SL Paper 2

A function f is given by $f(x) = (2x+2)(5-x^2)$.

The graph of the function $g(x) = 5^x + 6x - 6$ intersects the graph of f.

a. Find the **exact** value of each of the zeros of f. [3]

b.i. Expand the expression for $f(x)$. [1]

b.ii. Find $f'(x)$. [3]

c. Use your answer to part (b)(ii) to find the values of x for which f is increasing. [3]

d. **Draw** the graph of f for $-3 \leqslant x \leqslant 3$ and $-40 \leqslant y \leqslant 20$. Use a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 5 units on the y-axis. [4]

e. Write down the coordinates of the point of intersection. [2]

Markscheme

a. $-1,\ \sqrt{5},\ -\sqrt{5}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for –1 and each exact value seen. Award at most ***(A1)(A0)(A1)*** for use of 2.23606… instead of $\sqrt{5}$.

[3 marks]

b.i. $10x - 2x^3 + 10 - 2x^2$ ***(A1)***

Notes: The expansion may be seen in part (b)(ii).

[1 mark]

b.ii. $10 - 6x^2 - 4x$ ***(A1)*(ft)*(A1)*(ft)*(A1)*(ft)**

Notes: Follow through from part (b)(i). Award ***(A1)*(ft)** for each correct term. Award at most ***(A1)*(ft)*(A1)*(ft)*(A0)*** if extra terms are seen.

[3 marks]

c. $10 - 6x^2 - 4x > 0$ ***(M1)***

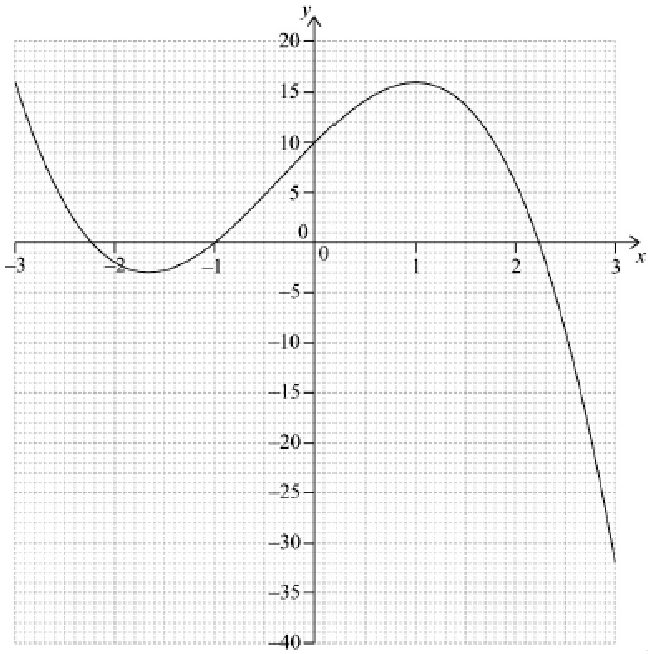
Notes: Award ***(M1)*** for their $f'(x) > 0$. Accept equality or weak inequality.

$-1.67 < x < 1 \left(-\frac{5}{3} < x < 1, \ -1.66666\ldots < x < 1\right)$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

Notes: Award ***(A1)*(ft)** for correct endpoints, ***(A1)*(ft)** for correct weak or strict inequalities. Follow through from part (b)(ii). Do not award any marks if there is no answer in part (b)(ii).

[3 marks]

d.



(A1)(A1)*(ft)*(A1)*(ft)*(A1)

Notes: Award ***(A1)*** for correct scale; axes labelled and drawn with a ruler.

Award ***(A1)*(ft)** for their correct x-intercepts in approximately correct location.

Award ***(A1)*** for correct minimum and maximum points in approximately correct location.

Award ***(A1)*** for a smooth continuous curve with approximate correct shape. The curve should be in the given domain.

Follow through from part (a) for the x-intercepts.

[4 marks]

e. $(1.49, \ 13.9) \ ((1.48702\ldots, \ 13.8714\ldots))$ ***(G1)*(ft)*(G1)*(ft)**

Notes: Award ***(G1)*** for 1.49 and ***(G1)*** for 13.9 written as a coordinate pair. Award at most ***(G0)(G1)*** if parentheses are missing. Accept $x = 1.49$ and $y = 13.9$. Follow through from part (b)(i).

[2 marks]

Examiners report

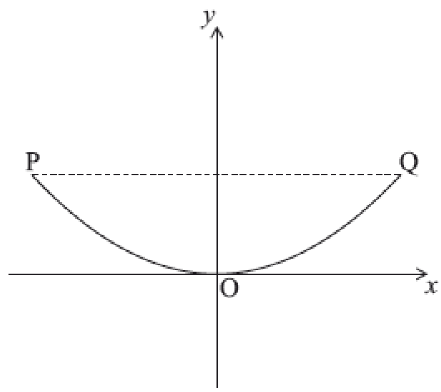
a. [N/A]

b.i. [N/A]

b.ii. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



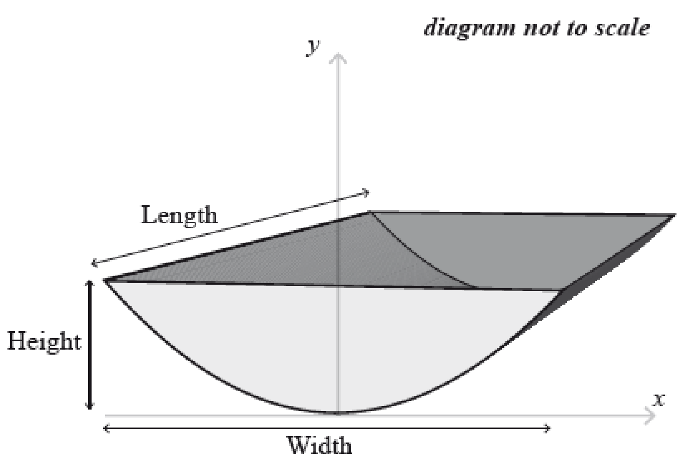
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

a. Write down the value of c. [1]

b. Find the value of a. [2]

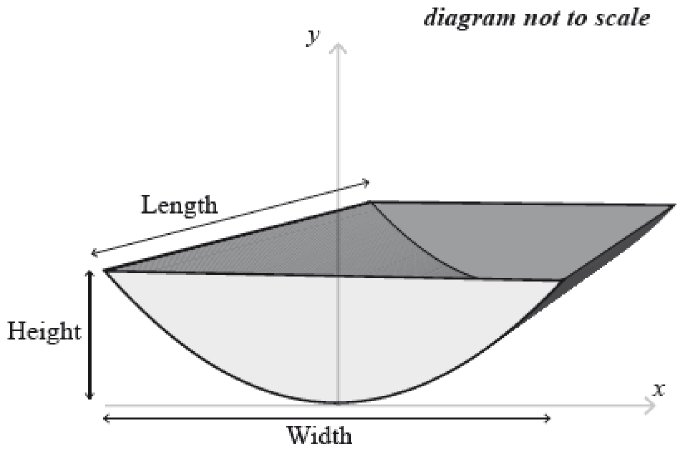
c. Hence write down the equation of the quadratic function which models the edge of the water tank. [1]

d. The water tank is shown below. It is partially filled with water. [2]



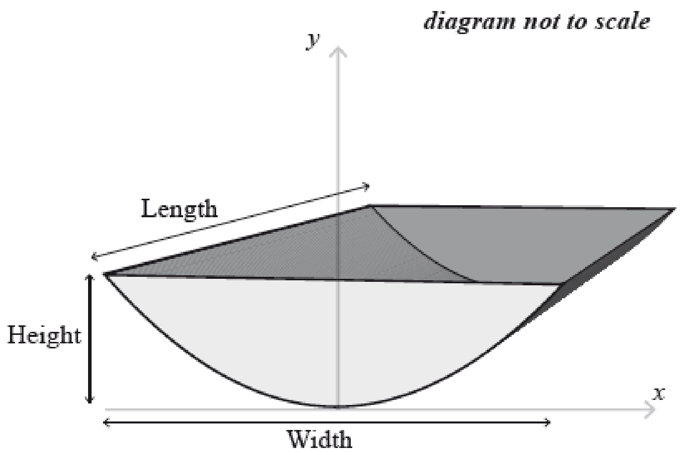
Calculate the value of *y* when $x = 2.4\text{ m}$.

e. The water tank is shown below. It is partially filled with water. [2]



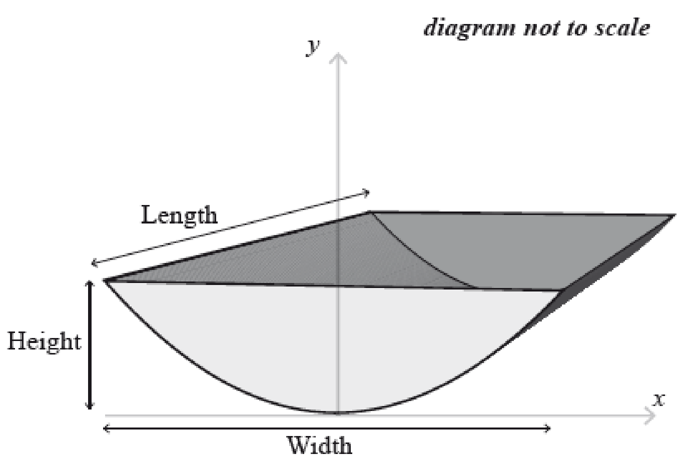
State what the value of x and the value of y represent for this water tank.

f. The water tank is shown below. It is partially filled with water. [2]



Find the value of x when the height of water in the tank is 0.9 m.

g. The water tank is shown below. It is partially filled with water. [2]



The water tank has a length of 5 m.

When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \text{m}^3$.

(ii) Calculate the percentage of water in the tank.

Markscheme

a. 0 ***(A1)(G1)***

[1 mark]

b. $1.8 = a(3)^2 + 0$ ***(M1)***

OR

$1.8 = a(-3)^2 + 0$ ***(M1)***

Note: Award ***(M1)*** for substitution of $y = 1.8$ or $x = 3$ and their value of c into equation. 0 may be implied.

$a = 0.2\ \left(\frac{1}{5}\right)$ ***(A1)*(ft)*(G1)***

Note: Follow through from their answer to part (a).

Award ***(G1)*** for a correct answer only.

[2 marks]

c. $y = 0.2x^2$ ***(A1)*(ft)**

Note: Follow through from their answers to parts (a) and (b).

Answer must be an equation.

[1 mark]

d. $0.2 \times (2.4)^2$ ***(M1)***

$= 1.15\ (\text{m})\ \ (1.152)$ ***(A1)*(ft)*(G1)***

Notes: Award ***(M1)*** for correctly substituted formula, ***(A1)*** for correct answer. Follow through from their answer to part (c).

Award ***(G1)*** for a correct answer only.

[2 marks]

e. y is the height ***(A1)***

positive value of x is half the width (*or equivalent*) ***(A1)***

[2 marks]

f. $0.9 = 0.2x^2$ ***(M1)***

Note: Award ***(M1)*** for setting their equation equal to 0.9.

$x = \pm 2.12\ (\text{m})\ \ \left(\pm\frac{3}{2}\sqrt{2},\ \pm\sqrt{4.5},\ \pm 2.12132\ldots\right)$ ***(A1)*(ft)*(G1)***

Note: Accept 2.12. Award ***(G1)*** for a correct answer only.

[2 marks]

g. (i) 2.55×5 ***(M1)***

Note: Award ***(M1)*** for correct substitution in formula.

$= 12.8\ (\text{m}^3)\ \ \left(12.75\ (\text{m}^3)\right)$ ***(A1)(G2)***

[2 marks]

(ii) $\frac{12.75}{36} \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct quotient multiplied by 100.

$= 35.4(\%)\ \ (35.4166\ldots)$ ***(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for $35.6(\%)(35.5555\ldots(\%))$.

Follow through from their answer to part (g)(i).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

A function is defined by $f(x) = \frac{5}{x^2} + 3x + c,\ x \neq 0,\ c \in \mathbb{Z}$.

a. Write down an expression for $f'(x)$. [4]

b. Consider the graph of *f*. The graph of *f* passes through the point P(1, 4). [2]

Find the value of *c*.

c, i. There is a local minimum at the point Q. [4]

Find the coordinates of Q.

c, ii. There is a local minimum at the point Q. [3]

Find the set of values of x for which the function is decreasing.

d, i. Let T be the tangent to the graph of f at P. [2]

Show that the gradient of T is –7.

d, ii. Let T be the tangent to the graph of f at P. [2]

Find the equation of T.

e. T intersects the graph again at R. Use your graphic display calculator to find the coordinates of R. [2]

Markscheme

a. $f'(x) = \frac{-10}{x^3} + 3$ ***(A1)(A1)(A1)(A1)***

Note: Award ***(A1)*** for –10, ***(A1)*** for x^3 (or x^{-3}), ***(A1)*** for 3, ***(A1)*** for no other constant term.

[4 marks]

b. 4 = 5 + 3 + c ***(M1)***

Note: Award ***(M1)*** for substitution in $f(x)$.

c = –4 ***(A1)(G2)***

[2 marks]

c, i. $f'(x) = 0$ ***(M1)***

$0 = \frac{-10}{x^3} + 3$ ***(A1)*(ft)**

(1.49, 2.72) (accept x = 1.49 y = 2.72) ***(A1)*(ft)*(A1)*(ft)*(G3)***

Notes: If answer is given as (1.5, 2.7) award ***(A0)(AP)(A1)***.

Award at most ***(M1)(A1)(A1)(A0)*** if parentheses not included. **(ft)** from their (a).

If no working shown award ***(G2)(G0)*** if parentheses are not included.

OR

Award ***(M2)*** for sketch, ***(A1)*(ft)*(A1)*(ft)** for correct coordinates. **(ft)** from their (b). ***(M2)(A1)*(ft)*(A1)*(ft)**

Note: Award at most ***(M2)(A1)*(ft)*(A0)*** if parentheses not included.

[4 marks]

c, ii. $0 < x < 1.49$ **OR** $0 < x \leq 1.49$ ***(A1)(A1)*(ft)*(A1)***

Notes: Award ***(A1)*** for 0, ***(A1)*(ft)** for 1.49 and ***(A1)*** for correct inequality signs.

(ft) from their *x* value in (c) (i).

[3 marks]

d, i For P(1, 4) $f'(1) = -10 + 3$ ***(M1)(A1)***

$= -7$ ***(AG)***

Note: Award ***(M1)*** for substituting $x = 1$ into their $f'(x)$. ***(A1)*** for $-10 + 3$.

-7 must be seen for ***(A1)*** to be awarded.

[2 marks]

d, ii $4 = -7 \times 1 + c \quad 11 = c$ ***(A1)***

$y = -7x + 11$ ***(A1)***

[2 marks]

e. Point of intersection is R(–0.5, 14.5) ***(A1)*(ft)*(A1)*(ft)*(G2)*(ft)**

Notes: Award ***(A1)*** for the *x* coordinate, ***(A1)*** for the *y* coordinate.

Allow **(ft)** from candidate's (d)(ii) equation and their (b) even with no working seen.

Award ***(A1)*(ft)*(A0)*** if brackets not included and not previously penalised.

[2 marks]

Examiners report

a. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Most candidates could score 4 marks.

b. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

A good number of candidates correctly substituted into the original function.

c, i A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Very few managed to answer this part algebraically. Those candidates who were aware that they could read the values from their GDC gained some easy marks.

c, ii A large number of candidates were not able to answer this question well because of the firstterm of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

Proved to be difficult for many candidates as increasing/decreasing intervals were not well understood by many.

d, i. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of T was −7. Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of T was a simple task.

d, ii. A large number of candidates were not able to answer this question well because of the firstterm of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of T was −7. Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of T was a simple task.

e. A large number of candidates were not able to answer this question well because of the first term of the function's negative index. Many candidates who did make an attempt lacked understanding of the topic.

For those who had part (d) (ii) correct, this allowed them to score easy marks. For the others, it proved a difficult task because their equation in (d) would not be a tangent.

A, a. Sketch the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$. Indicate clearly where the curve intersects the y-axis. [3]

A, b. Write down the equation of the asymptote of the graph of $y = 2^x$. [2]

A, c. On the same axes sketch the graph of $y = 3 + 2x - x^2$. Indicate clearly where this curve intersects the x and y axes. [3]

A, d. Using your graphic display calculator, solve the equation $3 + 2x - x^2 = 2^x$. [2]

A, e. Write down the maximum value of the function $f(x) = 3 + 2x - x^2$. [1]

A, f. Use Differential Calculus to verify that your answer to (e) is correct. [5]

B, a. The curve $y = px^2 + qx - 4$ passes through the point (2, −10). [2]

Use the above information to write down an equation in p and q.

B, b, i. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, −10) is 1. [2]

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

B, b, ii. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, −10) is 1. [1]

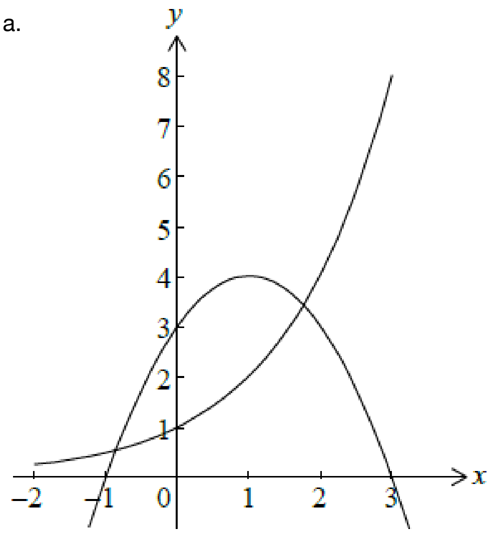
Hence, find a second equation in p and q.

B, c. The gradient of the curve $y = px^2 + qx - 4$ at the point (2, −10) is 1. [3]

Solve the equations to find the value of p and of q.

Markscheme

A, a.



(A1)(A1)(A1)

Note: Award ***(A1)*** for correct domain, ***(A1)*** for smooth curve, ***(A1)*** for y-intercept clearly indicated.

[3 marks]

A, b. $y = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for y = constant, ***(A1)*** for 0.

[2 marks]

A, c. **Note:** Award ***(A1)*** for smooth parabola,

(A1) for vertex (maximum) in correct quadrant.

(A1) for all clearly indicated intercepts $x = -1$, $x = 3$ and $y = 3$.

The final mark is to be applied very strictly. ***(A1)(A1)(A1)***

[3 marks]

A, d. $x = -0.857$ $x = 1.77$ ***(G1)(G1)***

Note: Award a maximum of ***(G1)*** if x and y coordinates are both given.

[2 marks]

A, e. 4 ***(G1)***

Note: Award ***(G0)*** for (1, 4).

[1 mark]

A, f. $f'(x) = 2 - 2x$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

$2 - 2x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their gradient function to zero.

$x = 1$ ***(A1)*****(ft)**

$f(1) = 3 + 2(1) - (1)^2 = 4$ ***(A1)***

Note: The final ***(A1)*** is for substitution of $x = 1$ into $f(x)$ and subsequent correct answer. Working must be seen for final ***(A1)*** to be awarded.

[5 marks]

B, a. $2^2 \times p + 2q - 4 = -10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the equation.

$4p + 2q = -6$ or $2p + q = -3$ ***(A1)***

Note: Accept equivalent simplified forms.

[2 marks]

B, b.i. $\frac{\mathrm{d}y}{\mathrm{d}x} = 2px + q$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

Award at most ***(A1)(A0)*** if any extra terms seen.

[2 marks]

B, b.ii. $4p + q = 1$ ***(A1)*****(ft)**

[1 mark]

B, c. $4p + 2q = -6$

$4p + q = 1$ ***(M1)***

Note: Award ***(M1)*** for sensible attempt to solve the equations.

$p = 2, q = -7$ ***(A1)(A1)*****(ft)*****(G3)***

[3 marks]

Examiners report

A, a. Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The most common error was using the incorrect domain.

A, b Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Many had little idea of asymptotes. Others did not write their answer as an equation.

A, c Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The intercepts being inexact or unlabelled was the most frequent cause of loss of marks.

A, d Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

Often, only one solution to the equation was given. Elsewhere, a lack of appreciation that the solutions were the x coordinates was a common mistake.

A, e Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

The maximum is the y coordinate only; again a common misapprehension was the answer “(1, 4)”.

A, f Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This was a major discriminator in the paper. Many candidates were unable to follow the analytic approach to finding a maximum point.

B, a Undoubtedly, this question caused the most difficulty in terms of its content. Where there was no alternative to using the calculus, the majority of candidates struggled. However, for those with a sound grasp of the topic, there were many very successful attempts.

This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

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This part was challenging to the majority, with a large number not attempting the question at all. However, there were a pleasing number of correct attempts that showed a fine understanding of the calculus.

Consider the function $f(x) = x^3 + \frac{48}{x}, x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f . [2]

f. Find the range of f . [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

Find the x-coordinate of this point.

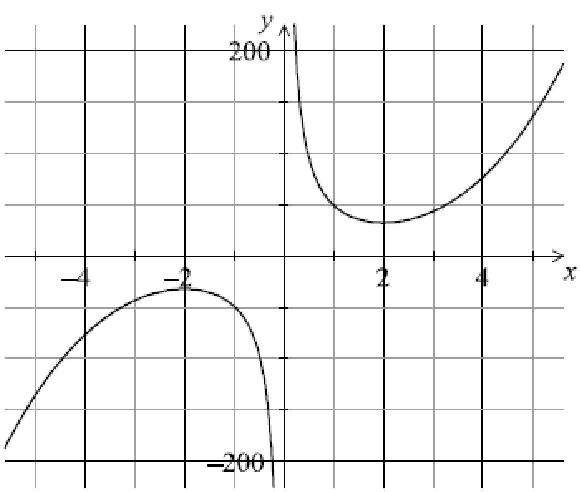
Markscheme

a. $f(2) = 2^3 + \frac{48}{2}$ ***(M1)***

$= 32$ ***(A1)(G2)***

[2 marks]

b.



(A1) for labels and some indication of scale in an appropriate window
(A1) for correct shape of the two unconnected and smooth branches
(A1) for maximum and minimum in approximately correct positions
(A1) for asymptotic behaviour at y-axis ***(A4)***

Notes: Please be rigorous.
The axes need not be drawn with a ruler.
The branches must be smooth: a single continuous line that does not deviate from its proper direction.
The position of the maximum and minimum points must be symmetrical about the origin.
The y-axis must be an asymptote for both branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

c. $f'(x) = 3x^2 - \frac{48}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $3x^2$, ***(A1)*** for -48 , ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

d. $f'(2) = 3(2)^2 - \frac{48}{(2)^2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of $x = 2$ into their derivative.

$= 0$ ***(A1)*(ft)*(G1)***

[2 marks]

e. $(-2, -32)$ or $x = -2, y = -32$ ***(G1)(G1)***

Notes: Award ***(G0)(G0)*** for $x = -32, y = -2$. Award at most ***(G0)(G1)*** if parentheses are omitted.

[2 marks]

f. $\{y \geqslant 32\} \cup \{y \leqslant -32\}$ ***(A1)(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** $y \geqslant 32$ or $y > 32$ seen, ***(A1)*(ft)** for $y \leqslant -32$ or $y < -32$, ***(A1)*** for weak (non-strict) inequalities used in both of the above.
Accept use of f in place of y. Accept alternative interval notation.
Follow through from their (a) and (e).
If domain is given award ***(A0)(A0)(A0)***.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for $[-200, -32]$, $[32, 200]$.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for $]-200, -32]$, $[32, 200[$.

[3 marks]

g. $f'(1) = -45$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for $f'(1)$ seen or substitution of $x = 1$ into their derivative. Follow through from their derivative if working is seen.

[2 marks]

h. $x = -1$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for equating their derivative to their -45 or for seeing parallel lines on their graph in the approximately correct position.

[2 marks]

Examiners report

a. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

b. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

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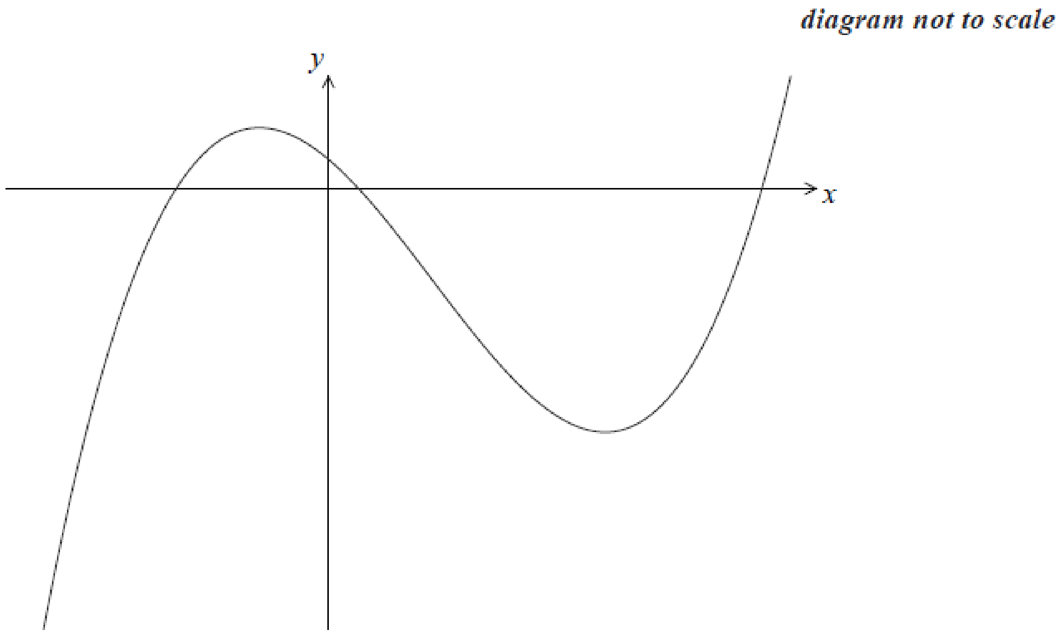
The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.



a. Write down the values of x where the graph of $f(x)$ intersects the x-axis. [3]

b. Write down $f'(x)$. [3]

c. Find the value of the local maximum of $y = f(x)$. [4]

d. Let P be the point where the graph of $f(x)$ intersects the y axis. [1]

Write down the coordinates of P.

e. Let P be the point where the graph of $f(x)$ intersects the y axis. [2]

Find the gradient of the curve at P.

f. The line, L, is the tangent to the graph of $f(x)$ at P. [2]

Find the equation of L in the form $y = mx + c$.

g. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [1]

Write down the gradient of the tangent at Q.

h. There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L. [3]

Calculate the x-coordinate of Q.

Markscheme

a. –1.10, 0.218, 3.13 ***(A1)(A1)(A1)***

[3 marks]

b. $f'(x) = 12x^2 - 18x - 12$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term and award maximum of ***(A1)(A1)*** if other terms seen.

[3 marks]

c. $f'(x) = 0$ ***(M1)***

$x = -0.5, 2$

$x = -0.5$ ***(A1)***

Note: If $x = -0.5$ not stated, can be inferred from working below.

$y = 4(-0.5)^3 - 9(-0.5)^2 - 12(-0.5) + 3$ ***(M1)***
$y = 6.25$ ***(A1)(G3)***

Note: Award ***(M1)*** for their value of x substituted into $f(x)$.

Award ***(M1)(G2)*** if sketch shown as method. If coordinate pair given then award ***(M1)(A1)(M1)(A0)***. If coordinate pair given with no working award ***(G2)***.

[4 marks]

d. (0, 3) ***(A1)***

Note: Accept $x = 0$, $y = 3$.

[1 mark]

e. $f'(0) = -12$ ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for substituting $x = 0$ into their derivative.

[2 marks]

f. Tangent: $y = -12x + 3$ ***(A1)*****(ft)*****(A1)(G2)***

Note: Award ***(A1)*****(ft)** for their gradient, ***(A1)*** for intercept = 3.

Award ***(A1)(A0)*** if y = not seen.

[2 marks]

g. -12 ***(A1)*****(ft)**

Note: Follow through from their part (e).

[1 mark]

h. $12x^2 - 18x - 12 = -12$ ***(M1)***

$12x^2 - 18x = 0$ ***(M1)***

$x = 1.5, 0$

At Q $x = 1.5$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(M1)(G2)*** for $12x^2 - 18x - 12 = -12$ followed by $x = 1.5$.

Follow through from their part (g).

[3 marks]

Examiners report

a. This question was either very well done – by the majority – or very poor and incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, though it is recognised that the differential calculus is one of the more problematic topics for the candidature.

It was however disappointing to note the number of candidates who do not use the GDC to good effect; in part (a) for example, the zeros were not found accurately due to “trace” being used; this is not a suitable approach – there is a built-in zero finder which should be used. Much of the question was accessible via a GDC approach, a sketch was given that could have been verified on the GDC; this was lost on many.

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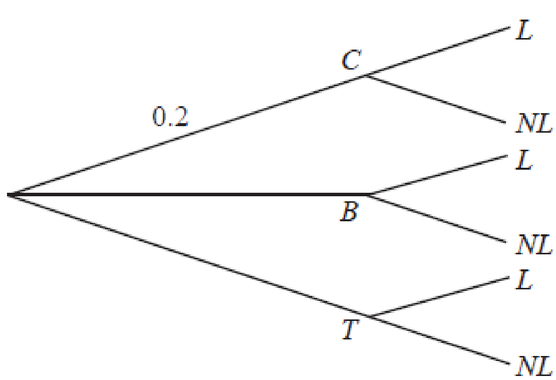
When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 % of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

*It is **not** necessary to use graph paper for this question.*

i.a. Copy the tree diagram below and fill in all the probabilities, where *NL* represents not late, to represent this information. [5]



i.b. Find the probability that Geraldine travels by bus and is late. [1]

i.c. Find the probability that Geraldine is late. [3]

i.d. Find the probability that Geraldine travelled by train, given that she is late. [3]

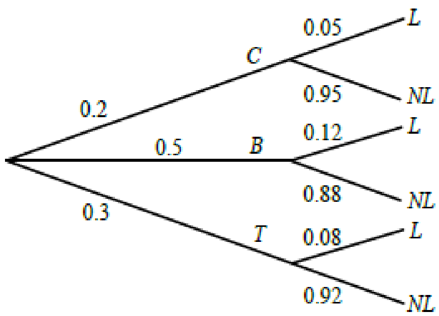
ii.a.Sketch the curve of the function $f(x)=x^3-2x^2+x-3$ for values of x from −2 to 4, giving the intercepts with both axes. [3]

ii.b.On the same diagram, sketch the line $y=7-2x$ and find the coordinates of the point of intersection of the line with the curve. [3]

ii.c.Find the value of the gradient of the curve where $x=1.7$. [2]

Markscheme

i.a.



*Award **(A1)** for 0.5 at B, **(A1)** for 0.3 at T, then **(A1)** for each correct pair. Accept fractions or percentages.* ***(A5)***

[5 marks]

i.b.0.06 (*accept* 0.5×0.12 *or* 6%) ***(A1)*(ft)**

[1 mark]

i.c.*for a relevant two-factor product, either* $C\times L$ *or* $T\times L$ ***(M1)***

for summing three two-factor products ***(M1)***

$(0.2\times0.05+0.06+0.3\times0.08)$

0.094 ***(A1)*(ft)*(G2)***

[3 marks]

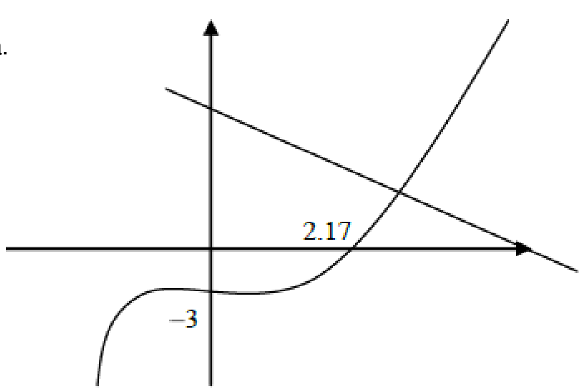
i.d. $\frac{0.3\times0.08}{0.094}$ ***(M1)(A1)*(ft)**

*award **(M1)** for substituted conditional probability formula seen, **(A1)**(ft) for correct substitution*

= 0.255 ***(A1)*(ft)*(G2)***

[3 marks]

ii.a.



(G3)

[3 marks]

ii.b*line drawn with **–ve** gradient and **+ve** y-intercept* ***(G1)***

(2.45, 2.11) ***(G1)(G1)***

[3 marks]

ii.c. $f'(1.7) = 3(1.7)^2 - 4(1.7) + 1$ ***(M1)***

award ***(M1)*** *for substituting in their* \(f' (x)\)

2.87 ***(A1)(G2)***

[2 marks]

Examiners report

i.a. This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

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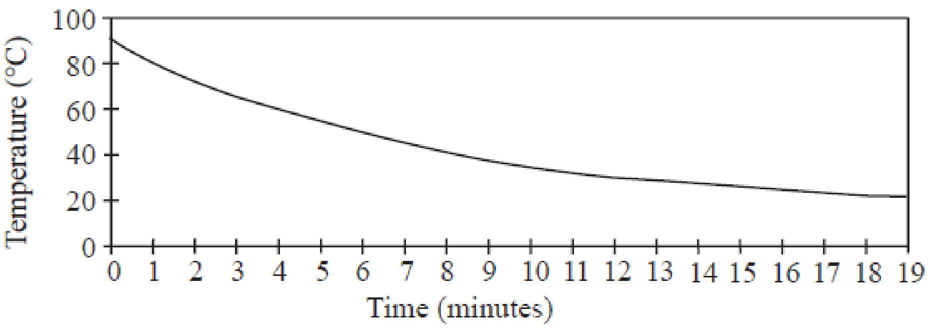
The following graph shows the temperature in degrees Celsius of Robert’s cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



Robert, who lives in the UK, travels to Belgium. The exchange rate is 1.37 euros to one British Pound (GBP) with a commission of 3 GBP, which is subtracted before the exchange takes place. Robert gives the bank 120 GBP.

i.a. Find the initial temperature of the coffee. [1]

i.b. Write down the equation of the horizontal asymptote. [1]

i.c. Find the room temperature. [1]

i.d. Find the temperature of the coffee after 10 minutes. [1]

i.e. Find the temperature of Robert’s coffee after being heated in the microwave for 30 **seconds** after it has reached the temperature in part (d). [3]

i.f. Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C. [4]

ii.a. Calculate **correct to 2 decimal places** the amount of euros he receives. [3]

ii.b. He buys 1 kilogram of Belgian chocolates at 1.35 euros per 100 g. [3]

Calculate the cost of his chocolates in GBP **correct to 2 decimal places**.

Markscheme

i.a. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 90°C ***(A1)***

[1 mark]

i.b. $y = 16$ ***(A1)***

[1 mark]

i.c. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 16°C **(ft)** *from answer to part (b)* ***(A1)*****(ft)**

[1 mark]

i.d. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

(UP) 25.4°C ***(A1)***

[1 mark]

i.e. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

for seeing $2^{0.75}$ *or equivalent* ***(A1)***

for multiplying their (d) by their $2^{0.75}$ ***(M1)***

(UP) 42.8°C ***(A1)*****(ft)*****(G2)***

[3 marks]

i.f. *Unit penalty **(UP)** is applicable in part (i)(a)(c)(d)(e) and (f)*

for seeing $20 \times 2^{1.5t} = 100$ ***(A1)***

for seeing a value of t between 1.54 *and* 1.56 *inclusive* ***(M1)(A1)***

(UP) 1.55 minutes or 92.9 seconds ***(A1)(G3)***

[4 marks]

ii.a. *Financial accuracy penalty **(FP)** is applicable in part (ii) **only**.*

$120 - 3 = 117$

(FP) 117×1.37 ***(A1)***

= 160.29 euros *(correct answer only)* ***(M1)***

*first **(A1)** for 117 seen, **(M1)** for multiplying by 1.37* ***(A1)(G2)***

[3 marks]

ii.b. *Financial accuracy penalty **(FP)** is applicable in part (ii) **only**.*

(FP) $\frac{13.5}{1.37}$ ***(A1)(M1)***

9.85 GBP *(answer correct to 2dp only)*

*first **(A1)** is for 13.5 seen, **(M1)** for dividing by 1.37* ***(A1)*****(ft)*****(G3)***

[3 marks]

Examiners report

i.a. Many candidates who had not lost a UP in question 2 lost one here. Parts (a), (c) and (d) were reasonably well tackled. Almost everybody had difficulty with the equation of the horizontal asymptote, a common answer being $y = 20$. Most of the candidates realised that 30 seconds was 0.5 minutes and calculated part (e) correctly. Part (f), solving an exponential equation, was a good discriminator. Trial and error was expected but many students did not think of doing this.

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ii.a. The financial part was the best done question in the paper and a large majority of candidates gained full marks here.

ii.b. The financial part was the best done question in the paper and a large majority of candidates gained full marks here.

Consider the function $f : x \mapsto \frac{kx}{2^x}$.

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x}$$

i.a. Given that $f(1) = 2$, show that $k = 4$. [2]

i.b. Write down the values of q and r for the following table. [2]

x	−1	0	1	2	4	8
$f(x)$	−8	0	2	q	1	r

i.c. As x increases from -1, the graph of $y = f(x)$ reaches a maximum value and then decreases, behaving asymptotically. [4]

Draw the graph of $y = f(x)$ for $-1 \leqslant x \leqslant 8$. Use a scale of 1 cm to represent 1 unit on both axes. The position of the maximum, \({\text{M}}\), the y-intercept and the asymptotic behaviour should be clearly shown.

i.d. Using your graphic display calculator, find the coordinates of M, the maximum point on the graph of $y = f(x)$. [2]

i.e. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. [2]

i.f. (i) Draw and label the line $y = 1$ on your graph. [4]

(ii) The equation $f(x) = 1$ has two solutions. One of the solutions is $x = 4$. Use your **graph** to find the other solution.

ii.a. Find $C'(x)$. [3]

ii.b. Show that the cost per person is a minimum when 10 people are invited to the party. [2]

ii.c. Calculate the minimum cost per person. [2]

Markscheme

i.a. $f(1) = \frac{k}{2^1}$ ***(M1)***

Note: ***(M1)*** for substituting $x = 1$ into the formula.

$\frac{k}{2} = 2$ ***(M1)***

Note: ***(M1)*** for equating to 2.

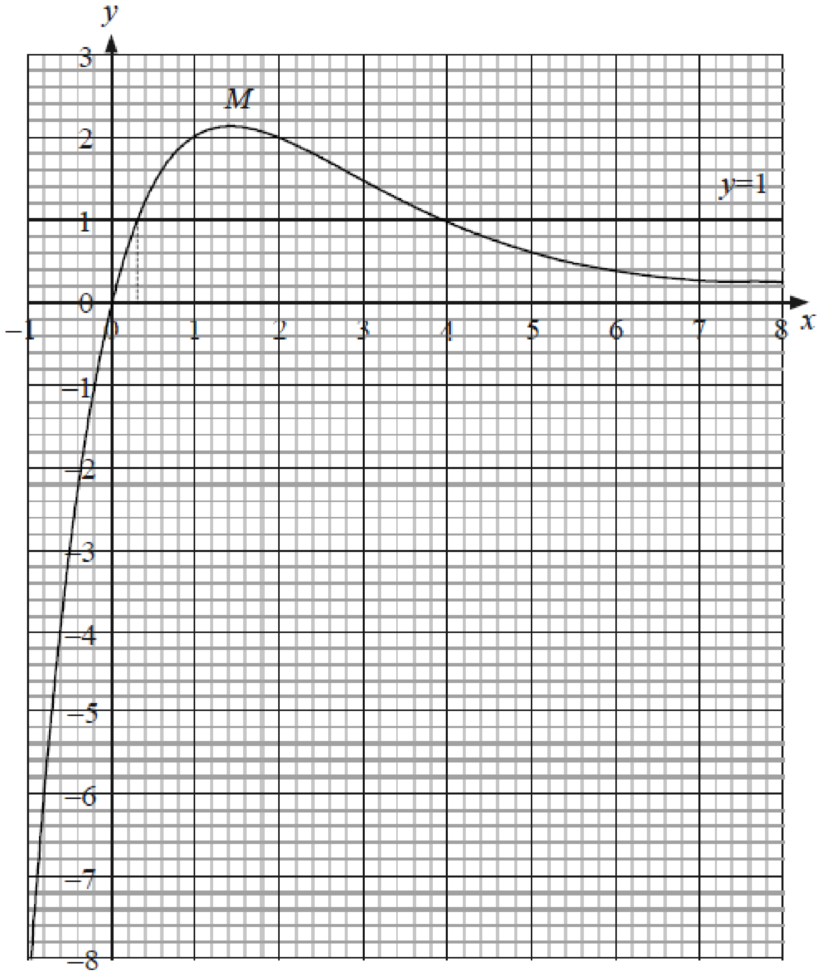
$k = 4$ ***(AG)***

[2 marks]

i.b. $q = 2$, $r = 0.125$ ***(A1)(A1)***

[2 marks]

i.c.



(A4)

Notes: ***(A1)*** for scales and labels.
(A1) for accurate smooth curve passing through $(0, 0)$ drawn at least in the given domain.
(A1) for asymptotic behaviour (curve must not go up or cross the x-axis).
(A1) for indicating the position of the maximum point.

[4 marks]

i.d. $\mathrm{M}\ (1.44,\ 2.12)$ ***(G1)(G1)***

Note: Brackets required, if missing award ***(G1)(G0)***. Accept $x = 1.44$ and $y = 2.12$.

[2 marks]

i.e. $y = 0$ ***(A1)(A1)***

Note: ***(A1)*** for '$y =$' provided the right hand side is a constant. ***(A1)*** for 0.

[2 marks]

i.f. (i) See graph ***(A1)(A1)***

Note: ***(A1)*** for correct line, ***(A1)*** for label.

(ii) $x = 0.3$ **(ft)** from candidate's graph. ***(A2)*****(ft)**

Notes: Accept ± 0.1 from their *x*. For 0.310 award ***(G1)(G0)***. For other answers taken from the GDC and not given correct to 3 significant figures award ***(G0)(AP)(G0)*** or ***(G1)(G0)*** if ***(AP)*** already applied.

[4 marks]

ii.a. $C'(x) = 1 - \frac{100}{x^2}$ ***(A1)(A1)(A1)***

Note: ***(A1)*** for 1, ***(A1)*** for -100 , ***(A1)*** for x^2 as denominator or x^{-2} as numerator. Award a maximum of ***(A2)*** if an extra term is seen.

[3 marks]

ii.b. For studying signs of the derivative at either side of $x = 10$ ***(M1)***

For saying there is a change of sign of the derivative ***(M1)(AG)***

OR

For putting $x = 10$ into C' and getting zero ***(M1)***

For clear sketch of the function or for mentioning that the function changes from decreasing to increasing at $x = 10$ ***(M1)(AG)***

OR

For solving $C'(x) = 0$ and getting 10 ***(M1)***

For clear sketch of the function or for mentioning that the function changes from decreasing to increasing at $x = 10$ ***(M1)(AG)***

Note: For a sketch with a clear indication of the minimum or for a table with values of x at either side of $x = 10$ award ***(M1)(M0)***.

[2 marks]

ii.c. $C(10) = 10 + \frac{100}{10}$ ***(M1)***

$C(10) = 20$ ***(A1)(G2)***

[2 marks]

Examiners report

i.a. There were many well drawn graphs using correctly scaled and labelled axes with a good curve drawn. A number of students did not label the maximum point. Although many students showed in their graph the asymptotic behaviour of the curve, they did not know how to describe the asymptote. It was noticed that some students were tracing the curve to find the coordinates of the maximum instead of finding the maximum directly. The intersection between the line $y = 1$ and the curve was not always read from their graph but from their GDC's graph.

i.b. There were many well drawn graphs using correctly scaled and labelled axes with a good curve drawn. A number of students did not label the maximum point. Although many students showed in their graph the asymptotic behaviour of the curve, they did not know how to describe the asymptote. It was noticed that some students were tracing the curve to find the coordinates of the maximum instead of finding the maximum directly. The intersection between the line $y = 1$ and the curve was not always read from their graph but from their GDC's graph.

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ii.a. Finding the derivative was done at least partially correctly by most of the candidates. However, using it to find the minimum and to justify why it is a minimum was troublesome for the majority of the candidates. Even those who used a graph in their reasoning neglected to mention the change from decreasing to increasing or to supply a sign diagram. Many candidates recovered in the last part of the question when finding the minimum cost.

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Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$

a. Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. [4]

b. A teacher asks her students to make some observations about the curve. [1]

Three students responded.
Nadia said *"The x-intercept of the curve is between −1 and zero".*
Rick said *"The curve is decreasing when x < 1 ".*
Paula said *"The gradient of the curve is less than zero between x = 1 and x = 2 ".*

State the name of the student who made an **incorrect** observation.

c. Find the value of y when $x = 1$. [2]

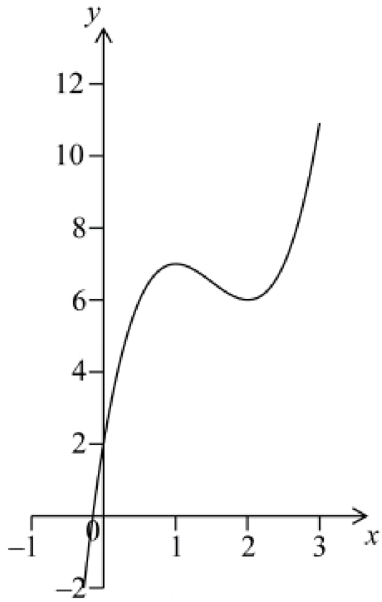
d. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$. [3]

e. Show that the stationary points of the curve are at $x = 1$ and $x = 2$. [2]

f. Given that $y = 2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. [3]

Markscheme

a.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct window (condone a window which is slightly off) and axes labels. An indication of window is necessary. −1 to 3 on the x-axis and −2 to 12 on the y-axis and a graph in that window.
(A1) for correct shape (curve having cubic shape and must be smooth).
(A1) for both stationary points in the 1st quadrant with approximate correct position,
(A1) for intercepts (negative x-intercept and positive y intercept) with approximate correct position.

[4 marks]

b. Rick ***(A1)***

Note: Award ***(A0)*** if extra names stated.

[1 mark]

c. $2(1)^3 - 9(1)^2 + 12(1) + 2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into equation.

$= 7$ ***(A1)(G2)***

[2 marks]

d. $6x^2 - 18x + 12$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

e. $6x^2 - 18x + 12 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to 0. If the derivative is not explicitly equated to 0, but a subsequent solving of their correct equation is seen, award ***(M1)***.

$6(x-1)(x-2)=0$ (or equivalent) ***(M1)***

Note: Award (M1) for correct factorization. The final ***(M1)*** is awarded only if answers are clearly stated.

Award ***(M0)(M0)*** for substitution of 1 and of 2 in their derivative.

$x = 1$, $x = 2$ ***(AG)***

[2 marks]

f. $6 < k < 7$ ***(A1)(A1)*****(ft)*****(A1)***

Note: Award ***(A1)*** for an inequality with 6, award ***(A1)*****(ft)** for an inequality with 7 from their part (c) provided it is greater than 6, ***(A1)*** for their correct strict inequalities. Accept]6, 7[or (6, 7).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

On Monday Paco goes to a running track to train. He runs the first lap of the track in 120 seconds. Each lap Paco runs takes him 10 seconds longer than his previous lap.

a. Find the time, in seconds, Paco takes to run his fifth lap. [3]

b. Paco runs his last lap in 260 seconds. [3]

Find how many laps he has run on Monday.

c. Find the **total** time, in **minutes,** run by Paco on Monday. [4]

d. On Wednesday Paco takes Lola to train. They both run the first lap of the track in 120 seconds. Each lap Lola runs takes 1.06 times as long as her previous lap. [3]

Find the time, in seconds, Lola takes to run her third lap.

e. Find the **total** time, in seconds, Lola takes to run her first four laps. [3]

f. Each lap Paco runs again takes him 10 seconds longer than his previous lap. After a certain number of laps Paco takes less time per lap than Lola. [3]

Find the number of the lap when this happens.

Markscheme

a. $120 + 10 \times 4$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted AP formula, ***(A1)*** for correct substitutions. Accept a list of 4 correct terms.

= 160 ***(A1)(G3)***

b. $120 + (n - 1) \times 10 = 260$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correctly substituted AP formula, ***(M1)*** for equating to 260. Accept a list of correct terms showing at least the 14th and 15th terms.

= 15 ***(A1)(G2)***

c. $\frac{15}{2}(120 + 260)$ **or** $\frac{15}{2}(2 \times 120 + (15 - 1) \times 10)$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substituted AP sum formula, ***(A1)*(ft)** for correct substitutions. Accept a sum of a list of 15 correct terms. Follow through from their answer to part (b).

2850 seconds ***(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for 2850 seen with no working shown.

47.5 minutes ***(A1)*(ft)*(G3)***

Notes: A final ***(A1)*(ft)** can be awarded for correct conversion from seconds into minutes of their incorrect answer. Follow through from their answer to part (b).

d. $120 \times 1.06^{3-1}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted GP formula, ***(A1)*** for correct substitutions. Accept a list of 3 correct terms.

= 135 (134.832) ***(A1)(G2)***

e. $S_4 = \frac{120(1.06^4 - 1)}{(1.06 - 1)}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted GP sum formula, ***(A1)*** for correct substitutions. Accept a sum of a list of 4 correct terms.

= 525 (524.953...) ***(A1)(G2)***

f. $120 + (n - 1) \times 10 < 120 \times 1.06^{n-1}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct left hand side, ***(M1)*** for correct right hand side. Accept an equation. Follow through from their expressions given in parts (a) and (d).

OR

List of at least 2 terms for both sequences (120, 130, … and 120, 127.2, …) ***(M1)***

List of correct 12th and 13th terms for both sequences (..., 230, 240 and …, 227.8, 241.5) ***(M1)***

OR

A sketch with a line and an exponential curve, ***(M1)***

An indication of the correct intersection point ***(M1)***

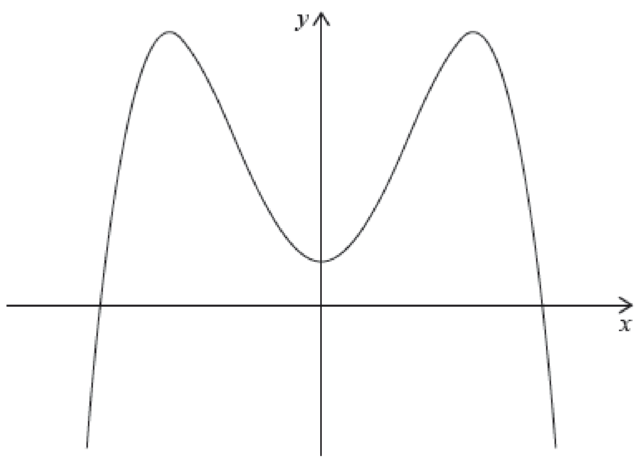
13th lap ***(A1)*(ft)*(G2)***

Note: Do not award the final ***(A1)*(ft)** if final answer is not a positive integer.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Consider the function $f(x) = -x^4 + ax^2 + 5$, where a is a constant. Part of the graph of $y = f(x)$ is shown below.



It is known that at the point where $x = 2$ the tangent to the graph of $y = f(x)$ is horizontal.

There are two other points on the graph of $y = f(x)$ at which the tangent is horizontal.

a. Write down the y-intercept of the graph. [1]

b. Find $f'(x)$. [2]

c.i. Show that $a = 8$. [2]

c.ii. Find $f(2)$. [2]

d.i. Write down the x-coordinates of these two points; [2]

d.ii. Write down the intervals where the gradient of the graph of $y = f(x)$ is positive. [2]

e. Write down the range of $f(x)$. [2]

f. Write down the number of possible solutions to the equation $f(x) = 5$. [1]

g. The equation $f(x) = m$, where $m \in \mathbb{R}$, has four solutions. Find the possible values of m. [2]

Markscheme

a. 5 ***(A1)***

Note: Accept an answer of $(0,\ 5)$.

[1 mark]

b. $(f'(x) =) - 4x^3 + 2ax$ ***(A1)(A1)***

Note: Award ***(A1)*** for $-4x^3$ and ***(A1)*** for $+2ax$. Award at most ***(A1)(A0)*** if extra terms are seen.

[2 marks]

c.i. $-4 \times 2^3 + 2a \times 2 = 0$ ***(M1)(M1)***

Note: Award ***(M1)*** for substitution of $x = 2$ into their derivative, ***(M1)*** for equating their derivative, written in terms of a, to 0 leading to a correct answer (note, the 8 does not need to be seen).

$a = 8$ ***(AG)***

[2 marks]

c.ii. $(f(2) =) - 2^4 + 8 \times 2^2 + 5$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $x = 2$ and $a = 8$ into the formula of the function.

21 ***(A1)(G2)***

[2 marks]

d.i. $(x =)\ -2,\ (x =)\ 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct solution. Award at most ***(A0)(A1)*(ft)** if answers are given as $(-2\ , 21)$ and $(0,\ 5)$ or $(-2,\ 0)$ and $(0,\ 0)$.

[2 marks]

d.ii $x < -2,\ 0 < x < 2$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for $x < -2$, follow through from part (d)(i) provided their value is negative.

Award ***(A1)*(ft)** for $0 < x < 2$, follow through only from their 0 from part (d)(i); 2 must be the upper limit.

Accept interval notation.

[2 marks]

e. $y \leqslant 21$ ***(A1)*(ft)*(A1)***

Notes: Award ***(A1)*(ft)** for 21 seen in an interval or an inequality, ***(A1)*** for "$y \leqslant$".

Accept interval notation.

Accept $-\infty < y \leqslant 21$ or $f(x) \leqslant 21$.

Follow through from their answer to part (c)(ii). Award at most ***(A1)*(ft)*(A0)*** if x is seen instead of y. Do not award the second ***(A1)*** if a (finite) lower limit is seen.

[2 marks]

f. 3 (solutions) ***(A1)***

[1 mark]

g. $5 < m < 21$ or equivalent ***(A1)*(ft)*(A1)***

Note: Award ***(A1)*(ft)** for 5 and 21 seen in an interval or an inequality, ***(A1)*** for correct strict inequalities. Follow through from their answers to parts (a) and (c)(ii).

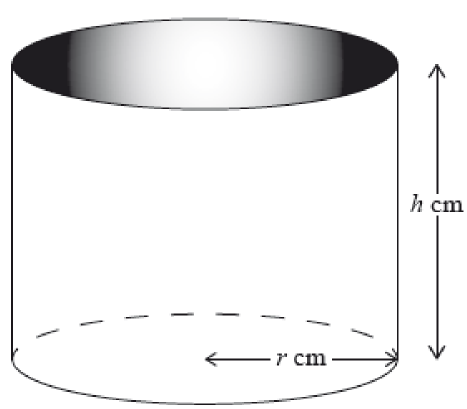
Accept interval notation.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c.i. [N/A]
c.ii. [N/A]
d.i. [N/A]
d.ii. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is $0.5\ \mathrm{m}^3$.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Markscheme

a. $(A =)\ \pi r^2 + 2\pi rh$ ***(A1)(A1)***

Note: Award ***(A1)*** for either πr^2 **OR** $2\pi rh$ seen. Award ***(A1)*** for two correct terms added together.

[2 marks]

b. $500\,000$ ***(A1)***

Notes: Units **not** required.

[1 mark]

c. $500\,000 = \pi r^2 h$ ***(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for $\pi r^2 h$ equating to their part (b).

Do not accept unless $V = \pi r^2 h$ is explicitly defined as their part (b).

[1 mark]

d. $A = \pi r^2 + 2\pi r\left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*(ft)*(M1)*** for rearranging part (c) to $\pi rh = \frac{500\,000}{r}$ and substituting for πrh in expression for A.

$A = \pi r^2 + \frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A = \pi r^2 + \frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

d. $A = \pi r^2 + 2\pi r\left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*(ft)*(M1)*** for rearranging part (c) to $\pi rh = \frac{500\,000}{r}$ and substituting for πrh in expression for A.

$A = \pi r^2 + \frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A = \pi r^2 + \frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

e. $2\pi r - \frac{1\,000\,000}{r^2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi r$, ***(A1)*** for $\frac{1}{r^2}$ or r^{-2}, ***(A1)*** for $-1\,000\,000$.

[3 marks]

f. $2\pi r - \frac{1\,000\,000}{r^2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (e) to zero.

$r^3 = \frac{1\,000\,000}{2\pi}$ **OR** $r = \sqrt[3]{\frac{1\,000\,000}{2\pi}}$ ***(M1)***

Note: Award ***(M1)*** for isolating r.

OR

sketch of derivative function ***(M1)***

with its zero indicated ***(M1)***

$(r =)\ 54.2\ (\text{cm})\ (54.1926\ldots)$ ***(A1)*(ft)*(G2)***

[3 marks]

g. $\pi(54.1926\ldots)^2 + \frac{1\,000\,000}{(54.1926\ldots)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (f) into the given equation.

$= 27\,700\ (\text{cm}^2)\ (27\,679.0\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

h. $\frac{27\,679.0\ldots}{2000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (g) by 2000.

$= 13.8395\ldots$ ***(A1)*(ft)**

Notes: Follow through from part (g).

14 (cans) ***(A1)*(ft)*(G3)***

Notes: Final ***(A1)*** awarded for rounding up their $13.8395\ldots$ to the next integer.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

Consider the sequence $u_1,\ u_2,\ u_3,\ \ldots,\ \text{u}_n,\ \ldots$ where

$$u_1 = 600,\ u_2 = 617,\ u_3 = 634,\ u_4 = 651.$$

The sequence continues in the same manner.

a. Find the value of u_{20}. [3]

b. Find the sum of the first 10 terms of the sequence. [3]

c. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

Find the exact value of v_{10}.

d. Now consider the sequence $v_1,\ v_2,\ v_3,\ \ldots,\ v_n,\ \ldots$ where [3]

$$v_1 = 3,\ v_2 = 6,\ v_3 = 12,\ v_4 = 24$$

This sequence continues in the same manner.

Find the sum of the first 8 terms of this sequence.

e. k is the smallest value of n for which v_n is greater than u_n. [3]

Calculate the value of k.

Markscheme

a. $600 + (20 - 1) \times 17$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 20 correct terms seen.

$= 923$ ***(A1)(G3)***

[3 marks]

b. $\frac{10}{2}[2 \times 600 + (10 - 1) \times 17]$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic series formula, ***(A1)*** for their correct substitutions. Follow through from part (a). For consistent use of geometric series formula in part (b) with the geometric sequence formula in part (a) award a maximum of ***(M1)(A1)(A0)*** since their final answer cannot be an integer.

OR

$u_{10} = 600 + (10 - 1)17 = 753$ ***(M1)***

$S_{10} = \frac{10}{2}(600 + \text{their } u_{10})$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted arithmetic sequence formula, ***(M1)*** for their correctly substituted arithmetic series formula. Follow through from part (a) and **within** part (b).

Note: If a list is used, award ***(M1)*** for at least 10 correct terms seen, award ***(A1)*** for these terms being added.

$= 6765$ (accept 6770) ***(A1)*(ft)*(G2)***

[3 marks]

c. 3×2^9 ***(M1)(A1)***

Note: Award ***(M1)*** for substituted geometric sequence formula, ***(A1)*** for correct substitutions. If a list is used, award ***(M1)*** for at least 6 correct terms seen, award ***(A1)*** for at least 8 correct terms seen.

$= 1536$ ***(A1)(G3)***

Note: Exact answer only. If both exact and rounded answer seen, award the final ***(A1)***.

[3 marks]

d. $\frac{3 \times (2^8 - 1)}{2 - 1}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted geometric series formula, ***(A1)*** for their correct substitutions. Follow through from part (c). If a list is used, award ***(M1)*** for at least 8 correct terms seen, award ***(A1)*** for these 8 correct terms being added. For consistent use of arithmetic series formula in part (d) with the arithmetic sequence formula in part (c) award a maximum of ***(M1)(A1)(A1)***.

$= 765$ ***(A1)*****(ft)*****(G2)***

[3 marks]

e. $3 \times 2^{k-1} > 600 + (k - 1)(17)$ ***(M1)***

Note: Award ***(M1)*** for their correct inequality; allow equation.

Follow through from parts (a) and (c). Accept sketches of the two functions as a valid method.

$k > 8.93648\ldots$ (may be implied) ***(A1)*****(ft)**

Note: Award ***(A1)*** for $8.93648\ldots$ seen. The GDC gives answers of -34.3 and 8.936 to the inequality; award ***(M1)(A1)*** if these are seen with working shown.

OR

$v_8 = 384$ $u_8 = 719$ ***(M1)***

$v_9 = 768$ $u_9 = 736$ ***(M1)***

Note: Award ***(M1)*** for v_8 and u_8 both seen, ***(M1)*** for v_9 and u_9 both seen.

$k = 9$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(G1)*** for $8.93648\ldots$ and -34.3 seen as final answer without working. Accept use of n.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

In the month before their IB Diploma examinations, eight male students recorded the number of hours they spent on social media.

For each student, the number of hours spent on social media (x) and the number of IB Diploma points obtained (y) are shown in the following table.

Hours on social media (x)	6	15	26	12	13	40	33	23
IB Diploma points (y)	43	33	27	36	39	17	20	33

Use your graphic display calculator to find

Ten female students also recorded the number of hours they spent on social media in the month before their IB Diploma examinations. Each of these female students spent between 3 and 30 hours on social media.

The equation of the regression line *y* on *x* for these ten female students is

$$y = -\frac{2}{3}x + \frac{125}{3}.$$

An eleventh girl spent 34 hours on social media in the month before her IB Diploma examinations.

a. On graph paper, draw a scatter diagram for these data. Use a scale of 2 cm to represent 5 hours on the x-axis and 2 cm to represent 10 points on the y-axis. [4]

b. (i) $\bar{x}$, the mean number of hours spent on social media; [2]

(ii) $\bar{y}$, the mean number of IB Diploma points.

c. Plot the point $(\bar{x},\ \bar{y})$ on your scatter diagram and label this point M. [2]

d. Write down the value of r, the Pearson's product–moment correlation coefficient, for these data. [2]

e. Write down the equation of the regression line y on x for these eight male students. [2]

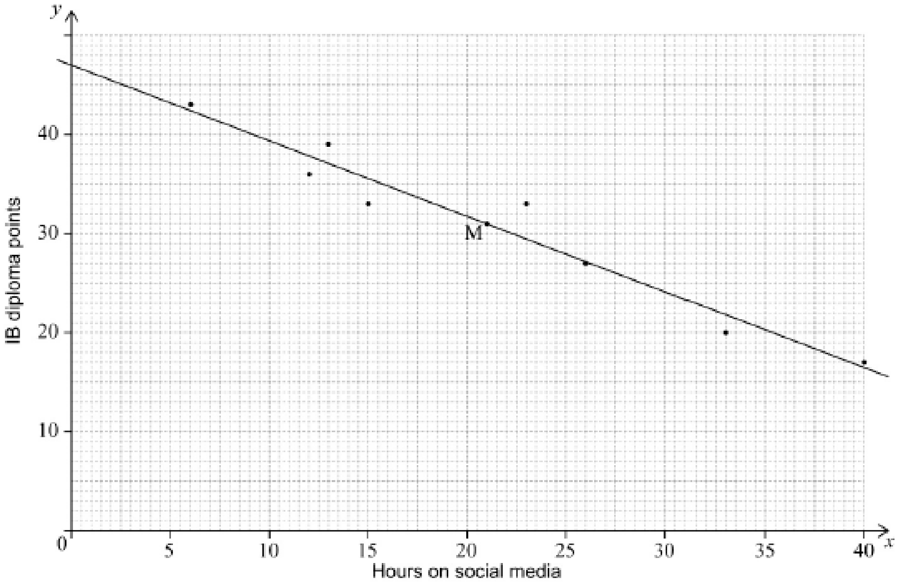
f. Draw the regression line, from part (e), on your scatter diagram. [2]

g. Use the given equation of the regression line to estimate the number of IB Diploma points that this girl obtained. [2]

h. Write down a reason why this estimate is not reliable. [1]

Markscheme

a.



(A4)

Notes: Award ***(A1)*** for correct scale and labelled axes.

Award ***(A3)*** for 7 or 8 points correctly plotted,

(A2) for 5 or 6 points correctly plotted,

(A1) for 3 or 4 points correctly plotted.

Award at most ***(A0)(A3)*** if axes reversed.

Accept x and y sufficient for labelling.

If graph paper is not used, award ***(A0)***.

If an inconsistent scale is used, award ***(A0).*** Candidates' points should be read from this scale **where possible** and awarded accordingly.

A scale which is too small to be meaningful (ie mm instead of cm) earns ***(A0)*** for plotted points.

[4 marks]

b. (i) $\bar{x} = 21$ ***(A1)***

(ii) $\bar{y} = 31$ ***(A1)***

[2 marks]

c. $(\bar{x},\ \bar{y})$ correctly plotted on graph ***(A1)*****(ft)**

this point labelled M ***(A1)***

Note: Follow through from parts (b)(i) and (b)(ii).

Only accept M for labelling.

[2 marks]

d. $-0.973\ (-0.973388\ldots)$ ***(G2)***

Note: Award ***(G1)*** for 0.973, without minus sign.

[2 marks]

e. $y = -0.761x + 47.0$ $(y = -0.760638\ldots x + 46.9734\ldots)$ ***(A1)(A1)(G2)***

Notes: Award ***(A1)*** for $-0.761x$ and ***(A1)*** $+47.0$. Award a maximum of ***(A1)(A0)*** if answer is not an equation.

[2 marks]

f. line on graph ***(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for **straight line** that passes through their M, ***(A1)*(ft)** for line (extrapolated if necessary) that passes through $(0,\ 47.0)$.

If M is not plotted or labelled, follow through from part (e).

[2 marks]

g. $y = -\frac{2}{3}(34) + \frac{125}{3}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

19 (points) ***(A1)(G2)***

[2 marks]

h. extrapolation ***(R1)***

OR

34 hours is outside the given range of data ***(R1)***

Note: Do not accept ‘outlier’.

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [5]

Using your answer to part (b), show that there is a second local minimum at $x = 3$.

d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [4]

Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [2]

Write down the coordinates of the local maximum.

f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$. [2]

Find the gradient of T.

g. The line L passes through the point $(2, -12)$ and is perpendicular to T. [5]

L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

Markscheme

a. $\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = -2$ in the function.

$= 4$ ***(A1)(G2)***

Note: If the coordinates $(-2,\ 4)$ are given as the answer award, at most, ***(M1)(A0)***. If no working shown award ***(G1)***.

If $x = -2,\ y = 4$ seen then award full marks.

[2 marks]

b. $3x^3 - 3x^2 - 18x$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term, award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

c. $f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3$ ***(M1)***

Note: Award ***(M1)*** for substitution in their $f'(x)$ of $x = 3$.

$= 0$ ***(A1)***

OR

$3x^3 - 3x^2 - 18x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their $f'(x)$ to zero.

$x = 3$ ***(A1)***

$f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0$ where $0 < x_1 < 3$ ***(M1)***

Note: Award ***(M1)*** for substituting a value of x_1 in the range $0 < x_1 < 3$ into their f' and showing it is negative (decreasing).

$f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0$ where $x_2 > 3$ ***(M1)***

Note: Award ***(M1)*** for substituting a value of x_2 in the range $x_2 > 3$ into their f' and showing it is positive (increasing).

OR

With or without a sketch:

Showing $f(x_1) > f(3)$ where $x_1 < 3$ and x_1 is close to 3. ***(M1)***

Showing $f(x_2) > f(3)$ where $x_2 > 3$ and x_2 is close to 3. ***(M1)***

Note: If a sketch of $f(x)$ is drawn **in this part of the question and** $x = 3$ is identified as a stationary point on the curve, then

(i) award, at most, ***(M1)(A1)(M1)(M0)*** if the stationary point has been found;

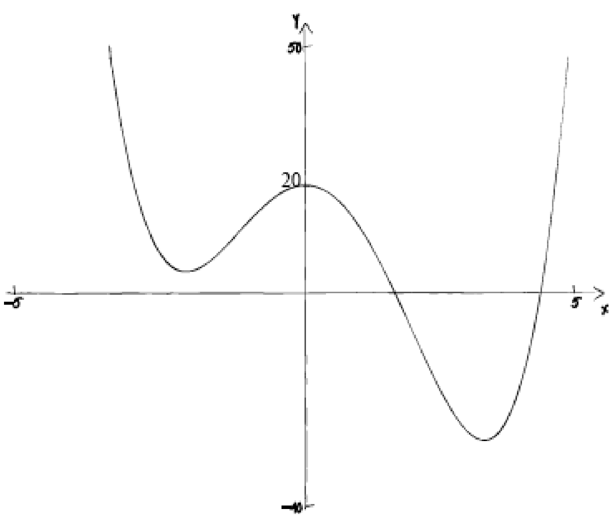
(ii) award, at most, ***(M0)(A0)(M1)(M0)*** if the stationary point has not been previously found.

Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum ***(R1)(AG)***

Note: Only award ***(R1)*** if the first two marks have been awarded *ie* $f'(3)$ has been shown to be equal to 0.

[5 marks]

d.



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labelled axes and indication of scale on both axes.

Award ***(A1)*** for smooth curve with correct shape.

Award ***(A1)*** for local minima in 2^{nd} and 4^{th} quadrants.

Award ***(A1)*** for *y* intercept $(0, 20)$ seen and labelled. Accept 20 on y-axis.

Do **not** award the third ***(A1)*** mark if there is a turning point on the x-axis.

If the derivative function is sketched then award, at most, ***(A1)(A0)(A0)(A0)***.

For a smooth curve (with correct shape) there should be **ONE** continuous thin line, no part of which is straight and no (one to many) mappings of x.

[4 marks]

e. $(0, 20)$ ***(G1)(G1)***

Note: If parentheses are omitted award ***(G0)(G1)***.

OR

$x = 0,\ y = 20$ ***(G1)(G1)***

Note: If the derivative function is sketched in part (d), award ***(G1)*(ft)*(G1)*(ft)** for $(-1.12, 12.2)$.

[2 marks]

f. $f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$ ***(M1)***

Notes: Award ***(M1)*** for substituting $x = 2$ into their $f'(x)$.

$= -24$ ***(A1)*(ft)*(G2)***

[2 marks]

g. (i) Gradient of perpendicular $= \frac{1}{24}$ $(0.0417, 0.041666\ldots)$ ***(A1)*(ft)*(G1)***

Note: Follow through from part (f).

(ii) $y + 12 = \frac{1}{24}(x - 2)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $(2, -12)$, ***(M1)*** for correct substitution of their perpendicular gradient into equation of line.

OR

$-12 = \frac{1}{24} \times 2 + d$ ***(M1)***

$d = -\frac{145}{12}$

$y = \frac{1}{24}x - \frac{145}{12}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $(2, -12)$ and gradient into equation of a straight line, ***(M1)*** for correct substitution of the perpendicular gradient and correct substitution of $d$into equation of line.

$b = -24,\ c = -290$ ***(A1)*(ft)*(A1)*(ft)*(G3)***

Note: Follow through from parts (f) and g(i).

To award **(ft)** marks, b and c must be integers.

Where candidate has used 0.042 from g(i), award ***(A1)*(ft)** for -288.

[5 marks]

Examiners report

a. Surprisingly, a correct method for substituting the value of –2 into the given function led many candidates to a variety of incorrect answers. This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x - 3$ into their answer to part (b). Once they had shown that there was a turning point at $x - 3$, candidates were not expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of –24. Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of L very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for b and the coordinates (2, –12) into the equation $x + by + c = 0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

b. Surprisingly, a correct method for substituting the value of –2 into the given function led many candidates to a variety of incorrect answers. This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x - 3$ into their answer to part (b). Once they had shown that there was a turning point at $x - 3$, candidates were not expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of –24. Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found

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Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

Markscheme

a. $\frac{-192}{x^3}+k$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for -192, ***(A1)*** for x^{-3}, ***(A1)*** for k (only).

b. at local minimum $f'(x)=0$ ***(M1)***

Note: Award ***(M1)*** for seeing $f'(x)=0$ (may be implicit in their working).

$\frac{-192}{4^3}+k=0$ ***(A1)***

$k=3$ ***(AG)***

Note: Award ***(A1)*** for substituting $x=4$ in their $f'(x)=0$, provided it leads to $k=3$. The conclusion $k=3$ must be seen for the ***(A1)*** to be awarded.

c. $\frac{96}{2^2}+3(2)$ ***(M1)***

Note: Award ***(M1)*** for substituting $x=2$ and $k=3$ in $f(x)$.

$=30$ ***(A1)(G2)***

d. $\frac{-192}{2^3}+3$ ***(M1)***

Note: Award ***(M1)*** for substituting $x=2$ and $k=3$ in their $f'(x)$.

$=-21$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a).

e. $y-30=\frac{1}{21}(x-2)$ ***(A1)*****(ft)*****(M1)***

Notes: Award ***(A1)*****(ft)** for their $\frac{1}{21}$ seen, ***(M1)*** for the correct substitution of their point and their normal gradient in equation of a line.

Follow through from part (c) and part (d).

OR

gradient of normal $=\frac{1}{21}$ ***(A1)*****(ft)**

$30=\frac{1}{21}\times 2+c$ ***(M1)***

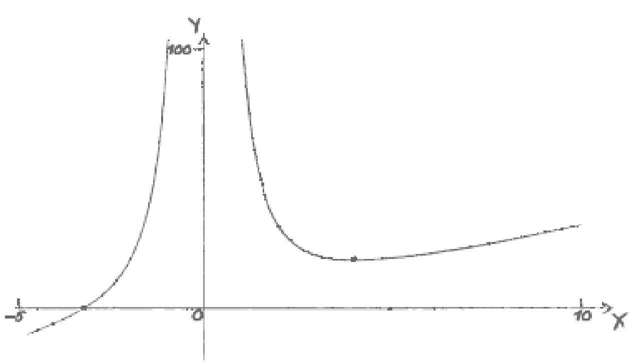
$c=29\frac{19}{21}$

$y=\frac{1}{21}x+29\frac{19}{21}\quad(y=0.0476x+29.904)$

$x-21y+628=0$ ***(A1)*****(ft)*****(G2)***

Notes: Accept equivalent answers.

f.



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; ***(A1)*** for a smooth curve with the correct shape (graph should not touch y-axis and should not curve away from the y-axis), on the given domain; ***(A1)*** for axis intercept in approximately the correct position (nearer -5 than zero); ***(A1)*** for local minimum in approximately the correct position (first quadrant, nearer the y-axis than $x = 10$).

If there is no scale, award a maximum of ***(A0)(A1)(A0)(A1)*** – the final ***(A1)*** being awarded for the zero and local minimum in approximately correct positions relative to each other.

g. $(-3.17,\ 0) \quad ((-3.17480\ldots,\ 0))$ ***(G1)(G1)***

Notes: If parentheses are omitted award ***(G0)(G1)*****(ft)**.

Accept $x = -3.17,\ y = 0$. Award ***(G1)*** for -3.17 seen.

h. $0 < x \leqslant 4 \text{ or } 0 < x < 4$ ***(A1)(A1)***

Notes: Award ***(A1)*** for correct end points of interval, ***(A1)*** for correct notation (note: lower inequality must be strict).

Award a maximum of ***(A1)(A0)*** if y or $f(x)$ used in place of x.

Examiners report

a. Differentiation of terms including negative indices remains a testing process; it will continue to be tested. There was, however, a noticeable improvement in responses compared to previous years. The parameter k was problematic for a number of candidates.

b. In part (b), the manipulation of the derivative to find the local minimum point caused difficulties for all but the most able; note that a GDC approach is not accepted in such questions and that candidates are expected to be able to apply the theory of the calculus as appropriate. Further, once a parameter is given, candidates are expected to use this value in subsequent parts.

c. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

d. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

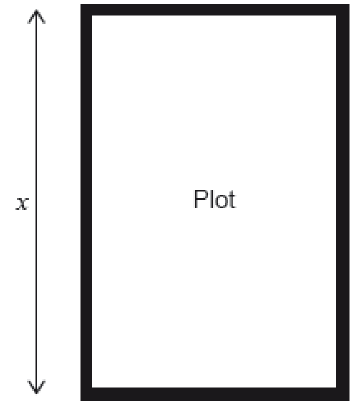
e. Part (e) discriminated at the highest level; the gradient of the normal often was not used, the form of the answer not given correctly.

f. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

g. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

h. Part (h) was not well attempted in the main; decreasing (and increasing) functions is a testing concept for the majority.

Violeta plans to grow flowers in a rectangular plot. She places a fence to mark out the perimeter of the plot and uses 200 metres of fence. The length of the plot is x metres.



Violeta places the fence so that the area of the plot is maximized.

By selling her flowers, Violeta earns 2 Bulgarian Levs (BGN) per square metre of the plot.

Violeta wants to invest her 5000 BGN.

A bank offers a nominal annual interest rate of 4%, compounded **half-yearly**.

Another bank offers an interest rate of $r\%$ compounded **annually**, that would allow her to double her money in 12 years.

a. Show that the width of the plot, in metres, is given by $100 - x$. [1]

b. Write down the area of the plot in terms of x. [1]

c. Find the value of x that maximizes the area of the plot. [2]

d. Show that Violeta earns 5000 BGN from selling the flowers grown on the plot. [2]

e.i. Find the amount of money that Violeta would have after 6 years. Give your answer correct to two decimal places. [3]

e.ii. Find how long it would take for the interest earned to be 2000 BGN. [3]

f. Find the lowest possible value for r. [2]

Markscheme

a. $\frac{200-2x}{2}$ (or equivalent) ***(M1)***

OR

$2x + 2y = 200$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for a correct expression leading to $100 - x$ (the $100 - x$ does not need to be seen). The 200 must be seen for the ***(M1)*** to be awarded. Do not accept $100 - x$ substituted in the perimeter of the rectangle formula.

$100 - x$ ***(AG)***

[1 mark]

b. $(\text{area} =)\ x(100 - x)$ **OR** $-x^2 + 100x$ (or equivalent) ***(A1)***

[1 mark]

c. $x = \frac{-100}{-2}$ **OR** $-2x + 100 = 0$ **OR** graphical method ***(M1)***

Note: Award ***(M1)*** for use of axis of symmetry formula or first derivative equated to zero or a sketch graph.

$x = 50$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b), provided *x* is positive and less than 100.

[2 marks]

d. $50(100 - 50) \times 2$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting their x into their formula for area (accept "50×50" for the substituted formula), and ***(M1)*** for multiplying by 2. Award at most ***(M0)(M1)*** if their calculation does not lead to 5000 (BGN), although the 5000 (BGN) does not need to be seen explicitly.

Substitution of 50 into area formula may be seen in part (c).

5000 (BGN) ***(AG)***

[2 marks]

e.i. $5000\left(1 + \frac{4}{2 \times 100}\right)^{2 \times 6}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

OR

$\text{N} = 6$

$\text{I\%} = 4$

$\text{PV} = -5000$

$\text{P/Y} = 1$

$\text{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for $\text{C/Y} = 2$ seen, ***(M1)*** for other correct entries.

OR

$\text{N} = 12$

$\text{I\%} = 4$

$\text{PV} = -5000$

$\text{P/Y} = 2$

$\text{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for $\mathrm{C/Y} = 2$ seen, ***(M1)*** for other correct entries.

6341.21 (BGN) ***(A1)(G3)***

Note: Award ***(A1)*** for correct answer, to two decimal places only.

Award ***(G2)*** for 6341.20 or a correct, unrounded final answer if no working is seen (6341.2089…).

[3 marks]

e.ii. $5000\left(1 + \frac{4}{2\times100}\right)^{2\times t} = 7000$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for using the compound interest formula with a variable for time, ***(A1)*****(ft)** for substituting the correct values and equating to 7000. Follow through for their "2" from part (e)(i).

OR

$\mathrm{I\%} = 4$

$\mathrm{PV} = (\pm)5000$

$\mathrm{FV} = \mp7000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.

Award ***(M1)*** for their C/Y from part (e)(i).

OR

$\mathrm{I\%} = 4$

$\mathrm{PV} = (\pm)5000$

$\mathrm{FV} = \mp7000$

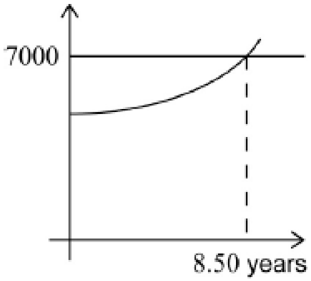
$\mathrm{P/Y} = 2$

$\mathrm{C/Y} = 2$ ***(M1)(A1)***

Note: Award ***(A1)*** for 7000 seen, ***(M1)*** for the other correct entries.

Award ***(M1)*** for their C/Y from part (e)(i).

OR



***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for a sketch with a straight line intercepted by appropriate curve, ***(A1)*(ft)** for numerical answer in the range of 8.4 and 8.5.

Follow through from their part (e)(i).

$t = 8.50 \text{ (years) } (8.49564\ldots)$ ***(A1)*(ft)*(G3)***

Note: Award only ***(A1)*** if 16.9912… is seen without working. If working is seen, award at most ***(M1)(A1)(A0)***.

[3 marks]

f. $5000\left(1 + \frac{r}{100}\right)^{12} = 10000$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into compound interest formula with 10 000 seen.

OR

$2 = \left(1 + \frac{r}{100}\right)^{12}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution and simplification of compound interest formula, equating to 2.

$r = 5.95\ (\%\ )\ (5.94630\ldots)$ ***(A1)(G2)***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e.i. [N/A]
e.ii. [N/A]
f. [N/A]

Consider the function $f(x) = 0.3x^3 + \frac{10}{x} + 2^{-x}$.

Consider a second function, $g(x) = 2x - 3$.

a. Calculate $f(1)$. [2]

b. Sketch the graph of $y = f(x)$ for $-7 \leqslant x \leqslant 4$ and $-30 \leqslant y \leqslant 30$. [4]

c. Write down the equation of the vertical asymptote. [2]

d. Write down the coordinates of the x-intercept. [2]

e. Write down the possible values of x for which $x < 0$ and $f'(x) > 0$. [2]

f. Find the solution of $f(x) = g(x)$. [2]

Markscheme

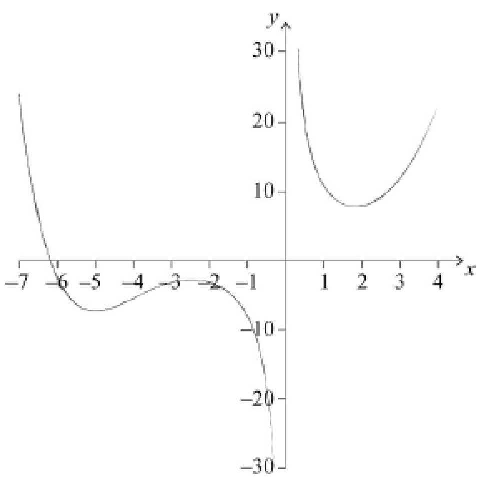
a. $0.3(1)^3 + \frac{10}{1} + 2^{-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into function.

$= 10.8$ ***(A1)(G2)***

[2 marks]

b.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for indication of correct window and labelled axes.

Award ***(A1)*** for correct shape and position for $x < 0$ (with the local maximum, local minimum and x-intercept in relative approximate location in 3^{rd} quadrant).

Award ***(A1)*** for correct shape and position for $x > 0$ (with the local minimum in relative approximate location in 1^{st} quadrant).

Award ***(A1)*** for smooth curve with indication of asymptote (graph should not touch y-axis and should not curve away from the y-axis). The asymptote is only assessed in this mark.

[4 marks]

c. $x = 0$ ***(A2)***

Note: Award ***(A1)*** for "$x = (\text{a constant})$" and ***(A1)*** for "$(\text{a constant}) = 0$".

The answer must be an equation.

[2 marks]

d. $(-6.18,\ 0)$ $(-6.17516\ldots,\ 0)$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct coordinate. Award ***(A0)(A1)*** if parentheses are missing.

[2 marks]

e. $-4.99 < x < -2.47$ $(-4.98688\ldots < x < -2.46635\ldots)$ ***(A1)(A1)***

Note: Award ***(A1)*** for both correct end points, ***(A1)*** for strict inequalities used with 2 endpoints.

[2 marks]

f. $0.3x^3 + \frac{10}{x} + 2^{-x} = 2x - 3$ ***(M1)***

Note: Award ***(M1)*** for equating the expressions for f and g or for the line $y = 2x - 3$ sketched (positive gradient, negative y-intercept) on their graph from part (a).

$(x =)\ -1.34$ $(-1.33650\ldots)$ ***(A1)(G2)***

Note: Award a maximum of ***(M1)(A0)*** or ***(G1)*** for coordinate pair seen as final answer.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Consider the functions $f(x) = \frac{2x+3}{x+4}$ and $g(x) = x + 0.5$.

a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. [6]

b. Write down the equation of the vertical asymptote. [2]

c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. [2]

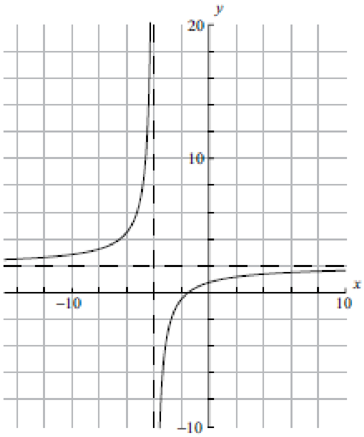
d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. [3]

e. Write down the gradient of the line $g(x) = x + 0.5$. [1]

f. The line L passes through the point with coordinates $(-2, -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. [3]

Markscheme

a.



(A6)

Notes: ***(A1)*** for labels and some idea of scale.
(A1) for x-intercept seen, ***(A1)*** for y-intercept seen in roughly the correct places (coordinates not required).
(A1) for vertical asymptote seen, ***(A1)*** for horizontal asymptote seen in roughly the correct places (equations of the lines not required).
(A1) for correct general shape.

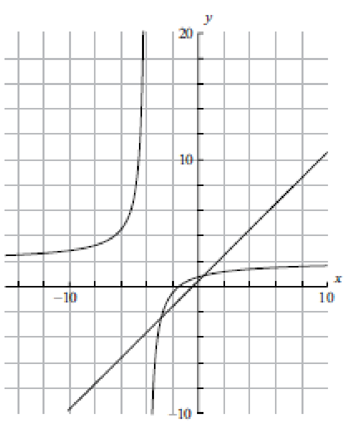
[6 marks]

b. $x = -4$ ***(A1)(A1)*(ft)**

Note: ***(A1)*** for $x =$, ***(A1)*(ft)** for -4.

[2 marks]

c.



(A1)(A1)

Note: ***(A1)*** for correct axis intercepts, ***(A1)*** for straight line

[2 marks]

d. $(-2.85078, -2.35078)$ OR $(0.35078, 0.85078)$ ***(G1)(G1)(A1)*(ft)**

Notes: ***(A1)*** for x-coordinate, ***(A1)*** for y-coordinate, ***(A1)*(ft)** for correct accuracy. Brackets required. If brackets not used award ***(G1)(G0)(A1)*(ft)**.
Accept $x = -2.85078$, $y = -2.35078$ or $x = 0.35078$, $y = 0.85078$.

[3 marks]

e. $\text{gradient} = 1$ ***(A1)***

[1 mark]

f. $\text{gradient of perpendicular} = -1$ ***(A1)*(ft)**

(can be implied in the next step)

$y = mx + c$

$-3 = -1 \times -2 + c$ ***(M1)***

$c = -5$

$y = -x - 5$ ***(A1)*(ft)*(G2)***

OR

$y + 3 = -(x + 2)$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for correct answer with no working at all but ***(A1)(G1)*** if the gradient is mentioned as -1 then correct answer with no further working.

[3 marks]

Examiners report

a. This was not very well done. The graph was often correct but was so small that it was difficult to check if axes intercepts were correct or not. Often the vertical asymptote looked as if it were joined to the rest of the graph. Very few of the candidates put a scale and/or labels on their axes.

b. Reasonably well done. Some put $y = -4$ while others omitted the minus sign.

c. Fairly well done – but once again too small to check the axes intercepts properly. Also, many candidates did not appear to have a ruler to draw the straight line.

d. Well done.

e. Most could find the gradient of the line.

f. Many forgot to find the gradient of the perpendicular line. Others had problems with the equation of a line in general.

Consider the function $g(x) = x^3 + kx^2 - 15x + 5$.

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

a. Find $g'(x)$. [3]

b.i. Show that $k = 6$. [2]

b.ii. Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [3]

c. Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

d.i. Find $g'(-1)$. [2]

d.ii. Hence justify that g is decreasing at $x = -1$. [1]

e. Find the y-coordinate of the local minimum. [2]

Markscheme

a. $3x^2 + 2kx - 15$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $3x^2$, ***(A1)*** for $2kx$ and ***(A1)*** for -15. Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

b.i. $21 = 3(2)^2 + 2k(2) - 15$ ***(M1)(M1)***

Note: Award ***(M1)*** for equating their derivative to 21. Award ***(M1)*** for substituting 2 into their derivative. The second ***(M1)*** should only be awarded if correct working leads to the final answer of $k = 6$.

Substituting in the known value, $k = 6$, invalidates the process; award ***(M0)(M0)***.

$k = 6$ ***(AG)***

[2 marks]

b.ii $g(2) = (2)^3 + (6)(2)^2 - 15(2) + 5\ (= 7)$ ***(M1)***

Note: Award ***(M1)*** for substituting 2 into g.

$7 = 21(2) + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 21, 2 and their 7 into gradient intercept form.

OR

$y - 7 = 21(x - 2)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 21, 2 and their 7 into gradient point form.

$y = 21x - 35$ ***(A1)*** ***(G2)***

[3 marks]

c. $3x^2 + 12x - 15 = 0$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for equating their part (a) (with $k = 6$ substituted) to zero.

$x = -5,\ x = 1$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Follow through from part (a).

[3 marks]

d.i. $3(-1)^2 + 12(-1) - 15$ ***(M1)***

Note: Award ***(M1)*** for substituting -1 into their derivative, with $k = 6$ substituted. Follow through from part (a).

$= -24$ ***(A1)*****(ft)** ***(G2)***

[2 marks]

d.ii $g'(-1) < 0$ (therefore g is decreasing when $x = -1$) ***(R1)***

[1 marks]

e. $g(1) = (1)^3 + (6)(1)^2 - 15(1) + 5$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting 6 and their 1 into g.

$= -3$ ***(A1)*****(ft)** ***(G2)***

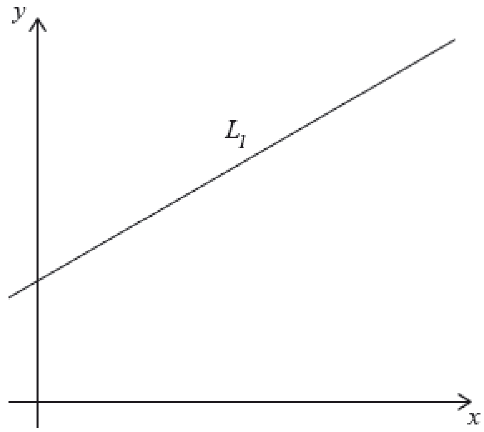
Note: Award, at most, (***M1)(A0)*** or ***(G1)*** if answer is given as a coordinate pair. Follow through from part (c).

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]
e. [N/A]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1,\ 4)$.

The point C has coordinates $(5,\ 12)$. M is the midpoint of AC.

The straight line, L_2, is perpendicular to AC and passes through M.

The point D is the intersection of L_1 and L_2.

The length of MD is $\frac{\sqrt{45}}{2}$.

The point B is such that ABCD is a rhombus.

a. Show that A lies on L_1. [2]

b. Find the coordinates of M. [2]

c. Find the length of AC. [2]

d. Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

e. Find the coordinates of D. [2]

f. Write down the length of MD correct to five significant figures. [1]

g. Find the area of ABCD. [3]

Markscheme

a. $2 \times 4 - 1 - 7 = 0$ (or equivalent) ***(R1)***

Note: For ***(R1)*** accept substitution of $x = 1$ or $y = 4$ into the equation followed by a confirmation that $y = 4$ or $x = 1$.

(since the point satisfies the equation of the line,) A lies on L_1 ***(A1)***

Note: Do not award ***(A1)(R0)***.

[2 marks]

b. $\frac{1+5}{2}$ **OR** $\frac{4+12}{2}$ seen ***(M1)***

Note: Award ***(M1)*** for at least one correct substitution into the midpoint formula.

$(3,\ 8)$ ***(A1)(G2)***

Notes: Accept $x = 3,\ y = 8$.

Award ***(M1)(A0)*** for $\left(\frac{1+5}{2},\ \frac{4+12}{2}\right)$.

Award ***(G1)*** for each correct coordinate seen without working.

[2 marks]

c. $\sqrt{(5-1)^2 + (12-4)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into distance between two points formula.

$= 8.94\ \left(4\sqrt{5},\ \sqrt{80},\ 8.94427\ldots\right)$ ***(A1)(G2)***

[2 marks]

d. gradient of $\mathrm{AC} = \frac{12-4}{5-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula.

$= 2$ ***(A1)***

Note: Award ***(M1)(A1)*** for gradient of $\mathrm{AC} = 2$ with or without working

gradient of the normal $= -\frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for the negative reciprocal of their gradient of AC.

$y - 8 = -\frac{1}{2}(x-3)$ **OR** $8 = -\frac{1}{2}(3) + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their point and gradient into straight line formula. This ***(M1)*** can **only** be awarded where $-\frac{1}{2}$ (gradient) is correctly determined as the gradient of the normal to AC.

$2y - 16 = -(x-3)$ **OR** $-2y + 16 = x - 3$ **OR** $2y = -x + 19$ ***(A1)***

Note: Award ***(A1)*** for correctly removing fractions, **but only** if their equation is equivalent to the given equation.

$2y + x - 19 = 0$ ***(AG)***

Note: The conclusion $2y + x - 19 = 0$ must be seen for the ***(A1)*** to be awarded.

Where the candidate has **shown** the gradient of the normal to $\text{AC} = -0.5$, award ***(M1)*** for $2(8) + 3 - 19 = 0$ and ***(A1)*** for (therefore) $2y + x - 19 = 0$.

Simply substituting $(3,\ 8)$ into the equation of L_2 with no other prior working, earns no marks.

[5 marks]

e. $(6,\ 6.5)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for 6, ***(A1)*** for 6.5. Award a maximum of ***(A1)(A0)*** if answers are not given as a coordinate pair. Accept $x = 6,\ y = 6.5$.

Award ***(M1)(A0)*** for an attempt to solve the two simultaneous equations $2y - x - 7 = 0$ and $2y + x - 19 = 0$ algebraically, leading to at least one incorrect or missing coordinate.

[2 marks]

f. 3.3541 ***(A1)***

Note: Answer must be to 5 significant figures.

[1 mark]

g. $2 \times \frac{1}{2} \times \sqrt{80} \times \frac{\sqrt{45}}{2}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct substitution into area of triangle formula.

If their triangle is a quarter of the rhombus then award ***(M1)*** for multiplying their triangle by 4.

If their triangle is a half of the rhombus then award ***(M1)*** for multiplying their triangle by 2.

OR

$\frac{1}{2} \times \sqrt{80} \times \sqrt{45}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for doubling MD to get the diagonal BD, ***(M1)*** for correct substitution into the area of a rhombus formula.

Award ***(M1)(M1)*** for $\sqrt{80}\times$ their (f).

$= 30$ ***(A1)*****(ft)*****(G3)***

Notes: Follow through from parts (c) and (f).

$8.94 \times 3.3541 = 29.9856\ldots$

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

The function $f(x)$ is defined by $f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0$.

a. Write down the equation of the vertical asymptote. [2]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of the function at $x = -1$. [2]

d. Using your answer to part (c), decide whether the function $f(x)$ is increasing or decreasing at $x = -1$. Justify your answer. [2]

e. Sketch the graph of $f(x)$ for $-10 \leqslant x \leqslant 10$ and $-20 \leqslant y \leqslant 20$. [4]

f. P_1 is the local maximum point and P_2 is the local minimum point on the graph of $f(x)$. [4]

Using your graphic display calculator, write down the coordinates of

(i) P_1 ;

(ii) P_2 .

g. Using your sketch from (e), determine the range of the function $f(x)$ for $-10 \leqslant x \leqslant 10$. [3]

Markscheme

a. $x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for $x = \text{constant}$, ***(A1)*** for 0.

[2 marks]

b. $f'(x) = 1.5 - \frac{6}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for 1.5, ***(A1)*** for -6, ***(A1)*** for x^{-2} . Award ***(A1)(A1)(A0)*** at most if any other term present.

[3 marks]

c. $1.5 - \frac{6}{(-1)}$ ***(M1)***

$= -4.5$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their derivative function.

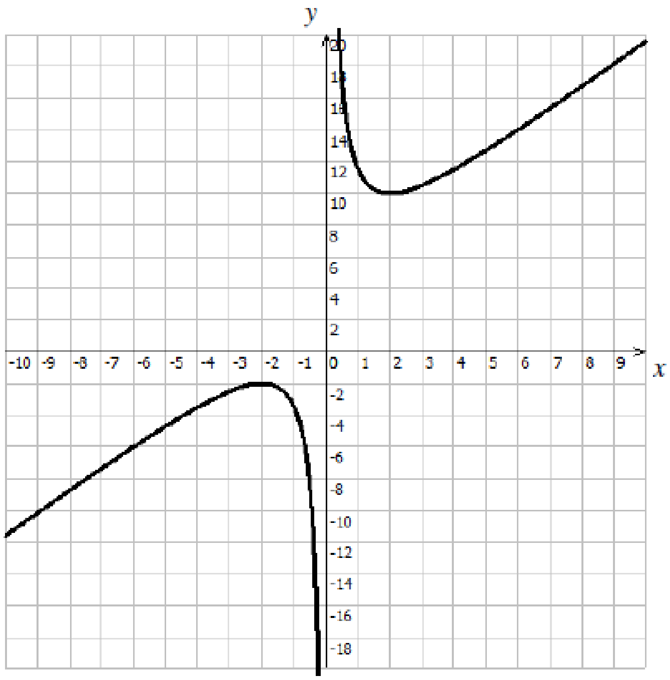
[2 marks]

d. Decreasing, the derivative (gradient or slope) is negative (at $x = -1$) ***(A1)(R1)*(ft)**

Notes: Do not award ***(A1)(R0)***. Follow through from their answer to part (c).

[2 marks]

e.



(A4)

Notes: Award ***(A1)*** for labels and some indication of scales and an appropriate window.

Award ***(A1)*** for correct shape of the two unconnected, and smooth branches.

Award ***(A1)*** for the maximum and minimum points in the approximately correct positions.

Award ***(A1)*** for correct asymptotic behaviour at $x = 0$.

Notes: Please be rigorous.

The axes need not be drawn with a ruler.

The branches must be smooth and single continuous lines that do not deviate from their proper direction.

The max and min points must be symmetrical about point $(0, 4)$.

The y-axis must be an asymptote for **both** branches.

[4 marks]

f. (i) $(-2, -2)$ or $x = -2, y = -2$ ***(G1)(G1)***

(ii) $(2, 10)$ or $x = 2, y = 10$ ***(G1)(G1)***

[4 marks]

g. $\{-2 \geqslant y\}$ or $\{y \geqslant 10\}$ ***(A1)(A1)*(ft)*(A1)***

Notes: *(A1)*(ft) for $y > 10$ or $y \geqslant 10$. ***(A1)*(ft)** for $y < -2$ or $y \leqslant -2$. ***(A1)*** for weak (non-strict) inequalities used in **both** of the above. Follow through from their (e) and (f).

[3 marks]

Examiners report

a. Part a) was either answered well or poorly.

b. Most candidates found the first term of the derivative in part b) correctly, but the rest of the terms were incorrect.

c. The gradient in c) was for the most part correctly calculated, although some candidates substituted incorrectly in $f(x)$ instead of in $f'(x)$.

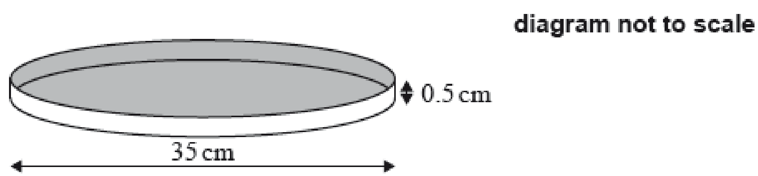
d. Part d) had mixed responses.

e. Lack of labels of the axes, appropriate scale, window, incorrect maximum and minimum and incorrect asymptotic behaviour were the main problems with the sketches in e).

f. Part f) was also either answered correctly or entirely incorrectly. Some candidates used the trace function on the GDC instead of the min and max functions, and thus acquired coordinates with unacceptable accuracy. Some were unclear that a point of local maximum may be positioned on the coordinate system "below" the point of local minimum, and exchanged the pairs of coordinates of those points in f(i) and f(ii).

g. Very few candidates were able to identify the range of the function in (g) irrespective of whether or not they had the sketches drawn correctly.

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.



A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19,\ t \geqslant 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

The pizza can be eaten once its temperature drops to 45 °C.

a. Calculate the volume of this pan. [3]

b. Find the radius of the sphere in cm, correct to one decimal place. [4]

c. Find the value of a. [2]

d. Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

e. Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

f. In the context of this model, state what the value of 19 represents. [1]

Markscheme

a. $(V =)\ \pi \times (17.5)^2 \times 0.5$ ***(A1)(M1)***

Notes: Award ***(A1)*** for 17.5 (or equivalent) seen.

Award ***(M1)*** for correct substitutions into volume of a cylinder formula.

$= 481\ \text{cm}^3\ (481.056\ldots\ \text{cm}^3,\ 153.125\pi\ \text{cm}^3)$ ***(A1)(G2)***

[3 marks]

b. $\frac{4}{3} \times \pi \times r^3 = 481.056\ldots$ ***(M1)***

Note: Award ***(M1)*** for equating **their** answer to part (a) to the volume of sphere.

$r^3 = \frac{3\times481.056\ldots}{4\pi}\ (= 114.843\ldots)$ ***(M1)***

Note: Award ***(M1)*** for correctly rearranging so r^3 is the subject.

$r = 4.86074\ldots\ (\text{cm})$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*** for correct unrounded answer seen. Follow through from part (a).

$= 4.9\ (\text{cm})$ ***(A1)*****(ft)*****(G3)***

Note: The final ***(A1)*****(ft)** is awarded for rounding their unrounded answer to one decimal place.

[4 marks]

c. $230 = a(2.06)^0 + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$a = 211$ ***(A1)(G2)***

[2 marks]

d. $(P =)\ 211 \times (2.06)^{-5} + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the function, $P(t)$. Follow through from part (c). The negative sign in the exponent is required for correct substitution.

$= 24.7$ (°C) $(24.6878\ldots$ (°C)) ***(A1)*(ft)*(G2)***

[2 marks]

e. $45 = 211 \times (2.06)^{-t} + 19$ ***(M1)***

Note: Award ***(M1)*** for equating 45 to the exponential equation and for correct substitution (follow through for their a in part (c)).

$(t =)\ 2.89711\ldots$ ***(A1)*(ft)*(G1)***

$174\ (\text{seconds})\ \ (173.826\ldots\ (\text{seconds}))$ ***(A1)*(ft)*(G2)***

Note: Award final ***(A1)*(ft)** for converting their $2.89711\ldots$ minutes into seconds.

[3 marks]

f. the temperature of the (dining) room ***(A1)***

OR

the lowest final temperature to which the pizza will cool ***(A1)***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The following table shows the number of bicycles, x, produced daily by a factory and their total production cost, y, in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

a. (i) Write down the Pearson's product–moment correlation coefficient, r, for these data. [4]

(ii) Hence comment on the result.

b. Write down the equation of the regression line y on x for these data, in the form $y = ax + b$. [2]

c. Estimate the total cost, **to the nearest USD**, of producing 13 bicycles on a particular day. [3]

d. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [2]

Explain why the factory does **not** make a profit when producing 13 bicycles on a particular day.

e. All the bicycles that are produced are sold. The bicycles are sold for 304 USD **each**. [5]

(i) Write down an expression for the total selling price of x bicycles.

(ii) Write down an expression for the **profit** the factory makes when producing x bicycles on a particular day.

(iii) Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit.

Markscheme

a. (i) $r = 0.985 \quad (0.984905\ldots)$ ***(G2)***

Notes: If unrounded answer is not seen, award ***(G1)(G0)*** for 0.99 or 0.984. Award ***(G2)*** for 0.98.

(ii) strong, positive ***(A1)(A1)***

b. $y = 259.909\ldots x + 698.648\ldots \quad (y = 260x + 699)$ ***(G1)(G1)***

Notes: Award ***(G1)*** for $260x$ and ***(G1)*** for 699. If the answer is not an equation award a maximum of ***(G1)(G0)***.

c. $y = 259.909\ldots \times 13 + 698.648\ldots$ ***(M1)***

Note: Award ***(M1)*** for substitution of 13 into their regression line equation from part (b).

$y = 4077.47\ldots$ ***(A1)*****(ft)*****(G2)***

$y = 4077 \text{ (USD)}$ ***(A1)*****(ft)**

Notes: Follow through from their answer to part (b). If rounded values from part (b) used, answer is 4079. Award the final ***(A1)*****(ft)** for a correct rounding to the nearest USD of their answer. The unrounded answer may not be seen.

If answer is 4077 and no working is seen, award ***(G2)***.

d. $13 \times 304 - (4077.47) = -125.477\ldots \quad (-125)$ **OR**

$4077.47 - (13 \times 304) = 125.477\ldots \quad (125)$ ***(M1)***

Notes: Award ***(M1)*** for calculating the difference between 13×304 and their answer to part (c).

If rounded values are used in equation, answer is -127.

profit is negative **OR** $\text{cost} > \text{sales}$ ***(A1)***

OR

$13 \times 304 = 3952$ ***(M1)***

Note: Award ***(M1)*** for calculating the price of 13 bikes.

$3952 < 4077.47$ ***(A1)*****(ft)**

Note: Award ***(A1)*** for showing 3952 is less than their part (c). This may be communicated in words. Follow through from part (c), but only if value is greater than 3952.

OR

$\frac{4077}{13} = 313.62$ ***(M1)***

Note: Award ***(M1)*** for calculating the cost of 1 bicycle.

$313.62 > 304$ ***(A1)*(ft)**

Note: Award ***(A1)*** for showing 313.62 is greater than 304. This may be communicated in words. Follow through from part (c), but only if value is greater than 304.

OR

$\frac{4077}{304} = 13.41$ ***(M1)***

Note: Award ***(M1)*** for calculating the number of bicycles that should have been be sold to cover total cost.

$13.41 > 13$ ***(A1)*(ft)**

Note: Award ***(A1)*** for showing 13.41 is greater than 13. This may be communicated in words. Follow through from part (c), but only if value is greater than 13.

e. (i) $304x$ ***(A1)***

(ii) $304x - (259.909\ldots x + 698.648\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for difference between their answers to parts (b) and (e)(i), ***(A1)*(ft)** for correct expression.

(iii) $304x - (259.909\ldots x + 698.648\ldots) > 0$ ***(M1)***

Notes: Award ***(M1)*** for comparing their expression in part (e)(ii) to 0. Accept an equation. Accept $3040x - y > 0$ or equivalent.

$x = 16 \text{ bicycles}$ ***(A1)*(ft)*(G2)***

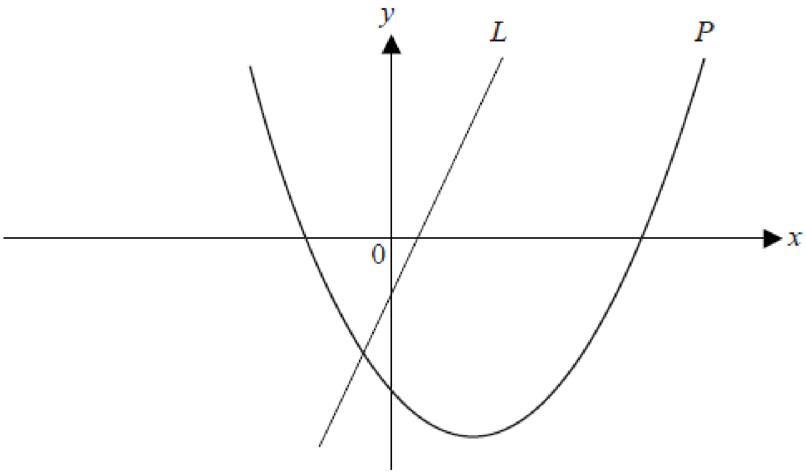
Notes: Follow through from their answer to part (b). Answer must be a positive integer greater than 13 for the ***(A1)*(ft)** to be awarded.

Award ***(G1)*** for an answer of 15.84.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The diagram below shows the graph of a line L passing through (1, 1) and (2 , 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



a. Find the gradient of the line *L*. [2]

b. Differentiate $f(x)$. [2]

c. Find the coordinates of the point where the tangent to *P* is parallel to the line *L*. [3]

d. Find the coordinates of the point where the tangent to *P* is perpendicular to the line *L*. [4]

e. Find [3]

(i) the gradient of the tangent to *P* at the point with coordinates (2, – 6).

(ii) the equation of the tangent to *P* at this point.

f. State the equation of the axis of symmetry of *P*. [1]

g. Find the coordinates of the vertex of *P* and state the gradient of the curve at this point. [3]

Markscheme

a. *for attempt at substituted* $\frac{ydistance}{xdistance}$ ***(M1)***

gradient = 2 ***(A1)(G2)***

[2 marks]

b. $2x - 3$ ***(A1)(A1)***

(A1) *for* $2x$, ***(A1)*** *for* -3

[2 marks]

c. *for their* $2x - 3 =$ *their gradient and attempt to solve* ***(M1)***

$x = 2.5$ ***(A1)*(ft)**

$y = -5.25$ (**(ft)** *from their x value)* ***(A1)*(ft)*(G2)***

[3 marks]

d. *for seeing* $\frac{-1}{their(a)}$ ***(M1)***

solving $2x - 3 = -\frac{1}{2}$ *(or their value)* ***(M1)***

x = 1.25 ***(A1)*(ft)*(G1)***

$y = -6.1875$ ***(A1)*(ft)*(G1)***

[4 marks]

e. (i) $2 \times 2 - 3 = 1$ *(**(ft)** from (b))* ***(A1)*(ft)*(G1)***

(ii) $y = mx + c$ *or equivalent method to find* $c \Rightarrow -6 = 2 + c$ ***(M1)***

$y = x - 8$ ***(A1)*(ft)*(G2)***

[3 marks]

f. $x = 1.5$ ***(A1)***

[1 mark]

g. *for substituting their answer to part (f) into the equation of the parabola* (1.5, –6.25) *accept x* = 1.5, *y* = –6.25 ***(M1)(A1)*(ft)*(G2)***

gradient is zero *(accept* $\frac{dy}{dx} = 0$*)* ***(A1)***

[3 marks]

Examiners report

a. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of $x = 1.5$.

Some forgot to write down that the gradient at the vertex was 0.

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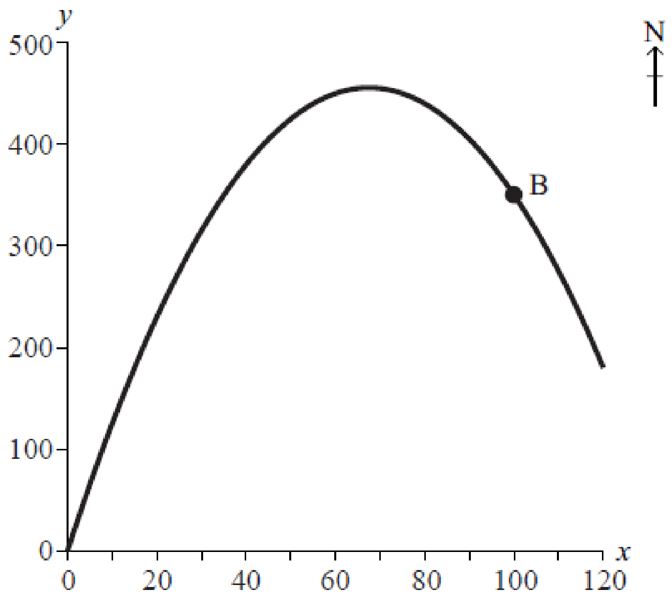
Some forgot to write down that the gradient at the vertex was 0.

The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where $x \geqslant 0,\ y \geqslant 0$

(x , y) are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. [3]

b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. [2]

c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. [4]

d. (i) Write down the midpoint of the line segment OB. [3]

(ii) Find the gradient of the line segment OB.

e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. [3]

Find the equation of Scott's road. Express your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{R}$.

f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. [2]

Markscheme

a. $y = -\frac{75^2}{10} + \frac{27}{2} \times 75$ ***(M1)***

Note: Award ***(M1)*** for substitution of 75 in the formula of the function.

= 450 ***(A1)***

Yes, point A is on the bike track. ***(A1)***

Note: Do not award the final ***(A1)*** if correct working is not seen.

b. $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{10} + \frac{27}{2} \left(\frac{\mathrm{d}y}{\mathrm{d}x} = -0.2x + 13.5 \right)$ ***(A1)(A1)***

Notes: Award ***(A1)*** for each correct term. If extra terms are seen award at most ***(A1)(A0)***. Accept equivalent forms.

c. $-\frac{2x}{10} + \frac{27}{2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative from part (b) to zero.

$x = 67.5$ ***(A1)*****(ft)**

Note: Follow through from their derivative from part (b).

$(\text{Their})\ 67.5 \neq 75$ ***(R1)***

Note: Award ***(R1)*** for a comparison of their 67.5 with 75. Comparison may be implied (*eg* 67.5 is the *x*-coordinate of the furthest north point).

OR

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2 \times (75)}{10} + \frac{27}{2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of 75 into their derivative from part (b).

$= -1.5$ ***(A1)*****(ft)**

Note: Follow through from their derivative from part (b).

$(\text{Their}) - 1.5 \neq 0$ ***(R1)***

Note: Award ***(R1)*** for a comparison of their –1.5 with 0. Comparison may be implied (*eg* The gradient of the parabola at the furthest north point (vertex) is 0).

Hence A is not the furthest north point. ***(A1)*****(ft)**

Note: Do not award ***(R0)(A1)*****(ft)**. Follow through from their derivative from part (b).

d. (i) M(50,175) ***(A1)***

Note: If parentheses are omitted award ***(A0)***. Accept *x* = 50, *y* = 175.

(ii) $\frac{350-0}{100-0}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in gradient formula.

$= 3.5 \left(\frac{350}{100}, \frac{7}{2}\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from (d)(i) if midpoint is used to calculate gradient. Award ***(G1)(G0)*** for answer 3.5*x* without working.

e. $y = 3.5x + 150$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for using their gradient from part (d), ***(A1)*****(ft)** for correct equation of line.

$3.5x - y = -150$ **or** $7x - 2y = -300$ (or equivalent) ***(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for expressing their equation in the form $ax + by = c$.

f. (18.4, 214) (18.3772..., 214.320...) ***(A1)*****(ft)*****(A1)*****(ft)*****(G2)*****(ft)**

Notes: Follow through from their equation in (e). Coordinates must be positive for follow through marks to be awarded. If parentheses are omitted and not already penalized in (d)(i) award at most ***(A0)(A1)*****(ft)**. If coordinates of the two intersection points are given award ***(A0)(A1)*****(ft)**. Accept *x* = 18.4, *y* = 214.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Give all answers in this question to the nearest whole currency unit.

Ying and Ruby each have 5000 USD to invest.

Ying invests his 5000 USD in a bank account that pays a nominal annual interest rate of 4.2 % **compounded yearly**. Ruby invests her 5000 USD in an account that offers a fixed interest of 230 USD each year.

a. Find the amount of money that Ruby will have in the bank after 3 years. [2]

b. Show that Ying will have 7545 USD in the bank at the end of 10 years. [3]

c. Find the number of complete years it will take for Ying's investment to first exceed 6500 USD. [3]

d. Find the number of complete years it will take for Ying's investment to exceed Ruby's investment. [3]

e. Ruby moves from the USA to Italy. She transfers 6610 USD into an Italian bank which has an exchange rate of 1 USD = 0.735 Euros. The bank charges 1.8 % commission. [4]

Calculate the amount of money Ruby will invest in the Italian bank after commission.

f. Ruby returns to the USA for a short holiday. She converts 800 Euros at a bank in Chicago and receives 1006.20 USD. The bank advertises an exchange rate of 1 Euro = 1.29 USD. [5]

Calculate the percentage commission Ruby is charged by the bank.

Markscheme

a. 5000 + 3 × 230 = 5690 ***(M1)(A1)(G2)***

Note: Accept alternative method.

[2 marks]

b. $\mathrm{A} = 5000\left(1 + \frac{4.2}{100}\right)^{10}$ *or equivalent* ***(M1)(A1)***

$= 7544.79\ldots$ ***(A1)***

$= 7545 \text{ USD}$ ***(AG)***

Note: Award ***(M1)*** for correct substituted compound interest formula, ***(A1)*** for correct substitutions, ***(A1)*** for unrounded answer seen.

If final line not seen award at most ***(M1)(A1)(A0)***.

[3 marks]

c. $5000(1.042)^n > 6500$ ***(M1)(A1)***

Notes: Award ***(M1)*** for setting up correct equation/inequality, ***(A1)*** for correct values.

Follow through from their formula in part (b).

OR

List of values seen with at least 2 terms ***(M1)***

Lists of values including at least the terms with $n = 6$ and $n = 7$ ***(A1)***

Note: Follow through from their formula in part (b).

OR

Sketch showing 2 graphs, one exponential, the other a horizontal line ***(M1)***

Point of intersection identified or vertical line ***(M1)***

Note: Follow through from their formula in part (b).

$n = 7$ ***(A1)*(ft)*(G2)***

[3 marks]

d. $5000(1.042)^n > 5000 + 230n$ ***(M1)(A1)***

Note: Award ***(M1)*** for setting up correct equation/inequality, ***(A1)*** for correct values.

OR

2 lists of values seen (at least 2 terms per list) ***(M1)***

Lists of values including at least the terms with $n = 5$ and $n = 6$ ***(A1)***

Note: One of the lists may be written under (c).

OR

Sketch showing 2 graphs of correct shape ***(M1)***

Point of intersection identified or vertical line ***(M1)***

$n = 6$ ***(A1)*(ft)*(G2)***

Note: Follow through from their formulae used in parts (a) and (b).

[3 marks]

e. 6610×0.735 ***(M1)***

$= 4858.35$ ***(A1)***

$4858.35 \times 0.982 (= 4770.8997...)$ ***(M1)***

= 4771 Euros ***(A1)*(ft)*(G3)***

Note: Accept alternative method.

[4 marks]

f. 800×1.29 (= 1032 USD) ***(M1)(A1)***

Note: Award ***(M1)*** for multiplying by 1.29, ***(A1)*** for 1032. Award ***(G2)*** for 1032 if product not seen.

$(1032 - 1006.20 = 25.8)$

$25.8 \times \frac{100}{1032}\%$ ***(A1)(M1)***

Note: Award ***(A1)*** for 25.8 seen, ***(M1)*** for multiplying by $\frac{100}{1032}$.

OR

$\frac{1006.20}{1032} = 0.975$ ***(M1)(A1)***

OR

$\frac{1006.20}{1032} \times 100 = 97.5$ ***(M1)(A1)***

$= 2.5\ \%$ ***(A1)(G3)***

Notes: If working not shown award ***(G3)*** for 2.5.

Accept alternative method.

[5 marks]

Examiners report

a. Most of the students read carefully the instruction written in the heading of the question and therefore gave their answers with the accuracy stated but some did not.

Simple interest was well done as well as compound interest with only a small minority of candidates making no progress. A number of students lost the answer mark in (b) for not showing the unrounded answer before writing the answer given. It is also important to mention that calculator commands are not accepted as correct working and therefore full marks are not awarded. Also, some candidates wrote their answers without showing any working leading to a number of marks being lost.

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f. Most of the students read carefully the instruction written in the heading of the question and therefore gave their answers with the accuracy stated but some did not.

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Daniel wants to invest $25 000 for a total of three years. There are two investment options.

Option One pays compound interest at a nominal annual rate of interest of 5 %, compounded **annually**.

Option Two pays compound interest at a nominal annual rate of interest of 4.8 %, compounded **monthly**.

An arithmetic sequence is defined as

$$u_n = 135 + 7n, \quad n = 1, 2, 3, \ldots$$

A.a Calculate the value of his investment at the end of the third year for each investment option, **correct to two decimal places**. [8]

A.b Determine Daniel’s best investment option. [1]

B.a Calculate u_1, the first term in the sequence. [2]

B.b Show that the common difference is 7. [2]

B.c S_n is the sum of the first n terms of the sequence. [3]

Find an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants.

B.d The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5. [2]

Show that the common ratio, r, of the geometric sequence is 1.5.

B.e T_n is the sum of the first n terms of the geometric sequence. [2]

Calculate T_7, the sum of the first seven terms of the geometric sequence.

B.f. T_n is the sum of the first n terms of the geometric sequence. [2]

Use your graphic display calculator to find the smallest value of n for which $T_n > S_n$.

Markscheme

A.a **Option 1:** Amount $= 25\,000\left(1+\frac{5}{100}\right)^3$ ***(M1)(A1)***

= $28\,940.63$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution in compound interest formula, ***(A1)*** for correct substitution. Give full credit for use of lists.

Option 2: Amount $= 25\,000\left(1 + \frac{4.8}{12(100)}\right)^{3\times 12}$ ***(M1)***

= $28\,863.81$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in the compound interest formula. Give full credit for use of lists.

[8 marks]

A.b. Option 1 is the best investment option. ***(A1)*(ft)**

[1 mark]

B.a. u_1 = 135 + 7(1) ***(M1)***

= 142 ***(A1)(G2)***

[2 marks]

B.b. u_2 = 135 + 7(2) = 149 ***(M1)***

d = 149 – 142 **OR** *alternatives* ***(M1)*(ft)**

d = 7 ***(AG)***

[2 marks]

B.c. $S_n = \frac{n[2(142)+7(n-1)]}{2}$ ***(M1)*(ft)**

Note: Award ***(M1)*** for correct substitution in correct formula.

$= \frac{n[277+7n]}{2}$ **OR** *equivalent* ***(A1)***

$= \frac{7n^2}{2} + \frac{277n}{2}$ $(= 3.5n^2 + 138.5n)$ ***(A1)(G3)***

[3 marks]

B.d. $20r^3 = 67.5$ ***(M1)***

$r^3 = 3.375$ **OR** $r = \sqrt[3]{3.375}$ ***(A1)***

$r = 1.5$ ***(AG)***

[2 marks]

B.e. $T_7 = \frac{20(1.5^7-1)}{(1.5-1)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in correct formula.

= 643 *(accept 643.4375)* ***(A1)(G2)***

[2 marks]

B.f. $\frac{20(1.5^n-1)}{(1.5-1)} > \frac{7n^2}{2} + \frac{277n}{2}$ ***(M1)***

Note: Award ***(M1)*** for an attempt using lists or for relevant graph.

$n = 10$ ***(A1)*(ft)*(G2)***

Note: Follow through from their (c).

[2 marks]

Examiners report

A.a For many, this question came as a welcome relief following the previous two questions. For those with a sound grasp of the topic, there were many very successful attempts.

A common error was to make all the comparisons using interest alone; though much credit was given for doing this, candidates should be aware of what is being asked for in the question.

Many did not understand the notion of monthly compounding periods.

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The final part was challenging to the great majority, with a large number not attempting it at all; only the highly competent reached the correct answer.

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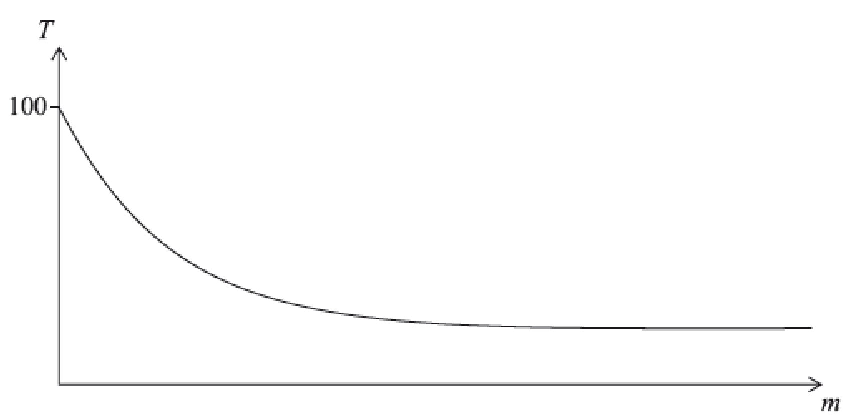
A cup of boiling water is placed in a room to cool. The temperature of the room is 20°C . This situation can be modelled by the exponential function $T = a + b(k^{-m})$, where T is the temperature of the water, in °C , and m is the number of minutes for which the cup has been placed in the room. A sketch of the situation is given as follows.



a. Explain why $a = 20$. [2]

b. Initially, at $m = 0$, the temperature of the water is 100°C. [2]

Find the value of b.

c. After being placed in the room for one minute, the temperature of the water is 84°C. [2]

Show that $k = 1.25$.

d. After being placed in the room for one minute, the temperature of the water is 84°C. [2]

Find the temperature of the water three minutes after it has been placed in the room.

e. After being placed in the room for one minute, the temperature of the water is 84°C. [2]

Find the total time needed for the water to reach a temperature of 35°C. Give your answer in minutes and seconds, correct to the nearest second.

Markscheme

a. the temperature of the water cannot fall below room temperature ***(R1)***

an (informal) explanation that as $m \to \infty,\ k^{-m} \to 0$ ***R1)***

OR

recognition that there is a horizontal asymptote at $y = a$ ***(R1)***

Note: Award ***(R1)*** for a contextual reason involving room temperature.

Award ***(R1)*** for a mathematical reason similar to one of the two alternatives.

b. $100 = 20 + b(k^0)$ ***(M1)***

Note: Award ***(M1)*** for substituting 100, 20 and 0.

$b = 80$ ***(A1)(G2)***

Note: The ***(A1)*** is awarded only if all working seen is consistent with the final answer of 80.

c. $84 = 20 + 80k^{-1}$ ***(M1)***

Note: Substituting $k = 1.25$ at any stage is an invalid method and is awarded ***(M0)(M0)***. Award ***(M1)*** for correctly substituting 84, 20 and their 80.

$\frac{64}{80} = k^{-1}$ ***(M1)***

$k = 1.25$ ***(AG)***

Note: Award ***(M1)*** for correct rearrangement that isolates k; $k = 1.25$ must be consistent with their working **and** the conclusion $k = 1.25$ must be seen.

d. $T = 20 + 80(1.25^{-3})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b) and $k = 1.25$.

$T = 61.0 \quad (60.96)$ ***(A1)*****(ft)*****(G2)***

e. $35 = 20 + 80(1.25^{-m})$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions into T. Follow through from part (b). Accept graphical solutions. Award ***(M1)*** for sketch of function.

$(m =)\ 7.50\ (\text{minutes}) \quad (7.50179\ldots)$ ***(A1)*****(ft)*****(G2)***

7 minutes and 30 seconds ***(A1)***

Note: Award the final ***(A1)*** for correct conversion of **their** m in minutes to minutes and seconds, but only if answer in minutes is explicitly shown.

Examiners report

a. Comments on some of the G2 forms indicated that teachers felt the presence of three parameters in the formula was inconsistent with the aims of Mathematical Studies; however, one parameter was given and a justification required, a second required knowledge only of $k^0=1$ and the third was also given. Candidates should have been exposed to graphs of this type in their classwork.

   The response of the candidature indicated that most were able to make some progress with the question, though for many the “show that” part was not attempted. As in previous sessions, comments were made about the use (or not) of logarithms in the final part – logarithms are not part of the Mathematical Studies SL syllabus (however, their correct use is never penalized), but efficient use of the GDC is very much part of a candidate’s “toolbox”. Questions of this nature – essentially requiring the use of the GDC as part of a problem solving exercise – will continue to be set.

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a. Antonio and Barbara start work at the same company on the same day. They each earn an annual salary of 8000 euros during the first year of employment. The company gives them a salary increase following the completion of each year of employment. Antonio is paid using plan A and Barbara is paid using plan B. [3]

Plan A: The annual salary increases by 450 euros each year.

Plan B: The annual salary increases by $5\,\%$ each year.

Calculate

i) Antonio's annual salary during his second year of employment;

ii) Barbara's annual salary during her second year of employment.

b. Write down an expression for [4]

i) Antonio's annual salary during his n th year of employment;

ii) Barbara's annual salary during her n th year of employment.

c. Determine the number of years for which Antonio's annual salary is greater than or equal to Barbara's annual salary. [2]

d. Both Antonio and Barbara plan to work at the company for a total of 15 years. [7]

i) Calculate the **total amount** that **Barbara** will be paid during these 15 years.

ii) Determine whether Antonio earns more than Barbara during these 15 years.

Markscheme

a. i) $8450\,(\text{euro})$ ***(A1)***

ii) 8000×1.05 ***(M1)***

Note: Award ***(M1)*** for 8000×1.05 **OR** $(8000 \times 0.05) + 8000$.

$= 8400\,(\text{euro})$ ***(A1)(G3)***

b. i) $8000 + 450\ (n - 1)\ \ (\text{accept } 450\,n + 7550)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in arithmetic sequence formula; ***(A1)*** for correct substitutions.

ii) $8000 \times 1.05^{(n-1)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in arithmetic sequence formula; ***(A1)*** for correct substitutions.

c. $8000 + 450\,(n - 1) \geqslant 8000 \times 1.05^{n-1}$ ***(M1)***

Note: Award ***(M1)*** for setting a correct inequality using their expressions for (b)(i) and (b)(ii). Accept an equation.

OR

list of at least 4 correct terms of each sequence ***(M1)***

Note: Award ***(M1)*** for correct lists corresponding to their answers for parts (b)(i) and (b)(ii).

6 ***(A1)*****(ft)*****(G2)***

Note: Value must be an integer for the final ***(A1)*** to be awarded. Follow through from parts (b)(i) and (b)(ii). Award ***(G1)*** for a final answer of $6.70018\ldots$ seen without working.

d. i) $S_{15}=\frac{8000\times(1.05^{15}-1)}{1.05-1}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into geometric series formula and ***(A1)*** for correct substitution of u_1 and their r from part (b)(ii). Follow through from part (b)(ii).

OR

$8000+8400+8820...+15839.45$ ***(M1)(A1)*(ft)**

Note: Follow through from part (b)(ii).

$=173\,000\,(\text{euro})\ \ (172629...)$ ***(A1)*(ft)*(G2)***

ii) $S_{15}=\frac{15}{2}(2\times8000+450\times14)$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substitution into arithmetic series formula and ***(A1)*** for correct substitution, using their first term and their last term from part (b)(i), or their u_1 and d. Follow through from part (b)(i).

OR

$8000+8450+8900...+14300$ ***(M1)(A1)*(ft)**

Note: Follow through from part (b)(i).

$=167\,000\,(\text{euro})\ \ (167\,250)$ ***(A1)*(ft)*(G2)***

Antonio does not earn more than Barbara

(his total salary will be less than Barbara's) ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for a final answer that is consistent with their part (d)(i) and (d)(ii). Accept "Barbara earns more". The final ***(A1)*** can only be awarded if two total salaries are seen.

Examiners report

a. Question 5: Arithmetic and Geometric progression

Most candidates calculated the salaries in the second year correctly. The most common error was to calculate the salaries for the third instead of the second year. In part (b) the use of n instead of $n-1$ was very common. For the geometric sequence often a ratio of 0.05 instead of 1.05 was used. Also many of the expressions given did not represent a geometric sequence. Candidates who used a list for part (c) did usually better than the ones that tried to solve an equation. In part (d) the sum of the arithmetic progression was done better than the geometric series. Many candidates calculated the 15th term of the progression and not the series. In general this question part was not answered well.

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Given $f(x) = x^2 - 3x^{-1}, x \in \mathbb{R}, -5 \leqslant x \leqslant 5, x \neq 0,$

A football is kicked from a point A (a, 0), 0 < a < 10 on the ground towards a goal to the right of A.

The ball follows a path that can be modelled by **part** of the graph

$y = -0.021x^2 + 1.245x - 6.01, x \in \mathbb{R}, y \geqslant 0.$

x is the horizontal distance of the ball from the origin

y is the height above the ground

Both x and y are measured in metres.

i.a. Write down the equation of the vertical asymptote. [1]

i.b. Find $f'(x)$. [2]

i.c. Using your graphic display calculator or otherwise, write down the coordinates of any point where the graph of $y = f(x)$ has zero gradient. [2]

i.d. Write down all intervals in the given domain for which $f(x)$ is increasing. [3]

ii.a. Using your graphic display calculator or otherwise, find the value of *a*. [1]

ii.b. Find $\frac{dy}{dx}$. [2]

ii.c. (i) Use your answer to part (b) to calculate the horizontal distance the ball has travelled from A when its height is a maximum. [4]

(ii) Find the maximum vertical height reached by the football.

ii.d. Draw a graph showing the path of the football from the point where it is kicked to the point where it hits the ground again. Use 1 cm to represent 5 m on the horizontal axis and 1 cm to represent 2 m on the vertical scale. [4]

ii.e. The goal posts are 35 m from **the point where the ball is kicked**. [2]

At what height does the ball pass over the goal posts?

Markscheme

i.a. equation of asymptote is *x* = 0 ***(A1)***

(Must be an equation.)

[1 mark]

i.b. $f'(x) = 2x + 3x^{-2}$ *(or equivalent)* ***(A1)*** *for each term* ***(A1)(A1)***

[2 marks]

i.c. stationary point (–1.14, 3.93) ***(G1)(G1)*****(ft)**

(-1,4) or similar error is awarded ***(G0)(G1)*****(ft)**. *Here and also as follow through in part (d) accept exact values* $-\left(\frac{3}{2}\right)^{\frac{1}{3}}$ *for the x coordinate and* $3\left(\frac{3}{2}\right)^{\frac{2}{3}}$ *for the y coordinate.*

OR $2x + \frac{3}{x^2} = 0$ *or equivalent*

Correct coordinates as above ***(M1)***

Follow through from candidate's $f'(x)$. ***(A1)*****(ft)**

[2 marks]

i.d. *In all alternative answers for (d), follow through from candidate's x coordinate in part (c).*

Alternative answers include:

–1.14 ≤ *x* < 0, 0 < *x* < 5 ***(A1)(A1)*****(ft)*****(A1)***

OR [–1.14,0), (0,5)

Accept alternative bracket notation for open interval] [. (Union of these sets is not correct, award ***(A2)*** *if all else is right in this case.)*

OR $-1.14 \leqslant x < 5, x \neq 0$

In all versions 0 ***must*** *be excluded* ***(A1)****. -1.14 must be the left bound . 5 must be the right bound* ***(A1)****. For* $x \geqslant -1.14$ *or* $x > -1.14$ *alone, award* ***(A1)****. For* $-1.4 \leqslant x < 0$ *together with* $x > 0$ *award* ***(A2)****.*

[3 marks]

ii.a. *a* = 5.30 *(3sf)* *(Allow (5.30, 0) but 5.3 receives an* ***(AP)****.)* ***(A1)***

[1 mark]

ii.b. $\frac{\mathrm{d}y}{\mathrm{d}x} = -0.042x + 1.245$ ***(A1)*** *for each term.* ***(A1)(A1)***

[2 marks]

ii.c. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

(i) Maximum value when $f'(x) = 0, -0.042x + 1.245 = 0,$ ***(M1)***

(M1) *is for either of the above but at least one must be seen.*

(*x* = 29.6.)

Football has travelled 29.6 – 5.30 = 24.3 m (3sf) horizontally. ***(A1)*****(ft)**

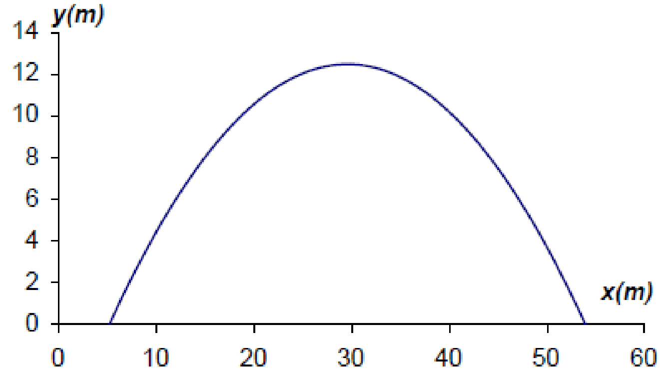
For answer of 24.3 m with no working or for correct subtraction of 5.3 from candidate's x-coordinate at the maximum (if not 29.6), award ***(A1)****(d).*

(UP) (ii) Maximum vertical height, *f* (29.6) = 12.4 m ***(M1)(A1)*****(ft)*****(G2)***

(M1) *is for substitution into f of a value seen in part (c)(i). f(24.3) with or without evaluation is awarded* ***(M1)(A0)****. For any other value without working, award* ***(G0)****. If lines are seen on the graph in part (d) award* ***(M1)*** *and then* ***(A1)*** *for candidate's value* ± 0.5 *(3sf not required.)*

[4 marks]

ii.d. (not to scale)



***(A1)(A1)(A1)*(ft)*(A1)*(ft)**

*Award **(A1)** for labels (units not required) and scale, **(A1)**(ft) for max*(29.6,12.4), ***(A1)*(ft)** for *x-intercepts at 5.30 and 53.9, (all coordinates can be within 0.5), **(A1)** for well-drawn parabola ending at the x-intercepts.*

[4 marks]

ii.e.

*Unit penalty **(UP)** is applicable where indicated in the left hand column.*

(UP) *f* (40.3) = 10.1 m (3sf).

*Follow through from (a). If graph used, award **(M1)** for lines drawn and **(A1)** for candidate's value* ± 0.5*. (3sf not required).* ***(M1)(A1)*(ft)*(G2)***

[2 marks]

Examiners report

i.a. (i) An attempt at part (a) was seen only rarely. If there was an attempt, it was often not a meaningful equation. If an equation was seen, sometimes it was for y, not x.

i.b. The derivative seemed manageable for many, though with the expected mis-handling of the negative power quite often. Parts (c) and (d) proved problematical. Marking of (d) was lenient and it was reaffirmed that testing of the concept in (d) will be done in a more straightforward context in future, when done at all.

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ii.a. (ii) Many candidates failed to recognise that extensive use of the GDC was intended for this question. An indicator of this was the choice of awkward coefficients. It is recognised that the context confused some candidates and that the horizontal shift was a bit disturbing for some.

Nevertheless, a lot of candidates could have earned more marks here if they had persevered. Many gave up on the graph, and elementary marks for scale and labels were lost unnecessarily.

As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

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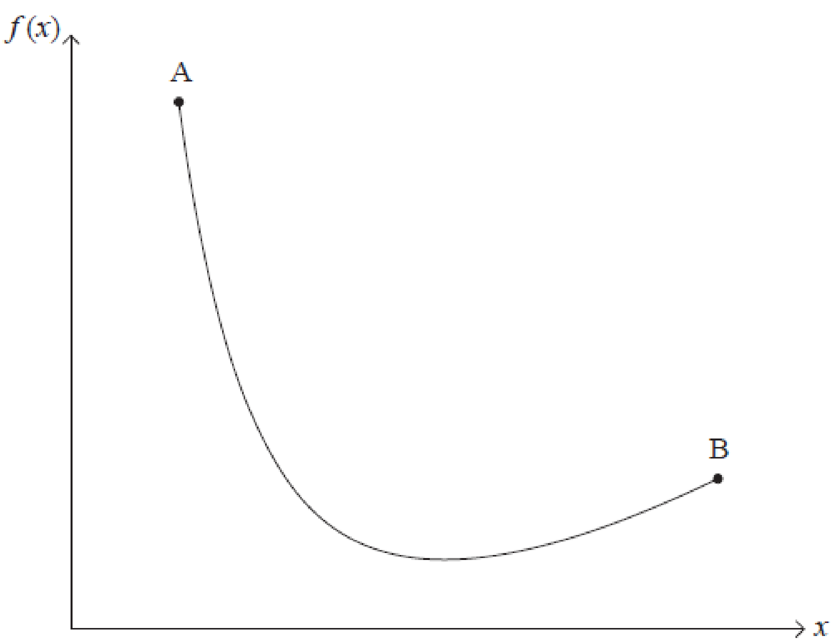
As this was the first time for the unit penalty, we were lenient about the units left off the labels but this is likely to change in the future.

The graph of the function $f(x) = \frac{14}{x} + x - 6$, for $1 \le x \le 7$ is given below.



a. Calculate $f(1)$. [2]

b. Find $f'(x)$. [3]

c. **Use your answer to part (b)** to show that the *x*-coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures. [3]

d. Find the range of f. [3]

e. Points A and B lie on the graph of f. The *x*-coordinates of A and B are 1 and 7 respectively. [1]

Write down the *y*-coordinate of B.

f. Points A and B lie on the graph of f . The *x*-coordinates of A and B are 1 and 7 respectively. [2]

Find the gradient of the straight line passing through A and B.

g. M is the midpoint of the line segment AB. [2]

Write down the coordinates of M.

h. *L* is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same *x*-coordinate as M. [2]

Find the gradient of *L*.

i. Find the equation of *L*. Give your answer in the form $y = mx + c$. [3]

Markscheme

a. $\frac{14}{(1)} + (1) - 6$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 1$ into f.

$= 9$ ***(A1)(G2)***

b. $-\frac{14}{x^2} + 1$ ***(A3)***

Note: Award ***(A1)*** for -14, ***(A1)*** for $\frac{14}{x^2}$ or for x^{-2}, ***(A1)*** for 1.

Award at most ***(A2)*** if any extra terms are present.

c. $-\frac{14}{x^2} + 1 = 0$ or $f'(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for equating **their** derivative in part (b) to 0.

$\frac{14}{x^2} = 1$ or $x^2 = 14$ or equivalent ***(M1)***

Note: Award ***(M1)*** for correct rearrangement of their equation.

$x = 3.74165...(\sqrt{14})$ ***(A1)***

$x = 3.7$ ***(AG)***

Notes: Both the unrounded and rounded answers must be seen to award the ***(A1)***. This is a “show that” question; appeals to their GDC are not accepted –award a maximum of ***(M1)(M0)(A0)***.

Specifically, $-\frac{14}{x^2} + 1 = 0$ followed by $x = 3.74165...$, $x = 3.7$ is awarded ***(M1)(M0)(A0)***.

d. $1.48 \leqslant y \leqslant 9$ ***(A1)(A1)*(ft)*(A1)***

Note: Accept alternative notations, for example [1.48,9]. ($x = \sqrt{14}$ leads to answer 1.48331...)

Note: Award ***(A1)*** for 1.48331…seen, accept 1.48378… from using the given answer $x = 3.7$, ***(A1)*(ft)** for their 9 from part (a) seen, ***(A1)*** for the correct notation for their interval (accept $\leqslant y \leqslant$ or $\leqslant f \leqslant$).

e. 3 ***(A1)***

Note: Do not accept a coordinate pair.

f. $\frac{3-9}{7-1}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the gradient formula.

$= -1$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answers to parts (a) and (e).

g. (4, 6) ***(A1)*(ft)*(A1)***

Note: Accept $x = 4$, $y = 6$. Award at most ***(A1)(A0)*** if parentheses not seen.

If coordinates reversed award ***(A0)(A1)*(ft)**.

Follow through from their answers to parts (a) and (e).

h. $-\frac{14}{4^2} + 1$ ***(M1)***

Note: Award ***(M1)*** for substitution into their gradient function.

Follow through from their answers to parts (b) and (g).

$= \frac{1}{8}(0.125)$ ***(A1)*(ft)*(G2)***

i. $y - 1.5 = \frac{1}{8}(x - 4)$ ***(M1)*(ft)*(M1)***

Note: Award ***(M1)*** for substituting their (4, 1.5) in any straight line formula,

(M1) for substituting their gradient in any straight line formula.

$y = \frac{x}{8} + 4$ ***(A1)*(ft)*(G2)***

Note: The form of the line has been specified in the question.

Examiners report

a. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y-coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.

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Consider the function $f(x) = 0.5x^2 - \frac{8}{x},\ x \neq 0.$

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of f at $x = -2$. [2]

d. Let T be the tangent to the graph of f at $x = -2$. [2]

Write down the equation of T.

e. Let T be the tangent to the graph of f at $x = -2$. [4]

Sketch the graph of f for $-5 \leqslant x \leqslant 5$ and $-20 \leqslant y \leqslant 20$.

f. Let T be the tangent to the graph of f at $x = -2$. [2]

Draw T on your sketch.

g. The tangent, T, intersects the graph of f at a second point, P. [2]

Use your graphic display calculator to find the coordinates of P.

Markscheme

a. $0.5 \times (-2)^2 - \frac{8}{-2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of $x = -2$ into the formula of the function.

6 ***(A1)(G2)***

b. $f'(x) = x + 8x^{-2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for x, ***(A1)*** for 8, ***(A1)*** for x^{-2} or $\frac{1}{x^2}$ (each term must have correct sign). Award at most ***(A1)(A1)(A0)*** if there are additional terms present or further incorrect simplifications are seen.

c. $f'(-2) = -2 + 8(-2)^{-2}$ ***(M1)***

Note: Award ***(M1)*** for $x = -2$ substituted into their $f'(x)$ from part (b).

$= 0$ ***(A1)*****(ft)*****(G2)***

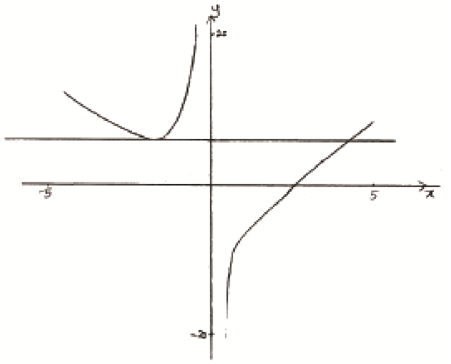
Note: Follow through from their derivative function.

d. $y = 6$ **OR** $y = 0x + 6$ **OR** $y - 6 = 0(x + 2)$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

Notes: Award ***(A1)*(ft)** for their gradient from part (c), ***(A1)*(ft)** for their answer from part (a). Answer must be an equation.

Award ***(A0)(A0)*** for $x = 6$.

e.



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labels and some indication of scales in the stated window. The point $(-2,\ 6)$ correctly labelled, or an x-value and a y-value on their axes in approximately the correct position, are acceptable indication of scales.

Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the y-axis).

Award ***(A1)*** for x-intercept in approximately the correct position.

Award ***(A1)*** for local minimum in the second quadrant.

f. Tangent to graph drawn approximately at $x = -2$ ***(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for straight line tangent to curve at approximately $x = -2$, with approximately correct gradient. Tangent must be straight for the ***(A1)*(ft)** to be awarded.

Award ***(A1)*(ft)** for (extended) line passing through approximately their y-intercept from (d). Follow through from their gradient in part (c) and their equation in part (d).

g. $(4,\ 6)$ **OR** $x = 4,\ y = 6$ ***(G1)*(ft)*(G1)*(ft)**

Notes: Follow through from their tangent from part (d). If brackets are missing then award ***(G0)(G1)*(ft).**

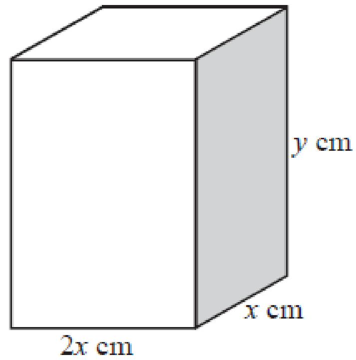
If line intersects their graph at more than one point (apart from $(-2,\ 6)$), follow through from the first point of intersection (to the right of -2).

Award ***(G0)(G0)*** for $(-2,\ 6)$.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2 .



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

Markscheme

i.a. $(3x - 2)(x + 5)$ ***(A1)(A1)***

[2 marks]

i.b. $(3x - 2)(x + 5) = 0$

$x = \frac{2}{3}$ or $x = -5$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G2)***

[2 marks]

i.c. $x = \frac{-13}{6}$ (-2.17) ***(A1)(A1)*****(ft)*****(G2)***

Note: ***(A1)*** is for $x =$, ***(A1)*** for value. **(ft)** if value is half way between roots in (b).

[2 marks]

i.d. Minimum $y = 3\left(\frac{-13}{6}\right)^2 + 13\left(\frac{-13}{6}\right) - 10$ ***(M1)***

Note: ***(M1)*** for substituting their value of x from (c) into $f(x)$.

$= -24.1$ ***(A1)(ft)(G2)***

[2 marks]

ii.a. $\text{Area} = 2(2x)x + 2xy + 2(2x)y$ ***(M1)(A1)***

Note: ***(M1)*** for using the correct surface area formula (which can be implied if numbers in the correct place). ***(A1)*** for using correct numbers.

$300 = 4x^2 + 6xy$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.b. $6xy = 300 - 4x^2$ ***(M1)***

$y = \frac{300-4x^2}{6x}$ *or* $\frac{150-2x^2}{3x}$ ***(A1)***

[2 marks]

ii.c. $\text{Volume} = x(2x)y$ ***(M1)***

$V = 2x^2\left(\frac{300-4x^2}{6x}\right)$ ***(A1)*****(ft)**

$= 100x - \frac{4}{3}x^3$ **(AG)**

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.d. $\frac{\mathrm{d}V}{\mathrm{d}x} = 100 - \frac{12x^2}{3}$ or $100 - 4x^2$ ***(A1)(A1)***

Note: ***(A1)*** for each term.

[2 marks]

ii.e. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column*

(i) For maximum $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ or $100 - 4x^2 = 0$ ***(M1)***

$x = 5$ ***(A1)*****(ft)**

$y = \frac{300-4(5)^2}{6(5)}$ or $\left(\frac{150-2(5)^2}{3(5)}\right)$ ***(M1)***

$= \frac{20}{3}$ ***(A1)*****(ft)**

(UP) (ii) $333\frac{1}{3}\ \text{cm}^3\ (333\ \text{cm}^3)$

Note: **(ft)** from their (e)(i) if working for volume is seen.

[5 marks]

Examiners report

i.a. Most candidates made a good attempt to factorise the expression.

i.b. Many gained both marks here from a correct answer or ft from the previous part.

i.c. Many used the formula correctly. Some forgot to put $x =$.

i.d. Most candidates found this value from their GDC.

ii.a. A good attempt was made to show the correct surface area.

ii.b. Many could rearrange the equation correctly.

ii.c. Although this was not a difficult question it probably looked complicated for the candidates and it was often left out.

ii.d. Those who reached this length could usually manage the differentiation.

ii.e. (i) Many found the correct value of x but not of y.

(ii) This was well done and again the units were included in most scripts.

Throughout this question *all* the numerical answers must be given correct to the nearest whole number.

a. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [4]

Find the number of students attending Park School in

(i) January 2001;

(ii) January 2003.

b. Park School started in January 2000 with 100 students. Every full year, there is an increase of 6% in the number of students. [2]

Show that the number of students attending Park School in January 2007 is 150.

c. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [2]

Find the number of students attending Grove School in January 2003.

d. Grove School had 110 students in January 2000. Every full year, the number of students is 10 more than in the previous year. [4]

Find the year in which the number of students attending Grove School will be first 60% **more than** in January 2000.

e. Each January, one of these two schools, the one that has more students, is given extra money to spend on sports equipment. [5]

(i) Decide which school gets the money in 2007. Justify your answer.

(ii) Find the first year in which Park School will be given this extra money.

Markscheme

a. (i) $100 \times 1.06 = 106$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for multiplying by 1.06 or equivalent. ***(A1)*** for correct answer.

(ii) $100 \times 1.06^3 = 119$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for multiplying by 1.06^3 or equivalent or for list of values. ***(A1)*** for correct answer.

[4 marks]

b. $100 \times 1.06^7 = 150.36\ldots = 150$ correct to the nearest whole ***(M1)(A1)(AG)***

Note: ***(M1)*** for correct formula or for list of values. ***(A1)*** for correct substitution or for 150 in the correct position in the list. Unrounded answer must be seen for the ***(A1)***.

[2 marks]

c. $110 + 3 \times 10 = 140$ ***(M1)(A1)(G2)***

Note: ***(M1)*** for adding 30 or for list of values. ***(A1)*** for correct answer.

[2 marks]

d. **In (d) and (e) follow through from (c) if consistent wrong use of correct AP formula.**

$110 + (n-1) \times 10 > 176$ ***(A1)(M1)***

$n = 8 \therefore \text{year } 2007$ ***(A1)(A1)*(ft)*(G2)***

Note: ***(A1)*** for 176 or 66 seen. ***(M1)*** for showing list of values and comparing them to 176 or for equating formula to 176 or for writing the inequality. If $n = 8$ not seen can still get ***(A2)*** for 2007. Answer $n = 8$ with no working gets ***(G1)***.

OR

$110 + n \times 10 > 176$ ***(A1)(M1)***

$n = 7 \therefore \text{year } 2007$ ***(A1)(A1)*(ft)*(G2)***

[4 marks]

e. **In (d) and (e) follow through from (c) if consistent wrong use of correct AP formula.**

(i) 180 ***(A1)*(ft)**

Grove School gets the money. ***(A1)*(ft)**

Note: ***(A1)*** for 180 seen. ***(A1)*** for correct answer.

(ii) $100 \times 1.06^{n-1} > 110 + (n-1) \times 10$ ***(M1)***

$n = 20 \therefore \text{year } 2019$ ***(A1)(A1)*(ft)*(G2)***

Note: ***(M1)*** for showing lists of values for each school and comparing them or for equating both formulae or writing the correct inequality. If $n = 20$ not seen can still get ***(A2)*** for 2019. Follow through with ratio used in (b) and/or formula used in (d).

OR

$100 \times 1.06^n > 110 + n \times 10$ ***(M1)***

$n = 19 \therefore \text{year } 2019$ ***(A1)(A1)*(ft)*(G2)***

OR

graphically

Note: ***(M1)*** for sketch of both functions on the same graph, ***(A1)*** for the intersection point, ***(A1)*** for correct answer.

[5 marks]

Examiners report

a. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

b. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

c. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

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e. This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

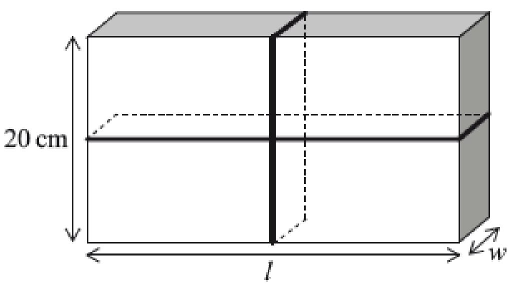
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is $3000\ \text{cm}^3$.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

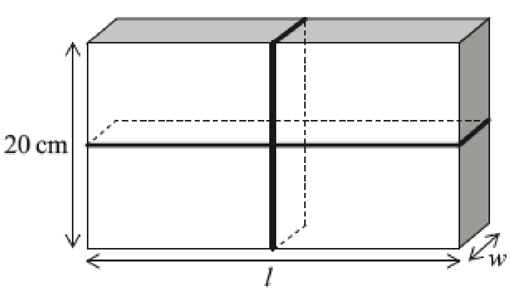
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

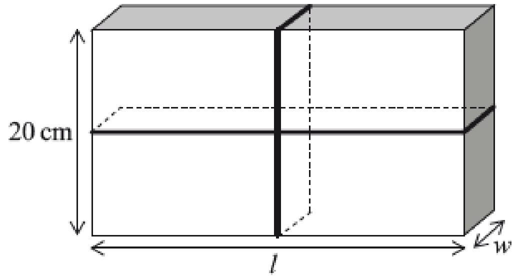
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



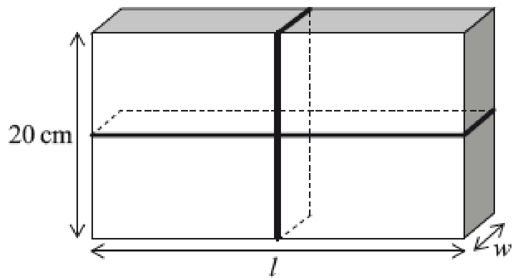
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



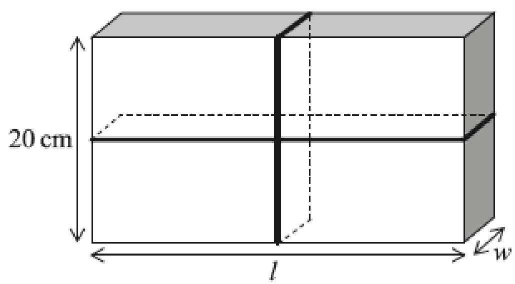
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



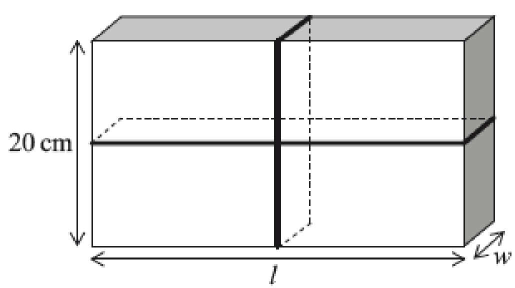
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



Write down the value, l, of the parcel for which the length of string is a minimum.

h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

Markscheme

a. $20lw$ **OR** $V = 20lw$ ***(A1)***

[1 mark]

b. $3000 = 20lw$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to 3000.

$l = \frac{3000}{20w}$ ***(M1)***

Note: Award ***(M1)*** for rearranging equation to make l subject of the formula. The above equation must be seen to award ***(M1)***.

OR

$150 = lw$ ***(M1)***

Note: Award ***(M1)*** for division by 20 on both sides. The above equation must be seen to award ***(M1)***.

$l = \frac{150}{w}$ ***(AG)***

[2 marks]

c. $S = 2l + 4w + 2(20)$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression for S.

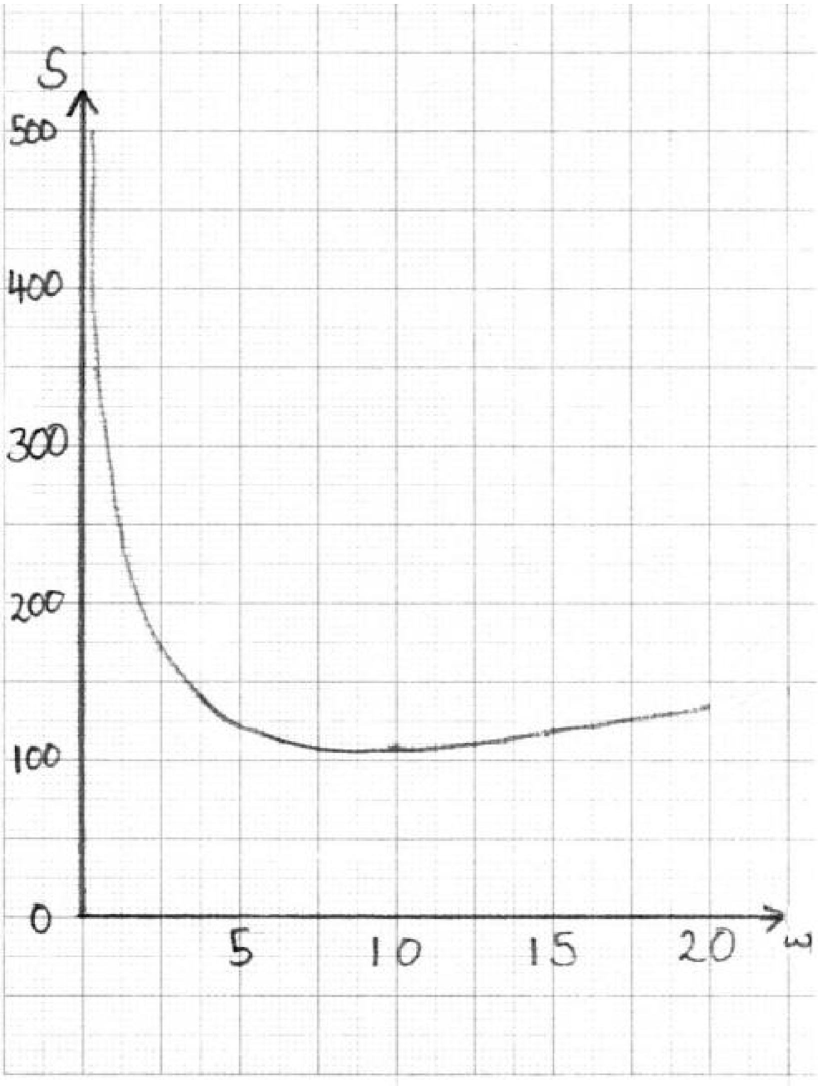
$2\left(\frac{150}{w}\right) + 4w + 2(20)$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into the expression for S. The above expression must be seen to award ***(M1)***.

$= 40 + 4w + \frac{300}{w}$ ***(AG)***

[2 marks]

d.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct scales, window and labels on axes, ***(A1)*** for approximately correct shape, ***(A1)*** for minimum point in approximately correct position, ***(A1)*** for asymptotic behaviour at $w = 0$.

Axes must be drawn with a ruler and labeled w and S.

For a smooth curve (with approximately correct shape) there should be **one** continuous thin line, no part of which is straight and no (one-to-many) mappings of w.

The S-axis must be an asymptote. The curve must not touch the S-axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

e. $4 - \frac{300}{w^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for 4, ***(A1)*** for -300, ***(A1)*** for $\frac{1}{w^2}$ or w^{-2}. If extra terms present, award at most ***(A1)(A1)(A0)***.

[3 marks]

f. $4 - \frac{300}{w^2} = 0$ **OR** $\frac{300}{w^2} = 4$ **OR** $\frac{\mathrm{d}S}{\mathrm{d}w} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to zero.

$w = 8.66\ \left(\sqrt{75},\ 8.66025\ldots\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to part (e).

[2 marks]

g. $17.3\ \left(\frac{150}{\sqrt{75}},\ 17.3205\ldots\right)$ ***(A1)*****(ft)**

Note: Follow through from their answer to part (f).

[1 mark]

h. $40 + 4\sqrt{75} + \frac{300}{\sqrt{75}}$ ***(M1)***

Note: Award ***(M1)*** for substitution of their answer to part (f) into the expression for S.

$= 110\ (\text{cm})\ \left(40 + 40\sqrt{3},\ 109.282\ldots\right)$ ***(A1)*****(ft)*****(G2)***

Note: Do not accept 109.

Follow through from their answers to parts (f) and (g).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

Consider the function $g(x) = bx - 3 + \frac{1}{x^2},\ x \neq 0$.

a. Write down the equation of the vertical asymptote of the graph of *y* = *g*(*x*) . [2]

b. Write down *g*′(*x*) . [3]

c. The line *T* is the tangent to the graph of *y* = *g*(*x*) at the point where *x* = 1. The gradient of *T* is 3. [2]

Show that *b* = 5.

d. The line T is the tangent to the graph of $y = g(x)$ at the point where $x = 1$. The gradient of T is 3. [3]

Find the equation of T.

e. Using your graphic display calculator find the coordinates of the point where the graph of $y = g(x)$ intersects the x-axis. [2]

f. (i) Sketch the graph of $y = g(x)$ for $-2 \le x \le 5$ and $-15 \le y \le 25$, indicating clearly your answer to part (e). [6]

(ii) Draw the line T on your sketch.

g. Using your graphic display calculator find the coordinates of the local minimum point of $y = g(x)$. [2]

h. Write down the interval for which $g(x)$ is increasing in the domain $0 < x < 5$. [2]

Markscheme

a. $x = 0$ ***(A1)(A1)***

Notes: Award ***(A1)*** for x=constant, ***(A1)*** for 0. Award ***(A0)(A0)*** if answer is not an equation.

[2 marks]

b. $b - \frac{2}{x^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for b, ***(A1)*** for −2, ***(A1)*** for $\frac{1}{x^3}$ (or x^{-3}). Award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

c. $3 = b - \frac{2}{(1)^3}$ ***(M1)(M1)***

Note: Award ***(M1)*** for substituting 1 into their gradient function, ***(M1)*** for equating their gradient function to 3.

$b = 5$ ***(AG)***

Note: Award at most ***(M1)(A0)*** if final line is not seen or b does not equal 5.

[2 marks]

d. $g(1) = 3$ **or** (1, 3) *(seen or implied from the line below)* ***(A1)***

$3 = 3 \times 1 + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their point (1, 3) and gradient 3 into equation $y = mx + c$. Follow through from their point of tangency.

$y = 3x$ ***(A1)*****(ft)*****(G2)***

OR

$y - 3 = 3(x - 1)$ ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for substitution of gradient 3 and their point (1, 3) into $y - y_1 = m(x - x_1)$, ***(A1)*****(ft)** for correct substitutions. Follow through from their point of tangency. Award at most ***(A1)(M1)(A0)*****(ft)** if further incorrect working seen.

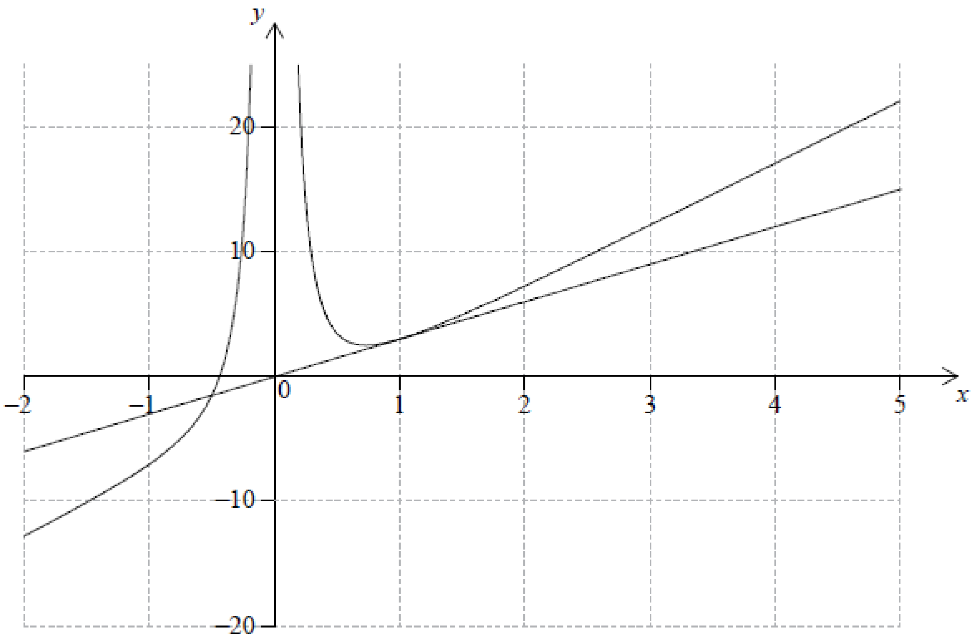
[3 marks]

e. (−0.439, 0) ((−0.438785..., 0)) ***(G1)(G1)***

Notes: If no parentheses award at most ***(G1)(G0)***. Accept $x = 0.439$, $y = 0$.

[2 marks]

f. (i)



Award ***(A1)*** for labels and some indication of scale in the stated window.

Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the y-axis)

Award ***(A1)*(ft)** for x-intercept consistent with their part (e).

Award ***(A1)*** for local minimum in the first quadrant. ***(A1)(A1)(A1)*(ft)*(A1)***

(ii) Tangent to curve drawn at approximately $x = 1$ ***(A1)(A1)***

Note: Award ***(A1)*** for a line tangent to curve approximately at $x = 1$. Must be a straight line for the mark to be awarded. Award ***(A1)*(ft)** for line passing through the origin. Follow through from their answer to part (d).

[6 marks]

g. (0.737, 2.53) ((0.736806..., 2.52604...)) ***(G1)(G1)***

Notes: Do not penalize for lack of parentheses if already penalized in (e). Accept $x = 0.737$, $y = 2.53$.

[2 marks]

h. $0.737 < x < 5$ **OR** (0.737;5) ***(A1)(A1)*(ft)**

Notes: Award ***(A1)*** for correct strict or weak inequalities with x seen if the interval is given as inequalities, ***(A1)*(ft)** for 0.737 and 5 or their value from part (g).

[2 marks]

Examiners report

a. This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation.

b. This question was moderately well answered. The concept of vertical asymptote in part (a) seemed to be problematic for a great number of candidates. In many cases students showed partial understanding of the vertical asymptote but found it difficult to write a correct equation. Finding the derivative in part (b) proved problematic as well. It seems that the presence of the parameter b in the function may have contributed to this.

c. This question was moderately well answered. In part (c) a great number of students substituted $b = 5$ in the equation of the function instead of substituting it in the equation of their derivative.

d. This question was moderately well answered. Very few students used the GDC to find the equation of the tangent at $x = 1$ in part (d).

e. This question was moderately well answered. Good use of the GDC was seen in part (e), although some students wrote the x-coordinates of the point of intersection and neglected to write the y-coordinate.

f. This question was moderately well answered. The sketch in part (f) was, for the most part, not well done. Often the axes labels were missing. Very few tangents to the curve at the correct point were seen. Often the intended tangent lines intersected the curve, which shows that candidates either did not know what a tangent is or did not make sense of the sketch.

g. This question was moderately well answered. Good use of the GDC was shown in part (g) for finding the coordinates of the minimum point.

h. This question was moderately well answered. Few acceptable answers were given in part (h).

Consider the function $f(x) = x^3 – 3\text{x}– 24x + 30$.

a. Write down $f(0)$. [1]

b. Find $f'(x)$. [3]

c. Find the gradient of the graph of $f(x)$ at the point where $x = 1$. [2]

d. (i) Use $f'(x)$ to find the x-coordinate of M and of N. [5]

(ii) Hence or otherwise write down the coordinates of M and of N.

e. Sketch the graph of $f(x)$ for $-5 \leqslant x \leqslant 7$ and $-60 \leqslant y \leqslant 60$. Mark clearly M and N on your graph. [4]

f. Lines L_1 and L_2 are parallel, and they are tangents to the graph of $f(x)$ at points A and B respectively. L_1 has equation $y = 21x + 111$. [6]

(i) Find the x-coordinate of A and of B.

(ii) Find the y-coordinate of B.

Markscheme

a. 30 ***(A1)***

[1 mark]

b. $f'(x) = 3x^2 – 6x – 24$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each term. Award at most ***(A1)(A1)*** if extra terms present.

[3 marks]

c. $f'(1) = -27$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for substituting $x = 1$ into their derivative.

[2 marks]

d. (i) $f'(x) = 0$

$3x^2 - 6x - 24 = 0$ ***(M1)***

$x = 4; x = -2$ ***(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(M1)*** for either $f'(x) = 0$ or $3x^2 - 6x - 24 = 0$ seen. Follow through from their derivative. Do not award the two answer marks if derivative not used.

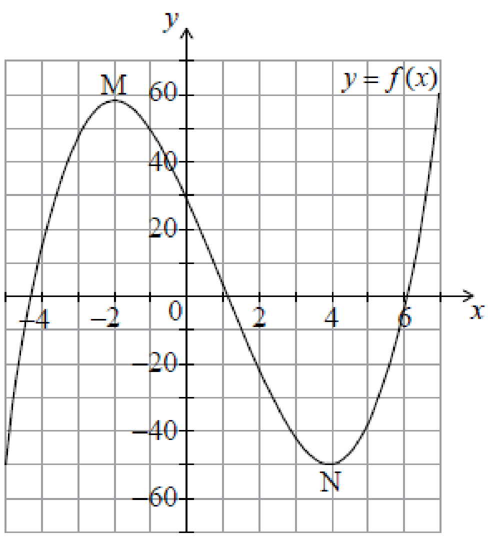
(ii) M(–2, 58) accept $x = -2$, $y = 58$ ***(A1)*(ft)**

N(4, – 50) accept $x = 4$, $y = -50$ ***(A1)*(ft)**

Note: Follow through from their answer to part (d) (i).

[5 marks]

e.



(A1) for window

(A1) for a smooth curve with the correct shape

(A1) for axes intercepts in approximately the correct positions

(A1) for M and N marked on diagram and in approximately correct position ***(A4)***

Note: If window is not indicated award at most ***(A0)(A1)(A0)(A1)*(ft)**.

[4 marks]

f. (i) $3x^2 - 6x - 24 = 21$ ***(M1)***

$3x^2 - 6x - 45 = 0$ ***(M1)***

$x = 5; x = -3$ ***(A1)*(ft)*(A1)*(ft)*(G3)***

Note: Follow through from their derivative.

OR

Award ***(A1)*** for L_1 drawn tangent to the graph of f on their sketch in approximately the correct position ($x = -3$), ***(A1)*** for a second tangent parallel to their L_1, ***(A1)*** for $x = -3$, ***(A1)*** for $x = 5$.   ***(A1)*(ft)*(A1)*(ft)*(A1)(A1)***

Note: If only $x = -3$ is shown without working award ***(G2)***. If both answers are shown irrespective of workingaward ***(G3)***.

(ii) $f(5) = -40$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for attempting to find the image of their $x = 5$. Award ***(A1)*** only for (5, −40). Follow through from their x-coordinate of B only if it has **been clearly identified** in (f) (i).

[6 marks]

Examiners report

a. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

b. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

c. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

d. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

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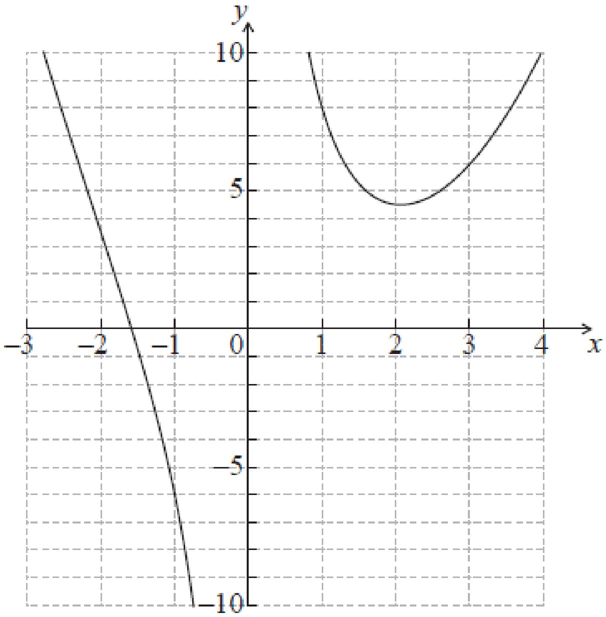
f. The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f(x)$ to find the x-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3-2x^2-9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the ***x*-coordinate** of B and of C .

Markscheme

a. (i) $x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for $x =$ a constant, ***(A1)*** for the constant in their equation being 0.

(ii) $-1.58\,(-1.58454\ldots)$ ***(G1)***

Note: Accept -1.6, do not accept -2 or -1.59.

(iii) $(2.06, 4.49)\ (2.06020\ldots, 4.49253\ldots)$ ***(G1)(G1)***

Note: Award at most ***(G1)(G0)*** if brackets not used. Award ***(G0)(G1)*(ft)** if coordinates are reversed.

Note: Accept $x = 2.06, y = 4.49$.

Note: Accept 2.1, do not accept 2.0 or 2. Accept 4.5, do not accept 5 or 4.50.

[5 marks]

b. $f'(x) = 2x - 2 - \frac{9}{x^2}$ ***(A1)(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2x$, ***(A1)*** for -2, ***(A1)*** for -9, ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A1)(A0)*** if there are extra terms present.

[4 marks]

c. $f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for taking the correct common denominator.

$= \frac{(2x^3-2x^2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for multiplying brackets or equivalent.

$= \frac{2x^3-2x^2-9}{x^2}$ ***(AG)***

Note: The final ***(M1)*** is not awarded if the given answer is not seen.

[2 marks]

d. $f'(1) = \frac{2(1)^3-2(1)-9}{(1)^2}$ ***(M1)***

$= -9$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into **given** (or their correct) $f'(x)$. There is no follow through for use of their incorrect derivative.

[2 marks]

e. $\frac{1}{9}$ ***(A1)*(ft)**

Note: Follow through from part (d).

[1 mark]

f. $y - 8 = \frac{1}{9}(x-1)$ ***(M1)(M1)***

Notes: Award ***(M1)*** for substitution of their gradient from (e), ***(M1)*** for substitution of given point. Accept all forms of straight line.

$y = \frac{1}{9}x + \frac{71}{9}$ $(y = 0.111111\ldots x + 7.88888\ldots)$ ***(A1)*(ft)*(G3)***

Note: Award the final ***(A1)*(ft)** for a correctly rearranged formula of **their** straight line in (f). Accept $0.11x$, do not accept $0.1x$. Accept 7.9, do not accept 7.88, do not accept 7.8.

[3 marks]

g. $-2.50, 3.61\ (-2.49545\ldots, 3.60656\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Notes: Follow through from their line L from part (f) even if no working shown. Award at most ***(A0)(A1)*****(ft)** if their correct coordinate pairs given.

Note: Accept -2.5, do not accept -2.49. Accept 3.6, do not accept 3.60.

[2 marks]

Examiners report

a. As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

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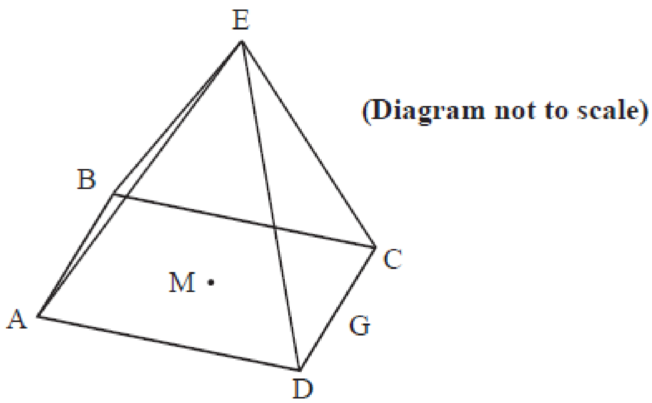
Mal is shopping for a school trip. He buys 50 tins of beans and 20 packets of cereal. The total cost is 260 Australian dollars (AUD).

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



i.a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1]

i.b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \text{AUD}$. [1]

Write down another equation to represent this information.

i.c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays $66\ \text{AUD}$. [2]

Find the cost of one tin of beans.

i.d. (i) Sketch the graphs of the two equations from parts (a) and (b). [4]

(ii) Write down the coordinates of the point of intersection of the two graphs.

ii.a. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1]

ii.b. Show that the height of the pyramid is $13.7\ \text{cm}$, to 3 significant figures. [2]

ii.c. Calculate [4]

(i) the length of EG;

(ii) the size of angle DEC.

ii.d. Find the total surface area of the pyramid. [2]

ii.e. Find the volume of the pyramid. [2]

Markscheme

i.a. $50b + 20c = 260$ ***(A1)***

[1 mark]

i.b. $12b + 6c = 66$ ***(A1)***

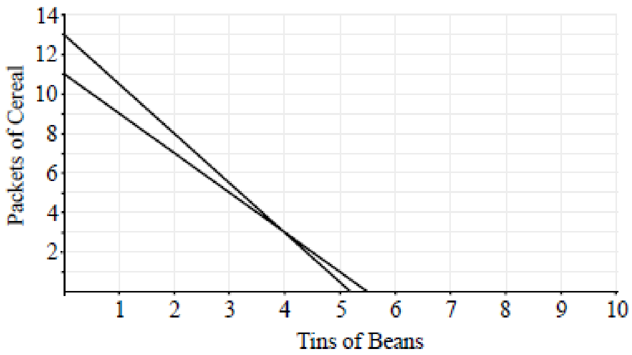
[1 mark]

i.c. Solve to get $b = 4$ ***(M1)(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for attempting to solve the equations simultaneously.

[2 marks]

i.d. (i)



(A1)(A1)(A1)

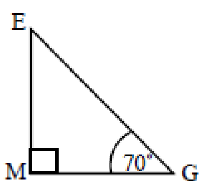
Notes: Award ***(A1)*** for labels and some idea of scale, ***(A1)*(ft)*(A1)*(ft)** for each line.
The axis can be reversed.

(ii) $(4, 3)$ or $(3, 4)$ ***(A1)*(ft)**

Note: Accept $b = 4$, $c = 3$

[4 marks]

ii.a.



(A1)

[1 mark]

ii.b. $\tan 70 = \frac{h}{5}$ ***(M1)***

$h = 5\tan 70 = 13.74$ ***(A1)***

$h = 13.7 \text{ cm}$ ***(AG)***

[2 marks]

ii.c. *Unit penalty* ***(UP)*** *is applicable in this part of the question where indicated in the left hand column.*

(i) $\text{EG}^2 = 5^2 + 13.7^2$ OR $5^2 + (5\tan 70)^2$ ***(M1)***

(UP) $\text{EG} = 14.6 \text{ cm}$ ***(A1)(G2)***

(ii) $\text{DEC} = 2 \times \tan^{-1}\left(\frac{5}{14.6}\right)$ ***(M1)***

$= 37.8^\circ$ ***(A1)*(ft)*(G2)***

[4 marks]

ii.d. *Unit penalty* ***(UP)*** *is applicable in this part of the question where indicated in the left hand column.*

$\text{Area} = 10 \times 10 + 4 \times 0.5 \times 10 \times 14.619$ ***(M1)***

(UP) $= 392 \text{ cm}^2$ ***(A1)*(ft)*(G2)***

[2 marks]

ii.e. *Unit penalty* ***(UP)*** *is applicable in this part of the question where indicated in the left hand column.*

$\text{Volume} = \frac{1}{3} \times 10 \times 10 \times 13.7$ ***(M1)***

(UP) $= 457 \text{ cm}^3$ (458 cm^3) ***(A1)(G2)***

[2 marks]

Examiners report

i.a. Most candidates managed to write down the equation.

i.b. Most candidates managed to write down the equation.

i.c. Many managed to find the correct answer and the others tried their best but made some mistake in the process.

i.d. (i) Few candidates sketched the graphs well. Few used a ruler.

(ii) Many candidates could not be awarded ft from their graph because the answer they gave was not possible.

ii.a. Very few correct drawings.

ii.b. Some managed to show this more by good fortune and ignoring their original triangle than by good reasoning.

ii.c. (i) Many found this as ft from the previous part. Some lost a UP here.

(ii) This was not well done. The most common answer was 40°.

ii.d. Many managed this or were awarded ft points.

ii.e. This was well done and most candidates also remembered their units on this part.

Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

a. (i) Write down the value of y when x is 2. [3]

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). [4]

c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

d. Let L_1 be the tangent to the curve at $x = 2$. [8]

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. [5]

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

Markscheme

a. (i) $y = 0$ ***(A1)***

(ii) $(0, -2)$ ***(A1)(A1)***

Note: Award ***(A1)(A0)*** if brackets missing.

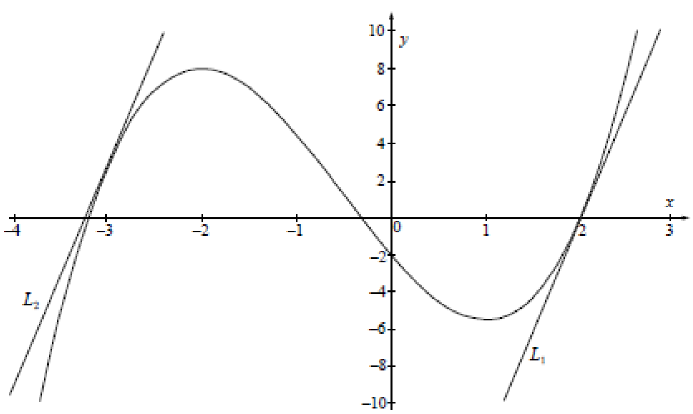
OR

$x = 0, y = -2$ ***(A1)(A1)***

Note: If coordinates reversed award ***(A0)(A1)*(ft)**. Two coordinates must be given.

[3 marks]

b.



(A4)

Notes: ***(A1)*** for appropriate window. Some indication of scale on the x-axis must be present (for example ticks). Labels not required. ***(A1)*** for smooth curve and shape, ***(A1)*** for maximum and minimum in approximately correct position, ***(A1)*** for x and y intercepts found in (a) in approximately correct position.

[4 marks]

c. $\frac{dy}{dx} = 3x^2 + 3x - 6$ ***(A1)(A1)(A1)***

Note: ***(A1)*** for each correct term. Award ***(A1)(A1)(A0)*** at most if any other term is present.

[3 marks]

d. (i) $3 \times 4 + 3 \times 2 - 6 = 12$ ***(M1)(A1)(AG)***

Note: ***(M1)*** for using the derivative and substituting $x = 2$. ***(A1)*** for correct (and clear) substitution. The 12 must be seen.

(ii) Gradient of L_2 is 12 (can be implied) ***(A1)***

$3x^2 + 3x - 6 = 12$ ***(M1)***

$x = -3$ ***(A1)(G2)***

Note: ***(M1)*** for equating the derivative to 12 or showing a sketch of the derivative together with a line at $y = 12$ or a table of values showing the 12 in the derivative column.

(iii) ***(A1)*** for L_1 correctly drawn at approx the correct point ***(A1)***

(A1) for L_2 correctly drawn at approx the correct point ***(A1)***

(A1) for 2 parallel lines ***(A1)***

Note: If lines are not labelled award at most ***(A1)(A1)(A0)***. Do not accept 2 horizontal or 2 vertical parallel lines.

[8 marks]

e. (i) $b = 1$ ***(G2)***

(ii) The curve is decreasing. ***(A1)***

Note: Accept any valid description.

(iii) $y = 8$ ***(A1)(A1)(G2)***

Note: ***(A1)*** for "$y =$ a constant", ***(A1)*** for 8.

[5 marks]

Examiners report

a. Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

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The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.

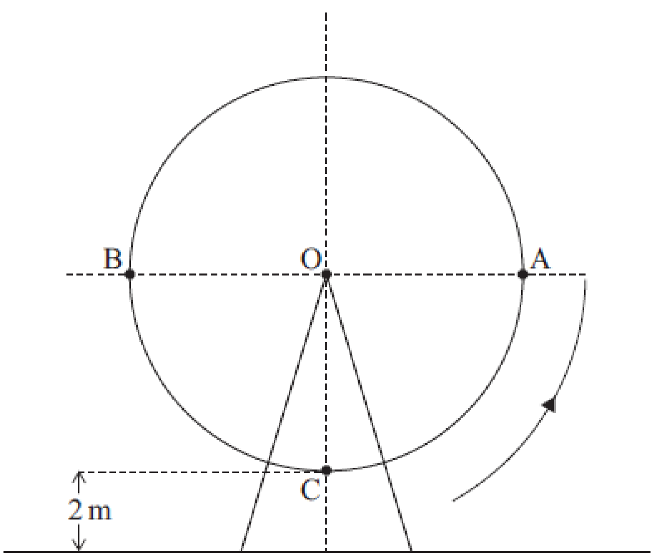


diagram not to scale

a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. [2]

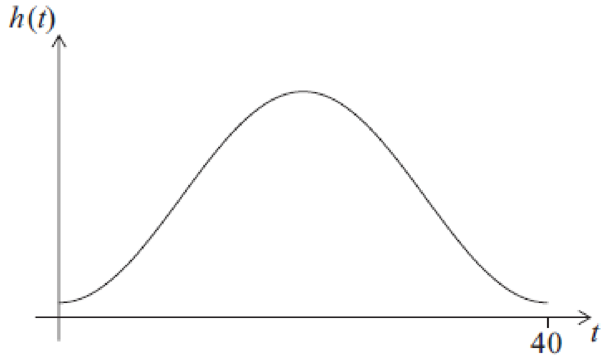
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2]

c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. [4]



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

Markscheme

a. (i) 14 m ***(A1)***

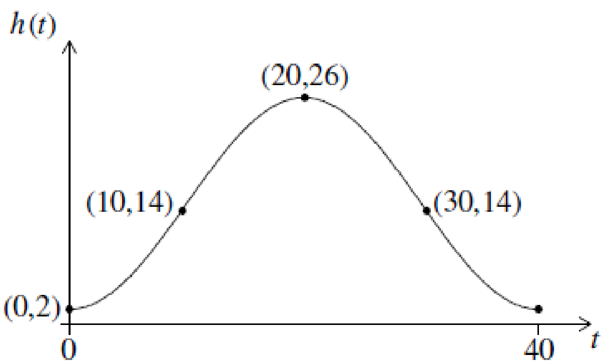
(ii) 26 m ***(A1)***

[2 marks]

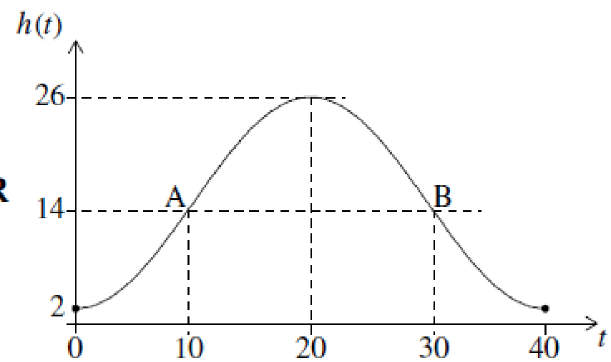
b. A:10, B:30 ***(A1)(A1)***

[2 marks]

c.



OR



(A1)*(ft)*(A1)*(ft)*(A1)

(ft)*(A1)*(ft)

Note: Award ***(A1)*(ft)** for coordinates of each point clearly indicated either by scale or by coordinate pairs. Points need not be labelled A and B in the second diagram. Award a maximum of ***(A1)(A0)(A1)*(ft)*(A1)*(ft)** if coordinates are reversed. Do not penalise reversed coordinates if this has already been penalised in Q4(a)(iii).

[4 marks]

Examiners report

a. Most candidates were able to start this question. Those of an average ability completed it to the end of part (c) and the best gained good success in the latter parts. Its purpose was to discriminate at the highest level and this it did.

Some concerns were raised on the G2 forms as to the appropriateness of this question. However, the MSSL course tries in part to link areas of the syllabus to "real-life" situations and address these. A look back to past years' examination papers, and to the syllabus documentation, should yield similar examples.

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A manufacturer claims that fertilizer has an effect on the height of rice plants. He measures the height of fertilized and unfertilized plants. The results are given in the following table.

Plant height	Fertilized plants	Unfertilized plants
>75 cm	115	80
$50 - 75$ cm	45	65
<50 cm	20	35

A chi-squared test is performed to decide if the manufacturer's claim is justified at the **1 %** level of significance.

The population of fleas on a dog after *t* days, is modelled by

$$N = 4 \times (2)^{\frac{t}{4}},\ t \geqslant 0$$

Some values of *N* are shown in the table below.

t	0	4	8	12	16	20
N	p	8	16	32	q	128

i, a.Write down the null and alternative hypotheses for this test. [2]

i, b.For the number of fertilized plants with height greater than 75 cm, show that the expected value is 97.5. [3]

i, c.Write down the value of χ^2_{calc}. [2]

i, d.Write down the number of degrees of freedom. [1]

i, f.Is the manufacturer's claim justified? Give a reason for your answer. [2]

ii, a.Write down the value of p. [1]

ii, a.Write down the value of q. [2]

ii, b. Using the values in the table above, draw the graph of N for $0 \le t \le 20$. Use 1 cm to represent 2 days on the horizontal axis and 1 cm to represent 10 fleas on the vertical axis. [6]

ii, c. **Use your graph** to estimate the number of days for the population of fleas to reach 55. [2]

Markscheme

i, a. H_0: The height of the rice plants is independent of the use of a fertilizer. ***(A1)***

Notes: For independent accept “not associated”, can accept “the use of a fertilizer has no effect on the height of the plants”.

Do not accept “not correlated”.

H_1: The height of the rice plants is not independent (dependent) of the use of fertilizer. ***(A1)*****(ft)**

Note: If H_0 and H_1 are reversed award ***(A0)(A1)*****(ft)**.

[2 marks]

i, b. $\frac{180 \times 195}{360}$ **or** $\frac{180}{360} \times \frac{195}{360} \times 360$ ***(A1)(A1)(M1)***

= 97.5 ***(AG)***

Notes: Award ***(A1)*** for numerator, ***(A1)*** for denominator ***(M1)*** for division.

If final 97.5 is not seen award at most ***(A1)(A0)(M1)***.

[3 marks]

i, c. $\chi^2_{calc} = 14.01 (14.0, 14)$ ***(G2)***

OR

If worked out by hand award ***(M1)*** for correct substituted formula with correct values, ***(A1)*** for correct answer. ***(M1)(A1)***

[2 marks]

i, d. 2 ***(A1)***

[1 mark]

i, f. $\chi^2_{calc} > \chi^2_{crit}$ ***(R1)***

The manufacturer's claim is justified. (or equivalent statement) ***(A1)***

Note: Do not accept ***(R0)(A1)***.

[2 marks]

ii, a.i. $p = 4$ ***(G1)***

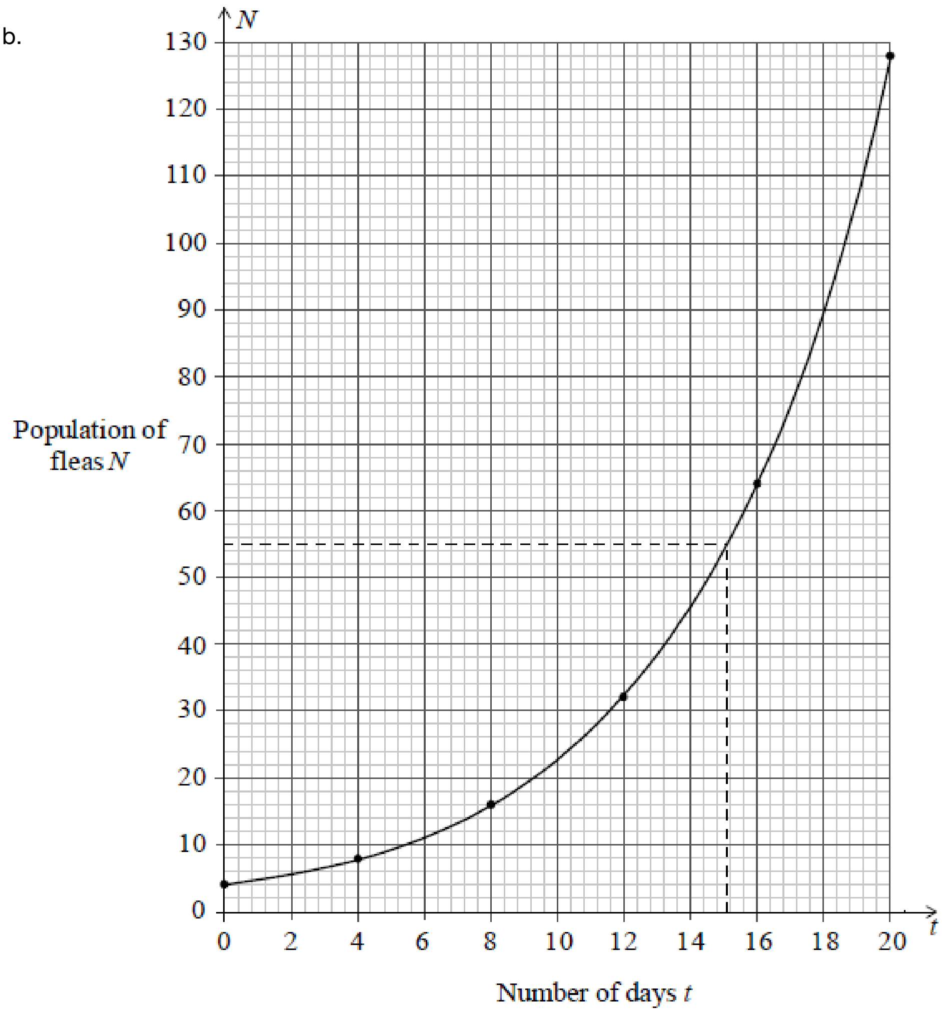
[1 mark]

ii, a. ii. $q = 4(2)^{\frac{16}{4}}$ ***(M1)***

$= 64$ ***(A1)(G2)***

[2 marks]

ii, b.



(A1)(A1)(A1) ***(A3)***

Notes: Award ***(A1)*** for *x* axis with correct scale and label, ***(A1)*** for *y* axis with correct scale and label.

Accept *x* and *y* for labels.

If *x* and *y* axis reversed award at most ***(A0)(A1)*****(ft)**.

(A1) for smooth curve.

Award ***(A3)*** for all 6 points correct, ***(A2)*** for 4 or 5 points correct, ***(A1)*** for 2 or 3 points correct, ***(A0)*** otherwise.

[6 marks]

ii, c. 15 (±0.8) ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for line drawn shown on graph, ***(A1)*****(ft)** from candidate's graph.

[2 marks]

Examiners report

i, a. It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of χ^2 at the 1% level of significance.

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ii, a, i. Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

ii, a, ii. Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

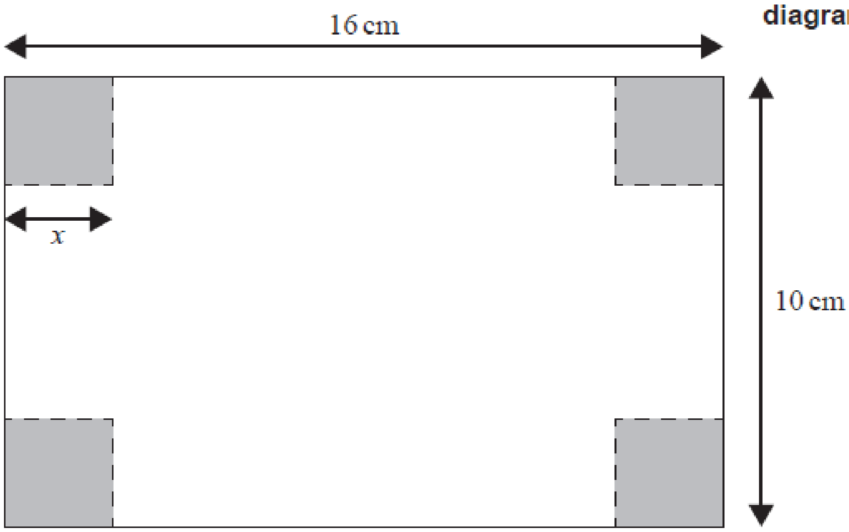
ii, b. Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

ii, c. Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

---

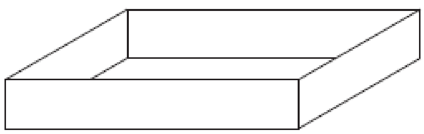
a. Hugo is given a rectangular piece of thin cardboard, 16 cm by 10 cm. He decides to design a tray with it. [2]

He removes from each corner the shaded squares of side $x\text{ cm}$, as shown in the following diagram.



The remainder of the cardboard is folded up to form the tray as shown in the following diagram.

diagram not to scale



Write down, **in terms of** x , the length and the width of the tray.

b. (i) State whether x can have a value of 5. Give a reason for your answer. [4]

(ii) Write down the interval for the possible values of x .

c. Show that the volume, $V \text{ cm}^3$, of this tray is given by [2]

$$V = 4x^3 - 52x^2 + 160x.$$

d. Find $\frac{dV}{dx}$. [3]

e. **Using your answer from part (d)**, find the value of x that maximizes the volume of the tray. [2]

f. Calculate the maximum volume of the tray. [2]

g. Sketch the graph of $V = 4x^3 - 51x^2 + 160x$, for the possible values of x found in part (b)(ii), and $0 \leqslant V \leqslant 200$. Clearly label the maximum point. [4]

Markscheme

a. $16 - 2x,\ 10 - 2x$ ***(A1)(A1)***

b. (i) no ***(A1)***

(when x is 5) the width of the tray will be zero / there is no short edge to fold / $10 - 2(5) = 0$ ***(R1)***

Note: Do not award ***(R0)(A1)***. Award the ***(R1)*** for reasonable explanation.

(ii) $0 < x < 5$ ***(A1)(A1)***

Note: Award ***(A1)*** for 0 and 5 seen, ***(A1)*** for correct strict inequalities (accept alternative notation). Award ***(A1)(A0)*** for “between 0 and 5” or “from 0 to 5”.
Do not accept a list of integers.

c. $V = (16 - 2x)(10 - 2x)(x)$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in volume of cuboid formula.

$= 160x - 32x^2 - 20x^2 + 4x^3$ (or equivalent) ***(M1)***

$= 160x - 52x^2 + 4x^3$ ***(AG)***

Note: Award ***(M1)*** for showing clearly the expansion and for simplifying the expression, and this **must** be seen to award second ***(M1)***. The ***(AG)*** line must be seen for the final ***(M1)*** to be awarded.

d. $12x^2 - 104x + 160$ (or equivalent) ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $12x^2$, ***(A1)*** for $-104x$ and ***(A1)*** for $+160$. If extra terms are seen award at most ***(A1)(A1)(A0)***.

e. $12x^2 - 104x + 160 = 0$ ***(M1)***

Note: Award ***(M1)*** for equating **their** derivative to 0.

$x = 2$ ***(A1)*****(ft)**

Note: Award ***(M1)*** for a sketch of their derivative in part (d), ***(A1)*****(ft)** for reading the x-intercept from their graph.
Award ***(M0)(A0)*** for $x = 2$ with no working seen.
Award at most ***(M1)(A0)*** if the answer is a pair of coordinates.
Award at most ***(M1)(A0)*** if the answer given is $x = 2$ **and** $x = \frac{20}{3}$
Follow through from their derivative in part (d). Award ***(A1)*****(ft)** only if answer is positive and less than 5.

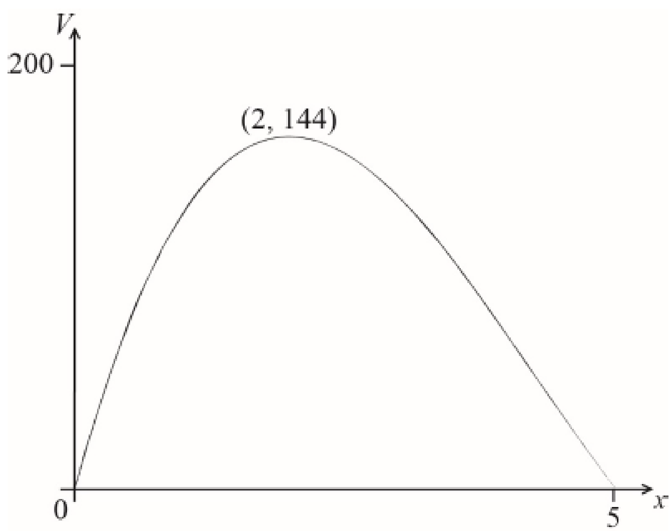
f. $4(2)^3 - 52(2)^2 + 160(2)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their answer to part (e) into volume formula.

$144\ (\text{cm}^3)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (d).

g.



(A1)(A1)**(ft)*****(A1)(A1)*****(ft)**

Note: Award ***(A1)*** for correctly labelled axes and window for V, *ie* $0 \leqslant V \leqslant 200$.
Award ***(A1)*****(ft)** for the correct domain $(0 < x < 5)$. Follow through from part (b)(ii) if a different domain is shown on graph.
Award ***(A1)*** for smooth curve with correct shape.
Award ***(A1)*****(ft)** for **their** maximum point indicated (coordinates, cross or dot *etc*.) in approximately correct place.
Follow through from parts (e) and (f) only if the maximum on their graph is different from $(2,\ 144)$.

Examiners report

a. Question 5: Differential calculus

In general, many candidates struggled in some parts. Most candidates who could state the dimensions of the box gave a reasonable justification of why x could not be 5. Very few candidates scored the two marks in part (d)(ii). Either their inequalities were not strict or their limits were incorrect or both. Some candidates stated the range of x as $1,\ 2,\ 3,\ 4$. The algebra in part (c) caused problems for a number of candidates. It seemed that there was a lack of understanding of what the question required. Some substituted $x = 2$ in the volume formula. A few candidates wrote the product of the length, width, height, omitting the appropriate brackets. Part (d) was well answered by most candidates. However, its application in the following part was not as good in part (e). In part (e) some candidates left both solutions for x, not appreciating the fact that one was outside the range. Others lost both marks as they did not show that they had used their derivative to part (d) as required by the question. Very few candidates scored full marks for the sketch in part (g). Not following the given instructions about the domain and range let most candidates down in this question.

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g. Question 5: Differential calculus

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The temperature in $^\circ$C of a pot of water removed from the cooker is given by $T(m) = 20 + 70 \times 2.72^{-0.4m}$, where m is the number of minutes after the pot is removed from the cooker.

a. Show that the temperature of the water when it is removed from the cooker is $90^\circ\mathrm{C}$. [2]

b. The following table shows values for m and $T(m)$. [9]

m	1	2	4	6	8	10
$T(m)$	66.9	51.4	34.1	26.3	22.8	s

(i) Write down the value of s.

(ii) Draw the graph of $T(m)$ for $0 \leqslant m \leqslant 10$. Use a scale of 1 cm to represent 1 minute on the horizontal axis and a scale of 1 cm to represent $10^\circ\mathrm{C}$ on the vertical axis.

(iii) **Use your graph** to find how long it takes for the temperature to reach $56^\circ\mathrm{C}$. Show your method clearly.

(iv) Write down the temperature approached by the water after a long time. Justify your answer.

c. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. [2]

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

Draw the graph of $S(m)$ on the same set of axes used for part (b).

d. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. [1]

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

Comment on the meaning of the constant 20 in the formula for $S(m)$ in relation to the temperature of the soup.

e. Consider the function $S(m) = 20m - 40$ for $2 \leqslant m \leqslant 6$. [4]

The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

(i) **Use your graph** to solve the equation $S(m) = T(m)$. Show your method clearly.

(ii) Hence describe by using inequalities the set of values of m for which $S(m) > T(m)$.

Markscheme

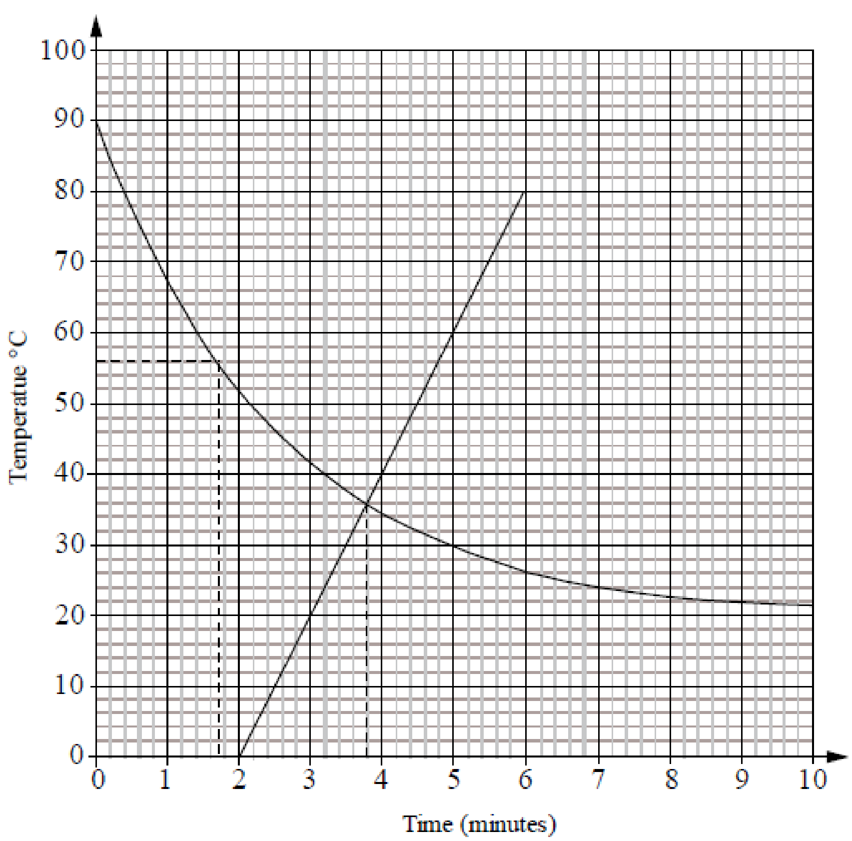
a. $T(0) = 20 + 70 \times 2.72^{-0.4 \times 0} = 90$ ***(M1)(A1)(AG)***

Note: ***(M1)*** for taking $m = 0$, ***(A1)*** for substituting 0 into the formula. For the ***A*** mark to be awarded 90 must be justified by correct method.

[2 marks]

b. (i) 21.3 ***(A1)***

(ii)



(A4)**(ft)**

Note: Scales and labels ***(A1)***. Smooth curve ***(A1)***. All points correct including the y-intercept ***(A2)***, 1 point incorrect ***(A1)***, otherwise ***(A0)***. Follow through from their value of s.

(iii) $m = 1.7 \text{ minutes}$ *(Accept* ± 0.2*)* ***(A2)*(ft)**

Note: Follow through from candidate's graph. Accept answers in minutes and seconds if consistent with graph. If answer incorrect and correct line(s) seen on graph award ***(M1)(A0)***.

(iv) 20°C ***(A1)*(ft)**

The curve behaves asymptotically to the line $y = 20$ or similar. ***(A1)***

OR

The room temperature is 20 or similar

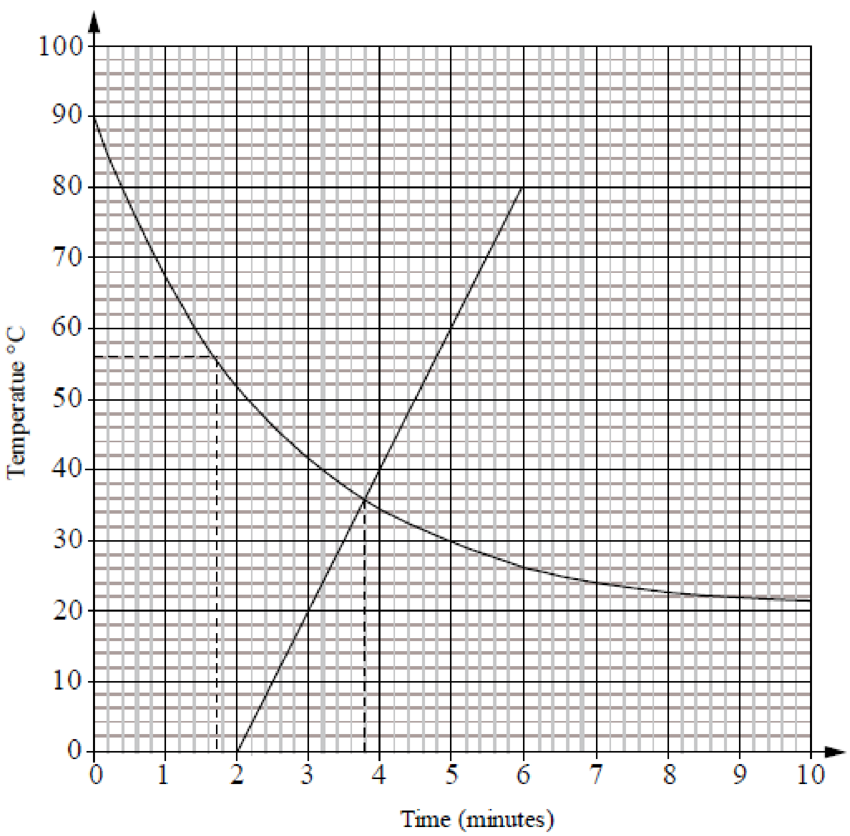
OR

When m is a very large number the term $70 \times 2.72^{-0.4\text{m}}$ tends to zero or similar.

Note: Follow through from their graph if appropriate.

[9 marks]

c.



(A1)(A1)

Notes: ***(A1)*** for correct line, ***(A1)*** for domain. If line not drawn on same set of axes award at most ***(A1)(A0)***.

[2 marks]

d. It indicates by how much the temperature increases per minute. ***(A1)***

[1 mark]

e. (i) $m = 3.8$ *(Accept* ± 0.1*)* ***(A2)*(ft)**

Note: Follow through from candidate's graph. Accept answers in minutes and seconds if consistent with graph. If answer incorrect and correct line(s) seen on graph award ***(M1)(A0)***.

(ii) $3.8 < m \leqslant 6$ ***(A1)(A1)*(ft)**

Note: ***(A1)*** for $m > 3.8$ and ***(A1)*** for $m \leqslant 6$. Follow through from candidate's answer to part (e)(i). If candidate was already penalized in (c) for domain and does not state $m \leqslant 6$ then award ***(A2)*****(ft)**.

Examiners report

a. Good marks were gained in this question, mainly from parts (a) to (d). Very few students answered the show that question using a backwards process. These students did not gain full marks. Labelled and neat exponential graphs were drawn. Marks were lost sometimes for starting the curve at point $(1,\ 66.9)$ instead of at $(0,\ 90)$. Also there were students using a ruler to help them joined up the points for which they lost one mark as the graph must be smooth. Candidates managed to find the time at which the temperature was 56. However, those students that gave their answer as a coordinate pair lost the answer mark. Some students justified the behaviour of the curve by mentioning the asymptote but the big majority said that the room temperature was 20 and it was awarded full marks. Most of the students drew the straight line correct and in the given domain. Those students that drew the line in a separate set of axes could not answer the last part of the question. This part question proved to be difficult for many students and so worked as a discriminating one. One of the most common errors seen in part (b)(iii) and (e)(i) was to give the answer as a point instead of giving just the first coordinate of that point.

b. Good marks were gained in this question, mainly from parts (a) to (d). Very few students answered the show that question using a backwards process. These students did not gain full marks. Labelled and neat exponential graphs were drawn. Marks were lost sometimes for starting the curve at point $(1,\ 66.9)$ instead of at $(0,\ 90)$. Also there were students using a ruler to help them joined up the points for which they lost one mark as the graph must be smooth. Candidates managed to find the time at which the temperature was 56. However, those students that gave their answer as a coordinate pair lost the answer mark. Some students justified the behaviour of the curve by mentioning the asymptote but the big majority said that the room temperature was 20 and it was awarded full marks. Most of the students drew the straight line correct and in the given domain. Those students that drew the line in a separate set of axes could not answer the last part of the question. This part question proved to be difficult for many students and so worked as a discriminating one. One of the most common errors seen in part (b)(iii) and (e)(i) was to give the answer as a point instead of giving just the first coordinate of that point.

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A deep sea diver notices that the intensity of light, I , below the surface of the ocean decreases with depth, d , according to the formula

$$I = k(1.05)^{-d},$$

where I is expressed as a percentage, d is the depth in metres below the surface and k is a constant.

The intensity of light at the surface is 100%.

a. Calculate the value of k . [2]

b. Find the intensity of light at a depth 25 m below the surface. [2]

c. To be able to see clearly, a diver needs the intensity of light to be at least 65%. [2]

Using your graphic display calculator, find the greatest depth below the surface at which she can see clearly.

d. The table below gives the intensity of light (correct to the nearest integer) at different depths. [4]

Depth (d)	0	10	20	50	80
Intensity (I)	100	61	38	9	2

Using this information draw the graph of I against d for $0 \leqslant d \leqslant 100$. Use a scale of 1 cm to represent 10 metres on the horizontal axis and 1 cm to represent 10% on the vertical axis.

e. Some sea creatures have adapted so they can see in low intensity light and cannot tolerate too much light. [2]

Indicate clearly on your graph the range of depths sea creatures could inhabit if they can tolerate between 5% and 35% of the light intensity at the surface.

Markscheme

a. $d = 0, k = 100$ ***(M1)(A1)(G2)***

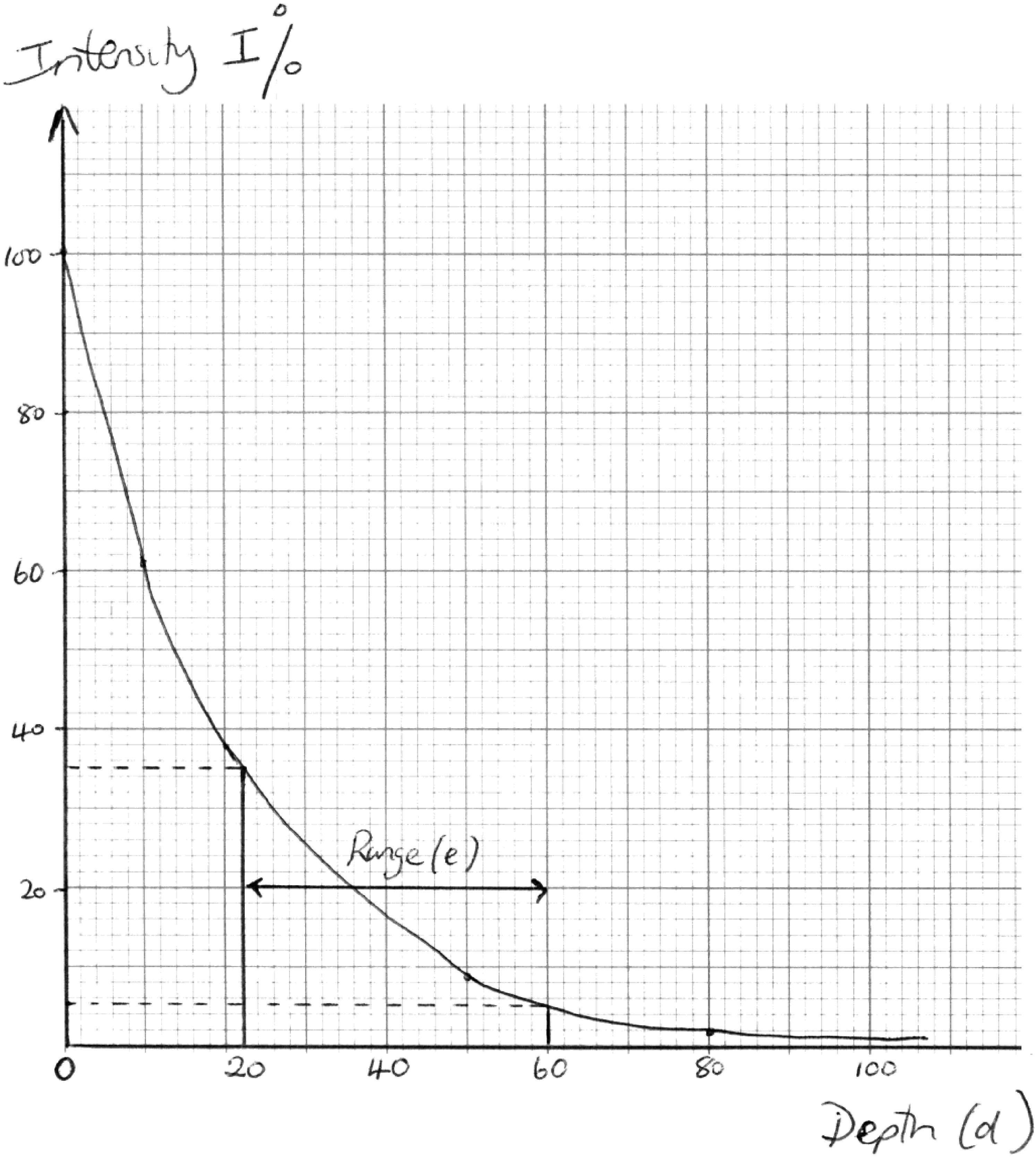
Note: Award ***(M1)*** for $d = 0$ seen.

b. $I = 100 \times (1.05)^{-25} = 29.5(\%)$ $(29.5302\ldots)$ ***(M1)(A1)*****(ft)*****(G2)***

c. $65 = 100 \times (1.05)^{-d}$ ***(M1)***

Note: Award ***(M1)*** for sketch with line drawn at $y = 65$.

$d = 8.83 \text{ (m)}$ $(8.82929\ldots)$ ***(A1)*****(ft)*****(G2)*** d.



(A1) for labels and scales
(A2) for all points correct, ***(A1)*** for 3 or 4 points correct
(A1) for smooth curve asymptotic to the x-axis ***(A4)***

e. Lines in approx correct positions **on** graph ***(M1)***

The range of values indicated (arrows or shading) $22–60\ \mathrm{m}$ ***(A1)***

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Consider the function $f\left(x\right) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is a constant.

The graph of the function passes through the point with coordinates (4 , 2).

P is the minimum point of the graph of *f*(*x*).

a. Find the value of *k*. [2]

b. Using your value of *k* , find *f* ′(*x*). [3]

c. **Use your answer** to part (b) to show that the minimum value of *f*(*x*) is −22 . [3]

d. Write down the **two** values of *x* which satisfy *f*(*x*) = 0. [2]

e. Sketch the graph of *y* = *f*(*x*) for 0 < *x* ≤ 6 and −30 ≤ *y* ≤ 60. [4]

Clearly indicate the minimum point P and the *x*-intercepts on your graph.

Markscheme

a. $\frac{48}{4} + k \times 4^2 - 58 = 2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of *x* = 4 and *y* = 2 into the function.

k = 3 ***(A1) (G2)***

[2 marks]

b. $\frac{-48}{x^2} + 6x$ ***(A1)(A1)(A1)*****(ft)** ***(G3)***

Note: Award ***(A1)*** for −48 , (A1) for x^{-2}, ***(A1)*****(ft)** for their 6*x*. Follow through from part (a). Award at most ***(A1)(A1)(A0)*** if additional terms are seen.

[3 marks]

c. $\frac{-48}{x^2} + 6x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (b) to zero.

x = 2 ***(A1)*****(ft)**

Note: Follow through from part (b). Award (***M1)(A1)*** for $\frac{-48}{\left(2\right)^2} + 6\left(2\right) = 0$ seen.

Award ***(M0)(A0)*** for *x* = 2 seen either from a graphical method or without working.

$\frac{48}{2} + 3 \times 2^2 - 58 \ \ (= -22)$ ***(M1)***

Note: Award ***(M1)*** for substituting their 2 into their function, but only if the final answer is −22. Substitution of the known result invalidates the process; award ***(M0)(A0)(M0)***.

−22 ***(AG)***

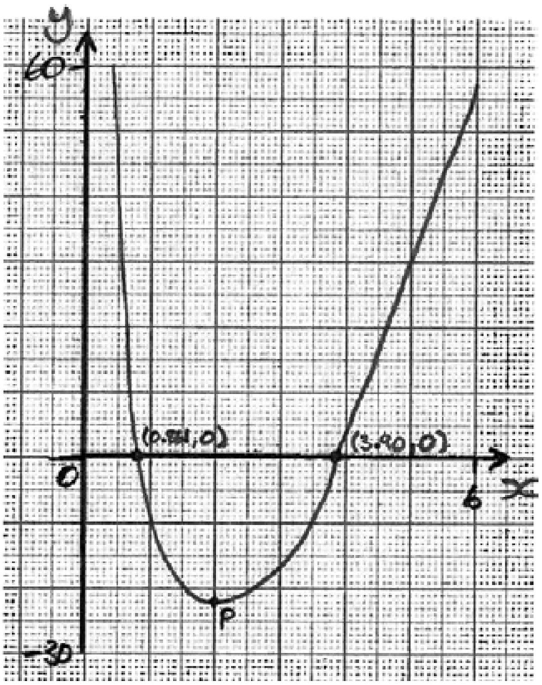
[3 marks]

d. 0.861 (0.860548…), 3.90 (3.90307…) ***(A1)*(ft)*(A1)*(ft) *(G2)***

Note: Follow through from part (a) but only if the answer is positive. Award at most ***(A1)*(ft)*(A0)*** if answers are given as coordinate pairs or if extra values are seen. The function *f*(*x*) only has two *x*-intercepts within the domain. Do not accept a negative *x*-intercept.

[2 marks]

e.



***(A1)(A1)*(ft)*(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*** for correct window. Axes must be labelled.
***(A1)*(ft)** for a smooth curve with correct shape and zeros in approximately correct positions relative to each other.
***(A1)*(ft)** for point P indicated in approximately the correct position. Follow through from their *x*-coordinate in part (c). ***(A1)*(ft)** for two *x*-intercepts identified on the graph and curve reflecting asymptotic properties.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

---

a. A function, f , is given by [2]

$$f(x) = 4 \times 2^{-x} + 1.5x - 5.$$

Calculate $f(0)$

b. Use your graphic display calculator to solve $f(x) = 0.$ [2]

c. Sketch the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 6$ and $-4 \leqslant y \leqslant 10$, showing the x and y intercepts. Use a scale of $2\,\text{cm}$ to represent 2 units on both the horizontal axis, x , and the vertical axis, y . [4]

d. The function f is the derivative of a function g . It is known that $g(1) = 3$. [4]

i) Calculate $g'(1)$.

ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 1$. Give your answer in the form $y = mx + c$.

Markscheme

a. $4 \times 2^{-0} + 1.5 \times 0 - 5$ ***(M1)***

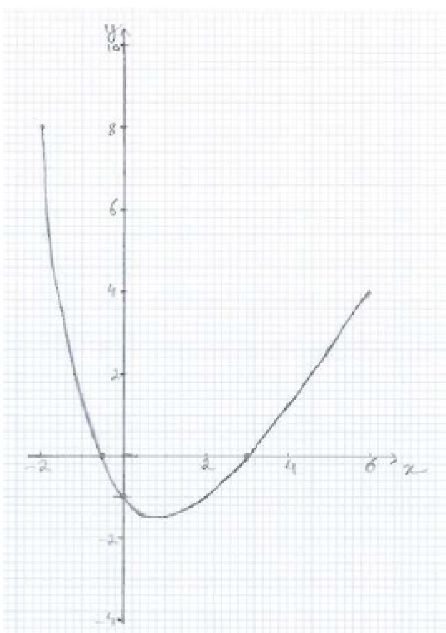
Note: Award ***(M1)*** for substitution of 0 into the expression for $f(x)$.

$= -1$ ***(A1)(G2)***

b. $-0.538\ \ (-0.537670\ldots)$ and 3 ***(A1)(A1)***

Note: Award at most ***(A0)(A1)*****(ft)** if answer is given as pairs of coordinates.

c.



(A1)(A1)(A1)**(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*** for labels and some indication of scale in the correct given window.

Award ***(A1)*** for smooth curve with correct general shape with $f(-2) > f(6)$ and minimum to the right of the y-axis.

Award ***(A1)*****(ft)** for correct y-intercept (consistent with their part (a)).

Award ***(A1)*****(ft)** for approximately correct x-intercepts (consistent with their part (b), one zero between -1 and 0, the other between 2.5 and 3.5).

d. i) $g'(1) = f(1) = 4 \times 2^{-1} + 1.5 - 5$ ***(M1)***

Note: Award ***(M1)*** for substitution of 1 into $f(x)$.

$= -1.5$ ***(A1)(G2)***

ii) $3 = -1.5 \times 1 + c$ **OR** $(y - 3) = -1.5\,(x - 1)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of gradient and the point $(1,\ 3)$ into the equation of a line. Follow through from (d)(i).

$y = -1.5x + 4.5$ ***(A1)*****(ft)*****(G2)***

Examiners report

a. Question 6: Functions and Calculus

Many candidates were able to calculate $f(0)$ although some had a problem with 2^0. Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

b. Question 6: Functions and Calculus

Many candidates were able to calculate $f(0)$ although some had a problem with 2^0. Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

c. Question 6: Functions and Calculus Many candidates were able to calculate $f(0)$ although some had a problem with 2^0. Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

d. Question 6: Functions and Calculus

Many candidates were able to calculate $f(0)$ although some had a problem with 2^0. Very few were able to find both roots in part (b), even when they represented correctly both zeros in their sketch. Many sketches were reasonably well drawn, presenting a smooth curve with correct points of intersection. Many lost a mark for not labelling their axes and the given scale was ignored by some, which was not penalized but often resulted in sketches which were difficult to read. The stronger candidates calculated $g'(1)$ correctly, but very few represented the gradient of the function at $x = 1$ correctly.

---

a. A distress flare is fired into the air from a ship at sea. The height, h , in metres, of the flare above sea level is modelled by the quadratic function [1]

$$h\left(t\right) = -0.2t^2 + 16t + 12\,,\; t \geqslant 0\,,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

Write down the height from which the flare was fired.

b. Find the height of the flare 15 seconds after it was fired. [2]

c. The flare fell into the sea k seconds after it was fired. [2]

Find the value of k .

d. Find $h'\left(t\right)$. [2]

e. i) Show that the flare reached its maximum height 40 seconds after being fired. [3]

ii) Calculate the maximum height reached by the flare.

f. The nearest coastguard can see the flare when its height is more than 40 metres above sea level. [3]

Determine the total length of time the flare can be seen by the coastguard.

Markscheme

a. $12\,(\mathrm{m})$ ***(A1)***

b. $(h\,(15) =) - 0.2 \times 15^2 + 16 \times 15 + 12$ ***(M1)***

Note: Award ***(M1)*** for substitution of 15 in expression for h.

$= 207\,(\mathrm{m})$ ***(A1)(G2)***

c. $h\,(k) = 0$ ***(M1)***

Note: Award ***(M1)*** for setting h to zero.

$(k =)\ 80.7\,(\mathrm{s})\ \ (80.7430)$ ***(A1)(G2)***

Note: Award at most ***(M1)(A0)*** for an answer including $K = -0.743$.
Award ***(A0)*** for an answer of 80 without working.

d. $h'\,(t) = -0.4t + 16$ ***(A1)(A1)***

Note: Award ***(A1)*** for $-0.4t$, (***A1)*** for 16. Award at most ***(A1)(A0)*** if extra terms seen. Do not accept x for t.

e. i) $-0.4t + 16 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative, from part (d), to zero, provided the correct conclusion is stated and consistent with their $h'\,(t)$.

OR

$t = \frac{-16}{2\times(-0.2)}$ ***(M1)***

Note: Award (***M1)*** for correct substitution into axis of symmetry formula, provided the correct conclusion is stated.

$t =\ 40\,(\mathrm{s})$ ***(AG)***

ii) $-0.2 \times 40^2 + 16 \times 40 + 12$ ***(M1)***

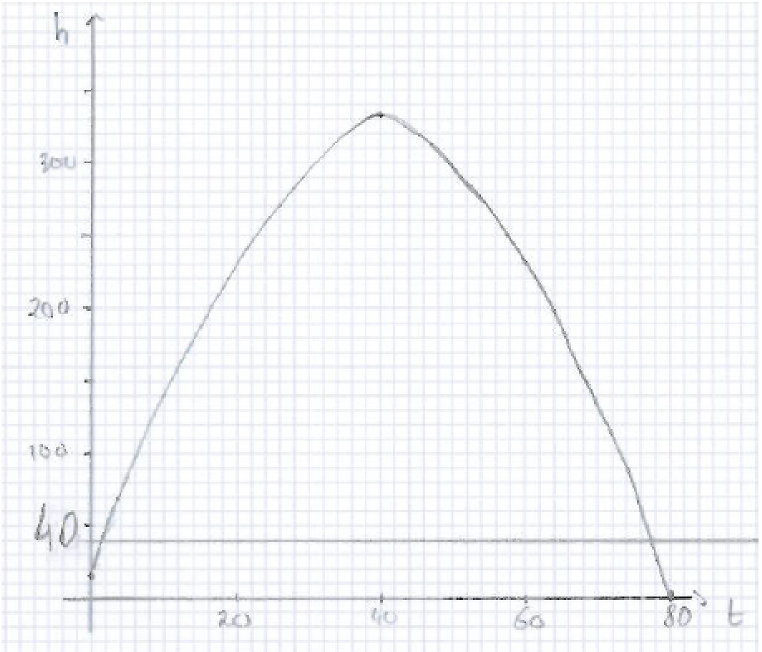
Note: Award ***(M1)*** for substitution of 40 in expression for h.

$= 332\,(\mathrm{m})$ ***(A1)(G2)***

f. $h\,(t) = 40$ ***(M1)***

Note: Award ***(M1)*** for setting h to 40. Accept inequality sign.

OR



M1

Note: Award ***(M1)*** for correct sketch. Indication of scale is not required.

$78.2 - 1.17\ (78.2099... \ \ - 1.79005...)$ ***(A1)***

Note: Award ***(A1)*** for 1.79 and 78.2 seen.

(total time $=$) $76.4\,(\mathrm{s})\ \ (76.4198...)$ ***(A1)(G2)***

Note: Award ***(G1)*** if the two endpoints are given as the final answer with no working.

Examiners report

a. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t. Some candidate seem to have a problem with the notation “$h\,(t) = \ldots$”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

b. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of “trial and error” of integer values for t. Some candidate seem to have a problem with the notation “$h\,(t) = \ldots$”, where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

c. Question 3: Quadratic function, problem solving.

Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of "trial and error" of integer values for t. Some candidate seem to have a problem with the notation "$h\left(t\right) = \ldots$", where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

d. Question 3: Quadratic function, problem solving.

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e. Question 3: Quadratic function, problem solving.

   Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of "trial and error" of integer values for t. Some candidate seem to have a problem with the notation "$h\left(t\right) = \ldots$", where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

f. Question 3: Quadratic function, problem solving.

   Parts (a) (finding the initial height) and (b) (finding the height after 15 seconds), were done very well by the majority of candidates. Many struggled to translate question (c) to find the (positive) zeros of the function, or did not write that down, losing a possible method mark. The derivative in part (d) was no problem for most; only very few used x instead of t. The maximum height reached was calculated correctly by the majority of candidates, but many lost the mark in part (e)(i) as they simply substituted 40 into their derivative or calculated the height at points close to 40. Only a few candidates showed correct method for part (f). Several were still able to obtain 2 marks as a result of "trial and error" of integer values for t. Some candidate seem to have a problem with the notation "$h\left(t\right) = \ldots$", where this is interpreted as $h \times t$, resulting in incorrect answers throughout.

Consider the function $f(x) = -\frac{1}{3}x^3 + \frac{5}{3}x^2 - x - 3$.

a. Sketch the graph of $y = f(x)$ for $-3 \le x \le 6$ and $-10 \le y \le 10$ showing clearly the axes intercepts and local maximum and minimum points. Use a scale of 2 cm to represent 1 unit on the x-axis, and a scale of 1 cm to represent 1 unit on the y-axis. [4]

b. Find the value of $f(-1)$. [2]

c. Write down the coordinates of the y-intercept of the graph of $f(x)$. [1]

d. Find $f'(x)$. [3]

e. Show that $f'(-1) = -\frac{16}{3}$. [1]

f. Explain what $f'(-1)$ represents. [2]

g. Find the equation of the tangent to the graph of $f(x)$ at the point where x is -1. [2]

h. Sketch the tangent to the graph of $f(x)$ at $x = -1$ on your diagram for (a). [2]

i. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [2]

Write down the value of

(i) a ;

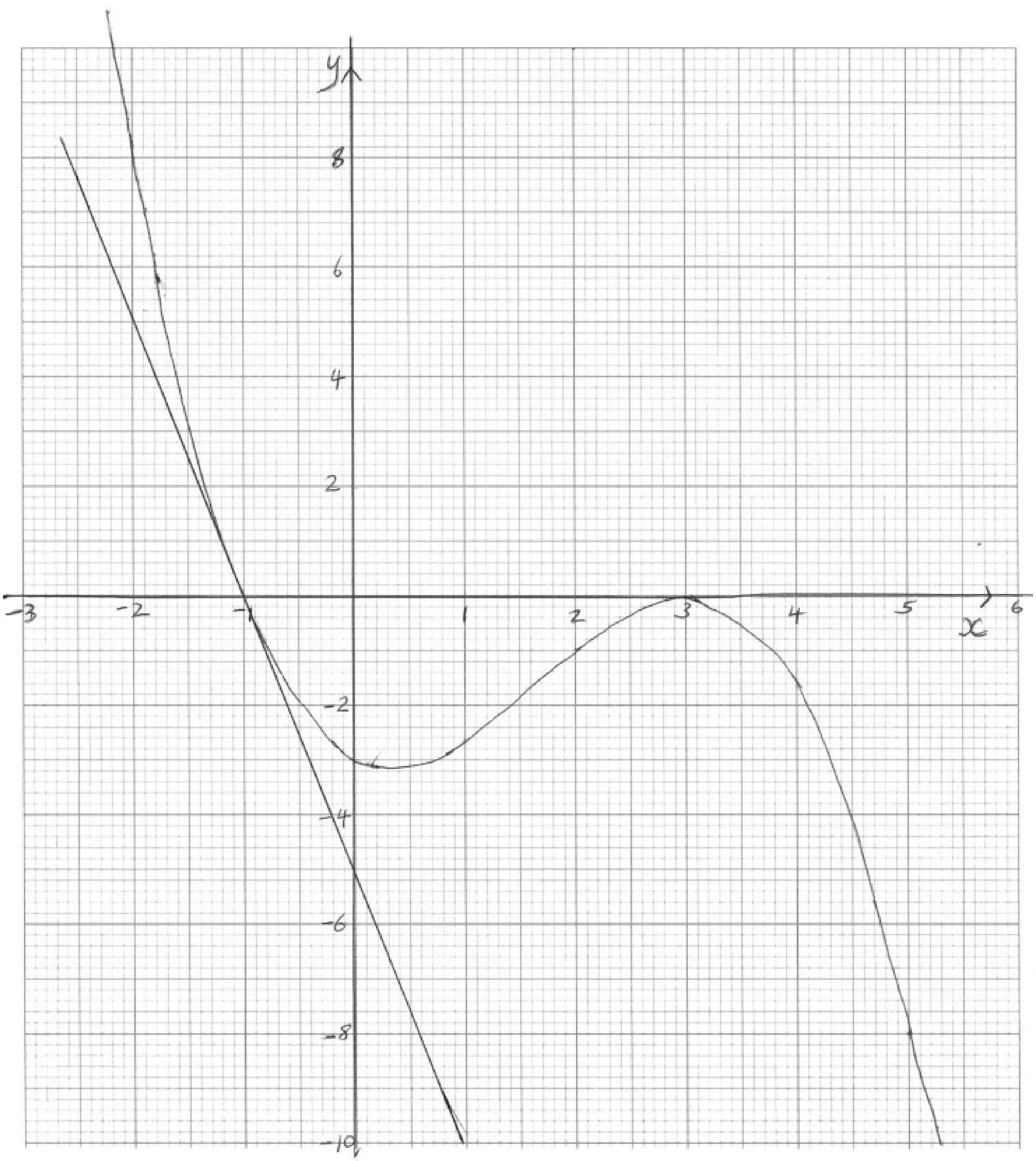
(ii) b .

j. P and Q are points on the curve such that the tangents to the curve at these points are horizontal. The x-coordinate of P is a, and the x-coordinate of Q is b, $b > a$. [1]

Describe the behaviour of $f(x)$ for $a < x < b$.

Markscheme

a.



(A1) for indication of window and labels. ***(A1)*** for smooth curve that does not enter the first quadrant, the curve must consist of one line only.

(A1) for x and y intercepts in approximately correct positions (allow ±0.5).

(A1) for local maximum and minimum in approximately correct position. (minimum should be $0 \le x \le 1$ and $-2 \le y \le -4$), the y-coordinate of the maximum should be 0 ± 0.5. ***(A4)***

[4 marks]

b. $-\frac{1}{3}(-1)^3 + \frac{5}{3}(-1)^2 - (-1) - 3$ ***(M1)***

Note: Award ***(M1)*** for substitution of –1 into $f(x)$

= 0 ***(A1)(G2)***

[2 marks]

c. (0, –3) ***(A1)***

OR

$x = 0, y = -3$ ***(A1)***

Note: Award ***(A0)*** if brackets are omitted.

[1 mark]

d. $f'(x) = -x^2 + \frac{10}{3}x - 1$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term. Award ***(A1)(A1)(A0)*** at most if there are extra terms.

[3 marks]

e. $f'(-1) = -(-1)^2 + \frac{10}{3}(-1) - 1$ ***(M1)***

$= -\frac{16}{3}$ ***(AG)***

Note: Award ***(M1)*** for substitution of $x = -1$ into correct derivative only. The final answer must be seen.

[1 mark]

f. $f'(-1)$ gives the gradient of the tangent to the curve at the point with $x = -1$. ***(A1)(A1)***

Note: Award ***(A1)*** for "gradient (of curve)", ***(A1)*** for "at the point with $x = -1$". Accept "the instantaneous rate of change of y" or "the (first) derivative".

[2 marks]

g. $y = -\frac{16}{3}x + c$ ***(M1)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation.

$0 = -\frac{16}{3} \times (-1) + c$

$c = -\frac{16}{3}$

$y = -\frac{16}{3}x - \frac{16}{3}$ ***(A1)(G2)***

Note: Accept $y = -5.33x - 5.33$.

OR

$(y - 0) = \frac{-16}{3}(x + 1)$ ***(M1)(A1)(G2)***

Note: Award ***(M1)*** for $-\frac{16}{3}$ substituted in equation, ***(A1)*** for correct equation. Follow through from their answer to part (b). Accept $y = -5.33(x + 1)$. Accept equivalent equations.

[2 marks]

h. ***(A1)*(ft)** for a tangent to their curve drawn.

***(A1)*(ft)** for their tangent drawn at the point $x = -1$.   ***(A1)*(ft)*(A1)*(ft)**

Note: Follow through from their graph. The tangent must be a straight line otherwise award at most ***(A0)(A1)***.

[2 marks]

i. (i) $a = \frac{1}{3}$ ***(G1)***

(ii) $b = 3$ ***(G1)***

Note: If *a* and *b* are reversed award ***(A0)(A1)***.

[2 marks]

j. *f* (*x*) is increasing   ***(A1)***

[1 mark]

Examiners report

a. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

Drawing/sketching graphs is perhaps the area of the course that results in the poorest responses. It is also the area of the course that results in the best. It is therefore the area of the course that good teaching can influence the most.

Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

b. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

In part (b) the answer could have been checked using the table on the GDC.

c. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

In part (c) **coordinates** were required.

d. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The responses to part (d) were generally correct.

e. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The "show that" nature of part (e) meant that the final answer had to be stated.

f. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

The interpretive nature of part (f) was not understood by the majority.

g. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

h. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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Candidates should:

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- Enter the formula into the GDC and use the table function to determine the points to be plotted.
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- Draw a curve rather than line segments; ensure that the curve is smooth.
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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

j. This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

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- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, t (minutes)	0	1	2	3	4	5	6
Temperature, y (°C)	94	54	34	24	k	16.5	15.25

a. Write down the decrease in the temperature of the coffee [3]

(i) during the first minute (between t = 0 and t =1) ;

(ii) during the second minute;

(iii) during the third minute.

b. Assuming the pattern in the answers to part (a) continues, show that $k = 19$. [2]

c. Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes. [4]

Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent 10 °C on the vertical axis.

d. The function that models the change in temperature of the coffee is $y = p\ (2^{-t}) + q$. [2]

(i) Use the values t = 0 and y = 94 to form an equation in p and q.

(ii) Use the values t =1 and y = 54 to form a second equation in p and q.

e. Solve the equations found in part (d) to find the value of p and the value of q. [2]

f. The graph of this function has a horizontal asymptote. [2]

Write down the equation of this asymptote.

g. George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression. [4]

Use the **seven** results in the table to write down

(i) the correlation coefficient;

(ii) the equation of the regression line y on t.

h. Use the equation of the regression line to estimate the temperature of the coffee at t = 3. [2]

i. Find the percentage error in this estimate of the temperature of the coffee at t = 3. [2]

Markscheme

a. (i) 40

(ii) 20

(iii) 10 ***(A3)***

Notes: Award ***(A0)(A1)*****(ft)*****(A1)*****(ft)** for −40, −20, −10.

Award ***(A1)(A0)(A1)*****(ft)** for 40, 60, 70 seen.

Award ***(A0)(A0)(A1)*****(ft)** for −40, −60, −70 seen.

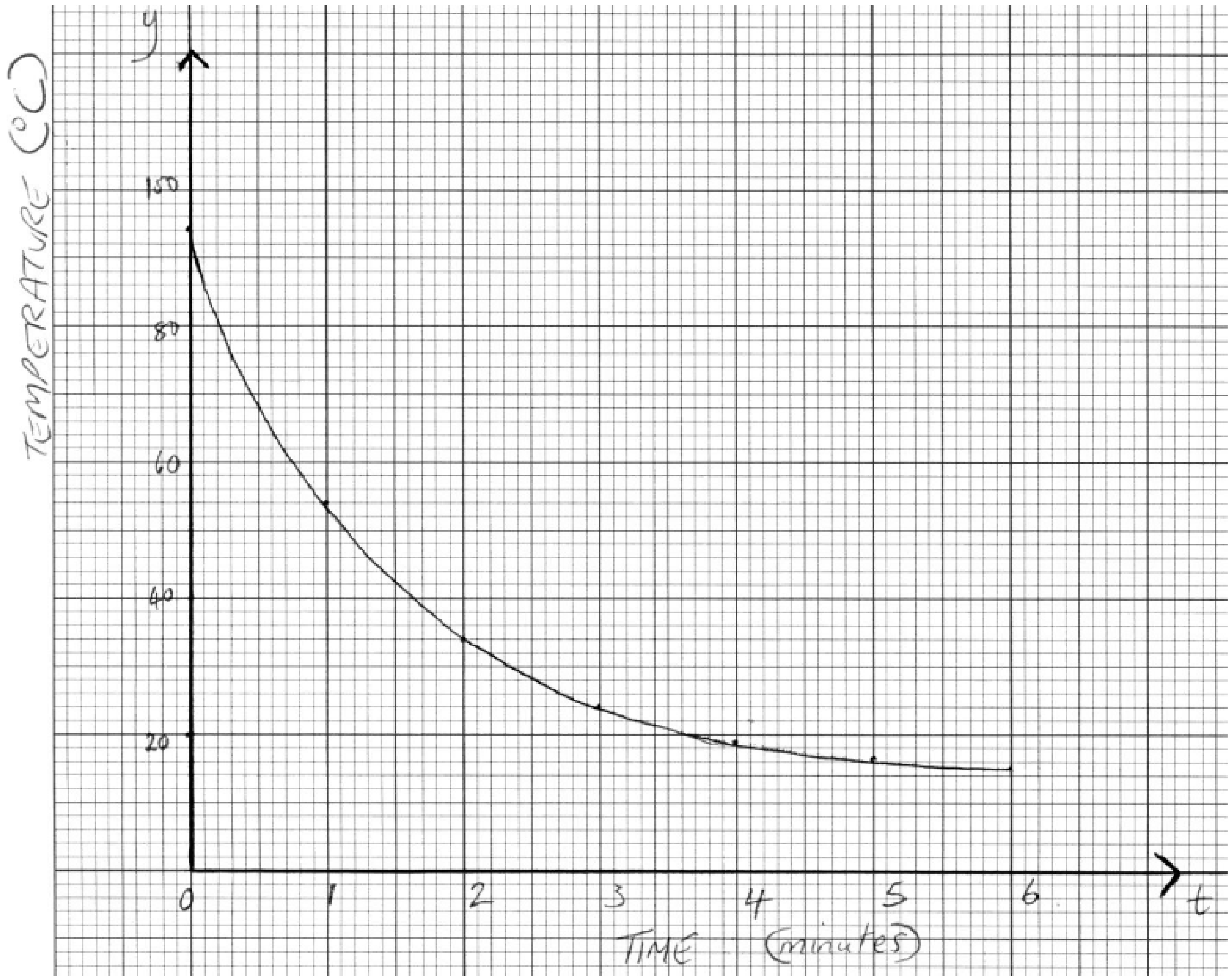
b. $24 - k = 5$ or equivalent ***(A1)(M1)***

Note: Award ***(A1)*** for 5 seen, ***(M1)*** for difference from 24 indicated.

$k = 19$ ***(AG)***

Note: If 19 is not seen award at most ***(A1)(M0)***.

c.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for scales and labelled axes (*t* or "time" and *y* or "temperature").

Accept the use of *x* on the horizontal axis only if "time" is also seen as the label.

Award ***(A2)*** for all seven points accurately plotted, award ***(A1)*** for **5 or 6** points accurately plotted, award ***(A0)*** for 4 points or fewer accurately plotted.

Award ***(A1)*** for smooth curve that passes through all points on domain [0, 6].

If graph paper is not used or one or more scales is missing, award a maximum of ***(A0)(A0)(A0)(A1)***.

d. (i) $94 = p + q$ ***(A1)***

(ii) $54 = 0.5p + q$ ***(A1)***

Note: The equations need not be simplified; accept, for example $94 = p(2^{-0}) + q$.

e. $p = 80$, $q = 14$ ***(G1)(G1)*****(ft)**

Note: If the equations have been incorrectly simplified, follow through even if no working is shown.

f. $y = 14$ ***(A1)(A1)*****(ft)**

Note: Award ***(A1)*** for y = a constant, ***(A1)*** for their 14. Follow through from part (e) only if their q lies between 0 and 15.25 inclusive.

g. (i) –0.878 (–0.87787...) ***(G2)***

Note: Award ***(G1)*** if −0.877 seen only. If negative sign omitted award a maximum of ***(A1)(A0)***.

(ii) y = −11.7*t* + 71.6 (y = −11.6517...*t* + 71.6336...) ***(G1)(G1)***

Note: Award ***(G1)*** for −11.7*t*, ***(G1)*** for 71.6.

If *y* = is omitted award at most ***(G0)(G1)***.

If the use of *x* in part (c) has **not** been penalized (the axis has been labelled "time") then award at most ***(G0)(G1)***.

h. −11.6517...(3) + 71.6339... ***(M1)***

Note: Award ***(M1)*** for correct substitution in their part (g)(ii).

= 36.7 (36.6785...) ***(A1)*(ft)*(G2)***

Note: Follow through from part (g). Accept 36.5 for use of the 3sf answers from part (g).

i. $\frac{36.6785...-24}{24} \times 100$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in percentage error formula.

= 52.8% (52.82738...) ***(A1)*(ft)*(G2)***

Note: Follow through from part (h). Accept 52.1% for use of 36.5.

Accept 52.9 % for use of 36.7. If partial working ($\times 100$ omitted) is followed by their correct answer award ***(M1)(A1)***. If partial working is followed by an incorrect answer award ***(M0)(A0)***. The percentage sign is not required.

Examiners report

a. Almost all candidates were able to score on the first parts of this question; errors occurring only when insufficient care was taken in reading what the question was asking for. The graph was usually well drawn, other than for those who have no idea what centimetres are.

The majority were able to determine the simultaneous equations, if only in unsimplified form; there was less success in solving these – though this is easily done via the GDC (the preferred approach) and the equation of the asymptote proved a discriminating task. The final parts, involving correlation and regression were largely independent of the previous parts and were accessible to most. Hopefully, contrasting the large percentage error with the value of the correlation coefficient will be valuable in class discussions. Given the many scripts that gave the value of the coefficient of determination as that of r , it seems better that the former is simply not taught.

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A biologist is studying the relationship between the number of chirps of the Snowy Tree cricket and the air temperature. He records the chirp rate, x, of a cricket, and the corresponding air temperature, T, in degrees Celsius.

The following table gives the recorded values.

Cricket's chirp rate, x, (chirps per minute)	20	40	60	80	100	120
Temperature, T (°C)	8.0	12.8	15.0	18.2	20.0	21.1

a. Draw the scatter diagram for the above data. Use a scale of 2 cm for 20 chirps on the horizontal axis and 2 cm for 4°C on the vertical axis. [4]

b. Use your graphic display calculator to write down the Pearson's product–moment correlation coefficient, r, between x and T. [2]

c. Interpret the relationship between x and T using your value of r. [2]

d. Use your graphic display calculator to write down the equation of the regression line T on x. Give the equation in the form $T = ax + b$. [2]

e. Calculate the air temperature when the cricket's chirp rate is 70. [2]

f. Given that $\bar{x} = 70$, draw the regression line T on x on your scatter diagram. [2]

g. A forest ranger uses her own formula for estimating the air temperature. She counts the number of chirps in 15 seconds, z, multiplies this number by 0.45 and then she adds 10. [1]

Write down the formula that the forest ranger uses for estimating the temperature, T.

Give the equation in the form $T = mz + n$.

h. A cricket makes 20 chirps in **15** seconds. [6]

For this chirp rate

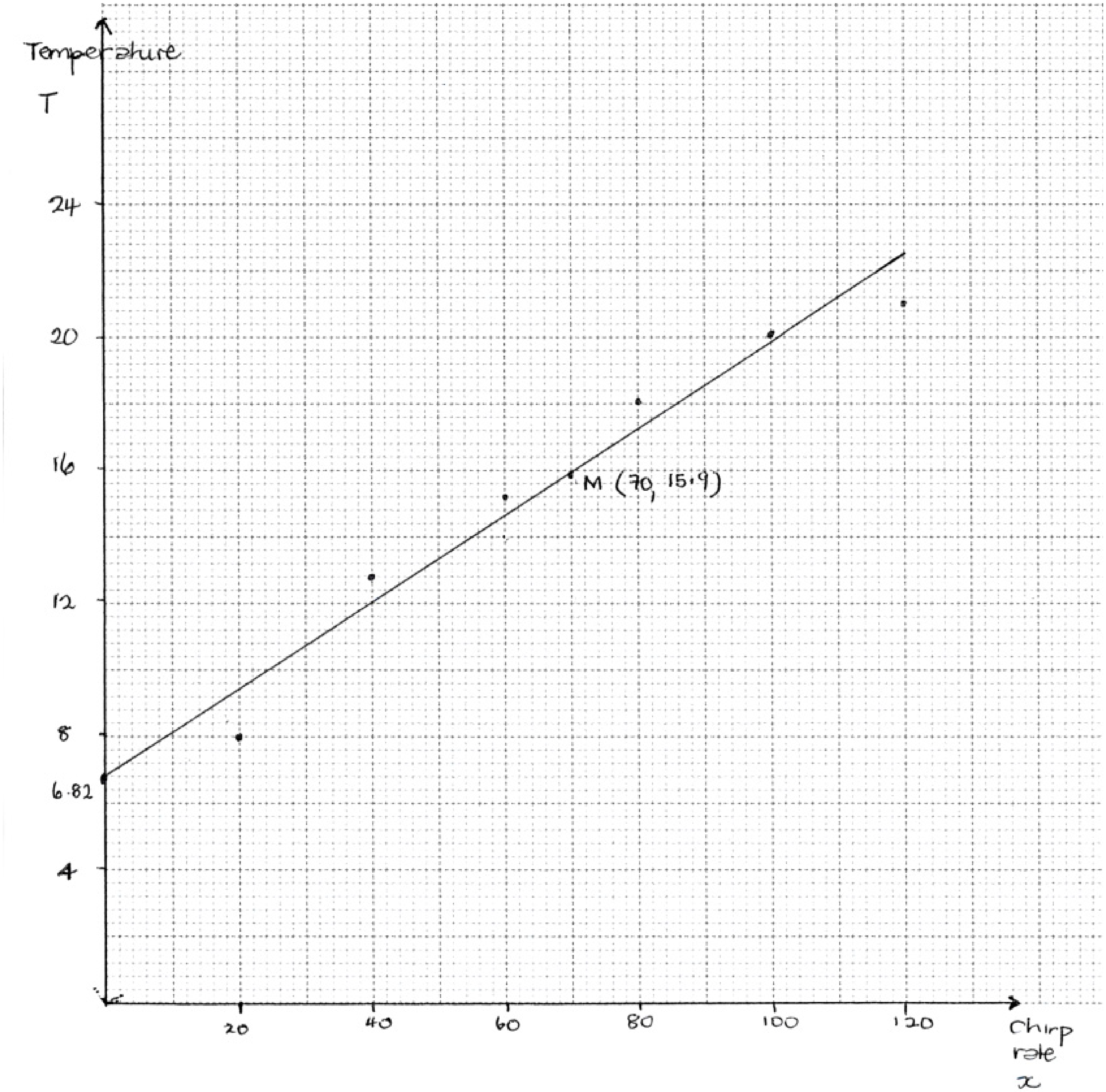
(i) calculate an estimate for the temperature, T, **using the forest ranger's formula**;

(ii) determine the actual temperature recorded by the biologist, **using the table above**;

(iii) calculate the percentage error in the forest ranger's estimate for the temperature, compared to the actual temperature recorded by the biologist.

Markscheme

a.



(A4)

Notes: Award ***(A1)*** for correct scales and labels.

Award ***(A3)*** for all six points correctly plotted,

(A2) for four or five points correctly plotted,

(A1) for two or three points correctly plotted.

Award at most ***(A0)(A3)*** if axes reversed.

Accept tolerance for T-axis.

b. $0.977 \quad (0.977324\ldots)$ ***(G2)***

Notes: Award ***(G1)*** for 0.97.

c. (Very) strong positive correlation ***(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*** for (very) strong, ***(A1)*** for positive.

Follow through from part (b).

d. $T = 0.129x + 6.82$ ***(G2)***

Notes: Award ***(G1)*** for $0.129x$, ***(G1)*** for $+6.82$.

Award a maximum of ***(G0)(G1)*** if the answer is not an equation.

e. $0.129 \times 70 + 6.82$ ***(M1)***

Note: Award ***(M1)*** for substitution of 70 into their equation of regression line.

OR

$\frac{8+12.8+\ldots+21.1}{6}$ ***(M1)***

$= 15.9\ (15.85)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (d) without working.

f. regression line through $(70,\ 15.9)$ ***(A1)*****(ft)**

Note: Accept 15.9 ± 0.2.

Follow through from part (e).

with T-intercept, 6.82 ***(A1)*****(ft)**

Note: Follow through from part (d). Accept 6.82 ± 0.2.

In case the regression line is not straight (ruler not used), award ***(A0)(A1)*****(ft)** if line passes through both their $(70,\ 15.9)$ and $(0,\ 6.82)$, otherwise award ***(A0)(A0)***.

Do not penalize if line does not intersect the T-axis.

g. $T = 0.45z + 10$ ***(A1)***

h. (i) $0.45(20) + 10$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of 20 into their formula from part (g).

$= 19 \quad (^\circ\mathrm{C})$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (g).

(ii) $= 18.2 \quad (^\circ\mathrm{C})$ ***(A1)***

(iii) $\left|\frac{19-18.2}{18.2}\right| \times 100\%$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution in the percentage error formula, ***(A1)*** for correct substitution.

$4.40\% \quad (4.39560\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Follow through from parts (h)(i) and (h)(ii).

Examiners report

a. [N/A]
[N/A]

b. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

Consider the function $f(x) = 3x + \frac{12}{x^2},\ x \neq 0.$

a. Differentiate $f(x)$ with respect to x. [3]

b. Calculate $f'(x)$ when $x = 1$. [2]

c. Use your answer to part (b) to decide whether the function, f , is increasing or decreasing at $x = 1$. Justify your answer. [2]

d. Solve the equation $f'(x) = 0$. [3]

e, i. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [2]

Write down the coordinates of P.

e, ii. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [1]

Write down the gradient of *T*.

e, iii. The graph of *f* has a local minimum at point P. Let *T* be the tangent to the graph of *f* at P. [2]

Write down the equation of *T*.

f. Sketch the graph of the function *f*, for $-3 \le x \le 6$ and $-7 \le y \le 15$. Indicate clearly the point P and any intercepts of the curve with the axes. [4]

g, i. On your graph draw and label the tangent *T*. [2]

g, ii. *T* intersects the graph of *f* at a second point. Write down the *x*-coordinate of this point of intersection. [1]

Markscheme

a. $f'(x) = 3 - \frac{24}{x^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for 3, ***(A1)*** for –24, ***(A1)*** for x^3 (or x^{-3}). If extra terms present award at most ***(A1)(A1)(A0)***.

[3 marks]

b. $f'(1) = -21$ ***(M1)(A1)*****(ft)*****(G2)***

Note: (ft) from their derivative only if working seen.

[2 marks]

c. Derivative (gradient, slope) is negative. Decreasing. ***(R1)(A1)*****(ft)**

Note: Do not award ***(R0)(A1)***.

[2 marks]

d. $3-\frac{24}{x^3}=0$ ***(M1)***

$x^3=8$ ***(A1)***

$x=2$ ***(A1)*****(ft)*****(G2)***

[3 marks]

e, i. (2, 9) (Accept x = 2, y = 9) ***(A1)(A1)(G2)***

Notes: (ft) from their answer in (d).

Award ***(A1)(A0)*** if brackets not included and not previously penalized.

[2 marks]

e, ii. 0 ***(A1)***

[1 mark]

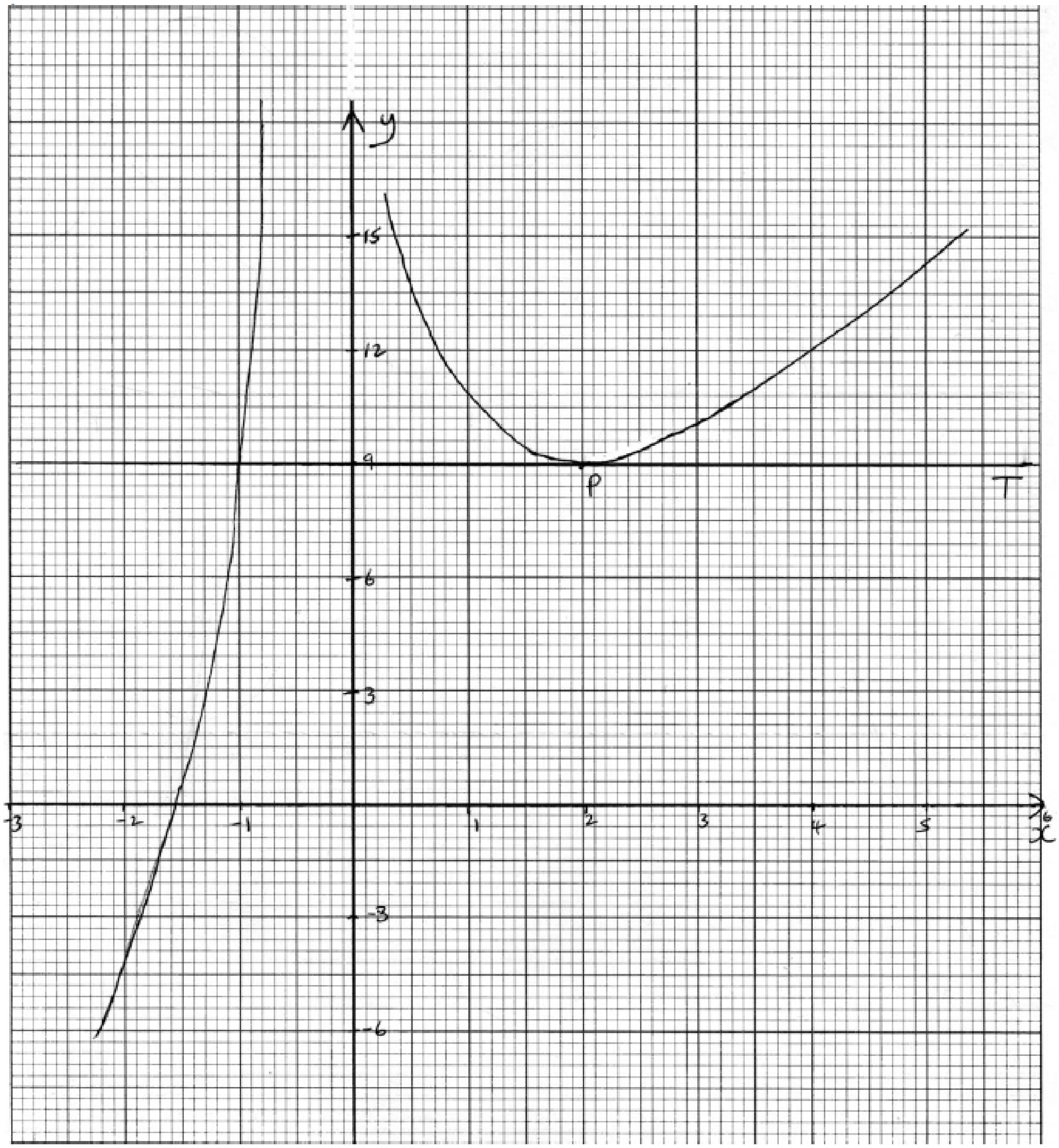
e, iii. y = 9 ***(A1)(A1)*****(ft)*****(G2)***

Notes: Award ***(A1)*** for y = constant, ***(A1)*** for 9.

Award ***(A1)*****(ft)** for their value of y in (e)(i).

[2 marks]

f.



(A4)

Notes: Award ***(A1)*** for labels and some indication of scale in the stated window.

Award ***(A1)*** for correct general shape (curve must be smooth and must not cross the *y*-axis).

Award ***(A1)*** for *x*-intercept seen in roughly the correct position.

Award ***(A1)*** for minimum (P).

[4 marks]

g, i. Tangent drawn at P (line must be a tangent and horizontal). ***(A1)***

Tangent labeled *T*. ***(A1)***

Note: (ft) from their tangent equation only if tangent is drawn and answer is consistent with graph.

[2 marks]

g, ii. $x = -1$ ***(G1)*****(ft)**

[1 mark]

Examiners report

a. Many students did not know the term "differentiate" and did not answer part (a).

b. However, the derivative was seen in (b) when finding the gradient at $x = 1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students.

c. [N/A]

d. Part (d) was handled well by some but many substituted $x = 0$ into $f'(x)$.

e, i. It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

e, ii. It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

e, iii. It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero.

f. There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.

g, i. Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

g, ii. Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.