Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2225–7106

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the function $f(x)=\dfrac{4x^3}{3}-16x$, where $x \in \mathbb{R}$.

The graph of $y=f(x)$ has a local minimum point at (p, q) where $p>0$.

Find the value of p and the value of q.

2. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

(b) Expand and simplify $(1+x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

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3. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = \mathrm{e}^x$, $y = -\mathrm{e}^x$, and the lines $x = -1$ and $x = 1$.

4. [Maximum mark: 6]

Consider events A and B such that $\mathrm{P}(A') = \mathrm{P}(A \cup B) = \frac{3}{4}$ and $\mathrm{P}(B|A) = \frac{2}{3}$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

5. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \dots, F_9, F_{10}$.

Picture frame F_1 has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame F_n, are each increased by 50% to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+$, $1 \le n \le 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\,\text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^a - 1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4\,\text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

6. [Maximum mark: 6]

The line L_1 has vector equation $\boldsymbol{r} = 4\boldsymbol{i} - \boldsymbol{k} + \lambda(a\boldsymbol{j} + \boldsymbol{k})$, where $a, \lambda \in \mathbb{R}$.

The line L_2 has vector equation $\boldsymbol{r} = \boldsymbol{i} - b\boldsymbol{k} + \mu(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$, where $b, \mu \in \mathbb{R}$.

The lines L_1 and L_2 are perpendicular and intersect at a unique point.

Find the value of a and the value of b.

7. [Maximum mark: 5]

Consider the complex number $z = 3^{\mathrm{i}-1}$.

(a) Write the integer 3 in the form e^{a} where $a \in \mathbb{R}$. [1]

(b) Hence, giving your answers in the form $p\cos(\ln q)$ where $p, q \in \mathbb{Q}^{+}$, find

(i) $\mathrm{Re}(z)$;

(ii) $\mathrm{Re}\left(\frac{1}{z}\right)$. [4]

8. [Maximum mark: 7]

Seema claims that $n > \log_2 n$ for $n \in \mathbb{Z}^+$.

(a) Show that $1 + \log_2 n \geq \log_2(n + 1)$ for $n \in \mathbb{Z}^+$. [2]

(b) Use mathematical induction and the result from part (a) to prove that Seema's claim is valid. [5]

9. [Maximum mark: 8]

Consider the homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x-y}{x+y}$, where $x > 0$ and $y \neq -x$.

It is given that $y = 0$ when $x = 2$.

By using the substitution $y = vx$, show that the solution of the differential equation is $x^2 - 2xy - y^2 = 4$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \le 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \le 40$. [3]

(e) Find the domain of $g \circ f$. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The plane Π_1 has equation $x + 2y + z = 0$ and the plane Π_2 has equation $x - y - 2z = 0$.

The acute angle between the planes Π_1 and Π_2 is θ.

(a) Show that $\theta = 60°$. [6]

A third plane Π_3 is perpendicular to both Π_1 and Π_2.

The unique point of intersection of all three planes is the point $\mathrm{R}(5, -5, 5)$.

(b) Find the Cartesian equation of Π_3. [4]

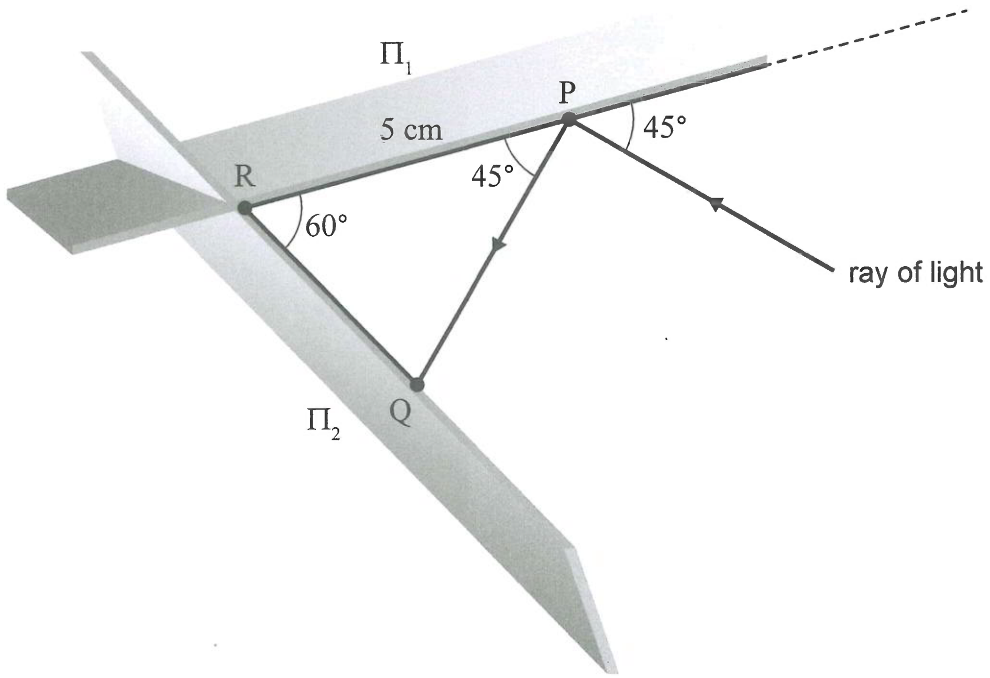
Each of the planes Π_1 and Π_2 contains a mirror.

A ray of light is directed towards the mirror in Π_1. The ray of light forms an angle of $45°$ with Π_1 and meets it at the point P.

The ray of light is then reflected towards the mirror in Π_2, and meets Π_2 at the point Q. The points P and Q are contained in Π_3.

It is given that $\mathrm{PR} = 5\,\mathrm{cm}$.

This information is shown on the following diagram.



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Do **not** write solutions on this page.

(Question 11 continued)

(c) (i) Using an appropriate compound angle identity, show that $\sin 75^{\circ} = \dfrac{\sqrt{2}+\sqrt{6}}{4}$.

(ii) Find QR, giving your answer in the form $p\left(\sqrt{q}-1\right)\text{cm}$ where $p, q, r \in \mathbb{Z}$. [7]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

Consider the family of functions $f_n(x) = \cos^n x$, where $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

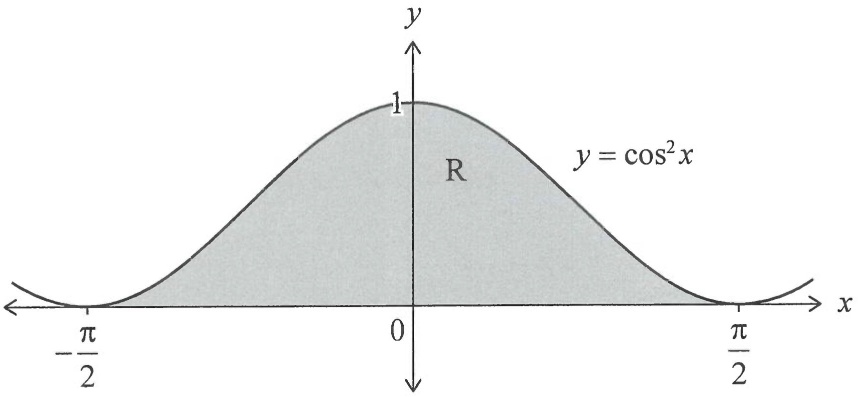
(a) By writing $\cos^n x$ as $\cos^{n-1} x \cos x$, show that

$$\int \cos^n x \, dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x \, dx - (n-1)\int \cos^n x \, dx \text{ for } n > 1.$$ [4]

(b) Hence, show that $\int f_n(x) dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int f_{n-2}(x) dx$ for $n > 1$. [2]

(c) Hence, find an expression for $\int \cos^4 x \, dx$, giving your answer in the form $p\cos^3 x \sin x + q\cos x \sin x + rx + c$ where $p, q, r \in \mathbb{Q}^+$. [4]

The region R is enclosed by the graph of $y = \cos^2 x$ and the x-axis where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, as shown in the following diagram.



The region R is rotated by 2π radians around the x-axis to form a solid of revolution.

(d) Find the volume of the solid. [4]

(e) (i) Find the Maclaurin series of $f_n(x)$ up to the term in x^2.

(ii) Hence or otherwise, find $\lim_{x \to 0} \frac{f_n(x) - 1}{x^2}$ in terms of n. [5]