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Diploma Programme
Programme du diplôme
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Markscheme

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Mathematics: analysis and approaches

Higher level

Paper 3

21 pages

International Baccalaureate
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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*

2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) attempts chain rule differentiation to find $f'(x)$ ***(M1)***

$f'(x)=\frac{2}{(2-x)^3}$ $\left(=(-1)(-2)(2-x)^{-3}\right)$ ***A1***

Note: Award ***(M1)*** for attempting chain rule differentiation on $\left(4-4x+x^2\right)^{-1}$ or attempting quotient rule differentiation on $\frac{1}{\left(4-4x+x^2\right)}$ $\left(=\frac{1}{(2-x)^2}\right)$.

Award ***A1*** for $f'(x)=\frac{2}{(2-x)^3}$ $\left(=(-1)(-2)(2-x)^{-3}\right)$.

[2 marks]

(ii) $g'(x)=2x$ ***(A1)***

$f'(x)g'(x)=\left(2(2-x)^{-3}\right)(2x)$ $\left(=\frac{2(2x)}{(2-x)^3}\right)$ (or equivalent) ***A1***

$=\frac{4x}{(2-x)^3}$ ***AG***

[2 marks]

continued…

Question 1 continued.

(iii)

Note: Award a maximum of ***(M1)A1(M1)A0FT*** from parts (a) (i) and (ii).

substitutes $f(x), g(x)$ and their $g'(x), f'(x)$ into the given expression ***(M1)***

EITHER

$f(x)g'(x)+g(x)f'(x)=2x(2-x)^{-2}+2x^2(2-x)^{-3}$ ***A1***

Note: Award ***A1*** if $f(x)g'(x)=2x(2-x)^{-2}$ and $g(x)f'(x)=2x^2(2-x)^{-3}$ are stated separately.

attempts to factorise their expression ***(M1)***

$=2x(2-x)^{-3}\left((2-x)+x\right)$ ***A1***

OR

$f(x)g'(x)+g(x)f'(x)=\dfrac{2x}{(2-x)^2}+\dfrac{2x^2}{(2-x)^3}$ ***A1***

Note: Award ***A1*** if $f(x)g'(x)=\dfrac{2x}{(2-x)^2}$ and $g(x)f'(x)=\dfrac{2x^2}{(2-x)^3}$ are stated separately.

attempts to form an expression with a common denominator ***(M1)***

Note: Award ***(M1)*** for $(2-x)^2(2-x)^3$ as a common denominator.

$=\dfrac{2x(2-x)}{(2-x)^3}+\dfrac{2x^2}{(2-x)^3}\left(=\dfrac{4x-2x^2+2x^2}{(2-x)^3}\right)$ ***A1***

THEN

$=\dfrac{4x}{(2-x)^3}$

AG

Note: Award marks as appropriate for attempting to find the derivative of $f(x)g(x)=\dfrac{x^2}{(2-x)^2}$ (or equivalent).

[4 marks]

continued…

Question 1 continued.

(b) **METHOD 1**

$f'(x)g'(x)-g(x)f'(x)=f(x)g'(x)$ ***(A1)***

$\left(f'(x)g'(x)-g(x)f'(x)-f(x)g'(x)=0\right)$

$f'(x)\left(g'(x)-g(x)\right)=f(x)g'(x)$ $\left(f'(x)\left(g(x)-g'(x)\right)=-f(x)g'(x)\right)$ ***A1***

$\dfrac{f'(x)}{f(x)}=\dfrac{g'(x)}{g'(x)-g(x)}$ ***AG***

Note: Award ***(A0)A0*** for use of $f(x)$ and $g(x)$ from part (a).

METHOD 2

$g'(x)=\dfrac{f'(x)g'(x)}{f(x)}-\dfrac{g(x)f'(x)}{f(x)}$

(A1)

$g'(x)=\dfrac{f'(x)}{f(x)}\left(g'(x)-g(x)\right)$

A1

$\dfrac{f'(x)}{f(x)}=\dfrac{g'(x)}{g'(x)-g(x)}$

AG

Note: Candidates may not show the steps exactly as shown above. Award ***(A0)A0*** for use of $f(x)$ and $g(x)$ from part (a).

continued…

Question 1 continued.

METHOD 3

$g'(x)=\frac{f(x)g'(x)}{f'(x)}+g(x)$ ***(A1)***

$\frac{f(x)}{f'(x)}=\frac{g'(x)-g(x)}{g'(x)}$ ***A1***

$\frac{f'(x)}{f(x)}=\frac{g'(x)}{g'(x)-g(x)}$ ***AG***

Note: Candidates may not show the steps exactly as shown above. Award ***(A0)A0*** for use of $f(x)$ and $g(x)$ from part (a).

METHOD 4

$\frac{f(x)}{f'(x)}+\frac{g(x)}{g'(x)}=1$ ***(A1)***

$\frac{f(x)}{f'(x)}=\frac{g'(x)-g(x)}{g'(x)}$ ***A1***

$\frac{f'(x)}{f(x)}=\frac{g'(x)}{g'(x)-g(x)}$ ***AG***

Note: Candidates may not show the steps exactly as shown above. Award ***(A0)A0*** for use of $f(x)$ and $g(x)$ from part (a).

[2 marks]

continued…

Question 1 continued.

(c) **METHOD 1**

Note: Condone the absence of '$\mathrm{d}x$' and the modulus sign throughout.
Only award the second ***A*** mark if the constant of integration has been dealt with correctly.

EITHER

$$\ln f(x) = \int \frac{g'(x)}{g'(x)-g(x)}\,\mathrm{d}x\ (+C)$$ ***A1***

Note: Condone the absence of '$+C$' when awarding the first ***A*** mark.

$$f(x) = \mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}\mathrm{e}^{C}\quad \left(f(x) = \mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x + C\right)}\right)$$ ***A1***

Note: Award ***A1*** for $f(x) = A\mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}$, where $A = \mathrm{e}^{C}$.

OR

$$\ln f(x)(+C) = \int \frac{g'(x)}{g'(x)-g(x)}\,\mathrm{d}x$$ ***A1***

Note: Condone the absence of '$+C$' when awarding the first ***A*** mark.

$$f(x) = \mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}\mathrm{e}^{-C}\quad \left(f(x)\mathrm{e}^{C} = \mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}, f(x) = \mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x - C\right)}\right)$$ ***A1***

Note: Award ***A1*** for $f(x) = A\mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}$, where $A = \mathrm{e}^{-C}$.

THEN

$$f(x) = A\mathrm{e}^{\left(\int \frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}$$ ***AG***

continued…

Question 1 continued.

METHOD 2

Note: Condone the absence of 'dx' throughout.

$$f'(x)-\frac{g'(x)}{g'(x)-g(x)}f(x)=0$$

Integrating factor: $\mathrm{e}^{\left(-\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}$ ***A1***

Note: Award ***A1*** for $f(x)\mathrm{e}^{\left(-\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}=C$.

$$\frac{d}{dx}\left[f(x)\mathrm{e}^{\left(-\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}\right]=0\Rightarrow f(x)\mathrm{e}^{\left(-\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}=A$$ ***A1***

Note: Award ***A1*** for $f(x)\mathrm{e}^{\left(-\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}=C$.

$$f(x)=A\mathrm{e}^{\left(\int\frac{g'(x)}{g'(x)-g(x)}\mathrm{d}x\right)}$$ ***AG***

[2 marks]

continued…

Question 1 continued.

(d) $g'(x) = xe^x + e^x$ (seen anywhere) ***(A1)***

attempts to find an expression for $\frac{g'(x)}{g'(x) - g(x)}$ ***(M1)***

$= \frac{xe^x + e^x}{e^x} \left(= \frac{e^x(x+1)}{e^x} \right)$

$= x + 1$ (as $e^x \neq 0$) ***(A1)***

attempts to integrate their $\frac{g'(x)}{g'(x) - g(x)}$ ***(M1)***

$\int (x+1)\,dx = \frac{1}{2}x^2 + x(+C)$

$f(x) = e^{\left(\frac{1}{2}x^2 + x\right)}$ ***A1***

Note: Award ***A0*** for $f(x) = e^{\left(\frac{1}{2}x^2 + x + C\right)}$ (or equivalent expressed with an arbitrary constant).

[5 marks]

continued…

Question 1 continued.

(e) $g'(x) = \cos x - \sin x$ (seen anywhere) ***(A1)***

attempts to find an expression for $\dfrac{g'(x)}{g'(x) - g(x)}$ ***(M1)***

$$= \frac{\cos x - \sin x}{\cos x - \sin x - \sin x - \cos x}\left(= \frac{\sin x - \cos x}{2\sin x}\right)$$

$= \dfrac{1}{2} - \dfrac{1}{2}\cot x$ (as $\sin x \neq 0$) OR $= \dfrac{1}{2} - \dfrac{1}{2}\dfrac{\cos x}{\sin x}$ (as $\sin x \neq 0$) ***A1***

$$f(x) = \mathrm{e}^{\int\left(\frac{1}{2} - \frac{1}{2}\cot x\right)\mathrm{d}x}$$

attempts to find the indefinite integral of $(\pm k)\cot x$ OR $(\pm k)\dfrac{\cos x}{\sin x}$ ***(M1)***

Note: As $|\sin x| = \sin x$ for $0 < x < \pi$, condone the presence or omission of the modulus sign throughout a candidate's solution.
Condone the presence of an arbitrary constant except when awarding the final ***A*** mark.

$$\int\left(\frac{1}{2} - \frac{1}{2}\cot x\right)\mathrm{d}x = \frac{x}{2} - \frac{1}{2}\ln|\sin x|(+C)\ \left(= \frac{1}{2}\left(x - \ln|\sin x|(+C)\right)\right)$$ ***A1***

$$f(x) = \mathrm{e}^{\frac{x}{2}}\,\mathrm{e}^{-\frac{1}{2}\ln|\sin x|}\left(\mathrm{e}^{C}\right)$$

$$= \mathrm{e}^{\frac{x}{2}}\,\mathrm{e}^{\ln\sqrt{\frac{1}{\sin x}}}\left(\mathrm{e}^{C}\right)\ \left(= \mathrm{e}^{\frac{x}{2}}\,\mathrm{e}^{\frac{1}{2}\ln\left(\frac{1}{\sin x}\right)}\left(\mathrm{e}^{C}\right), = \sqrt{\mathrm{e}^{x - \ln(\sin x)}}\left(\mathrm{e}^{C}\right)\right)$$ ***A1***

$$= \mathrm{e}^{\frac{x}{2}}\sqrt{\frac{1}{\sin x}}$$

$= \sqrt{\mathrm{e}^{x}\operatorname{cosec} x}\ \left(= \sqrt{\dfrac{\mathrm{e}^{x}}{\sin x}}\right)$ (where $h(x) = \dfrac{1}{\sin x}$) ***A1***

[7 marks]

Total [24 marks]

2. (a) 6^3 OR $6\times6\times6$ ***A1***

Note: Accept a labelled diagram that clearly illustrates correct application of the multiplication principle leading to 216.

$=216$ ***AG***

[1 mark]

(b) **EITHER**

attempts to find Δ ***(M1)***

$\Delta\left(=\left(4^2-4(1)(4)\right)\right)=0$ ***A1***

OR

attempts to solve $x^2+4x+4=0$ ***(M1)***

$\left((x+2)^2=0\Rightarrow\right)x=-2$ ***A1***

OR

attempts to express $x^2+4x+4(=0)$ as a perfect square ***(M1)***

$(x+2)^2\,(=0)$ is a perfect square ***A1***

OR

a graph of $y=x^2+4x+4$ with the vertex touching the x- axis at $x=-2$ ***A2***

THEN

graph of f has only one x-intercept ***AG***

[2 marks]

continued…

Question 2 continued.

Note: In parts (c) – (f), $(a,b,c)=(1,2,1)$, for example, represents an ordered 3-tuple $a=1, b=2$ and $c=1$.

(c) recognizes that $b^2-4ac=0$ (or equivalent) ***(M1)***

EITHER

attempts to use $\frac{b^2}{ac}=4$ $\left(\frac{b^2}{4}=ac\right)$ ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) OR one value of ac from $ac=1,4$ or 9 only (seen anywhere) ***(A1)***

OR

attempts to find a possible value of b ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) ***(A1)***

OR

recognizes that b^2 must be a multiple of 4 OR b must be a multiple of 2 ***(M1)***

determines one value of b from $b=2,4$ or 6 only (seen anywhere) ***(A1)***

OR

attempts to find a possible value of ac ***(M1)***

determines one value of ac from $ac=1,4$ or 9 only (seen anywhere) ***(A1)***

THEN

$b=2$ and $ac=1$:

$(a,b,c)=(1,2,1)$ OR 1 possible way OR $\frac{1}{216}$ ***A1***

$b=4$ and $ac=4$:

$(a,b,c)=(1,4,4),(4,4,1),(2,4,2)$ OR 3 possible ways OR $\frac{3}{216}$ ***A1***

$b=6$ and $ac=9$:

$(a,b,c)=(3,6,3)$ OR 1 possible way OR $\frac{1}{216}$ ***A1***

therefore the required probability is $\frac{1}{216}+\frac{3}{216}+\frac{1}{216}$

$=\frac{5}{216}$ ***AG***

[6 marks]

continued…

Question 2 continued.

(d) recognizes that $b^2 - 4ac > 0$ (or equivalent eg. $\frac{b^2}{4} > ac$) ***(M1)***

maximum value of b^2 is 36 OR maximum value of ac is 8 ***(A1)***

Note: The above ***(A1)*** is independent of the ***(M1)***.

$ac = 1, 2, 3, 4, 5, 6, 8$ ***A1***

[3 marks]

(e) (i) $ac = 1 \ (b^2 > 4)$

$b = 3, 4, 5, 6$ OR 1×4 (quadratics) OR $6 - 2$ (quadratics) ***A1***

there are four quadratic functions ***AG***

[1 mark]

(ii)

$ac = 2 \ (b^2 > 8)$

$b = 3, 4, 5, 6$ ***(A1)***

Note: Award ***(A1)*** for referencing their result shown in part (e) (i).

EITHER

$(a, b, c) = (1,3,2), (1,4,2), (1,5,2), (1,6,2), (2,3,1), (2,4,1), (2,5,1), (2,6,1)$ ***A1***

Note: Award ***A1*** for listing the eight quadratic expressions.

OR

2×4 (quadratics) ***A1***

THEN

there are eight quadratics functions ***AG***

[2 marks]

continued…

Question 2 continued.

(f) **METHOD 1**

varies ac $(ac \neq 1,2)$ and determines possible values of b such that $\Delta > 0$ ***(M1)***

correctly determines one of the following five cases ***(A1)***

correctly determines a further two of the following five cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $ac = 3$ $\left(b^2 > 12 \Rightarrow b = 4,5,6\right)$

$(a,b,c) = (1,4,3),(1,5,3),(1,6,3),(3,4,1),(3,5,1),(3,6,1)$ OR

6 possible ways OR $\dfrac{6}{216}$

case 2: $ac = 4$ $\left(b^2 > 16 \Rightarrow b = 5,6\right)$

$(a,b,c) = (1,5,4),(1,6,4),(2,5,2),(2,6,2),(4,5,1),(4,6,1)$ OR

6 possible ways OR $\dfrac{6}{216}$

case 3: $ac = 5$ $\left(b^2 > 20 \Rightarrow b = 5,6\right)$

$(a,b,c) = (1,5,5),(1,6,5),(5,5,1),(5,6,1)$ OR 4 possible ways OR $\dfrac{4}{216}$

case 4: $ac = 6$ $\left(b^2 > 24 \Rightarrow b = 5,6\right)$

$(a,b,c) = (1,5,6),(2,5,3),(3,5,2),(6,5,1),(1,6,6),(2,6,3),(3,6,2),(6,6,1)$

OR 8 possible ways OR $\dfrac{8}{216}$

case 5: $ac = 8$ $\left(b^2 > 32 \Rightarrow b = 6\right)$

$(a,b,c) = (2,6,4),(4,6,2)$ OR 2 possible ways OR $\dfrac{2}{216}$

adds their probabilities ***(M1)***

Note: Award ***(M1)*** for adding at least 3 of their probabilities (denominator 216).

$$(p =)\ \frac{4}{216} + \frac{8}{216} + \frac{6}{216} + \frac{6}{216} + \frac{4}{216} + \frac{8}{216} + \frac{2}{216}$$

$$(= 0.0185... + 0.0370... + 0.0277... + 0.0277... + 0.0185... + 0.0370... + 0.0092...)$$

$$= \frac{38}{216} \quad \left(= \frac{19}{108}, = 0.176\right)$$ ***A1***

continued…

Question 2 continued.

METHOD 2

varies $b^2\left(\neq 1,4\right)$ OR $b\left(\neq 1,2\right)$ and determines possible values of ac such that $\Delta > 0$ ***(M1)***

correctly determines one of the following four cases ***(A1)***

correctly determines another case from the following four cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $b^2 = 9$ $\left(b = 3\right)$ $\left(ac = 1,2\right)$

$\left(a,b,c\right) = \left(1,3,1\right),\left(1,3,2\right),\left(2,3,1\right)$ OR 3 possible ways OR $\frac{3}{216}$

case 2: $b^2 = 16$ $\left(b = 4\right)$ $\left(ac = 1,2,3\right)$

$\left(a,b,c\right) = \left(1,4,1\right),\left(1,4,2\right),\left(2,4,1\right),\left(1,4,3\right),\left(3,4,1\right)$ OR 5 possible ways OR $\frac{5}{216}$

case 3: $b^2 = 25$ $\left(b = 5\right)$ $\left(ac = 1,2,3,4,5,6\right)$

$\left(a,b,c\right) = \left(1,5,1\right),\left(1,5,2\right),\left(2,5,1\right),\left(1,5,3\right),\left(3,5,1\right),\left(1,5,4\right),\left(2,5,2\right)$
$\left(4,5,1\right),\left(1,5,5\right),\left(5,5,1\right),\left(1,5,6\right),\left(2,5,3\right),\left(3,5,2\right),\left(6,5,1\right)$

OR 14 possible ways OR $\frac{14}{216}$

case 4: $b^2 = 36$ $\left(b = 6\right)$ $\left(ac = 1,2,3,4,5,6,8\right)$

$\left(a,b,c\right) = \left(1,6,1\right),\left(1,6,2\right),\left(2,6,1\right),\left(1,6,3\right),\left(3,6,1\right),\left(1,6,4\right),\left(2,6,2\right),\left(4,6,1\right)$
$\left(1,6,5\right),\left(5,6,1\right),\left(1,6,6\right),\left(2,6,3\right),\left(3,6,2\right),\left(6,6,1\right),\left(2,6,4\right),\left(4,6,2\right)$

OR 16 possible ways OR $\frac{16}{216}$

adds their probabilities ***(M1)***

Note: Award ***(M1)*** for adding at least 3 of their probabilities (denominator 216).

$(p =)\ \frac{3}{216} + \frac{5}{216} + \frac{14}{216} + \frac{16}{216}$

$\left(= 0.013889... + 0.023148... + 0.064815... + 0.074074...\right)$

$= \frac{38}{216}\ \ (= \frac{19}{108}, = 0.176)$ ***A1***

continued…

Question 2 continued.

METHOD 3

varies b^2 OR b and determines possible values of ac such that $\Delta < 0$ ***(M1)***

correctly determines two of the following six cases ***(A1)***

correctly determines a further two of the following six cases ***(A1)***

correctly determines the remaining two cases ***(A1)***

case 1: $b^2 = 1$ $(b = 1)$ 36 possible ways OR $\frac{36}{216}$

case 2: $b^2 = 4$ $(b = 2)$ 35 possible ways OR $\frac{35}{216}$

case 3: $b^2 = 9$ $(b = 3)$ 33 possible ways OR $\frac{33}{216}$

case 4: $b^2 = 16$ $(b = 4)$ 28 possible ways OR $\frac{28}{216}$

case 5: $b^2 = 25$ $(b = 5)$ 22 possible ways OR $\frac{22}{216}$

case 6: $b^2 = 36$ $(b = 6)$ 19 possible ways OR $\frac{19}{216}$

$(p =)\ 1 - \left(\frac{36}{216} + \frac{35}{216} + \frac{33}{216} + \frac{28}{216} + \frac{22}{216} + \frac{19}{216} + \frac{5}{216}\right)$ ***(M1)***

$$\left(\begin{array}{l} = 1 - \\ (0.16666... + 0.16203... + 0.15277... + 0.12962... + 0.10185... + 0.087962... + 0.023148...) \end{array}\right)$$

Note: Award ***(M1)*** for adding at least 3 of their probabilities inside the above bracket (denominator 216).

$= \frac{38}{216}$ $(= \frac{19}{108}, = 0.176)$ ***A1***

[6 marks]

continued…

Question 2 continued.

(g) recognizes that $4Z^2 - 4 > 0$ ($Z^2 > 1$) ***(M1)***

probability of two x-intercepts is

EITHER

$\mathrm{P}(|Z| > 1)$ ***(A1)***

OR

$\mathrm{P}(Z < -1)$ or $\mathrm{P}(Z > 1)$ (can be shown on a labelled diagram) ***(A1)***

$= 0.158655... + 0.158655...$

OR

$1 - \mathrm{P}(-1 \le Z \le 1)$ (can be shown on a labelled diagram) ***(A1)***

$= 1 - 0.682689...$

THEN

$= 0.317310...$

$= 0.317$ ***A1***

[3 marks]

(h) attempts to solve $X_1 > 0.5$ for Z ***(M1)***

$-1.25 < Z \le -1$ ***(A1)(A1)***

Note: Award ***(M1)(A1)*** for obtaining $Z = -1.25$ from solving $X_1 = 0.5$ and award ***(A1)*** for stating the correct inequality.
Award ***(M1)(A1)(A1)*** for $-1.25 < Z < -1$.
Award ***(M1)(A1)(A0)*** for $-1.25 < Z$.
Award ***(M1)*** for rearranging to form $-\sqrt{Z^2 - 1} = Z + 0.5$ and then attempting to square both sides $Z^2 - 1 = (Z + 0.5)^2$ $(= Z^2 + Z + 0.25)$.

attempts to calculate their $\mathrm{P}(X_1, X_2 \text{ both } > 0.5)$ ***(M1)***

$\mathrm{P}(-1.25 < Z \le -1) = 0.053005...$ ***(A1)***

attempts to calculate their $\mathrm{P}(X_1, X_2 \text{ both } > 0.5 \mid x\text{-intercepts})$ ***(M1)***

$= \dfrac{0.053005...}{0.317310...}$

$= 0.167$ ***A1***

[7 marks]

Total [31 marks]