SL Paper 1

A liquid is heated so that after 20 seconds of heating its temperature, T , is 25°C and after 50 seconds of heating its temperature is 37°C .

The temperature of the liquid at time t can be modelled by $T = at + b$, where t is the time in seconds after the start of heating.
Using this model one equation that can be formed is $20a + b = 25$.

a. Using the model, write down a second equation in a and b . [2]

b. Using your graphic display calculator or otherwise, find the value of a and of b . [2]

c. Use the model to predict the temperature of the liquid 60 seconds after the start of heating. [2]

Markscheme

a. $50a + b = 37$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $50a + b$, ***(A1)*** for $= 37$.

b. $a = 0.4, b = 17$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Notes: Award ***(M1)*** for attempt to solve their equations if this is done analytically. If the GDC is used, award **(ft)** even if no working seen.

c. $T = 0.4(60) + 17$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their values and 60 into equation for T.

$T = 41\ (^\circ\text{C})$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (b).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A plumber in Australia charges 90 AUD per hour for work, plus a fixed cost. His total charge is represented by the cost function $C = 60 + 90t$, where t is in hours.

a. Write down the fixed cost. [1]

b. It takes $3\frac{1}{2}$ hours to complete a job for Paula. Find the total cost. [2]

c. Steve received a bill for 510 AUD. Calculate the time it took the plumber to complete the job. [3]

Markscheme

a. AUD 60 ***(A1)*** ***(C1)***

[1 mark]

b. C = 60 + 90(3.5) = AUD 375 ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for correct substitution of 3.5.

[2 marks]

c.

Note: Unit penalty (UP) applies in this part

510 = 60 + 90t ***(M1)(A1)***

(UP) t = 5h (hours, hrs) ***(A1)*** ***(C3)***

Note: Award ***(M1)*** for setting formula = to any number.

(A1) for 510 seen.

[3 marks]

Examiners report

a. The majority of candidates answered this question correctly. Some gave an answer of 150 AUD.

b. The majority of candidates answered this question correctly.

c. The majority of candidates answered this question correctly. Very few lost a unit penalty mark in this part.

The straight line, L, has equation $2y - 27x - 9 = 0$.

a. Find the gradient of L. [2]

b. Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L. [1]

Write down $f'(x)$.

c, i. Find the x coordinate of the point at which the tangent must be drawn. [2]

c, ii. Write down the value of $f(x)$ at this point. [1]

Markscheme

a. $y = 13.5x + 4.5$ ***(M1)***

Note: Award ***(M1)*** for $13.5x$ seen.

gradient = 13.5 ***(A1)*** ***(C2)***

[2 marks]

b. $4x^3$ ***(A1)*** ***(C1)***

[1 mark]

c, i. $4x^3 = 13.5$ ***(M1)***

Note: Award ***(M1)*** for equating their answers to (a) and (b).

$x = 1.5$ ***(A1)*(ft)**

[2 marks]

c, ii. $\frac{81}{16}$ (5.0625, 5.06) ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for substitution of their (c)(i) into $x4$ with working seen.

[1 mark]

Examiners report

a. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Most were successful in this part.

b. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

This part was usually well attempted.

c, i. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

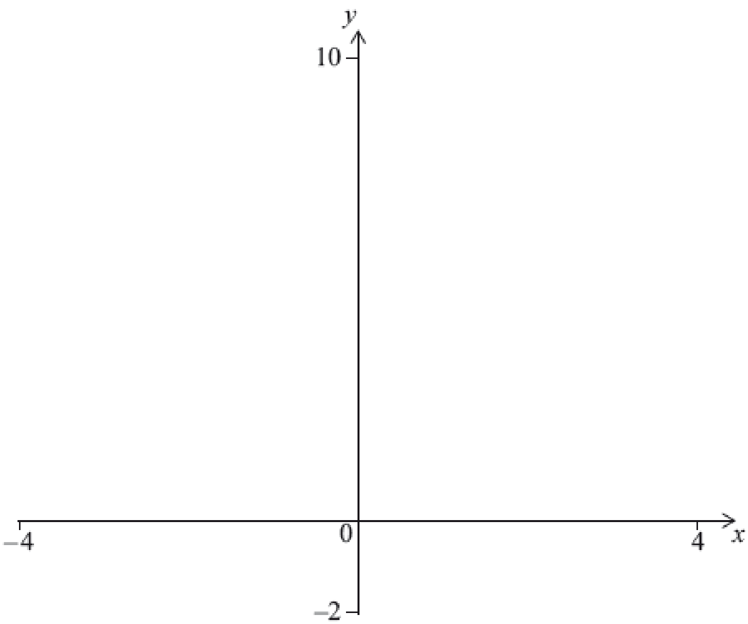
c, ii. The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

Only the best candidates succeeded in this part.

Consider the two functions, f and g, where

$f(x) = \frac{5}{x^2+1}$
$g(x) = (x-2)^2$

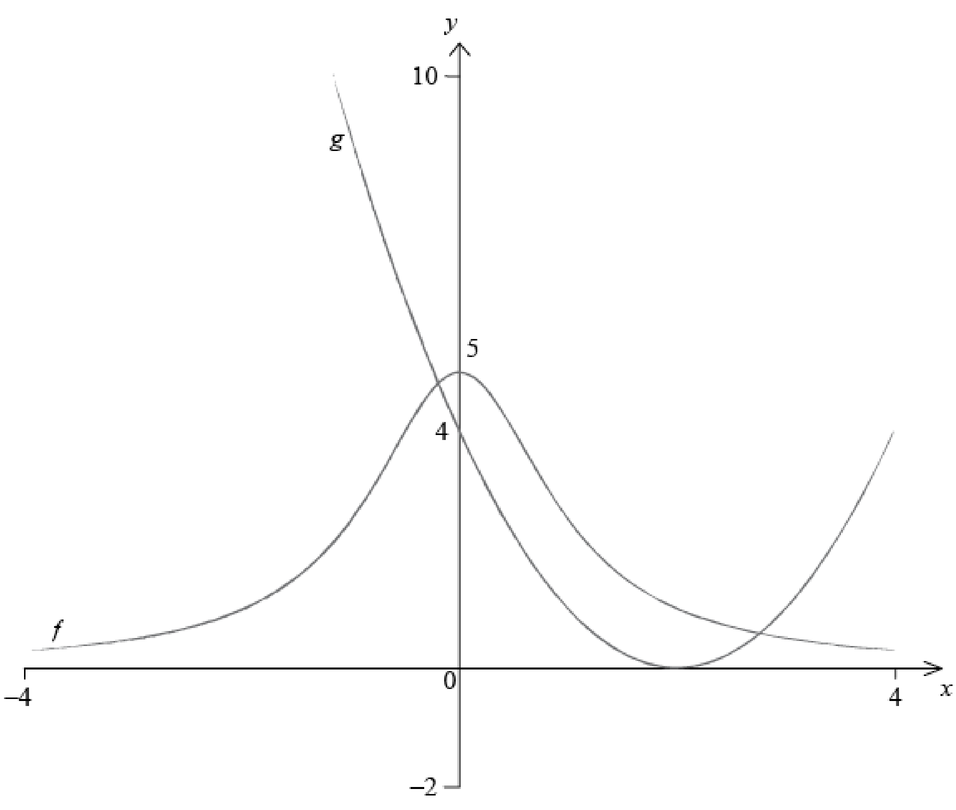
a. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Indicate clearly the points where each graph intersects the y-axis. [4]



b. Use your graphic display calculator to solve $f(x) = g(x)$. [2]

Markscheme

a.



$f(x)$: a smooth curve symmetrical about *y*-axis, $f(x) > 0$ ***(A1)***

Note: If the graph crosses the *x*-axis award ***(A0)***.

Intercept at their numbered $y = 5$ ***(A1)***

Note: Accept clear scale marks instead of a number.

$g(x)$: a smooth parabola with axis of symmetry at about $x = 2$ (the 2 does not need to be numbered) and $g(x) \geqslant 0$ ***(A1)***

Note: Right hand side must not be higher than the maximum of $f(x)$ at $x = 4$.

Accept the quadratic correctly drawn beyond $x = 4$.

Intercept at their numbered $y = 4$ ***(A1)*** ***(C4)***

Note: Accept clear scale marks instead of a number.

[4 marks]

b. $-0.195, 2.76$ $(-0.194808\ldots, 2.761377\ldots)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Award ***(A0)(A1)*(ft)** if both coordinates are given.

Follow through only if $f(x) = \frac{5}{x^2} + 1$ is sketched; the solutions are $-0.841, 3.22$ $(-0.840913\ldots, 3.217747...)$

[2 marks]

Examiners report

a. Many candidates attempted this question but relatively few were awarded the full six marks. Although they were asked to indicate clearly where the graph met the axes, many did not do this. Some entered the functions incorrectly into their calculator. A common error in part (b) was to give ordered pairs and therefore were not awarded the final mark.

b. Many candidates attempted this question but relatively few were awarded the full six marks. Although they were asked to indicate clearly where the graph met the axes, many did not do this. Some entered the functions incorrectly into their calculator. A common error in part (b) was to give ordered pairs and therefore were not awarded the final mark.

A quadratic function f is given by $f(x) = ax^2 + bx + c$. The points $(0,\ 5)$ and $(-4,\ 5)$ lie on the graph of $y = f(x)$.

The y-coordinate of the minimum of the graph is 3.

a. Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

b. Write down the value of c. [1]

c. Find the value of a and of b. [3]

Markscheme

a. $x = -2$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$ (a constant) and ***(A1)*** for -2.

[2 marks]

b. $(c =)\ 5$ ***(A1)*** ***(C1)***

[1 mark]

c. $-\frac{b}{2a} = -2$

$a(-2)^2 - 2b + 5 = 3$ or equivalent

$a(-4)^2 - 4b + 5 = 5$ or equivalent

$2a(-2) + b = 0$ or equivalent ***(M1)***

Note: Award ***(M1)*** for two of the above equations.

$a = 0.5$ ***(A1)*(ft)**

$b = 2$ ***(A1)*(ft)** ***(C3)***

Note: Award at most ***(M1)(A1)*(ft)*(A0)*** if the answers are reversed.

Follow through from parts (a) and (b).

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

Maria owns a cheese factory. The amount of cheese, in kilograms, Maria sells in one week, Q, is given by

$$Q = 882 - 45p,$$

where p is the price of a kilogram of cheese in euros (EUR).

Maria earns $(p-6.80)$ EUR for each kilogram of cheese sold.

To calculate her weekly profit W, in EUR, Maria multiplies the amount of cheese she sells by the amount she earns per kilogram.

a. Write down how many kilograms of cheese Maria sells in one week if the price of a kilogram of cheese is 8 EUR. [1]

b. Find how much Maria earns in one week, from selling cheese, if the price of a kilogram of cheese is 8 EUR. [2]

c. Write down an expression for W in terms of p. [1]

d. Find the price, p, that will give Maria the highest weekly profit. [2]

Markscheme

a. 522 (kg) ***(A1)*** ***(C1)***

[1 mark]

b. $522(8-6.80)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for multiplying their answer to part (a) by $(8-6.80)$.

626 (EUR) (626.40) ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

c. $(W=)\ (882-45p)(p-6.80)$ ***(A1)***

OR

$(W=)-45p^2+1188p-5997.6$ ***(A1)*** ***(C1)***

[1 mark]

d. sketch of W with some indication of the maximum ***(M1)***

OR

$-90p+1188=0$ ***(M1)***

Note: Award ***(M1)*** for equating the correct derivative of their part (c) to zero.

OR

$(p=)\ \frac{-1188}{2\times(-45)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the formula for axis of symmetry.

$(p =)\ 13.2\ (\mathrm{EUR})$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their part (c), if the value of p is such that $6.80 < p < 19.6$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.2 \times (0.87)^t,\ t \geqslant 0$$

where D is the amount of the drug in the bloodstream in mg per litre $(\mathrm{mg\,l^{-1}})$ and t is the time in hours.

a. Write down the amount of the drug in the bloodstream at $t = 0$. [1]

b. Calculate the amount of the drug in the bloodstream after 3 hours. [2]

c. Use your graphic display calculator to determine the time it takes for the amount of the drug in the bloodstream to decrease to $0.333\ \mathrm{mg l^{-1}}$. [3]

Markscheme

a. $1.2\ (\mathrm{mg\,l^{-1}})$ ***(A1)*** ***(C1)***

[1 mark]

b. $1.2 \times (0.87)^3$ ***(M1)***

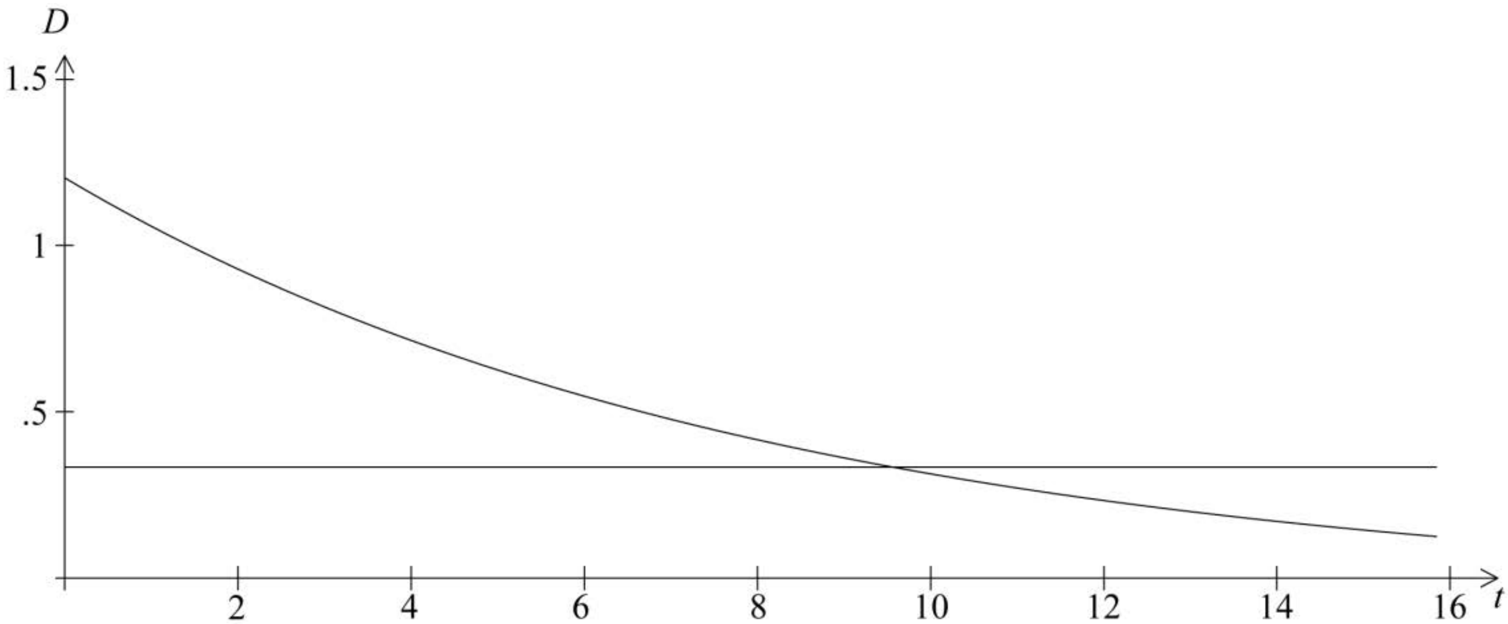
Note: Award ***(M1)*** for correct substitution into given formula.

$= 0.790\ (\mathrm{mg\,l^{-1}})\ (0.790203\ldots)$ ***(A1)*** ***(C2)***

[2 marks]

c. $1.2 \times 0.87^t = 0.333$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation.



(M1)

Notes: Some indication of scale is to be shown, for example the window used on the calculator.

Accept alternative methods.

9.21 (hours) ($9.20519\ldots$, 9 hours 12 minutes, 9:12) ***(A1) (C3)***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The number of bacteria in a colony is modelled by the function

$$N(t) = 800 \times 3^{0.5t},\ t \geqslant 0,$$

where N is the number of bacteria and t is the time in hours.

a. Write down the number of bacteria in the colony at time $t = 0$. [1]

b. Calculate the number of bacteria present at 2 hours and 30 minutes. Give your answer correct to the nearest hundred bacteria. [3]

c. Calculate the time, in hours, for the number of bacteria to reach 5500. [2]

Markscheme

a. 800 ***(A1) (C1)***

b. $800 \times 3^{(0.5 \times 2.5)}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted formula.

$= 3158.57...$ ***(A1)***

$= 3200$ ***(A1)*** ***(C3)***

Notes: Final ***(A1)*** is given for correctly rounding ***their*** answer. This may be awarded regardless of a preceding ***(A0)***.

c. $5500 = 800 \times 3^{(0.5 \times t)}$ ***(M1)***

Notes: Award ***(M1)*** for equating function to 5500. Accept correct alternative methods.

$= 3.51$ hours (3.50968...) ***(A1)*** ***(C2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

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a. A company sells fruit juices in cylindrical cans, each of which has a volume of $340\,\mathrm{cm}^3$. The surface area of a can is $A\,\mathrm{cm}^2$ and is given by the formula [3]

$A = 2\pi r^2 + \frac{680}{r}$,

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

Find $\frac{\mathrm{d}A}{\mathrm{d}r}$

b. Calculate the value of r that minimizes the surface area of a can. [3]

Markscheme

a. $\left(\frac{\mathrm{d}A}{\mathrm{d}r}\right) = 4\pi r - \frac{680}{r^2}$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $4\pi r$ (accept $12.6r$), ***(A1)*** for -680, ***(A1)*** for $\frac{1}{r^2}$ or r^{-2}

Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

b. $4\pi r - \frac{680}{r^2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their $\frac{\mathrm{d}A}{\mathrm{d}r}$ to zero.

$4\pi r^3 - 680 = 0$ ***(M1)***

Note: Award ***(M1)*** for initial correct rearrangement of the equation. This may be assumed if $r^3 = \frac{680}{4\pi}$ or $r = \sqrt[3]{\frac{680}{4\pi}}$ seen.

OR

sketch of A with some indication of minimum point ***(M1)(M1)***

Note: Award ***(M1)*** for sketch of A, ***(M1)*** for indication of minimum point.

OR

sketch of $\frac{\mathrm{d}A}{\mathrm{d}r}$ with some indication of zero ***(M1)(M1)***

Note: Award ***(M1)*** for sketch of $\frac{\mathrm{d}A}{\mathrm{d}r}$, ***(M1)*** for indication of zero.

$(r =)\ 3.78\,(\mathrm{cm}) \quad (3.78239...)$ ***(A1)(ft) (C3)***

Note: Follow through from part (a).

Examiners report

a. Question 15: Optimization

Many candidates were able to differentiate in part (a), but then were unable to relate this to part (b). However, it seemed that many more had not studied the calculus at all.

b. Question 15: Optimization

Many candidates were able to differentiate in part (a), but then were unable to relate this to part (b). However, it seemed that many more had not studied the calculus at all.

Given the function $f(x) = 2 \times 3^x$ for $-2 \leqslant x \leqslant 5$,

a. find the range of f. [4]

b. find the value of x given that $f(x) = 162$. [2]

Markscheme

a. $f(-2) = 2 \times 3^{-2}$ ***(M1)***

$= \frac{2}{9}(0.222)$ ***(A1)***

$f(5) = 2 \times 3^5$

$= 486$ ***(A1)***

Range $\frac{2}{9} \leqslant f(x) \leqslant 486$ **OR** $\left[\frac{2}{9},\ 486\right]$ ***(A1) (C4)***

Note: Award ***(M1)*** for correct substitution of –2 or 5 into $f(x)$, ***(A1)(A1)*** for each correct end point.

[4 marks]

b. $2 \times 3^x = 162$ ***(M1)***

$x = 4$ ***(A1) (C2)***

[2 marks]

Examiners report

a. Part (a) proved to be difficult to gain the maximum marks as, although candidates could find the end points, they did not seem to be able to identify the range of the function. Many students gave a list of values for the range, which indicates that this concept was not understood well.

b. This question was generally answered well in part (b).

A potato is placed in an oven heated to a temperature of 200°C.

The temperature of the potato, in °C, is modelled by the function $p(t) = 200 - 190(0.97)^t$, where t is the time, in minutes, that the potato has been in the oven.

a. Write down the temperature of the potato at the moment it is placed in the oven. [2]

b. Find the temperature of the potato half an hour after it has been placed in the oven. [2]

c. After the potato has been in the oven for k minutes, its temperature is 40°C. [2]

Find the value of k.

Markscheme

a. ***For parts (a) and (b) only, the first time a correct answer has incorrect or missing units, the final (A1) is not awarded.***

$200 - 190(0.97)^0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$= 10\,^\circ\mathrm{C}$ ***(A1)*** ***(C2)***

Note: Units are required.

b. ***For parts (a) and (b) only, the first time a correct answer has incorrect or missing units, the final (A1) is not awarded.***

$200 - 190(0.97)^{30}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$= 124^\circ\mathrm{C}\ (123.808\ldots^\circ\ \mathrm{C})$ ***(A1)*** ***(C2)***

Note: Units are required, unless already omitted in part (a).

c. $200 - 190(0.97)^k = 40$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$k = 5.64\ \text{(minutes)}\ (5.64198\ldots)$ ***(A1)*** ***(C2)***

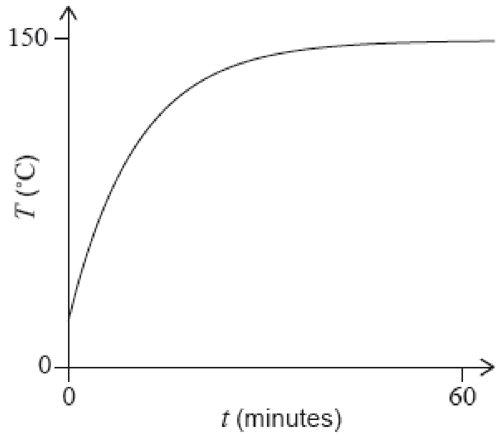
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Sejah placed a baking tin, that contained cake mix, in a preheated oven in order to bake a cake. The temperature in the centre of the cake mix, T, in degrees Celsius (°C) is given by

$$T(t) = 150 - a \times (1.1)^{-t}$$

where t is the time, in minutes, since the baking tin was placed in the oven. The graph of T is shown in the following diagram.



The temperature in the centre of the cake mix was 18 °C when placed in the oven.

The baking tin is removed from the oven 15 minutes after the temperature in the centre of the cake mix has reached 130 °C.

a. Write down what the value of 150 represents in the context of the question. [1]

b. Find the value of a. [2]

c. Find the total time that the baking tin is in the oven. [3]

Markscheme

a. the temperature in the oven ***(A1)***

OR

the maximum possible temperature of the cake mix ***(A1) (C1)***

Note: Award ***(A0)*** for "the maximum temperature".

[1 mark]

b. $18 = 150 - a(\times 1.1^{\circ})$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 18 and 0. Substitution of 0 can be implied.

$(a) = 132$ ***(A1)*** ***(C2)***

[2 marks]

c. $150 - 132 \times 1.1^{-t} = 130$ ***(M1)***

Note: Award ***(M1)*** for substituting their *a* and equating to 130. Accept an inequality.

Award ***(M1)*** for a sketch of the horizontal line on the graph.

$t = 19.8\ (19.7992\ldots)$ ***(A1)*(ft)**

Note: Follow through from part (b).

34.8 (minutes) (34.7992…, 34 minutes 48 seconds) ***(A1)*(ft)** ***(C3)***

Note: Award the final ***(A1)*** for adding 15 minutes to their t value.

In part (c), award ***(C2)*** for a final answer of 19.8 with no working.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the function $f\left(x\right) = \frac{x^4}{4}$.

a. Find *f*'(*x*) [1]

b. Find the gradient of the graph of *f* at $x = -\frac{1}{2}$. [2]

c. Find the *x*-coordinate of the point at which the **normal** to the graph of *f* has gradient $-\frac{1}{8}$. [3]

Markscheme

a. x^3 ***(A1) (C1)***

Note: Award ***(A0)*** for $\frac{4x^3}{4}$ and not simplified to x^3.

[1 mark]

b. $\left(-\frac{1}{2}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $-\frac{1}{2}$ into their derivative.

$-\frac{1}{8}$ (−0.125) ***(A1)*(ft) *(C2)***

Note: Follow through from their part (a).

[2 marks]

c. $x^3 = 8$ ***(A1)(M1)***

Note: Award ***(A1)*** for 8 seen maybe seen as part of an equation $y = 8x + c$, ***(M1)*** for equating their derivative to 8.

$(x =)\ 2$ ***(A1) (C3)***

Note: Do not accept (2, 4).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Shiyun bought a car in 1999. The value of the car V, in USD, is depreciating according to the exponential model

$$V = 25000 \times 1.5^{-0.2t},\ t \geqslant 0$$

where t is the time, in years, that Shiyun has owned the car.

a. Write down the value of the car when Shiyun bought it. [1]

b. Calculate the value of the car three years after Shiyun bought it. Give your answer correct to **two decimal places**. [2]

c. Calculate the time for the car to depreciate to half of its value since Shiyun bought it. [3]

Markscheme

a. $25000\ \text{USD}$ ***(A1) (C1)***

[1 mark]

b. $25000 \times 1.5^{-0.6}$ ***(M1)***

$19601.32\ \text{USD}$ ***(A1) (C2)***

[2 marks]

c. $12500 = 25000 \times 1.5^{-0.2t}$ ***(A1)*(ft)*(M1)***

Notes: Award ***(A1)*(ft)** for 12500 seen. Follow through from their answer to part (a). Award ***(M1)*** for equating their half value to $25000 \times 1.5^{-0.2t}$. Allow the use of an inequality.

OR

Graphical method (sketch):

***(A1)*(ft)** for $y = 12500$ seen on the sketch. Follow through from their answer to part (a). ***(A1)*(ft)**

(M1) for the exponent model and an indication of their intersection with their horizontal line. ***(M1)***

8.55 ***(A1)*(ft)** ***(C3)***

[3 marks]

Examiners report

a. A substituted value of $t = 1$ in part (a) saw many incorrect answers of 23052.70 for this part of the question. Part (b) was better attempted with many correct answers seen. Many candidates picked up the first two marks of part (c) equating a correct expression to half their answer found in part (a). Many though did not seem to know the correct process of using their GDC to find the required answer. Much *trial and improvement* was seen here with varying degrees of success.

b. A substituted value of $t = 1$ in part (a) saw many incorrect answers of 23052.70 for this part of the question. Part (b) was better attempted with many correct answers seen. Many candidates picked up the first two marks of part (c) equating a correct expression to half their answer found in part (a). Many though did not seem to know the correct process of using their GDC to find the required answer. Much *trial and improvement* was seen here with varying degrees of success.

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Consider the quadratic function, $f(x) = px(q - x)$, where p and q are positive integers.

The graph of $y = f(x)$ passes through the point $(6,\ 0)$.

a. Calculate the value of q. [2]

b. The vertex of the function is $(3,\ 27)$. [2]

Find the value of p.

c. The vertex of the function is $(3,\ 27)$. [2]

Write down the range of f.

Markscheme

a. $0 = p(6)(q - 6)$ ***(M1)***

$q = 6$ ***(A1)***

OR

$f(x) = -px^2 + pqx$

$3 = \frac{-pq}{-2p}$ ***(M1)***

$q = 6$ ***(A1)***

OR

$f(x) = -px^2 + pqx$

$f'(x) = pq - 2px$ ***(M1)***

$pq - 2p(3) = 0$

$q = 6$ ***(A1)*** ***(C2)***

b. $27 = p(3)(6 - 3)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of the vertex $(3,\ 27)$ **and** *their* q into or equivalent $f(x) = px(q - x)$ or equivalent.

$p = 3$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a).

c. $y \leqslant 27 \quad (f(x) \leqslant 27)$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for $y \leqslant \ (\text{or } f(x) \leqslant)$, ***(A1)*** for 27 as part of an inequality.

Accept alternative notation: $(-\infty,\ 27],\ ]-\infty,\ 27]$.

Award ***(A0)(A1)*** for $[27,\ -\infty)$.

Award ***(A0)(A0)*** for $(-\infty,\ \infty)$.

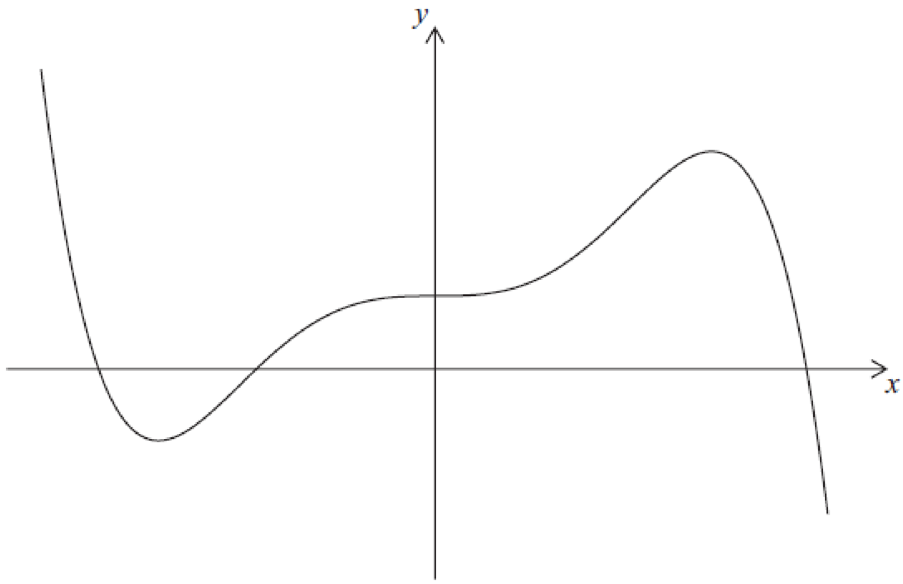
Examiners report

a. This question was left unanswered by many candidates. For candidates who attempted the question, one method mark was often awarded for a correct equation resulting from substitution of the point (6, 0). Many were unable to find the value of *q*, and therefore did not continue to find the value of *p*.

b. Many were unable to find the value of *q*, and therefore did not continue to find the value of *p*.

c. Part (c) was poorly attempted, although the range was independent of the values of *p* and *q*. The most common error was confusion between domain and range, resulting in an answer of (-∞, ∞).

A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \leqslant x \leqslant 1.5$ and $-6 \leqslant y \leqslant 6$.



a. Write down $f'(x)$. [2]

b. Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$. [2]

c. Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. [2]

Markscheme

a. $f'(x) = 15x^2 - 15x^4$ ***(A1)(A1)*** ***(C2)***

Note: Award a maximum of ***(A1)(A0)*** if extra terms seen.

b. $f'(1) = 0$ ***(M1)***

Note: Award ***(M1)*** for $f'(x) = 0$.

$y = 3$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

c. $(-1.38, 3)$ $(-1.38481\ldots, 3)$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to parts (a) and (b).

Note: Accept $x = -1.38, y = 3$ ($x = -1.38481\ldots, y = 3$) .

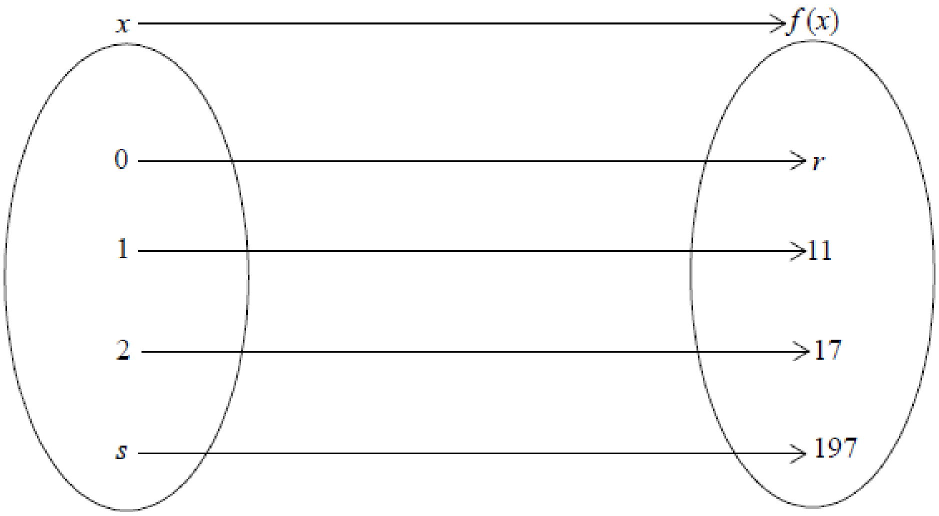
Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

A function $f(x) = p \times 2^x + q$ is defined by the mapping diagram below.



a. Find the value of [3]

(i) p ;

(ii) q .

b. Write down the value of r . [1]

c. Find the value of s . [2]

Markscheme

a. (i) $2p + q = 11$ and $4p + q = 17$ ***(M1)***

Note: Award ***(M1)*** for either two correct equations or a correct equation in one unknown equivalent to $2p = 6$.

$p = 3$ ***(A1)***

(ii) $q = 5$ ***(A1)*** ***(C3)***

Notes: If only one value of p and q is correct and no working shown, award ***(M0)(A1)(A0)***.

[3 marks]

b. $r = 8$ ***(A1)*****(ft)** ***(C1)***

Note: Follow through from their answers for p and q irrespective of whether working is seen.

[1 mark]

c. $3 \times 2^s + 5 = 197$ ***(M1)***

Note: Award ***(M1)*** for setting the correct equation.

$s = 6$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their values of p and q.

[2 marks]

Examiners report

a. Candidates both understood how to interpret a mapping diagram correctly and did very well on this question or the question was very poorly answered or not answered at all. Writing down two simultaneous equations in part (a) proved to be elusive to many and this prevented further work on this question. Candidates who were able to find values of p and q (correct or otherwise) invariably made a good attempt at finding the value of s in part (c).

b. Candidates both understood how to interpret a mapping diagram correctly and did very well on this question or the question was very poorly answered or not answered at all. Writing down two simultaneous equations in part (a) proved to be elusive to many and this prevented further work on this question. Candidates who were able to find values of p and q (correct or otherwise) invariably made a good attempt at finding the value of s in part (c).

c. Candidates both understood how to interpret a mapping diagram correctly and did very well on this question or the question was very poorly answered or not answered at all. Writing down two simultaneous equations in part (a) proved to be elusive to many and this prevented further work on this question. Candidates who were able to find values of p and q (correct or otherwise) invariably made a good attempt at finding the value of s in part (c).

The number of fish, N, in a pond is decreasing according to the model

$$N(t) = ab^{-t} + 40, \quad t \geqslant 0$$

where a and b are positive constants, and t is the time in months since the number of fish in the pond was first counted.

At the beginning 840 fish were counted.

a. Find the value of a. [2]

b. After 4 months 90 fish were counted. [3]

Find the value of b.

c. The number of fish in the pond will **not** decrease below p. [1]

Write down the value of p.

Markscheme

a. $ab^0 + 40 = 840$ ***(M1)***

Note: Award ***(M1)*** for substituting $t = 0$ and equating to 840.

$a = 800$ ***(A1)(C2)***

b. $800b^{-4} + 40 = 90$ ***(M1)***

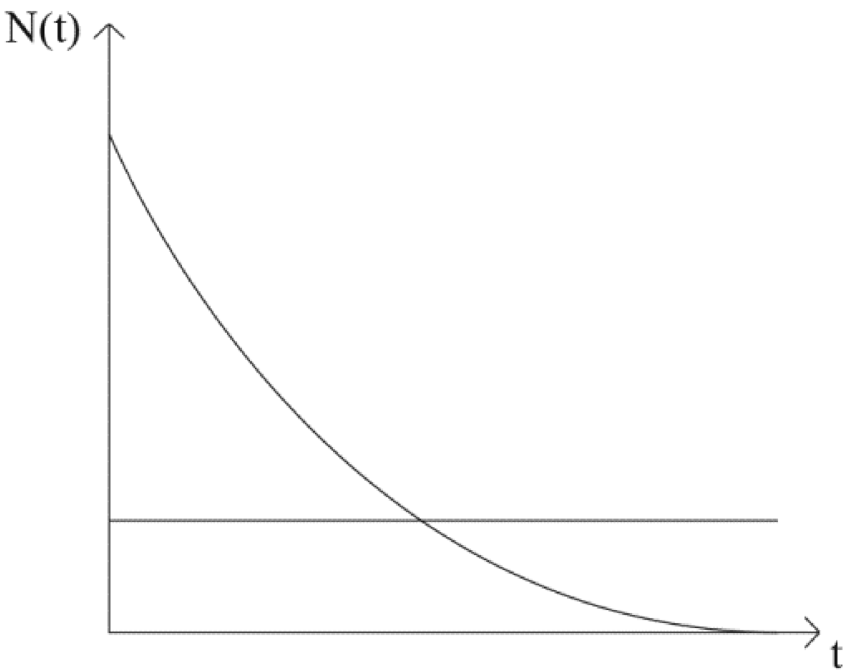
Note: Award ***(M1)*** for correct substitution of their a (from part (a)) **and** 4 in the formula of the function and equating to 90.

$b^4 = 16$ **OR** $\frac{1}{b^4} = \frac{1}{16}$ **OR** $b = \sqrt[4]{16}$ **OR** $b = 16^{\frac{1}{4}}$ ***(M1)***

Notes: Award second ***(M1)*** for correctly rearranging their equation and eliminating the negative index (see above examples).

Accept $\frac{800}{50}$ in place of 16.

OR



(M1)(M1)

Notes: Award ***(M1)*** for a decreasing exponential and a horizontal line that are both in the first quadrant, and ***(M1)*** for their graphs intersecting.

For graphs drawn in both first and second quadrants award at most ***(M1)(M0)***.

$b = 2$ ***(A1)***(ft) ***(C3)***

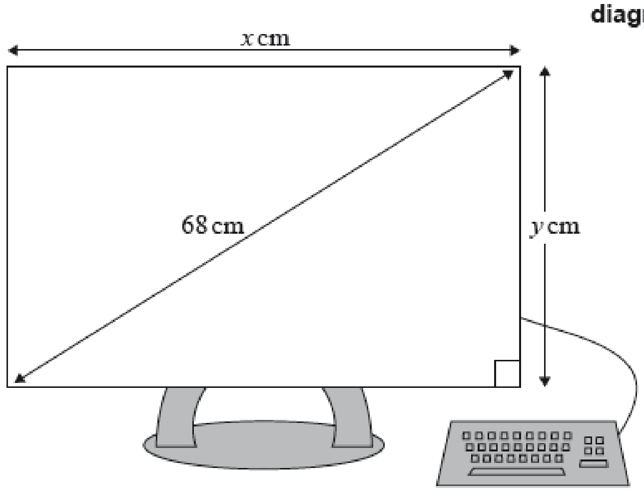
Note: Follow through from their answer to part (a) only if a is positive.

c. 40 ***(A1)*** ***(C1)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The size of a computer screen is the length of its diagonal. Zuzana buys a rectangular computer screen with a size of 68 cm, a height of y cm and a width of x cm, as shown in the diagram.



The ratio between the height and the width of the screen is 3:4.

a. Use this information to write down an equation involving x and y. [1]

b. Use this ratio to write down y in terms of x. [2]

c. Find the value of x and of y. [3]

Markscheme

a. $x^2 + y^2 = 68^2$ (or 4624 or equivalent) ***(A1)*** ***(C1)***

[1 mark]

b. $\frac{y}{x} = \frac{3}{4}$ ***(M1)***

Note: Award ***(M1)*** for a correct equation.

$y = \frac{3}{4}x \ (y = 0.75x)$ ***(A1)*** ***(C2)***

[2 marks]

c. $x^2 + \left(\frac{3}{4}x\right)^2 = 68^2 \ \left(\text{or } x^2 + \frac{9}{16}x^2 = 4624 \text{ or equivalent}\right)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their expression for y into their answer to part (a). Accept correct substitution of x in terms of y.

$x = 54.4 \text{ (cm)}, y = 40.8 \text{ (cm)}$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***(C3)***

Note: Follow through from parts (a) and (b) as long as $x > 0$ and $y > 0$.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Gabriella purchases a new car.

The car's value in dollars, V, is modelled by the function

$$V(t) = 12870 - k(1.1)^t,\ t \geqslant 0$$

where t is the number of years since the car was purchased and k is a constant.

After two years, the car's value is $9143.20.

This model is defined for $0 \leqslant t \leqslant n$. At n years the car's value will be zero dollars.

a. Write down, and simplify, an expression for the car's value when Gabriella purchased it. [2]

b. Find the value of k. [2]

c. Find the value of n. [2]

Markscheme

a. $12870 - k(1.1)^0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into $V(t)$.

$= 12870 - k$ ***(A1)*** ***(C2)***

Note: Accept $12870 - 3080$ **OR** 9790 for a final answer.

[2 marks]

b. $9143.20 = 12870 - k(1.1)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into $V(t)$.

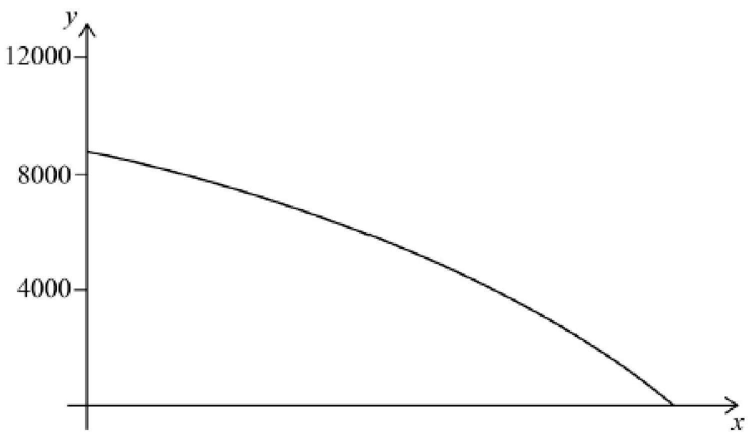
$(k =)\ 3080$ ***(A1)*** ***(C2)***

[2 marks]

c. $12870 - 3080(1.1)^n = 0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into $V(t)$.

OR



(M1)

Note: Award ***(M1)*** for a correctly shaped curve with some indication of scale on the vertical axis.

$(n =)\ 15.0\ (15.0033\ldots)$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (b).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Factorise the expression $x^2 - 3x - 10$. [2]

b. A function is defined as $f(x) = 1 + x^3$ for $x \in \mathbb{Z}$, $-3 \leqslant x \leqslant 3$. [4]

(i) List the elements of the domain of $f(x)$.

(ii) Write down the range of $f(x)$.

Markscheme

a. $(x - 5)(x + 2)$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $(x + 5)(x - 2)$, ***(A0)*** otherwise. If equation is equated to zero and solved by factorizing award ***(A1)*** for both correct factors, followed by ***(A0)***.

[2 marks]

b. (i) $-3, -2, -1, 0, 1, 2, 3$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A2)*** for all correct answers seen and no others. Award ***(A1)*** for 3 correct answers seen.

(ii) -26, -7, 0,1, 2, 9, 28 ***(A1)(A1)*** ***(C2)***

Note: Award ***(A2)*** for all correct answers seen and no others. Award ***(A1)*** for 3 correct answers seen. If domain and range are interchanged award ***(A0)*** for (b)(i) and ***(A1)*(ft)*(A1)*(ft)** for (b)(ii).

[4 marks]

Examiners report

a. It was surprising how many candidates could not factorise this expression. Of those that could some went on to find the zeros of a quadratic equation which was not what the question was asking. Some confused domain and range and many did not write down all the values when they did know domain and range.

b. It was surprising how many candidates could not factorise this expression. Of those that could some went on to find the zeros of a quadratic equation which was not what the question was asking. Some confused domain and range and many did not write down all the values when they did know domain and range.

Consider the quadratic function $f\left(x\right) = ax^2 + bx + 22.$

The equation of the line of symmetry of the graph $y = f\left(x\right)\text{ is }x = 1.75.$

The graph intersects the *x*-axis at the point (–2 , 0).

a. Using only this information, write down an equation in terms of *a* and *b*. [1]

b. Using this information, write down a second equation in terms of *a* and *b*. [1]

c. Hence find the value of *a* and of *b*. [2]

d. The graph intersects the *x*-axis at a second point, P. [2]

Find the *x*-coordinate of P.

Markscheme

a. $1.75 = \frac{-b}{2a}$ (or equivalent) ***(A1) (C1)***

Note: Award ***(A1)*** for $f\left(x\right) = {\left(1.75\right)}^2a + 1.75b$ or for $y = {\left(1.75\right)}^2a + 1.75b + 22$ or for $f\left(1.75\right) = {\left(1.75\right)}^2a + 1.75b + 22$.

[1 mark]

b. ${\left(-2\right)}^2 \times a + \left(-2\right) \times b + 22 = 0$ (or equivalent) ***(A1) (C1)***

Note: Award ***(A1)*** for ${\left(-2\right)}^2 \times a + \left(-2\right) \times b + 22 = 0$ seen.

Award ***(A0)*** for $y = (-2)^2 \times a + (-2) \times b + 22$.

[1 mark]

c. $a = -2$, $b = 7$ ***(A1)*(ft)*(A1)*(ft) *(C2)***

Note: Follow through from parts (a) and (b).
Accept answers(s) embedded as a coordinate pair.

[2 marks]

d. $-2x^2 + 7x + 22 = 0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of *a* and *b* into equation and setting to zero. Follow through from part (c).

(x =) 5.5 ***(A1)*(ft) *(C2)***

Note: Follow through from parts (a) and (b).

OR

x-coordinate = 1.75 + (1.75 − (−2)) ***(M1)***

Note: Award ***(M1)*** for correct use of axis of symmetry and given intercept.

(x =) 5.5 ***(A1) (C2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

---

a. The golden ratio, r , was considered by the Ancient Greeks to be the perfect ratio between the lengths of two adjacent sides of a rectangle. The exact value of r is $\frac{1+\sqrt{5}}{2}$. [2]

Write down the value of r

i) correct to 5 significant figures;

ii) correct to 2 decimal places.

b. Phidias is designing rectangular windows with adjacent sides of length x metres and y metres. The area of each window is $1\ \mathrm{m}^2$. [1]

Write down an equation to describe this information.

c. Phidias designs the windows so that the ratio between the longer side, y , and the shorter side, x , is the golden ratio, r. [1]

Write down an equation in y , x and r to describe this information.

d. Find the value of x . [2]

Markscheme

a. i) 1.6180 ***(A1)***

ii) 1.62 ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a)(i).

b. $xy = 1$ ***(A1)*** ***(C1)***

c. $\frac{y}{x} = r$ **OR** $\frac{y}{x} = \frac{1+\sqrt{5}}{2}$ **OR** equivalent ***(A1)*** ***(C1)***

Note: Accept $\frac{y}{x} =$ their part (a)(i) or (a)(ii).

d. $x^2r = 1$ or eqivalent ***(M1)***

$x = 0.786 \ (0.78615...)$ ***(A1)*(ft)** ***(C2)***

Note: Award ***(M1)*** for substituting their part (c) into their equation from part (b). Follow through from parts (a), (b) and (c). Use of $r = 1.62$ gives $0.785674...$

Examiners report

a. Question 13: Golden ratio

This question was partially answered by all but the best candidates. Parts (a) and (b) yielded the most success. Only the best candidates were successful in part (d).

b. Question 13: Golden ratio

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c. Question 13: Golden ratio

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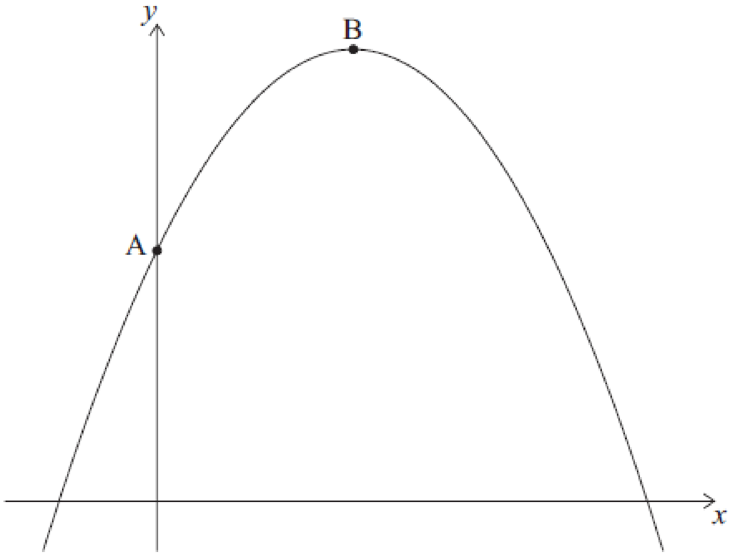
d. Question 13: Golden ratio

This question was partially answered by all but the best candidates. Parts (a) and (b) yielded the most success. Only the best candidates were successful in part (d).

The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the y-axis at point A(0, 5) and has its vertex at point B(2, 9).



a. Write down the value of *c*. [1]

b. Find the value of *b*. [2]

c. Find the *x*-intercepts of the graph of *f* . [2]

d. Write down $f(x)$ in the form $f(x) = -(x-p)(x+q)$. [1]

Markscheme

a. 5 ***(A1)*** ***(C1)***

b. $\frac{-b}{2(-1)} = 2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in axis of symmetry formula.

OR

$y = 5 + bx - x^2$

$9 = 5 + b(2) - (2)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 9 and 2 into their quadratic equation.

$(b =)4$ ***(A1)*(ft)*** ***(C2)***

Note: Follow through from part (a).

c. 5, –1 ***(A1)*(ft)*(A1)*(ft)*** ***(C2)***

Notes: Follow through from parts (a) and (b), irrespective of working shown.

d. $f(x) = -(x-5)(x+1)$ ***(A1)*(ft)*** ***(C1)***

Notes: Follow through from part (c).

Examiners report

a. Many candidates did not see the connection between the x-intercepts and the factored form of a quadratic function. The syllabus explicitly sates that the graphs of quadratics should be linked to solutions of quadratic equations by factorizing and vice versa. This was one of the most challenging questions for candidates.

b. Many candidates did not see the connection between the x-intercepts and the factored form of a quadratic function. The syllabus explicitly sates that the graphs of quadratics should be linked to solutions of quadratic equations by factorizing and vice versa. This was one of the most challenging questions for candidates.

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A building company has many rectangular construction sites, of varying widths, along a road.

The area, A, of each site is given by the function

$$A(x) = x(200 - x)$$

where x is the **width** of the site in metres and $20 \leqslant x \leqslant 180$.

a. Site S has a width of 20 m. Write down the area of S. [1]

b. Site T has the same area as site S, but a different width. Find the width of T. [2]

c. When the width of the construction site is b metres, the site has a maximum area. [2]

(i) Write down the value of b.

(ii) Write down the maximum area.

d. The range of $A(x)$ is $m \leqslant A(x) \leqslant n$. [1]

Hence write down the value of m and of n.

Markscheme

a. $3600\ (\mathrm{m}^2)$ ***(A1)(C1)***

b. $x(200 - x) = 3600$ ***(M1)***

Note: Award ***(M1)*** for setting up an equation, equating to their 3600.

$180\ (\mathrm{m})$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

c. (i) $100\ (\mathrm{m})$ ***(A1)*** ***(C1)***

(ii) $10\,000\ (\mathrm{m}^2)$ ***(A1)*(ft)*(C1)***

Note: Follow through from their answer to part (c)(i).

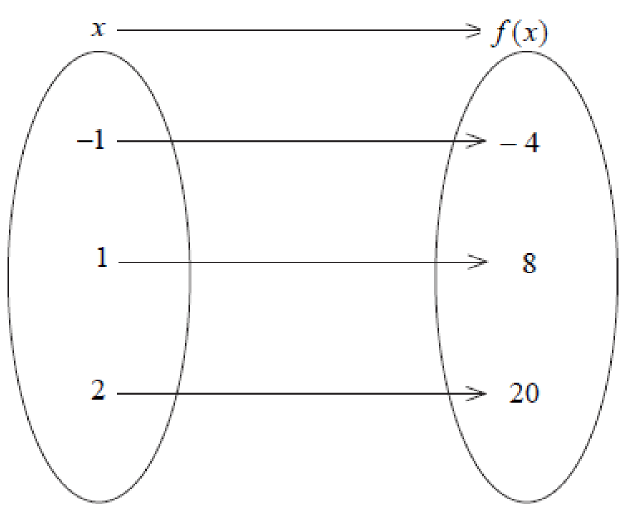
d. $m = 3600$ **and** $n = 10\,000$ ***(A1)*(ft)** ***(C1)***

Notes: Follow through from part (a) and part (c)(ii), but only if their m is less than their n. Accept the answer $3600 \leqslant A \leqslant 10\,000$.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.



a. Use the mapping diagram to write down **two** equations in terms of *a* and *b*. [2]

b.i. Find the value of *a*. [1]

b.ii. Find the value of *b*. [1]

c. Calculate the *x*-coordinate of the vertex of the graph of *f* (*x*). [2]

Markscheme

a. 4*a* + 2*b* = 20

a + *b* = 8 ***(A1)***

$a - b = -4$ ***(A1)*** ***(C2)***

Note: Award ***(A1)(A1)*** for any two of the given or equivalent equations.

[2 marks]

b.i. $a = 2$ ***(A1)*(ft)**

[1 mark]

b.ii. $b = 6$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their (a).

[1 mark]

c. $x = -\frac{6}{2(2)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in correct formula.

$= -1.5$ ***(A1)*(ft)** ***(C2)***

[2 marks]

Examiners report

a. Most candidates attempted this question but very few of them completed it entirely. A number of students wrote incorrect equations in part (a), which shows that the mapping diagram was poorly understood and read. Part (c) proved to be difficult for many who didn't know how to find the x-coordinate of the vertex of the graph of the function. Some students gave the two coordinates instead of the x-coordinate only.

b.i. Most candidates attempted this question but very few of them completed it entirely. A number of students wrote incorrect equations in part (a), which shows that the mapping diagram was poorly understood and read. Part (c) proved to be difficult for many who didn't know how to find the x-coordinate of the vertex of the graph of the function. Some students gave the two coordinates instead of the x-coordinate only.

b.ii. Most candidates attempted this question but very few of them completed it entirely. A numberof students wrote incorrect equations in part (a), which shows that the mapping diagram was poorly understood and read. Part (c) proved to be difficult for many who didn't know how to find the x-coordinate of the vertex of the graph of the function. Some students gave the two coordinates instead of the x-coordinate only.

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An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius (°C).

a. At 150°C the length of the iron bar is 180 mm. [1]

Write down an equation that shows this information.

b. At 210°C the length of the iron bar is 181.5 mm. [1]

Write down an equation that shows this second piece of information.

c. At 210°C the length of the iron bar is 181.5 mm. [4]

Hence, find the length of the iron bar at 40°C.

Markscheme

a. $180 = 150m + c$ (or equivalent) ***(A1)*** ***(C1)***

b. $181.5 = 210m + c$ (or equivalent) ***(A1)*** ***(C1)***

c. $m = 0.25,\ c = 176.25$ ***(A1)(A1)*(ft)**

Note: Follow through from part (a) and part (b), irrespective of working shown.

$L = 0.025(4) + 176.25$ ***(M1)***

Note: Award ***(M1)*** for substitution of their m, their c and 40 into equation.

$L = 177 \quad (177.25)\ (\text{mm})$ ***(A1)*(ft)** ***(C4)***

Note: Follow through, within **part (c)**, from their m and c only if working shown.

Examiners report

a. The equations in part (a) and (b) were given correctly by the vast majority of the candidates.

b. The equations in part (a) and (b) were given correctly by the vast majority of the candidates.

c. Part (c) was in most cases either completely correct or awarded no marks at all. Only few were able to find the values of *m* and *c*, and therefore the length at 40°C. Part (c) was often left open or answered incorrectly. A common answer was *L = 40m + c* . Very few partial correct responses were given. Some candidates managed a correct 3 sf answer by intelligent guessing. As the question was not structured asking for the m and c values explicitly, not many candidates made an attempt to find those values. Very few seemed to realize they could find those values using their GDC. An attempt to use simultaneous equations was the most common approach.

Consider the quadratic function $y = f(x)$, where $f(x) = 5 - x + ax^2$.

a. It is given that $f(2) = -5$. Find the value of a . [2]

b. Find the equation of the axis of symmetry of the graph of $y = f(x)$. [2]

c. Write down the range of this quadratic function. [2]

Markscheme

a. $-5 = 5 - (2) + a(2)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in equation.

$(a =) -2$ ***(A1)*** ***(C2)***

[2 marks]

b. $x = -\frac{1}{4}$ (–0.25) ***(A1)(A1)*(ft)** ***(C2)***

Notes: Follow through from their part (a). Award ***(A1)(A0)*(ft)** for “ x = constant”. Award ***(A0)(A1)*(ft)** for $y = -\frac{1}{4}$.

[2 marks]

c. $f(x) \le 5.125$ ***(A1)(A1)*(ft)** ***(C2)***

Notes: Award ***(A1)*** for $f(x) \le$ (accept y). Do not accept strict inequality. Award ***(A1)*(ft)** for 5.125 (accept 5.13). Accept other correct notation, for example, $(-\infty, 5.125]$. Follow through from their answer to part (b).

[2 marks]

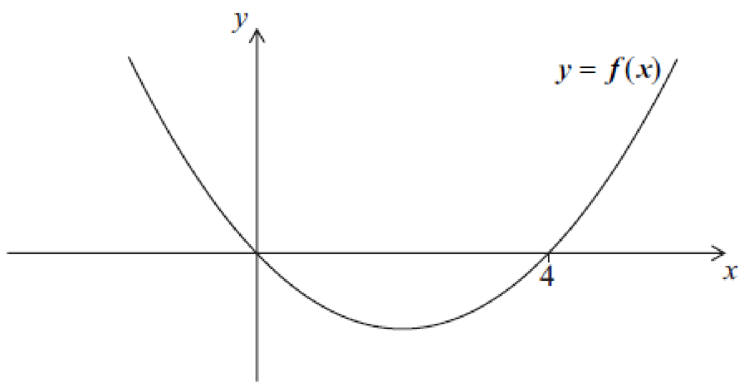
Examiners report

a. This question proved to be quite a discriminator with a significant number of candidates achieving, at most, one or two marks in part (a). Part (b) was tested in the May 2012 series of examinations but the same errors were prevalent here as they were then. A number of candidates simply wrote the equation of the axis of symmetry in terms of y rather than x or just wrote down a numerical value rather than an equation. Expressions for the required range in part (c) fared little better with again much confusion between the variables x and y. A strict inequality was required at the turning point and a mark was lost where this was not indicated. Alternative forms for the range such as $(-\infty, 5.125]$ were, of course, accepted.

b. This question proved to be quite a discriminator with a significant number of candidates achieving, at most, one or two marks in part (a). Part (b) was tested in the May 2012 series of examinations but the same errors were prevalent here as they were then. A number of candidates simply wrote the equation of the axis of symmetry in terms of y rather than x or just wrote down a numerical value rather than an equation. Expressions for the required range in part (c) fared little better with again much confusion between the variables x and y. A strict inequality was required at the turning point and a mark was lost where this was not indicated. Alternative forms for the range such as $(-\infty , 5.125]$ were, of course, accepted.

c. This question proved to be quite a discriminator with a significant number of candidates achieving, at most, one or two marks in part (a). Part (b) was tested in the May 2012 series of examinations but the same errors were prevalent here as they were then. A number of candidates simply wrote the equation of the axis of symmetry in terms of y rather than x or just wrote down a numerical value rather than an equation. Expressions for the required range in part (c) fared little better with again much confusion between the variables x and y. A strict inequality was required at the turning point and a mark was lost where this was not indicated. Alternative forms for the range such as $(-\infty, 5.125]$ were, of course, accepted.

The following is the graph of the quadratic function $y = f(x)$.



a. Write down the solutions to the equation $f(x) = 0$. [2]

b. Write down the equation of the axis of symmetry of the graph of $f(x)$. [2]

c. The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution. [1]

d. The minimum value for y is – 4. Write down the range of $f(x)$. [1]

Markscheme

a. $x = 0, x = 4$ ***(A1)(A1) (C2)***

Notes: Accept 0 and 4.

[2 marks]

b. $x = 2$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for x = constant, ***(A1)*** for 2.

[2 marks]

c. $x = -2$ ***(A1) (C1)***

Note: Accept –2.

[1 mark]

d. $y \geqslant -4\ (f(x) \geqslant -4)$ ***(A1) (C1)***

Notes: Accept alternative notations.

Award ***(A0)*** for use of strict inequality.

[1 mark]

Examiners report

a. A number of candidates left out this question which indicated that this topic was either entirely unfamiliar, that this topic of the syllabus had perhaps not been taught, or was barely familiar. A few candidates wrote down coordinate pairs when asked for a solution to the equation. A number of candidates wrote down the formula for the equation of the axis of symmetry without being able to substitute values for a and b. When given the minimum value of the graph a small number of candidates could identify the range of the function correctly. Overall this question proved to be difficult with its demands for reading and interpreting the graph, and dealing with additional information about the quadratic function given in the different parts.

b. A number of candidates left out this question which indicated that this topic was either entirely unfamiliar, that this topic of the syllabus had perhaps not been taught, or was barely familiar. A few candidates wrote down coordinate pairs when asked for a solution to the equation. A number of candidates wrote down the formula for the equation of the axis of symmetry without being able to substitute values for a and b. When given the minimum value of the graph a small number of candidates could identify the range of the function correctly. Overall this question proved to be difficult with its demands for reading and interpreting the graph, and dealing with additional information about the quadratic function given in the different parts.

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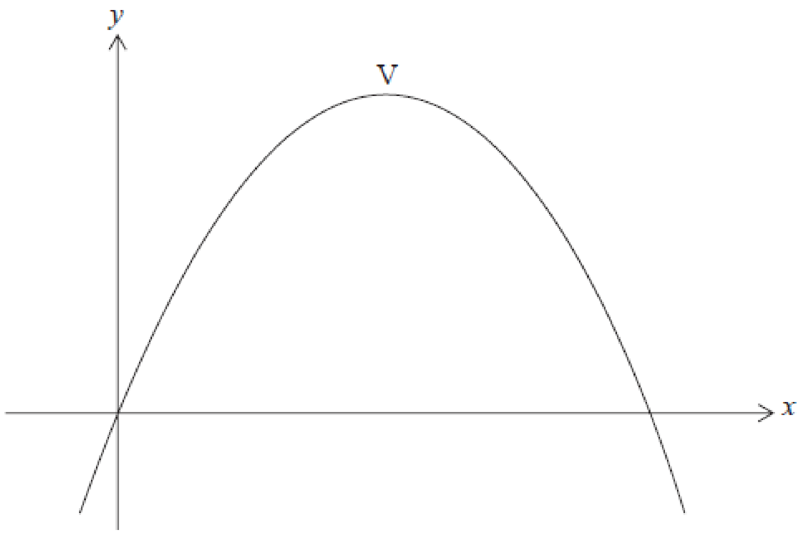
d. A number of candidates left out this question which indicated that this topic was either entirely unfamiliar, that this topic of the syllabus had perhaps not been taught, or was barely familiar. A few candidates wrote down coordinate pairs when asked for a solution to the equation. A number of candidates wrote down the formula for the equation of the axis of symmetry without being able to substitute values for a and b.

When given the minimum value of the graph a small number of candidates could identify the range of the function correctly. Overall this question proved to be difficult with its demands for reading and interpreting the graph, and dealing with additional information about the quadratic function given in the different parts.

A quadratic curve with equation $y = ax(x - b)$ is shown in the following diagram.



The x-intercepts are at (0, 0) and (6, 0), and the vertex V is at $(h, 8)$.

a. Find the value of h. [2]

b. Find the equation of the curve. [4]

Markscheme

a. $\frac{0+6}{2} = 3 \quad h = 3$ ***(M1)(A1)*** ***(C2)***

Note: Award ***(M1)*** for any correct method.

[2 marks]

b. $y = ax(x-6)$ ***(A1)***

$8 = 3a(-3)$ ***(A1)*(ft)**

$a = -\frac{8}{9}$ ***(A1)*(ft)**

$y = -\frac{8}{9}x(x-6)$ ***(A1)*(ft)**

Notes: Award ***(A1)*** for correct substitution of $b = 6$ into equation.

Award ***(A1)*(ft)** for substitution of their point V into the equation.

OR

$y = a(x-3)^2 + 8$ ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for correct substitution of their *h* into the equation.

$0 = a(6-3)^2 + 8$ ***OR*** $0 = a(0-3)^2 + 8$ ***(A1)***

Note: Award ***(A1)*** for correct substitution of an *x* intercept.

$a = -\frac{8}{9}$ ***(A1)*(ft)**

$y = -\frac{8}{9}(x-3)^2 + 8$ ***(A1)*(ft)** ***(C4)***

[4 marks]

Examiners report

a. Most candidates successfully found h but very few could find the equation of the curve.

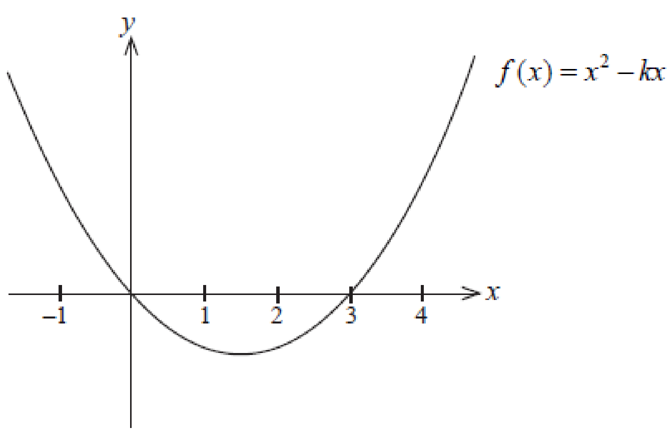
b. This question appeared to be the most difficult question on the paper.

---

a. Factorise the expression $x^2 - kx$. [1]

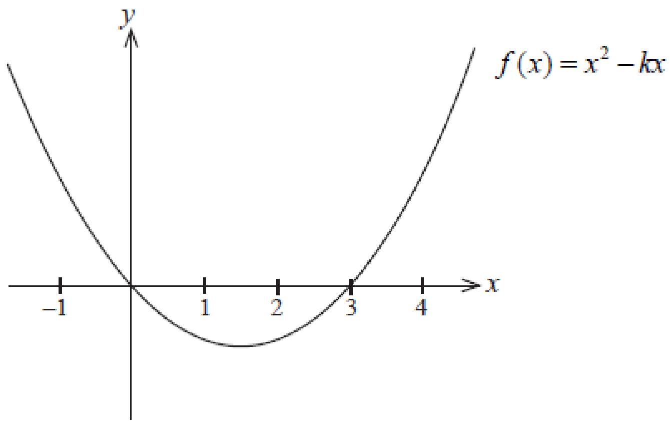
b. Hence solve the equation $x^2 - kx = 0$. [1]

c. The diagram below shows the graph of the function $f(x) = x^2 - kx$ for a particular value of k. [1]



Write down the value of k for this function.

d. The diagram below shows the graph of the function $f(x) = x^2 - kx$ for a particular value of k. [3]



Find the minimum value of the function $y = f(x)$.

Markscheme

a. $x(x - k)$ ***(A1)*** ***(C1)***

[1 mark]

b. $x = 0$ **or** $x = k$ ***(A1)*** ***(C1)***

Note: Both correct answers only.

[1 mark]

c. $k = 3$ ***(A1)*** ***(C1)***

[1 mark]

d. $\text{Vertex at } x = \frac{-(-3)}{2(1)}$ ***(M1)***

Note: ***(M1)*** for correct substitution in formula.

$x = 1.5$ ***(A1)*(ft)**

$y = -2.25$ ***(A1)*(ft)**

OR

$f'(x) = 2x - 3$ ***(M1)***

Note: ***(M1)*** for correct differentiation.

$x = 1.5$ ***(A1)*(ft)**
$y = -2.25$ ***(A1)*(ft)**

OR

for finding the midpoint of their 0 and 3 ***(M1)***
$x = 1.5$ ***(A1)*(ft)**
$y = -2.25$ ***(A1)*(ft)**

Note: If final answer is given as $(1.5, -2.25)$ award a maximum of ***(M1)(A1)(A0)***

[3 marks]

Examiners report

a. This question was poorly answered by all but the best candidates. The links between the parts were not made. The idea of the line of symmetry for the graph was seldom investigated. The “minimum value of the function” was often incorrectly given as a coordinate pair.

b. This question was poorly answered by all but the best candidates. The links between the parts were not made. The idea of the line of symmetry for the graph was seldom investigated. The “minimum value of the function” was often incorrectly given as a coordinate pair.

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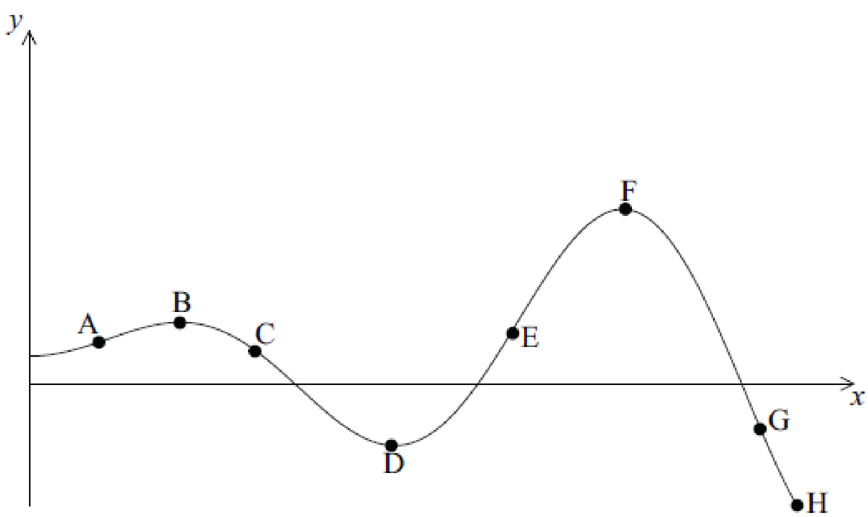
d. This question was poorly answered by all but the best candidates. The links between the parts were not made. The idea of the line of symmetry for the graph was seldom investigated. The “minimum value of the function” was often incorrectly given as a coordinate pair.

Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labelled points on the curve

a. that are local maximum points; [1]

b. where the function attains its least value; [1]

c. where the function attains its greatest value; [1]

d. where the gradient of the tangent to the curve is positive; [1]

e. where $f(x) > 0$ and $f'(x) < 0$. [2]

Markscheme

a. B, F ***(C1)***

b. H ***(C1)***

c. F ***(C1)***

d. A, E ***(C1)***

e. C ***(C2)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

---

a. Consider the function $f(x) = ax^2 + c$. [1]

Find $f'(x)$

b. Point $\mathrm{A}(-2,\ 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6 . [3]

Find the value of a .

c. Find the value of c . [2]

Markscheme

a. $2ax$ ***(A1) (C1)***

Note: Award ***(A1)*** for $2ax$. Award ***(A0)*** if other terms are seen.

b. $2a(-2) = -6$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $x = -2$ in their gradient function, ***(M1)*** for equating their gradient function to -6 . Follow through from part (a).

$(a =)1.5\ \left(\frac{3}{2}\right)$ ***(A1)*(ft) *(C3)***

c. $\text{their } 1.5 \times (-2)^2 + c = 5$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their a and point A. Follow through from part (b).

$(c =) - 1$ ***(A1)*(ft) *(C2)***

Examiners report

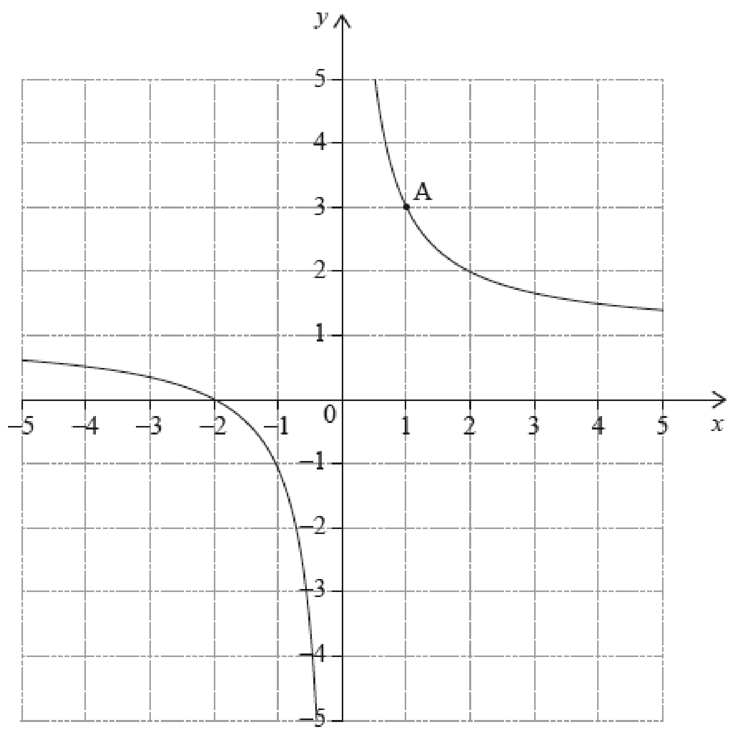
a. Question 11: Equation of tangent

Part (a) was generally well answered.

b. In part (b), many candidates substituted the value of the function, rather than its gradient; this was usually correctly followed through into part (c).

c. In part (b), many candidates substituted the value of the function, rather than its gradient; this was usually correctly followed through into part (c).

The diagram shows part of the graph of a function $y = f(x)$. The graph passes through point $\mathrm{A}(1,\ 3)$.



The tangent to the graph of $y = f(x)$ at A has equation $y = -2x + 5$. Let N be the normal to the graph of $y = f(x)$ at A.

a. Write down the value of $f(1)$. [1]

b. Find the equation of N. Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$. [3]

c. Draw the line N on the diagram above. [2]

Markscheme

a. 3 ***(A1) (C1)***

Notes: Accept $y = 3$

[1 mark]

b. $3 = 0.5(1) + c$ **OR** $y - 3 = 0.5(x - 1)$ ***(A1)(A1)***

Note: Award ***(A1)*** for correct gradient, ***(A1)*** for correct substitution of $\mathrm{A}(1,\ 3)$ in the equation of line.

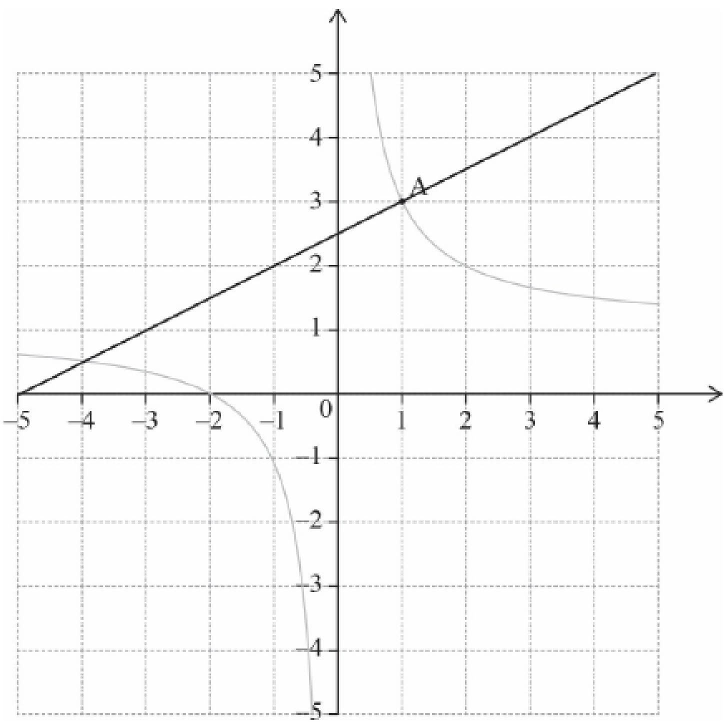
$x - 2y + 5 = 0$ or any integer multiple ***(A1)*(ft)** ***(C3)***

Note: Award ***(A1)*(ft)** for their equation correctly rearranged in the indicated form.

The candidate's answer **must** be an equation for this mark.

[3 marks]

c.



(M1)(A1)*(ft)** ***(C2)

Note: Award ***M1)*** for a straight line, with positive gradient, passing through $(1,\ 3)$, ***(A1)*(ft)** for line (or extension of their line) passing approximately through 2.5 or their intercept with the y-axis.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Water has a lower boiling point at higher altitudes. The relationship between the boiling point of water (T) and the height above sea level (h) can be described by the model $T = -0.0034h + 100$ where T is measured in degrees Celsius (°C) and h is measured in metres from sea level.

a. Write down the boiling point of water at sea level. [1]

b. Use the model to calculate the boiling point of water at a height of 1.37 km above sea level. [3]

c. Water boils at the top of Mt. Everest at 70 °C. [2]

Use the model to calculate the height above sea level of Mt. Everest.

Markscheme

a. 100 °C ***(A1)*** ***(C1)***

[1 mark]

b. $T = -0.0034 \times 1370 + 100$ ***(A1)(M1)***

Note: Award ***(A1)*** for 1370 seen, ***(M1)*** for substitution of their *h* into the equation.

95.3 °C (95.342) ***(A1)*** ***(C3)***

Notes: If their *h* is incorrect award at most ***(A0)(M1)(A0)***. If their *h* = 1.37 award at most ***(A0)(M1)(A1)*****(ft)**.

[3 marks]

c. $70 = -0.0034h + 100$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted equation.

h = 8820 m (8823.52...) ***(A1)*** ***(C2)***

Note: The answer is 8820 m (or 8.82 km.) units are required.

[2 marks]

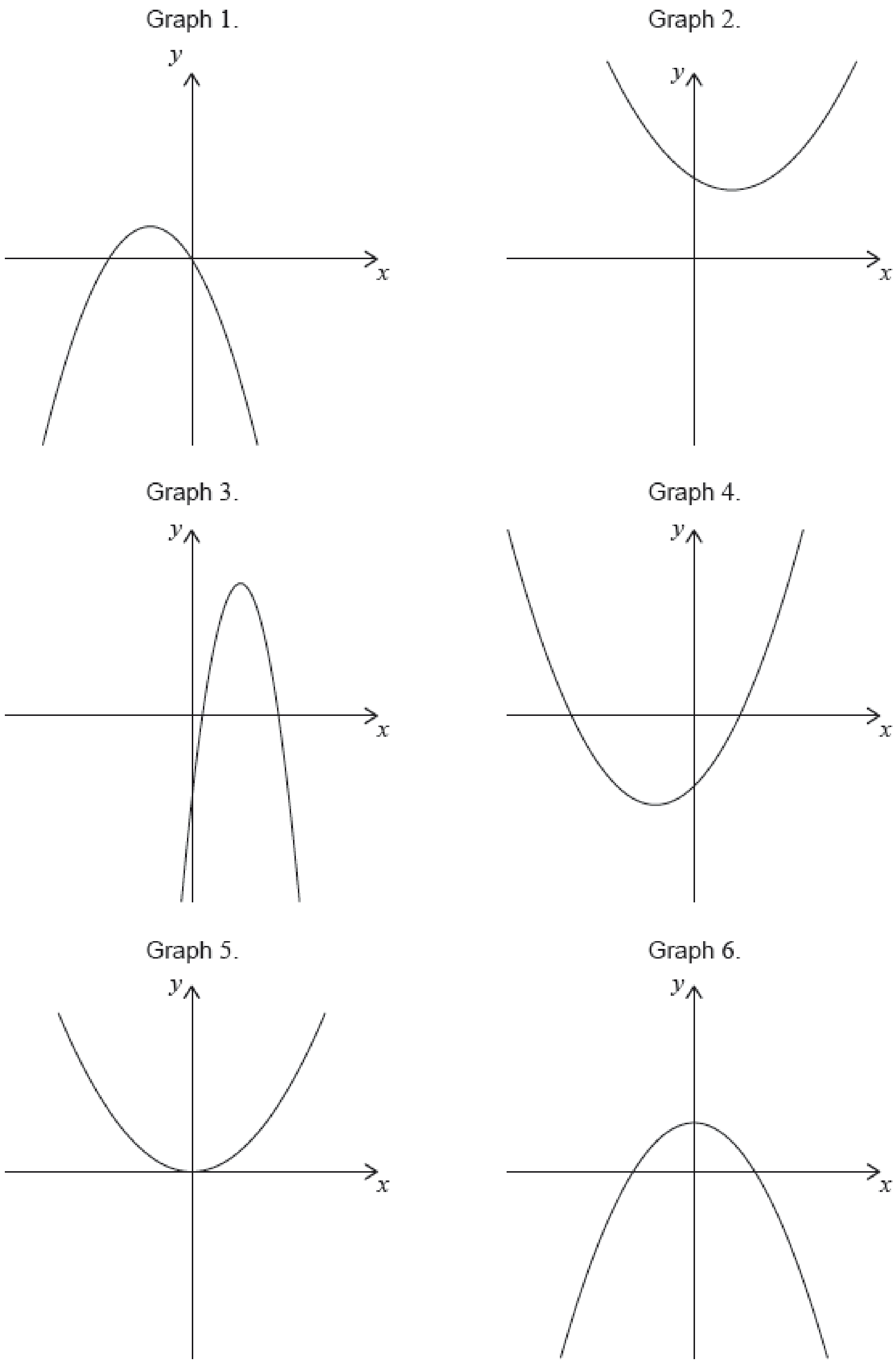
Examiners report

a. The majority of the candidates showed they were able to substitute values into the model. The most common mistake was to neglect converting 1.37 km into metres. Some candidates did not appreciate the practical considerations of this question; Mount Everest will never be less than one metre high. It is important to remind students to check their answers in terms of the context of the information given.

b. The majority of the candidates showed they were able to substitute values into the model. The most common mistake was to neglect converting 1.37 km into metres. Some candidates did not appreciate the practical considerations of this question; Mount Everest will never be less than one metre high. It is important to remind students to check their answers in terms of the context of the information given.

c. The majority of the candidates showed they were able to substitute values into the model. The most common mistake was to neglect converting 1.37 km into metres. Some candidates did not appreciate the practical considerations of this question; Mount Everest will never be less than one metre high. It is important to remind students to check their answers in terms of the context of the information given.

Consider the following graphs of quadratic functions.



The equation of each of the quadratic functions can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

Each of the sets of conditions for the constants a, b and c, in the table below, corresponds to one of the graphs above.

Write down the number of the corresponding graph next to each set of conditions.

Conditions	Graph number
$a > 0$, $b < 0$, $c > 0$	
$a < 0$, $b = 0$, $c > 0$	
$a < 0$, $b > 0$, $c < 0$	
$a > 0$, $b = 0$, $c = 0$	
$a > 0$, $b > 0$, $c < 0$	
$a < 0$, $b < 0$, $c = 0$	

Markscheme

Conditions	Graph Number
$a>0, b<0, c>0$	2
$a<0, b=0, c>0$	6
$a<0, b>0, c<0$	3
$a>0, b=0, c=0$	5
$a>0, b>0, c<0$	4
$a<0, b<0, c=0$	1

(A1)(A1)(A1)(A1)(A1)(A1) ***(C6)***

Note: Award ***(A1)*** for each correct entry.

[6 marks]

Examiners report

[N/A]

The following function models the growth of a bacteria population in an experiment,

$$P(t) = A \times 2^t, \ t \geq 0$$

where A is a constant and t is the time, in hours, since the experiment began.

Four hours after the experiment began, the bacteria population is 6400.

a. Find the value of A. [2]

b. Interpret what A represents in this context. [1]

c. Find the time since the experiment began for the bacteria population to be equal to $40A$. [3]

Markscheme

a. $6400 = A \times 2^4$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 4 and 6400 in equation.

$(A =)\ 400$ ***(A1) (C2)***

[2 marks]

b. the initial population **OR** the population at the start of experiment ***(A1) (C1)***

[1 mark]

c. $40A = A \times 2^t$ **OR** $40 \times 400 = 400 \times 2^t$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into equation. Follow through with their A from part (a).

$40 = 2^t$ ***(M1)***

Note: Award ***(M1)*** for simplifying.

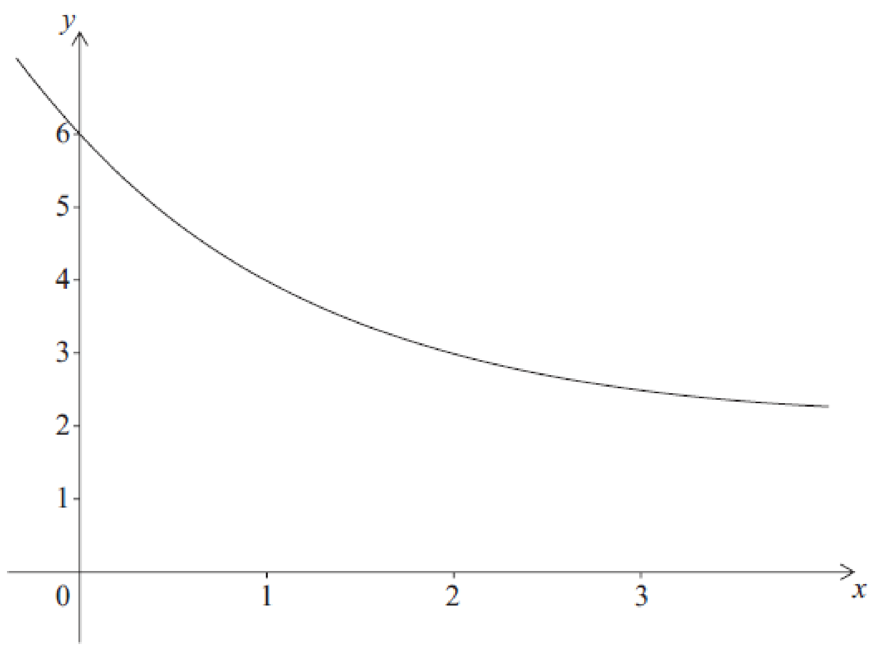
5.32 (5.32192…) (hours) **OR** 5 hours 19.3 (19.3156…) minutes ***(A1) (C3)***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the function $f(x) = p(0.5)^x + q$ where p and q are constants. The graph of $f(x)$ passes through the points $(0,\ 6)$ and $(1,\ 4)$ and is shown below.



a. Write down two equations relating *p* and *q*. [2]

b. Find the value of *p* and of *q*. [2]

c. Write down the equation of the horizontal asymptote to the graph of *f* (*x*). [2]

Markscheme

a. $p + q = 6$ ***(A1)***

$0.5p + q = 4$ ***(A1)*** ***(C2)***

Note: Accept correct equivalent forms of the equations.

[2 marks]

b. $p = 4$, $q = 2$ ***(A1)(A1)*(ft)** ***(C2)***

Notes: If both answers are incorrect, award ***(M1)*** for attempt at solving simultaneous equations.

[2 marks]

c. $y = 2$ ***(A1)(A1)*****(ft)** ***(C2)***

Notes: Award ***(A1)*** for "y = a constant", ***(A1)*****(ft)** for 2. Follow through from their value for q as long as their constant is greater than 2 and less than 6.

An equation must be seen for any marks to be awarded.

[2 marks]

Examiners report

a. A significant number of candidates found it difficult to identify and write two equations that relate p and q. Many of those who wrote the equations were unable to solve them or use their GDC to find the values of p and q in part b). Although the question in part c) was quite standard, there were many errors in the responses. Many students wrote $x = 2$ or only 2 instead of $y = 2$.

b. A significant number of candidates found it difficult to identify and write two equations that relate p and q. Many of those who wrote the equations were unable to solve them or use their GDC to find the values of p and q in part b). Although the question in part c) was quite standard, there were many errors in the responses. Many students wrote $x = 2$ or only 2 instead of $y = 2$.

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---

a. A population of mosquitoes decreases exponentially. The size of the population, P , after t days is modelled by [1]

$P = 3200 \times 2^{-t} + 50$, where $t \geqslant 0$.

Write down the **exact** size of the initial population.

b. Find the size of the population after 4 days. [2]

c. Calculate the time it will take for the size of the population to decrease to 60. [2]

d. The population will stabilize when it reaches a size of k . [1]

Write down the value of k .

Markscheme

a. 3250 ***(A1)*** ***(C1)***

b. $3200 \times 2^{-4} + 50$ ***(M1)***

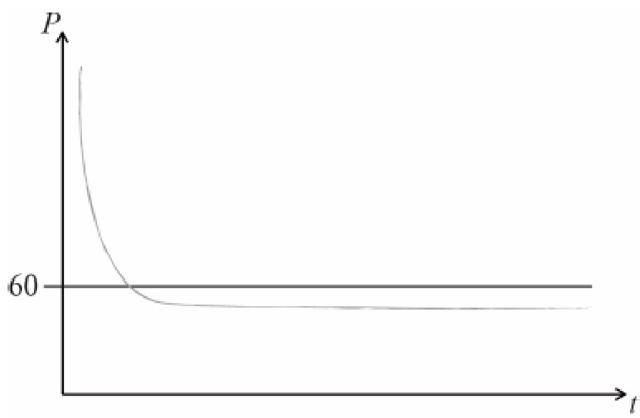
Note: Award ***(M1)*** for substituting t into exponential equation.

$= 250$ ***(A1) (C2)***

c. $3200 \times 2^{-t} + 50 = 60$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation used in part (b).

OR



(M1)

Note: Award ***(M1)*** for a decreasing exponential graph intersecting a horizontal line.

$(t =)\ 8.32\ (8.32192\ldots)$ (days) ***(A1) (C2)***

Note: Accept a final answer of "8 days, 7 hours and 44 minutes", or equivalent. Award **(M0)(A0)** for an answer of 8 days with no working

d. 50 ***(A1) (C1)***

Examiners report

a. Question 13: Exponential model.

Most candidates were able to correctly substitute values into the given exponential model but only the stronger ones found a correct answer. It was expected that candidates would use their calculator to solve the exponential equation rather than use logarithms which is not in the syllabus. The concept of the population stabilizing (horizontal asymptote) was not widely understood.

b. Question 13: Exponential model.

Most candidates were able to correctly substitute values into the given exponential model but only the stronger ones found a correct answer. It was expected that candidates would use their calculator to solve the exponential equation rather than use logarithms which is not in the syllabus. The concept of the population stabilizing (horizontal asymptote) was not widely understood.

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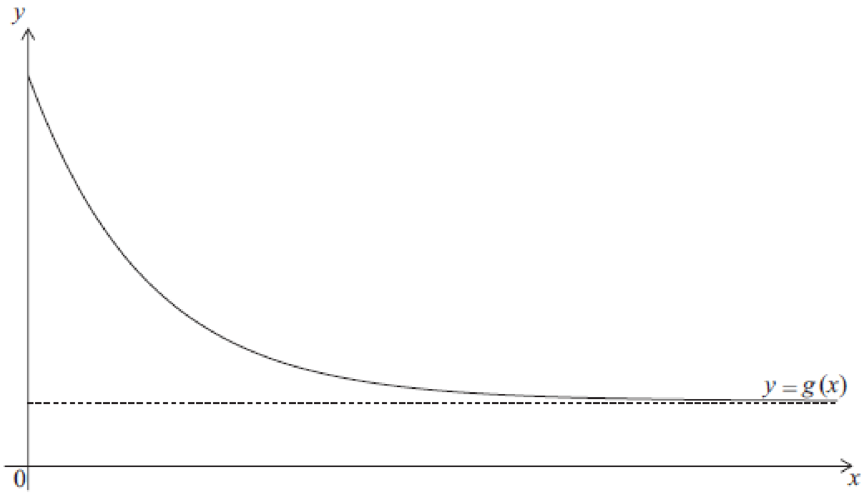
Most candidates were able to correctly substitute values into the given exponential model but only the stronger ones found a correct answer. It was expected that candidates would use their calculator to solve the exponential equation rather than use logarithms which is not in the syllabus. The concept of the population stabilizing (horizontal asymptote) was not widely understood.

The function $g(x)$ is defined as $g(x) = 16 + k(c^{-x})$ where $c > 0$.

The graph of the function g is drawn below on the domain $x \geqslant 0$.



The graph of g intersects the y-axis at (0, 80) .

a. Find the value of k . [2]

b. The graph passes through the point (2, 48) . [2]

Find the value of c .

c. The graph passes through the point (2, 48) . [2]

Write down the equation of the horizontal asymptote to the graph of $y = g(x)$.

Markscheme

a. $80 = 16 + k(c^0)$ ***(M1)***

$k = 64$ ***(A1)*** ***(C2)***

[2 marks]

b. $48 = 16 + 64(c^{-2})$ ***(M1)***

Note: Award ***(M1)*** for substitution of their k and (2, 48) into the equation for $g(x)$.

$c = \sqrt{2}\ (1.41) \quad (1.41421\ldots)$ ***(A1)*(ft)** ***(C2)***

Notes: Award ***(M1)(A1)*(ft)** for $c = \pm\sqrt{2}$. Follow through from their answer to part (a).

[2 marks]

c. $y = 16$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $y =$ a constant, ***(A1)*** for 16.

[2 marks]

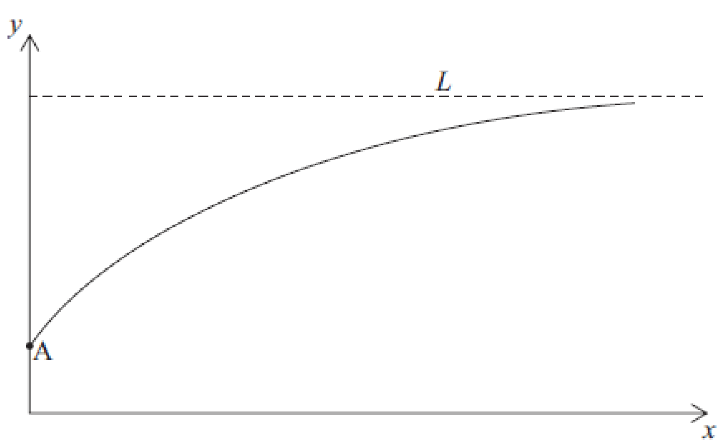
Examiners report

a. This was perhaps the most difficult question on the paper. Being the last question some candidates may have felt that they were under pressure to complete and many scripts showed no attempt at an answer to this question. The response by the upper quartile of candidates was quite encouraging with many achieving at least 4 of the 6 marks available. For the rest, many fell at the first hurdle and were unable to obtain a value of k. This, in turn, led to problems in finding c. For a large number of candidates the only mark that they achieved was identifying that the asymptote was a linear equation in y.

b. This was perhaps the most difficult question on the paper. Being the last question some candidates may have felt that they were under pressure to complete and many scripts showed no attempt at an answer to this question. The response by the upper quartile of candidates was quite encouraging with many achieving at least 4 of the 6 marks available. For the rest, many fell at the first hurdle and were unable to obtain a value of k. This, in turn, led to problems in finding c. For a large number of candidates the only mark that they achieved was identifying that the asymptote was a linear equation in y.

c. This was perhaps the most difficult question on the paper. Being the last question some candidates may have felt that they were under pressure to complete and many scripts showed no attempt at an answer to this question. The response by the upper quartile of candidates was quite encouraging with many achieving at least 4 of the 6 marks available. For the rest, many fell at the first hurdle and were unable to obtain a value of k. This, in turn, led to problems in finding c. For a large number of candidates the only mark that they achieved was identifying that the asymptote was a linear equation in y.

Consider the function $f(x) = 1.25 - a^{-x}$, where a is a positive constant and $x \geqslant 0$. The diagram shows a sketch of the graph of f . The graph intersects the y-axis at point A and the line L is its horizontal asymptote.



a. Find the y-coordinate of A . [2]

b. The point $(2,\ 1)$ lies on the graph of $y = f(x)$. Calculate the value of a . [2]

c. The point $(2,\ 1)$ lies on the graph of $y = f(x)$. Write down the equation of L . [2]

Markscheme

a. $y = 1.25 - a^0 \quad 1.25 - 1$ ***(M1)***

$= 0.25$ ***(A1) (C2)***

Note: Award ***(M1)(A1)*** for $(0, 0.25)$.

[2 marks]

b. $1 = 1.25 - a^{-2}$ ***(M1)***

$a = 2$ ***(A1) (C2)***

[2 marks]

c. $y = 1.25$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for $y =$ “a constant”, ***(A1)*** for 1.25.

[2 marks]

Examiners report

a. Very few candidates showed working and subsequently lost marks due to this. Many candidates seemed to forget that $a^0 = 1$ and not 0.

b. Very few candidates showed working and subsequently lost marks due to this. Many candidates seemed to forget that $a^0 = 1$ and not 0.

c. Very few candidates showed working and subsequently lost marks due to this. Many candidates seemed to forget that $a^0 = 1$ and not 0.

Consider the graph of the function $f(x) = \frac{3}{x} - 2, \ \ x \neq 0.$

a. Write down the equation of the vertical asymptote. [2]

b. Write down the equation of the horizontal asymptote. [2]

c. Calculate the value of *x* for which *f*(*x*) = 0 . [2]

Markscheme

a. *x* = 0 ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for *x* = “a constant” ***(A1)*** for = 0. Award ***(A0)(A0)*** for an answer of “0”.

[2 marks]

b. *f*(*x*) = −2 (*y* = −2) ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for y = “a constant” ***(A1)*** for = −2. Award ***(A0)(A0)*** for an answer of “−2”.

[2 marks]

c. $\frac{3}{x} - 2 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating *f*(*x*) to 0.

$(x =)\ \frac{3}{2}\quad (1.5)$ ***(A1) (C2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. A population of 200 rabbits was introduced to an island. One week later the number of rabbits was 210. The number of rabbits, N , can be modelled by the function [2]

$$N(t) = 200 \times b^t,\ t \geqslant 0\,,$$

where t is the time, in weeks, since the rabbits were introduced to the island.

Find the value of b .

b. Calculate the number of rabbits on the island after 10 weeks. [2]

c. An ecologist estimates that the island has enough food to support a maximum population of 1000 rabbits. [2]

Calculate the number of weeks it takes for the rabbit population to reach this maximum.

Markscheme

a. $210 = 200 \times b^1$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into equation.

$(b =)\ 1.05$ ***(A1)*** ***(C2)***

b. 200×1.05^{10} ***(M1)***

Note: Award ***(M1)*** for correct substitution into formula. Follow through from part (a).

$= 325$ ***(A1)*(ft)** ***(C2)***

Note: The answer must be an integer.

c. $200 \times 1.05^t = 1000$ ***(M1)***

$t = 33.0\ \ (32.9869...)$ ***(A1)*(ft)** ***(C2)***

Note: Award ***(M1)*** for setting up the equation. Accept alternative methods such as $t = \frac{\log\,(5)}{\log\,(1.05)}$, or a sketch of $y = 200 \times 1.05^t$ and $y = 1000$ with indication of point of intersection. Follow through from (a).

Examiners report

a. Question 14: Exponential Function

This question was well-answered by the majority of candidates.

Part (a) was generally accessible, unless subtraction was used in the rearrangement of the formula.

b. Part (b) required an integer value.

c. Part (c) saw good use of the GDC – with many sketch graphs being shown on paper (this is to be encouraged) – as well as attempts using logarithms. Use of the GDC is to be encouraged as its efficient use is a mandatory part of the course. Logarithms are not discouraged – but they are not a necessary component of the course and it is easy to construct equations that are not accessible to solution by logarithms.

A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it is first started.

a. Find the number of teenagers who started the rumour. [2]

b. Write down the number of teenagers who have heard the rumour five hours after it is first started. [1]

c. Determine the length of time it would take for 150 teenagers to have heard the rumour. **Give your answer correct to the nearest minute.** [3]

Markscheme

a. $N = 2 \times (1.81)^{0.7 \times 0}$ ***(M1)***

$N = 2$ ***(A1)*** ***(C2)***

Notes: Award ***(M1)*** for correct substitution of $t = 0$.
Award ***(A1)*** for correct answer.

[2 marks]

b. 16.0 (3 s.f) ***(A1)*** ***(C1)***

Note: Accept 16 and 15.

[1 mark]

c. $150 = 2 \times (1.81)^{0.7t}$ ***(M1)***

$t = 10.39...$ h ***(A1)***

$t = 624$ minutes ***(A1)*****(ft)*** ***(C3)***

Notes: Accept 10 hours 24 minutes. Accept alternative methods.

Award last ***(A1)*(ft)** for correct rounding to the nearest minute of their answer.

Unrounded answer must be seen so that the follow through can be awarded.

[3 marks]

Examiners report

a. Parts (a) and (b) were confidently answered with many candidates correctly finding the number who started the rumour and also the number involved after 5 hours. A common mistake was to let $t = 0$ but not evaluate the expression correctly.

b. Parts (a) and (b) were confidently answered with many candidates correctly finding the number who started the rumour and also the number involved after 5 hours. A common mistake was to let $t = 0$ but not evaluate the expression correctly.

c. Very few candidates could answer part (c). With the working shown, it was obvious candidates could correctly state the equation, but could not use their calculators to find the value of t.

Jashanti is saving money to buy a car. The price of the car, in US Dollars (USD), can be modelled by the equation

$$P = 8500\,(0.95)^t.$$

Jashanti's savings, in USD, can be modelled by the equation

$$S = 400t + 2000.$$

In both equations t is the time in months since Jashanti started saving for the car.

Jashanti does not want to wait too long and wants to buy the car two months after she started saving. She decides to ask her parents for the extra money that she needs.

a. Write down the amount of money Jashanti saves per month. [1]

b. Use your graphic display calculator to find how long it will take for Jashanti to have saved enough money to buy the car. [2]

c. Calculate how much extra money Jashanti needs. [3]

Markscheme

a. 400 (USD) ***(A1)*** ***(C1)***

[1 mark]

b. $8500\,(0.95)^t = 400 \times t + 2000$ ***(M1)***

Note: Award ***(M1)*** for equating $8500(0.95)^t$ to $400 \times t + 2000$ or for comparing the difference between the two expressions to zero or for showing a sketch of both functions.

$(t =)\ 8.64\ (\text{months})\ \ (8.6414\ldots\ (\text{months}))$ ***(A1) (C2)***

Note: Accept 9 months.

[2 marks]

c. $8500(0.95)^2 - (400 \times 2 + 2000)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $t = 2$ into equation for P, ***(M1)*** for finding the difference between a value/expression for P and a value/expression for S. The first ***(M1)*** is implied if 7671.25 seen.

4870 (USD) (4871.25) ***(A1) (C3)***

Note: Accept 4871.3.

[3 marks]

Examiners report

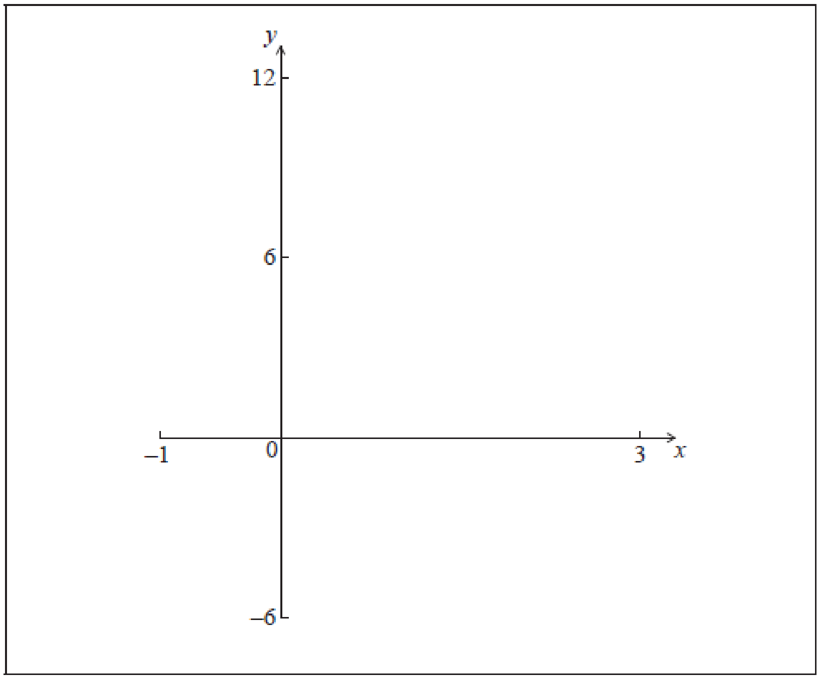
a. [N/A]
b. [N/A]
c. [N/A]

The x-coordinate of the minimum point of the quadratic function $f(x) = 2x^2 + kx + 4$ is $x = 1.25$.

a. (i) Find the value of k . [4]

(ii) Calculate the y-coordinate of this minimum point.

b. Sketch the graph of $y = f(x)$ for the domain $-1 \leqslant x \leqslant 3$. [2]



Markscheme

a. (i) $1.25 = -\frac{k}{2(2)}$ ***(M1)***

OR

$f'(x) = 4x + k = 0$ ***(M1)***

Note: Award ***(M1)*** for setting the gradient function to zero.

$k = -5$ ***(A1)*** ***(C2)***

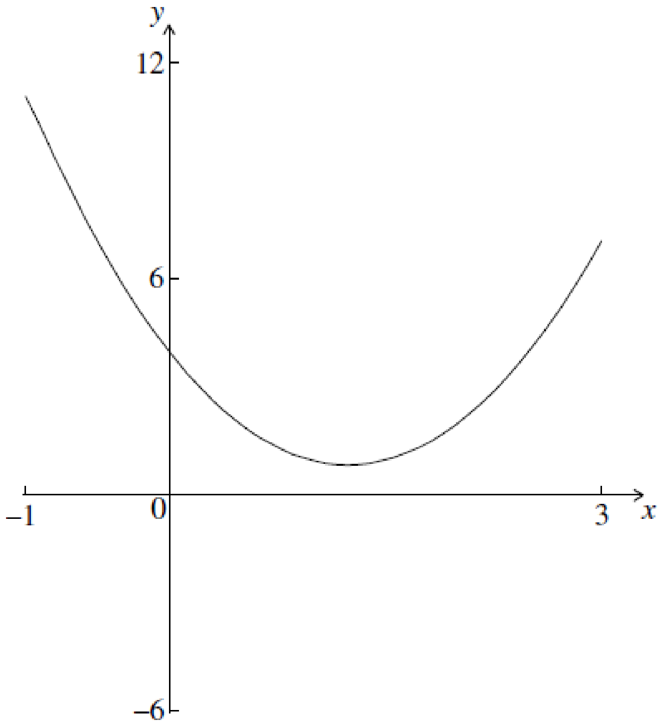
(ii) $2(1.25)^2 - 5(1.25) + 4$ ***(M1)***

$= 0.875$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from their k.

[4 marks]

b.



(A1)*(ft)*(A1)*(ft)** ***(C2)

Notes: Award ***(A1)*(ft)** for a curve with correct concavity consistent with their k passing through (0, 4).

***(A1)*(ft)** for minimum in approximately the correct place. Follow through from their part (a).

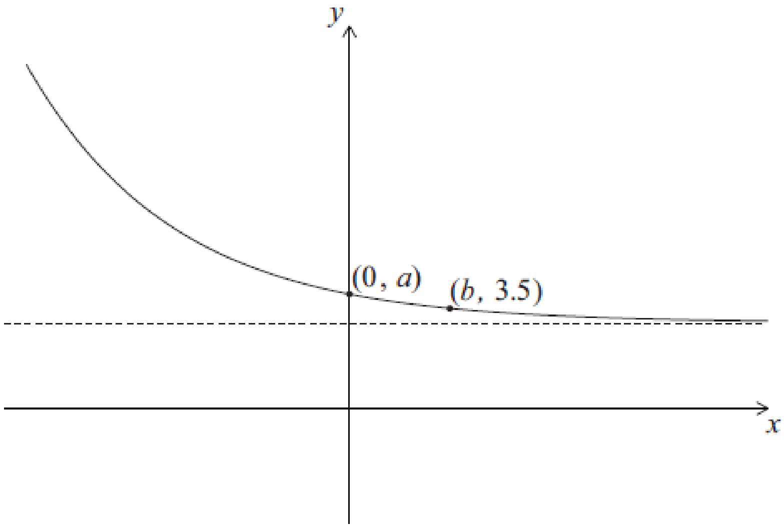
[2 marks]

Examiners report

a. This question was not answered well at all except by the more able. Indeed, of the lower quartile of candidates, the maximum mark achieved was only 1. Of those that did make a successful attempt at the question, very few used the fact that $1.25 = -\frac{k}{2(2)}$ preferring instead to differentiate and equate to zero. But such candidates were in the minority as substituting $x = 1.25$ into the given quadratic and equating to zero produced the popular, but erroneous, answer of -5.7. Recovery was possible for the next two marks if this incorrect value had been seen to be substituted into the correct quadratic, along with $x = 1.25$ to arrive at an answer of 0. This would have given (M1)(A1)(ft). However, candidates who had an answer of $k = -5.7$ in part (a)(i), invariably showed no working in part (ii) and consequently earned no marks here. Irrespective of incorrect working in part (a), the quadratic function clearly passes through (0, 4) and has a minimum at $x = 1.25$. Using this information, a minority of candidates picked up at least one of the two marks in part (b).

b. This question was not answered well at all except by the more able. Indeed, of the lower quartile of candidates, the maximum mark achieved was only 1. Of those that did make a successful attempt at the question, very few used the fact that $1.25 = -\frac{k}{2(2)}$ preferring instead to differentiate and equate to zero. But such candidates were in the minority as substituting $x = 1.25$ into the given quadratic and equating to zero produced the popular, but erroneous, answer of -5.7. Recovery was possible for the next two marks if this incorrect value had been seen to be substituted into the correct quadratic, along with $x = 1.25$ to arrive at an answer of 0. This would have given (M1)(A1)(ft). However, candidates who had an answer of $k = -5.7$ in part (a)(i), invariably showed no working in part (ii) and consequently earned no marks here. Irrespective of incorrect working in part (a), the quadratic function clearly passes through (0, 4) and has a minimum at $x = 1.25$. Using this information, a minority of candidates picked up at least one of the two marks in part (b).

The diagram shows part of the graph of $y = 2^{-x} + 3$, and its horizontal asymptote. The graph passes through the points (0, a) and (b, 3.5).



a. Find the value of [4]

(i) a ;

(ii) b .

b. Write down the equation of the horizontal asymptote to this graph. [2]

Markscheme

a. (i) $2^0 + 3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$= 4$ ***(A1)*** ***(C2)***

(ii) $3.5 = 2^{-b} + 3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$b = 1$ ***(A1)*** ***(C2)***

[4 marks]

b. $y = 3$ ***(A1)(A1)*** ***(C2)***

Notes: y = constant (other than 3) award ***(A1)(A0)***.

[2 marks]

Examiners report

a. Most candidates answered parts (a) i and ii correctly, however a large number of candidates could not find the correct equation for part (b).

b. Most candidates answered parts (a) i and ii correctly, however a large number of candidates could not find the correct equation for part (b).

In an experiment it is found that a culture of bacteria triples in number every four hours. There are 200 bacteria at the start of the experiment.

Hours	0	4	8	12	16
No. of bacteria	200	600	a	5400	16200

a. Find the value of a. [1]

b. Calculate how many bacteria there will be after one day. [2]

c. Find how long it will take for there to be two million bacteria. [3]

Markscheme

a. $a = 1800$ ***(A1)*** ***(C1)***

[1 mark]

b. 200×3^6 (or 16200×9) $= 145800$ ***(M1)(A1)*** ***(C2)***

[2 marks]

c. $200 \times 3^n = 2 \times 10^6$ (where n is each 4 hour interval) ***(M1)***

Note: Award ***(M1)*** for attempting to set up the equation or writing a list of numbers.

$3^n = 10^4$

$n = 8.38$ (8.383613097) *correct answer only* ***(A1)***

$\text{Time} = 33.5$ hours *(accept 34, 35 or 36 if previous* ***A*** *mark awarded)* ***(A1)*****(ft)** ***(C3)***

Note: ***(A1)*****(ft)** for correctly multiplying their answer by 4. If 34, 35 or 36 seen, or $32 - 36$ seen, award ***(M1)(A0)(A0)***.

[3 marks]

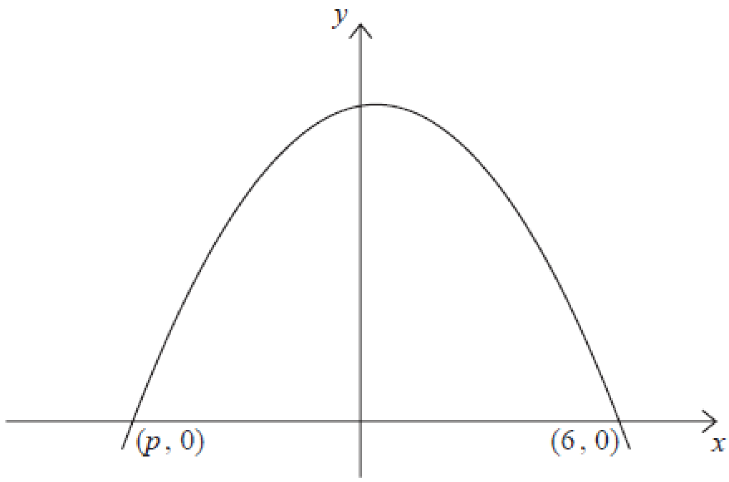
Examiners report

a. Parts (a) and (b) were answered well with most candidates attempting and gaining marks. Very few candidates gained maximum marks for part (c) with most using a list to find the number of hours rather than the formula.

b. Parts (a) and (b) were answered well with most candidates attempting and gaining marks. Very few candidates gained maximum marks for part (c) with most using a list to find the number of hours rather than the formula.

c. Parts (a) and (b) were answered well with most candidates attempting and gaining marks. Very few candidates gained maximum marks for part (c) with most using a list to find the number of hours rather than the formula.

The diagram below shows the graph of a quadratic function. The graph passes through the points (6, 0) and (p, 0). The maximum point has coordinates (0.5, 30.25).



a. Calculate the value of *p*. [2]

b. Given that the quadratic function has an equation $y = -x^2 + bx + c$ where $b,\ c \in \mathbb{Z}$, find b and c. [4]

Markscheme

a. $\frac{(p+6)}{2} = 0.5$ ***(M1)***

$p = -5$ ***(A1)*** ***(C2)***

[2 marks]

b. $\frac{-b}{2(-1)} = 0.5$ ***(M1)***

$b = 1$ ***(A1)***

$-0.5^2 + 0.5 + c = 30.25$ ***(M1)***

$c = 30$ ***(A1)*****(ft)**

Note: Follow through from their value of *b*.

OR

$y = (6 - x)(5 + x)$ ***(M1)***

$= 30 + x - x^2$ ***(A1)***

$b = 1,\ c = 30$ ***(A1)(A1)*****(ft)** ***(C4)***

Note: Follow through from their value of *p* in part (a).

[4 marks]

Examiners report

a. This question was one of the most difficult in this paper. Many students left this question blank, showed incorrect working or gave answers without any preceding working.

b. This question was one of the most difficult in this paper. Many students left this question blank, showed incorrect working or gave answers without any preceding working.

The number of cells, C, in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.

The number of cells in the culture at 16:00 on Monday is 53.

a. Use the above information to write down two equations in p and q ; [2]

b. Use the above information to calculate the value of p and of q ; [2]

c. Use the above information to find the number of cells in the culture at 22:00 on Monday. [2]

Markscheme

a. $p + q = 47$ ***(A1)***

$4p + q = 53$ ***(A1)*** ***(C2)***

[2 marks]

b. Reasonable attempt to solve their equations ***(M1)***

$p = 2, q = 45$ ***(A1)*** ***(C2)***

Note: Accept only the answers $p = 2, q = 45$.

[2 marks]

c. $C = 2 \times 2^{0.5(10)} + 45$ ***(M1)***

$C = 109$ ***(A1)*(ft)** ***(C2)***

Note: Award ***(M1)*** for substitution of 10 into the formula with their values of p and q.

[2 marks]

Examiners report

a. Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

The concept of the zero index was not understood by many.

b. Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

The use of the GDC was (as always) expected in solving the simultaneous equations.

c. Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

Working was required for follow through in this part.

The graph of a quadratic function has y-intercept 10 and **one** of its x-intercepts is 1.

The x-coordinate of the vertex of the graph is 3.

The equation of the quadratic function is in the form $y = ax^2 + bx + c$.

a. Write down the value of c. [1]

b. Find the value of a and of b. [4]

c. Write down the second x-intercept of the function. [1]

Markscheme

a. 10 ***(A1)*** ***(C1)***

Note: Accept $(0,\ 10)$.

[1 mark]

b. $3 = \frac{-b}{2a}$

$0 = a(1)^2 + b(1) + c$

$10 = a(6)^2 + b(6) + c$

$0 = a(5)^2 + b(5) + c$ ***(M1)(M1)***

Note: Award ***(M1)*** for each of the above equations, provided they are not equivalent, up to a maximum of ***(M1)(M1)***. Accept equations that substitute their 10 for c.

OR

sketch graph showing given information: intercepts $(1,\ 0)$ and $(0,\ 10)$ and line $x = 3$ ***(M1)***

$y = a(x-1)(x-5)$ ***(M1)***

Note: Award ***(M1)*** for $(x-1)(x-5)$ seen.

$a = 2$ ***(A1)*(ft)**

$b = -12$ ***(A1)*(ft)** ***(C4)***

Note: Follow through from part (a).

If it is not clear which is a and which is b award at most ***(A0)(A1)(ft)***.

[4 marks]

c. 5 ***(A1)*** ***(C1)***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The amount of electrical charge, C, stored in a mobile phone battery is modelled by $C(t) = 2.5 - 2^{-t}$, where t, in hours, is the time for which the battery is being charged.

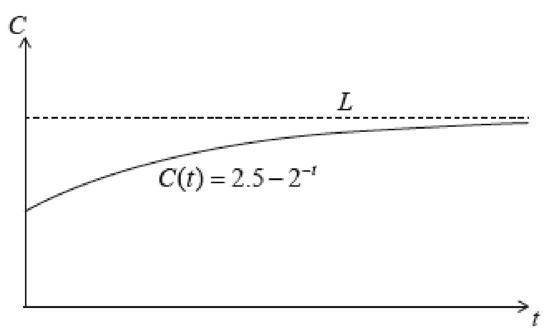


diagram not to scale

a. Write down the amount of electrical charge in the battery at $t = 0$. [1]

b. The line L is the horizontal asymptote to the graph. [2]

Write down the equation of L.

c. To download a game to the mobile phone, an electrical charge of 2.4 units is needed. [3]

Find the time taken to reach this charge. Give your answer correct to the nearest minute.

Markscheme

a. 1.5 ***(A1)*** ***(C1)***

[1 mark]

b. $C = 2.5$ (accept $y = 2.5$) ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for $C\ (\text{or } y) =$ a positive constant, ***(A1)*** for the constant $= 2.5$.

Answer must be an equation.

[2 marks]

c. $2.4 = 2.5 - 2^{-t}$ ***(M1)***

Note: Award ***(M1)*** for setting the equation equal to 2.4 or for a horizontal line drawn at approximately $C = 2.4$.

Allow x instead of t.

OR

$-t\ln(2) = \ln(0.1)$ ***(M1)***

$t = 3.32192\ldots$ ***(A1)***

$t = 3 \text{ hours and } 19 \text{ minutes } (199 \text{ minutes})$ ***(A1)*(ft)** ***(C3)***

Note: Award the final ***(A1)*(ft)** for correct conversion of **their** time in hours to the nearest minute.

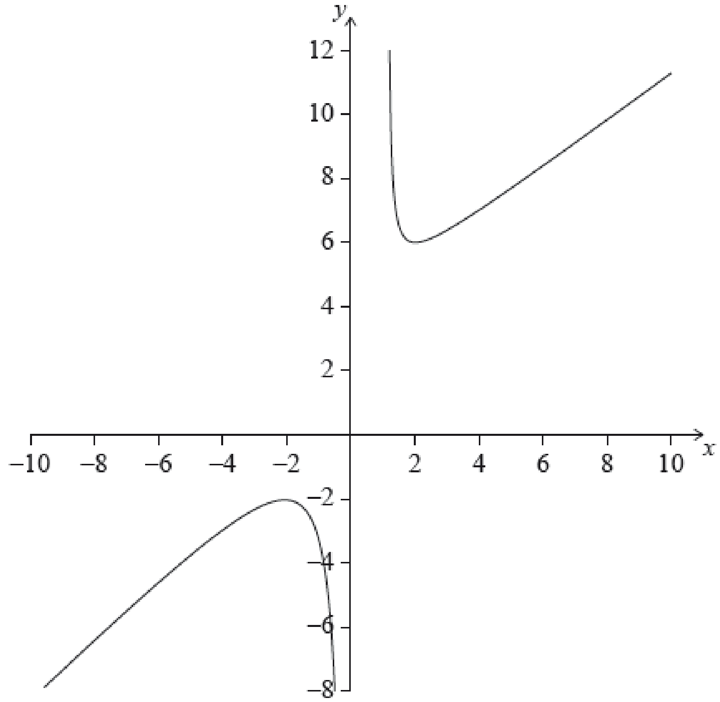
[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The function f is of the form $f(x) = ax + b + \frac{c}{x}$, where a , b and c are positive integers.

Part of the graph of $y = f(x)$ is shown on the axes below. The graph of the function has its local maximum at $(-2,\ -2)$ and its local minimum at $(2,\ 6)$.



a. Write down the domain of the function. [2]

b.i. Draw the line $y = -6$ on the axes. [1]

b.ii. Write down the number of solutions to $f(x) = -6$. [1]

c. Find the range of values of k for which $f(x) = k$ has no solution. [2]

Markscheme

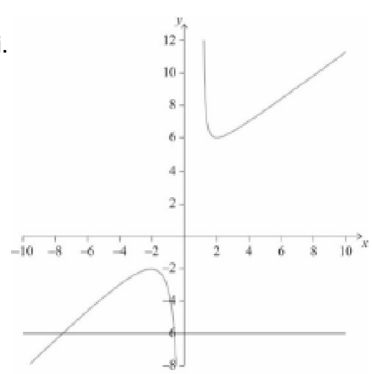
a. $(x \in \mathbb{R}),\ x \neq 0$ ***(A2)*** ***(C2)***

Note: Accept equivalent notation. Award ***(A1)(A0)*** for $y \neq 0$.

Award ***(A1)*** for a clear statement that demonstrates understanding of the meaning of domain. For example, $\mathrm{D} : (-\infty,\ 0) \cup (1,\ \infty)$ should be awarded ***(A1)(A0)***.

[2 marks]

b.i.



(A1) ***(C1)***

Note: The command term “Draw” states: “A ruler (straight edge) should be used for straight lines”; do not accept a freehand $y = -6$ line.

[1 mark]

b.ii. 2 ***(A1)*(ft)** ***(C1)***

Note: Follow through from part (b)(i).

[1 mark]

c. $-2 < k < 6$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for both end points correct and ***(A1)*** for correct **strict** inequalities.

Award at most ***(A1)(A0)*** if the stated variable is different from k or y for example $-2 < x < 6$ is ***(A1)(A0)***.

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]

A computer virus spreads according to the exponential model

$$N = 200 \times (1.9)^{0.85t},\ t \geqslant 0$$

where N is the number of computers infected, and t is the time, in hours, after the initial infection.

a. Calculate the number of computers infected after 6 hours. [2]

b. Calculate the time for the number of infected computers to be greater than $1\,000\,000$. [4]

Give your answer correct to the nearest hour.

Markscheme

a. $200 \times (1.9)^{0.85 \times 6}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into given formula.

$= 5280$ ***(A1)*** ***(C2)***

Note: Accept 5281 or 5300 but no other answer.

[2 marks]

b. $1\,000\,000 < 200 \times (1.9)^{0.85t}$ ***(M1)(M1)***

Note: Award ***(M1)*** for setting up the inequality (accept an equation), and ***(M1)*** for $1\,000\,000$ seen in the inequality or equation.

$t = 15.6 \quad (15.6113\ldots)$ ***(A1)***

16 hours ***(A1)*(ft)*** ***(C4)***

Note: The final ***(A1)*(ft)** is for rounding **up** their answer to the nearest hour.

Award ***(C3)*** for an answer of 15.6 with no working.

Accept $1\,000\,001$ in an equation.

[4 marks]

Examiners report

a. This question was answered very well, although some candidates were not awarded the final mark because the answer was not an integer number of computers. In part (b), some candidates neglected to give their answer correct to the nearest hour and lost the final mark.

b. This question was answered very well, although some candidates were not awarded the final mark because the answer was not an integer number of computers. In part (b), some candidates neglected to give their answer correct to the nearest hour and lost the final mark.

Passengers of Flyaway Airlines can purchase tickets for either Business Class or Economy Class.

On one particular flight there were 154 passengers.

Let x be the number of Business Class passengers and y be the number of Economy Class passengers on this flight.

On this flight, the cost of a ticket for each Business Class passenger was 320 euros and the cost of a ticket for each Economy Class passenger was 85 euros. The total amount that Flyaway Airlines received for these tickets was $14\,970$ euros.

The airline's finance officer wrote down the total amount received by the airline for these tickets as $14\,270$ euros.

a. Use the above information to write down an equation in x and y. [1]

b. Use the information about the cost of tickets to write down a second equation in x and y. [1]

c. Find the value of x and the value of y. [2]

d. Find the percentage error. [2]

Markscheme

a. $x + y = 154$ ***(A1)*** ***(C1)***

[1 mark]

b. $320x + 85y = 14\,970$ ***(A1)*** ***(C1)***

[1 mark]

c. $x = 8,\ y = 146$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Note: Follow through from parts (a) and (b) irrespective of working seen, **but only** if both values are positive integers.

Award ***(M1)(A0)*** for a reasonable attempt to solve simultaneous equations algebraically, leading to at least one incorrect or missing value.

[2 marks]

d. $\left|\frac{14270-14970}{14970}\right| \times\ 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into percentage error formula.

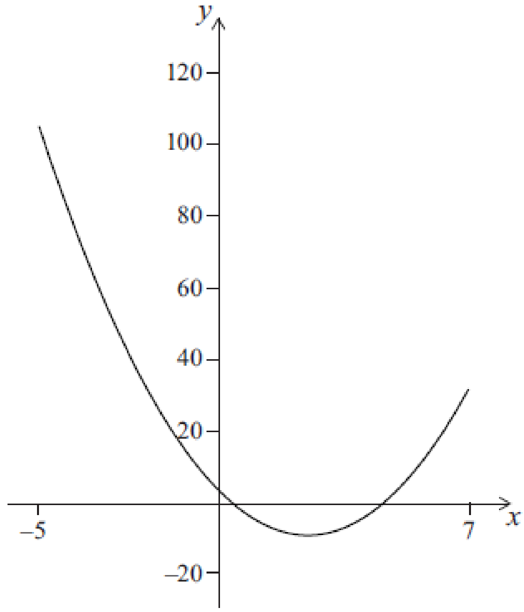
$= 4.68(\%)\ (4.67601\ldots)$ ***(A1) (C2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The graph of $y = 2x^2 - rx + q$ is shown for $-5 \leqslant x \leqslant 7$.



The graph cuts the y axis at (0, 4).

a. Write down the value of q. [1]

b. The axis of symmetry is x = 2.5. [2]

Find the value of r.

c. The axis of symmetry is x = 2.5. [1]

Write down the minimum value of y.

d. The axis of symmetry is x = 2.5.x [2]

Write down the range of y.

Markscheme

a. $q = 4$ ***(A1)*** ***(C1)***

[1 mark]

b. $2.5 = \frac{r}{4}$ ***(M1)***

$r = 10$ ***(A1)*** ***(C2)***

[2 marks]

c. –8.5 ***(A1)*(ft)** ***(C1)***

[1 mark]

d. $-8.5 \leqslant y \leqslant 104$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Notes: Award ***(A1)*(ft)** for their answer to part (c) with correct inequality signs, ***(A1)*(ft)** for 104. Follow through from their values of q and r.

Accept 104 ±2 if read from graph.

[2 marks]

Examiners report

a. This question was not well answered with few candidates gaining full marks. Many candidates could find the value of q but not r. Although many found the minimum value of y, they could not find the maximum value of the function or express the range correctly.

b. This question was not well answered with few candidates gaining full marks. Many candidates could find the value of q but not r. Although many found the minimum value of y, they could not find the maximum value of the function or express the range correctly.

c. This question was not well answered with few candidates gaining full marks. Many candidates could find the value of q but not r. Although many found the minimum value of y, they could not find the maximum value of the function or express the range correctly.

d. This question was not well answered with few candidates gaining full marks. Many candidates could find the value of q but not r. Although many found the minimum value of y, they could not find the maximum value of the function or express the range correctly.

A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3,\ x \neq 0.$

a. Write down the derivative of f. [3]

b. Find the point on the graph of f at which the gradient of the tangent is equal to 6. [3]

Markscheme

a. $12x^2 - \frac{6}{x^3}$ or equivalent ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $12x^2$, ***(A1)*** for -6 and ***(A1)*** for $\frac{1}{x^3}$ or x^{-3}. Award at most ***(A1)(A1)(A0)*** if additional terms seen.

[3 marks]

b. $12x^2 - \frac{6}{x^3} = 6$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to 6.

$(1,\ 4)$ **OR** $x = 1,\ y = 4$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***(C3)***

Note: A frequent wrong answer seen in scripts is $(1,\ 6)$ for this answer with correct working award ***(M1)(A0)(A1)*** and if there is no working award ***(C1)***.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

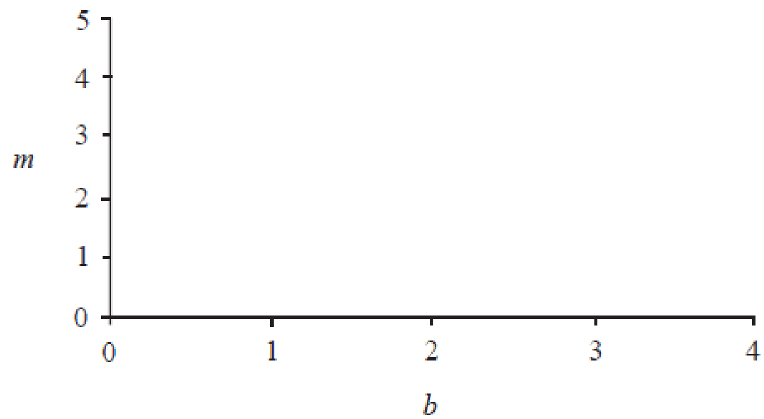
A store sells bread and milk. On Tuesday, 8 loaves of bread and 5 litres of milk were sold for \$21.40. On Thursday, 6 loaves of bread and 9 litres of milk were sold for \$23.40.

If $b =$ the price of a loaf of bread and $m =$ the price of one litre of milk, Tuesday's sales can be written as $8b + 5m = 21.40$.

a. Using simplest terms, write an equation in *b* and *m* for Thursday's sales. [2]

b. Find *b* and *m*. [2]

c. Draw a sketch, in the space provided, to show how the prices can be found graphically. [2]



Markscheme

a. Thursday's sales, $6b + 9m = 23.40$ ***(A1)***

$2b + 3m = 7.80$ ***(A1)*** ***(C2)***

[2 marks]

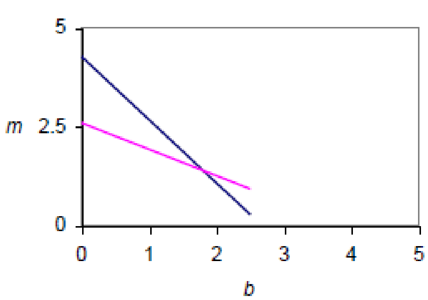
b. $m = 1.40$ *(accept 1.4)* ***(A1)*(ft)**

$b = 1.80$ *(accept 1.8)* ***(A1)*(ft)**

*Award **(A1)**(d) for a reasonable attempt to solve by hand and answer incorrect.* ***(C2)***

[2 marks]

c.



***(A1)(A1)*(ft)**

***(A1)** each for two reasonable straight lines. The intersection point must be approximately correct to earn both marks, otherwise penalise at least one line.*

Note: The follow through mark is for candidate's line from (a). ***(C2)***

[2 marks]

Examiners report

a. a) Nearly all the candidates were able to write the equation but very few simplified it.

b. b) A majority of candidates were able to find the values of b and m. Some used the right method but made arithmetical errors, many of which were due to them using the method of substitution which involved fractions. GDC use was expected.

c. c) A majority of candidates did not attempt this part. For those who did, very few were able to sketch the graph correctly. Common errors were to plot the point (1.4, 1.8) or draw a straight line through that point and the origin.

A hotel has a rectangular swimming pool. Its length is x metres, its width is y metres and its perimeter is 44 metres.

a. Write down an equation for x and y. [1]

b. The area of the swimming pool is $112\mathrm{m}^2$. [1]

Write down a second equation for x and y.

c. Use your graphic display calculator to find the value of x and the value of y. [2]

d. An Olympic sized swimming pool is 50 m long and 25 m wide. [2]

Determine the area of the hotel swimming pool as a percentage of the area of an Olympic sized swimming pool.

Markscheme

a. $2x + 2y = 44$ ***(A1) (C1)***

Note: Accept equivalent forms.

b. $xy = 112$ ***(A1) (C1)***

c. $8,\ 14$ ***(A1)*(ft)*(A1)*(ft)** ***(C2)***

Notes: Accept $x = 8, y = 14$ **OR** $x = 14, y = 8$

Follow through from their answers to parts (a) and (b) only if both values are positive.

d. $\frac{112}{1250} \times 100$ ***(M1)***

Note: Award ***(M1)*** for 112 divided by 1250.

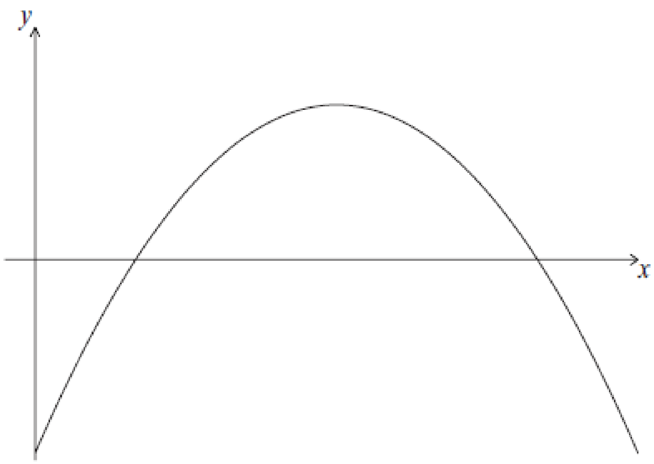
$= 8.96$ ***(A1) (C2)***

Note: Do not penalize if percentage sign seen.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Part of the graph of the quadratic function *f* is given in the diagram below.



On this graph one of the x-intercepts is the point (5, 0). The x-coordinate of the maximum point is 3.

The function *f* is given by $f(x) = -x^2 + bx + c$, where $b, c \in \mathbb{Z}$

a. Find the value of [3]

(i) b ;

(ii) c .

b. The domain of *f* is $0 \le x \le 6$. [3]

Find the range of *f* .

Markscheme

a. (i) $3 = \frac{-b}{-2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in formula.

OR

$-1 + b + c = 0$

$-25 + 5b + c = 0$

$-24 + 4b = 0$ ***(M1)***

Note: Award ***(M1)*** for setting up 2 correct simultaneous equations.

OR

$-2x + b = 0$ ***(M1)***

Note: Award ***(M1)*** for correct derivative of $f(x)$ equated to zero.

$b = 6$ ***(A1)*** ***(C2)***

(ii) $0 = -(5)^2 + 6 \times 5 + c$

$c = -5$ ***(A1)*(ft)** ***(C1)***

Note: Follow through from their value for *b*.

Note: Alternatively candidates may answer part (a) using the method below, and not as two separate parts.

$(x-5)(-x+1)$ ***(M1)***

$-x^2 + 6x - 5$ ***(A1)***

$b = 6\ c = -5$ ***(A1)*** ***(C3)***

[3 marks]

b. $-5 \le y \le 4$ ***(A1)*(ft)*(A1)*(ft)*(A1)*** ***(C3)***

Notes: Accept [–5, 4]. Award ***(A1)*(ft)** for –5, ***(A1)*(ft)** for 4. ***(A1)*** for inequalities in the correct direction or brackets with values in the correct order or a clear word statement of the range. Follow through from their part (a).

[3 marks]

Examiners report

a. Question 11 proved to be the most problematic of the whole paper. Many candidates attempted this question but were not able to set up a system of equations to find the value of b or use the formula $x = \frac{-b}{2a}$. From the working seen, many candidates did not understand the non-standard notation for the domain, with a number believing it to be a coordinate pair. This was taken into careful consideration by the senior examiners when setting the grade boundaries for this paper.

b. Question 11 proved to be the most problematic of the whole paper. Many candidates attempted this question but were not able to set up a system of equations to find the value of b or use the formula $x = \frac{-b}{2a}$. From the working seen, many candidates did not understand the non-standard notation for the domain, with a number believing it to be a coordinate pair. This was taken into careful consideration by the senior examiners when setting the grade boundaries for this paper.

In an experiment, a number of fruit flies are placed in a container. The population of fruit flies, *P* , increases and can be modelled by the function

$$P(t) = 12 \times 3^{0.498t},\ t \geqslant 0,$$

where *t* is the number of days since the fruit flies were placed in the container.

a.i. Find the number of fruit flies which were placed in the container. [2]

a.ii. Find the number of fruit flies that are in the container after 6 days. [2]

b. The maximum capacity of the container is 8000 fruit flies. [2]

Find the number of days until the container reaches its maximum capacity.

Markscheme

a.i. $12 \times 3^{0.498\times 0}$ ***(M1)***

Note: Award ***(M1)*** for substituting zero into the equation.

= 12 ***(A1) (C2)***

[2 marks]

a.ii. $12 \times 3^{0.498\times 6}$ ***(M1)***

Note: Award ***(M1)*** for substituting 6 into the equation.

320 ***(A1) (C2)***

Note: Accept an answer of 319.756… or 319.

[2 marks]

b. $8000 = 12 \times 3^{0.498\times t}$ ***(M1)***

Note: Award ***(M1)*** for equating equation to 8000.
Award ***(M1)*** for a sketch of P(*t*) intersecting with the straight line *y* = 8000.

= 11.9 (11.8848…) ***(A1) (C2)***

Note: Accept an answer of 11 or 12.

[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]

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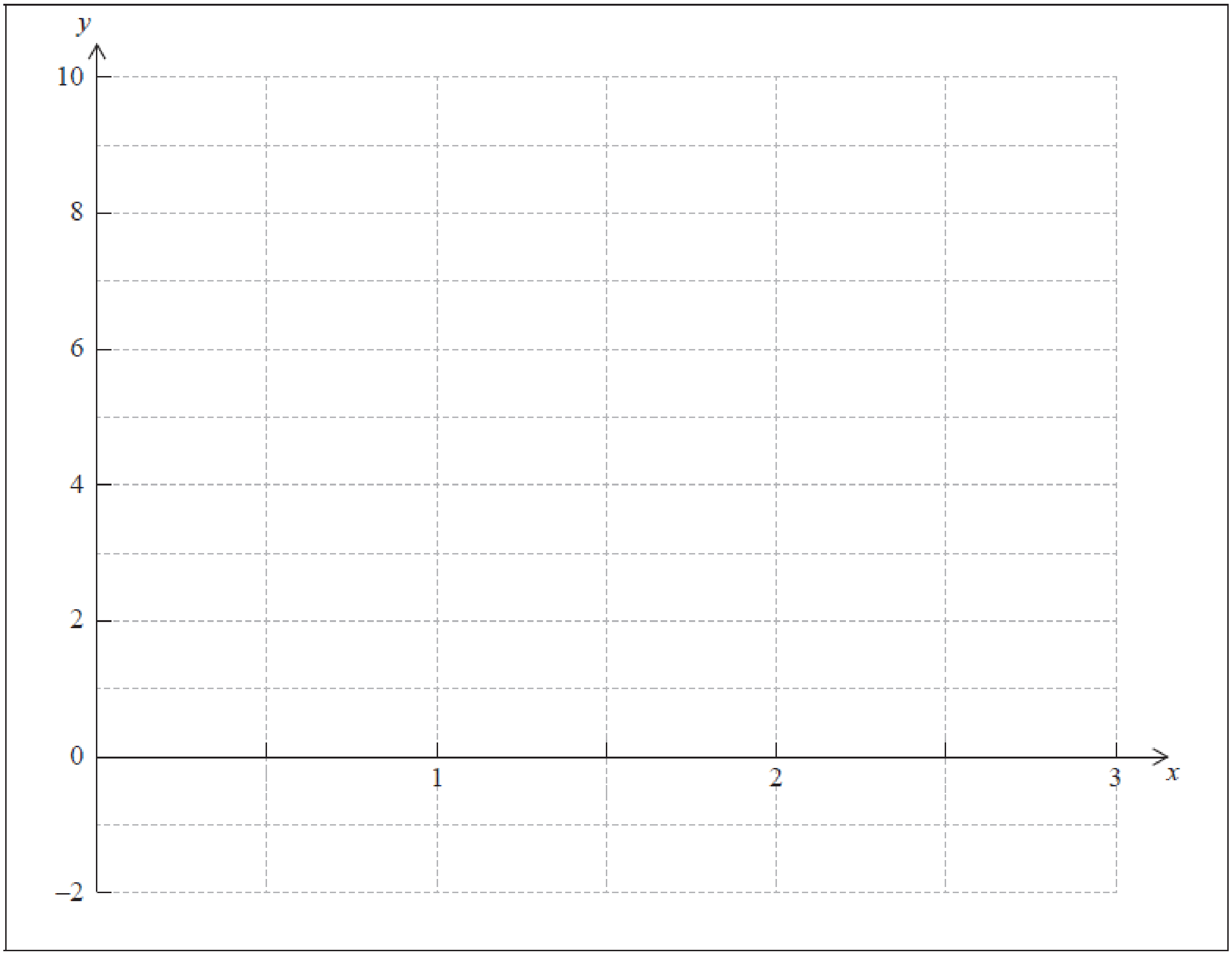
a. On the grid below sketch the graph of the function $f(x) = 2(1.6)^x$ for the domain $0 \leqslant x \leqslant 3$. [2]



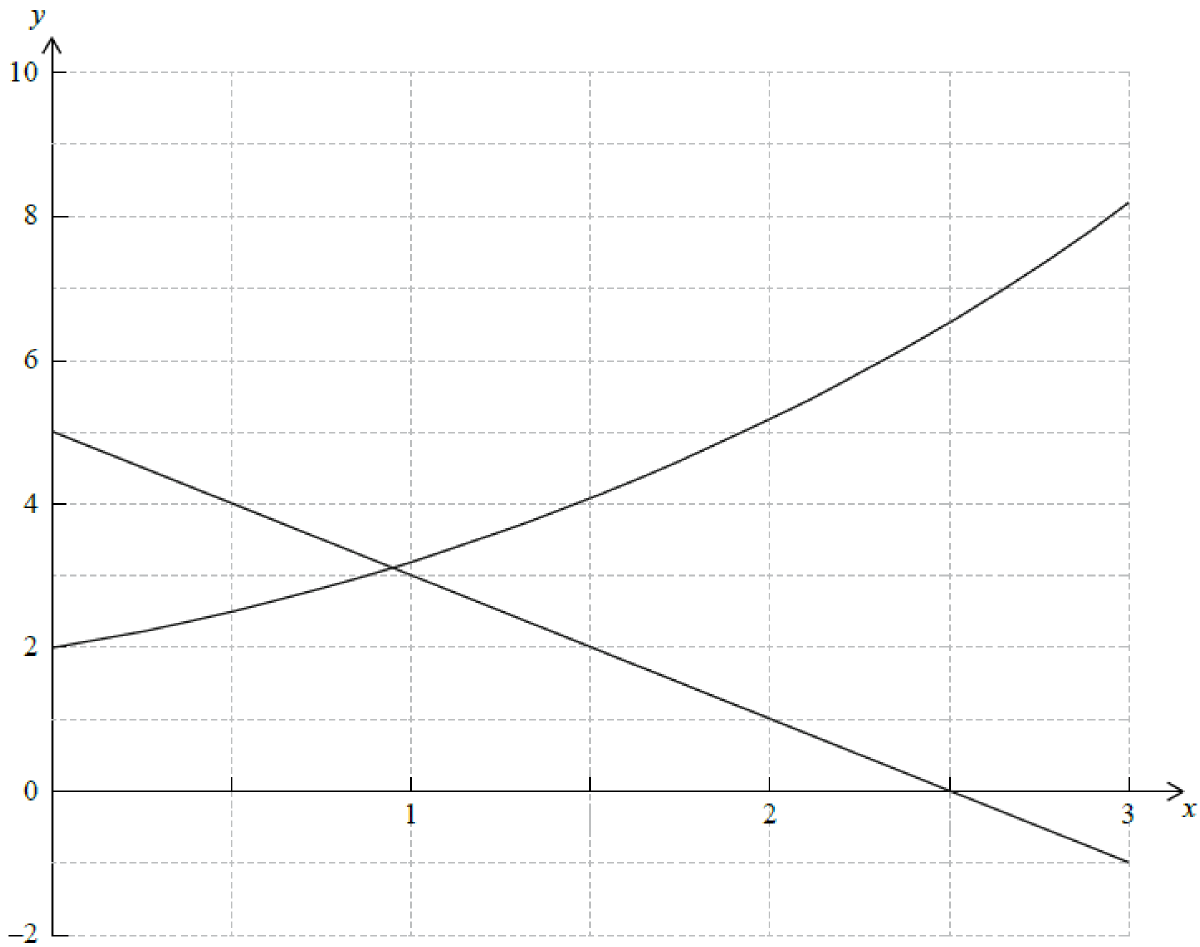
b. Write down the coordinates of the y-intercept of the graph of $y = f(x)$. [1]

c. On the grid draw the graph of the function $g(x) = 5 - 2x$ for the domain $0 \leqslant x \leqslant 3$. [2]

d. Use your graphic display calculator to solve $f(x) = g(x)$. [1]

Markscheme

a.



Note: Award ***(A1)*** correct endpoints, ***(A1)*** for smooth curve. ***(A1)(A1)*** ***(C2)***

b. $(0,2)$ ***(A1)*** ***(C1)***

Note: Accept $x=0, y=2$

c. Straight line in the given domain ***(A1)***

Axes intercepts in the correct positions ***(A1)*** ***(C2)***

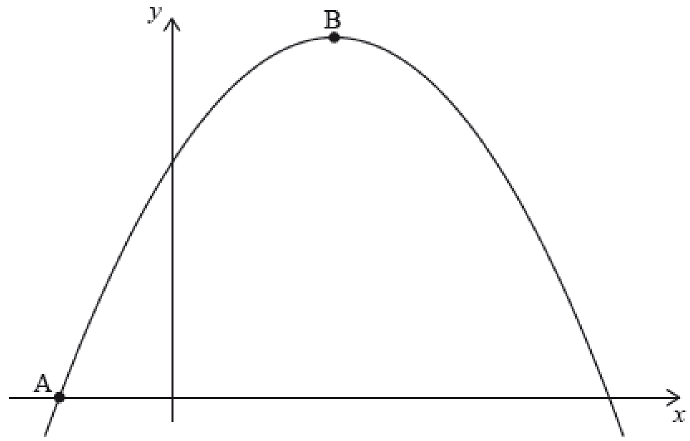
d. $x=0.943\,(0.94259\dots)$ ***(A1)*** ***(C1)***

Note: Award ***(A0)*** if y-coordinate given.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The graph of the quadratic function $f(x)=c+bx-x^2$ intersects the x-axis at the point $\mathrm{A}(-1,\ 0)$ and has its vertex at the point $\mathrm{B}(3,\ 16)$.



a. Write down the equation of the axis of symmetry for this graph. [2]

b. Find the value of b. [2]

c. Write down the range of $f(x)$. [2]

Markscheme

a. $x = 3$ ***(A1)(A1)*** ***(C2)***

Note: Award ***(A1)*** for $x =$ constant, ***(A1)*** for the constant being 3.

The answer must be an equation.

[2 marks]

b. $\frac{-b}{2(-1)} = 3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into axis of symmetry formula.

OR

$b - 2x = 0$ ***(M1)***

Note: Award ***(M1)*** for correctly differentiating and equating to zero.

OR

$c + b(-1) - (-1)^2 = 0$ (or equivalent)

$c + b(3) - (3)^2 = 16$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for correct substitution of $(-1,\ 0)$ and $(3,\ 16)$ in the original quadratic function.

$(b =)\ 6$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

c. $(-\infty,\ 16]$ **OR** $]-\infty,\ 16]$ ***(A1)(A1)***

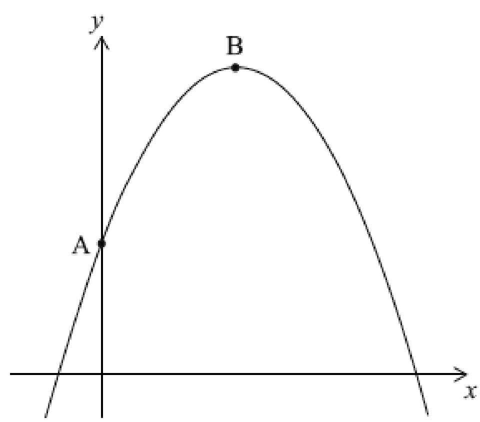
Note: Award ***(A1)*** for two correct interval endpoints, ***(A1)*** for left endpoint excluded **and** right endpoint included.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the y-axis at point A (0, 5) and has its vertex at point B (4, 13).



a. Write down the value of c. [1]

b. By using the coordinates of the vertex, B, or otherwise, write down **two** equations in a and b. [3]

c. Find the value of a and of b. [2]

Markscheme

a. 5 ***(A1)*** ***(C1)***

[1 mark]

b. *at least one of the following equations required*

$a(4)^2 + 4b + 5 = 13$

$4 = -\frac{b}{2a}$

$a(8)^2 + 8b + 5 = 5$ ***(A2)(A1)*** ***(C3)***

Note: Award ***(A2)(A0)*** for one correct equation, or its equivalent, and ***(C3)*** for any two correct equations.

Follow through from part (a).

The equation $a(0)^2 + b(0) = 5$ earns no marks.

[3 marks]

c. $a = -\frac{1}{2},\ b = 4$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***(C2)***

Note: Follow through from their equations in part (b), but only if their equations lead to unique solutions for a and b.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the functions $f(x) = x + 1$ and $g(x) = 3^x - 2$.

a. Write down [2]

(i) the x-intercept of the graph of $y = f(x)$;

(ii) the y-intercept of the graph of $y = g(x)$.

b. Solve $f(x) = g(x)$. [2]

c. Write down the interval for the values of x for which $f(x) > g(x)$. [2]

Markscheme

a. (i) $(-1,\ 0)$ ***(A1)***

Note: Accept -1.

(ii) $(0,\ -1)$ ***(A1)*** ***(C2)***

Note: Accept -1.

b. $(x =)\ -2.96\ (-2.96135\ldots)$ ***(A1)***

$(x =)\ 1.34\ (1.33508\ldots)$ ***(A1)*** ***(C2)***

c. $-2.96 < x < 1.34 \quad \textbf{OR} \quad]-2.96,\ 1.34[\quad \textbf{OR} \quad (-2.96,\ 1.34)$ ***(A1)*****(ft)*****(A1)*** ***(C2)***

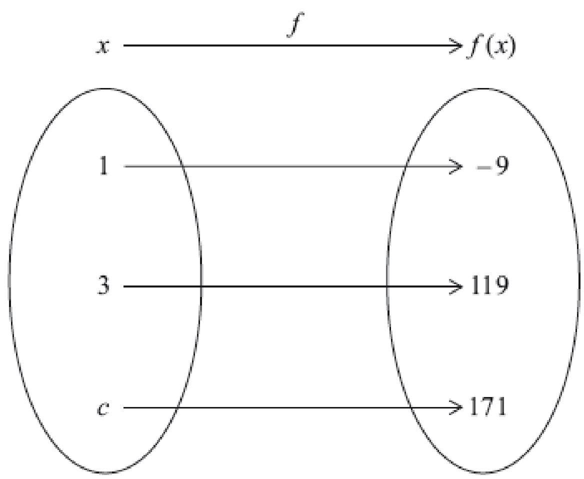
Notes: Award ***(A1)*****(ft)** for both correct endpoints of the interval, ***(A1)*** for correct strict inequalities (or correct open interval notation).

Follow through from part (b).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A quadratic function $f : x \mapsto ax^2 + b$, where a and $b \in \mathbb{R}$ and $x \geqslant 0$, is represented by the mapping diagram.



a. Using the mapping diagram, write down two equations in terms of a and b. [2]

b. Solve the equations to find the value of [2]

(i) a;

(ii) b.

c. Find the value of c. [2]

Markscheme

a. $a(1)^2 + b = -9$ ***(A1)***

$a(3)^2 + b = 119$ ***(A1)*** ***(C2)***

Note: Accept equivalent forms of the equations.

[2 marks]

b. (i) $a = 16$ ***(A1)*(ft)**

(ii) $b = -25$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a) irrespective of whether working is seen.

If working is seen follow through from part (i) to part (ii).

[2 marks]

c. $16c^2 - 25 = 171$ ***(M1)***

Note: Award ***(M1)*** for correct quadratic with their a and b substituted.

$c = 3.5$ ***(A1)*(ft)** ***(C2)***

Note: Accept x instead of c.

Follow through from part (b).

Award ***(A1)*** only, for an answer of ± 3.5 with or without working.

[2 marks]

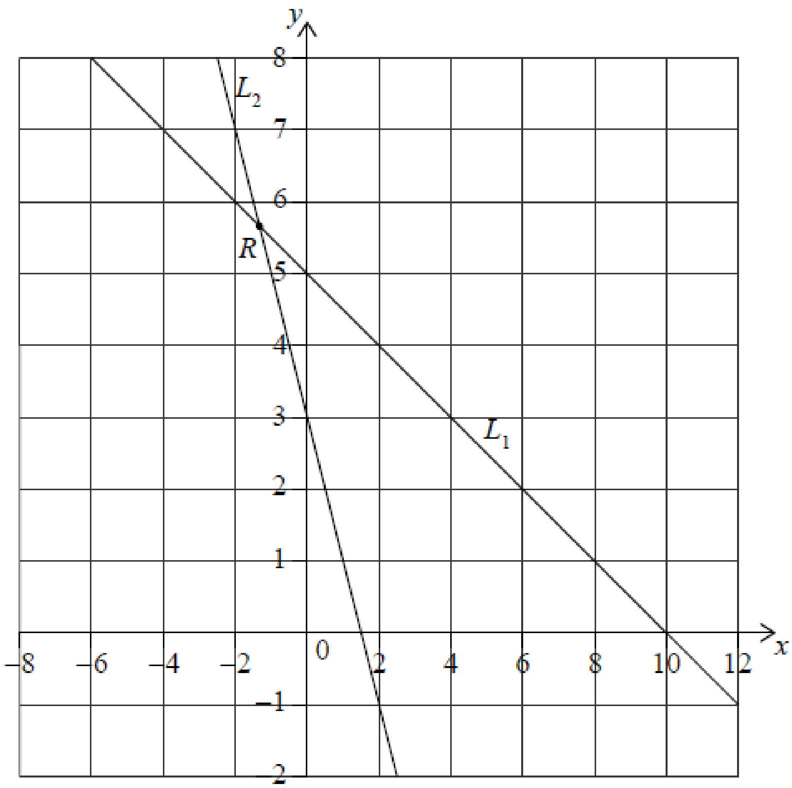
Examiners report

a. This question was answered reasonably well with many candidates able to write down the two equations and solve them for a and b. Errors such as mistaking the equation given for $3a^2 + b = 119$ meant that marks were lost even though the candidates appeared to know what they needed to do. Most candidates who were able to set up the equation in part (c) solved it correctly. Follow through marks were awarded to many candidates for correct working with their substituted values from part (b).

b. This question was answered reasonably well with many candidates able to write down the two equations and solve them for a and b. Errors such as mistaking the equation given for $3a^2 + b = 119$ meant that marks were lost even though the candidates appeared to know what they needed to do. Most candidates who were able to set up the equation in part (c) solved it correctly. Follow through marks were awarded to many candidates for correct working with their substituted values from part (b).

c. This question was answered reasonably well with many candidates able to write down the two equations and solve them for a and b. Errors such as mistaking the equation given for $3a^2 + b = 119$ meant that marks were lost even though the candidates appeared to know what they needed to do. Most candidates who were able to set up the equation in part (c) solved it correctly. Follow through marks were awarded to many candidates for correct working with their substituted values from part (b).

Consider the straight lines L_1 and L_2. R is the point of intersection of these lines.



The equation of line L_1 is $y = ax + 5$.

The equation of line L_2 is $y = -2x + 3$.

a. Find the value of a. [2]

b. Find the coordinates of R. [2]

c. Line L_3 is parallel to line L_2 and passes through the point (2, 3). [2]

Find the equation of line L_3. Give your answer in the form $y = mx + c$.

Markscheme

a. $0 = 10a + 5$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting any point from L_1 into the equation.

OR

$\frac{0-5}{10-0}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting any two points on L_1 into the gradient formula.

$-\frac{5}{10}\left(-\frac{1}{2},\ -0.5\right)$ ***(A1) (C2)***

[2 marks]

b. $(-1.33,\ 5.67)\ \left(\left(-\frac{4}{3},\ \frac{17}{13}\right),\ \left(-1\frac{1}{3},\ 5\frac{2}{3}\right),\ (-1.33333\ldots,\ 5.66666\ldots)\right)$ ***(A1)*(ft)*(A1)*(ft) *(C2)***

Note: Award ***(A1)*** for x-coordinate and ***(A1)*** for y-coordinate. Follow through from their part (a). Award ***(A1)(A0)*** if brackets are missing. Accept $x = -1.33$, $y = 5.67$.

[2 marks]

c. $3 = -2(2) + c$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting –2 and the given point into the equation of a line.

$y = -2x + 7$ ***(A1) (C2)***

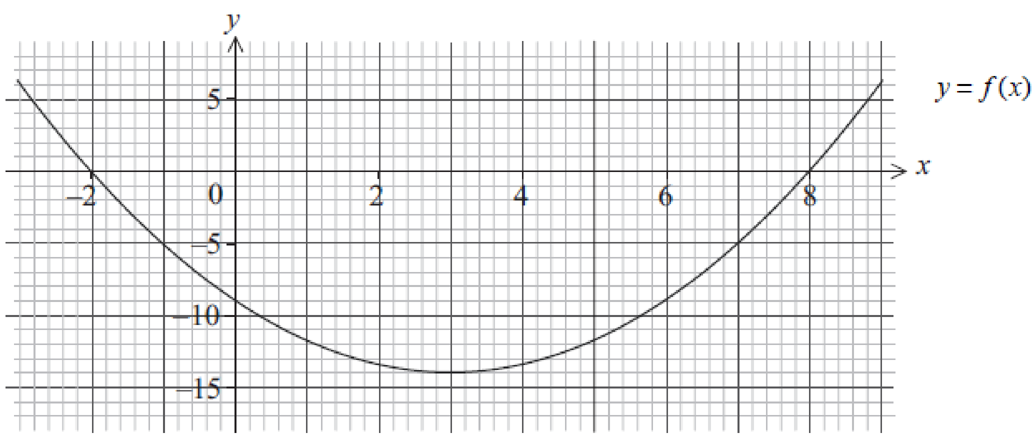
Note: Award ***(A0)*** if the equation is not written in the form $y = mx + c$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The graph of a quadratic function $y = f(x)$ is given below.



a. Write down the equation of the axis of symmetry. [2]

b. Write down the coordinates of the minimum point. [2]

c. Write down the range of $f(x)$. [2]

Markscheme

a. $x = 3$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)*** for " $x =$ " ***(A1)*** for 3.

The mark for $x =$ is not awarded unless a constant is seen on the other side of the equation.

[2 marks]

b. (3, −14) (Accept $x = 3$, $y = -14$) ***(A1)(ft)(A1)*** ***(C2)***

Note: Award ***(A1)(A0)*** for missing coordinate brackets.

[2 marks]

c. $y \geq -14$ ***(A1)(A1)*(ft)** ***(C2)***

Notes: Award ***(A1)*** for y ≥ , ***(A1)*(ft)** for −14.

Accept alternative notation for intervals.

[2 marks]

Examiners report

a. The lack of the answer, “ $x = 3$ ”, expressed as an equation was a common fault.

b. Misreading the y coordinate was the common error.

c. This part proved challenging for many; there was confusion between domain and range for many, the incorrect inequality also was a common error. It is the accepted practice in examinations that if a domain is not specified, then it is taken as the real numbers.

A small manufacturing company makes and sells x machines each month. The monthly cost C , in dollars, of making x machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income I , in dollars, obtained by selling x machines is given by

$$I(x) = 150x - 0.6x^2.$$

$P(x)$ is the monthly profit obtained by selling x machines.

a. Find $P(x)$. [2]

b. Find the number of machines that should be made and sold each month to maximize $P(x)$. [2]

c. Use your answer to part (b) to find the selling price of **each machine** in order to maximize $P(x)$. [2]

Markscheme

a. $P(x) = I(x) - C(x)$ ***(M1)***

$= -x^2 + 150x - 2600$ ***(A1)*** ***(C2)***

b. $-2x + 150 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting $P'(x) = 0$.

OR

Award ***(M1)*** for sketch of $P(x)$ and maximum point identified. ***(M1)***
$x = 75$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from their answer to part (a).

c. $\frac{7875}{75}$ ***(M1)***

Note: Award ***(M1)*** for 7875 seen.

$= 105$ ***(A1)*(ft)** ***(C2)***

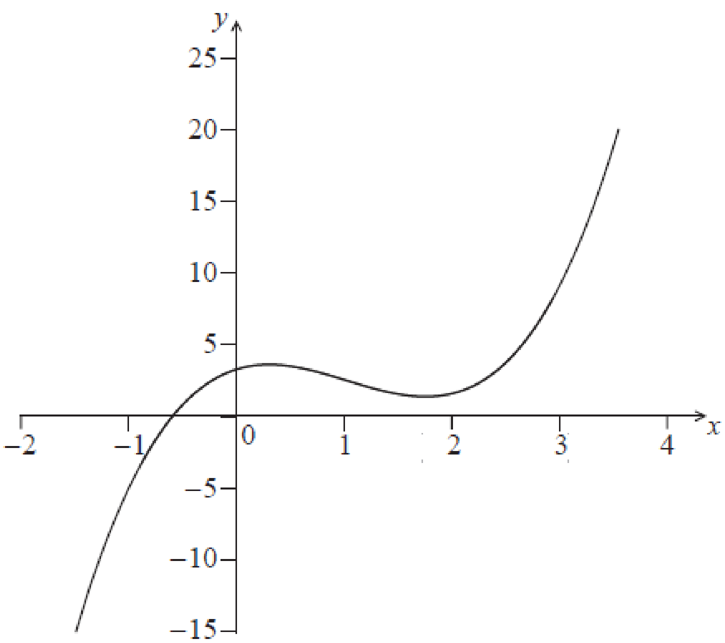
Note: Follow through from their answer to part (b).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Consider the function $f(x) = x^3 - 3x^2 + 2x + 2$. Part of the graph of f is shown below. [3]



Find $f'(x)$.

b. There are two points at which the gradient of the graph of f is 11. Find the x-coordinates of these points. [3]

Markscheme

a. $(f'(x) =)\ 3x^2 - 6x + 2$ ***(A1)(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for $3x^2$, ***(A1)*** for $-6x$ and ***(A1)*** for $+2$.
Award at most ***(A1)(A1)(A0)*** if there are extra terms present.

b. $11 = 3x^2 - 6x + 2$ ***(M1)***

Note: Award ***(M1)*** for equating their answer from part (a) to 11, this may be implied from $0 = 3x^2 - 6x - 9$.

$(x =)\ -1\ ,\ \ (x =)\ 3$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***(C3)***

Note: Follow through from part (a).
If final answer is given as coordinates, award at most ***(M1)(A0)(A1)*****(ft)** for $(-1,\ -4)$ and $(3,\ 8)$.

Examiners report

a. Question 15: Differential calculus.

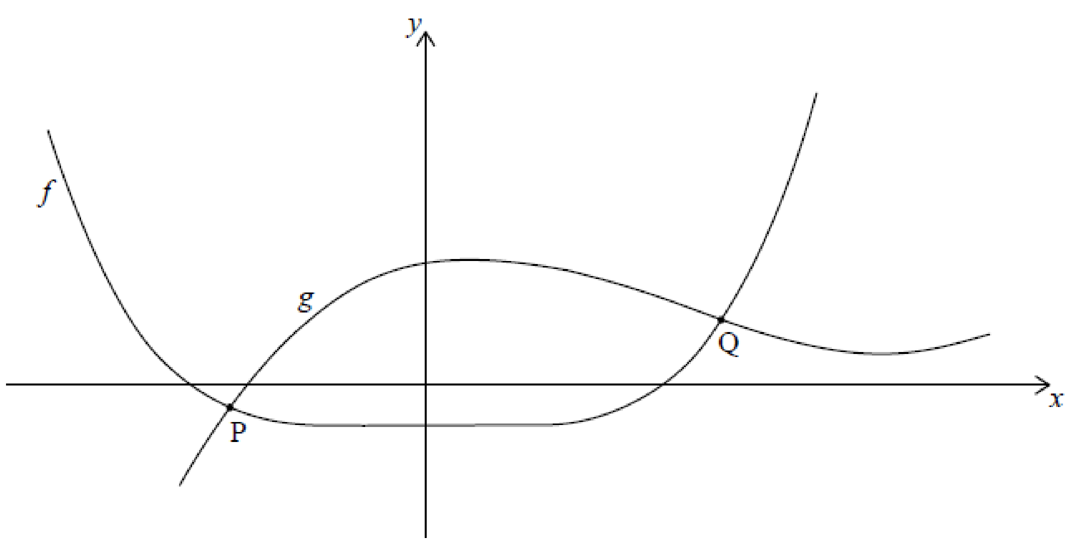
Many candidates correctly differentiated the cubic equation. Most candidates were unable to use differential calculus to find the point where a cubic function had a specified gradient.

b. Question 15: Differential calculus.

Many candidates correctly differentiated the cubic equation. Most candidates were unable to use differential calculus to find the point where a cubic function had a specified gradient.

Consider the functions $f\left(x\right) = x^4 - 2$ and $g\left(x\right) = x^3 - 4x^2 + 2x + 6$

The functions intersect at points P and Q. Part of the graph of $y = f\left(x\right)$ and part of the graph of $y = g\left(x\right)$ are shown on the diagram.



a. Find the range of *f*. [2]

b. Write down the *x*-coordinate of P and the *x*-coordinate of Q. [2]

c. Write down the values of *x* for which $f\left(x\right) > g\left(x\right)$. [2]

Markscheme

a. $[-2,\ \infty[$ or $[-2,\ \infty)$ **OR** $f(x) \geqslant -2$ or $y \geqslant -2$ **OR** $-2 \leqslant f(x) < \infty$ ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for –2 and ***(A1)*** for completely correct mathematical notation, including weak inequalities. Accept $f \geqslant -2$.

[2 marks]

b. –1 and 1.52 (1.51839…) ***(A1)(A1) (C2)***

Note: Award ***(A1)*** for –1 and ***(A1)*** for 1.52 (1.51839).

[2 marks]

c. $x < -1,\ \ x > 1.52$ **OR** $(-\infty,\ -1) \cup (1.52,\ \infty)$. ***(A1)*(ft)*(A1)*(ft) *(C2)***

Note: Award ***(A1)*(ft)** for **both** critical values in inequality or range statements such as $x < -1,\ (-\infty,\ -1),\ x > 1.52$ or $(1.52,\ \infty)$.

Award the second ***(A1)*(ft)** for correct strict inequality statements used with their critical values. If an incorrect use of strict and weak inequalities has already been penalized in (a), condone weak inequalities for this second mark and award ***(A1)*(ft)**.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A factory produces shirts. The cost, *C*, in Fijian dollars (FJD), of producing *x* shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most *s* shirts.

a. Find the cost of producing 70 shirts. [2]

b. Find the value of *s*. [2]

c. Find the number of shirts produced when the cost of production is lowest. [2]

Markscheme

a. $(70 - 75)^2 + 100$ ***(M1)***

Note: Award ***(M1)*** for substituting in $x = 70$.

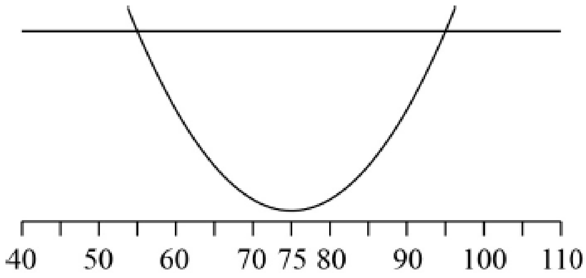
125 ***(A1) (C2)***

[2 marks]

b. $(s - 75)^2 + 100 = 500$ ***(M1)***

Note: Award ***(M1)*** for equating $C(x)$ to 500. Accept an inequality instead of =.

OR



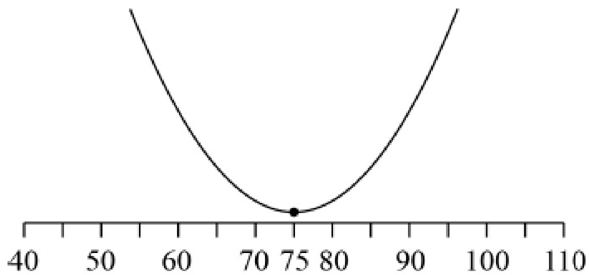
(M1)

Note: Award ***(M1)*** for sketching correct graph(s).

(s =) 95 ***(A1) (C2)***

[2 marks]

c.



(M1)

Note: Award ***(M1)*** for an attempt at finding the minimum point using graph.

OR

$\frac{95+55}{2}$ ***(M1)***

Note: Award ***(M1)*** for attempting to find the mid-point between their part (b) and 55.

OR

($C'(x)$ =) $2x - 150 = 0$ ***(M1)***

Note: Award ***(M1)*** for an attempt at differentiation that is correctly equated to zero.

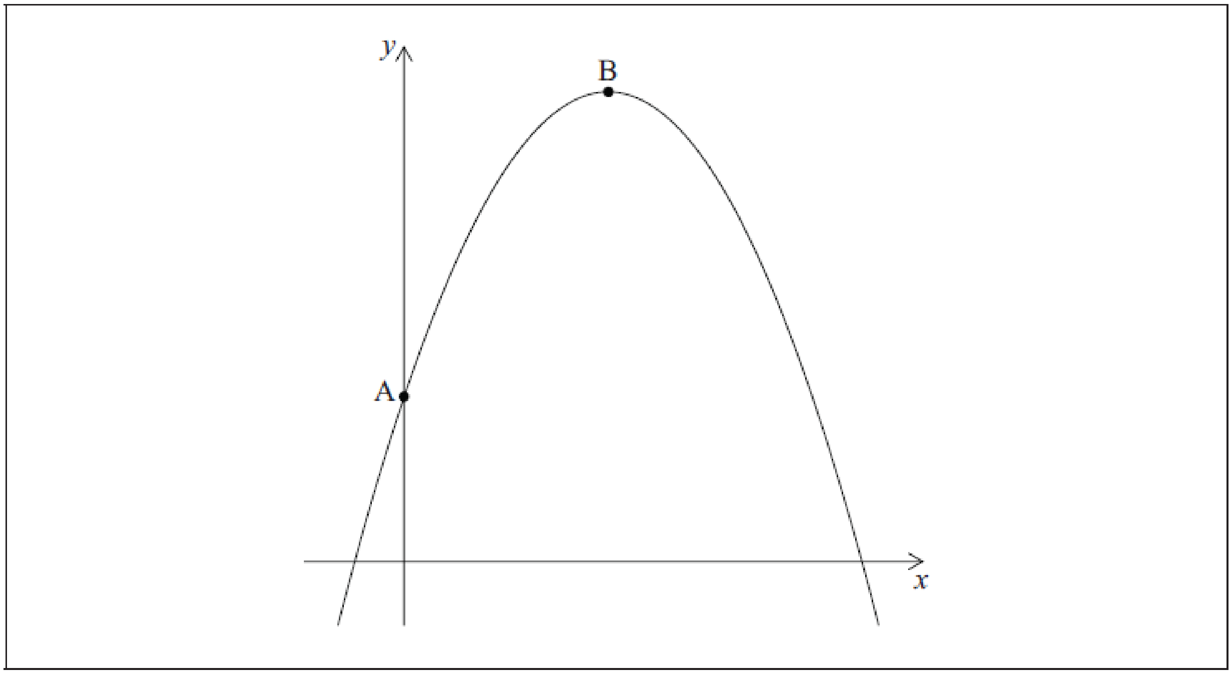
75 ***(A1) (C2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The graph of the quadratic function $f(x) = 3 + 4x - x^2$ intersects the y-axis at point A and has its vertex at point B .



a. Find the coordinates of B . [3]

b. Another point, C , which lies on the graph of $y = f(x)$ has the same y-coordinate as A . [3]

(i) Plot and label C on the graph above.

(ii) Find the x-coordinate of C .

Markscheme

a. $x = -\frac{4}{-2}$ ***(M1)***

$x = 2$ ***(A1)***

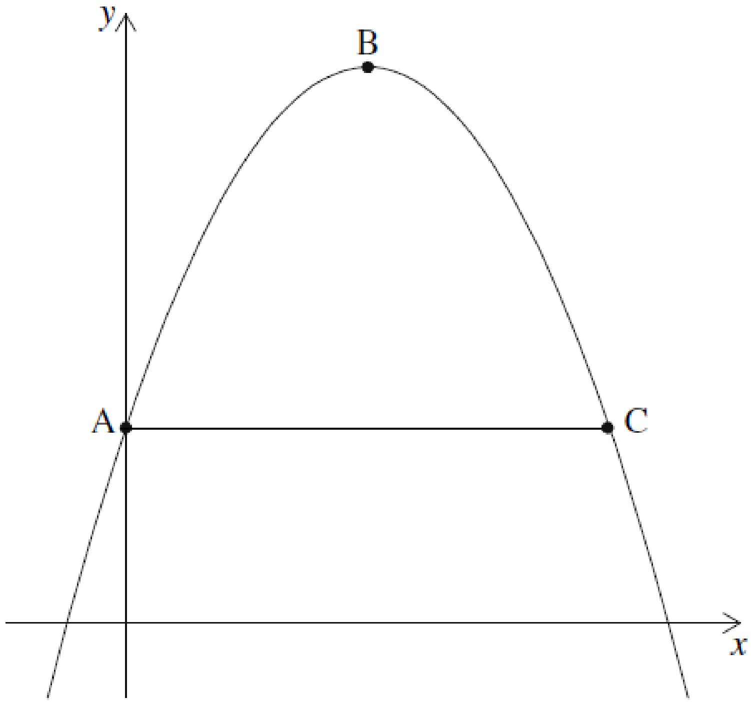
OR

$\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 2x$ ***(M1)***
$x = 2$ ***(A1)***
$(2, 7)$ or $x = 2, y = 7$ ***(A1)*** ***(C3)***

Notes: Award ***(M1)(A1)(A0)*** for 2, 7 without parentheses.

[3 marks]

b. (i) C labelled in correct position on graph ***(A1)*** ***(C1)***



(ii) $3 = 3 + 4x - x^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $y = 3$ into quadratic.

$(x =)4$ ***(A1)*** ***(C2)***

OR

Using symmetry of graph $x = 2 + 2$. ***(M1)***

Note: Follow through from their x-coordinate of the vertex.

$(x =)4$ ***(A1)*(ft)** ***(C2)***

[3 marks]

Examiners report

a. In part b, the point C was sometimes not labelled or not shown on the graph provided. Candidates using their GDC to find the coordinates of the vertex needed to translate their calculator answer to the exact mathematical answer. Answers of $(1.9, 7)$ or $(2.1, 7)$ did not achieve the maximum number of marks. A common response in part c was to give $(4, 3)$, with no working shown. This incurred a penalty of one mark. The correct answer to this question was $x = 4$.

b. In part b, the point C was sometimes not labelled or not shown on the graph provided. Candidates using their GDC to find the coordinates of the vertex needed to translate their calculator answer to the exact mathematical answer. Answers of $(1.9, 7)$ or $(2.1, 7)$ did not achieve the maximum number of marks.

a. Consider the numbers 2, $\sqrt{3}$, $-\frac{2}{3}$ and the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$. [3]

Complete the table below by placing a tick in the appropriate box if the number is an element of the set, and a cross if it is not.

		$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
(i)	2				
(ii)	$\sqrt{3}$				
(iii)	$-\frac{2}{3}$				

b. A function f is given by $f(x) = 2x^2 - 3x$, $x \in \{-2, 2, 3\}$. [1]

Write down the range of function f.

Markscheme

a.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
2	✓	✓	✓	✓
$\sqrt{3}$	✗	✗	✗	✓
$-\frac{2}{3}$	✗	✗	✓	✓

(A1)(A1)(A1) ***(C3)***

Note: Accept any symbol for ticks. Do not penalise if the other boxes are left blank.

[3 marks]

b. $\text{Range} = \{2, 9, 14\}$ ***(A1)*(ft)** ***(C1)***

Note: Brackets not required.

[1 mark]

Examiners report

a. There was a lack of familiarity with number systems and mappings - it was surprising to see how few knew what a mapping diagram involved.

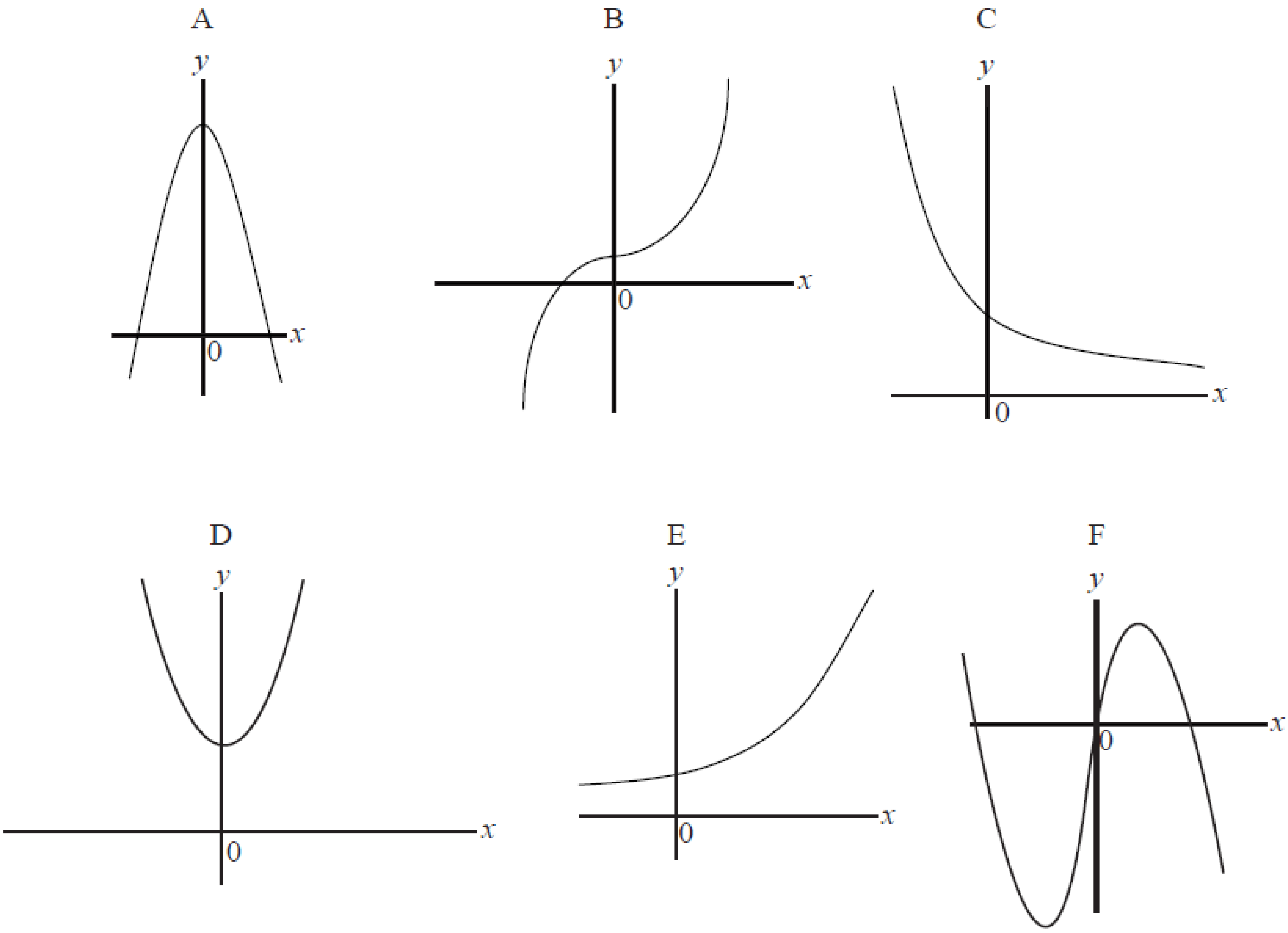
Part (c) (range) was also poorly answered with many giving an interval although they had correctly worked out the values for the function.

b. There was a lack of familiarity with number systems and mappings - it was surprising to see how few knew what a mapping diagram involved.

Part (b) (range) was also poorly answered with many giving an interval although they had correctly worked out the values for the function.

The following curves are sketches of the graphs of the functions given below, but in a different order. Using your graphic display calculator, match the equations to the curves, writing your answers in the table below.

(the diagrams are not to scale)



	Function	Graph label
(i)	$y = x^3 + 1$	
(ii)	$y = x^2 + 3$	
(iii)	$y = 4 - x^2$	
(iv)	$y = 2^x + 1$	
(v)	$y = 3^{-x} + 1$	
(vi)	$y = 8x - 2x^2 - x^3$	

Markscheme

(i) B ***(A1)***

(ii) D ***(A1)***

(iii) A ***(A1)***

(iv) E ***(A1)***

(v) C ***(A1)***

(vi) F ***(A1)*** ***(C6)***

[6 marks]

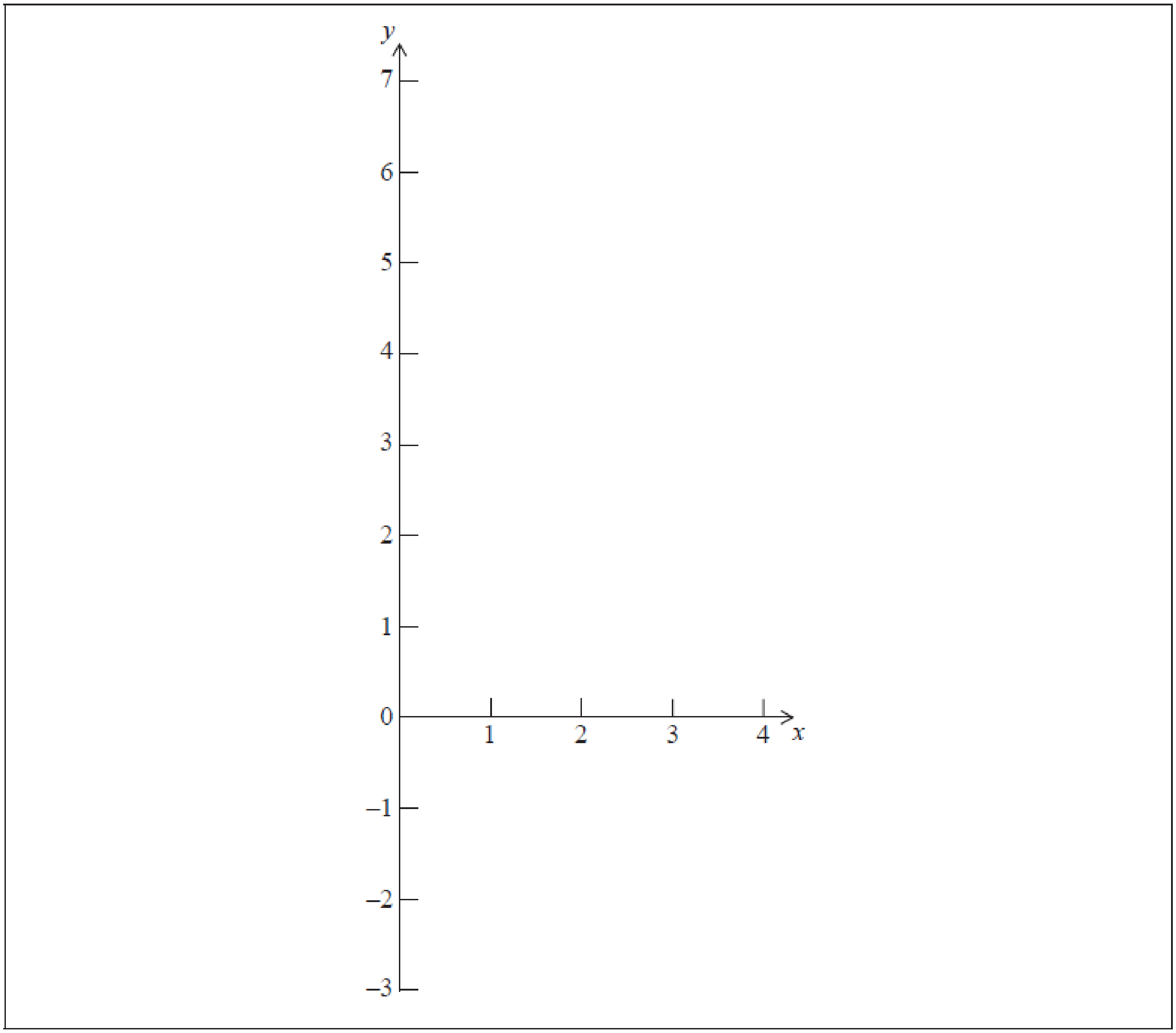
Examiners report

Nearly all the candidates scored 6 marks for this question.Without any working shown it was difficult to say where the errors might have arisen from the few candidates who did not score full marks. However, it was obvious that the candidates were using their GDC's to graph the functions.

$y = f(x)$ is a quadratic function. The graph of $f(x)$ intersects the y-axis at the point A(0, 6) and the x-axis at the point B(1, 0). The vertex of the graph is at the point C(2, –2).

a. Write down the equation of the axis of symmetry. [2]

b. Sketch the graph of $y = f(x)$ on the axes below for $0 \le x \le 4$. Mark clearly on the sketch the points A , B , and C. [3]



c. The graph of $y = f(x)$ intersects the x-axis for a second time at point D. [1]

Write down the x-coordinate of point D.

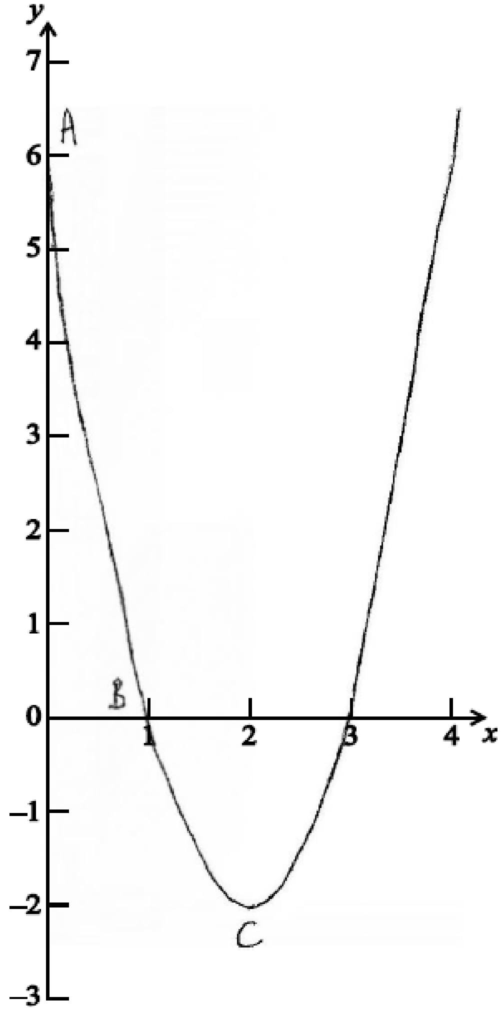
Markscheme

a. $x = 2$ ***(A1)(A1)*** ***(C2)***

Notes: Award ***(A1)(A0)*** for " x = constant" (other than 2). Award ***(A0)(A1)*** for $y = 2$. Award ***(A0)(A0)*** for only seeing 2. Award ***(A0)(A0)*** for $2 = -b / 2a$.

[2 marks]

b.



(A1) for correctly plotting and labelling A, B and C

(A1) for a smooth curve passing through the three given points

(A1) for completing the symmetry of the curve over the **domain given**. ***(A3)*** ***(C3)***

Notes: For ***A*** marks to be awarded for the curve, each segment must be a reasonable attempt at a continuous curve. If straight line segments are used, penalise once only in the last two marks.

[3 marks]

c. 3 ***(A1)*(ft)*** ***(C1)***

Notes: ***(A0)*** for coordinates. Accept $x = 3$ or D = 3 .

[1 mark]

Examiners report

a. (a) Identifying '2' and leaving this as the answer was not sufficient for any marks in this part of the question as was simply leaving the equation $2 + \frac{-b}{2a}$.

b. In part (b) whilst much good work was seen by some candidates in sketching the correct curve, others failed to recognise the symmetry, joined the given points with straight lines or simply drew curved segments which were far from smooth.

c. Part (c) required, for one mark, the writing down of the x-coordinate of the point D. A significant number of candidates, including very able candidates, lost this mark by writing down (3,0).

The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geqslant 0$.

a.i. On the axes below sketch the graph of *f* (*x*) and show the behaviour of the curve as *x* increases. [3]

a.ii. Write down the coordinates of any intercepts with the axes. [1]

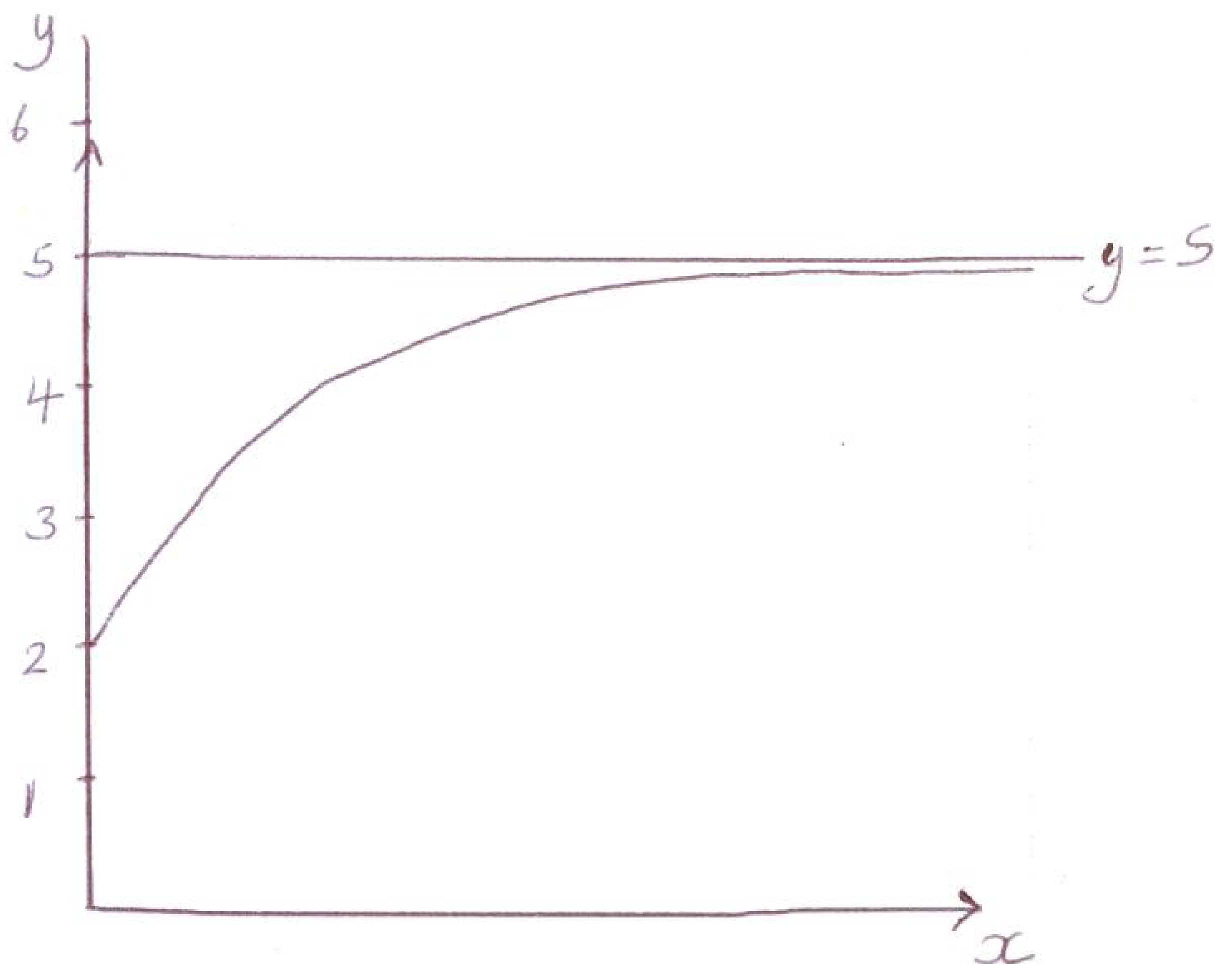
onbekend.png

b. Draw the line *y* = 5 on your sketch. [1]

c. Write down the number of solutions to the equation *f* (*x*) = 5 . [1]

Markscheme

a.i.



(A1)(A1)(A1)

Notes: Award ***(A1)*** for labels and scale on *y*-axis.
Award ***(A1)*** for smooth increasing curve in the given domain.
Award ***(A1)*** for asymptote implied ($\text{gradient} \to 0$).

[3 marks]

a.ii. (0, 2) *accept* $x = 0, y = 2$ ***(A1)*** ***(C4)***

Note: If incorrect domain used and both (0, 2) and (−0.737, 0) seen award ***(A1)*****(ft)**.

[1 mark]

b. line passing through (0, 5), parallel to x axis and not intersecting their graph. ***(A1)*** ***(C1)***

[1 mark]

c. zero ***(A1)*** ***(C1)***

[1 mark]

Examiners report

a.i. Most candidates attempted this question and many gained 3 or 4 marks. All made an attempt at sketching the graph which demanded that students used their GDC. Many candidates failed to label their graphs and to give an indication of scale, and lost one mark in part (a). Some did not pay attention to the domain and sketched the graph in a different region. A significant number could also write down the coordinates of the y-intercept, although some wrote only $y = 2$ instead of giving the two coordinates. Almost all could draw the line $y = 5$ on the sketch but many could not find the answer for the number of solutions to the equation given in part c). Some candidates lost time in an attempt to draw this graph accurately on graph paper, which was not the intended task. Most candidates attempted this question, which clearly indicated that the time given for the paper was sufficient.

a.ii. Most candidates attempted this question and many gained 3 or 4 marks. All made an attemptat sketching the graph which demanded that students used their GDC. Many candidates failed to label their graphs and to give an indication of scale, and lost one mark in part (a). Some did not pay attention to the domain and sketched the graph in a different region. A significant number could also write down the coordinates of the y-intercept, although some wrote only$y = 2$ instead of giving the two coordinates. Almost all could draw the line $y = 5$ on thesketch but many could not find the answer for the number of solutions to the equation given in part c). Some candidates lost time in an attempt to draw this graph accurately on graphpaper, which was not the intended task. Most candidates attempted this question, which clearly indicated that the time given for the paper was sufficient.

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---

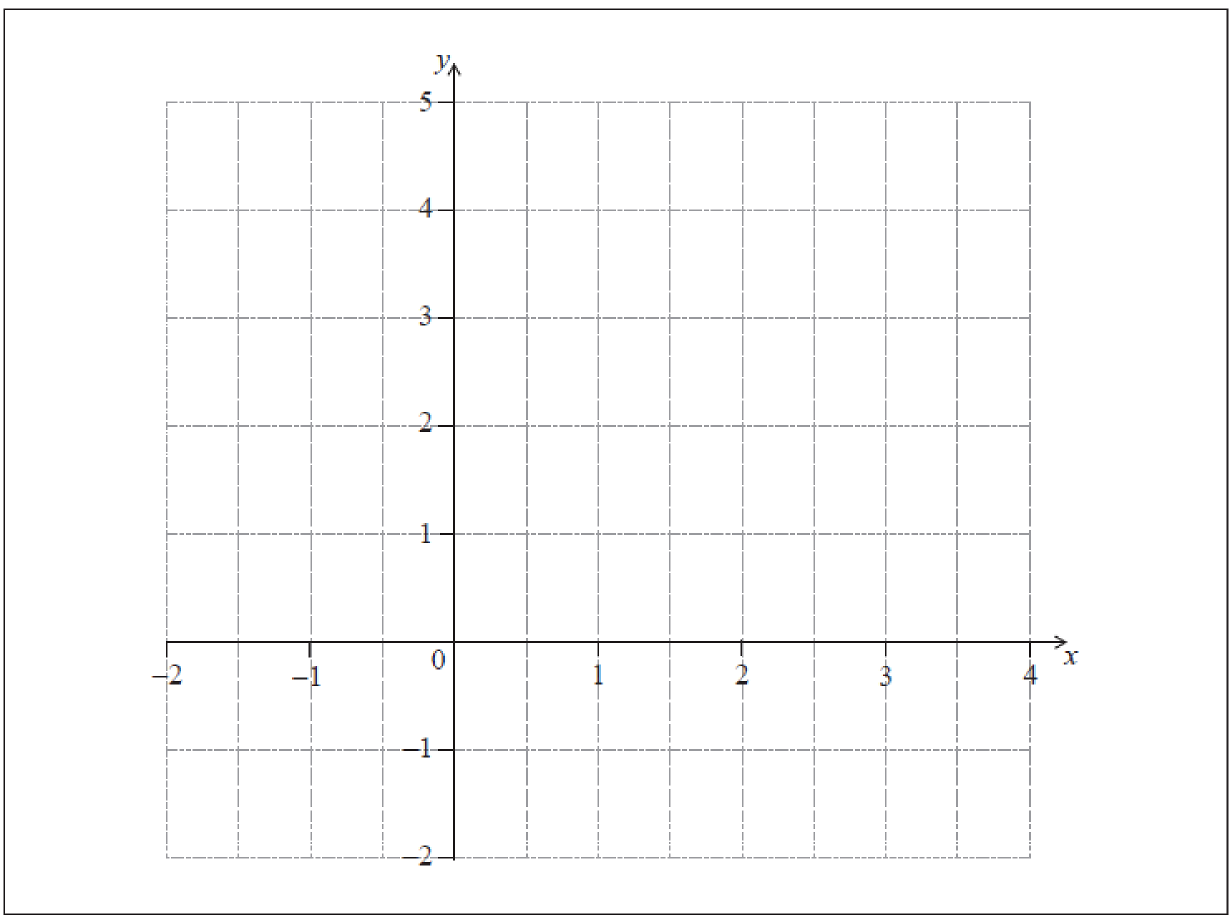
a. Consider the curve $y = 1 + \frac{1}{2x},\ x \neq 0.$ [3]

For this curve, write down

i) the value of the x-intercept;

ii) the equation of the vertical asymptote.

b. Sketch the curve for $-2 \leqslant x \leqslant 4$ on the axes below. [3]



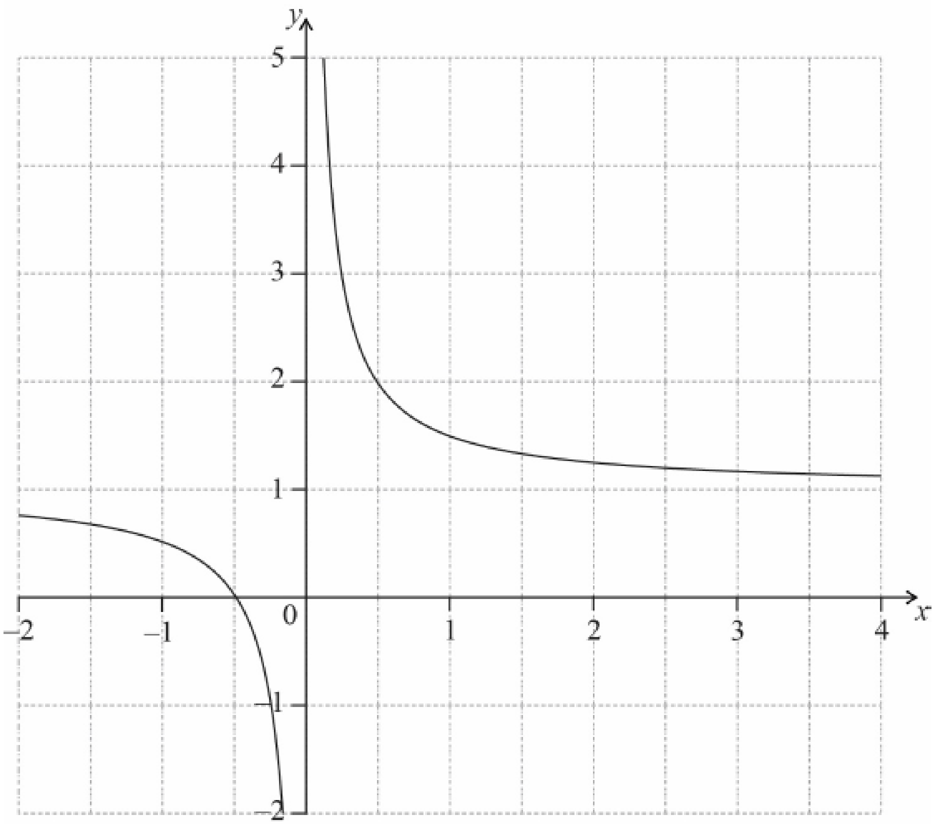
Markscheme

a. i) $(x =) - 0.5\ \left(-\frac{1}{2}\right)$ ***(A1)***

ii) $x = 0$ ***(A1)(A1)*** ***(C3)***

Note: Award ***(A1)*** for "$x =$" and ***(A1)*** for "0" seen as part of an equation.

b.



(A1)*(ft)*(A1)(ft)(A1) ***(C3)***

Note: Award ***(A1)*(ft)** for correct x-intercept, ***(A1)*(ft)** for asymptotic behaviour at y-axis, ***(A1)*** for approximately correct shape (cannot intersect the horizontal asymptote of $y = 1$). Follow through from part (a).

Examiners report

a. Question 8: Rational function.

Few candidates could find the x-intercept of the rational function. Many candidates did appreciate that the curve does not cross the asymptote. Often the candidates wrote down the equation of the horizontal asymptote rather than the equation of the vertical asymptote. The most frequent incorrect sketch was that of $y = \frac{1}{2}x + 1$ suggesting that the candidate did not understand that the curve $y = 1 + \frac{1}{2x}$ is not linear and had taken insufficient care in entering the function into the calculator. Some candidates that appreciated the shape of the curve did not earn marks on account of the poor quality of their sketches, which either crossed, or veered away from, the asymptotes.

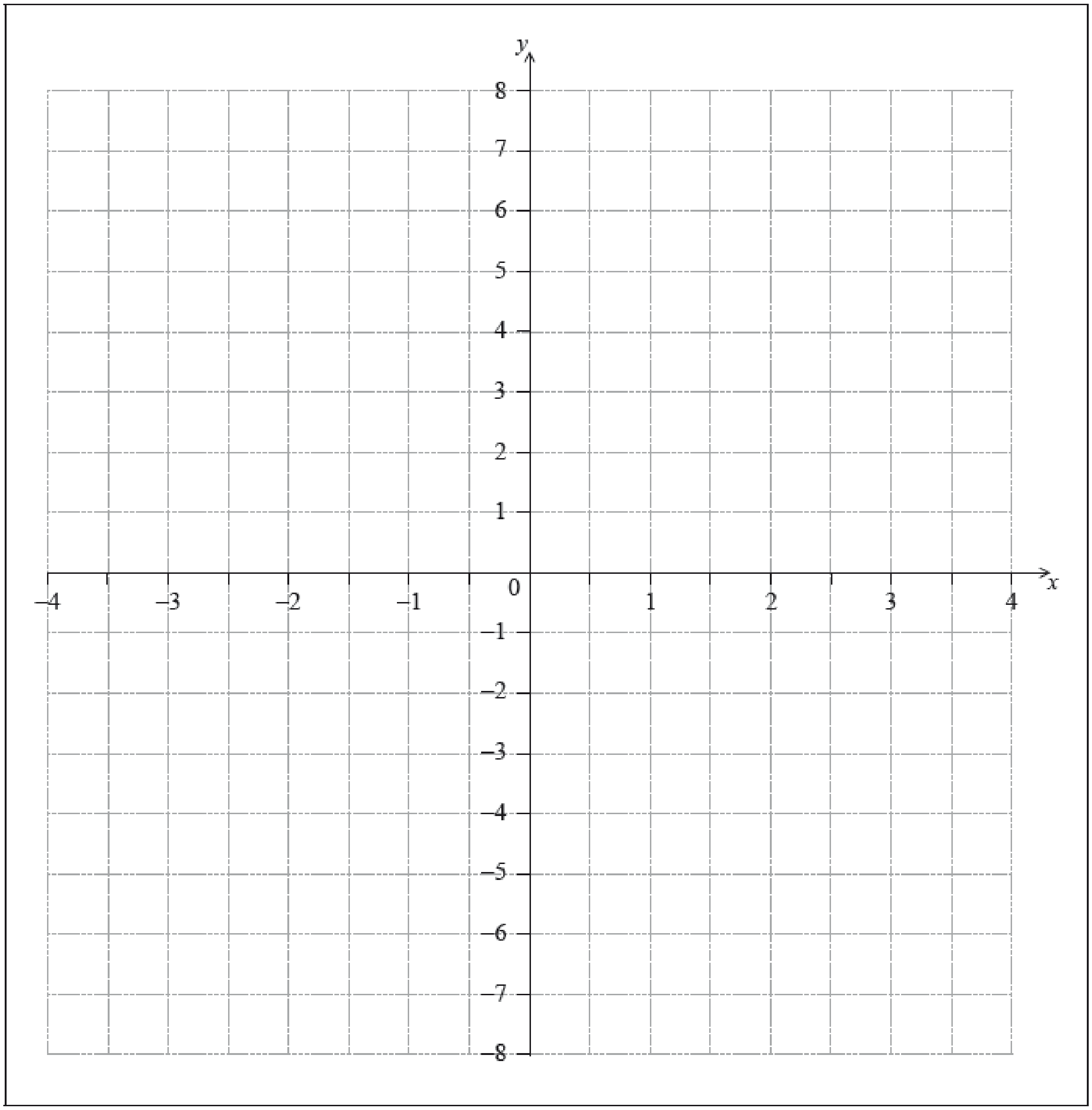
b. Question 8: Rational function.

Few candidates could find the x-intercept of the rational function. Many candidates did appreciate that the curve does not cross the asymptote. Often the candidates wrote down the equation of the horizontal asymptote rather than the equation of the vertical asymptote. The most frequent incorrect sketch was that of $y = \frac{1}{2}x + 1$ suggesting that the candidate did not understand that the curve $y = 1 + \frac{1}{2x}$ is not linear and had taken insufficient care in entering the function into the calculator. Some candidates that appreciated the shape of the curve did not earn marks on account of the poor quality of their sketches, which either crossed, or veered away from, the asymptotes.

The axis of symmetry of the graph of a quadratic function has the equation $x = -\frac{1}{2}$

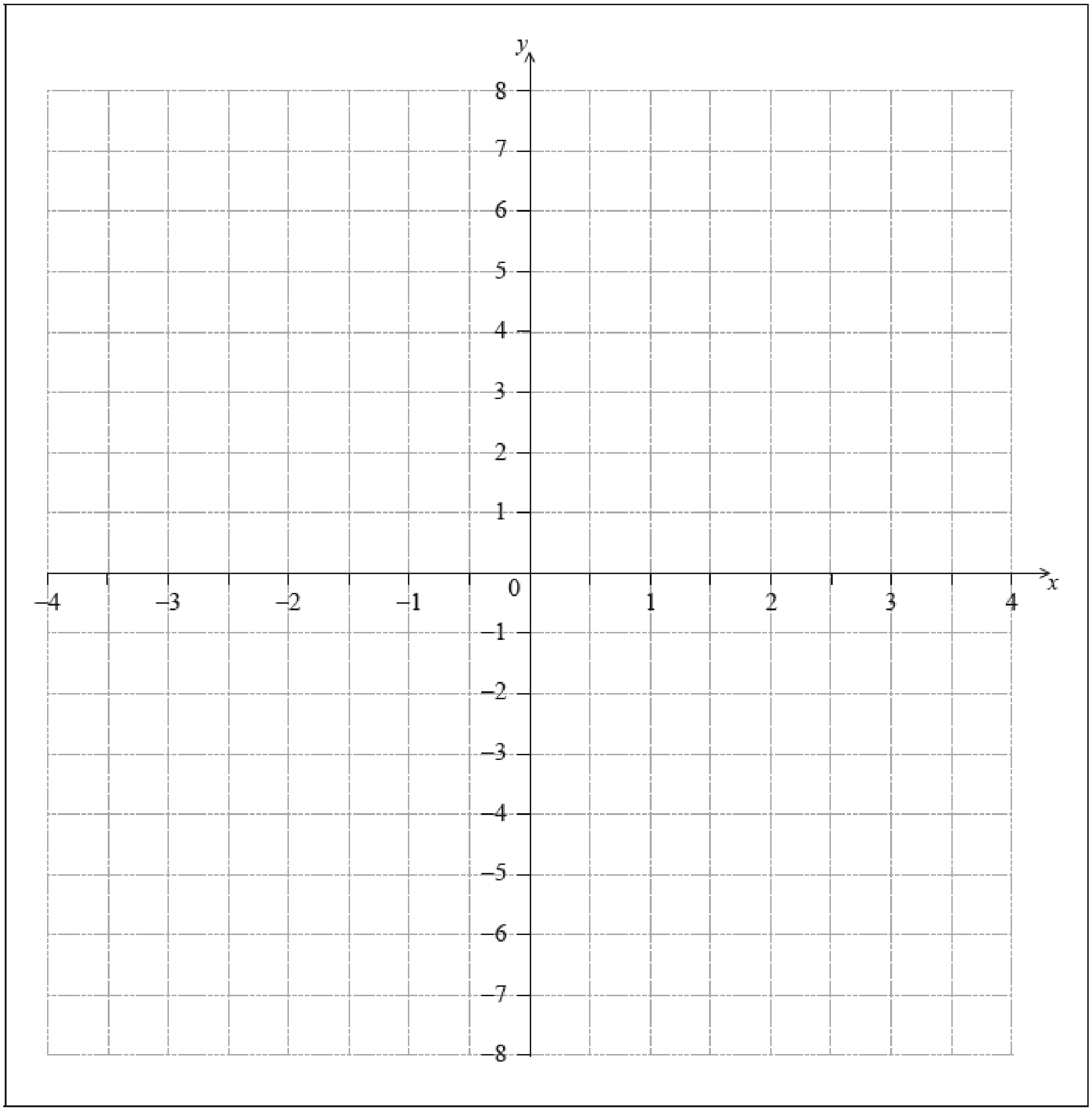
.

a. Draw the axis of symmetry on the following axes. [1]



The graph of the quadratic function intersects the *x*-axis at the point N(2, 0) . There is a second point, M, at which the graph of the quadratic function intersects the *x*-axis.

a. Draw the axis of symmetry on the following axes. [1]



b. The graph of the quadratic function intersects the x-axis at the point $\mathrm{N}(2, 0)$. There is a second point, M, at which the graph of the quadratic function intersects the x-axis. [1]

Clearly mark and label point M on the axes.

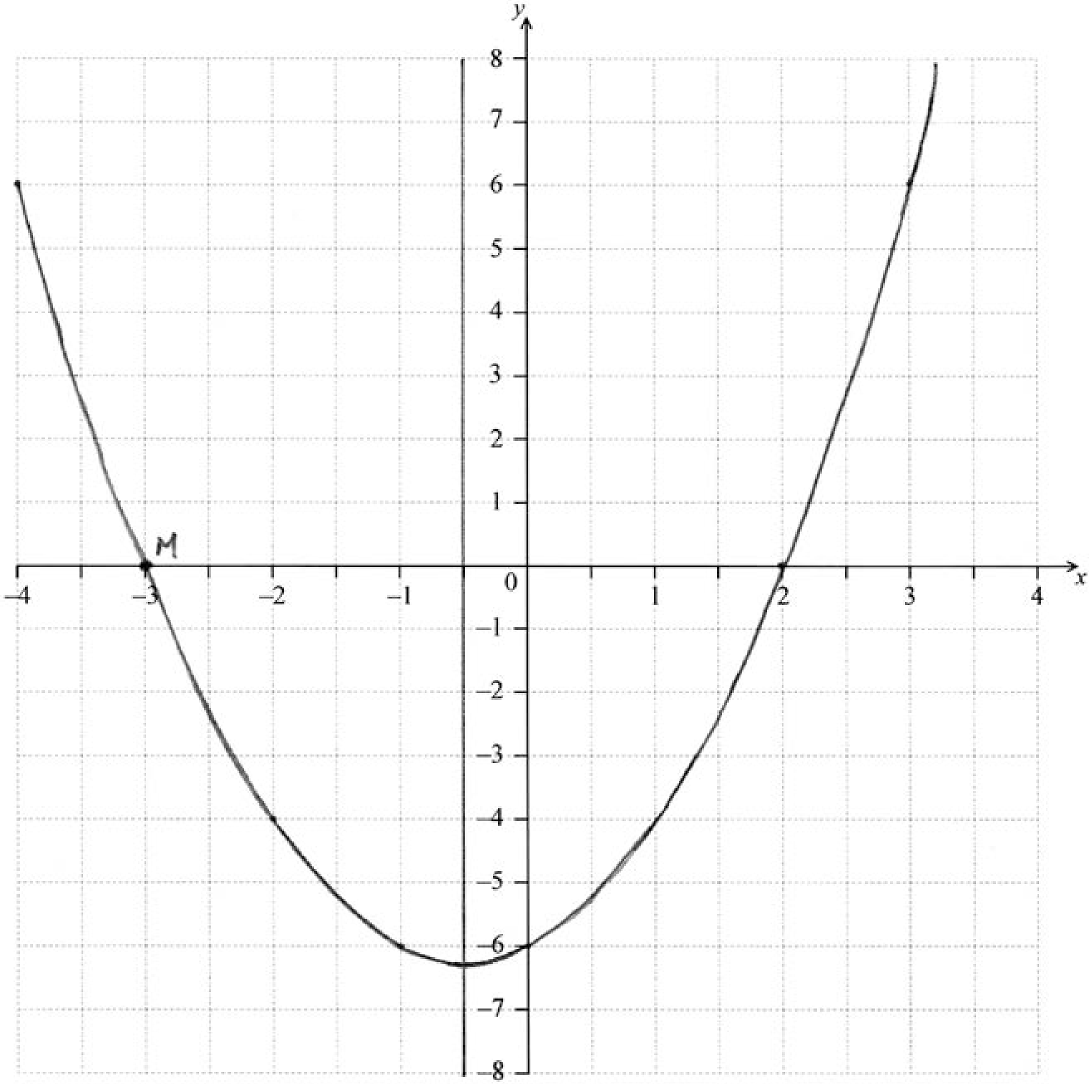
c. (i) Find the value of b and the value of c. [4]

(ii) Draw the graph of the function on the axes.

Markscheme

a.

vertical straight line which may be dotted passing through $\left(-\frac{1}{2},\ 0\right)$ ***(A1) (C1)*** a.



vertical straight line which may be dotted passing through $\left(-\frac{1}{2},\ 0\right)$ ***(A1)*** ***(C1)***

b. point $\mathrm{M}\ (-3,\ 0)$ correctly marked on the x-axis ***(A1)*****(ft)** ***(C1)***

Note: Follow through from part (a).

c. (i) $b = 1, c = -6$ ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Follow through from (b).

(ii) smooth parabola passing through M and N ***(A1)*****(ft)**

Note: Follow through from their point M from part (b).

parabola passing through $(0,\ -6)$ and symmetrical about $x = -0.5$ ***(A1)*****(ft)** ***(C4)***

Note: Follow through from part (c)(i).

If parabola is not smooth and not concave up award at most ***(A1)(A0)***.

Examiners report

a. [N/A]

a. [N/A]

[N/A]

b. [N/A]