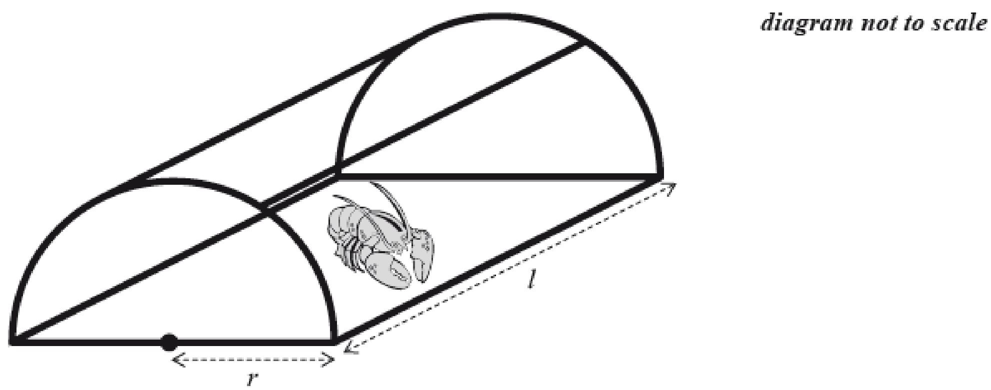
SL Paper 2

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is 0.75 m^3. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is 0.75 m^3. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is 0.75 m^3. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is 0.719 m, correct to three significant figures.

f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

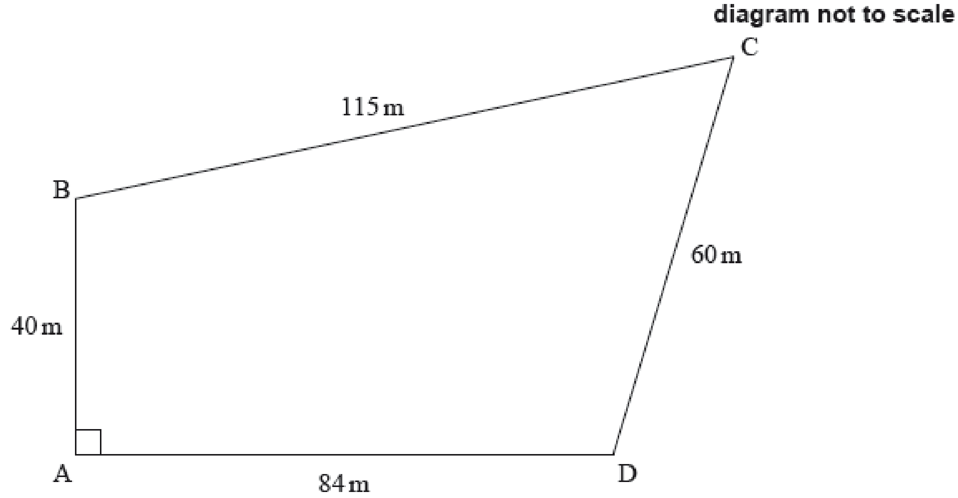
g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $\mathrm{AB} = 40\ \mathrm{m}$, $\mathrm{BC} = 115\ \mathrm{m}$, $\mathrm{CD} = 60\ \mathrm{m}$, $\mathrm{AD} = 84\ \mathrm{m}$ and angle $\mathrm{B\hat{A}D} = 90°$.

This information is shown on the diagram.



The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(\mathrm{AB}+\mathrm{CD})(\mathrm{AD}+\mathrm{BC})}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

a. Show that $\mathrm{BD} = 93\ \mathrm{m}$ correct to the nearest metre. [2]

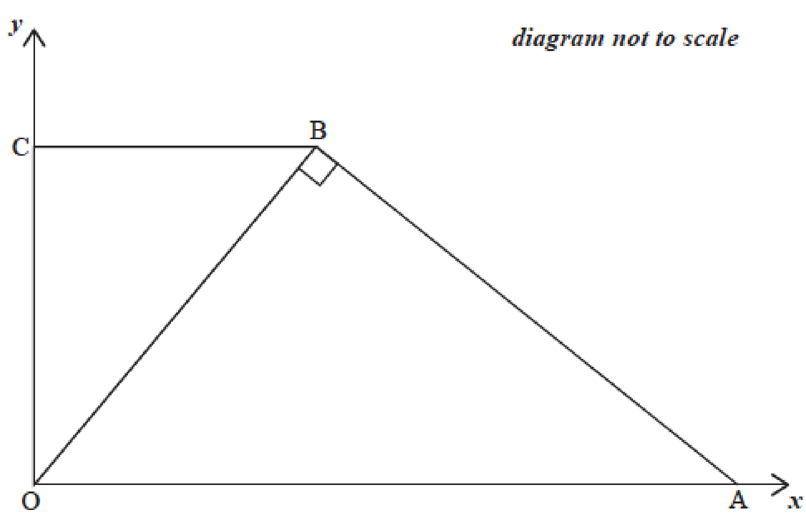
b. Calculate angle $\mathrm{B\hat{C}D}$. [3]

c. Find the area of ABCD. [4]

d.i. Calculate Abdallah's estimate for the area. [2]

d.ii. Find the percentage error in Abdallah's estimate. [2]

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



a. The equation of the line BC is $y = 4$. [1]

Write down the coordinates of point C.

b. The x-coordinate of point B is a. [2]

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

c. Point A lies on the x-axis and the line AB is perpendicular to line OB. [4]

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

d. The area of triangle AOB is **three times** the area of triangle OBC. [3]

Find an expression, **in terms of *a***, for

(i) the area of triangle OBC;

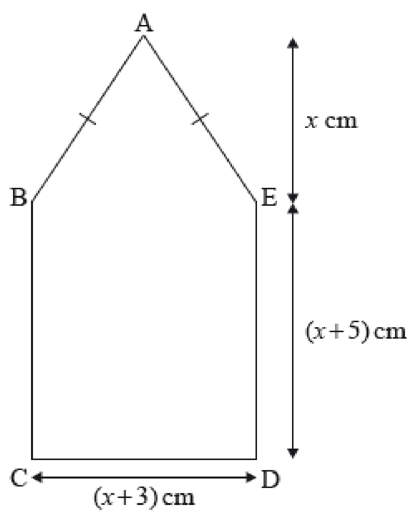
(ii) the *x*-coordinate of point A.

e. Calculate the value of a. [2]

The base of an electric iron can be modelled as a pentagon ABCDE, where:

BCDE is a rectangle with sides of length $(x+3)$ cm and $(x+5)$ cm;
ABE is an isosceles triangle, with $\text{AB} = \text{AE}$ and a height of x cm;
the area of ABCDE is $222\ \text{cm}^2$.

diagram not to scale



Insulation tape is wrapped around the perimeter of the base of the iron, ABCDE.

F is the point on AB such that $\text{BF} = 8\ \text{cm}$. A heating element in the iron runs in a straight line, from C to F.

a.i. Write down an **equation** for the area of ABCDE using the above information. [2]

a.ii. Show that the equation in part (a)(i) simplifies to $3x^2 + 19x - 414 = 0$. [2]

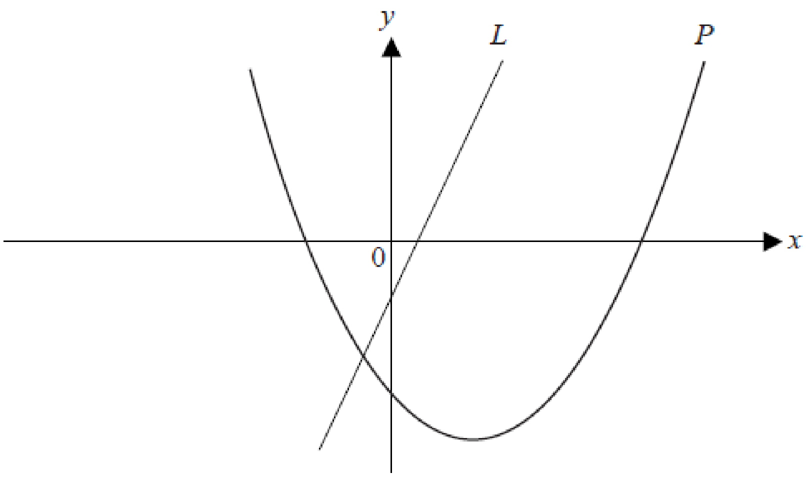
b. Find the length of CD. [2]

c. Show that angle $\mathrm{B\hat{A}E} = 67.4°$, correct to one decimal place. [3]

d. Find the length of the perimeter of ABCDE. [3]

e. Calculate the length of CF. [4]

The diagram below shows the graph of a line L passing through (1, 1) and (2 , 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



a. Find the gradient of the line *L*. [2]

b. Differentiate $f(x)$. [2]

c. Find the coordinates of the point where the tangent to *P* is parallel to the line *L*. [3]

d. Find the coordinates of the point where the tangent to *P* is perpendicular to the line *L*. [4]

e. Find [3]

(i) the gradient of the tangent to *P* at the point with coordinates (2, – 6).

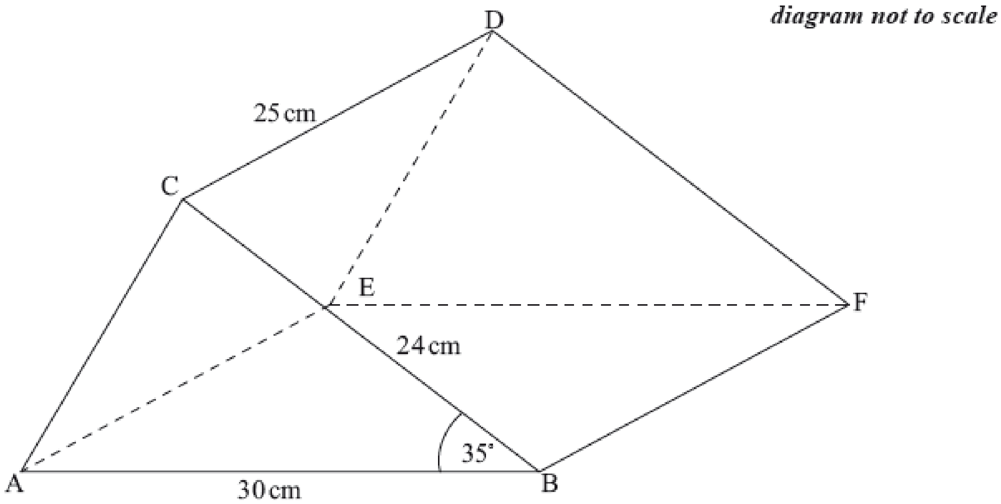
(ii) the equation of the tangent to *P* at this point.

f. State the equation of the axis of symmetry of *P*. [1]

g. Find the coordinates of the vertex of *P* and state the gradient of the curve at this point. [3]

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.

$\mathrm{AB} = 30\ \mathrm{cm},\ \mathrm{BC} = 24\ \mathrm{cm},\ \mathrm{CD} = 25\ \mathrm{cm}$ and angle $\mathrm{A\hat{B}C} = 35°$.



a. Calculate the length of AC. [3]

b. Calculate the area of triangle ABC. [3]

c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\ \mathrm{cm}^3$, correct to three significant figures. [2]

d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Show that the total surface area of one block is $2190\ \mathrm{cm}^2$, correct to three significant figures. [3]

f. The blocks are to be painted. One litre of paint will cover $22\ \mathrm{m}^2$. [3]

Calculate the number of litres required to paint all 2600 blocks.

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

a. (i) Write down the value of y when x is 2. [3]

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). [4]

c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

d. Let L_1 be the tangent to the curve at $x = 2$. [8]

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. [5]

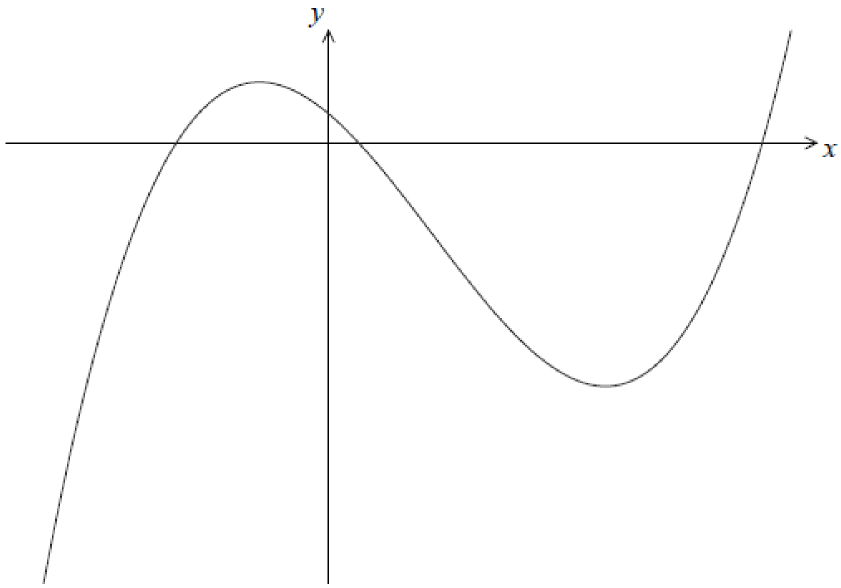
(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

diagram not to scale



a. Write down the values of x where the graph of $f(x)$ intersects the x-axis. [3]

b. Write down $f'(x)$. [3]

c. Find the value of the local maximum of $y = f(x)$. [4]

d. Let P be the point where the graph of $f(x)$ intersects the y axis. [1]

Write down the coordinates of P.

e. Let P be the point where the graph of $f(x)$ intersects the y axis. [2]

Find the gradient of the curve at P.

f. The line, L, is the tangent to the graph of $f(x)$ at P. [2]

Find the equation of L in the form $y = mx + c$.

g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [1]

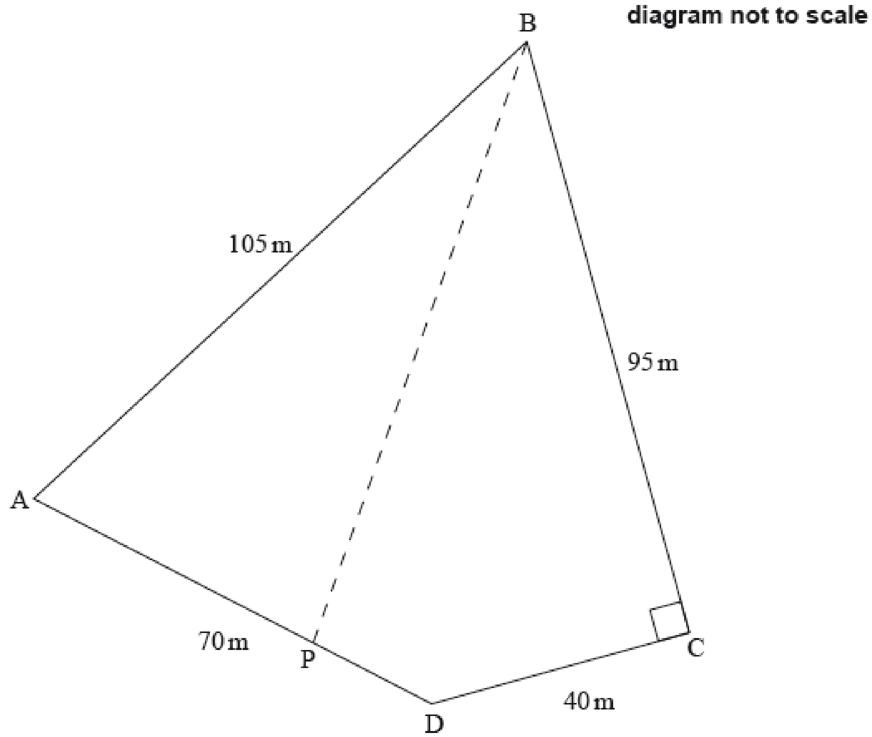
Write down the gradient of the tangent at Q.

h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [3]

Calculate the x-coordinate of Q.

A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$\mathrm{AB} = 105\ \mathrm{m}$, $\mathrm{BC} = 95\ \mathrm{m}$, $\mathrm{CD} = 40\ \mathrm{m}$, $\mathrm{DA} = 70\ \mathrm{m}$ and angle $\mathrm{DCB} = 90^{\circ}$.



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

a. the length of BD; [2]

b. the size of angle DAB; [3]

c. the area of triangle ABD; [3]

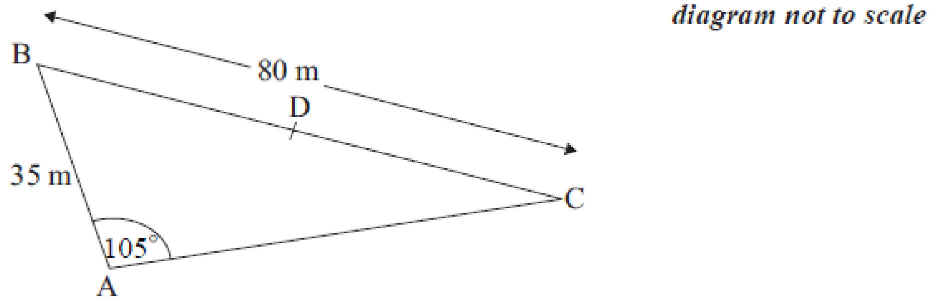
d. the area of quadrilateral ABCD; [2]

e. the length of AP; [3]

f. the length of the fence, BP. [3]

A farmer has a triangular field, ABC, as shown in the diagram.

AB = 35 m, BC = 80 m and BÂC = 105°, and D is the midpoint of BC.



a. Find the size of BĈA. [3]

b. Calculate the length of AD. [5]

c. The farmer wants to build a fence around ABD. [2]

Calculate the total length of the fence.

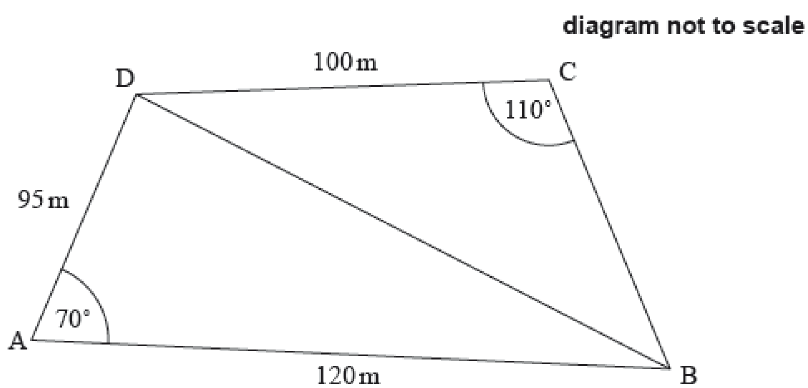
d. The farmer wants to build a fence around ABD. [2]

The farmer pays 802.50 USD for the fence. Find the cost per metre.

e. Calculate the area of the triangle ABD. [3]

f. A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. [3]

The quadrilateral ABCD represents a park, where $\mathrm{AB} = 120\ \mathrm{m}$, $\mathrm{AD} = 95\ \mathrm{m}$ and $\mathrm{DC} = 100\ \mathrm{m}$. Angle DAB is 70° and angle DCB is 110°. This information is shown in the following diagram.



A straight path through the park joins the points B and D.

A new path, CE, is to be built such that E is the point on BD closest to C.

The section of the park represented by triangle DCE will be used for a charity race. A track will be marked along the sides of this section.

a. Find the length of the path BD. [3]

b. Show that angle DBC is 48.7°, correct to three significant figures. [3]

c. Find the area of the park. [4]

d. Find the length of the path CE. [2]

e. Calculate the total length of the track. [3]

A boat race takes place around a triangular course, ABC, with $\mathrm{AB} = 700\ \mathrm{m}$, $\mathrm{BC} = 900\ \mathrm{m}$ and angle $\mathrm{ABC} = 110^{\circ}$. The race starts and finishes at point A.

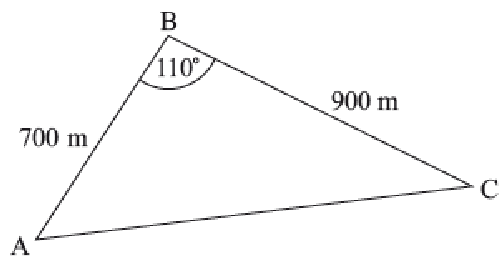


diagram not to scale

a. Calculate the total length of the course. [4]

b. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

c. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Find the size of angle ACB.

d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Calculate the area that must be kept clear of boats.

e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Determine, giving a reason, whether the course complies with the safety regulations.

f. The race is filmed from a helicopter, H, which is flying vertically above point A. [2]

The angle of elevation of H from B is 15°.

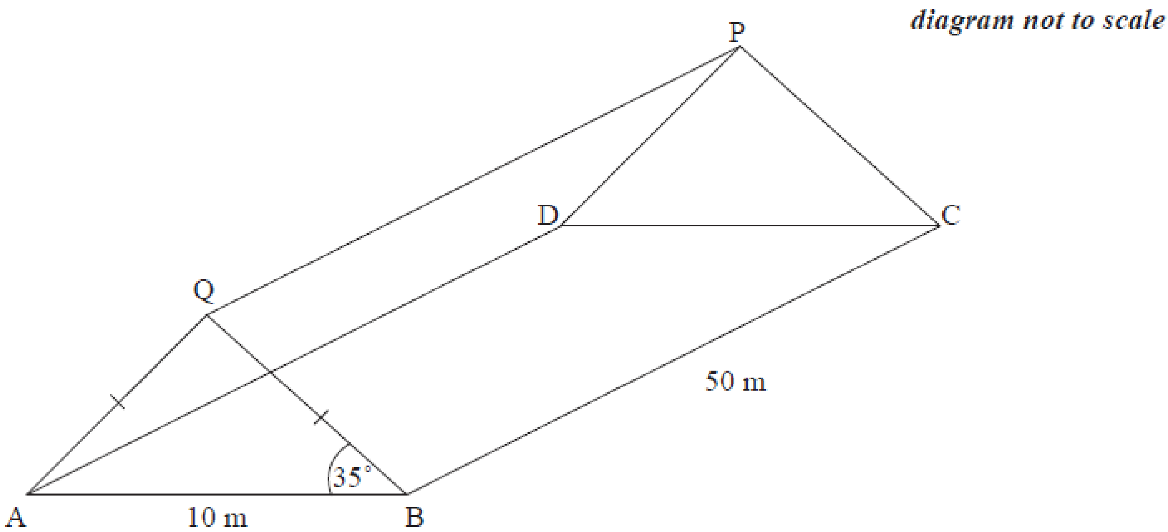
Calculate the vertical height, AH, of the helicopter above A.

g. The race is filmed from a helicopter, H, which is flying vertically above point A. [3]

The angle of elevation of H from B is 15°.

Calculate the maximum possible distance from the helicopter to a boat on the course.

A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is 35°.



a. Write down the size of angle AQB. [1]

b. Calculate the length of AQ. [3]

c. Calculate the length of AC. [2]

d. Show that the length of CQ is 50.37 m, correct to 4 significant figures. [2]

e. Find the size of the angle AQC. [3]

f. Calculate the total area of the glass needed to construct [5]

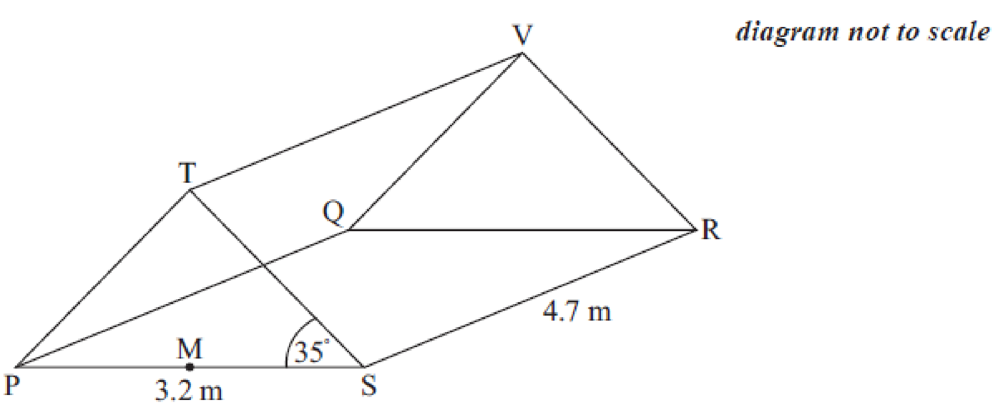
(i) the two rectangular faces of the greenhouse;

(ii) the two triangular faces of the greenhouse.

g. The cost of one square metre of glass used to construct the greenhouse is 4.80 USD. [3]

Calculate the cost of glass to make the greenhouse. Give your answer correct to the nearest 100 USD.

A tent is in the shape of a triangular right prism as shown in the diagram below.



The tent has a rectangular base PQRS .

PTS and QVR are isosceles triangles such that PT = TS and QV = VR .

PS is 3.2 m , SR is 4.7 m and the angle TSP is 35 °.

a. Show that the length of side ST is 1.95 m, correct to 3 significant figures. [3]

b. Calculate the area of the triangle PTS. [3]

c. Write down the area of the rectangle STVR. [1]

d. Calculate the **total** surface area of the tent, including the base. [3]

e. Calculate the volume of the tent. [2]

f. A pole is placed from V to M, the midpoint of PS. [4]

Find in metres,

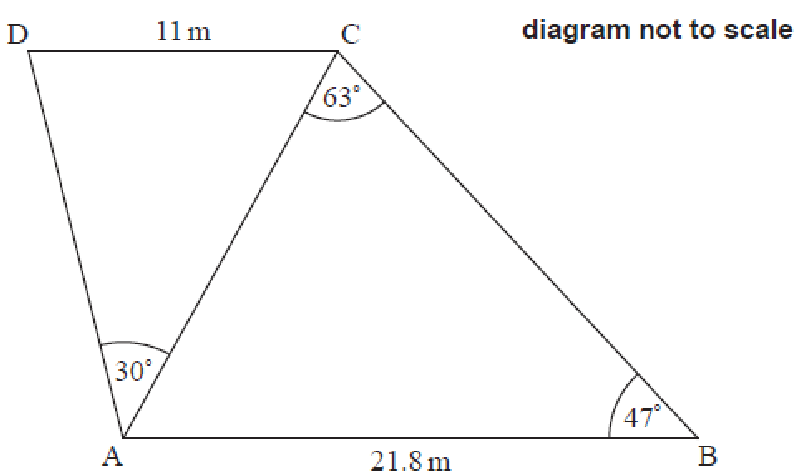
(i) the height of the tent, TM;

(ii) the length of the pole, VM.

g. Calculate the angle between VM and the base of the tent. [2]

---

a. A playground, when viewed from above, is shaped like a quadrilateral, ABCD, where $\mathrm{AB} = 21.8\,\mathrm{m}$ and $\mathrm{CD} = 11\,\mathrm{m}$. Three of the internal angles have been measured and angle $\mathrm{ABC} = 47^\circ$, angle $\mathrm{ACB} = 63^\circ$ and angle $\mathrm{CAD} = 30^\circ$. This information is represented in the following diagram. [3]



Calculate the distance AC.

b. Calculate angle ADC. [3]

c. There is a tree at C, perpendicular to the ground. The angle of elevation to the top of the tree from D is 35°. [2]

Calculate the height of the tree.

d. Chavi estimates that the height of the tree is $6\,\mathrm{m}$. [2]

Calculate the percentage error in Chavi's estimate.

e. Chavi is celebrating her birthday with her friends on the playground. Her mother brings a 2 litre bottle of orange juice to share among them. [3]

She also brings **cone-shaped** paper cups.

Each cup has a vertical height of 10 cm and the top of the cup has a diameter of 6 cm.

Calculate the volume of one paper cup.

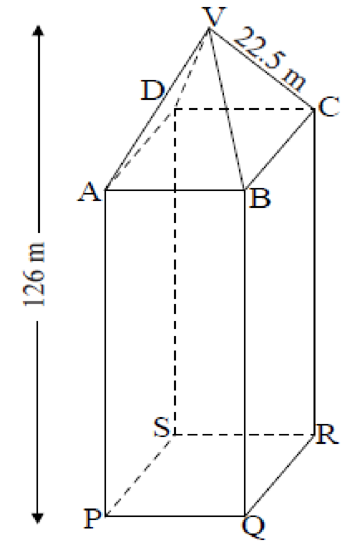
f. Calculate the maximum number of cups that can be completely filled with the 2 litre bottle of orange juice. [3]

The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

diagram not to scale



a.i. Write down the length of VA in metres. [1]

a.ii. Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. [1]

b. Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. [2]

c. Calculate the length of AC in metres. [3]

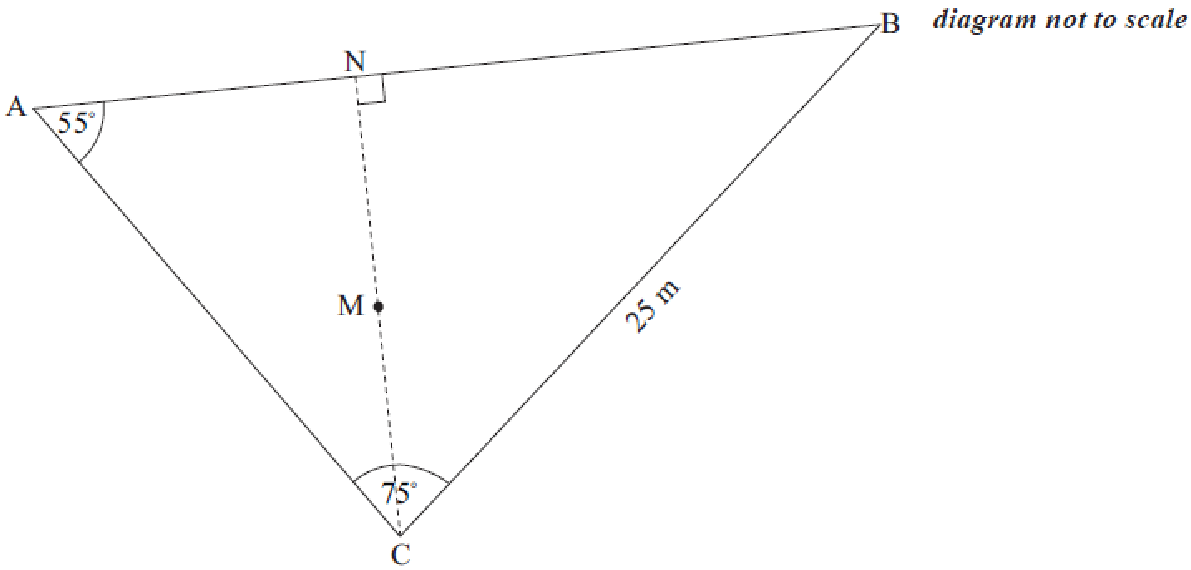
d. Show that the length of BC is 19.1 metres correct to 3 significant figures. [2]

e. Calculate the volume of the tower. [4]

f. To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg. [3]

Calculate the weight of air in the tower.

The diagram represents a small, triangular field, ABC , with $\mathrm{BC} = 25\ \mathrm{m}$, angle $\mathrm{BAC} = 55^\circ$ and angle $\mathrm{ACB} = 75^\circ$.



a. Write down the size of angle ABC. [1]

b. Calculate the length of AC. [3]

c. Calculate the area of the field ABC. [3]

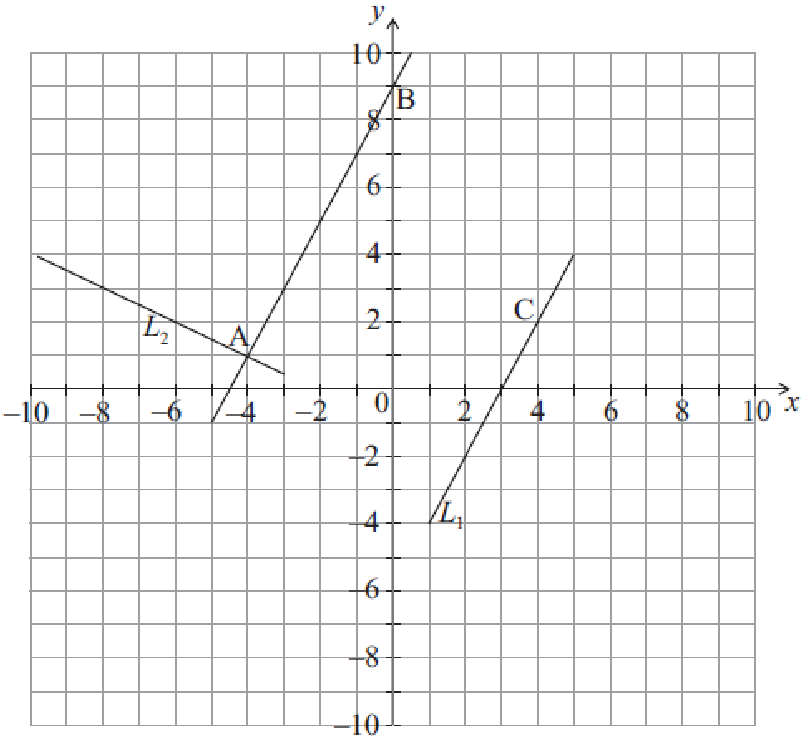
d. N is the point on AB such that CN is perpendicular to AB. M is the midpoint of CN. [3]

Calculate the length of NM.

e. A goat is attached to one end of a rope of length 7 m. The other end of the rope is attached to the point M. [5]

Decide whether the goat can reach point P, the midpoint of CB. Justify your answer.

The points A (−4, 1), B (0, 9) and C (4, 2) are plotted on the diagram below. The diagram also shows the lines AB, L_1 and L_2.



a. Find the gradient of AB. [2]

b. L_1 passes through C and is parallel to AB. [1]

Write down the y-intercept of L_1.

c. L_2 passes through A and is perpendicular to AB. [3]

Write down the equation of L_2. Give your answer in the form ***ax* + *by* + *d* = 0** where *a*, *b* and *d* $\in \mathbb{Z}$.

d. Write down the coordinates of the point D, the intersection of L_1 and L_2. [1]

e. There is a point R on L_1 such that ABRD is a rectangle. [2]

Write down the coordinates of R.

f. The distance between A and D is $\sqrt{45}$. [4]

(i) Find the distance between D and R .

(ii) Find the area of the triangle BDR .

Consider the functions $f(x) = \frac{2x+3}{x+4}$ and $g(x) = x + 0.5$.

a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. [6]

b. Write down the equation of the vertical asymptote. [2]

c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. [2]

d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. [3]

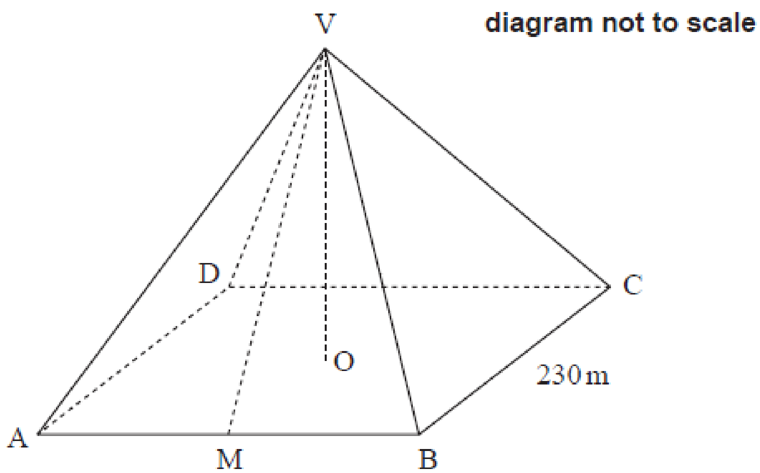
e. Write down the gradient of the line $g(x) = x + 0.5$. [1]

f. The line L passes through the point with coordinates $(-2,\ -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. [3]

---

a. The Great Pyramid of Giza in Egypt is a right pyramid with a square base. The pyramid is made of solid stone. The sides of the base are $230\,\mathrm{m}$ long. The diagram below represents this pyramid, labelled VABCD. [3]

V is the vertex of the pyramid. O is the centre of the base, ABCD . M is the midpoint of AB. Angle $\mathrm{ABV} = 58.3^\circ$.



Show that the length of VM is 186 metres, correct to three significant figures.

b. Calculate the height of the pyramid, VO . [2]

c. Find the volume of the pyramid. [2]

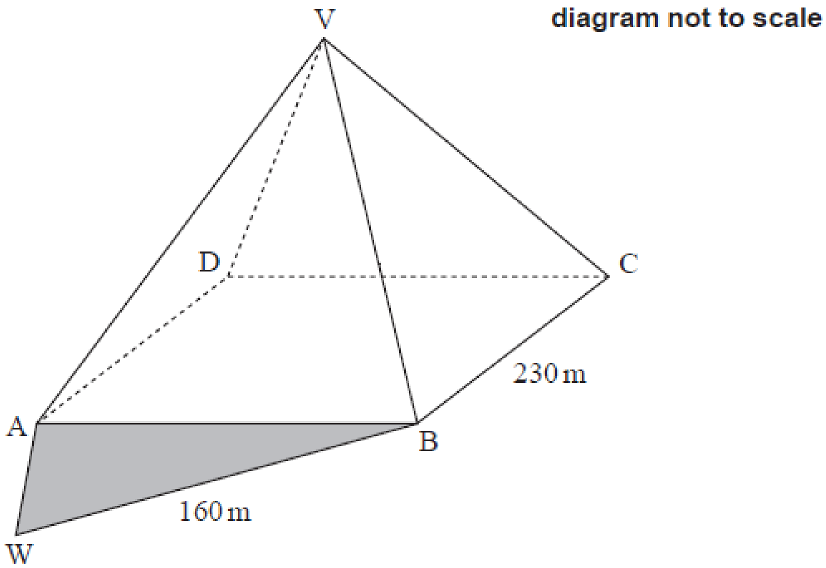
d. Write down your answer to part (c) in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Ahmad is a tour guide at the Great Pyramid of Giza. He claims that the amount of stone used to build the pyramid could build a wall 5 metres high and 1 metre wide stretching from Paris to Amsterdam, which are $430\,\mathrm{km}$ apart. [4]

Determine whether Ahmad's claim is correct. Give a reason.

f. Ahmad and his friends like to sit in the pyramid's shadow, ABW, to cool down. [6]

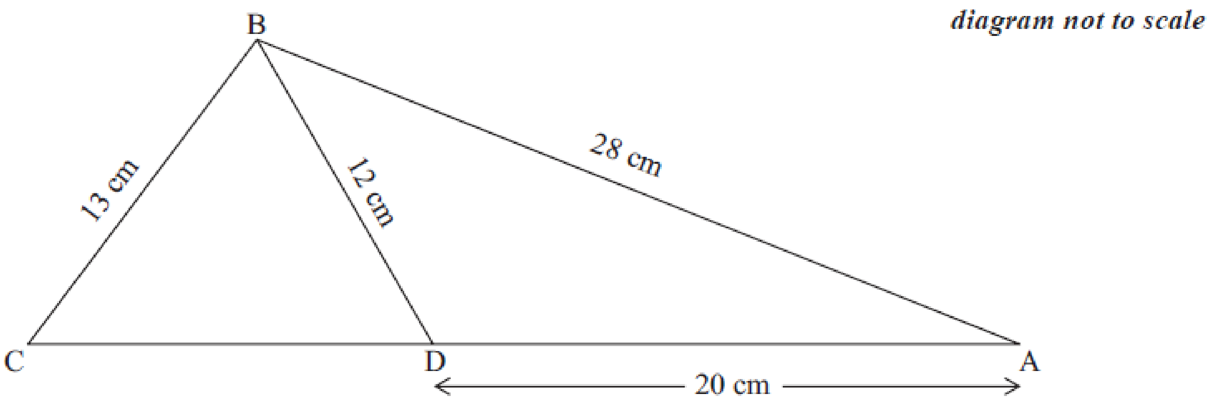
At mid-afternoon, $\mathrm{BW} = 160\,\mathrm{m}$ and angle $\mathrm{ABW} = 15^\circ$.



i) Calculate the length of AW at mid-afternoon.

ii) Calculate the area of the shadow, ABW, at mid-afternoon.

The diagram shows triangle ABC in which $\mathrm{AB} = 28$ cm, $\mathrm{BC} = 13$ cm, $\mathrm{BD} = 12$ cm and $\mathrm{AD} = 20$ cm.



a. Calculate the size of angle ADB. [3]

b. Find the area of triangle ADB. [3]

c. Calculate the size of angle BCD. [4]

d. Show that the triangle ABC is not right angled. [4]

An office block, ABCPQR, is built in the shape of a triangular prism with its "footprint", ABC, on horizontal ground. $\mathrm{AB} = 70$ m, $\mathrm{BC} = 50$ m and $\mathrm{AC} = 30$ m. The vertical height of the office block is 120 m .

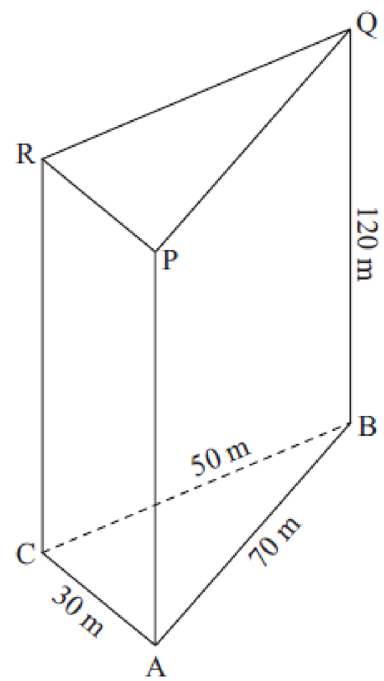


diagram not to scale

a. Calculate the size of angle ACB. [3]

b. Calculate the area of the building's footprint, ABC. [3]

c. Calculate the volume of the office block. [2]

d. To stabilize the structure, a steel beam must be made that runs from point C to point Q. [2]

Calculate the length of CQ.

e. Calculate the angle CQ makes with BC. [2]

In the diagram below A, B and C represent three villages and the line segments AB, BC and CA represent the roads joining them. The lengths of AC and CB are 10 km and 8 km respectively and the size of the angle between them is 150°.

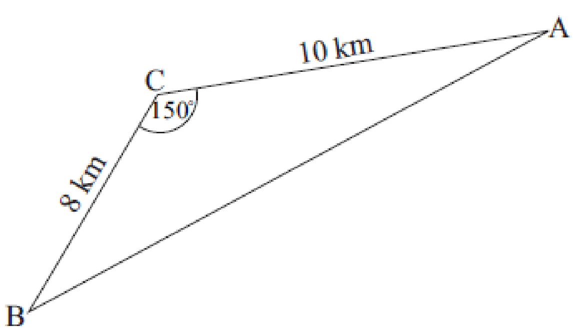


diagram not to scale

a. Find the length of the road AB. [3]

b. Find the size of the angle CAB. [3]

c. Village D is halfway between A and B. A new road perpendicular to AB and passing through D is built. Let T be the point where this road cuts AC. This information is shown in the diagram below. [1]

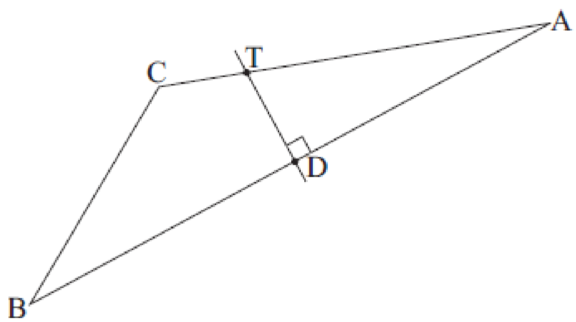


diagram not to scale

Write down the distance from A to D.

d. Show that the distance from D to T is 2.06 km correct to three significant figures. [2]

e. A bus starts and ends its journey at A taking the route AD to DT to TA. [3]

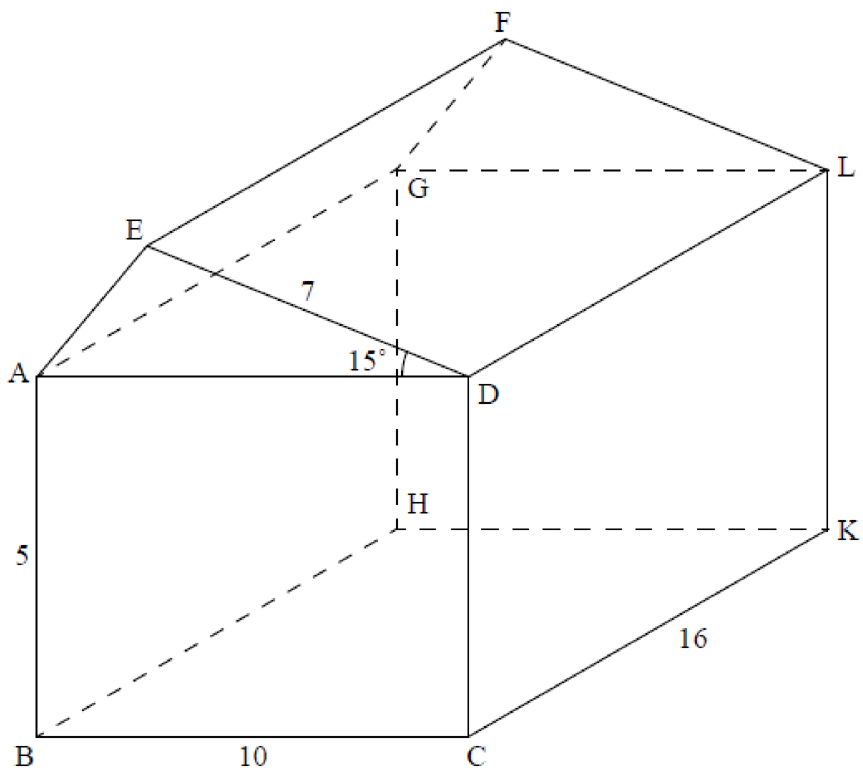
Find the total distance for this journey.

f. The average speed of the bus while it is moving on the road is 70 km h^{-1}. The bus stops for **5 minutes** at each of D and T . [4]

Estimate the time taken by the bus to complete its journey. Give your answer correct to the nearest minute.

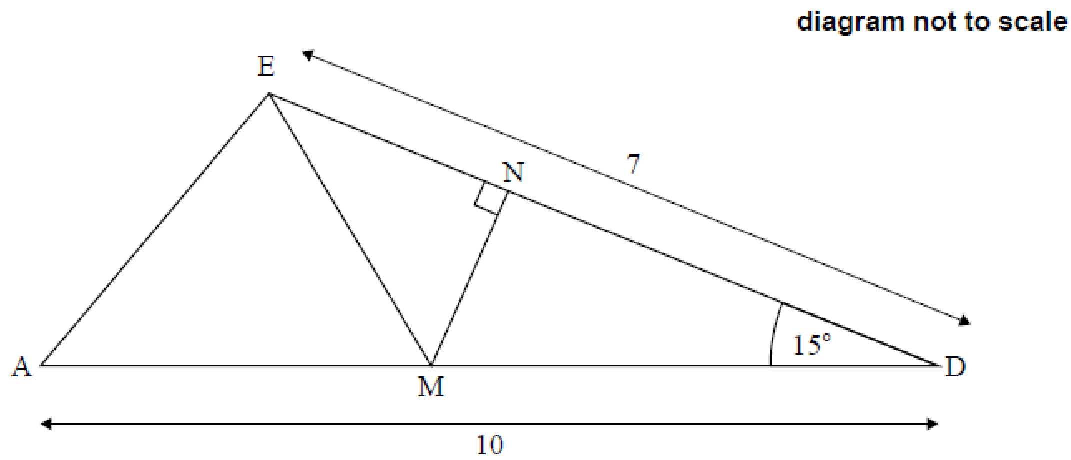
Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

Farmer Brown believes that N is the midpoint of ED.

a. Calculate the area of triangle EAD. [3]

b. Calculate the **total** volume of the barn. [3]

c. Calculate the length of MN. [2]

d. Calculate the length of AE. [3]

e. Show that Farmer Brown is incorrect. [3]

f. Calculate the **total** length of metal required for one support. [4]

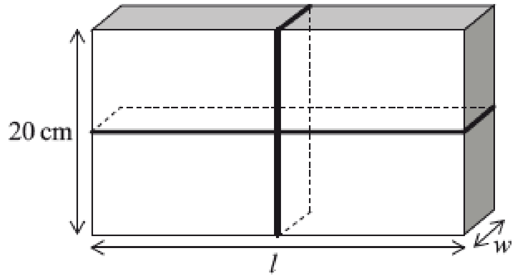
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is $3000\ \text{cm}^3$.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

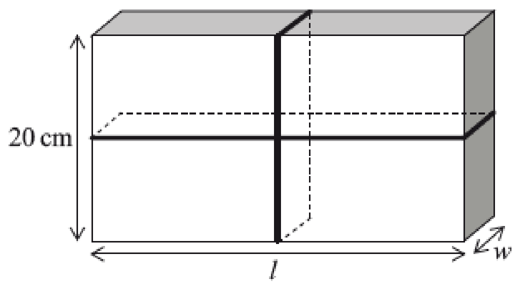
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

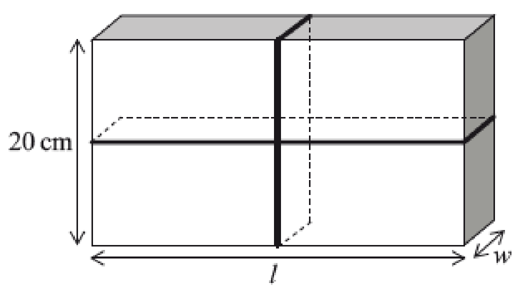
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



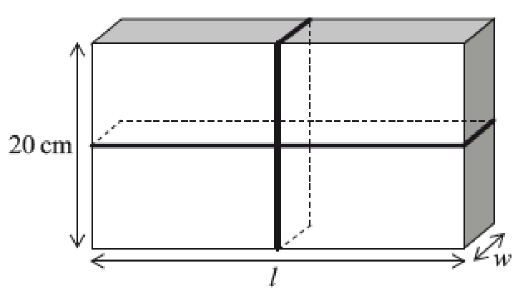
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



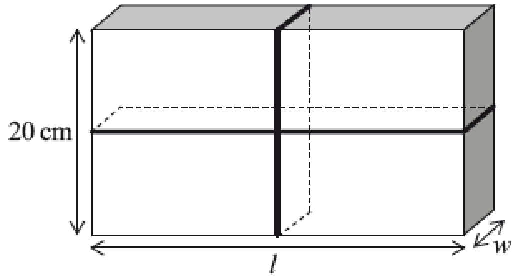
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



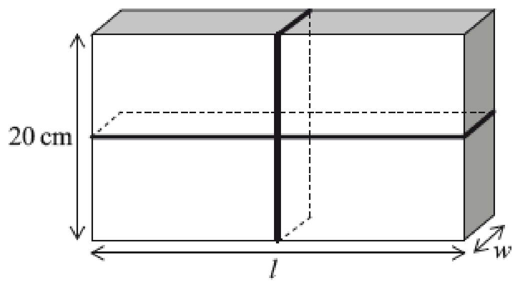
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



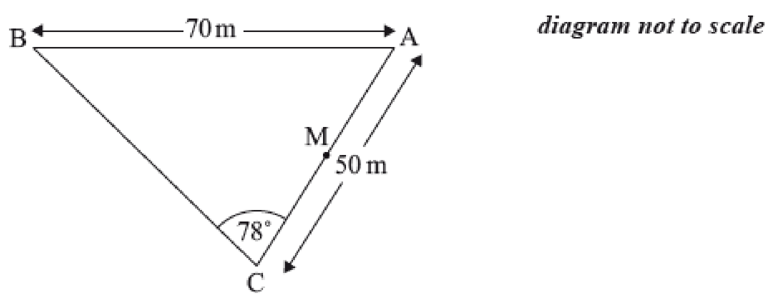
Write down the value, l, of the parcel for which the length of string is a minimum.

h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78°.



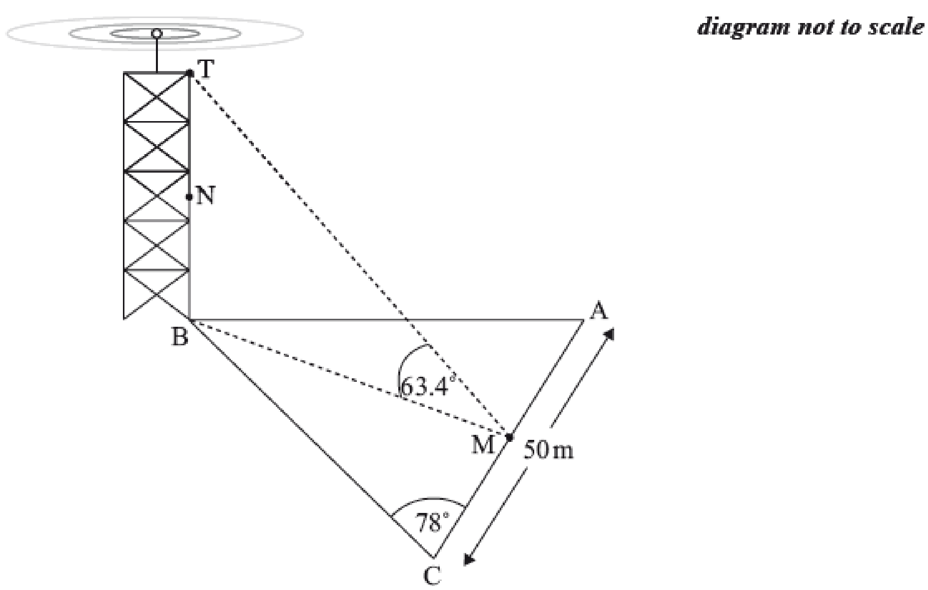
a. Find the size of angle ABC. [3]

b. Find the area of the triangular field. [4]

c. M is the midpoint of AC. [3]

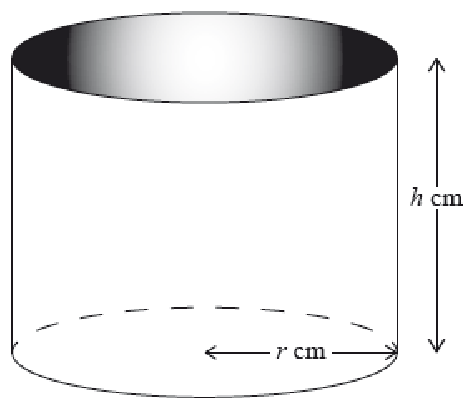
Find the length of BM.

d. A vertical mobile phone mast, TB, is built next to the field with its base at B. The angle of elevation of T from M is 63.4°. N is the midpoint of the mast. [5]



Calculate the angle of elevation of N from M.

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is $0.5\ \mathrm{m}^3$.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\ 000\ 000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

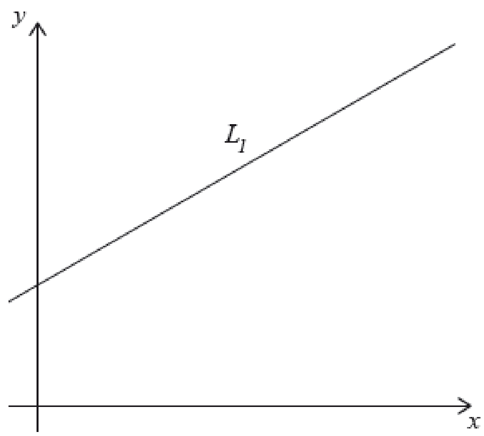
e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1,\ 4)$.

The point C has coordinates $(5,\ 12)$. M is the midpoint of AC.

The straight line, L_2, is perpendicular to AC and passes through M.

The point D is the intersection of L_1 and L_2.

The length of MD is $\frac{\sqrt{45}}{2}$.

The point B is such that ABCD is a rhombus.

a. Show that A lies on L_1. [2]

b. Find the coordinates of M. [2]

c. Find the length of AC. [2]

d. Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

e. Find the coordinates of D. [2]

f. Write down the length of MD correct to five significant figures. [1]

g. Find the area of ABCD. [3]

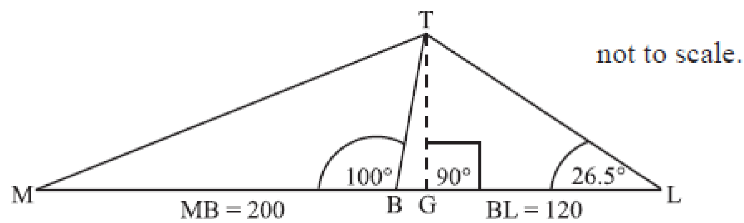
Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

An old tower (BT) leans at 10° away from the vertical (represented by line TG).

The base of the tower is at B so that $\mathrm{M\hat{B}T} = 100°$.

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be $\mathrm{B\hat{L}T} = 26.5°$.



i.a. Calculate the volume of the cylinder **in cm^3**. Give your answer correct to **two** decimal places. [3]

i.b. The cylinder is used for storing tennis balls. Each ball has a **radius** of 3.25 cm. [1]

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

i.c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls. [4]

(ii) Convert your answer to (c) (i) into cubic metres.

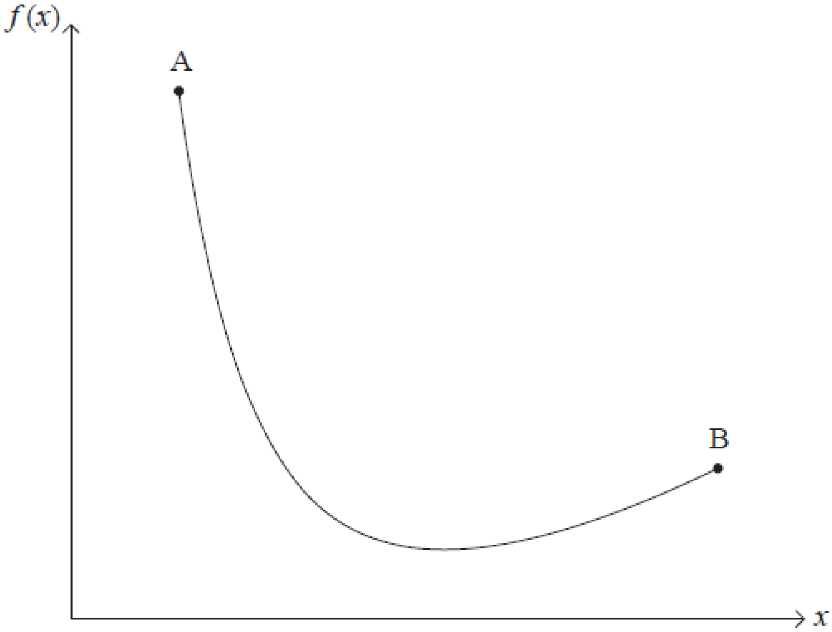
ii.a. (i) Find the value of angle $\mathrm{B\hat{T}L}$. [5]

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

ii.b. Calculate the vertical height TG of the top of the tower. [2]

ii.c. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. [3]

The graph of the function $f(x) = \frac{14}{x} + x - 6$, for $1 \le x \le 7$ is given below.



a. Calculate $f(1)$. [2]

b. Find $f'(x)$. [3]

c. **Use your answer to part (b)** to show that the *x*-coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures. [3]

d. Find the range of f. [3]

e. Points A and B lie on the graph of f. The *x*-coordinates of A and B are 1 and 7 respectively. [1]

Write down the *y*-coordinate of B.

f. Points A and B lie on the graph of f . The *x*-coordinates of A and B are 1 and 7 respectively. [2]

Find the gradient of the straight line passing through A and B.

g. M is the midpoint of the line segment AB. [2]

Write down the coordinates of M.

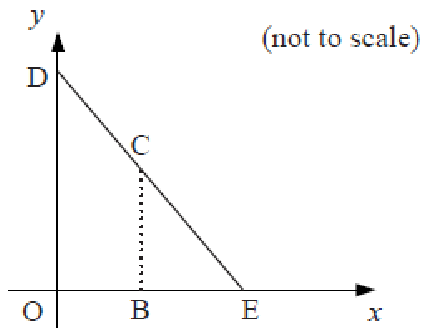
h. *L* is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same *x*-coordinate as M. [2]

Find the gradient of *L*.

i. Find the equation of *L*. Give your answer in the form $y = mx + c$. [3]

On the coordinate axes below, D is a point on the y-axis and E is a point on the x-axis. O is the origin. The equation of the line DE is $y + \frac{1}{2}x = 4$

.



a. Write down the coordinates of point E. [2]

b. C is a point on the line DE. B is a point on the x-axis such that BC is parallel to the y-axis. The x-coordinate of C is t. [2]

Show that the y-coordinate of C is $4-\frac{1}{2}t$.

c. OBCD is a trapezium. The y-coordinate of point D is 4. [3]

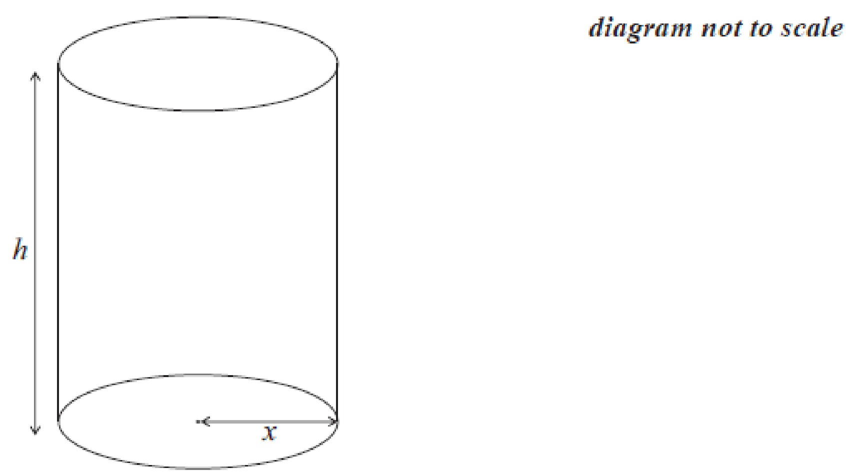
Show that the area of OBCD is $4t-\frac{1}{4}t^2$.

d. The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. [1]

e. (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d). [4]

(ii) Hence find the correct value for t. Give a reason for your answer.

A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x, the radius of the base of the can in cm.



The volume of the dog food cans is 600 cm^3.

a. Show that $h=\frac{600}{\pi x^2}$. [2]

b.i. Find an expression for the curved surface area of the can, in terms of *x*. Simplify your answer. [2]

b.ii. Hence write down an expression for *A*, the total surface area of the can, in terms of *x*. [2]

c. Differentiate *A* in terms of *x*. [3]

d. Find the value of x that makes A a minimum. [3]

e. Calculate the minimum total surface area of the dog food can. [2]

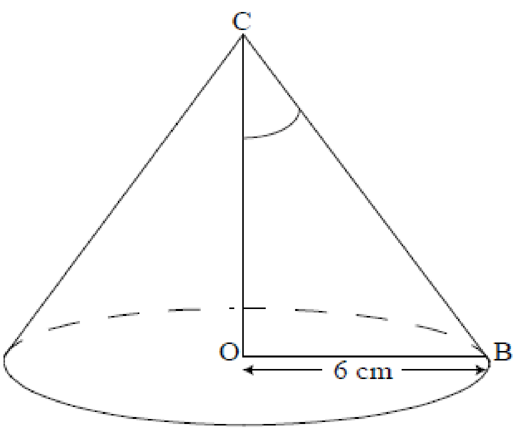
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

a. Find the area of the base of the cylinder. [2]

b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. [2]

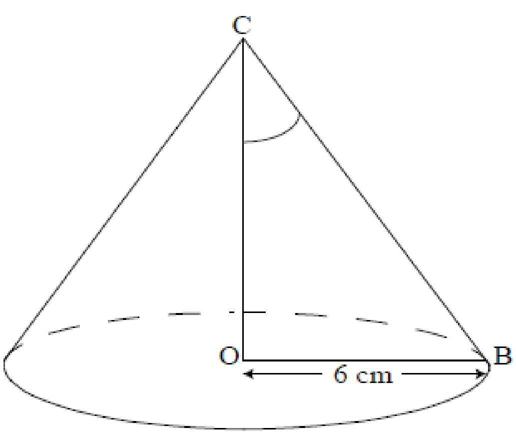
c. Find the total surface area of the cylinder. [3]

d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [3]



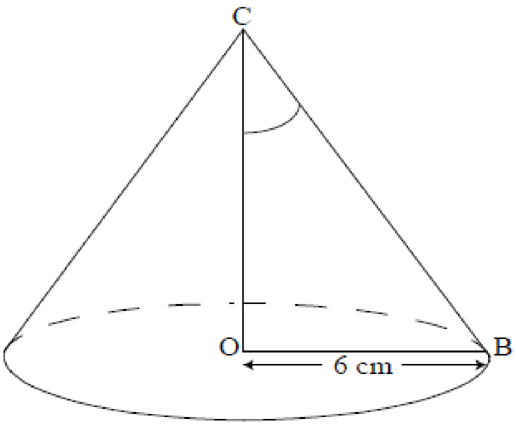
Find the height, OC, of the cone.

e. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



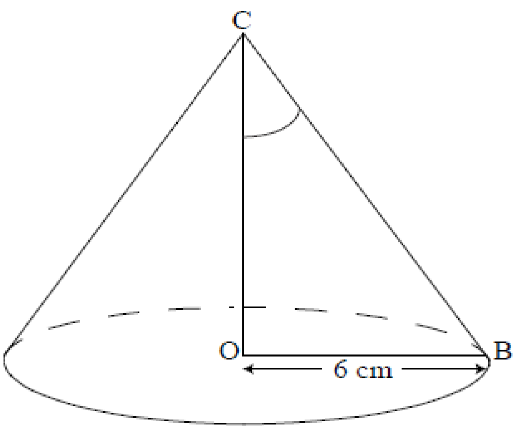
Find the size of angle BCO.

f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



Find the slant height, CB.

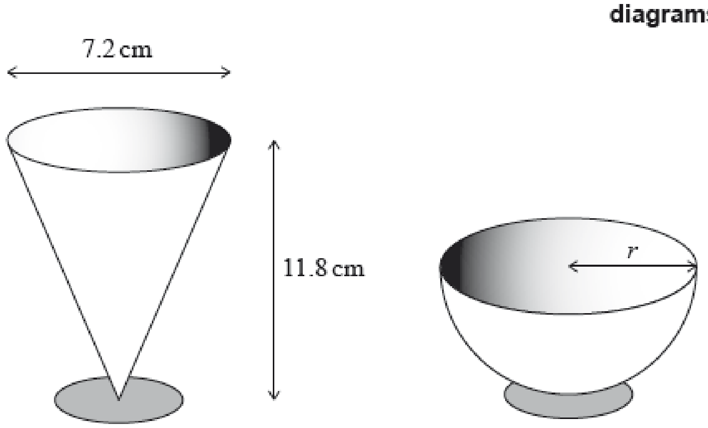
g. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [4]



Find the total surface area of the cone.

A restaurant serves desserts in glasses in the shape of a cone and in the shape of a hemisphere. The diameter of a cone shaped glass is 7.2 cm and the height of the cone is 11.8 cm as shown.

diagrams not to scale

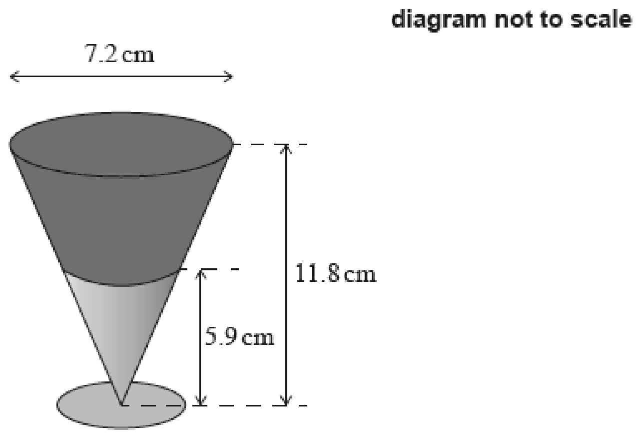


The volume of a hemisphere shaped glass is $225\ \text{cm}^3$.

The restaurant offers two types of dessert.

The **regular dessert** is a hemisphere shaped glass completely filled with chocolate mousse. The cost, to the restaurant, of the chocolate mousse for one regular dessert is $1.89.

The **special dessert** is a cone shaped glass filled with two ingredients. It is first filled with orange paste to half of its height and then with chocolate mousse for the remaining volume.



The cost, to the restaurant, of $100\ \mathrm{cm}^3$ of orange paste is \$7.42.

A chef at the restaurant prepares 50 desserts; x regular desserts and y special desserts. The cost of the ingredients for the 50 desserts is \$111.44.

a. Show that the volume of a cone shaped glass is $160\ \mathrm{cm}^3$, correct to 3 significant figures. [2]

b. Calculate the radius, r, of a hemisphere shaped glass. [3]

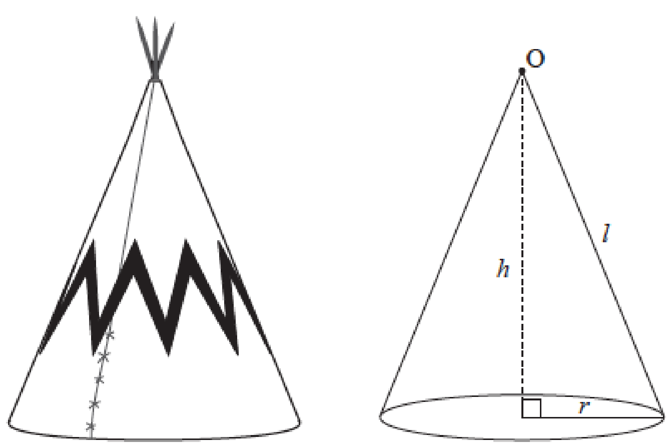
c. Find the cost of $100\ \mathrm{cm}^3$ of chocolate mousse. [2]

d. Show that there is $20\ \mathrm{cm}^3$ of orange paste in each special dessert. [2]

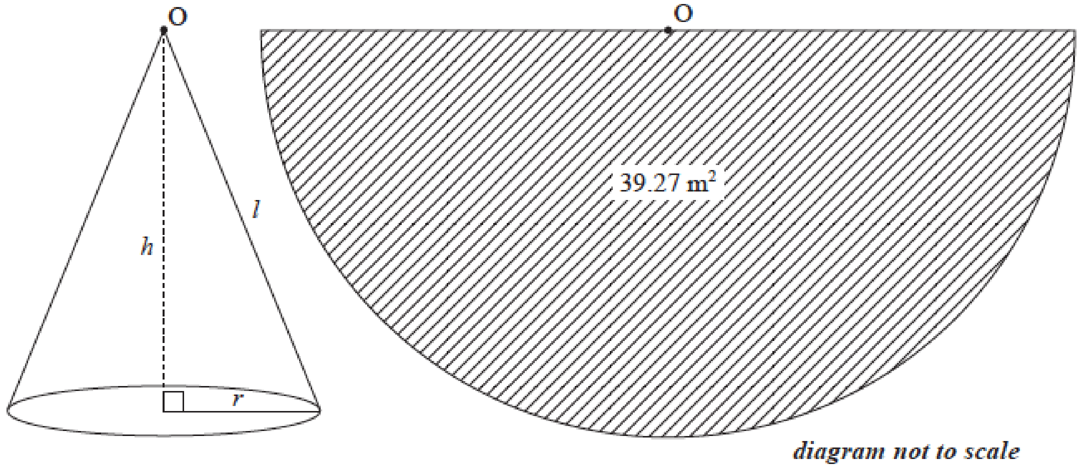
e. Find the total cost of the ingredients of one special dessert. [2]

f. Find the value of x. [3]

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is $39.27\ \mathrm{m}^2$, and has the shape of a semicircle, as shown in the following diagram.



a. Show that the slant height, l, is 5 m, correct to the nearest metre. [2]

b. (i) Find the circumference of the base of the cone. [6]

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

c. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

e. A company designs cone-shaped tents to resemble the traditional tepees. [2]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

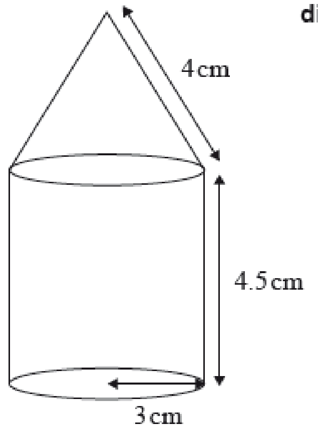
These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

The following diagram shows a perfume bottle made up of a cylinder and a cone.

diagram not to scale



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. [6]

(ii) Calculate the volume of the perfume bottle.

b. The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part. [2]

Find the height of the perfume in the bottle.

c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. [4]

Find the **total** surface area of the perfume bottle.

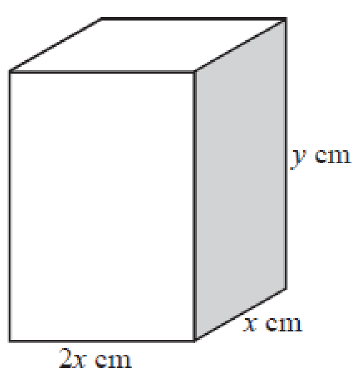
d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2. [4]

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. [2]

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

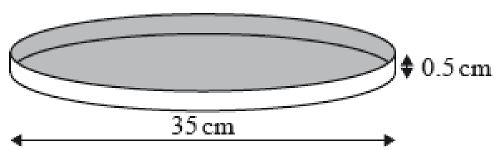
ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.

diagram not to scale



A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19,\ t \geqslant 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

The pizza can be eaten once its temperature drops to 45 °C.

a. Calculate the volume of this pan. [3]

b. Find the radius of the sphere in cm, correct to one decimal place. [4]

c. Find the value of a. [2]

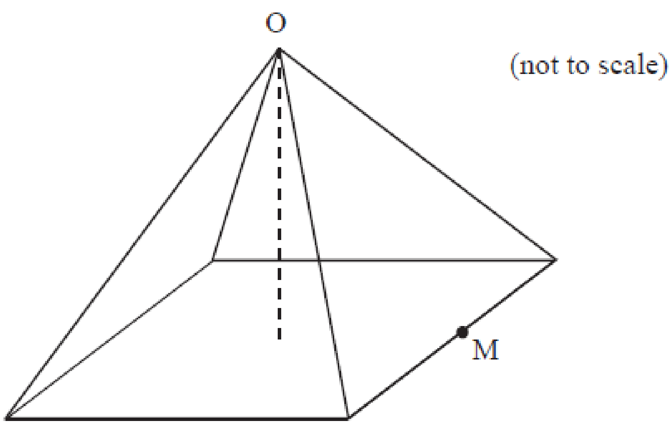
d. Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

e. Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

f. In the context of this model, state what the value of 19 represents. [1]

Amir needs to construct an isosceles triangle ABC whose area is 100 cm^2. The equal sides, AB and BC, are 20 cm long.

Sylvia is making a square-based pyramid. Each triangle has a base of length 12 cm and a height of 10 cm.



i.a. Angle ABC is acute. Show that the angle ABC is 30°. [2]

i.b. Find the length of AC. [3]

ii.a. Show that the **height** of the pyramid is 8 cm. [2]

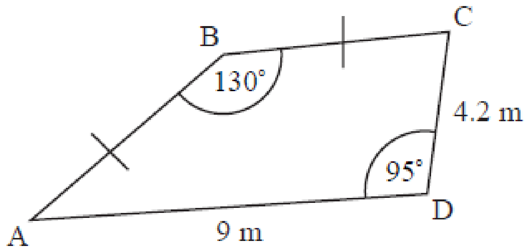
ii.b. M is the midpoint of the base of one of the triangles and O is the apex of the pyramid. [3]

Find the angle that the line MO makes with the base of the pyramid.

ii.c. Calculate the volume of the pyramid. [2]

ii.d. Daniel wants to make a rectangular prism with the same volume as that of Sylvia's pyramid. The base of his prism is to be a square of side 10 cm. Calculate the height of the prism. [2]

The quadrilateral ABCD shown below represents a sandbox. AB and BC have the same length. AD is 9 m long and CD is 4.2 m long. Angles ADC and ABC are 95° and 130° respectively.



a. Find the length of AC. [3]

b. (i) Write down the size of angle BCA. [4]

(ii) Calculate the length of AB.

c. Show that the area of the sandbox is $31.1\ \mathrm{m}^2$ correct to 3 s.f. [4]

d. The sandbox is a prism. Its edges are $40\ \mathrm{cm}$ high. The sand occupies one third of the volume of the sandbox. Calculate the volume of sand in the sandbox. [3]

Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [5]

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [4]

Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [2]

Write down the coordinates of the local maximum.

f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$. [2]

Find the gradient of T.

g. The line L passes through the point $(2, -12)$ and is perpendicular to T. [5]

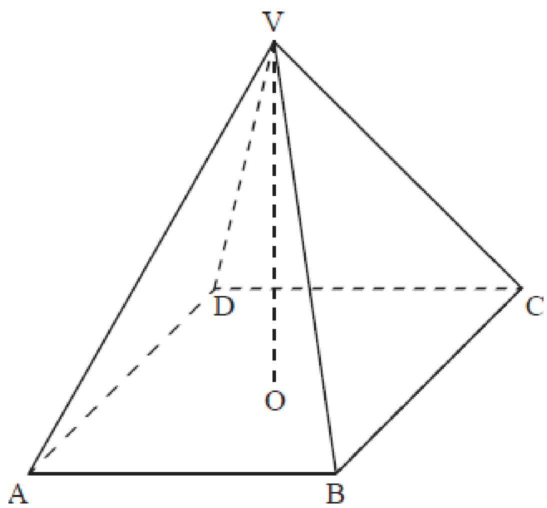
L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



a. Calculate the volume of the pyramid. [2]

b. The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid. [2]

c. Show that the length of the sloping edge VC of the pyramid is 3.6 cm. [4]

d. Calculate the angle at the vertex, $\mathrm{B\hat{V}C}$. [3]

e. Calculate the total surface area of the pyramid. [4]

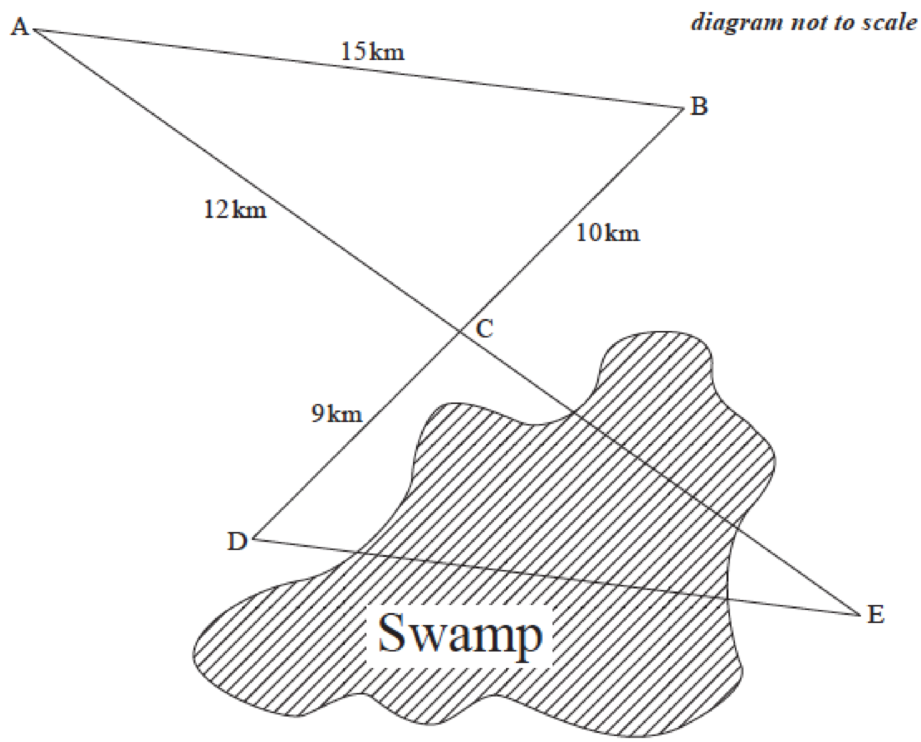
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor’s information.



a. (i) Find the size of angle ACB. [4]

(ii) Show that the size of angle DCE is 85.5°, correct to one decimal place.

b. The surveyor measures the size of angle CDE to be twice that of angle DEC. [5]

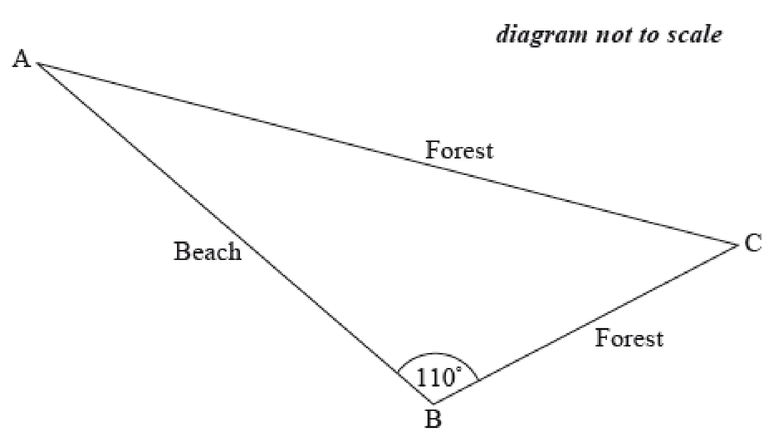
(i) Using angle $\mathrm{DCE} = 85.5^{\circ}$, find the size of angle DEC.

(ii) Find the length of DE.

c. Calculate the area of triangle DEC. [4]

A cross-country running course consists of a beach section and a forest section. Competitors run from A to B, then from B to C and from C back to A.

The running course from A to B is along the beach, while the course from B, through C and back to A, is through the forest.
The course is shown on the following diagram.



Angle ABC is 110°.
It takes Sarah 5 minutes and 20 seconds to run from A to B at a speed of $3.8\ \mathrm{ms}^{-1}$.

a. Using ‘*distance* = *speed* $\times$ *time*’, show that the distance from A to B is 1220 metres correct to 3 significant figures. [2]

b. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [1]

Calculate the speed, in ms^{-1}, that Sarah runs from B to C.

c. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the distance, in metres, from $\mathbf{C}$ **to** $\mathbf{A}$.

d. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [2]

Calculate the total distance, in metres, of the cross-country running course.

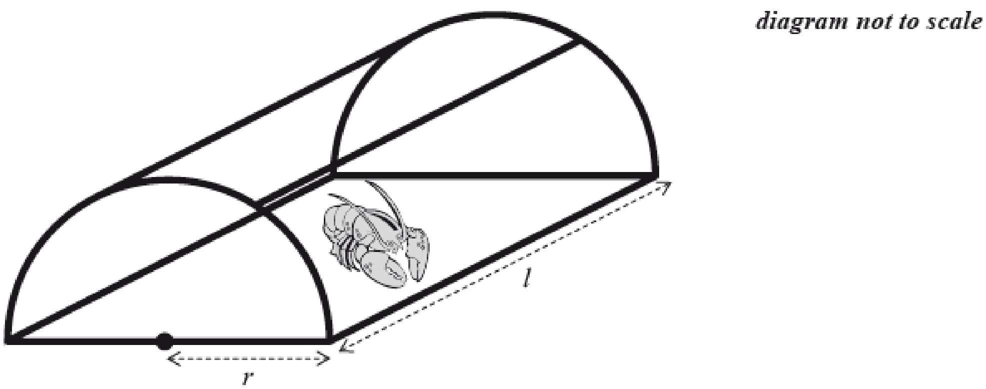
e. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Find the size of angle BCA.

f. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the area of the cross-country course bounded by the lines AB, BC and CA.

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is 0.719 m, correct to three significant figures.

f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

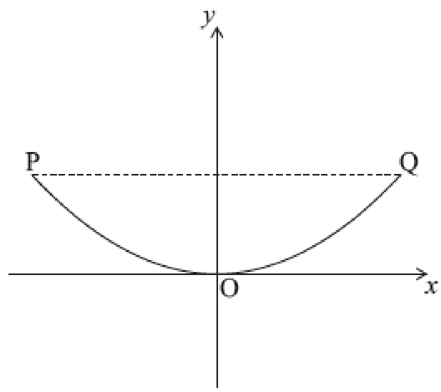
Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



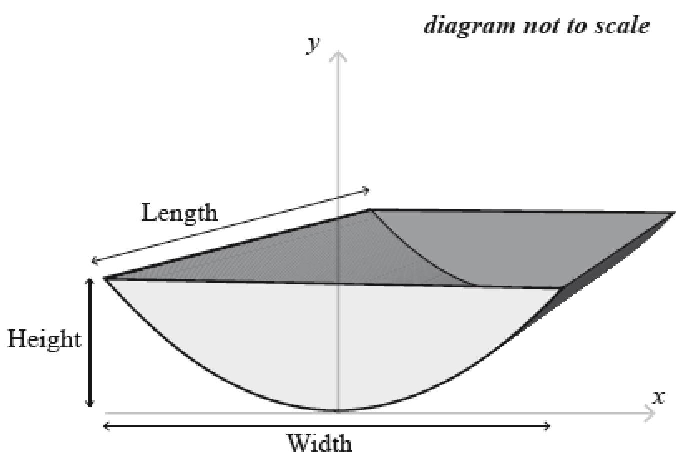
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

a. Write down the value of c. [1]

b. Find the value of a. [2]

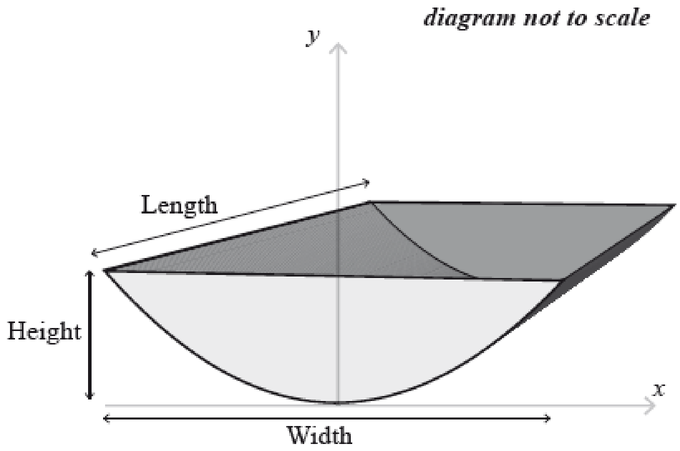
c. Hence write down the equation of the quadratic function which models the edge of the water tank. [1]

d. The water tank is shown below. It is partially filled with water. [2]



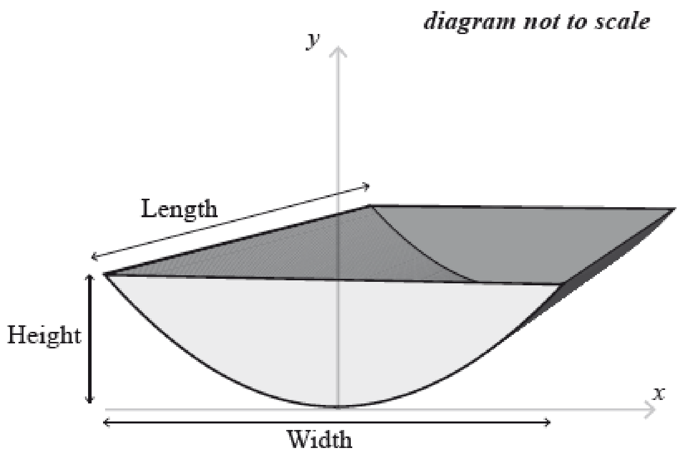
Calculate the value of y when $x = 2.4$ m.

e. The water tank is shown below. It is partially filled with water. [2]



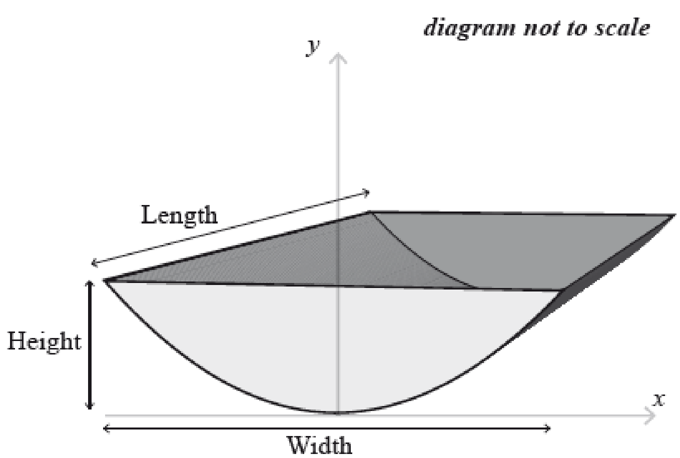
State what the value of x and the value of y represent for this water tank.

f. The water tank is shown below. It is partially filled with water. [2]



Find the value of x when the height of water in the tank is 0.9 m.

g. The water tank is shown below. It is partially filled with water. [2]



The water tank has a length of 5 m.

When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \mathrm{m}^3$.

(ii) Calculate the percentage of water in the tank.

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

The diagram below shows a square based right pyramid. ABCD is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.

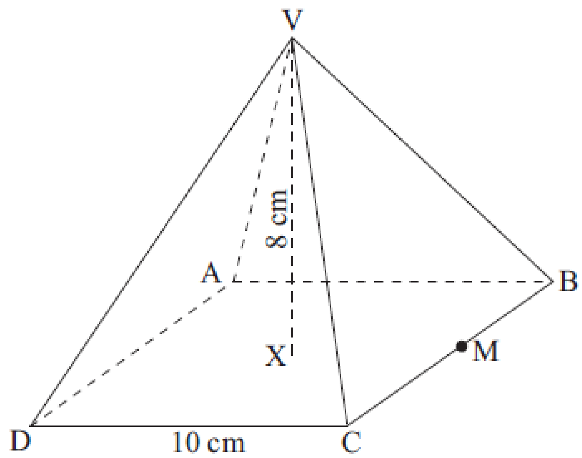


diagram not to scale

In a mountain region there appears to be a relationship between the number of trees growing in the region and the depth of snow in winter. A set of 10 areas was chosen, and in each area the number of trees was counted and the depth of snow measured. The results are given in the table below.

Number of trees (x)	**Depth of snow in cm (y)**
45	30
75	50
66	40
27	25
44	30
28	5
60	35
35	20
73	45
47	25

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115°. A diagram is shown below.

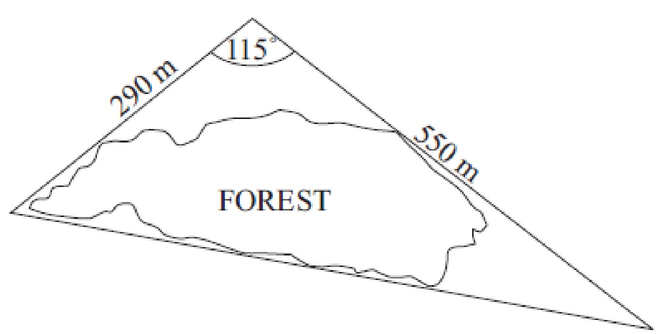


diagram not to scale

A, a Write down the length of XM. [1]

A, a, ii Use your graphic display calculator to find the standard deviation of the number of trees. [1]

A, b Calculate the length of VM. [2]

A, c. Calculate the angle between VM and ABCD. [2]

B, a. Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m. [4]

B, b. Calculate the area enclosed by the path that goes around the forest. [3]

B, d. Inside the forest a second path forms the three sides of another triangle named ABC. Angle BAC is 53°, AC is 180 m and BC is 230 m. [4]

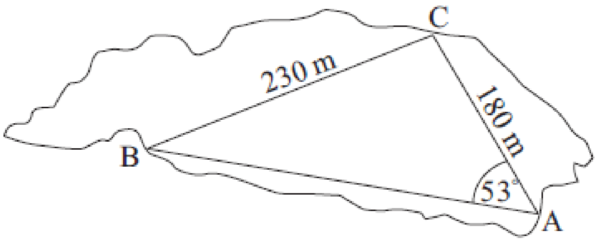


diagram not to scale

Calculate the size of angle ACB.

Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f . [2]

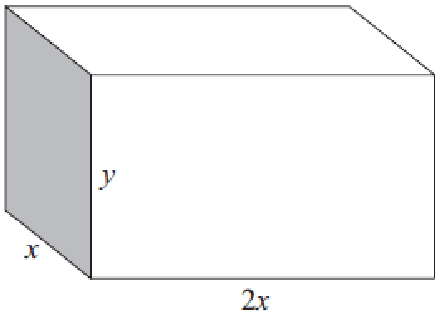
f. Find the range of f . [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x-coordinate of this point.

A shipping container is to be made with six rectangular faces, as shown in the diagram.



The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

a. Show that $y = 12 - 3x$. [3]

b. Show that the volume V m^3 of the container is given by [2]

$V = 24x^2 - 6x^3$

c. Find $\frac{dV}{dx}$. [2]

d. Find the value of x for which V is a maximum. [3]

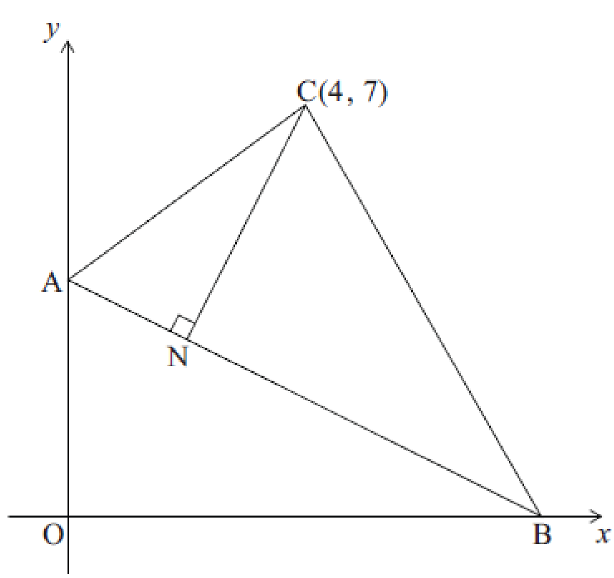
e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2 . Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.



a.i. Find the coordinates of A. [1]

a.ii. Find the coordinates of B. [1]

b. Show that the distance between A and B is 8.94 correct to 3 significant figures. [2]

c.i. N lies on the line AB. The line CN is perpendicular to the line AB. [3]

Find the gradient of CN.

c.ii. N lies on the line AB. The line CN is perpendicular to the line AB. [2]

Find the equation of CN.

d. N lies on the line AB. The line CN is perpendicular to the line AB. [3]

Calculate the coordinates of N.

e. It is known that AC = 5 and BC = 8.06. [3]

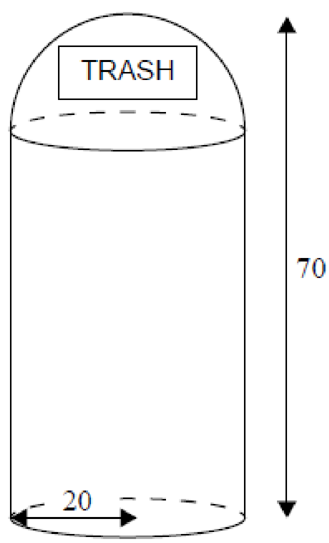
Calculate the size of angle ACB.

f. It is known that AC = 5 and BC = 8.06. [3]

Calculate the area of triangle ACB.

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

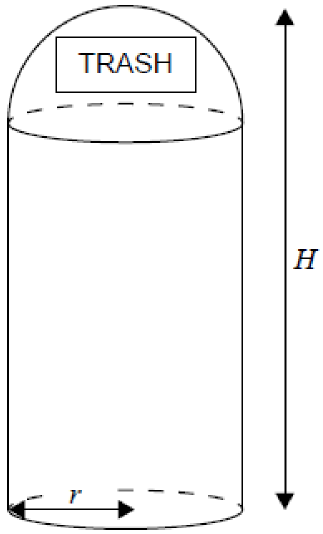


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder. [1]

b. Find the total volume of the trash can. [4]

c. Find the height of the **cylinder**, h , of the new trash can, in terms of r. [2]

d. Show that the volume, V cm^3 , of the new trash can is given by [3]

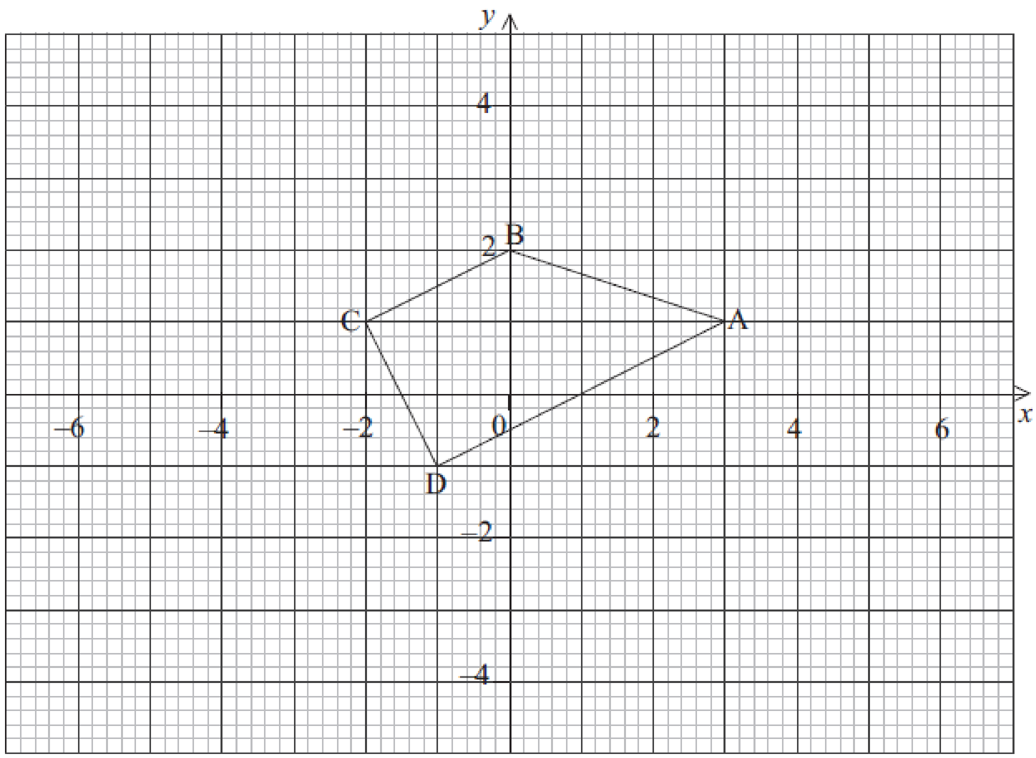
$V = 110\pi r^3$.

e. Using your graphic display calculator, find the value of r which maximizes the value of V. [2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can. [4]

State whether the designer's claim is correct. Justify your answer.

The vertices of quadrilateral ABCD as shown in the diagram are A (3, 1), B (0, 2), C (–2, 1) and D (–1, –1).



a. Calculate the gradient of line CD. [2]

b. Show that line AD is perpendicular to line CD. [2]

c. Find the equation of line CD. Give your answer in the form $ax + by = c$ where $a,\ b,\ c \in \mathbb{Z}$. [3]

d. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. [2]

Find the coordinates of E.

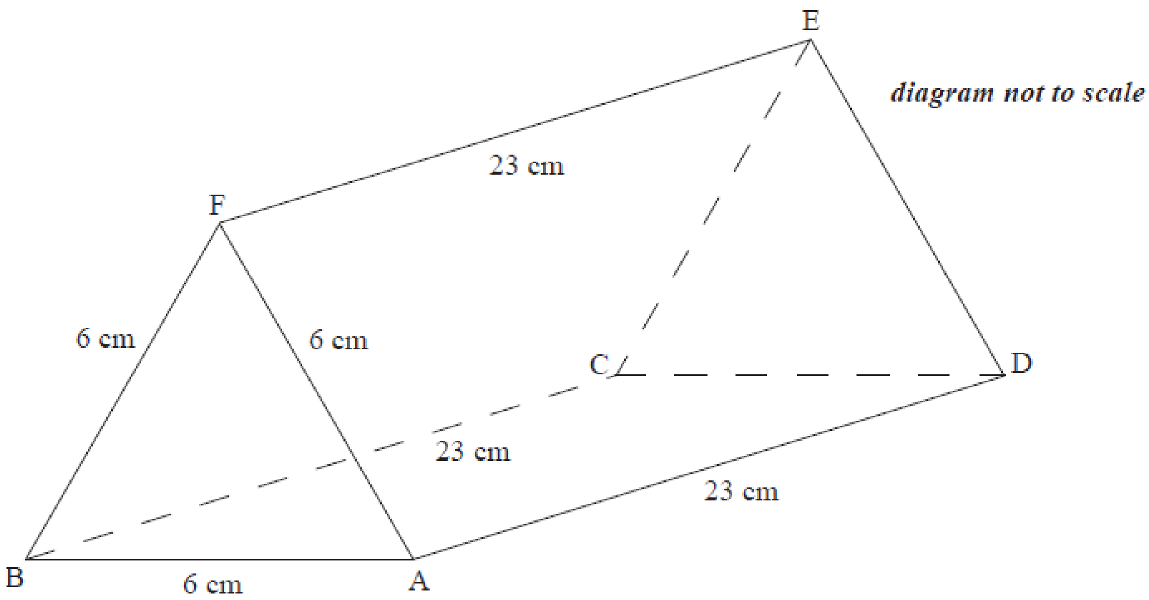
e. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. [2]

Find the distance between A and D.

f. The distance between D and E is $\sqrt{20}$. [2]

Find the area of triangle ADE.

A chocolate bar has the shape of a triangular right prism ABCDEF as shown in the diagram. The ends are equilateral triangles of side 6 cm and the length of the chocolate bar is 23 cm.



a, i.Write down the size of angle BAF. [1]

a, ii.Hence or otherwise find the area of the triangular end of the chocolate bar. [3]

b. Find the total surface area of the chocolate bar. [3]

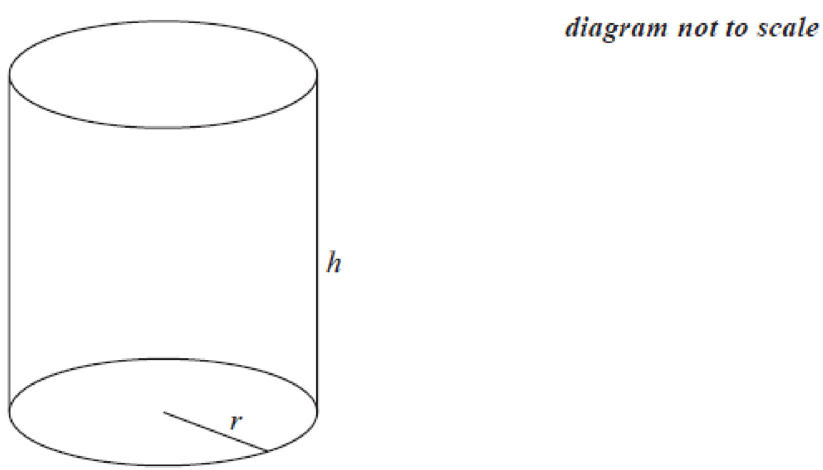
c. It is known that 1 cm^3 of this chocolate weighs 1.5 g. Calculate the weight of the chocolate bar. [3]

d. A different chocolate bar made with the same mixture also has the shape of a triangular prism. The ends are triangles with sides of length 4 cm, 6 cm and 7 cm. [3]

Show that the size of the angle between the sides of 6 cm and 4 cm is 86.4° correct to 3 significant figures.

e. The weight of this chocolate bar is 500 g. Find its length. [4]

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.



Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of $8000\ \text{cm}^3$. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

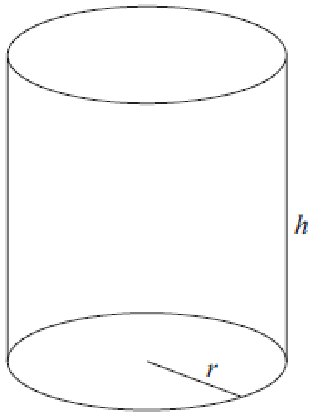


diagram not to scale

Let the radius of the base of Merryn's wastepaper bin be r , and let its height be h .

a. Calculate [7]

(i) the area of the base of the wastepaper bin;

(ii) the height, h , of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason. [2]

c. Write down an equation in h and r , using the given volume of the bin. [1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2]

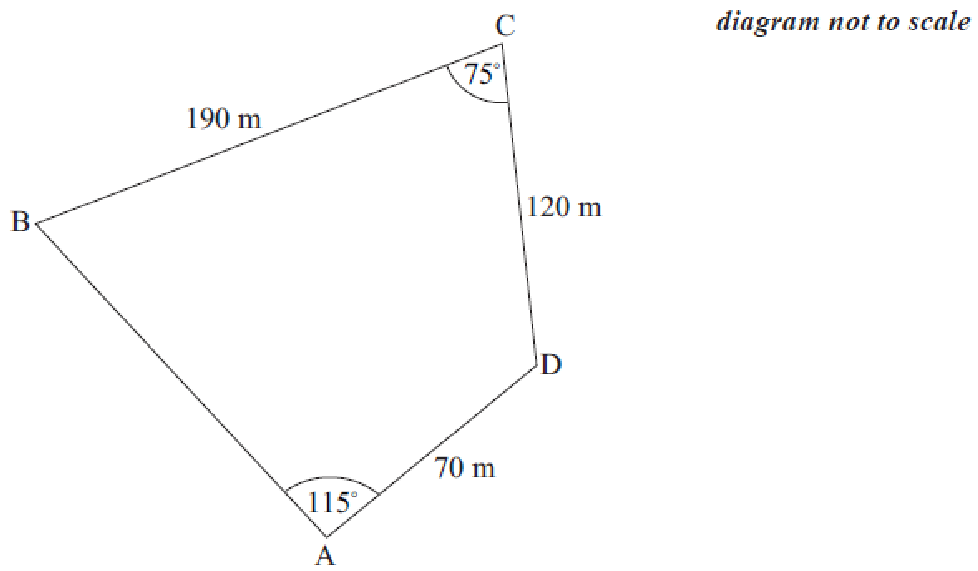
e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. [5]

(ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. [2]

Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is $190\ \text{m}$, the length of CD is $120\ \text{m}$, the length of AD is $70\ \text{m}$, the size of angle BCD is 75° and the size of angle BAD is 115° .



Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

a. Calculate the length of the fence. [3]

b. The fence costs 17 USD per metre to build. [2]

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

c. Show that the size of angle ABD is 18.8° , correct to three significant figures. [3]

d. Calculate the area of triangle ABD . [4]

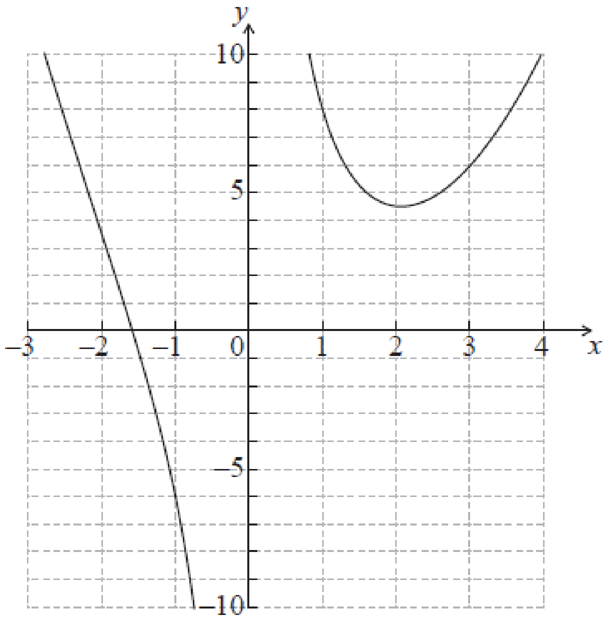
e. She sells the land for 120 USD per square metre. [2]

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

f. Pauline invests 300000 USD from the sale of the land in a bank that pays compound interest compounded annually. [4]

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3-2x^2-9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

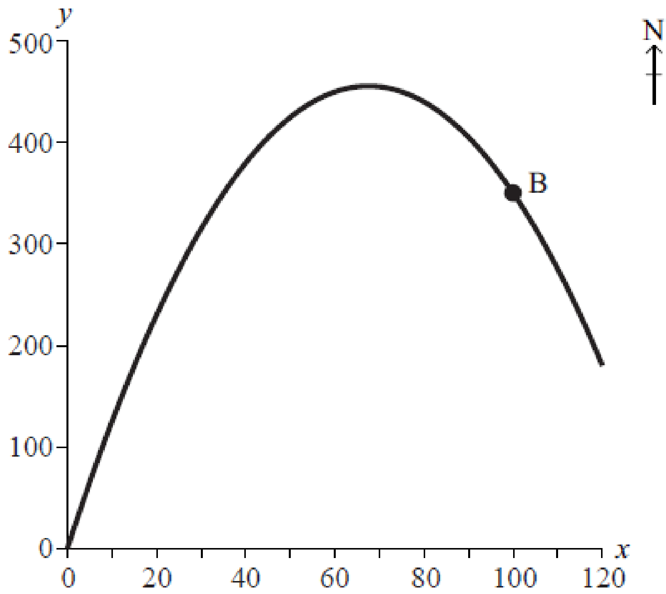
g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the **x-coordinate** of B and of C .

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where $x \geqslant 0,\ y \geqslant 0$

(x , y) are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. [3]

b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. [2]

c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. [4]

d. (i) Write down the midpoint of the line segment OB. [3]

(ii) Find the gradient of the line segment OB.

e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. [3]

Find the equation of Scott's road. Express your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{R}$.

f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. [2]

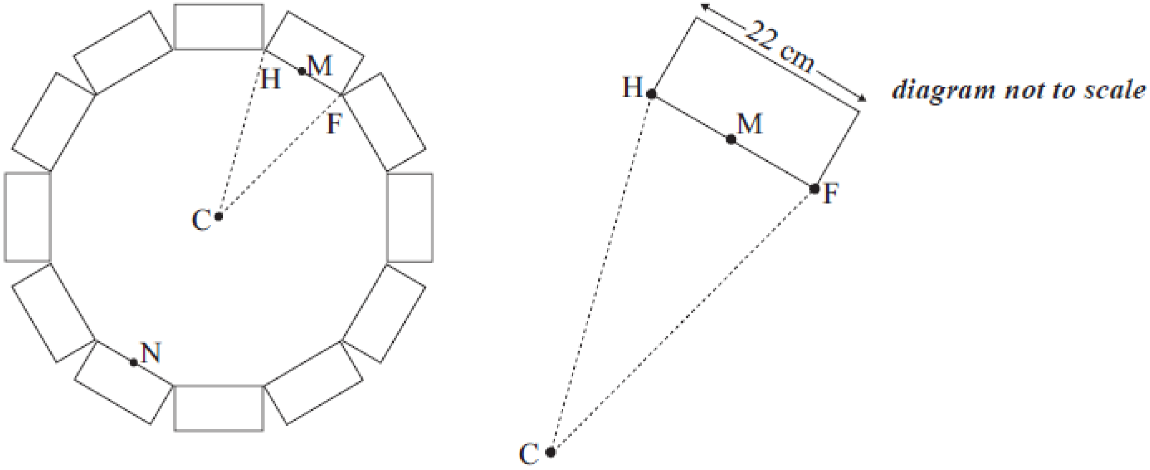
A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.

The gardener decides to have a triangular lawn ABC, instead of paving, in the middle of the rectangular area, as shown in the diagram below.

onbekend.png

The distance AB is 4 metres, AC is 6 metres and angle BAC is 40°.

In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.



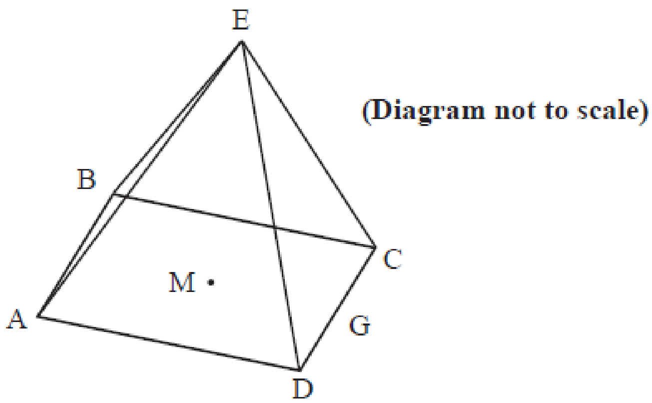
The garden bed has an area of 5419 cm^2. It is covered with soil to a depth of 2.5 cm.

It is estimated that 1 kilogram of soil occupies 514 cm^3.

a.i.	Calculate the total area to be paved. Give your answer in cm^2.	[3]
a.ii.	Write down the area of each brick.	[1]
a.iii	Find how many bricks are required to pave the total area.	[2]
b.i.	Find the length of BC.	[3]
b.ii.	Hence write down the perimeter of the triangular lawn.	[1]
b.iii	Calculate the area of the lawn.	[2]
b.iv	Find the percentage of the rectangular area which is to be lawn.	[3]
c.i.	Find the angle FCH.	[2]
c.ii.	Calculate the distance MN from one side of the garden bed to the other, passing through C.	[3]
d.	Find the volume of soil used.	[2]
e.	Find the number of kilograms of soil required for this garden bed.	[2]

Mal is shopping for a school trip. He buys 50 tins of beans and 20 packets of cereal. The total cost is 260 Australian dollars (AUD).

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



i.a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1]

i.b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \mathrm{AUD}$. [1]

Write down another equation to represent this information.

i.c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \mathrm{AUD}$. [2]

Find the cost of one tin of beans.

i.d. (i) Sketch the graphs of the two equations from parts (a) and (b). [4]

(ii) Write down the coordinates of the point of intersection of the two graphs.

ii.a. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1]

ii.b. Show that the height of the pyramid is $13.7\ \mathrm{cm}$, to 3 significant figures. [2]

ii.c. Calculate [4]

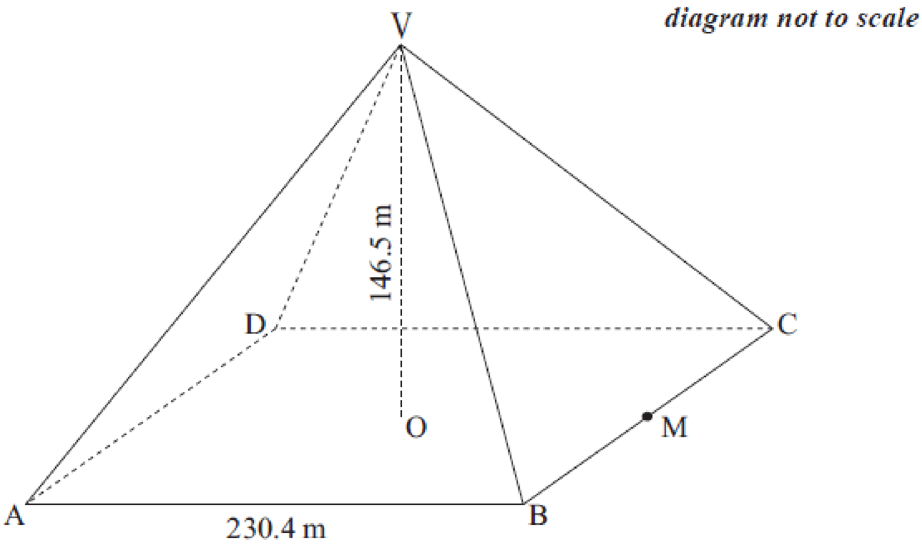
(i) the length of EG;

(ii) the size of angle DEC.

ii.d. Find the total surface area of the pyramid. [2]

ii.e. Find the volume of the pyramid. [2]

The Great Pyramid of Cheops in Egypt is a square based pyramid. The base of the pyramid is a square of side length 230.4 m and the vertical height is 146.5 m. The Great Pyramid is represented in the diagram below as ABCDV . The vertex V is directly above the centre O of the base. M is the midpoint of BC.



a. (i) Write down the length of OM . [3]

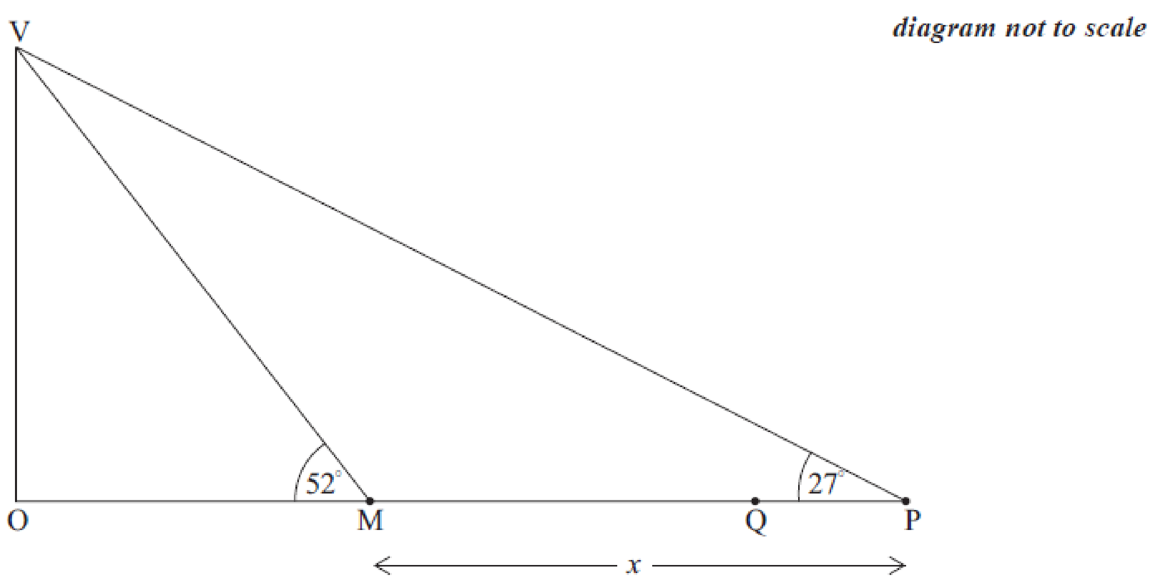
(ii) Find the length of VM .

b. Find the area of triangle VBC . [2]

c. Calculate the volume of the pyramid. [2]

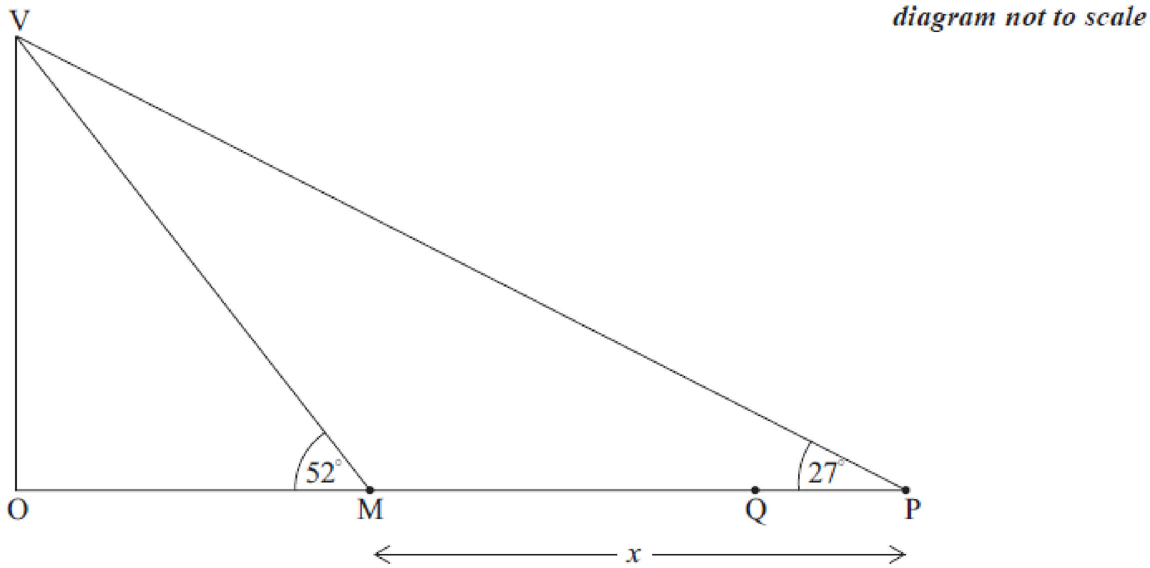
d. Show that the angle between the line VM and the base of the pyramid is 52° correct to 2 significant figures. [2]

e. Ahmed is at point P , a distance *x* metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [1]



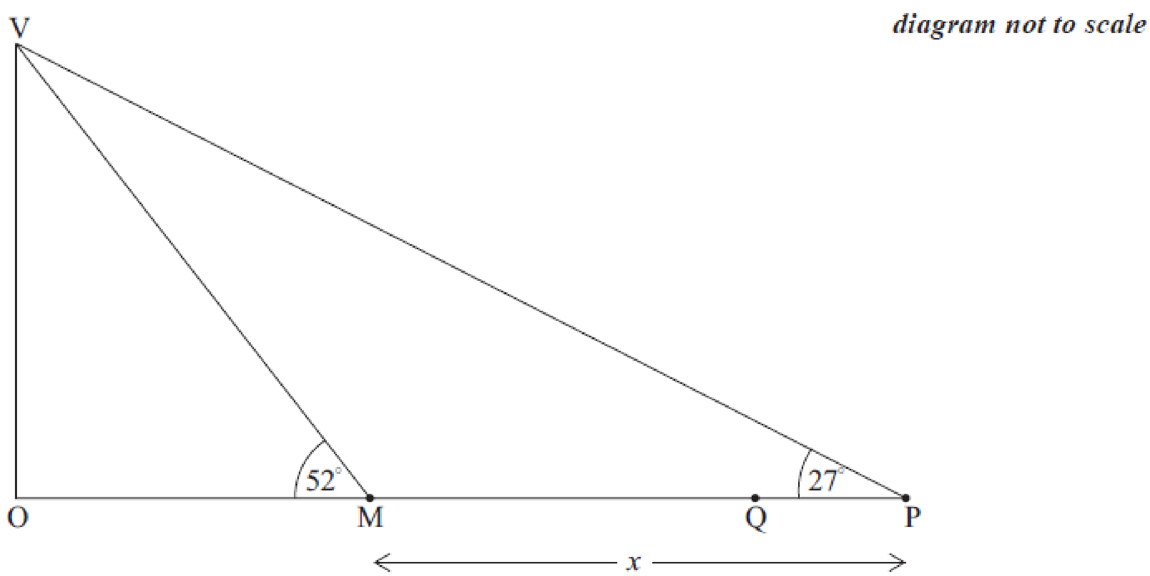
Write down the size of angle VMP .

f. Ahmed is at point P , a distance *x* metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [4]



Using your value of VM from part (a)(ii), find the value of x.

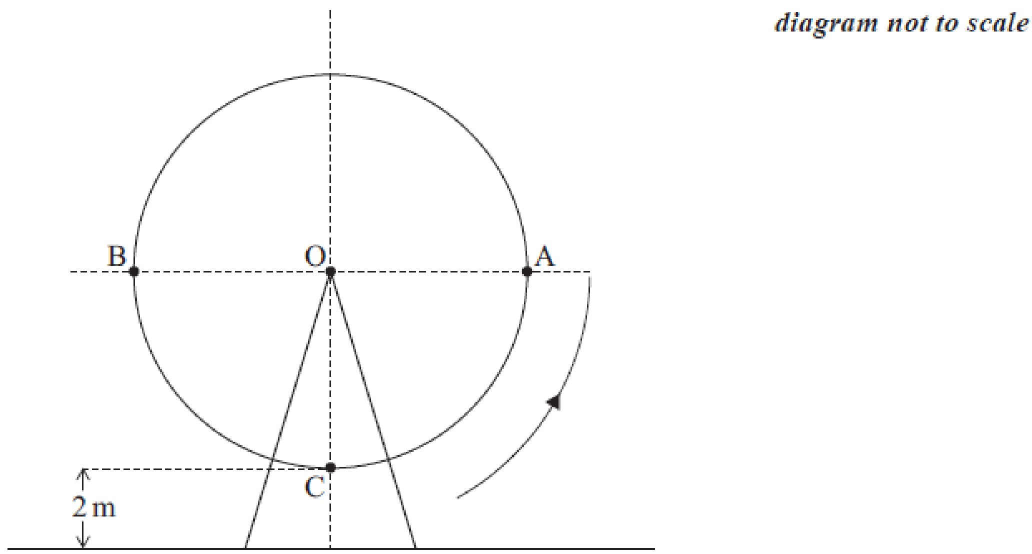
g. Ahmed is at point P , a distance x metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [4]



Ahmed walks 50 m from P to Q.

Find the length of QV, the distance from Ahmed to the vertex of the pyramid.

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.



a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. [2]

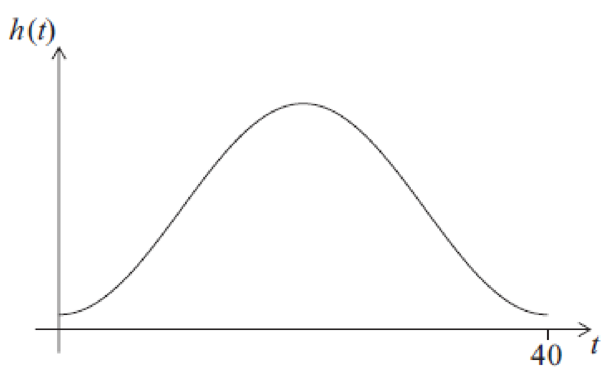
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

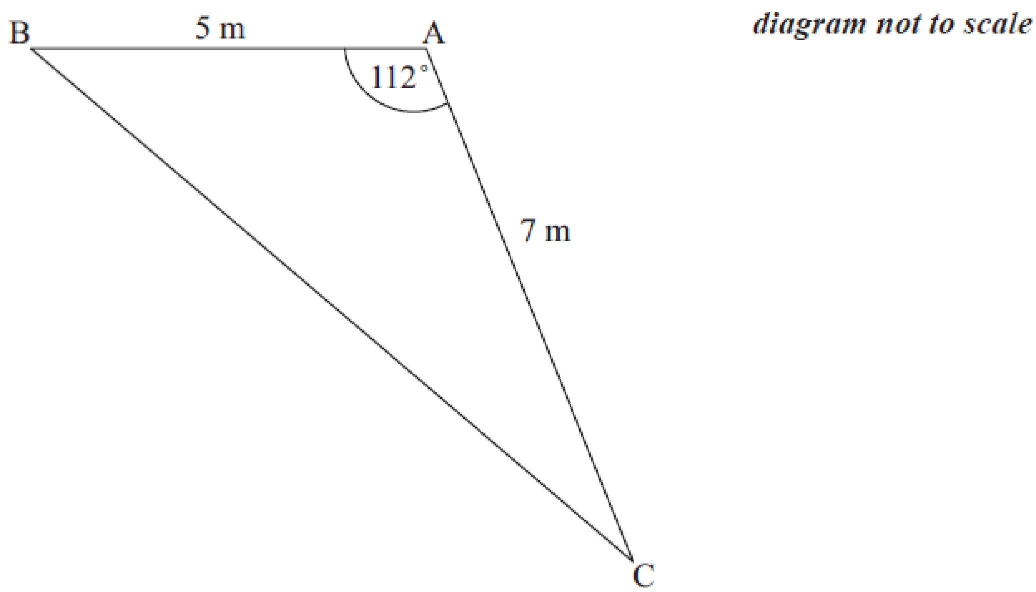
b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2]

c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. [4]



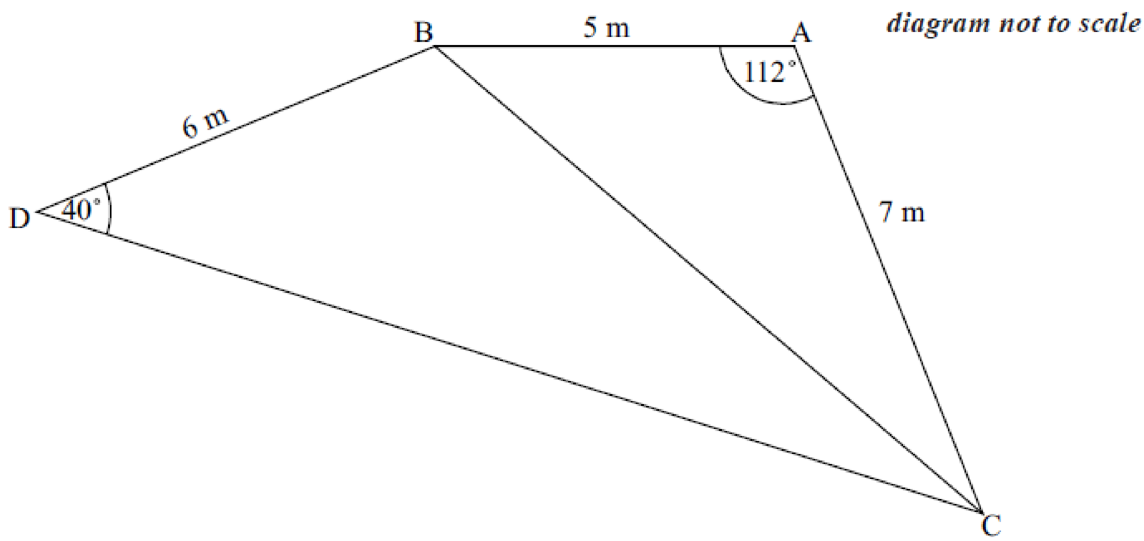
Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

A contractor is building a house. He first marks out three points A , B and C on the ground such that AB = 5 m , AC = 7 m and angle BAC = 112 °.



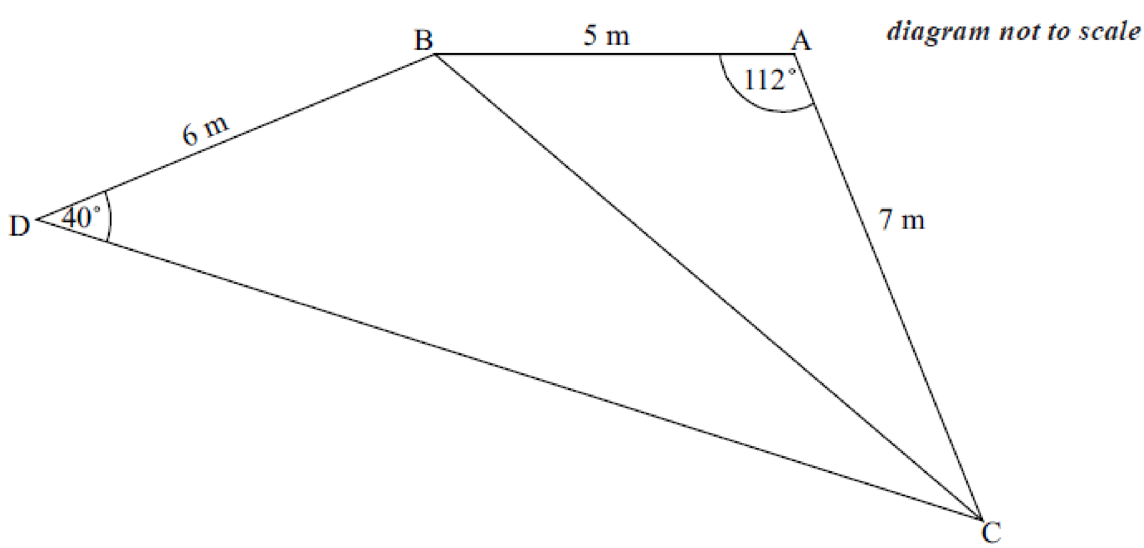
a. Find the length of BC. [3]

b. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



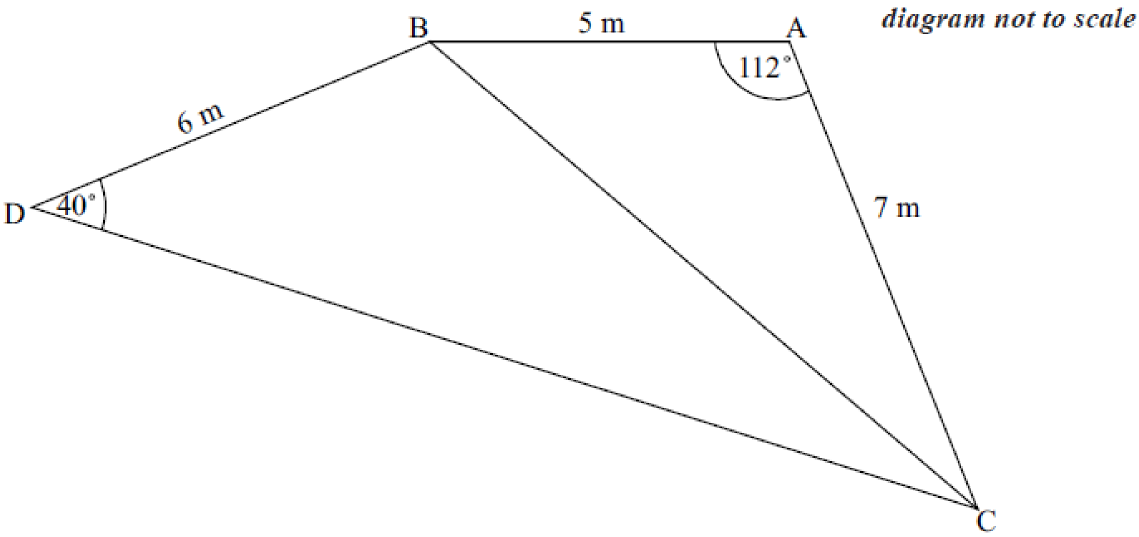
Find the size of angle DBC .

c. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



Find the area of the quadrilateral ABDC.

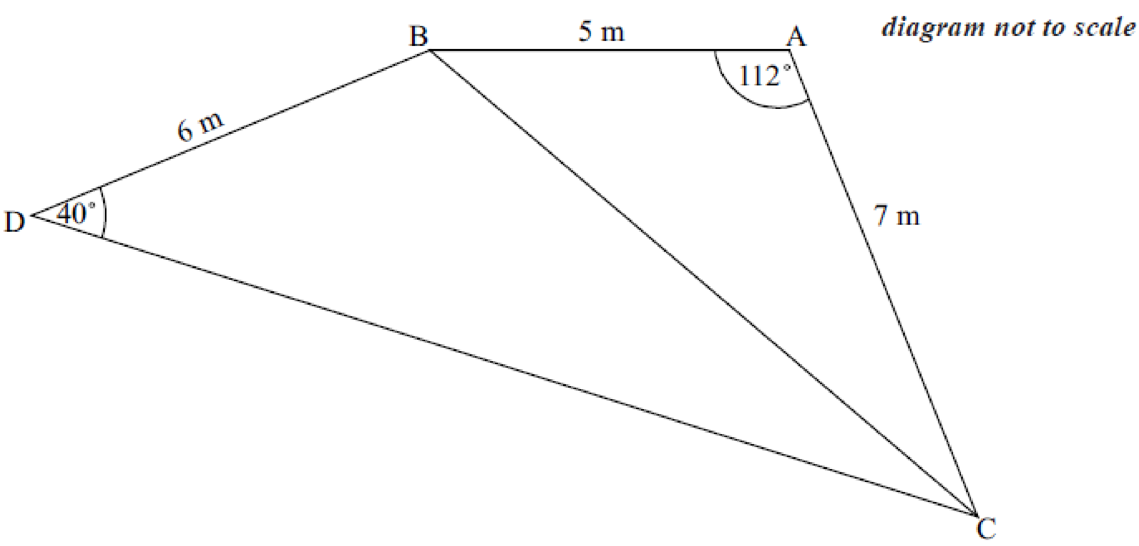
d. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [3]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

Find the volume of the soil removed. Give your answer in $\mathbf{m^3}$.

e. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [5]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm.

(i) Find the volume of a drum. Give your answer in $\mathbf{m^3}$.

(ii) Find the minimum number of drums required to transport the soil removed.

Francesca is a chef in a restaurant. She cooks eight chickens and records their masses and cooking times. The mass m of each chicken, in kg, and its cooking time t, in minutes, are shown in the following table.

Mass m (kg)	**Cooking time t (minutes)**
1.5	62
1.6	75
1.8	82
1.9	83
2.0	86
2.1	87
2.1	91
2.3	98

a. Draw a scatter diagram to show the relationship between the mass of a chicken and its cooking time. Use 2 cm to represent 0.5 kg on the horizontal axis and 1 cm to represent 10 minutes on the vertical axis. [4]

b. Write down for this set of data [2]

(i) the mean mass, $\bar{m}$;

(ii) the mean cooking time, $\bar{t}$.

c. Label the point $\mathrm{M}(\bar{m}, \bar{t})$ on the scatter diagram. [1]

d. Draw the line of best fit on the scatter diagram. [2]

e. Using your line of best fit, estimate the cooking time, in minutes, for a 1.7 kg chicken. [2]

f. Write down the Pearson's product–moment correlation coefficient, r . [2]

g. Using your value for r , comment on the correlation. [2]

h. The cooking time of an additional 2.0 kg chicken is recorded. If the mass and cooking time of this chicken is included in the data, the correlation is weak. [2]

(i) Explain how the cooking time of this additional chicken might differ from that of the other eight chickens.

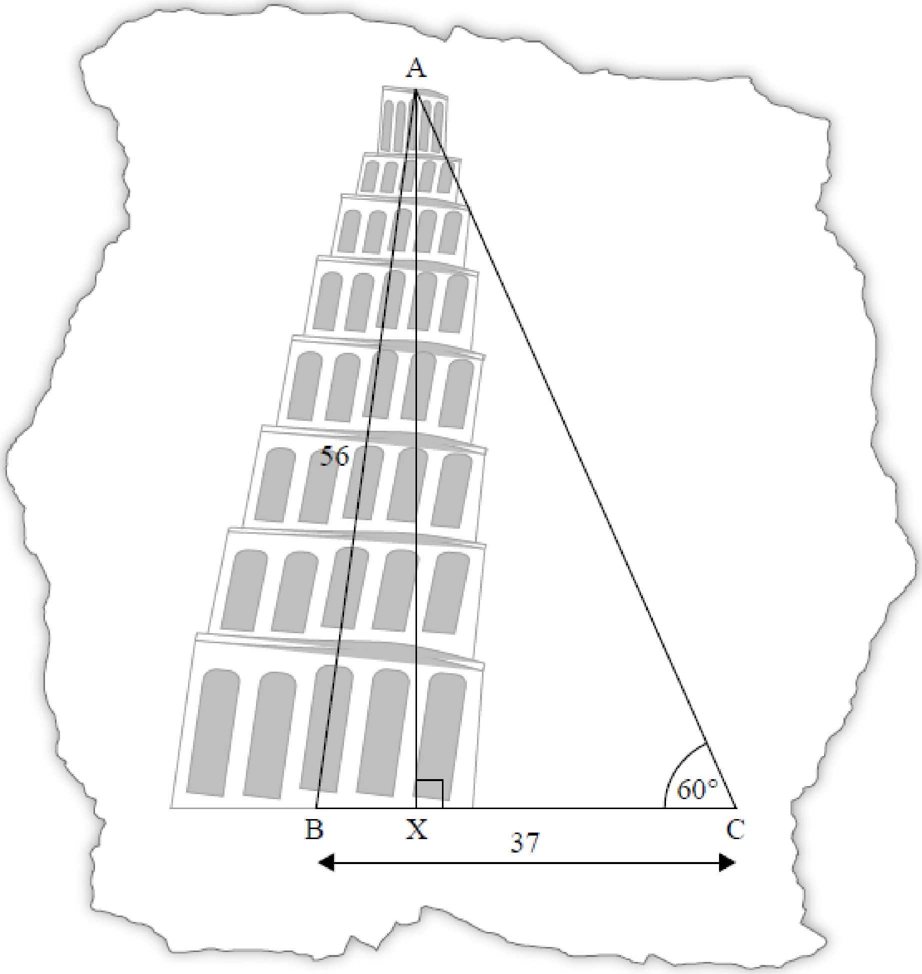
(ii) Explain how a new line of best fit might differ from that drawn in part (d).

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



Giovanni’s tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

a.i. Use Giovanni’s diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. [5]

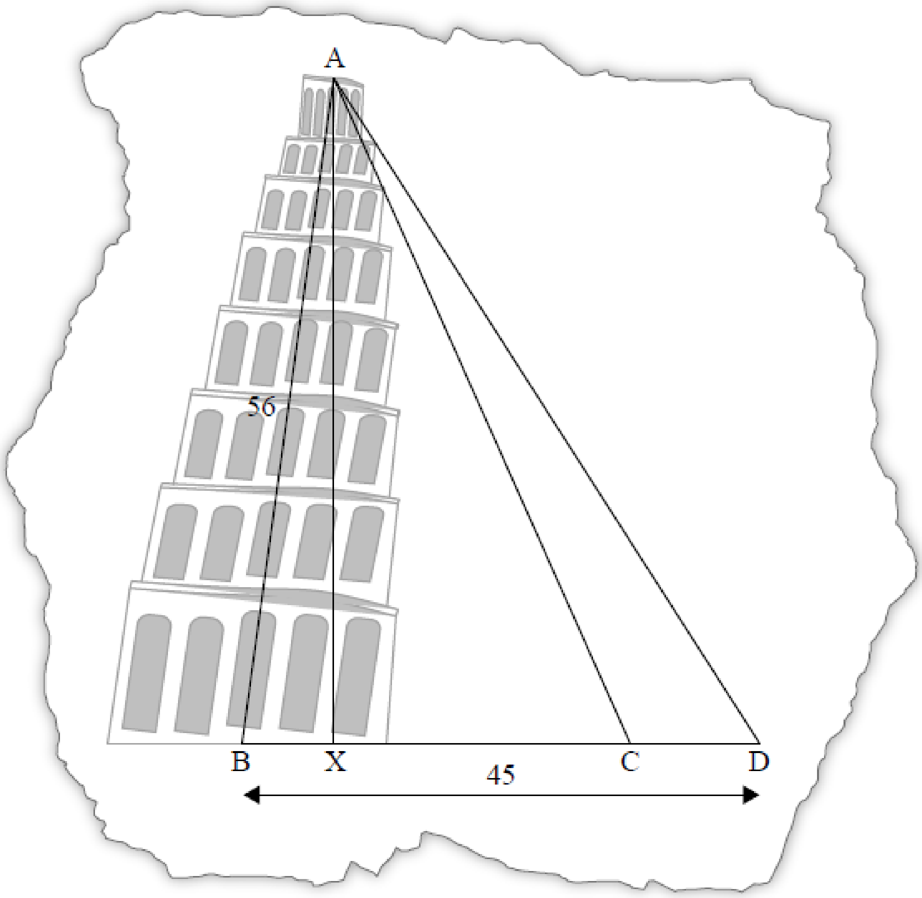
a.ii. Use Giovanni's diagram to calculate the length of AX. [2]

a.iii Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. [2]

b. Find the percentage error on Giovanni’s diagram. [2]

c. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. [3]

diagram not to scale



Find the angle of elevation of A from D.