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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Tuesday 12 November 2013 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL** *information booklet* is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 18]*

The table shows the distance, in km, of eight regional railway stations from a city centre terminus and the price, in $, of a return ticket from each regional station to the terminus.

Distance in km (x)	3	15	23	42	56	62	74	93
Price in $ (y)	5	24	43	56	68	74	86	100

(a) Draw a scatter diagram for the above data. Use a scale of 1 cm to represent 10 km on the x-axis and 1 cm to represent $10 on the y-axis. *[4]*

(b) Use your graphic display calculator to find

    (i) $\bar{x}$, the mean of the distances;

    (ii) $\bar{y}$, the mean of the prices. *[2]*

(c) Plot and label the point $M(\bar{x}, \bar{y})$ on your scatter diagram. *[1]*

(d) Use your graphic display calculator to find

    (i)   the product–moment correlation coefficient, r;

    (ii)   the equation of the regression line y on x. *[3]*

(e) Draw the regression line y on x on your scatter diagram. *[2]*

A ninth regional station is 76 km from the city centre terminus.

(f) Use the equation of the regression line to estimate the price of a return ticket to the city centre terminus from this regional station. **Give your answer correct to the nearest $.** *[3]*

(g) Give a reason why it is valid to use your regression line to estimate the price of this return ticket. *[1]*

The actual price of the return ticket is $80.

(h) **Using your answer to part (f)**, calculate the percentage error in the estimated price of the ticket. *[2]*

2. *[Maximum mark: 16]*

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.
AB = 30 cm, BC = 24 cm, CD = 25 cm and angle $\hat{ABC} = 35°$.

diagram not to scale
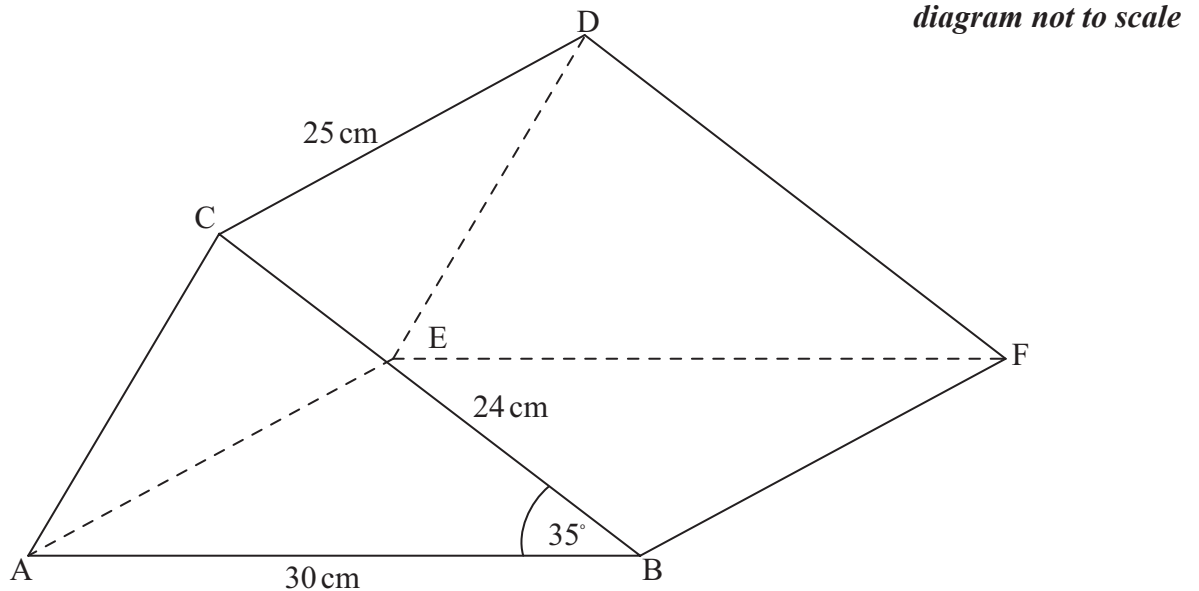
(a) Calculate the length of AC. *[3]*

(b) Calculate the area of triangle ABC. *[3]*

(c) Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\,\text{cm}^3$, correct to three significant figures. *[2]*

(d) Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$. *[2]*

(e) Show that the total surface area of one block is $2190\,\text{cm}^2$, correct to three significant figures. *[3]*

The blocks are to be painted. One litre of paint will cover $22\,\text{m}^2$.

(f) Calculate the number of litres required to paint all 2600 blocks. *[3]*

Turn over

3. *[Maximum mark: 17]*

A group of 120 women in the USA were asked whether they had visited the continents of Europe (E) or South America (S) or Asia (A).

> 7 had visited all three continents
> 28 had visited Europe only
> 22 had visited South America only
> 16 had visited Asia only
> 15 had visited Europe and South America but had not visited Asia
> x had visited South America and Asia but had not visited Europe
> $2x$ had visited Europe and Asia but had not visited South America
> 20 had not visited any of these continents

(a) Draw a Venn diagram, using sets labelled E, S and A, to show this information. *[5]*

(b) Calculate the value of x. *[2]*

(c) Explain, in words, the meaning of $(E \cup S) \cap A'$. *[2]*

(d) Write down $n\big((E \cup S \cup A)'\big)$. *[1]*

(e) Find the probability that a woman selected at random from the group had visited Europe. *[2]*

(f) Find the probability that a woman selected at random from the group had visited Europe, given that she had visited Asia. *[2]*

Two women from the group are selected at random.

(g) Find the probability that both women selected had visited South America. *[3]*

4.   *[Maximum mark: 23]*

Consider the function $f(x) = \dfrac{3}{4}x^4 - x^3 - 9x^2 + 20$.

(a)   Find $f(-2)$.   *[2]*

(b)   Find $f'(x)$.   *[3]*

The graph of the function $f(x)$ has a local minimum at the point where $x = -2$.

(c)   Using your answer to part (b), show that there is a second local minimum at $x = 3$.   *[5]*

(d)   Sketch the graph of the function $f(x)$ for $-5 \leq x \leq 5$ and $-40 \leq y \leq 50$. Indicate on your
      sketch the coordinates of the *y*-intercept.   *[4]*

(e)   Write down the coordinates of the local maximum.   *[2]*

Let *T* be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$.

(f)   Find the gradient of *T*.   *[2]*

The line *L* passes through the point $(2, -12)$ and is perpendicular to *T*.
L has equation $x + by + c = 0$, where *b* and $c \in \mathbb{Z}$.

(g)   Find

      (i)   the gradient of *L*;

      (ii)   the value of *b* and the value of *c*.   *[5]*

5. *[Maximum mark: 16]*

Give all answers in this question correct to two decimal places.

Arthur lives in London. On 1st August 2008 Arthur paid 37 500 euros (EUR) for a new car from Germany. The price of the same car in London was 34 075 British pounds (GBP).

The exchange rate on 1st August 2008 was $1\,\text{EUR} = 0.7234\,\text{GBP}$.

(a) Calculate, **in GBP**, the price that Arthur paid for the car. *[2]*

(b) Write down, in GBP, the amount of money Arthur saved by buying the car in Germany. *[1]*

On 1st August 2008 Arthur invested the money he saved in a bank that paid 4.5 % annual simple interest.

(c) Calculate, in GBP, the **value** of Arthur's investment on 1st August 2012. *[3]*

Between 1st August 2008 and 1st August 2012 Arthur's car depreciated at an annual rate of 9 % of its current value.

(d) Calculate the value, in GBP, of Arthur's car on 1st August **2009**. *[3]*

(e) Show that the value of Arthur's car on 1st August **2012** was 18 600 GBP, correct to the nearest 100 GBP. *[3]*

On 1st August 2012 Arthur sold his car for 18 600 GBP and bought a new car from Germany for 30 500 EUR. He used the 18 600 GBP and the value of the investment he made on 1st August 2008 to buy the new car.

The exchange rate on 1st August 2012 was $1\,\text{EUR} = 0.8694\,\text{GBP}$.

(f) Calculate the amount of money remaining, **in EUR**, after the car had been bought. *[4]*