IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

N06/5/MATSD/SP2/ENG/TZ0/XX+

88067402

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Friday 3 November 2006 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 18]*

(i) Amos travels to school either by car or by bicycle. The probability of being late for school is $\frac{1}{10}$ if he travels by car and $\frac{1}{5}$ if he travels by bicycle. On any particular day he is equally likely to travel by car or by bicycle.

(a) Draw a probability tree diagram to illustrate this information. *[4 marks]*

(b) Find the probability that

(i) Amos will travel by car and be late. *[2 marks]*

(ii) Amos will be late for school. *[3 marks]*

(c) Given that Amos is late for school, find the probability that he travelled by bicycle. *[3 marks]*

(ii) Given a universal set $U = \{\text{cars}\}$, $S = \{\text{sports cars}\}$, $G = \{\text{green sports cars}\}$.

(a) Draw a Venn diagram to illustrate this information. *[3 marks]*

(b) Shade the set $S \cap G'$ on your diagram. *[1 mark]*

(c) Write in words the meaning of $S \cap G'$. *[2 marks]*

2. *[Maximum mark: 14]*

Three points are given A(0, 4), B(6, 0) and C(8, 3).

(a) Calculate the gradient (slope) of line AB. *[2 marks]*

(b) Find the coordinates of the midpoint, M, of the line AC. *[2 marks]*

(c) Calculate the length of line AC. *[2 marks]*

(d) Find the equation of the line BM giving your answer in the form $ax + by + d = 0$ where a, b and $d \in \mathbb{Z}$. *[5 marks]*

(e) State whether the line AB is perpendicular to the line BC showing clearly your working and reasoning. *[3 marks]*

3. *[Maximum mark: 18]*

In an experiment a vertical spring was fixed at its upper end. It was stretched by hanging different weights on its lower end. The length of the spring was then measured. The following readings were obtained.

Load (kg) x	0	1	2	3	4	5	6	7	8
Length (cm) y	23.5	25	26.5	27	28.5	31.5	34.5	36	37.5

(a) Plot these pairs of values on a scatter diagram taking 1 cm to represent 1 kg on the horizontal axis and 1 cm to represent 2 cm on the vertical axis. *[4 marks]*

(b) (i) Write down the mean value of the load ($\bar{x}$). *[1 mark]*

(ii) Write down the standard deviation of the load. *[1 mark]*

(iii) Write down the mean value of the length ($\bar{y}$). *[1 mark]*

(iv) Write down the standard deviation of the length. *[1 mark]*

(c) Plot the mean point ($\bar{x}$, $\bar{y}$) on the scatter diagram. Name it L. *[1 mark]*

It is given that the covariance S_{xy} is 12.17.

(d) (i) Write down the correlation coefficient, ***r***, for these readings. *[1 mark]*

(ii) Comment on this result. *[2 marks]*

(e) Find the equation of the regression line of y on x. *[2 marks]*

(f) Draw the line of regression on the scatter diagram. *[2 marks]*

(g) (i) Using your diagram or otherwise, estimate the length of the spring when a load of 5.4 kg is applied. *[1 mark]*

(ii) Malcolm uses the equation to claim that a weight of 30 kg would result in a length of 62.8 cm. Comment on his claim. *[1 mark]*

4. *[Maximum mark: 20]*

(i) The first three terms of an arithmetic sequence are

$$2k+3,\ 5k-2 \text{ and } 10k-15.$$

(a) Show that $k = 4$. *[3 marks]*

(b) Find the values of the first three terms of the sequence. *[1 mark]*

(c) Write down the value of the common difference. *[1 mark]*

(d) Calculate the 20^{th} term of the sequence. *[2 marks]*

(e) Find the sum of the first 15 terms of the sequence. *[2 marks]*

(ii) The diagram below shows a child's toy which is made up of a circular hoop, centre O, radius 7 cm. The hoop is suspended in a horizontal plane by three equal strings XA, XB, and XC. Each string is of length 25 cm. The points A, B and C are equally spaced round the circumference of the hoop and X is vertically above the point O.

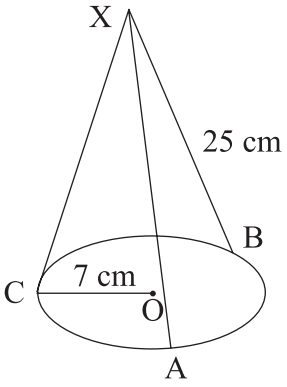

diagram not to scale

(a) Calculate the length of XO. *[2 marks]*

(b) Find the angle, in degrees, between any string and the horizontal plane. *[2 marks]*

(c) Write down the size of angle AOB. *[1 mark]*

(d) Calculate the length of AB. *[3 marks]*

(e) Find the angle between strings XA and XB. *[3 marks]*

5. *[Maximum mark: 20]*

A farmer has a rectangular enclosure with a straight hedge running down one side. The area of the enclosure is $162\ \text{m}^2$. He encloses this area using x metres of the hedge on one side as shown on the diagram below.

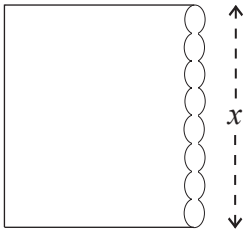

diagram not to scale

(a) If he uses y metres of fencing to complete the enclosure, show that $y = x + \frac{324}{x}$. *[3 marks]*

The farmer wishes to use the least amount of fencing.

(b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

(c) Find the value of x which makes y a minimum. *[3 marks]*

(d) Calculate this minimum value of y. *[2 marks]*

(e) Using $y = x + \frac{324}{x}$ find the values of ***a*** and ***b*** in the following table.

x	6	9	12	18	24	27	36
y	60	45	39	***a***	37.5	***b***	45

[2 marks]

(f) Draw an accurate graph of this function using a horizontal scale starting at 0 and taking 2 cm to represent 10 metres, and a vertical scale **starting at 30** with 4 cm to represent 10 metres. *[5 marks]*

(g) Write down the values of x for which y increases. *[2 marks]*