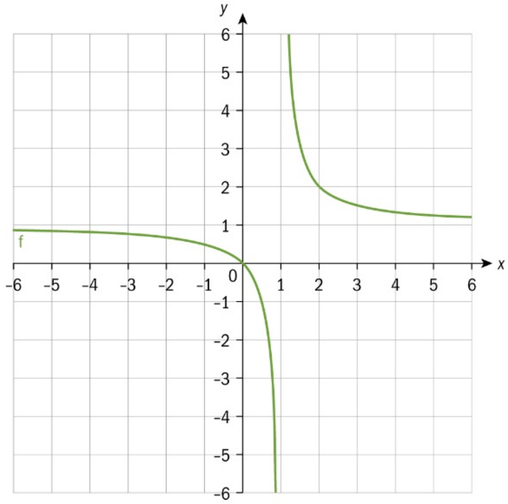
ii Find the possible values of r. (4)

The sum of all the terms of sequence is 9.

c State which value of r leads to this sum. Justify your answer. (2)

d Find the first term of the sequence. (2)

9 P1: The following diagram shows the graph of a function f defined for $x \neq 1$.



a State

i the range of f

ii the equations of the asymptotes to the graph of f. (3)

b On the same axes, sketch and label

i the graph of $g(x) = f(x-2)+1$

ii the graph of $h(x) = \dfrac{1}{f(x)}$. (3)

10 P1: The diagram below shows two consecutive rows of Pascal Triangle.

1 **4** ***a*** ***b*** **1**

1 ***c*** **10** ***d*** ***e*** **1**

a Write down the values of a, b, c, d and e. (5)

b Find the coefficient of x^3 in the expansion $(2x-1)^5$. (3)

11 P2: Let $f(x) = \dfrac{x+2}{5}$ and $g(x) = \dfrac{3x-1}{x-2}$.

a State the largest possible domain of g. (1)

b Find an expression for the inverse of f. (3)

c Hence

i determine $g\left(f^{-1}(x)\right)$ **ii** state the domain of $g \circ f^{-1}$. (3)

The line $y = k$ is an asymptote of the graph of g.

d Find the value of k. (2)

e Solve the inequality $\left|\dfrac{3x-1}{x-2}\right| \geq \dfrac{x+2}{5}$ (6)

12 P1: Consider the function $f(x) = kx(x-6)^2$, where $k > 0$.

a Find an expression for $f(2)$ in terms of k . (2)

b Find an expression for $f'(x)$ in terms of k . (3)

c Hence show that the graph of the function f has a local maximum at $x = 2$. (2)

Given that $f(3) = 54$,

d i Find the value of k

ii Write down the coordinates of the local maximum of f . (3)

e Sketch the graph of the function f for $0 \le x \le 7$. (2)

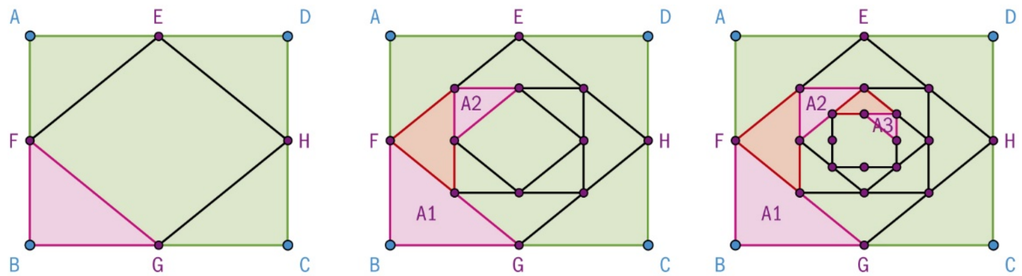
Let T be the tangent to the graph of the function f at $x = 3$.

f Find the gradient of T . (2)

The line L passes through the point (2, 10) and is perpendicular to T.

g Find an equation for L in the form $ax + by + c = 0$, $a, b, c \in \mathbb{Z}$. (3)

13 P1: A rectangle ABCD has dimensions 16 cm by 12 cm. In a Mathematics class, students are instructed to construct a sequence of right-angled triangles by successively dividing the sides of quadrilaterals as shown in the diagram below.



Stage 1 Stage 2 Stage 3

The midpoints of the sides of ABCD are joined to form a quadrilateral EFGH and four triangles, one of which is shown in red (Stage 1). Then the process is repeated twice, as shown in stages 2 and 3, to obtain right-angled triangles that are shown in red.

a**Explain why the quadrilateral EFGH is a rhombus. (2)

b***Find the values of the areas A_1, A_2 and A_3 of the right-angled triangles shown in the diagram. (4)

c**Suppose that the process of obtaining these right-angled triangles is repeated indefinitely. Find an expression for the area A_n. (2)

d**Show that the sum to infinity of $A_1 + A_2 + \dots = 32\ \text{cm}^2$ (2)

14 P1: Let $h = f \circ g$ for functions f and g, where $f(3) = 5$, $f'(3) = -2$, $g(2) = 3$ and $g'(2) = 4$.

a Show that h is decreasing at $x = 2$. (3)

b Find the equation of the tangent to the graph of h at $x = 2$. (3)

15 P1: Consider the family of rational functions defined by $r_k(x) = \dfrac{2kx}{x-k}$, $k \neq 0$.

a Find an expression for the inverse of r_k. (4)

The horizontal asymptote of the graph of $r_k(x)$ is $y = 4$.

b Find the value of k. (2)

c For the value of k found in part **b**, state the domain of the inverse of r_k. (1)

16 P2: A particle, A, is moving along a straight line. The displacement, s metres, of A after t seconds since its motion began is given by $s(t) = \dfrac{1}{4}t^4 - 2t^3 + 3t^2 + t$.

a Sketch the graph of $s = s(t)$, for $0 \le t \le 6.5$ with s on the vertical axis and t on the horizontal. Mark on your sketch the local maximum and minimum points, and the intercepts with the t-axis. (4)

b State the range of $s = s(t)$ for $0 \le t \le 6.5$. (2)

c Find an expression for the velocity $v = v(t)$ of A. (2)

d Write down the intervals of time for which the magnitude of the velocity of the particle is increasing. (2)

e Hence state, with a reason, the time at which the velocity of A is at a *local* maximum. (2)

Answers

1 a $50 \times 12 \times 3 = \$1800$ M1A1

b i $1 + 11 \times 3 = \$34$ M1A1

ii $\dfrac{2 \times 1 + 35 \times 3}{2} \times 36 = \$1926$ M1A1

c i $20 \times 1.05^{11} = \$34$ (nearest dollar) M1A1

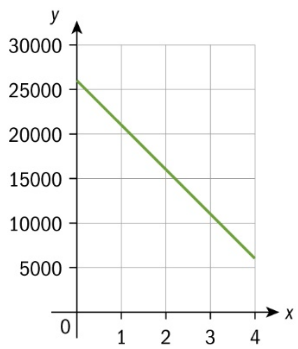
ii $\dfrac{1.05^{36} - 1}{1.05 - 1} \times 20 = \$1917$ (nearest dollar) M1A1

d Option B A1

2 a $26\,000 - 6000 = \$20\,000$ A1

b $\dfrac{20000}{4} = \$5000$ M1A1

c y – intercept at 26 000; A1 Straight line with end-point at (4,6000)



A1

3 a $b = 1$ and $c = -4$ A1A1

b $4 = a(3-1)^2 - 4$ M1A1

$a = 2$ A1

4 a $21 = 15 + 2d$ M1

$d = 3$ A1

b $15 = u_1 + 2 \times 3$ M1

$u_1 = 9$ A1

c $S_{50} = \dfrac{2 \times 9 + 49 \times 3}{2} \times 50$ M1A1

$S_{50} = 4125$ A1

5 a $f'(x) = (3x^2 + 1)'(1 - 2x) + (3x^2 + 1)(1 - 2x)'$ M1

$f'(x) = 6x(1 - 2x) - 2(3x^2 + 1)$ A1

$f'(x) = -18x^2 + 6x - 2$ A1

b $g'(x) = 2(2x + 3)(x^2 + 3x)$ M1A1

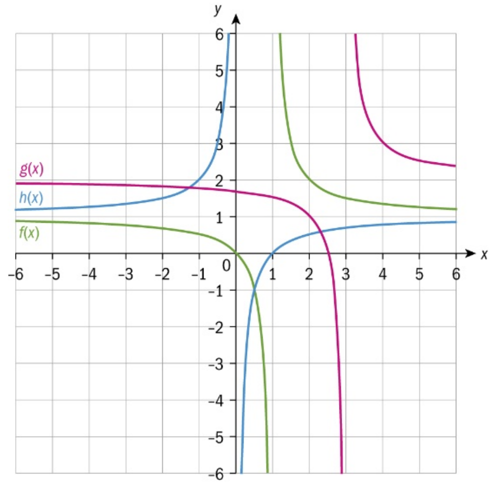
c $h'(x) = \dfrac{(4x-1)'(2-x) - (4x-1)(2-x)'}{(2-x)^2}$ M1

$h'(x) = \dfrac{4(2-x) + (4x-1)}{(2-x)^2}$ A1

$h'(x) = \dfrac{7}{(2-x)^2}$ A1

6	**a**		$v = 24 - 1.6t$ m s^{-1}, $a = -1.6$ m s^{-1}	M1A1A1
	b	**i**	$v_0 = 24$ m s^{-1}	A1
		ii	1.6 m s^{-2}	A1
	c		Use of GDC to find maximum	M1
			180 m after 15 seconds	A1A1
	d		Use of GDC to find zero, or double result from **c**	M1
			30 seconds	A1
	e		Find intersection with line $v = 90$	M1
			4.39 s and 25.6 s	A1A1
7	**a**	**i**	$S_1 = 6$	A1
		ii	$S_2 = 16$	A1
	b		$u_2 = S_2 - S_1$	M1
			$u_2 = 10$	A1
	c		$u_1 = S_1 = 6$	R1
			$d = u_2 - u_1 = 4$	A1
	d		$u_n = 6 + 4(n-1)$	M1A1
			$u_n = 4n + 2$	AG
	e		$S_n = 6n + 2(n-1)n$	A1
			Use GDC to solve $6n + 2(n-1)n > 400n + 200$	M1A1
			199	A1
8	**a**	**i**	$\frac{6}{a+2}$ and $\frac{a+7}{6}$	A2
		ii	$\frac{6}{a+2} = \frac{a+7}{6}$	M1
			$(a+7)(a+2) = 36$	A1
			$a^2 + 9a - 22 = 0$	AG
	b	**i**	Attempt to solve $a^2 + 9a - 22 = 0$	M1
			$a = 2, a = -11$	A1
		ii	$r = -\frac{2}{3}, r = \frac{3}{2}$	A1A1
	c		$r = -\frac{2}{3}$, the common ratio must have absolute value less than 1.	R1
	d		$\dfrac{u_1}{1+\frac{2}{3}} = 9$	M1
			$u_1 = 15$	A1
9	**a**	**i**	$y \neq 1$	A1
		ii	$x = 1, y = 1$	A1A1

b Attempt to transform the graph of f A1



10 a $a = 6, b = 4, c = e = 5, d = 10$ A1A1A1A1A1

b $10 \times 2^3 \times (-1)^2 = 80$ M1A1A1

11 a $x \neq 2$ A1

b $x = \dfrac{y+2}{5}$ M1

Solve for y M1

$f^{-1}(x) = 5x - 2$ A1

c **i** $g\left(f^{-1}(x)\right) = \dfrac{3(5x-2)-1}{(5x-2)-2}$ M1

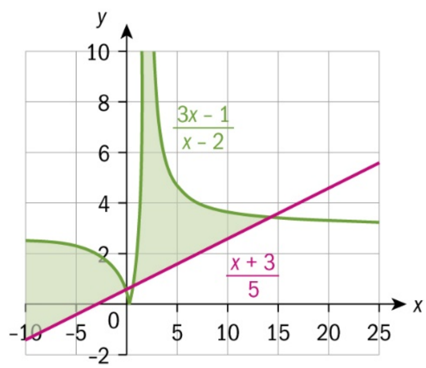
$g\left(f^{-1}(x)\right) = \dfrac{15x-7}{5x-4}$ A1

ii $x \neq \dfrac{4}{5}$ A1

d $\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} \dfrac{3x-1}{x-2} = 3$ M1

$k = 3$ A1

e



Labelled axes with appropriate scale A1

Correct graphs A1A1

$x \leq -0.071, \quad 0.660 \leq x < 2, \quad 2 < x \leq 14.1$ A1A1A1

12 a $f(2) = 2k(2-6)^2 = 32k$ M1A1

b $f'(x) = k(x-6)^2 + 2kx(x-6)$ M1A1

$f'(x) = k(x-6)(3x-6)$ A1

c $f'(2) = 0$ A1

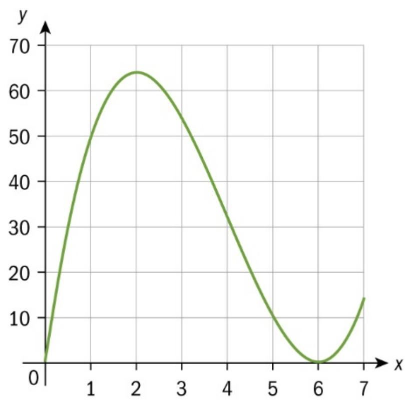
$f'(2^-) > 0$ and $f'(2^+) < 0$ (or considers second derivative) R1

f has a local maximum at $x = 2$ AG

d **i** $3k(3-6)^2 = 54 \Rightarrow k = 2$ M1A1

ii (2, 64) A1

e



Correct zeros A1

Correct shape with one maximum at $x = 2$ A1

f $f'(3) = 2(3-6)(3\times 3-6)$ M1

$f'(3) = -18$ A1

g $y - 10 = \frac{1}{18}(x-2)$ M1A1

$x - 18y + 178 = 0$ A1

13 a All the sides are equal as they are hypotenuses of equal right-angled triangles; R1

the diagonals are parallel to the sides of the rectangle ABCD and are therefore perpendicular. R1

So EFGH is a rhombus. AG

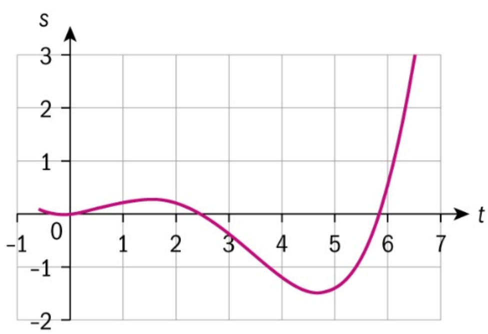
b $A_1 = \frac{8\times 6}{2} = 24$; A1

$A_2 = 24\times\frac{1}{4} = 6$; $A_3 = 6\times\frac{1}{4} = \frac{3}{2}$ M1A1A1

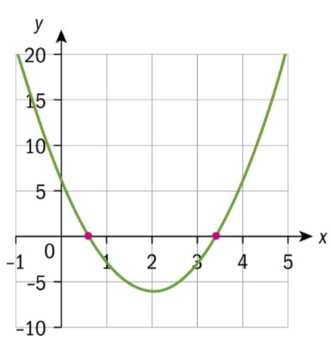
c $A_n = 24\times\left(\frac{1}{4}\right)^{n-1} = \frac{96}{4^n}$ M1A1

d $S = \frac{24}{1-\frac{1}{4}} = 32$ M1A1AG

14 a	$h'(2) = f'(g(2))g'(2)$	M1
	$= f'(3) \times 4 = -8$	A1
	< 0	R1
	h is decreasing at $x = 2$	AG
b	$h(2) = f(g(2)) = f(3) = 5$	A1
	$y - 5 = -8(x - 2)$	M1
	$y = -8x + 21$	A1
15 a	$x = \dfrac{2ky}{y - k}$	M1
	$xy - kx = 2ky \Rightarrow y = \dfrac{kx}{x - 2k}$	M1A1
	$r_k^{-1}(x) = \dfrac{kx}{x - 2k}$	A1
b	$\lim\limits_{x \to \infty} r_k(x) = 4 \Rightarrow 2k = 4$	M1
	$k = 2$	A1
c	$x \neq 4$	A1
16 a	Max (1.47,2.77) Min (4.67,-14.7)	A1A1
	Zeros: 0 and 2.46	A1
	shape and correct domain	A1



b	$-14.7 \le t \le 66.0$	M1A1
c	$v(t) = t^3 - 6t^2 + 6t + 1$	M1A1
d	$0 < t < 0.586$ and $3.41 < t < 6.5$	M1A1
e		



the accelaration changes sign from positive to negative at 0.586 seconds, so the velocity is at a local maximum then. M1A1

Exam practice: chapters 1 – 10

1 P2: Ben practises playing the Oboe daily.

The time (in minutes) he spends on daily practice over 28 days is as follows.

10, 15, 30, 35, 40, 40, 45, 55, 60, 62, 64, 64, 66, 68,

70, 70, 72, 75, 75, 80, 82, 84, 90, 90, 105, 110, 120, 180

a Find the median time (2)

b Find the lower quartile (2)

c Find the upper quartile (2)

d Find the range (2)

e Determine whether there are any outliers in the data (4)

f Draw a box and whisker diagram for the above data, marking any outliers as required.

2 P1: Find the range of values of k for which the equation $3kx^2 + k\sqrt{3}x + 3 = 0$ has two real roots. (6)

3 P1: Consider the quadratic functions $f(x) = x^2$ and $g(x) = \frac{1}{2}x^2 - 3x + \frac{1}{2}$.

a Express $g(x)$ in the form $g(x) = a(x-h)^2 + k$. (4)

b Find the coordinates of the vertex of the graph of $y = g(x)$. (1)

c The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$ by a sequence of three transformations. Describe each transformation in turn. (3)

4 P1: Consider the functions $f(x) = \frac{1}{3}x + 12$ and $g(x) = x^2$.

a Find the value of $(f \circ g)(2\sqrt{3})$. (2)

b Find an expression for $f^{-1}(x)$. (2)

c Solve the equation $f^{-1} \circ g(x) = 0$. (3)

5 P2: Jake and Elisa are given a mathematics problem.

The probability that Jake can solve it is 0.35.

If Jake has solved it, the probability that Elisa can solve it is 0.6, otherwise it is 0.45.

a Draw a tree diagram to illustrate the above situation, showing clearly the probabilities on each branch. (3)

b Find the probability that neither student solves the problem. (2)

c Find the probability that at least one of the students can solve the problem. (2)

d Find the probability that Jake solves the problem, given than Elisa has. (4)

6 P1: In the binomial expansion of $\left(2x^3-\frac{1}{x}\right)^8$, find the coefficient of the term containing x^{12}. (6)

7 P1: Consider the function $f(x)=\frac{2}{3x-1}$, $x\neq\frac{1}{3}$, $x\in\mathbb{R}$.

a State the range of f. (1)

b Sketch the graph of $y=f(x)$, stating clearly the equations of any asymptotes.

State also the coordinates of any points of intersection with the x and y axes. (5)

c Find the inverse function $f^{-1}(x)$. (3)

8 P1: Consider the function $f(x)=\frac{5-8x}{4x+3}$ $x\neq-\frac{3}{4}$, $x\in\mathbb{R}$.

a Write down the equations of the two asymptotes on the graph of $y=f(x)$. (2)

b State the range of f. (1)

c Find an expression for $f\circ f(x)$, giving your answer in the form $f\circ f(x)=\frac{ax+b}{cx+d}$.

State also the domain of $f\circ f(x)$. (5)

9 P1: A population of ferrets has mean age 5.25 years and standard deviation 1.2 years.

a Find the mean age of the same ferrets 3 years later. (2)

b Find the standard deviation of the same ferrets 2 years later. (2)

10 P2: The following raw data is a list the height of flowers (in cm) in Eve's garden:

26.5, 53.2, 27.5, 33.6, 44.6, 39.5, 24.9, 45.1, 47.8, 39.3, 33.1, 38.7, 44.1, 22.3, 44.1, 30.5, 25.5, 35.9, 37.1, 40.2, 23.3, 36.2, 34.8, 37.3

a Copy and complete the following grouped frequency table:

height, (x cm)	frequency
$20\le x<25$	
$25\le x<30$	
$30\le x<35$	
$35\le x<40$	
$40\le x<45$	
$45\le x<50$	
$50\le x<55$	

(3)

b Find an estimate for the mean height, using the frequency table. (2)

c Find an estimate for the variance, using the frequency table. (2)

d Find an estimate for the standard deviation, using the frequency table. (2)

Eve's neighbour's garden was also surveyed. It was found that the flowers in the neighbour's garden had a mean height of 32.1 cm and standard deviation 7.83 cm.

e Compare the heights of the flowers in both gardens, drawing specific conclusions.

11 P1: 'Icicles creamery' decided to analyse their ice-cream sales to see if there was any correlation between sales and the average outdoor temperature for that particular month.

The following data was collected:

Month	**Mean temperature** $(^\circ C)$	**Sales** $(\$)$
January	3	350
February	4	650
March	9	900
April	11	920
May	17	1080
June	22	1200
July	25	1260
August	29	1390
September	19	1220
October	11	880
November	8	770
December	6	500

a Draw a scatter diagram to represent the data. (2)

b Describe the correlation between mean outdoor temperature and sales of ice-cream.(1)

c Comment on whether you can conclude, from this data, that outdoor temperature affects ice-cream sales. (2)

12 P2: Neeve conducts a test to determine if there is any correlation between a person's age and the number of hours they watch television per week.

Age	8	42	17	81	45	14	39	42	31	40	28	24
No. of hours	20	15	30	2	25	28	19	14	16	21	26	20

a Find Pearson's product-moment correlation coefficient (r) for this data. (2)

b Interpret the value of Pearson's product-moment correlation coefficient (r) in the context of the question. (1)

c Find the equation of the y on x regression line. (2)

d Using your result from part c), determine the number hours per week that a 60-year-old might be expected to watch television. (2)

From her data, Neeve concludes that a person's age affects the number of hours per week that they tend to watch television.

e Explain whether or not this is a valid conclusion. If not, suggest what conclusion Neeve could draw from these results. (3)

13 P1: **a** Show that $\log_{16} 4 = \frac{1}{2}$. (3)

b Hence or otherwise, solve the equation $\log_{16}(x-4) - \log_{16}(x-12) = \frac{1}{2}$. Give your answer as an exact value. (4)

14 P1: Find the value of the definite integral $\int_1^6 \frac{dx}{\sqrt{3x-2}}$. (5)

15 P2: **a** Sketch a graph of the region bounded by the curves $y = e^{\frac{x}{2}}$ and $y = 10 - x^2$. (3)

b Write down a definite integral representing the area of the region drawn. (3)

c Find the size of the bounded area using a GDC. (1)

16 P1: Find the indefinite integral $\int \frac{(x-6)^2}{x^2}\,dx$. (6)

17 P1: Consider the function $f(x) = x - 4\sqrt{x},\ x \geq 0$.

a Find an expression for $f'(x)$. (2)

b Find an expression for $f''(x)$. (1)

c Find the coordinates of any turning point(s) on the curve and determine their nature. (5)

d Sketch the graph of $y = f(x)$, showing clearly the coordinates of any turning points and intersections with axes. (6)

18 P2: Consider the function defined by $f(x) = \frac{x^2}{2x^3 - 1},\ x \neq \sqrt[3]{\frac{1}{2}}$.

a Find an expression for $f'(x)$. (3)

b Find the equation of the tangent to the curve $y = f(x)$ at the point where $x = 1$. (4)

c Find the coordinates of the points on the curve $y = f(x)$ where the gradient is zero. (6)

d Without the use of technology, determine the range of values of x for which $f(x)$ is an increasing function. To gain full marks, you must show all your working. (4)

19 P1: A and B are events such that $P(A) = 0.3$, $P(B) = 0.65$ and $P(A \cup B) = 0.7$.

By drawing a Venn diagram (or otherwise), find:

a $P(A \cap B)$ (2)

b $P(A \cup B')$ (2)

c $P(A' \cap B)'$ (2)

20 P2: The first four terms of an arithmetic sequence are 80, 99, 118, 137.

a Find the 30th term in the sequence. (3)

b Find the sum to 13 terms. (2)

c Find the least number of terms required so that the sum exceeds 50 000. To gain full marks, you must show all of your working. (5)

21 P1: Solve the equation $5^{2x} - 5^{x+2} + 100 = 0$. Give all solutions in exact form. (7)

Answers

1 **a** 69 minutes M1A1

b 50 minutes M1A1

c 83 minutes M1A1

d 170 minutes M1A1

e Interquartile range is $83 - 50 = 33$ A1

$Q_3 + 1.5 \times \text{IQ range} = 83 + 1.5 \times 33$ M1

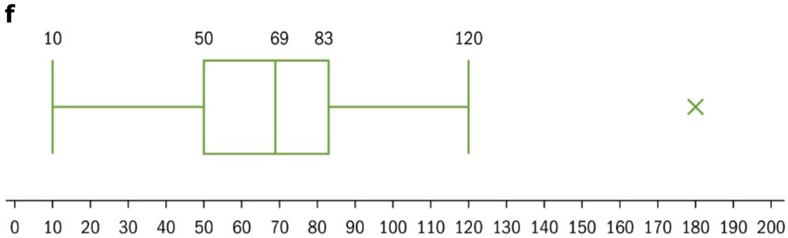
$= 132.5$

$Q_1 - 1.5 \times \text{IQ range} = 50 - 1.5 \times 33$ M1

$= 0.5$

Therefore 180 is an outlier A1

f



M1A1A1

2 Two real roots implies $b^2 - 4ac > 0$ M1

$\left(k\sqrt{3}\right)^2 - 4(3k)(3) > 0$ A1

$3k^2 - 36k > 0$

$k^2 - 12k > 0$

$k(k - 12) > 0$ M1

Critical values are $k = 0$ and $k = 12$ A1

Solution is $k < 0$ or $k > 12$ A1A1

3 **a** $g(x) = \frac{1}{2}x^2 - 3x + \frac{1}{2} = \frac{1}{2}\left[x^2 - 6x + 1\right]$ M1

$= \frac{1}{2}\left[(x-3)^2 - 9 + 1\right]$ M1A1

$= \frac{1}{2}\left[(x-3)^2 - 8\right] = \frac{1}{2}(x-3)^2 - 4$ A1

b $(3, -4)$ A1

c Horizontal translation 3 units to the right A1

Vertical stretch scale factor $\frac{1}{2}$ A1

Vertical translation down 4 units A1

4 **a** $(f \circ g)\left(2\sqrt{3}\right) = f(12) = 16$ M1A1

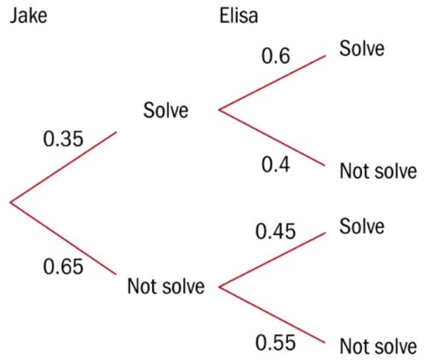
b $y = \frac{1}{3}x + 12 \Rightarrow x = 3(y - 12)$ M1

So $f^{-1}(x) = 3(x - 12)$ A1

c $f^{-1} \circ g(x) = 3\left(x^2 - 12\right)$ M1A1

$f^{-1} \circ g(x) = 0 \Rightarrow x = \pm 2\sqrt{3}$ A1

5 **a**



M1A1A1

b $0.65 \times 0.55 = 0.3575$ M1A1

c $1-(0.65 \times 0.55) = 0.6425$ M1A1

d $\dfrac{P(\text{Jake and Elisa solve})}{P(\text{Elisa solve})} = \dfrac{0.35 \times 0.6}{(0.35 \times 0.6)+(0.65 \times 0.45)}$ M1A1A1

$= 0.418$ A1

6 The general term in the expansion is ${}^8C_r\left(2x^3\right)^r\left(-\dfrac{1}{x}\right)^{8-r}$ M1

$= (-1)^{8-r}\,{}^8C_r\left(\dfrac{2^r x^{3r}}{x^{8-r}}\right) = (-1)^{8-r}\,{}^8C_r\left(2^r x^{4r-8}\right)$ A1

Therefore $4r-8=12$ M1

So $r=5$ A1

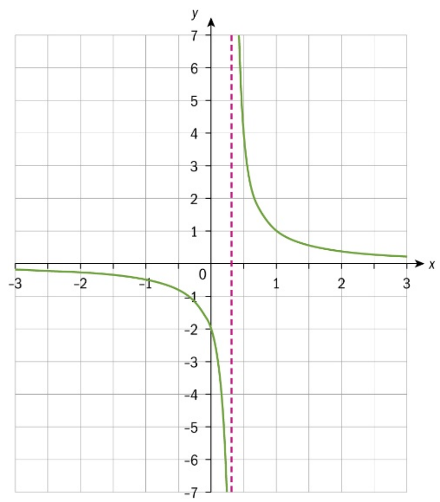
So the term required is ${}^8C_5\left(2x^3\right)^5\left(-\dfrac{1}{x}\right)^3 = -56 \times 2^5 \times x^{12}$

The coefficient is therefore -56×2^5 A1

$= -1792$ A1

7 **a** Range is $f(x) \neq 0$, $(f(x) \in \mathbb{R})$ A1

b



M1A1A1

Asymptotes $x = \dfrac{1}{3}$ and $y = 0$ A1

Intersects $y-$ axis at $(0,-2)$ A1

c $y = \dfrac{2}{3x-1}$

$y(3x-1) = 2$ M1

$3xy - y = 2$

$3xy = 2 + y$

$x = \dfrac{2+y}{3y}$

$f^{-1}(x) = \dfrac{2+x}{3x} \quad \left(\text{or } f^{-1}(x) = \dfrac{1}{3} + \dfrac{2}{3x}\right)$ A1A1

8 a $x = -\dfrac{3}{4}$ A1

$y = -2$ A1

b Range is $f(x) \neq -2$, $(f(x) \in \mathbb{R})$ A1

c $f \circ f(x) = \dfrac{5 - 8\left(\dfrac{5-8x}{4x+3}\right)}{4\left(\dfrac{5-8x}{4x+3}\right)+3}$ M1A1

$= \dfrac{5(4x+3) - 8(5-8x)}{4(5-8x) + 3(4x+3)}$ A1

$= \dfrac{20x + 15 - 40 + 64x}{20 - 32x + 12x + 9} = \dfrac{84x - 25}{29 - 20x}$ A1

$x \neq \dfrac{29}{20}$, $(x \in \mathbb{R})$ A1

9 a $5.25 + 3 = 8.25$ years M1A1

b 1.2 years M1A1

10 a

height, (x cm)	frequency
$20 \leq x < 25$	4
$25 \leq x < 30$	2
$30 \leq x < 35$	4
$35 \leq x < 40$	7
$40 \leq x < 45$	4
$45 \leq x < 50$	2
$50 \leq x < 55$	1

M1A1A1

b Using GDC, mean height $= 35.6$ cm M1A1

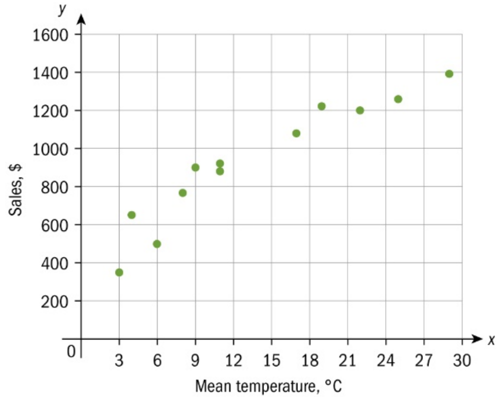
c Using GDC, variance $= 68.3$ cm2 M1A1

d Using GDC, SD $= 8.27$ cm M1A1

e On average, the neighbour's garden's flowers had a slightly lower height compared to Eve's. R1

The neighbour's flowers also had a smaller standard deviation, indicating they were grown to a more consistent length. R1R1

11 a



M1A1

b (strong) positive correlation A1

c You cannot be sure that outdoor temperature affects ice-cream sales, as correlation does not imply causation. R1

The graph demonstrates that there is a link between outdoor temperature and ice-cream sales, but you cannot say that one causes the other as there may be other factors involved. R1

12 a Use of GDC to give $r = -0.775$ M1A1

b This is a strong negative correlation. i.e. as a person's age increases, the number of hours they watch TV decreases. R1

c Use of GDC to give $y = -0.306x + 30.1$ M1A1

d $y = -0.306 \times 60 + 30.1$ M1

$= 11.7$ hours A1

e Neeve is incorrect. A1

A value of $r = -0.775$ indicates a strong negative correlation between a person's age and the hours per week they watch TV. R1

However, you cannot say this is causal. R1

(i.e. You cannot conclude that your age affects the amount of TV you watch.)

13 a $\log_{16} 4 = \dfrac{1}{\log_4 16}$ M1

$= \dfrac{1}{\log_4 4^2}$ M1

$= \dfrac{1}{2\log_4 4} = \dfrac{1}{2}$ A1

b $\log_{16}(x-4) - \log_{16}(x-12) = \dfrac{1}{2}$

$\log_{16}(x-4) - \log_{16}(x-12) = \log_{16} 4$ M1

$\log_{16}\left(\dfrac{x-4}{x-12}\right) = \log_{16} 4$ M1

$\dfrac{x-4}{x-12} = 4$ A1

$x - 4 = 4x - 48$

$3x = 44$

$x = \dfrac{44}{3}$ A1

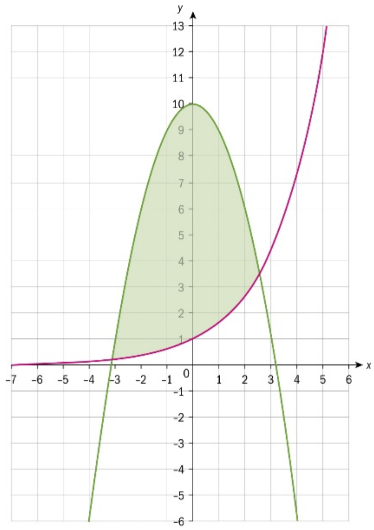
14 $\int_1^6 \frac{dx}{\sqrt{3x-2}} = \int_1^6 (3x-2)^{-\frac{1}{2}}\, dx$ M1

$= \frac{2}{3}\left[(3x-2)^{\frac{1}{2}}\right]_1^6$ M1A1

$= \frac{2}{3}(4-1)$ M1

$= 2$ A1

15 a



M1A1A1

b $\int_{-3.129}^{2.538} \left(10 - x^2 - e^{\frac{x}{2}}\right) dx$

Correct limits A1

Correct expression A1

All correct A1

c 34.3 A1

16 $\int \frac{(x-6)^2}{x^2}\, dx = \int \frac{x^2 - 12x + 36}{x^2}\, dx$ M1A1

$= \int 1 - \frac{12}{x} + 36x^{-2}\, dx$ M1A1

$= x - 12\ln|x| - \frac{36}{x} + c$ M1A1

17 a $f(x) = x - 4x^{\frac{1}{2}}$

$f'(x) = 1 - 2x^{-\frac{1}{2}}$ M1A1

b $f''(x) = x^{-\frac{3}{2}}$ A1

c Attempting to solve $f'(x) = 0$ M1

$1 - 2x^{-\frac{1}{2}} = 0$

$1 - \frac{2}{\sqrt{x}} = 0$

$x = 4$ A1

$f(4) = 4 - 4\left(4^{\frac{1}{2}}\right) = -4$ A1

Therefore $(4,-4)$ is the only turning point

$f''(4) = 4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{8}$ A1

$\frac{1}{8} > 0$. Therefore a minimum point R1

d Intersects $y-$ axis at $(0,0)$ A1

Intersects $x-$ axis where $x - 4x^{\frac{1}{2}} = 0$ M1

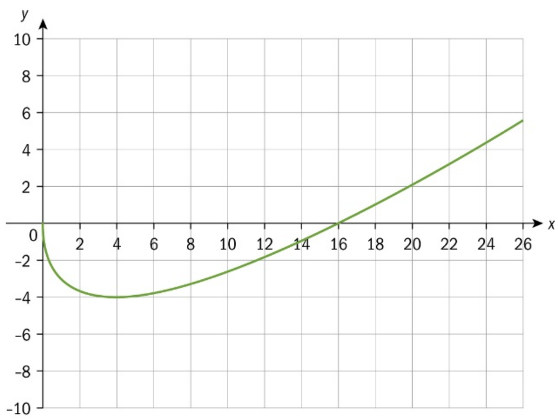
$x\left(1 - 4x^{-\frac{1}{2}}\right) = 0$ M1

$1 - 4x^{-\frac{1}{2}} = 0$ or $x = 0$

$1 - \frac{4}{\sqrt{x}} = 0$

$x = 16$ A1

So crosses $x-$ axis at $(16,0)$



A1 shape; A1 minimum; A1intercepts

18 a Attempt to use quotient rule M1

$f'(x) = \frac{2x\left(2x^3 - 1\right) - x^2\left(6x^2\right)}{\left(2x^3 - 1\right)^2} = \frac{-2x^4 - 2x}{\left(2x^3 - 1\right)^2}$ A1A1

b At $x = 1$, $y = 1$ A1

$f'(1) = \frac{-4}{1} = -4$ A1

Equation is $y - 1 = -4(x - 1)$ M1A1

Or $y = 5 - 4x$

c Setting $f'(x) = 0$ M1

$\frac{-2x^4 - 2x}{\left(2x^3 - 1\right)^2} = 0$

$-2x^4 - 2x = 0$ M1

$2x^4 + 2x = 0$

$2x\left(x^3 + 1\right) = 0$ M1

So $x = 0$ or $x = -1$ A1

Coordinates are $(0,0)$ and $\left(-1, -\frac{1}{3}\right)$ A1A1

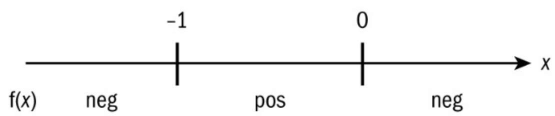
d Setting $f'(x) > 0$ M1

$-2x^4 - 2x > 0$

$2x^4 + 2x < 0$

$2x(x^3 + 1) < 0$

Critical values are $x = -1,\ x = 0$ A1



(or similar method to consider sign of $f(x)$ either side of critical values) M1

So solution is $-1 < x < 0$ A1

19 a $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.3 + 0.65 - 0.7 = 0.25$ M1A1

b $P(A \cup B') = P(A) + (1 - P(A \cup B)) = 0.3 + (1 - 0.7) = 0.6$ M1A1

c This is the same region as described in part b R1

so has probability 0.6. A1

20 a $u_{30} = u_1 + (n-1)d$ M1

$= 80 + 29 \times 19$ A1

$= 631$ A1

b $S_{13} = \frac{n}{2}\left[2u_1 + (n-1)d\right]$ M1

$= \frac{13}{2}\left[160 + 12 \times 19\right]$

$= 2522$ A1

c Require $S_n > 50\,000$

Consider $\frac{n}{2}\left[160 + 19(n-1)\right] = 50\,000$ M1

$19n^2 + 141n - 100\,000 = 0$ A1

Valid attempt to solve three term quadratic M1

$n = \frac{-141 \pm 2760.4}{38}$

$n > 0$, so $n = \frac{-141 + 2760.4}{38} = 68.9$ A1

So require 69 terms. A1

21 $5^{2x} - 25 \times 5^x + 100 = 0$ M1

Substitute $u = 5^x$ (or recognise as a quadratic in 5^x) M1

$u^2 - 25u + 100 = 0$ A1

$(u - 20)(u - 5) = 0$ or other valid attempt to solve M1

$u = 20$ or $u = 5$ A1

$5^x = 20 \Rightarrow x = \log_5 20$ (or $x = \frac{\ln 20}{\ln 5}$ or similar) A1

$5^x = 5 \Rightarrow x = 1$ A1

Exam practice: chapters 1 – 14

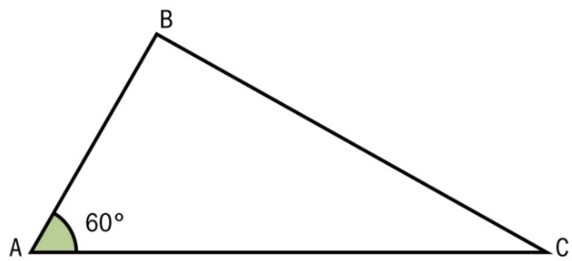
1 P2: In a geometric series, the second term is 24 and the fifth term is 1.536.

a Find the common ratio for the series. (5)

b Find the first term in the series. (2)

c Find the sum to infinity of the series. (2)

2 P1: Triangle ABC has sides $AB = 4\sqrt{2} - 3$, $AC = 4\sqrt{2} + 3$ and $\angle BAC = 60^\circ$



a Find the exact area of the triangle. (3)

b Find the exact length of BC. (4)

3 P2: A study was conducted to determine if there was any correlation between a person's age (x) and their reaction time (T). A number of people were tested, and the mean reaction time calculated for each age is shown in the table below.

Age x (to the nearest 10 years)	10	20	30	40	60	70	80
Mean reaction time (T seconds)	0.125	0.148	0.166	0.221	0.231	0.270	0.341

a Find Pearson's product-moment correlation coefficient (r) for this data. (2)

b Use your result from part (a) to describe the strength of the correlation between x and T. (1)

c Find the equation of the regression line of T on x. (2)

d Using the above data, determine an estimate for the reaction time of a 50-year-old. (2)

e Explain why would it be unwise to use your answer to part c) to determine the reaction time of a 90-year-old. (1)

4 P1: Find the term in x^3 in the binomial expansion of $(2-x)(3+x)^5$ (5)

5 P1: Sanju invests $\$500$ in a savings account earning compound interest at a rate of 2.9%

At the same time, Pranav invests $\$800$ in a different account, earning compound interest at a rate of 2%. After N years, both Sanju and Pranav will have the same amount in their respective accounts.

a Show that $1.075^N = 1.6$ (3)

b Hence find the least number of complete years they must wait for Sanju to have more money that Pranav. (4)

6 P1: Two functions are given by

$$f(x) = \frac{5x-3}{2x+6}\ ,\ x \neq -3\ ,\ x \in \mathbb{R} \text{ and } g(x) = \sqrt{x}\ ,\ x > 0\ ,\ x \in \mathbb{R}.$$

a Write down the equation of the vertical asymptote of the graph of $y = f(x)$. (1)

b Write down the equation of the horizontal asymptote of the graph of $y = f(x)$. (1)

c Sketch the graph of $y = f(x)$. On your sketch, mark the horizontal and vertical asymptotes as dashed lines. (3)

d Write down the range of the function $f(x)$. (1)

e Write down an expression for the function $g \circ f(x)$. (1)

f Find the domain of $g \circ f(x)$. (2)

7 P1: Points O, A and B are given by the coordinates $O(0,0)$, $A(-12,18)$ and $B(18,-2)$.

a Find the equation of the line AB, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. (4)

b Hence find the area of the triangle bounded by the line AB and the coordinate axes. (3)

8 P1: **a** Solve the inequality $(x+4)(3-x) > 0$. (3)

b Solve the inequality $2x^2 - 11x + 9 < 0$. (4)

c Hence, solve simultaneously the inequalities $(x+4)(3-x) > 0$ and $2x^2 - 11x + 9 < 0$. (1)

9 P1: A quadratic function $y = ax^2 + bx + c$ intersects the points with coordinates $(0,18)$, $(-1,27)$ and $(3,3)$.

a Find the values of a, b and c. (5)

b Use the discriminant and your answer to part (a) to determine the number of solutions to the equation $ax^2 + bx + c = 0$. (3)

c Express y in the form $y = p(x-q)^2 + r$ and hence write down the coordinates of the vertex of the graph. (3)

10 P1: Solve the equation $\log_{10} x = \dfrac{45}{\log_{10} x} - 12$. Give your answers as powers of 10. (5)

11 P1: Find the equation of the normal to the curve $y = \dfrac{2x+\sqrt{x}}{x\sqrt{x}}$ at the point on the curve where $x = 4$.

Give your answer in the form $ax+by+c=0$ where a,b,c are integers. (8)

12 P1: The line $y = k$ is a tangent to the curve $y = e^{x}(1-x)$.

Determine the value of k . (6)

13 P1: Given $\int_{2}^{a} \dfrac{10}{5x+4}\,dx = 4\ln 2$, $(a>2)$, find an exact value for a . (7)

14 P1: Solve the equation $\cos 2\theta = 3\cos\theta - 2$, giving all solutions in the range $0 \le \theta < 2\pi$. (7)

15 P2: Alison walks to school every day. The time she takes to walk to school is modelled as a normal distribution, with mean 36 minutes and standard deviation 3.12 minutes.

a Find the probability that on any randomly selected day, Alison's journey to school takes longer than 40 minutes. (2)

b Find the probability that on any randomly selected day, Alison's journey takes between 34 and 38 minutes. (4)

c If the probability that Alison walks for longer than M minutes is 0.015, find the value of M . (3)

d Given that Alison walks to school on 195 days of the year, find the number of days on which she can expect to reach school in under 30 minutes. (4)

16 P2: Approximately 4% of eggs produced and sold by a local farm are cracked.

Jerry buys 24 eggs from the farm.

a Find the probability that exactly two of Jerry's eggs are cracked. (2)

b Find the probability that Jerry buys no more than four cracked eggs. (2)

c Find the probability that Jerry buys at least two cracked eggs . (2)

d Find the variance of the number of cracked eggs. (2)

17 P1: Two events A and B are independent. It is given that $P(A)=0.3$ and $P(B)=0.8$.

a State, with a reason, whether events A and B are mutually exclusive. (2)

b Find the probability of the event:

i $A \cap B$

ii $A \cup B$

iii $A \mid B'$

iv $A' \cap B$ (8)

18 P1: By using the substitution $u = 1+\cos 2x$, show that $\int_{0}^{\frac{\pi}{3}} \dfrac{\sin 2x}{1+\cos 2x}\,dx = \ln 2$. (8)

19 P1: Consider the functions $f(x) = x^2 - 25 \quad (x \le 0, x \in \mathbb{R})$ and $g(x) = 4 - x \quad (x \in \mathbb{R})$.

a Find an expression for $g^{-1}(x)$, stating clearly the domain. (2)

b Find an expression for $f^{-1}(x)$, stating clearly the domain. (5)

c Solve the equation $f \circ g(x) = 0$. (4)

20 P2: Over the course of a single December day in Limassol, Cyprus, the highest temperature was found to be 22° C. The lowest temperature was 12° C, which occurred at 0300 hours.

If t is the number of hours since midnight, the temperature T may be modelled by the equation $T = A\sin(B(t - C)) + D$.

a Find the values of A, B, C and D. (9)

b Hence, by using technology, find for how many hours the daily temperature is above 20° C. (4)

21 P1: The first four terms in an arithmetic series are given by

$\log_2 343,\ \log_2 x,\ \log_2 y,\ \log_2 1331$

Find the value of x (9)

22 P1: The velocity, v m s^{-1}, of an object moving along a horizontal line at time t seconds is given by $v = \cos t - \sin t$ where t is in radians, and $0 \le t \le \pi$.

a Find the maximum speed of the object. Give your answer in exact form. (7)

b Find the displacement of the object after π seconds. (4)

c Find the exact value of the total distance travelled by the object after π seconds.

(7)

Answers

1 a $ar = 24$ A1

$ar^4 = 1.536$ A1

Attempt to solve simultaneously M1

$\frac{ar^4}{ar} = \frac{1.536}{24}$

$r^3 = 0.064$ A1

$r = 0.4$ A1

b $a = \frac{24}{r} = \frac{24}{0.4} = 60$ M1A1

c $S_\infty = \frac{a}{1-r}$ M1

$= \frac{60}{1-0.4} = \frac{60}{0.6} = 100$ A1

2 a Attempt to use $\frac{1}{2}bc\sin A$ for area M1

Area $= \frac{1}{2} \times \left(4\sqrt{2}-3\right) \times \left(4\sqrt{2}+3\right) \times \sin 60^\circ$ A1

$= \frac{1}{2} \times 23 \times \frac{\sqrt{3}}{2}$

$= \frac{23\sqrt{3}}{4}$ A1

b Attempt to use cosine rule M1

$AC^2 = \left(4\sqrt{2}-3\right)^2 + \left(4\sqrt{2}+3\right)^2 - 2\left(4\sqrt{2}-3\right)\left(4\sqrt{2}+3\right)\cos 60^\circ$ A1

$= 32 + 9 - 24\sqrt{2} + 32 + 9 + 24\sqrt{2} - 2 \times 23 \times \frac{1}{2}$ A1

$= 82 - 23 \ = 59$

So $AC = \sqrt{59}$ A1

3 a Use of GDC to give M1

$r = 0.9675$ A1

b This is a strong positive correlation. R1

c Use of GDC to give: $T = 0.00277x + 0.0919$ M1A1

d $T = 0.00277 \times 50 + 0.0919 = 0.230$ seconds M1A1

e 90 years lies outside the range of data we are given, which would therefore involve extrapolation of data. R1

4 We require $2 \times \binom{5}{2} 3^2 x^3 + (-x) \times \binom{5}{2} 3^3 x^2$ M1A1A1

$= 180x^3 - 270x^3$ A1

$= -90x^3$ A1

5 a $500 \times 1.029^N = 800 \times 1.02^N$ M1A1

$\frac{1.029^N}{1.02^N} = \frac{800}{500}$ A1

$\left(\frac{1.029}{1.02}\right)^N = \frac{8}{5}$

$1.00882^N = 1.6$

b Attempts values for N (or use of logarithms) M1

$1.075^6 = 1.54$ A1

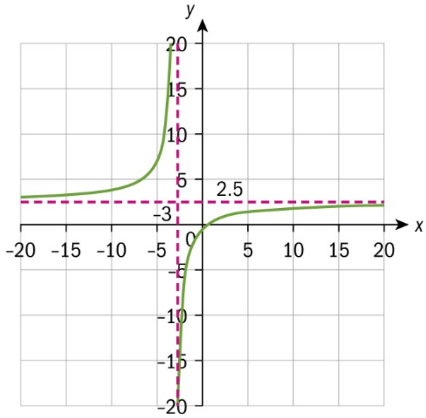
$1.075^7 = 1.66$ A1

So they must wait 7 years A1

6 a $x = -3$ A1

b $y = \frac{5}{2}$ A1

c A1 left-hand branch; A1 right-hand branch; A1 asymptotes



d $f(x) \neq \frac{5}{2}$, $(f(x) \in \mathbb{R})$ A1

e $g \circ f(x) = \sqrt{\frac{5x-3}{2x+6}}$ A1

f $x < -3$, $x \geq \frac{3}{5}$, $(x \in \mathbb{R})$ A1A1

7 a $m = \frac{18-(-2)}{-12-18} = \frac{20}{-30} = -\frac{2}{3}$ M1A1

Equation is $y + 2 = -\frac{2}{3}(x - 18)$ M1A1

$3y + 6 = -2x + 36$

$2x + 3y - 30 = 0$ A1

b AB crosses axes at $(15, 0)$ and $(0, 10)$ A1

Area is therefore $\frac{1}{2} \times 15 \times 10 = 75$ units². M1A1

8 a Using sketch of $y = (x+4)(3-x)$ M1

Chooses 'inside values' M1

Solution is $-4 < x < 3$ A1

b $2x^2 - 11x + 9 < 0$

$(2x-9)(x-1) < 0$ M1

Using sketch of $y = (2x-9)(x-1)$ M1

Chooses 'inside values' M1

Solution is $1 < x < \frac{9}{2}$ A1

c Comparing answers from **a** and **b** gives: $1 < x < 3$ A1

9 **a** $c = 18$ A1

$27 = a - b + 18$ M1

$3 = 9a + 3b + 18$

Attempt to solve simultaneously M1

$a = 1$, $b = -8$ A1A1

b $b^2 - 4ac = (-8)^2 - 4 \times 1 \times 18 = -8$ M1A1

< 0 therefore no solutions R1

c $y = x^2 - 8x + 18$

$= (x-4)^2 - 16 + 18$ M1

$= (x-4)^2 + 2$ A1

Therefore vertex is at the point $(4, 2)$ A1

10 $(\log_{10} x)^2 = 45 - 12\log_{10} x$ M1

$(\log_{10} x)^2 + 12\log_{10} x - 45 = 0$

$(\log_{10} x - 3)(\log_{10} x + 15) = 0$ (or other valid method to solve the quadratic) M1A1

$\log_{10} x - 3 = 0$ or $\log_{10} x + 15 = 0$

$\log_{10} x = 3$ $\log_{10} x = -15$

$x = 10^3$ $x = 10^{-15}$ A1

11 $y = \dfrac{2}{\sqrt{x}} + \dfrac{1}{x}$

$x = 4 \Rightarrow y = \dfrac{5}{4}$ A1

$\dfrac{dy}{dx} = -x^{-\frac{3}{2}} - x^{-2}$ M1A1

At $x = 4$, $\dfrac{dy}{dx} = -\dfrac{1}{8} - \dfrac{1}{16} = -\dfrac{3}{16}$ A1

Required gradient is therefore $\dfrac{16}{3}$ M1

Equation is $y - \dfrac{5}{4} = \dfrac{16}{3}(x-4)$ M1A1

So $64x - 12y - 241 = 0$ A1

12 $y = k$ must intersect $y = e^x(1-x)$ at a maximum or minimum point. R1

$\dfrac{dy}{dx} = -e^x + (1-x)e^x = -xe^x$ M1A1

$\dfrac{dy}{dx} = 0 \Rightarrow x = 0$ M1A1

At $x = 0$, $y = 1$ A1

So $k = 1$

13 $\int_2^a \frac{10}{5x+4}\,dx = \left[2\ln\lvert 5x+4\rvert\right]_2^a$	M1A1
$= 2\ln(5a+4) - 2\ln 14$	M1
$= 2\ln\left(\frac{5a+4}{14}\right)$	M1
$4\ln 2 = 2\ln 4$	A1
$\frac{5a+4}{14} = 4$	M1
$a = \frac{52}{5}$	A1
14 $2\cos^2\theta - 1 = 3\cos\theta - 2$	M1
$2\cos^2\theta - 3\cos\theta + 1 = 0$	
$(2\cos\theta - 1)(\cos\theta - 1) = 0$	M1A1
$\cos\theta = \frac{1}{2}$ or $\cos\theta = 1$	A1
$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \theta = \frac{5\pi}{3}$	A1A1
$\cos\theta = 1 \Rightarrow \theta = 0$	A1
15 a $X \sim N(36, 3.12^2)$	
$P(X > 40) = P\left(Z > \frac{40-36}{3.12}\right)$	M1
$= 0.1$	A1
b $P(34 < X < 38) = P\left(\frac{34-36}{3.12} < Z < \frac{38-36}{3.12}\right)$	M1
$= P(-0.641 < Z < 0.641)$	A1
$= P(Z < 0.641) - P(Z < -0.641)$	M1
$= 0.739 - 0.261$	
$= 0.478$	A1
c $P\left(Z > \frac{M-36}{3.12}\right) = 0.015$	M1
$P\left(Z < \frac{M-36}{3.12}\right) = 0.985$	
$\frac{M-36}{3.12} = 2.170$	A1
$\Rightarrow M = 42.77$	
$\Rightarrow M = 42$ minutes, 46 seconds	A1
d $P(X < 30) = P\left(Z < \frac{30-36}{3.12}\right)$	M1
$= P(Z < -1.923)$	
$= 0.027$	A1
$195 \times 0.027 = 5.3$	M1
Therefore the expected number of days is 5.	A1

16 a Let X represent the number of cracked eggs Jerry buys.

$X \sim B(24, 0.04)$

$P(X = 2) = {}^{24}C_2 (0.04)^2 (0.96)^{22}$ or uses GDC directly M1

$= 0.180$ A1

b $P(X \le 4) = 0.998$ M1A1

c $P(X \ge 2) = 0.249$ M1A1

d $\text{Var}(X) = np(1-p)$ M1

$= 24 \times 0.04 \times 0.96$

$= 0.9216$ A1

17 a If they were mutually exclusive, then $P(A \cap B) = 0$, A1

but since they are independent, we have $P(A \cap B) = P(A)P(B) \neq 0$.

Therefore we have a contradiction, and so A and B are not mutually exclusive. R1

b **i** $P(A \cap B) = P(A)P(B) = 0.3 \times 0.8 = 0.24$ M1A1

ii $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.8 - 0.24 = 0.86$ M1A1

iii $P(A \mid B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{P(B')} = \dfrac{0.3 - 0.24}{0.2} = \dfrac{0.06}{0.2} = 0.3$ M1A1

iv $P(A' \cap B) = P(B) - P(A \cap B) = 0.8 - 0.24 = 0.56$ M1A1

18 $\dfrac{\mathrm{d}u}{\mathrm{d}x} = -2\sin 2x$ A1

$x = \dfrac{\pi}{3} \Rightarrow u = \dfrac{1}{2}$ and $x = 0 \Rightarrow u = 2$ A1

$\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \cos 2x}\,\mathrm{d}x = -\frac{1}{2}\int_2^{\frac{1}{2}} \frac{\mathrm{d}u}{u}$ M1A1

$= -\dfrac{1}{2}\left[\ln u\right]_2^{\frac{1}{2}}$ A1

$= -\dfrac{1}{2}\left(\ln\dfrac{1}{2} - \ln 2\right)$ M1

$= -\dfrac{1}{2}(-\ln 2 - \ln 2)$ A1

$= \dfrac{1}{2}(2\ln 2)$ A1

$= \ln 2$

19 a $g^{-1}(x) = 4 - x$, $x \in \mathbb{R}$ A1A1

b $y = x^2 - 25$

Attempt to make x the subject M1

$y + 25 = x^2$

$x = \pm\sqrt{y + 25}$ A1

Since domain of f is $x \le 0$, it follows that range of $f^{-1}(x)$ is $y \le 0$ (R1)

and hence $f^{-1}(x) = -\sqrt{x + 25}$, $x \ge -25$ $(x \in \mathbb{R})$ A1A1

c $f \circ g(x) = (4-x)^2 - 25$ M1

Attempt to solve $(4-x)^2 - 25 = 0$ M1

$4 - x = \pm 5$

Since domain of *f* is $x \le 0$, it follows that domain of $f \circ g(x)$ is $4 - x \le 0$ R1

Hence $4 - x = -5 \Rightarrow x = 9$ A1

20 a $D = \dfrac{22+12}{2} = 17$ M1A1

$A = \dfrac{22-12}{2} = 5$ M1A1

The period is $\dfrac{360}{B} = 24$ M1

Therefore $B = 15$ A1

So $T = 5\sin(15(t-C)) + 17$

At $(3,12)$, $12 = 5\sin(15(t-C)) + 17$ M1

$-1 = \sin(15(3-C))$

$15(3-C) = -90$ A1

$C = 9$ A1

Therefore $T = 5\sin(15(t-9)) + 17$

b Solving $T = 5\sin(15(t-9)) + 17$ and $T = 20$ by GDC M1

Solutions are $T = 18.54$ and $T = 11.46$ A1A1

$18.54 - 11.46 = 7.08$ hours (7 hours 5 minutes) A1

21 $a = \log_2 343$ A1

$a + 3d = \log_2 1331$ A1

Solve simultaneously to find d M1

$\log_2 343 + 3d = \log_2 1331$

$3d = \log_2 1331 - \log_2 343$

$3d = \log_2\left(\dfrac{1331}{343}\right)$

$d = \dfrac{1}{3}\log_2\left(\dfrac{1331}{343}\right)$ A1

$d = \log_2\left(\dfrac{1331}{343}\right)^{\frac{1}{3}}$ M1

$d = \log_2\left(\dfrac{11}{7}\right)$ A1

So $\log_2 x = a + d$ M1

$= \log_2 343 + \log_2\left(\dfrac{11}{7}\right) = \log_2\left(343 \times \dfrac{11}{7}\right)$ A1

$= \log_2(49 \times 11) = \log_2 539$ A1

So $x = 539$

22 a $\dfrac{dv}{dt} = -\sin t - \cos t$ M1A1

Setting $\dfrac{dv}{dt} = 0$ M1

$\tan t = -1$ A1

$t = \dfrac{3\pi}{4}$ A1

$v_{MAX} = \cos\dfrac{3\pi}{4} - \sin\dfrac{3\pi}{4}$

$= -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}$

$= -\sqrt{2}$ A1

So maximum speed is $\sqrt{2}$ m s^{-1} A1

b $s = \int_0^{\pi} (\cos t - \sin t)\ dt$ M1

$= [\sin t + \cos t]_0^{\pi}$ A1

$= -1 - 1$ M1

$= -2$ A1

c $v = 0 \Rightarrow \tan t = 1 \Rightarrow t = \dfrac{\pi}{4}$ M1A1

So total distance $= \left|\int_0^{\frac{\pi}{4}} (\cos t - \sin t)\ dt\right| + \left|\int_{\frac{\pi}{4}}^{\pi} (\cos t - \sin t)\ dt\right|$ M1A1

$= \left|[\sin t + \cos t]_0^{\frac{\pi}{4}}\right| + \left|[\sin t + \cos t]_{\frac{\pi}{4}}^{\pi}\right|$ A1

$= \sqrt{2} - 1 + \left|-1 - \sqrt{2}\right|$ M1A1

$= \sqrt{2} - 1 + 1 + \sqrt{2}$

$= 2\sqrt{2}$ A1