OXFORD IB DIPLOMA PROGRAMME

PRACTICE EXAM PAPERS

MATHEMATICS: ANALYSIS AND APPROACHES

STANDARD LEVEL

COURSE COMPANION

ENHANCED ONLINE

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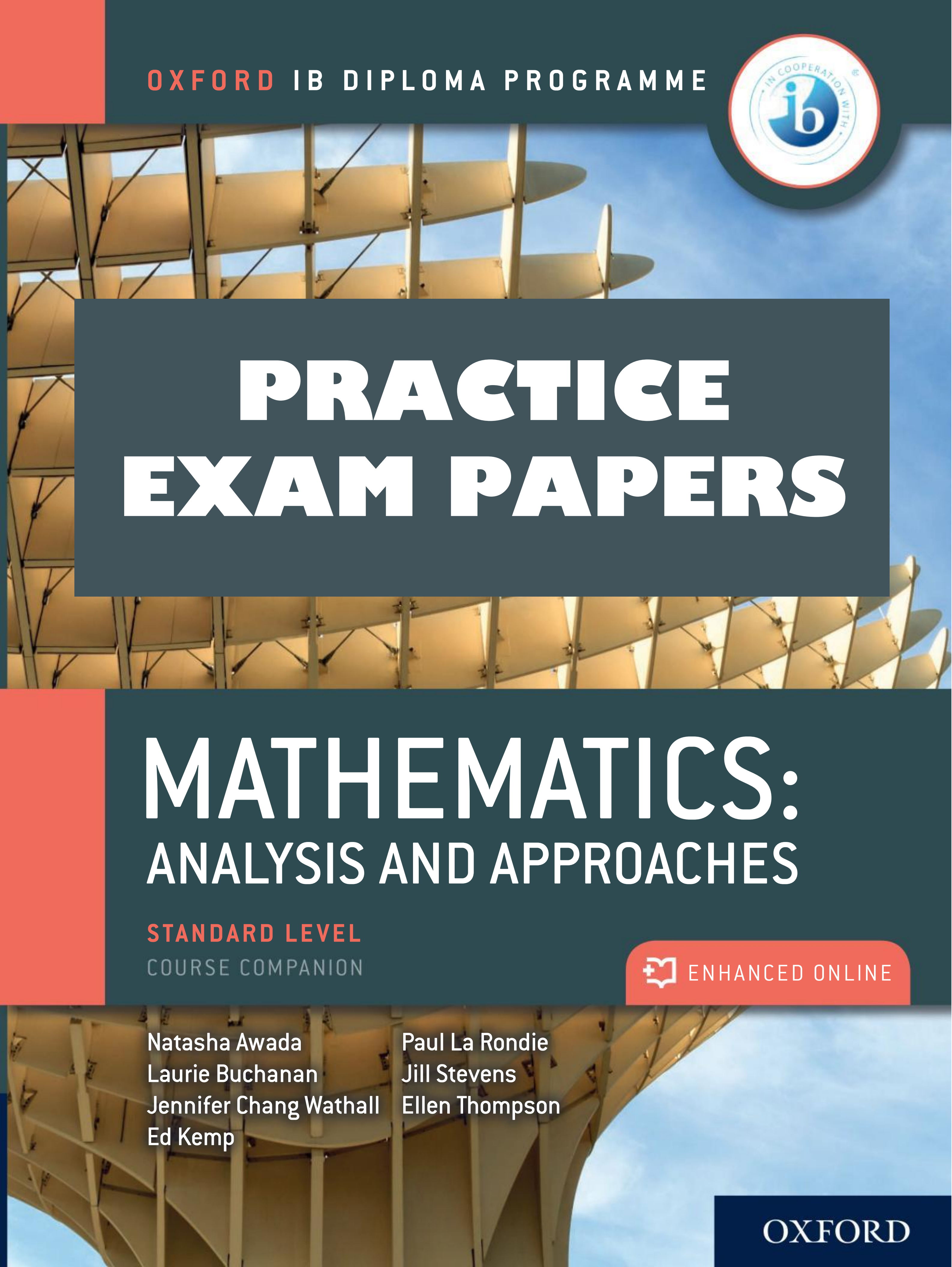


Paper 1

Time allowed: 1 hour 30 minutes

Maximum number of marks: 80 marks

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You are not allowed to use a calculator for this paper.

Section A

1 [Maximum mark: 6]

a Write down the number of elements in each of the following sets:

i $\varnothing$ **ii** $\{\varnothing,\{\varnothing\}\}$ **iii** $\{\varnothing,\{\varnothing\},\{\{\varnothing\}\}\}$. [3]

b **i** Write down the next set in this sequence of sets.

ii State how many elements in this set are members of the set of Natural numbers. [3]

2 [Maximum mark: 8]

The random variable X satisfies the $N(\mu,\sigma^2)$ distribution. It is known that $P(X \le 7) = 0.3$.

a Find $P(X > 7)$. [2]

b Let $Z = \dfrac{X-\mu}{\sigma}$.

i State the distribution that Z will satisfy.

ii Find $P\left(Z < \dfrac{7-\mu}{\sigma}\right)$. [3]

c Two independent readings of X are to be taken. Calculate the probability that exactly one of these readings is less than or equal to 7. [3]

3 [Maximum mark: 5]

A set of data, with values given in ascending order, is as follows:

$2, 3, 4, 6, 7, 9, 10, 11, 14, 17, x$.

a Given that x is an outlier, find an inequality that x must satisfy. [3]

b If this outlier is removed, state how the mean will change (without calculating it). [1]

c In general, if outliers are removed from a set of data, state what will happen to the standard deviation. [1]

4 [Maximum mark: 8]

A function is given by $f(x) = \sqrt{x-3}$.

a Find

i the largest possible domain of $f(x)$

ii the range of $f(x)$ for the domain found in part (a). [2]

b Find the inverse function $f^{-1}(x)$, stating its domain and range. [6]

5 [Maximum mark: 5]

Find the constant term in the expansion of $\left(x + \frac{2}{x}\right)^4$. [5]

6 [Maximum mark: 8]

Let $y(x) = 3x^2 - 24x + 57$.

a Write $y(x)$ in the form $a(x-b)^2 + c$. [3]

b Hence, write down the minimum point of this function. [2]

c Hence, solve $y(x) = 36$. [3]

Section B

7 [Maximum mark: 12]

A curve is given by the equation $y = x^3 + 1$.

a Find the equation of the tangent to the curve at the point where $x = 1$. [7]

b Find the equation of the normal to the curve at the point where $x = 1$. [5]

8 [Maximum mark: 16]

a Solve the equation $\cos^2 x + \frac{3}{2}\sin x - \frac{3}{2} = 0$, for $0^\circ \le x \le 360^\circ$. [7]

b Without using calculus, explain why $\cos^2 x + \frac{3}{2}\sin x$ is always strictly smaller than $\frac{5}{2}$. [4]

c By analogy with part **a**, solve $\sin^2 x + \frac{3}{2}\cos x - \frac{3}{2} = 0$, for $0^\circ \le x \le 360^\circ$. [5]

9 [Maximum mark: 12]

For two sets A and B, the following information is given:

$\mathrm{P}(A) = 0.25$, $\mathrm{P}(B) = 0.4$, $\mathrm{P}(A' \cap B') = 0.45$

a Determine, with a reason, whether A and B are mutually exclusive. [5]

b Determine, with a reason, whether A and B are independent. [3]

c Find $\mathrm{P}(A \cap B')$. [2]

d Find $\mathrm{P}((A \cap B)|A)$. [2]

Markscheme

Section A

1 a i 1	A1
ii 2	A1
iii 3	A1
	[3 marks]
b i $\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\{\{\varnothing\}\}\}\}$	A2
ii none	A1
	[3 marks]
	[Total: 6 marks]
2 a $1-0.3=0.7$	(M1) A1
	[2 marks]
b i $N(0,1^2)$	A1 A1
ii 0.3	A1
	[3 marks]
c Need $X_1 > 7$ and $X_2 < 7$, or vice versa.	R1
Probability is $2 \times 0.7 \times 0.3 = 0.42$	M1 A1
	[3 marks]
	[Total: 8 marks]
3 a IQR = 14-4=10	A1
so $x>14+1.5\times10 \Rightarrow x>29$	M1 A1
	[3 marks]
b The mean will decrease.	R1
	[1 mark]
c The standard deviation will decrease.	R1
	[1 mark]
	[Total: 5 marks]
4 a i Domain $\{x \in \mathbb{R} \mid x \geq 3\}$ **ii** Range $\{y \in \mathbb{R} \mid y \geq 0\}$	A1 A1
	[2 marks]
b Inverse given by $x = \sqrt{y-3} \Rightarrow y = x^2 + 3$	M1 A1 A1
So $f^{-1}(x) = x^2 + 3$; domain $\{x \in \mathbb{R} \mid x \geq 0\}$; range $\{y \in \mathbb{R} \mid y \geq 3\}$	A1 A1 A1
	[6 marks]
	[Total: 8 marks]
5 General term is $\binom{4}{r} x^{4-r} \left(\frac{2}{x}\right)^r = \binom{4}{r} x^{4-2r} (2^r)$	(M1)
For constant term, we require $4-2r=0 \Rightarrow r=2$	(R1) A1
Constant term is $\binom{4}{2} x^2 \left(\frac{2}{x}\right)^2 = 6 \times 4 = 24$	A1 A1
	[5 marks]
	[Total: 5 marks]
6 a $y = 3(x^2 - 8x) + 57 = 3(x-4)^2 - 48 + 57$	M1 A1
$y = 3(x-4)^2 + 9$	A1
	[3 marks]

b minimum point is $(4,9)$ A1 A1

[2 marks]

c $3(x-4)^2+9=36 \Rightarrow (x-4)^2=9 \Rightarrow x-4=\pm 3$ M1

$x=7 \text{ or } 1$ A1 A1

[3 marks]

[Total: 8 marks]

Section B

7 **a** $\frac{dy}{dx}=3x^2 \Rightarrow \left.\frac{dy}{dx}\right|_{x=1}=3$ (M1) A1

Equation of tangent is $y=3x+c$ M1

Curve passes through $(1,2)$ A1

$2=3+c \Rightarrow c=-1$ (M1) A1

Equation of tangent is $y=3x-1$ A1

[7 marks]

b Gradient of normal is $-\frac{1}{3}$ A1

Equation of normal is $y=-\frac{1}{3}x+d$; through $(1,2)$ M1

$2=-\frac{1}{3}+d \Rightarrow d=\frac{7}{3}$ (M1) A1

Equation of normal is $y=-\frac{1}{3}x+\frac{7}{3}$ (or equivalent form) A1

[5 marks]

[Total: 12 marks]

8 **a** $\cos^2 x+\frac{3}{2}\sin x-\frac{3}{2}=0 \Rightarrow 1-\sin^2 x+\frac{3}{2}\sin x-\frac{3}{2}=0 \Rightarrow \sin^2 x-\frac{3}{2}\sin x+\frac{1}{2}=0$ M1 A1

$\Rightarrow \left(\sin x-\frac{1}{2}\right)(\sin x-1)=0 \Rightarrow \sin x=\frac{1}{2} \text{ or } 1$ M1 A1

$x=30^\circ, 150^\circ \text{ or } 90^\circ$ A1 A1 A1

[7 marks]

b Since $\sin x$ and $\cos x$ are both between -1 and $+1$, $\cos^2 x \le 1$ and $\frac{3}{2}\sin x \le \frac{3}{2}$ R1 A1

Hence $\cos^2 x+\frac{3}{2}\sin x \le \frac{5}{2}$ A1

and since $\cos x=\pm 1$ and $\sin x=1$ cannot happen for the same value of x R1

then $\cos^2 x+\frac{3}{2}\sin x < \frac{5}{2}$ AG

[4 marks]

This is the same equation as in **a** with $\sin x$ and $\cos x$ reversed, so the solutions will be

given by $\cos x=\frac{1}{2} \text{ or } 1$ R1

$x=60^\circ, 300^\circ, 0^\circ \text{ or } 360^\circ$ A1 A1 A1 A1

[5 marks]

[Total: 16 marks]

9 **a** $P(A' \cap B')=0.45 \Rightarrow P((A \cup B)')=0.45 \Rightarrow P(A \cup B)=0.55$ M1 A1

Using $P(A \cup B)=P(A)+P(B)-P(A \cap B)$, $0.55=0.25+0.4-P(A \cap B) \Rightarrow P(A \cap B)=0.1$

M1 A1

$\mathrm{P}(A \cap B) \neq 0$ so they are not mutually exclusive. R1

[5 marks]

b $\mathrm{P}(A) \times \mathrm{P}(B) = 0.25 \times 0.4 = 0.1$ M1 A1

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ so they are independent R1

[3 marks]

c $\mathrm{P}(A \cap B') = \mathrm{P}(A) - \mathrm{P}(A \cap B) = 0.25 - 0.1 = 0.15$ M1 A1

[2 marks]

$\mathrm{P}((A \cap B)|A) = \dfrac{\mathrm{P}(A \cap B \cap A)}{\mathrm{P}(A)} = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)} = \dfrac{0.1}{0.25} = 0.4$ M1 A1

[2 marks]

[Total: 12 marks]

Paper 2

Time allowed: 1 hour 30 minutes

Maximum number of marks: 80 marks

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphic display calculator for this paper.

Section A

1 [Maximum mark: 8]

An arithmetic progression has a second term of 5 and an eleventh term of 41.

a Find the first term and the common difference. [3]

b **i** Find and simplify a formula for the nth term, u_n .

ii Hence find the value of n for which $u_n = 549$. [2]

c **i** Find and simplify a formula for the sum of the first n terms, S_n .

ii Hence find the smallest value of n for which $S_n > 1000$. [3]

2 [Maximum mark: 5]

In this question give monetary answers to 2 decimal places.

Anna is going to invest money in a bank which pays 5% compound interest per year, compounded annually.

a Calculate how much Anna should deposit now, so that it will be worth £1000 in 10 years' time. [2]

Doris also invests money into an account paying 5% compound interest per year, compounded annually. She is going to deposit $£X$ in the bank initially. In 5 years' time she will take out £1000, and leave the remainder in for another 6 years. At this time, she will then take out another £1000, which will leave her account empty.

b Calculate the value of X . [3]

3 [Maximum mark: 8]

a Write down the value of $\int_0^1 x^2\sqrt{x^3+1}\,dx$, giving your answer to 4 decimal places. [3]

b **i** Find $\int x^2\sqrt{x^3+1}\,dx$.

ii Hence find the **exact** value of $\int_0^1 x^2\sqrt{x^3+1}\,dx$, simplifying your answer as far as possible.

[6]

4 [Maximum mark: 8]

a If $y=5x^4$, find a linear expression connecting $\log y$ and $\log x$. [3]

b By taking logarithms to base 10 of both sides, solve the equation $3^{4x}=5^{2x-1}$.

Give your answer as *both*

i an exact form, and

ii correct to 4 significant figures. [5]

5 [Maximum mark: 6]

A triangle ABC has sides of lengths $a=4$, $b=5$ and $c=6$. Find, in degrees, the sizes of angles $\hat{A}, \hat{B}$ and $\hat{C}$. [6]

6 [Maximum mark: 5]

Solve the equation $e^{2x}+e^x-20=0$, giving your answers in **exact** form. [5]

Section B

7 [Maximum mark: 12]

Paired bivariate data (x,y) is collected from 11 students, where x is their time to swim 100 m (measured in seconds) and y is their time to run 200 m (also measured in seconds). The data is given in the following table:

x	100	81	120	104	180	200	152	102	94	131	142
y	40	35	44	39	51	60	48	40	37	43	47

a Calculate the Pearson product moment correlation coefficient (r) for this data, and state what this value of r implies about the relationship between the swimming and running times. [5]

b **i** Calculate the equation of the linear regression line of y on x .

ii Write down the mean point $(\bar{x},\bar{y})$ that the linear regression line of y on x must pass through.

iii A twelfth student had a swimming time of 110 seconds. Estimate their running time. [5]

c State two reasons why the equation found in part b i should not be used to estimate the swimming time of a student with a running time of 23 seconds. [2]

8 [Maximum mark: 14]

The masses of a species of large tortoises are normally distributed with a mean of μ kg and a standard deviation of 2 kg. It has been discovered that 6.68% of these tortoises have a mass greater than 13 kg.

a Calculate the value of μ. [8]

There are a very large number of these tortoises on an island. A sample of 100 of them is at random. Since they do not run very fast, it can be assumed that their masses are independent of one another.

b **i** Find the probability that exactly 6 of the sample have a mass greater than 13 kg.

ii Find the probability that at least 3 of the sample have a mass greater than 13 kg. [6]

9 [Maximum mark: 14]

A rational function is given by $y(x) = \dfrac{x-3}{4x+2}, x \neq -\dfrac{1}{2}$.

a Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence show that the graph of $y(x)$ is always increasing. [3]

b Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence comment on the concavity of the graph of $y(x)$. [4]

c For the graph of $y(x)$, write down the equations of

i the vertical asymptote

ii the horizontal asymptote. [2]

d For the graph of $y(x)$, write down the equations of

i the coordinates of the x-axis intercept

ii the coordinates of the y-axis intercept. [2]

e Sketch the graph of this function, showing the information that has been obtained in parts **a** to **e**. [3]

Markscheme

Section A

1 a $a+d=5,\quad a+10d=41$		M1
Solving gives $a=1,\ d=4$	A1	A1
	[3 Marks]	
b i $u_n=1+(n-1)4=4n-3$		A1
ii solving $4n-3=549$ gives $n=138$		A1
	[2 marks]	
c i $S_n=\frac{n}{2}(2+(n-1)4)=2n^2-n$		A1
ii solving (e.g. with "table" on GDC) $2n^2-n>1000$ gives $n=23$ ($S_n=1035$)	M1	A1
	[3 marks]	
	[Total: 8 marks]	

2 a $Y(1.05)^{10}=1000\Rightarrow Y=£613.91$ (2 d.p.)	M1	A1
	[2 marks]	
b Require $(1.05^5X-1000)1.05^6=1000$		M1
$\Rightarrow 1.05^{11}X=(1+1.05^6)1000\Rightarrow X=1368.21$ (2 d.p.)	A1	A1
	[3 marks]	
	[Total: 5 marks]	

3 a 0.4063 (4 d.p.)			A2
		[2 marks]	
b i by inspection or substitution $\frac{2}{9}(x^3+1)^{\frac{3}{2}}+c$		(M1)	A2
ii $\left(\frac{2}{9}(2)^{\frac{3}{2}}\right)-\left(\frac{2}{9}\right)=\frac{4\sqrt{2}-2}{9}$	M1	A1	A1
		[6 marks]	
		[Total: 8 marks]	

4 a $\log y=\log(5x^4)=\log 5+\log x^4$	M1	A1
So $\log y=\log 5+4\log x$		A1
	[3 marks]	
b $\log(3^{4x})=\log(5^{2x-1})\Rightarrow 4x\log 3=(2x-1)\log 5$	M1	A1
$\Rightarrow(4\log 3-2\log 5)x=-\log 5$		A1
i $x=\frac{\log 5}{2\log 5-4\log 3}$		A1
ii -1.369 (4 s.f.)		A1
	[5 marks]	
	[Total: 8 marks]	

5 $6^2=4^2+5^2-2\times4\times5\cos C\Rightarrow 5=40\cos C\Rightarrow C=82.819\ldots$ (or another angle)	M1
$C=82.8°$ (3 s.f.)	A1
$\frac{5}{\sin B}=\frac{6}{\sin 82.819\ldots}\Rightarrow B=55.771$ (or correct use of sine rule with other angles/sides)	M1
$B=55.8°$ (3 s.f.)	A1
$A=180-82.819\ldots-55.771\ldots=41.41\ldots$	M1

$A = 41.4°$ (3 s.f.) A1

[6 marks]
[Total: 6 marks]

6 $(e^x)^2 + e^x - 20 = 0 \Rightarrow (e^x + 5)(e^x - 4) = 0$ M1 M1

$e^x = -5$ has no solutions and hence $e^x = 4$ R1 A1

$\Rightarrow x = \ln 4$ A1

[5 marks]
[Total: 5 marks]

Section B

7 a $r = 0.979$ (3 s.f.) A2

Strong, positive, linear correlation A1 A1 A1

[5 marks]

b i $y = 0.187x + 20.1$ A1 A1

ii $(128, 44)$ A1

iii $y = 0.187(110) + 20.1 = 40.7$ (3 s.f.) M1 A1

[5 marks]

c The line of x on y should be used instead when estimating a swimming time from a running time; R1

Using the line to estimate a swimming time when the running time is 23 seconds would be extrapolation a long way away from the given data. R1

[2 marks]
[Total: 12 marks]

8 a $X \sim N(\mu, 2^2)$ $P(X > 13) = 0.0668$ $P(X \le 13) = 0.9332$ M1 A1

$Z = \frac{X - \mu}{2} \sim N(0, 1^2)$ R1

$\text{P}\left(Z \le \frac{13 - \mu}{2}\right) = 0.9332$ M1 A1

$\frac{13 - \mu}{2} = 1.500\ldots$ M1 A1

$\mu = 10$ A1

[8 marks]

b i $Y \sim B(100, 0.0668)$ $P(Y = 6) = 0.159$ (3 s.f.) M1 A2

ii $P(Y \ge 3) = 1 - P(Y \le 2) = 0.967$ (3 s.f.) M1 A2

[6 marks]
[Total: 14 marks]

9 a $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x+2) - (x-3)4}{(4x+2)^2} = \frac{14}{(4x+2)^2}$ M1 A1

Since $\frac{14}{(4x+2)^2} > 0$ for all x, so the graph of $y(x)$ is always increasing. R1

[3 marks]

b $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-112}{(4x+2)^3}$ M1 A1

If $x > -\frac{1}{2}$ then $(4x+2)^3 > 0$ and hence $\frac{d^2y}{dx^2} = \frac{-112}{(4x+2)^3} < 0$,

so the graph is concave down. R1

If $x < -\frac{1}{2}$ then $(4x+2)^3 < 0$ and hence $\frac{d^2y}{dx^2} = \frac{-112}{(4x+2)^3} > 0$,

so the graph is concave up. R1

[4 marks]

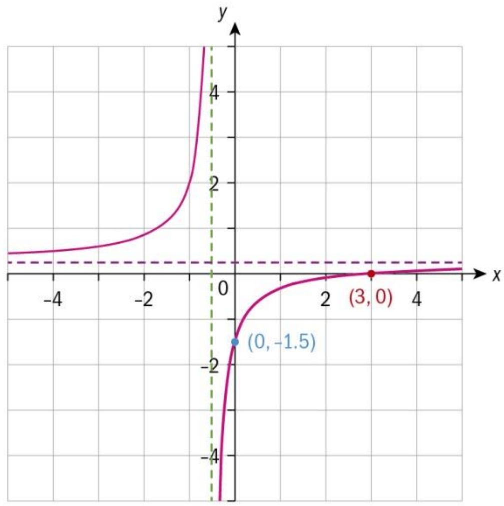
c **i** $x = -\frac{1}{2}$ **ii** $y = \frac{1}{4}$ A1 A1

[2 marks]

d **i** $(3, 0)$ **ii** $\left(0, -\frac{3}{2}\right)$ A1 A1

[2 marks]

e



A1 for $y(x)$; A1 for asymptotes; A1 for axes intercepts

[3 marks]

[Total: 14 marks]