

## Self-assessment answers: 14 Further differentiation

1. (a)  $10(2x + 1)^4$  [1 mark]

(b)  $-6 \cos^2(2x) \sin(2x)$  [3 marks]

2.  $\frac{dy}{dx} = \frac{-2x^2 e^{-2x} - 2xe^{-2x}}{x^4}$

$$\frac{-2e^{-2x}(x+1)}{x^3}$$

$= 0$  when  $x = -1$  (as  $e^{-2x} \neq 0$ ) [5 marks]

3. (a)  $\text{Area} = 2\sqrt{208\pi h} + \frac{416}{h}$  [4 marks]

(b)  $h = 6.42 \text{ cm}$  (3SF) [3 marks]

4. (a)  $f'(x) = \sin(ax) + ax \cos(ax)$

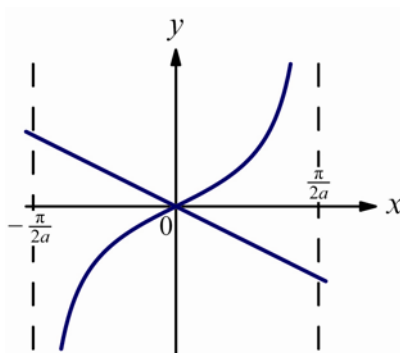
$$f''(x) = a \cos(ax) + a \cos(ax) - a^2 x \sin(ax)$$

$$= 2a \cos(ax) - a^2 x \sin(ax)$$

(b) (i)  $f'(x) = 0 \Rightarrow \sin(ax) = -ax \cos(ax)$

$$\Rightarrow \tan(ax) = -ax$$

(ii)



The only intersection is  $x = 0$ .

$$(iii) f(0) = 0$$

$$f''(0) = 2a > 0$$

So  $(0, 0)$  is a minimum point.

$$(c) f''(x) + 4f(x) = 2a \cos(ax)$$

$$\Leftrightarrow 2a \cos(ax) - a^2 x \sin(ax) + 4x \sin x = 2a \cos(ax)$$

$$\Leftrightarrow 2a \cos(ax) + (4 - a^2)x \sin x = 2a \cos(ax)$$

This is satisfied when  $4 - a^2 = 0$ .

$$a > 0 \therefore a = 2$$

[14 marks]