HL Paper 1

A and B are independent events such that $\mathrm{P}(A) = \mathrm{P}(B) = p,\ p \neq 0$.

a. Show that $\mathrm{P}(A \cup B) = 2p - p^2$. [2]

b. Find $\mathrm{P}(A|A \cup B)$ in simplest form. [4]

Markscheme

a. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ***(M1)***

$= p + p - p^2$ ***A1***

$= 2p - p^2$ ***AG***

[2 marks]

b. $\mathrm{P}(A|A \cup B) = \frac{\mathrm{P}(A \cap (A \cup B))}{\mathrm{P}(A \cup B)}$ ***(M1)***

Note: Allow $\mathrm{P}(A \cap A \cup B)$ if seen on the numerator.

$= \frac{\mathrm{P}(A)}{\mathrm{P}(A \cup B)}$ ***(A1)***

$= \frac{p}{2p - p^2}$ ***A1***

$= \frac{1}{2 - p}$ ***A1***

[4 marks]

Examiners report

a. Part (a) posed few problems. Part (b) was possibly a good discriminator for the 4/5 candidates. Some were aware of an alternative (useful) form for the conditional probability, but were unable to interpret $P\left(A \cap (A \cup B)\right)$. Large numbers of fully correct answers were seen.

b. Part (a) posed few problems. Part (b) was possibly a good discriminator for the 4/5 candidates. Some were aware of an alternative (useful) form for the conditional probability, but were unable to interpret $P\left(A \cap (A \cup B)\right)$. Large numbers of fully correct answers were seen.

The ten numbers $x_1,\ x_2,\ \ldots,\ x_{10}$ have a mean of 10 and a standard deviation of 3.

Find the value of $\sum_{i=1}^{10}(x_i-12)^2$.

Markscheme

EITHER

let $y_i = x_i - 12$

$\bar{x} = 10 \Rightarrow \bar{y} = -2$ ***M1A1***

$\sigma_x = \sigma_y = 3$ ***A1***

$\frac{\sum_{i=1}^{10} y_i^2}{10} - \bar{y}^2 = 9$ ***M1A1***

$\sum_{i=1}^{10} y_i^2 = 10(9+4) = 130$ ***A1***

OR

$\sum_{i=1}^{10}(x_i-12)^2 = \sum_{i=1}^{10} x_i^2 - 24\sum_{i=1}^{10} x_i + 144\sum_{i=1}^{10} 1$ ***M1A1***

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{10} x_i = 100$ ***A1***

$\sigma_x = 3,\ \frac{\sum_{i=1}^{10} x_i^2}{10} - \bar{x}^2 = 9$ ***(M1)***

$\Rightarrow \sum_{i=1}^{10} x_i^2 = 10(9+100)$ ***A1***

$\sum_{i=1}^{10}(x_i-12)^2 = 1090 - 2400 + 1440 = 130$ ***A1***

[6 marks]

Examiners report

Very few candidates answered this question well, but among those a variety of nice approaches were seen. Most candidates though revealed an inability to deal with sigma expressions, especially $\sum_{i=1}^{i=10} 144$. Some tried to use expectation algebra but could not then relate those results to sigma expressions (often the factor 10 was forgotten). In a few cases candidates attempted to show the result using particular examples.

The continuous random variable *X* has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \leqslant x \leqslant 6 \\ 0, & \text{otherwise} \end{cases}.$$

a. Find the value of k. [4]

b.i. By considering the graph of *f* write down the mean of X; [1]

b.ii. By considering the graph of *f* write down the median of X; [1]

b.iii. By considering the graph of *f* write down the mode of X. [1]

c.i. Show that $P(0 \leqslant X \leqslant 2) = \frac{1}{4}$. [4]

c.ii. Hence state the interquartile range of X. [2]

d. Calculate $P(X \leqslant 4 | X \geqslant 3)$. [2]

Markscheme

a. attempt to equate integral to 1 (may appear later) ***M1***

$$k\int_0^6 \sin\left(\frac{\pi x}{6}\right)\mathrm{d}x = 1$$

correct integral ***A1***

$$k\left[-\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^6 = 1$$

substituting limits ***M1***

$$-\frac{6}{\pi}(-1-1) = \frac{1}{k}$$

$k = \frac{\pi}{12}$ ***A1***

[4 marks]

b.i. mean $= 3$ ***A1***

Note: Award ***A1A0A0*** for three equal answers in $(0,\ 6)$.

[1 mark]

b.ii. median $= 3$ ***A1***

Note: Award ***A1A0A0*** for three equal answers in $(0,\ 6)$.

[1 mark]

b.iii. mode $= 3$ ***A1***

Note: Award ***A1A0A0*** for three equal answers in $(0,\ 6)$.

[1 mark]

c.i. $\frac{\pi}{12}\int_0^2 \sin\left(\frac{\pi x}{6}\right)\mathrm{d}x$ ***M1***

$= \frac{\pi}{12}\left[-\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^2$ ***A1***

Note: Accept without the $\frac{\pi}{12}$ at this stage if it is added later.

$\frac{\pi}{12}\left[-\frac{6}{\pi}\left(\cos\frac{\pi}{3}-1\right)\right]$ ***M1***

$=\frac{1}{4}$ ***AG***

[4 marks]

c.ii. from (c)(i) $Q_1 = 2$ ***(A1)***

as the graph is symmetrical about the middle value $x = 3 \Rightarrow Q_3 = 4$ ***(A1)***

so interquartile range is

$4-2$

$=2$ ***A1***

[3 marks]

d. $P(X \leqslant 4|X \geqslant 3) = \frac{P(3\leqslant X\leqslant 4)}{P(X\geqslant 3)}$

$=\frac{\frac{1}{4}}{\frac{1}{2}}$ ***(M1)***

$=\frac{1}{2}$ ***A1***

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
b.iii. [N/A]
c.i. [N/A]
c.ii. [N/A]
d. [N/A]

The discrete random variable X has probability distribution:

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	a

(a) Find the value of a.
(b) Find $\mathrm{E}(X)$.
(c) Find $\mathrm{Var}(X)$.

Markscheme

(a) $\frac{1}{6}+\frac{1}{2}+\frac{3}{10}+a=1 \Rightarrow a=\frac{1}{30}$ ***A1***

(b) $\mathrm{E}(X)=\frac{1}{2}+2\times\frac{3}{10}+3\times\frac{1}{30}$ ***M1***

$= \frac{6}{5}$ ***A1***

Note: Do not award ***FT*** marks if a is outside [0, 1].

[2 marks]

(c) $\mathrm{E}(X^2) = \frac{1}{2} + 2^2 \times \frac{3}{10} + 3^2 \times \frac{1}{30} = 2$ ***(A1)***

attempt to apply $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$ ***M1***

$\left(= 2 - \frac{36}{25}\right) = \frac{14}{25}$ ***A1***

[3 marks]

Total [6 marks]

Examiners report

This was very well answered and many fully correct solutions were seen. A small number of candidates made arithmetic mistakes in part a) and thus lost one or two accuracy marks. A few also seemed unaware of the formula $\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$ and resorted to seeking an alternative, sometimes even attempting to apply a clearly incorrect $\mathrm{Var}(X) = \sum (x_i - \mu)^2$.

The discrete random variable *X* has the following probability distribution, where *p* is a constant.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	p	$0.5 - p$	0.25	0.125	p^3

a. Find the value of *p*. [2]

b.i. Find *μ*, the expected value of *X*. [2]

b.ii. Find P(*X* > *μ*). [2]

Markscheme

a. equating sum of probabilities to 1 ($p + 0.5 - p + 0.25 + 0.125 + p^3 = 1$) ***M1***

$p^3 = 0.125 = \frac{1}{8}$

$p = 0.5$ ***A1***

[2 marks]

b.i. $\mu = 0 \times 0.5 + 1 \times 0 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.125$ ***M1***

$= 1.375 \left(= \frac{11}{8}\right)$ ***A1***

[2 marks]

b.ii. $P(X > \mu) = P(X = 2) + P(X = 3) + P(X = 4)$ ***(M1)***

$= 0.5$ ***A1***

Note: Do not award follow through ***A*** marks in (b)(i) from an incorrect value of p.

Note: Award ***M*** marks in both (b)(i) and (b)(ii) provided no negative probabilities, and provided a numerical value for μ has been found.

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]

On Saturday, Alfred and Beatrice play 6 different games against each other. In each game, one of the two wins. The probability that Alfred wins any one of these games is $\frac{2}{3}$.

a. Show that the probability that Alfred wins exactly 4 of the games is $\frac{80}{243}$. [3]

b. (i) Explain why the total number of possible outcomes for the results of the 6 games is 64. [4]

(ii) By expanding $(1 + x)^6$ and choosing a suitable value for x, prove

$$64 = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$$

(iii) State the meaning of this equality in the context of the 6 games played.

c. The following day Alfred and Beatrice play the 6 games again. Assume that the probability that Alfred wins any one of these games is still $\frac{2}{3}$. [9]

(i) Find an expression for the probability Alfred wins 4 games on the first day and 2 on the second day. Give your answer in the form $\binom{6}{r}^2 \left(\frac{2}{3}\right)^s \left(\frac{1}{3}\right)^t$ where the values of r, s and t are to be found.

(ii) Using your answer to (c) (i) and 6 similar expressions write down the probability that Alfred wins a total of 6 games over the two days as the sum of 7 probabilities.

(iii) Hence prove that $\binom{12}{6} = \binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2$.

d. Alfred and Beatrice play n games. Let A denote the number of games Alfred wins. The expected value of A can be written as $\mathrm{E}(A) = \sum_{r=0}^{n} r \binom{n}{r} \frac{a^r}{b^n}$. [6]

(i) Find the values of a and b.

(ii) By differentiating the expansion of $(1 + x)^n$, prove that the expected number of games Alfred wins is $\frac{2n}{3}$.

Markscheme

a. $B\left(6, \frac{2}{3}\right)$ ***(M1)***

$p(4) = \binom{6}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2$ ***A1***

$\binom{6}{4} = 15$ ***A1***

$= 15 \times \frac{2^4}{3^6} = \frac{80}{243}$ ***AG***

[3 marks]

b. (i) 2 outcomes for each of the 6 games or $2^6 = 64$ ***R1***

(ii) $(1+x)^6 = \binom{6}{0} + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6$ ***A1***

Note: Accept ${}^nC_r$ notation or $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$

setting $x = 1$ in both sides of the expression ***R1***

Note: Do not award ***R1*** if the right hand side is not in the correct form.

$64 = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}$ ***AG***

(iii) the total number of outcomes = number of ways Alfred can win no games, plus the number of ways he can win one game *etc.* ***R1***

[4 marks]

c. (i) Let $P(x,\ y)$ be the probability that Alfred wins x games on the first day and y on the second.

$P(4, 2) = \binom{6}{4} \times \left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 \times \binom{6}{2} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^4$ ***M1A1***

$\binom{6}{2}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6$ or $\binom{6}{4}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6$ ***A1***

$r = 2$ or 4, $s = t = 6$

(ii) P(Total = 6) =

P(0, 6) + P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) + P(6, 0) ***(M1)***

$= \binom{6}{0}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 + \binom{6}{1}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 + \ldots + \binom{6}{6}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6$ ***A2***

$= \frac{2^6}{3^{12}} \left(\binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 \right)$

Note: Accept any valid sum of 7 probabilities.

(iii) use of $\binom{6}{i} = \binom{6}{6-i}$ ***(M1)***

(can be used either here or in (c)(ii))

P(wins 6 out of 12) $= \binom{12}{6} \times \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)^6 = \frac{2^6}{3^{12}} \binom{12}{6}$ ***A1***

$= \frac{2^6}{3^{12}} \left(\binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 \right) = \frac{2^6}{3^{12}} \binom{12}{6}$ ***A1***

therefore $\binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 = \binom{12}{6}$ ***AG***

[9 marks]

d. (i) $\mathrm{E}(A)=\sum_{r=0}^{n} r\binom{n}{r}\left(\frac{2}{3}\right)^{r}\left(\frac{1}{3}\right)^{n-r}=\sum_{r=0}^{n} r\binom{n}{r}\frac{2^{r}}{3^{n}}$

($a = 2$, $b = 3$) ***M1A1***

Note: ***M0A0*** for $a = 2$, $b = 3$ without any method.

(ii) $n(1+x)^{n-1}=\sum_{r=1}^{n}\binom{n}{r}rx^{r-1}$ ***A1A1***

(sigma notation not necessary)

(if sigma notation used also allow lower limit to be $r = 0$)

let $x = 2$ ***M1***

$n3^{n-1}=\sum_{r=1}^{n}\binom{n}{r}r2^{r-1}$

multiply by 2 and divide by 3^n ***(M1)***

$\frac{2n}{3}=\sum_{r=1}^{n}\binom{n}{r}r\frac{2^{r}}{3^{n}}\left(=\sum_{r=0}^{n}\binom{n}{r}\frac{2^{r}}{3^{n}}\right)$ ***AG***

[6 marks]

Examiners report

a. This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(a) Candidates need to be aware how to work out binomial coefficients without a calculator

b. This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(b) (ii) A surprising number of candidates chose to work out the values of all the binomial coefficients (or use Pascal's triangle) to make a total of 64 rather than simply putting 1 into the left hand side of the expression.

c. This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

d. This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(d) This was poorly done. Candidates were not able to manipulate expressions given using sigma notation.

Consider the following functions:

$$f(x)=\frac{2x^{2}+3}{75},\ x\geqslant 0$$

$$g(x)=\frac{|3x-4|}{10},\ x\in\mathbb{R}\,.$$

a. State the range of f and of g . [2]

b. Find an expression for the composite function $f\circ g(x)$ in the form $\frac{ax^{2}+bx+c}{3750}$, where $a,\ b$ and $c\in\mathbb{Z}$. [4]

c. (i) Find an expression for the inverse function $f^{-1}(x)$. [4]

(ii) State the domain and range of f^{-1}.

d. The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$. [6]

By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X, stating your reasons clearly.

e. Using this probability distribution, calculate the mean of X . [2]

Markscheme

a. $f(x) \geqslant \frac{1}{25}$ ***A1***

$g(x) \in \mathbb{R},\ g(x) \geqslant 0$ ***A1***

[2 marks]

b. $f \circ g(x) = \frac{2\left(\frac{3x-4}{10}\right)^2+3}{75}$ ***M1A1***

$= \frac{\frac{2(9x^2-24x+16)}{100}+3}{75}$ ***(A1)***

$= \frac{9x^2-24x+166}{3750}$ ***A1***

[4 marks]

c. (i) **METHOD 1**

$y = \frac{2x^2+3}{75}$

$x^2 = \frac{75y-3}{2}$ ***M1***

$x = \sqrt{\frac{75y-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form $f^{-1}(x) =$.

METHOD 2

$y = \frac{2x^2+3}{75}$

$x = \frac{2y^2+3}{75}$ ***M1***

$y = \sqrt{\frac{75x-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form $f^{-1}(x) =$.

(ii) domain: $x \geqslant \frac{1}{25}$; range: $f^{-1}(x) \geqslant 0$ ***A1***

[4 marks]

d. probabilities from $f(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

probabilities from $g(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

only in the case of $f(x)$ does $\sum P(X=x) = 1$, hence only $f(x)$ can be used as a probability mass function ***A2***

[6 marks]

e. $E(x) = \sum x \cdot \mathrm{P}(X = x)$ ***M1***

$= \frac{5}{75} + \frac{22}{75} + \frac{63}{75} + \frac{140}{75} = \frac{230}{75} \left(= \frac{46}{15}\right)$ ***A1***

[2 marks]

Examiners report

a. In (a), the ranges were often given incorrectly, particularly the range of *g* where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for $f \circ g(x)$. Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of *X*.

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In a population of rabbits, 1% are known to have a particular disease. A test is developed for the disease that gives a positive result for a rabbit that **does** have the disease in 99% of cases. It is also known that the test gives a positive result for a rabbit that **does not** have the disease in 0.1% of cases. A rabbit is chosen at random from the population.

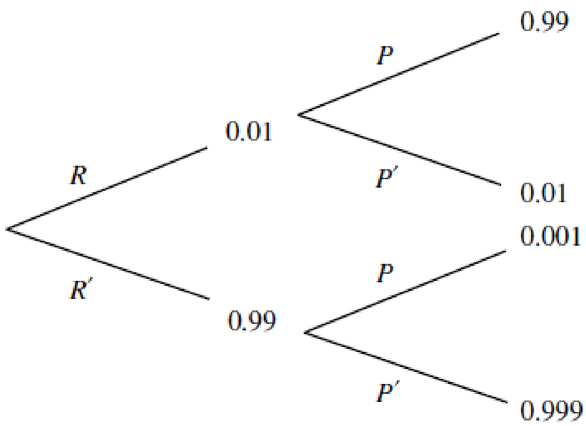
a. Find the probability that the rabbit tests positive for the disease. [2]

b. Given that the rabbit tests positive for the disease, show that the probability that the rabbit does not have the disease is less than 10 %. [3]

Markscheme

a. R is 'rabbit with the disease'

P is 'rabbit testing positive for the disease'



$\mathrm{P}(P) = P(R \cap P) + P(R' \cap P)$

$= 0.01 \times 0.99 + 0.99 \times 0.001$ ***M1***

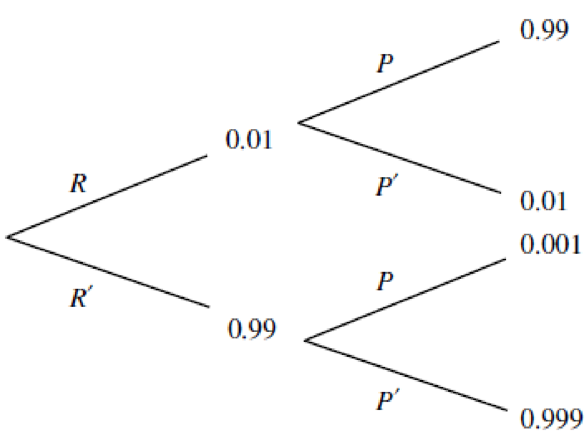
$= 0.01089 (= 0.0109)$ ***A1***

Note: Award ***M1*** for a correct tree diagram with correct probability values shown.

[2 marks]

b. R is 'rabbit with the disease'

P is 'rabbit testing positive for the disease'



$P(R'|P) = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.01 \times 0.99} \left(= \frac{0.00099}{0.01089}\right)$ ***M1A1***

$\frac{0.00099}{0.01089} < \frac{0.001}{0.01} = 10\%$ (or other valid argument) ***R1***

[3 marks]

Examiners report

a. There was a mixed performance in this question with some candidates showing good understanding of probability and scoring well and many others showing no understanding of conditional probability and difficulties in working with decimals. Very few candidates were able to provide a valid argument to justify their answer to part (b).

b. There was a mixed performance in this question with some candidates showing good understanding of probability and scoring well and many others showing no understanding of conditional probability and difficulties in working with decimals. Very few candidates were able to provide a valid argument to justify their answer to part (b).

Let A and B be events such that $\mathrm{P}(A) = 0.6,\ \mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A|B) = 0.6$.

Find $\mathrm{P}(B)$.

Markscheme

EITHER

Using $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ ***A1***

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to obtain $0.8 = 0.6 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***A1***

Substituting $0.6\mathrm{P}(B) = \mathrm{P}(A \cap B)$ into above equation ***M1***

OR

As $\mathrm{P}(A|B) = \mathrm{P}(A)$ then A and B are independent events ***M1R1***

Using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A) \times \mathrm{P}(B)$ ***A1***

to obtain $0.8 = 0.6 + \mathrm{P}(B) - 0.6 \times \mathrm{P}(B)$ ***A1***

THEN

$0.8 = 0.6 + 0.4\mathrm{P}(B)$ ***A1***

$\mathrm{P}(B) = 0.5$ ***A1*** ***N1***

[6 marks]

Examiners report

This question was generally well done, with a few candidates spotting an opportunity to use results for the *independent events* A and B.

A bag contains three balls numbered 1, 2 and 3 respectively. Bill selects one of these balls at random and he notes the number on the selected ball.

He then tosses that number of fair coins.

a. Calculate the probability that no head is obtained. [3]

b. Given that no head is obtained, find the probability that he tossed two coins. [3]

Markscheme

a. P(no heads from *n* coins tossed) = 0.5^n ***(A1)***

P(no head) = $\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8}$ ***M1***

$= \frac{7}{24}$ ***A1***

[3 marks]

b. $\mathrm{P}(2 \mid \text{no heads}) = \frac{\mathrm{P}(2 \text{ coins and no heads})}{\mathrm{P}(\text{no heads})}$ ***M1***

$= \frac{\frac{1}{12}}{\frac{7}{24}}$ ***A1***

$= \frac{2}{7}$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Events A and B are such that $\mathrm{P}(A) = 0.2$ and $\mathrm{P}(B) = 0.5$.

a. Determine the value of $\mathrm{P}(A \cup B)$ when [4]

(i) A and B are mutually exclusive;

(ii) A and B are independent.

b. Determine the range of possible values of $\mathrm{P}\left(A|B\right)$. [3]

Markscheme

a. (i) use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ ***(M1)***

$\mathrm{P}(A \cup B) = 0.2 + 0.5$

$= 0.7$ ***A1***

(ii) use of $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ***(M1)***

$\mathrm{P}(A \cup B) = 0.2 + 0.5 - 0.1$

$= 0.6$ ***A1***

[4 marks]

b. $\mathrm{P}\left(A|B\right) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

$\mathrm{P}\left(A|B\right)$ is a maximum when $\mathrm{P}(A \cap B) = \mathrm{P}(A)$

$\mathrm{P}\left(A|B\right)$ is a minimum when $\mathrm{P}(A \cap B) = 0$

$0 \le \mathrm{P}\left(A|B\right) \le 0.4$ ***A1A1A1***

Note: ***A1*** for each endpoint and ***A1*** for the correct inequalities.

[3 marks]

Total [7 marks]

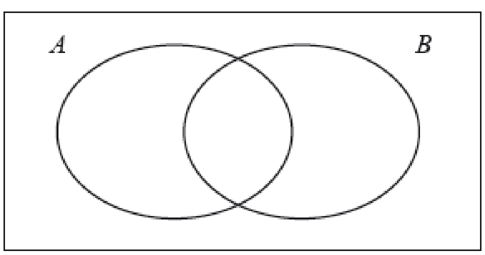
Examiners report

a. This part was generally well done.

b. Disappointingly, many candidates did not seem to understand the meaning of the word ‘range’ in this context.

Two events A and B are such that $\mathrm{P}(A \cap B') = 0.2$ and $\mathrm{P}(A \cup B) = 0.9$.

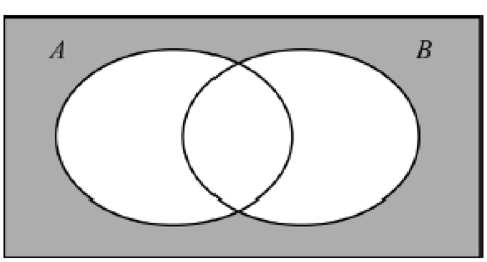
a. On the Venn diagram shade the region $A' \cap B'$. [1]



b. Find $\mathrm{P}(A'|B')$. [4]

Markscheme

a.



A1

[1 mark]

b. $P(A'|B') = \frac{P(A' \cap B')}{P(B')}$ ***(M1)***

$P(B') = 0.1 + 0.2 = 0.3$ ***(A1)***

$P(A' \cap B') = 0.1$ ***(A1)***

$P(A'|B') = \frac{0.1}{0.3} = \frac{1}{3}$ ***A1***

[4 marks]

Examiners report

a. Part (a) was well done.

b. In part (b) some candidates were unable to write down the conditional probability formula. Some then failed to realise that part (a) was designed to help them work out $P(A' \cap B')$ and instead incorrectly assumed independence.

A and B are two events such that $\mathrm{P}(A) = 0.25,\ \mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.

a. Find $\mathrm{P}(A \cap B)$. [2]

b. Determine whether events A and B are independent. [2]

Markscheme

a. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = 0.25 + 0.6 = 0.7$ ***M1***

$= 0.15$ ***A1***

[2 marks]

b. **EITHER**

$\mathrm{P}(A)\mathrm{P}(B)(= 0.25 \times 0.6) = 0.15$ ***A1***

$= \mathrm{P}(A \cap B)$ so independent ***R1***

OR

$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{0.15}{0.6} = 0.25$ ***A1***

$= \mathrm{P}(A)$ so independent ***R1***

Note: Allow follow through for incorrect answer to (a) that will result in events being dependent in (b).

[2 marks]

Total [4 marks]

Examiners report

a. [N/A]
b. [N/A]

Two unbiased tetrahedral (four-sided) dice with faces labelled 1, 2, 3, 4 are thrown and the scores recorded. Let the random variable *T* be the maximum of these two scores.

The probability distribution of *T* is given in the following table.

t	1	2	3	4
$\mathrm{P}(T = t)$	$\frac{1}{16}$	a	b	$\frac{7}{16}$

a. Find the value of *a* and the value of *b*. [3]

b. Find the expected value of *T*. [2]

Markscheme

a. $a = \frac{3}{16}$ and $b = \frac{5}{16}$ ***(M1)A1A1***

[3 marks]

Note: Award ***M1*** for consideration of the possible outcomes when rolling the two dice.

b. $\mathrm{E}\left(T\right) = \frac{1+6+15+28}{16} = \frac{25}{8} (= 3.125)$ ***(M1)A1***

Note: Allow follow through from part (a) even if probabilities do not add up to 1.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

A biased coin is weighted such that the probability of obtaining a head is $\frac{4}{7}$. The coin is tossed 6 times and X denotes the number of heads observed. Find the value of the ratio $\frac{\mathrm{P}(X=3)}{\mathrm{P}(X=2)}$.

Markscheme

recognition of $X \sim \mathrm{B}\left(6, \frac{4}{7}\right)$ ***(M1)***

$\mathrm{P}(X = 3) = \binom{6}{3}\left(\frac{4}{7}\right)^3\left(\frac{3}{7}\right)^3 \left(= 20 \times \frac{4^3 \times 3^3}{7^6}\right)$ ***A1***

$\mathrm{P}(X = 2) = \binom{6}{3}\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^4 \left(= 15 \times \frac{4^2 \times 34}{7^6}\right)$ ***A1***

$\frac{P(X=3)}{P(X=2)} = \frac{80}{45} \left(= \frac{16}{9}\right)$ ***A1***

[4 marks]

Examiners report

Many correct answers were seen to this and the majority of candidates recognised the need to use a Binomial distribution. A number of candidates, although finding the correct expressions for $\mathrm{P}(X = 3)$ and $\mathrm{P}(X = 4)$, were unable to perform the required simplification.

In a particular city 20 % of the inhabitants have been immunized against a certain disease. The probability of infection from the disease among those immunized is $\frac{1}{10}$, and among those not immunized the probability is $\frac{3}{4}$. If a person is chosen at random and found to be infected, find the probability that this person has been immunized.

Markscheme

tree diagram ***(M1)***

$\mathrm{P}(\mathrm{I}|\mathrm{D}) = \frac{\mathrm{P}(\mathrm{D}|\mathrm{I}) \times \mathrm{P}(\mathrm{I})}{\mathrm{P}(\mathrm{D})}$ ***(M1)***

$= \frac{0.1 \times 0.2}{0.1 \times 0.2 + 0.8 \times 0.75}$ ***A1A1A1***

$\left(= \frac{0.02}{0.62}\right) = \frac{1}{31}$ ***A1***

Note: Alternative presentation of results: ***M1*** for labelled tree; ***A1*** for initial branching probabilities, 0.2 and 0.8; ***A1*** for at least the relevant second branching probabilities, 0.1 and 0.75; ***A1*** for the 'infected' end-point probabilities, 0.02 and 0.6; ***M1A1*** for the final conditional probability calculation.

[6 marks]

Examiners report

Candidates who drew a tree diagram, the majority, usually found the correct answer.

A mathematics test is given to a class of 20 students. One student scores 0, but all the other students score 10.

a. Find the mean score for the class. [2]

b. Write down the median score. [1]

c. Write down the number of students who scored [2]

(i) above the mean score;

(ii) below the median score.

Markscheme

a. $\bar{x} = \frac{1 \times 0 + 19 \times 10}{20} = 9.5$ ***(M1)A1***

[2 marks]

b. median is 10 ***A1***

[1 mark]

c. (i) 19 ***A1***

(ii) 1 ***A1***

[2 marks]

Total [5 marks]

Examiners report

a. Well done.

b. Well done.

c. Both parts well done.

At a nursing college, 80 % of incoming students are female. College records show that 70 % of the incoming females graduate and 90 % of the incoming males graduate. A student who graduates is selected at random. Find the probability that the student is male, giving your answer as a fraction in its lowest terms.

Markscheme

$\mathrm{P}\ M|G = \frac{\mathrm{P}(M \cap G)}{\mathrm{P}(G)}$ ***(M1)***

$= \frac{0.2 \times 0.9}{0.2 \times 0.9 + 0.8 \times 0.7}$ ***M1A1A1***

$= \frac{0.18}{0.74}$

$= \frac{9}{37}$ ***A1***

[5 marks]

Examiners report

Most candidates answered this question successfully. Some made arithmetic errors.

The probability distribution of a discrete random variable X is defined by

$\mathrm{P}(X=x)=cx(5-x),\ x=1, 2, 3, 4$.

(a) Find the value of c.

(b) Find $E(X)$.

Markscheme

(a) Using $\sum \mathrm{P}(X=x)=1$ ***(M1)***

$4c+6c+6c+4c=1 \quad (20c=1)$ ***A1***

$c=\frac{1}{20} \quad (=0.05)$ ***A1 N1***

(b) Using $\mathrm{E}(X)=\sum x\mathrm{P}(X=x)$ ***(M1)***

$=(1\times 0.2)+(2\times 0.3)+(3\times 0.3)+(4\times 0.2)$ ***(A1)***

$=2.5$ ***A1 N1***

Notes: Only one of the first two marks can be implied.

Award ***M1A1A1*** if the x values are averaged only if symmetry is explicitly mentioned.

[6 marks]

Examiners report

This question was generally well done, but a few candidates tried integration for part (b).

A continuous random variable X has the probability density function

$$f(x)=\begin{cases} k\sin x, & 0\leqslant x\leqslant \frac{\pi}{2} \\ 0, & \text{otherwise.}\end{cases}$$

a. Find the value of k. [2]

b. Find $\mathrm{E}(X)$. [5]

c. Find the median of X. [3]

Markscheme

a. $k\int_0^{\frac{\pi}{2}} \sin x\mathrm{d}x=1$ ***M1***

$k[-\cos x]_0^{\frac{\pi}{2}} = 1$

$k = 1$ ***A1***

[2 marks]

b. $\mathrm{E}(X) = \int_0^{\frac{\pi}{2}} x \sin x \mathrm{d}x$ ***M1***

integration by parts ***M1***

$[-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \mathrm{d}x$ ***A1A1***

$= 1$ ***A1***

[5 marks]

c. $\int_0^M \sin x \mathrm{d}x = \frac{1}{2}$ ***M1***

$[-\cos x]_0^M = \frac{1}{2}$ ***A1***

$\cos M = \frac{1}{2}$

$M = \frac{\pi}{3}$ ***A1***

Note: accept arccos $\frac{1}{2}$

[3 marks]

Examiners report

a.

Most candidates scored maximum marks on this question. A few candidates found k = –1.

b.

Most candidates scored maximum marks on this question. A few candidates found k = –1.

c.

Most candidates scored maximum marks on this question. A few candidates found k = –1.

---

a. Mobile phone batteries are produced by two machines. Machine A produces 60% of the daily output and machine B produces 40%. It is found by testing that on average 2% of batteries produced by machine A are faulty and 1% of batteries produced by machine B are faulty. [6]

(i) Draw a tree diagram clearly showing the respective probabilities.

(ii) A battery is selected at random. Find the probability that it is faulty.

(iii) A battery is selected at random and found to be faulty. Find the probability that it was produced by machine A.

b. In a pack of seven transistors, three are found to be defective. Three transistors are selected from the pack at random without replacement. [6]

The discrete random variable X represents the number of defective transistors selected.

(i) Find $\mathrm{P}(X = 2)$.

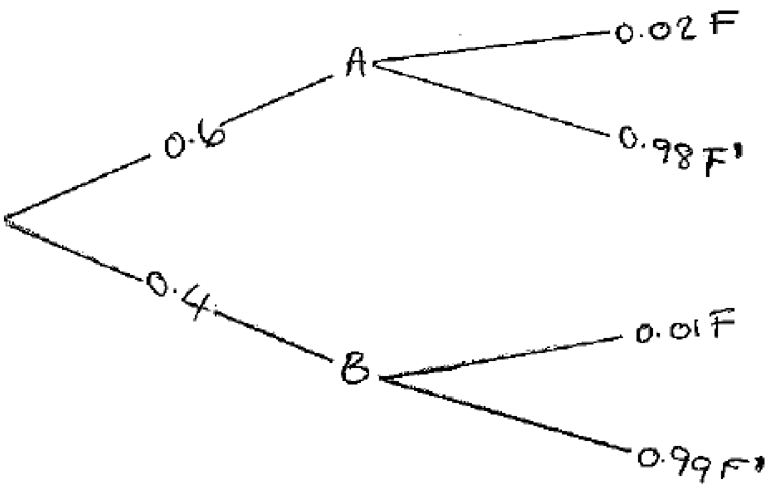
(ii) **Copy** and complete the following table:

x	0	1	2	3
$P(X=x)$				

(iii) Determine $E(X)$.

Markscheme

a. (i)



A1A1

Note: Award ***A1*** for a correctly labelled tree diagram and ***A1*** for correct probabilities.

(ii) $P(F) = 0.6 \times 0.02 + 0.4 \times 0.01$ ***(M1)***

$= 0.016$ ***A1***

(iii) $P(A|F) = \frac{P(A \cap F)}{P(F)}$

$= \frac{0.6 \times 0.02}{0.016} \left(= \frac{0.012}{0.016}\right)$ ***M1***

$= 0.75$ ***A1***

[6 marks]

b. (i) **METHOD 1**

$P(X=2) = \frac{{}^3C_2 \times {}^4C_1}{{}^7C_3}$ ***(M1)***

$= \frac{12}{35}$ ***A1***

METHOD 2

$\frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} \times 3$ ***(M1)***

$= \frac{12}{35}$ ***A1***

(ii)

x	0	1	2	3
$P(X=x)$	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

A2

Note: Award ***A1*** if $\frac{4}{35}$, $\frac{18}{35}$ or $\frac{1}{35}$ is obtained.

(iii) $E(X) = \sum xP(X=x)$

$E(X) = 0 \times \frac{4}{35} + 1 \times \frac{18}{35} + 2 \times \frac{12}{35} + 3 \times \frac{1}{35}$ ***M1***

$= \frac{45}{35} = \left(\frac{9}{7}\right)$ ***A1***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]

The faces of a fair six-sided die are numbered 1, 2, 2, 4, 4, 6. Let X be the discrete random variable that models the score obtained when this die is rolled.

a. Complete the probability distribution table for X. [2]

x				
$P(X=x)$				

b. Find the expected value of X. [2]

Markscheme

a.

x	1	2	4	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

b. $E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{3} + 4 \times \frac{1}{3} + 6 \times \frac{1}{6}$ ***(M1)***

$= \frac{19}{6} \left(= 3\frac{1}{6}\right)$ ***A1***

Note: If the probabilities in (a) are not values between 0 and 1 or lead to $E(X) > 6$ award ***M1A0*** to correct method using the incorrect probabilities; otherwise allow ***FT*** marks.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} ae^{-x}, & 0 \leqslant x \leqslant 1 \\ 0, & \text{otherwise.} \end{cases}$$

a. State the mode of X . [1]

b. Determine the value of a . [3]

c. Find $\mathrm{E}(X)$. [4]

Markscheme

a. 0 ***A1***

[1 mark]

b. $\int_0^1 f(x)dx = 1$ ***(M1)***

$\Rightarrow a = \frac{1}{\int_0^1 e^{-x}dx}$

$\Rightarrow a = \frac{1}{[-e^{-x}]_0^1}$

$\Rightarrow a = \frac{e}{e-1}$ (or equivalent) ***A1***

Note: Award first ***A1*** for correct integration of $\int e^{-x}dx$.

This ***A1*** is independent of previous ***M*** mark.

[3 marks]

c. $\mathrm{E}(X) = \int_0^1 xf(x)dx \left(= a\int_0^1 xe^{-x}dx\right)$ ***M1***

attempt to integrate by parts ***M1***

$= a[-xe^{-x} - e^{-x}]_0^1$ ***(A1)***

$= a\left(\frac{e-2}{e}\right)$

$= \frac{e-2}{e-1}$ (or equivalent) ***A1***

[4 marks]

Examiners report

a. A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression $-xe^{-x} + e^{-x}$.

b. A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression $-xe^{-x} + e^{-x}$.

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Events A and B are such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.4$.

a. Find the value of $\mathrm{P}(A \cup B)$ when [4]

(i) A and B are mutually exclusive;

(ii) A and B are independent.

b. Given that $\mathrm{P}(A \cup B) = 0.6$, find $\mathrm{P}(A|B)$. [3]

Markscheme

a. (i) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) = 0.7$ ***A1***

(ii) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ***(M1)***

$= 0.3 + 0.4 - 0.12 = 0.58$ ***A1***

[4 marks]

b. $\mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B)$

$= 0.3 + 0.4 - 0.6 = 0.1$ ***A1***

$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$= \frac{0.1}{0.4} = 0.25$ ***A1***

[3 marks]

Examiners report

a. Most candidates attempted this question and answered it well. A few misconceptions were identified (eg $\mathrm{P}(A \cup B) = \mathrm{P}(A)\mathrm{P}(B)$). Many candidates were unsure about the meaning of independent events.

b. Most candidates attempted this question and answered it well. A few misconceptions were identified (eg $\mathrm{P}(A \cup B) = \mathrm{P}(A)\mathrm{P}(B)$). Many candidates were unsure about the meaning of independent events.

A random variable has a probability density function given by

$$f(x) = \begin{cases} kx(2-x), & 0 \leqslant x \leqslant 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that $k = \frac{3}{4}$.

(b) Find $\mathrm{E}(X)$.

Markscheme

(a) $\int_0^2 kx(2-x)\mathrm{d}x = 1$ ***M1A1***

Note: Award ***M1*** for LHS and ***A1*** for setting = 1 at any stage.

$\left[\frac{2k}{2}x^2 - \frac{k}{3}x^3\right]_0^2 = 1$ ***A1***

$k\left(4 - \frac{8}{3}\right) = 1$ ***A1***

$k = \frac{3}{4}$ ***AG***

(b) $\mathrm{E}(X) = \frac{3}{4}\int_0^2 x^2(2-x)\mathrm{d}x$ ***(M1)***

= 1 ***A1***

Note: Accept answers that indicate use of symmetry.

[6 marks]

Examiners report

The integration was particularly well done in this question. A number of students treated the distribution as discrete. On the whole a) was done well once the distribution was recognized although there was a certain amount of fudging to achieve the result. A significant number of students did not initially set the integral equal to 1. Very few noted the symmetry of the distribution in b).

A box contains four red balls and two white balls. Darren and Marty play a game by each taking it in turn to take a ball from the box, without replacement. The first player to take a white ball is the winner.

a. Darren plays first, find the probability that he wins. [4]

b. The game is now changed so that the ball chosen is replaced after each turn. [3]

   Darren still plays first.

   Show that the probability of Darren winning has not changed.

Markscheme

a. probability that Darren wins $\mathrm{P}(W) + \mathrm{P}(RRW) + \mathrm{P}(RRRRW)$ ***(M1)***

   Note: Only award ***M1*** if three terms are seen or are implied by the following numerical equivalent.

   Note: Accept equivalent tree diagram for method mark.

$= \frac{2}{6} + \frac{4}{6} \bullet \frac{3}{5} \bullet \frac{2}{4} + \frac{4}{6} \bullet \frac{3}{5} \bullet \frac{2}{4} \bullet \frac{1}{3} \bullet \frac{2}{2} \quad \left(= \frac{1}{3} + \frac{1}{5} + \frac{1}{15}\right)$ ***A2***

Note: ***A1*** for two correct.

$= \frac{3}{5}$ ***A1***

[4 marks]

b. **METHOD 1**

the probability that Darren wins is given by

$\mathrm{P}(W) + \mathrm{P}(RRW) + \mathrm{P}(RRRRW) + \ldots$ ***(M1)***

Note: Accept equivalent tree diagram with correctly indicated path for method mark.

$\mathrm{P}\ (\text{Darren Win}) = \frac{1}{3} + \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{1}{3} + \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{2}{3} \bullet \frac{1}{3} + \ldots$

or $= \frac{1}{3}\left(1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \ldots\right)$ ***A1***

$= \frac{1}{3}\left(\frac{1}{1-\frac{4}{9}}\right)$ ***A1***

$= \frac{3}{5}$ ***AG***

METHOD 2

$\mathrm{P}\ (\text{Darren wins}) = \mathrm{P}$

$\mathrm{P} = \frac{1}{3} + \frac{4}{9}\mathrm{P}$ ***M1A2***

$\frac{5}{9}\mathrm{P} = \frac{1}{3}$

$\mathrm{P} = \frac{3}{5}$ ***AG***

[3 marks]

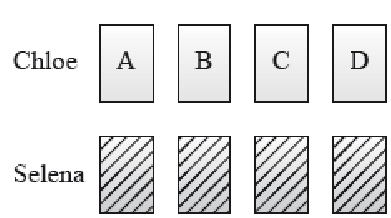
Total [7 marks]

Examiners report

a. [N/A]
b. [N/A]

Chloe and Selena play a game where each have four cards showing capital letters A, B, C and D.

Chloe lays her cards face up on the table in order A, B, C, D as shown in the following diagram.



Selena shuffles her cards and lays them face down on the table. She then turns them over one by one to see if her card matches with Chloe's card directly above.
Chloe wins if **no** matches occur; otherwise Selena wins.

Chloe and Selena repeat their game so that they play a total of 50 times.

Suppose the discrete random variable *X* represents the number of times Chloe wins.

a. Show that the probability that Chloe wins the game is $\frac{3}{8}$. [6]

b.i. Determine the mean of *X*. [3]

b.ii. Determine the variance of *X*. [2]

Markscheme

a. **METHOD 1**

number of possible "deals" $= 4! = 24$ ***A1***

consider ways of achieving "no matches" (Chloe winning):

Selena could deal B, C, D (*ie*, 3 possibilities)

as her first card ***R1***

for each of these matches, there are only 3 possible combinations for the remaining 3 cards ***R1***

so no. ways achieving no matches $= 3 \times 3 = 9$ ***M1A1***

so probability Chloe wins $= \frac{9}{23} = \frac{3}{8}$ ***A1AG***

METHOD 2

number of possible "deals" $= 4! = 24$ ***A1***

consider ways of achieving a match (Selena winning)

Selena card A can match with Chloe card A, giving 6 possibilities for this happening ***R1***

if Selena deals B as her first card, there are only 3 possible combinations for the remaining 3 cards. Similarly for dealing C and dealing D ***R1***

so no. ways achieving one match is $= 6 + 3 + 3 + 3 = 15$ ***M1A1***

so probability Chloe wins $= 1 - \frac{15}{24} = \frac{3}{8}$ ***A1AG***

METHOD 3

systematic attempt to find number of outcomes where Chloe wins (no matches)

(using tree diag. or otherwise) ***M1***

9 found ***A1***

each has probability $\frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1$ ***M1***

$= \frac{1}{24}$ ***A1***

their 9 multiplied by their $\frac{1}{24}$ ***M1A1***

$= \frac{3}{8}$ ***AG***

[6 marks]

b.i. $X \sim \mathrm{B}\left(50,\ \frac{3}{8}\right)$ ***(M1)***

$\mu = np = 50 \times \frac{3}{8} = \frac{150}{8}\ \left(= \frac{75}{4}\right)\ (= 18.75)$ ***(M1)A1***

[3 marks]

b.ii $\sigma^2 = np(1-p) = 50 \times \frac{3}{8} \times \frac{5}{8} = \frac{750}{64}\ \left(= \frac{375}{32}\right)\ (= 11.7)$ ***(M1)A1***

[2 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]

A biased coin is tossed five times. The probability of obtaining a head in any one throw is p.

Let X be the number of heads obtained.

a. Find, in terms of p, an expression for $\mathrm{P}(X = 4)$. [2]

b. (i) Determine the value of p for which $\mathrm{P}(X = 4)$ is a maximum. [6]

(ii) For this value of p, determine the expected number of heads.

Markscheme

a. $X \sim \mathrm{B}(5,\ p)$ ***(M1)***

$\mathrm{P}(X = 4) = \binom{5}{4} p^4(1-p)$ (or equivalent) ***A1***

[2 marks]

b. (i) $\frac{\mathrm{d}}{\mathrm{d}p}(5p^4 - 5p^5) = 20p^3 - 25p^4$ ***M1A1***

$5p^3(4-5p) = 0 \Rightarrow p = \frac{4}{5}$ ***M1A1***

Note: Do not award the final ***A1*** if $p = 0$ is included in the answer.

(ii) $\mathrm{E}(X) = np = 5\left(\frac{4}{5}\right)$ ***(M1)***

$= 4$ ***A1***

[6 marks]

Examiners report

a. This question was generally very well done and posed few problems except for the weakest candidates.

b. This question was generally very well done and posed few problems except for the weakest candidates.

A batch of 15 DVD players contains 4 that are defective. The DVD players are selected at random, one by one, and examined. The ones that are checked are not replaced.

a. What is the probability that there are exactly 3 defective DVD players in the first 8 DVD players examined? [4]

b. What is the probability that the 9^{th} DVD player examined is the 4^{th} defective one found? [3]

Markscheme

a. **METHOD 1**

$\mathrm{P}(3 \text{ defective in first } 8) = \binom{8}{3} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} \times \frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8}$ ***M1A1A1***

Note: Award ***M1*** for multiplication of probabilities with decreasing denominators.

Award ***A1*** for multiplication of correct eight probabilities.

Award ***A1*** for multiplying by $\binom{8}{3}$.

$= \frac{56}{195}$ ***A1***

METHOD 2

$\mathrm{P}(3 \text{ defective DVD players from } 8) = \frac{\binom{4}{3}\binom{11}{5}}{\binom{15}{8}}$ ***M1A1***

Note: Award ***M1*** for an expression of this form containing three combinations.

$= \frac{\frac{4!}{3!1!} \times \frac{11!}{5!6!}}{\frac{15!}{8!7!}}$ ***M1***

$= \frac{56}{195}$ ***A1***

[4 marks]

b. $\mathrm{P}(9^{\text{th}} \text{ selected is } 4^{\text{th}} \text{ defective player} | 3 \text{ defective in first } 8) = \frac{1}{7}$ ***(A1)***

$\mathrm{P}(9^{\text{th}} \text{ selected is } 4^{\text{th}} \text{ defective player}) = \frac{56}{195} \times \frac{1}{7}$ ***M1***

$= \frac{8}{195}$ ***A1***

[3 marks]

Examiners report

a. There were two main methods used to complete this question, the most common being a combinations approach. Those who did this coped well with the factorial simplification. Many who did not manage the first part were able to complete the second part successfully.

b. There were two main methods used to complete this question, the most common being a combinations approach. Those who did this coped well with the factorial simplification. Many who did not manage the first part were able to complete the second part successfully.

At a skiing competition the mean time of the first three skiers is 34.1 seconds. The time for the fourth skier is then recorded and the mean time of the first four skiers is 35.0 seconds. Find the time achieved by the fourth skier.

Markscheme

total time of first 3 skiers $= 34.1 \times 3 = 102.3$ ***(M1)A1***

total time of first 4 skiers $= 35.0 \times 4 = 140.0$ ***A1***

time taken by fourth skier $= 140.0 - 102.3 = 37.7 \text{ (seconds)}$ ***A1***

[4 marks]

Examiners report

This was done successfully by almost all candidates.

Four numbers are such that their mean is 13, their median is 14 and their mode is 15. Find the four numbers.

Markscheme

using the sum divided by 4 is 13 ***M1***

two of the numbers are 15 ***A1***

(as median is 14) we need a 13 ***A1***

fourth number is 9 ***A1***

numbers are 9, 13, 15, 15 ***N4***

[4 marks]

Examiners report

[N/A]

A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} c(x - x^2), & 0 \leqslant x \leqslant 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine c.

(b) Find $\mathrm{E}(X)$.

Markscheme

(a) the total area under the graph of the pdf is unity ***(A1)***

area $= c\int_0^1 x - x^2 \mathrm{d}x$

$= c\left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$ ***A1***

$= \frac{c}{6}$

$\Rightarrow c = 6$ ***A1***

(b) $\mathrm{E}(X) = 6\int_0^1 x^2 - x^3 \mathrm{d}x$ ***(M1)***

$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2}$ ***A1***

Note: Allow an answer obtained by a symmetry argument.

[5 marks]

Examiners report

Most candidates made a meaningful attempt at this question with many gaining the correct answers. One or two candidates did not attempt this question at all.

The probability density function of the random variable X is defined as

$$f(x) = \begin{cases} \sin x, & 0 \leqslant x \leqslant \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{E}(X)$.

Markscheme

$\int_0^{\frac{\pi}{2}} x \sin x \mathrm{d}x$ ***M1***

$= [-x\cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \mathrm{d}x$ ***M1(A1)***

Note: Condone the absence of limits or wrong limits to this point.

$= [-x\cos x + \sin x]_0^{\frac{\pi}{2}}$ ***A1***

$= 1$ ***A1***

[5 marks]

Examiners report

It was pleasing to note how many candidates recognised the expression that needed to be integrated and successfully used integration by parts to reach the correct answer.

Two players, A and B, alternately throw a fair six-sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

Markscheme

P(six in first throw) $= \frac{1}{6}$ ***(A1)***

P(six in third throw) $= \frac{25}{36} \times \frac{1}{6}$ ***(M1)(A1)***

P(six in fifth throw)$= \left(\frac{25}{36}\right)^2 \times \frac{1}{6}$

P(A obtains first six) $= \frac{1}{6} + \frac{25}{36} \times \frac{1}{6} + \left(\frac{25}{36}\right)^2 \times \frac{1}{6} + \ldots$ ***(M1)***

recognizing that the common ratio is $\frac{25}{36}$ ***(A1)***

P(A obtains first six) $= \frac{\frac{1}{6}}{1-\frac{25}{36}}$ (by summing the infinite GP) ***M1***

$= \frac{6}{11}$ ***A1***

[7 marks]

Examiners report

This question proved difficult to the majority of the candidates although a few interesting approaches to this problem have been seen. Candidates who started the question by drawing a tree diagram were more successful, although a number of these failed to identify the geometric series.

Two events A and B are such that $\mathrm{P}(A \cup B) = 0.7$ and $\mathrm{P}(A|B') = 0.6$.

Find $\mathrm{P}(B)$.

Markscheme

Note: Be aware that an unjustified assumption of independence will also lead to P(B) = 0.25, but is an invalid method.

METHOD 1

$\mathrm{P}(A'|B') = 1 - \mathrm{P}(A|B') = 1 - 0.6 = 0.4$ ***M1A1***

$\mathrm{P}(A'|B') = \frac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$

$\mathrm{P}(A' \cap B') = \mathrm{P}\left((A \cup B)'\right) = 1 - 0.7 = 0.3$ ***A1***

$0.4 = \frac{0.3}{\mathrm{P}(B')} \Rightarrow \mathrm{P}(B') = 0.75$ ***(M1)A1***

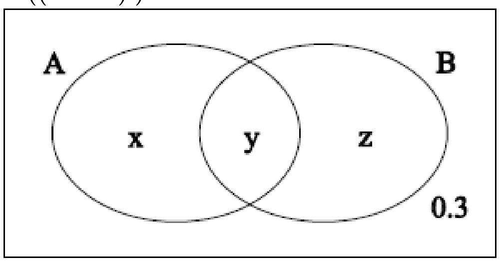
$\mathrm{P}(B) = 0.25$ ***A1***

(this method can be illustrated using a tree diagram)

[6 marks]

METHOD 2

$\mathrm{P}\left((A \cup B)'\right) = 1 - 0.7 = 0.3$ ***A1***



$\mathrm{P}(A|B') = \frac{x}{x+0.3} = 0.6$ ***M1A1***

$x = 0.6x + 0.18$

$0.4x = 0.18$

$x = 0.45$ ***A1***

$\mathrm{P}(A \cup B) = x + y + z$

$\mathrm{P}(B) = y + z = 0.7 - 0.45$ ***(M1)***

$= 0.25$ ***A1***

[6 marks]

METHOD 3

$\frac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')} = 0.6$ (or $\mathrm{P}(A \cap B') = 0.6\mathrm{P}(B')$ ***M1***

$\mathrm{P}(A \cap B') = \mathrm{P}(A \cup B) - \mathrm{P}(B)$ ***M1A1***

$\mathrm{P}(B') = 1 - \mathrm{P}(B)$

$0.7 - \mathrm{P}(B) = 0.6 - 0.6\mathrm{P}(B)$ ***M1(A1)***

$0.1 = 0.4\mathrm{P}(B)$

$\mathrm{P}(B) = \frac{1}{4}$ ***A1***

[6 marks]

Examiners report

There is a great variety of ways to approach this question and there were plenty of very good solutions produced, all of which required an insight into the structure of conditional probability. A few candidates unfortunately assumed independence and so did not score well.

Consider two events A and A defined in the same sample space.

Given that $\mathrm{P}(A \cup B) = \frac{4}{9}$, $\mathrm{P}(B|A) = \frac{1}{3}$ and $\mathrm{P}(B|A') = \frac{1}{6}$,

a. Show that $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$. [3]

b. (i) show that $\mathrm{P}(A) = \frac{1}{3}$; [6]

(ii) hence find $\mathrm{P}(B)$.

Markscheme

a. **METHOD 1**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B) - \mathrm{P}(A \cap B)$ ***M1A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

METHOD 2

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A|B) \times \mathrm{P}(B)$ ***M1***

$= \mathrm{P}(A) + (1 - \mathrm{P}(A|B)) \times \mathrm{P}(B)$

$= \mathrm{P}(A) + \mathrm{P}(A'|B) \times \mathrm{P}(B)$ ***A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

[3 marks]

b. (i) use $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ and $\mathrm{P}(A' \cap B) = \mathrm{P}(B|A')\mathrm{P}(A')$ ***(M1)***

$\frac{4}{9} = \mathrm{P}(A) + \frac{1}{6}(1 - \mathrm{P}(A))$ ***A1***

$8 = 18\mathrm{P}(A) + 3(1 - \mathrm{P}(A))$ ***M1***

$\mathrm{P}(A) = \frac{1}{3}$ ***AG***

(ii) **METHOD 1**

$\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$ ***M1***

$= \mathrm{P}(B|A)\mathrm{P}(A) + \mathrm{P}(B|A')\mathrm{P}(A')$ ***M1***

$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{2}{3} = \frac{2}{9}$ ***A1***

METHOD 2

$\mathrm{P}(A \cap B) = \mathrm{P}(B|A)\mathrm{P}(A) \Rightarrow \mathrm{P}(A \cap B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ ***M1***

$\mathrm{P}(B) = \mathrm{P}(A \cup B) + \mathrm{P}(A \cap B) - \mathrm{P}(A)$ ***M1***

$\mathrm{P}(B) = \frac{4}{9} + \frac{1}{9} - \frac{1}{3} = \frac{2}{9}$ ***A1***

[6 marks]

Examiners report

a. [N/A]

b. [N/A]

A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ ae^{-ax}, & x \geqslant 0. \end{cases}$$

It is known that $\mathrm{P}(X < 1) = 1 - \frac{1}{\sqrt{2}}$.

(a) Show that $a = \frac{1}{2}\ln 2$.

(b) Find the median of X.

(c) Calculate the probability that $X < 3$ given that $X > 1$.

Markscheme

(a) $\int_0^1 ae^{-ax}\mathrm{d}x = 1 - \frac{1}{\sqrt{2}}$ ***M1A1***

$[-e^{-ax}]_0^1 = 1 - \frac{1}{\sqrt{2}}$ ***M1A1***

$-e^{-a} + 1 = 1 - \frac{1}{\sqrt{2}}$ ***A1***

Note: Accept e^0 instead of 1.

$e^{-a} = \frac{1}{\sqrt{2}}$

$e^a = \sqrt{2}$

$a = \ln 2^{\frac{1}{2}}$ $\left(\text{accept } -a = \ln 2^{-\frac{1}{2}}\right)$ ***A1***

$a = \frac{1}{2}\ln 2$ ***AG***

[6 marks]

(b) $\int_0^M ae^{-ax}\mathrm{d}x = \frac{1}{2}$ ***M1A1***

$[-e^{-ax}]_0^M = \frac{1}{2}$ ***A1***

$-e^{-Ma} + 1 = \frac{1}{2}$

$e^{-Ma} = \frac{1}{2}$ ***A1***

$Ma = \ln 2$

$M = \frac{\ln 2}{a} = 2$ ***A1***

[5 marks]

(c) $\mathrm{P}(1 < X < 3) = \int_1^3 ae^{-ax}\mathrm{d}x$ ***M1A1***

$= -e^{-3a} + e^{-a}$ ***A1***

$\mathrm{P}(X < 3|X > 1) = \frac{\mathrm{P}(1<X<3)}{\mathrm{P}(X>1)}$ ***M1A1***

$= \frac{-e^{-3a}+e^{-a}}{1-\mathrm{P}(X<1)}$ ***A1***

$= \frac{-e^{-3a}+e^{-a}}{\frac{1}{\sqrt{2}}}$ ***A1***

$= \sqrt{2}(-e^{-3a} + e^{-a})$

$= \sqrt{2}\left(-2^{-\frac{3}{2}} + 2^{-\frac{1}{2}}\right)$ ***A1***

$= \frac{1}{2}$ ***A1***

Note: Award full marks for $\mathrm{P}(X < 3/X > 1) = \mathrm{P}(X < 2) = \frac{1}{2}$ or quoting properties of exponential distribution.

[9 marks]

Total [20 marks]

Examiners report

Many candidates did not attempt this question and many others were clearly not familiar with this topic. On the other hand, most of the candidates who were familiar with continuous random variables and knew how to start the questions were successful and scored well in parts (a) and (b). The most common errors were in the integral of e^{-at}, having the limits from $-\infty$ to 1, confusion over powers and signs ('-' sometimes just disappeared). Understanding of conditional probability was poor and marks were low in part (c). A small number of candidates from a small number of schools coped very competently with the algebra throughout the question.

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a. The random variable X has the Poisson distribution $\mathrm{Po}(m)$. Given that $\mathrm{P}(X > 0) = \frac{3}{4}$, find the value of m in the form $\ln a$ where a is an integer. [3]

b. The random variable Y has the Poisson distribution $\mathrm{Po}(2m)$. Find $\mathrm{P}(Y > 1)$ in the form $\frac{b - \ln c}{c}$ where b and c are integers. [4]

Markscheme

a. $\mathrm{P}(X > 0) = 1 - \mathrm{P}(X = 0)$ ***(M1)***

$\Rightarrow 1 - \mathrm{e}^{-m} = \frac{3}{4}$ or equivalent ***A1***

$\Rightarrow m = \ln 4$ ***A1***

[3 marks]

b. $\mathrm{P}(Y > 1) = 1 - \mathrm{P}(Y = 0) - \mathrm{P}(Y = 1)$ ***(M1)***

$= 1 - \mathrm{e}^{-2\ln 4} - \mathrm{e}^{-2\ln 4} \times 2\ln 4$ ***A1***

recognition that $2\ln 4 = \ln 16$ ***(A1)***

$\mathrm{P}(Y > 1) = \frac{15 - \ln 16}{16}$ ***A1***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

Events A and B are such that $\mathrm{P}(A) = \frac{2}{5}$, $\mathrm{P}(B) = \frac{11}{20}$ and $\mathrm{P}(A|B) = \frac{2}{11}$.

(a) Find $\mathrm{P}(A \cap B)$.

(b) Find $\mathrm{P}(A \cup B)$.

(c) State with a reason whether or not events A and B are independent.

Markscheme

(a) $\mathrm{P}(A \cap B) = \mathrm{P}(A|B) \times P(B)$

$\mathrm{P}(A \cap B) = \frac{2}{11} \times \frac{11}{20}$ ***(M1)***

$= \frac{1}{10}$ ***A1***

[2 marks]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cup B) = \frac{2}{5} + \frac{11}{20} - \frac{1}{10}$ ***(M1)***

$= \frac{17}{20}$ ***A1***

[2 marks]

(c) No – events A and B are not independent ***A1***

EITHER

$\mathrm{P}(A|B) \neq \mathrm{P}(A)$ ***R1***

$\left(\frac{2}{11} \neq \frac{2}{5}\right)$

OR

$\mathrm{P}(A) \times \mathrm{P}(B) \neq \mathrm{P}(A \cap B)$

$\frac{2}{5} \times \frac{11}{20} = \frac{11}{50} \neq \frac{1}{10}$ ***R1***

Note: The numbers are required to gain ***R1*** in the '**OR**' method only.

Note: Do not award ***A1R0*** in either method.

[2 marks]

Total [6 marks]

Examiners report

[N/A]

The random variable T has the probability density function

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), \quad -1 \leqslant t \leqslant 1.$$

Find

(a) $\mathrm{P}(T = 0)$;

(b) the interquartile range.

Markscheme

(a) Any consideration of $\int_0^0 f(x)\mathrm{d}x$ ***(M1)***

0 ***A1*** ***N2***

(b) **METHOD 1**

Let the upper and lower quartiles be a and $-a$

$\frac{\pi}{4}\int_a^1 \cos\frac{\pi t}{2}\mathrm{d}t = 0.25$ ***M1***

$\Rightarrow \left[\frac{\pi}{4}\times\frac{2}{\pi}\sin\frac{\pi t}{2}\right]_a^1 = 0.25$ ***A1***

$\Rightarrow \left[\frac{1}{2}\sin\frac{\pi t}{2}\right]_a^1 = 0.25$

$\Rightarrow \left[\frac{1}{2}-\frac{1}{2}\sin\frac{\pi a}{2}\right] = 0.25$ ***A1***

$\Rightarrow \frac{1}{2}\sin\frac{\pi a}{2} = \frac{1}{4}$

$\Rightarrow \sin\frac{\pi a}{2} = \frac{1}{2}$

$\frac{\pi a}{2} = \frac{\pi}{6}$

$a = \frac{1}{3}$ ***A1***

Since the function is symmetrical about $t = 0$,

interquartile range is $\frac{1}{3}-\left(-\frac{1}{3}\right) = \frac{2}{3}$ ***R1***

METHOD 2

$\frac{\pi}{4}\int_{-a}^{a}\cos\frac{\pi t}{2}\mathrm{d}t = 0.5 = \frac{\pi}{2}\int_0^a \cos\frac{\pi t}{2}\mathrm{d}t$ ***M1A1***

$\Rightarrow \left[\sin\frac{a\pi}{2}\right] = 0.5$ ***A1***

$\Rightarrow \frac{a\pi}{2} = \frac{\pi}{6}$

$\Rightarrow a = \frac{1}{3}$ ***A1***

The interquartile range is $\frac{2}{3}$ ***R1***

[7 marks]

Examiners report

All but the best candidates struggled with part (a). The vast majority either did not attempt it or let $t = 1$. There was no indication from any of the scripts that candidates wasted an undue amount of time in trying to solve part (a). Many candidates attempted part (b), but few had a full understanding of the situation and hence were unable to give wholly correct answers.

The continuous variable X has probability density function

$$f(x) = \begin{cases} 12x^2(1-x), & 0 \leqslant x \leqslant 1 \\ 0, & \text{otherwise.} \end{cases}$$

a. Determine $\mathrm{E}(X)$. [3]

b. Determine the mode of X . [3]

Markscheme

a. $\mathrm{E}(X)=\int_0^1 12x^3(1-x)\mathrm{d}x$ ***M1***

$=12\left[\frac{x^4}{4}-\frac{x^5}{5}\right]_0^1$ ***A1***

$=\frac{3}{5}$ ***A1***

[3 marks]

b. $f'(x)=12(2x-3x^2)$ ***A1***

at the mode $f'(x)=12(2x-3x^2)=0$ ***M1***

therefore the mode $=\frac{2}{3}$ ***A1***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

Find the coordinates of the point of intersection of the planes defined by the equations $x+y+z=3,\ x-y+z=5$ and $x+y+2z=6$.

Markscheme

METHOD 1

for eliminating one variable from two equations ***(M1)***

eg, $\begin{cases}(x+y+z=3)\\ 2x+2z=8\\ 2x+3z=11\end{cases}$ ***A1A1***

for finding correctly one coordinate

eg, $\begin{cases}(x+y+z=3)\\ (2x+2z=8)\\ z=3\end{cases}$ ***A1***

for finding correctly the other two coordinates ***A1***

$\Rightarrow\begin{cases}x=1\\ y=-1\\ z=3\end{cases}$

the intersection point has coordinates $(1,\ -1,\ 3)$

METHOD 2

for eliminating two variables from two equations or using row reduction ***(M1)*** eg, $\begin{cases} (x+y+z=3) \\ -2=2 \\ z=3 \end{cases}$ **or** $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ ***A1A1***

for finding correctly the other coordinates ***A1A1***

$\Rightarrow \begin{cases} x=1 \\ y=-1 \\ (z=3) \end{cases}$ **or** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$

the intersection point has coordinates $(1,\ -1,\ 3)$

METHOD 3

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -2$ ***(A1)***

attempt to use Cramer's rule ***M1***

$x = \dfrac{\begin{vmatrix} 3 & 1 & 1 \\ 5 & -1 & 1 \\ 6 & 1 & 2 \end{vmatrix}}{-2} = \frac{-2}{-2} = 1$ ***A1***

$y = \dfrac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 5 & 1 \\ 1 & 6 & 2 \end{vmatrix}}{-2} = \frac{2}{-2} = -1$ ***A1***

$z = \dfrac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 5 \\ 1 & 1 & 6 \end{vmatrix}}{-2} = \frac{-6}{-2} = 3$ ***A1***

Note: Award ***M1*** only if candidate attempts to determine at least one of the variables using this method.

[5 marks]

Examiners report

[N/A]

John removes the labels from three cans of tomato soup and two cans of chicken soup in order to enter a competition, and puts the cans away. He then discovers that the cans are identical, so that he cannot distinguish between cans of tomato soup and chicken soup. Some weeks later he decides to have a can of chicken soup for lunch. He opens the cans at random until he opens a can of chicken soup. Let Y denote the number of cans he opens.

Find

(a) the possible values of Y,

(b) the probability of each of these values of Y,

(c) the expected value of Y.

Markscheme

(a) 1, 2, 3, 4 ***A1***

(b) $\mathrm{P}(Y=1)=\frac{2}{5}$ ***A1***
$\mathrm{P}(Y=2)=\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$ ***A1***
$\mathrm{P}(Y=3)=\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}=\frac{1}{5}$ ***A1***
$\mathrm{P}(Y=4)=\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}=\frac{1}{10}$ ***A1***

(c) $\mathrm{E}(Y)=1\times\frac{2}{5}+2\times\frac{3}{10}+3\times\frac{1}{5}+4\times\frac{1}{10}$ ***M1***
$=2$ ***A1***
[7 marks]

Examiners report

Candidates found this question challenging with only better candidates gaining the correct answers. A number of students assumed incorrectly that the distribution was either Binomial or Geometric.

A football team, Melchester Rovers are playing a tournament of five matches.

The probabilities that they win, draw or lose a match are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively.

These probabilities remain constant; the result of a match is independent of the results of other matches. At the end of the tournament their coach Roy loses his job if they lose three **consecutive** matches, otherwise he does not lose his job. Find the probability that Roy loses his job.

Markscheme

METHOD 1

to have 3 consecutive losses there must be exactly 5, 4 or 3 losses

the probability of exactly 5 losses (must be 3 consecutive) is $\left(\frac{1}{3}\right)^5$ ***A1***

the probability of exactly 4 losses (with 3 consecutive) is $4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$ ***A1A1***

Note: First ***A1*** is for the factor 4 and second ***A1*** for the other 2 factors.

the probability of exactly 3 losses (with 3 consecutive) is $3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$ ***A1A1***

Note: First ***A1*** is for the factor 3 and second ***A1*** for the other 2 factors.

(Since the events are mutually exclusive)

the total probability is $\frac{1+8+12}{3^5}=\frac{21}{243}\ \left(=\frac{7}{81}\right)$ ***A1***

[6 marks]

METHOD 2

Roy loses his job if

A – first 3 games are all lost (so the last 2 games can be any result)

B – first 3 games are not all lost, but middle 3 games are all lost (so the first game is not a loss and the last game can be any result)

or C – first 3 games are not all lost, middle 3 games are not all lost but last 3 games are all lost, (so the first game can be any result but the second game is not a loss)

for A 4^{th} & 5^{th} games can be anything

$\mathrm{P}(A) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$ ***A1***

for B 1^{st} game not a loss & 5^{th} game can be anything ***(R1)***

$\mathrm{P}(B) = \frac{2}{3} \times \left(\frac{1}{3}\right)^3 = \frac{2}{81}$ ***A1***

for C 1^{st} game anything, 2^{nd} game not a loss ***(R1)***

$\mathrm{P}(C) = 1 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^3 = \frac{2}{81}$ ***A1***

(Since the events are mutually exclusive)

total probability is $\frac{1}{27} + \frac{2}{81} + \frac{2}{81} = \frac{7}{81}$ ***A1***

Note: In both methods all the ***A*** marks are independent.

Note: If the candidate misunderstands the question and thinks that it is asking for exactly 3 losses award ***A1 A1*** and ***A1*** for an answer of $\frac{12}{243}$ as in the last lines of Method 1.

[6 marks]

Total [6 marks]

Examiners report

If a script has lots of numbers with the wrong final answer and no explanation of method it is not going to gain many marks. Working has to be explained. The counting strategy needs to be decided on first. Some candidates misunderstood the context and tried to calculate exactly 3 consecutive losses. Not putting a non-loss as $\frac{2}{3}$ caused unnecessary work.

The random variable X has probability density function f where

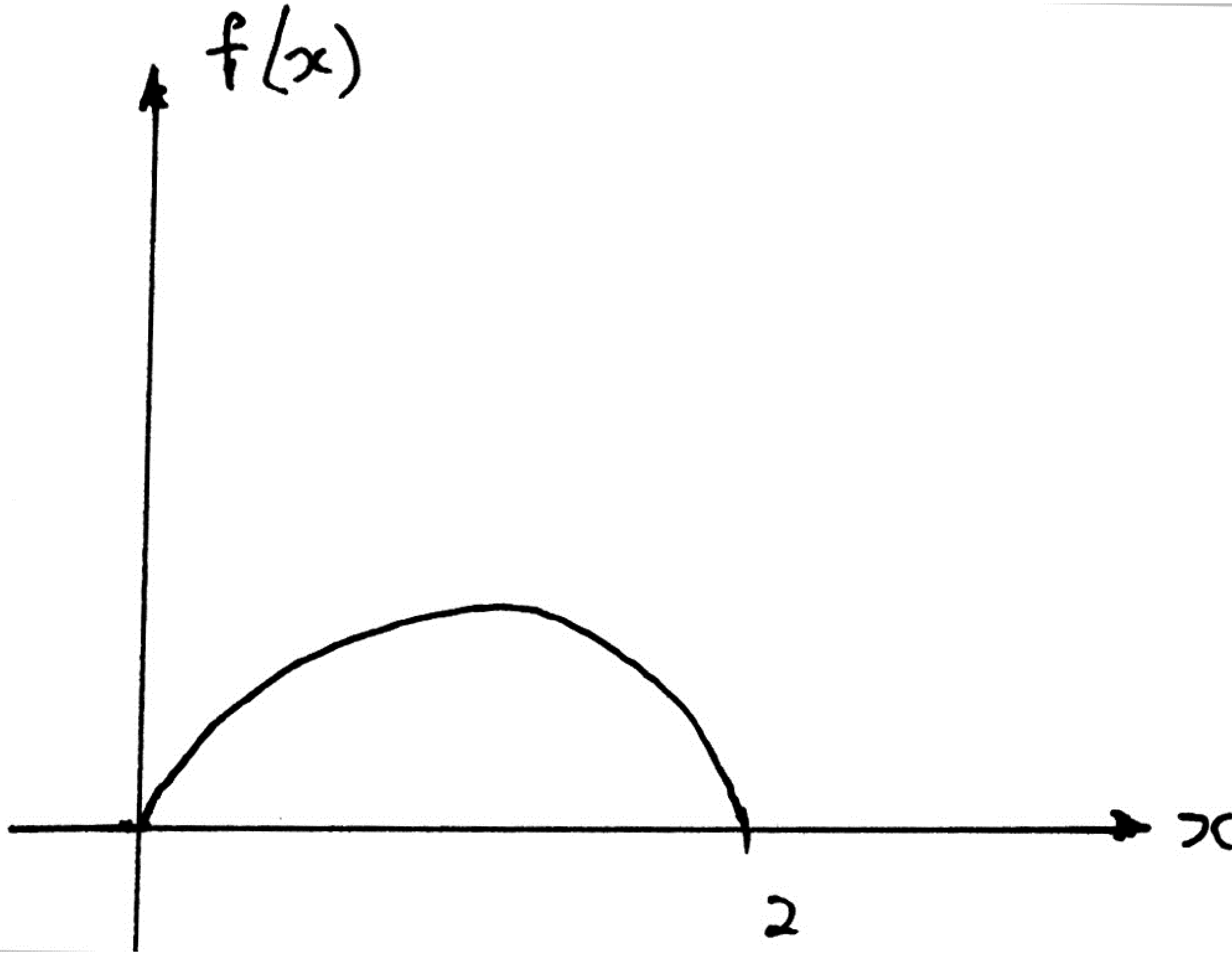
$$f(x) = \begin{cases} kx(x+1)(2-x), & 0 \leqslant x \leqslant 2 \\ 0, & \text{otherwise} . \end{cases}$$

a. Sketch the graph of the function. You are not required to find the coordinates of the maximum. [1]

b. Find the value of k . [5]

Markscheme

a.



A1

Note: Award ***A1*** for intercepts of 0 and 2 and a concave down curve in the given domain .

Note: Award ***A0*** if the cubic graph is extended outside the domain [0, 2] .

[1 mark]

b. $\int_0^2 kx(x+1)(2-x)\mathrm{d}x = 1$ ***(M1)***

Note: The correct limits and =1 must be seen but may be seen later.

$k\int_0^2 (-x^3+x^2+2x)\mathrm{d}x = 1$ ***A1***

$k\left[-\frac{1}{4}x^4+\frac{1}{3}x^3+x^2\right]_0^2 = 1$ ***M1***

$k\left(-4+\frac{8}{3}+4\right) = 1$ ***(A1)***

$k = \frac{3}{8}$ ***A1***

[5 marks]

Examiners report

a. Most candidates completed this question well. A number extended the graph beyond the given domain.

b. Most candidates completed this question well. A number extended the graph beyond the given domain.

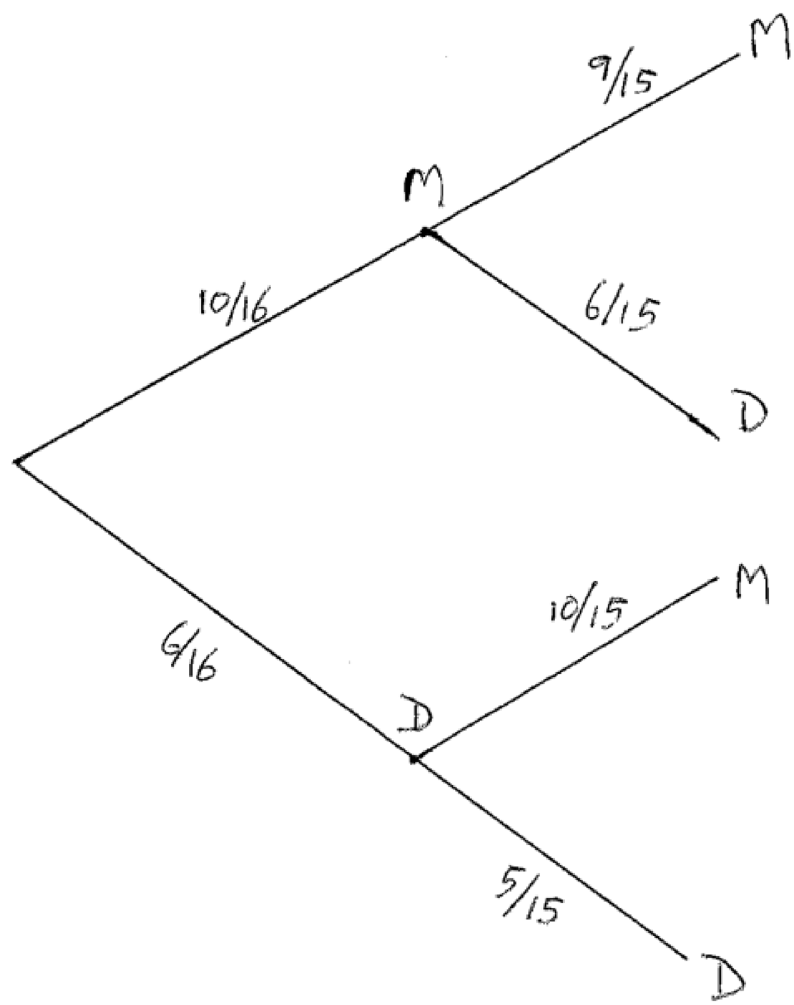
Tim and Caz buy a box of 16 chocolates of which 10 are milk and 6 are dark. Caz randomly takes a chocolate and eats it. Then Tim randomly takes a chocolate and eats it.

a. Draw a tree diagram representing the possible outcomes, clearly labelling each branch with the correct probability. [3]

b. Find the probability that Tim and Caz eat the same type of chocolate. [2]

Markscheme

a.



A1A1A1

[3 marks]

Note: Award ***A1*** for the initial level probabilities, ***A1*** for each of the second level branch probabilities.

b. $\frac{10}{16} \times \frac{9}{15} + \frac{6}{16} \times \frac{5}{15}$ ***(M1)***

$= \frac{120}{240} \left(= \frac{1}{2}\right)$ ***A1***

[2 marks]

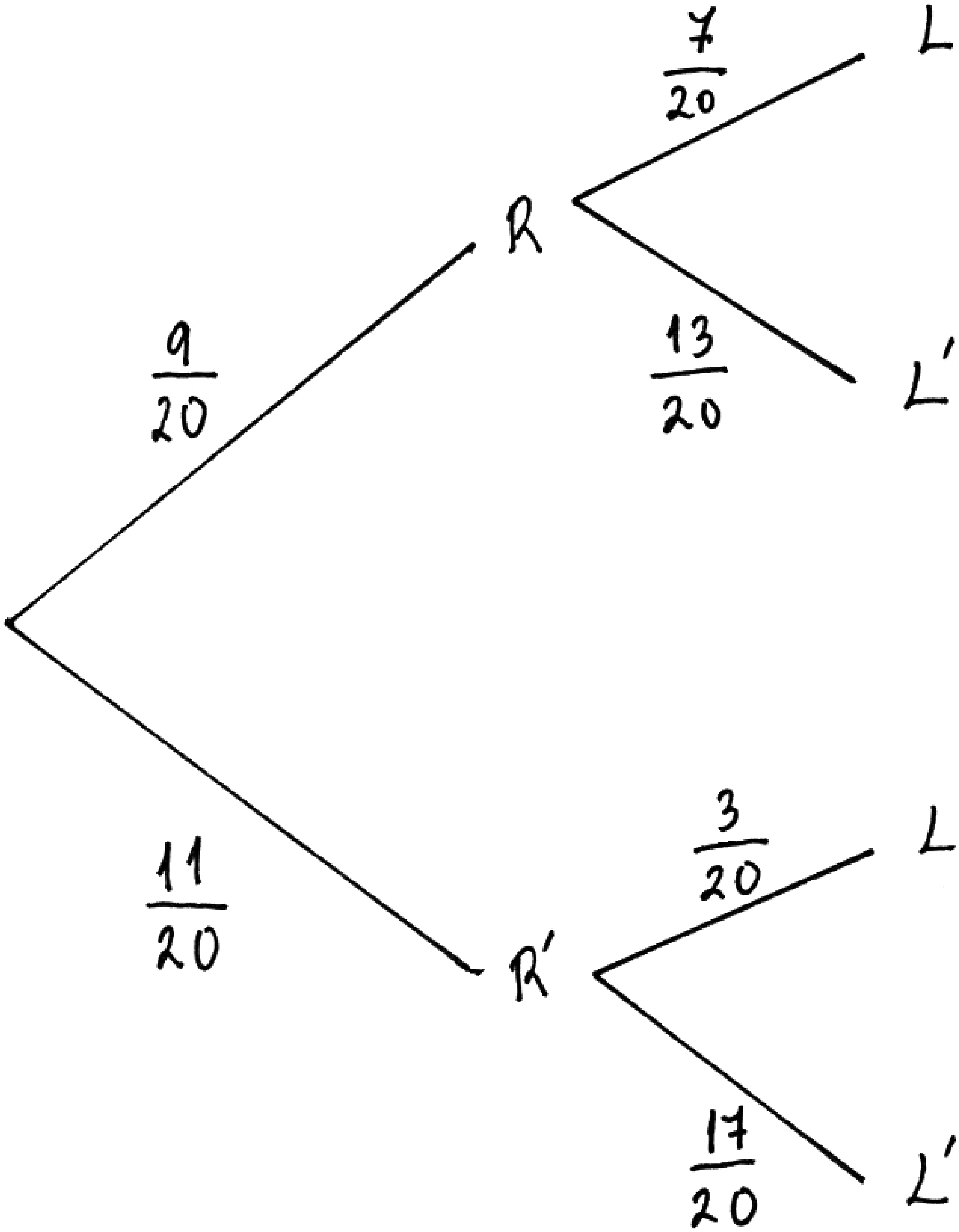
Examiners report

a. Generally well done. A few candidates didn't take account of the fact that Caz ate the chocolate, so didn't replace it. A few candidates made arithmetic errors in calculating the probability.

b. Generally well done. A few candidates didn't take account of the fact that Caz ate the chocolate, so didn't replace it. A few candidates made arithmetic errors in calculating the probability.

Jenny goes to school by bus every day. When it is not raining, the probability that the bus is late is $\frac{3}{20}$. When it is raining, the probability that the bus is late is $\frac{7}{20}$. The probability that it rains on a particular day is $\frac{9}{20}$. On one particular day the bus is late. Find the probability that it is not raining on that day.

Markscheme



(A1)

$\mathrm{P}(R' \cap L) = \frac{11}{20} \times \frac{3}{20}$ ***A1***

$\mathrm{P}(L) = \frac{9}{20} \times \frac{7}{20} + \frac{11}{20} \times \frac{3}{20}$ ***A1***

$\mathrm{P}(R'|L) = \frac{\mathrm{P}(R' \cap L)}{\mathrm{P}(L)}$ ***(M1)***

$= \frac{33}{96} \left(= \frac{11}{32}\right)$ ***A1***

[5 marks]

Examiners report

This question was generally well answered with candidates who drew a tree diagram being the most successful.

On a particular day, the probability that it rains is $\frac{2}{5}$. The probability that the "Tigers" soccer team wins on a day when it rains is $\frac{2}{7}$ and the probability that they win on a day when it does not rain is $\frac{4}{7}$.

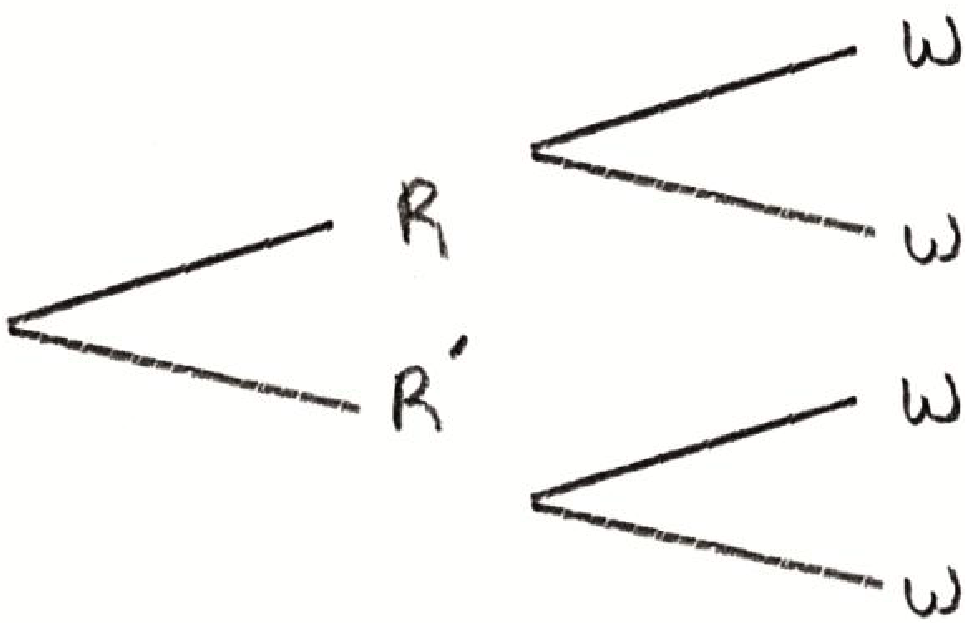
a. Draw a tree diagram to represent these events and their outcomes. [1]

b. What is the probability that the "Tigers" soccer team wins? [2]

c. Given that the "Tigers" soccer team won, what is the probability that it rained on that day? [2]

Markscheme

a. let R be "it rains" and W be "the 'Tigers' soccer team win"



A1

[1 mark]

b. $\mathrm{P}(W) = \frac{2}{5} \times \frac{2}{7} + \frac{3}{5} \times \frac{4}{7}$ ***(M1)***

$= \frac{16}{35}$ ***A1***

[2 marks]

c. $\mathrm{P}(R|W) = \frac{\frac{2}{5} \times \frac{2}{7}}{\frac{16}{35}}$ ***(M1)***

$= \frac{1}{4}$ ***A1***

[2 marks]

Examiners report

a. This question was well answered in general.

b. This question was well answered in general.

c. This question was well answered in general.

A continuous random variable T has probability density function f defined by

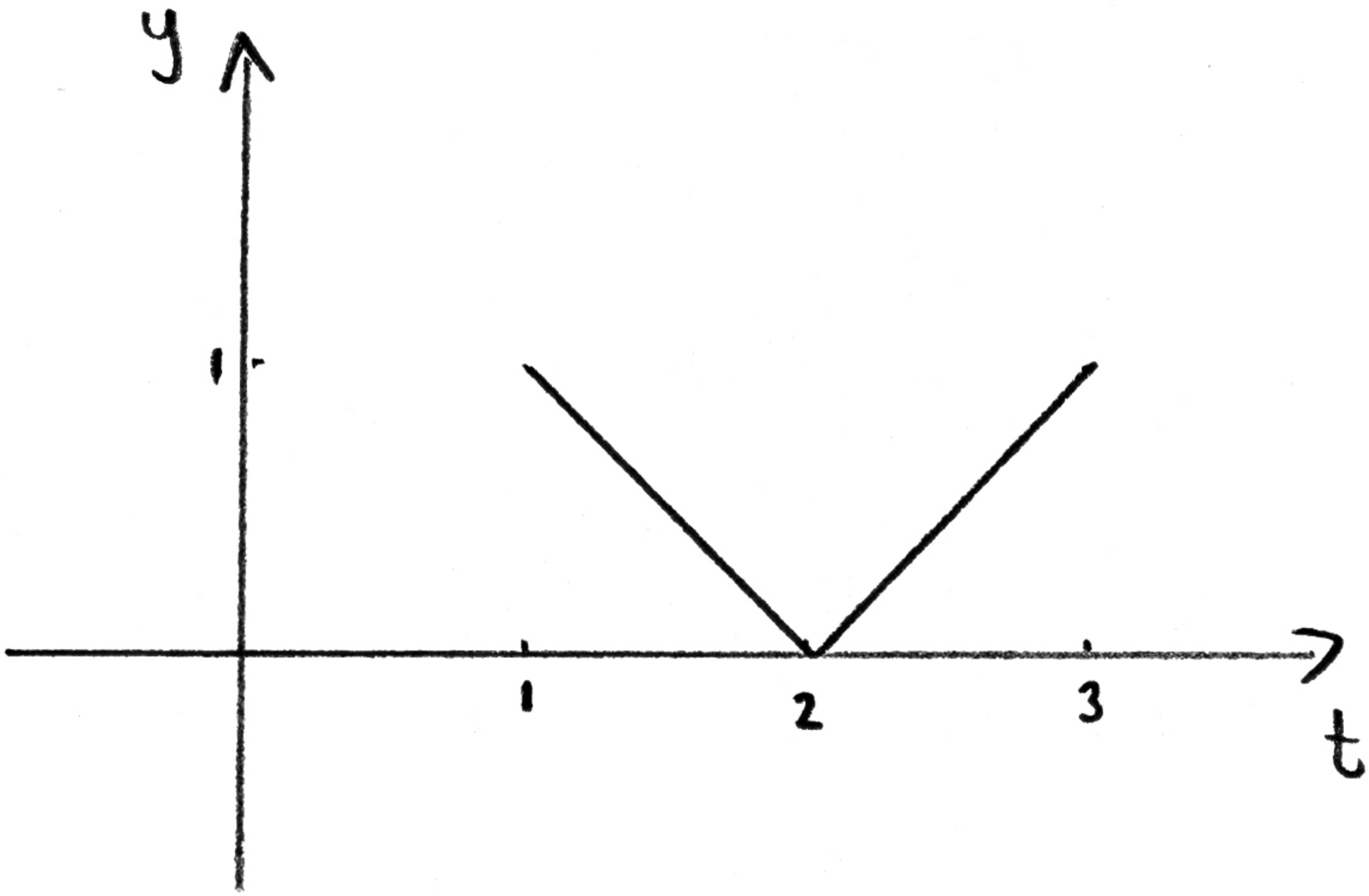
$$f(t) = \begin{cases} |2-t|, & 1 \le t \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

a. Sketch the graph of $y = f(t)$. [2]

b. Find the interquartile range of T. [4]

Markscheme

a.



$|2-t|$ correct for $[1,\ 2]$ ***A1***

$|2-t|$ correct for $[2,\ 3]$ ***A1***

b. **EITHER**

let q_1 be the lower quartile and let q_3 be the upper quartile

let $d = 2 - q_1$ $(= q_3 - 2)$ and so $\mathrm{IQR} = 2d$ by symmetry

use of area formulae to obtain $\frac{1}{2}d^2 = \frac{1}{4}$

(or equivalent) ***M1A1***

$d = \frac{1}{\sqrt{2}}$ or the value of at least one q. ***A1***

OR

let q_1 be the lower quartile

consider $\int_1^{q_1} (2 - t)\mathrm{d}t = \frac{1}{4}$ ***M1A1***

obtain $q_1 = 2 - \frac{1}{\sqrt{2}}$ ***A1***

THEN

$\mathrm{IQR} = \sqrt{2}$ ***A1***

Note: Only accept this final answer for the ***A1***.

[4 marks]

Total [6 marks]

Examiners report

a. The sketched graphs were mostly acceptable, but sometimes scrappy.

b. Most candidates had some idea about the upper and lower quartiles, but some were rather vague about how to calculate them for this probability density function. Even those who integrated for the lower quartile often made algebraic mistakes in calculating its value.