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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

8822 – 7201

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Please **do not** write on this page.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

2. [Maximum mark: 5]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^\circ\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\ t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

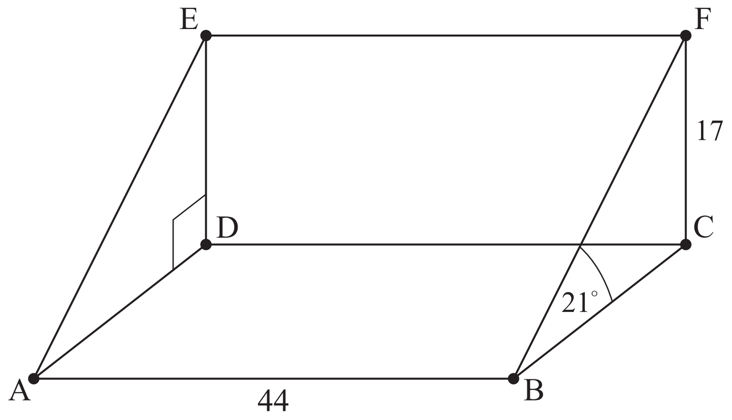
(b) Write down the room temperature. [1]

(c) Given that $T^{-1}(50) = k$, find the value of k. [2]

3. [Maximum mark: 5]

An artificial ski slope can be modelled as a triangular prism, as shown in the diagram. Rectangle ABCD is horizontal, and rectangle CDEF is vertical.

diagram not to scale



The maximum height of the ski slope, CF, is 17 metres and the steepest angle of the ski slope, $\text{F}\hat{\text{B}}\text{C}$, is 21°.

(a) Calculate the length of [BF]. [2]

The width of the base of the ski slope, AB, is 44 metres. Mayumi skis in a straight line, starting from point E and finishing at the base of the ski slope.

(b) Find the value of the least steep angle that Mayumi can ski. [3]

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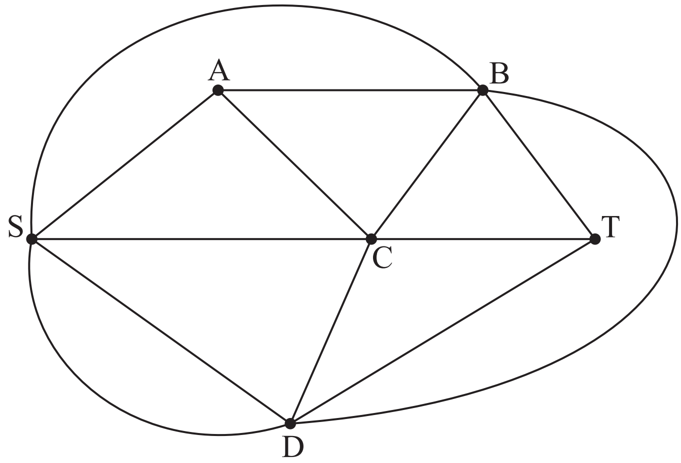
4. [Maximum mark: 7].

In a competition, a contestant has to move through a maze to find treasure. A graph of the maze is shown below, where each edge represents a corridor in the maze. The contestant starts at S and the treasure is located at T.



(a) Complete the adjacency matrix, $\boldsymbol{M}$, for the graph. [2]

	S	A	B	C	D	T
S	0	1	1	1	☐	0
A	1	0	1	1	☐	0
B	1	1	0	1	1	1
C	1	1	1	0	1	1
D	☐	☐	1	1	0	1
T	0	0	1	1	1	0

The competition rules state that the contestant can walk along a maximum of four corridors.

(b) Find the number of walks from S to T with a maximum of 4 edges. [4]

(c) Explain why the number of ways the contestant can reach the treasure is less than the answer to part (b). [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

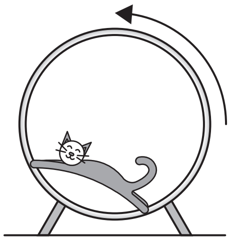
(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

6. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and $a,\ b,\ c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

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7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR } 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR } 10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account. Express your answer correct to the nearest EUR. [3]

8. [Maximum mark: 7]

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

(a) Find the coordinates of M. [3]

(b) Find the acute angle between the two lines. [4]

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9. [Maximum mark: 8]

The transformation T is represented by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$.

A pentagon with an area of $12\,\text{cm}^2$ is transformed by T.

(a) Find the area of the image of the pentagon. [2]

Under the transformation T, the image of point X has coordinates $(2t-3,\ 6-5t)$, where $t \in \mathbb{R}$.

(b) Find, in terms of t, the coordinates of X. [6]

10. [Maximum mark: 7]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

The star Proxima Centauri has a greater magnitude than the planet Neptune. The difference in their magnitudes is 3.2.

(c) Find how many times brighter Neptune is compared to Proxima Centauri. [3]

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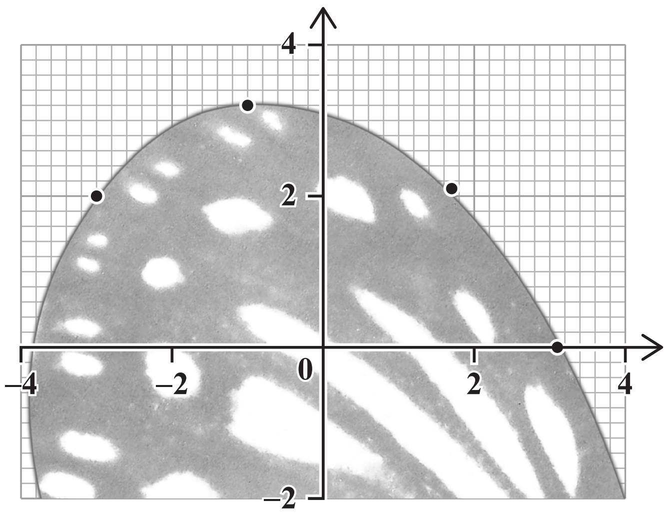
11. [Maximum mark: 5]

Gloria wants to model the curved edge of a butterfly wing. She inserts a photo of the wing into her graphing software and finds the coordinates of four points on the edge of the wing.



x	y
−3	2
−1	3.2
1.7	2.1
3.1	0

Gloria thinks a cubic curve will be a good model for the butterfly wing.

(a) Find the equation of the cubic regression curve for this data. [2]

For the photo of a second butterfly wing, Gloria finds the equation of the regression curve is $y = 0.0083x^3 - 0.075x^2 - 0.58x + 2.2$.

Gloria realizes that her photo of the second butterfly is an enlargement of the life-size butterfly, scale factor 2 and centred on $(0, 0)$.

(b) Find the equation of the cubic curve that models the life-size wing. [3]

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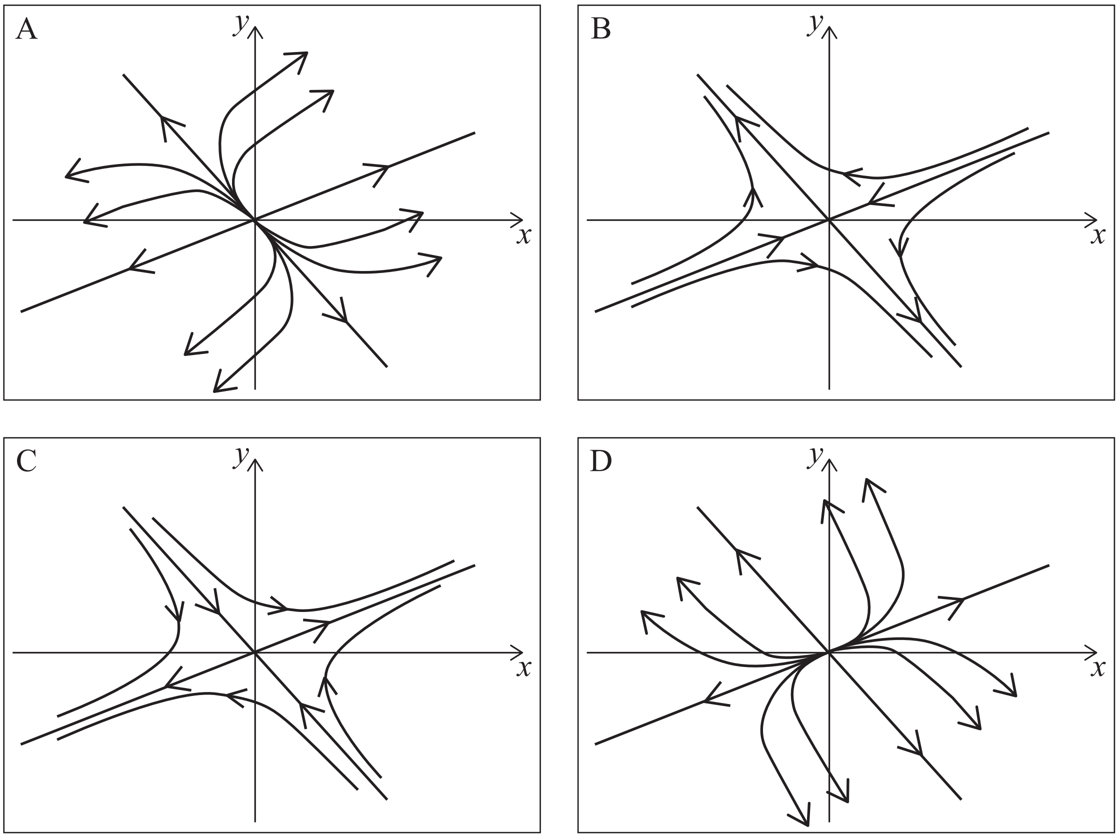
Turn over

12. [Maximum mark: 5]

Four possible phase portraits for the coupled differential equations $\frac{dx}{dt} = ax + by$ and $\frac{dy}{dt} = cx + dy$ are shown, labelled A, B, C and D.



The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has eigenvalues λ_1 and λ_2.

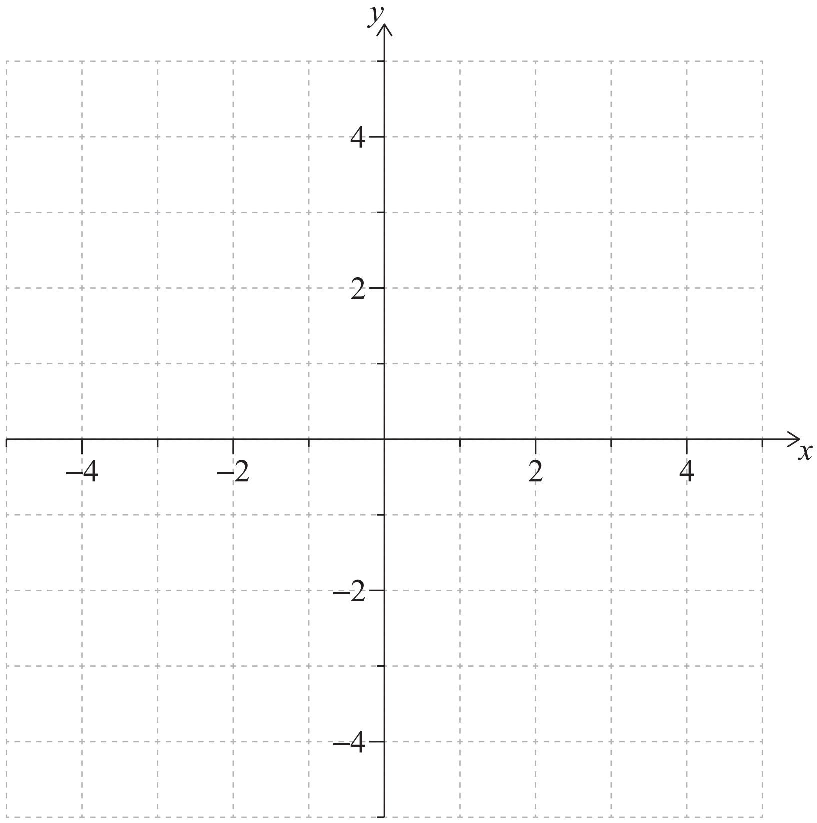
(a) Complete the following table by writing down the letter of the phase portrait that best matches the description. [3]

Description	Phase portrait
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	

(This question continues on the following page)

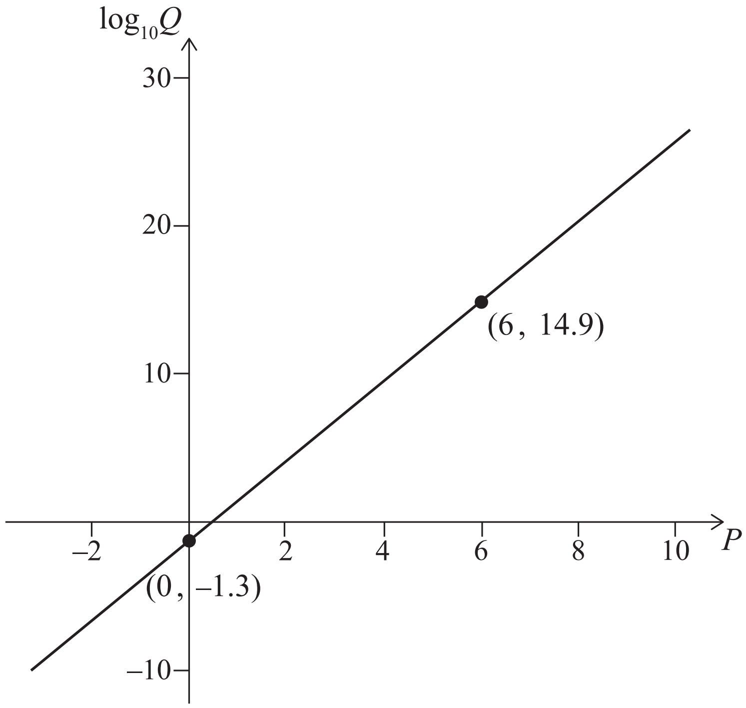
(Question 12 continued)

(b) On the following axes, sketch the phase portrait that corresponds to $\lambda_1 = -2 + 3\mathrm{i}$ and $\lambda_2 = -2 - 3\mathrm{i}$, given that $\frac{\mathrm{d}y}{\mathrm{d}t} = -12$ at $(3,\ 0)$. [2]



13. [Maximum mark: 6]

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10}Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, -1.3)$ and $(6, 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]

(This question continues on the following page)

(Question 13 continued)

14. [Maximum mark: 9]

A particle moves such that its velocity, v metres per second, at time t seconds, is given by $v = t\sin(t^2)$.

(a) Find an expression for the acceleration of the particle. [2]

(b) Hence, or otherwise, find its greatest acceleration for $0 \leq t \leq 8$. [2]

The particle starts at the origin.

(c) Find an expression for the displacement of the particle. [3]

(d) Hence show that the particle never has a negative displacement. [2]

15. [Maximum mark: 5]

An electrical circuit contains a capacitor. The charge on the capacitor, q Coulombs, at time t seconds, satisfies the differential equation

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + 5\frac{\mathrm{d}q}{\mathrm{d}t} + 20q = 200 .$$

Initially $q = 1$ and $\frac{\mathrm{d}q}{\mathrm{d}t} = 8$.

Use Euler's method with $h = 0.1$ to estimate the maximum charge on the capacitor during the first second.

16. [Maximum mark: 8]

The principal of a school is concerned that only 30% of her students are choosing healthy options from the school canteen. She organizes a campaign to promote healthy eating and decides to test if the campaign has increased the number of students choosing healthy options. She assumes that a student's choice is independent of other students' choices.

(a) Write down suitable hypotheses for this test. [2]

The principal decides to take a random sample of 80 students. She will reject the null hypothesis if at least 31 students choose a healthy option.

(b) Find the probability that she makes a Type I error. [3]

In fact, the campaign led to 40% of her students choosing a healthy option.

(c) Find the probability that she makes a Type II error. [3]

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17. [Maximum mark: 8]

The time of sunrise, R hours after midnight, in Taipei can be modelled by

$$R = 1.08\cos(0.0165t + 0.413) + 4.94,$$

where t is the day of the year 2021 (for example, $t = 2$ represents 2 January 2021).

The time of sunset, S hours after midnight, in Taipei can be modelled by

$$S = 1.15\cos(0.0165t - 2.97) + 18.9.$$

The number of daylight hours, D, in Taipei during 2021 can be modelled by

$$D = a\cos(0.0165t + b) + c.$$

(a) Find the value of a, of b and of c. [6]

(b) Hence, or otherwise, find the largest number of daylight hours in Taipei during 2021 and the day of the year on which this occurs. [2]

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References:

11. Fleur, 2019. *photo-1560263816-d704d83cce0f.* [image online] Available at: https://unsplash.com/photos/SE2zTdS1MNo [Accessed 8 February 2022]. Source adapted.

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