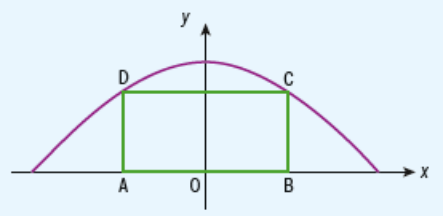
Chapter 13 / **Example 9**

Optimization with derivatives

Derivatives are useful in optimization problems, such as the maximizing or minimizing profits, costs, areas, volumes or distances.

A rectangle is inscribed under the curve $f(x)=2\cos\left(\frac{1}{2}x\right)$, for $-\pi \le x \le \pi$. Points A and B lie on the x-axis, and points C and D lie on the curve, as shown in the diagram.



The coordinates of B are $(x, 0)$ and the coordinates of C are $\left(x, 2\cos\left(\frac{1}{2}x\right)\right)$.

a Write expressions for the lengths AB and BC in terms of x.

b Write an expression for the area of the rectangle, $A(x)$, in terms of x.

c Find $A'(x)$.

d Use your answer from part **c** to find the value of x for which the area is a maximum.

e Use your GDC to plot a graph of $y = A(x)$ and verify your answer from part **d**. Find the maximum area of the rectangle.

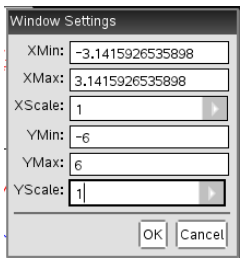
Open a new document and add a Graphs page.

Press menu 4:Window/Zoom | 1:Window Settings…

Set the axes to show $-\pi \le x \le \pi$ with a scale of 1 and $-6 \le y \le 6$ with a scale of 1

Use tab to navigate through the settings.

Press enter when you have finished.



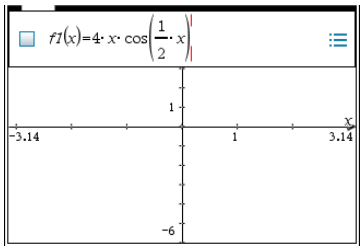
$AB = 2x$ and $BC = 2\cos\left(\frac{1}{2}x\right)$

$A(x) = 4x\cos\left(\frac{1}{2}x\right)$

Press tab to display the entry line '$f1(x)=$' is displayed.

Type $4x\cos\left(\frac{1}{2}x\right)$ using trig to enter cos and ctrl to enter $\frac{1}{2}$.

Press enter.



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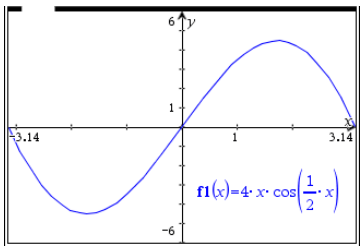
The GDC displays the function $A(x) = 4x\cos\left(\frac{1}{2}x\right)$



$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$

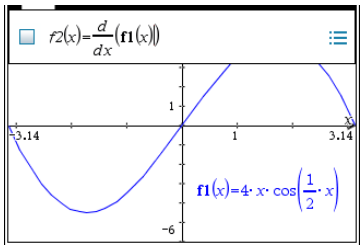
To graph the derivative function

Press tab to display the entry line '$f2(x)=$' is displayed.

Press menu and select $\frac{d}{d\square}\square$

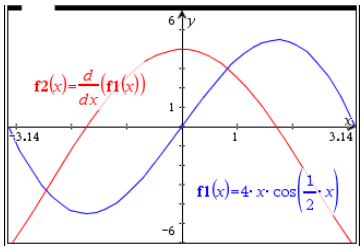
Type X in the denominator and the function $f1(x)$

Press enter.



The GDC displays $A(x) = 4x\cos\left(\frac{1}{2}x\right)$ as $f1(x)$ and

$A'(x) = -2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$ as $f2(x)$.



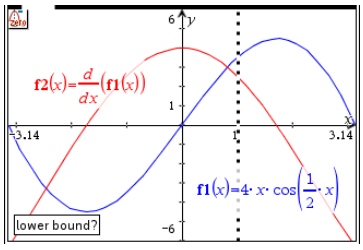
To find the zero of $f2(x)$ press menu 6:Analyse Graph | 1:Zero

Select graph $f2$

You will need to give the lower and upper bounds of the region that includes the zero.

The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the zero.

Click the touchpad.



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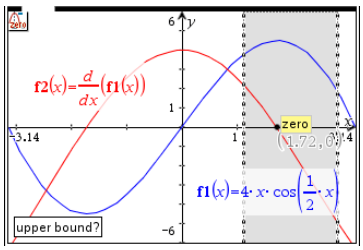
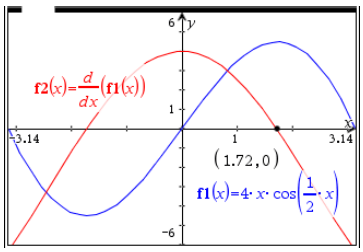
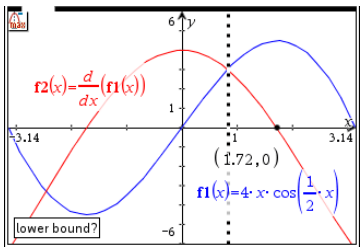
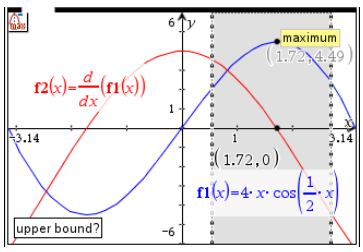
Optimization with derivatives

The GDC shows another line and asks you to set the upper bound. Use the touchpad to move the line so that the region between the lower and upper bounds contains the zero. When the region contains the zero, the calculator will display the word 'zero' in a box. Click the touchpad.	
$A'(x) = 0$ when x = 1.72.	
To find the maximum press menu 6:Analyse Graph | 3:Maximum Select graph $f1$ You will need to give the lower and upper bounds of the region that includes the vertex. The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the vertex. Click the touchpad.	
The GDC shows another line and asks you to set the upper bound. Use the touchpad to move the line so that the region between the lower and upper bounds contains the vertex. When the region contains the zero, the calculator will display the word 'maximum' in a box. Click the touchpad.	
The GDC displays the vertex and verifies that the maximum occurs when $x = 1.72$. The maximum area is 4.49.