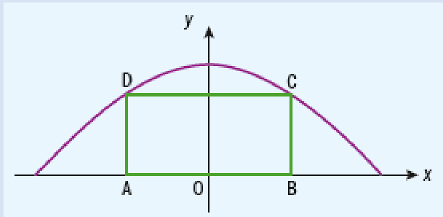
Chapter 13 / **Example 9**

Optimization with derivatives

Derivatives are useful in optimization problems, such as the maximizing or minimizing profits, costs, areas, volumes or distances.

A rectangle is inscribed under the curve $f(x) = 2\cos\left(\frac{1}{2}x\right)$, for $-\pi \le x \le \pi$. Points A and B lie on the x-axis, and points C and D lie on the curve, as shown in the diagram.
The coordinates of B are $(x, 0)$ and the coordinates of C are $\left(x, 2\cos\left(\frac{1}{2}x\right)\right)$.



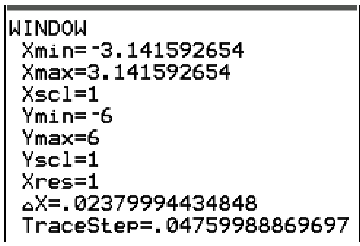
a Write expressions for the lengths AB and BC in terms of x.
b Write an expression for the area of the rectangle, $A(x)$, in terms of x.
c Find $A'(x)$.
d Use your answer from part **c** to find the value of x for which the area is a maximum.
e Use your GDC to plot a graph of $y = A(x)$ and verify your answer from part **d**. Find the maximum area of the rectangle.

Press [f2] [window]

Set the axes to show $-\pi \le x \le \pi$ with a scale of 1 and $-6 \le y \le 6$ with a scale of 1

Use [▼] [▲] to navigate through the settings.

Press [f1] [y=] when you have finished.

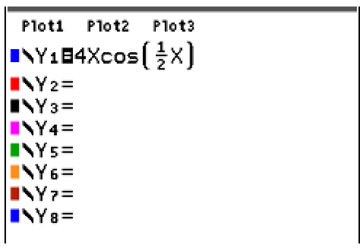


$AB = 2x$ and $BC = 2\cos\left(\frac{1}{2}x\right)$

$A(x) = 4x\cos\left(\frac{1}{2}x\right)$

Type $4x\cos\left(\frac{1}{2}x\right)$ using [illegible] [f1] 1:n/d to enter $\frac{1}{2}$.

Press [enter].



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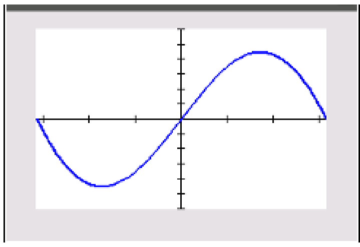
Press graph.

The GDC displays the function $A(x) = 4x\cos\left(\frac{1}{2}x\right)$.



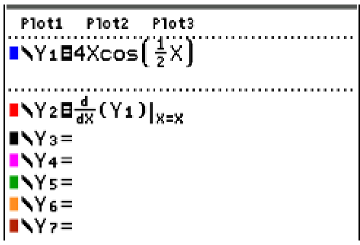
$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$

To graph the derivative function

In Y_2 press

Type X in the denominator and the function Y_1 by pressing alpha [f4] 1:Y_1. Type X as the value of x.

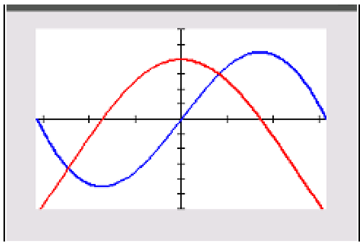
Press enter.



Press graph.

The GDC displays $A(x) = 4x\cos\left(\frac{1}{2}x\right)$ as $f1(x)$ and

$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$ as $f2(x)$.



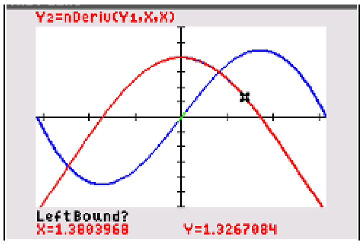
To find the zero of Y_2 press 2nd [f4] [calc] 2:zero

Use ▲ the select the graph Y_2.

You will need to give the left and right bounds of the region that includes the zero.

The GDC shows a point on the curve and asks you to set the left bound. Move the point using ▶ ◀ and choose a position to the left of the zero.

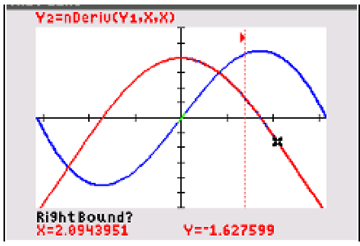
Press enter.



The GDC shows a line where you have set the left bound and a point on the curve.

Move the point using ▶ ◀ and choose a position to the right of the zero.

When the region contains the zero, Press enter.



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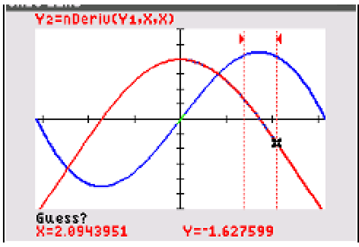
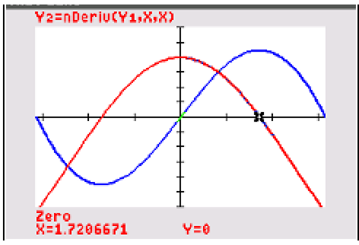
The GDC requires an initial guess for the position of the zero. Choose the default position.

Press [enter].



$A'(x) = 0$ when $x = 1.72$.



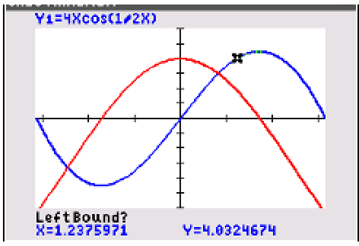
To find the maximum press [2nd] [f4] [calc] 4:maximum

Use [▲] the select the graph Y_1.

You will need to give the left and right bounds of the region that includes the vertex.

The GDC shows point on the curve and asks you to set the left bound. Move the point using [▶] [◀] and choose a position to the left of the vertex.

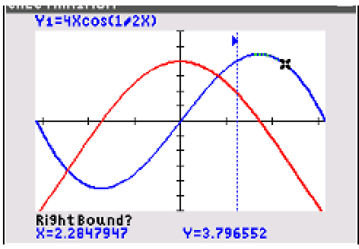
Press [enter].



The GDC shows a line where you have set the left bound and a point on the curve.

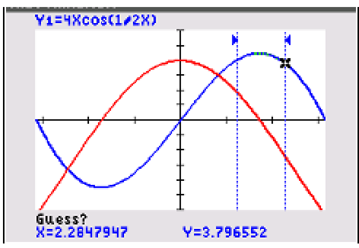
Move the point using [▶] [◀] and choose a position to the right of the vertex.

When the region contains the vertex, Press [enter].



The GDC requires an initial guess for the position of the zero. Choose the default position.

Press [enter].



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The GDC displays the vertex and verifies that the maximum occurs when $x = 1.72$. The maximum area is 4.49.