YOUR PRACTICE PAPER

ANALYSIS AND APPROACHES

STANDARD LEVEL
FOR IBDP MATHEMATICS

ANSWERS

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- 4 Sets of Practice Papers
- Distributions of Exam Topics
- Exam Format Analysis
- Comprehensive Formula List

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AA SL Practice Set 1 Paper 1 Solution

Section A

1. (a) $m + 0.2 = 0.6$ (M1) for valid approach

$m = 0.4$ A1 N2

[2]

(b) $n + 0.4 + 0.2 + 0.1 = 1$ (A1) for substitution

$n = 0.3$ A1 N2

[2]

(c) $\text{P}(B') = 0.4 + 0.3$ (M1) for valid approach

$\text{P}(B') = 0.7$ A1 N2

[2]

2. (a) The mean

$= \frac{300}{15}$ (M1) for valid approach

$= 20$ A1 N2

[2]

(b) (i) -40 A1 N1

(ii) The new variance

$= (-2)^2(9)$ (M1) for valid approach

$= 36$ A1 N2

(iii) 6 A1 N1

[4]

3. (a) The gradient of L_1

$= \frac{32-0}{24-8}$ (M1) for valid approach

$= 2$

The equation of L_1:

$y - 0 = 2(x-8)$ A1

$y = 2x - 16$

$2x - y - 16 = 0$ A1 N2

[3]

(b) $2 \times -\frac{1}{-a} = -1$ (M1) for valid approach

$2 = -a$

$a = -2$ A1 N2

[2]

4. (a) L.H.S.

$= (2n+1)^2 + (2n+3)^2 + (2n+5)^2$

$= 4n^2 + 4n + 1 + 4n^2 + 12n + 9 + 4n^2 + 20n + 25$ M1A1

$= 12n^2 + 36n + 35$

$= 12n^2 + 36n + 33 + 2$ M1

$= 3(4n^2 + 12n + 11) + 2$

$=$R.H.S. AG N0

[3]

(b) $2n+1$, $2n+3$ and $2n+5$ are three consecutive odd numbers. R1

$(2n+1)^2 + (2n+3)^2 + (2n+5)^2$
$= 3(4n^2 + 12n + 11) + 2$ A1

Also $3(4n^2 + 12n + 11)$ is a multiple of 3. R1

Thus, the sum of the squares of any three consecutive odd numbers is greater than a multiple of 3 by 2. AG N0

[3]

5. $f(x) = px^3 + qx^2 - 2x + 1$

$f'(x) = p(3x^2) + q(2x) - 2(1) + 0$ (A1) for correct derivatives

$f'(x) = 3px^2 + 2qx - 2$

$f'(1) = -1 \div -\frac{1}{15}$

$\therefore 3p(1)^2 + 2q(1) - 2 = 15$ (M1) for setting equation

$3p + 2q = 17$

$2q = 17 - 3p$ A1

$f^{-1}(41) = 2$

$\therefore\ f(2) = 41$ (M1) for valid approach

$p(2)^3 + q(2)^2 - 2(2) + 1 = 41$ A1

$8p + 4q - 3 = 41$

$\therefore 8p + 2(17 - 3p) - 3 = 41$ (M1) for substitution

$8p + 34 - 6p - 3 = 41$

$2p = 10$

$p = 5$ A1

$\therefore q = \frac{17 - 3(5)}{2}$

$q = 1$ A1 N5

[8]

6. $kx^2 + (8 + k)x - 1 = 0$ has no real roots.

$\therefore \Delta < 0$ R1

$b^2 - 4ac < 0$ (M1) for valid approach

$(8 + k)^2 - 4(k)(-1) < 0$ A1

$64 + 16k + k^2 + 4k < 0$ (A1) for correct approach

$k^2 + 20k + 64 < 0$ (A1) for correct inequality

$(k + 16)(k + 4) < 0$ (A1) for factorization

$\therefore -16 < k < -4$ A2 N5

[8]

Section B

7. (a) $y = 20 - 4x$ A1 N1

[1]

(b) $V = (4x)(2x)(20 - 4x)$ (M1) for valid approach

$V = 8x^2(20 - 4x)$

$V = 160x^2 - 32x^3$ A1 N2

[2]

(c) $\frac{dV}{dx} = 160(2x) - 32(3x^2)$ (A1) for correct derivatives

$\frac{dV}{dx} = 320x - 96x^2$ A1 N2

[2]

(d) $\frac{dV}{dx} = 0$ (M1) for setting equation

$\therefore 320x - 96x^2 = 0$ A1

$32x(10 - 3x) = 0$ (A1) for factorization

$x = 0$ (*Rejected*) or $x = \frac{10}{3}$ A1 N3

By the first derivative test, M1A1

x	$0 < x < \frac{10}{3}$	$x = \frac{10}{3}$	$x > \frac{10}{3}$
$\frac{dV}{dx}$	$+$	0	$-$

Thus, V attains its maximum at $x = \frac{10}{3}$. R1 N0

[7]

(e) The maximum volume

$= 160\left(\frac{10}{3}\right)^2 - 32\left(\frac{10}{3}\right)^3$ (M1) for substitution

$= \frac{16000}{9} - \frac{32000}{27}$

$= \frac{16000}{27}\ \text{cm}^3$ A1 N2

[2]

(f) $\frac{20}{3}\ \text{cm}$ A1 N1

[1]

8. (a) (i) $\{y : 0 \le y \le 1, y \in \mathbb{R}\}$ A2 N2

(ii) $f(x) = 1$

$\therefore \cos^4 x = 1$

$\cos^2 x = -1$ (*Rejected*) or $\cos^2 x = 1$ (M1) for valid approach

$\cos x = -1$ or $\cos x = 1$

$x = \pi$ or $x = 0$, $x = 2\pi$ (A1) for correct values

Thus, there are 3 solutions. A1 N2

[5]

(b) $f'(x) = (4\cos^3 x)(-\sin x)$ (A1) for chain rule

$f'(x) = -4\sin x \cos^3 x$ A1 N2

[2]

(c) The total area of the regions

$= \int_0^{\pi} (\cos^4 x)(2\sin x)\mathrm{d}x$ (A1) for definite integral

Let $u = \cos x$

$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x \Rightarrow (-1)\mathrm{d}u = \sin x\mathrm{d}x$ (A1) for substitution

$x = \pi \Rightarrow u = \cos \pi = -1$

$x = 0 \Rightarrow u = \cos 0 = 1$

$= \int_1^{-1} -2u^4 \mathrm{d}u$ M1A1

$= \left[-\frac{2}{5}u^5\right]_1^{-1}$ A1

$= -\frac{2}{5}(-1)^5 - \left(-\frac{2}{5}(1)^5\right)$ (M1) for substitution

$= \frac{4}{5}$ A1 N4

[7]

9. (a) (i) $a = \dfrac{37-(-5)}{2}$ M1A1

$a = 21$ AG N0

(ii) $b = \dfrac{2\pi}{2(11-2)}$ (M1) for valid approach

$b = \dfrac{\pi}{9}$ A1 N2

(iii) $d = \dfrac{37+(-5)}{2}$ (M1) for valid approach

$d = 16$ A1 N2

(iv) $c = -2.5$ A1 N1

[7]

(b) The coordinates of P'

$= (3(2)+17, 37+8)$ A1

$= (23, 45)$ A1 N1

[2]

(c) Translation of $\begin{pmatrix} -12 \\ -20 \end{pmatrix}$ followed by A2 N2

a horizontal stretch of scale factor $\dfrac{1}{3}$ A1 N1

[3]

AA SL Practice Set 1 Paper 2 Solution

Section A

1. (a) $y = 3x + 7$

$\Rightarrow x = 3y + 7$ (A1) for correct approach

$3y = x - 7$

$y = \dfrac{x-7}{3}$

$\therefore f^{-1}(x) = \dfrac{x-7}{3}$ A1 N2

[2]

(b) $(f \circ g)(x)$

$= 3g(x) + 7$ (A1) for substitution

$= 3(2\sqrt{x}) + 7$

$= 6\sqrt{x} + 7$ A1 N2

[2]

(c) $(f \circ g)(529)$

$= 6\sqrt{529} + 7$ (M1) for substitution

$= 145$ A1 N2

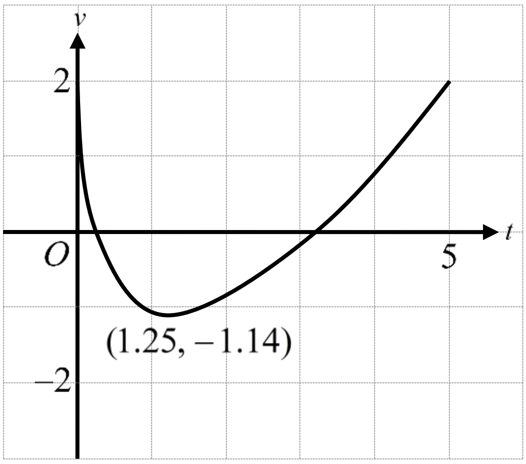
[2]

2. (a) For approximately correct shape A1

For correct minimum point A1

For approximately correct endpoints A1 N3

[3]



(b) (i) $d=\int_0^5 |v(t)|\,\mathrm{d}t$ (M2) for valid approach

$d=\int_0^5 \left|2.5t-5.6\sqrt{t}+2\right|\mathrm{d}t$ A1 N3

(ii) $d=4.084252067\text{ m}$

$d=4.08\text{ m}$ A1 N1

[4]

3. (a) The volume

$=\frac{1}{3}\pi r^2 h$ (M1) for valid approach

$=\frac{1}{3}\pi(18)^2(18)$

$=6107.256119$ (A1) for correct value

$=6110$

$=6.11\times10^3\text{ cm}^3$ A1 N3

[3]

(b) $V=27\left(\frac{2}{3}\pi R^3\right)$ (M1) for setting equation

$16(6107.256119)=18\pi R^3$ (A1) for substitution

$R^3=1728$

$R=12$ A1

The ratio

$=18:12$

$=3:2$ A1 N3

[4]

4. (a) $r = \frac{5.4}{4.5}$ (M1) for valid approach

$r = 1.2$ A1 N2

[2]

(b) $S_{12} = \frac{4.5(1.2^{12} - 1)}{1.2 - 1}$ (A1) for substitution

$S_{12} = 178.1122601$

$S_{12} = 178$ A1 N2

[2]

(c) $u_n < 678$

$4.5 \cdot 1.2^{n-1} < 678$

$4.5 \cdot 1.2^{n-1} - 678 < 0$ (M1) for valid approach

By considering the graph of $y = 4.5 \cdot 1.2^{n-1} - 678$,

$n < 28.50673$. A1

Thus, the greatest value of n is 28. A1 N2

[3]

5. The general term

$= 2ax\binom{17}{r}(1)^{17-r}(3ax^2)^r$ (M1) for valid expansion

$= 2\binom{17}{r}3^r a^{r+1}x^{2r+1}$

$2r + 1 = 9$ (A1) for correct equation

$2r = 8$

$r = 4$ (A1) for correct value

The required term

$= 2\binom{17}{4}3^4 a^{4+1}x^{2(4)+1}$

$= 385560a^5x^9$ (A1) for correct term

$385560a^5 = -385560$ (M1) for setting equation

$a^5 = -1$

$a = -1$ A1 N3

[6]

6.	(a)	$20P_1 - 17P_0 = 0$	
		$\therefore 20(P_0 e^{k(1)}) - 17P_0 = 0$	A1
		$20e^k - 17 = 0$	
		$e^k = 0.85$	M1
		$k = \ln 0.85$	AG N0
			[2]
	(b)	$\frac{P_t}{P_0} \le 0.5$	
		$\therefore \frac{P_0 e^{(\ln 0.85)t}}{P_0} \le 0.5$	(A1) for correct inequality
		$e^{(\ln 0.85)t} \le 0.5$	(A1) for correct approach
		$(\ln 0.85)t \le \ln 0.5$	
		$(\ln 0.85)t - \ln 0.5 \le 0$	A1
		By considering the graph of $y = (\ln 0.85)t - \ln 0.5$, $t \ge 4.2650243$.	(M1) for valid approach
		Thus, the least number of whole years is 43.	A1 N3
			[5]

Section B

7.	(a)	$a = -0.176$	A1	N1	
		$b = 15260$	A1	N1	
					[2]
	(b)	The estimated insurance cost			
		$= -0.176(32500) + 15260$	(A1) for substitution		
		$= \$9540$	A1	N2	
					[2]
	(c)	The insurance cost			
		$= 9540 \times (1 - 2.5\%)^4$	(M1)(A1) for valid approach		
		$= 9540 \times 0.975^4$	(A1) for simplification		
		$= \$8621.182477$			
		$= \$8620$	A1	N2	
					[4]
	(d)	$9540 \times (1 - 2.5\%)^t = 6500$	(M1) for setting equation		
		$9540 \times 0.975^t - 6500 = 0$	(A1) for simplification		
		By considering the graph of			
		$y = 9540 \times 0.975^t - 6500$, $t = 15.154997$.	(A1) for correct value		
		Thus, the year is 2036.	A1	N2	
					[4]

8. (a) The required probability

$= \mathrm{P}(T \le 24)$ — (M1) for valid approach

$= 0.9452007106$

$= 0.945$ — A1 N2

[2]

(b) $\mathrm{P}(U \le 48) = 0.99494$

$\mathrm{P}\left(Z \le \frac{48-\mu}{7}\right) = 0.99494$ — (M1) for standardization

$\frac{48-\mu}{7} = 2.571701859$ — A1

$48-\mu = 18.00191301$

$\mu = 29.99808699$

$\mu = 30.0$ — A1 N3

[3]

(c) The required probability

$= \mathrm{P}(U \le 36)$ — R1

$= 0.8043925789$ — A1

Thus, for all school buses departing at $8:24\ \mathrm{am}$, 80.439% of them will arrive at school on time. — AG N0

[2]

(d) The required probability

$= 1 - \mathrm{P}(T \le 12)\mathrm{P}(U \le 48)$

$- \mathrm{P}(12 < T \le 24)\mathrm{P}(U \le 36)$ — M1A1

$= 1 - (0.2118553337)(0.99494)$

$-(0.7333453769)(0.80439)$ — (A2) for correct values

$= 0.1993209666$

$= 0.199$ — A1 N3

[5]

(e) The expected number

$= (20)(0.1993209666)$ — (A1) for correct formula

$= 3.986419331$

$= 3.99$ — A1 N2

[2]

9.	(a)	$AB^2 = r^2 + r^2 - 2(r)(r)\cos 2\alpha$	A1	
		$AB^2 = 2r^2 - 2r^2\cos 2\alpha$		
		$AB = \sqrt{2r^2 - 2r^2\cos 2\alpha}$	A1	
		$AB = \sqrt{2r^2(1-\cos 2\alpha)}$		
		$AB = r\sqrt{2(1-\cos 2\alpha)}$	AG	N0
				[2]
	(b)	The arc length ACB		
		$= (r)(2\alpha)$	A1	
		$= 2r\alpha$		
		$\therefore P$		
		$= 2r\alpha + r\sqrt{2(1-\cos 2\alpha)}$	M1	
		$= 2r\alpha + r\sqrt{2(1-(1-2\sin^2\alpha))}$	A1	
		$= 2r\alpha + r\sqrt{2(2\sin^2\alpha)}$	A1	
		$= 2r\alpha + r\sqrt{4\sin^2\alpha}$		
		$= 2r\alpha + 2r\sin\alpha$	A1	
		$= 2r(\alpha + \sin\alpha)$	AG	N0
				[5]
	(c)	(i) $\theta = 1.1060602$		
		$\theta = 1.11$	A1	N1
		(ii) $\theta = 0.7897927$		
		$\theta = 0.790$	A1	N1
				[2]
	(d)	$1.5(2r) < P < 2(2r)$	M1A1	
		$\therefore 1.5(2r) < 2r(\alpha + \sin\alpha) < 2(2r)$		
		$1.5 < \alpha + \sin\alpha < 2$	(A1) for correct inequality	
		$1.5 < f(\alpha) < 2$		
		By using (c), $0.7897927 < \alpha < 1.1060602$.	(M1) for valid approach	
		$\therefore 0.790 < \alpha < 1.11$	A1	N3
				[5]

AA SL Practice Set 2 Paper 1 Solution

Section A

1. (a) $12+f+10+16+24=80$ (M1) for setting equation

$f=18$ A1 N2

[2]

(b) (i) The median

$=\frac{3+4}{2}$ (M1) for valid approach

$=3.5$ A1 N2

(ii) 5 A1 N1

(iii) The interquartile range

$=\frac{5+5}{2}-\frac{2+2}{2}$ (M1) for valid approach

$=3$ A1 N2

[5]

2. (a) (i) 7 A1 N1

(ii) 1 A1 N1

[2]

(b) $(f\circ g)(x)$

$=(g(x))^2$ (A1) for substitution

$=(3-4x)^2$

$=9-24x+16x^2$ A1 N2

[2]

(c) $y=3-4x$

$\Rightarrow x=3-4y$ (A1) for correct approach

$4y=3-x$

$y=\frac{3-x}{4}$

$\therefore g^{-1}(x)=\frac{3-x}{4}$ A1 N2

[2]

3. (a) $g'(x) = 4\cos 2x$

$g(x) = \int 4\cos 2x \, dx$ (M1) for indefinite integral

Let $u = 2x$
$\frac{du}{dx} = 2 \Rightarrow du = 2dx$

(A1) for substitution

$g(x) = \int 2\cos u \, du$

$g(x) = 2\sin u + C$ A1

$g(x) = 2\sin 2x + C$

$\therefore 7 = 2\sin 2\left(\frac{\pi}{4}\right) + C$ (M1) for substitution

$7 = 2\sin\frac{\pi}{2} + C$

$7 = 2 + C$

$C = 5$

$\therefore g(x) = 2\sin 2x + 5$ A1 N4

[5]

(b) 5 A1 N1

[1]

4. (a) R.H.S.

$= \frac{1\times 49}{1\times 49} + \frac{2\times 7}{7\times 7} + \frac{5}{49}$ M1

$= \frac{49+14+5}{49}$ A1

$= \frac{68}{49} = \text{L.H.S.}$

$\therefore \frac{68}{49} = 1 + \frac{2}{7} + \frac{5}{49}$ AG N0

[2]

(b) R.H.S.

$= \frac{1\times (m+2)^2}{1\times (m+2)^2} + \frac{2\times (m+2)}{(m+2)\times (m+2)} + \frac{5}{(m+2)^2}$ M1

$= \frac{(m^2+4m+4)+(2m+4)+5}{(m+2)^2}$ M1A1

$= \frac{m^2+6m+9+4}{(m+2)^2}$

$= \frac{(m+3)^2+4}{(m+2)^2} = \text{L.H.S.}$

$\therefore \frac{(m+3)^2+4}{(m+2)^2} \equiv 1 + \frac{2}{m+2} + \frac{5}{(m+2)^2}$ for $m \neq -2$ AG N0

[3]

5. $9\log_{27}(x+1) = 1 + \log_3(3+x+x^2)$

$\frac{9\log_3(x+1)}{\log_3 27} = \log_3 3 + \log_3(3+x+x^2)$ (M1)(A1) for change of base

$\frac{9\log_3(x+1)}{3} = \log_3 3(3+x+x^2)$ (A1) for correct approach

$3\log_3(x+1) = \log_3 3(3+x+x^2)$

$\log_3(x+1)^3 = \log_3 3(3+x+x^2)$ A1

$\therefore (x+1)^3 = 3(3+x+x^2)$ M1

$x^3+3x^2+3x+1 = 9+3x+3x^2$

$x^3 = 8$ A1

$x = \sqrt[3]{8}$

$x = 2$ A1 N4

[7]

6. (a) The discriminant of $f(x)$

$= b^2 - 4ac$

$= (8-p)^2 - 4\left(1 + 2p - \frac{3}{8}p^2\right)(-2)$ M1A1

$= 64 - 16p + p^2 + 8 + 16p - 3p^2$ A1

$= 72 - 2p^2$ AG N0

[3]

(b) $f(x) = 0$ has two equal roots

$\therefore 72 - 2p^2 = 0$ (M1) for setting equation

$2p^2 = 72$

$p^2 = 36$

$p = -6$ or $p = 6$ A2 N3

[3]

(c) $p = 6$

$\therefore \left(1 + 2(6) - \frac{3}{8}(6)^2\right)x^2 + (8-6)x - 2 = 0$ (M1) for setting equation

$-\frac{1}{2}x^2 + 2x - 2 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$ A1 N2

[2]

Section B

7. (a) $f'(x) = -\frac{1}{2}x^2 + 5x$

$f(x) = \int\left(-\frac{1}{2}x^2 + 5x\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x) = -\frac{1}{2}\left(\frac{1}{3}x^3\right) + 5\left(\frac{1}{2}x^2\right) + C$ A1

$f(x) = -\frac{1}{6}x^3 + \frac{5}{2}x^2 + C$

$\therefore -\frac{26}{3} = -\frac{1}{6}(0)^3 + \frac{5}{2}(0)^2 + C$ (M1) for substitution

$C = -\frac{26}{3}$

$\therefore f(x) = -\frac{1}{6}x^3 + \frac{5}{2}x^2 - \frac{26}{3}$ A1

$f(2) = -\frac{1}{6}(2)^3 + \frac{5}{2}(2)^2 - \frac{26}{3}$ (M1) for substitution

$f(2) = -\frac{4}{3} + 10 - \frac{26}{3}$

$f(2) = 0$ A1 N4

[6]

(b) $f''(x) = -\frac{1}{2}(2x) + 5(1)$ (A1) for correct derivatives

$f''(x) = -x + 5$

$f''(x) = 0$

$\therefore -x + 5 = 0$ (M1) for setting equation

$x = 5$ A1

$f(5) = -\frac{1}{6}(5)^3 + \frac{5}{2}(5)^2 - \frac{26}{3}$ (M1) for substitution

$f(5) = -\frac{125}{6} + \frac{375}{6} - \frac{52}{6}$

$f(5) = 33$

Thus, the coordinates of P are $(5, 33)$. A1 N4

[5]

(c) The graph of f is concave up

$\therefore f''(x) > 0$ (A1) for correct inequality

$-x + 5 > 0$

$x < 5$ A1 N2

[2]

8.	(a)	$2r+h=20$	(A1) for correct approach	
		$2r=20-h$		
		$r=10-\frac{1}{2}h$	A1 N2	
				[2]
	(b)	$V=\pi r^2 h$		
		$V=\pi\left(10-\frac{1}{2}h\right)^2 h$	(A1) for substitution	
		$V=100\pi h-10\pi h^2+\frac{1}{4}\pi h^3$	A1 N2	
				[2]
	(c)	$Q=(3)(2\pi rh)+(4)(\pi r^2)$	M1A1	
		$Q=6\pi\left(10-\frac{1}{2}h\right)h+4\pi\left(10-\frac{1}{2}h\right)^2$	M1	
		$Q=60\pi h-3\pi h^2+400\pi-40\pi h+\pi h^2$	A1	
		$Q=400\pi+20\pi h-2\pi h^2$		
		$Q=2\pi(200+10h-h^2)$	AG N0	
				[4]
	(d)	$\frac{dQ}{dh}=2\pi(0+10(1)-2h)$	(A1) for correct derivatives	
		$\frac{dQ}{dh}=4\pi(5-h)$	A1	
		$\frac{dQ}{dh}=0$	(M1) for setting equation	
		$\therefore 4\pi(5-h)=0$	A1	
		$h=5$	A1	
		The maximum value of Q		
		$=2\pi(200+10(5)-(5)^2)$	(M1) for substitution	
		$=450\pi$	A1 N4	
				[7]

9. (a) $r=\dfrac{20\cos^4\alpha}{30\cos^2\alpha}$ (M1) for valid approach

$r=\dfrac{2}{3}\cos^2\alpha$ A1 N2

[2]

(b) $\pi\le\alpha\le\dfrac{4}{3}\pi$

$\therefore\cos\pi\le\cos\alpha\le\cos\dfrac{4}{3}\pi$ (M1) for valid approach

$-1\le\cos\alpha\le-\dfrac{1}{2}$

$\dfrac{1}{4}\le\cos^2\alpha\le1$

$\dfrac{1}{6}\le\dfrac{2}{3}\cos^2\alpha\le\dfrac{2}{3}$

$\therefore\dfrac{1}{6}\le r\le\dfrac{2}{3}$ A1 N2

[2]

(c) $S_\infty=\dfrac{30\cos^2\alpha}{1-\dfrac{2}{3}\cos^2\alpha}$ A1

$S_\infty=\dfrac{30\cos^2\alpha}{\sin^2\alpha+\cos^2\alpha-\dfrac{2}{3}\cos^2\alpha}$ M1

$S_\infty=\dfrac{30\cos^2\alpha}{\sin^2\alpha+\dfrac{1}{3}\cos^2\alpha}$ A1

$S_\infty=\dfrac{30}{\tan^2\alpha+\dfrac{1}{3}}$ A1

$S_\infty=\dfrac{90}{3\tan^2\alpha+1}$ AG N0

[4]

(d) $\pi \le \alpha \le \frac{4}{3}\pi$

$\therefore \tan\pi \le \tan\alpha \le \tan\frac{4}{3}\pi$ M1

$0 \le \tan\alpha \le \sqrt{3}$

$0 \le \tan^2\alpha \le 3$ A1

$0 \le 3\tan^2\alpha \le 9$

$1 \le 3\tan^2\alpha + 1 \le 10$

$\therefore \frac{1}{10} \le \frac{1}{3\tan^2\alpha + 1} \le 1$ A1

$9 \le S_\infty \le 90$ A1

When $\alpha = \frac{4}{3}\pi$,

$S_\infty = \frac{90}{3\tan^2\left(\frac{4}{3}\pi\right) + 1}$ M1

$S_\infty = 9$

Thus, S_∞ attains its minimum at $\alpha = \frac{4}{3}\pi$. AG N0

[5]

AA SL Practice Set 2 Paper 2 Solution

Section A

1. (a) $(3, 5)$ A2 N2

[2]

(b) $g(x) = -(x-3)^2 + 5$ A2 N2

[2]

(c) $(-3, 5)$ A2 N2

[2]

2. (a) $d = 6 - 4.9$ (M1) for valid approach

$d = 1.1$ A1 N2

[2]

(b) $u_1 = 4.9 - 1.1$ (M1) for valid approach

$u_1 = 3.8$ A1 N2

[2]

(c) $S_{38} = \frac{38}{2}\left[2(3.8) + (38-1)(1.1)\right]$ (A1) for substitution

$S_{38} = 917.7$ A1 N2

[2]

3.

$$\left(kx-\frac{4}{x}\right)^8=(kx)^8+\binom{8}{1}(kx)^7\left(-\frac{4}{x}\right)+\binom{8}{2}(kx)^6\left(-\frac{4}{x}\right)^2$$
$$+\binom{8}{3}(kx)^5\left(-\frac{4}{x}\right)^3+\binom{8}{4}(kx)^4\left(-\frac{4}{x}\right)^4+\ldots$$

(M1)(A1) for correct approach

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8+8k^7x^7\left(-\frac{4}{x}\right)+28k^6x^6\left(\frac{16}{x^2}\right)$$
$$+56k^5x^5\left(-\frac{64}{x^3}\right)+70k^4x^4\left(\frac{256}{x^4}\right)+\ldots$$

(A1) for simplification

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8-32k^7x^6+448k^6x^4$$
$$-3584k^5x^2+17920k^4+\ldots$$

A1

$\therefore 448k^6:17920k^4=9:40$ A1

$$\frac{448k^6}{17920k^4}=\frac{9}{40}$$

$$\frac{k^2}{40}=\frac{9}{40}$$

$k=-3$ or $k=3$ (*Rejected*) A1 N4

[6]

4. (a) $A=2\pi r^2+2\pi rh+2\pi r^2$ (M2) for setting equation

$135\pi=4\pi r^2+2\pi r(3.5)$ (A1) for substitution

$135=4r^2+7r$

$4r^2+7r-135=0$ (M1) for quadratic equation

$(4r+27)(r-5)=0$

$4r+27=0$ or $r-5=0$

$r=-\frac{27}{4}$ (*Rejected*) or $r=5\text{ mm}$ A1 N3

[5]

(b) The volume

$=\frac{4}{3}\pi r^3+\pi r^2h$ (M1) for valid approach

$=\frac{4}{3}\pi(5)^3+\pi(5)^2(3.5)$

$=798.4881328\text{ mm}^3$

$=798\text{ mm}^3$ A1 N2

[2]

5. $X \sim \mathrm{B}\left(5, \dfrac{2p}{p+2p+10}\right)$ (R1) for correct distribution

The standard deviation of X

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(1-\dfrac{2p}{3p+10}\right)}$ (A1) for substitution

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)}$

$\therefore \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)} > \dfrac{11}{10}$ (M1) for valid approach

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) > \dfrac{121}{100}$ M1

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100} > 0$ A1

By considering the graph of

$y = 5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100}$,

$5.3435147 < p < 25.443002$.

Thus, the greatest value of p is 25. A1 N3

[6]

6. $v = \int (8-8t)\,\mathrm{d}t$ (M1) for indefinite integral

$v = 8t - 8\left(\dfrac{1}{2}t^2\right) + C$ A1

$v = 8t - 4t^2 + C$

The initial velocity

$= 8(0) - 4(0)^2 + C$ (M1) for valid approach

$= C$

The difference between the velocities is $4\ \mathrm{ms}^{-1}$

$\therefore 8t - 4t^2 + C = C + 4$ or $\therefore 8t - 4t^2 + C = C - 4$ (A1) for correct approach

$4t^2 - 8t + 4 = 0$ or $4t^2 - 8t - 4 = 0$ A2

$4(t-1)^2 = 0$ or $t = \dfrac{-(-8) \pm \sqrt{(-8) - 4(4)(-4)}}{2(4)}$

$t = 1$ or $t = 2.414213562$, $t = -0.4142135624$ (*Rejected*)

$\therefore m = 1$ or $m = 2.41$ A2 N5

[8]

Section B

7. (a) $a = 5.6$ A1 N1

$b = 34.8$ A1 N1

[2]

(b) The estimated hardness

$= 5.6(6.3) + 34.8$ (A1) for substitution

$= 70.08$ A1 N2

[2]

(c) (i) The required probability

$= \frac{108}{120}$ (M1) for valid approach

$= \frac{9}{10}$ A1 N2

(ii) The required probability

$= \frac{120-56}{120}$ (M1) for valid approach

$= \frac{8}{15}$ A1 N2

(iii) The required probability

$= \frac{120-108}{120-56}$ (M1) for valid approach

$= \frac{3}{16}$ A1 N2

[6]

(d) $\left(\frac{120-q}{120}\right)\left(\frac{120-q-1}{120-1}\right) = \frac{29}{476}$ (A1) for correct equation

$\left(\frac{120-q}{120}\right)\left(\frac{119-q}{119}\right) = \frac{29}{476}$

$(120-q)(119-q) = 870$

$(120-q)(119-q) - 870 = 0$

By considering the graph of

$y = (120-q)(119-q) - 870$, $q = 90$ or

$q = 149$ (*Rejected*).

$\therefore q = 90$ A1 N2

[2]

8. (a) (i) $\cos A\hat{C}B = \dfrac{r^2 + (1.5r)^2 - (1.75r)^2}{2(r)(1.5r)}$ M1A1

$\cos A\hat{C}B = \dfrac{0.1875r^2}{3r^2}$ A1

$\cos A\hat{C}B = 0.0625$ AG N0

(ii) $\sin A\hat{C}B = \sqrt{1-\cos^2 A\hat{C}B}$

$\sin A\hat{C}B = \sqrt{1-0.0625^2}$ (A1) for substitution

$\sin A\hat{C}B = 0.9980449639$

$\sin A\hat{C}B = 0.998$ A1 N2

[5]

(b) $\dfrac{1}{2}(r)(1.5r)\sin A\hat{C}B = 7$ (M1) for setting equation

$(0.75r^2)(0.9980449639) = 7$

$r^2 = 9.35161608$

$r = 3.058041216$

$r = 3.06$ A1 N2

[2]

(c) (i) $\dfrac{\sin A\hat{B}C}{AC} = \dfrac{\sin A\hat{C}B}{AB}$ (M1) for sine rule

$\dfrac{\sin A\hat{B}C}{1.5r} = \dfrac{0.9980449639}{1.75r}$ (A1) for substitution

$\sin A\hat{B}C = 0.8554671119$

$A\hat{B}C = 1.026452178 \text{ rad}$

$A\hat{B}C = 1.03 \text{ rad}$ A1 N3

(ii) The area of the sector BDC

$= \dfrac{1}{2}(3.058041216)^2(\pi - 1.026452178)$ (A1) for substitution

$= 9.88999084$

$= 9.89$ A1 N2

[5]

9. (a) $P(L > 59.2) = 0.12$ (M1) for valid approach

$P\left(Z > \dfrac{59.2-\mu}{3.5}\right) = 0.12$ (A1) for correct approach

$\dfrac{59.2-\mu}{3.5} = 1.174986791$ A1

$59.2-\mu = 4.11245377$

$\mu = 55.08754623$

$\mu = 55.1$ A1 N3

[4]

(b) $P(L < q) = 0.55$

$P\left(Z < \dfrac{q-55.08754623}{3.5}\right) = 0.55$ (A1) for correct approach

$\dfrac{q-55.08754623}{3.5} = 0.1256613375$

$q-55.08754623 = 0.4398146813$

$q = 55.52736091$ A1

$\therefore q = 55.5$ A1 N2

[3]

(c) (i) $X \sim B(10, 0.55)$ (R1) for correct distribution

$E(X) = (10)(0.55)$ (A1) for substitution

$E(X) = 5.5$ A1 N2

(ii) $P(X > 5) = 1 - P(X \le 5)$ (M1) for valid approach

$P(X > 5) = 1 - 0.4955954083$ A1

$P(X > 5) = 0.5044045917$

$P(X > 5) = 0.504$ A1 N2

[6]

(d) $m\left(\dfrac{55\%}{55\%+33\%}\right)(0.8) + m\left(\dfrac{33\%}{55\%+33\%}\right)(1.1)$ (M1)(A1) for correct approach

$= (949)(1000)$

$0.5m + 0.4125m = 949000$ A1

$0.9125m = 949000$

$m = 1040000$ A1 N3

[4]

AA SL Practice Set 3 Paper 1 Solution

Section A

1. (a) The equation of the axis of symmetry:

$x = -\frac{-20}{2(2)}$ (A1) for substitution

$x = 5$ A1 N2

[2]

(b) (i) 2 A1 N1

(ii) 5 A1 N1

(iii) $k = 2(5)^2 - 20(5) + 60$ (M1)(A1) for substitution

$k = 10$ A1 N2

[5]

2. (a) The common difference

$= 95 - 100$ (M1) for valid approach

$= -5$ A1 N2

[2]

(b) The fifteenth term

$= 100 + (15-1)(-5)$ (A1) for substitution

$= 30$ A1 N2

[2]

(c) The sum of the first fifteen terms

$= \frac{15}{2}\left[2(100) + (15-1)(-5)\right]$ (A1) for substitution

$= 975$ A1 N2

[2]

3. (a) The gradient of L_1 is 2. A1 N1

The y-intercept of L_1 is -20. A1 N1

[2]

(b) The gradient of L_2 is $-\frac{1}{2}$. (A1) for correct value

The equation of L_2:

$y-(-20)=-\frac{1}{2}(x-0)$ A1

$y+20=-\frac{1}{2}x$

$2y+40=-x$

$x+2y+40=0$ A1 N2

[3]

4. (a) (i) 4 A1 N1

(ii) $\frac{1}{3}$ A1 N1

(iii) -1 A1 N1

[3]

(b) $\log_{27}x+\frac{8}{3}=\log_4 256+\log_{125}5+\log_\pi\frac{1}{\pi}$

$\log_{27}x+\frac{8}{3}=4+\frac{1}{3}-1$ (M1) for substitution

$\log_{27}x=\frac{2}{3}$

$x=27^{\frac{2}{3}}$ (A1) for correct approach

$x=(3^3)^{\frac{2}{3}}$

$x=3^2$

$x=9$ A1 N3

[3]

5. $\left(1-\frac{3}{4}x\right)^n(1+2nx)^3$

$=\left(1+\binom{n}{1}\left(-\frac{3}{4}x\right)+\cdots\right)\left(1+\binom{3}{1}(2nx)+\cdots\right)$ (M1) for valid expansion

$=\left(1+(n)\left(-\frac{3}{4}x\right)+\cdots\right)(1+(3)(2nx)+\cdots)$ (A1) for correct approach

$=\left(1-\frac{3}{4}nx+\cdots\right)(1+6nx+\cdots)$ A2

The coefficient of x

$=(1)(6n)+\left(-\frac{3}{4}n\right)(1)$ (A1) for correct approach

$=\frac{21}{4}n$

$\therefore \frac{21}{4}n=\frac{105}{4}$ (M1) for setting equation

$n=5$ A1 N5

[7]

6. $-3\sqrt{3}\le f(x)\le 3\sqrt{3}$

$-3\sqrt{3}\le 6\sin 2x\le 3\sqrt{3}$

$-\frac{\sqrt{3}}{2}\le \sin 2x\le \frac{\sqrt{3}}{2}$ A1

$\therefore \sin\left(-\frac{\pi}{3}\right)\le \sin 2x\le \sin\frac{\pi}{3}$,

$\sin\left(\pi-\frac{\pi}{3}\right)\le \sin 2x\le \sin\left(\pi+\frac{\pi}{3}\right)$ or (A2) for correct ranges

$\sin\left(2\pi-\frac{\pi}{3}\right)\le \sin 2x\le \sin\left(2\pi+\frac{\pi}{3}\right)$

$-\frac{\pi}{3}\le 2x\le\frac{\pi}{3}$, $\frac{2\pi}{3}\le 2x\le\frac{4\pi}{3}$ or $\frac{5\pi}{3}\le 2x\le\frac{7\pi}{3}$ A1

$-\frac{\pi}{6}\le x\le\frac{\pi}{6}$, $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ or $\frac{5\pi}{6}\le x\le\frac{7\pi}{6}$ (M1) for valid approach

$\therefore 0\le x\le\frac{\pi}{6}$, $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ or $\frac{5\pi}{6}\le x\le\frac{7\pi}{6}$ A3 N4

[8]

Section B

7. (a) (i) The number of girls

$=35+45-50$ (M1) for valid approach

$=30$ A1 N2

(ii) 5 A1 N1

[3]

(b) (i) $\frac{9}{10}$ A1 N1

(ii) The required probability

$=\frac{30}{50}\div\frac{9}{10}$ (A1) for substitution

$=\frac{2}{3}$ A1 N2

[3]

(c) The required probability

$=\left(\frac{5}{50}\right)\left(\frac{4}{49}\right)$ (M1) for valid approach

$=\frac{2}{245}$ A1 N2

[2]

(d) (i) $\mathrm{P}(G\cap V)=\frac{30}{50}$ A1

$\because \mathrm{P}(G\cap V)\neq 0$ R1

Thus, G and V are not mutually exclusive. AG N0

(ii) $\mathrm{P}(G)=\frac{35}{50}$ A1

$\mathrm{P}(V)=\frac{45}{50}$

$\mathrm{P}(G)\cdot\mathrm{P}(V)=\frac{63}{100}$ A1

$\because \mathrm{P}(G)\cdot\mathrm{P}(V)\neq\mathrm{P}(G\cap V)$ R1

Thus, G and V are not independent. AG N0

[5]

8. (a) $f''(x) = k(2x) - 12(1) - 0$ (A1) for correct derivatives

$f''(x) = 2kx - 12$ A1 N2

[2]

(b) $f''(1.5) = 0$ M1

$\therefore 2k(1.5) - 12 = 0$ A1

$3k = 12$ A1

$k = 4$ AG N0

[3]

(c) $f'(4) = 4(4)^2 - 12(4) - 40$ (M1) for substitution

$f'(4) = -24$ A1

The slope of the normal

$= \dfrac{-1}{-24}$ (A1) for correct approach

$= \dfrac{1}{24}$

The equation of the normal:

$y - \dfrac{13}{6} = \dfrac{1}{24}(x - 4)$ M1A1

$y - \dfrac{13}{6} = \dfrac{1}{24}x - \dfrac{1}{6}$

$y = \dfrac{1}{24}x + 2$ A1 N3

[6]

(d) $f''(5) = 2(4)(5) - 12$ M1

$f''(5) = 28$ A1

$f''(5) > 0$ R1

Thus, the graph of f has a local minimum at $x = 5$. AG N0

[3]

9. (a) $g(x) - f(x) = 0$

$e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right) = 0$ (M1) for valid approach

$e^{\frac{1}{2}\sqrt{x}}\left(1 - \sin\left(\frac{\pi}{3}x\right)\right) = 0$

$1 - \sin\left(\frac{\pi}{3}x\right) = 0$

$\sin\left(\frac{\pi}{3}x\right) = 1$ A1

$\frac{\pi}{3}x = \frac{\pi}{2}$, $\frac{\pi}{3}x = \frac{5\pi}{2}$ or $\frac{\pi}{3}x = \frac{9\pi}{2}$ (A1) for correct values

$x = \frac{3}{2}$, $x = \frac{15}{2}$ or $x = \frac{27}{2}$ A3 N3

[6]

(b) (i) $\frac{\pi}{3}x_n = \frac{\pi}{2} + (n-1)(2\pi)$ A1

$x_n = \frac{3}{2} + 6(n-1)$

$x_{n+1} - x_n$

$= \left(\frac{3}{2} + 6((n+1)-1)\right) - \left(\frac{3}{2} + 6(n-1)\right)$ M1

$x_{n+1} - x_n = \left(\frac{3}{2} + 6n\right) - \left(\frac{3}{2} + 6n - 6\right)$

$x_{n+1} - x_n = 6$ A1

The differences between each pair of consecutive terms are equal to 6. Thus, $x_1, x_2, x_3, \ldots$ is an arithmetic sequence. AG N0

(ii) $x_n = \frac{3}{2} + 6n - 6$

$x_n = 6n - \frac{9}{2}$ A1 N1

[4]

(c) Note that $x_2 = \frac{15}{2}$ and $x_3 = \frac{27}{2}$.

$f(x) = 0$

$e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right) = 0$ M1

$\sin\left(\frac{\pi}{3}x\right) = 0$

$\frac{\pi}{3}x = 3\pi$ or $\frac{\pi}{3}x = 4\pi$

$x = 9$ or $x = 12$ (A1) for correct values

$\therefore R = \int_{\frac{15}{2}}^{9} \left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right)\right) dx + \int_{9}^{12} e^{\frac{1}{2}\sqrt{x}} dx$

$+ \int_{12}^{\frac{27}{2}} \left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \sin\left(\frac{\pi}{3}x\right)\right) dx$ A2 N3

[4]

AA SL Practice Set 3 Paper 2 Solution

Section A

1. (a) $x=3$ A2 N2

[2]

(b) The y-intercept

$=\frac{4}{3-0}+\frac{2}{3}e^{0}$ (M1) for valid approach

$=\frac{4}{3}+\frac{2}{3}$

$=2$ A1 N2

[2]

(c) $f'(2)=8.9260422$ A1

$f'(2)=8.93$ A1 N2

[2]

2. (a) (i) 6 A1 N1

(ii) 6 A1 N1

(iii) The range

$=18-3$ (M1) for valid approach

$=15$ A1 N2

[4]

(b) (i) The mean

$$=\frac{(3)(12)+(6)(20)+(9)(12)+(12)(8)+(15)(4)+(18)(4)}{12+20+12+8+4+4}$$ (M1) for valid approach

$=8.2$ A1 N2

(ii) The variance

$=4.308131846^{2}$ (M1) for valid approach

$=18.6$ A1 N2

[4]

3. The common ratio

$=\frac{9600}{12000}$ (M1) for valid approach

$=0.8$ A1

$u_n > 96$ (M1) for setting inequality

$\therefore 12000\times 0.8^{n-1} > 96$ A1

$12000\times 0.8^{n-1} - 96 > 0$

By considering the graph of $y = 12000\times 0.8^{n-1} - 96$,

$n < 22.637702$. (M1) for valid approach

Thus, the greatest value of n is 22. A1 N4

[6]

4. (a) $f(x) = g(x)$

$\pi e^{-x^2} = 1 + \frac{1}{\pi e^{-x^2}}$ (M1) for setting equation

$\pi e^{-x^2} - 1 - \frac{e^{x^2}}{\pi} = 0$

By considering the graph of $y = \pi e^{-x^2} - 1 - \frac{e^{x^2}}{\pi}$,

$x = -0.814566$ or $x = 0.8145662$.

$\therefore a = -0.815$, $b = 0.815$ A2 N3

[3]

(b) The required area

$=\int_{-0.814566}^{0.8145662}(f(x) - g(x))\mathrm{d}x$ (A1) for correct integral

$=\int_{-0.814566}^{0.8145662}\left(\pi e^{-x^2} - 1 - \frac{e^{x^2}}{\pi}\right)\mathrm{d}x$

$=1.890606422$ (A1) for correct value

$=1.89$ A1 N3

[3]

5. (a) (i) $\frac{1}{2}$ A1 N1

(ii) 3 A1 N1

(iii) -4 A1 N1

[3]

(b) The coordinates of P'

$=\left(\frac{2}{2}+3, -5(8-4)\right)$ (A2) for correct approach

$=(4, -20)$ A2 N2

[4]

6. Let H be the height (in cm) of a tree.

$\text{P}(H<400)=0.2119$

$\text{P}\left(Z<\frac{400-\mu}{25}\right)=0.2119$ (M1) for standardization

$\frac{400-\mu}{25}=-0.7998460519$ A1

$400-\mu=-19.9961513$

$\mu=419.9961513$ A1

$\text{P}(H>394)=0.8507942645$ (A1) for correct value

$\therefore \text{P}(H>394+r)=0.8507942645-0.5$ A1

$\text{P}(H>394+r)=0.3507942645$

$394+r=429.5755765$ (A1) for correct value

$r=35.5755765$

$r=35.6$ A1 N4

[7]

Section B

7. (a) $a = -0.0807147258$

$a = -0.0807$ A1 N1

$b = 3.177202711$

$b = 3.18$ A1 N1

[2]

(b) $\log y = -0.0807147258\sqrt{9} + 3.177202711$ (M1) for valid approach

$\log y = 2.935058534$

$y = 10^{2.935058534}$ (M1) for valid approach

$y = 861.1098035$

$y = 861$ A1 N3

[3]

(c) $\log y = -0.0807147258\sqrt{x} + 3.177202711$

$y = 10^{-0.0807147258\sqrt{x}+3.177202711}$ (M1) for valid approach

$y = 10^{-0.0807147258\sqrt{x}} \cdot 10^{3.177202711}$ (A1) for correct approach

$y = 10^{3.177202711} \cdot (10^{-0.0807147258})^{\sqrt{x}}$ A1

$k = 10^{3.177202711}$ (A1) for correct approach

$k = 1503.843735$

$k = 1500$ A1 N2

$m = 10^{-0.0807147258}$ (A1) for correct approach

$m = 0.8303960491$

$m = 0.830$ A1 N2

[7]

8. (a) (i) By considering the graph of $y=\sin\left(\frac{\pi}{6}x\right)-\cos\left(\frac{\pi}{6}x\right)$, the coordinates of the maximum point and the minimum point are $(4.5000008, 1.4142136)$ and $(10.500001, -1.414214)$ respectively. (A2) for correct approach

Thus, the function is increasing when $0 \le x < 4.50$ or $10.5 < x \le 12$. A2 N4

(ii) $4.50 < x < 10.5$ A1 N1

[5]

(b) (i) $a=\frac{1.4142136-(-1.414214)}{2}$ (M1) for valid approach

$a=1.4142138$

$a=1.41$ A1 N2

(ii) Note that $f(0)=-1$.

$-1=1.4142138\sin\left(\frac{\pi}{6}(0+h)\right)$ (M1) for setting equation

$-0.7071066624=\sin\left(\frac{\pi}{6}h\right)$ (A1) for correct approach

$\frac{\pi}{6}h=5.497787312$ or -0.7853979954 (A1) for correct approach

$h=10.50000032$ (*Rejected*) or

$h=-1.499999679$ A1

$\therefore h=-1.50$ A1 N3

[7]

9. (a) $\cos\theta = \frac{\mathrm{AB}}{r}$

$\mathrm{AB} = r\cos\theta$ A1 N1

[1]

(b) $\sin\theta = \frac{\mathrm{AE}}{r}$

$\mathrm{AE} = r\sin\theta$ A1

The area of the triangle ABE

$= \frac{(\mathrm{AB})(\mathrm{AE})}{2}$

$= \frac{(r\cos\theta)(r\sin\theta)}{2}$ M1

$= \frac{1}{2}r^2\sin\theta\cos\theta$ A1

$= \frac{1}{2}r^2\left(\frac{1}{2}\sin 2\theta\right)$ A1

$= \frac{r^2\sin 2\theta}{4}$ AG N0

[4]

(c) $\mathrm{A\hat{E}B} + \mathrm{B\hat{E}C} + \mathrm{C\hat{E}D} = \pi$ M1

$\left(\frac{\pi}{2} - \theta\right) + \mathrm{B\hat{E}C} + \left(\frac{\pi}{2} - \theta\right) = \pi$ A1

$\pi - 2\theta + \mathrm{B\hat{E}C} = \pi$

$\mathrm{B\hat{E}C} = 2\theta$ AG N0

[2]

(d) ABCD is a square

$\therefore \text{AB} = 2\text{AE}$ — (M1) for valid approach

$r\cos\theta = 2r\sin\theta$

$\cos\theta - 2\sin\theta = 0$ — (A1) for correct equation

By considering the graph of $y = \cos\theta - 2\sin\theta$,

$\theta = 0.4636476$. — A1

The area of the sector EBC

$= \frac{1}{2}r^2(2(0.4636476))$ — (A1) for substitution

$= 0.4636476r^2$

$\therefore 0.4636476r^2 = k\left(\frac{r^2\sin 2(0.4636476)}{4}\right)$ — M1A1

$k = 0.4636476\left(\frac{4}{\sin 2(0.4636476)}\right)$

$k = 2.318238031$

$k = 2.32$ — A1 N4

[7]

(e) $r^2\theta = 3\left(\frac{r^2\sin 2\theta}{4}\right)$ — (A1) for correct equation

$\theta - \frac{3}{4}\sin 2\theta = 0$

By considering the graph of $y = \theta - \frac{3}{4}\sin 2\theta$,

$\theta = 0.7478908 \text{ rad}$.

$\therefore \theta = 0.748 \text{ rad}$ — A1 N2

[2]

AA SL Practice Set 4 Paper 1 Solution

Section A

1. (a) The area of the shaded region

$= \frac{1}{2}(20)^2(1.5)$ (A1) for substitution

$= 300 \text{ cm}^2$ A1 N2

[2]

(b) The arc length ABC

$= (20)(1.5)$ (A1) for substitution

$= 30 \text{ cm}$ A1 N2

[2]

(c) The required perimeter

$= 2\pi(20) - 30 + 20 + 20$ (M1) for valid approach

$= (40\pi + 10) \text{ cm}$ A1 N2

[2]

2. (a) $\log_4 64$

$= \log_4 4^3$ (A1) for correct approach

$= 3$ A1 N2

[2]

(b) $\log_{12} 36 + \log_{12} 4$

$= \log_{12} 144$ (A1) for correct approach

$= \log_{12} 12^2$

$= 2$ A1 N2

[2]

(c) $\log_2 11 - \log_2 88$

$= \log_2 \frac{1}{8}$ (A1) for correct approach

$= \log_2 2^{-3}$

$= -3$ A1 N2

[2]

3. (a) $f'(x) = 3e^{3x+1}$ A1 N1

$f''(x) = 9e^{3x+1}$ A1 N1

$f^{(3)}(x) = 27e^{3x+1}$ A1 N1

[3]

(b) $f^{(n)}(x) = 3^n e^{3x+1}$ A3 N3

[3]

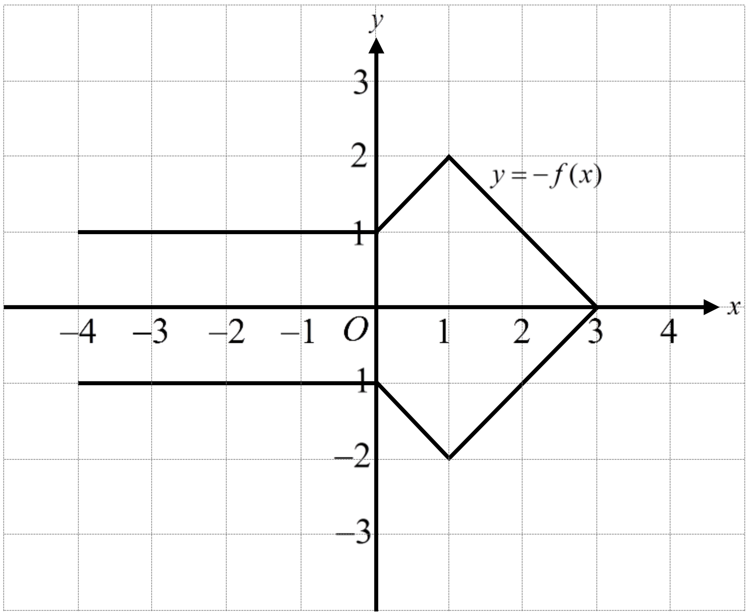
(c) $f^{(6)}\left(-\frac{1}{3}\right) = 729$ A1 N1

[1]

4. (a) For correct x-intercept and y-intercept A1

For two correct points $(-4, 1)$ and $(1, 2)$ A1 N2

[2]



(b) $p = 2$ A2 N2

$q = -1$ A2 N2

[4]

5. (a) $u_9 = 6\ln 2$

$\therefore \ln 0.25 + (9-1)(\ln D) = 6\ln 2$	(A1) for correct equation
$\ln 0.25 + 8\ln D = \ln 64$	(A1) for correct approach
$8\ln D = \ln 64 - \ln 0.25$	
$8\ln D = \ln 256$	(A1) for correct approach
$8\ln D = \ln 2^8$	(M1) for valid approach
$8\ln D = 8\ln 2$	
$\therefore D = 2$	A1 N3

[5]

(b) The sum of the first seven terms

$= \frac{7}{2}\left[2\ln 0.25 + (7-1)(\ln 2)\right]$	(A1) for substitution
$= 7\ln 2^{-2} + 21\ln 2$	A1
$= -14\ln 2 + 21\ln 2$	
$= 7\ln 2$	A1 N2

[3]

6. (a)

$a = 2(-\sin \pi t)(\pi) + 0$	(A1) for correct derivatives
$a = -2\pi \sin \pi t$	A1 N2

[2]

(b)

$s = \int (2\cos \pi t + \pi)\mathrm{d}t$	(M1) for indefinite integral
$s = \int 2\cos \pi t \mathrm{d}t + \int \pi \mathrm{d}t$	
Let $u = \pi t$; $\frac{\mathrm{d}u}{\mathrm{d}t} = \pi \Rightarrow \frac{1}{\pi}\mathrm{d}u = \mathrm{d}t$	(A1) for substitution
$s = \int \frac{2}{\pi}\cos u \mathrm{d}u + \int \pi \mathrm{d}t$	
$s = \frac{2}{\pi}\sin u + \pi t + C$	A1
$s = \frac{2}{\pi}\sin \pi t + \pi t + C$	
$\therefore -3 = \frac{2}{\pi}\sin 0 + 0 + C$	(M1) for substitution
$C = -3$	
$\therefore s = \frac{2}{\pi}\sin \pi t + \pi t - 3$	A1 N4

[5]

Section B

7. (a) $x-4=7$ (M1) for valid approach

$x=11$ A1 N2

[2]

(b) The number of people

$=\frac{20+40}{4}$ (M1) for valid approach

$=15$ A1 N2

[2]

(c) The mean number of hours

$=\frac{120}{20}$ (M1) for substitution

$=6$ A1 N2

[2]

(d) (i) The total number of hours

$=(60)(9)$ (M1) for valid approach

$=540$ A1 N2

(ii) The mean number of hours

$=\frac{540-120}{40}$ (M1)(A1) for correct approach

$=10.5$ A1 N2

[5]

(e) (i) The required mean

$=10.5+1.5$ (M1) for valid approach

$=12$ A1 N2

(ii) The required variance

$=2^2$ (M1)(A1) for correct approach

$=4$ A1 N2

[5]

8. (a) (i) The required probability

$= \frac{3}{n}$ A1 N1

(ii) The required probability

$= \left(\frac{n-3}{n}\right)\left(\frac{n-4}{n-1}\right)\left(\frac{3}{n-2}\right)$ (A1) for correct approach

$= \frac{3(n-3)(n-4)}{n(n-1)(n-2)}$ A1 N2

[3]

(b) The required probability

$= \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$ (A1) for correct approach

$= \frac{1}{8}$ A1 N2

[2]

(c) The game is fair if the expected gain is zero, which is equivalent to the expected amount of money earns back equals to \$10. R1

$$\therefore \left(\frac{3}{10}\right)(10) + \left(\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\right)(10)$$

$$+\left(\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\right)(25x) + \left(\frac{1}{8}\right)(21x)$$ M1A2

$$+\left(1 - \frac{3}{10} - \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) - \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right) - \frac{1}{8}\right)(0) = 10$$

$3 + \frac{7}{3} + \frac{35}{8}x + \frac{21}{8}x = 10$ M1A1

$7x = \frac{14}{3}$ A1

$x = \frac{2}{3}$ AG N0

[7]

9. (a) $f'(x)=\left(\frac{1}{x^2+4}\right)(2x+0)$ (A2) for correct derivatives

$f'(2)=\frac{2(2)}{2^2+4}$ (M1) for substitution

$f'(2)=\frac{1}{2}$ A1 N3

[4]

(b) (i) 2 A1 N1

(ii) 1 A1 N1

[2]

(c) $h(x)=(f\circ g)(x)$

$h(x)=f(g(x))$

$h'(x)=f'(g(x))\cdot g'(x)$ (A1) for chain rule

The slope of tangent

$=h'(5)$

$=f'(g(5))\cdot g'(5)$

$=f'(2)\cdot g'(5)$ (M1) for valid approach

$=\left(\frac{1}{2}\right)(1)$

$=\frac{1}{2}$ A1

$h(5)=f(g(5))$

$h(5)=f(2)$ (M1) for valid approach

$h(5)=\ln(2^2+4)$

$h(5)=\ln 8$

The equation of tangent:

$y-\ln 8=\frac{1}{2}(x-5)$ A1

$y-\ln 8=\frac{1}{2}x-\frac{5}{2}$

$y=\frac{1}{2}x+\left(\ln 8-\frac{5}{2}\right)$ A1 N4

[6]

AA SL Practice Set 4 Paper 2 Solution

Section A

1. (a) (i) $r = 0.956518027$ A1

$r = 0.957$ A1 N2

(ii) $a = 2.022727273$

$a = 2.02$ A1 N1

$b = -75.9469697$

$b = -75.9$ A1 N1

[4]

(b) The estimated final exam score

$= 2.022727273(84) - 75.9469697$ (A1) for substitution

$= 93.96212123$

$= 94.0$ A1 N2

[2]

2. (a) (i) $(3, -127)$ A2 N2

(ii) $f(x) = 3(x-3)^2 - 127$ A2 N2

[4]

(b) $3x^2 - 18x - 100 = -52$

$3x^2 - 18x - 48 = 0$ (A1) for correct equation

$3(x+2)(x-8) = 0$

$x = -2$ or $x = 8$ A2 N3

[3]

3. (a) $P(W > m) = 0.087$

$m = 4.343908413$ (A1) for correct value

$m = 4.34$ A1 N2

[2]

(b) $P(W > 4.5 | W > 4.343908413)$

$= \frac{P(W > 4.5 \cap W > 4.343908413)}{P(W > 4.343908413)}$ (A1) for correct approach

$= \frac{P(W > 4.5)}{P(W > 4.343908413)}$ M1

$= \frac{0.0630016205}{0.087}$ A1

$= 0.7241565576$

$= 0.724$ A1 N2

[4]

4. (a) $(g \circ f)(x)$

$= 2(f(x))^2 - 5$ (A1) for substitution

$= 2(e^x)^2 - 5$

$= 2e^{2x} - 5$ A1 N2

[2]

(b) (i) $(g \circ f)(x) = x^3$

$2e^{2x} - 5 = x^3$

$2e^{2x} - 5 - x^3 = 0$ (A1) for correct equation

By considering the graph of

$y = 2e^{2x} - 5 - x^3$, $x = -1.702369$ or

$x = 0.4683121$ (*Rejected*)

$\therefore x = -1.70$ A1 N2

(ii) $f(\sqrt[3]{p}) = g^{-1}(p)$

$(g \circ f)(\sqrt[3]{p}) = (p)$ (M1) for valid approach

$\therefore \sqrt[3]{p} = -1.702369$ (A1) for correct approach

$p = -4.933567865$

$p = -4.93$ A1 N3

[5]

5. (a) The common ratio r

$= \dfrac{3k^2 - 4k^3}{k^2}$ (M1) for valid approach

$= 3 - 4k$ A1 N2

[2]

(b) S_∞ exists if $-1 < r < 1$. R1

$\therefore -1 < 3 - 4k < 1$ M1

$-1 < 4k - 3 < 1$

$2 < 4k < 4$ A1

$\dfrac{1}{2} < k < 1$ AG N0

[3]

(c) $800rS_\infty + 243 = 0$

$\therefore 800(3-4k)\left(\dfrac{k^2}{1-(3-4k)}\right) + 243 = 0$ (M1) for setting equation

$800(3-4k)k^2 + 243(4k-2) = 0$

By considering the graph of

$y = 800(3-4k)k^2 + 243(4k-2)$,

$k = -0.492582$ (*Rejected*),

$k = 0.3425823$ (*Rejected*) or $k = 0.9$.

$\therefore k = 0.9$ A2 N3

[3]

6. The general term

$= \binom{9}{r}\left(\frac{x}{h^2}\right)^{9-r}\left(-\frac{h}{x^2}\right)^r$ (M1) for valid expansion

$= \binom{9}{r}(-1)^r h^{3r-18} x^{9-3r}$

$9-3r=0$ (A1) for correct equation

$3r=9$

$r=3$ (A1) for correct value

The required term

$= \binom{9}{3}(-1)^3 h^{3(3)-18} x^{9-3(3)}$

$= -\frac{84}{h^9}$ (A1) for correct term

$-\frac{84}{h^9} = -\frac{21}{65536}$ (M1) for setting equation

$h^9 = 262144$

$h=4$ A1 N3

[6]

Section B

7. (a) The height of a high tide

$=1.9+4.3$ (M1) for valid approach

$=6.2\text{ m}$ A1 N2

[2]

(b) p is negative as the first turning point is a minimum point. R1

$p=-\frac{4.3}{2}$ A1

$p=-2.15$ AG N0

[2]

(c) (i) The period

$=13.75-2.75$ (M1) for valid approach

$=11$ hours (A1) for correct value

$\therefore q=\frac{2\pi}{11}$ A1 N3

(ii) $r=\frac{6.2+1.9}{2}$ (M1) for valid approach

$r=4.05$ A1 N2

[5]

(d) 4 January 2021 implies $24\le t<48$. (M1) for valid approach

$t=13.75+3(11)$

$t=46.75$ (A1) for correct value

Thus, the time is $22:45$. A1 N3

[3]

8. (a) $\text{B}\hat{\text{A}}\text{C}$

$= \pi - 0.88 - 1.23$ (M1) for valid approach

$= 1.031592654$ A1

$\frac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}} = \frac{\text{BC}}{\sin \text{B}\hat{\text{A}}\text{C}}$ (M1) for sine rule

$\frac{\text{AB}}{\sin 1.23} = \frac{20}{\sin 1.031592654}$ (A1) for substitution

$\text{AB} = 21.96641928 \text{ cm}$

$\text{AB} = 22.0 \text{ cm}$ A1 N3

[5]

(b) (i) $\text{AB}^2 = \text{OA}^2 + \text{OB}^2 - 2(\text{OA})(\text{OB})\cos \text{A}\hat{\text{O}}\text{B}$ M1

$\text{AB}^2 = r^2 + r^2 - 2(r)(r)\cos \text{A}\hat{\text{O}}\text{B}$ A1

$\text{AB}^2 = 2r^2 - 2r^2 \cos \text{A}\hat{\text{O}}\text{B}$

$\text{AB}^2 = 2r^2(1 - \cos \text{A}\hat{\text{O}}\text{B})$ A1

$r^2 = \frac{\text{AB}^2}{2(1 - \cos \text{A}\hat{\text{O}}\text{B})}$ AG N0

(ii) $\text{A}\hat{\text{O}}\text{B} = 2\text{A}\hat{\text{C}}\text{B}$

$\text{A}\hat{\text{O}}\text{B} = 2.46 \text{ rad}$ (A1) for correct value

$\therefore r^2 = \frac{21.96641928^2}{2(1 - \cos 2.46)}$ (A1) for substitution

$r = 11.65341128$

$r = 11.7$ A1 N3

[6]

(c) The required sum

$= \pi(11.65341128)^2 - \frac{1}{2}(21.96641928)(20)\sin 0.88$ M1A1

$= 257.3308144 \text{ cm}^2$

$= 257 \text{ cm}^2$ A1 N1

[3]

9. (a) (i) $a_1(t) = \dfrac{20-30}{2-0}$ M1A1

$a_1(t) = -5$ AG N0

(ii) $v_1(t) = -5t + 30$ A2 N2

[4]

(b) The total distance the marble travelled

$= \int_0^2 |v_1(t)|\,\mathrm{d}t$ (M1) for valid approach

$= \int_0^2 |-5t+30|\,\mathrm{d}t$ (A1) for correct formula

$= 50$ cm A1 N2

[3]

(c) (i) $v_2(2) = 20$

$\therefore 20e^{b-0.2(2)} = 20$ M1

$e^{b-0.4} = 1$

$b - 0.4 = 0$ A1

$b = 0.4$ AG N0

(ii) $\int_2^c |v_2(t)|\,\mathrm{d}t = 50$

$\int_2^c 20e^{0.4-0.2t}\,\mathrm{d}t = 50$ (M1) for setting equation

Let $u = 0.4 - 0.2t$
$\dfrac{\mathrm{d}u}{\mathrm{d}t} = -0.2 \Rightarrow -100\mathrm{d}u = 20\mathrm{d}t$
$t = c \Rightarrow u = 0.4 - 0.2c$
$t = 2 \Rightarrow u = 0.4 - 0.2(2) = 0$

(A1) for substitution

$\int_0^{0.4-0.2c} -100e^u\,\mathrm{d}u = 50$ A1

$\left[-100e^u\right]_0^{0.4-0.2c} = 50$

$e^{0.4-0.2c} - e^0 = -0.5$ (M1) for substitution

$e^{0.4-0.2c} = 0.5$

$0.4 - 0.2c = \ln 0.5$

$0.4 - \ln 0.5 = 0.2c$

$c = 5.465735903$

$c = 5.47$ A1 N3

[7]