OXFORD IB DIPLOMA PROGRAMME

PRIOR LEARNING SUPPORT

MATHEMATICS: ANALYSIS AND APPROACHES

STANDARD LEVEL

COURSE COMPANION

ENHANCED ONLINE

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Solving linear equations

'Solve an equation' means 'find the value of the unknown variable' (the letter).

Rearrange the equation so that the unknown variable *x* becomes the subject of the equation. To keep the equation 'balanced' always do the same to both sides.

Example 1

Solve the equation 3x + 5 = 17

Answer

3x + 5 = 17

3x + 5 - 5 = 17 – 5 *subtract 5*

3x = 12

$\frac{3x}{3} = \frac{12}{3}$ *divide by 3*

x = 4

Example 2

Solve the equation 4(x - 5) = 8

Answer

4(x - 5) = 8

$\frac{4(x-5)}{4} = \frac{8}{4}$ *divide by 4*

x - 5 = 2

x - 5 + 5 = 2 + 5 *add 5*

x = 7

Example 3

Solve the equation 7 - 3x = 1

Answer

7 - 3x = 1

7 - 3x - 7 = 1 – 7 *subtract 7*

-3x = -6

$\frac{-3x}{-3} = \frac{-6}{-3}$ *divide by -3*

x = 2

Example 4

Solve the equation $3(2 + 3x) = 5(4 - x)$

Answer

$3(2 + 3x) = 5(4 - x)$

$6 + 9x = 20 - 5x$

$6 + 9x + 5x = 20 - 5x + 5x$ add 5x

$6 + 14x = 20$

$6 + 14x - 6 = 20 - 6$ subtract 6

$14x = 14$

$\frac{14x}{14} = \frac{14}{14}$ divide by 14

$x = 1$

Exercise

Solve these equations.

1 $3x - 10 = 2$

2 $\frac{x}{2} + 5 = 7$

3 $5x + 4 = -11$

4 $3(x + 3) = 18$

5 $4(2x - 5) = 20$

6 $\frac{2}{5}(3x - 7) = 8$

7 $21 - 6x = 9$

8 $12 = 2 - 5x$

9 $2(11 - 3x) = 4$

10 $4(3 + x) = 3(9 - 2x)$

11 $2(10 - 2x) = 4(3x + 1)$

12 $\frac{5x + 2}{3} = \frac{3x + 10}{4}$

Answers

1 x = 4 **2** x = 4 **3** x = -3 **4** x = 3

5 x = 5 **6** x = 9 **7** x = -2.5 **8** x = -2

9 x = 3 **10** x = 1.5 **11** x = 1 **12** x = 2

Fractions

Using a GDC, you can either enter a fraction using the fraction template $\frac{\square}{\square}$ or by using the divide key ÷.

Take care - you will sometimes need to use brackets.

Example 1

a Evaluate

$\frac{1}{2}+\frac{3}{8}\times\frac{4}{9}$ × *before* +

$=\frac{1}{2}+\frac{1}{6}$

$=\frac{4}{6}$ *simplify*

$=\frac{2}{3}$

b Evaluate

$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}}$ *evaluate numerator and denominator first*

$=\frac{\frac{5}{6}}{\frac{1}{6}}$

$= 5$

Exercise

1 Calculate

a $\frac{1}{2}+\frac{3}{4}\times\frac{5}{9}$ **b** $\frac{2}{3}\div\frac{5}{6}\times1\frac{1}{3}$ **c** $\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$ **d** $\frac{1-\left(\frac{2}{3}\right)^5}{1-\frac{2}{3}}$

Answers

1 **a** $\frac{11}{12}$ **b** $\frac{16}{15}$ **c** 1 **d** $\frac{211}{81}$ or $2\frac{49}{81}$

Order of operations

There are several versions of the rules for the order of operations.

BEDMAS: Brackets, exponents, division, multiplication, addition, subtraction.

BIDMAS: Brackets, indices, division, multiplication, addition, subtraction.

BEMDAS: Brackets, exponents, multiplication, division, addition, subtraction.

BODMAS: Brackets, orders, division, multiplication, addition, subtraction.

BOMDAS: Brackets, orders, multiplication, division, addition, subtraction.

PEMDAS: Parentheses, exponents, multiplication, division, addition, subtraction.

They all amount to the same

- Brackets or parentheses are calculated first.
- Next come exponents, indices or orders.
- Then multiplication and division, in order from left to right.
- Finally additions and subtractions.

A fraction line or the line above a square root counts as a bracket too.

Your GDC follows the rules, so if you enter a calculation correctly you should get the correct answers.

The GDC shows divisions as fractions, which makes the order of operations clearer.

Simple calculators, like the ones on phones, do not always follow the calculation rules.

Example 1

Evaluate $\dfrac{11+(-1)^2}{4-(3-5)}$

$= \dfrac{11+1}{4-(-2)}$ *brackets first*

$= \dfrac{12}{6}$ *simplify numerator and denominator*

$= 2$

Example 2

Evaluate $\dfrac{-3+\sqrt{9-8}}{4}$

$= \dfrac{-3+\sqrt{1}}{4}$ *simplify the terms inside the square root*

$= \dfrac{-3+1}{4}$ *evaluate the root*

$= \dfrac{-2}{4}$

$= -\dfrac{1}{2}$ *simplify the numerator and denominator*

On your GDC you can either use templates for the fractions and roots or you can use brackets.

Exercise

Do the questions by hand first, then check your answers with your GDC.

1 Calculate

a 12 - 5 + 4 **b** 6 ÷ 3 × 5 **c** 4 + 2 x 3 - 2

d 8 - 6 ÷ 3 × 2 **e** 4 + (3 - 2) **f** (7 + 2) ÷ 3

g (1 + 4) × (8 - 4) **h** 1 - 3 + 5 × (2 - 1)

2 Find

a $\dfrac{6+9}{4-1}$ **b** $\dfrac{2\times 9}{3\times 4}$ **c** $\dfrac{2-(3+4)}{4\times(2-3)}$ **d** $\dfrac{6\times 5\times 4}{3\times 2-1}$

3 Determine

a $3 \times (-2)^2$ **b** $2^2 \times 3^3 \times 5$ **c** $4 \times (5-3)^2$ **d** $(-3)^2 - 2^2$

4 Calculate

a $\sqrt{3^2+4^2}$ **b** $\left(\sqrt{4}\right)^3$ **c** $\sqrt{4^3}$ **d** $\sqrt{2+\sqrt{2+2}}$

5 Find

a $\sqrt{\dfrac{13^2-\left(3^2+4^2\right)}{2\times 18}}$ **b** $2\sqrt{\dfrac{3+5^2}{7}}$ **c** $2\left(3^2-4(-2)\right)-\left(2-\sqrt{7-3}\right)$

Answers

1 **a** 11 **b** 10 **c** 8 **d** 4

e 5 **f** 3 **g** 16 **f** 3

2 **a** 5 **b** $\frac{3}{2}$ **c** $\frac{5}{4}$ **d** 24

3 **a** 12 **b** 540 **c** 16 **d** 5

4 **a** 5 **b** 8 **c** 8 **d** 2

5 **a** 2 **b** 4 **c** 34

Substituting into formulae

Substitution means putting numbers in place of letters. It is important to follow the order of operations.

When using formulae, you can use your GDC to do the calculation for you. You should still show your working.

1 Find the formula you are going to use (from the formula booklet, from the question or from memory) and write it down.

2 Identify the values that you are going to substitute into the formula.

3 Write out the formula with the values substituted for the letters.

4 Enter the formula into your calculator. Use templates to make the formula look the same on your GDC as it is on paper.

5 If you think it is necessary, use brackets. It is better to have too many brackets than too few!

6 Write down, with units if necessary, the result from your calculator (to the required accuracy).

Example 1

x and y are linked by the formula $y = \dfrac{x^2 + 1}{2\sqrt{x + 1}}$.

Find y when x is 3.1

Answer

$y = \dfrac{3.1^2 + 1}{2\sqrt{3.1 + 1}}$ Write the formula with 3.1 instead of x.

$y = 2.62$

Exercise

1 If $a = 2.3$, $b = 4.1$ and $c = 1.7$, find d where $d = \dfrac{3a^2 + 2\sqrt{b}}{ac + b}$

2 If $b = 8.2$, $c = 7.5$ and $A = 27°$, find a where $a = \sqrt{b^2 + c^2 - 2bc\cos A}$

3 If $u_1 = 10.2$, $r = 0.75$ and $n = 14$, find the value of S, where $S = u_1 \dfrac{1 - r^n}{1 - r}$

Answers

1 2.487

2 3.728

3 40.073

Coordinates

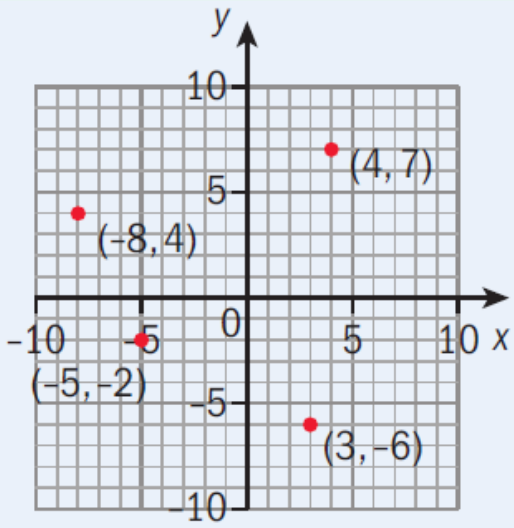
Coordinates describe the position of points in the plane. Horizontal positions are shown on the *x*-axis and vertical positions on the *y*-axis.

Example

Draw axes for and $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Plot the points with coordinates: (4, 7), (3, -6), (-5, -2) and (-8, 4).

Answer

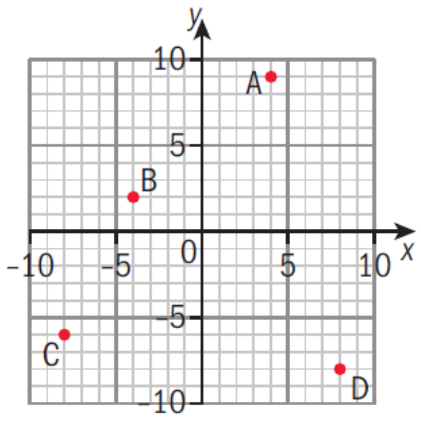


Exercise

1 Draw axes for $-8 \leq x \leq 8$ and $-5 \leq y \leq 10$.

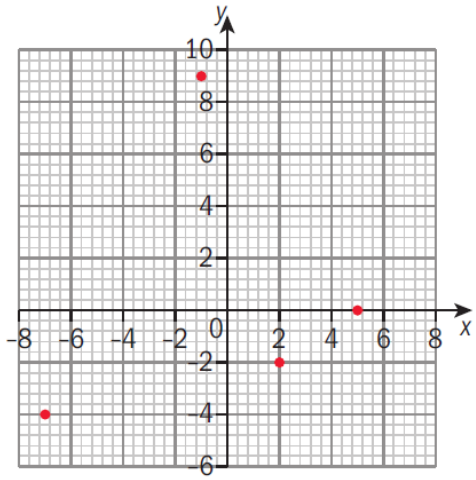
Plot the points with coordinates: (5, 0), (2, -2), (-7, -4) and (-1, 9).

2 Write down the coordinates of the points shown in this diagram.



Answers

1



2 A(4, 9), B(-4, 2), C(-8, -6), D(8, -8)

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Example 1

Solve the equation 3x + 5 = 17

Answer

3x + 5 = 17

3x + 5 - 5 = 17 – 5 *subtract 5*

3x = 12

$\frac{3x}{3} = \frac{12}{3}$ *divide by 3*

x = 4

Example 2

Solve the equation 4(x - 5) = 8

Answer

4(x - 5) = 8

$\frac{4(x-5)}{4} = \frac{8}{4}$ *divide by 4*

x - 5 = 2

x - 5 + 5 = 2 + 5 *add 5*

x = 7

Example 3

Solve the equation 7 - 3x = 1

Answer

7 - 3x = 1

7 - 3x - 7 = 1 – 7 *subtract 7*

-3x = -6

$\frac{-3x}{-3} = \frac{-6}{-3}$ *divide by -3*

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Example 4

Solve the equation $3(2 + 3x) = 5(4 - x)$

Answer

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$6 + 9x = 20 - 5x$

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Solve these equations.

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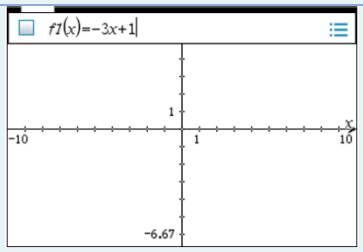
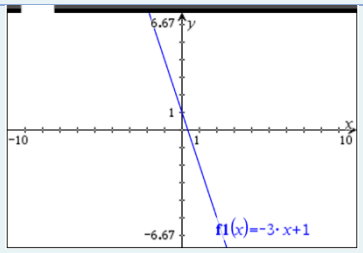
Graphing linear functions

Example

Draw the graph of the function $y = -3x + 1$

Answer

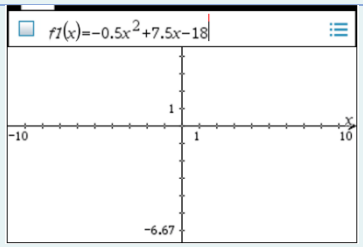
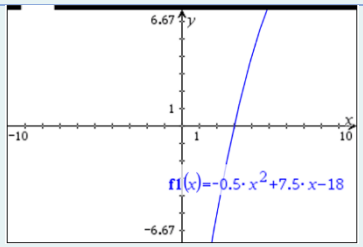
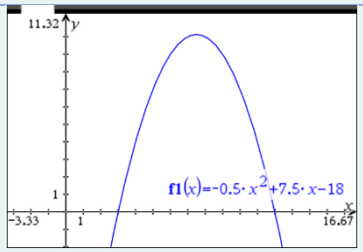
Open a new document and add a Graphs page. The entry line is displayed at the top of the work area. The default graph type is function, so '$f1(x)=$' is displayed. The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$. Type $-3x+1$ and press enter.	
The GDC displays the first straight-line graph $f1(x) = -3x + 1$	

Drawing a quadratic graph

Example

Draw the graph of $f(x)=-0.5x^2+7.5x-18$ and display using suitable axes.

Answer

Open a new document and add a Graphs page. The entry line is displayed at the top of the work area. The default graph type is function, so '*f1*(*x*)=' is displayed. The default axes are $-10 \le x \le 10$ and $-6.67 \le y \le 6.67$. Type $-0.5x^2+7.5x-18$ and press enter.	
The GDC displays the graph $f1(x)=-0.5x^2+7.5x-18$ with the default axes.	
Click and hold the touchpad somewhere on the white area of the screen. You should see the cursor change to ✋. Drag the axes. This is called panning. When you have a better view of the curve, click the touchpad again (or press esc).	

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Expanding brackets and factorizing

The distributive law is used to expand brackets and factorize expressions.

$a(h + c) = ah + ac$

Example 1

Expand $2y(3x + 5y - z)$

Answer

$2y\ (3x + 5y - z) = 2y \cdot 3x + 2y \cdot 5y + 2y(-z)$

$= 6xy + 10y^2 - 2yz$

Example 2

Factorise $6x^2y - 9xy + 12xz^2$

Answer

$6x^2y - 9xy + 12x\,z^2 = 3x\,(2xy - 3y + 4z^2)$

Look for a common factor. Write this outside the bracket. Find the terms inside the bracket by dividing each term by the common factor.

Exercise

1 Expand

a $3x(x - 2)$ **b** $\frac{x}{y}\left(x^2y - y^2 + x\right)$ **c** $a(b - 2c) + b(2a + b)$

2 Factorize

a $3pq - 6p^2q^3r$ **b** $12ac^2 + 15bc - 3c^2$ **c** $2a^2bc + 3ab^2c - 5abc^2$

Answers

1 **a** $3x^2 - 2$ **b** $x^3 - xy + \frac{x^2}{y}$ **c** $ab - 2ac + 2ab + b^2$

2 **a** $3pq(1 - 2pq^2r)$ **b** $3c(4ac + 5b - c)$ **c** $abc(2a + 3b - 5c)$

Factorizing quadratic expressions

It is possible to express a quadratic expression as the product of two linear expressions.

$(x + 2)(x + 5) = x^2 + 7x + 10$ 10 is the product of 2 and 5 and 7 is the sum of 2 and 5

$(x + 6)(x - 4) = x^2 + 2x - 24$ -24 is the product of 6 and -4 and 2 is the sum of 6 and -4

To factorize quadratics of the form $ax^2 + bx + c$, where the coefficient of x^2 is 1, look for pairs of factors of c whose sum is b.

Example 1

Factorize

a $x^2 - 15x + 14$

b $x^2 + 5x + 6$

c $x^2 - 5x - 24$

Answers

a $x^2 - 15x + 14 = (x - 1)(x - 14)$

Factors of 14	*Sum of factors*
1 and 14	*15*
-1 and -14	***-15***
2 and 7	*9*
-2 and -7	*-9*

b $x^2 + 5x + 6 = (x + 2)(x + 3)$

Factors of 6	*Sum of factors*
1 and 6	*7*
-1 and -6	*-7*
2 and 3	***5***
-2 and -3	*-5*

c $x^2 - 5x - 24 = (x + 3)(x - 8)$

Factors of 24	*Sum of factors*
1 and -24	*15*
-1 and 24	***-15***
2 and -12	*9*
-2 and 12	*-9*
3 and -8	**-5**
-3 and 8	*5*
4 and -6	*-2*
-4 and 6	*2*

To factorize quadratics of the form $ax^2 + bx + c$, where $a \neq 0$, use trial and error to find the correct pair of factors.

Try factors that give the correct product for the first and last terms, until you find the one that gives the correct product for the middle term.

Example 2

Factorize

a $2x^2 + 5x + 3$

b $6x^2 + x - 15$

Answers

a $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

Factors of $2x^2$: $2x$, x

Factors of 3: 1, 3; -1, -3

Possible factors	*Linear term*
(2x + 1)(x + 3)	*6x + 1x = 7x*
(2x – 1)(x – 3)	*-6x – 1x = -7x*
(2x + 3)(x + 1)	***2x + 3x = 5x***
(2x – 3)(x – 1)	*-2x – 3x = -5x*

b $6x^2 + x - 15 = (2x - 3)(3x + 5)$

Factors of $6x^2$: 6x, x; 2x, 3x

Factors of -15: 1, -15; -1, 15; 3, -5; -3, 5

Possible factors	*Linear term*
(6x + 1)(x – 15)	*-90x + 1x = -89x*
(6x – 1)(x + 15)	*90x – 1x = 89x*
(6x + 3)(x – 5)	*-30x + 3x = -27x*
(6x – 3)(x + 5)	*30x – 3x = 27x*
(2x + 1)(3x – 15)	*-30x + 3x = -27x*
(2x – 1)(3x + 15)	*30x – 3x = 27x*
(2x + 3)(3x – 5)	*-10x + 9x = -x*
(2x – 3)(3x + 5)	***10x – 9x = x***

Exercise

1 Factorize these quadratic expressions.

a $x^2 + 11x + 28$ **b** $x^2 - 14x + 13$ **c** $x^2 - x - 20$

d $x^2 + 2x - 8$ **e** $x^2 + 13x + 36$ **f** $x^2 - 7x - 18$

2 **a** $2x^2 - 9x + 9$ **b** $3x^2 + 7x + 2$ **c** $5x^2 - 17x + 6$

d $4x^2 - x - 3$ **e** $3x^2 - 7x - 6$ **f** $14x^2 - 17x + 5$

Answers

1 **a** $(x + 4)(x + 7)$ **b** $(x - 1)(x - 13)$ **c** $(x + 4)(x - 5)$

d $(x + 4)(x - 2)$ **e** $(x + 4)(x + 9)$ **f** $(x + 2)(x - 9)$

2 **a** $(2x - 3)(x - 3)$ **b** $(3x + 1)(x + 2)$ **c** $(5x - 2)(x - 3)$

d $(4x + 3)(x - 1)$ **e** $(3x + 2)(x - 3)$ **f** $(7x - 5)(2x - 1)$

Factorizing the difference of two squares

Remember that $a^2 - b^2 = (a + b)(a - b)$.

Example 1

Factorize

a $x^2 - 16$

b $9x^2 - 25y^2$

Answers

a $x^2 - 16 = (x + 4)(x - 4)$

$a^2 = x^2$, so $a = x$

$b^2 = 16$, so $b = 4$

Substitute values into $(a + b)(a - b)$.

b $9x^2 - 25y^2 = (3x + 5y)(3x - 5y)$

$a^2 = 9x^2$, so $a = 3x$

$b^2 = 25y^2$, so $b = 5y$

Substitute values into $(a + b)(a - b)$.

Exercise

Factorize these quadratic expressions.

a $x^2 - 9$ **b** $x^2 - 100$ **c** $4x^2 - 81$

d $25x^2 - 1$ **e** $m^2 - n^2$ **f** $16x^2 - 49y^2$

Answers

a $(x - 3)(x + 3)$ **b** $(x - 10)(x + 10)$ **c** $(2x - 9)(2x + 9)$

d $(5x + 1)(5x - 1)$ **e** $(m + n)(m - n)$ **f** $(4x - 7)(4x + 7)$

Answers

Solving linear equations

'Solve an equation' means 'find the value of the unknown variable' (the letter).

Rearrange the equation so that the unknown variable *x* becomes the subject of the equation. To keep the equation 'balanced' always do the same to both sides.

Example 1

Solve the equation 3x + 5 = 17

Answer

3x + 5 = 17

3x + 5 - 5 = 17 – 5 *subtract 5*

3x = 12

$\frac{3x}{3} = \frac{12}{3}$ *divide by 3*

x = 4

Example 2

Solve the equation 4(x - 5) = 8

Answer

4(x - 5) = 8

$\frac{4(x-5)}{4} = \frac{8}{4}$ *divide by 4*

x - 5 = 2

x - 5 + 5 = 2 + 5 *add 5*

x = 7

Example 3

Solve the equation 7 - 3x = 1

Answer

7 - 3x = 1

7 - 3x - 7 = 1 – 7 *subtract 7*

-3x = -6

$\frac{-3x}{-3} = \frac{-6}{-3}$ *divide by -3*

x = 2

Example 4

Solve the equation $3(2 + 3x) = 5(4 - x)$

Answer

$3(2 + 3x) = 5(4 - x)$

$6 + 9x = 20 - 5x$

$6 + 9x + 5x = 20 - 5x + 5x$ add 5x

$6 + 14x = 20$

$6 + 14x - 6 = 20 - 6$ subtract 6

$14x = 14$

$\frac{14x}{14} = \frac{14}{14}$ divide by 14

$x = 1$

Exercise

Solve these equations.

1 $3x - 10 = 2$

2 $\frac{x}{2} + 5 = 7$

3 $5x + 4 = -11$

4 $3(x + 3) = 18$

5 $4(2x - 5) = 20$

6 $\frac{2}{5}(3x - 7) = 8$

7 $21 - 6x = 9$

8 $12 = 2 - 5x$

9 $2(11 - 3x) = 4$

10 $4(3 + x) = 3(9 - 2x)$

11 $2(10 - 2x) = 4(3x + 1)$

12 $\frac{5x + 2}{3} = \frac{3x + 10}{4}$

Answers

1 x = 4 **2** x = 4 **3** x = -3 **4** x = 3

5 x = 5 **6** x = 9 **7** x = -2.5 **8** x = -2

9 x = 3 **10** x = 1.5 **11** x = 1 **12** x = 2

Finding the gradient of a line given two points

The gradient of a line is $\frac{Rise}{Run}$ which is $\frac{\text{The change in } y}{\text{The change in } x}$.

Given two points (x_1, y_1) and (x_2, y_2), $\frac{\text{The change in } y}{\text{The change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1

Find the gradient of the line joining (-3, -2) and (4, 1).

Answer

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{4 - (-3)} = \frac{3}{7}$$

Exercise

Find the gradient of the line through each pair of points.

1 (19, -16) and (-7, -15)

2 (1, -19) and (-2, -7)

3 (-4, 7) and (-6, -4)

4 (20, 8) and (9, 16)

5 (17, -1 3) and (17, 7)

6 (14, 3) and (1, 3)

7 (3, 0) and (-11, -15)

8 (19, -2) and (-11, 10)

9 (6, -10) and (-15, 15)

10 (12, -18) and (18, -18)

Answers

1 $-\frac{1}{26}$ **2** -4 **3** 4.5 **4** $-\frac{8}{11}$

5 undefined **6** 0 **7** $\frac{15}{14}$ **8** -0.4

9 $-\frac{25}{21}$ **10** 0

Answers

Statistical graphs

In a statistical investigation we collect information, known as **data**.

To represent this data in a clear way we can use graphs. Three types of statistical graph are bar charts, pie charts and pictograms.

Bar charts

A bar chart is a graph made from rectangles, or bars, of equal width whose length is proportional to the quantity they represent, or frequency. Sometimes we leave a small gap between the bars.

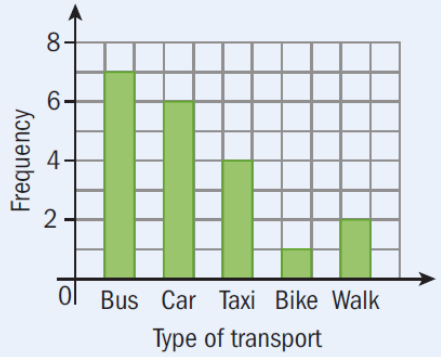
Example 1

Juliene collected some data about the ways in which her class travel to school.

Type of transport	Bus	Car	Taxi	Bike	Walk
Frequency	7	6	4	1	2

Represent this information in a bar chart.

Answer



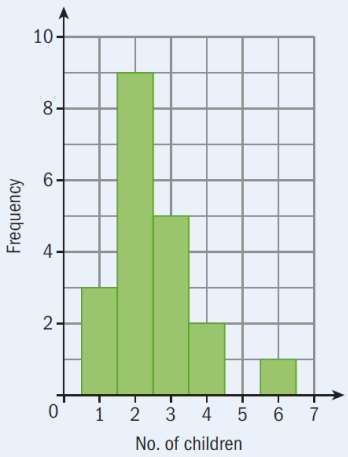
Example 2

Lakshmi collected data from the same class about the number of children in each of their families.

No. of children	1	2	3	4	6
Frequency	3	9	5	2	1

Represent this information in a bar chart.

Answer



Pie charts

A **pie chart** is a circle divided into sectors, like slices from a pie.

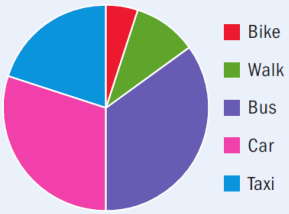
The sector angles are proportional to the quantities they represent.

Example 3

Use Juliene's data from Example 71 to construct a pie chart.

Answer

Type of transport	Frequency		Sector angle
Bus	7	$\frac{7}{20} \times 360°$	126°
Car	6	$\frac{6}{20} \times 360°$	108°
Taxi	4	$\frac{4}{20} \times 360°$	72°
Bike	1	$\frac{1}{20} \times 360°$	18°
Walk	2	$\frac{2}{20} \times 360°$	36°



The total of the frequencies is 20. The total angle for the whole circle is 360°.

Start by drawing a radius and then measure, with your protractor, each angle in turn. The total of the sector angles should be 360°.

Pictograms

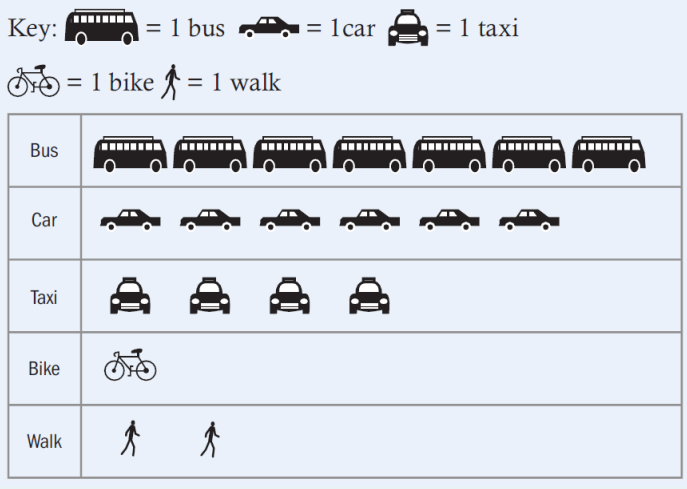
Pictograms are similar to bar charts, except that pictures are used.

The number of pictures is proportional to the quantity they represent. The pictures can be relevant to the items they show or just a simple character such as an asterisk.

Example 4

Use Juliene's data from Example 1 to construct a pictogram.

Answer



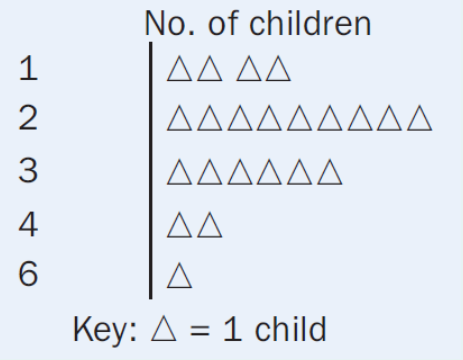
In this pictogram, different symbols are used for each category but the symbols describe the category as well.

Example 5

Use this data on the number of children in a sample of families to construct a pictogram.

Number of children	1	2	3	4	6
Frequency	4	9	6	2	1

Answer



Exercise

1 Adam carried out a survey of the cars passing by his window on the road outside. He noted the colors of the cars that passed by for 10 minutes and collected the following data.

Color	Black	Red	Blue	Green	Silver	White
Frequency	12	6	10	7	14	11

Draw a bar chart, a pie chart and a pictogram to represent the data.

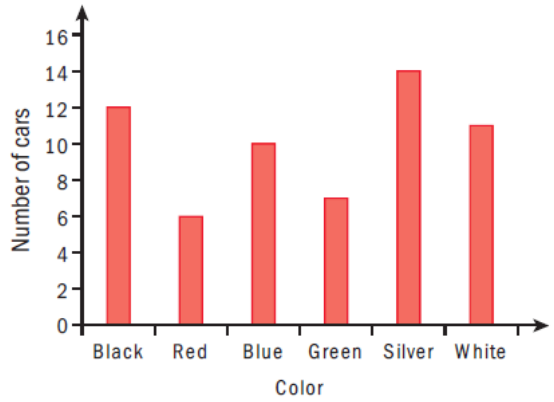
2 Ida asked the members of her class how many times they had visited the cinema in the past month. She collected the following data.

Number of times visited	1	2	3	4	8	12
Number of students	4	7	4	3	1	1

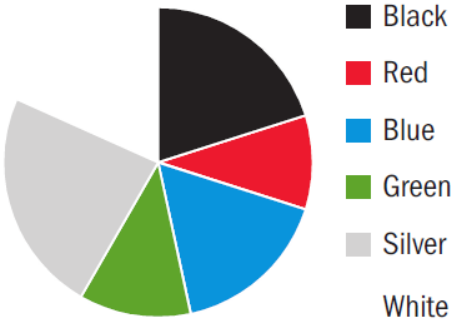
Draw a bar chart, a pie chart and a pictogram to represent the data.

Answers

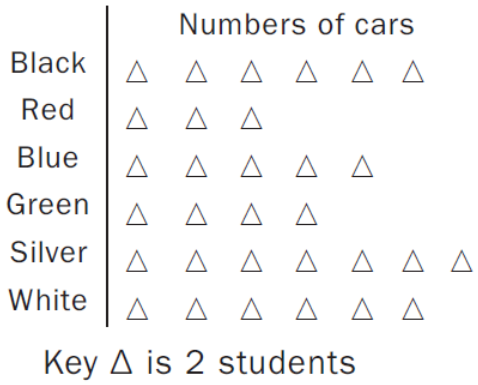
1 Bar graph to show colors of cars



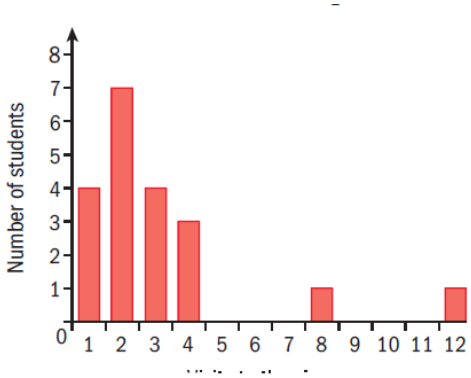
Pie Chart to show colours of cars



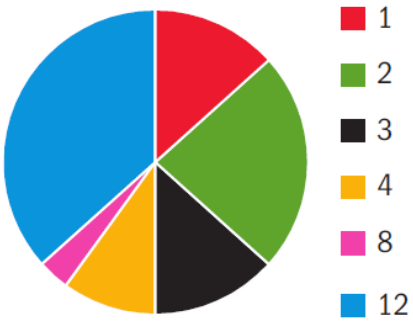
Pictogram to show colours of cars



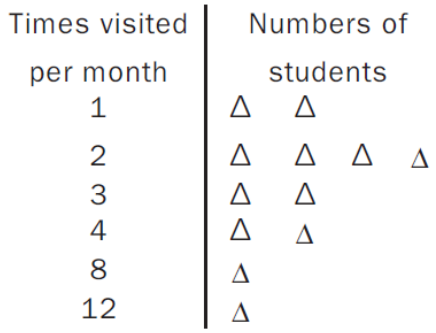
2 Bar graph to show number of times Ida's classmates had visited the cinema per month



Pie chart to show number of visits to the cinema by Ida's classmates



Pictogram to show number of visits to the cinema by Ida's classmates



Measures of central tendency

A measure of central tendency, or average, describes a typical value for a set of data.

There are three common types of average:

- The mode - this is the data value that occurs most often.
- The median - this is the middle item when the data is arranged in order of size.
- The mean - this is what most people mean when they use the word "average". It is found by adding up all of the data and dividing by the number of pieces of data.

Example 1

Find **a** the mode **b** the median and **c** the mean of this data set:

2, 5, 4, 9, 1, 3, 2, 6, 9, 2, 5, 1 3, 4

Answers

a The mode is 2 *2 occurs the most often*

b 1, 2, 2, 2, 3, 4, 4, 5, 5, 6, 9, 9, 1 3 *Write them in order and find the middle one*

The median is 4

c Mean $= \frac{1 + 2 + 2 + 2 + 3 + 4 + 4 + 5 + 5 + 6 + 9 + 9 + 13}{13}$ *Add them all together. There are 13 pieces of data, so divide by 13.*

$= \frac{65}{13} = 5$

Exercise

1 Find **a** the mode **b** the median and **c** the mean of

a 1, 4, 1 , 5, 6, 7, 3, 1 , 8

b 4, 7, 5, 12, 5, -3, -2

c 2, 3, 8, 2, 1, 7, 9, 8, 5

d 25, 28, 29, 21, 25, 20, 27

e 7.4, 10.2, 12.5, 6.8, 10.2

2 Fifteen students were asked how many brothers and sisters they had. The results were:

2, 2, 1, 0, 3, 5, 2, 1 ,1, 0, 1, 4, 1, 0, 2.

Find **a** the mode, **b** the median and **c** the mean number of brothers and sisters.

3 My last nine homework scores, marked out of 10, were:

8, 7, 9, 10, 8, 9, 6, 8, 7

Find **a** the mode **b** the median and **c** the mean homework score.

4 A sprinter's times in seconds for the 40 m dash were:

5.1 3, 4.82, 5.25, 4.94, 5.06, 4.82, 5.12

Find **a** the mode, **b** the median and **c** the mean of the times.

5 Seven farmers own different numbers of chickens.

These numbers are:

253, 78, 497, 166, 710, 497 and 599

Find **a** the mode, **b** the median and **c** the mean number of chickens.

Answers

1 **a** mode=1, median=4, mean=4 **b** mode=5, median=5, mean=4

c mode=2 and 8, median=5, mean=5 **d** mode=25, median=25, mean=25

e mode=10.2, median=10.2, mean=9.42

2 **a** 1 **b** 1 **c** 1 .67

3 **a** 8 **b** 8 **c** 9

4 **a** 4.82 **b** 5.06 **c** 5.02

5 **a** 497 **b** 497 **c** 400

Exponential expressions

Repeated multiplication can be written as an exponential expression.

For example, squaring a number $3 \times 3 = 3^2$ or $5.42 \times 5.42 = 5.42^2$.

If we multiply a number by itself three times then the exponential expression is a cube. For example

$4.6 \times 4.6 \times 4.6 = 4.6^3$.

You can also use exponential expressions for larger integer values.

So, for example, $3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$.

Where the exponent is not a positive integer, these rules apply:

$a^0 = 1$, $a \neq 0$ and $a^{-n} = \frac{1}{a^n}$

Example

Write down the values of 10^2, 10^3, 10^1, 10^0, 10^{-2}, 10^{-3}.

Answer

$10^2 = 10 \times 10 = 100$

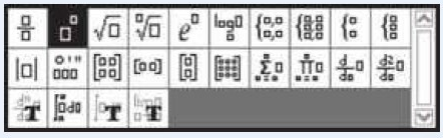
$10^3 = 10 \times 10 \times 10 = 1000$

$10^1 = 10$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

To evaluate an exponential function with the GDC use either the $\wedge$ *key or the template key* $\left|\square\right|\{\square$ *and the exponent template.*



Exercise

Evaluate these expressions.

1 **a** $2^3 + 3^2$ **b** $4^2 \times 3^2$ **c** 2^6

2 **a** 5^0 **b** 3^{-2} **c** 2^{-4}

3 **a** 3.5^5 **b** 0.495^{-2} **c** $2\frac{(1-0.02)^{10}}{1-0.02}$

Answers

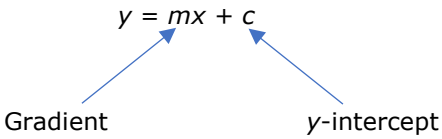
1 **a** 17 **b** 144 **c** 64

2 **a** 0 **b** $\frac{1}{9}$ **c** $\frac{1}{16}$

3 **a** 525.219 **b** 4.081 **c** 2.488

Equations of lines

A straight line is defined by a linear equation of the form.



This is called the gradient-intercept form.

Example 1

Find the equation of the line with gradient 3 passing through (0, 4)

Answer

The line is $y = 3x + 4$ *The y-intercept is 4. The gradient is 3.*

Exercise

1 Find the equation of each line

a gradient 4 passing through (0, 6)

b gradient -2 passing through (0, 2)

c gradient $\frac{1}{3}$ passing through (0, -5)

d gradient $-\frac{2}{5}$ passing through $\left(0, \frac{4}{5}\right)$

2 State the gradient and *y*-intercept of each line.

a $y = 3x$

b $y = -5x + 4$

c $y = \frac{1}{2}x - 8$

d $y = 3(x - 3)$

Answers

1 **a** $y = 4x + 6$ **b** $y = -2x + 2$ **c** $y = \frac{1}{3}x - 5$ **d** $y = -\frac{2}{5}x + \frac{4}{5}$

2 **a** gradient 3, y-intercept (0, 0)

b gradient -5, y-intercept (0, 4)

c gradient $\frac{1}{2}$, y-intercept (0, -8)

d gradient 3, y-intercept (0, -9)

Using the gradient formula to find the equation of a line

Consider a line with a fixed point (x_1, y_1) and a general point (x, y).

Then $m = \frac{y - y_1}{x - x_1}$

or $y - y_1 = m(x - x_1)$.

Example 1

Find the equation of the line with gradient $m = 3$ passing through $(x, y) = (6, 12)$.

Answer

$y - y_1 = m(x - x_1)$

$y - 12 = 3(x - 6)$

$y - 12 = 3x - 18$

$y = 3x - 6$

Exercise

Find the equation of each line in gradient-intercept form

1 Gradient 3, passing through (1, 5)

2 Gradient 4, passing through (5, 11)

3 Gradient 2.5, passing through (4, 12)

4 Gradient $\frac{1}{2}$, passing through (12, 20)

5 Gradient 5, passing through (-2, -13)

6 Gradient -3, passing through (1, 1)

7 Gradient -2, passing through (-3, -1)

8 Gradient $-\frac{1}{2}$, passing though (-4, -3)

Answers

1 $y = 3x + 2$ **2** $y = 4x + 9$ **3** $y = 2.5x + 2$ **4** $y = 0.5x + 1\ 4$

5 $y = 5x - 3$ **6** $y = -3x + 4$ **7** $y = 2x - 5$ **8** $y = -0.5x + 1$

Fractions

Using a GDC, you can either enter a fraction using the fraction template $\frac{\square}{\square}$ or by using the divide key ÷.

Take care - you will sometimes need to use brackets.

Example 1

a Evaluate

$\frac{1}{2}+\frac{3}{8}\times\frac{4}{9}$ × *before* +

$=\frac{1}{2}+\frac{1}{6}$

$=\frac{4}{6}$ *simplify*

$=\frac{2}{3}$

b Evaluate

$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}}$ *evaluate numerator and denominator first*

$=\frac{\frac{5}{6}}{\frac{1}{6}}$

$= 5$

Exercise

1 Calculate

a $\frac{1}{2}+\frac{3}{4}\times\frac{5}{9}$ **b** $\frac{2}{3}\div\frac{5}{6}\times1\frac{1}{3}$ **c** $\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}$ **d** $\frac{1-\left(\frac{2}{3}\right)^5}{1-\frac{2}{3}}$

Answers

1 **a** $\frac{11}{12}$ **b** $\frac{16}{15}$ **c** 1 **d** $\frac{211}{81}$ or $2\frac{49}{81}$

Order of operations

There are several versions of the rules for the order of operations.

BEDMAS: Brackets, exponents, division, multiplication, addition, subtraction.

BIDMAS: Brackets, indices, division, multiplication, addition, subtraction.

BEMDAS: Brackets, exponents, multiplication, division, addition, subtraction.

BODMAS: Brackets, orders, division, multiplication, addition, subtraction.

BOMDAS: Brackets, orders, multiplication, division, addition, subtraction.

PEMDAS: Parentheses, exponents, multiplication, division, addition, subtraction.

They all amount to the same

- Brackets or parentheses are calculated first.
- Next come exponents, indices or orders.
- Then multiplication and division, in order from left to right.
- Finally additions and subtractions.

A fraction line or the line above a square root counts as a bracket too.

Your GDC follows the rules, so if you enter a calculation correctly you should get the correct answers.

The GDC shows divisions as fractions, which makes the order of operations clearer.

Simple calculators, like the ones on phones, do not always follow the calculation rules.

Example 1

Evaluate $\dfrac{11+(-1)^2}{4-(3-5)}$

$= \dfrac{11+1}{4-(-2)}$ *brackets first*

$= \dfrac{12}{6}$ *simplify numerator and denominator*

$= 2$

Example 2

Evaluate $\dfrac{-3+\sqrt{9-8}}{4}$

$= \dfrac{-3+\sqrt{1}}{4}$ *simplify the terms inside the square root*

$= \dfrac{-3+1}{4}$ *evaluate the root*

$= \dfrac{-2}{4}$

$= -\dfrac{1}{2}$ *simplify the numerator and denominator*

On your GDC you can either use templates for the fractions and roots or you can use brackets.

Exercise

Do the questions by hand first, then check your answers with your GDC.

1 Calculate

a 12 - 5 + 4 **b** 6 ÷ 3 × 5 **c** 4 + 2 x 3 - 2

d 8 - 6 ÷ 3 × 2 **e** 4 + (3 - 2) **f** (7 + 2) ÷ 3

g (1 + 4) × (8 - 4) **h** 1 - 3 + 5 × (2 - 1)

2 Find

a $\dfrac{6+9}{4-1}$ **b** $\dfrac{2\times 9}{3\times 4}$ **c** $\dfrac{2-(3+4)}{4\times(2-3)}$ **d** $\dfrac{6\times 5\times 4}{3\times 2-1}$

3 Determine

a $3 \times (-2)^2$ **b** $2^2 \times 3^3 \times 5$ **c** $4 \times (5-3)^2$ **d** $(-3)^2 - 2^2$

4 Calculate

a $\sqrt{3^2+4^2}$ **b** $\left(\sqrt{4}\right)^3$ **c** $\sqrt{4^3}$ **d** $\sqrt{2+\sqrt{2+2}}$

5 Find

a $\sqrt{\dfrac{13^2-\left(3^2+4^2\right)}{2\times 18}}$ **b** $2\sqrt{\dfrac{3+5^2}{7}}$ **c** $2\left(3^2-4(-2)\right)-\left(2-\sqrt{7-3}\right)$

Answers

1 **a** 11 **b** 10 **c** 8 **d** 4

e 5 **f** 3 **g** 16 **f** 3

2 **a** 5 **b** $\frac{3}{2}$ **c** $\frac{5}{4}$ **d** 24

3 **a** 12 **b** 540 **c** 16 **d** 5

4 **a** 5 **b** 8 **c** 8 **d** 2

5 **a** 2 **b** 4 **c** 34

Percentages

A percentage is a way of expressing a fraction or a ratio as part of a hundred.

For example 25% means 25 parts out of 100.

As a fraction, 25% = $\frac{25}{100} = \frac{1}{4}$.

As a decimal, 25% = 0.25.

Example 1

Lara's mark in her maths test was 25 out of 40. What was her mark as a percentage?

Answer

$\frac{25}{40} \times 100 = 62.5\%$ *Write the mark as a fraction.*

Multiply by 100.

Use your GDC.

Example 2

There are 80 students taking the IB in a school. 15% take Maths Standard level. How many students is this?

Answer

Method 1:

$\frac{15}{100} \times 80 = 12$ *Write the percentage as a fraction out of a hundred and then multiply by 80.*

Method 2:

15% = 0.15 *Write the percentage as a decimal.*

$0.15 \times 80 = 12$ *Multiply by 80.*

Exercise

1 Write as percentages

a 13 students from a class of 25

b 14 marks out of 20

2 Find the value of

a 7% of 32 CHF

b 4.5% of 12.00 GBP

c 25% of 750.28 EUR

d 130% of 8000 JPY

Answers

1 **a** 52% **b** 70%

2 **a** 2.24 CHF **b** 0.54 GBP **c** 187.57 EUR **d** 10400 JPY

Exponential expressions

Repeated multiplication can be written as an exponential expression.

For example, squaring a number $3 \times 3 = 3^2$ or $5.42 \times 5.42 = 5.42^2$.

If we multiply a number by itself three times then the exponential expression is a cube. For example

$4.6 \times 4.6 \times 4.6 = 4.6^3$.

You can also use exponential expressions for larger integer values.

So, for example, $3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$.

Where the exponent is not a positive integer, these rules apply:

$a^0 = 1$, $a \neq 0$ and $a^{-n} = \frac{1}{a^n}$

Example

Write down the values of 10^2, 10^3, 10^1, 10^0, 10^{-2}, 10^{-3}.

Answer

$10^2 = 10 \times 10 = 100$

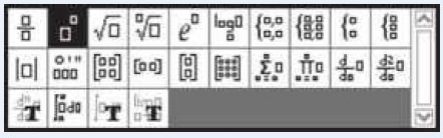
$10^3 = 10 \times 10 \times 10 = 1000$

$10^1 = 10$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

To evaluate an exponential function with the GDC use either the $\wedge$ *key or the template key* $\left|\square\right|\{\square$ *and the exponent template.*



Exercise

Evaluate these expressions.

1 **a** $2^3 + 3^2$ **b** $4^2 \times 3^2$ **c** 2^6

2 **a** 5^0 **b** 3^{-2} **c** 2^{-4}

3 **a** 3.5^5 **b** 0.495^{-2} **c** $2\frac{(1-0.02)^{10}}{1-0.02}$

Answers

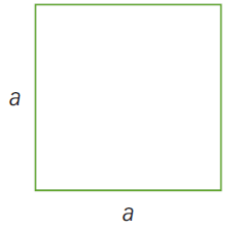
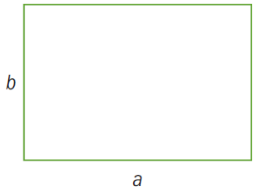
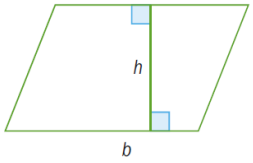
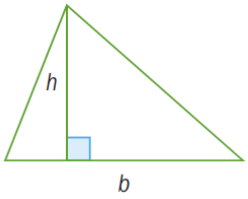
1 **a** 17 **b** 144 **c** 64

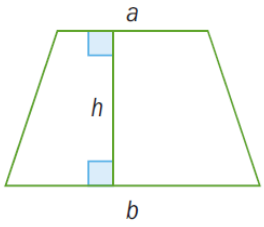
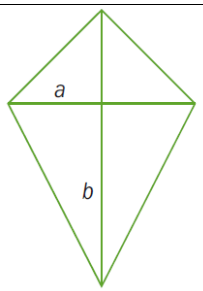
2 **a** 0 **b** $\frac{1}{9}$ **c** $\frac{1}{16}$

3 **a** 525.219 **b** 4.081 **c** 2.488

Area

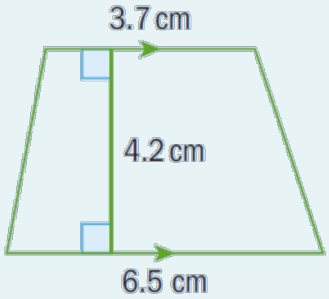
These are the formulae for the areas of a number of plane.

Square	Rectangle	Parallelogram	Triangle
			
$A = a^2$	$A = ab$	$A = bh$	$A = \frac{1}{2}bh$

Trapezium	Kite	Circle
		
$A = \frac{1}{2}(a+b)h$	$A = \frac{1}{2}ab$	$A = \pi r^2$

Example 1

Find the area of this shape.

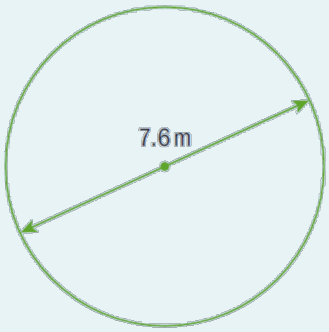


Answer

Area $= \frac{1}{2}(3.7+6.5)(4.2) = 21.42$ cm^2

Example 2

Find the area of this shape giving your answer to 3 significant figures.



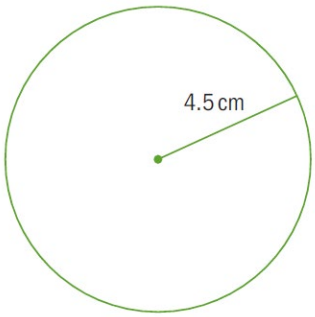
Answer

Area = $\pi(7.6)^2 = 181$ cm^2 (3 sf)

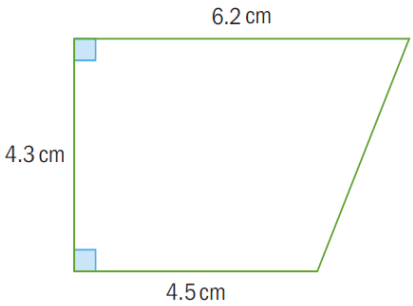
Exercise

Find the areas of these shapes. Give your answer to 3 significant figures.

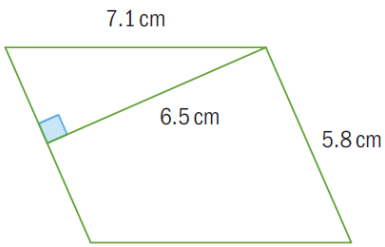
1



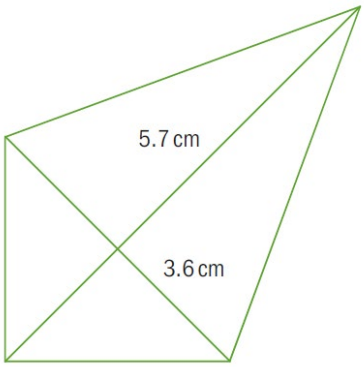
2



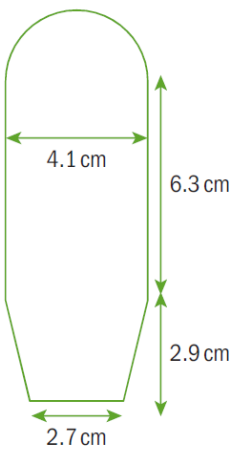
3



4



5



Answers

1 63.6 cm^2 **2** 23.0 cm^2 **3** 37.7 cm^2 **4** 20.5 cm^2

5 42.3 cm^2

Expanding brackets and factorizing

The distributive law is used to expand brackets and factorize expressions.

$a(h + c) = ah + ac$

Example 1

Expand $2y(3x + 5y - z)$

Answer

$2y\ (3x + 5y - z) = 2y \cdot 3x + 2y \cdot 5y + 2y(-z)$

$= 6xy + 10y^2 - 2yz$

Example 2

Factorise $6x^2y - 9xy + 12xz^2$

Answer

$6x^2y - 9xy + 12x\,z^2 = 3x\,(2xy - 3y + 4z^2)$

Look for a common factor. Write this outside the bracket. Find the terms inside the bracket by dividing each term by the common factor.

Exercise

1 Expand

a $3x(x - 2)$ **b** $\frac{x}{y}\left(x^2y - y^2 + x\right)$ **c** $a(b - 2c) + b(2a + b)$

2 Factorize

a $3pq - 6p^2q^3r$ **b** $12ac^2 + 15bc - 3c^2$ **c** $2a^2bc + 3ab^2c - 5abc^2$

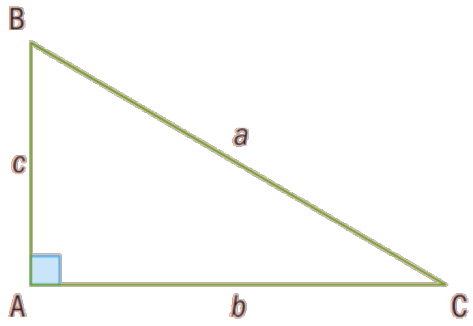
Answers

1 **a** $3x^2 - 2$ **b** $x^3 - xy + \frac{x^2}{y}$ **c** $ab - 2ac + 2ab + b^2$

2 **a** $3pq(1 - 2pq^2r)$ **b** $3c(4ac + 5b - c)$ **c** $abc(2a + 3b - 5c)$

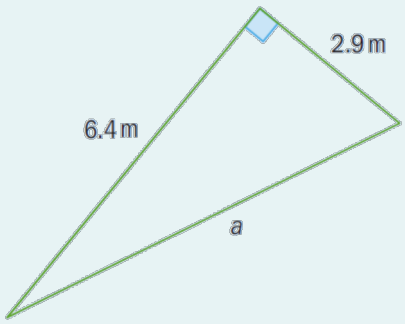
Pythagoras' theorem

In a right-angled triangle *ABC* with sides *a*, *b* and *c*, where *a* is the hypotenuse: $a^2 = b^2 + c^2$



Example 1

Find the length marked *a*.



Answer

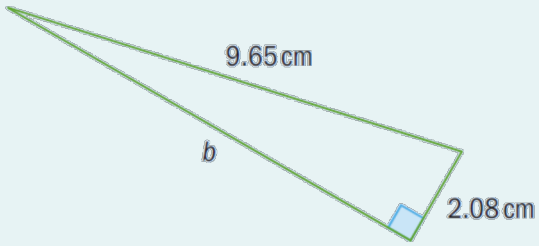
$a^2 = 6.4^2 + 2.9^2$

$a = \sqrt{6.4^2 + 2.9^2} = 7.03$ cm

You can use Pythagoras' Theorem to calculate the length of one side of a right-angled triangle when you know the other two.

Example 2

Find the length marked *b*.



Answer

$9.65^2 = b^2 + 2.08^2$

$b^2 = 9.65^2 - 2.08^2$

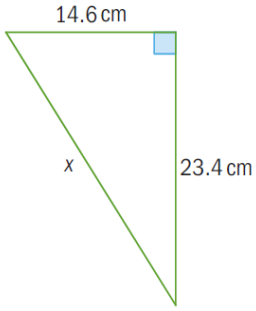
$a = \sqrt{9.65^2 - 2.08^2} = 9.42$ cm

Check your answer by making sure that the hypotenuse is the longest side of the triangle.

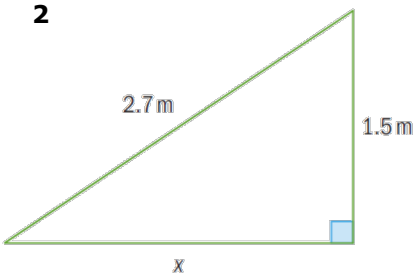
Exercise

In each diagram, find the length of the side marked x. Give your answer to 3 significant figures.

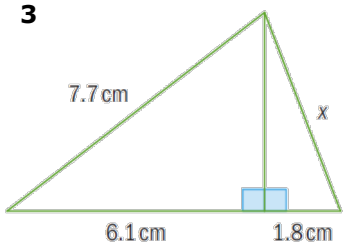
1



2



3



Answers

1 27.6 cm

2 2.24 cm

3 5.03 cm

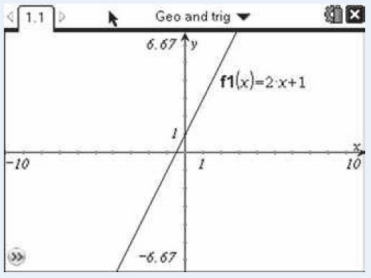
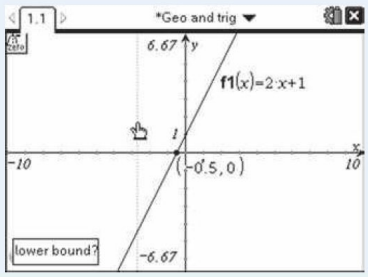
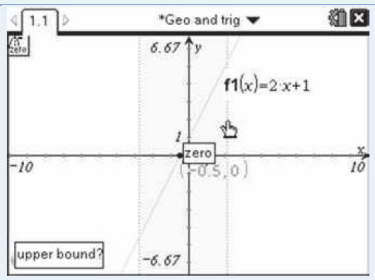
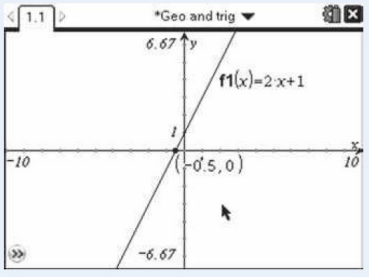
Finding a zero

The x-intercept is known as a zero of the function. At the x-intercept, $y = 0$.

Example 1

Find the zero of $y = 2x + 1$

Answer

First draw the graph of y = 2x + 1	
Press menu 6:Analyze Graph | 1 :Zero Press enter To find the zero you need to give the lower and upper bounds of a region that includes the zero. The GDC shows a line and asks you to set the lower bound. Move the line using the touchpad and choose a position to the left of the zero. Click the touchpad.	
The GDC shows another line and asks you to set the upper bound. Use the touchpad to move the line so that the region between the upper and lower bounds contains the zero. When the region contains the zero, the calculator will display the word 'zero' in a box. Click the touchpad.	
The GDC displays the zero of the function $y = 2x + 1$ at the point (.0.5, 0).	

Measures of central tendency

A measure of central tendency, or average, describes a typical value for a set of data.

There are three common types of average:

- The mode - this is the data value that occurs most often.
- The median - this is the middle item when the data is arranged in order of size.
- The mean - this is what most people mean when they use the word "average". It is found by adding up all of the data and dividing by the number of pieces of data.

Example 1

Find **a** the mode **b** the median and **c** the mean of this data set:

2, 5, 4, 9, 1, 3, 2, 6, 9, 2, 5, 1 3, 4

Answers

a The mode is 2 *2 occurs the most often*

b 1, 2, 2, 2, 3, 4, 4, 5, 5, 6, 9, 9, 1 3 *Write them in order and find the middle one*

The median is 4

c Mean $= \frac{1 + 2 + 2 + 2 + 3 + 4 + 4 + 5 + 5 + 6 + 9 + 9 + 13}{13}$ *Add them all together. There are 13 pieces of data, so divide by 13.*

$= \frac{65}{13} = 5$

Exercise

1 Find **a** the mode **b** the median and **c** the mean of

a 1, 4, 1 , 5, 6, 7, 3, 1 , 8

b 4, 7, 5, 12, 5, -3, -2

c 2, 3, 8, 2, 1, 7, 9, 8, 5

d 25, 28, 29, 21, 25, 20, 27

e 7.4, 10.2, 12.5, 6.8, 10.2

2 Fifteen students were asked how many brothers and sisters they had. The results were:

2, 2, 1, 0, 3, 5, 2, 1 ,1, 0, 1, 4, 1, 0, 2.

Find **a** the mode, **b** the median and **c** the mean number of brothers and sisters.

3 My last nine homework scores, marked out of 10, were:

8, 7, 9, 10, 8, 9, 6, 8, 7

Find **a** the mode **b** the median and **c** the mean homework score.

4 A sprinter's times in seconds for the 40 m dash were:

5.1 3, 4.82, 5.25, 4.94, 5.06, 4.82, 5.12

Find **a** the mode, **b** the median and **c** the mean of the times.

5 Seven farmers own different numbers of chickens.

These numbers are:

253, 78, 497, 166, 710, 497 and 599

Find **a** the mode, **b** the median and **c** the mean number of chickens.

Answers

1 **a** mode=1, median=4, mean=4 **b** mode=5, median=5, mean=4

c mode=2 and 8, median=5, mean=5 **d** mode=25, median=25, mean=25

e mode=10.2, median=10.2, mean=9.42

2 **a** 1 **b** 1 **c** 1 .67

3 **a** 8 **b** 8 **c** 9

4 **a** 4.82 **b** 5.06 **c** 5.02

5 **a** 497 **b** 497 **c** 400

Solving linear equations

'Solve an equation' means 'find the value of the unknown variable' (the letter).

Rearrange the equation so that the unknown variable *x* becomes the subject of the equation. To keep the equation 'balanced' always do the same to both sides.

Example 1

Solve the equation 3x + 5 = 17

Answer

3x + 5 = 17

3x + 5 - 5 = 17 – 5 *subtract 5*

3x = 12

$\frac{3x}{3} = \frac{12}{3}$ *divide by 3*

x = 4

Example 2

Solve the equation 4(x - 5) = 8

Answer

4(x - 5) = 8

$\frac{4(x-5)}{4} = \frac{8}{4}$ *divide by 4*

x - 5 = 2

x - 5 + 5 = 2 + 5 *add 5*

x = 7

Example 3

Solve the equation 7 - 3x = 1

Answer

7 - 3x = 1

7 - 3x - 7 = 1 – 7 *subtract 7*

-3x = -6

$\frac{-3x}{-3} = \frac{-6}{-3}$ *divide by -3*

x = 2

Example 4

Solve the equation $3(2 + 3x) = 5(4 - x)$

Answer

$3(2 + 3x) = 5(4 - x)$

$6 + 9x = 20 - 5x$

$6 + 9x + 5x = 20 - 5x + 5x$ add 5x

$6 + 14x = 20$

$6 + 14x - 6 = 20 - 6$ subtract 6

$14x = 14$

$\frac{14x}{14} = \frac{14}{14}$ divide by 14

$x = 1$

Exercise

Solve these equations.

1 $3x - 10 = 2$

2 $\frac{x}{2} + 5 = 7$

3 $5x + 4 = -11$

4 $3(x + 3) = 18$

5 $4(2x - 5) = 20$

6 $\frac{2}{5}(3x - 7) = 8$

7 $21 - 6x = 9$

8 $12 = 2 - 5x$

9 $2(11 - 3x) = 4$

10 $4(3 + x) = 3(9 - 2x)$

11 $2(10 - 2x) = 4(3x + 1)$

12 $\frac{5x + 2}{3} = \frac{3x + 10}{4}$

Answers

1 x = 4 **2** x = 4 **3** x = -3 **4** x = 3

5 x = 5 **6** x = 9 **7** x = -2.5 **8** x = -2

9 x = 3 **10** x = 1.5 **11** x = 1 **12** x = 2