Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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13 pages

2225–7112

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The following table shows the number of hours of play time, x, and sleep time, y, for a group of six children, over the period of one week.

Play time (x)	11	13	14	17	22	24
Sleep time (y)	62	65	68	75	84	87

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

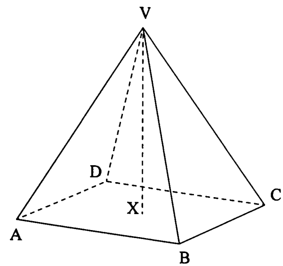
(b) Use the equation of the regression line to estimate the sleep time of a child whose weekly play time is 20 hours. [2]

2. [Maximum mark: 6]

The following diagram shows a square-based right-pyramid with vertex $V(1, 7, 0)$.

Point $X(-3, 4, 2)$ is the centre of the base ABCD.

diagram not to scale



(a) Find VX. [2]

The square base has side length 5 cm.

(b) Find AC. [2]

(c) Find the size of the angle between the edge [VC] and the base of the pyramid. [2]

3. [Maximum mark: 6]

The derivative of a function f is given by $f'(x) = 4 + 2x - 3e^x$, where $x \in \mathbb{R}$.

(a) Find the values of x for which f is decreasing. [3]

(b) Find the values of x for which the graph of f is concave-up. [3]

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4. [Maximum mark: 6]

Alex purchases a car for €30000. The value of the car depreciates at 15% per annum.

(a) Find the value of the car after ten years. Give your answer to two decimal places. [2]

Alex invests €50000 in a bank account that pays a compound interest rate of 1.5% per month.

Inflation over the same time period was 0.8% per month.

(b) Find the number of months required for the real value of the investment to first exceed €55000. [4]

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5. [Maximum mark: 7]

A particle P moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of P, at time t seconds is given by $v(t) = e^{-\sin t}\cos(2t)$, for $0 \leq t \leq 5$.

(a) Find the maximum speed of P. [2]

(b) Find the total distance travelled by P. [2]

(c) Find the acceleration when P changes direction for the **second** time. [3]

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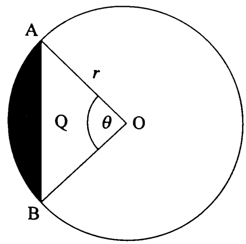
6. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and $\text{A}\hat{\text{O}}\text{B} = \theta$ radians.

Sector OAB is divided into two regions, a shaded segment P and a triangle Q.



The area of the shaded segment P is $12.8\,\text{cm}^2$.

The areas of P and Q are in the ratio $3:5$.

Find the value of r.

7. [Maximum mark: 7]

A geometric sequence has first term 80 and fourth term 0.74088.

(a) Find the second term. [3]

The first two terms of this geometric sequence are also the first term and eleventh term respectively, of an arithmetic sequence.

Let S_n denote the sum of the first n terms of the arithmetic sequence.

(b) Find the greatest value of S_n, giving your answer to two decimal places. [4]

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8. [Maximum mark: 7]

The marks obtained by students in a class quiz are shown in the following table where $p, q \in \mathbb{Z}^+$.

Marks	**Frequency**
20	12
35	q
p	8

The mean and variance of the marks are 31 and 124 respectively.

Find the value of p and the value of q.

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9. [Maximum mark: 8]

A line L_1 has vector equation $\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ where $t \in \mathbb{R}$.

The plane Π_1 contains the line L_1 and passes through the point $(2, 1, 5)$.

(a) Show that the Cartesian equation of the plane Π_1 is $x + y - z = -2$. [4]

Consider the three planes

$$\Pi_1 : x + y - z = -2$$
$$\Pi_2 : 2x + by - z = 3$$
$$\Pi_3 : x - y + 2z = d$$

where $b, d \in \mathbb{Q}^+$.

The three planes intersect in a line.

(b) Find the value of b and the value of d. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

At Adam's Apple Orchard the weights of apples, W, in grams, are normally distributed with a mean 175 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen apple weighs less than 170 grams. [2]

(b) It is found that 20% of the apples weigh more than w grams. Find w, correct to four significant figures. [2]

All orchards classify an apple as premium when its weight is between 170 and 185 grams.

(c) Find the percentage of apples that are classified as premium at Adam's Apple Orchard. [2]

After orders are completed, there are many apples left over. Boxes are filled with randomly chosen left-over apples. Each box contains 40 apples.

(d) Find the probability that a randomly chosen box contains at least 30 premium apples. [3]

(e) If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples. [2]

At a neighbouring orchard the weights of apples, M, in grams, are normally distributed with mean μ and standard deviation σ. It is known that:

- 82% of their apples are classified as premium
- the percentage of apples that weigh less than 170 grams is twice the percentage of apples that weigh more than 185 grams.

(f) Find the value of μ. [6]

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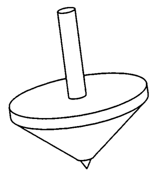
Do **not** write solutions on this page.

11. [Maximum mark: 14]

A mathematics class of 15 students plays a game which requires three equal size teams.

(a) Find the total number of ways that the three teams can be chosen. [3]

The game involves the spinning of a top.



The time, T, in minutes that the spinning top is in motion can be modelled by the probability density function f where

$$f(t)=\begin{cases} kte^{-3t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and $k \in \mathbb{Z}^{+}$.

(b) Show that $\int_0^a f(t)\,\mathrm{d}t = \frac{k}{9}\left[1-(3a+1)\mathrm{e}^{-3a}\right]$, where $a \in \mathbb{R}^{+}$. [4]

(c) (i) Use l'Hôpital's rule to find $\lim_{x\to\infty}(3x+1)\mathrm{e}^{-3x}$.

(ii) Hence, by considering $\lim_{a\to\infty}\int_0^a f(t)\,\mathrm{d}t$, find the value of k. [5]

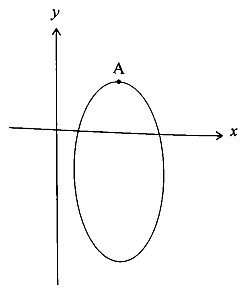
(d) Find the median length of time that a spinning top is in motion. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 22]

The curve C has equation $4x^2 + y^2 - 24x + 4y + 20 = 0$.

The following diagram shows C with a maximum point at A.



(a) Use implicit differentiation to show that $\frac{dy}{dx} = \frac{4(3-x)}{y+2}$. [4]

(b) Hence, determine the domain of C. Give your answer in the form $3-\sqrt{a} \le x \le 3+\sqrt{a}$, where $a \in \mathbb{Z}^+$. [4]

(c) Find (x_A, y_A), the coordinates of A. [3]

A line $y = mx$ is a tangent to C, where $m \in \mathbb{Z}$.

(d) Find the possible values of m. [4]

The line $y = -4x$ touches C at point B.

(e) Find y_B, the y-coordinate of B. [3]

The region bounded by the curve C, the y-axis and the lines $y = y_A$ and $y = y_B$, is rotated 360° about the y-axis to form a solid of revolution.

(f) Find the volume of the solid formed. [4]