Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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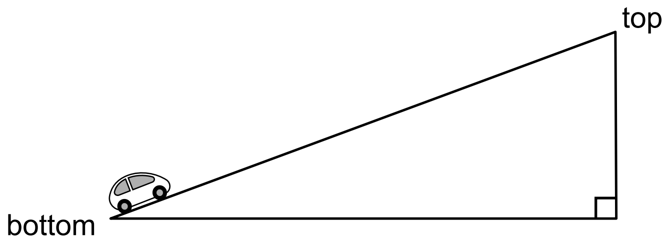
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 4]

One of the steepest straight roads in the world is in Dunedin, New Zealand.

This road is 161 m long, and the angle of elevation from the bottom of the road to the top is $16.3°$.

diagram not to scale



(a) Label the diagram with the given values for the road length and the angle of elevation. [2]

(b) Find the vertical change in height from the bottom of the road to the top. [2]

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2. [Maximum mark: 7]

The scores on a test, out of 7 points, for 220 students are shown in the following table.

Score	1	2	3	4	5	6	7
Frequency	11	24	26	35	61	43	20

(a) Find the mean **and** standard deviation of these test scores. Give your answers correct to four significant figures. [3]

(b) The scores are multiplied by ten. Write down the new mean and standard deviation. [2]

(c) After the scores have been multiplied by ten, 30 points are added to each of them. Write down the new mean and standard deviation. [2]

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3. [Maximum mark: 5]

Tickets to enter a theme park are priced at a dollars for adults and c dollars for children.

A school group of 6 adults and 50 children paid a total of $\$1292$.

A family of 2 adults and 3 children paid a total of $\$130$.

(a) Write down **two** equations that represent this information. [2]

(b) Hence, find the price of

(i) an adult ticket

(ii) a child ticket. [2]

Rounded to the **nearest thousand**, there were $101\,000$ visitors at the theme park last year.

(c) Write down the lower bound for the number of visitors last year. [1]

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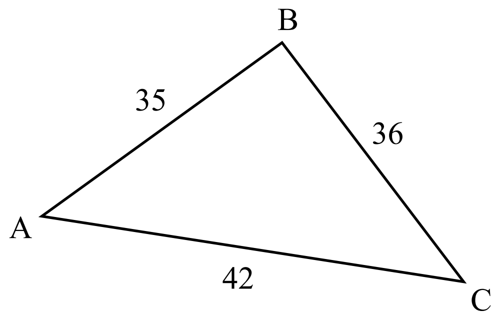
4. [Maximum mark: 5]

Consider the following triangle, ABC, such that $\text{AB} = 35\,\text{cm}$, $\text{BC} = 36\,\text{cm}$, and $\text{CA} = 42\,\text{cm}$.

diagram not to scale



(a) Find the value of $\text{C}\hat{\text{A}}\text{B}$. [3]

(b) Find the area of the triangle ABC. [2]

5. [Maximum mark: 7]

The total cost, $C(d)$, in Canadian dollars (CAD), to hire a bicycle for d days from *Pedal Paradise* is given by the function

$$C(d) = 60d + 10\,,\ d \geq 3\,,\ d \in \mathbb{Z}\,.$$

The total cost includes a fixed charge to hire both a helmet and a repair kit.

(a) State, in context, what the values 60 and 10 represent. [2]

(b) Calculate the cost of hiring a bicycle for 5 days. [2]

Hema hires a bicycle from *Pedal Paradise*.

(c) Write down the minimum number of days she can hire the bicycle. [1]

(d) Given that $C^{-1}(1270) = k$, find the value of k. [2]

6. [Maximum mark: 6]

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120\mathrm{e}^{-0.000121t}$$

where t is the time since the plant died.

(a) Write down the initial mass of the radioactive carbon. [1]

(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

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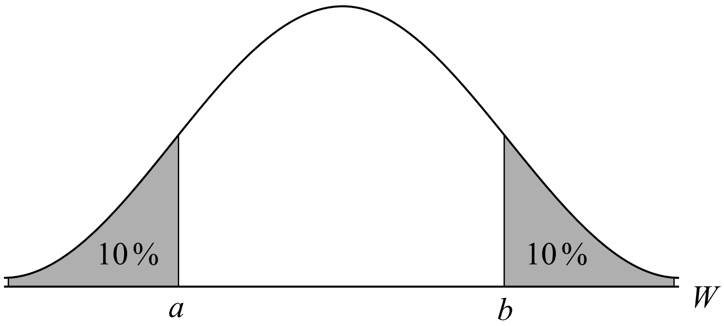
7. [Maximum mark: 7]

The mass, W, of Manx cats is normally distributed with a mean of $4.5\,\text{kg}$ and a standard deviation of $0.4\,\text{kg}$.

(a) A Manx cat is selected at random. Calculate the probability this cat's mass is more than $3.5\,\text{kg}$. [2]

The following curve represents this distribution. It is known that $\mathrm{P}(W < a) = 0.1$ and $\mathrm{P}(W > b) = 0.1$.



(b) Find the value of

(i) a

(ii) b. [3]

(c) Two Manx cats are selected at random from a large population. Find the probability that they both have a mass less than $3.5\,\text{kg}$. [2]

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8. [Maximum mark: 5]

On 1 January in a particular year, Eva invests $\$25\,000$ in a new bank account. The account earns 5% simple interest, on the original $\$25\,000$, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Eva's account will be greater than $\$44\,000$ for the first time.

(b) Find the value of k. [3]

9. [Maximum mark: 7]

The records at a driving school show that 55% of students pass their driving test on the first attempt.

A group of 20 students take their driving test for the first time.

As part of its quality control, the driving school uses the model $X \sim \mathrm{B}\,(20\,,\,0.55)$, where X is the number of students who pass the driving test.

(a) Calculate the

(i) mean of X

(ii) variance of X. [2]

(b) Find the probability that

(i) exactly 14 students pass the test

(ii) fewer than 5 students pass the test. [4]

(c) State one assumption that the driving school makes in using this model. [1]

10. [Maximum mark: 6]

When Daniel retires, he invests $\$400\,000$ in an annuity fund that earns interest at a nominal rate of 4.5% per year, compounded monthly.

Daniel then withdraws $\$3600$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 5 years. [3]

(b) Calculate how many times Daniel is able to make these withdrawals. [3]

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11. [Maximum mark: 7]

A fair game is played where points are scored as follows:

- A win scores w points.
- A draw scores 0 points.
- A loss scores -5 points.

Let X be the number of points scored during a game. The probability distribution is shown.

x	w	0	-5
$\mathrm{P}(X=x)$	0.25	0.4	p

(a) Find the value of p. [2]

The game is played 60 times.

(b) Find the expected number of losses. [2]

(c) Calculate the value of w, given that the game is fair. [3]

12. [Maximum mark: 7]

Consider the curve $y = 4x^3 - \frac{2}{x^2}$.

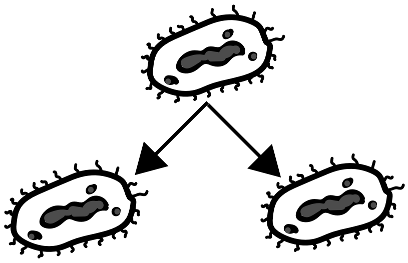
(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Write down the gradient of the curve at $x = 1$. [1]

(c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

13. [Maximum mark: 7]

A type of bacteria reproduces by **dividing in two** every 10 minutes.



There were 1250 bacteria in a colony 10 minutes after the start of an experiment.

The following table is used to estimate the number of bacteria, u_n, for this colony.

The values of u_n form the terms of a sequence.

n	1	2	3		9		k
Time in minutes	10	20	30		90		$10k$
Number of bacteria, u_n	1250	2500	5000		…		…

(a) Complete the table by adding the two missing values. [3]

As the number of bacteria increases from 1250 to 2500, the total number of **bacterial divisions** is 1250.

(b) (i) Find the value of n when the number of bacteria is 1.28×10^6.

(ii) Hence or otherwise, find the total number of bacterial divisions as the number of bacteria increases from 1250 to 1.28×10^6. Give your answer correct to the nearest thousand bacterial divisions. [4]

(This question continues on the following page)

(Question 13 continued)

Please **do not** write on this page.

Answers written on this page
will not be marked.