SL Paper 2

Give your answers to parts (a) to (e) to the nearest dollar.

On Hugh's 18th birthday his parents gave him options of how he might receive his monthly allowance for the next two years.

Option A $60 each month for two years

Option B $10 in the first month, $15 in the second month, $20 in the third month, increasing by $5 each month for two years

Option C $15 in the first month and increasing by 10% each month for two years

Option D Investing $1500 at a bank at the beginning of the first year, with an interest rate of 6% per annum, **compounded monthly**.

Hugh does not spend any of his allowance during the two year period.

a. If Hugh chooses **Option A**, calculate the total value of his allowance at the end of the two year period. [2]

b. If Hugh chooses **Option B**, calculate [5]

(i) the amount of money he will receive in the 17th month;

(ii) the total value of his allowance at the end of the two year period.

c. If Hugh chooses **Option C**, calculate [5]

(i) the amount of money Hugh would receive in the 13th month;

(ii) the total value of his allowance at the end of the two year period.

d. If Hugh chooses **Option D**, calculate the total value of his allowance at the end of the two year period. [3]

e. State which of the options, A, B, C or D, Hugh should choose to give him the greatest total value of his allowance at the end of the two year period. [1]

f. Another bank guarantees Hugh an amount of $\$1750$ after two years of investment if he invests $1500 at this bank. The interest is **compounded annually**. [3]

Calculate the interest rate per annum offered by the bank.

Markscheme

a. ***The first time an answer is not given to the nearest dollar in parts (a) to (e), the final (A1) in that part is not awarded.***

60×24 ***(M1)***

Note: Award ***(M1)*** for correct product.

$= 1440$ ***(A1)(G2)***

[2 marks]

b. ***The first time an answer is not given to the nearest dollar in parts (a) to (e), the final (A1) in that part is not awarded.***

(i) $10+(17-1)(5)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitution.

$=90$ ***(A1)(G2)***

(ii) $\frac{24}{2}(2(10)+(24-1)(5))$ ***(M1)***

OR

$\frac{24}{2}(10+125)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in arithmetic series formula.

$=1620$ ***(A1)*(ft)*(G1)***

Note: Follow through from part (b)(i).

[5 marks]

c. ***The first time an answer is not given to the nearest dollar in parts (a) to (e), the final (A1) in that part is not awarded.***

(i) $15(1.1)^{12}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted geometric sequence formula, ***(A1)*** for correct substitutions.

$=47$ ***(A1)(G2)***

Note: Award ***(M1)(A1)(A0)*** for 47.08.

Award ***(G1)*** for 47.08 if workings are not shown.

(ii) $\frac{15(1.1^{24}-1)}{1.1-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in geometric series formula.

$=1327$ ***(A1)*(ft)*(G1)***

Note: Follow through from part (c)(i).

[5 marks]

d.

The first time an answer is not given to the nearest dollar in parts (a) to (e), the final (A1) in that part is not awarded.

$1500\left(1+\frac{6}{100(12)}\right)^{12(2)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest formula, ***(A1)*** for correct substitutions.

OR

$N = 2$

$I\% = 6$

$PV = 1500$

$P/Y = 1$

$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for other correct entries.

OR

$N = 24$

$I\% = 6$

$PV = 1500$

$P/Y = 12$

$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for other correct entries.

$= 1691$ ***(A1)(G2)***

[3 marks]

e. ***The first time an answer is not given to the nearest dollar in parts (a) to (e), the final (A1) in that part is not awarded.***

Option D ***(A1)*(ft)**

Note: Follow through from their parts (a), (b), (c) and (d). Award ***(A1)*(ft)** only if values for the four options are seen and only if their answer is consistent with their parts (a), (b), (c) and (d).

[1 mark]

f. $1750 = 1500\left(1 + \frac{r}{100}\right)^2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest formula equated to 1750, ***(A1)*** for correct substitutions into formula.

OR

$N = 2$

$PV = 1500$

$FV = -1750$

$P/Y = 1$

$C/Y = 1$ ***(A1)(M1)***

Note: Award ***(A1)*** for $FV = 1750$ seen, ***(M1)*** for other correct entries.

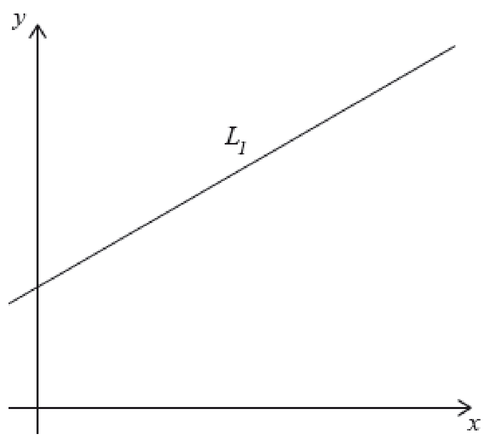
$= 8.01\% \ (8.01234\ldots\%, \ 0.0801)$ ***(A1)(G2)***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1, \ 4)$.

The point C has coordinates $(5, \ 12)$. M is the midpoint of AC.

The straight line, L_2, is perpendicular to AC and passes through M.

The point D is the intersection of L_1 and L_2.

The length of MD is $\frac{\sqrt{45}}{2}$.

The point B is such that ABCD is a rhombus.

a. Show that A lies on L_1. [2]

b. Find the coordinates of M. [2]

c. Find the length of AC. [2]

d. Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

e. Find the coordinates of D. [2]

f. Write down the length of MD correct to five significant figures. [1]

g. Find the area of ABCD. [3]

Markscheme

a. $2 \times 4 - 1 - 7 = 0$ (or equivalent) ***(R1)***

Note: For ***(R1)*** accept substitution of $x = 1$ or $y = 4$ into the equation followed by a confirmation that $y = 4$ or $x = 1$.

(since the point satisfies the equation of the line,) A lies on L_1 ***(A1)***

Note: Do not award ***(A1)(R0)***.

[2 marks]

b. $\frac{1+5}{2}$ **OR** $\frac{4+12}{2}$ seen ***(M1)***

Note: Award ***(M1)*** for at least one correct substitution into the midpoint formula.

$(3,\ 8)$ ***(A1)(G2)***

Notes: Accept $x = 3,\ y = 8$.

Award ***(M1)(A0)*** for $\left(\frac{1+5}{2},\ \frac{4+12}{2}\right)$.

Award ***(G1)*** for each correct coordinate seen without working.

[2 marks]

c. $\sqrt{(5-1)^2 + (12-4)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into distance between two points formula.

$= 8.94\ \left(4\sqrt{5},\ \sqrt{80},\ 8.94427\ldots\right)$ ***(A1)(G2)***

[2 marks]

d. gradient of $\mathrm{AC} = \frac{12-4}{5-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula.

$= 2$ ***(A1)***

Note: Award ***(M1)(A1)*** for gradient of $\mathrm{AC} = 2$ with or without working

gradient of the normal $= -\frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for the negative reciprocal of their gradient of AC.

$y - 8 = -\frac{1}{2}(x - 3)$ **OR** $8 = -\frac{1}{2}(3) + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their point and gradient into straight line formula. This ***(M1)*** can **only** be awarded where $-\frac{1}{2}$ (gradient) is correctly determined as the gradient of the normal to AC.

$2y - 16 = -(x - 3)$ **OR** $-2y + 16 = x - 3$ **OR** $2y = -x + 19$ ***(A1)***

Note: Award ***(A1)*** for correctly removing fractions, **but only** if their equation is equivalent to the given equation.

$2y + x - 19 = 0$ ***(AG)***

Note: The conclusion $2y + x - 19 = 0$ must be seen for the ***(A1)*** to be awarded.

Where the candidate has **shown** the gradient of the normal to $\mathrm{AC} = -0.5$, award ***(M1)*** for $2(8) + 3 - 19 = 0$ and ***(A1)*** for (therefore) $2y + x - 19 = 0$.

Simply substituting $(3,\ 8)$ into the equation of L_2 with no other prior working, earns no marks.

[5 marks]

e. $(6,\ 6.5)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for 6, ***(A1)*** for 6.5. Award a maximum of ***(A1)(A0)*** if answers are not given as a coordinate pair. Accept $x = 6,\ y = 6.5$.

Award ***(M1)(A0)*** for an attempt to solve the two simultaneous equations $2y - x - 7 = 0$ and $2y + x - 19 = 0$ algebraically, leading to at least one incorrect or missing coordinate.

[2 marks]

f. 3.3541 ***(A1)***

Note: Answer must be to 5 significant figures.

[1 mark]

g. $2 \times \frac{1}{2} \times \sqrt{80} \times \frac{\sqrt{45}}{2}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct substitution into area of triangle formula.

If their triangle is a quarter of the rhombus then award ***(M1)*** for multiplying their triangle by 4.

If their triangle is a half of the rhombus then award ***(M1)*** for multiplying their triangle by 2.

OR

$\frac{1}{2} \times \sqrt{80} \times \sqrt{45}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for doubling MD to get the diagonal BD, ***(M1)*** for correct substitution into the area of a rhombus formula.

Award ***(M1)(M1)*** for $\sqrt{80}\times$ their (f).

$= 30$ ***(A1)*****(ft)*****(G3)***

Notes: Follow through from parts (c) and (f).

$8.94 \times 3.3541 = 29.9856\ldots$

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

The natural numbers: 1, 2, 3, 4, 5… form an arithmetic sequence.

A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

i.a. State the values of u_1 and d for this sequence. [2]

i.b. Use an appropriate formula to show that the sum of the natural numbers from 1 to n is given by $\frac{1}{2}n(n+1)$. [2]

i.c. Calculate the sum of the natural numbers from 1 to 200. [2]

ii.a. The sum of the first n terms of G_1 is 29 524. Find n. [3]

ii.b A second geometric progression G_2 has the form $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}\ldots$ [1]

ii.c. Calculate the sum of the first 10 terms of G_2. [2]

ii.d. Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures. [1]

ii.e. Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2, 3\frac{1}{3}, 9\frac{1}{9}, 27\frac{1}{27}\ldots$ [3]

Give your answer **correct to one decimal place**.

Markscheme

i.a. $u_1 = d = 1.$ ***(A1)(A1)***

[2 marks]

i.b. Sum is $\frac{1}{2}n(2u_1 + d(n-1))$ or $\frac{1}{2}n(u_1 + u_n)$ ***(M1)***

*Award **(M1)** for either sum formula seen, even without substitution.*

So sum is $\frac{1}{2}n(2 + (n-1)) = \frac{1}{2}n(n+1)$ ***(A1)(AG)***

*Award **(A1)** for substitution of $u_1 = 1 = d$ or $u_1 = 1$ and $u_n = n$ with simplification where appropriate. $\frac{1}{2}n(n+1)$ must be seen to award this **(A1)**.*

[2 marks]

i.c. $\frac{1}{2}(200)(201) = 20100$ ***(M1)(A1)(G2)***

***(M1)** is for correct formula with correct numerical input. Original sum formula with u, d and n can be used.*

[2 marks]

ii.a. $\frac{1-3^n}{1-3} = 29524$ ***(M1)(A1)***

***(M1)** for correctly substituted formula on one side, **(A1)** for = 29524 on the other side.*

n = 10. ***(A1)(G2)***

*Trial and error is a valid method. Award **(M1)** for at least $\frac{1-3^{10}}{1-3}$ seen and then **(A1)** for = 29524, **(A1)** for $n = 10$. For only unproductive trials with $n \neq 10$, award **(M1)** and then **(A1)** if the evaluation is correct.*

[3 marks]

ii.b. Common ratio is $\frac{1}{3}$, (0.333 (3sf) or 0.3) ***(A1)***

Accept 'divide by 3'.

[1 mark]

ii.c. $\frac{1-\left(\frac{1}{3}\right)^{10}}{1-\frac{1}{3}}$ ***(M1)***

= 1.50 (3sf) ***(A1)*(ft)*(G1)***

*1.5 and $\frac{3}{2}$ receive **(A0)(AP)** if **AP** not yet used Incorrect formula seen in (a) or incorrect value in (b) can follow through to (c). Can award **(M1)** for $1 + \left(\frac{1}{3}\right) + \left(\frac{1}{9}\right) + \ldots\ldots$*

[2 marks]

ii.d. Both $\left(\frac{1}{3}\right)^{10}$ and $\left(\frac{1}{3}\right)^{1000}$ (or those numbers divided by 2/3) are 0 when corrected to 3sf, so they make no difference to the final answer. ***(R1)***

*Accept any valid explanation but please note: statements which only convey the idea of convergence are not enough for **(R1)**. The reason must show recognition that the convergence is adequately fast (though this might be expressed in a much less technical manner).*

[1 mark]

ii.e. The sequence given is $G_1 + G_2$ ***(M1)***

The sum is 29 524 + 1.50 ***(A1)*(ft)**

= 29 525.5 ***(A1)*(ft)*(G2)***

*The **(M1)** is implied if the sum of the two numbers is seen. Award **(G1)** for 29 500 with no working. **(M1)** can be awarded for* $2 + 3\frac{1}{3} + \ldots$ *Award final **(A1)** only for answer given correct to 1dp.*

[3 marks]

Examiners report

i.a. (i) Identification of u_1 and d was fine. In (b) many candidates failed to recognise the need for a general proof and simply gave an example substitution. Part (c) was well done.

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ii.a. (ii) Too many candidates here failed to swap to the GP formulae. Those who did know what was happening here often performed the calculations well and got decent marks. The explanations in (d) were often unsatisfactory but some allowance was made for the language difficulties encountered by candidates writing in a 2^{nd} or higher language. The last part (e) of the question, intended as a high-grade discriminator performed that task very well.

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Daniel wants to invest $25 000 for a total of three years. There are two investment options.

Option One pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Two pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \ldots$$

A.a Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. [8]

A.b Determine Daniel's best investment option. [1]

B.a Calculate u_1, the first term in the sequence. [2]

B.b Show that the common difference is 7. [2]

B.c S_n is the sum of the first n terms of the sequence. [3]

Find an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants.

B.d The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5. [2]

Show that the common ratio, r, of the geometric sequence is 1.5.

B.e T_n is the sum of the first n terms of the geometric sequence. [2]

Calculate T_7, the sum of the first seven terms of the geometric sequence.

B.f. T_n is the sum of the first n terms of the geometric sequence. [2]

Use your graphic display calculator to find the smallest value of n for which $T_n > S_n$.

Markscheme

A.a **Option 1:** Amount $= 25\,000\left(1 + \frac{5}{100}\right)^3$ ***(M1)(A1)***

$= 28\,940.63$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution. Give full credit for use of lists.

Option 2: Amount $= 25\,000\left(1 + \frac{4.8}{12(100)}\right)^{3\times 12}$ ***(M1)***

$= 28\,863.81$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in the compound interest formula. Give full credit for use of lists.

[8 marks]

A.b. Option 1 is the best investment option. ***(A1)*(ft)**

[1 mark]

B.a. u_1 = 135 + 7(1) ***(M1)***

= 142 ***(A1)(G2)***

[2 marks]

B.b. u_2 = 135 + 7(2) = 149 ***(M1)***

d = 149 – 142 **OR** *alternatives* ***(M1)*(ft)**

d = 7 ***(AG)***

[2 marks]

B.c. $S_n = \frac{n[2(142)+7(n-1)]}{2}$ ***(M1)*(ft)**

Note: Award ***(M1)*** for correct substitution in correct formula.

$= \frac{n[277+7n]}{2}$ **OR** *equivalent* ***(A1)***

$= \frac{7n^2}{2} + \frac{277n}{2}$ (= $3.5n^2 + 138.5n$) ***(A1)(G3)***

[3 marks]

B.d. $20r^3 = 67.5$ ***(M1)***

$r^3 = 3.375$ **OR** $r = \sqrt[3]{3.375}$ ***(A1)***

r = 1.5 ***(AG)***

[2 marks]

B.e. $T_7 = \frac{20(1.5^7-1)}{(1.5-1)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in correct formula.

= 643 *(accept 643.4375)* ***(A1)(G2)***

[2 marks]

B.f. $\frac{20(1.5^n-1)}{(1.5-1)} > \frac{7n^2}{2} + \frac{277n}{2}$ ***(M1)***

Note: Award ***(M1)*** for an attempt using lists or for relevant graph.

n = 10 ***(A1)*(ft)*(G2)***

Note: Follow through from their (c).

[2 marks]

Examiners report

A.a For many, this question came as a welcome relief following the previous two questions. For those with a sound grasp of the topic, there were many very successful attempts.

A common error was to make all the comparisons using interest alone; though much credit was given for doing this, candidates should be aware of what is being asked for in the question.

Many did not understand the notion of monthly compounding periods.

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For many, finding an expression for S_n in (c) was problematical.

The final part was challenging to the great majority, with a large number not attempting it at all; only the highly competent reached the correct answer.

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Give all answers in this question correct to two decimal places.

Part A

Estela lives in Brazil and wishes to exchange 4000 BRL (Brazil reals) for GBP (British pounds). The exchange rate is 1.00 BRL = 0.3071 GBP. The bank charges 3 % commission on the amount in BRL.

Give all answers in this question correct to two decimal places.

Part B

Daniel invests \$1000 in an account that offers a nominal annual interest rate of 3.5 % **compounded half yearly**.

A.a Find, **in BRL**, the amount of money Estela has after commission. [2]

A.b Find, in GBP, the amount of money Estela receives. [2]

A.c After her trip to the United Kingdom Estela has 400 GBP left. At the airport she changes this money back into BRL. The exchange rate is now 1.00 BRL = 0.3125 GBP. [2]

Find, in BRL, the amount of money that Estela should receive.

A.d Estela actually receives 1216.80 BRL after commission. [1]

Find, in BRL, the commission charged to Estela.

A.e The commission rate is t % . Find the value of t. [2]

B.a Show that after three years Daniel will have $1109.70 in his account, correct to two decimal places. [3]

B.b Write down the interest Daniel receives after three years. [1]

Markscheme

A.a 4000 × 0.97 = 3880.00 (3880) ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for multiplication of correct numbers.

OR

3 % of 4000 = 120 ***(A1)***

4000 – 120 = 3880.00 (3880) ***(A1)(G2)***

[2 marks]

A.b 3880 × 0.3071 = 1191.55 ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for multiplication of correct numbers. Follow through from their answer to part (a).

[2 marks]

A.c $\frac{400}{0.3125}$ ***(M1)***

= 1280.00 (1280) ***(A1)(G2)***

Note: Award ***(M1)*** for division of correct numbers.

[2 marks]

A.d 63.20 ***(A1)*****(ft)**

Note: Follow through (their (c) –1216.80).

[1 mark]

A.e $t = \frac{63.20 \times 100}{1280}$ ***(M1)***

t = 4.94 ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answers to parts (c) and (d).

[2 marks]

B.a $\mathrm{A} = 1000\left(1 + \frac{3.5}{2 \times 100}\right)^6 = 1109.7023\ldots$ ***(M1)(A1)(A1)***

= 1109.70 ***(AG)***

Notes: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for unrounded answer. If 1109.70 not seen award at most ***(M1)(A1)(A0)***.

OR

$I = 1000\left(1 + \frac{3.5}{2 \times 100}\right)^{6} - 1000 = 109.7023$ ***(M1)(A1)***

A = 1109.7023... ***(A1)***

= 1109.70 ***(AG)***

Note: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for unrounded answer.

[3 marks]

B.b 109.70 ***(A1)***

Note: No follow through here.

[1 mark]

Examiners report

A.a Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

A.b Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

A.c Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

A.d Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

A.e Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

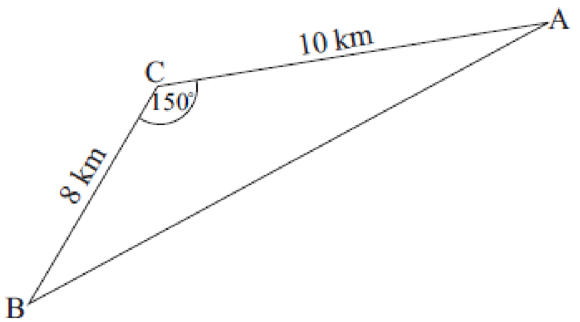
B.a Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

Most of the students used the correct formula but not all made the correct substitution. From those that made the correct substitution, very few showed the unrounded answer. Part (b) was well done. In part (c) the majority did not put the interest (only) in the formula but the total amount \$1109.70.

B.b Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

Most of the students used the correct formula but not all made the correct substitution. From those that made the correct substitution, very few showed the unrounded answer. Part (b) was well done. In part (c) the majority did not put the interest (only) in the formula but the total amount \$1109.70.

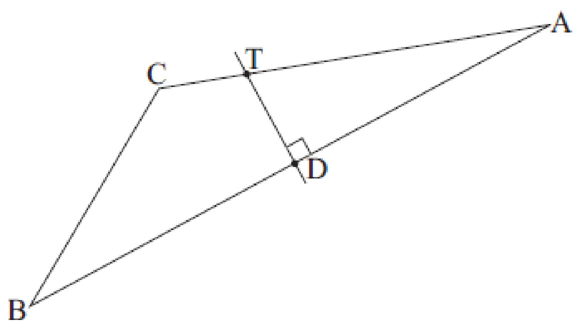
In the diagram below A, B and C represent three villages and the line segments AB, BC and CA represent the roads joining them. The lengths of AC and CB are 10 km and 8 km respectively and the size of the angle between them is 150°.



a. Find the length of the road AB. [3]

b. Find the size of the angle CAB. [3]

c. Village D is halfway between A and B. A new road perpendicular to AB and passing through D is built. Let T be the point where this road cuts AC. This information is shown in the diagram below. [1]



Write down the distance from A to D.

d. Show that the distance from D to T is 2.06 km correct to three significant figures. [2]

e. A bus starts and ends its journey at A taking the route AD to DT to TA. [3]

Find the total distance for this journey.

f. The average speed of the bus while it is moving on the road is 70 km h^{-1}. The bus stops for **5 minutes** at each of D and T . [4]

Estimate the time taken by the bus to complete its journey. Give your answer correct to the nearest minute.

Markscheme

a. $AB^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 150°$ ***(M1)(A1)***

AB = 17.4 km ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

[3 marks]

b. $\frac{8}{\sin \mathrm{C\hat{A}B}} = \frac{17.4}{\sin 150^{\circ}}$ ***(M1)(A1)***

$\mathrm{C\hat{A}B} = 13.3^{\circ}$ ***(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer. Follow through from their answer to part (a).

[3 marks]

c. AD = 8.70 km (8.7 km) ***(A1)*****(ft)**

Note: Follow through from their answer to part (a).

[1 mark]

d. DT = tan (13.29...°) × 8.697... = 2.0550... ***(M1)(A1)***

= 2.06 ***(AG)***

Notes: Award ***(M1)*** for correct substitution in the correct formula, award ***(A1)*** for the unrounded answer seen. If 2.06 not seen award at most ***(M1)(AO)***.

[2 marks]

e. $\sqrt{8.70^2 + 2.06^2} + 8.70 + 2.06$ ***(A1)(M1)***

= 19.7 km ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*** for AT, ***(M1)*** for adding the three sides of the triangle ADT, ***(A1)*****(ft)** for answer. Follow through from their answer to part (c).

[3 marks]

f. $\frac{19.7}{70} \times 60 + 10$ ***(M1)(M1)***

= 26.9 ***(A1)*****(ft)**

Note: Award ***(M1)*** for time on road in minutes, ***(M1)*** for adding 10, ***(A1)*****(ft)** for unrounded answer. Follow through from their answer to (e).

= 27 (nearest minute) ***(A1)*****(ft)*****(G3)***

Note: Award ***(A1)*****(ft)** for their unrounded answer given to the nearest minute.

[4 marks]

Examiners report

a. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

b. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

c. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

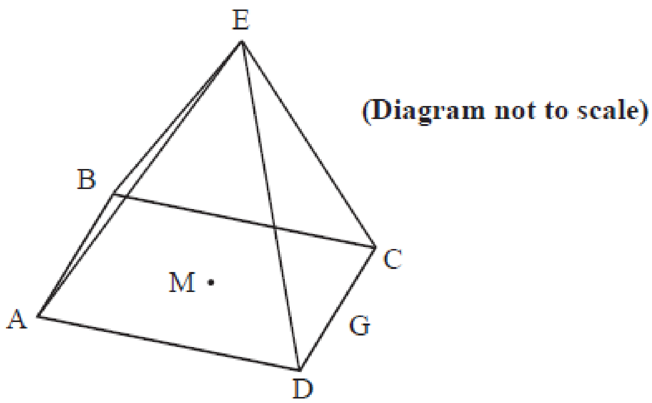
d. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

e. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

f. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

Mal is shopping for a school trip. He buys 50 tins of beans and 20 packets of cereal. The total cost is 260 Australian dollars (AUD).

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



i.a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1]

i.b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \mathrm{AUD}$. [1]

Write down another equation to represent this information.

i.c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \mathrm{AUD}$. [2]

Find the cost of one tin of beans.

i.d. (i) Sketch the graphs of the two equations from parts (a) and (b). [4]

(ii) Write down the coordinates of the point of intersection of the two graphs.

ii.a. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1]

ii.b. Show that the height of the pyramid is $13.7\ \mathrm{cm}$, to 3 significant figures. [2]

ii.c. Calculate [4]

(i) the length of EG;

(ii) the size of angle DEC.

ii.d. Find the total surface area of the pyramid. [2]

ii.e. Find the volume of the pyramid. [2]

Markscheme

i.a. $50b + 20c = 260$ ***(A1)***

[1 mark]

i.b. $12b + 6c = 66$ ***(A1)***

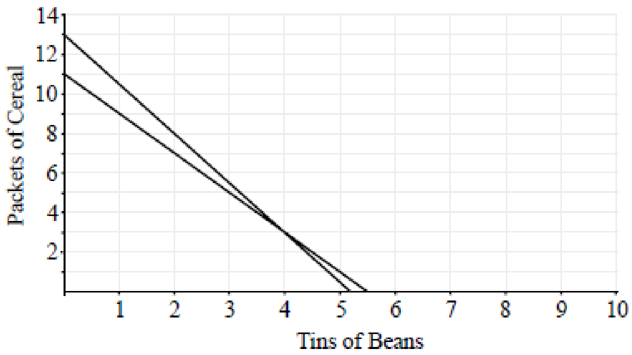
[1 mark]

i.c. Solve to get $b = 4$ ***(M1)(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for attempting to solve the equations simultaneously.

[2 marks]

i.d.(i)



(A1)(A1)(A1)

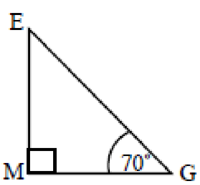
Notes: Award ***(A1)*** for labels and some idea of scale, ***(A1)*(ft)*(A1)*(ft)** for each line.
The axis can be reversed.

(ii) $(4, 3)$ or $(3, 4)$ ***(A1)*(ft)**

Note: Accept $b = 4$, $c = 3$

[4 marks]

ii.a.



(A1)

[1 mark]

ii.b. $\tan 70 = \frac{h}{5}$ ***(M1)***

$h = 5 \tan 70 = 13.74$ ***(A1)***

$h = 13.7 \text{ cm}$ ***(AG)***

[2 marks]

ii.c. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

(i) $\text{EG}^2 = 5^2 + 13.7^2$ OR $5^2 + (5 \tan 70)^2$ ***(M1)***

(UP) $\text{EG} = 14.6 \text{ cm}$ ***(A1)(G2)***

(ii) $\text{DEC} = 2 \times \tan^{-1}\left(\frac{5}{14.6}\right)$ ***(M1)***

$= 37.8^\circ$ ***(A1)*(ft)*(G2)***

[4 marks]

ii.d. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

$\text{Area} = 10 \times 10 + 4 \times 0.5 \times 10 \times 14.619$ ***(M1)***

(UP) $= 392 \text{ cm}^2$ ***(A1)*(ft)*(G2)***

[2 marks]

ii.e. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

$\text{Volume} = \frac{1}{3} \times 10 \times 10 \times 13.7$ ***(M1)***

(UP) $= 457 \text{ cm}^3$ (458 cm^3) ***(A1)(G2)***

[2 marks]

Examiners report

i.a. Most candidates managed to write down the equation.

i.b. Most candidates managed to write down the equation.

i.c. Many managed to find the correct answer and the others tried their best but made some mistake in the process.

i.d. (i) Few candidates sketched the graphs well. Few used a ruler.

(ii) Many candidates could not be awarded ft from their graph because the answer they gave was not possible.

ii.a. Very few correct drawings.

ii.b. Some managed to show this more by good fortune and ignoring their original triangle than by good reasoning.

ii.c. (i) Many found this as ft from the previous part. Some lost a UP here.

(ii) This was not well done. The most common answer was 40°.

ii.d. Many managed this or were awarded ft points.

ii.e. This was well done and most candidates also remembered their units on this part.

Rosa joins a club to prepare to run a marathon. During the first training session Rosa runs a distance of 3000 metres. Each training session she increases the distance she runs by 400 metres.

A marathon is 42.195 kilometres.

In the kth training session Rosa will run further than a marathon for the first time.

Carlos joins the club to lose weight. He runs 7500 metres during the first month. The distance he runs increases by 20% each **month**.

a.i. Write down the distance Rosa runs in the third training session; [1]

a.ii. Write down the distance Rosa runs in the nth training session. [2]

b. Find the value of k. [2]

c. Calculate the total distance, in **kilometres**, Rosa runs in the first 50 training sessions. [4]

d. Find the distance Carlos runs in the fifth month of training. [3]

e. Calculate the total distance Carlos runs in the first year. [3]

Markscheme

a.i. 3800 m ***(A1)***

[1 mark]

a.ii. $3000 + (n - 1)400\text{ m}$ **OR** $2600 + 400n\text{ m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into arithmetic sequence formula, ***(A1)*** for correct substitution.

[2 marks]

b. $3000 + (k - 1)400 > 42195$ ***(M1)***

Notes: Award ***(M1)*** for their correct inequality. Accept $3 + (k - 1)0.4 > 42.195$.

Accept $=$ **OR** $\geqslant$. Award ***(M0)*** for $3000 + (k - 1)400 > 42.195$.

$(k =)\ 99$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a)(ii), but only if k is a positive integer.

[2 marks]

c. $\frac{50}{2}(2 \times 3000 + (50 - 1)(400))$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into sum of an arithmetic series formula, ***(A1)*****(ft)** for correct substitution.

$640\,000\text{ m}$ ***(A1)***

Note: Award ***(A1)*** for their $640\,000$ seen.

$= 640\text{ km}$ ***(A1)*****(ft)*****(G3)***

Note: Award ***(A1)*****(ft)** for correctly converting their answer in metres to km; this can be awarded independently from previous marks.

OR

$\frac{50}{2}(2 \times 3 + (50 - 1)(0.4))$ ***(M1)(A1)*****(ft)*****(A1)***

Note: Award ***(M1)*** for substitution into sum of an arithmetic series formula, ***(A1)*****(ft)** for correct substitution, ***(A1)*** for correctly converting 3000 m and 400 m into km.

$= 640\text{ km}$ ***(A1)(G3)***

[4 marks]

d. $7500 \times 1.2^{5-1}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into geometric series formula, ***(A1)*** for correct substitutions.

$= 15\,600 \text{ m } (15\,552 \text{ m})$ ***(A1)(G3)***

OR

$7.5 \times 1.2^{5-1}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into geometric series formula, ***(A1)*** for correct substitutions.

$= 15.6 \text{ km}$ ***(A1)(G3)***

[3 marks]

e. $\frac{7500(1.2^{12}-1)}{1.2-1}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution into sum of a geometric series formula, ***(A1)*** for correct substitutions. Follow through from their ratio (r) in part (d). If $r < 1$ (distance does not increase) or the final answer is unrealistic (*eg* $r = 20$), do not award the final ***(A1)***.

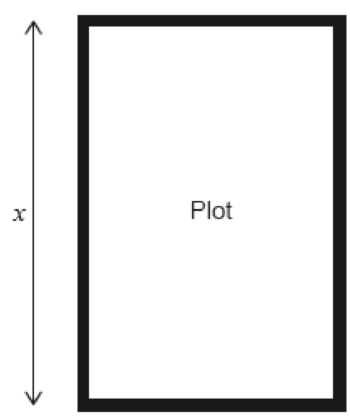
$= 297\,000 \text{ m } (296\,853\ldots \text{ m}, \ 297 \text{ km})$ ***(A1)(G2)***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Violeta plans to grow flowers in a rectangular plot. She places a fence to mark out the perimeter of the plot and uses 200 metres of fence. The length of the plot is x metres.



Violeta places the fence so that the area of the plot is maximized.

By selling her flowers, Violeta earns 2 Bulgarian Levs (BGN) per square metre of the plot.

Violeta wants to invest her 5000 BGN.

A bank offers a nominal annual interest rate of 4%, compounded **half-yearly**.

Another bank offers an interest rate of $r\%$ compounded **annually**, that would allow her to double her money in 12 years.

a. Show that the width of the plot, in metres, is given by $100 - x$. [1]

b. Write down the area of the plot in terms of x. [1]

c. Find the value of x that maximizes the area of the plot. [2]

d. Show that Violeta earns 5000 BGN from selling the flowers grown on the plot. [2]

e.i. Find the amount of money that Violeta would have after 6 years. Give your answer correct to two decimal places. [3]

e.ii. Find how long it would take for the interest earned to be 2000 BGN. [3]

f. Find the lowest possible value for r. [2]

Markscheme

a. $\frac{200-2x}{2}$ (or equivalent) ***(M1)***

OR

$2x + 2y = 200$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for a correct expression leading to $100 - x$ (the $100 - x$ does not need to be seen). The 200 must be seen for the ***(M1)*** to be awarded. Do not accept $100 - x$ substituted in the perimeter of the rectangle formula.

$100 - x$ ***(AG)***

[1 mark]

b. $(\text{area} =)\ x(100 - x)$ **OR** $-x^2 + 100x$ (or equivalent) ***(A1)***

[1 mark]

c. $x = \frac{-100}{-2}$ **OR** $-2x + 100 = 0$ **OR** graphical method ***(M1)***

Note: Award ***(M1)*** for use of axis of symmetry formula or first derivative equated to zero or a sketch graph.

$x = 50$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b), provided *x* is positive and less than 100.

[2 marks]

d. $50(100-50)\times 2$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting their x into their formula for area (accept "50×50" for the substituted formula), and ***(M1)*** for multiplying by 2. Award at most ***(M0)(M1)*** if their calculation does not lead to 5000 (BGN), although the 5000 (BGN) does not need to be seen explicitly.

Substitution of 50 into area formula may be seen in part (c).

5000 (BGN) ***(AG)***

[2 marks]

e.i. $5000\left(1+\frac{4}{2\times 100}\right)^{2\times 6}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

OR

$\mathrm{N}=6$

$\mathrm{I\%}=4$

$\mathrm{PV}=-5000$

$\mathrm{P/Y}=1$

$\mathrm{C/Y}=2$ ***(M1)(A1)***

Note: Award ***(A1)*** for $\mathrm{C/Y}=2$ seen, ***(M1)*** for other correct entries.

OR

$\mathrm{N}=12$

$\mathrm{I\%}=4$

$\mathrm{PV}=-5000$

$\mathrm{P/Y}=2$

$\mathrm{C/Y}=2$ ***(M1)(A1)***

Note: Award ***(A1)*** for $\mathrm{C/Y}=2$ seen, ***(M1)*** for other correct entries.

6341.21 (BGN) ***(A1)(G3)***

Note: Award ***(A1)*** for correct answer, to two decimal places only.

Award ***(G2)*** for 6341.20 or a correct, unrounded final answer if no working is seen (6341.2089…).

[3 marks]

e.ii. $5000\left(1+\frac{4}{2\times 100}\right)^{2\times t}=7000$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for using the compound interest formula with a variable for time, ***(A1)*(ft)** for substituting the correct values and equating to 7000. Follow through for their "2" from part (e)(i).

OR

$\mathrm{I\%} = 4$

$\mathrm{PV} = (\pm)5000$

$\mathrm{FV} = \mp 7000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.

Award ***(M1)*** for their C/Y from part (e)(i).

OR

$\mathrm{I\%} = 4$

$\mathrm{PV} = (\pm)5000$

$\mathrm{FV} = \mp 7000$

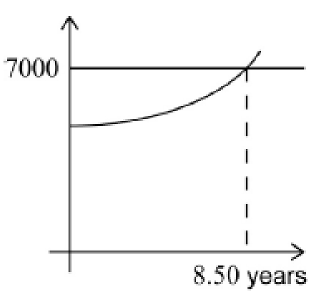
$\mathrm{P/Y} = 2$

$\mathrm{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.

Award ***(M1)*** for their C/Y from part (e)(i).

OR



***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for a sketch with a straight line intercepted by appropriate curve, ***(A1)*(ft)** for numerical answer in the range of 8.4 and 8.5.

Follow through from their part (e)(i).

$t = 8.50\ (\text{years})\ (8.49564\ldots)$ ***(A1)*(ft)*(G3)***

Note: Award only ***(A1)*** if 16.9912… is seen without working. If working is seen, award at most ***(M1)(A1)(A0)***.

[3 marks]

f. $5000\left(1+\frac{r}{100}\right)^{12} = 10000$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into compound interest formula with 10 000 seen.

OR

$2 = \left(1 + \frac{r}{100}\right)^{12}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution and simplification of compound interest formula, equating to 2.

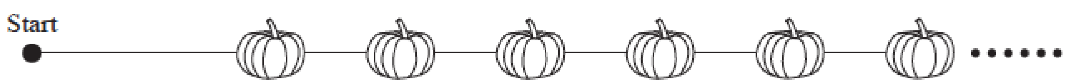
$r = 5.95\ (\%\ )\ (5.94630\ldots)$ ***(A1)(G2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e.i. [N/A]
e.ii. [N/A]
f. [N/A]

In a game, *n* small pumpkins are placed 1 metre apart in a straight line. Players start 3 metres before the first pumpkin.



Each player **collects** a single pumpkin by picking it up and bringing it back to the start. The nearest pumpkin is collected first. The player then collects the next nearest pumpkin and the game continues in this way until the signal is given for the end.

Sirma runs to get each pumpkin and brings it back to the start.

a. Write down the distance, a_1, in metres that she has to run in order to **collect** the first pumpkin. [1]

b. The distances she runs to **collect** each pumpkin form a sequence $a_1,\ a_2,\ a_3, \ldots$. [2]

(i) Find a_2.

(ii) Find a_3.

c. Write down the common difference, d, of the sequence. [1]

d. The final pumpkin Sirma **collected** was 24 metres from the start. [5]

(i) Find the total number of pumpkins that Sirma **collected**.

(ii) Find the total distance that Sirma ran to **collect** these pumpkins.

e. Peter also plays the game. When the signal is given for the end of the game he has run 940 metres. [3]

Calculate the total number of pumpkins that Peter **collected**.

f. Peter also plays the game. When the signal is given for the end of the game he has run 940 metres. [2]

Calculate Peter's distance from the start when the signal is given.

Markscheme

a. $6\ (\mathrm{m})$ ***(A1)(G1)***

b. (i) 8 ***(A1)*(ft)**

(ii) 10 ***(A1)*(ft)*(G2)***

Note: Follow through from part (a).

c. $2\ (\mathrm{m})$ ***(A1)*(ft)**

Note: Follow through from parts (a) and (b).

d. (i) $2 \times 24 = 6 + 2(n-1)$ **OR** $24 = 3 + (n-1)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in arithmetic sequence formula.

$n = 22$ ***(A1)*(ft)*(G1)***

Note: Follow through from parts (a) and (c).

(ii) $\frac{(6+48)}{2} \times 22$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution in arithmetic series formula, ***(A1)*(ft)** for correct substitution.

$= 594$ ***(A1)*(ft)*(G2)***

Note: Follow through from parts (a) and (d)(i).

e. $\frac{[2\times6+2(n-1)]\times n}{2} = 940$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substitution in arithmetic series formula, ***(A1)*** for their correct substituted formula equated to 940. Follow through from parts (a) and (c).

$n^2 + 5n - 940 = 0$

$n = 28.2611\ldots$

$n = 28$ ***(A1)*(ft)*(G2)***

f. $\frac{[2\times6+2(28-1)]\times28}{2}$ ***(M1)***

Notes: Award ***(M1)*** for substituting their 28 into the arithmetic series formula.

$= 16\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
[N/A]

e. [N/A]

Part A

The Green Park Amphitheatre was built in the form of a horseshoe and has 20 rows. The number of seats in each row increase by a fixed amount, d, compared to the number of seats in the previous row. The number of seats in the sixth row, u_6, is 100, and the number of seats in the tenth row, u_{10}, is 124. u_1 represents the number of seats in the first row.

Part B

Frank is at the amphitheatre and receives a text message at 12:00. Five minutes later he forwards the text message to three people. Five minutes later, those three people forward the text message to three new people. **Assume this pattern continues and each time the text message is sent to people who have not received it before.**

The number of new people who receive the text message forms a geometric sequence

$$1, 3, \ldots$$

A.a (i) Write an equation for u_6 in terms of d and u_1. [2]

(ii) Write an equation for u_{10} in terms of d and u_1.

A.b Write down the value of [2]

(i) d ;

(ii) u_1 .

A.c Find the **total** number of seats in the amphitheatre. [3]

A.d A few years later, a **second** level was added to increase the amphitheatre's capacity by another 1600 seats. Each row has four more seats than the previous row. The first row on this level has 70 seats. [4]

Find the number of rows on the second level of the amphitheatre.

B.a Write down the next two terms of this geometric sequence. [1]

B.b Write down the common ratio of this geometric sequence. [1]

B.c Calculate the number of people who will receive the text message at 12:30. [2]

B.d Calculate the **total** number of people who will have received the text message by 12:30. [2]

B.e Calculate the exact time at which a total of 29 524 people will have received the text message. [3]

Markscheme

A.a (i) $u_1 + 5d = 100$ ***(A1)***

(ii) $u_1 + 9d = 124$ ***(A1)***

[2 marks]

A.b(i) 6 ***(G1)*(ft)**

(ii) 70 ***(G1)*(ft)**

Notes: Follow through from their equations in parts (a) and (b) even if working not seen. Their answers must be integers. Award ***(M1)(A0)*** for an attempt to solve two equations analytically.

[2 marks]

A.c $S_{20} = \frac{20}{2}(2 \times 70 + (20 - 1) \times 6)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted sum of AP formula, ***(A1)*(ft)** for their correct substituted values.

= 2540 ***(A1)*(ft)*(G2)***

Note: Follow through from their part (b).

[3 marks]

A.d $\frac{n}{2}(2 \times 70 + (n - 1) \times 4) = 1600$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sum of AP formula, ***(A1)*** for their correct substituted values.

$4n^2 + 136n - 3200 = 0$ ***(M1)***

Note: Award ***(M1)*** for this equation (or other equivalent expanded quadratic) seen, may be implied if correct final answer seen.

$n = 16$ ***(A1)(G3)***

Note: Do not award the final ***(A1)*** for $n = 16, -50$ given as final answer, award ***(G2)*** if $n = 16, -50$ given as final answer without working.

[4 marks]

B.a 9, 27 ***(A1)***

[1 mark]

B.b 3 ***(A1)***

[1 mark]

B.c 1×3^6 ***(M1)***

= 729 ***(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for correctly substituted GP formula. Follow through from their answer to part (b).

[2 marks]

B.d $\frac{1(3^7-1)}{(3-1)}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted GP formula. Accept sum 1+ 3 + 9 + 27 + ... + 729. If lists are used, award ***(M1)*** for correct list that includes 1093. (1, 4, 13, 40, 121, 364, 1093, 3280…)

= 1093 ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (b). For consistent use of $n = 6$ from part (c) (243) to part (d) leading to an answer of 364, treat as double penalty and award ***(M1)(A1)*(ft)** if working is shown.

[2 marks]

B.e $\frac{1(3^n-1)}{(3-1)} = 29524$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted GP formula. If lists are used, award ***(M1)*** for correct list that includes 29524. (1, 4, 13, 40, 121, 364, 1093, 3280, 9841, 29524, 88573...). Accept alternative methods, for example continuation of sum in part (d).

$n = 10$ ***(A1)*(ft)**

Note: Follow through from their answer to part (b).

Exact time = 12:45 ***(A1)*(ft)*(G2)***

[3 marks]

Examiners report

A.a **Part A: Arithmetic**

The contextual nature of this question posed problems for many, though there were many fine attempts. Failure to discriminate between the sequence and series formulas was the cause of the most errors. The final part saw many able to substitute into the formula for the series, but then unable to continue. The use of the GDC in such situations is encouraged; either by graphing each side of the equation and drawing the resultant sketch or by the solver function.

A.b **Part A: Arithmetic**

The contextual nature of this question posed problems for many, though there were many fine attempts. Failure to discriminate between the sequence and series formulas was the cause of the most errors. The final part saw many able to substitute into the formula for the series, but then unable to continue. The use of the GDC in such situations is encouraged; either by graphing each side of the equation and drawing the resultant sketch or by the solver function.

A.c **Part A: Arithmetic**

The contextual nature of this question posed problems for many, though there were many fine attempts. Failure to discriminate between the sequence and series formulas was the cause of the most errors. The final part saw many able to substitute into the formula for the series, but then unable to continue. The use of the GDC in such situations is encouraged; either by graphing each side of the equation and drawing the resultant sketch or by the solver function.

A.d **Part A: Arithmetic**

The contextual nature of this question posed problems for many, though there were many fine attempts. Failure to discriminate between the sequence and series formulas was the cause of the most errors. The final part saw many able to substitute into the formula for the series, but then unable to continue. The use of the GDC in such situations is encouraged; either by graphing each side of the equation and drawing the resultant sketch or by the solver function.

B.a **Part B: Geometric**

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

B.b **Part B: Geometric**

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

B.c **Part B: Geometric**

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

B.d **Part B: Geometric**

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

B.e **Part B: Geometric**

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

The following diagram shows a perfume bottle made up of a cylinder and a cone.

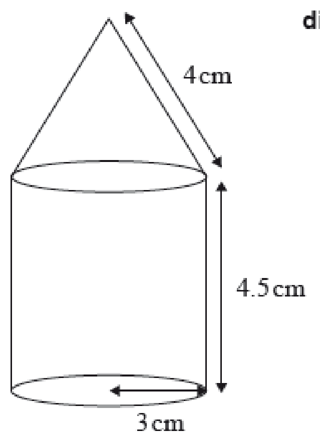


diagram not to scale

The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. [6]

(ii) Calculate the volume of the perfume bottle.

b. The bottle contains $125\ \mathrm{cm}^3$ of perfume. The bottle is **not** full and all of the perfume is in the cylinder part. [2]

Find the height of the perfume in the bottle.

c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. [4]

Find the **total** surface area of the perfume bottle.

d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of $7\ \text{cm}^2$. [4]

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. [2]

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

Markscheme

a. (i) $x^2 + 3^2 = 4^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' formula.

Accept correct alternative method using trigonometric ratios.

$x = 2.64575\ldots$ ***(A1)***

$x = 2.65\ (\text{cm})$ ***(AG)***

Note: The unrounded and rounded answer must be seen for the ***(A1)*** to be awarded.

OR

$\sqrt{4^2 - 3^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' formula.

$= \sqrt{7}$ ***(A1)***

$= 2.65\ (\text{cm})$ ***(AG)***

Note: The exact answer must be seen for the final ***(A1)*** to be awarded.

(ii) $\pi \times 3^2 \times 4.5 + \frac{1}{3}\pi \times 3^2 \times 2.65$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a cylinder formula, ***(M1)*** for correct substitution into the volume of a cone formula, ***(M1)*** for adding both of their volumes.

$= 152\ \text{cm}^3 \quad (152.210\ldots\ \text{cm}^3,\ 48.45\pi\ \text{cm}^3)$ ***(A1)(G3)***

b. $\pi 3^2 h = 125$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a cylinder formula.

Accept alternative methods. Accept 4.43 $(4.42913\ldots)$ from using rounded answers in $h = \frac{125 \times 4.5}{127}$.

$h = 4.42\ (\text{cm}) \quad (4.42097\ldots\ (\text{cm}))$ ***(A1)(G2)***

c. $2\pi \times 3 \times 4.5 + \pi \times 3 \times 4 + \pi \times 3^2$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into curved surface area of a cylinder formula, ***(M1)*** for correct substitution into the curved surface area of a cone formula, ***(M1)*** for adding the area of the base of the cylinder to the other two areas.

$= 151 \text{ cm}^2 \quad (150.796\ldots \text{ cm}^2,\ 48\pi \text{ cm}^2)$ ***(A1)(G3)***

d. $\frac{150.796\ldots}{7} \times 3$ ***(M1)(M1)***

Notes: Award ***(M1)*** for dividing their answer to (c) by 7, ***(M1)*** for multiplying by 3. Accept equivalent methods.

$= 64.63 \text{ (ZAR)}$ ***(A1)*(ft)*(G2)***

Notes: The ***(A1)*** is awarded for their correct answer, correctly rounded to 2 decimal places. Follow through from their answer to part (c). If rounded answer to part (c) is used the answer is 64.71 (ZAR).

e. $\frac{325}{13.03}$ ***(M1)***

Note: Award ***(M1)*** for dividing 325 by 13.03.

$= 24.94 \text{ (EUR)}$ ***(A1)(G2)***

Note: The ***(A1)*** is awarded for the correct answer rounded to 2 decimal places, unless already penalized in part (d).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A geometric sequence has second term 12 and fifth term 324.

Consider the following propositions

p: The number is a multiple of five.

q: The number is even.

r: The number ends in zero.

i, a. Calculate the value of the common ratio. [4]

i, b. Calculate the 10$^{\text{th}}$ term of this sequence. [3]

i, c. The k^{th} term is the first term which is greater than 2000. Find the value of k. [3]

ii, a. Write in words $(q \wedge \neg r) \Rightarrow \neg p$. [3]

ii, b, i. Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [4]

Write this statement in symbolic form.

ii, b, ii. Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [2]

Write the contrapositive of this statement in symbolic form.

Markscheme

i, a$u_1r^4 = 324$ ***(A1)***

$u_1r = 12$ ***(A1)***

$r^3 = 27$ ***(M1)***

$r = 3$ ***(A1)(G3)***

Note: Award at most ***(G3)*** for trial and error.

[4 marks]

i, b$4 \times 3^9 = 78732$ **or** $12 \times 3^8 = 78732$ ***(A1)(M1)(A1)*(ft)*(G3)***

Note: Award ***(A1)*** for $u_1 = 4$ if $n = 9$, **or** $u_1 = 12$ if $n = 8$, ***(M1)*** for correctly substituted formula.

(ft) from their (a).

[3 marks]

i, c$4 \times 3^{k-1} > 2000$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in correct formula. Accept an equation.

$k > 6$ ***(A1)***

$k = 7$ ***(A1)*(ft)*(G2)***

Notes: If second line not seen award ***(A2)*** for correct answer. **(ft)** from their (a).

Accept a list, must see at least **3 terms** including the 6th and 7th.

Note: If arithmetic sequence formula is used consistently in parts (a), (b) and (c), award ***(A0)(A0)(M0)(A0)*** for (a) and **(ft)** for parts (b) and (c).

[3 marks]

ii, aIf the number is even and the number does not end in zero, (then) the number is not a multiple of five. ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for "if…(then)", ***(A1)*** for "the number is even and the number does not end in zero", ***(A1)*** for the number is not a multiple of 5.

[3 marks]

ii, b$(p \wedge \neg q) \Rightarrow \neg r$ ***(A1)(A1)(A1)(A1)***

(A1) *for* $\Rightarrow$, ***(A1)*** *for* $\wedge$, ***(A1)*** *for* p and $\neg q$, ***(A1)*** *for* $\neg r$

Note: If parentheses not present award at most ***(A1)(A1)(A1)(A0)***.

[4 marks]

ii, b, ii. $r \Rightarrow (\neg p \vee q)$ **OR** $r \Rightarrow \neg(p \wedge \neg q)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for reversing the order, ***(A1)*** for negating the statements on both sides.

If parentheses not present award at most ***(A1)*(ft)*(A0)***.

Do not penalise twice for missing parentheses in (i) and (ii).

[2 marks]

Examiners report

i, a. An easy ratio to find and the majority of candidates found $r = 3$, though many had trouble showing the appropriate method, thus losing marks.

i, b. A fairly straightforward part for most candidates.

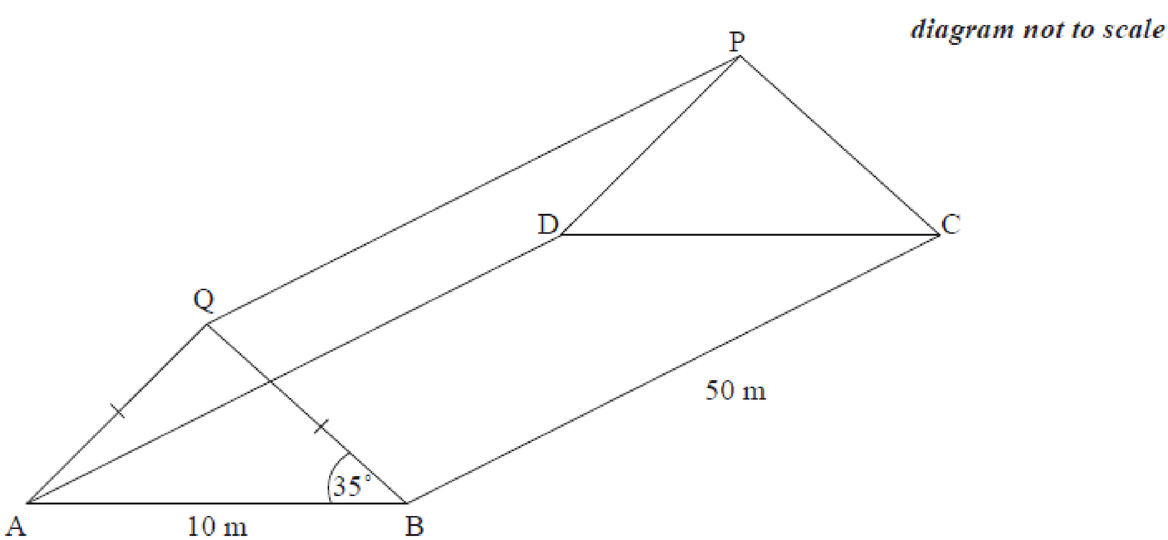
i, c. The majority found $k - 7$; many without supporting work which lost them a mark. Where candidates had difficulty in this part, it was generally a case of poor algebraic skills.

ii, a. This question on logic was straightforward for most candidates who scored full marks for parts (a) and (b) (i). A few omitted the brackets in part (b).

ii, b. This question on logic was straightforward for most candidates who scored full marks for parts (a) and (b) (i). A few omitted the brackets in part (b).

ii, b, ii. Very poorly answered with many candidates scoring just one mark. The main error was to open the bracket and not use the "or".

A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is 35°.



a. Write down the size of angle AQB. [1]

b. Calculate the length of AQ. [3]

c. Calculate the length of AC. [2]

d. Show that the length of CQ is 50.37 m, correct to 4 significant figures. [2]

e. Find the size of the angle AQC. [3]

f. Calculate the total area of the glass needed to construct [5]

(i) the two rectangular faces of the greenhouse;

(ii) the two triangular faces of the greenhouse.

g. The cost of one square metre of glass used to construct the greenhouse is 4.80 USD. [3]

Calculate the cost of glass to make the greenhouse. Give your answer correct to the nearest 100 USD.

Markscheme

a. 110° ***(A1)***

b. $\frac{AQ}{\sin 35^\circ} = \frac{10}{\sin 110^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule formula, ***(A1)*** for their correct substitutions.

OR

$AQ = \frac{5}{\cos 35^\circ}$ ***(A1)(M1)***

Note: Award ***(A1)*** for 5 seen, ***(M1)*** for correctly substituted trigonometric ratio.

$AQ = 6.10$ (6.10387...) ***(A1)*****(ft)*****(G2)***

Notes: Follow through from their answer to part (a).

c. $AC^2 = 10^2 + 50^2$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted Pythagoras formula.

$AC = 51.0(\sqrt{2600}, 50.9901...)$ ***(A1)(G2)***

d. $QC^2 = (6.10387...)^2 + (50)^2$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted Pythagoras formula.

$QC = 50.3711...$ ***(A1)***

$= 50.37$ ***(AG)***

Note: Both the unrounded and rounded answers must be seen to award ***(A1)***.

If 6.10 is used then 50.3707... is the unrounded answer.

For an incorrect follow through from part (b) award a maximum of ***(M1)(A0)*** – the given answer must be reached to award the final ***(A1)(AG)***.

e. $\cos AQC = \frac{(6.10387...)^2+(50.3711...)^2-(50.9901...)^2}{2(6.10387...)(50.3711...)}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*****(ft)** for their correct substitutions.

= 92.4° $(92.3753...^\circ)$ ***(A1)*****(ft)*****(G2)***

Notes: Follow through from their answers to parts (b), (c) and (d). Accept 92.2 if the 3 sf answers to parts (b), (c) and (d) are used.

Accept 92.5° $(92.4858...^\circ)$ if the 3 sf answers to parts (b), (c) and 4 sf answers to part (d) used.

f. (i) $2(50 \times 6.10387...)$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted rectangular area formula, the area of one rectangle is not sufficient.

= 610 m^2 (610.387...) ***(A1)*****(ft)*****(G2)***

Notes: Follow through from their answer to part (b).

The answer is 610 m^2. The units are required.

(ii) Area of triangular face $= \frac{1}{2} \times 10 \times 6.10387... \times \sin 35^\circ$ ***(M1)(A1)*****(ft)**

OR

Area of triangular face $= \frac{1}{2} \times 6.10387... \times 6.10387... \times \sin 110^\circ$ ***(M1)(A1)*****(ft)**

$= 17.5051...$

Note: Award ***(M1)*** for substituted triangle area formula, ***(A1)*****(ft)** for correct substitutions.

OR

(Height of triangle) $= (6.10387...)^2 - 5^2$

$= 3.50103...$

Area of triangular face $= \frac{1}{2} \times 10 \times their\ height$

$= 17.5051...$

Note: Award ***(M1)*** for substituted triangle area formula, ***(A1)*****(ft)** for correctly substituted area formula. If 6.1 is used, the height is 3.49428... and the area of both triangular faces 34.9 m^2

Area of both triangular faces = 35.0 m^2 (35.0103...) ***(A1)(ft)(G2)***

Notes: The answer is 35.0 m^2. The units are required. Do not penalize if already penalized in part (f)(i). Follow through from their part (b).

g. (610.387... + 35.0103...) × 4.80 ***(M1)***

= 3097.90... ***(A1)*****(ft)**

Notes: Follow through from their answers to parts (f)(i) and (f)(ii).

Accept 3096 if the 3 sf answers to part (f) are used.

= 3100 ***(A1)*****(ft)*****(G2)***

Notes: Follow through from their unrounded answer, irrespective of whether it is correct. Award ***(M1)(A2)*** if working is shown and 3100 seen without the unrounded answer being given.

Examiners report

a. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

b. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

c. Most candidates were able to recognize sine rule, substitute correctly and reach the required result.

d. Most candidates were able to recognize sine rule, substitute correctly and reach the required result. The use of Pythagoras’ theorem was also successful, the major source of error being the lack of unrounded and rounded answers in part (d).

Again, most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

e. Most candidates were able to recognize sine rule, substitute correctly and reach the required result. Part (e) was less well answered, due in part to the triangle being in three dimensions. However, all three sides had either been asked for in previous parts or given and all that was required was a sketch of a triangle with the vertices labelled; such a diagram was never on any script and this technique should be encouraged.

Again, most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

f. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

g. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated “concrete base”.

Leanne goes fishing at her favourite pond. The pond contains four different types of fish: bream, flathead, whiting and salmon. The fish are either undersized or normal. This information is shown in the table below.

Size / Type of fish	Bream	Flathead	Whiting	Salmon	Total
Undersized	3	12	18	9	42
Normal	0	11	24	13	48
Total	3	23	42	22	

a. Write down the total number of fish in the pond. [1]

b. Leanne catches a fish. [7]

Find the probability that she

(i) catches an undersized bream;

(ii) catches either a flathead or an undersized fish or both;

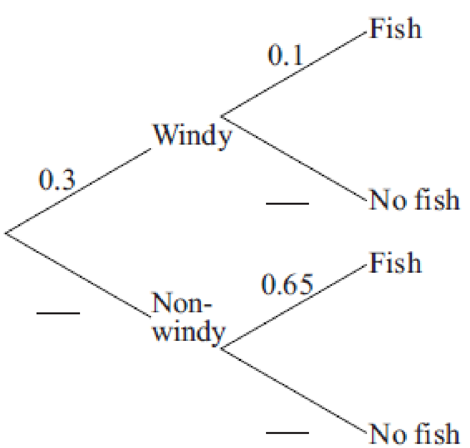
(iii) does not catch an undersized whiting;

(iv) catches a whiting given that the fish was normal.

c. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Copy and complete the probability tree diagram below.



d. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [2]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that it is windy and Leanne catches a fish on a particular day.

e. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that Leanne catches a fish on a particular day.

f. Use your answer to part (e) to calculate the probability that Leanne catches a fish on two consecutive days. [2]

g. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Given that Leanne catches a fish on a particular day, calculate the probability that the day was windy.

Markscheme

a. 90 ***(A1)***

[1 mark]

b. (i) $\frac{3}{90}(0.0\bar{3},\ 0.0333,\ 0.0333...,\ 3.\bar{3}\%,\ 3.33\%)$ ***(A1)*(ft)**

Note: For the denominator follow through from their answer in part (a).

(ii) $\frac{53}{90}(0.5\bar{8},\ 0.588...,\ 0.589,\ 58.\bar{8}\%,\ 58.9\%)$ ***(A1)(A1)*(ft)*(G2)***

Notes: Award ***(A1)*** for the numerator. ***(A1)*(ft)** for denominator. For the denominator follow through from their answer in part (a).

(iii) $\frac{72}{90}(0.8,\ 80\%)$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

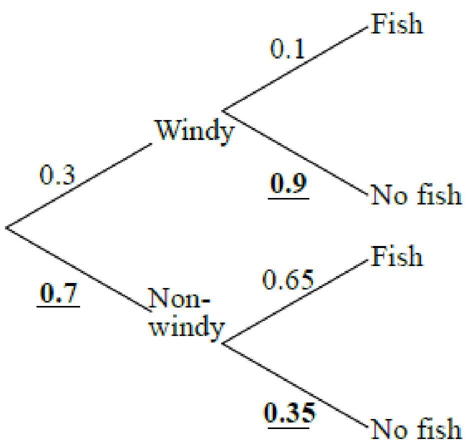
Notes: Award ***(A1)*(ft)** for the numerator, (their part (a) –18) ***(A1)*(ft)** for denominator. For the denominator follow through from their answer in part (a).

(iv) $\frac{24}{48}(0.5,\ 50\%)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for numerator, ***(A1)*** for denominator.

[7 marks]

c.



(A1)(A1)(A1)

Notes: Award ***(A1)*** for each correct entry. Tree diagram must be seen for marks to be awarded.

[3 marks]

d. $0.3 \times 0.1 = 0.03\left(\frac{3}{100}\right)$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for correct product seen.

[2 marks]

e. $0.3 \times 0.1 + 0.7 \times 0.65$ ***(M1)(M1)***

Notes: Award ***(M1)*** for 0.7×0.65 (or 0.455) seen, ***(M1)*** for adding their 0.03. Follow through from their answers to parts (c) and (d).

$= 0.485\left(\frac{485}{1000}, \frac{97}{200}\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their tree diagram and their answer to part (d).

[3 marks]

f. 0.485×0.485 ***(M1)***

$0.235 \left(\frac{9409}{40000}, 0.235225\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (e).

[2 marks]

g. $\frac{0.03}{0.485}$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substituted conditional probability formula, ***(A1)*(ft)** for their (d) as numerator and their (e) as denominator.

$0.0619 \left(\frac{6}{97}, 0.0618556...\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their parts (d) and (e).

[3 marks]

Examiners report

a. (a) Most candidates found this correctly although a few wrote 180 instead of 90.

b. (b) This was also answered well. The main errors were putting 65/90 in part (ii) and 24/90 in part (iv).

c. (c) The tree diagram was completed correctly in most scripts. It appears that some candidates may have answered this on their question paper and this was not sent to the scanning centre with the answer papers.

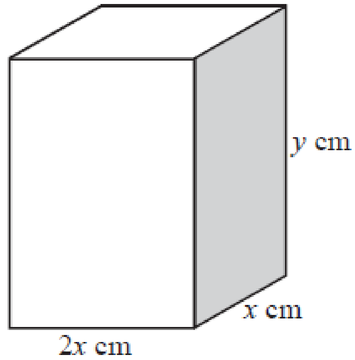
d. (d) Many answered this correctly. Some added instead of multiplying.

e. (e) Surprisingly well answered. Again some added and multiplied in the wrong place.

f. (f) Most candidates added here and then divided by 2 rather than multiplying.

g. (g) This was badly done with very few correct answers seen.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

Markscheme

i.a. $(3x - 2)(x + 5)$ ***(A1)(A1)***

[2 marks]

i.b. $(3x - 2)(x + 5) = 0$

$x = \frac{2}{3}$ or $x = -5$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

[2 marks]

i.c. $x = \frac{-13}{6}$ (-2.17) ***(A1)(A1)*(ft)*(G2)***

Note: ***(A1)*** is for $x =$, ***(A1)*** for value. **(ft)** if value is half way between roots in (b).

[2 marks]

i.d. Minimum $y = 3\left(\frac{-13}{6}\right)^2 + 13\left(\frac{-13}{6}\right) - 10$ ***(M1)***

Note: ***(M1)*** for substituting their value of x from (c) into $f(x)$.

$= -24.1$ ***(A1)*(ft)*(G2)***

[2 marks]

ii.a. $\text{Area} = 2(2x)x + 2xy + 2(2x)y$ ***(M1)(A1)***

Note: ***(M1)*** for using the correct surface area formula (which can be implied if numbers in the correct place). ***(A1)*** for using correct numbers.

$300 = 4x^2 + 6xy$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.b. $6xy = 300 - 4x^2$ ***(M1)***

$y = \frac{300-4x^2}{6x}$ *or* $\frac{150-2x^2}{3x}$ ***(A1)***

[2 marks]

ii.c. $\text{Volume} = x(2x)y$ ***(M1)***

$V = 2x^2\left(\frac{300-4x^2}{6x}\right)$ ***(A1)*(ft)**

$= 100x - \frac{4}{3}x^3$ **(AG)**

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.d. $\frac{\mathrm{d}V}{\mathrm{d}x} = 100 - \frac{12x^2}{3}$ or $100 - 4x^2$ ***(A1)(A1)***

Note: ***(A1)*** for each term.

[2 marks]

ii.e. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column*

(i) For maximum $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ or $100 - 4x^2 = 0$ ***(M1)***

$x = 5$ ***(A1)*(ft)**

$y = \frac{300-4(5)^2}{6(5)}$ or $\left(\frac{150-2(5)^2}{3(5)}\right)$ ***(M1)***

$= \frac{20}{3}$ ***(A1)*(ft)**

(UP) (ii) $333\frac{1}{3}\ \mathrm{cm}^3\ (333\ \mathrm{cm}^3)$

Note: **(ft)** from their (e)(i) if working for volume is seen.

[5 marks]

Examiners report

i.a. Most candidates made a good attempt to factorise the expression.

i.b. Many gained both marks here from a correct answer or ft from the previous part.

i.c. Many used the formula correctly. Some forgot to put $x =$.

i.d. Most candidates found this value from their GDC.

ii.a. A good attempt was made to show the correct surface area.

ii.b. Many could rearrange the equation correctly.

ii.c. Although this was not a difficult question it probably looked complicated for the candidates and it was often left out.

ii.d. Those who reached this length could usually manage the differentiation.

ii.e. (i) Many found the correct value of x but not of y.

(ii) This was well done and again the units were included in most scripts.

Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

a. Find the amount of money that Ruby will have in the bank after 3 years. [2]

b. Show that Ying will have 7545 USD in the bank at the end of 10 years. [3]

c. Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. [3]

d. Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. [3]

e. Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 Euros. The bank charges 1.8 % commission. [4]

Calculate the amount of money Ruby will invest in the Italian bank after commission.

f. Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro = 1.29 USD. [5]

Calculate the percentage commission Ruby is charged by the bank.

Markscheme

a. 5000 + 3 × 230 = 5690 ***(M1)(A1)(G2)***

Note: Accept alternative method.

[2 marks]

b. $\mathrm{A} = 5000\left(1 + \frac{4.2}{100}\right)^{10}$ *or equivalent* ***(M1)(A1)***

$= 7544.79\ldots$ ***(A1)***

$= 7545\text{ USD}$ ***(AG)***

Note: Award ***(M1)*** for correct substituted compound interest formula, ***(A1)*** for correct substitutions, ***(A1)*** for unrounded answer seen.

If final line not seen award at most ***(M1)(A1)(A0)***.

[3 marks]

c. $5000(1.042)^n > 6500$ ***(M1)(A1)***

Notes: Award ***(M1)*** for setting up correct equation/inequality, ***(A1)*** for correct values.

Follow through from their formula in part (b).

OR

List of values seen with at least 2 terms ***(M1)***

Lists of values including at least the terms with $n = 6$ and $n = 7$ ***(A1)***

Note: Follow through from their formula in part (b).

OR

Sketch showing 2 graphs, one exponential, the other a horizontal line ***(M1)***

Point of intersection identified or vertical line ***(M1)***

Note: Follow through from their formula in part (b).

$n = 7$ ***(A1)*****(ft)*****(G2)***

[3 marks]

d. $5000(1.042)^n > 5000 + 230n$ ***(M1)(A1)***

Note: Award ***(M1)*** for setting up correct equation/inequality, ***(A1)*** for correct values.

OR

2 lists of values seen (at least 2 terms per list) ***(M1)***

Lists of values including at least the terms with $n = 5$ and $n = 6$ ***(A1)***

Note: One of the lists may be written under (c).

OR

Sketch showing 2 graphs of correct shape ***(M1)***

Point of intersection identified or vertical line ***(M1)***

$n = 6$ ***(A1)*(ft)*(G2)***

Note: Follow through from their formulae used in parts (a) and (b).

[3 marks]

e. 6610×0.735 ***(M1)***

$= 4858.35$ ***(A1)***

$4858.35 \times 0.982 (= 4770.8997...)$ ***(M1)***

$= 4771$ Euros ***(A1)*(ft)*(G3)***

Note: Accept alternative method.

[4 marks]

f. 800×1.29 (= 1032 USD) ***(M1)(A1)***

Note: Award ***(M1)*** for multiplying by 1.29, ***(A1)*** for 1032. Award ***(G2)*** for 1032 if product not seen.

$(1032 - 1006.20 = 25.8)$

$25.8 \times \frac{100}{1032}\%$ ***(A1)(M1)***

Note: Award ***(A1)*** for 25.8 seen, ***(M1)*** for multiplying by $\frac{100}{1032}$.

OR

$\frac{1006.20}{1032} = 0.975$ ***(M1)(A1)***

OR

$\frac{1006.20}{1032} \times 100 = 97.5$ ***(M1)(A1)***

$= 2.5\ \%$ ***(A1)(G3)***

Notes: If working not shown award ***(G3)*** for 2.5.

Accept alternative method.

[5 marks]

Examiners report

a. Most of the students read carefully the instruction written in the heading of the question and therefore gave their answers with the accuracy

stated but some did not.

Simple interest was well done as well as compound interest with only a small minority of candidates making no progress. A number of students lost the answer mark in (b) for not showing the unrounded answer before writing the answer given. It is also important to mention that calculator commands are not accepted as correct working and therefore full marks are not awarded. Also, some candidates wrote their answers without showing any working leading to a number of marks being lost.

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It was nice to see many students recovering after part (d) and to gain full marks in the last two parts of the question.

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It was nice to see many students recovering after part (d) and to gain full marks in the last two parts of the question.

Give all answers in this question correct to the ***nearest*** *dollar.*

Clara wants to buy some land. She can choose between two different payment options. Both options require her to pay for the land in **20** monthly installments.

Option 1: The first installment is $\$2500$. Each installment is $\$200$ more than the one before.

Option 2: The first installment is $\$2000$. Each installment is 8% more than the one before.

a. If Clara chooses option 1, [7]

(i) write down the values of the second and third installments;

(ii) calculate the value of the final installment;

(iii) show that the **total amount** that Clara would pay for the land is $\$88000$.

b. If Clara chooses option 2, [4]

(i) find the value of the second installment;

(ii) show that the value of the fifth installment is $\$2721$.

c. The price of the land is $\$80000$. In option 1 her total repayments are $\$88000$ over the **20** months. Find the annual rate of simple interest which gives this total. [4]

d. Clara knows that the **total amount** she would pay for the land is not the same for both options. She wants to spend the least amount of money. Find how much she will save by choosing the cheaper option. [4]

Markscheme

a. (i) Second installment $= \$2700$ ***(A1)***

Third installment $= \$2900$ ***(A1)***

(ii) Final installment $= 2500 + 200 \times 19$ ***(M1)(A1)***

Note: ***(M1)*** for substituting in correct formula or listing, ***(A1)*** for correct substitutions.

$= \$6300$ ***(A1)(G2)***

(iii) Total amount $= \frac{20}{2}(2500 + 6300)$

OR

$\frac{20}{2}(5000 + 19 \times 200)$ ***(M1)(A1)***

Note: ***(M1)*** for substituting in correct formula or listing, ***(A1)*** for correct substitution.

$= \$88000$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[7 marks]

b. (i) Second installment $2000 \times 1.08 = \$2160$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for multiplying by 1.08 or equivalent, ***(A1)*** for correct answer.

(ii) Fifth installment $= 2000 \times 1.08^4 = 2720.98 = \$2721$ ***(M1)(A1)(AG)***

Notes: ***(M1)*** for correct formula used with numbers from the problem. ***(A1)*** for correct substitution. The $2720.9\ldots$ must be seen for the ***(A1)*** mark to be awarded. Accept list of 5 correct values. If values are rounded prematurely award ***(M1)(A0)(AG)***.

[4 marks]

c. Interest is $= \$8000$ ***(A1)***

$80000 \times \frac{r}{100} \times \frac{20}{12} = 8000$ ***(M1)(A1)***

Note: ***(M1)*** for attempting to substitute in simple interest formula, ***(A1)*** for correct substitution.

Simple Interest Rate $= 6\%$ ***(A1)(G3)***

Note: Award ***(G3)*** for answer of 6% with no working present if interest is also seen award ***(A1)*** for interest and ***(G2)*** for correct answer.

[4 marks]

d. *Financial accuracy penalty* ***(FP)*** *is applicable where indicated in the left hand column.*

(FP) Total amount for option 2 $= 2000\frac{(1-1.08^{20})}{(1-1.08)}$ ***(M1)(A1)***

Note: ***(M1)*** for substituting in correct formula, ***(A1)*** for correct substitution.

$= \$91523.93\ (= \$91524)$ ***(A1)***
$91523.93 - 88000 = \$3523.93 = \$3524$ to the nearest dollar ***(A1)*****(ft)*****(G3)***

Note: Award ***(G3)*** for an answer of $\$3524$ with no working. The difference follows through from the sum, if reasonable. Award a maximum of ***(M1)(A0)(A0)(A1)*****(ft)** if candidate has treated option 2 as an arithmetic sequence and has followed through into their common difference. Award a maximum of ***(M1)(A1)(A0)*****(ft)*****(A0)*** if candidate has consistently used 0.08 in (b) and (d).

[4 marks]

Examiners report

a. This question was answered correctly by many. Candidates were able to restart if they failed to complete a particular part. Many candidates wasted much time because their understanding was limited to a recursive method and hence wrote out all the terms rather than using the formula for the nth term or sum. A surprising number of students were not able to use the simple interest formula for a period which was not a whole number of years. Also hardly anyone knew to calculate interest first before substituting into the formula. Many students who attempted part (d) lost a point due to FP. A number of students rounded their answers prematurely to the nearest dollar.

b. This question was answered correctly by many. Candidates were able to restart if they failed to complete a particular part. Many candidates wasted much time because their understanding was limited to a recursive method and hence wrote out all the terms rather than using the formula for the nth term or sum. A surprising number of students were not able to use the simple interest formula for a period which was not a whole number of years. Also hardly anyone knew to calculate interest first before substituting into the formula. Many students who attempted part (d) lost a point due to FP. A number of students rounded their answers prematurely to the nearest dollar.

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A geometric sequence has 1024 as its first term and 128 as its fourth term.

Consider the arithmetic sequence $1,\ 4,\ 7,\ 10,\ 13,\ \ldots$

A.a Show that the common ratio is $\frac{1}{2}$. [2]

A.b Find the value of the eleventh term. [2]

A.c Find the sum of the first eight terms. [3]

A.d Find the number of terms in the sequence for which the **sum** first exceeds 2047.968. [3]

B.a Find the value of the eleventh term. [2]

B.b The sum of the first n terms of this sequence is $\frac{n}{2}(3n-1)$. [6]

(i) Find the sum of the first 100 terms in this arithmetic sequence.

(ii) The sum of the first n terms is 477.

(a) Show that $3n^2 - n - 954 = 0$.

(b) Using your graphic display calculator or otherwise, find the number of terms, n .

Markscheme

A.a $1024r^3 = 128$ ***(M1)***

$r^3 = \frac{1}{8}$ **or** $r = \sqrt[3]{\frac{1}{8}}$ ***(M1)***

$r = \frac{1}{2}\ (0.5)$ ***(AG)***

Notes: Award at most ***(M1)(M0)*** if last line not seen. Award ***(M1)(M0)*** if 128 is found by repeated multiplication (division) of 1024 by $0.5\ (2)$.

[2 marks]

A.b. 1024×0.5^{10} ***(M1)***

Notes: Award ***(M1)*** for correct substitution into correct formula. Accept an equivalent method.

1 ***(A1)(G2)***

[2 marks]

A.c. $S_8 = \frac{1024\left(1-\left(\frac{1}{2}\right)^8\right)}{1-\frac{1}{2}}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the correct formula, ***(A1)*** for correct substitution.

OR

(A1) for complete and correct list of eight terms ***(A1)***

(M1) for their eight terms added ***(M1)***

$S_8 = 2040$ ***(A1)(G2)***

[3 marks]

A.d. $\frac{1024\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} > 2047.968$ ***(M1)(M1)*****(ft)**

Notes: Award ***(M1)*** for correct substitution into the correct formula for the sum, ***(M1)*** for comparing to 2047.968 . Accept equation. Follow through from their expression for the sum used in part (c).

OR

If a list is used: $S_{15} = 2047.9375$ ***(M1)***

$S_{16} = 2047.96875$ ***(M1)***

$n = 16$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their expression for the sum used in part (c).

[3 marks]

B.a. $\text{common difference} = 3$ (may be implied) ***(A1)***

$u_{11} = 31$ ***(A1)(G2)***

[2 marks]

B.b. (i) $\frac{100}{2}(3 \times 100 - 1)$ **OR** $\frac{100(2+99\times3)}{2}$ ***(M1)***

14950 ***(A1)(G2)***

(ii) (a) $\frac{n}{2}(3n-1) = 477$ **OR** $\frac{n}{2}(2+3(n-1)) = 477$ ***(M1)***

$3n^2 - n = 954$ ***(M1)***

$3n^2 - n - 954 = 0$ ***(AG)***

Notes: Award second ***(M1)*** for correct removal of denominator or brackets and no further incorrect working seen. Award at most

(M1)(M0) if last line not seen.

(b) 18 ***(G2)***

Note: If both solutions to the quadratic equation are seen and the correct value is not identified as the required answer, award ***(G1)(G0)***.

[6 marks]

Examiners report

A.a **Part A: Geometric sequences/series**

The majority of the candidates were not able to offer a satisfactory justification in a) and only scored 1 mark.

A.b **Part A: Geometric sequences/series**

Parts b) and c) were mostly well answered.

A.c **Part A: Geometric sequences/series**

Parts b) and c) were mostly well answered.

A.d **Part A: Geometric sequences/series**

The responses to part d) were often weak. Those candidates who set up the equation scored two marks but very few of them were able to reach the correct final answer.

B.a **Part B: Arithmetic sequences/series**

Parts a), and b)(i) were mostly answered correctly.

B.b **Part B: Arithmetic sequences/series**

Parts a), and b)(i) were mostly answered correctly. Parts b)(ii)a) and b)(ii)b) were poorly answered. Many candidates did not know how to approach the "show that" question. A few were able to solve the quadratic equation using the GDC. Those who attempted to solve it without the GDC generally failed to find the correct answer.

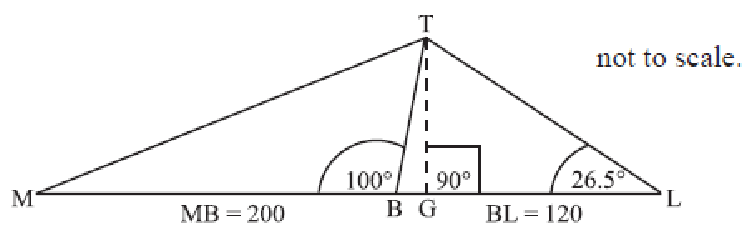
Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

An old tower (BT) leans at 10° away from the vertical (represented by line TG).

The base of the tower is at B so that $\text{M}\hat{\text{B}}\text{T} = 100^\circ$.

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be $\text{B}\hat{\text{L}}\text{T} = 26.5^\circ$.



i.a. Calculate the volume of the cylinder **in cm^3**. Give your answer correct to **two** decimal places. [3]

i.b. The cylinder is used for storing tennis balls. Each ball has a **radius** of 3.25 cm. [1]

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

i.c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls. [4]

(ii) Convert your answer to (c) (i) into cubic metres.

ii.a. (i) Find the value of angle $\mathrm{B\hat{T}L}$. [5]

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

ii.b. Calculate the vertical height TG of the top of the tower. [2]

ii.c. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. [3]

Markscheme

i.a. $\pi \times 3.25^2 \times 39$ ***(M1)(A1)***

(= 1294.1398)

Answer 1294.14 (cm^3)(2dp) ***(A1)*****(ft)*****(G2)***

__(UP)__ not applicable in this part due to wording of question. __(M1)__ is for substituting appropriate numbers from the problem into the correct formula, even if the units are mixed up. __(A1)__ is for correct substitutions or correct answer with more than 2dp in cubic centimetres seen. Award __(G1)__ for answer to > 2dp with no working and no attempt to correct to 2dp. Award __(M1)(A0)(A1)(ft)__ for $\pi \times 32.5^2 \times 39\ \mathrm{cm}^3$ (= 129413.9824) = 129413.98

Use of $\pi = \frac{22}{7}$ **or** 3.142 *etc is premature rounding and is awarded at most __(M1)(A1)(A0)__ or __(M1)(A0)(A1)(ft)__ depending on whether the intermediate value is seen or not. For all other incorrect substitutions, award __(M1)(A0)__ and only follow through the 2 dp correction if the intermediate answer to more decimal places is seen. Answer given as a multiple of π is awarded at most __(M1)(A1)(A0).__ As usual, an* **unsubstituted** *formula followed by correct answer only receives the G marks.*

[3 marks]

i.b. 39/6.5 = 6 ***(A1)***

[1 mark]

i.c. *Unit penalty __(UP)__ is applicable where indicated in the left hand column.*

(UP) (i) Volume of one ball is $\frac{4}{3}\pi \times 3.25^3\ \mathrm{cm}^3$ ***(M1)***

$\text{Volume of air} = \pi \times 3.25^2 \times 39 - 6 \times \frac{4}{3}\pi \times 3.25^3 = 431\ \mathrm{cm}^3$ ***(M1)(A1)*****(ft)*****(G2)***

Award first __(M1)__ for substituted volume of sphere formula or for numerical value of sphere volume seen (143.79… or 45.77… $\times\pi$*). Award second __(M1)__ for subtracting candidate's sphere volume multiplied by their answer to (b). Follow through from parts (a) and (b) only, but negative or zero answer is always awarded __(A0)__*__(ft)__

(UP) (ii) 0.000431m^3 or 4.31×10^{-4} m^3 ***(A1)*****(ft)**

[4 marks]

ii.a. *Unit penalty __(UP)__ is applicable where indicated in the left hand column.*

(i) $\text{Angle } \mathrm{B\widehat{T}L} = 180 - 80 - 26.5$ or $180 - 90 - 26.5 - 10$ ***(M1)***

$= 73.5^\circ$ ***(A1)(G2)***

(ii) $\frac{BT}{\sin(26.5^\circ)} = \frac{120}{\sin(73.5^\circ)}$ ***(M1)(A1)*****(ft)**

(UP) BT = 55.8 m (3sf) ***(A1)*****(ft)**

[5 marks]

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)(ft)****.*

The answers are (all 3sf)

(ii)(a) – 124 m ***(A0)*****(ft)**

(ii)(b) 123 m ***(A0)***

(ii)(c) 313 m ***(A0)***

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)*****(ft)**

ii.b. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

TG = 55.8sin(80°) or 55.8cos(10°) ***(M1)***

(UP)* =** 55.0 m (3sf) ***(A1)**(ft)*****(G2)***

Apply ***(AP)*** *if 0 missing*

[2 marks]

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)(ft)****.*

The answers are (all 3sf)

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(ii)(b) 123 m ***(A0)***

(ii)(c) 313 m ***(A0)***

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)*****(ft)**

ii.c. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

$\mathrm{MT}^2 = 200^2 + 55.8^2 - 2 \times 200 \times 55.8 \times \cos(100^\circ)$ ***(M1)(A1)*****(ft)**

(UP) MT = 217 m (3sf) ***(A1)*****(ft)**

Follow through only from part (ii)(a)(ii). Award marks at discretion for any valid alternative method.

[3 marks]

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)(ft)****.*

The answers are (all 3sf)

(ii)(a) – 124 m ***(A0)*****(ft)**

(ii)(b) 123 m ***(A0)***

(ii)(c) 313 m ***(A0)***

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)*****(ft)**

Examiners report

i.a. (i) Many candidates incurred the new one-off unit penalty here. Too many ignored the call for two decimal places and some extrapolated that instruction to later parts (which was clearly not intended). There was the predictable confusion of using radius instead of diameter. Another common error was to divide the cylinder volume by that of the ball, to decide how many would fit. Some follow-through was allowed later from this error, however, this led to zero or negligible air volume, which was clearly ridiculous.

Choice and use of the formulae for volumes was often competent but the conversion to cubic metres was very badly done. Almost no correct answers were seen at all.

i.b. (i) Many candidates incurred the new one-off unit penalty here. Too many ignored the call for two decimal places and some extrapolated that instruction to later parts (which was clearly not intended). There was the predictable confusion of using radius instead of diameter. Another common error was to divide the cylinder volume by that of the ball, to decide how many would fit. Some follow-through was allowed later from this error, however, this led to zero or negligible air volume, which was clearly ridiculous.

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Choice and use of the formulae for volumes was often competent but the conversion to cubic metres was very badly done. Almost no correct answers were seen at all.

ii.a. (ii) Candidates were often sloppy in reading the information. In particular, despite the statement BL = 120 clearly written, many took GL as 120. Triangle TBL was often taken as right-angled. Angle BTL presented few problems, though sometimes the method was very long-winded. Candidates often managed part (a) then went awry in later parts. Many unit penalties were applied, if not already used in questions 1 or 2.

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Consider the sequence $u_1,\ u_2,\ u_3,\ \ldots,\ \mathrm{u}_n,\ \ldots$ where

$$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$$

The sequence continues in the same manner.

a. Find the value of u_{20}. [3]

b. Find the sum of the first 10 terms of the sequence. [3]

c. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

Find the exact value of v_{10}.

d. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

Find the sum of the first 8 terms of this sequence.

e. k is the smallest value of n for which v_n is greater than u_n. [3]

Calculate the value of k.

Markscheme

a. $600 + (20 - 1) \times 17$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 20 correct terms seen.

$= 923$ ***(A1)(G3)***

[3 marks]

b. $\frac{10}{2}[2 \times 600 + (10 - 1) \times 17]$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic series formula, ***(A1)*** for their correct substitutions. Follow through from part (a). For consistent use of geometric series formula in part (b) with the geometric sequence formula in part (a) award a maximum of ***(M1)(A1)(A0)*** since their final answer cannot be an integer.

OR

$u_{10} = 600 + (10 - 1)17 = 753$ ***(M1)***

$S_{10} = \frac{10}{2}(600 + \text{their } u_{10})$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted arithmetic sequence formula, ***(M1)*** for their correctly substituted arithmetic series formula. Follow through from part (a) and **within** part (b).

Note: If a list is used, award ***(M1)*** for at least 10 correct terms seen, award ***(A1)*** for these terms being added.

$= 6765$ (accept 6770) ***(A1)*****(ft)*****(G2)***

[3 marks]

c. 3×2^9 ***(M1)(A1)***

Note: Award ***(M1)*** for substituted geometric sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 8 correct terms seen.

$= 1536$ ***(A1)(G3)***

Note: Exact answer only. If both exact and rounded answer seen, award the final ***(A1)***.

[3 marks]

d. $\frac{3\times(2^8-1)}{2-1}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted geometric series formula, ***(A1)*** for their correct substitutions. Follow through from part (c). If a list is used, award ***(M1)*** for at least 8 correct terms seen, award ***(A1)*** for these 8 correct terms being added. For consistent use of arithmetic series formula in part (d) with the arithmetic sequence formula in part (c) award a maximum of ***(M1)(A1)(A1)***.

$= 765$ ***(A1)*****(ft)*****(G2)***

[3 marks]

e. $3 \times 2^{k-1} > 600 + (k-1)(17)$ ***(M1)***

Note: Award ***(M1)*** for their correct inequality; allow equation.

Follow through from parts (a) and (c). Accept sketches of the two functions as a valid method.

$k > 8.93648\ldots$ (may be implied) ***(A1)*****(ft)**

Note: Award ***(A1)*** for $8.93648\ldots$ seen. The GDC gives answers of -34.3 and 8.936 to the inequality; award ***(M1)(A1)*** if these are seen with working shown.

OR

$v_8 = 384 \quad u_8 = 719$ ***(M1)***

$v_9 = 768 \quad u_9 = 736$ ***(M1)***

Note: Award ***(M1)*** for v_8 and u_8 both seen, ***(M1)*** for v_9 and u_9 both seen.

$k = 9$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(G1)*** for $8.93648\ldots$ and -34.3 seen as final answer without working. Accept use of n.

[3 marks]

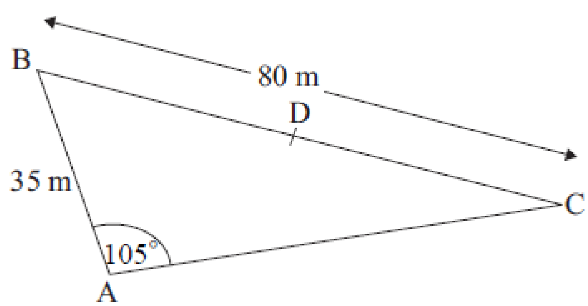
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A farmer has a triangular field, ABC, as shown in the diagram.

AB = 35 m, BC = 80 m and BÂC = 105°, and D is the midpoint of BC.

diagram not to scale



a. Find the size of BĈA. [3]

b. Calculate the length of AD. [5]

c. The farmer wants to build a fence around ABD. [2]

Calculate the total length of the fence.

d. The farmer wants to build a fence around ABD. [2]

The farmer pays 802.50 USD for the fence. Find the cost per metre.

e. Calculate the area of the triangle ABD. [3]

f. A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. [3]

Markscheme

a. $\frac{\sin \mathrm{BCA}}{35} = \frac{\sin 105^\circ}{80}$ ***(M1)(A1)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*** for correct substitutions.

$\mathrm{B\hat{C}A} = 25.0^\circ$ ***(A1)(G2)***

[3 marks]

b. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

Length BD = 40 m ***(A1)***

Angle ABC = 180° – 105° – 25° = 50° ***(A1)*(ft)**

Note: (ft) from their answer to (a).

$AD^2 = 35^2 + 40^2 - (2 \times 35 \times 40 \times \cos 50°)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*(ft)** for correct substitutions.

(UP) AD = 32.0 m ***(A1)*(ft)*(G3)***

Notes: If 80 is used for BD award at most ***(A0)(A1)*(ft)*(M1)(A1)*(ft)*(A1)*(ft)** for an answer of 63.4 m.

If the angle ABC is incorrectly calculated **in this part** award at most ***(A1)(A0)(M1)(A1)*(ft)*(A1)*(ft)**.

If angle BCA is used award at most ***(A1)(A0)(M1)(A0)(A0)***.

[5 marks]

c. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

length of fence = 35 + 40 + 32 ***(M1)***

(UP) = 107 m ***(A1)*(ft)*(G2)***

Note: ***(M1)*** for adding 35 + 40 + their (b).

[2 marks]

d. cost per metre $= \frac{802.50}{107}$ ***(M1)***

Note: Award ***(M1)*** for dividing 802.50 by their (c).

cost per metre = 7.50 USD (7.5 USD) (USD not required) ***(A1)*(ft)*(G2)***

[2 marks]

e. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

Area of ABD $= \frac{1}{2} \times 35 \times 40 \times \sin 50°$ ***(M1)***

= 536.2311102 ***(A1)*(ft)**

(UP) = 536 m^2 ***(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*(ft)** for correct substitution, **(ft)** from their value of BD and their angle ABC in (b).

[3 marks]

f. Volume = 0.03 × 536 ***(A1)(M1)***

= 16.08

= 16.1 ***(A1)*(ft)*(G2)***

Note: Award ***(A1)*** for 0.03, ***(M1)*** for correct formula. **(ft)** from their (e).

If 3 is used award at most ***(A0)(M1)(A0)***.

[3 marks]

Examiners report

a. This was a simple application of non-right angled trigonometry and most candidates answered it well. Some candidates lost marks in both parts due to the incorrect setting of the calculators. Those that did not score well overall primarily used Pythagoras.

b. This was a simple application of non-right angled trigonometry and most candidates answered it well. Some candidates lost marks in both parts due to the incorrect setting of the calculators. Those that did not score well overall primarily used Pythagoras.

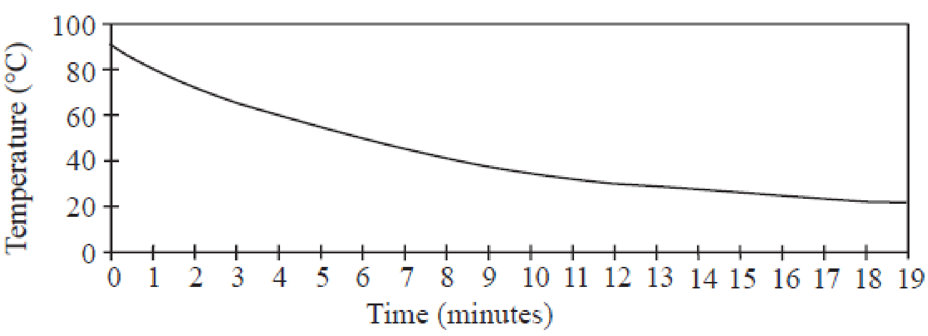
c. Most candidates scored full marks, many by follow through from an incorrect part (b). The main error was using the value for BC and not BD.

d. Most candidates scored full marks, many by follow through from an incorrect part (b). The main error was using the value for BC and not BD.

e. Done well; again some candidates used the right-angled formula.

f. This part was poorly done; many candidates unable to convert 3 cm to 0.03 m. A significant number used the wrong formula, multiplying their answer by 1/3.

The following graph shows the temperature in degrees Celsius of Robert’s cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

i.a. Find the initial temperature of the coffee. [1]

i.b. Write down the equation of the horizontal asymptote. [1]

i.c. Find the room temperature. [1]

i.d. Find the temperature of the coffee after 10 minutes. [1]

i.e. Find the temperature of Robert's coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3]

i.f. Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C. [4]

ii.a. Calculate **correct to 2 decimal places** the amount of euros he receives. [3]

ii.b. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3]

Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

Markscheme

i.a. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 90°C ***(A1)***

[1 mark]

i.b. $y = 16$ ***(A1)***

[1 mark]

i.c. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 16°C **(ft)** *from answer to part (b)* ***(A1)*(ft)**

[1 mark]

i.d. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 25.4°C ***(A1)***

[1 mark]

i.e. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

for seeing $2^{0.75}$ *or equivalent* ***(A1)***

for multiplying their (d) by their $2^{0.75}$ ***(M1)***

(UP) 42.8°C ***(A1)*(ft)*(G2)***

[3 marks]

i.f. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

for seeing $20 \times 2^{1.5t} = 100$ ***(A1)***

for seeing a value of t between 1.54 *and* 1.56 *inclusive* ***(M1)(A1)***

(UP) 1.55 minutes or 92.9 seconds ***(A1)(G3)***

[4 marks]

ii.a. *Financial accuracy penalty **(FP)** is applicable in part (ii) **only**.*

$120 - 3 = 117$

(FP) 117×1.37 ***(A1)***

= 160.29 euros *(correct answer only)* ***(M1)***

*first **(A1)** for 117 seen, **(M1)** for multiplying by 1.37* ***(A1)(G2)***

[3 marks]

ii.b. *Financial accuracy penalty* ***(FP)*** *is applicable in part (ii)* ***only****.*

(FP) $\frac{13.5}{1.37}$ ***(A1)(M1)***

9.85 GBP *(answer correct to 2dp only)*

first ***(A1)*** *is for 13.5 seen,* ***(M1)*** *for dividing by 1.37* ***(A1)*****(ft)*****(G3)***

[3 marks]

Examiners report

i.a. Many candidates who had not lost a UP in question 2 lost one here. Parts (a), (c) and (d) were reasonably well tackled. Almost everybody had difficulty with the equation of the horizontal asymptote, a common answer being $y = 20$. Most of the candidates realised that 30 seconds was 0.5 minutes and calculated part (e) correctly. Part (f), solving an exponential equation, was a good discriminator. Trial and error was expected but many students did not think of doing this.

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ii.a.The financial part was the best done question in the paper and a large majority of candidates gained full marks here.

ii.b.The financial part was the best done question in the paper and a large majority of candidates gained full marks here.

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

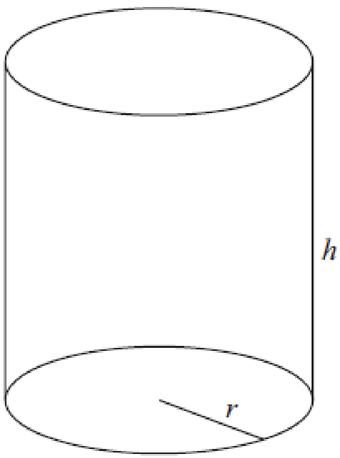


diagram not to scale

Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

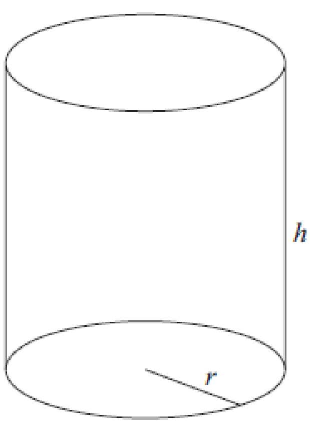


diagram not to scale

Let the radius of the base of Merryn's wastepaper bin be r , and let its height be h .

a. Calculate [7]

(i) the area of the base of the wastepaper bin;

(ii) the height, h , of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason. [2]

c. Write down an equation in h and r , using the given volume of the bin. [1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2]

e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. [5]

(ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. [2]

Markscheme

a. (i) $\text{Area} = \pi(5)^2$ ***(M1)***

$= 78.5\ (\text{cm}^2)$ $(78.5398\ldots)$ ***(A1)(G2)***

Note: Accept 25π .

(ii) $8000 = 78.5398\ldots \times h$ ***(M1)***
$h = 102\ (\text{cm})$ $(101.859\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (a)(i).

(iii) $\text{Area} = \pi(5)^2 + 2\pi(5)(101.859\ldots)$ ***(M1)(M1)***

Note: Award ***(M1)*** for their substitution in curved surface area formula, ***(M1)*** for addition of their two areas.

$= 3280\ (\text{cm}^2)$ $(3278.53\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answers to parts (a)(i) and (ii).

b. No, it is too tall/narrow. ***(A1)*(ft)*(R1)***

Note: Follow through from their value for h.

c. $8000 = \pi r^2 h$ ***(A1)***

d. $A = \pi r^2 + 2\pi r\left(\frac{8000}{\pi r^2}\right)$ ***(A1)(M1)***

Note: Award ***(A1)*** for correct rearrangement of **their** part (c), ***(M1)*** for substitution of **their** rearrangement into area formula.

$= \pi r^2 + \frac{16000}{r}$ ***(AG)***

e. $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r - 16000r^{-2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi r$, ***(A1)*** for -16000 ***(A1)*** for r^{-2} . If an extra term is present award at most ***(A1)(A1)(A0)***.

f. (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ ***(M1)***

$2\pi r^3 - 16000 = 0$ ***(M1)***

$r = 13.7 \text{ cm } (13.6556\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (e).

(ii) $h = \frac{8000}{\pi(13.65\ldots)^2}$ ***(M1)***

$= 13.7 \text{ cm } (13.6556\ldots)$ ***(A1)*(ft)**

Note: Accept 13.6 if 13.7 used.

g. Yes or No, accompanied by a consistent and sensible reason. ***(A1)(R1)***

Note: Award ***(A0)(R0)*** if no reason is given.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

The lengths (l) in centimetres of 100 copper pipes at a local building supplier were measured. The results are listed in the table below.

Length l (cm)	Frequency
17.5	12
32.5	26
47.5	32
62.5	21
77.5	9

a. Write down the mode. [1]

b. Using your graphic display calculator, write down the value of [4]

(i) the mean;

(ii) the standard deviation;

(iii) the median.

c. Find the interquartile range. [2]

d. Draw a box and whisker diagram for this data, on graph paper, using a scale of 1 cm to represent 5 cm. [4]

e. Sam estimated the value of the mean of the measured lengths to be 43 cm. [2]

Find the percentage error of Sam's estimated mean.

Markscheme

a. 47.5 (cm) ***(A1)***

b. (i) 45.85 (cm) ***(G2)***

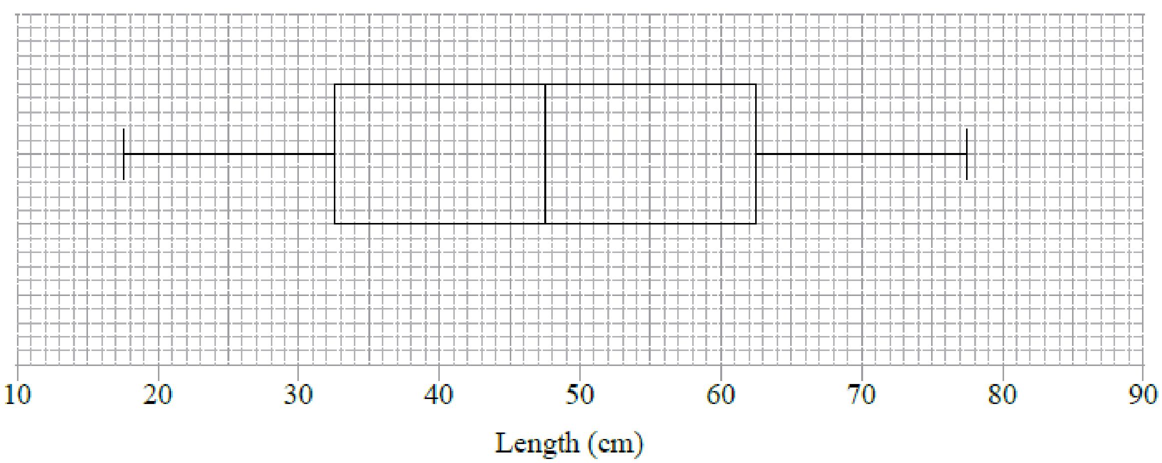
Note: Accept 45.9 .

(ii) $17.1\ (17.0888\ldots)$ ***(G1)***
(iii) 47.5 (cm) ***(G1)***

c. $62.5 - 32.5 = 30$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for correct quartiles seen.

d.



(A1) for correct label and scale
***(A1)*(ft)** for correct median
***(A1)*(ft)** for correct quartiles and box
(A1) for endpoints at 17.5 and 77.5 joined to box by straight lines ***(A1)(A1)*(ft)*(A1)*(ft)*(A1)***

Notes: The final ***(A1)*** is lost if the lines go through the box. Follow through from their parts (b) and (c).

e. $\varepsilon = \left|\frac{43-45.85}{45.85}\right| \times 100\%$ **(M1)**

Note: Award ***(M1)*** for their correct substitution in $\%$ error formula.

$= 6.22\% \, (6.21592\ldots)$ ***(A1)(ft)(G2)***

Notes: Follow through from their answer to part (b)(i). Accept 6.32% with use of 45.9 .

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The sum of the first n terms of an arithmetic sequence is given by $S_n = 6n + n^2$.

a. Write down the value of [2]

(i) S_1;

(ii) S_2.

b. The n^{th} term of the arithmetic sequence is given by u_n. [1]

Show that $u_2 = 9$.

c. The n^{th} term of the arithmetic sequence is given by u_n. [2]

Find the common difference of the sequence.

d. The n^{th} term of the arithmetic sequence is given by u_n. [2]

Find u_{10}.

e. The n^{th} term of the arithmetic sequence is given by u_n. [3]

Find the lowest value of n for which u_n is greater than 1000.

f. The n^{th} term of the arithmetic sequence is given by u_n. [2]

There is a value of n for which

$$u_1 + u_2 + \ldots + u_n = 1512.$$

Find the value of n.

Markscheme

a. (i) $S_1 = 7$ ***(A1)***

(ii) $S_2 = 16$ ***(A1)***

b. $(u_2 =)\ 16 - 7 = 9$ ***(M1)(AG)***

Note: Award ***(M1)*** for subtracting 7 from 16. The 9 must be seen.

OR

$16 - 7 - 7 = 2$

$(u_2 =)\ 7 + (2 - 1)(2) = 9$ ***(M1)(AG)***

Note: Award ***(M1)*** for subtracting twice 7 from 16 and for correct substitution in correct arithmetic sequence formula.

The 9 must be seen.

Do not accept: $9 - 7 = 2,\ u_2 = 7 + (2 - 1)(2) = 9.$

c. $u_1 = 7$ ***(A1)*(ft)**

$d = 2\ (= 9 - 7)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their S_1 in part (a)(i).

d. $7 + 2 \times (10 - 1)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the correct arithmetic sequence formula. Follow through from **their** parts (a)(i) and (c).

$= 25$ ***(A1)*(ft)*(G2)***

Note: Award ***(A1)*(ft)** for their correct tenth term.

e. $7 + 2 \times (n - 1) > 1000$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their correct expression for the n^{th} term, ***(M1)*** for comparing their expression to 1000. Accept an equation. Follow through from their parts (a)(i) and (c).

$n = 498$ ***(A1)*(ft)*(G2)***

Notes: Answer must be a natural number.

f. $6n + n^2 = 1512$ **OR** $\frac{n}{2}(14 + 2(n - 1)) = 1512$ **OR**

$S_n = 1512$ **OR** $7 + 9 + \ldots + u_n = 1512$ ***(M1)***

Notes: Award ***(M1)*** for equating the sum of the first n terms to 1512. Accept a sum of at least the first 7 correct terms.

$n = 36$ ***(A1)(G2)***

Note: If $n = 36$ is seen without working, award ***(G2)***. Award a maximum of ***(M1)(A0)*** if -42 is also given as a solution.

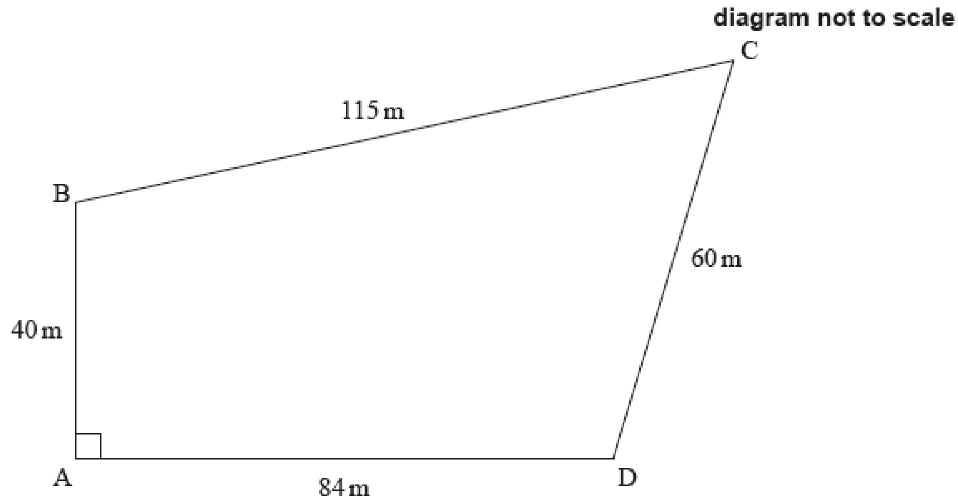
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $\mathrm{AB} = 40\ \mathrm{m}$, $\mathrm{BC} = 115\ \mathrm{m}$, $\mathrm{CD} = 60\ \mathrm{m}$, $\mathrm{AD} = 84\ \mathrm{m}$ and angle $\mathrm{B\hat{A}D} = 90^\circ$.

This information is shown on the diagram.



The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(\text{AB}+\text{CD})(\text{AD}+\text{BC})}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

a. Show that $\text{BD} = 93\text{ m}$ correct to the nearest metre. [2]

b. Calculate angle $\text{B}\hat{\text{C}}\text{D}$. [3]

c. Find the area of ABCD. [4]

d.i. Calculate Abdallah's estimate for the area. [2]

d.ii. Find the percentage error in Abdallah's estimate. [2]

Markscheme

a. $\text{BD}^2 = 40^2 + 84^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras.

Accept correct substitution into cosine rule.

$\text{BD} = 93.0376\ldots$ ***(A1)***

$= 93$ ***(AG)***

Note: Both the rounded and unrounded value must be seen for the ***(A1)*** to be awarded.

[2 marks]

b. $\cos C = \frac{115^2+60^2-93^2}{2\times 115\times 60}$ $(93^2 = 115^2 + 60^2 - 2\times 115\times 60\times \cos C)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine formula, ***(A1)*** for correct substitutions.

$= 53.7^\circ\ (53.6679\ldots^\circ)$ ***(A1)(G2)***

[3 marks]

c. $\frac{1}{2}(40)(84) + \frac{1}{2}(115)(60)\sin(53.6679\ldots)$ ***(M1)(M1)(A1)*(ft)**

Note: Award ***(M1)*** for correct substitution into right-angle triangle area. Award ***(M1)*** for substitution into area of triangle formula and ***(A1)*(ft)** for correct substitution.

$= 4460\ \mathrm{m}^2\ (4459.30\ldots\ \mathrm{m}^2)$ ***(A1)*(ft)*(G3)***

Notes: Follow through from part (b).

[4 marks]

d.i. $\frac{(40+60)(84+115)}{4}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the area formula used by ‘Ancient Egyptians’.

$= 4980\ \mathrm{m}^2\ (4975\ \mathrm{m}^2)$ ***(A1)(G2)***

[2 marks]

d.ii. $\left|\frac{4975 - 4459.30\ldots}{4459.30\ldots}\right| \times 100$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into percentage error formula.

$= 11.6\ (\%)\ (11.5645\ldots)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from parts (c) and (d)(i).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]

Give all your numerical answers correct to two decimal places.

On 1 January 2005, Daniel invested 30000 AUD at an annual **simple** interest rate in a *Regular Saver* account. On 1 January 2007, Daniel had 31650 AUD in the account.

b. On 1 January 2005, Rebecca invested 30000 AUD in a *Supersaver* account at a nominal annual rate of 2.5% **compounded annually**. [3]

Calculate the amount in the *Supersaver* account after two years.

c. On 1 January 2005, Rebecca invested 30000 AUD in a *Supersaver* account at a nominal annual rate of 2.5% **compounded annually**. [3]

Find the number of complete years since 1 January 2005 it would take for the amount in Rebecca's account to exceed the amount in Daniel's account.

d. On 1 January 2007, Daniel reinvested 80% of the money from the *Regular Saver* account in an *Extra Saver* account at a nominal annual rate of 3% **compounded quarterly**. [5]

(i) Calculate the amount of money reinvested by Daniel on the 1 January 2007.

(ii) Find the number of complete years it will take for the amount in Daniel's *Extra Saver* account to exceed 30000 AUD.

Markscheme

b. $\text{Amount} = 30000\left(1+\frac{2.5}{100}\right)^2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

31518.75 AUD ***(A1)(G2)***

OR

$\text{I} = 30000\left(1+\frac{2.5}{100}\right)^2 - 30000$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

31518.75 AUD ***(A1)(G2)***

[3 marks]

c. $\text{Rebecca's amount} = 30000\left(1+\frac{2.5}{100}\right)^n$

$\text{Daniel's amount} = 30000 + \frac{30000\times2.75\times n}{100}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution in the correct formula for the two amounts, ***(A1)*** for correct substitution. Follow through from their expressions used in part (a) and/or part (b).

OR

2 lists of values seen (at least 2 terms per list) ***(M1)***

lists of values including at least the terms with $n = 8$ and $n = 9$ ***(A1)*(ft)**

For $n = 8$ $\quad \text{CI} = 36552.09 \quad \text{SI} = 36600$

For $n = 9$ $\quad \text{CI} = 37465.89 \quad \text{SI} = 37425$

Note: Follow through from their expressions used in part (a) or/and (b).

OR

Sketch showing 2 graphs, one exponential and the other straight line ***(M1)*** point of intersection identified ***(M1)***

Note: Follow through from their expressions used in part (a) or/and (b).

$n = 9$ ***(A1)*(ft)*(G2)***

Note: Answer 8.57 without working is awarded ***(G1)***.

Note: Accept comparison of interests instead of the total amounts in the two accounts.

[3 marks]

d. (i) $0.80 \times 31650 = 25320$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for correct use of percentages.

(ii) $25320\left(1 + \frac{3}{4 \times 100}\right)^{4n} > 30000$ ***(M1)(M1)*(ft)**

Notes: Award ***(M1)*** for correct left-hand side of the inequality, ***(M1)*** for comparison to 30000. Accept equation. Follow through from their answer to part (d) (i).

OR

List of values from their $25320\left(1 + \frac{3}{4 \times 100}\right)^{4n}$ seen (at least 2 terms) ***(M1)***

Their correct values for $n = 5$ (29401.18) and $n = 6$ (30293) seen ***(A1)*(ft)**

Note: Follow through from their answer to (d) (i).

OR

Sketch showing 2 graphs – an exponential and a horizontal line ***(M1)***

Point of intersection identified or vertical line drawn ***(M1)***

Note: Follow through from their answer to (d) (i).

$n = 6$ ***(A1)*(ft)*(G2)***

Note: Award ***(G1)*** for answer 5.67 with no working.

[5 marks]

Examiners report

b. Part b) was well done.

c. Parts c) and d) were not answered well. Marks were gained by candidates who showed detailed working. Many candidates had difficulty working with the compound interest formula where the interest was compounded quarterly. Correct final answers in parts c) and d) were rare.

d. Parts c) and d) were not answered well. Marks were gained by candidates who showed detailed working. Many candidates had difficulty working with the compound interest formula where the interest was compounded quarterly. Correct final answers in parts c) and d) were rare.

a. Prachi is on vacation in the United States. She is visiting the Grand Canyon. [2]

When she reaches the top, she drops a coin down a cliff. The coin falls down a distance of 5 metres during the first second, 15 metres during the next second, 25 metres during the third second and continues in this way. The distances that the coin falls during each second forms an arithmetic sequence.

(i) Write down the common difference, d , of this arithmetic sequence.

(ii) Write down the distance the coin falls during the fourth second.

b. Calculate the distance the coin falls during the 15th second. [2]

c. Calculate the **total** distance the coin falls in the first 15 seconds. Give your answer in kilometres. [3]

d. Prachi drops the coin from a height of 1800 metres above the ground. [3]

Calculate the time, to the nearest second, the coin will take to reach the ground.

e. Prachi visits a tourist centre nearby. It opened at the start of 2015 and in the first year there were $17\,000$ visitors. The number of people who visit the tourist centre is expected to increase by $10\,\%$ each year. [2]

Calculate the number of people expected to visit the tourist centre in 2016.

f. Calculate the total number of people expected to visit the tourist centre during the first 10 years since it opened. [3]

Markscheme

a. (i) $10\,(\mathrm{m})$ ***(A1)***

(ii) $35\,(\mathrm{m})$ ***(A1)*****(ft)**

Note: Follow through from part (a)(i).

b. $5 + 14 \times 10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic sequence formula. A list of **their** 10 correct terms (excluding those given in question and the 35 from part (a)(ii)) must be seen for the ***(M1)*** to be awarded.

$= 145\,(\mathrm{m})$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their value for d.

If a list is used, award ***(A1)*** for their 15^{th} term.

c. $\frac{15}{2}(2 \times 5 + 14 \times 10)$ **OR** $\frac{15}{2}(5 + 145)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic series formula. Follow through from their part (a)(i). Accept a list added together until the 15th term.

$= 1125\ (\mathrm{m})$ ***(A1)*****(ft)**

Note: Follow through from parts (a) and (b).

$= 1.13\ (\mathrm{km})\ (1.125\,(\mathrm{km}))$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*****(ft)** for correctly converting **their** metres to kilometres, irrespective of method used. To award the last ***(A1)*****(ft)** in follow through, the candidate's answer in metres must be seen.

d. $\frac{n}{2}(2\times 5+(n-1)10)=1800$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic series formula equated to 1800. Follow through from their part (a)(i). Accept a list of terms that shows clearly the 18th second and 19th second distances.
Correct use of kinematics equations is a valid method.

$n=18.97$ ***(A1)*(ft)**

19 (seconds) ***(A1)*(ft)*(G2)***

Note: Award ***(A1)*(ft)** for correct unrounded value for n. The second ***(A1)*(ft)** is awarded for the correct rounding off of their value for n to the nearest second if their unrounded value is seen.
Award ***(M1)(A2)*(ft)** for their 19 if method is shown. Unrounded value for n may not be seen. Follow through from their u_I and d only if workings are shown.

OR

$1125+155+165+175+185=1805$ ***(M1)***

Note: Award ***(M1)*** for adding the terms until reaching 1800.

$(n=)\,19$ ***(A2)*(ft)**

Note: In this method, follow through from their d from part (a) and their 1125 from part (c).

e. $17\,000\ (1.1)$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for multiplying $17\,000$ by 1.1 or equivalent.

$=18\,700$ ***(A1)(G2)***

f. $S_{10}=\frac{17\,000\,(1.1^{10}-1)}{1.1-1}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into the geometric series formula, ***(A1)*(ft)** for correct substitution. Award ***(A1)*(ft)** for a list of their correct 10 terms, ***(M1)*** for adding their 10 terms.

$271\,000\ \ (270\,936)$ ***(A1)*(ft)(G2)**

Note: Follow through from their 1.1 in part (e).

Examiners report

a. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

b. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

c. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

d. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

e. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

f. Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

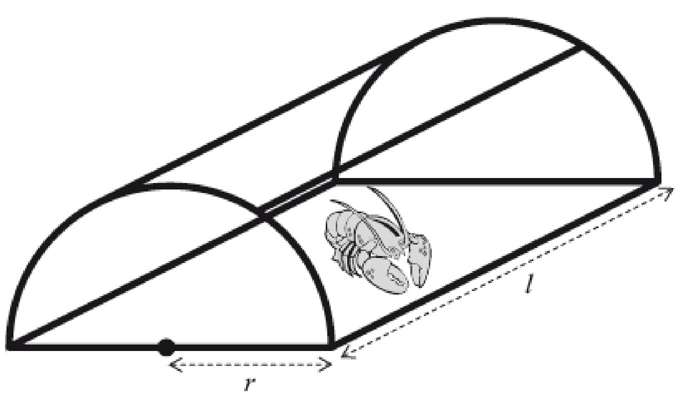


diagram not to scale

The semicircular ends each have radius r and the support rods each have length l.

Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is $0.75\ \text{m}^3$. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is $0.75\ \text{m}^3$. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is $0.75\ \text{m}^3$. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is $0.719\ \text{m}$, correct to three significant figures.

f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

Markscheme

a. $2\pi r + 4r + 4l$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2\pi r$ ("π" must be seen), ***(A1)*** for $4r$, ***(A1)*** for $4l$. Accept equivalent forms. Accept $T = 2\pi r + 4r + 4l$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

b. $0.75 = \frac{\pi r^2 l}{2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for their formula equated to 0.75, ***(A1)*** for l substituted into volume of cylinder formula, ***(A1)*** for volume of cylinder formula divided by 2.

If "π" not seen in part (a) accept use of 3.14 or greater accuracy. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

c. $T = 2\pi r + 4r + 4\left(\frac{1.5}{\pi r^2}\right)$ ***(A1)*(ft)*(A1)***

$= (2\pi + 4)r + \frac{6}{\pi r^2}$ ***(AG)***

Notes: Award ***(A1)*(ft)** for correct rearrangement of their volume formula in part (b) seen, award ***(A1)*** for the correct substituted formula for T. The final line must be seen, with no incorrect working, for this second ***(A1)*** to be awarded.

[2 marks]

d. $\frac{\mathrm{d}T}{\mathrm{d}r} = 2\pi + 4 - \frac{12}{\pi r^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi + 4$, ***(A1)*** for $\frac{-12}{\pi}$, ***(A1)*** for r^{-3}.

Accept 10.3 (10.2832…) for $2\pi + 4$, accept -3.82 $-3.81971\ldots$ for $\frac{-12}{\pi}$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

e. $2\pi + 4 - \frac{12}{\pi r^3} = 0$ **OR** $\frac{\mathrm{d}T}{\mathrm{d}r} = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative equal to zero.

$r = 0.718843\ldots$ **OR** $\sqrt[3]{0.371452\ldots}$ **OR** $\sqrt[3]{\frac{12}{\pi(2\pi+4)}}$ **OR** $\sqrt[3]{\frac{3.81971}{10.2832\ldots}}$ ***(A1)***

$r = 0.719\ (\mathrm{m})$ ***(AG)***

Note: The rounded and unrounded or formulaic answers must be seen for the final ***(A1)*** to be awarded. The use of 3.14 gives an unrounded answer of $r = 0.719039\ldots$.

[2 marks]

f. $0.75 = \frac{\pi \times (0.719)^2 l}{2}$ ***(M1)***

Note: Award ***(M1)*** for substituting 0.719 into their volume formula. Follow through from part (b).

$l = 0.924\ (\mathrm{m})\ \ (0.923599\ldots)$ ***(A1)*****(ft)*****(G2)***

[2 marks]

g. $T = (2\pi + 4) \times 0.719 + \frac{6}{\pi (0.719)^2}$ ***(M1)***

Notes: Award ***(M1)*** for substituting 0.719 in their expression for T. Accept alternative methods, for example substitution of their l and 0.719 into their part (a) (for which the answer is 11.08961024). Follow through from their answer to part (a).

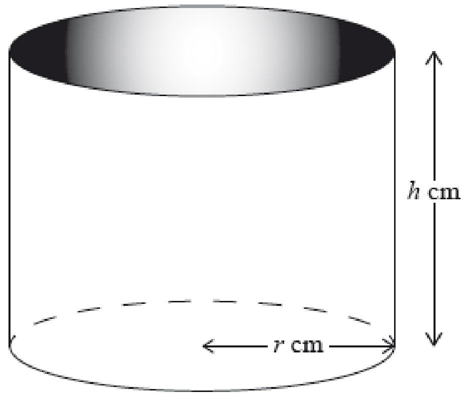
$= 11.1\ (\mathrm{m})\ \ (11.0880\ldots)$ ***(A1)*****(ft)*****(G2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
[N/A]

g. [N/A]

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is $0.5\ \mathrm{m}^3$.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Markscheme

a. $(A =)\ \pi r^2 + 2\pi rh$ ***(A1)(A1)***

Note: Award ***(A1)*** for either πr^2 **OR** $2\pi rh$ seen. Award ***(A1)*** for two correct terms added together.

[2 marks]

b. $500\,000$ ***(A1)***

Notes: Units **not** required.

[1 mark]

c. $500\,000 = \pi r^2 h$ ***(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for $\pi r^2 h$ equating to their part (b).

Do not accept unless $V = \pi r^2 h$ is explicitly defined as their part (b).

[1 mark]

d. $A = \pi r^2 + 2\pi r \left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*(ft)*(M1)*** for rearranging part (c) to $\pi r h = \frac{500\,000}{r}$ and substituting for $\pi r h$ in expression for A.

$A = \pi r^2 + \frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A = \pi r^2 + \frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

d. $A = \pi r^2 + 2\pi r \left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*(ft)*(M1)*** for rearranging part (c) to $\pi r h = \frac{500\,000}{r}$ and substituting for $\pi r h$ in expression for A.

$A = \pi r^2 + \frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A = \pi r^2 + \frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

e. $2\pi r - \frac{1\,000\,000}{r^2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi r$, ***(A1)*** for $\frac{1}{r^2}$ or r^{-2}, ***(A1)*** for $-1\,000\,000$.

[3 marks]

f. $2\pi r - \frac{1\,000\,000}{r^2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (e) to zero.

$r^3 = \frac{1\,000\,000}{2\pi}$ **OR** $r = \sqrt[3]{\frac{1\,000\,000}{2\pi}}$ ***(M1)***

Note: Award ***(M1)*** for isolating r.

OR

sketch of derivative function ***(M1)***

with its zero indicated ***(M1)***

$(r =)\ 54.2\ (\text{cm})\ (54.1926\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 marks]

g. $\pi(54.1926\ldots)^2 + \frac{1\,000\,000}{(54.1926\ldots)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (f) into the given equation.

$= 27\,700\ (\text{cm}^2)\ (27\,679.0\ldots)$ ***(A1)*****(ft)*****(G2)***

[2 marks]

h. $\frac{27\,679.0\ldots}{2000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (g) by 2000.

$= 13.8395\ldots$ ***(A1)*****(ft)**

Notes: Follow through from part (g).

14 (cans) ***(A1)*****(ft)*****(G3)***

Notes: Final ***(A1)*** awarded for rounding up their $13.8395\ldots$ to the next integer.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

On Monday Paco goes to a running track to train. He runs the first lap of the track in 120 seconds. Each lap Paco runs takes him 10 seconds longer than his previous lap.

a. Find the time, in seconds, Paco takes to run his fifth lap. [3]

b. Paco runs his last lap in 260 seconds. [3]

Find how many laps he has run on Monday.

c. Find the **total** time, in **minutes,** run by Paco on Monday. [4]

d. On Wednesday Paco takes Lola to train. They both run the first lap of the track in 120 seconds. Each lap Lola runs takes 1.06 times as long as her previous lap. [3]

Find the time, in seconds, Lola takes to run her third lap.

e. Find the **total** time, in seconds, Lola takes to run her first four laps. [3]

f. Each lap Paco runs again takes him 10 seconds longer than his previous lap. After a certain number of laps Paco takes less time per lap than Lola. [3]

Find the number of the lap when this happens.

Markscheme

a. $120 + 10 \times 4$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted AP formula, ***(A1)*** for correct substitutions. Accept a list of 4 correct terms.

= 160 ***(A1)(G3)***

b. $120 + (n - 1) \times 10 = 260$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correctly substituted AP formula, ***(M1)*** for equating to 260. Accept a list of correct terms showing at least the 14^{th} and 15^{th} terms.

= 15 ***(A1)(G2)***

c. $\frac{15}{2}(120 + 260)$ **or** $\frac{15}{2}(2 \times 120 + (15 - 1) \times 10)$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substituted AP sum formula, ***(A1)*(ft)** for correct substitutions. Accept a sum of a list of 15 correct terms. Follow through from their answer to part (b).

2850 seconds ***(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for 2850 seen with no working shown.

47.5 minutes ***(A1)*(ft)*(G3)***

Notes: A final ***(A1)*(ft)** can be awarded for correct conversion from seconds into minutes of their incorrect answer. Follow through from their answer to part (b).

d. $120 \times 1.06^{3-1}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted GP formula, ***(A1)*** for correct substitutions. Accept a list of 3 correct terms.

= 135 (134.832) ***(A1)(G2)***

e. $S_4 = \frac{120(1.06^4 - 1)}{(1.06 - 1)}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted GP sum formula, ***(A1)*** for correct substitutions. Accept a sum of a list of 4 correct terms.

= 525 (524.953...) ***(A1)(G2)***

f. $120 + (n - 1) \times 10 < 120 \times 1.06^{n-1}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct left hand side, ***(M1)*** for correct right hand side. Accept an equation. Follow through from their expressions given in parts (a) and (d).

OR

List of at least 2 terms for both sequences (120, 130, … and 120, 127.2, …) ***(M1)***

List of correct 12$^{\text{th}}$ and 13$^{\text{th}}$ terms for both sequences (..., 230, 240 and …, 227.8, 241.5) ***(M1)***

OR

A sketch with a line and an exponential curve, ***(M1)***

An indication of the correct intersection point ***(M1)***

13$^{\text{th}}$ lap ***(A1)*(ft)*(G2)***

Note: Do not award the final ***(A1)*(ft)** if final answer is not a positive integer.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
[N/A]

f.

Consider the function $f(x) = x^3 + \frac{48}{x}, x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f . [2]

f. Find the range of f . [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x-coordinate of this point.

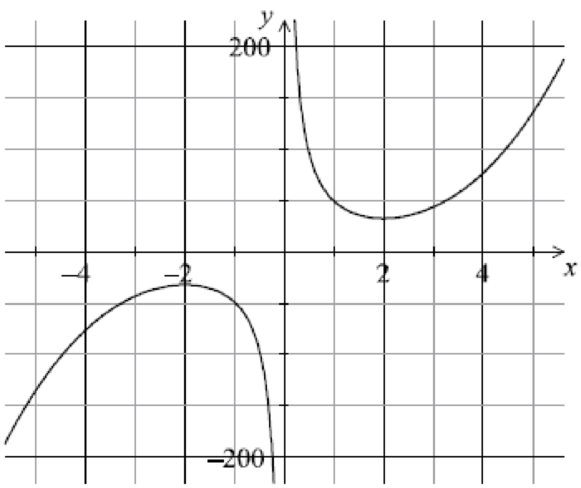
Markscheme

a. $f(2) = 2^3 + \frac{48}{2}$ ***(M1)***

$= 32$ ***(A1)(G2)***

[2 marks]

b.



(A1) for labels and some indication of scale in an appropriate window
(A1) for correct shape of the two unconnected and smooth branches
(A1) for maximum and minimum in approximately correct positions
(A1) for asymptotic behaviour at y-axis ***(A4)***

Notes: Please be rigorous.
The axes need not be drawn with a ruler.
The branches must be smooth: a single continuous line that does not deviate from its proper direction.
The position of the maximum and minimum points must be symmetrical about the origin.
The y-axis must be an asymptote for both branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

c. $f'(x) = 3x^2 - \frac{48}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $3x^2$, ***(A1)*** for -48 , ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

d. $f'(2) = 3(2)^2 - \frac{48}{(2)^2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of $x = 2$ into their derivative.

$= 0$ ***(A1)*****(ft)*****(G1)***

[2 marks]

e. $(-2,\ -32)$ or $x = -2, y = -32$ ***(G1)(G1)***

Notes: Award ***(G0)(G0)*** for $x = -32, y = -2$. Award at most ***(G0)(G1)*** if parentheses are omitted.

[2 marks]

f. $\{y \geqslant 32\} \cup \{y \leqslant -32\}$ ***(A1)(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*****(ft)** $y \geqslant 32$ or $y > 32$ seen, ***(A1)*****(ft)** for $y \leqslant -32$ or $y < -32$, ***(A1)*** for weak (non-strict) inequalities used in both of the above.
Accept use of f in place of y. Accept alternative interval notation.
Follow through from their (a) and (e).
If domain is given award ***(A0)(A0)(A0)***.
Award ***(A0)(A1)*****(ft)*****(A1)*****(ft)** for $[-200,\ -32]$, $[32,\ 200]$.
Award ***(A0)(A1)*****(ft)*****(A1)*****(ft)** for $]-200,\ -32]$, $[32,\ 200[$.

[3 marks]

g. $f'(1) = -45$ ***(M1)(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for $f'(1)$ seen or substitution of $x = 1$ into their derivative. Follow through from their derivative if working is seen.

[2 marks]

h. $x = -1$ ***(M1)(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for equating their derivative to their -45 or for seeing parallel lines on their graph in the approximately correct position.

[2 marks]

Examiners report

a. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the "window".

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

b. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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It was also evident that some centres do not teach the differential calculus.

f. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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It was also evident that some centres do not teach the differential calculus.

g. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

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h. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

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The Brahma chicken produces eggs with weights in grams that are normally distributed about a mean of 55 g with a standard deviation of 7 g. The eggs are classified as small, medium, large or extra large according to their weight, as shown in the table below.

Size	Weight (g)
Small	Weight < 53
Medium	53 ≤ Weight < 63
Large	63 ≤ Weight < 73
Extra Large	Weight ≥ 73

a. Sketch a diagram of the distribution of the weight of Brahma chicken eggs. On your diagram, show clearly the boundaries for the classification of the eggs. [3]

b. An egg is chosen at random. Find the probability that the egg is [4]

(i) medium;

(ii) extra large.

c. There is a probability of 0.3 that a randomly chosen egg weighs more than w grams. [2]

Find w .

d. The probability that a Brahma chicken produces a large size egg is 0.121. Frank's Brahma chickens produce 2000 eggs each month. [2]

Calculate an estimate of the number of large size eggs produced by Frank's chickens each month.

e. The selling price, in US dollars (USD), of each size is shown in the table below. [3]

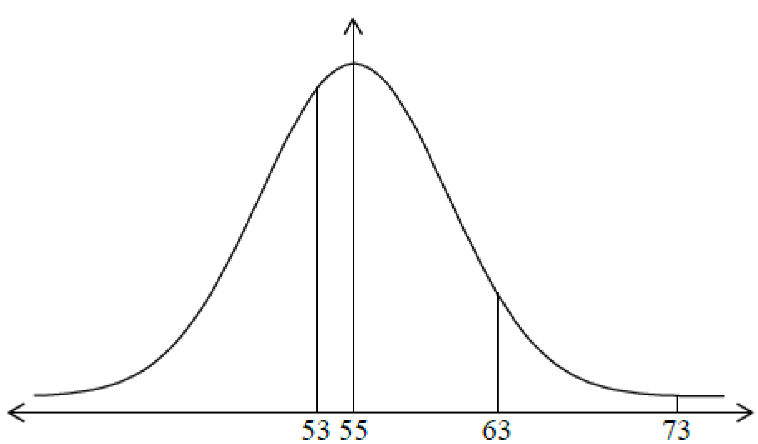
Size	Selling price (USD)
Small	0.30
Medium	0.50
Large	0.65
Extra Large	0.80

The probability that a Brahma chicken produces a small size egg is 0.388.

Estimate the monthly income, in USD, earned by selling the 2000 eggs. Give your answer correct to two decimal places.

Markscheme

a.



(A1) for normal curve with mean of 55 indicated
(A1) for three lines in approximately the correct position
(A1) for labels on the three lines ***(A1)(A1)(A1)***

b. (i) $\mathrm{P}(53 \leqslant \text{Weight} < 63) = 0.486$ $(0.485902\ldots)$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for correct region indicated on labelled diagram.

(ii) $\mathrm{P}(\text{Weight} > 73) = 0.00506$ (0.00506402) ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for correct region indicated on labelled diagram.

c. $\mathrm{P}(\text{Weight} > w) = 0.3$ ***(M1)***

$w = 58.7$ $(58.6708\ldots)$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct region indicated on labelled diagram.

d. Expected number of large size eggs

$= 2000(0.121)$ ***(M1)***
$= 242$ ***(A1)(G2)***

e. Expected income

$= 2000 \times 0.30 \times 0.388 + 2000 \times 0.50 \times 0.486 + 2000 \times 0.65 \times 0.121 + 2000 \times 0.80 \times 0.00506$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correct products, ***(M1)*** for addition of 4 terms.

$= 884.20 \text{ USD}$ ***(A1)*(ft)*(G3)***

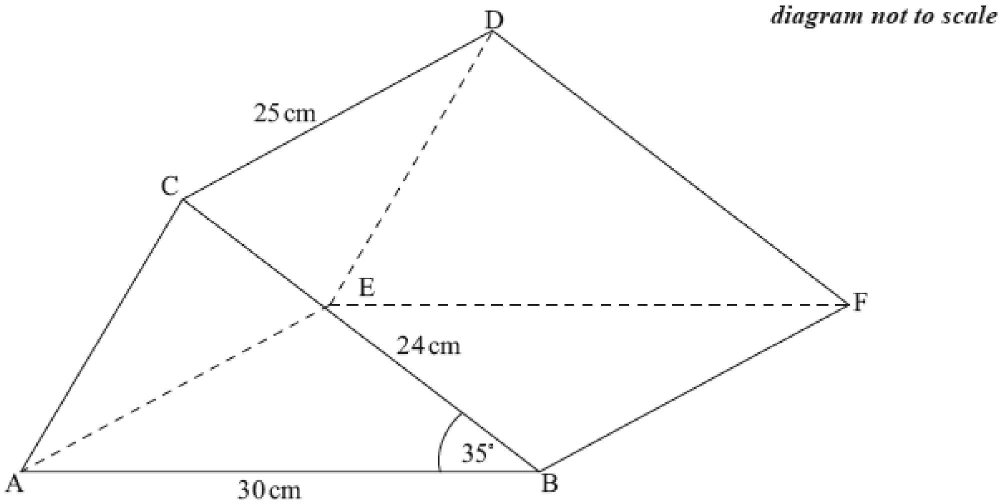
Note: Follow through from part (b).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.

$\mathrm{AB} = 30 \text{ cm},\ \mathrm{BC} = 24 \text{ cm},\ \mathrm{CD} = 25 \text{ cm}$ and angle $\mathrm{A\hat{B}C} = 35^\circ$.



a. Calculate the length of AC. [3]

b. Calculate the area of triangle ABC. [3]

c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\ \mathrm{cm}^3$, correct to three significant figures. [2]

d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Show that the total surface area of one block is $2190\ \mathrm{cm}^2$, correct to three significant figures. [3]

f. The blocks are to be painted. One litre of paint will cover $22\ \mathrm{m}^2$. [3]

Calculate the number of litres required to paint all 2600 blocks.

Markscheme

a. $\mathrm{AC}^2 = 30^2 + 24^2 - 2 \times 30 \times 24 \times \cos 35^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula,
(A1) for correct substitutions.

$\mathrm{AC} = 17.2\ \mathrm{cm} \quad (17.2168\ldots)$ ***(A1)(G2)***

Notes: Use of radians gives $52.7002\ldots$ Award ***(M1)(A1)(A0)***.
No marks awarded in this part of the question where candidates assume that angle $\mathrm{ACB} = 90^\circ$.

[3 marks]

b. ***Units are required in part (b).***

Area of triangle $\mathrm{ABC} = \frac{1}{2} \times 24 \times 30 \times \sin 35^\circ$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution into area formula, ***(A1)*** for correct substitutions.

Special Case: Where a candidate has assumed that angle $\text{ACB} = 90^\circ$ in part (a), award ***(M1)(A1)*** for a correct alternative substituted formula for the area of the triangle $\left(ie\ \frac{1}{2} \times \text{base} \times \text{height}\right)$.

$= 206\ \text{cm}^2 \quad (206.487\ldots\text{cm}^2)$ ***(A1)(G2)***

Notes: Use of radians gives negative answer, $-154.145\ldots$ Award ***(M1)(A1)(A0)***.

Special Case: Award ***(A1)*****(ft)** where the candidate has arrived at an area which is correct to the standard rounding rules from their lengths (units required).

[3 marks]

c. $206.487\ldots \times 25 \times 2600$ ***(M1)***

Note: Award ***(M1)*** for multiplication of their answer to part (b) by 25 and 2600.

$13\,421\,688.61$ ***(A1)***

Note: Accept unrounded answer of $13\,390\,000$ for use of 206.

$13\,400\,000$ ***(AG)***

Note: The final ***(A1)*** cannot be awarded unless both the unrounded and rounded answers are seen.

[2 marks]

d. 1.34×10^7 ***(A2)***

Notes: Award ***(A2)*** for the correct answer.

Award ***(A1)(A0)*** for 1.34 and an incorrect index value.

Award ***(A0)(A0)*** for any other combination (including answers such as 13.4×10^6).

[2 marks]

e. $2 \times 206.487\ldots + 24 \times 25 + 30 \times 25 + 17.2168\ldots \times 25$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplication of their answer to part (b) by 2 for area of two triangular ends, ***(M1)*** for three correct rectangle areas using 24, 30 and their 17.2.

$2193.26\ldots$ ***(A1)***

Note: Accept 2192 for use of 3 sf answers.

2190 ***(AG)***

Note: The final ***(A1)*** cannot be awarded unless both the unrounded and rounded answers are seen.

[3 marks]

f. $\frac{2190 \times 2600}{22 \times 10\,000}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for multiplication by 2600 and division by 22, ***(M1)*** for division by $10\,000$.

The use of 22 may be implied *ie* division by 2200 would be acceptable.

25.9 litres $(25.8818\ldots)$ ***(A1)(G2)***

Note: Accept 26.

[3 marks]

Examiners report

a. Some candidates assumed that triangle ACB was a right angled triangle with angle $\mathrm{ACB} = 90^\circ$. Such candidates earned no marks for part (a) but were able to recover most of the marks in the remainder of the question. For those candidates who correctly used the cosine rule for part (a), most achieved all 3 marks for this part and used a correct formula for the area of the triangle in part (b) to obtain at least 2 marks for this part. The final mark was not awarded, however, if no units or the incorrect units were given. Parts (c) and (e) were generally well done with many candidates showing the unrounded answer before the required answer. Part (f) proved to be quite problematic for many candidates. Whilst many were able to earn a method mark for $\frac{2190 \times 2600}{22}$, a significant number of these candidates were unable to convert the units correctly and very few correct answers were seen. Indeed, the most popular answer seemed to be 2590 litres.

b. Some candidates assumed that triangle ACB was a right angled triangle with angle $\mathrm{ACB} = 90^\circ$. Such candidates earned no marks for part (a) but were able to recover most of the marks in the remainder of the question. For those candidates who correctly used the cosine rule for part (a), most achieved all 3 marks for this part and used a correct formula for the area of the triangle in part (b) to obtain at least 2 marks for this part. The final mark was not awarded, however, if no units or the incorrect units were given. Parts (c) and (e) were generally well done with many candidates showing the unrounded answer before the required answer. Part (f) proved to be quite problematic for many candidates. Whilst many were able to earn a method mark for $\frac{2190 \times 2600}{22}$, a significant number of these candidates were unable to convert the units correctly and very few correct answers were seen. Indeed, the most popular answer seemed to be 2590 litres.

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Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

Markscheme

a. $\frac{-192}{x^3} + k$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for -192, ***(A1)*** for x^{-3}, ***(A1)*** for k (only).

b. at local minimum $f'(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for seeing $f'(x) = 0$ (may be implicit in their working).

$\frac{-192}{4^3} + k = 0$ ***(A1)***

$k = 3$ ***(AG)***

Note: Award ***(A1)*** for substituting $x = 4$ in their $f'(x) = 0$, provided it leads to $k = 3$. The conclusion $k = 3$ must be seen for the ***(A1)*** to be awarded.

c. $\frac{96}{2^2} + 3(2)$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in $f(x)$.

$= 30$ ***(A1)(G2)***

d. $\frac{-192}{2^3} + 3$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in their $f'(x)$.

$= -21$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a).

e. $y - 30 = \frac{1}{21}(x - 2)$ ***(A1)*****(ft)*****(M1)***

Notes: Award ***(A1)*****(ft)** for their $\frac{1}{21}$ seen, ***(M1)*** for the correct substitution of their point and their normal gradient in equation of a line.

Follow through from part (c) and part (d).

OR

gradient of normal $= \frac{1}{21}$ ***(A1)*****(ft)**

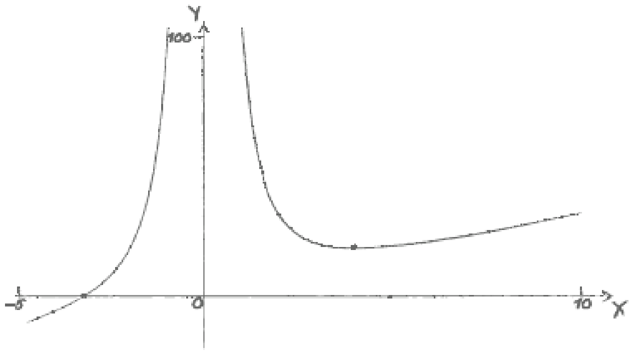
$30 = \frac{1}{21} \times 2 + c$ ***(M1)***

$c = 29\frac{19}{21}$

$y = \frac{1}{21}x + 29\frac{19}{21} \quad (y = 0.0476x + 29.904)$

$x - 21y + 628 = 0$ ***(A1)*****(ft)*****(G2)***

Notes: Accept equivalent answers.

f. 

(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; ***(A1)*** for a smooth curve with the correct shape (graph should not touch y-axis and should not curve away from the y-axis), on the given domain; ***(A1)*** for axis intercept in approximately the correct position (nearer -5 than zero); ***(A1)*** for local minimum in approximately the correct position (first quadrant, nearer the y-axis than $x = 10$).

If there is no scale, award a maximum of ***(A0)(A1)(A0)(A1)*** – the final ***(A1)*** being awarded for the zero and local minimum in approximately correct positions relative to each other.

g. $(-3.17,\ 0) \quad ((-3.17480\ldots,\ 0))$ ***(G1)(G1)***

Notes: If parentheses are omitted award ***(G0)(G1)*****(ft)**.

Accept $x = -3.17,\ y = 0$. Award ***(G1)*** for -3.17 seen.

h. $0 < x \leqslant 4$ or $0 < x < 4$ ***(A1)(A1)***

Notes: Award ***(A1)*** for correct end points of interval, ***(A1)*** for correct notation (note: lower inequality must be strict).

Award a maximum of ***(A1)(A0)*** if y or $f(x)$ used in place of x.

Examiners report

a. Differentiation of terms including negative indices remains a testing process; it will continue to be tested. There was, however, a noticeable improvement in responses compared to previous years. The parameter k was problematic for a number of candidates.

b. In part (b), the manipulation of the derivative to find the local minimum point caused difficulties for all but the most able; note that a GDC approach is not accepted in such questions and that candidates are expected to be able to apply the theory of the calculus as appropriate. Further, once a parameter is given, candidates are expected to use this value in subsequent parts.

c. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

d. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

e. Part (e) discriminated at the highest level; the gradient of the normal often was not used, the form of the answer not given correctly.

f. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

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h. Part (h) was not well attempted in the main; decreasing (and increasing) functions is a testing concept for the majority.

Cedric wants to buy an €8000 car. The car salesman offers him a loan repayment option of a 25 % deposit followed by 12 equal monthly payments of €600 .

a. Write down the amount of the deposit. [1]

b. Calculate the total cost of the loan under this repayment scheme. [2]

c. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €x in the first month and €y in every following month until the €8000 is repaid. [1]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation $x + 11y = 3500$. The total amount that Cedric's mother receives after **24** months is €7100.

Write down a second equation involving x and y.

d. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €x in the first month and €y in every following month until the €8000 is repaid. [2]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation $x + 11y = 3500$. The total amount that Cedric's mother receives after **24** months is €7100.

Write down the value of x and the value of y.

e. Cedric's mother decides to help him by giving him an interest free loan of €8000 to buy the car. She arranges for him to repay the loan by paying her €x in the first month and €y in every following month until the €8000 is repaid. [3]

The total amount that Cedric's mother receives after **12** months is €3500. This can be written using the equation $x + 11y = 3500$. The total amount that Cedric's mother receives after **24** months is €7100.

Calculate the number of months it will take Cedric's mother to receive the €8000.

f. Cedric decides to buy a cheaper car for €6000 and invests the remaining €2000 at his bank. The bank offers two investment options over three years. [5]

Option A: Compound interest at an annual rate of 8 %.

Option B: Compound interest at a nominal annual rate of 7.5 % , **compounded monthly**.

Express each answer in part (f) to the nearest euro.

Calculate the value of his investment at the end of three years if he chooses

(i) Option A;

(ii) Option B.

Markscheme

a. 2000 (euros) ***(A1)***

[1 mark]

b. $2000 + 12 \times 600$ ***(M1)***

Note: Award ***(M1)*** for addition of two correct terms.

9200 (euros) ***(A1)*****(ft)*****(G2)***

Note: Follow through from their part (a).

[2 marks]

c. $x + 23y = 7100$ ***(A1)***

[1 mark]

d. $x = 200$, $y = 300$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G2)***

[2 marks]

e. $200 + n \times 300 = 8000$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation. Follow through from their x and y found in part (d).

$n = 26$ ***(A1)*****(ft)***

26 + 1 = 27 (months) ***(A1)*****(ft)*****(G3)***

Notes: Middle line $n = 26$ may be implied if correct answer given. The final ***(A1)*****(ft)** is for adding 1 to their value of n (even if it is incorrect). Follow through from their part (d). If the final answer is not a positive integer award at most ***(M1)(A1)*****(ft)*****(A0)***. Award ***(G2)*** for final answer of 26.

OR

$\frac{8000-7100}{300} + 24$ ***(M1)(A1)***

Note: Award ***(M1)*** for division of difference by their value of y, ***(A1)*** for 24 seen.

27 (months) ***(A1)*****(ft)*****(G3)***

Note: Follow through from their value of y.

[3 marks]

f. (i) $2000\left(1 + \frac{8}{100}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in compound interest formula.

2519 (euros) ***(A1)(G2)***

Note: If the answer is not given to the nearest euro award at most ***(M1)(A0)***.

(ii) $2000\left(1+\frac{7.5}{100\times12}\right)^{3\times12}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitutions.

2503 (euros) ***(A1)(G2)***

Note: If the answer is not given to the nearest euro, award at most ***(M1)(A1)(A0)***, provided this has not been penalized in part (f)(i).

[5 marks]

Examiners report

a. (a) Most candidates managed to answer this correctly.

b. (b) On the whole this was well answered but some candidates gave 7200 as their final answer.

c. (c) Some candidates found this surprisingly difficult, others gave the answer as $x + 24y = 7100$.

d. (d) Many managed to find the correct answers for x and y even though their answer to part (c) was not correct. Others received follow through marks.

e. (e) The most common answer here was 26 months.

f. (f) Part (i) was well done but there were fewer correct answers seen for part (ii). Some candidates used 6000 instead of 2000, others did not give their answer to the nearest euro and others kept the same interest rate for both parts of the question.

The following table shows the average body weight, x, and the average weight of the brain, y, of seven species of mammal. Both measured in kilograms (kg).

Species	Average body weight, x (kg)	Average weight of the brain, y (kg)
Cat	3	0.026
Cow	465	0.423
Donkey	187	0.419
Giraffe	529	0.680
Goat	28	0.115
Jaguar	100	0.157
Sheep	56	0.175

The average body weight of grey wolves is 36 kg.

In fact, the average weight of the brain of grey wolves is 0.120 kg.

The average body weight of mice is 0.023 kg.

a. Find the range of the average body weights for these seven species of mammal. [2]

b.i. For the data from these seven species calculate r, the Pearson's product–moment correlation coefficient; [2]

b.ii. For the data from these seven species describe the correlation between the average body weight and the average weight of the brain. [2]

c. Write down the equation of the regression line y on x, in the form $y = mx + c$. [2]

d. Use your regression line to estimate the average weight of the brain of grey wolves. [2]

e. Find the percentage error in your estimate in part (d). [2]

f. State whether it is valid to use the regression line to estimate the average weight of the brain of mice. Give a reason for your answer. [2]

Markscheme

a. $529 - 3$ ***(M1)***

$= 526 \text{ (kg)}$ ***(A1)(G2)***

[2 marks]

b.i. $0.922\ (0.921857\ldots)$ ***(G2)***

[2 marks]

b.ii. (very) strong, positive ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Follow through from part (b)(i).

[2 marks]

c. $y = 0.000986x + 0.0923$ $(y = 0.000985837\ldots x + 0.0923391\ldots)$ ***(A1)(A1)***

Note: Award ***(A1)*** for $0.000986x$, ***(A1)*** for 0.0923.

Award a maximum of ***(A1)(A0)*** if the answer is not an equation in the form $y = mx + c$.

[2 marks]

d. $0.000985837\ldots(36) + 0.0923391\ldots$ ***(M1)***

Note: Award ***(M1)*** for substituting 36 into their equation.

$0.128\ (\mathrm{kg})\ \ (0.127829\ldots\ (\mathrm{kg}))$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c). The final ***(A1)*** is awarded only if their answer is positive.

[2 marks]

e. $\left|\frac{0.127829\ldots - 0.120}{0.120}\right| \times 100$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into percentage error formula.

$6.52\ (\%)\ \ (6.52442...\ (\%))$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (d). Do not accept a negative answer.

[2 marks]

f. Not valid ***(A1)***

the mouse is smaller/lighter/weighs less than the cat (lightest mammal) ***(R1)***

OR

as it would mean the mouse’s brain is heavier than the whole mouse ***(R1)***

OR

0.023 kg is outside the given data range. ***(R1)***

OR

Extrapolation ***(R1)***

Note: Do not award ***(A1)(R0)***. Do not accept percentage error as a reason for validity.

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

In a college 450 students were surveyed with the following results

150 have a television

205 have a computer

220 have an iPhone

75 have an iPhone and a computer

60 have a television and a computer

70 have a television and an iPhone

40 have all three.

a. Draw a Venn diagram to show this information. Use *T* to represent the set of students who have a television, *C* the set of students who have a computer and *I* the set of students who have an iPhone. [4]

b. Write down the number of students that [2]

(i) have a computer only;

(ii) have an iPhone and a computer but no television.

c. Write down $n[T \cap (C \cup I)']$. [1]

d. Calculate the number of students who have none of the three. [2]

e. Two students are chosen at random from the 450 students. Calculate the probability that [6]

(i) neither student has an iPhone;

(ii) only one of the students has an iPhone.

f. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3]

In the first 10 months they collect 13 050 dollars.

(i) Write down a second equation in *x* and *y* to represent this information.

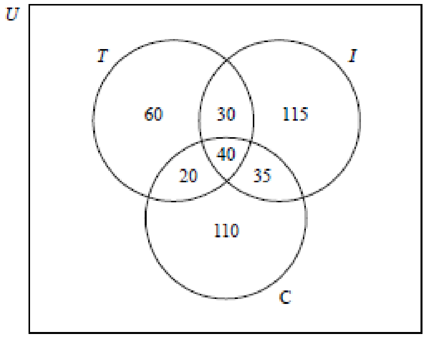
(ii) Write down the value of *x* and of *y* .

g. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$. [3]

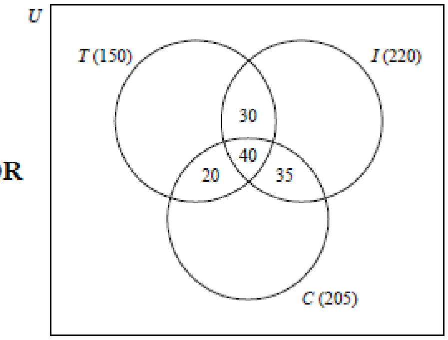
In the first 10 months they collect 13 050 dollars.

Calculate the number of months that it will take the students to collect 49 500 dollars.

Markscheme

a. 

OR



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labelled sets *T*, *C*, and *I* included inside an enclosed universal set. (Label *U* is not essential.) Award ***(A1)*** for central entry 40. ***(A1)*** for 20, 30 and 35 in the other intersecting regions. ***(A1)*** for 60, 110 and 115 or *T*(150), *C*(205), *I*(220).

[4 marks]

b. ***In parts (b), (c) and (d) follow through from their diagram.***

(i) 110 ***(A1)*(ft)**

(ii) 35 ***(A1)*(ft)**

[2 marks]

c. ***In parts (b), (c) and (d) follow through from their diagram.***

60 ***(A1)*(ft)**

[2 marks]

d. ***In parts (b), (c) and (d) follow through from their diagram.***

450 – (60 + 20 + 40 + 30 + 115 + 35 + 110) ***(M1)***

Note: Award ***(M1)*** for subtracting all their values from 450.

= 40 ***(A1)*(ft)*(G2)***

[2 marks]

e. (i) $\frac{230}{450} \times \frac{229}{449}$ ***(A1)(M1)***

Note: Award ***(A1)*** for correct fractions, ***(M1)*** for multiplying their fractions.

$\frac{52670}{202050} \left(\frac{5267}{20205}, \ 0.261, \ 26.1\% \right) (0.26067\ldots)$ ***(A1)(G2)***

Note: Follow through from their Venn diagram in part (a).

(ii) $\frac{220}{450} \times \frac{230}{449} + \frac{230}{450} \times \frac{220}{449}$ ***(A1)(A1)***

Note: Award ***(A1)*** for addition of their products, ***(A1)*** for two correct products.

OR

$\frac{230}{450} \times \frac{220}{449} \times 2$ ***(A1)(A1)***

Notes: Award ***(A1)*** for their product of two fractions multiplied by 2, ***(A1)*** for correct product of two fractions multiplied by 2. Award ***(A0)(A0)*** if correct product is seen not multiplied by 2.

$\frac{2024}{4041} (0.501,\ 50.1\%\ )(0.50086\ldots)$ ***(A1)(G2)***

Note: Follow through from their Venn diagram in part (a) and/or their 230 used in part (e)(i).

Note: For consistent use of replacement in parts (i) and (ii) award at most ***(A0)(M1)(A0)*** in part (i) and ***(A1)*(ft)*(A1)(A1)*(ft)** in part (ii).

[6 marks]

f. (i) $x + 9y = 13050$ ***(A1)***

(ii) $x = 900$ ***(A1)*(ft)**

$y = 1350$ ***(A1)*(ft)**

Notes: Follow through from their equation in (f)(i). Do not award ***(A1)*(ft)** if answer is negative. Award ***(M1)(A0)*** for an attempt at solving simultaneous equations algebraically but incorrect answer obtained.

[3 marks]

g. $49500 = 900 + 1350n$ ***(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for setting up correct equation. Follow through from candidate's part (f).

$n = 36$ ***(A1)*(ft)**

The total number of months is 37. ***(A1)*(ft)*(G2)***

Note: Award ***(G1)*** for 36 seen as final answer with no working. The value of n must be a positive integer for the last two ***(A1)*(ft)** to be awarded.

OR

$49500 = 900 + 1350(n - 1)$ ***(A2)*(ft)**

Notes: Award ***(A2)*(ft)** for setting up correct equation. Follow through from candidate's part (f).

$n = 37$ ***(A1)*(ft)*(G2)***

Note: The value of n must be a positive integer for the last ***(A1)*(ft)** to be awarded.

[3 marks]

Examiners report

a. The question was moderately well answered. The majority of candidates answered part (a) and at least parts of (b), and (d).

b. The question was moderately well answered. The majority of candidates answered part (a) and at least parts of (b), and (d).

c. The question was moderately well answered. Part (c) proved to be difficult, as it required understanding and interpreting set notation.

d. The question was moderately well answered. The majority of candidates answered part (a) and at least parts of (b), and (d).

e. The question was moderately well answered. Part (e) was rarely answered in its entirety.

f. The question was moderately well answered. Part (f) was answered by many candidates, but most of them offered a partial answer to part (g); a typical response was 36 instead of 37.

g. The question was moderately well answered. Part (f) was answered by many candidates, but most of them offered a partial answer to part (g); a typical response was 36 instead of 37.

Consider the function $f(x) = x^3 – 3x– 24x + 30$.

a. Write down $f(0)$. [1]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2]

d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. [5]

(ii) Hence or otherwise write down the coordinates of M and of N.

e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. [4]

f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. [6]

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

Markscheme

a. 30 ***(A1)***

[1 mark]

b. $f'(x) = 3x^2 – 6x – 24$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each term. Award at most ***(A1)(A1)*** if extra terms present.

[3 marks]

c. $f'(1) = –27$ ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for substituting $x = 1$ into their derivative.

[2 marks]

d. (i) $f'(x) = 0$

$3x^2 – 6x – 24 = 0$ ***(M1)***

$x = 4$; $x = –2$ ***(A1)*****(ft)*****(A1)*****(ft)***

Notes: Award ***(M1)*** for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen. Follow through from their derivative. Do not award the two answer marks if derivative not used.

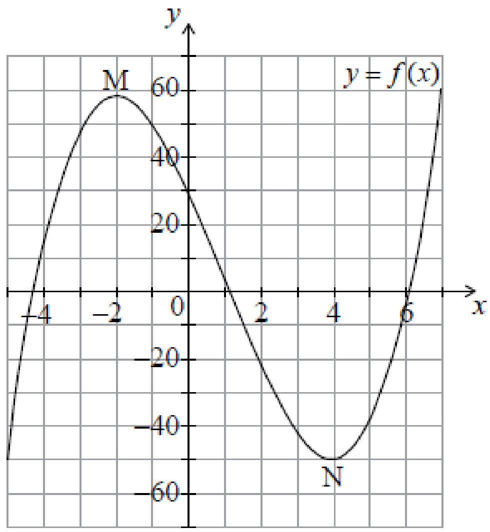
(ii) M(–2, 58) accept $x = -2$, $y = 58$ ***(A1)*(ft)**

N(4, – 50) accept $x = 4$, $y = -50$ ***(A1)*(ft)**

Note: Follow through from their answer to part (d) (i).

[5 marks]

e.



(A1) for window

(A1) for a smooth curve with the correct shape

(A1) for axes intercepts in approximately the correct positions

(A1) for M and N marked on diagram and in approximately correct position ***(A4)***

Note: If window is not indicated award at most ***(A0)(A1)(A0)(A1)*(ft)**.

[4 marks]

f. (i) $3x^2 - 6x - 24 = 21$ ***(M1)***

$3x^2 - 6x - 45 = 0$ ***(M1)***

$x = 5$; $x = -3$ ***(A1)*(ft)*(A1)*(ft)*(G3)***

Note: Follow through from their derivative.

OR

Award ***(A1)*** for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), ***(A1)*** for a second tangent parallel to their L_1, ***(A1)*** for $x = -3$, ***(A1)*** for $x = 5$. ***(A1)*(ft)*(A1)*(ft)*(A1)(A1)***

Note: If only $x = -3$ is shown without working award ***(G2)*.** If both answers are shown irrespective of workingaward ***(G3)***.

(ii) $f(5) = -40$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for attempting to find the image of their $x = 5$. Award ***(A1)*** only for (5, –40). Follow through from their x-coordinate of B only if it has **been clearly identified** in (f) (i).

[6 marks]

Examiners report

a. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

b. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

c. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

d. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

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f. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X) , bread (Y) and fruit (Z). It is found that

10 students had all three

17 students had bread and fruit only

15 students had cereal and fruit only

12 students had cereal and bread only

13 students had only bread

8 students had only cereal

9 students had only fruit

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

	3 or fewer meals per day	4 or 5 meals per day	More than 5 meals per day	Total
Male	15	25	15	55
Female	12	20	13	45
Total	27	45	28	100

A χ^2 test is carried out at the 5 % level of significance.

A.aRepresent this information on a Venn diagram. [4]

A.bFind the number of students who had none of the three choices for breakfast. [2]

A.cWrite down the percentage of students who had fruit for breakfast. [2]

A.dDescribe in words what the students in the set $X \cap Y'$ had for breakfast. [2]

A.eFind the probability that a student had **at least** two of the three choices for breakfast. [2]

A.f.Two students are chosen at random. Find the probability that both students had all three choices for breakfast. [3]

B.aWrite down the null hypothesis, H_0, for this test. [1]

B.bWrite down the number of degrees of freedom for this test. [1]

B.cWrite down the critical value for this test. [1]

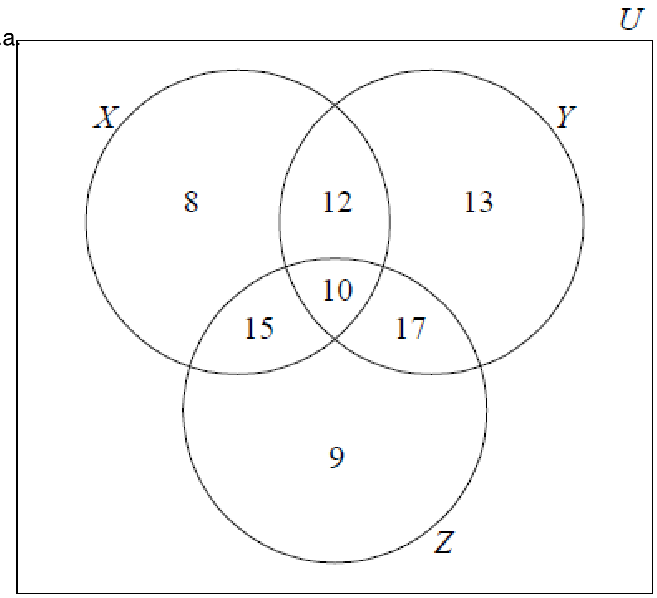
B.dShow that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer. [2]

B.eUse your graphic display calculator to find the χ^2_{calc} for this data. [2]

B.f.Decide whether H_0 must be accepted. Justify your answer. [2]

Markscheme

A.a.



(A1) for rectangle and three intersecting circles

(A1) for 10, ***(A1)*** for 8, 13 and 9, ***(A1)*** for 12, 15 and 17 ***(A4)***

[4 marks]

A.b. 100 – (9 +12 +13 +15 +10 +17 + 8) =16 ***(M1)(A1)*****(ft)*****(G2)***

Note: Follow through from their diagram.

[2 marks]

A.c. $\frac{51}{100}\,(0.51)$ ***(A1)*****(ft)**

= 51% ***(A1)*****(ft)*****(G2)***

Note: Follow through from their diagram.

[2 marks]

A.d. **Note:** The following statements are correct. Please note that the connectives are important. It is not the same (had cereal) and (not bread) and (had cereal) or (not bread). The parentheses are not needed but are there to facilitate the understanding of the propositions.

(had cereal) and (did not have bread)

(had cereal only) or (had cereal and fruit only)

(had either cereal or (fruit and cereal)) and (did not have bread) ***(A1)(A1)***

Notes: If the statements are correct but the connectives are wrong then award at most ***(A1)(A0)***. For the statement (had only cereal) and (cereal and fruit) award ***(A1)(A0)***. For the statement had cereal and fruit award ***(A0)(A0)***.

[2 marks]

A.e. $\frac{54}{100}\,(0.54,\ 54\ \%\ )$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*****(ft)** for numerator, follow through from their diagram, ***(A1)*****(ft)** for denominator. Follow through from total or denominator used in part (c).

[2 marks]

A.f. $\frac{10}{100} \times \frac{9}{99} = \frac{1}{110} (0.00909,\ 0.909\ \%)$ ***(A1)*(ft)*(M1)(A1)*(ft)*(G2)***

Notes: Award ***(A1)*(ft)** for their correct fractions, ***(M1)*** for multiplying two fractions, ***(A1)*(ft)** for their correct answer. Answer 0.009 with no working receives no marks. Follow through from denominator in parts (c) and (e) and from their diagram.

[3 marks]

B.a. H_0 : The (average) number of meals per day a student has and gender are independent ***(A1)***

Note: For “independent” accept “not associated” but do not accept “not related” or “not correlated”.

[1 mark]

B.b. 2 ***(A1)***

[1 mark]

B.c. 5.99 (accept 5.991) ***(A1)*(ft)**

Note: Follow through from their part (b).

[1 mark]

B.d. $\frac{28 \times 45}{100} = 12.6 = 13$ or $\frac{28}{100} \times \frac{25}{100} \times 100 = 12.6 = 13$ ***(M1)(A1)(AG)***

Notes: Award ***(M1)*** for correct formula and ***(A1)*** for correct substitution. Unrounded answer must be seen for the ***(A1)*** to be awarded.

[2 marks]

B.e. 0.0321 ***(G2)***

Note: For 0.032 award ***(G1)(G1)(AP)***. For 0.03 with no working award ***(G0)***.

[2 marks]

B.f. $0.0321 < 5.99$ or $0.984 > 0.05$ ***(R1)***

accept H_0 ***(A1)*(ft)**

Note: If reason is incorrect both marks are lost, do not award ***(R0)(A1)***.

[2 marks]

Examiners report

A.a. This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what Y was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100}$, $\frac{10}{100} \times \frac{10}{100}$ or $\frac{10}{100} + \frac{9}{99}$.

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B.a. In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use

of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the χ^2_{calc} through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

B.b.In general this part question was well answered. The major concerns of the examining team were the following:

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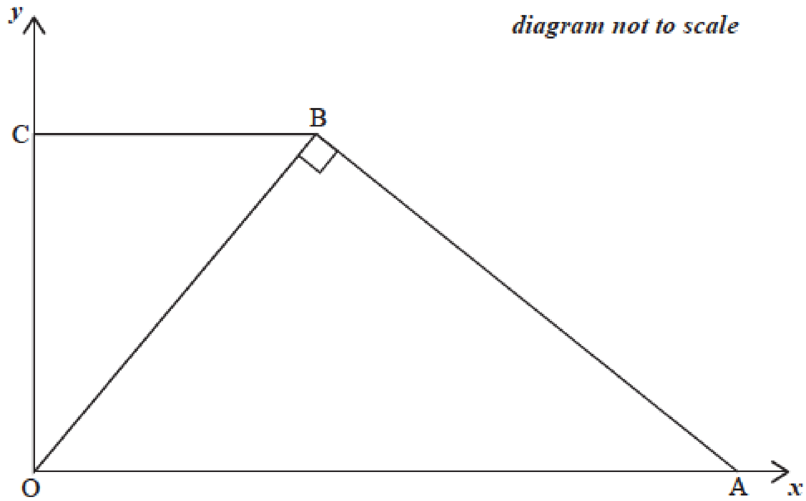
The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



a. The equation of the line BC is $y = 4$. [1]

Write down the coordinates of point C.

b. The x-coordinate of point B is a. [2]

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

c. Point A lies on the x-axis and the line AB is perpendicular to line OB. [4]

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

d. The area of triangle AOB is **three times** the area of triangle OBC. [3]

Find an expression, **in terms of *a***, for

(i) the area of triangle OBC;

(ii) the *x*-coordinate of point A.

e. Calculate the value of a. [2]

Markscheme

a. $(0,\ 4)$ ***(A1)***

Notes: Accept $x = 0,\ y = 4$.

b. (i) $(a,\ 4)$ ***(A1)*(ft)**

Notes: Follow through from part (a).

(ii) $\frac{4}{a}$ ***(A1)*(ft)**

Note: Follow through from part (b)(i).

c. (i) $-\frac{a}{4}$ ***(A1)*(ft)**

Note: Follow through from part (b)(ii).

(ii) $y=-\frac{a}{4}x+c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their gradient from part (c)(i) in the equation.

$4=-\frac{a}{4}\times a+c$

$c=\frac{1}{4}\times a^2+4$

$y=-\frac{a}{4}x+\frac{1}{4}a^2+4$ ***(A1)***

OR

$y-4=-\frac{a}{4}(x-a)$ ***(M1)***

Note: Award ***(M1)*** for substitution of their gradient from part (c)(i) in the equation.

$y=-\frac{ax}{4}+\frac{a^2}{4}+4$ ***(A1)***

$4y=-ax+a^2+16$

$4y+ax-a^2-16=0$ ***(AG)***

Note: Both the simplified and the not simplified equations must be seen for the final ***(A1)*** to be awarded.

d. (i) $2a$ ***(A1)***

(ii) $\frac{4x}{2}=3\times 2a$ ***(M1)***

Note: Award ***(M1)*** for correct equation.

$x=3a$ ***(A1)*(ft)**

Note: Follow through from part (d)(i).

OR

$0-4=-\frac{a}{4}(x-a)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their gradient and the coordinates of their point into the equation of a line.

$\frac{16}{a}=x-a$

$x=a+\frac{16}{a}$ ***(A1)*(ft)**

Note: Follow through from parts (b)(i) and (c)(i).

OR

$4\times 0+ax-a^2-16=0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of the coordinates of $\mathrm{A}(x,\ 0)$ into the equation of line AB.

$ax-a^2-16=0$

$x=a+\frac{16}{a}$ **OR** $x=\frac{(a^2+16)}{a}$ ***(A1)(G1)***

e. $4(0)+a(3a)-a^2-16=0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $3a$ from part (d)(ii) into the equation of line AB.

OR

$\frac{1}{2}\left(a + \frac{16}{a}\right) \times 4 = 3\left(\frac{4a}{2}\right)$ ***(M1)***

Note: Award ***(M1)*** for area of triangle AOB (with their substituted $a + \frac{16}{a}$ and 4) equated to three times their area of triangle AOB.

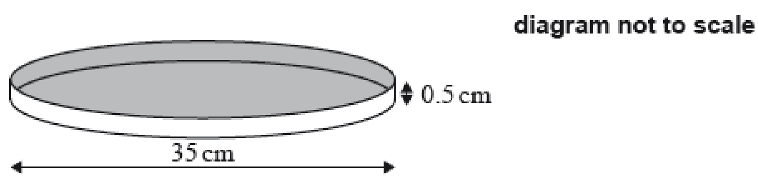
$a = 2.83 \quad \left(2.82842..., \ 2\sqrt{2}, \ \sqrt{8}\right)$ ***(A1)(ft)(G1)***

Note: Follow through from parts (d)(i) and (d)(ii).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.



A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19, \ t \geqslant 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

The pizza can be eaten once its temperature drops to 45 °C.

a. Calculate the volume of this pan. [3]

b. Find the radius of the sphere in cm, correct to one decimal place. [4]

c. Find the value of a. [2]

d. Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

e. Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

f. In the context of this model, state what the value of 19 represents. [1]

Markscheme

a. $(V =)\ \pi \times (17.5)^2 \times 0.5$ ***(A1)(M1)***

Notes: Award ***(A1)*** for 17.5 (or equivalent) seen.

Award ***(M1)*** for correct substitutions into volume of a cylinder formula.

$= 481\ \text{cm}^3\ (481.056\ldots\ \text{cm}^3,\ 153.125\pi\ \text{cm}^3)$ ***(A1)(G2)***

[3 marks]

b. $\frac{4}{3} \times \pi \times r^3 = 481.056\ldots$ ***(M1)***

Note: Award ***(M1)*** for equating **their** answer to part (a) to the volume of sphere.

$r^3 = \frac{3 \times 481.056\ldots}{4\pi}\ (= 114.843\ldots)$ ***(M1)***

Note: Award ***(M1)*** for correctly rearranging so r^3 is the subject.

$r = 4.86074\ldots\ (\text{cm})$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*** for correct unrounded answer seen. Follow through from part (a).

$= 4.9\ (\text{cm})$ ***(A1)*****(ft)*****(G3)***

Note: The final ***(A1)*****(ft)** is awarded for rounding their unrounded answer to one decimal place.

[4 marks]

c. $230 = a(2.06)^0 + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$a = 211$ ***(A1)(G2)***

[2 marks]

d. $(P =)\ 211 \times (2.06)^{-5} + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the function, $P(t)$. Follow through from part (c). The negative sign in the exponent is required for correct substitution.

$= 24.7$ (°C) $(24.6878\ldots$ (°C)) ***(A1)*(ft)*(G2)***

[2 marks]

e. $45 = 211 \times (2.06)^{-t} + 19$ ***(M1)***

Note: Award ***(M1)*** for equating 45 to the exponential equation and for correct substitution (follow through for their a in part (c)).

$(t =)\ 2.89711\ldots$ ***(A1)*(ft)*(G1)***

$174\ (\text{seconds})\ \ (173.826\ldots\ (\text{seconds}))$ ***(A1)*(ft)*(G2)***

Note: Award final ***(A1)*(ft)** for converting their $2.89711\ldots$ minutes into seconds.

[3 marks]

f. the temperature of the (dining) room ***(A1)***

OR

the lowest final temperature to which the pizza will cool ***(A1)***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

---

a. Antonio and Barbara start work at the same company on the same day. They each earn an annual salary of 8000 euros during the first year of employment. The company gives them a salary increase following the completion of each year of employment. Antonio is paid using plan A and Barbara is paid using plan B. [3]

Plan A: The annual salary increases by 450 euros each year.

Plan B: The annual salary increases by $5\,\%$ each year.

Calculate

i) Antonio’s annual salary during his second year of employment;

ii) Barbara’s annual salary during her second year of employment.

b. Write down an expression for [4]

i) Antonio’s annual salary during his n th year of employment;

ii) Barbara’s annual salary during her n th year of employment.

c. Determine the number of years for which Antonio’s annual salary is greater than or equal to Barbara’s annual salary. [2]

d. Both Antonio and Barbara plan to work at the company for a total of 15 years. [7]

i) Calculate the **total amount** that **Barbara** will be paid during these 15 years.

ii) Determine whether Antonio earns more than Barbara during these 15 years.

Markscheme

a. i) $8450\,(\text{euro})$ ***(A1)***

ii) 8000×1.05 ***(M1)***

Note: Award ***(M1)*** for 8000×1.05 **OR** $(8000 \times 0.05) + 8000$.

$= 8400\,(\text{euro})$ ***(A1)(G3)***

b. i) $8000 + 450\ (n-1)\ \ (\text{accept } 450\,n + 7550)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in arithmetic sequence formula; ***(A1)*** for correct substitutions.

ii) $8000 \times 1.05^{(n-1)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in arithmetic sequence formula; ***(A1)*** for correct substitutions.

c. $8000 + 450\,(n-1) \geqslant 8000 \times 1.05^{n-1}$ ***(M1)***

Note: Award ***(M1)*** for setting a correct inequality using their expressions for (b)(i) and (b)(ii). Accept an equation.

OR

list of at least 4 correct terms of each sequence ***(M1)***

Note: Award ***(M1)*** for correct lists corresponding to their answers for parts (b)(i) and (b)(ii).

6 ***(A1)*(ft)*(G2)***

Note: Value must be an integer for the final ***(A1)*** to be awarded. Follow through from parts (b)(i) and (b)(ii). Award ***(G1)*** for a final answer of $6.70018\ldots$ seen without working.

d. i) $S_{15} = \frac{8000 \times (1.05^{15} - 1)}{1.05 - 1}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into geometric series formula and ***(A1)*** for correct substitution of u_1 and their r from part (b)(ii). Follow through from part (b)(ii).

OR

$8000 + 8400 + 8820\ldots + 15839.45$ ***(M1)(A1)*(ft)**

Note: Follow through from part (b)(ii).

$= 173\,000\,(\text{euro})\ \ (172629\ldots)$ ***(A1)*(ft)*(G2)***

ii) $S_{15} = \frac{15}{2}(2 \times 8000 + 450 \times 14)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into arithmetic series formula and ***(A1)*** for correct substitution, using their first term and their last term from part (b)(i), or their u_1 and d. Follow through from part (b)(i).

OR

$8000 + 8450 + 8900\ldots + 14300$ ***(M1)(A1)*(ft)**

Note: Follow through from part (b)(i).

$= 167\,000\,(\text{euro})\ \ (167\,250)$ ***(A1)*(ft)*(G2)***

Antonio does not earn more than Barbara

(his total salary will be less than Barbara's) ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for a final answer that is consistent with their part (d)(i) and (d)(ii). Accept "Barbara earns more". The final ***(A1)*** can only be awarded if two total salaries are seen.

Examiners report

a. Question 5: Arithmetic and Geometric progression

Most candidates calculated the salaries in the second year correctly. The most common error was to calculate the salaries for the third instead of the second year. In part (b) the use of n instead of $n-1$ was very common. For the geometric sequence often a ratio of 0.05 instead of 1.05 was used. Also many of the expressions given did not represent a geometric sequence. Candidates who used a list for part (c) did usually better than the ones that tried to solve an equation. In part (d) the sum of the arithmetic progression was done better than the geometric series. Many candidates calculated the 15th term of the progression and not the series. In general this question part was not answered well.

b. Question 5: Arithmetic and Geometric progression

Most candidates calculated the salaries in the second year correctly. The most common error was to calculate the salaries for the third instead of the second year. In part (b) the use of n instead of $n-1$ was very common. For the geometric sequence often a ratio of 0.05 instead of 1.05 was used. Also many of the expressions given did not represent a geometric sequence. Candidates who used a list for part (c) did usually better than the ones that tried to solve an equation. In part (d) the sum of the arithmetic progression was done better than the geometric series. Many candidates calculated the 15th term of the progression and not the series. In general this question part was not answered well.

c. Question 5: Arithmetic and Geometric progression

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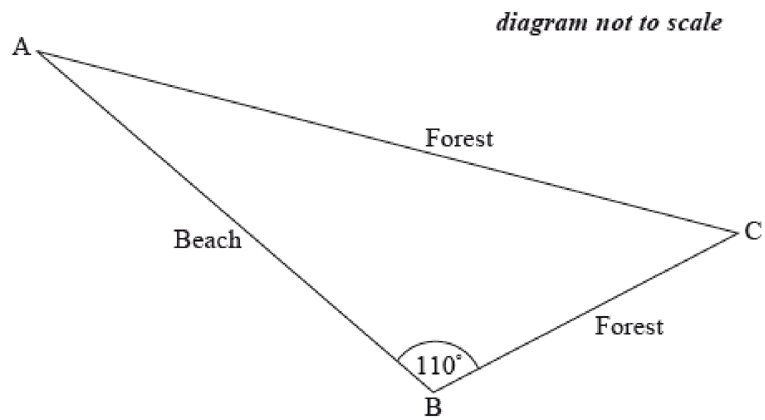
d. Question 5: Arithmetic and Geometric progression

Most candidates calculated the salaries in the second year correctly. The most common error was to calculate the salaries for the third instead of the second year. In part (b) the use of n instead of $n-1$ was very common. For the geometric sequence often a ratio of 0.05 instead of 1.05 was used. Also many of the expressions given did not represent a geometric sequence. Candidates who used a list for part (c) did usually better than the ones that tried to solve an equation. In part (d) the sum of the arithmetic progression was done better than the geometric series. Many candidates calculated the 15th term of the progression and not the series. In general this question part was not answered well.

A cross-country running course consists of a beach section and a forest section. Competitors run from A to B, then from B to C and from C back to A.

The running course from A to B is along the beach, while the course from B, through C and back to A, is through the forest.
The course is shown on the following diagram.



Angle ABC is 110°.

It takes Sarah 5 minutes and 20 seconds to run from A to B at a speed of $3.8\ \text{ms}^{-1}$.

a. Using '*distance* = *speed* $\times$ *time*', show that the distance from A to B is 1220 metres correct to 3 significant figures. [2]

b. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [1]

Calculate the speed, in ms^{-1}, that Sarah runs from B to C.

c. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the distance, in metres, from **$\mathbf{C}$ to $\mathbf{A}$**.

d. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [2]

Calculate the total distance, in metres, of the cross-country running course.

e. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Find the size of angle BCA.

f. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the area of the cross-country course bounded by the lines AB, BC and CA.

Markscheme

a. 3.8×320 ***(A1)***

Note: Award ***(A1)*** for 320 or equivalent seen.

$= 1216$ ***(A1)***

$= 1220\ (\text{m})$ ***(AG)***

Note: Both unrounded and rounded answer must be seen for the final ***(A1)*** to be awarded.

[2 marks]

b. $\frac{850}{303}\ (\text{ms}^{-1})\ (2.81, 2.80528\ldots)$ ***(A1)(G1)***

[1 mark]

c. $\mathrm{AC}^2 = 1220^2 + 850^2 - 2(1220)(850)\cos 110°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitutions.

$\mathrm{AC} = 1710\ (\mathrm{m})\ (1708.87\ldots)$ ***(A1)(G2)***

Notes: Accept $1705\ (1705.33\ldots)$.

[3 marks]

d. $1220 + 850 + 1708.87\ldots$ ***(M1)***

$= 3780\ (\mathrm{m})\ (3778.87\ldots)$ ***(A1)*****(ft)*****(G1)***

Notes: Award ***(M1)*** for adding the three sides. Follow through from their answer to part (c). Accept $3771\ (3771.33\ldots)$.

[2 marks]

e. $\frac{\sin C}{1220} = \frac{\sin 110°}{1708.87\ldots}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*****(ft)** for correct substitutions. Follow through from their part (c).

$C = 42.1°\ (42.1339\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Accept $41.9°, 42.0°, 42.2°, 42.3°$.

OR

$\cos C = \frac{1708.87\ldots^2 + 850^2 - 1220^2}{2 \times 1708.87\ldots \times 850}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*****(ft)** for correct substitutions. Follow through from their part (c).

$C = 42.1°\ (42.1339\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Accept $41.2°, 41.8°, 42.4°$.

[3 marks]

f. $\frac{1}{2} \times 1220 \times 850 \times \sin 110°$ ***(M1)(A1)*****(ft)**

OR

$\frac{1}{2} \times 1708.87\ldots \times 850 \times \sin 42.1339\ldots°$ ***(M1)(A1)*****(ft)**

OR

$\frac{1}{2} \times 1220 \times 1708.87\ldots \times \sin 27.8661\ldots°$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into area formula, ***(A1)*(ft)** for correct substitution.

$= 487\,000\ \mathrm{m}^2\ (487\,230\ldots\ \mathrm{m}^2)$ ***(A1)*(ft)*(G2)***

Notes: The answer is $487\,000\ \mathrm{m}^2$, **units are required**.

Accept $486\,000\ \mathrm{m}^2\ (485\,633\ldots\ \mathrm{m}^2)$.

If workings are not shown and units omitted, award ***(G1)*** for $487\,000$ or $486\,000$.

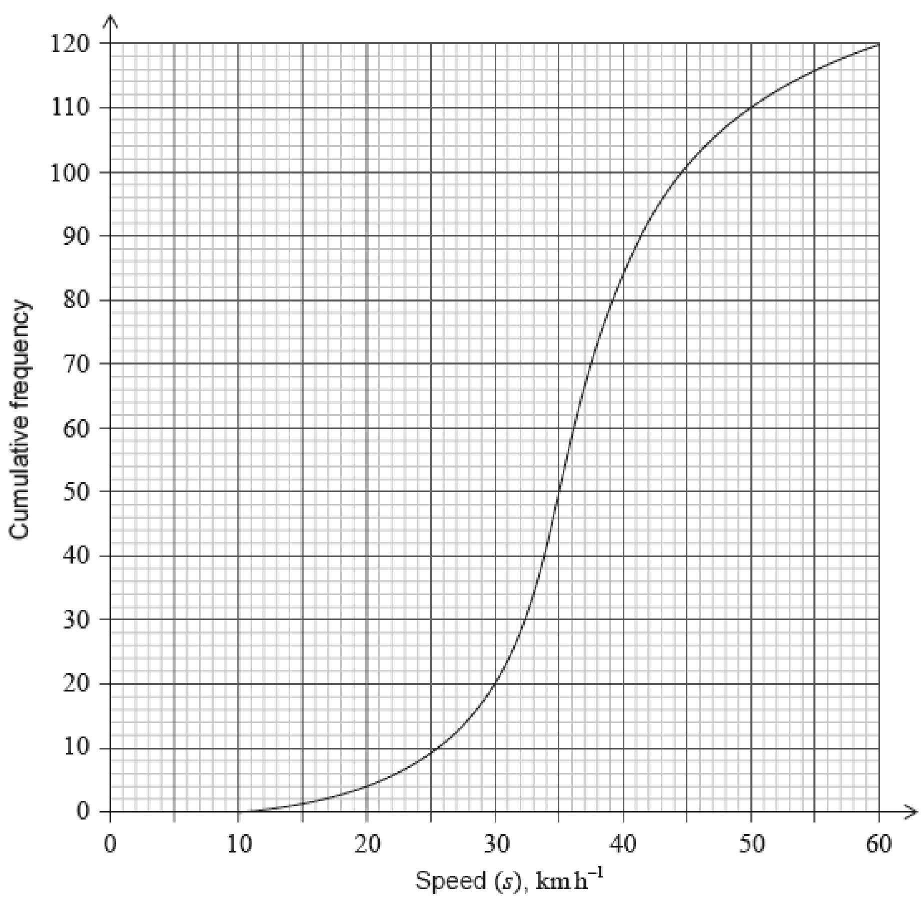
Follow through from parts (c) and (e).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The cumulative frequency graph shows the speed, s, in $\mathrm{km\,h^{-1}}$, of 120 vehicles passing a hospital gate.



a. Estimate the minimum possible speed of one of these vehicles passing the hospital gate. [1]

b. Find the median speed of the vehicles. [2]

c. Write down the 75^{th} percentile. [1]

d. Calculate the interquartile range. [2]

e. The speed limit past the hospital gate is $50\ \text{km h}^{-1}$. [2]

Find the number of these vehicles that exceed the speed limit.

f. The table shows the speeds of these vehicles travelling past the hospital gate. [2]

Speed of Vehicles	Number of Vehicles
$0 < s \le 10$	0
$10 < s \le 20$	p
$20 < s \le 30$	16
$30 < s \le 40$	64
$40 < s \le 50$	26
$50 < s \le 60$	q

Find the value of p and of q.

g. The table shows the speeds of these vehicles travelling past the hospital gate. [2]

Speed of Vehicles	Number of Vehicles
$0 < s \le 10$	0
$10 < s \le 20$	p
$20 < s \le 30$	16
$30 < s \le 40$	64
$40 < s \le 50$	26
$50 < s \le 60$	q

(i) Write down the modal class.

(ii) Write down the mid-interval value for this class.

h. The table shows the speeds of these vehicles travelling past the hospital gate. [3]

Speed of Vehicles	Number of Vehicles
$0 < s \le 10$	0
$10 < s \le 20$	p
$20 < s \le 30$	16
$30 < s \le 40$	64
$40 < s \le 50$	26
$50 < s \le 60$	q

Use your graphic display calculator to calculate an estimate of

(i) the mean speed of these vehicles;

(ii) the standard deviation.

i. It is proposed that the speed limit past the hospital gate is reduced to $40\ \text{km}\,\text{h}^{-1}$ from the current $50\ \text{km}\,\text{h}^{-1}$. [2]

Find the percentage of these vehicles passing the hospital gate that **do not** exceed the current speed limit but **would** exceed the new speed limit.

Markscheme

a. $10\ (\text{km}\,\text{h}^{-1})$ ***(A1)***

b. 36 ***(G2)***

c. 41.5 ***(G1)***

d. $41.5 - 32.5$ ***(M1)***

$= 9\ (\pm 1)$ ***(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for quartiles seen. Follow through from part (c).

e. $120 - 110$ ***(M1)***

$= 10$ ***(A1)(G2)***

Note: Award ***(M1)*** for 110 seen.

f. $p = 4 \quad q = 10$ ***(A1)*(ft)*(A1)*(ft)**

Note: Follow through from part (e).

g. (i) $30 < s \leqslant 40$ ***(A1)***

(ii) 35 ***(A1)*(ft)**

Note: Follow through from part (g)(i).

h. (i) $36.8\ (\text{km}\,\text{h}^{-1})\quad (36.8333)$ ***(G2)*(ft)**

Notes: Follow through from part (f).

(ii) $8.85\quad (8.84904\ldots)$ ***(G1)*(ft)**

Note: Follow through from part (f), irrespective of working seen.

i. $\frac{26}{120} \times 100$ ***(M1)***

Note: Award ***(M1)*** for $\frac{26}{120} \times 100$ seen.

$= 21.7\ (\%)\quad \left(21.6666\ldots,\ 21\frac{2}{3},\ \frac{65}{3}\right)$ ***(A1)(G2)***

Examiners report

a. For the great majority, this was a straightforward and accessible question. There were many, however, who had no appreciation of medians, percentiles and quartiles – all straightforward concepts. Most were able to read from the graph, using correctly the scales; only the weakest misinterpreting these. Calculation of the mean and standard deviation are expected to be completed using the graphic display calculator (GDC) – formulae are no longer required and the covariance will **not** be given in questions. Many candidates, however, were unable to calculate the mean and standard deviation of a (grouped) frequency distribution, instead treating the data as raw; comments on the G2 forms from schools indicated that some teachers were also unable to do this and advice must be sought.

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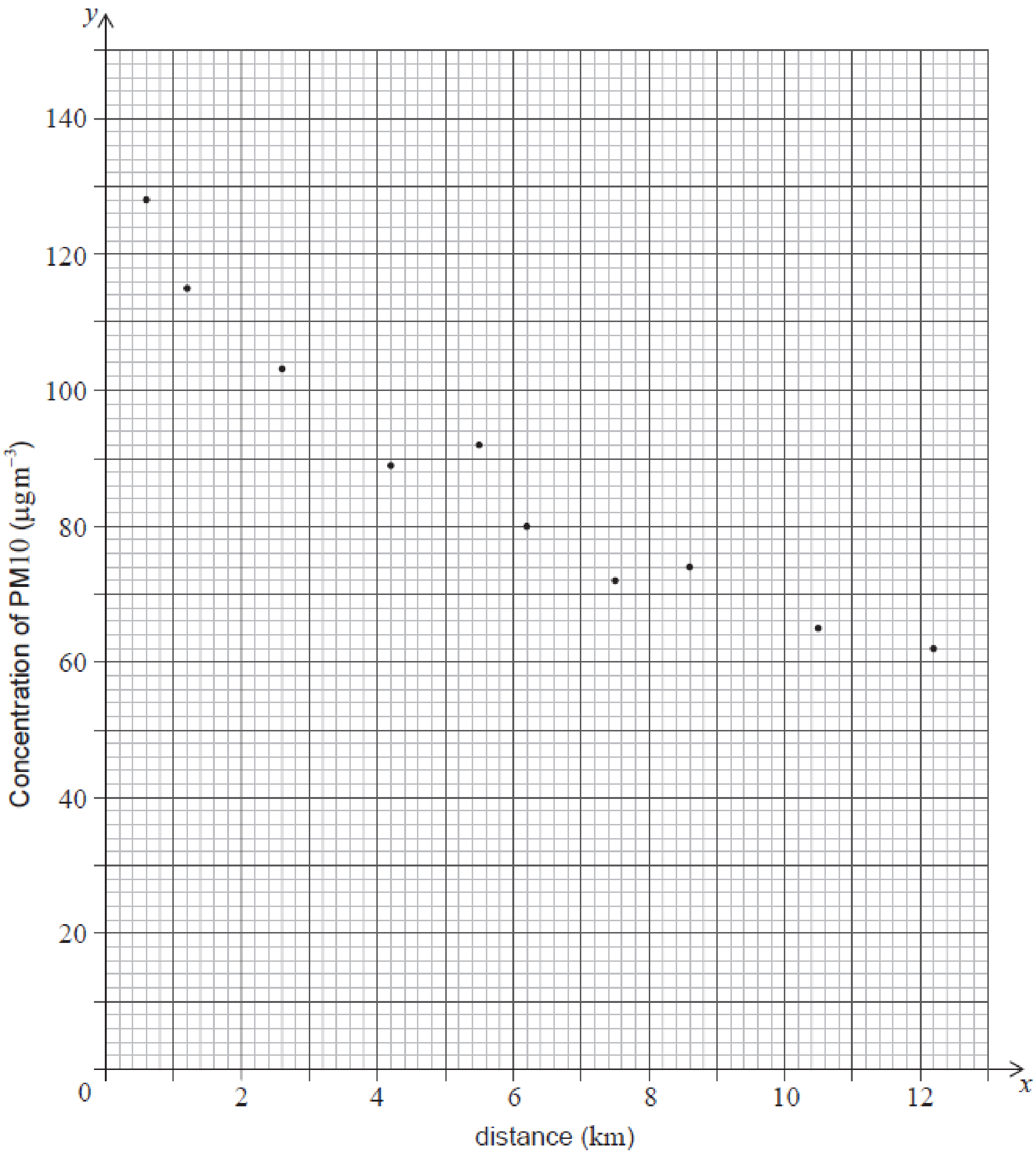
a. For an ecological study, Ernesto measured the average concentration (y) of the fine dust, $\mathrm{PM10}$, in the air at different distances (x) from a power plant. His data are represented on the following scatter diagram. The concentration of $\mathrm{PM10}$ is measured in micrograms per cubic metre and the distance is measured in kilometres. [2]



His data are also listed in the following table.

Distance (x)	0.6	1.2	2.6	a	5.5	6.2	7.5	8.6	10.5	12.2
Concentration of PM10 (y)	128	115	103	89	92	80	72	b	65	62

Use the scatter diagram to find the value of a and of b in the table.

b. Calculate [4]

i) $\bar{x}$, the mean distance from the power plant;

ii) $\bar{y}$, the mean concentration of $\mathrm{PM10}$;

iii) r , the Pearson's product–moment correlation coefficient.

c. Write down the equation of the regression line y on x . [2]

d. Ernesto's school is located $14\,\mathrm{km}$ from the power plant. He uses the equation of the regression line to estimate the concentration of $\mathrm{PM10}$ in the air at his school. [4]

i) Calculate the value of Ernesto's estimate.

ii) State whether Ernesto’s estimate is reliable. Justify your answer.

Markscheme

a. $a = 4.2\,;\ b = 74$ ***(A1)(A1)***

b. i) $5.91\ (\mathrm{km})$ ***(A1)*(ft)**

ii) 88 (micrograms per cubic metre) ***(A1)*(ft)**

Note: Follow through from part (a) irrespective of working seen.

iii) $-0.956\ \ (-0.955528\ldots)$ ***(G2)*(ft)**

Note: Follow through from part (a) irrespective of working seen.

c. $y = -5.39x + 120\ \ (y = -5.38955\ldots x + 119.852\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for -5.39. Award ***(A1)*(ft)** for 120. If answer is not an equation award at most ***(A1)*(ft)*(A0)***. Follow through from part (a) irrespective of working seen.

d. i) $-5.38955\ldots \times 14 + 119.852\ldots$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into their regression line.

$= 44.4\ (44.3984\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c). Accept $44.5\ \ (44.54)$ from use of 3 significant figure values.

ii) Ernesto’s estimate is not reliable ***(A1)***

this is extrapolation ***(R1)***

OR

$14\,\mathrm{km}$ is not within the range (outside the domain) of distances given ***(R1)***

Note: Do not accept “14 is too high” or “14 is an outlier” or “result not valid/not reliable” if explanation not given. Do not award ***(A1)(R0)***. Do not accept reasoning based on the strength of r.

Examiners report

a. Question 1: Reading scatter diagram, mean, correlation and regression line.

The majority of the candidates scored very well on this question. There were only a few candidates who read the diagram incorrectly. The most common mistake in parts (b), (c) and (d)(i) were rounding errors, sometimes resulting in candidates losing follow-through marks when working was not presented. Part (d)(ii) was answered incorrectly by most candidates. The most common incorrect answer was based on strong correlation. Some commented on the trend of decreasing PM10 values for increasing distances, showing lack of understanding about extrapolation.

b. Question 1: Reading scatter diagram, mean, correlation and regression line.

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d. Question 1: Reading scatter diagram, mean, correlation and regression line.

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Throughout this question *all* the numerical answers must be given correct to the nearest whole number.

a. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [4]

Find the number of students attending Park School in

(i) January 2001;

(ii) January 2003.

b. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [2]

Show that the number of students attending Park School in January 2007 is 150.

c. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [2]

Find the number of students attending Grove School in January 2003.

d. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [4]

Find the year in which the number of students attending Grove School will be first 60% **more than** in January 2000.

e. Each January, one of these two schools, the one that has more students, is given extra money to spend on sports equipment. [5]

(i) Decide which school gets the money in 2007. Justify your answer.

(ii) Find the first year in which Park School will be given this extra money.

Markscheme

a. (i) $100 \times 1.06 = 106$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for multiplying by 1.06 or equivalent. ***(A1)*** for correct answer.

(ii) $100 \times 1.06^3 = 119$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for multiplying by 1.06^3 or equivalent or for list of values. ***(A1)*** for correct answer.

[4 marks]

b. $100 \times 1.06^7 = 150.36\ldots = 150$ correct to the nearest whole ***(M1)(A1)(AG)***

Note: ***(M1)*** for correct formula or for list of values. ***(A1)*** for correct substitution or for 150 in the correct position in the list. Unrounded answer must be seen for the ***(A1)***.

[2 marks]

c. $110 + 3 \times 10 = 140$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for adding 30 or for list of values. ***(A1)*** for correct answer.

[2 marks]

d. **In (d) and (e) follow through from (c) if consistent wrong use of correct AP formula.**

$110 + (n-1) \times 10 > 176$ ***(A1)(M1)***

$n = 8 \therefore \text{year } 2007$ ***(A1)(A1)*****(ft)*****(G2)***

Note: ***(A1)*** for 176 or 66 seen. ***(M1)*** for showing list of values and comparing them to 176 or for equating formula to 176 or for writing the inequality. If $n = 8$ not seen can still get ***(A2)*** for 2007. Answer $n = 8$ with no working gets ***(G1)***.

OR

$110 + n \times 10 > 176$ ***(A1)(M1)***

$n = 7 \therefore \text{year } 2007$ ***(A1)(A1)*****(ft)*****(G2)***

[4 marks]

e. **In (d) and (e) follow through from (c) if consistent wrong use of correct AP formula.**

(i) 180 ***(A1)*****(ft)**

Grove School gets the money. ***(A1)*****(ft)**

Note: ***(A1)*** for 180 seen. ***(A1)*** for correct answer.

(ii) $100 \times 1.06^{n-1} > 110 + (n-1) \times 10$ ***(M1)***

$n = 20 \therefore \text{year } 2019$ ***(A1)(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for showing lists of values for each school and comparing them or for equating both formulae or writing the correct inequality. If $n = 20$ not seen can still get ***(A2)*** for 2019. Follow through with ratio used in (b) and/or formula used in (d).

OR

$100 \times 1.06^n > 110 + n \times 10$ ***(M1)***

$n = 19 \therefore \text{year } 2019$ ***(A1)(A1)*****(ft)*****(G2)***

OR

graphically

Note: ***(M1)*** for sketch of both functions on the same graph, ***(A1)*** for the intersection point, ***(A1)*** for correct answer.

[5 marks]

Examiners report

a. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

b. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

c. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

d. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

e. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

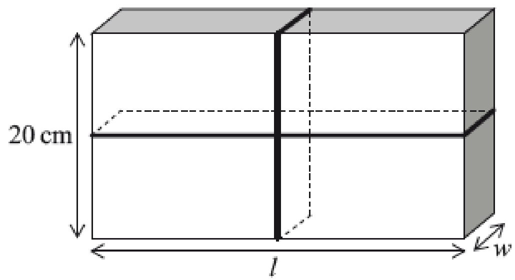
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is 3000 cm^3.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

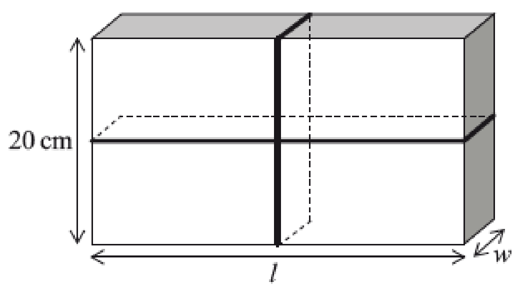
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

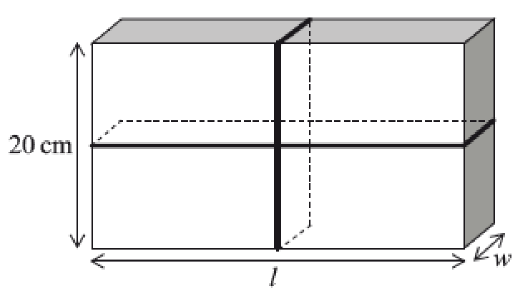
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



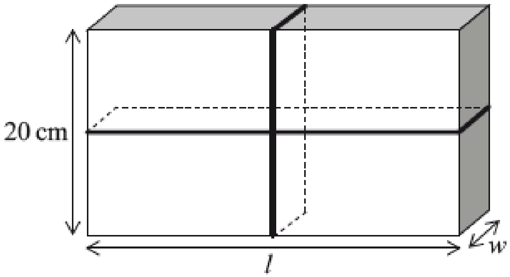
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



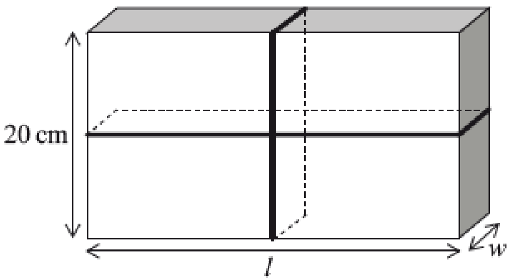
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



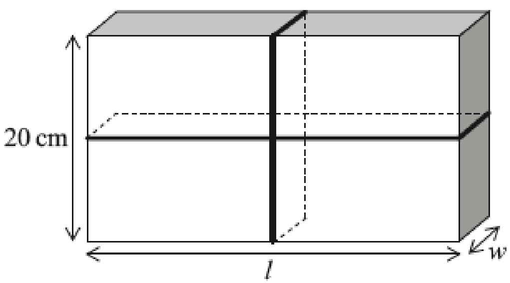
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



Write down the value, l, of the parcel for which the length of string is a minimum.

h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

Markscheme

a. $20lw$ **OR** $V = 20lw$ ***(A1)***

[1 mark]

b. $3000 = 20lw$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to 3000.

$l = \frac{3000}{20w}$ ***(M1)***

Note: Award ***(M1)*** for rearranging equation to make l subject of the formula. The above equation must be seen to award ***(M1)***.

OR

$150 = lw$ ***(M1)***

Note: Award ***(M1)*** for division by 20 on both sides. The above equation must be seen to award ***(M1)***.

$l = \frac{150}{w}$ ***(AG)***

[2 marks]

c. $S = 2l + 4w + 2(20)$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression for S.

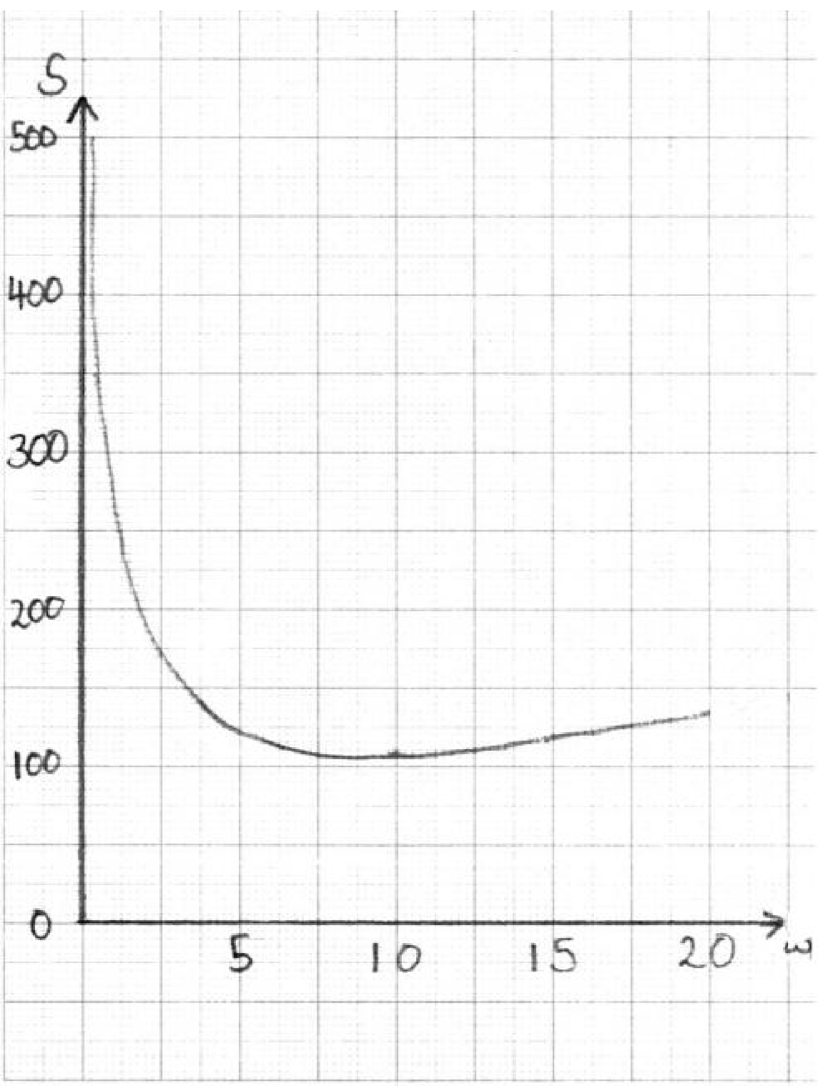
$2\left(\frac{150}{w}\right) + 4w + 2(20)$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into the expression for S. The above expression must be seen to award ***(M1)***.

$= 40 + 4w + \frac{300}{w}$ ***(AG)***

[2 marks]

d.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct scales, window and labels on axes, ***(A1)*** for approximately correct shape, ***(A1)*** for minimum point in approximately correct position, ***(A1)*** for asymptotic behaviour at $w = 0$.

Axes must be drawn with a ruler and labeled w and S.

For a smooth curve (with approximately correct shape) there should be **one** continuous thin line, no part of which is straight and no (one-to-many) mappings of w.

The S-axis must be an asymptote. The curve must not touch the S-axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

e. $4 - \frac{300}{w^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for 4, ***(A1)*** for -300, ***(A1)*** for $\frac{1}{w^2}$ or w^{-2}. If extra terms present, award at most ***(A1)(A1)(A0)***.

[3 marks]

f. $4 - \frac{300}{w^2} = 0$ **OR** $\frac{300}{w^2} = 4$ **OR** $\frac{\mathrm{d}S}{\mathrm{d}w} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to zero.

$w = 8.66 \left(\sqrt{75},\ 8.66025\ldots\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to part (e).

[2 marks]

g. $17.3 \left(\frac{150}{\sqrt{75}},\ 17.3205\ldots\right)$ ***(A1)*****(ft)**

Note: Follow through from their answer to part (f).

[1 mark]

h. $40 + 4\sqrt{75} + \frac{300}{\sqrt{75}}$ ***(M1)***

Note: Award ***(M1)*** for substitution of their answer to part (f) into the expression for S.

$= 110\ (\mathrm{cm})\ \left(40 + 40\sqrt{3},\ 109.282\ldots\right)$ ***(A1)*****(ft)*****(G2)***

Note: Do not accept 109.

Follow through from their answers to parts (f) and (g).

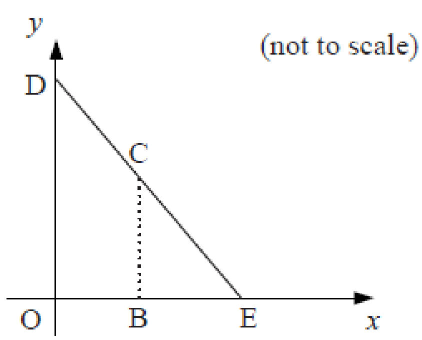
[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

On the coordinate axes below, D is a point on the y-axis and E is a point on the x-axis. O is the origin. The equation of the line DE is $y + \frac{1}{2}x = 4$

.



a. Write down the coordinates of point E. [2]

b. C is a point on the line DE. B is a point on the x-axis such that BC is parallel to the y-axis. The x-coordinate of C is t. [2]

Show that the y-coordinate of C is $4 - \frac{1}{2}t$.

c. OBCD is a trapezium. The y-coordinate of point D is 4. [3]

Show that the area of OBCD is $4t - \frac{1}{4}t^2$.

d. The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. [1]

e. (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d). [4]

(ii) Hence find the correct value for t. Give a reason for your answer.

Markscheme

a. $\mathrm{E}(8, 0)$ ***(A1)(A1)***

Notes: Brackets required but do not penalize again if mark lost in **Q4** (i)(d). If missing award ***(A1)(A0)***.
Accept $x = 8$, $y = 0$
Award ***(A1)*** for $x = 8$

b. $y + \frac{1}{2}t = 4$ ***(M1)(M1)***

Note: ***(M1)*** for the equation of the line seen. ***(M1)*** for substituting t.

$y = 4 - \frac{1}{2}t$ ***(AG)***

Note: Final line must be seen or previous ***(M1)*** mark is lost.

[2 marks]

c. $\text{Area} = \frac{1}{2} \times (4 + 4 - \frac{1}{2}t) \times t$ ***(M1)(A1)***

Note: ***(M1)*** for substituting in correct formula, ***(A1)*** for correct substitution.

$= \frac{1}{2} \times (8 - \frac{1}{2}t) \times t = \frac{1}{2}(8t - \frac{1}{2}t^2)$ ***(A1)***
$= 4t - \frac{1}{4}t^2$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[3 marks]

d. $4t - \frac{1}{4}t^2 = 9.75$ or any equivalent form. ***(A1)***

[1 mark]

e. (i) $t = 3$ or $t = 13$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

Note: Follow through from candidate's equation to part (d). Award ***(A0)(A1)*(ft)** for $(3, 0)$ and $(13, 0)$.

(ii) t must be a value between 0 and 8 then $t = 3$

Note: Accept B is between O and E. Do not award ***(R0)(A1)***.

Examiners report

a. A number of candidates did not attempt this question worth 12 marks but the majority answered this question partially and were able to gain some marks. Parts (a) and (b) were mostly well done. Very few candidates managed to answer part (c) well; this part of the question required good algebra along with a clear understanding of the situation given in the diagram. Many recovered then in (d) when they were asked to write down the quadratic equation. Solving the equation was not always found to be easy. Use of the GDC was expected but many used the formula. The correct solution, $t = 3$, was chosen in the last part of the question. However, their justification was often false causing them to lose both the reasoning and the answer mark.

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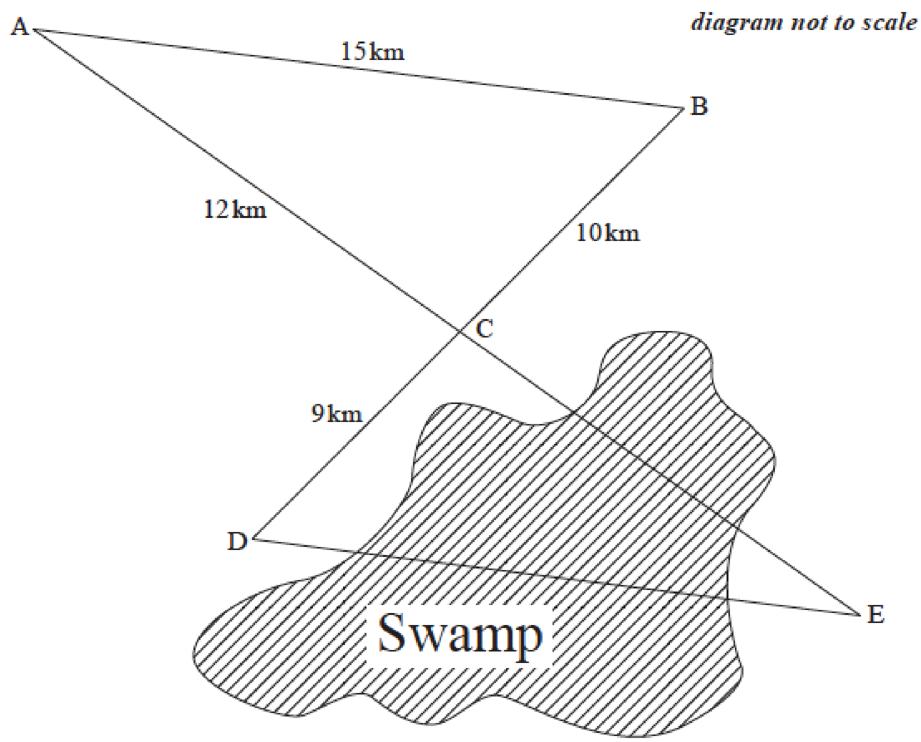
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor’s information.



a. (i) Find the size of angle ACB. [4]

(ii) Show that the size of angle DCE is 85.5°, correct to one decimal place.

b. The surveyor measures the size of angle CDE to be twice that of angle DEC. [5]

(i) Using angle $\mathrm{DCE} = 85.5^\circ$, find the size of angle DEC.

(ii) Find the length of DE.

c. Calculate the area of triangle DEC. [4]

Markscheme

a. (i) $\cos \mathrm{A\hat{C}B} = \frac{10^2+12^2-15^2}{2\times10\times12}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule,

(A1) for correct substitution.

$\mathrm{A\hat{C}B} = 85.5^\circ \quad (85.4593\ldots)$ ***(A1)(G2)***

(ii) $\mathrm{D\hat{C}E} = \mathrm{A\hat{C}B} \quad \text{and} \quad \mathrm{A\hat{C}B} = 85.5^\circ \quad (85.4593\ldots^\circ)$ ***(A1)***

OR

$\mathrm{B\hat{C}E} = 180^\circ - 85.5^\circ = 94.5^\circ \quad \text{and} \quad \mathrm{D\hat{C}E} = 180^\circ - 94.5^\circ = 85.5^\circ$ ***(A1)***

Notes: Both reasons must be seen for the ***(A1)*** to be awarded.

$\mathrm{D\hat{C}E} = 85.5^\circ$ ***(AG)***

b. (i) $\mathrm{D\hat{E}C} = \frac{180^\circ - 85.5^\circ}{3}$ ***(M1)***

$\hat{DEC} = 31.5°$ ***(A1)(G2)***

(ii) $\frac{\sin(31.5°)}{9} = \frac{\sin(85.5°)}{DE}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$DE = 17.2\ (km)(17.1718\ldots)$. ***(A1)*(ft)*(G2)***

c. $0.5 \times 17.1718\ldots \times 9 \times \sin(63°)$ ***(A1)*(ft)*(M1)(A1)*(ft)**

Note: Award ***(A1)*(ft)** for 63 seen, ***(M1)*** for substituted triangle area formula, ***(A1)*(ft)** for $0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$.

OR

$(\text{triangle height} =)\ 9 \times \sin(63°)$ ***(A1)*(ft)*(A1)*(ft)**

$0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$ ***(M1)***

Note: Award ***(A1)*(ft)** for 63 seen, ***(A1)*(ft)** for correct triangle height with their angle CDE, ***(M1)*** for $0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$.

$= 68.9\ km^2\quad (68.8509\ldots)$ ***(A1)*(ft)*(G3)***

Notes: Units are required for the last ***(A1)*(ft)** mark to be awarded.

Follow through from parts (b)(i) and (b)(ii).

Follow through from their angle CDE **within this part of the question**.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following table shows the number of bicycles, x, produced daily by a factory and their total production cost, y, in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

a. (i) Write down the Pearson's product–moment correlation coefficient, r, for these data. [4]

(ii) Hence comment on the result.

b. Write down the equation of the regression line y on x for these data, in the form $y = ax + b$. [2]

c. Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. [3]

d. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [2]

Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day.

e. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [5]

(i) Write down an expression for the total selling price of x bicycles.

(ii) Write down an expression for the **profit** the factory makes when producing x bicycles on a particular day.

(iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit.

Markscheme

a. (i) $r = 0.985 \quad (0.984905\ldots)$ ***(G2)***

Notes: If unrounded answer is not seen, award ***(G1)(G0)*** for 0.99 or 0.984. Award ***(G2)*** for 0.98.

(ii) strong, positive ***(A1)(A1)***

b. $y = 259.909\ldots x + 698.648\ldots \quad (y = 260x + 699)$ ***(G1)(G1)***

Notes: Award ***(G1)*** for $260x$ and ***(G1)*** for 699. If the answer is not an equation award a maximum of ***(G1)(G0)***.

c. $y = 259.909\ldots \times 13 + 698.648\ldots$ ***(M1)***

Note: Award ***(M1)*** for substitution of 13 into their regression line equation from part (b).

$y = 4077.47\ldots$ ***(A1)*(ft)*(G2)***

$y = 4077 \text{ (USD)}$ ***(A1)*(ft)**

Notes: Follow through from their answer to part (b). If rounded values from part (b) used, answer is 4079. Award the final ***(A1)*(ft)** for a correct rounding to the nearest USD of their answer. The unrounded answer may not be seen.

If answer is 4077 and no working is seen, award ***(G2)***.

d. $13 \times 304 - (4077.47) = -125.477\ldots \quad (-125)$ **OR**

$4077.47 - (13 \times 304) = 125.477\ldots \quad (125)$ ***(M1)***

Notes: Award ***(M1)*** for calculating the difference between 13×304 and their answer to part (c).

If rounded values are used in equation, answer is -127.

profit is negative **OR** $\text{cost} > \text{sales}$ ***(A1)***

OR

$13 \times 304 = 3952$ ***(M1)***

Note: Award ***(M1)*** for calculating the price of 13 bikes.

$3952 < 4077.47$ ***(A1)*(ft)**

Note: Award ***(A1)*** for showing 3952 is less than their part (c). This may be communicated in words. Follow through from part (c), but only if value is greater than 3952.

OR

$\frac{4077}{13} = 313.62$ ***(M1)***

Note: Award ***(M1)*** for calculating the cost of 1 bicycle.

$313.62 > 304$ ***(A1)*(ft)**

Note: Award ***(A1)*** for showing 313.62 is greater than 304. This may be communicated in words. Follow through from part (c), but only if value is greater than 304.

OR

$\frac{4077}{304} = 13.41$ ***(M1)***

Note: Award ***(M1)*** for calculating the number of bicycles that should have been be sold to cover total cost.

$13.41 > 13$ ***(A1)*(ft)**

Note: Award ***(A1)*** for showing 13.41 is greater than 13. This may be communicated in words. Follow through from part (c), but only if value is greater than 13.

e. (i) $304x$ ***(A1)***

(ii) $304x - (259.909\ldots x + 698.648\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for difference between their answers to parts (b) and (e)(i), ***(A1)*(ft)** for correct expression.

(iii) $304x - (259.909\ldots x + 698.648\ldots) > 0$ ***(M1)***

Notes: Award ***(M1)*** for comparing their expression in part (e)(ii) to 0. Accept an equation. Accept $3040x - y > 0$ or equivalent.

$x = 16$ bicycles ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to part (b). Answer must be a positive integer greater than 13 for the ***(A1)*(ft)** to be awarded.

Award ***(G1)*** for an answer of 15.84.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Give all answers in this question correct to two decimal places.

Arthur lives in London. On 1^{st} August 2008 Arthur paid $37\,500$ euros (EUR) for a new car from Germany. The price of the same car in London was $34\,075$ British pounds (GBP).
The exchange rate on 1^{st} August 2008 was $1\,\text{EUR} = 0.7234\,\text{GBP}$.

a. Calculate, **in GBP**, the price that Arthur paid for the car. [2]

b. Write down, in GBP, the amount of money Arthur saved by buying the car in Germany. [1]

d. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. [3]

Calculate the value, in GBP, of Arthur's car on 1^{st} August **2009**.

e. Between 1^{st} August 2008 and 1^{st} August 2012 Arthur's car depreciated at an annual rate of 9% of its current value. [3]

Show that the value of Arthur's car on 1^{st} August **2012** was $18\,600\ \mathrm{GBP}$, correct to the nearest $100\ \mathrm{GBP}$.

Markscheme

a. ***The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.***

$37\,500 \times 0.7234$ ***(M1)***

$= 27\,127.50$ ***(A1)(G2)***

[2 marks]

b. ***The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.***

6947.50 ***(A1)*(ft)*(G1)***

Note: Follow through from part (a) irrespective of whether working is seen.

[1 mark]

d. ***The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.***

$27\,127.50 \times 0.91$ ***(A1)(M1)***

Note: Award ***(A1)*** for 0.91 seen or equivalent, ***(M1)*** for their $27\,127.50$ multiplied by 0.91

OR

$27\,127.50 - 0.09 \times 27\,127.50$ ***(A1)(M1)***

Note: Award ***(A1)*** for $0.09 \times 27\,127.50$ seen, and ***(M1)*** for $27\,127.50 - 0.09 \times 27\,127.50$.

$= 24\,686.03$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a).

[3 marks]

e. ***The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.***

$27\,127.50 \times \left(1 - \frac{9}{100}\right)^4$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substituted compound interest formula, ***(A1)*(ft)** for correct substitution.

Follow through from part (a).

OR

$27\,127.50 \times (0.91)^4$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substituted geometric sequence formula, ***(A1)*****(ft)** for correct substitution.

Follow through from part (a).

OR (lists (i))

$24\,686.03,\ 22\,464.28\ldots,\ 20\,442.50\ldots,\ 18\,602.67\ldots$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for at least the 2^{nd} term correct (calculated from their $(\text{a}) \times 0.91$). Award ***(A1)*****(ft)** for four correct terms (rounded or unrounded).

Follow through from part (a).

Accept list containing the last three terms only ($24\,686.03$ may be implied).

OR (lists(ii))

$27\,127.50 - (2441.47\ldots + 2221.74\ldots + 2021.79\ldots + 1839.82\ldots)$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for subtraction of four terms from $27\,127.50$.

Award ***(A1)*** for four correct terms (rounded or unrounded).

Follow through from part (a).

$= 18\,602.67$ ***(A1)***

$= 18\,600$ ***(AG)***

Note: The final ***(A1)*** is not awarded unless both the unrounded and rounded answers are seen.

[3 marks]

Examiners report

a. Despite the fact that "Give all answers in this question correct to two decimal places" was written in bold at the top of the question, many candidates lost one (and only one) mark for giving at least one answer to only a single decimal place. There was a lot of reading in this question and some candidates seemed to lose their way as their solution developed and, as a consequence, lost marks in the latter part of the question. A significant number of candidates obtained nearly full marks for parts (a) through to (d). The marks which tended to not be awarded were not giving the required answer to two decimal places and not adding the amount invested onto the interest earned in part (c). Indeed, many candidates were able to correctly determine the depreciated value of the car on 1st August 2009 by simply finding 91% of the original price. However, part (e) proved to be elusive for many candidates as some simply treated the problem as a 'reverse simple interest problem' and subtracted 9% for each of a further 3 years. As a consequence, erroneous answers of the form 17,361.60, from

$(27127.50 \times (1 - 0.09 \times 4))$, were often conveniently ignored and rounded to the required answer of 18,600 GBP. Such a method earned no marks at all. There was a lot of information given in the stem to the last part of the question and, as a consequence, many candidates were unable to achieve full marks here. There was certainly a great deal of confusion as to what to divide by 0.8694 (seeing $\frac{18\,600+8198.05-30\,500}{0.86944} = -4258.05$ was not uncommon) and even introducing the original exchange rate of 0.7234 caused confusion. As a further example, an incorrect value carried forward from part (c) (1,250.55) led to a negative result. Provided the method was correct (despite an incorrect value carried forward), the three method marks were awarded. However, the negative result of –7,667.53 should have flagged to the candidate that something was wrong somewhere and this could only be in the current part of the question or part (c).

b. Despite the fact that “Give all answers in this question correct to two decimal places” was written in bold at the top of the question, many candidates lost one (and only one) mark for giving at least one answer to only a single decimal place. There was a lot of reading in this question and some candidates seemed to lose their way as their solution developed and, as a consequence, lost marks in the latter part of the question. A significant number of candidates obtained nearly full marks for parts (a) through to (d). The marks which tended to not be awarded were not giving the required answer to two decimal places and not adding the amount invested onto the interest earned in part (c). Indeed, many candidates were able to correctly determine the depreciated value of the car on 1st August 2009 by simply finding 91% of the original price. However, part (e) proved to be elusive for many candidates as some simply treated the problem as a ‘reverse simple interest problem’ and subtracted 9% for each of a further 3 years. As a consequence, erroneous answers of the form 17,361.60, from $(27127.50 \times (1 - 0.09 \times 4))$, were often conveniently ignored and rounded to the required answer of 18,600 GBP. Such a method earned no marks at all. There was a lot of information given in the stem to the last part of the question and, as a consequence, many candidates were unable to achieve full marks here. There was certainly a great deal of confusion as to what to divide by 0.8694 (seeing $\frac{18\,600+8198.05-30\,500}{0.86944} = -4258.05$ was not uncommon) and even introducing the original exchange rate of 0.7234 caused confusion. As a further example, an incorrect value carried forward from part (c) (1,250.55) led to a negative result. Provided the method was correct (despite an incorrect value carried forward), the three method marks were awarded. However, the negative result of –7,667.53 should have flagged to the candidate that something was wrong somewhere and this could only be in the current part of the question or part (c).

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Consider the functions $f(x) = \frac{2x+3}{x+4}$ and $g(x) = x + 0.5$.

a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. [6]

b. Write down the equation of the vertical asymptote. [2]

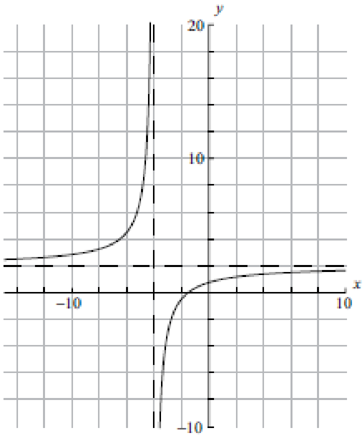
c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. [2]

d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. [3]

e. Write down the gradient of the line $g(x) = x + 0.5$. [1]

f. The line L passes through the point with coordinates $(-2,\ -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. [3]

Markscheme

a. 

(A6)

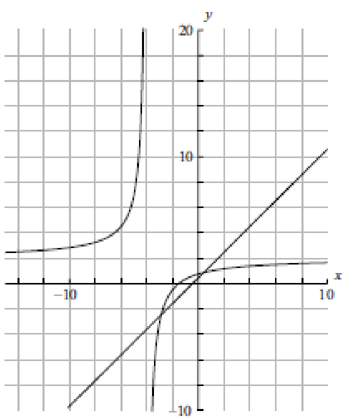
Notes: ***(A1)*** for labels and some idea of scale.
(A1) for x-intercept seen, ***(A1)*** for y-intercept seen in roughly the correct places (coordinates not required).
(A1) for vertical asymptote seen, ***(A1)*** for horizontal asymptote seen in roughly the correct places (equations of the lines not required).
(A1) for correct general shape.

[6 marks]

b. $x = -4$ ***(A1)(A1)*****(ft)**

Note: ***(A1)*** for $x =$, ***(A1)*****(ft)** for -4.

[2 marks]

c. 

(A1)(A1)

Note: ***(A1)*** for correct axis intercepts, ***(A1)*** for straight line

[2 marks]

d. $(-2.85078, -2.35078)$ OR $(0.35078, 0.85078)$ ***(G1)(G1)(A1)*****(ft)**

Notes: ***(A1)*** for x-coordinate, ***(A1)*** for y-coordinate, ***(A1)*****(ft)** for correct accuracy. Brackets required. If brackets not used award ***(G1)(G0)(A1)*****(ft)**.
Accept $x = -2.85078$, $y = -2.35078$ or $x = 0.35078$, $y = 0.85078$.

[3 marks]

e. $\text{gradient} = 1$ ***(A1)***

[1 mark]

f. $\text{gradient of perpendicular} = -1$ ***(A1)*****(ft)**

(can be implied in the next step)

$y = mx + c$

$-3 = -1 \times -2 + c$ ***(M1)***

$c = -5$

$y = -x - 5$ ***(A1)*****(ft)*****(G2)***

OR

$y + 3 = -(x + 2)$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for correct answer with no working at all but ***(A1)(G1)*** if the gradient is mentioned as -1 then correct answer with no further working.

[3 marks]

Examiners report

a. This was not very well done. The graph was often correct but was so small that it was difficult to check if axes intercepts were correct or not. Often the vertical asymptote looked as if it were joined to the rest of the graph. Very few of the candidates put a scale and/or labels on their axes.

b. Reasonably well done. Some put $y = -4$ while others omitted the minus sign.

c. Fairly well done – but once again too small to check the axes intercepts properly. Also, many candidates did not appear to have a ruler to draw the straight line.

d. Well done.

e. Most could find the gradient of the line.

f. Many forgot to find the gradient of the perpendicular line. Others had problems with the equation of a line in general.

The table shows the distance, in km, of eight regional railway stations from a city centre terminus and the price, in $, of a return ticket from each regional station to the terminus.

Distance in km (x)	3	15	23	42	56	62	74	93
Price in \$ (y)	5	24	43	56	68	74	86	100

a. Draw a scatter diagram for the above data. Use a scale of 1 cm to represent 10 km on the x-axis and 1 cm to represent $\$10$ on the y-axis. [4]

b. Use your graphic display calculator to find [2]

(i) $\bar{x}$, the mean of the distances;

(ii) $\bar{y}$, the mean of the prices.

c. Plot and label the point $\mathrm{M}\ (\bar{x},\ \bar{y})$ on your scatter diagram. [1]

d. Use your graphic display calculator to find [3]

(i) the product–moment correlation coefficient, r ;

(ii) the equation of the regression line y on x.

e. Draw the regression line y on x on your scatter diagram. [2]

f. A ninth regional station is 76 km from the city centre terminus. [3]

Use the equation of the regression line to estimate the price of a return ticket to the city centre terminus from this regional station. **Give your answer correct to the nearest $\$$.**

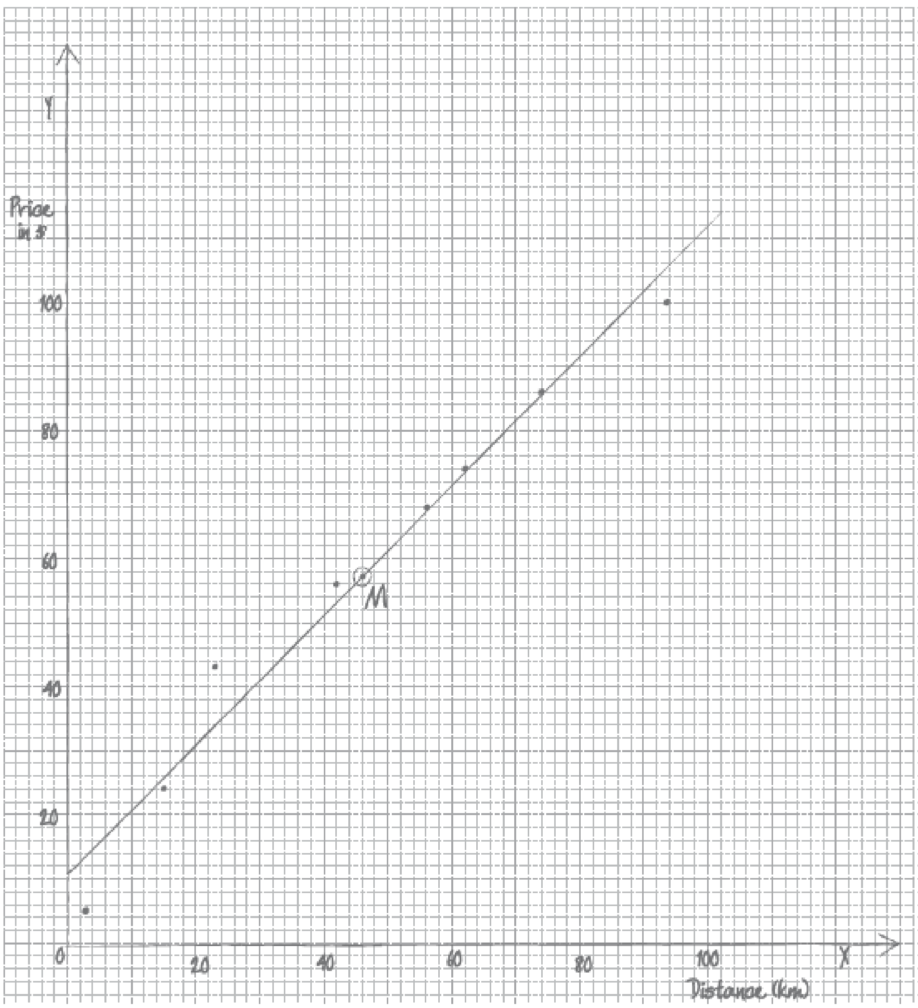
g. Give a reason why it is valid to use your regression line to estimate the price of this return ticket. [1]

h. The actual price of the return ticket is $\$80$. [2]

Using your answer to part (f), calculate the percentage error in the estimated price of the ticket.

Markscheme

a.



(A4)

Notes: Award ***(A1)*** for correct scale and labels (accept x and y).

Award ***(A3)*** for 7 or 8 points plotted correctly.

Award ***(A2)*** for 5 or 6 points plotted correctly.

Award ***(A1)*** for 3 or 4 points plotted correctly.

Award at most ***(A1)(A2)*** if points are joined up.

If axes are reversed, award at most ***(A0)(A3)***.

If graph paper is not used, award at most ***(A1)(A0)***.

[4 marks]

b. (i) $(\bar{x} =)\ 46$ ***(G1)***

(ii) $(\bar{y} =)\ 57$ ***(G1)***

[2 marks]

c. $\mathrm{M}(46, 57)$ plotted and labelled on the scatter diagram ***(A1)*(ft)**

Notes: Follow through from their part (b).

Accept $(\bar{x},\ \bar{y})$ as the label.

[1 mark]

d. (i) $0.986\ \ (0.986322...)$ ***(G1)***

(ii) $y = 1.01x + 10.3\ \ (y = 1.01431\ldots x + 10.3412\ldots)$ ***(G1)(G1)***

Notes: Award ***(G1)*** for $1.01x$, ***(G1)*** for 10.3.

Award ***(G1)(G0)*** if not written in the form of an equation.

OR

$(y - 57) = 1.01(x - 46)\ \ (y - 57 = 1.01431...(x - 46))$ ***(G1)(G1)*(ft)**

Note: Award ***(G1)*** for 1.01, ***(G1)*** for their 57 and 46.

[3 marks]

e. straight line drawn on the scatter diagram ***(A1)*(ft)*(A1)*(ft)**

Notes: The line must be straight for either of the two marks to be awarded.

Award ***(A1)*(ft)** passing through their M plotted in (c).

Award ***(A1)*(ft)** for correct y-intercept (between 9 and 12).

Follow through from their y-intercept found in part (d).

If part (d) is used, award ***(A1)*(ft)** for their intercept (± 1).

[2 marks]

f. $y = 1.01431... \times 76 + 10.3412\ldots$ ***(M1)***

Note: Award ***(M1)*** for substitution of 76 into their regression line.

$= 87.4295\ldots$ ***(A1)*(ft)**

Note: Follow through from part (d). If 3 sf values are used the value is 87.06.

$\$87$ ***(A1)*(ft)*(G2)***

Notes: The final ***(A1)*** is awarded for their answer given correct to the nearest dollar.

Method, followed by the answer of 87 earns ***(M1)(G2)***. It is not necessary to see the interim step.

Where the candidate uses their graph instead of the equation, and arrives at an answer other than 87, award, at most, ***(G1)*(ft)**.

If the candidate uses their graph and arrives at the required answer of 87, award ***(G2)*(ft)**.

[3 marks]

g. 76 is within the range of distances given in the data **OR** the correlation coefficient is close to 1. ***(R1)***

Notes: Award ***(R1)*** if **either** condition is given.

Sufficient to indicate that 76 is 'within the data range' and the correlation is 'strong'.

Allow r^2 close to 1.

Do **not** accept "within the range of prices".

[1 mark]

h. $\text{Percentage error} = \frac{87-80}{80} \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into formula.

8.75% ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to part (f).

Accept either the rounded or unrounded answer to part (f).

If no integer value seen in part (f), follow through from their unrounded answer to part (f).

Answer must be positive.

[2 marks]

Examiners report

a. This question was very well attempted by a significant majority of candidates. Many good and accurate attempts at plotting a scatter diagram were seen in part (a). However, a minority of candidates chose not to use graph paper but instead used their answer book. These candidates achieved, at most, one mark for that part question. Many correct answers were seen in parts (b) and (d) reflecting good use of the graphic display calculator. Whilst many candidates realized that the line of regression passes through the point *M*, a significant number of candidates seemed to draw their line 'by eye' rather than using the equation found in part (d) and, as a consequence for many, their straight line (or projected line) did not fall within the required tolerances for the second mark. Many candidates understood the requirements for part (f) and full marks were seen on a majority of scripts. Those candidates, however, who used their graph instead scored, at most, two marks here. Many candidates seemed to be well-drilled in giving a suitable reason in part (f) and 'within the data range' or a 'strong correlation' were frequently seen. Percentage error caused very few problems for candidates and many correct answers were seen in part (h).

b. This question was very well attempted by a significant majority of candidates. Many good and accurate attempts at plotting a scatter diagram were seen in part (a). However, a minority of candidates chose not to use graph paper but instead used their answer book. These candidates achieved, at most, one mark for that part question. Many correct answers were seen in parts (b) and (d) reflecting good use of the graphic display calculator. Whilst many candidates realized that the line of regression passes through the point *M*, a significant number of candidates seemed to draw their line 'by eye' rather than using the equation found in part (d) and, as a consequence for many, their straight line (or projected line) did not fall within the required tolerances for the second mark. Many candidates understood the requirements for part (f) and full marks were seen on a majority of scripts. Those candidates, however, who used their graph instead scored, at most, two marks here. Many candidates seemed to be well-drilled in giving a suitable reason in part (f) and 'within the data range' or a 'strong correlation' were frequently seen. Percentage error caused very few problems for candidates and many correct answers were seen in part (h).

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Give your answers to parts (b), (c) and (d) to the nearest whole number.

Harinder has 14 000 US Dollars (USD) to invest for a period of five years. He has two options of how to invest the money.

Option A: Invest the full amount, in USD, in a fixed deposit account in an American bank.

The account pays a nominal annual interest rate of $r\%$, **compounded yearly**, for the five years. The bank manager says that this will give Harinder a return of 17500 USD.

Option B: Invest the full amount, in Indian Rupees (INR), in a fixed deposit account in an Indian bank. The money must be converted from USD to INR before it is invested.

The exchange rate is 1 USD = 66.91 INR.

The account in the Indian bank pays a nominal annual interest rate of 5.2 % **compounded monthly**.

a. Calculate the value of *r*. [3]

b. Calculate 14 000 USD in INR. [2]

c. Calculate the amount of this investment, in INR, in this account after five years. [3]

d. Harinder chose option B. At the end of five years, Harinder converted this investment back to USD. The exchange rate, at that time, was 1 USD = 67.16 INR. [3]

Calculate how much **more** money, in USD, Harinder earned by choosing option B instead of option A.

Markscheme

a. $17500 = 14000\left(1 + \frac{r}{100}\right)^5$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the compound interest formula, ***(A1)*** for correct substitution. Award at most ***(M1)(A0)*** if not equated to 17500.

OR

N = 5

PV = ±14000

FV = ∓17500

P/Y = 1

C/Y = 1 ***(A1)(M1)***

Note: Award ***(A1)*** for *C/Y* = 1 seen, ***(M1)*** for **all** other correct entries. *FV* and *PV* must have opposite signs.

= 4.56 (%) (4.56395… (%)) ***(A1) (G3)***

[3 marks]

b. 14000 × 66.91 ***(M1)***

Note: Award ***(M1)*** for multiplying 14000 by 66.91.

936740 (INR) ***(A1) (G2)***

Note: Answer must be given to the nearest whole number.

[2 marks]

c. $936740 \times \left(1 + \frac{5.2}{12 \times 100}\right)^{12 \times 5}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into the compound interest formula, ***(A1)*(ft)** for their correct substitution.

OR

N = 60

I% = 5.2

PV = ±936740

P/*Y*= 12

C/*Y*= 12 ***(A1)(M1)***

Note: Award ***(A1)*** for *C*/*Y* = 12 seen, ***(M1)*** for **all** other correct entries.

OR

N = 5

I% = 5.2

PV = ±936740

P/*Y*= 1

C/*Y*= 12 ***(A1)(M1)***

Note: Award ***(A1)*** for *C*/*Y* = 12 seen, ***(M1)*** for **all** other correct entries

= 1214204 (INR) ***(A1)*(ft) *(G3)***

Note: Follow through from part (b). Answer must be given to the nearest whole number.

[3 marks]

d. $\frac{1214204}{67.16}$ ***(M1)***

Note: Award ***(M1)*** for dividing their (c) by 67.16.

$\left(\frac{1214204}{67.16}\right) - 17500 = 579$ (USD) ***(M1)(A1)*(ft) *(G3)***

Note: Award ***(M1)*** for finding the difference between their conversion and 17500. Answer must be given to the nearest whole number. Follow through from part (c).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Daniel grows apples and chooses at random a sample of 100 apples from his harvest.

He measures the diameters of the apples to the nearest cm. The following table shows the distribution of the diameters.

Diameter (to the nearest cm)	5	6	7	8	9
Frequency	15	27	33	17	8

a. Using your graphic display calculator, write down the value of [3]

(i) the mean of the diameters in this sample;

(ii) the standard deviation of the diameters in this sample.

b. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [3]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

Calculate the percentage of **small** apples in Daniel's harvest.

c. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

Of the apples harvested, 5% are **large** apples.

Find the value of a.

d. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	$6.5 \le$ Diameter $< a$
Large	Diameter $\ge a$

Find the percentage of **medium** apples.

e. Daniel assumes that the diameters of all of the apples from his harvest are normally distributed with a mean of 7 cm and a standard deviation of 1.2 cm. He classifies the apples according to their diameters as shown in the following table. [2]

Classification	Diameter (cm)
Small	Diameter < 6.5
Medium	6.5 ≤ Diameter < a
Large	Diameter ≥ a

This year, Daniel estimates that he will grow $100\,000$ apples.

Estimate the number of **large** apples that Daniel will grow this year.

Markscheme

a. (i) 6.76 (cm) ***(G2)***

Notes: Award ***(M1)*** for an attempt to use the formula for the mean with a least two rows from the table.

(ii) $1.14\text{ (cm)}\quad(1.14122\ldots\text{ (cm)})$ ***(G1)***

b. $\mathrm{P}(\text{diameter} < 6.5) = 0.338\quad(0.338461)$ ***(M1)(A1)***

Notes: Award ***(M1)*** for attempting to use the normal distribution to find the probability **or** for correct region indicated on labelled diagram. Award ***(A1)*** for correct probability.

$33.8(\%)$ ***(A1)*(ft)*(G3)***

Notes: Award ***(A1)*(ft)** for converting their probability into a percentage.

c. $\mathrm{P}(\text{diameter} \geqslant a) = 0.05$ ***(M1)***

Note: Award ***(M1)*** for attempting to use the normal distribution to find the probability **or** for correct region indicated on labelled diagram.

$a = 8.97\text{ (cm)}\quad(8.97382\ldots)$ ***(A1)(G2)***

d. $100 - (5 + 33.8461\ldots)$ ***(M1)***

Note: Award ***(M1)*** for subtracting "$5+$ their part (b)" from 100 **or** ***(M1)*** for attempting to use the normal distribution to find the probability $\mathrm{P}\left(6.5 \leqslant \text{diameter} < \text{their part (c)}\right)$ **or** for correct region indicated on labelled diagram.

$= 61.2(\%)\quad(61.1538\ldots(\%))$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to part (b). Percentage symbol is not required. Accept $61.1(\%)$ $(61.1209\ldots(\%))$ if 8.97 used.

e. $100\,000 \times 0.05$ ***(M1)***

Note: Award ***(M1)*** for multiplying by 0.05 (or 5%).

$= 5000$ ***(A1)(G2)***

Examiners report

a. [N/A]
b. [N/A]
[N/A]

d. [N/A]
e. [N/A]

A boat race takes place around a triangular course, ABC, with $\mathrm{AB} = 700\ \mathrm{m}$, $\mathrm{BC} = 900\ \mathrm{m}$ and angle $\mathrm{ABC} = 110^\circ$. The race starts and finishes at point A.

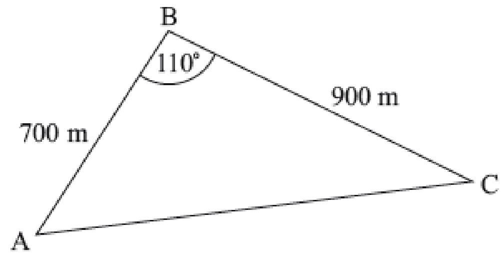

diagram not to scale

a. Calculate the total length of the course. [4]

b. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

c. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Find the size of angle ACB.

d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Calculate the area that must be kept clear of boats.

e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Determine, giving a reason, whether the course complies with the safety regulations.

f. The race is filmed from a helicopter, H, which is flying vertically above point A. [2]

The angle of elevation of H from B is 15°.

Calculate the vertical height, AH, of the helicopter above A.

g. The race is filmed from a helicopter, H, which is flying vertically above point A. [3]

The angle of elevation of H from B is 15°.

Calculate the maximum possible distance from the helicopter to a boat on the course.

Markscheme

a. $\mathrm{AC}^2 = 700^2 + 900^2 - 2 \times 700 \times 900 \times \cos 110^\circ$ ***(M1)(A1)***

$\mathrm{AC} = 1315.65\ldots$ ***(A1)(G2)***

length of course $= 2920\ (\mathrm{m}) \quad (2915.65\ldots\ \mathrm{m})$ ***(A1)***

Notes: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

Award ***(G3)*** for $2920 \quad (2915.65\ldots)$ seen without working.

The final ***(A1)*** is awarded for adding 900 and 700 to their AC irrespective of working seen.

b. $\frac{2915.65}{1.5}$ ***(M1)***

Note: Award ***(M1)*** for their length of course divided by 1.5.

Follow through from part (a).

$= 1943.76\ldots \text{ (seconds)}$ ***(A1)*(ft)**

$= 32 \text{ (minutes)}$ ***(A1)*(ft)*(G2)***

Notes: Award the final ***(A1)*** for correct conversion of **their** answer in seconds to minutes, correct to the nearest minute.

Follow through from part (a).

c. $\frac{700}{\sin \mathrm{ACB}} = \frac{1315.65\ldots}{\sin 110^\circ}$ ***(M1)(A1)*(ft)**

OR

$\cos \mathrm{ACB} = \frac{900^2 + 1315.65\ldots^2 - 700^2}{2 \times 900 \times 1315.65\ldots}$ ***(M1)(A1)*(ft)**

$\mathrm{ACB} = 30.0^\circ \quad (29.9979\ldots^\circ)$ ***(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for substitution into sine rule or cosine rule formula, ***(A1)*** for their correct substitution, ***(A1)*** for correct answer.

Accept 29.9° for sine rule and 29.8° for cosine rule from use of correct three significant figure values. Follow through from their answer to (a).

d. $\frac{1}{2} \times 700 \times 900 \times \sin 110^\circ$ ***(M1)(A1)***

Note: Accept $\frac{1}{2} \times \text{their AC} \times 900 \times \sin(\text{their ACB})$. Follow through from parts (a) and (c).

$= 296000 \text{ m}^2 \quad (296003 \text{ m}^2)$ ***(A1)(G2)***

Notes: Award ***(M1)*** for substitution into area of triangle formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

Award ***(G1)*** if 296000 is seen without units or working.

e. $\sin 29.9979\ldots = \frac{\text{distance}}{900}$ ***(M1)***

$(\text{distance} =)\ 450 \text{ (m)} \quad (449.971\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c).

OR

$\frac{1}{2} \times \text{distance} \times 1315.65\ldots = 296003$ ***(M1)***

$(\text{distance} =)\ 450 \text{ (m)} \quad (449.971\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a) and part (d).

450 is greater than 375, thus the course complies with the safety regulations ***(R1)***

Notes: A comparison of their area from (d) and the area resulting from the use of 375 as the perpendicular distance is a valid approach and should be given full credit. Similarly a comparison of angle ACB and $\sin^{-1}\left(\frac{375}{900}\right)$ should be given full credit.

Award ***(R0)*** for correct answer without any working seen. Award ***(R1)*(ft)** for a justified reason consistent with their working.

Do not award ***(M0)(A0)(R1)***.

f. $\tan 15^{\circ}=\frac{\mathrm{AH}}{700}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trig formula.

$\mathrm{AH}=188\ (\mathrm{m})\quad(187.564\ldots)$ ***(A1)*****(ft)*****(G2)***

g. $\mathrm{HC}^2=187.564\ldots^2+1315.65\ldots^2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into Pythagoras, ***(A1)*** for their $1315.65\ldots$ and their $187.564\ldots$ correctly substituted in formula.

$\mathrm{HC}=1330\ldots\ (\mathrm{m})\quad(1328.95\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to parts (a) and (f).

Examiners report

a. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

b. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

c. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

d. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again

successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

e. In part (e), unless evidence was presented, reasoning marks did not accrue; the interpretative nature of this part was a significant discriminator in determining the quality of a response.

f. There were many instances of parts (f) and (g) being left blank and angle of elevation is still not well understood. Again, the interpretative nature of part (g) – even when part (f) was correct – caused difficulties

g. There were many instances of parts (f) and (g) being left blank and angle of elevation is still not well understood. Again, the interpretative nature of part (g) – even when part (f) was correct – caused difficulties

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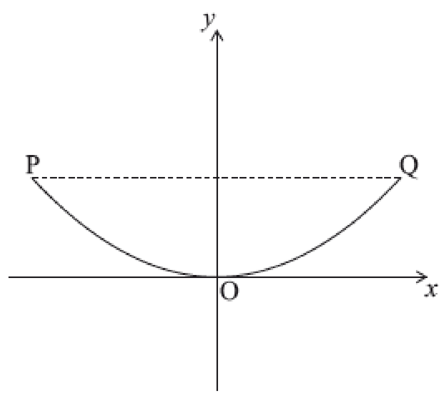
The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



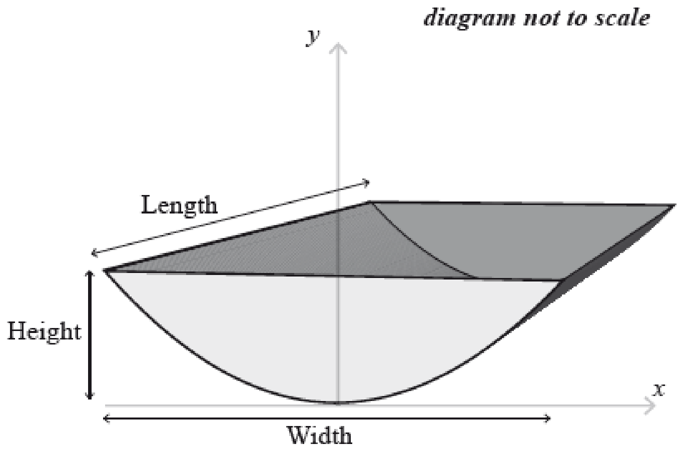
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

a. Write down the value of c. [1]

b. Find the value of a. [2]

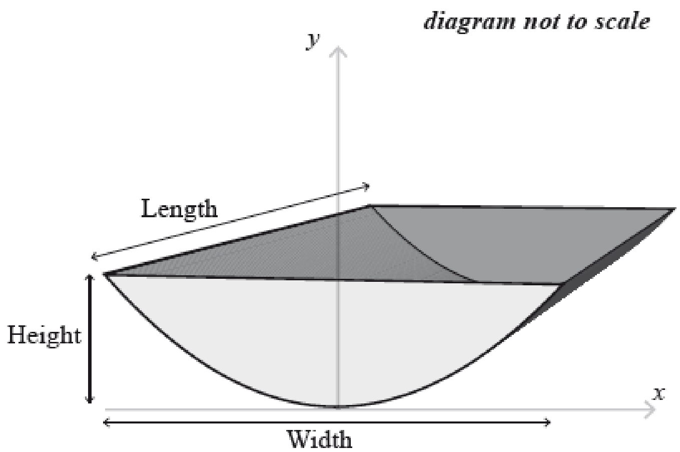
c. Hence write down the equation of the quadratic function which models the edge of the water tank. [1]

d. The water tank is shown below. It is partially filled with water. [2]



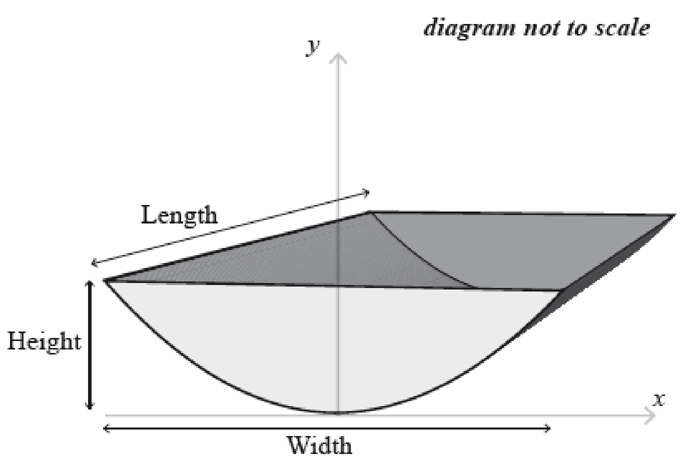
Calculate the value of y when $x = 2.4$ m.

e. The water tank is shown below. It is partially filled with water. [2]



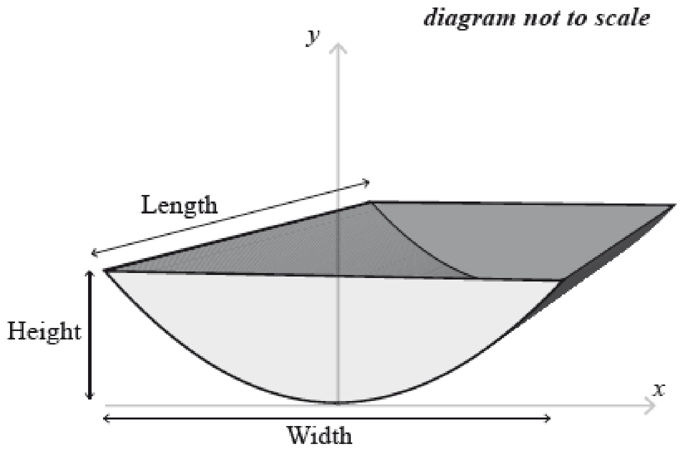
State what the value of x and the value of y represent for this water tank.

f. The water tank is shown below. It is partially filled with water. [2]



Find the value of x when the height of water in the tank is 0.9 m.

g. The water tank is shown below. It is partially filled with water. [2]



The water tank has a length of 5 m.

When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \mathrm{m}^3$.

(ii) Calculate the percentage of water in the tank.

Markscheme

a. 0 ***(A1)(G1)***

[1 mark]

b. $1.8 = a(3)^2 + 0$ ***(M1)***

OR

$1.8 = a(-3)^2 + 0$ ***(M1)***

Note: Award ***(M1)*** for substitution of $y = 1.8$ or $x = 3$ and their value of c into equation. 0 may be implied.

$a = 0.2\ \left(\frac{1}{5}\right)$ ***(A1)*****(ft)*****(G1)***

Note: Follow through from their answer to part (a).

Award ***(G1)*** for a correct answer only.

[2 marks]

c. $y = 0.2x^2$ ***(A1)*****(ft)**

Note: Follow through from their answers to parts (a) and (b).

Answer must be an equation.

[1 mark]

d. $0.2 \times (2.4)^2$ ***(M1)***

$= 1.15\ (\text{m})\ \ (1.152)$ ***(A1)*(ft)*(G1)***

Notes: Award ***(M1)*** for correctly substituted formula, ***(A1)*** for correct answer. Follow through from their answer to part (c).

Award ***(G1)*** for a correct answer only.

[2 marks]

e. y is the height ***(A1)***

positive value of x is half the width (*or equivalent*) ***(A1)***

[2 marks]

f. $0.9 = 0.2x^2$ ***(M1)***

Note: Award ***(M1)*** for setting their equation equal to 0.9.

$x = \pm 2.12\ (\text{m})\ \ \left(\pm\frac{3}{2}\sqrt{2},\ \pm\sqrt{4.5},\ \pm 2.12132\ldots\right)$ ***(A1)*(ft)*(G1)***

Note: Accept 2.12. Award ***(G1)*** for a correct answer only.

[2 marks]

g. (i) 2.55×5 ***(M1)***

Note: Award ***(M1)*** for correct substitution in formula.

$= 12.8\ (\text{m}^3)\ \ \left(12.75\ (\text{m}^3)\right)$ ***(A1)(G2)***

[2 marks]

(ii) $\frac{12.75}{36} \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct quotient multiplied by 100.

$= 35.4(\%)\ \ (35.4166\ldots)$ ***(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for $35.6(\%)(35.5555\ldots(\%))$.

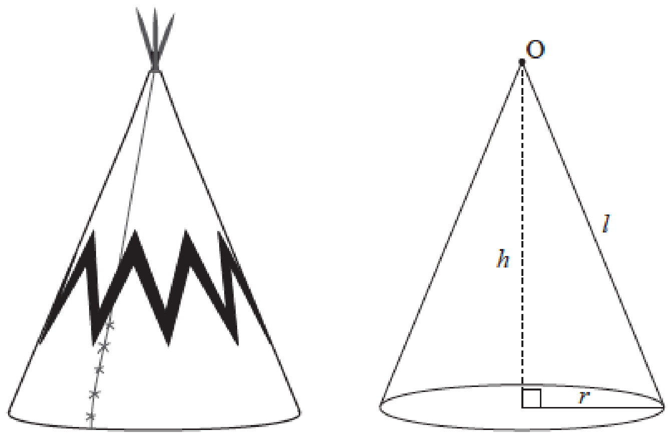
Follow through from their answer to part (g)(i).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \mathrm{m}^2$, and has the shape of a semicircle, as shown in the following diagram.

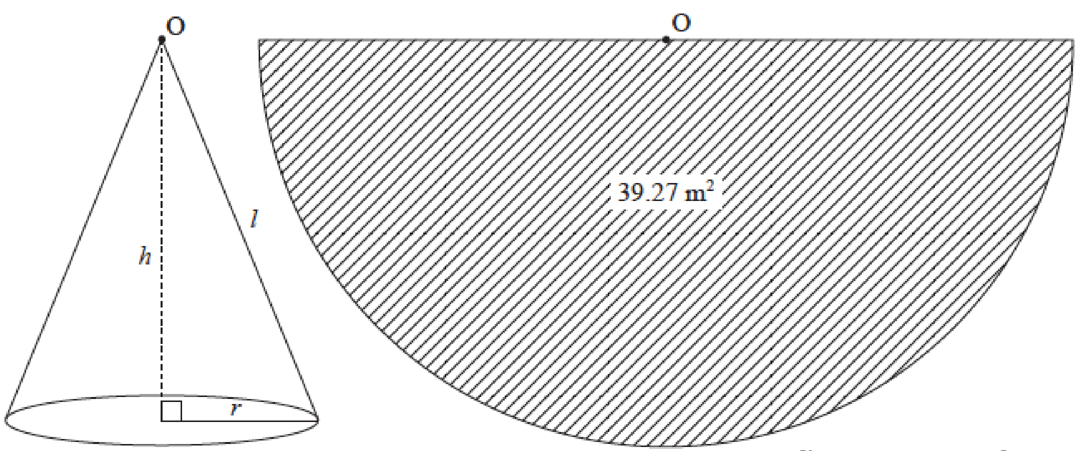


diagram not to scale

a. Show that the slant height, l, is 5 m, correct to the nearest metre. [2]

b. (i) Find the circumference of the base of the cone. [6]

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

c. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

e. A company designs cone-shaped tents to resemble the traditional tepees. [2]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

Markscheme

a. $\frac{\pi l^2}{2} = 39.27$ ***(M1)(A1)***

Note: Award ***(M1)*** for equating the formula for area of a semicircle to 39.27, award ***(A1)*** for correct substitution of l into the formula for area of a semicircle.

$l = 5\ (\mathrm{m})$ ***(AG)***

b. (i) $5 \times \pi$ ***(M1)***

$= 15.7 \quad (15.7079...,\ 5\pi)\ (\mathrm{m})$ ***(A1)(G2)***

(ii) $2\pi r = 15.7079\ldots$ **OR** $5\pi r = 39.27$ ***(M1)***

$(r =)\ 2.5\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b)(i).

(iii) $(h^2 =)\ 5^2 - 2.5^2$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into Pythagoras' theorem. Follow through from part (b)(ii).

$(h =)\ 4.33\ (4.33012\ldots)\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

c. $9.33 - 2 \times r$ ***(A1)***

d. $V = \frac{\pi r^2}{3} \times (9.33 - 2r)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula.

$V = 3.11\pi r^2 - \frac{2}{3}\pi^3$ ***(AG)***

e. $6.22\pi r - 2\pi r^2$ ***(A1)(A1)***

Notes: Award ***(A1)*** for $6.22\pi r$, ***(A1)*** for $-2\pi r^2$.

If extra terms present, award at most ***(A1)(A0)***.

f. (i) $6.22\pi r - 2\pi r^2 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative from part (e) to 0.

$r = 3.11\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

Notes: Award ***(A1)*** for identifying 3.11 as the answer.

Follow through from their answer to part (e).

(ii) $\frac{1}{3}\pi(3.11)^3$ **OR** $3.11\pi(3.11)^2 - \frac{2}{3}\pi(3.11)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the correct volume formula.

$31.5\ (\mathrm{m}^3)(31.4999\ldots)$ ***(A1)*(ft)*(G2)***

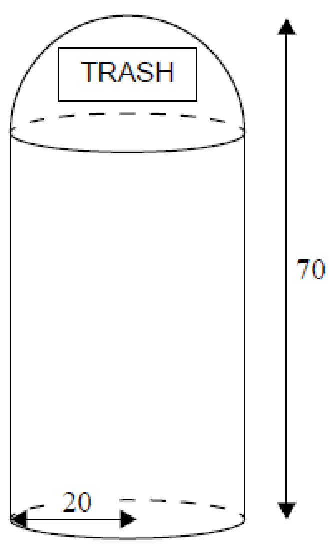
Note: Follow through from their answer to part (f)(i).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

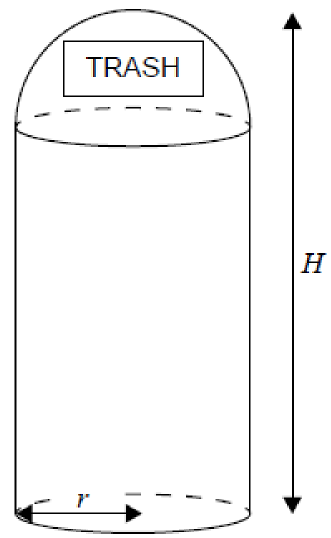


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of *H* cm and a base radius of *r* cm.

diagram not to scale



There is a design constraint such that *H* + 2*r* = 110 cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder. [1]

b. Find the total volume of the trash can. [4]

c. Find the height of the **cylinder**, *h* , of the new trash can, in terms of *r*. [2]

d. Show that the volume, *V* cm^3 , of the new trash can is given by [3]

$V = 110\pi r^3$.

e. Using your graphic display calculator, find the value of *r* which maximizes the value of *V*. [2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can. [4]

State whether the designer's claim is correct. Justify your answer.

Markscheme

a. 50 (cm) ***(A1)***

[1 mark]

b. $\pi \times 50 \times 20^2 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for their correctly substituted volume of cylinder, ***(M1)*** for correctly substituted volume of sphere formula, ***(M1)*** for halving the substituted volume of sphere formula. Award at most ***(M1)(M1)(M0)*** if there is no addition of the volumes.

$= 79600 \ \left(\text{cm}^3\right) \ \left(79587.0\ldots\left(\text{cm}^3\right)\ , \ \frac{76000}{3}\pi\right)$ ***(A1)(ft) (G3)***

Note: Follow through from part (a).

[4 marks]

c. $h = H - r$ (or equivalent) ***OR*** $H = 110 - 2r$ ***(M1)***

Note: Award ***(M1)*** for writing h in terms of H and r or for writing H in terms of r.

(h =) $110 - 3r$ ***(A1) (G2)***

[2 marks]

d. $(V =)\ \frac{2}{3}\pi r^3 + \pi r^2 \times (110 - 3r)$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for volume of hemisphere, ***(M1)*** for correct substitution of their h into the volume of a cylinder, ***(M1)*** for addition of two correctly substituted volumes leading to the given answer. Award at most ***(M1)(M1)(M0)*** for subsequent working that does not lead to the given answer. Award at most ***(M1)(M1)(M0)*** for substituting $H = 110 - 2r$ as their h.

$V = 110\pi r^2 - \frac{7}{3}\pi r^3$ ***(AG)***

[3 marks]

e. (r =) 31.4 (cm) (31.4285… (cm)) ***(G2)***

OR

$(\pi)\left(220r - 7r^2\right) = 0$ ***(M1)***

Note: Award ***(M1)*** for setting the correct derivative equal to zero.

(r =) 31.4 (cm) (31.4285… (cm)) ***(A1)***

[2 marks]

f. $(V =)\ 110\pi(31.4285\ldots)^3 - \frac{7}{3}\pi(31.4285\ldots)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their 31.4285… into the given equation.

= 114000 (113781…) ***(A1)*(ft)**

Note: Follow through from part (e).

(increase in capacity =) $\frac{113.781\ldots - 79587.0\ldots}{79587.0\ldots} \times 100 = 43.0\ (\%)$ ***(R1)*(ft)**

Note: Award ***(R1)*(ft)** for finding the correct percentage increase from their two volumes.

OR

1.4 × 79587.0… = 111421.81… ***(R1)*(ft)**

Note: Award ***(R1)*(ft)** for finding the capacity of a trash can 40% larger than the original.

Claim is correct ***(A1)*(ft)**

Note: Follow through from parts (b), (e) and within part (f). The final ***(R1)(A1)*(ft)** can be awarded for their correct reason and conclusion. Do not award ***(R0)(A1)*(ft)**.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

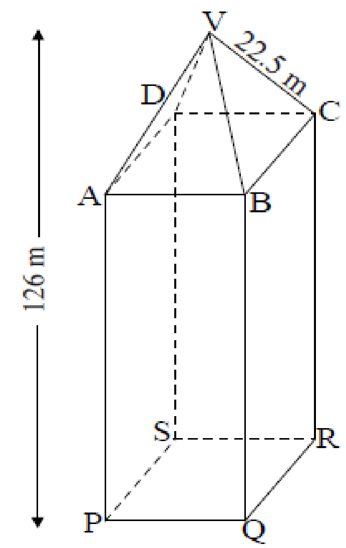


diagram not to scale

a.i. Write down the length of VA in metres. [1]

a.ii. Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. [1]

b. Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. [2]

c. Calculate the length of AC in metres. [3]

d. Show that the length of BC is 19.1 metres correct to 3 significant figures. [2]

e. Calculate the volume of the tower. [4]

f. To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg. [3]

Calculate the weight of air in the tower.

Markscheme

a.i. 22.5 (m) ***(A1)***

[1 mark]

a.ii. onbekend.png ***(A1)***

[1 mark]

b. $h = 22.5 \quad \sin 53.1°$ ***(M1)***

$= 17.99$ ***(A1)***

$= 18.0$ ***(AG)***

Note: Unrounded answer must be seen for ***(A1)*** to be awarded.

Accept 18 as ***(AG)***.

[2 marks]

c. $\mathrm{AC} = 2\sqrt{22.5^2 - 17.99...^2}$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 2, ***(M1)*** for correct substitution into formula.

OR

AC = 2(22.5)cos53.1° ***(M1)(M1)***

Notes: Award ***(M1)*** for correct use of cosine trig ratio, ***(M1)*** for multiplying by 2.

OR

$AC^2 = 22.5^2 + 22.5^2 - 2(22.5)(22.5)\cos 73.8°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine formula, ***(A1)*** for correct substitutions.

OR

$\frac{\mathrm{AC}}{\sin(73.8°)} = \frac{22.5}{\sin(53.1°)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine formula, ***(A1)*** for correct substitutions.

AC = 27.0 ***(A1)(G2)***

[3 marks]

d. $\mathrm{BC} = \sqrt{13.5^2 + 13.5^2}$ ***(M1)***

= 19.09 ***(A1)***

= 19.1 ***(AG)***

OR

$x^2 + x^2 = 27^2$ ***(M1)***

$2x^2 = 27^2$ ***(A1)***

BC = 19.09… ***(A1)***

= 19.1 ***(AG)***

Notes: Unrounded answer must be seen for ***(A1)*** to be awarded.

[2 marks]

e. Volume = Pyramid + Cuboid

$= \frac{1}{3}(18)(19.1^2) + (108)(19.1^2)$ ***(A1)(M1)(M1)***

Note: Award ***(A1)*** for 108, the height of the cuboid seen. Award ***(M1)*** for correctly substituted volume of cuboid and ***(M1)*** for correctly substituted volume of pyramid.

= $41\,588$ *(41 553 if* $2(13.5^2)$ *is used)*

= $41\,600\text{ m}^3$ ***(A1)*(ft)*(G3)***

[4 marks]

f. Weight of air = $41\,600 \times 1.2 \times 0.9$ ***(M1)(M1)***

= $44\,900\text{ kg}$ ***(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for their part (e) × 1.2, ***(M1)*** for × 0.9.

Award at most ***(M1)(M1)(A0)*** if the volume of the cuboid is used.

[3 marks]

Examiners report

a.i. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

a.ii. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

b. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

c. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

d. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

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f. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

A, a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. [3]

A, b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2]

A, c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3]

A, d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2]

A, e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1]

A, f. Use Differential Calculus to verify that your answer to (e) is correct. [5]

B, a. The curve $y = px^2 + qx - 4$ passes through the point (2, –10). [2]

Use the above information to write down an equation in p and q.

B, b, i. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [2]

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

B, b, ii. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [1]

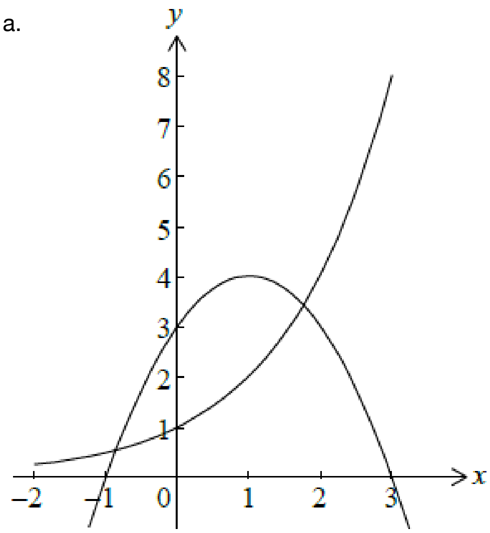
Hence, find a second equation in p and q.

B, c. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, –10) is 1. [3]

Solve the equations to find the value of p and of q.

Markscheme

A, a.



(A1)(A1)(A1)

Note: Award ***(A1)*** for correct domain, ***(A1)*** for smooth curve, ***(A1)*** for *y*-intercept clearly indicated.

[3 marks]

A, b. $y = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for y = constant, ***(A1)*** for 0.

[2 marks]

A, c. **Note:** Award ***(A1)*** for smooth parabola,

(A1) for vertex (maximum) in correct quadrant.

(A1) for all clearly indicated intercepts $x = -1$, $x = 3$ and $y = 3$.

The final mark is to be applied very strictly. ***(A1)(A1)(A1)***

[3 marks]

A, d. $x = -0.857$ $x = 1.77$ ***(G1)(G1)***

Note: Award a maximum of ***(G1)*** if *x* and *y* coordinates are both given.

[2 marks]

A, e. 4 ***(G1)***

Note: Award ***(G0)*** for (1, 4).

[1 mark]

A, f. $f'(x) = 2 - 2x$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

$2 - 2x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their gradient function to zero.

$x = 1$ ***(A1)*(ft)**

$f(1) = 3 + 2(1) - (1)^2 = 4$ ***(A1)***

Note: The final ***(A1)*** is for substitution of *x* = 1 into $f(x)$ and subsequent correct answer. Working must be seen for final ***(A1)*** to be awarded.

[5 marks]

B, a. $2^2 \times p + 2q - 4 = -10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the equation.

$4p + 2q = -6$ or $2p + q = -3$ ***(A1)***

Note: Accept equivalent simplified forms.

[2 marks]

B, b.i. $\frac{\mathrm{d}y}{\mathrm{d}x} = 2px + q$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

[2 marks]

B, b.ii. $4p + q = 1$ ***(A1)*(ft)**

[1 mark]

B, c. $4p + 2q = -6$

$4p + q = 1$ ***(M1)***

Note: Award ***(M1)*** for sensible attempt to solve the equations.

$p = 2, q = -7$ ***(A1)(A1)*(ft)*(G3)***

[3 marks]

Examiners report

A, a. Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The most common error was using the incorrect domain.

A, b Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Many had little idea of asymptotes. Others did not write their answer as an equation.

A, c Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The intercepts being inexact or unlabelled was the most frequent cause of loss of marks.

A, d Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Often, only one solution to the equation was given. Elsewhere, a lack of appreciation that the solutions were the x coordinates was a common mistake.

A, e Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The maximum is the y coordinate only; again a common misapprehension was the answer "(1, 4)".

A, f Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This was a major discriminator in the paper. Many candidates were unable to follow the analytic approach to finding a maximum point.

B, a Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

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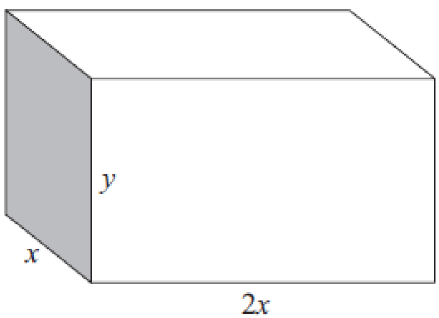
B, b, ii Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

B, c Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

A shipping container is to be made with six rectangular faces, as shown in the diagram.



The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

a. Show that $y = 12 - 3x$. [3]

b. Show that the volume V m^3 of the container is given by [2]

$V = 24x^2 - 6x^3$

c. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

d. Find the value of x for which V is a maximum. [3]

e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

Markscheme

a. $4(2x) + 4y + 4x = 48$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation.

$12x + 4y = 48$ ***(M1)***

Note: Award ***(M1)*** for simplifying (can be implied).

$y = \frac{48-12x}{4}$ **OR** $3x + y = 12$ ***(A1)***

$y = 12 - 3x$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[3 marks]

b. $V = 2x \times x \times (12 - 3x)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into volume equation, ***(A1)*** for correct substitution.

$= 24x^2 - 6x^3$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[2 marks]

c. $\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

[2 marks]

d. $48x - 18x^2 = 0$ ***(M1)(M1)***

Note: Award ***(M1)*** for using their derivative, ***(M1)*** for equating their answer to part (c) to 0.

OR

(M1) for sketch of $V = 24x^2 - 6x^3$, ***(M1)*** for the maximum point indicated ***(M1)(M1)***

OR

(M1) for sketch of $\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$, ***(M1)*** for the positive root indicated ***(M1)(M1)***

$2.67 \left(\frac{24}{9},\ \frac{8}{3},\ 2.66666...\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their part (c).

[3 marks]

e. $V = 24 \times \left(\frac{8}{3}\right)^2 - 6 \times \left(\frac{8}{3}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for substitution of their value from part (d) into volume equation.

$56.9(\mathrm{m}^3) \left(\frac{512}{9},\ 56.8888...\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to part (d).

[2 marks]

f. $\text{length} = \frac{16}{3}$ ***(A1)*****(ft)*****(G1)***

Note: Follow through from their answer to part (d). Accept 5.34 from use of 2.67

$\text{height} = 12 - 3 \times \left(\frac{8}{3}\right) = 4$ ***(M1)(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for substitution of their answer to part (d), ***(A1)*****(ft)** for answer. Accept 3.99 from use of 2.67.

[3 marks]

g. $\mathrm{SA} = 2 \times \frac{16}{3} \times 4 + 2 \times \frac{8}{3} \times 4 + 2 \times \frac{16}{3} \times \frac{8}{3}$ ***(M1)***

OR

$\mathrm{SA} = 4\left(\frac{8}{3}\right)^2 + 6 \times \frac{8}{3} \times 4$ ***(M1)***

Note: Award ***(M1)*** for substitution of their values from parts (d) and (f) into formula for surface area.

92.4 (m^2) (92.4444...(m^2)) ***(A1)***

Note: Accept 92.5 (92.4622...) from use of 3 sf answers.

$\text{Number of tins} = \frac{92.4444...}{15 \times 4} (= 1.54)$ ***(M1)***

[4 marks]

Note: Award ***(M1)*** for division of their surface area by 60.

2 tins required ***(A1)*****(ft)**

Note: Follow through from their answers to parts (d) and (f).

Examiners report

a. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x.

b. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x.

(b) Similar comment as for part (a) although more candidates made an attempt at finding the Volume.

c. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(c) This part was very well done.

d. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(d) Not many correct answers seen. Many candidates graphed the wrong equation and found 1.333 as their answer.

e. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(e) Some managed to gain follow through marks for this part.

f. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(f) Again here follow through marks were gained by those who attempted it.

g. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(g) Very few correct answers for the surface area were seen. Most candidates thought that there were 4 equal faces 2 xy and 2 faces xy. Some managed to get follow through marks for the last part if they divided by 60.

Beartown has three local newspapers: *The Art Journal*, *The Beartown News*, and *The Currier*.

A survey shows that

32 % of the town's population read *The Art Journal*,

46 % read *The Beartown News*,

54 % read *The Currier*,

3 % read *The Art Journal* and *The Beartown News* **only**,

8 % read *The Art Journal* and *The Currier* **only**,

12 % read *The Beartown News* and *The Currier* **only**, and

5 % of the population reads **all** three newspapers.

a. Draw a Venn diagram to represent this information. Label A the set that represents *The Art Journal* readers, B the set that represents *The Beartown News* readers, and C the set that represents *The Currier* readers. [4]

b. Find the percentage of the population that does not read any of the three newspapers. [2]

c. Find the percentage of the population that reads exactly one newspaper. [2]

d. Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*. [2]

e. A local radio station states that 83 % of the population reads either *The Beartown News* or *The Currier*. [2]

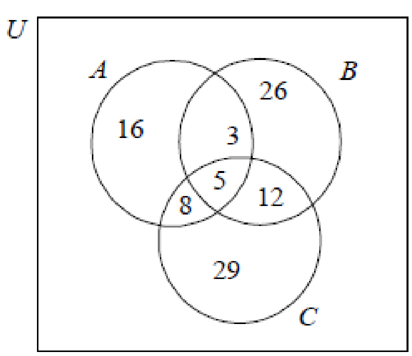
Use your Venn diagram to decide whether the statement is true. Justify your answer.

f. The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local newspapers. [4]

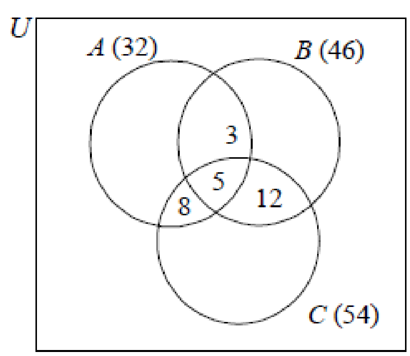
Find the percentage error in this claim.

Markscheme

a.



OR



(A1) for three circles and a rectangle (U need not be seen)

(A1) for 5

(A1) for 3, 8 and 12

(A1) for 16, 26 and 29 **OR** 32, 46, 54 placed outside the circles. ***(A4)***

Note: Accept answers given as decimals or fractions.

[4 marks]

b. 100 – (16 + 26 + 29) – (8 + 5 + 3 + 12) ***(M1)***

100 – 71 – 28

Note: Award ***(M1)*** for correct expression. Accept equivalent expressions, for example 100 – 71 – 28 or 100 – (71 + 28).

= 1 ***(A1)*****(ft)*****(G2)***

Note: Follow through from their Venn diagram but only if working is seen.

[2 marks]

c. 16 + 26 + 29 ***(M1)***

Note: Award ***(M1)*** for 16, 26, 29 seen.

= 71 ***(A1)*****(ft)*****(G2)***

Note: Follow through from their Venn diagram but only if working is seen.

[2 marks]

d. 16 + 3 + 26 ***(M1)***

Note: Award ***(M1)*** for their 16, 3, 26 seen.

= 45 ***(A1)*****(ft)*****(G2)***

Note: Follow through from their Venn diagram but only if working is seen.

[2 marks]

e. True ***(A1)*****(ft)**

100 – (1 –16) = 83 ***(R1)*****(ft)**

OR

46 + 54 – 17 = 83 ***(R1)*****(ft)**

Note: Do not award ***(A1)(R0)***. Follow through from their Venn diagram.

[2 marks]

f. 28% of 120000 ***(M1)***

= 33600 ***(A1)***

$\% \text{ error} = \frac{(34000-33600)}{33600} \times 100$ ***(M1)***

Note: Award ***(M1)*** for 28 seen (may be implied by 33600 seen), award ***(M1)*** for correct substitution of **their** 33600 in the percentage error formula. If an error is made in calculating 33600 award a maximum of ***(M1)(A0)(M1)(A0)***, the final accuracy mark is lost.

OR

$\frac{34000}{120000} \times 100$ ***(M1)***

= 28.3(28.3333…) ***(A1)***

$\% \text{ error} = \frac{(28.3333...-28)}{28} \times 100$ ***(M1)***

= 1.19% (1.19047...) ***(A1)*****(ft)*****(G3)***

Note: % sign not required. Accept 1.07 (1.0714…) with use of 28.3. 1.18 with use of 28.33 and 1.19 with use of 28.333. Award ***(G3)*** for 1.07, 1.18 or 1.19 seen without working.

[4 marks]

Examiners report

a. This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
- the **incorrect** interpretation of “either ...or” as “exclusive or”. It is of the utmost importance to note that the ambiguity of the “or” statement will be removed and “exclusive or” signalled by the phrase “either ...or....**but not both**”. Otherwise, “inclusive or” must always be assumed.

A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

b. This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
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A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

c. This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
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A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

d. This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
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- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
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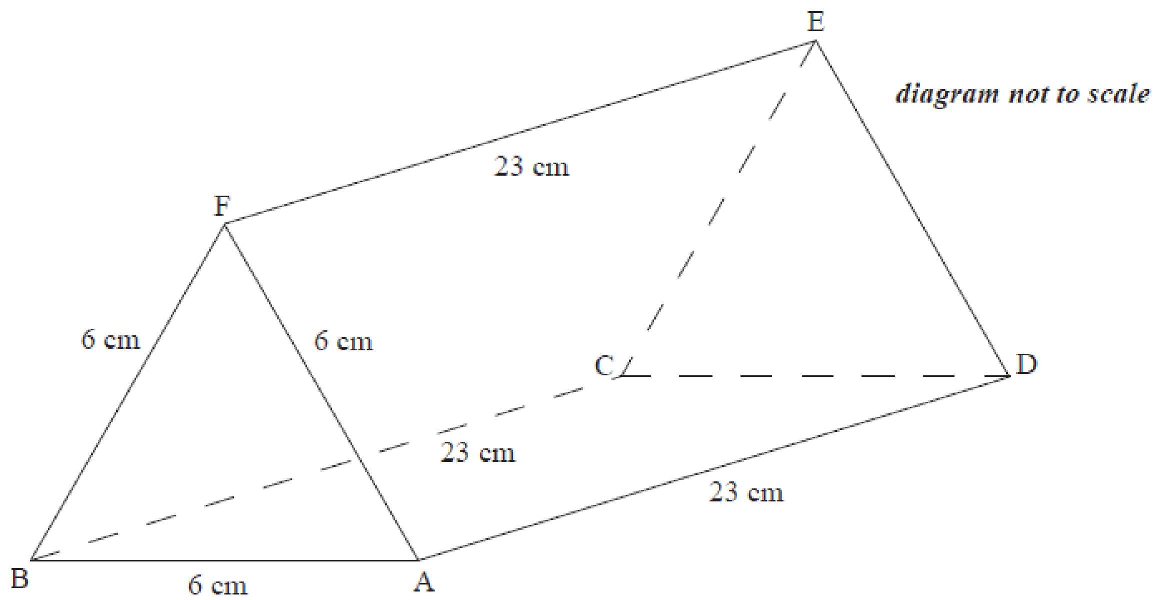
A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

f. This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
- the **incorrect** interpretation of “either ...or” as “exclusive or”. It is of the utmost importance to note that the ambiguity of the “or” statement will be removed and“exclusive or” signalled by the phrase “either ...or....**but not both**”. Otherwise, “inclusive or” must always be assumed.

A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

A chocolate bar has the shape of a triangular right prism ABCDEF as shown in the diagram. The ends are equilateral triangles of side 6 cm and the length of the chocolate bar is 23 cm.



a, i. Write down the size of angle BAF. [1]

a, ii. Hence or otherwise find the area of the triangular end of the chocolate bar. [3]

b. Find the total surface area of the chocolate bar. [3]

c. It is known that 1 cm^3 of this chocolate weighs 1.5 g. Calculate the weight of the chocolate bar. [3]

d. A different chocolate bar made with the same mixture also has the shape of a triangular prism. The ends are triangles with sides of length 4 cm, 6 cm and 7 cm. [3]

Show that the size of the angle between the sides of 6 cm and 4 cm is 86.4° correct to 3 significant figures.

e. The weight of this chocolate bar is 500 g. Find its length. [4]

Markscheme

a, i. 60° ***(A1)***

[1 mark]

a, ii. ***Unit penalty (UP) applies in this part***

$\text{Area} = \frac{6 \times 6 \times \sin 60^\circ}{2}$ ***(M1)(A1)***

(UP) = 15.6 cm^2 $(9\sqrt{3})$ ***(A1)(ft)(G2)***

Note: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct values. Accept alternative correct methods.

[3 marks]

b. ***Unit penalty (UP) applies in this part***

$\text{Surface Area} = 15.58 \times 2 + 23 \times 6 \times 3$ ***(M1)(M1)***

Note: Award ***(M1)*** for two terms with 2 and 3 respectively, ***(M1)*** for 23×6 (138).

(UP) Surface Area = 445 cm^2 ***(A1)*(ft)*(G2)***

[3 marks]

c. ***Unit penalty (UP) applies in this part***

$\text{weight} = 1.5 \times 15.59 \times 23$ ***(M1)(M1)***

Note: Award ***(M1)*** for finding the volume, ***(M1)*** for multiplying their volume by 1.5.

(UP) weight = 538 g ***(A1)*(ft)*(G3)***

[3 marks]

d. $\cos\alpha = \frac{4^2+6^2-7^2}{2\times4\times6}$ ***(M1)(A1)***

Note: Award ***(M1)*** for using cosine rule with values from the problem, ***(A1)*** for correct substitution.

$\alpha = 86.41\ldots$ ***(A1)***

$\alpha = 86.4^\circ$ ***(AG)***

Note: 86.41… must be seen for final ***(A1)*** to be awarded.

[3 marks]

e. ***Unit penalty (UP) applies in this part***

$l \times \frac{4\times6\times\sin 86.4^\circ}{2} \times 1.5 = 500$ ***(M1)(A1)(M1)***

Notes: Award ***(M1)*** for finding an expression for the volume, ***(A1)*** for correct substitution, ***(M1)*** for multiplying the volume by 1.5 and equating to 500, or for equating the volume to $\frac{500}{1.5}$.

If formula for volume is not correct but consistent with that in (c) award at most ***(M1)(A0)*(ft)*(M1)(A0)***.

(UP) l = 27.8 cm ***(A1)(G3)***

[4 marks]

Examiners report

a, i. It was pleasing to show candidate working throughout this question. Follow through marks could be awarded when incorrect answers were given. Many candidates incorrectly calculated the weight of the chocolate bar by multiplying the surface area by 1.5g. Also a large number of students incorrectly used the formula for the volume of a pyramid rather than for a prism. Most candidates were successful in their use of the cosine rule but did not give the answer before it was rounded to 86.4, resulting in the loss of the final A mark. The last part acted as a clear discriminator, very few students were able to find the correct length of the new chocolate bar. Most students used units correctly.

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Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is $190\ \mathrm{m}$, the length of CD is $120\ \mathrm{m}$, the length of AD is $70\ \mathrm{m}$, the size of angle BCD is 75° and the size of angle BAD is 115° .

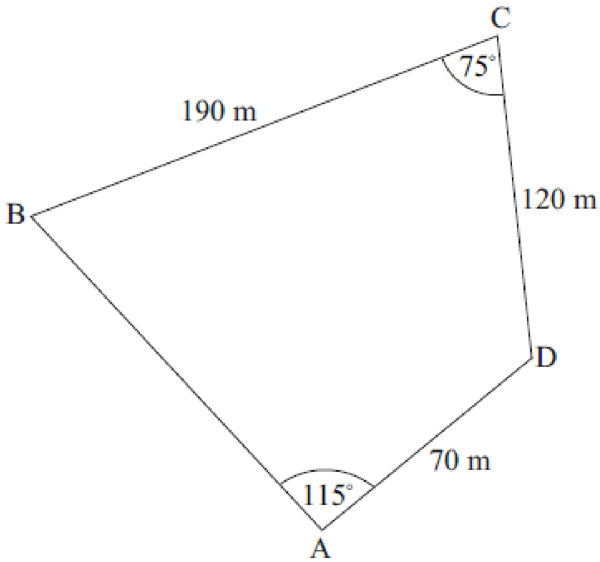


diagram not to scale

Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

a. Calculate the length of the fence. [3]

b. The fence costs 17 USD per metre to build. [2]

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

c. Show that the size of angle ABD is 18.8° , correct to three significant figures. [3]

d. Calculate the area of triangle ABD . [4]

e. She sells the land for 120 USD per square metre. [2]

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

f. Pauline invests 300000 USD from the sale of the land in a bank that pays compound interest compounded annually. [4]

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

Markscheme

a. $\mathrm{BD}^2 = 190^2 + 120^2 - 2(190)(120)\cos 75^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine formula, ***(A1)*** for correct substitution.

$= 197$ m ***(A1)(G2)***

Note: If radians are used award a maximum of ***(M1)(A1)(A0)***.

[3 marks]

b. $\text{cost} = 196.717\ldots \times 17$ ***(M1)***

$= 3344\ \text{USD}$ ***(A1)(ft)(G2)***

Note: Accept 3349 from 197.

[2 marks]

c. $\frac{\sin(\text{ABD})}{70} = \frac{\sin(115°)}{196.7}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine formula, ***(A1)*** for correct substitution.

$= 18.81° \ldots$ ***(A1)*(ft)**
$= 18.8°$ ***(AG)***

Notes: Both the unrounded and rounded answers must be seen for the final ***(A1)*** to be awarded. Follow through from their (a). If 197 is used the unrounded answer is $= 18.78° \ldots$

[3 marks]

d. $\text{angle BDA} = 46.2°$ ***(A1)***

$\text{Area} = \frac{70 \times (196.717\ldots) \times \sin(46.2°)}{2}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted area formula, ***(A1)*** for correct substitution.

$\text{Area ABD} = 4970 \text{ m}^2$ ***(A1)*(ft)*(G2)***

Notes: If 197 used answer is 4980.

Notes: Follow through from (a) only. Award ***(G2)*** if there is no working shown and $46.2°$ not seen. If $46.2°$ seen without subsequent working, award ***(A1)(G2)***.

[4 marks]

e. $4969.38 \ldots \times 120$ ***(M1)***

$= 596327 \text{ USD}$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their (d).

[2 marks]

f. $300000\left(1 + \frac{r}{100}\right)^{15} = 600000$ or equivalent ***(A1)(M1)(A1)***

Notes: Award ***(A1)*** for 600000 seen or implied by alternative formula, ***(M1)*** for substituted CI formula, ***(A1)*** for correct substitutions.

$r = 4.73$ ***(A1)*(ft)*(G3)***

Notes: Award ***(G3)*** for 4.73 with no working. Award ***(G2)*** for 4.7 with no working.

[4 marks]

Examiners report

a. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

b. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

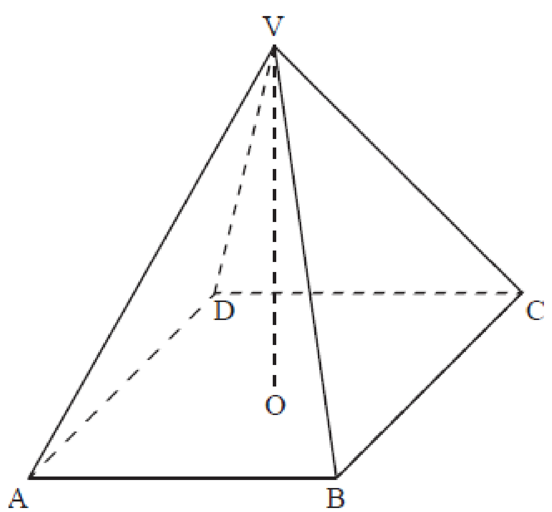
c. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

d. Again, most candidates used the appropriate area formula – however, some did not appreciate the purpose of the given answer and were unable to complete the question accurately.

e. Again, most candidates used the appropriate area formula – however, some did not appreciate the purpose of the given answer and were unable to complete the question accurately.

f. The final part, in which compound interest was again asked for, tested most candidates but there were many successful attempts using either the GDC's finance package or correct use of the formula. Care must be taken with the former to show some indication of the values to be used in the context of the question. With the latter approach marks were again lost due to a lack of appreciation of the difference between interest and value.

ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



a. Calculate the volume of the pyramid. [2]

b. The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid. [2]

c. Show that the length of the sloping edge VC of the pyramid is 3.6 cm. [4]

d. Calculate the angle at the vertex, $\text{B}\hat{\text{V}}\text{C}$. [3]

e. Calculate the total surface area of the pyramid. [4]

Markscheme

a. *Unit penalty **(UP)** is applicable in question parts (a), (b) and (e) **only**.*

$\text{V} = \frac{1}{3} \times 3.2^2 \times 2.8$ ***(M1)***

***(M1)** for substituting in correct formula*

(UP) = 9.56 cm^3 ***(A1)(G2)***

[2 marks]

b. *Unit penalty **(UP)** is applicable in question parts (a), (b) and (e) **only**.*

9.56×9.3 ***(M1)***

(UP) = 88.9 grams ***(A1)*(ft)*(G2)***

[2 marks]

c. $\frac{1}{2}\text{base} = 1.6 \text{ seen}$ ***(M1)***

*award **(M1)** for halving base*

$\text{OC}^2 = 1.6^2 + 1.6^2 = 5.12$ ***(A1)***

*award **(A1)** for one correct use of Pythagoras*

$5.12 + 2.8^2 = 12.96 = \text{VC}^2$ ***(M1)***

*award **(M1)** for using Pythagoras again to find VC^2*

VC = 3.6 **AG**

*award **(A1)** for* 3.6 *obtained from* 12.96 *only (not* 12.95…*)* ***(A1)***

OR

$\text{AC}^2 = 3.2^2 + 3.2^2 = 20.48$ ***(A1)***

*award **(A1)** for one correct use of Pythagoras*

({\text{OC}} = \frac{1}{2} \sqrt{20.48}\) (= 2.26...) ***(M1)***

*award **(M1)** for halving AC*

$2.8^2 + (2.26...)^2 = \text{VC}^2 = 12.96$ ***(M1)***

*award **(M1)** for using Pythagoras again to find VC^2*

VC = 3.6 **AG** ***(A1)***

*award **(A1)** for* 3.6 *obtained from* 12.96 *only (not* 12.95…*)*

[4 marks]

d. $3.2^2 = 3.6^2 + 3.6^2 - 2 \times (3.6)(3.6) \cos \text{B}\hat{\text{V}}\text{C}$ ***(M1)(A1)***

$\text{B}\hat{\text{V}}\text{C} = 52.8^\circ$ *(no **(ft)** here)* ***(A1)(G2)***

*award **(M1)** for substituting in correct formula, **(A1)** for correct substitution*

OR

$\sin \text{B}\hat{\text{V}}\text{M} = \frac{1.6}{3.6}$ where *M* is the midpoint of BC ***(M1)(A1)***

$\text{B}\hat{\text{V}}\text{C} = 52.8^\circ$ *(no **(ft)** here)* ***(A1)***

[3 marks]

e. *Unit penalty **(UP)** is applicable in question parts (a), (b) and (e) **only**.*

$4 \times \frac{1}{2}(3.6)^2 \times \sin(52.8^\circ) + (3.2)^2$ ***(M1)(M1)(M1)***

*award **(M1)** for $\times 4$, **(M1)** for substitution in relevant triangle area, ($\frac{1}{2}(3.2)(2.8)$ gets **(M0)**)*

***(M1)** for $+(3.2)^2$*

(UP) = 30.9 cm^2 **((ft)** *from their (d))* ***(A1)*****(ft)*****(G2)***

[4 marks]

Examiners report

a. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

b. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

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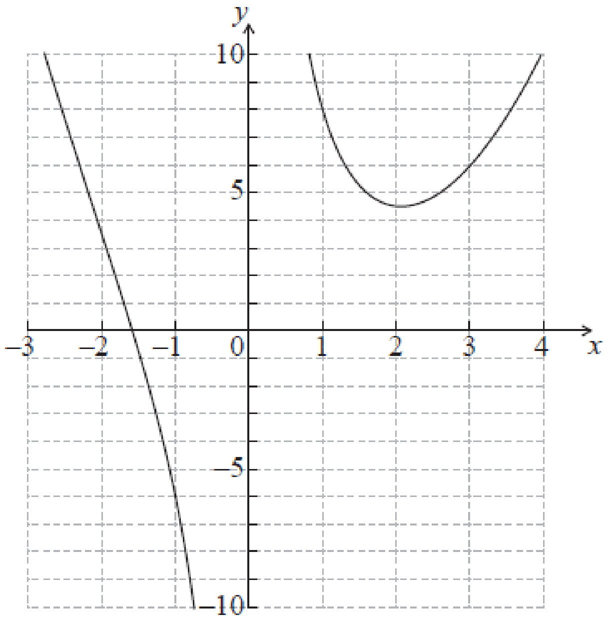
The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3 - 2x^2 - 9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the **x-coordinate** of B and of C .

Markscheme

a. (i) $x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for $x =$ a constant, ***(A1)*** for the constant in their equation being 0.

(ii) $-1.58\,(-1.58454\ldots)$ ***(G1)***

Note: Accept -1.6, do not accept -2 or -1.59.

(iii) $(2.06, 4.49)$ $(2.06020\ldots, 4.49253\ldots)$ ***(G1)(G1)***

Note: Award at most ***(G1)(G0)*** if brackets not used. Award ***(G0)(G1)*****(ft)** if coordinates are reversed.

Note: Accept $x = 2.06,\, y = 4.49$.

Note: Accept 2.1, do not accept 2.0 or 2. Accept 4.5, do not accept 5 or 4.50.

[5 marks]

b. $f'(x) = 2x - 2 - \frac{9}{x^2}$ ***(A1)(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2x$, ***(A1)*** for -2, ***(A1)*** for -9, ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A1)(A0)*** if there are extra terms present.

[4 marks]

c. $f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for taking the correct common denominator.

$= \frac{(2x^3-2x^2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for multiplying brackets or equivalent.

$= \frac{2x^3-2x^2-9}{x^2}$ ***(AG)***

Note: The final ***(M1)*** is not awarded if the given answer is not seen.

[2 marks]

d. $f'(1) = \frac{2(1)^3-2(1)-9}{(1)^2}$ ***(M1)***

$= -9$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into **given** (or their correct) $f'(x)$. There is no follow through for use of their incorrect derivative.

[2 marks]

e. $\frac{1}{9}$ ***(A1)*(ft)**

Note: Follow through from part (d).

[1 mark]

f. $y - 8 = \frac{1}{9}(x - 1)$ ***(M1)(M1)***

Notes: Award ***(M1)*** for substitution of their gradient from (e), ***(M1)*** for substitution of given point. Accept all forms of straight line.

$y = \frac{1}{9}x + \frac{71}{9}$ $(y = 0.111111\ldots x + 7.88888\ldots)$ ***(A1)*(ft)*(G3)***

Note: Award the final ***(A1)*(ft)** for a correctly rearranged formula of **their** straight line in (f). Accept $0.11x$, do not accept $0.1x$. Accept 7.9, do not accept 7.88, do not accept 7.8.

[3 marks]

g. $-2.50,\, 3.61$ $(-2.49545\ldots,\, 3.60656\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Notes: Follow through from their line L from part (f) even if no working shown. Award at most ***(A0)(A1)*(ft)** if their correct coordinate pairs given.

Note: Accept -2.5, do not accept -2.49. Accept 3.6, do not accept 3.60.

[2 marks]

Examiners report

a. As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

   As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

   Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

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   Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

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Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

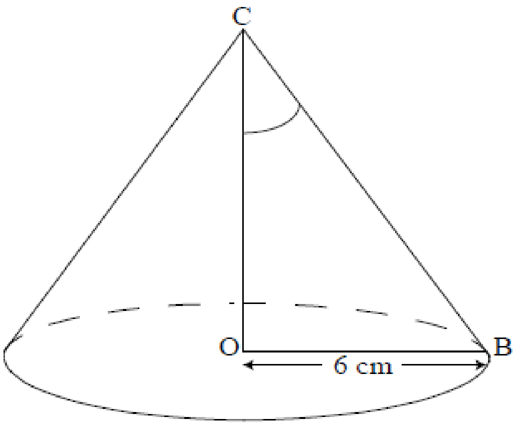
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

a. Find the area of the base of the cylinder. [2]

b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. [2]

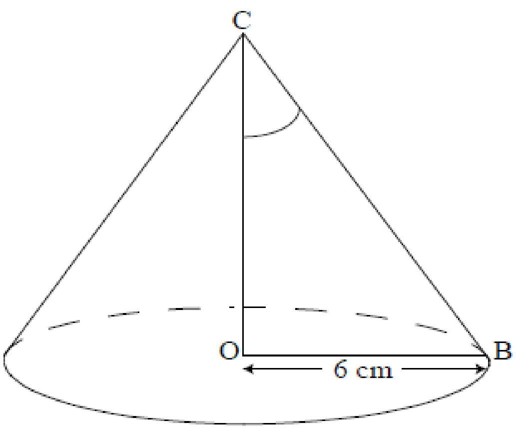
c. Find the total surface area of the cylinder. [3]

d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [3]



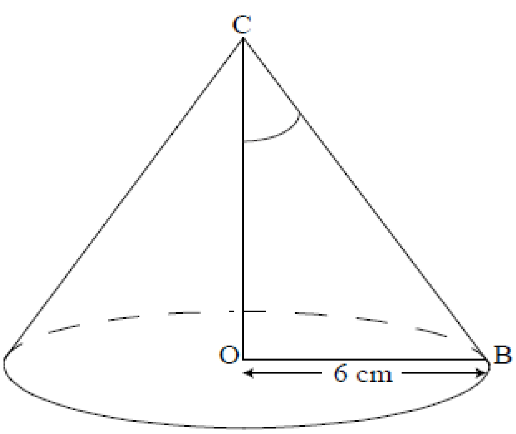
Find the height, OC, of the cone.

e. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



Find the size of angle BCO.

f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



Find the slant height, CB.

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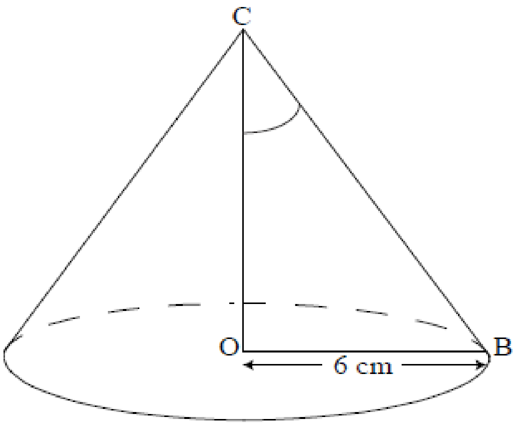

Find the total surface area of the cone.

Markscheme

a. $\pi \times 4^2$ ***(M1)***

= 50.3 (16π) cm^2 (50.2654...) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in area formula. The answer is 50.3 cm^2, the units are required.

[2 marks]

b. 50.265...× 8 ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula.

= 402.123... ***(A1)***
= 402 (cm^3) ***(AG)***

Note: Both the unrounded and the rounded answer must be seen for the ***(A1)*** to be awarded. The units are **not** required

[2 marks]

c. $2 \times \pi \times 4 \times 8 + 2 \times \pi \times 4^2$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in the curved surface area formula, ***(M1)*** for adding the area of their two bases.

= 302 cm^2 (96π cm^2) (301.592...) ***(A1)*(ft)*(G2)***

Notes: The answer is 302 cm^2, the units are required. Do not penalise for missing or incorrect units if penalised in part (a). Follow through from their answer to part (a).

[3 marks]

d. $\frac{1}{3}\pi \times 6^2 \times \mathrm{OC} = 402$ ***(M1)(M1)***

Note: Award ***(M1)*** for correctly substituted volume formula, ***(M1)*** for equating to 402 (402.123…).

$\mathrm{OC} = 10.7\ (\mathrm{cm}) \left(10\frac{2}{3},\ 10.6666...\right)$ ***(A1)(G2)***

[3 marks]

e. $\tan \mathrm{BCO} = \frac{6}{10.66...}$ ***(M1)***

Note: Award ***(M1)*** for use of correct tangent ratio.

$\mathrm{B\hat{C}O} = 29.4^\circ$ (29.3577...) ***(A1)*(ft)*(G2)***

Notes: Accept 29.3° (29.2814...) if 10.7 is used. An acceptable alternative method is to calculate CB first and then angle BCO. Allow follow through from parts (d) and (f). Answers range from 29.2° to 29.5°.

[2 marks]

f. $\mathrm{CB} = \sqrt{6^2 + (10.66...)^2}$ ***(M1)***

OR

$\sin 29.35...^\circ = \frac{6}{\mathrm{CB}}$ ***(M1)***

OR

$\cos 29.35...^\circ = \frac{10.66...}{\mathrm{CB}}$ ***(M1)***

CB = 12.2 (cm) (12.2383...) ***(A1)*****(ft)*****(G2)***

Note: Accept 12.3 (12.2674...) if 10.7 (and/or 29.3) used. Follow through from part (d) or part (e) as appropriate.

[2 marks]

g. $\pi \times 6 \times 12.2383... + \pi \times 6^2$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in curved surface area formula, ***(M1)*** for correct substitution in area of circle formula, ***(M1)*** for addition of the two areas.

= 344 cm^2 (343.785...) ***(A1)*****(ft)*****(G3)***

Note: The answer is 344 cm^2, the units are required. Do not penalise for missing or incorrect units if already penalised in either part (a) or (c). Accept 345 cm^2 if 12.3 is used and 343 cm^2 if 12.2 is used. Follow through from their part (f).

[4 marks]

Examiners report

a. This question was either very well done – by the majority – or very poorly (but not both). Many incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, since it was that the formulas for cones were not well understood. Further, the idea of "total surface area" was a mystery to many – a slavish reliance of formulas, irrespective of context, led to many errors and a consequent loss of marks.

The invariance of volume for solids and liquids that provided the link in this question was not understood by many, but was felt to be an appropriate subject for an examination.

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Alex and Kris are riding their bicycles together along a bicycle trail and note the following distance markers at the given times.

Time (t hours)	1	2	3	4	5	6	7
Distance (d km)	57	65	72	81	89	97	107

a. Draw a scatter diagram of the data. Use 1 cm to represent 1 hour and 1 cm to represent 10 km. [3]

b.i. Write down for this set of data the mean time, $\bar{t}$. [1]

b.ii.Write down for this set of data the mean distance, $\bar{d}$. [1]

c. Mark and label the point $M(\bar{t}\,,\ \bar{d}\,)$ on your scatter diagram. [2]

d. Draw the line of best fit on your scatter diagram. [2]

e. **Using your graph**, estimate the time when Alex and Kris pass the 85 km distance marker. Give your answer correct to **one decimal place**. [2]

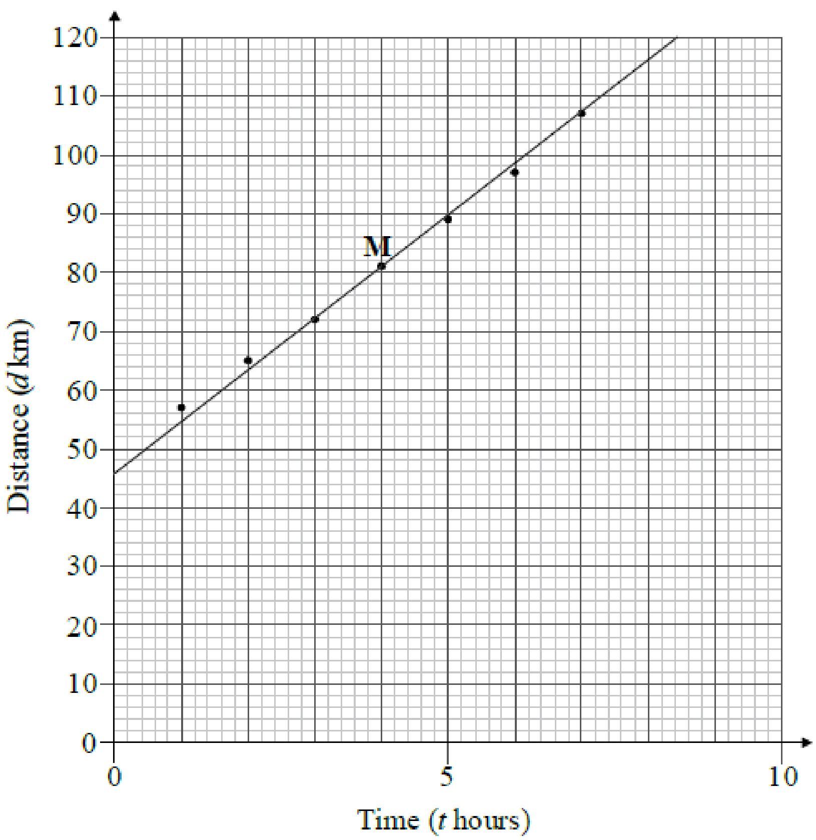
f. Write down the equation of the regression line for the data given. [2]

g.i. **Using your equation** calculate the distance marker passed by the cyclists at 10.3 hours. [2]

g.ii.Is this estimate of the distance reliable? Give a reason for your answer. [2]

Markscheme

a.



(A1)(A2)

Notes: Award ***(A1)*** for axes labelled with d and t and correct scale, ***(A2)*** for 6 or 7 points correctly plotted, ***(A1)*** for 4 or 5 points, ***(A0)*** for 3 or less points correctly plotted. Award at most ***(A1)(A1)*** if points are joined up. If axes are reversed award at most ***(A0)(A2)***

[3 marks]

b.i.$\bar{t}\ = 4$ ***(G1)***

[1 mark]

b.ii.

$\bar{d}\ = 81.1\left(\frac{568}{7}\right)$ ***(G1)***

Note: If answers are the wrong way around award in (i) ***(G0)*** and in (ii) ***(G1)*(ft)**.

[1 mark]

c. Point marked and labelled with M or $\bar{t}$, $\bar{d}$ on their graph ***(A1)*(ft)*(A1)*(ft)**

[2 marks]

d. Line of best fit drawn that passes through their M and (0, 48) ***(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for straight line that passes through their M, ***(A1)*** for line (extrapolated if necessary) that passes through (0, 48).

Accept error of ±3. If ruler not used award a maximum of ***(A1)*(ft)*(A0)***.

[2 marks]

e. 4.5h (their answer ±0.2) ***(M1)(A1)*(ft)*(G2)***

Note: Follow through from their graph. If method shown by some indication on graph of point but answer is incorrect, award ***(M1)(A0)***.

[2 marks]

f. $d = 8.25t + 48.1$ ***(G1)(G1)***

Notes: Award ***(G1)*** for 8.25, ***(G1)*** for 48.1.

Award at most ***(G1)(G0)*** if $d =$ (*or* $y =$) is not seen.

Accept $d - 81.1 = 8.25(t - 4)$ or equivalent.

[2 marks]

g.i. $d = 8.25 \times 10.3 + 48.1$ ***(M1)***

$d = 133$ km ***(A1)*(ft)*(G2)***

[2 marks]

g.ii. No ***(A1)***

Outside the set of values of t or equivalent. ***(R1)***

Note: Do not award ***(A1)(R0)***.

[2 marks]

Examiners report

a. This question was well answered by most of the candidates. Diagrams were in general well drawn except for some students that reversed the axes or did not use the stated scales. They were able to use the GDC to find the means and the equation of the regression line. Very few students could take the correct decision in (g) (ii) by stating that the value was outside the range of the data set. The majority inclined their answers towards the context of the question and forgot what they had been taught about how wrong extrapolation can be.

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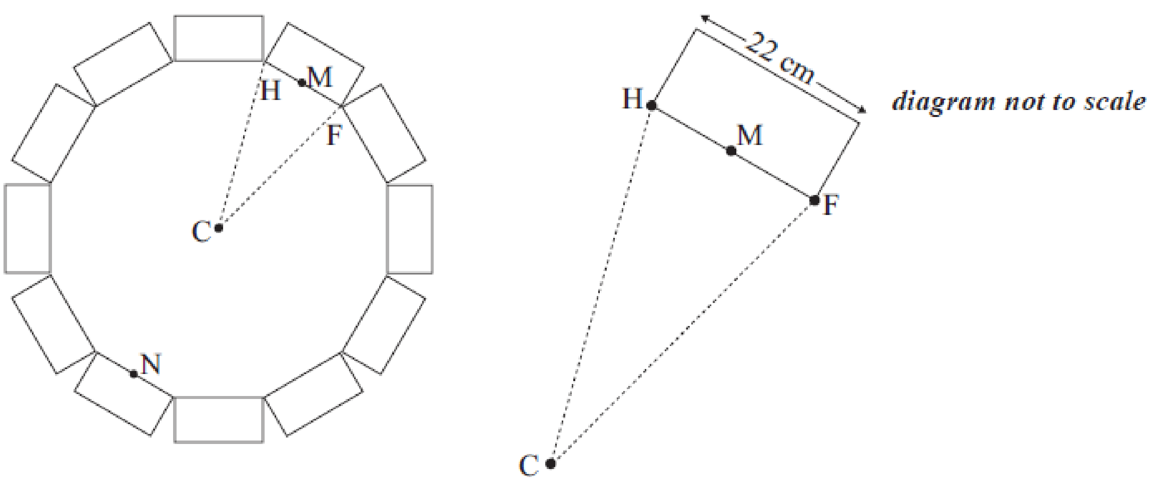
A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.

The gardener decides to have a triangular lawn ABC, instead of paving, in the middle of the rectangular area, as shown in the diagram below.

onbekend.png

The distance AB is 4 metres, AC is 6 metres and angle BAC is 40°.

In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.



The garden bed has an area of 5419 cm^2. It is covered with soil to a depth of 2.5 cm.

It is estimated that 1 kilogram of soil occupies 514 cm^3.

a.i. Calculate the total area to be paved. Give your answer in cm^2. [3]

a.ii. Write down the area of each brick. [1]

a.iii Find how many bricks are required to pave the total area. [2]

b.i. Find the length of BC. [3]

b.ii. Hence write down the perimeter of the triangular lawn. [1]

b.iii. Calculate the area of the lawn. [2]

b.iv. Find the percentage of the rectangular area which is to be lawn. [3]

c.i. Find the angle FCH. [2]

c.ii. Calculate the distance MN from one side of the garden bed to the other, passing through C. [3]

d. Find the volume of soil used. [2]

e. Find the number of kilograms of soil required for this garden bed. [2]

Markscheme

a.i. 15.4 × 5.5 ***(M1)***

84.7 m^2 ***(A1)***

= 847000 cm^2 ***(A1)(G3)***

Note: Award ***(G2)*** if 84.7 m^2 seen with no working.

OR

1540 × 550 ***(A1)(M1)***

= 847000 cm^2 ***(A1)*(ft)*(G3)***

Note: Award ***(A1)*** for both dimensions converted correctly to cm, ***(M1)*** for multiplication of both dimensions. ***(A1)*(ft)** for correct product of their sides in cm.

[3 marks]

a.ii. 242 cm^2 (0.0242 m^2) ***(A1)***

[1 marks}

a.iii. $\frac{15.4}{0.22} = 70$ ***(M1)***

$\frac{5.5}{0.11} = 50$

$70 \times 50 = 3500$ ***(A1)(G2)***

OR

$\frac{847000}{242} = 3500$ ***(M1)(A1)*(ft)*(G2)***

Note: Follow through from parts (a) (i) and (ii).

[2 marks]

b.i. $BC^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 40^\circ$ ***(M1)(A1)***

$BC = 3.90 \text{ m}$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*** for correct substitutions, ***(A1)*** for correct answer.

[3 marks]

b.ii. perimeter = 13.9 m ***(A1)*****(ft)*****(G1)***

Notes: Follow through from part (b) (i).

[1 mark]

b.iii.

$\text{Area} = \frac{1}{2} \times 4 \times 6 \times \sin 40^\circ$ ***(M1)***

= 7.71 m^2 ***(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for correct formula and correct substitution, ***(A1)*****(ft)** for correct answer.

[2 marks]

b.iv. $\frac{7.713}{84.7} \times 100\ \% = 9.11\ \%$ ***(A1)(M1)(A1)*****(ft)*****(G2)***

Notes: Accept 9.10 %.

Award ***(A1)*** for both measurements correctly written in the same unit, ***(M1)*** for correct method, ***(A1)*****(ft)** for correct answer.

Follow through from (b) (iii) and from consistent error in conversion of units throughout the question.

[3 marks]

c.i. $\frac{360^\circ}{12}$ ***(M1)***

$= 30^\circ$ ***(A1)(G2)***

[2 marks]

c.ii. $MN = 2 \times \frac{11}{\tan 15}$ ***(A1)*****(ft)*****(M1)***

OR

$MN = 2 \times 11 \tan 75^\circ$

$MN = 82.1 \text{ cm}$ ***(A1)*****(ft)*****(G2)***

Notes: Award ***(A1)*** for 11 and 2 seen (implied by 22 seen), ***(M1)*** for dividing by tan15 (or multiplying by tan 75).

Follow through from their angle in part (c) (i).

[3 marks]

d. volume = 5419 × 2.5 ***(M1)***

= 13500 cm^3 ***(A1)(G2)***

[2 marks]

e. $\frac{13547.34...}{514} = 26.4$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for dividing their part (d) by 514.

Accept 26.3.

[2 marks]

Examiners report

a.i. Part (a) was well done except for the fact that very few students were able to convert correctly from m^2 to cm^2 and this was very disappointing.

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b.i. In part (b) the cosine rule and the area of a triangle were well done. In some cases units were missing and therefore a unit penalty was applied.

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c.i. Part (c) was clearly the most difficult one for the students. The general impression was that they did not read the diagram in detail. A number of candidates could not distinguish the circle from the triangle and hence used an incorrect method to find the radius.

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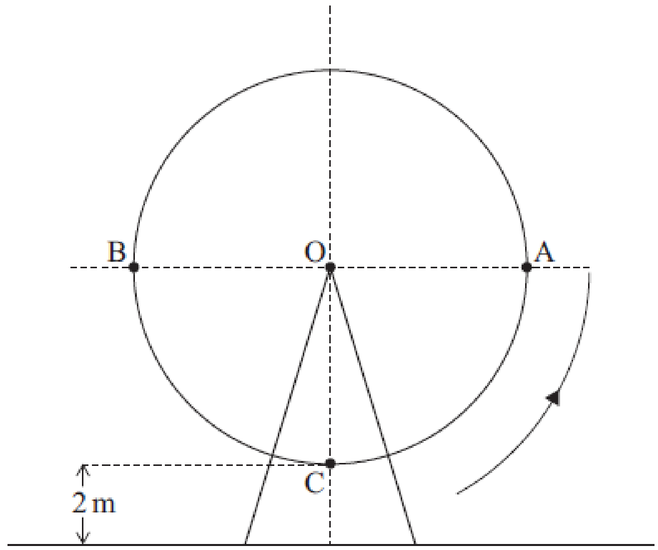
d. It was pleasing to see candidates recovering well to get full marks for the last two parts.

e. It was pleasing to see candidates recovering well to get full marks for the last two parts.

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.



a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. [2]

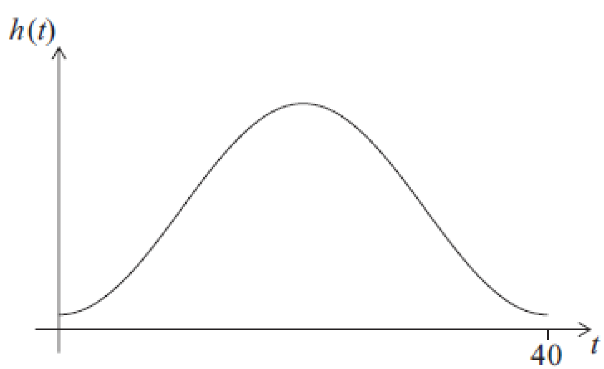
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2]

c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. [4]



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

Markscheme

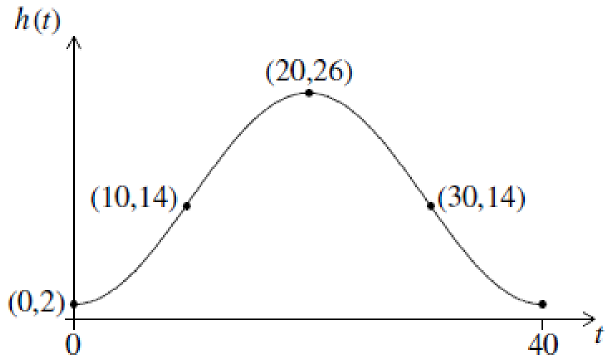
a. (i) 14 m ***(A1)***

(ii) 26 m ***(A1)***

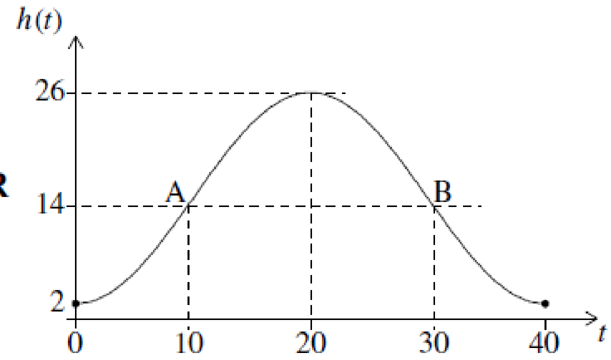
[2 marks]

b. A:10, B:30 ***(A1)(A1)***

[2 marks]

c. 

OR



(A1)*(ft)*(A1)*(ft)*(A1)

(ft)*(A1)*(ft)

Note: Award ***(A1)*(ft)** for coordinates of each point clearly indicated either by scale or by coordinate pairs. Points need not be labelled A and B in the second diagram. Award a maximum of ***(A1)(A0)(A1)*(ft)*(A1)*(ft)** if coordinates are reversed. Do not penalise reversed coordinates if this has already been penalised in Q4(a)(iii).

[4 marks]

Examiners report

a. Most candidates were able to start this question. Those of an average ability completed it to the end of part (c) and the best gained good success in the latter parts. Its purpose was to discriminate at the highest level and this it did.

Some concerns were raised on the G2 forms as to the appropriateness of this question. However, the MSSL course tries in part to link areas of the syllabus to "real-life" situations and address these. A look back to past years' examination papers, and to the syllabus documentation, should yield similar examples.

b. Most candidates were able to start this question. Those of an average ability completed it to the end of part (c) and the best gained good success in the latter parts. Its purpose was to discriminate at the highest level and this it did.

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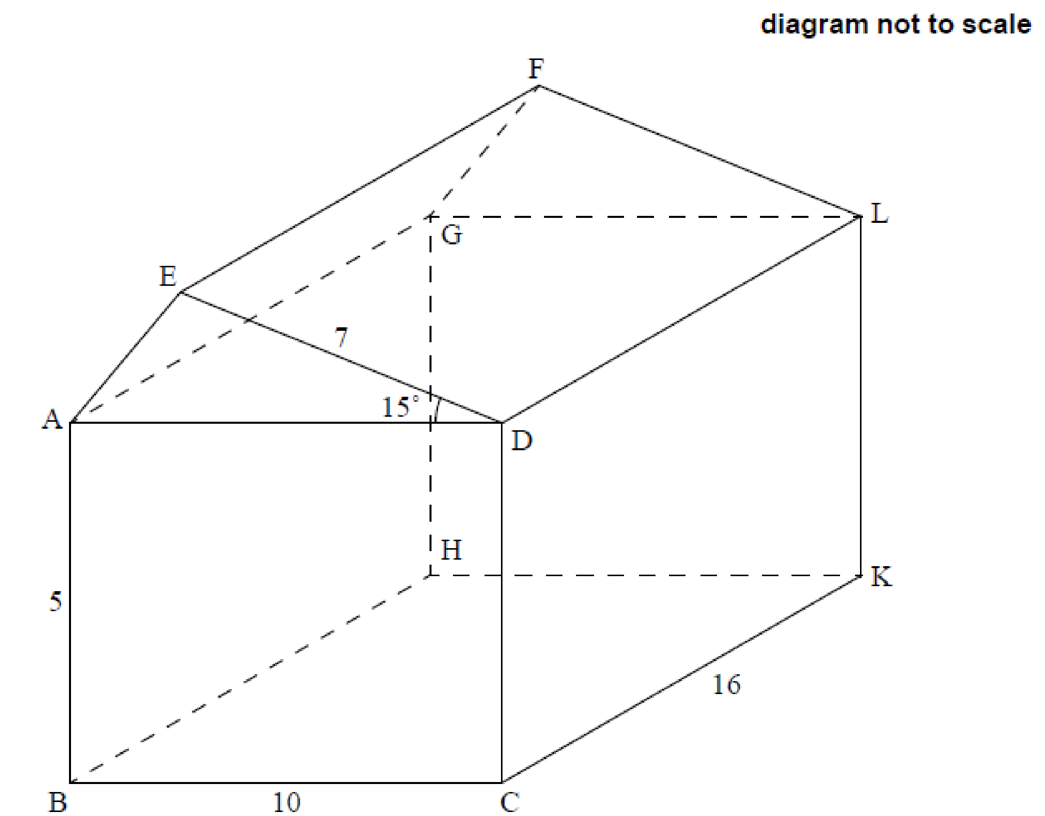
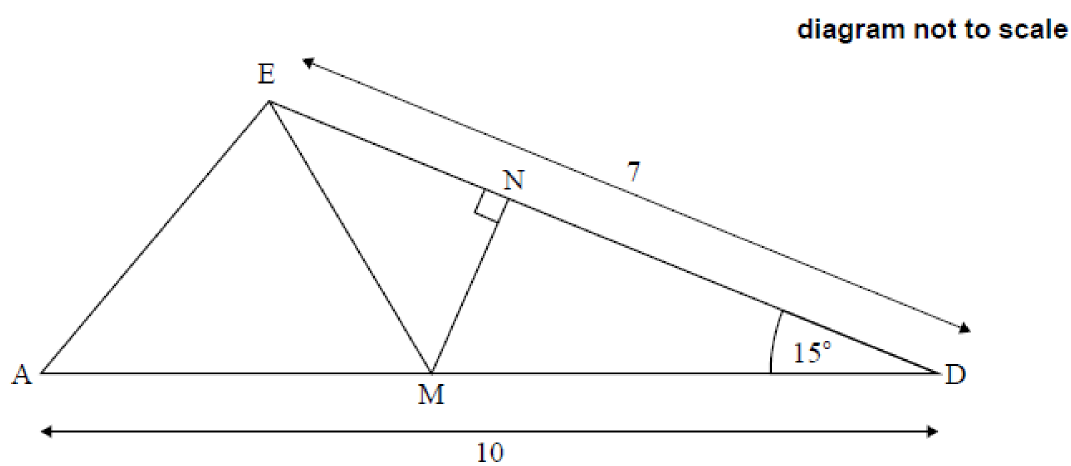
Some concerns were raised on the G2 forms as to the appropriateness of this question. However, the MSSL course tries in part to link areas of the syllabus to "real-life" situations and address these. A look back to past years' examination papers, and to the syllabus documentation, should yield similar examples.

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

Farmer Brown believes that N is the midpoint of ED.

a. Calculate the area of triangle EAD. [3]

b. Calculate the **total** volume of the barn. [3]

c. Calculate the length of MN. [2]

d. Calculate the length of AE. [3]

e. Show that Farmer Brown is incorrect. [3]

f. Calculate the **total** length of metal required for one support. [4]

Markscheme

a. (Area of EAD =) $\frac{1}{2} \times 10 \times 7 \times \sin 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area of a triangle formula, **(A1)** for correct substitution. Award ***(M0)(A0)(A0)*** if EAD or AED is considered to be a right-angled triangle.

= 9.06 m^2 (9.05866… m^2) ***(A1) (G3)***

[3 marks]

b. (10 × 5 × 16) + (9.05866… × 16) ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into volume of a cuboid, ***(M1)*** for adding the correctly substituted volume of their triangular prism.

= 945 m^3 (944.938… m^3) ***(A1)*****(ft)** ***(G3)***

Note: Follow through from part (a).

[3 marks]

c. $\frac{\text{MN}}{5} = \sin 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trigonometric equation.

(MN =) 1.29(m) (1.29409… (m)) ***(A1) (G2)***

[2 marks]

d. (AE^2 =) $10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, and ***(A1)*** for correct substitution.

(AE =) 3.71(m) (3.71084… (m)) ***(A1) (G2)***

[3 marks]

e. $ND^2 = 5^2 - (1.29409\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{1.29409\ldots}{\text{ND}} = \tan 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into tangent.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{\text{ND}}{5} = \cos 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine.

(ND =) 4.83 (4.82962…) ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$ND^2 = 1.29409\ldots^2 + 5^2 - 2 \times 1.29409\ldots \times 5 \times \cos 75°$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule.

(ND =) 4.83 (4.82962…) ***(A1)*(ft)**

Note: Follow through from part (c).

4.82962… ≠ 3.5 (ND ≠ 3.5) ***(R1)*(ft)**

OR

4.82962… ≠ 2.17038… (ND ≠ NE) ***(R1)*(ft)**

(hence Farmer Brown is incorrect)

Note: Do not award ***(M0)(A0)(R1)*(ft)**. Award ***(M0)(A0)(R0)*** for a correct conclusion without any working seen.

[3 marks]

f. $(EM^2 =)\ 1.29409\ldots^2 + (7 - 4.82962\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into Pythagoras theorem.

OR

$(EM^2 =)\ 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule formula.

(EM =) 2.53(m) (2.52689...(m)) ***(A1)*(ft) *(G2)*(ft)**

Note: Follow through from parts (c), (d) and (e).

(Total length =) 2.52689… + 3.71084… + 1.29409… +10 + 7 ***(M1)***

Note: Award ***(M1)*** for adding their EM, their parts (c) and (d), and 10 and 7.

= 24.5 (m) (24.5318… (m)) ***(A1)*(ft) *(G4)***

Note: Follow through from parts (c) and (d).

[4 marks]

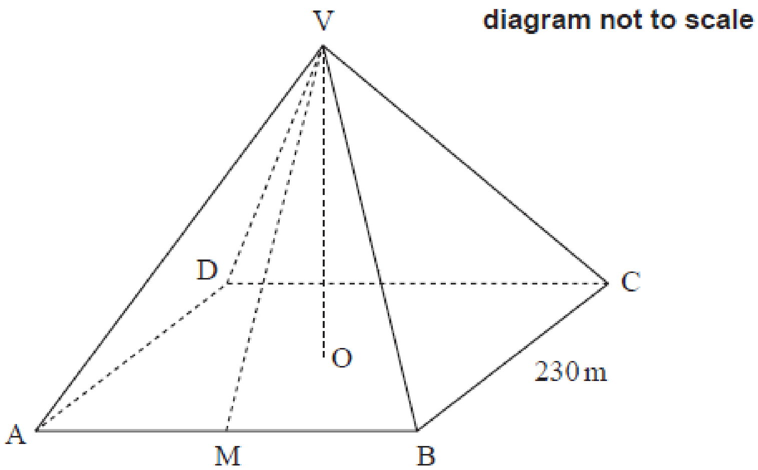
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

---

a. The Great Pyramid of Giza in Egypt is a right pyramid with a square base. The pyramid is made of solid stone. The sides of the base are $230\,\mathrm{m}$ long. The diagram below represents this pyramid, labelled VABCD. [3]

V is the vertex of the pyramid. O is the centre of the base, ABCD . M is the midpoint of AB. Angle $\mathrm{ABV} = 58.3°$.



Show that the length of VM is 186 metres, correct to three significant figures.

b. Calculate the height of the pyramid, VO . [2]

c. Find the volume of the pyramid. [2]

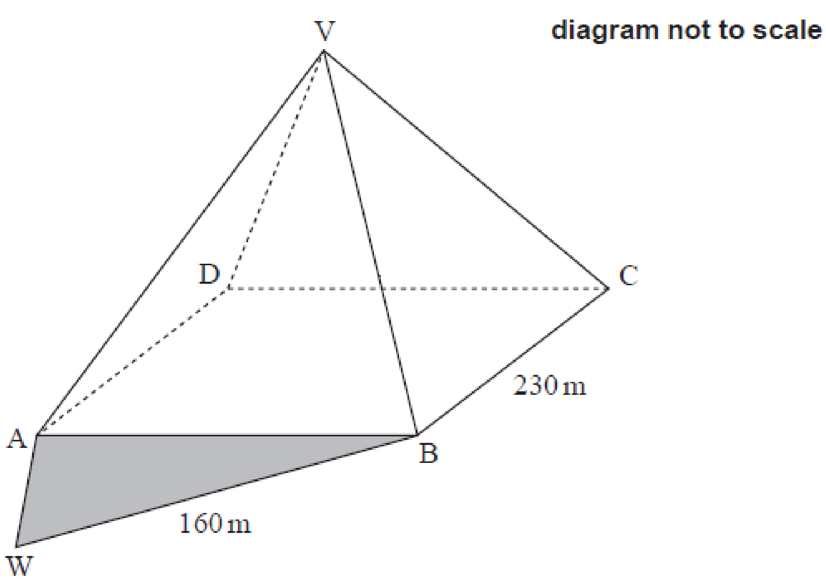
d. Write down your answer to part (c) in the form $a \times 10^{k}$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Ahmad is a tour guide at the Great Pyramid of Giza. He claims that the amount of stone used to build the pyramid could build a wall 5 metres high and 1 metre wide stretching from Paris to Amsterdam, which are $430\,\mathrm{km}$ apart. [4]

Determine whether Ahmad's claim is correct. Give a reason.

f. Ahmad and his friends like to sit in the pyramid's shadow, ABW, to cool down. [6]

At mid-afternoon, $\mathrm{BW} = 160\,\mathrm{m}$ and angle $\mathrm{ABW} = 15^{\circ}$.



i) Calculate the length of AW at mid-afternoon.

ii) Calculate the area of the shadow, ABW, at mid-afternoon.

Markscheme

a. $\tan(58.3) = \frac{\mathrm{VM}}{115}$ **OR** $115 \times \tan(58.3^{\circ})$ ***(A1)(M1)***

Note: Award ***(A1)*** for 115 $\left(ie\ \frac{230}{2}\right)$ seen, ***(M1)*** for correct substitution into trig formula.

$(\mathrm{VM} =)\ 186.200\,(\mathrm{m})$ ***(A1)***

$(\mathrm{VM} =)\ 186\,(\mathrm{m})$ ***(AG)***

Note: Both the rounded and unrounded answer must be seen for the final ***(A1)*** to be awarded.

b. $\mathrm{VO}^2 + 115^2 = 186^2$ **OR** $\sqrt{186^2 - 115^2}$ **(M1)**

Note: Award ***(M1)*** for correct substitution into Pythagoras formula. Accept alternative methods.

$(\mathrm{VO} =)\ 146\,(\mathrm{m})\ (146.188...)$ ***(A1)(G2)***

Note: Use of full calculator display for VM gives $146.443...\,(\mathrm{m})$.

c. **Units are required in part (c)**

$\frac{1}{3}(230^2 \times 146.188...)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in volume formula. Follow through from part (b).

$= 2\,580\,000\,\mathrm{m}^3\ (2\,577\,785...\,\mathrm{m}^3)$ ***(A1)*(ft)*(G2)***

Note: The answer is $2\,580\,000\,\mathrm{m}^3$, the units are required. Use of $\mathrm{OV} = 146.442$ gives $2582271...\,\mathrm{m}^3$

Use of $\mathrm{OV} = 146$ gives $2574466...\,\mathrm{m}^3$.

d. $2.58 \times 10^6\,(\mathrm{m}^3)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for 2.58 and ***(A1)*(ft)** for $\times 10^6$.

Award ***(A0)(A0)*** for answers of the type: $2.58 \times 10^5\,(\mathrm{m}^3)$.

Follow through from part (c).

e. the volume of a wall would be $430\,000 \times 5 \times 1$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume formula.

$2150000\,(\mathrm{m}^3)$ ***(A1)(G2)***

which is less than the volume of the pyramid ***(R1)(ft)***

Ahmad is correct. ***(A1)(ft)***

OR

the length of the wall would be $\frac{\text{their part (c)}}{5 \times 1 \times 1000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (c) by 5000.

$516\,(\mathrm{km})$ ***(A1)*(ft)*(G2)***

which is more than the distance from Paris to Amsterdam ***(R1)(ft)***

Ahmad is correct. ***(A1)(ft)***

Note: Do not award final ***(A1)*** without an explicit comparison. Follow through from part (c) or part (d). Award ***(R1)*** for reasoning that is consistent with their working in part (e); comparing two volumes, or comparing two lengths.

f. **Units are required in part (f)(ii).**

i) $\mathrm{AW}^2 = 160^2 + 230^2 - 2 \times 160 \times 230 \times \cos(15^\circ)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$\mathrm{AW} = 86.1\,(\mathrm{m})\ (86.0689...)$ ***(A1)(G2)***

Note: Award ***(M0)(A0)(A0)*** if BAW or AWB is considered to be a right angled triangle.

ii) $\text{area} = \frac{1}{2} \times 230 \times 160 \times \sin(15^\circ)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area formula, ***(A1)*** for correct substitutions.

$= 4760\,\mathrm{m}^2 \quad (4762.27\ldots\,\mathrm{m}^2)$ ***(A1)(G2)***

Note: The answer is $4760\,\mathrm{m}^2$, the units are required.

Examiners report

a. Question 4: Trigonometry, volume and area.

Many were able to write a correct trig ratio for part (a). The most common error was not to write the unrounded or the rounded answer. Some incorrectly used the given value of 186 in their proof. Part (b) was mostly answered correctly, with only a few candidates using Pythagoras' Theorem incorrectly. Most candidates used the correct formula to calculate the volume of the pyramid, but some did not find the correct area for the base of the pyramid. Some lost a mark for missing or for incorrect units. Even with an incorrect answer for part (c), candidates did very well on part (d). In part (e) some excellent justifications were given. However, many struggled to convert kilometres to metres, others were confused and compared surface area instead of volume. Some thought the volumes needed to be the same. For part (f) candidates often assumed a right angle at BAW or BWA. When they used the sine and cosine rule, this was mostly done correctly.

b. Question 4: Trigonometry, volume and area.

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d. Question 4: Trigonometry, volume and area.

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A contractor is building a house. He first marks out three points A , B and C on the ground such that AB = 5 m , AC = 7 m and angle BAC = 112 °.

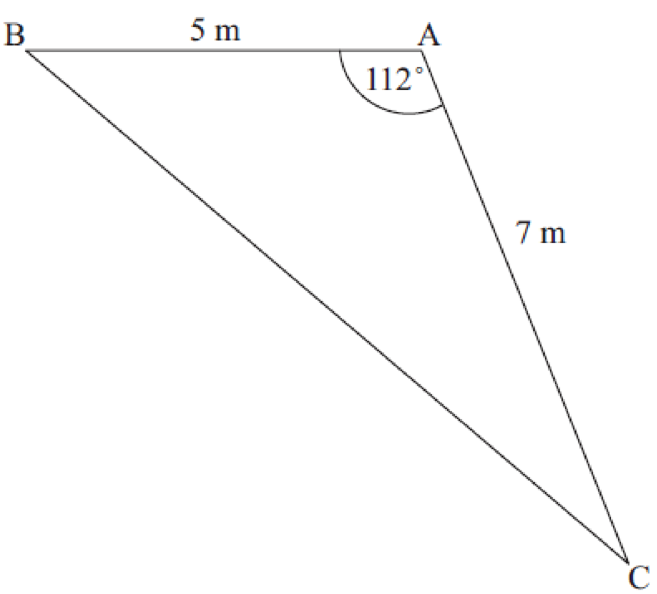
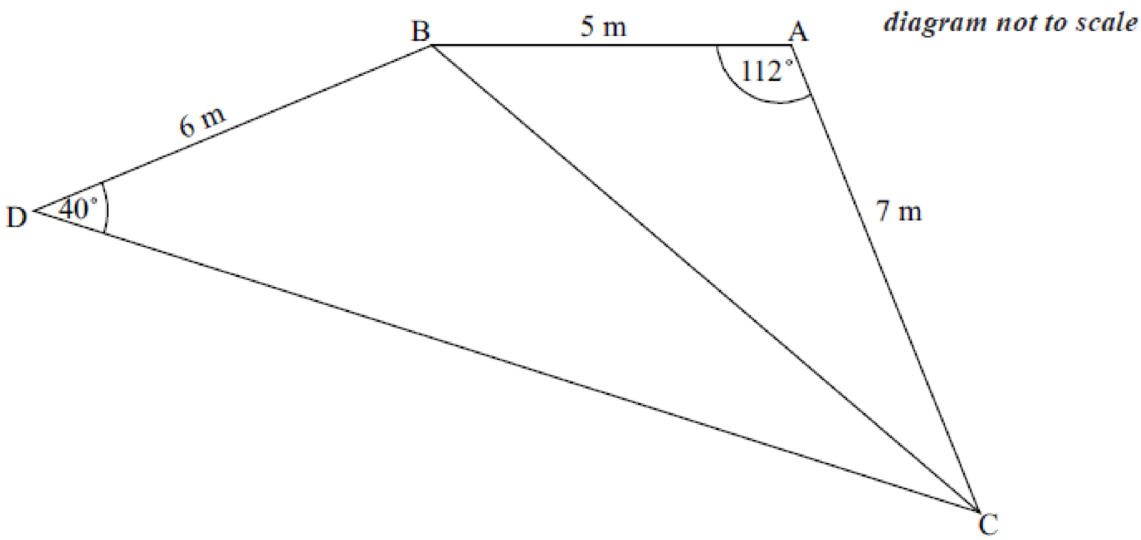


diagram not to scale

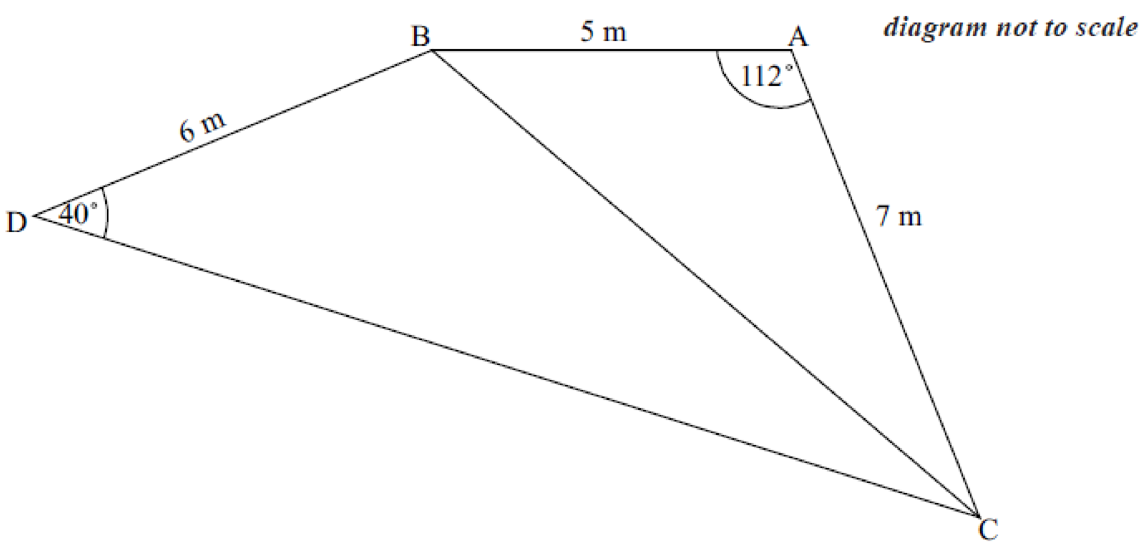
a. Find the length of BC. [3]

b. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



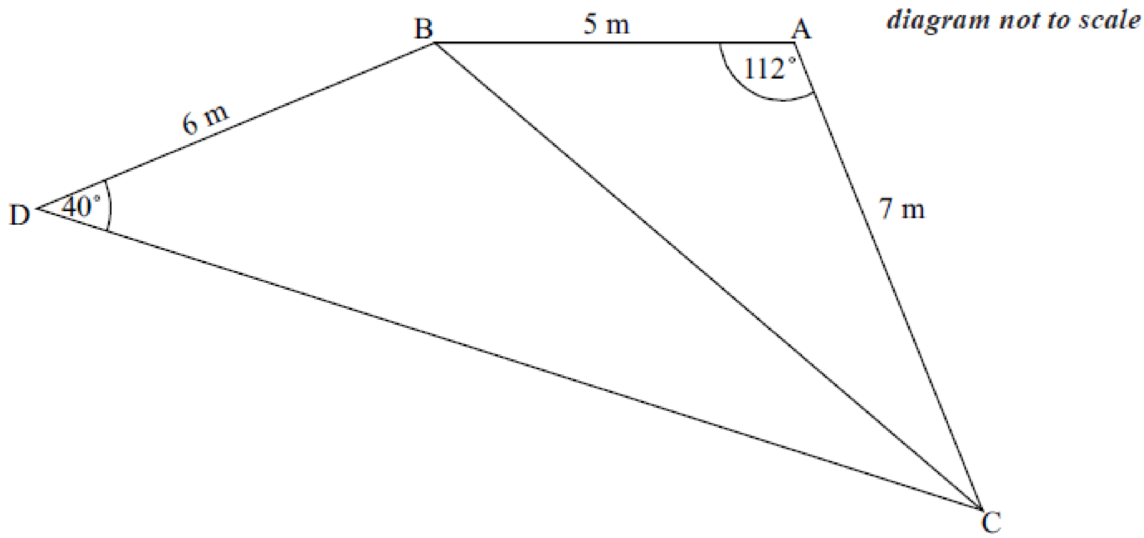
Find the size of angle DBC .

c. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



Find the area of the quadrilateral ABDC.

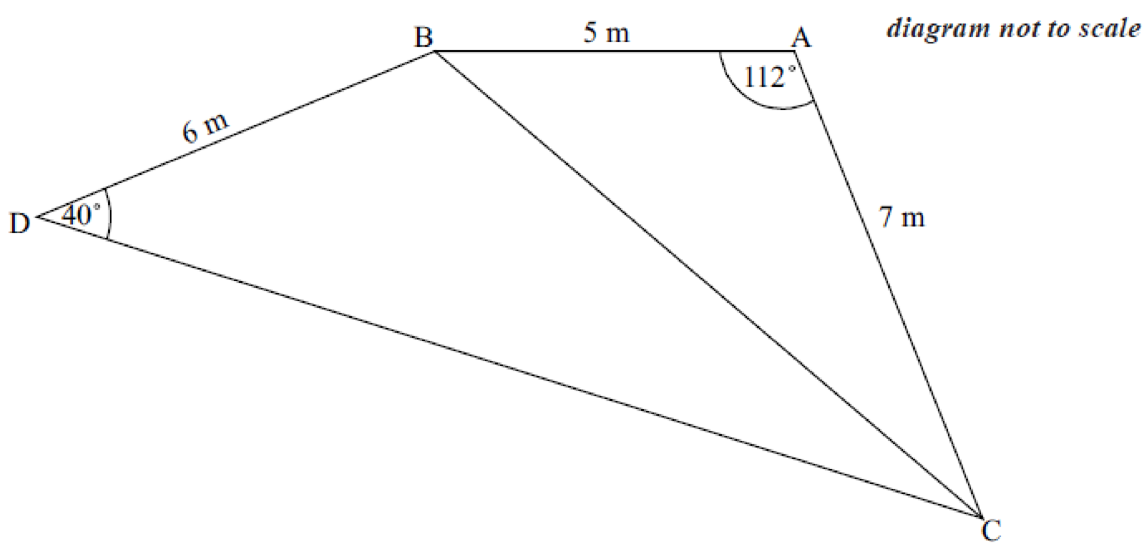
d. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [3]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

Find the volume of the soil removed. Give your answer in $\mathbf{m^3}$.

e. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [5]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm.

(i) Find the volume of a drum. Give your answer in $\mathbf{m^3}$.

(ii) Find the minimum number of drums required to transport the soil removed.

Markscheme

a. ***Units are required in part (c) only.***

$BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 112°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in cosine formula, ***(A1)*** for correct substitutions.

BC = 10.0 (m) (10.0111...) ***(A1)(G2)***

Note: If radians are used, award at most ***(M1)(A1)(A0)***.

[3 marks]

b. ***Units are required in part (c) only.***

$\frac{\sin 40^\circ}{10.0111...} = \frac{\sin D\hat{C}B}{6}$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substitution in sine formula, ***(A1)*(ft)** for their correct substitutions. Follow through from their part (a).

$D\hat{C}B$ = 22.7° (22.6589...) ***(A1)*(ft)**

Notes: Award ***(A2)*** for 22.7° seen without working. Use of radians results in unrealistic answer. Award a maximum of ***(M1)(A1)*(ft)*(A0)*(ft)**. Follow through from their part (a).

$D\hat{C}B$ = 117° (117.341...) ***(A1)*(ft)*(G3)***

Notes: Do not penalize if use of radians was already penalized in part (a). Follow through from their answer to part (a).

OR

From use of cosine formula

DC = 13.8(m) (13.8346…) (***A1)*(ft)**

Note: Follow through from their answer to part (a).

$\frac{\sin \alpha}{13.8346...} = \frac{\sin 40^\circ}{10.0111...}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the correct sine formula.

α = 62.7° (62.6589) ***(A1)*(ft)**

Note: Accept 62.5° from use of 3sf.

$D\hat{B}C$ = 117(117.341...) ***(A1)*(ft)**

Note: Follow through from their part (a). Use of radians results in unrealistic answer, award a maximum of ***(A1)(M1)(A0)(A0)***.

[4 marks]

c. ***Units are required in part (c) only.***

$\text{ABDC} = \frac{1}{2}(5)(7)\sin 112^\circ + \frac{1}{2}(6)(10.0111...)\sin 117.341...^\circ$ ***(M1)(A1)*(ft)*(M1)*N**

Note: Award ***(M1)*** for substitution in both **triangle** area formulae, ***(A1)*(ft)** for their correct substitutions, ***(M1)*** for seen or implied addition of their two **triangle** areas. Follow through from their answer to part (a) and (b).

= 42.9 m^2 (42.9039...) ***(A1)*(ft)*(G3)***

Notes: Answer is 42.9 m^2 *i.e.* **the units are required** for the final ***(A1)*(ft)** to be awarded. Accept 43.0 m^2 from using 3sf answers to parts (a) and (b). Do not penalize if use of radians was previously penalized.

[4 marks]

d. ***Units are required in part (c) only.***

$42.9039... \times 0.5$ ***(M1)(M1)***

Note: Award ***(M1)*** for 0.5 seen (or equivalent), ***(M1)*** for multiplication of their answer in part (c) with their value for depth.

$= 21.5$ (m^3) (21.4519...) ***(A1)*(ft)*(G3)***

Note: Follow through from their part (c) only if working is seen. Do not penalize if use of radians was previously penalized. Award at most ***(A0)(M1)(A0)*(ft)** for multiplying by 50.

[3 marks]

e. ***Units are required in part (c) only.***

(i) $\pi(0.15)^2(0.4)$ ***(M1)(A1)***

OR

$\pi \times 15^2 \times 40$ (28274.3...) ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution in the correct volume formula. ***(A1)*** for correct substitutions.

$= 0.0283$ (m^3) (0.0282743..., 0.09π)

(ii) $\frac{21.4519...}{0.0282743...}$ ***(M1)***

Note: Award ***(M1)*** for correct division of their volumes.

$= 759$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their parts (d) and (e)(i). Accept 760 from use of 3sf answers. Answer must be a positive integer for the final ***(A1)*(ft)** mark to be awarded.

[5 marks]

Examiners report

a. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

b. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students.

Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

c. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

d. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

e. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

A random sample of 167 people who own mobile phones was used to collect data on the amount of time they spent per day using their phones. The results are displayed in the table below.

Time spent per day (t minutes)	$0 \le t < 15$	$15 \le t < 30$	$30 \le t < 45$	$45 \le t < 60$	$60 \le t < 75$	$75 \le t < 90$
Number of people	21	32	35	41	27	11

Manuel conducts a survey on a random sample of 751 people to see which television programme type they watch most from the following: Drama, Comedy, Film, News. The results are as follows.

	Drama	Comedy	Film	News
Males under 25	22	65	90	35
Males 25 and over	36	54	67	17
Females under 25	22	59	82	15
Females 25 and over	64	39	38	46

Manuel decides to ignore the ages and to test at the 5 % level of significance whether the most watched programme type is independent of **gender.**

i.a. State the modal group. [1]

i.b. Use your graphic display calculator to calculate approximate values of the mean and standard deviation of the time spent per day on these mobile phones. [3]

i.c. On graph paper, draw a fully labelled histogram to represent the data. [4]

ii.a. Draw a table with 2 rows and 4 columns of data so that Manuel can perform a chi-squared test. [3]

ii.b. State Manuel's null hypothesis and alternative hypothesis. [1]

ii.c. Find the expected frequency for the number of females who had 'Comedy' as their most-watched programme type. Give your answer to the nearest whole number. [2]

ii.d. Using your graphic display calculator, or otherwise, find the chi-squared statistic for Manuel's data. [3]

ii.e. (i) State the number of degrees of freedom available for this calculation. [3]

(ii) State his conclusion.

Markscheme

i.a. $45 \leqslant t < 60$ ***(A1)***

[1 mark]

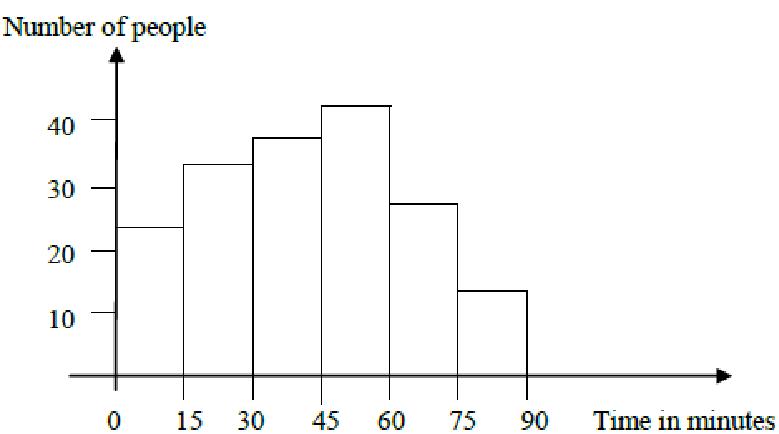
i.b. *Unit penalty **(UP)** is applicable in question part (i)(b) **only**.*

(UP) 42.4 minutes ***(G2)***

21.6 minutes ***(G1)***

[3 marks]

i.c.



(A4)

[4 marks]

ii.a.

	Drama	Comedy	Film	News
Males	58	119	157	52
Females	86	98	120	61

(M1)(M1)(A1)

[3 marks]

ii.b. H_0: favourite TV programme is independent of gender or no association between favourite TV programme and gender

H_1: favourite TV programme is dependent on gender *(must have both)* ***(A1)***

[1 mark]

ii.c. $\frac{365 \times 217}{751}$ ***(M1)***

$= 105$ ***(A1)*(ft)*(G2)***

[2 marks]

ii.d. 12.6 *(accept* 12.558*)* ***(G3)***

[3 marks]

ii.e.(i) 3 ***(A1)***

(ii) reject H_0 *or equivalent statement (e.g. accept H_1)* ***(A1)*****(ft)**

[3 marks]

Examiners report

i.a. Many candidates who had survived the previous two unit penalties, fell here with omission of units for the mean and standard deviation. The modal group was answered well. Part (b), finding the mean and standard deviation by GDC, was answered very poorly. Most did put the midpoints in one list and the frequencies in a second list but then either used the 2-Var stats button or 1-var stats button but only named L1 instead of L1, L2. Candidates who showed midpoints in their working did at least score a method mark.

i.b. Many candidates who had survived the previous two unit penalties, fell here with omission of units for the mean and standard deviation. The modal group was answered well. Part (b), finding the mean and standard deviation by GDC, was answered very poorly. Most did put the midpoints in one list and the frequencies in a second list but then either used the 2-Var stats button or 1-var stats button but only named L1 instead of L1, L2. Candidates who showed midpoints in their working did at least score a method mark.

i.c. Many candidates who had survived the previous two unit penalties, fell here with omission of units for the mean and standard deviation. The modal group was answered well. Part (b), finding the mean and standard deviation by GDC, was answered very poorly. Most did put the midpoints in one list and the frequencies in a second list but then either used the 2-Var stats button or 1-var stats button but only named L1 instead of L1, L2. Candidates who showed midpoints in their working did at least score a method mark.

ii.a. The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

ii.b. The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

ii.c. The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

ii.d. The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

ii.e. The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

15 like all three fruits

22 like mangoes and bananas

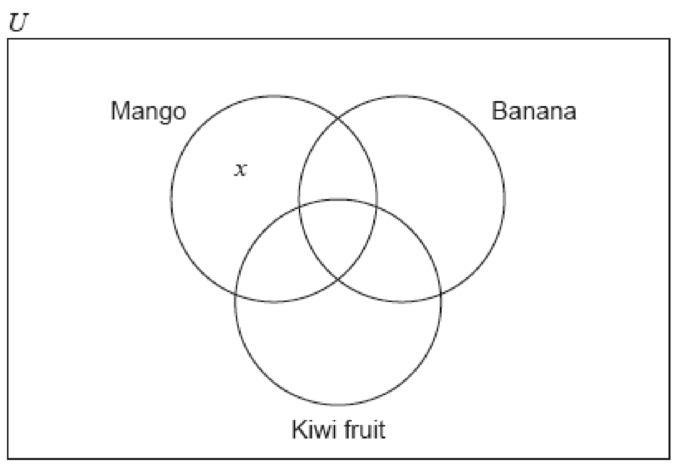
33 like mangoes and kiwi fruits

27 like bananas and kiwi fruits

8 like none of these three fruits

x like **only** mangoes

a. **Copy** the following Venn diagram and correctly insert all values from the above information. [3]



b. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Complete your Venn diagram from part (a) with this additional information **in terms of** x.

c. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Find the value of x.

d. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [2]

Write down the number of customers who like

(i) mangoes;

(ii) mangoes or bananas.

e. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [4]

A customer is chosen at random from the 100 customers. Find the probability that this customer

(i) likes none of the three fruits;

(ii) likes only two of the fruits;

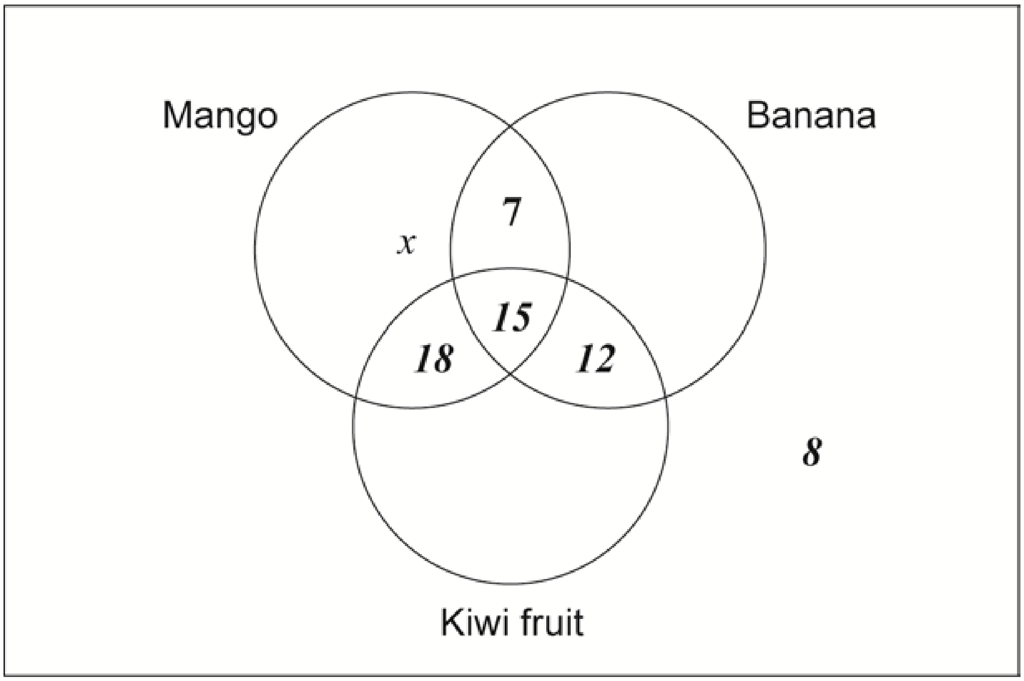
(iii) likes all three fruits given that the customer likes mangoes and bananas.

f. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas. [3]

Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

Markscheme

a. U



(A1)(A1)(A1)

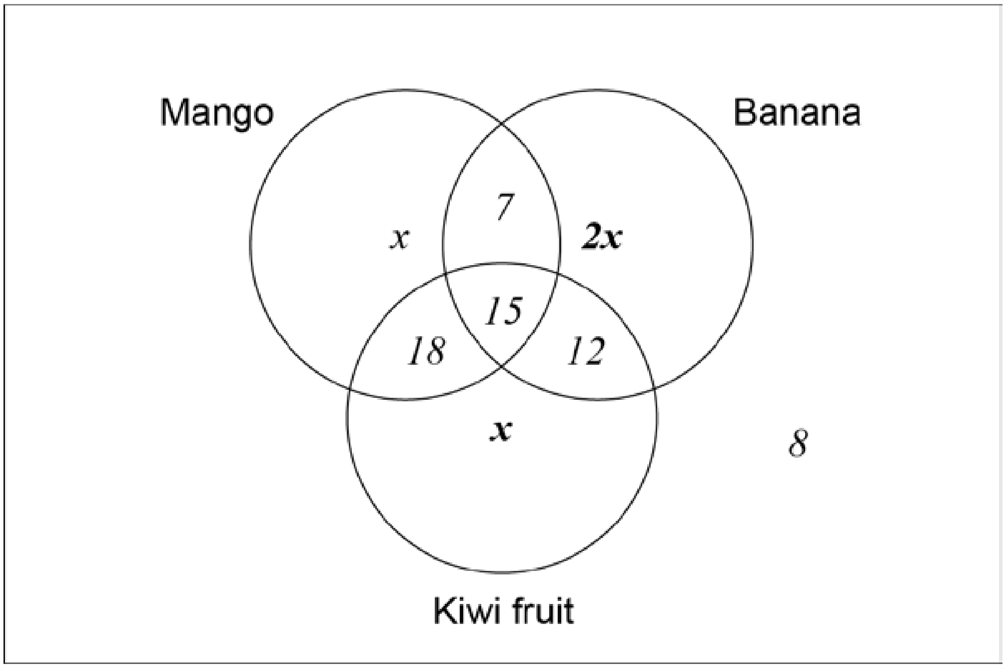
Notes: Award ***(A1)*** for 15 in the correct place.

Award ***(A1)*** for 7, 18 and 12 seen in the correct places.

Award ***(A1)*** for 8 in the correct place.

Award at most ***(A0)(A1)(A1)*** if diagram is missing the rectangle.

b. U



(A1)(A1)

Notes: Award ***(A1)*** for x seen in the correct places.

Award ***(A1)*** for $2x$ seen in the correct place.

Award ***(A0)(A1)*(ft)** if x and $2x$ are replaced by 10 and 20 respectively.

c. $2x + x + x + 15 + 8 + 7 + 18 + 12 = 100 \quad (4x + 60 = 100 \text{ or equivalent})$ ***(M1)***

Note: Award ***(M1)*** for equating the sum of the elements of their Venn diagram to 100. Equating to 100 may be implied.

$(x =)\ 10$ ***(A1)*(ft)*(G2)***

Note: Follow through from their Venn diagram. The answer must be a positive integer.

d. (i) 50 ***(A1)*(ft)**

(ii) 82 ***(A1)*(ft)**

Note: Follow through from their answer to part (c) and their Venn diagram.

Award ***(A0)*(ft)*(A1)*(ft)** if answer is $\frac{50}{100}$ and $\frac{82}{100}$.

e. (i) $\frac{8}{100}$ $\left(\frac{2}{25};\ 0.08;\ 8\%\right)$ ***(A1)***

Note: Correct answer only. There is no follow through.

(ii) $\frac{37}{100}$ $(0.37,\ 37\%)$ ***(A1)*(ft)**

Note: Follow through from their Venn diagram.

(iii) $\frac{15}{22}$ $(0.681;\ 0.682;\ 68.2\%)$ $(0.681818\ldots)$ ***(A1)(A1)*(ft)*(G2)***

Notes: Award ***(A1)*** for numerator, ***(A1)*(ft)** for denominator, follow through from their Venn diagram. Award ***(A0)(A0)*** if answer is given as incorrect reduced fraction without working.

f. $\frac{8}{100} \times \frac{7}{99}$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for correct fractions, follow through from their answer to part (e)(i), ***(M1)*** for multiplying their fractions.

$= \frac{56}{9900}$ $\left(\frac{14}{2477}, 0.00565656\ldots, 0.00566, 0.0056, 0.566\%\right)$ ***(A1)*(ft)*(G2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

A new café opened and during the first week their profit was $60.

The café's profit increases by $10 every week.

A new tea-shop opened at the same time as the café. During the first week their profit was also $60.

The tea-shop's profit increases by 10 % every week.

a. Find the café's profit during the 11th week. [3]

b. Calculate the café's **total** profit for the first 12 weeks. [3]

c. Find the tea-shop's profit during the 11th week. [3]

d. Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

e. In the *m*th week the tea-shop's **total** profit exceeds the café's **total** profit, for the first time since they both opened. [4]

Find the value of *m*.

Markscheme

a. 60 + 10 × 10 ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the arithmetic sequence formula, ***(A1)*** for correct substitution.

= ($) 160 ***(A1)(G3)***

[3 marks]

b. $\frac{12}{2}(2 \times 60 + 11 \times 10)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituting the arithmetic series formula, ***(A1)*(ft)** for correct substitution. Follow through from their first term and common difference in part (a).

= ($) 1380 ***(A1)*(ft)*(G2)***

[3 marks]

c. 60×1.1^{10} ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the geometric progression nth term formula, ***(A1)*** for correct substitution.

= ($) 156 (155.624…) ***(A1)(G3)***

Note: Accept the answer if it rounds correctly to 3 sf, as per the accuracy instructions.

[3 marks]

d. $\frac{60\left(1.1^{12}-1\right)}{1.1-1}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituting the geometric series formula, ***(A1)*(ft)** for correct substitution. Follow through from part (c) for their first term and common ratio.

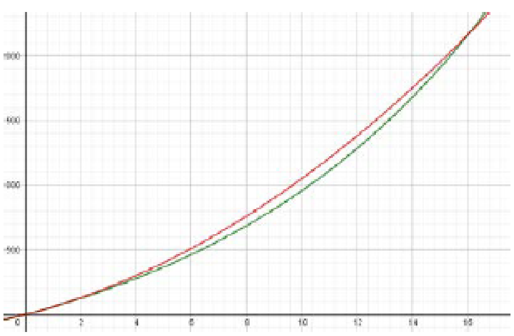
= ($)1280 (1283.05…) ***(A1)*(ft)*(G2)***

[3 marks]

e. $\frac{60(1.1^{n}-1)}{1.1-1} > \frac{n}{2}(2 \times 60 + (n-1) \times 10)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correctly substituted geometric and arithmetic series formula with n (accept other variable for "n"), ***(M1)*** for comparing their expressions consistent with their part (b) and part (d).

OR

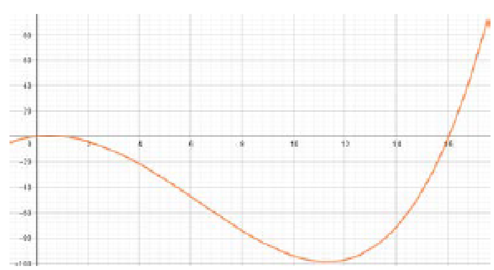


(M1)(M1)

Note: Award ***(M1)*** for two curves with approximately correct shape drawn in the first quadrant, ***(M1)*** for one point of intersection with approximate correct position.

Accept alternative correct sketches, such as

OR



Award ***(M1)*** for a curve with approximate correct shape drawn in the 1st (or 4th) quadrant and all above (or below) the x-axis, ***(M1)*** for one point of intersection with the x-axis with approximate correct position.

17 ***(A2)*(ft)*(G3)***

Note: Follow through from parts (b) and (d).
An answer of 16 is incorrect. Award at most ***(M1)(M1)(A0)(A0)*** with working seen. Award ***(G0)*** if final answer is 16 without working seen.

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

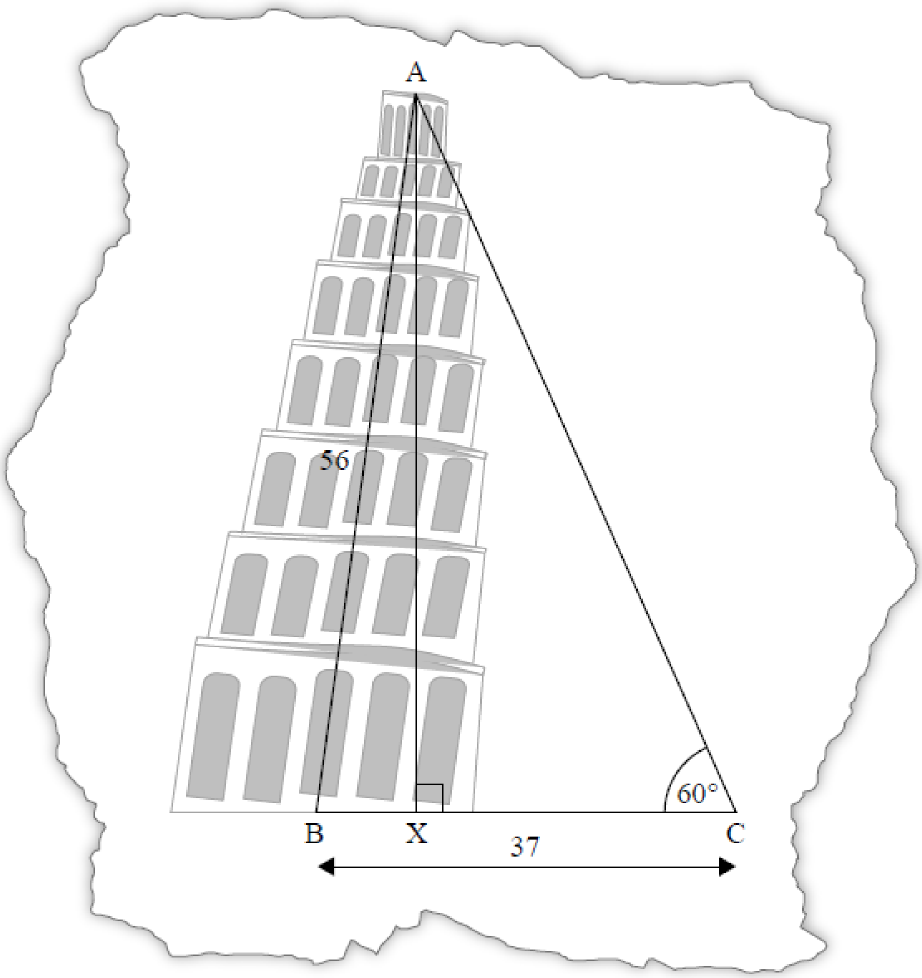
c. [N/A]
d. [N/A]
e. [N/A]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



Giovanni's tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

a.i. Use Giovanni's diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. [5]

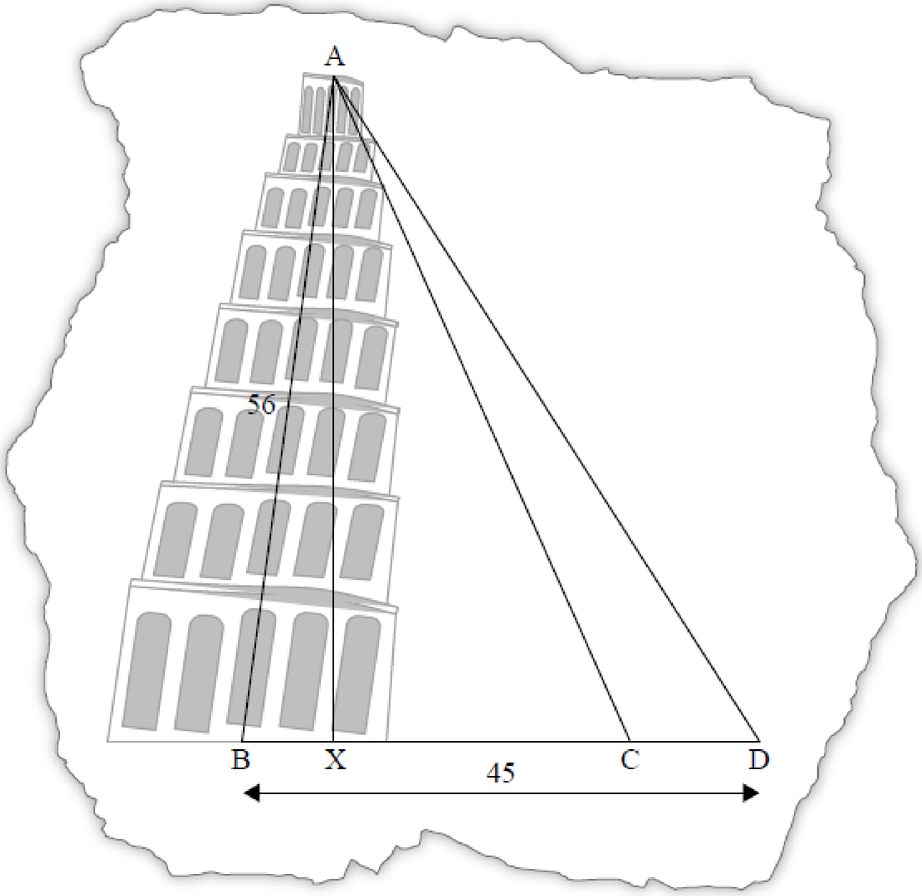
a.ii. Use Giovanni's diagram to calculate the length of AX. [2]

a.iii. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. [2]

b. Find the percentage error on Giovanni's diagram. [2]

c. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. [3]

diagram not to scale



Find the angle of elevation of A from D.

Markscheme

a.i. $\frac{\sin \mathrm{BAC}}{37} = \frac{\sin 60}{56}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the sine rule formula, ***(A1)*** for correct substitution.

angle $\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$ = 34.9034…° ***(A1)***

Note: Award ***(A0)*** if unrounded answer does not round to 35. Award ***(G2)*** if 34.9034… seen without working.

angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ = 180 − (34.9034… + 60) ***(M1)***

Note: Award ***(M1)*** for subtracting their angle BAC + 60 from 180.

85.0965…° ***(A1)***

85° ***(AG)***

Note: Both the unrounded and rounded value must be seen for the final ***(A1)*** to be awarded. If the candidate rounds 34.9034...° to 35° while substituting to find angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$, the final ***(A1)*** can be awarded but **only** if both 34.9034...° and 35° are seen.
If 85 is used as part of the workings, award at most ***(M1)(A0)(A0)(M0)(A0)(AG)***. This is the reverse process and not accepted.

a.ii. sin 85… × 56 ***(M1)***

= 55.8 (55.7869…) (m) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

a.iii. $\sqrt{56^2 - 55.7869\ldots^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the Pythagoras theorem formula. Follow through from part (a)(ii).

OR

cos(85) × 56 ***(M1)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

= 4.88 (4.88072…) (m) ***(A1)*(ft)*(G2)***

Note: Accept 4.73 (4.72863…) (m) from using their 3 s.f answer. Accept equivalent methods.

[2 marks]

b. $\left|\frac{4.88-3.9}{3.9}\right| \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

= 25.1 (25.1282) (%) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a)(iii).

[2 marks]

c. $\tan^{-1}\left(\frac{55.7869\ldots}{40.11927\ldots}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their 40.11927… seen. Award ***(M1)*** for correct substitution into trigonometric ratio.

OR

(37 − 4.88072…)^2 + 55.7869…^{2}

(AC =) 64.3725…

64.3726…^{2} + 8^2 − 2 × 8 × 64.3726… × cos120

(AD =) 68.7226…

$\frac{\sin 120}{68.7226\ldots} = \frac{\sin \mathrm{A}\hat{\mathrm{D}}\mathrm{C}}{64.3725\ldots}$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their correct values seen, ***(M1)*** for correct substitution into the sine formula.

= 54.3° (54.2781…°) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a). Accept equivalent methods.

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
a.iii. [N/A]
b. [N/A]
c. [N/A]

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a. A distress flare is fired into the air from a ship at sea. The height, h , in metres, of the flare above sea level is modelled by the quadratic function [1]

$$h\left(t\right) = -0.2t^2 + 16t + 12\,,\ t \geqslant 0\,,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

Write down the height from which the flare was fired.

b. Find the height of the flare 15 seconds after it was fired. [2]

c. The flare fell into the sea k seconds after it was fired. [2]

Find the value of k .

d. Find $h'(t)$. [2]

e. i) Show that the flare reached its maximum height 40 seconds after being fired. [3]

ii) Calculate the maximum height reached by the flare.

f. The nearest coastguard can see the flare when its height is more than 40 metres above sea level. [3]

Determine the total length of time the flare can be seen by the coastguard.

Markscheme

a. $12\,(\mathrm{m})$ ***(A1)***

b. $(h(15) =) - 0.2 \times 15^2 + 16 \times 15 + 12$ ***(M1)***

Note: Award ***(M1)*** for substitution of 15 in expression for h.

$= 207\,(\mathrm{m})$ ***(A1)(G2)***

c. $h(k) = 0$ ***(M1)***

Note: Award ***(M1)*** for setting h to zero.

$(k =)\ 80.7\,(\mathrm{s})\ \ (80.7430)$ ***(A1)(G2)***

Note: Award at most ***(M1)(A0)*** for an answer including $K = -0.743$.
Award ***(A0)*** for an answer of 80 without working.

d. $h'(t) = -0.4t + 16$ ***(A1)(A1)***

Note: Award ***(A1)*** for $-0.4t$, ***(A1)*** for 16. Award at most ***(A1)(A0)*** if extra terms seen. Do not accept x for t.

e. i) $-0.4t + 16 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative, from part (d), to zero, provided the correct conclusion is stated and consistent with their $h'(t)$.

OR

$t = \frac{-16}{2\times(-0.2)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into axis of symmetry formula, provided the correct conclusion is stated.

$t = \ 40\,(\mathrm{s})$ ***(AG)***

ii) $-0.2 \times 40^2 + 16 \times 40 + 12$ ***(M1)***

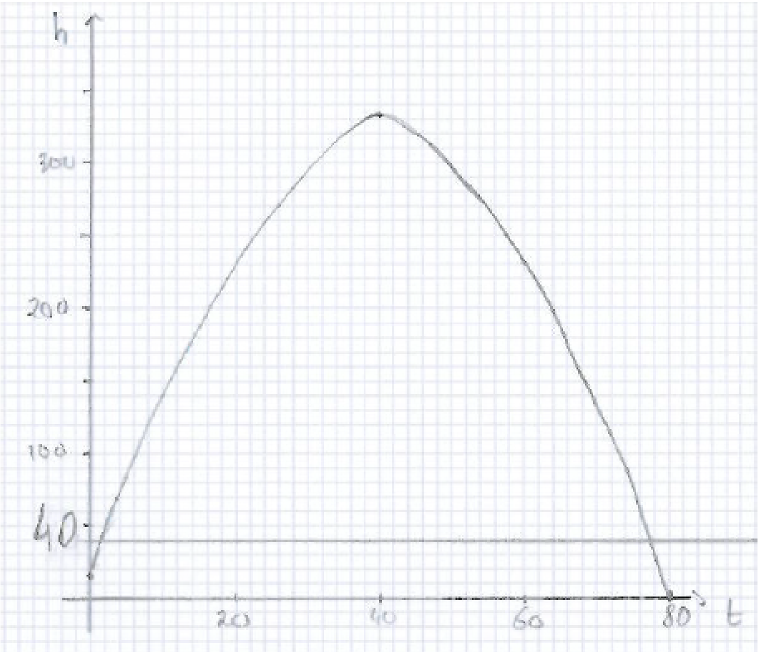
Note: Award ***(M1)*** for substitution of 40 in expression for h.

$= 332\,(\mathrm{m})$ ***(A1)(G2)***

f. $h(t) = 40$ ***(M1)***

Note: Award ***(M1)*** for setting h to 40. Accept inequality sign.

OR



M1

Note: Award ***(M1)*** for correct sketch. Indication of scale is not required.

$78.2 - 1.17\ (78.2099... \ \ - 1.79005...)$ ***(A1)***

Note: Award ***(A1)*** for 1.79 and 78.2 seen.

(total time $=$) $76.4\,(\mathrm{s})\ \ (76.4198...)$ ***(A1)(G2)***

Note: Award ***(G1)*** if the two endpoints are given as the final answer with no working.

Examiners report

a. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t. Some candidate seem to have a problem with the notation “$h\left(t\right) = \ldots$”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

b. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t. Some candidate seem to have a problem with the notation “$h\left(t\right) = \ldots$”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

c. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t. Some candidate seem to have a problem with the notation “$h\left(t\right) = \ldots$”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

d. Question 3: Quadratic function, problem solving.

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George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, t (minutes)	0	1	2	3	4	5	6
Temperature, y (°C)	94	54	34	24	k	16.5	15.25

a. Write down the decrease in the temperature of the coffee [3]

(i) during the first minute (between t = 0 and t =1) ;

(ii) during the second minute;

(iii) during the third minute.

b. Assuming the pattern in the answers to part (a) continues, show that $k = 19$. [2]

c. Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes. [4]

Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent 10 °C on the vertical axis.

d. The function that models the change in temperature of the coffee is $y = p\,(2^{-t}) + q$. [2]

(i) Use the values $t = 0$ and $y = 94$ to form an equation in p and q.

(ii) Use the values $t = 1$ and $y = 54$ to form a second equation in p and q.

e. Solve the equations found in part (d) to find the value of p and the value of q. [2]

f. The graph of this function has a horizontal asymptote. [2]

Write down the equation of this asymptote.

g. George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression. [4]

Use the **seven** results in the table to write down

(i) the correlation coefficient;

(ii) the equation of the regression line y on t.

h. Use the equation of the regression line to estimate the temperature of the coffee at $t = 3$. [2]

i. Find the percentage error in this estimate of the temperature of the coffee at $t = 3$. [2]

Markscheme

a. (i) 40

(ii) 20

(iii) 10 ***(A3)***

Notes: Award ***(A0)(A1)*(ft)*(A1)*(ft)** for −40, −20, −10.

Award ***(A1)(A0)(A1)*(ft)** for 40, 60, 70 seen.

Award ***(A0)(A0)(A1)*(ft)** for −40, −60, −70 seen.

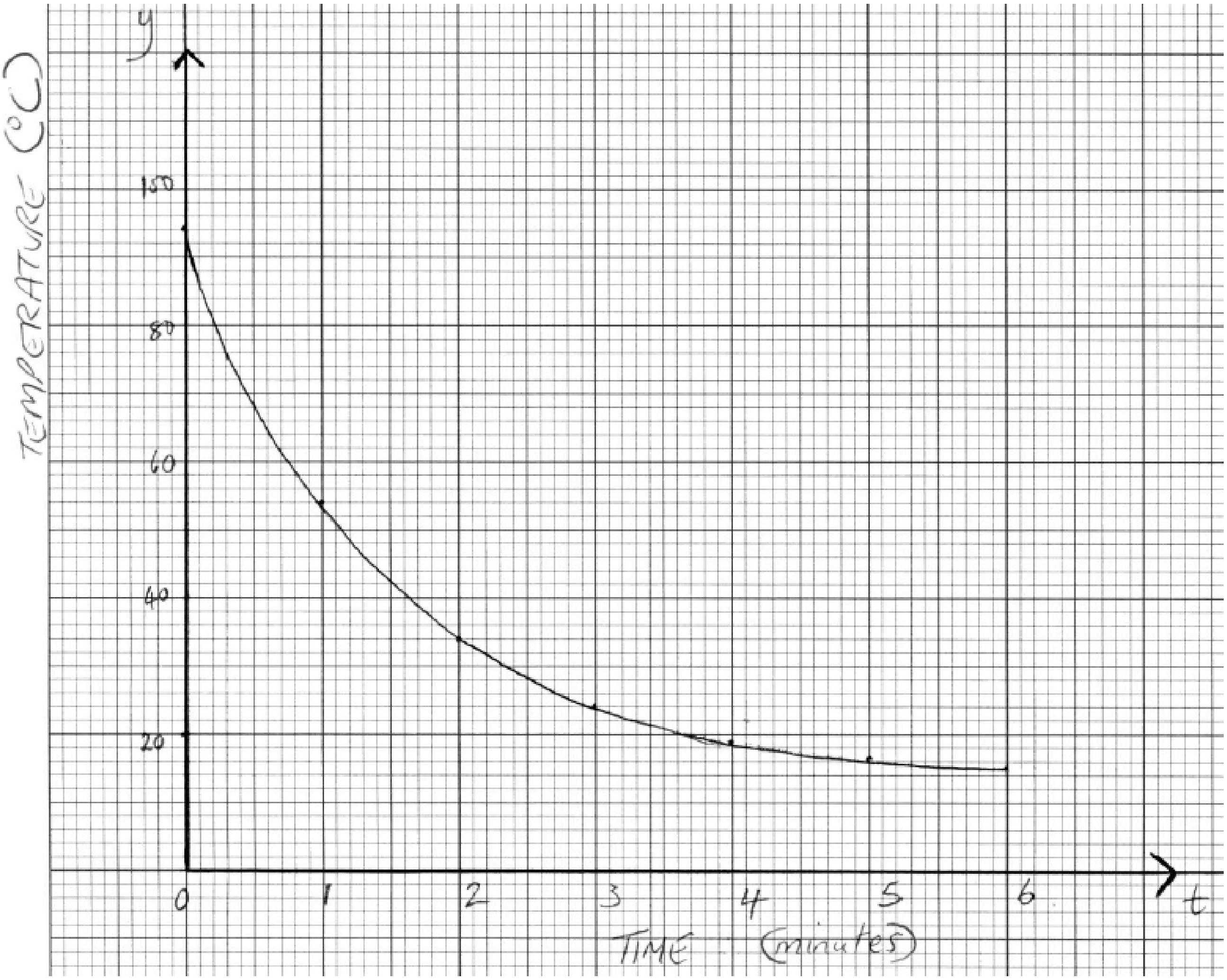
b. $24 - k = 5$ or equivalent ***(A1)(M1)***

Note: Award ***(A1)*** for 5 seen, ***(M1)*** for difference from 24 indicated.

$k = 19$ ***(AG)***

Note: If 19 is not seen award at most ***(A1)(M0)***.

c.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for scales and labelled axes (*t* or “time” and *y* or “temperature”).

Accept the use of *x* on the horizontal axis only if “time” is also seen as the label.

Award ***(A2)*** for all seven points accurately plotted, award ***(A1)*** for **5 or 6** points accurately plotted, award ***(A0)*** for 4 points or fewer accurately plotted.

Award ***(A1)*** for smooth curve that passes through all points on domain [0, 6].

If graph paper is not used or one or more scales is missing, award a maximum of ***(A0)(A0)(A0)(A1)***.

d. (i) $94 = p + q$ ***(A1)***

(ii) $54 = 0.5p + q$ ***(A1)***

Note: The equations need not be simplified; accept, for example $94 = p(2^{-0}) + q$.

e. *p* = 80, *q* = 14 ***(G1)(G1)*****(ft)**

Note: If the equations have been incorrectly simplified, follow through even if no working is shown.

f. *y* = 14 ***(A1)(A1)*****(ft)**

Note: Award ***(A1)*** for *y* = a constant, ***(A1)*** for their 14. Follow through from part (e) only if their *q* lies between 0 and 15.25 inclusive.

g. (i) –0.878 (–0.87787...) ***(G2)***

Note: Award ***(G1)*** if –0.877 seen only. If negative sign omitted award a maximum of ***(A1)(A0)***.

(ii) y = −11.7t + 71.6 (y = −11.6517...t + 71.6336...) ***(G1)(G1)***

Note: Award ***(G1)*** for −11.7t, ***(G1)*** for 71.6.

If y = is omitted award at most ***(G0)(G1)***.

If the use of x in part (c) has **not** been penalized (the axis has been labelled "time") then award at most ***(G0)(G1)***.

h. −11.6517...(3) + 71.6339... ***(M1)***

Note: Award ***(M1)*** for correct substitution in their part (g)(ii).

= 36.7 (36.6785...) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (g). Accept 36.5 for use of the 3sf answers from part (g).

i. $\frac{36.6785...-24}{24} \times 100$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in percentage error formula.

= 52.8% (52.82738...) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (h). Accept 52.1% for use of 36.5.

Accept 52.9 % for use of 36.7. If partial working ($\times 100$ omitted) is followed by their correct answer award ***(M1)(A1)***. If partial working is followed by an incorrect answer award ***(M0)(A0)***. The percentage sign is not required.

Examiners report

a. Almost all candidates were able to score on the first parts of this question; errors occurring only when insufficient care was taken in reading what the question was asking for. The graph was usually well drawn, other than for those who have no idea what centimetres are.

The majority were able to determine the simultaneous equations, if only in unsimplified form; there was less success in solving these – though this is easily done via the GDC (the preferred approach) and the equation of the asymptote proved a discriminating task. The final parts, involving correlation and regression were largely independent of the previous parts and were accessible to most. Hopefully, contrasting the large percentage error with the value of the correlation coefficient will be valuable in class discussions. Given the many scripts that gave the value of the coefficient of determination as that of r , it seems better that the former is simply not taught.

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Consider the function $f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3$.

a. Sketch the graph of $y = f(x)$ for $-3 \le x \le 6$ and $-10 \le y \le 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x-axis, and a scale of 1 cm to represent 1 unit on the y-axis. [4]

b. Find the value of $f(-1)$. [2]

c. Write down the coordinates of the y-intercept of the graph of $f(x)$. [1]

d. Find $f'(x)$. [3]

e. Show that $f'(-1) = -\frac{16}{3}$. [1]

f. Explain what $f'(-1)$ represents. [2]

g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is –1. [2]

h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). [2]

i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [2]

Write down the value of

(i) a ;

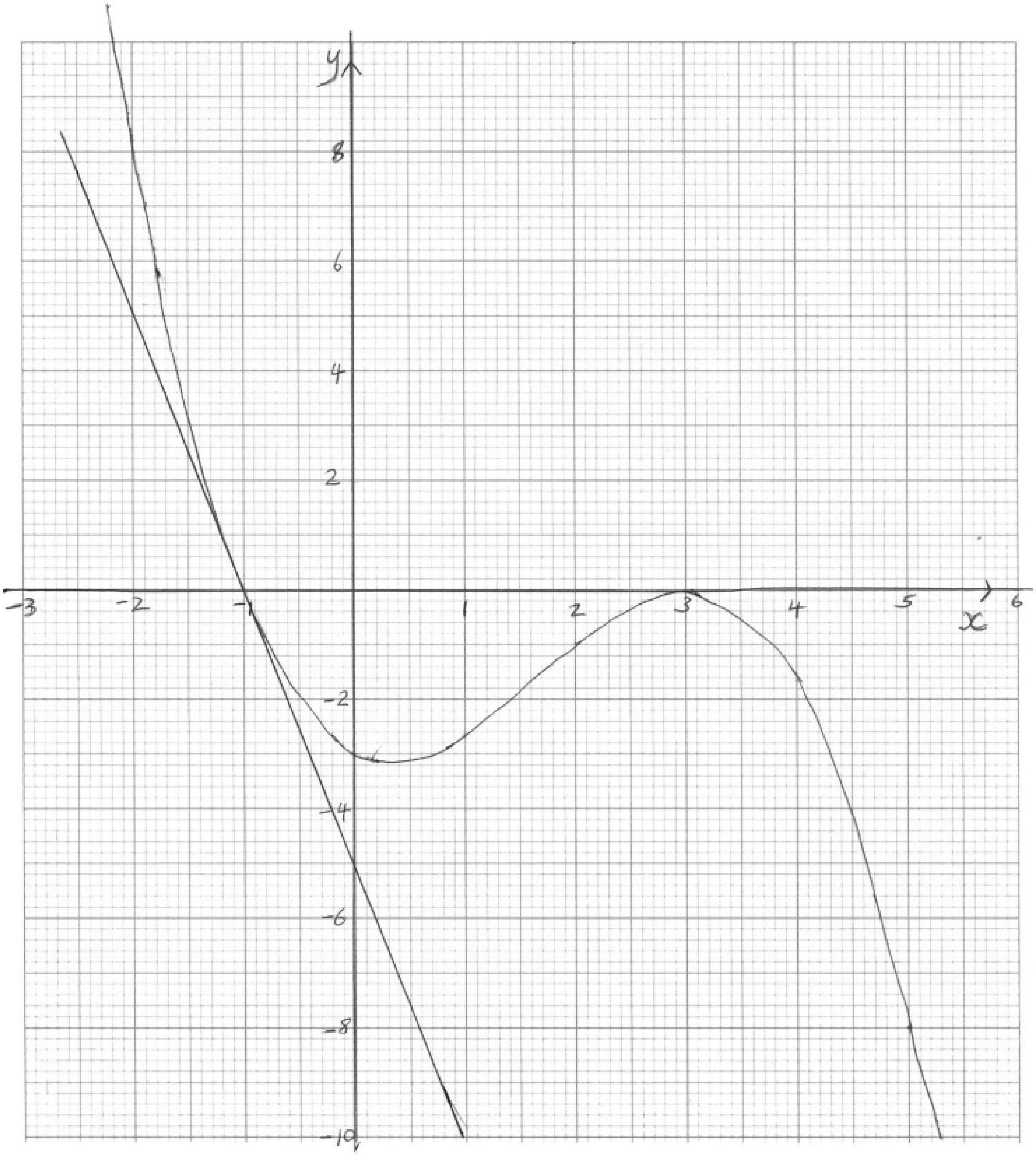
(ii) b .

j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [1]

Describe the behaviour of $f(x)$ for $a < x < b$.

Markscheme

a.



(A1) for indication of window and labels. ***(A1)*** for smooth curve that does not enter the first quadrant, the curve must consist of one line only.

(A1) for x and y intercepts in approximately correct positions (allow ±0.5).

(A1) for local maximum and minimum in approximately correct position. (minimum should be $0 \le x \le 1$ and $-2 \le y \le -4$), the y-coordinate of the maximum should be 0 ± 0.5. ***(A4)***

[4 marks]

b. $-\frac{1}{3}(-1)^3 + \frac{5}{3}(-1)^2 - (-1) - 3$ ***(M1)***

Note: Award ***(M1)*** for substitution of –1 into $f(x)$

$= 0$ ***(A1)(G2)***

[2 marks]

c. (0, –3) ***(A1)***

OR

$x = 0, y = -3$ ***(A1)***

Note: Award ***(A0)*** if brackets are omitted.

[1 mark]

d. $f'(x) = -x^2 + \frac{10}{3}x - 1$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award ***(A1)(A1)(A0)*** at most if there are extra terms.

[3 marks]

e. $f'(-1) = -(-1)^2 + \frac{10}{3}(-1) - 1$ ***(M1)***

$= -\frac{16}{3}$ ***(AG)***

Note: Award ***(M1)*** for substitution of $x = -1$ into correct derivative only. The final answer must be seen.

[1 mark]

f. *f* '(–1) gives the gradient of the tangent to the curve at the point with $x = -1$. ***(A1)(A1)***

Note: Award ***(A1)*** for "gradient (of curve)", ***(A1)*** for "at the point with $x = -1$". Accept "the instantaneous rate of change of *y*" or "the (first) derivative".

[2 marks]

g. $y = -\frac{16}{3}x + c$ ***(M1)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation.

$0 = -\frac{16}{3} \times (-1) + c$

$c = -\frac{16}{3}$

$y = -\frac{16}{3}x - \frac{16}{3}$ ***(A1)(G2)***

Note: Accept $y = -5.33x - 5.33$.

OR

$(y - 0) = \frac{-16}{3}(x + 1)$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation, ***(A1)*** for correct equation. Follow through from their answer to part (b). Accept $y = -5.33\ (x + 1)$. Accept equivalent equations.

[2 marks]

h. ***(A1)*****(ft)** for a tangent to their curve drawn.

(A1)**(ft)** for their tangent drawn at the point $x = -1$. ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Follow through from their graph. The tangent must be a straight line otherwise award at most ***(A0)(A1)***.

[2 marks]

i. (i) $a = \frac{1}{3}$ ***(G1)***

(ii) $b = 3$ ***(G1)***

Note: If *a* and *b* are reversed award ***(A0)(A1)***.

[2 marks]

j. *f* (*x*) is increasing ***(A1)***

[1 mark]

Examiners report

a. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

Drawing/sketching graphs is perhaps the area of the course that results in the poorest responses. It is also the area of the course that results in the best. It is therefore the area of the course that good teaching can influence the most.

Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

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In part (b) the answer could have been checked using the table on the GDC.

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In part (c) **coordinates** were required.

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The responses to part (d) were generally correct.

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The "show that" nature of part (e) meant that the final answer had to be stated.

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The interpretive nature of part (f) was not understood by the majority.

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Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

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Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

A biologist is studying the relationship between the number of chirps of the Snowy Tree cricket and the air temperature. He records the chirp rate, x, of a cricket, and the corresponding air temperature, T, in degrees Celsius.

The following table gives the recorded values.

Cricket's chirp rate, x, (chirps per minute)	20	40	60	80	100	120
Temperature, T (°C)	8.0	12.8	15.0	18.2	20.0	21.1

a. Draw the scatter diagram for the above data. Use a scale of 2 cm for 20 chirps on the horizontal axis and 2 cm for 4°C on the vertical axis. [4]

b. Use your graphic display calculator to write down the Pearson's product–moment correlation coefficient, r, between x and T. [2]

c. Interpret the relationship between x and T using your value of r. [2]

d. Use your graphic display calculator to write down the equation of the regression line T on x. Give the equation in the form $T = ax + b$. [2]

e. Calculate the air temperature when the cricket's chirp rate is 70. [2]

f. Given that $\bar{x} = 70$, draw the regression line T on x on your scatter diagram. [2]

g. A forest ranger uses her own formula for estimating the air temperature. She counts the number of chirps in 15 seconds, z, multiplies this number by 0.45 and then she adds 10. [1]

Write down the formula that the forest ranger uses for estimating the temperature, T.

Give the equation in the form $T = mz + n$.

h. A cricket makes 20 chirps in **15** seconds. [6]

For this chirp rate

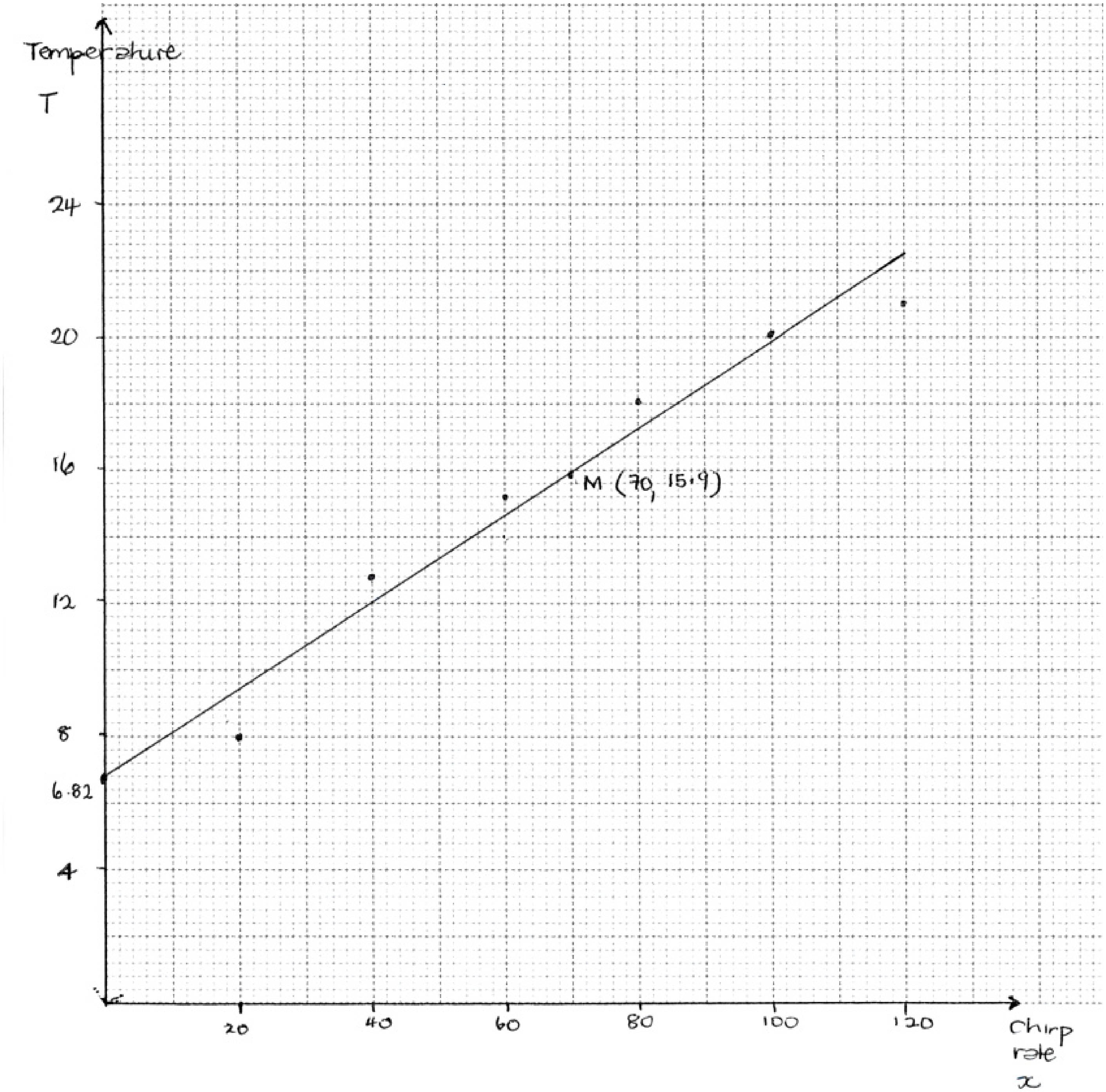
(i) calculate an estimate for the temperature, T, **using the forest ranger's formula**;

(ii) determine the actual temperature recorded by the biologist, **using the table above**;

(iii) calculate the percentage error in the forest ranger's estimate for the temperature, compared to the actual temperature recorded by the biologist.

Markscheme

a.



(A4)

Notes: Award ***(A1)*** for correct scales and labels.

Award ***(A3)*** for all six points correctly plotted,

(A2) for four or five points correctly plotted,

(A1) for two or three points correctly plotted.

Award at most ***(A0)(A3)*** if axes reversed.

Accept tolerance for T-axis.

b. $0.977 \quad (0.977324\ldots)$ ***(G2)***

Notes: Award ***(G1)*** for 0.97.

c. (Very) strong positive correlation ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*** for (very) strong, ***(A1)*** for positive.

Follow through from part (b).

d. $T = 0.129x + 6.82$ ***(G2)***

Notes: Award ***(G1)*** for $0.129x$, ***(G1)*** for $+6.82$.

Award a maximum of ***(G0)(G1)*** if the answer is not an equation.

e. $0.129 \times 70 + 6.82$ ***(M1)***

Note: Award ***(M1)*** for substitution of 70 into their equation of regression line.

OR

$\frac{8+12.8+\ldots+21.1}{6}$ ***(M1)***

$= 15.9\ (15.85)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (d) without working.

f. regression line through $(70,\ 15.9)$ ***(A1)*(ft)**

Note: Accept 15.9 ± 0.2.

Follow through from part (e).

with T-intercept, 6.82 ***(A1)*(ft)**

Note: Follow through from part (d). Accept 6.82 ± 0.2.

In case the regression line is not straight (ruler not used), award ***(A0)(A1)*(ft)** if line passes through both their $(70,\ 15.9)$ and $(0,\ 6.82)$, otherwise award ***(A0)(A0)***.

Do not penalize if line does not intersect the T-axis.

g. $T = 0.45z + 10$ ***(A1)***

h. (i) $0.45(20) + 10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 20 into their formula from part (g).

$= 19 \quad (^\circ\mathrm{C})$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (g).

(ii) $= 18.2 \quad (^\circ\mathrm{C})$ ***(A1)***

(iii) $\left|\frac{19-18.2}{18.2}\right| \times 100\%$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution in the percentage error formula, ***(A1)*** for correct substitution.

$4.40\% \quad (4.39560\ldots)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from parts (h)(i) and (h)(ii).

Examiners report

a. [N/A]
[N/A]

b. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]