

Self-assessment: 15 Further integration

1. The part of the curve with equation $y = \sqrt{\sin 3x}$ between $x = 0$ and $x = \pi$ is rotated 2π radians around the x -axis. Find the exact value of the resulting volume of revolution.

(accessible to students on the path to grade 3 or 4) [5 marks]

2. Use the substitution $u = x - 1$ to find $\int x^2 \sqrt{x-1} \, dx$.

(accessible to students on the path to grade 5 or 6) [6 marks]

3. Use a suitable substitution to evaluate $\int_1^3 \frac{2x-1}{(x^2-x+5)^3} \, dx$.

(accessible to students on the path to grade 5 or 6) [6 marks]

4. A particle moves so that its velocity (measured in ms^{-1}) depends on time (measured in seconds) according to the equation $v = 3e^{-2t} \sin t$ for $t \geq 0$.

- (a) Find the maximum velocity of the particle.
- (b) Find the acceleration of the particle when $t = 3$.
- (c) Find the distance travelled by the particle in the first three seconds of motion.

(accessible to students on the path to grade 3 or 4)

- (d) Initially the particle is at the origin. Find an equation for the displacement in terms of time.

(accessible to students on the path to grade 5 or 6)

[13 marks]