

Practice paper 2

Time allowed: 2 hours

- Answer all the questions
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

- Find:

- [3 marks]*

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- [5 marks]
[2 marks]

This image shows a full page of a document template designed for handwritten notes or essays. It features approximately 30 evenly spaced, thin horizontal grey lines across the entire width of the page. The lines are uniform in thickness and color, providing a guide for writing without being distracting. There are no margins, headers, footers, or other markings present on the page.

- 3** A random variable X is normally distributed with mean and variance both equal to a . Given that $P(X < 2) = 0.3$, find the value of a .

[4 marks]

[illegible]

4 Consider the function defined by $f(x) = \frac{1-x^2}{x}$

- a** Find the first and second derivatives and hence show that the graph of f has no maxima, no minima or points of inflexion.
- b** Hence sketch the graph of f , showing clearly the intercepts and any asymptotes.

[5 marks]

[3 marks]

[illegible]

- [6 marks]

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- 6** Given that $(1+x)^5(1+ax)^6 \equiv 1+bx+10x^2+\dots+a^6x^{11}$, find the values of integers a and b .

[6 marks]

[illegible]

7

[6 marks]

[illegible]

- [6 marks]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 Consider the sequence (u_n) defined by
$$\begin{cases} u_1 = 2 \\ u_{n+1} = \frac{u_n + 1}{3}, n \in \mathbb{Z}^+ \end{cases}$$

Investigate the numerical behaviour of the terms of the sequence

and deduce that $u_n = \frac{3^{2^{-n}} + 1}{2}$

[8 marks]

[illegible]

- [4 marks]

[4 marks]

[illegible]

Handwriting practice lines consisting of 30 horizontal dotted lines.

- [3 marks]

[3 marks]

- [3 marks]

[4 marks]

- [4 marks]

[6 marks]

- f** Consider the plane π defined by the equation $x + y + z = 0$. Find the coordinates of the point P on the plane π that is at the same distance from the points A, B and C.

This image shows a full page of a document template designed for handwritten notes or essays. It features approximately 28 evenly spaced, thin grey horizontal lines across the entire width of the page. The margins are consistent on all sides, providing ample space for writing. There are no pre-printed questions, headings, or other markings on the page.

Handwriting practice lines consisting of 30 horizontal dotted lines.

12 Let $f(x) = \cos(2x) + 1$ and $g(x) = \frac{e^x + e^{-x}}{2}$

- a** Show that both functions are even. [3 marks]
- b** Find the derivatives $f'(x)$ and $g'(x)$. [4 marks]
- c** Show that both derivative functions are odd. [2 marks]
- d** Sketch the curves $y = f(x)$ and $y = g(x)$ and find their points of intersection. [4 marks]
- e** Show that the tangents to the curves $y = f(x)$ and $y = g(x)$ at the point of intersection in the first quadrant have equations $y = -1.95x + 2.53$ and $y = 0.719x + 0.751$ respectively. [3 marks]
- f** Find the area of the region enclosed by all four tangents to the curves $y = f(x)$ and $y = g(x)$ at the points of intersection. [3 marks]
- g** The region enclosed by the curves $y = f(x)$ and $y = g(x)$ is rotated by 2π about the x -axis. Find the volume of revolution generated. [4 marks]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]