HAESE MATHEMATICS

Mathematics

for the international student

6

MYP 1

second edition

Michael Haese
Sandra Haese
Mark Humphries
Edward Kemp
Pamela Vollmar

for use with

IB Middle Years Programme

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Specialists in mathematics publishing

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MATHEMATICS FOR THE INTERNATIONAL STUDENT 6
MYP 1 second edition

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FOREWORD

MYP 1 second edition has been designed and written for the IB Middle Years Program (MYP) Mathematics framework.

This book may also be used as a general textbook at about 6th Grade level in classes where students complete a rigorous course in mathematics. We have developed this book independently of the International Baccalaureate Organization (IBO) in consultation with experienced teachers of IB Mathematics. The text is not endorsed by the IBO.

It is not our intention that each chapter be worked through in full. Teachers must select carefully, according to the abilities and prior knowledge of their students, to make the most efficient use of time and give as thorough coverage of content as possible.

Each chapter begins with an Opening Problem, offering an insight into the application of the mathematics that will be studied in the chapter. Important information and key notes are highlighted, while worked examples provide step-by-step instructions with concise and relevant explanations. Discussions, Activities, Investigations, Puzzles, and Research exercises are used throughout the chapters to develop understanding, problem solving, and reasoning, within an interactive environment.

We understand the emphasis that the IB MYP places on the six Global Contexts, and in response there are online links to ideas for projects and investigations to help busy teachers (see p. 10).

Frequent use of the interactive online features should nurture a much deeper understanding and appreciation of mathematical concepts. The inclusion of our Self Tutor software (see p. 4) is intended to help students who have been absent from classes or who experience difficulty understanding the material.

The book contains many problems to cater for a range of student abilities and interests, and efforts have been made to contextualise problems so that students can see the practical applications of the mathematics they are studying.

We welcome your feedback. Email: info@haesemathematics.com.au
Web: www.haesemathematics.com.au

PMH, SHH, MH, EK, PV

ACKNOWLEDGEMENTS

The authors and publishers would like to thank all those teachers who have read proofs and offered advice and encouragement.

EXTENSION QUESTIONS

Extension questions throughout the textbook are marked in red.

ONLINE FEATURES

There are a range of interactive features which are available online.
With the purchase of a new hard copy textbook, you will gain 15 months subscription to our online product.
This subscription can be renewed annually for a small fee.

COMPATIBILITY

For iPads, tablets, and other mobile devices, the interactive features may not work. However, the electronic version of the textbook and additional chapters can be viewed online using any of these devices.

REGISTERING

You will need to register to access the online features of this textbook.
Visit www.haesemathematics.com.au/register and follow the instructions. Once you have registered, you can:

- activate your electronic textbook
- use your account to make additional purchases.

To activate your electronic textbook, contact Haese Mathematics. On providing proof of purchase, your electronic textbook will be activated. **It is important that you keep your receipt as proof of purchase.**

For general queries about registering and licence keys:

- Visit our Snowflake help page: http://snowflake.haesemathematics.com.au/help
- Contact Haese Mathematics: info@haesemathematics.com.au

ONLINE VERSION OF THE TEXTBOOK

The entire text of the book can be viewed online, allowing you to leave your textbook at school.

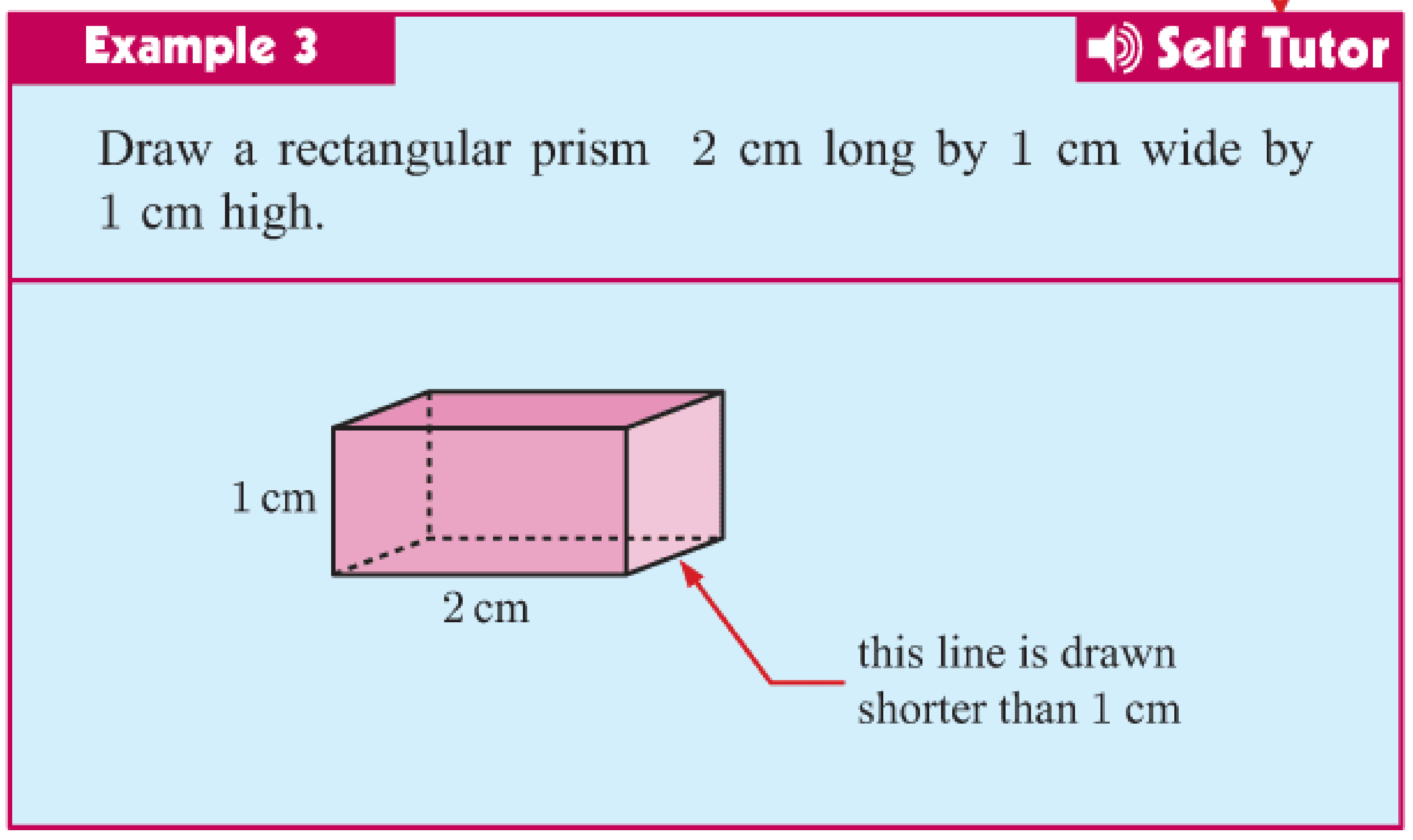
SELF TUTOR

Self Tutor is an exciting feature of this book.

The 🔊 **Self Tutor** icon on each worked example denotes an active online link.

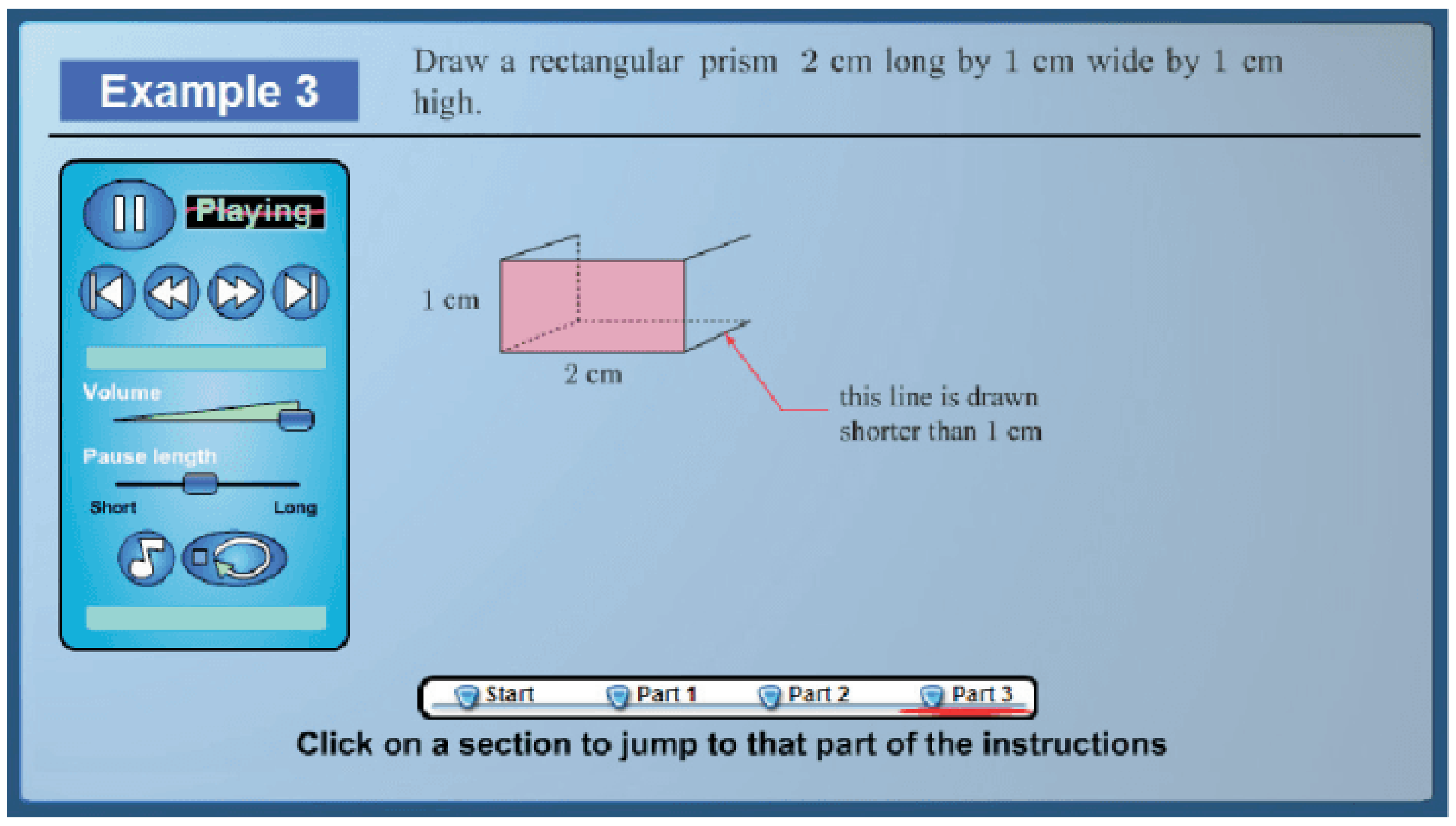
> Simply ‘click’ on the 🔊 **Self Tutor** (or anywhere in the example box) to access the worked example, with a teacher’s voice explaining each step necessary to reach the answer.
>
> Play any line as often as you like. See how the basic processes come alive using movement and colour on the screen.

For example:



Example 3 **Self Tutor**

Draw a rectangular prism 2 cm long by 1 cm wide by 1 cm high.



Example 3 Draw a rectangular prism 2 cm long by 1 cm wide by 1 cm high.

Click on a section to jump to that part of the instructions

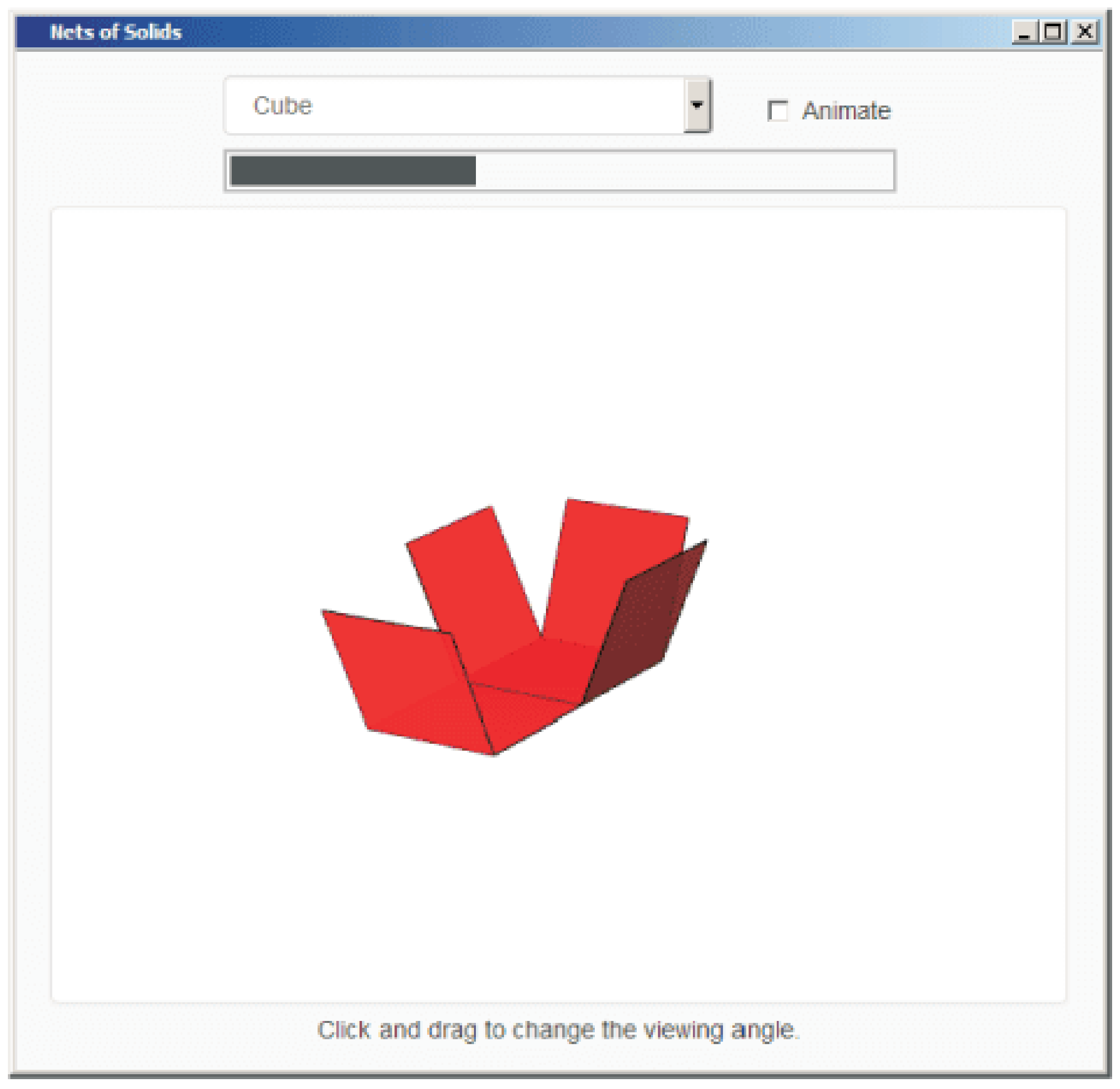
See **Chapter 5**, **Geometric shapes**, p. 97

INTERACTIVE LINKS

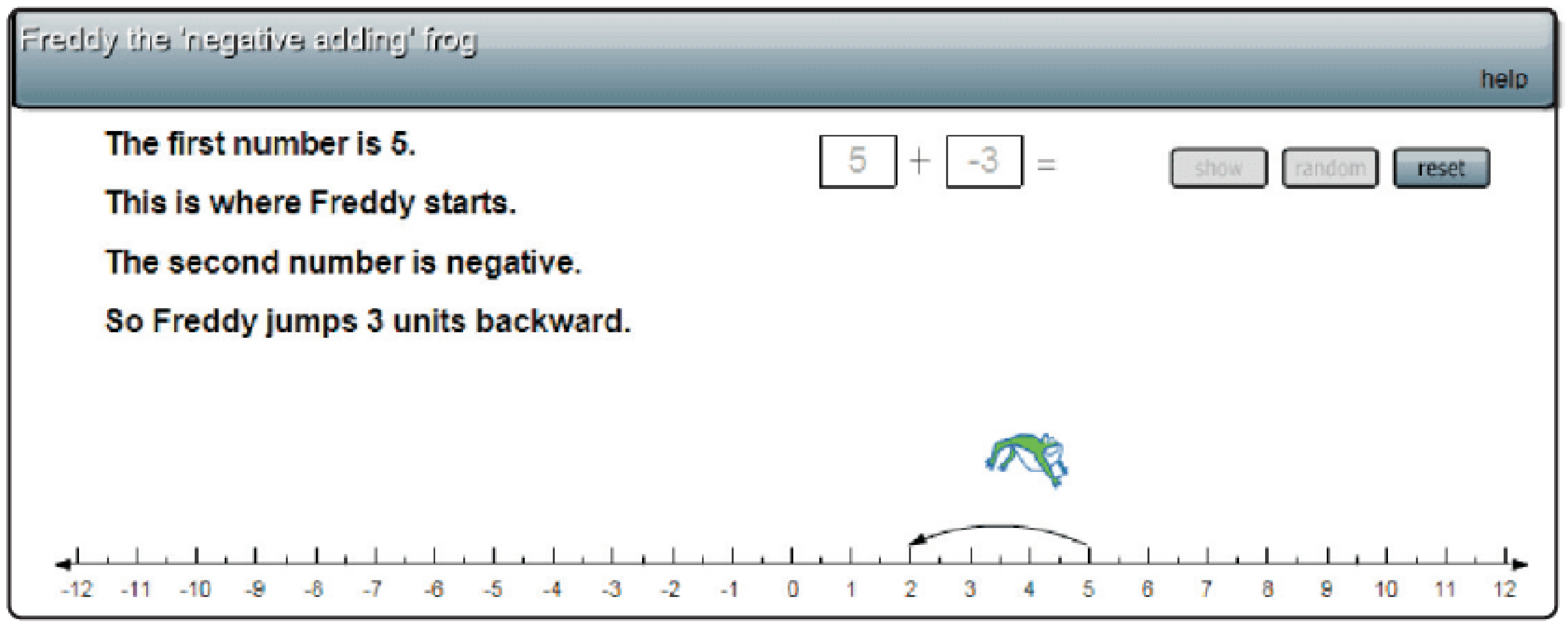
Throughout your electronic textbook, you will find interactive links to:

- Statistics packages
- Geometry packages
- Games
- Demonstrations
- Printable pages

CLICK ON THESE ICONS ONLINE



Click and drag to change the viewing angle.



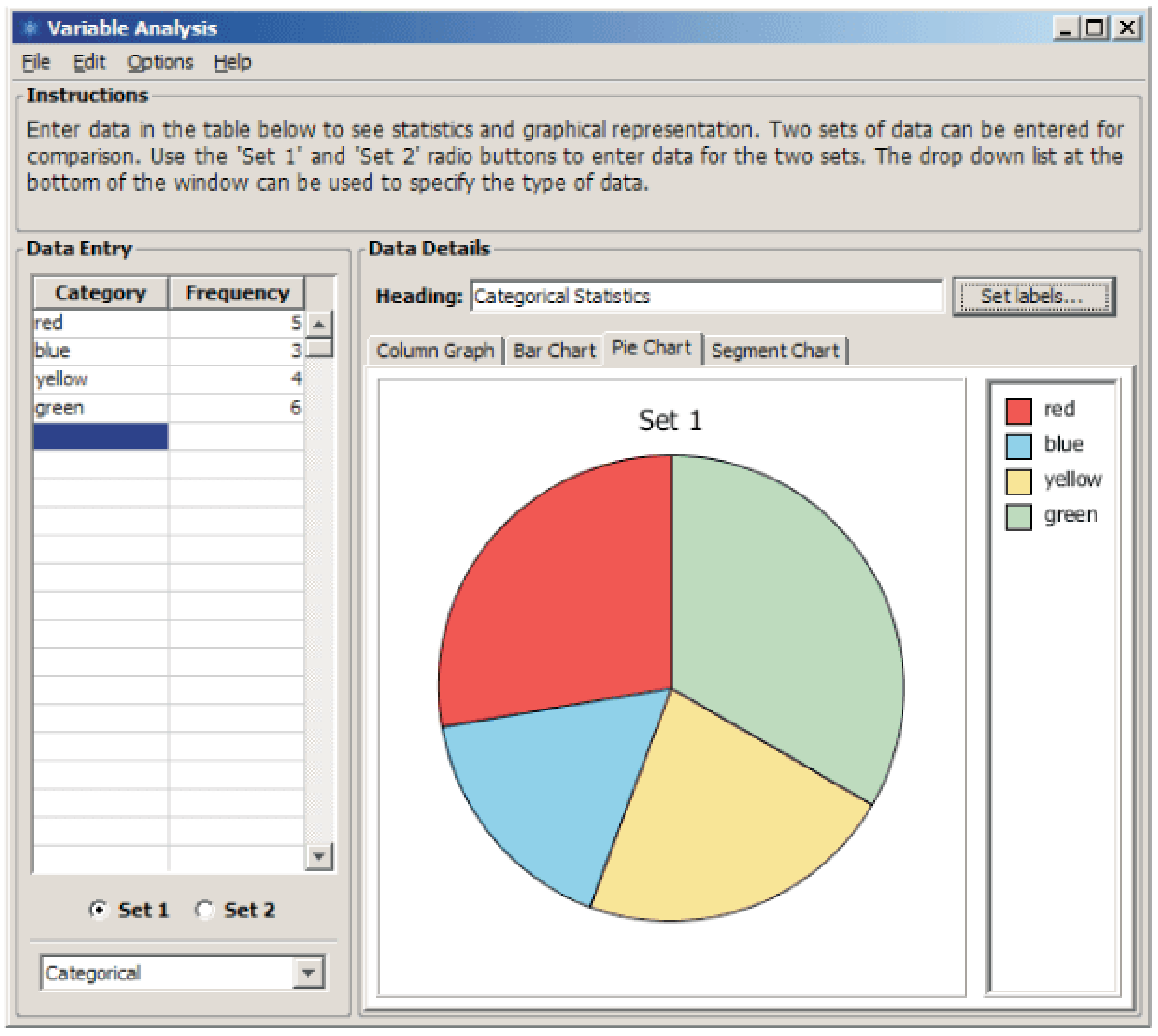
The first number is 5.

This is where Freddy starts.

The second number is negative.

So Freddy jumps 3 units backward.

5 + -3 =



Instructions

Enter data in the table below to see statistics and graphical representation. Two sets of data can be entered for comparison. Use the 'Set 1' and 'Set 2' radio buttons to enter data for the two sets. The drop down list at the bottom of the window can be used to specify the type of data.

Data Entry

Category	Frequency
red	5
blue	3
yellow	4
green	6

● Set 1 ○ Set 2

Categorical

Data Details

Heading: Categorical Statistics

Enter the size of the angle marked red to set sail for the next fishing spot.

$10

degrees

Actual angle: 0°

Angle guessed: 0°

Error: 0°

TABLE OF CONTENTS

COMMAND WORDS

This is a list of the command words used in this series of textbooks. It is probably not complete, but should certainly contain the most common and important words students will meet in the course.

We understand that in English, words often have multiple meanings, but we have tried to use our command words in the simplest mathematical context. For example, when we use the word *reflect*, it should always be in the context of a geometrical transformation, rather than to reflect on your experiences. This is to help students for whom English is a second language as best we can. Please contact us at info@haesemathematics.com.au if you have any complaints or suggestions in this area.

MORE COMMAND WORDS

Command Word	Description	Examples
Check	You should check your answer is correct by using a different method from what you did originally. This may involve using a calculator, or by substituting a numerical answer back into the original equation it came from.	Check your answer ....
Determine	You need to make a calculation or perform another mathematical procedure in order to obtain a result. The answer cannot generally be found by inspection only. See also: Find, Hence	Determine the cost of ....
Discuss	You should write your thoughts on the topic in sentence form. You do not generally need to write an essay, but your answer must be complete. The number of marks allocated to the question should indicate how much you are expected to write. Your discussion may include observations, explanations, and conclusions.	Discuss the reliability of your estimate .... Discuss the behaviour of the function ....
Draw	You may be asked to draw a graph, diagram, or geometrical figure. You do not need to include construction lines, but you may be asked to draw the diagram to scale. It should be accurate enough for observations to be made. All known information should be marked on the diagram.	Draw the graph of ....
Estimate	We may decide to find an estimate or approximation of a value if the exact answer cannot be found, or if we only need to have an idea of its size in round figures, or if we want to quickly check that an answer is reasonable. You will often be asked to estimate an answer to a particular number of significant figures or decimal places. See also: Round	Estimate the value of ....
Explain	You need to provide reasons for your answer or to justify a conclusion you have made.	Explain your answer ....
Express	You need to write your answer in a particular form. This may be a sentence, a fraction, a decimal, or a percentage, or else using particular units, or to a particular degree of accuracy.	Express as a percentage:

Command Word	Description	Examples
Find	You need to calculate a particular value, or provide an expression or formula with a given property. The answer cannot generally be found by inspection. You may be required to provide reasons for your answer.	Find the value of .... Find an expression for ....
Hence	You must use the information given or found in the previous part(s) of the question to answer this part. If you use another method to answer the question, you will lose marks.	Hence find .... Hence show that ....
Measure	Most commonly you will be asked to use a ruler to find a length, or a protractor to find the size of an angle. Sometimes you may be asked to use another measuring device to measure mass, time, or speed.	Measure each side .... Measure these angles ....
Predict	Observe a pattern in a sequence of numbers or diagrams to form a general conclusion.	Predict a formula for ....
Round	Rather than giving an exact answer, you should approximate it to the given accuracy. This may be to the nearest thousand or ten or whole number, or a number of significant figures or decimal places.	Round your answer to ....
Show	This is a less formal version of "prove". You may not need to produce a complete argument as to why something is true (unless reasons are asked for) and you may not need to consider all cases. It may be sufficient to verify a result by substitution.	Show that ....
Simplify	In algebra, you are expected to remove the brackets and collect the "like" terms in an expression. You may also be asked to simplify a fraction, ratio, or rate so it involves the smallest possible whole numbers and is therefore easiest to work with.	Simplify by collecting like terms ....
Sketch	You need to illustrate the general features of an object or function. Your drawing does not need to be to scale, but it must show the general trends and the key features. It must be appropriately labelled.	Sketch the graph of ....
Solve	Find the value(s) of the variable(s) which make a given equation or set of equations true.	Solve for x:
State	You are only required to give an answer, not to explain it or discuss it. In general, the answer will be found by inspection only.	State the y-intercept of the given graph ....
Use	You need to follow the instructions or method that follows. You will lose marks if you do not, or at the very least cost yourself time.	Use technology to .... Use the cosine rule to ....
Write	This command is usually used to tell you what form your answer is to be given in.	Write as a single fraction .... Write in decimal form ....

GLOBAL CONTEXTS

The International Baccalaureate Middle Years Programme focuses teaching and learning through six Global Contexts:

- Identities and relationships
- Orientation in space and time
- Personal and cultural expression
- Scientific and technical innovation
- Globalisation and sustainability
- Fairness and development

Click on the heading to access the online link.

The Global Contexts are intended as a focus for developing connections between different subject areas in the curriculum, and to promote an understanding of the interrelatedness of different branches of knowledge and the coherence of knowledge as a whole.

Global context

click here

Cicadas

Statement of inquiry:	Mathematics can be used to explain occurrences in nature.
Global context:	Orientation in space and time
Key concept:	Form
Related concepts:	Generalisation, Pattern
Objectives:	Investigating patterns, Applying mathematics in real-life contexts
Approaches to learning:	Communication, Research

There are six projects in this book, one for each of the Global Contexts:

Chapter 2:	Whole numbers p. 48	**FAMILY TREES** Identities and relationships
Chapter 4:	Number properties p. 81	**CICADAS** Orientation in space and time
Chapter 5:	Geometric shapes p. 103	**PLATONIC SOLIDS** Scientific and technical innovation
Chapter 8:	Measurement p. 176	**CALCULATING YOUR CARBON FOOTPRINT** Globalisation and sustainability
Chapter 15:	Statistics p. 304	**CLOTHING SIZES** Fairness and development
Chapter 16:	Transformations p. 325	**THE ISHTAR GATE** Personal and cultural expression

Each project contains a series of questions, divided into:

- Factual questions (in green)
- Conceptual questions (in blue)
- Debatable questions (in red).

These questions should help guide the unit of work.

The projects are also accompanied by the general descriptor and a task-specific descriptor for each of the relevant assessment criteria, to help teachers assess the unit of work.

Chapter 1

Number systems

Contents:

OPENING PROBLEM

Look at this group of stars.

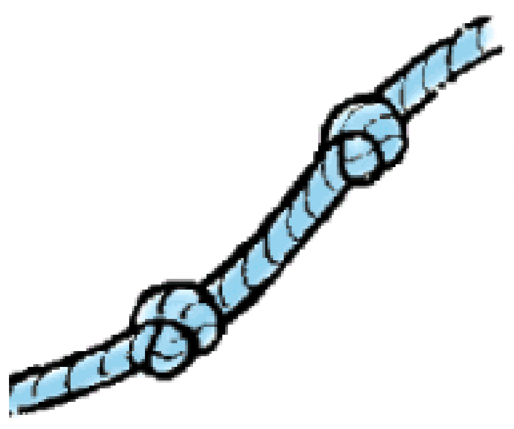
Things to think about:

- **a** How many stars are there? How would you write down the number of stars?
- **b** How many *digits* are in the number you have written?
- **c** Is the order in which the digits are written important?
- **d** What does each digit represent?

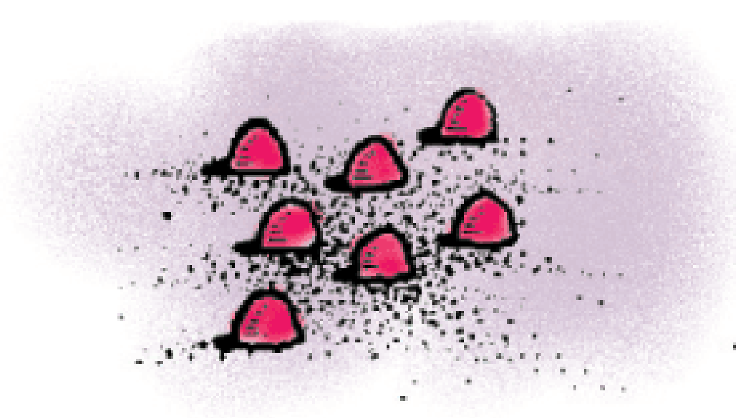
A DIFFERENT NUMBER SYSTEMS

In ancient times, people used items to represent numbers:

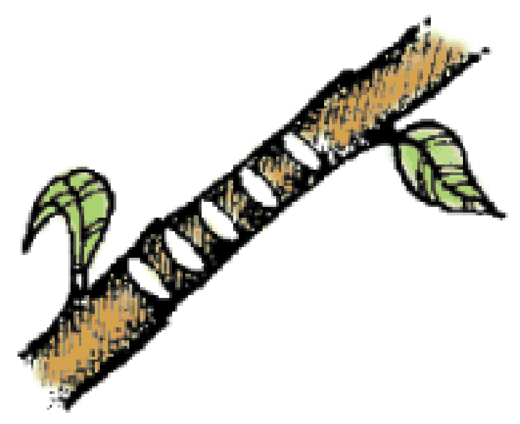
scratches on a cave wall showed the number of new moons since the buffalo herd came through



knots on a rope showed the number of corn rows planted



pebbles on the sand showed the number of traps set for fish



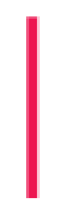
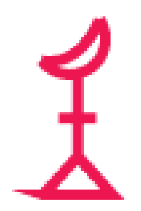
notches cut on a branch showed the number of new lambs born

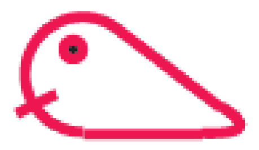
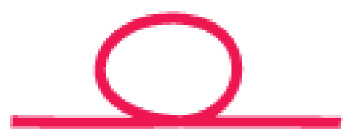
In time, humans learned to write numbers more efficiently. They did this by developing **number systems**.

THE EGYPTIAN NUMBER SYSTEM

Archaeological evidence suggests that the Ancient Egyptians used a detailed number system at least 5600 years ago. The symbols used to represent numbers were pictures of everyday things. These symbols are called **hieroglyphics**, which means sacred picture writings.

The Egyptians used a tally system based on the number ten. Ten of one symbol could be replaced by one of another symbol. We call this a **base ten system**.

1	10	100	1000
			
staff	hock	scroll	lotus flower

10 000	100 000	1 000 000	10 000 000
			
bent stick	burbay fish	astonished man	religious symbol

The value of a number could be found by adding the values of the symbols used. The order in which the symbols were written down did not affect the value of the number represented.

So, ∩∩∩||||| or |||||∩∩∩ would still represent 35.

The Egyptian number system did not have **place values**.

Example 1 🔊 **Self Tutor**

a What number is represented by 𓆼∩∩∩|||||| ?

b Write the Egyptian symbols for 324.

a $\underbrace{\text{𓆼}}_{1000}\ \underbrace{\text{∩∩∩}}_{\text{3 lots of 10}}\ \underbrace{\text{||||||}}_{\text{6 lots of 1}} = 1036$

b $324 = \underbrace{\text{𓍢𓍢𓍢}}_{\text{3 lots of 100}}\ \underbrace{\text{∩∩}}_{\text{2 lots of 10}}\ \underbrace{\text{||||}}_{\text{4 lots of 1}}$

EXERCISE 1A.1

1 Write down the number represented by:

a ∩∩∩∩∩∩∩|||||

b 𓍢𓍢𓍢∩∩∩∩

c 𓆼𓍢𓍢𓍢𓍢𓍢𓍢||||||||

d 𓆼𓆼𓆼𓆼𓍢∩∩∩∩∩∩||

e 𓂭𓂭𓂭𓆼𓍢𓍢𓍢𓍢∩∩|||||||

f 𓇳𓁨𓂭𓂭𓆼𓍢𓍢𓆼𓇳𓆐𓆼𓍢𓍢∩∩|||

2 Write the Egyptian symbols for:

a 42
b 50
c 108
d 213
e 730
f 2404
g 5007
h 6142
i 10 422
j 234 124
k 3 005 103
l 14 020 531

3 Which two of these symbol combinations have the same value?

A 𓍢∩∩||||

B ∩𓍢|||∩∩|

C 𓍢𓍢∩||∩∩

D 𓍢||||∩∩∩∩

E ||∩∩𓍢||∩

F ∩𓍢||𓍢||∩

4 In our number system today, three symbols are used to write the number 999. How many Egyptian symbols are needed to write 999?

THE ANCIENT GREEK OR ATTIC SYSTEM

The Ancient Greeks saw the need to include a symbol for 5. This symbol was combined with the symbols for 10, 100, and 1000 to make 50, 500, and 5000.

Some examples of Ancient Greek numbers are given below.

1 |
2 |
3 |||
4 ||||
5 Γ
6 Γ|
7 Γ||
8 Γ|||
9 Γ||||
10 Δ
20 ΔΔ
30 ΔΔΔ
50 𐅄
60 𐅄Δ
100 H
400 HHHH
500 𐅅
700 𐅅HH
1000 X
5000 𐅆

Δ, H, and X are combined with the symbol Γ for 5 to make 50, 500, and 5000.

Example 2 **Self Tutor**

a What number is represented by 𐅆X𐅅HH𐅄ΔΔΔ| ?

b Write the Ancient Greek symbols for 825.

a $\underbrace{\text{𐅆X}}_{6000}\ \underbrace{\text{𐅅HH}}_{700}\ \underbrace{\text{𐅄ΔΔΔ}}_{80}\ \underbrace{|}_{1} = 6781$

b $825 = \underbrace{\text{𐅅HHH}}_{800}\ \underbrace{\text{ΔΔ}}_{20}\ \underbrace{\text{Γ}}_{5}$

EXERCISE 1A.2

1 Write down the number represented by:

a Δ|||
b ΔΔΓ|
c HH||||
d XXXHHΔΔΔΔ
e 𐅅HHΔΔ|||
f 𐅆HH𐅄Γ||||

2 Write the Ancient Greek symbols for:

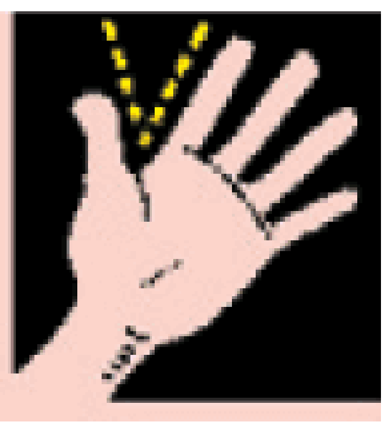
a 14
b 31
c 27
d 53
e 68
f 99
g 555
h 4082
i 5601
j 7264

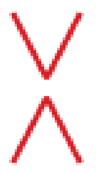
3 How many symbols are needed to write 999 in Ancient Greek?

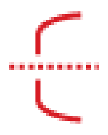
ROMAN NUMERALS

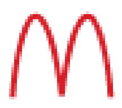
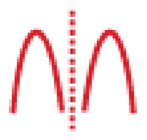
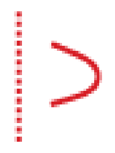
Like the Greeks, the Romans used a number for 5.

The first four numbers could be represented by the fingers on one hand, so the V formed by the thumb and forefinger of an open hand represented 5.



Two Vs joined together  became two lots of 5, so 10 was represented by X.

C represented 100, and half a  or L became 50.

1000 was represented by an  . With a little imagination,  split in half and turned on its side is  , so D became half a thousand or 500.

Some examples of Roman numerals are shown below.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	
20	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**500**	**1000**
XX	**XXX**	**XL**	**L**	**LX**	**LXX**	**LXXX**	**XC**	**C**	**D**	**M**

Notice that:

- When a smaller number comes *after* a larger number, the numbers are *added*.
  For example, VI is $5 + 1 = 6$.
- When a smaller number comes *before* a larger number, the smaller number is *subtracted* from the larger number.
  For example, IV is $5 - 1 = 4$.

Therefore, the order in which the numerals are written is important.

There were also rules for using smaller numerals before larger numerals:

- I could only appear before V or X.
- X could only appear before L or C.
- C could only appear before D or M.

999 was therefore not written as IM, but as CMXCIX.

Larger numerals were formed by placing a stroke above the symbol. This stroke made the number 1000 times larger.

5000	**10 000**	**50 000**	**100 000**	**500 000**	**1 000 000**
$\overline{\mathrm{V}}$	$\overline{\mathrm{X}}$	$\overline{\mathrm{L}}$	$\overline{\mathrm{C}}$	$\overline{\mathrm{D}}$	$\overline{\mathrm{M}}$

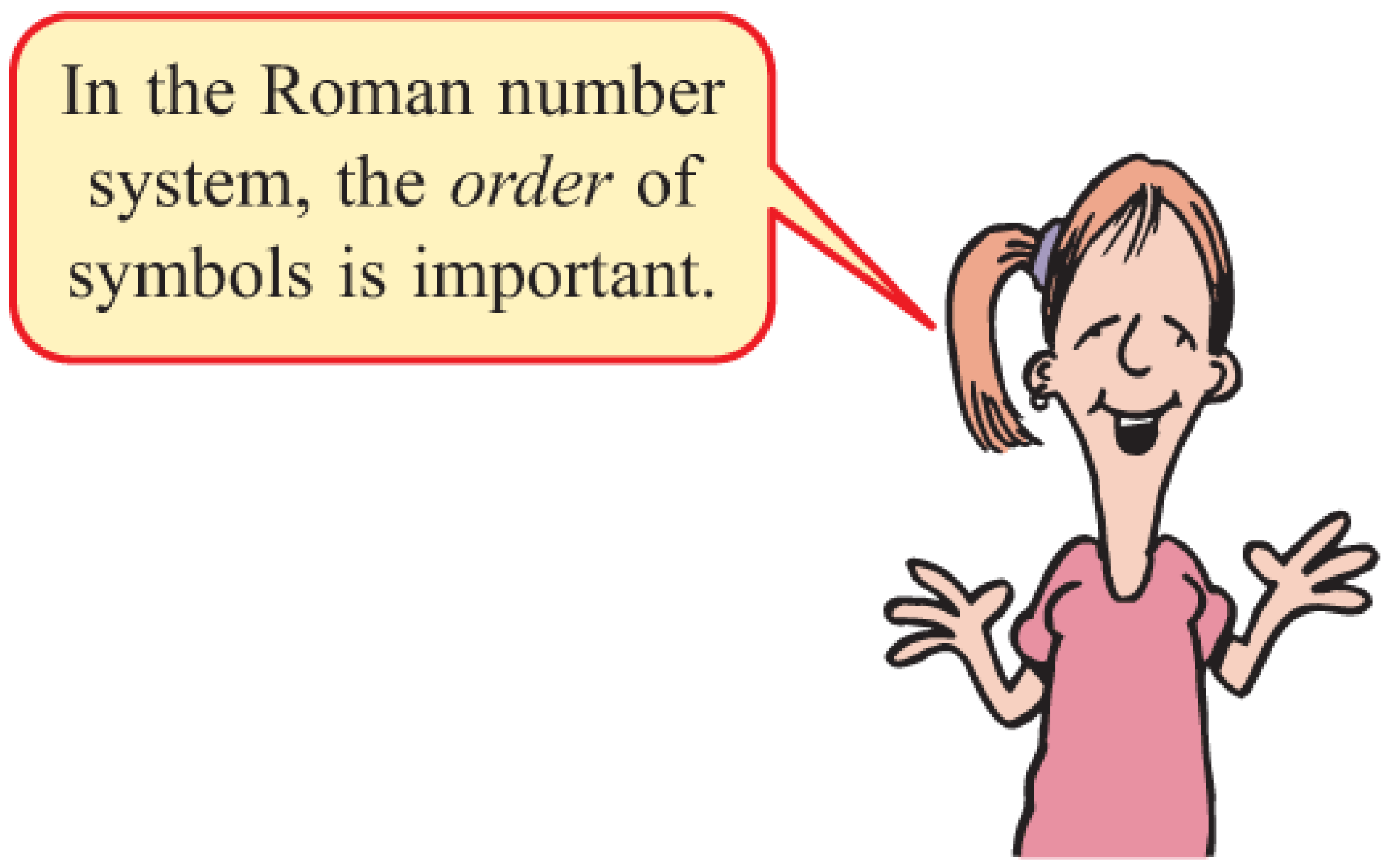
Example 3 **Self Tutor**

a What number is represented by DCXCIV?

b Write the Roman numerals for 47.

a $\underbrace{\mathrm{D\,C}}_{600}\underbrace{\mathrm{X\,C}}_{90}\underbrace{\mathrm{I\,V}}_{4} = 694$

b $47 = \underbrace{\mathrm{X\,L}}_{40}\underbrace{\mathrm{V\,I\,I}}_{7}$



EXERCISE 1A.3

1 Write down the number represented by:

a VIII **b** XIV **c** XVI **d** XXXI **e** CX
f LXXXI **g** CXXV **h** CCXVI **i** LXII **j** MCLVI
k $\overline{\text{D}}\,\overline{\text{L}}\,\text{DCV}$ **l** DCCXX **m** CDXIX **n** $\overline{\text{D}}\,\overline{\text{L}}\,\overline{\text{V}}\,\text{DI}$ **o** $\overline{\text{M}}\,\overline{\text{M}}\,\overline{\text{C}}\,\overline{\text{C}}\,\overline{\text{C}}$

2 Write the following numbers in Roman numerals:

a 18 **b** 34 **c** 65 **d** 141 **e** 279
f 902 **g** 1046 **h** 2551 **i** 6032 **j** 31 967

3 Which Roman numeral less than one hundred is written using the greatest number of symbols?

4 What is the highest Roman numeral between M and MM which uses only two symbols?

5 Use Roman numerals to answer the following questions.

- **a** Each week Octavius sharpens CCCLIV swords for his general. How many will he need to sharpen if the general doubles his order?
- **b** To finish his courtyard, Claudius needs to pay for CL pavers at VIII denarii each. How much will it cost Claudius?

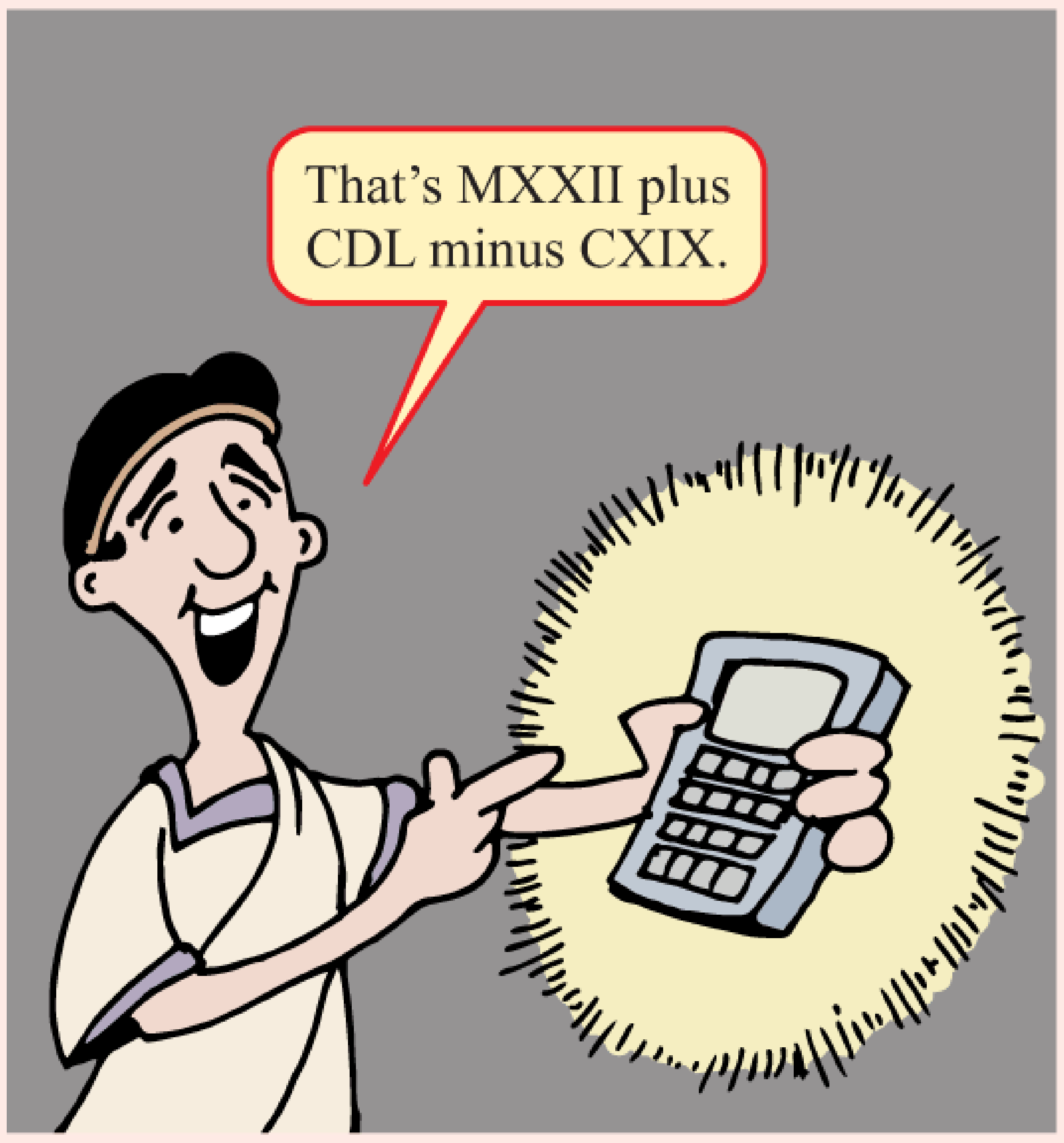
The **denarius** (plural: **denarii**) was the unit of currency used by the Romans.

ACTIVITY 1 IF YOU LIVED IN ROMAN TIMES

What to do:

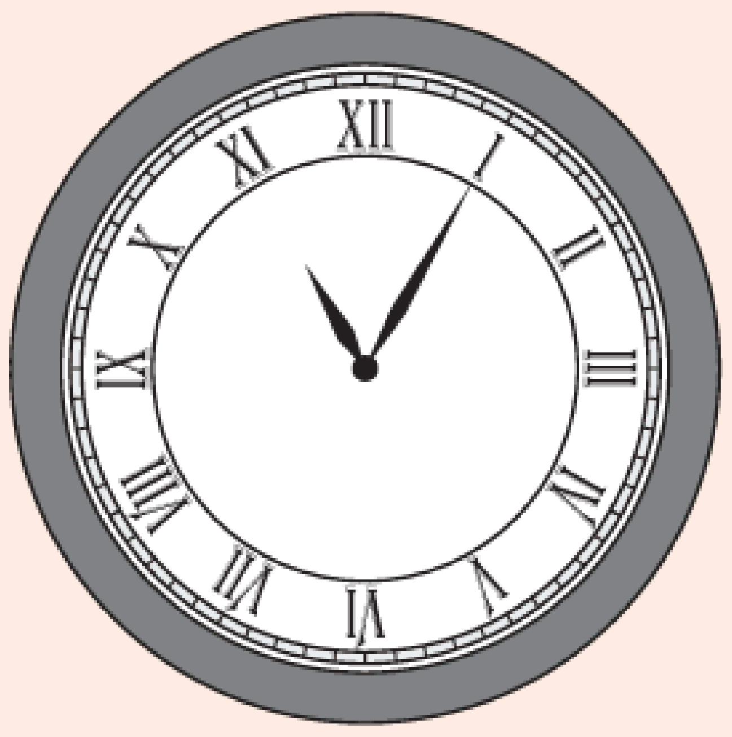
1 Use Roman numerals to write:
- **a** your house number and postcode
- **b** your height in centimetres
- **c** your phone number
- **d** the number of students in your class
- **e** the width of your desk in centimetres.

2 Use a calendar to help you write in Roman numerals:
- **a** your date of birth, for example $\underbrace{\text{XXI}}_{\text{day}}$ - $\underbrace{\text{XI}}_{\text{month}}$ - $\underbrace{\text{MCMXLVI}}_{\text{year}}$
- **b** the year in which you will turn:
  i XV **ii** L **iii** XXI **iv** C



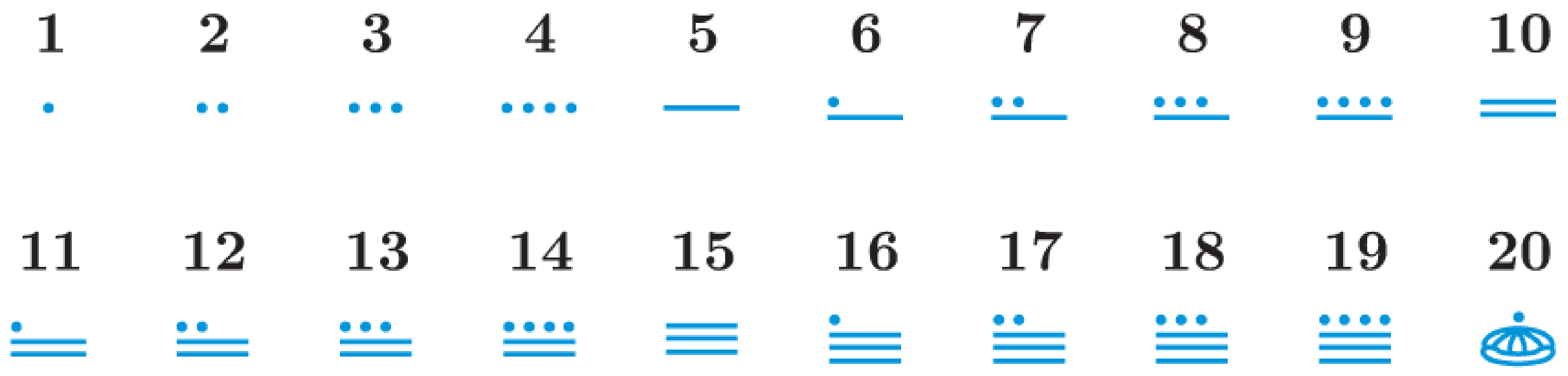
DISCUSSION

- Where do we still see Roman numerals today?
- Is it important to use Roman numerals?



THE MAYAN SYSTEM

The Mayans originally used pebbles and sticks to represent numbers. They later recorded them as dots and strokes. A stroke represented the number 5.

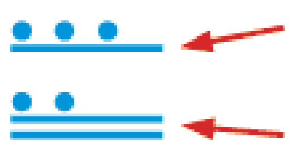


Unlike the Egyptians and Romans, the Mayans created a **place value** by placing one symbol *above* the other, with the highest place value on top.

The number system we are familiar with uses base 10. This means that the number 92 represents 9 lots of 10, plus 2 lots of 1.

In contrast, the Mayan system used base 20.

For example:



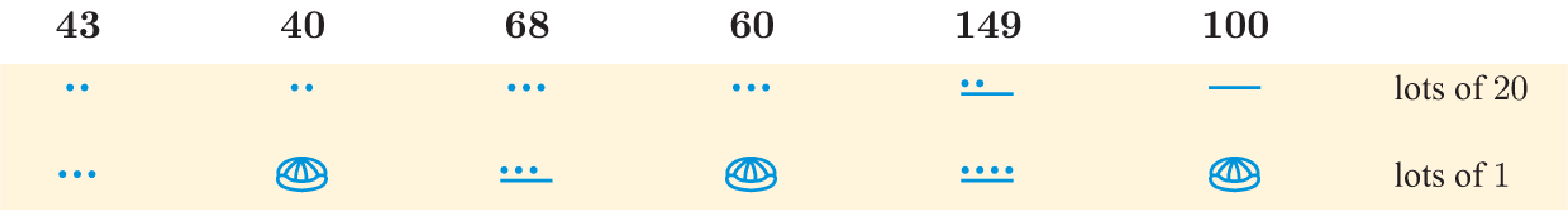
this upper part represents 8 lots of 20 or 160

this lower part represents 12 lots of 1 or 12

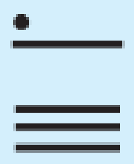
So, the number represented is 172

The Mayans also recognised the need for a number **zero** to show the difference between 'lots of 1' and 'lots of 20'. The symbol  , which represented a mussel shell, works like our zero.

Here are some more examples of Mayan symbols:

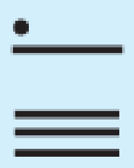


Example 4 **Self Tutor**

a What number is represented by  ?

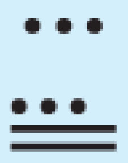
b Write the Mayan symbols for 73.

a 6 lots of 20 →
15 lots of 1 →



$6 \times 20 = 120$
$15 \times 1 = 15$

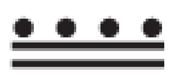
The number represented is 135

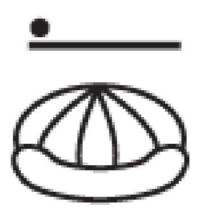
b 73 = 

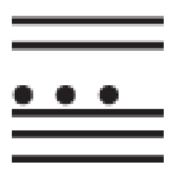
3 lots of 20
13 lots of 1

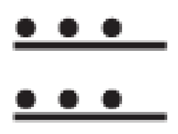
EXERCISE 1A.4

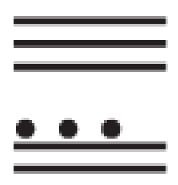
1 Write down the number represented by:

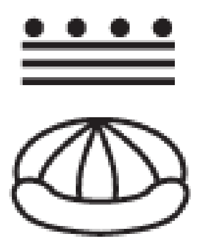
a 

b 

c 

d 

e 

f 

2 Write these numbers using Mayan symbols:

a 23 **b** 50 **c** 99 **d** 105 **e** 217 **f** 303

THE CHINESE - JAPANESE SYSTEM

The Chinese and Japanese use a base 10 place value system.

Their symbols are:

1	2	3	4	5	6
一	二	三	四	五	六
7	**8**	**9**	**10**	**100**	**1000**
七	八	九	十	百	千

For place values bigger than units, the symbols are used in pairs one on top of the other. However, you do not need to put a 一 (1) before 十 (10), 百 (100), or 千 (1000).

Example 5 **Self Tutor**

a Find the number represented by:

i 五百六十三

ii 四千九百八十三

b Write the Chinese-Japanese symbols for 2047.

a **i**

5 lots of 100 { 五 百 } = 563
6 lots of 10 { 六 十 }
3 { 三 }

ii

4 lots of 1000 { 四 千 } = 4983
9 lots of 100 { 九 百 }
8 lots of 10 { 八 十 }
3 { 三 }

b 2047 =

二 千 } 2 lots of 1000
四 十 } 4 lots of 10
七 } 7

EXERCISE 1A.5

1 Find the number represented by:

a 七百六十五

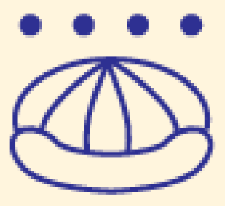
b 三千二百四十八

c 九千九百九十九

2 Write these numbers using Chinese-Japanese symbols:

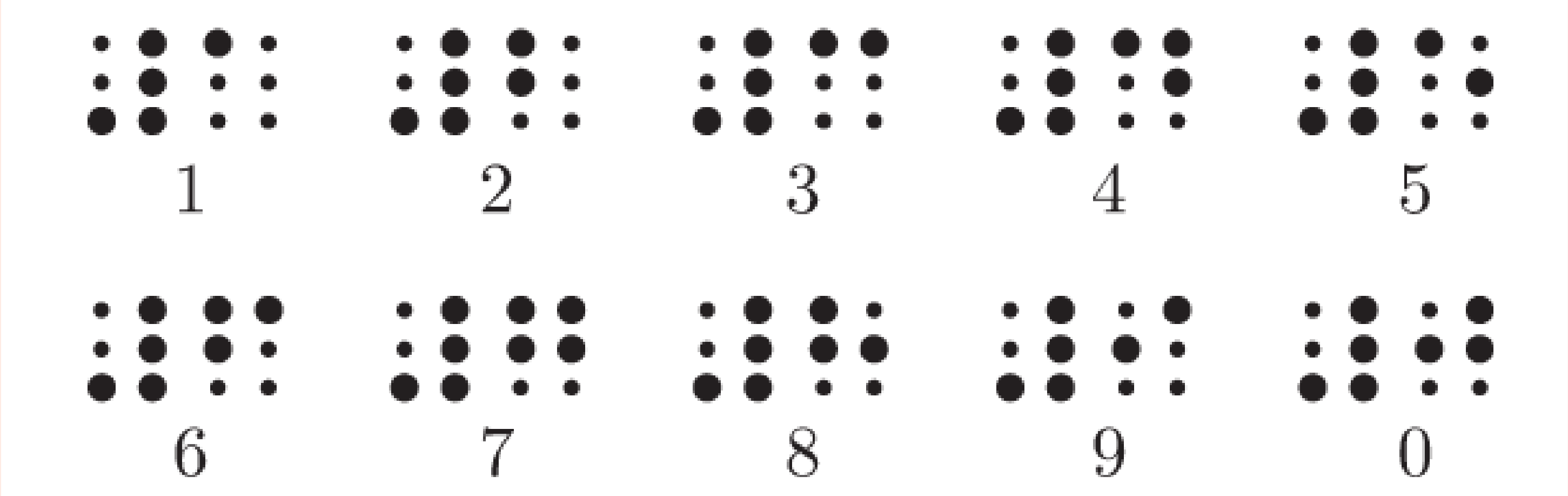
a 497 **b** 8400 **c** 1111

3 Copy and complete:

	Words	*Numeral*	*Roman*	*Egyptian*	*Mayan*	*Chinese-Japanese*
a	thirty seven	37				
b						
c			CLIX			
d						
e						二百九

RESEARCH **OTHER WAYS OF COUNTING**

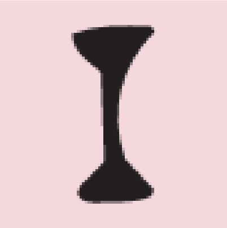
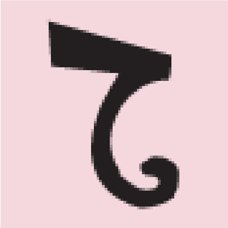
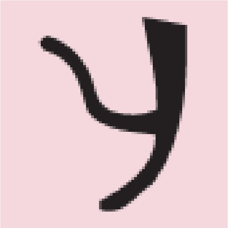
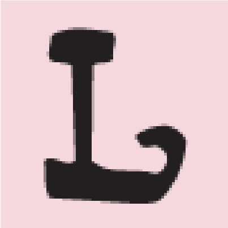
1. How did the Ancient Egyptians and Mayans represent numbers larger than 1000?
2. Did the Egyptians use a symbol for zero?
3. What are **Braille** numbers? What do they feel like?
4. How do deaf people ‘sign’ numbers?



B THE HINDU-ARABIC NUMBER SYSTEM

The number system we use today was developed in India about 2000 years ago. It was introduced to European nations by Arab traders about 1000 years later. The system is therefore called the **Hindu-Arabic** number system.

The marks we use to represent numbers are called **numerals**. They are formed using the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, which are known as **digits**.

ordinal number	one	two	three	four	five	six	seven	eight	nine
Hindu-Arabic numeral									
modern numeral	1	2	3	4	5	6	7	8	9

The digits 3 and 8 can be used to form the numeral 38 for the number ‘thirty eight’, and the numeral 83 for the number ‘eighty three’. The order of symbols is therefore important.

The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, ... are called **natural numbers**. They are also **whole numbers**, because they have no fractional or decimal part.

There is no largest natural number, so we say the set of all natural numbers is **infinite**.

The Hindu-Arabic system is more efficient than the number systems used by the Egyptians, Romans, and Mayans.

- It uses only **10 digits** to construct all the natural numbers.
- It has a **place value system** where each digit has a different value depending on the column it is written in.
- It uses the digit **0** to show an empty place value.

PLACE VALUES

The **place** or position of a digit in a number determines its value. You should be familiar with the place values shown in the table:

units	1
tens	10
hundreds	100
thousands	1000
ten thousands	10 000
hundred thousands	100 000

Using these place values, we see that 6794 is really

6 thousands + 7 hundreds + 9 tens + 4 units

or six thousand, seven hundred, and ninety four.

thousands	hundreds	tens	units
6	7	9	4

Example 6 **Self Tutor**

What number is represented by the digit 7 in:

a 374 **b** 5709 **c** 127 624?

a In 374, the 7 represents '7 lots of 10' or 70.

b In 5709, the 7 represents '7 lots of 100' or 700.

c In 127 624, the 7 represents '7 lots of 1000' or 7000.

EXERCISE 1B.1

1 Identify the digit in the 'tens' column of the following numbers:

a 57 **b** 3249 **c** 709 **d** 67 310

2 Identify the digit in the 'hundreds' column of the following numbers:

a 923 **b** 5076 **c** 531 740 **d** 909 800

3 Write down the number represented by the digit 8 in:

a 38 **b** 81 **c** 458 **d** 847
e 1981 **f** 8247 **g** 2861 **h** 28 902
i 60 008 **j** 84 019 **k** 78 794 **l** 189 964

4 Write down the place value of the 3, the 5, and the 8 in each of the following:

a 53 486 **b** 3508 **c** 50 083 **d** 805 340

5 Write down the place value of the 1, the 4, and the 7 in each of the following:

a 7014 **b** 91 487 **c** 43 761 **d** 154 978

EXPANDED FORM

When a number is written as a numeral, we call it **simplest form**.

When a number is written as a sum, we call it **expanded form**.

For example,
$$\begin{aligned} 6794 &= 6000 + 700 + 90 + 4 \\ &= 6 \times 1000 + 7 \times 100 + 9 \times 10 + 4 \qquad \{\text{expanded form}\} \end{aligned}$$

Example 7 **Self Tutor**

a Write $3 \times 10\,000 + 4 \times 1000 + 8 \times 10 + 5 \times 1$ in simplest form.

b Write 9602 in expanded form.

a $3 \times 10\,000 + 4 \times 1000 + 8 \times 10 + 5 \times 1 = 34\,085$

b $9602 = 9 \times 1000 + 6 \times 100 + 2 \times 1$

EXERCISE 1B.2

1 Write in simplest form:

- **a** $8 \times 10 + 6 \times 1$
- **b** $6 \times 100 + 7 \times 10 + 4 \times 1$
- **c** $9 \times 1000 + 6 \times 100 + 3 \times 10 + 8 \times 1$
- **d** $5 \times 10\,000 + 2 \times 100 + 4 \times 10$
- **e** $2 \times 10\,000 + 7 \times 1000 + 3 \times 1$
- **f** $3 \times 100 + 5 \times 100\,000 + 7 \times 10 + 5 \times 1$
- **g** $2 \times 100 + 7 \times 10\,000 + 3 \times 1000 + 9 \times 10 + 8 \times 1$
- **h** $8 \times 100\,000 + 9 \times 1000 + 3 \times 100 + 2 \times 1$

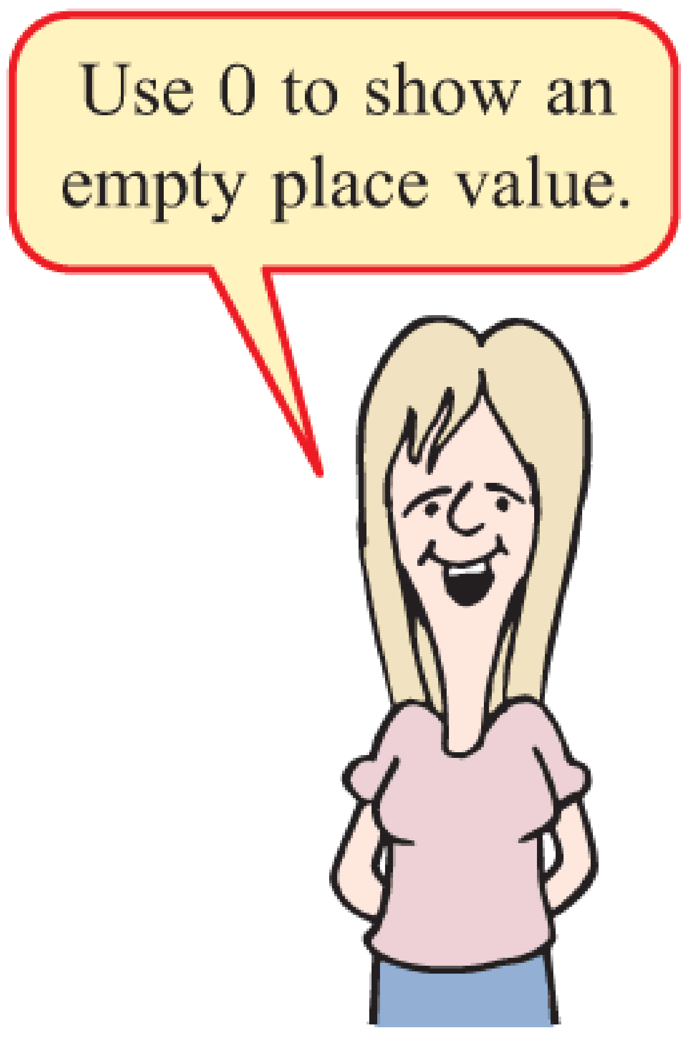
2 Write in expanded form:

- **a** 975
- **b** 680
- **c** 3874
- **d** 9083
- **e** 56 742
- **f** 75 007
- **g** 600 829
- **h** 354 718

DEMO

3 Write in numeral form:

- **a** twenty seven
- **b** eighty
- **c** six hundred and eight
- **d** one thousand and sixteen
- **e** eight thousand two hundred
- **f** nineteen thousand five hundred and thirty eight
- **g** seventy five thousand four hundred and three
- **h** six hundred and two thousand eight hundred and eighteen.



4 What number is:

- **a** one less than eight
- **b** two more than eleven
- **c** four more than seventeen
- **d** one less than three hundred
- **e** seven greater than four thousand
- **f** 3 less than 10 000?

ACTIVITY 2

PRINTABLE CARDS

This Activity is done in pairs. Each student will need a set of cards with the digits 0, 1, 2, ..., 9. The students should take it in turns to be "student A" and "student B".

What to do:

1. Student A selects four to six cards, and uses them to make a four to six digit number.
2. Student B says the number out loud, or writes out the number fully in words.
3. Student A asks student B the place value of any of the digits used.
4. Student B asks student A to rearrange the digits to form the largest number possible.

PUZZLE

When the natural numbers are spelled out in words, there is only one number whose letters are in alphabetical order.

Which number is it?

one
two
three
four
five
⋮

C BIG NUMBERS

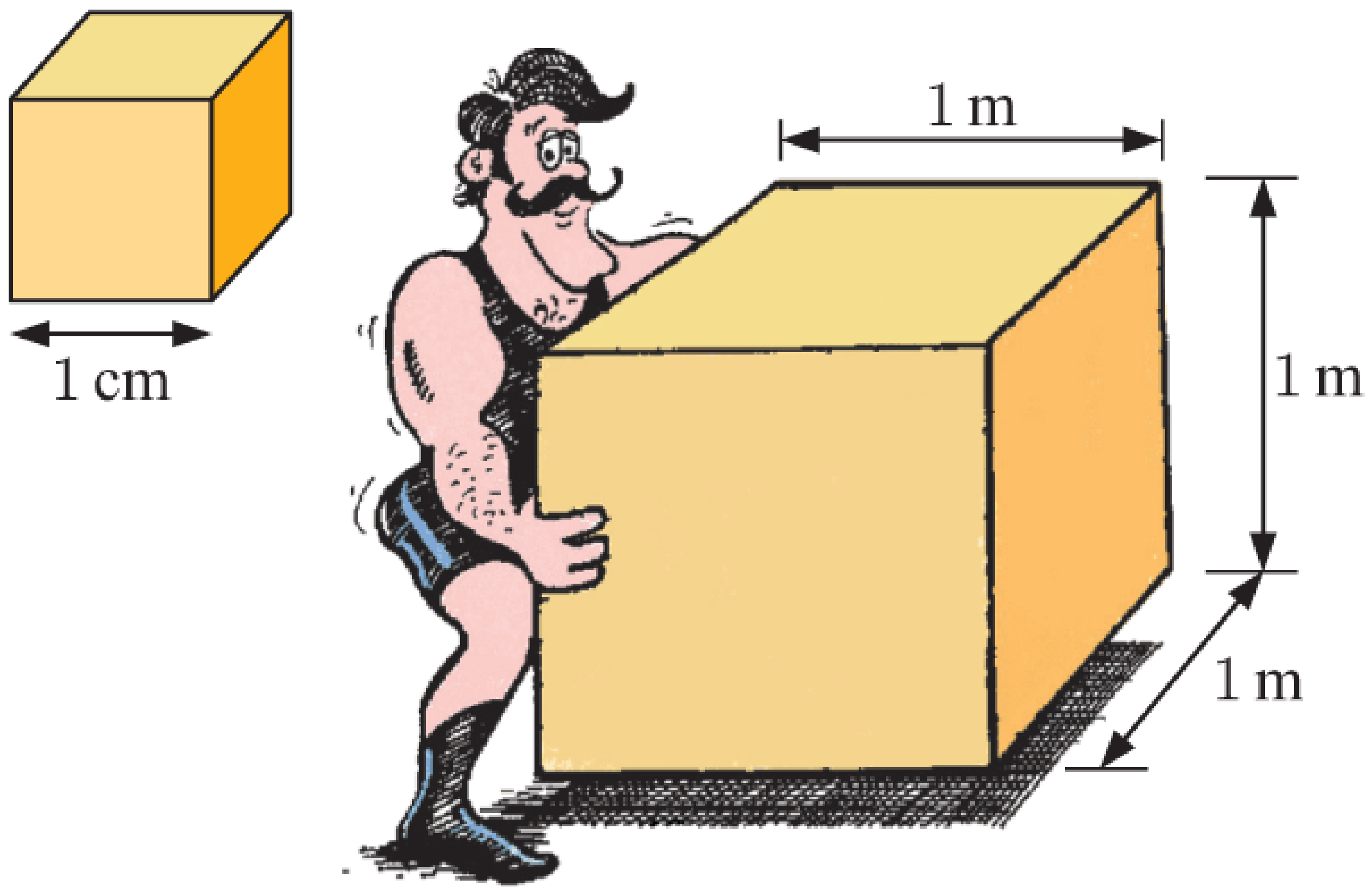
We can extend the place value system beyond hundred thousands to write even bigger numbers.

MILLIONS

A **million** is 1000 thousand, or 1 000 000.

Consider a small cube with side length 1 centimetre.

One million of these small cubes would fill a cube with side length 1 metre.



When we write big numbers on a place value chart, we arrange the digits into units, thousands, and millions.

For example:

Millions			*Thousands*			*Units*		
hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
	5	3	4	7	9	6	8	2

The number displayed in this place value chart is 53 million, 479 thousand, 682.

When we write the number as a numeral, we use a comma or a space to separate the groups: 53 479 682.

In words, we say: fifty three million, four hundred and seventy nine thousand,
six hundred and eighty two.

EXERCISE 1C.1

1 In the number 53 479 682, the digit 9 has the value 9000 and the digit 3 has the value 3 000 000. Give the value of the:

a 8 **b** 5 **c** 6 **d** 4 **e** 7 **f** 2

2 Write the value of each digit in the following numbers:

a 3 648 597 **b** 34 865 271 **c** 293 148 756

3 Write in words:

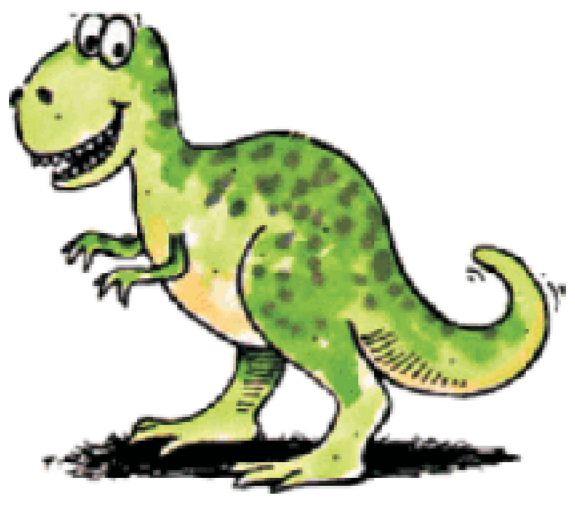
a 5 784 214 **b** 43 029 306 **c** 198 003 620

4 Read the following stories about large numbers. Write each large number using numerals.

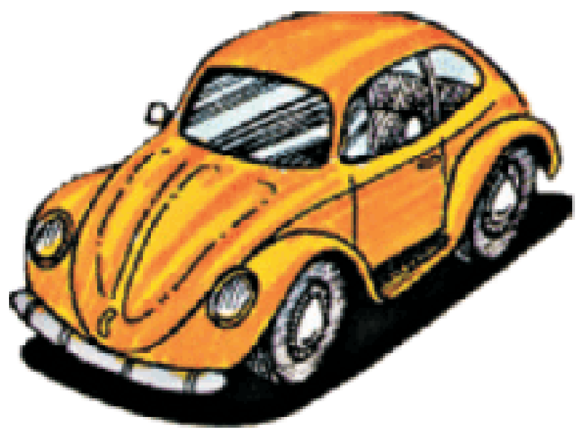
a A heart beating at a rate of 70 beats per minute would beat about thirty seven million times in a year.

b A hamburger chain bought two hundred million bread buns and used seventeen million kilograms of beef in one year.

c The Jurassic era was about one hundred and fifty million years ago.



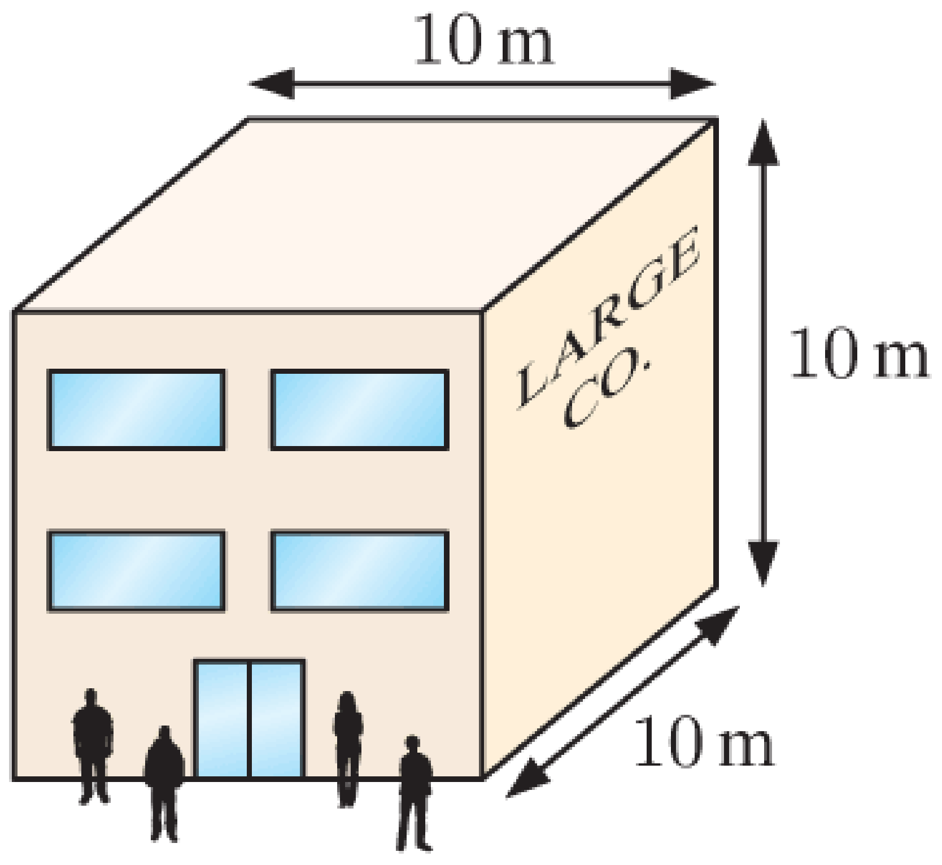
d A total of twenty one million, two hundred and forty thousand, six hundred and fifty seven Volkswagen 'Beetles' had been built by the end of 1995.



BILLIONS AND TRILLIONS

A **billion** is 1000 million or 1 000 000 000.

One billion of the small 1 cm cubes would fill a cube with side length 10 metres. This cube would be the size of a large building!



A **trillion** is 1000 billion or 1 000 000 000 000.

Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
	6	3	5	8	4	2	0	1	5	7	1	9	2	6

The number in the place value chart is 63 trillion, 584 billion, 201 million, 571 thousand, 926.

EXERCISE 1C.2

1 Write in a place value chart:

a 31 827 406 593 **b** 4 908 275 623 115 **c** 32 403 976 813 214

2 Write in words:

a 2 005 017 369 **b** 30 508 457 112 **c** 7 026 411 080 943

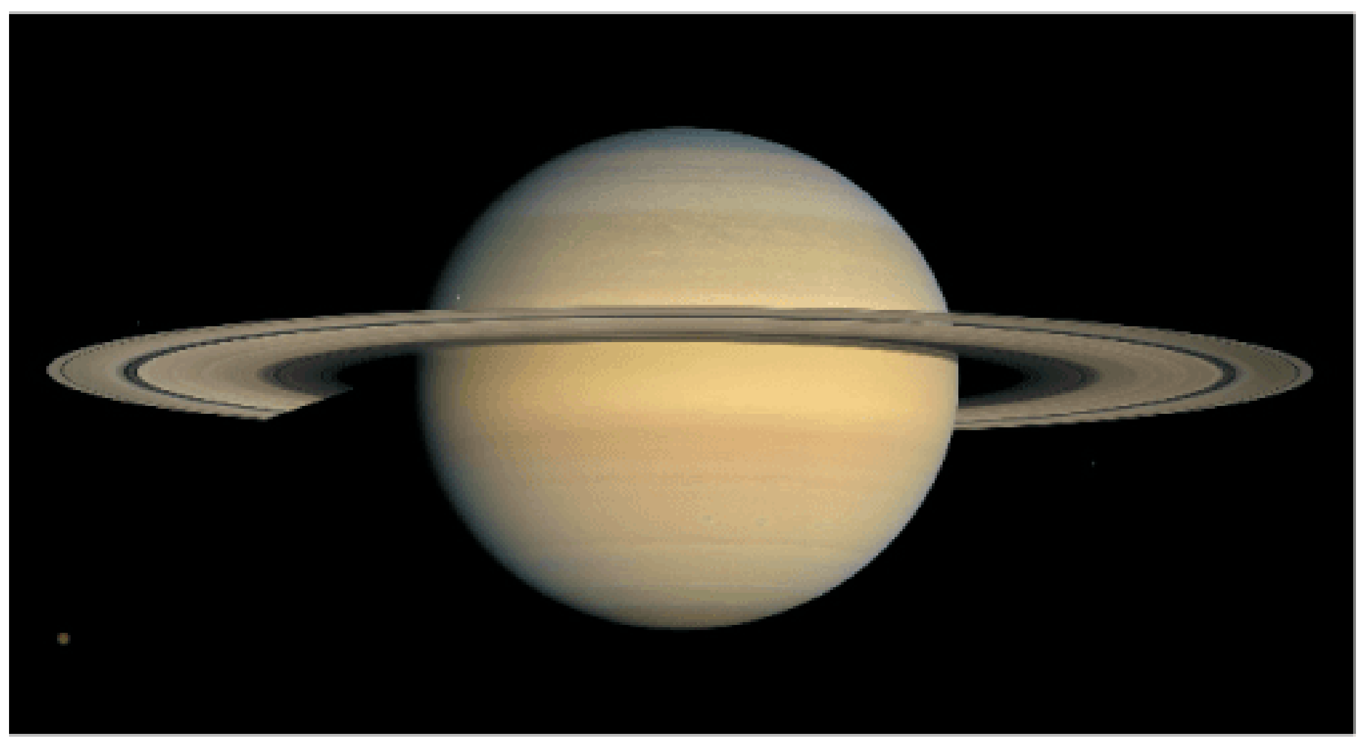
3 Read the following stories about large numbers. Write each large number using numerals.

a Saturn is approximately one billion, four hundred and twenty seven million kilometres from the Sun.

b There are about three billion, eight hundred and forty three million email accounts currently in use.

c The estimated population of the world at the end of 2012 was seven billion, sixty two million, one hundred and eighty six thousand, three hundred and twenty.

d One terabyte of data is one trillion, ninety nine billion, five hundred and eleven million, six hundred and twenty seven thousand, seven hundred and seventy six bytes.



PUZZLE NUMBER SEARCH PROBLEMS

Click on the icon to obtain these printable puzzles.

NUMBER SEARCH

KEY WORDS USED IN THIS CHAPTER

- Ancient Greek system
- digit
- infinite
- natural number
- place value
- whole number
- billion
- Egyptian system
- Mayan system
- number system
- Roman numeral
- Chinese-Japanese system
- Hindu-Arabic system
- million
- numeral
- trillion

REVIEW SET 1A

1 Write the numbers represented by the Ancient Greek symbols:

a Η 𐅄 Δ Γ **b** Χ Χ 𐅅 Η Δ Δ Δ Ι Ι Ι Ι

2 Write using Egyptian symbols:

a 27 **b** 569

3 Write the numbers represented by the Roman numerals:

a XVIII **b** LXXIX

4 Write using the Mayan system:

a 46 **b** 273

5 Write the numbers represented by the Chinese-Japanese symbols:

a 四百七十六 **b** 三百五十九

6 Identify the digit in the 'thousands' column of:

a 36 520 **b** 416 098 **c** 8 172 360

7 Write 17 304 in expanded form.

8 What number is:

a five more than eighteen **b** nine less than one thousand?

9 Write in words:

a 6 317 694 **b** 7 805 036 527

10 The average person will travel five million, eight hundred and ninety thousand metres in a lifetime. Write this number using numerals.

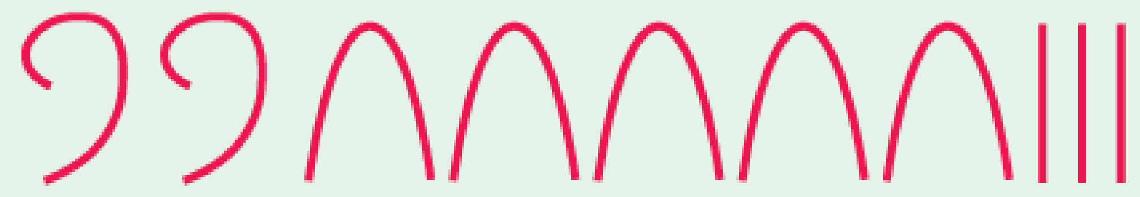
REVIEW SET 1B

1 Write using Ancient Greek symbols:

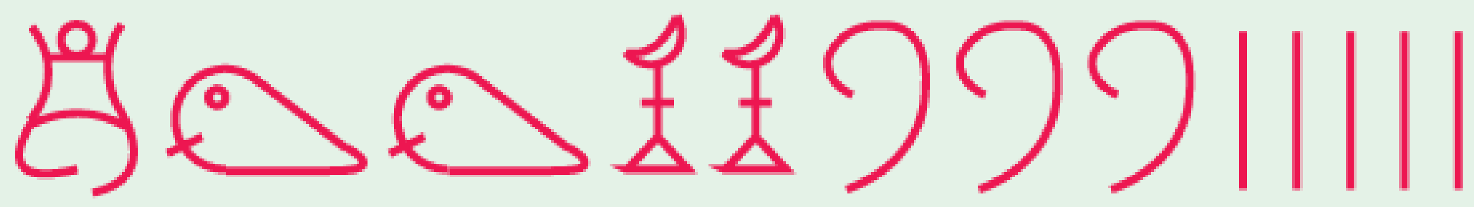
a 78 **b** 245

2 Write the numbers represented by the Egyptian symbols:

a



b



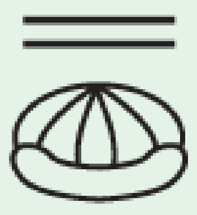
3 Which Roman numeral between 100 and 200 uses the greatest number of symbols?

4 Write using the Chinese-Japanese system:

a 386 **b** 2113

5 Write the numbers represented by the Mayan symbols:

a



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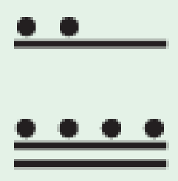


6 Give the number represented by the digit 4 in:

a 3409 **b** 41 076 **c** 490 036

7 Write down the place value of the 2, the 5, and the 9 in:

a 15 249 **b** 96 527 **c** 527 693

8 Express $2 \times 1000 + 4 \times 100 + 9 \times 10 + 7 \times 1$ in simplest form.

9 Write in a place value chart:

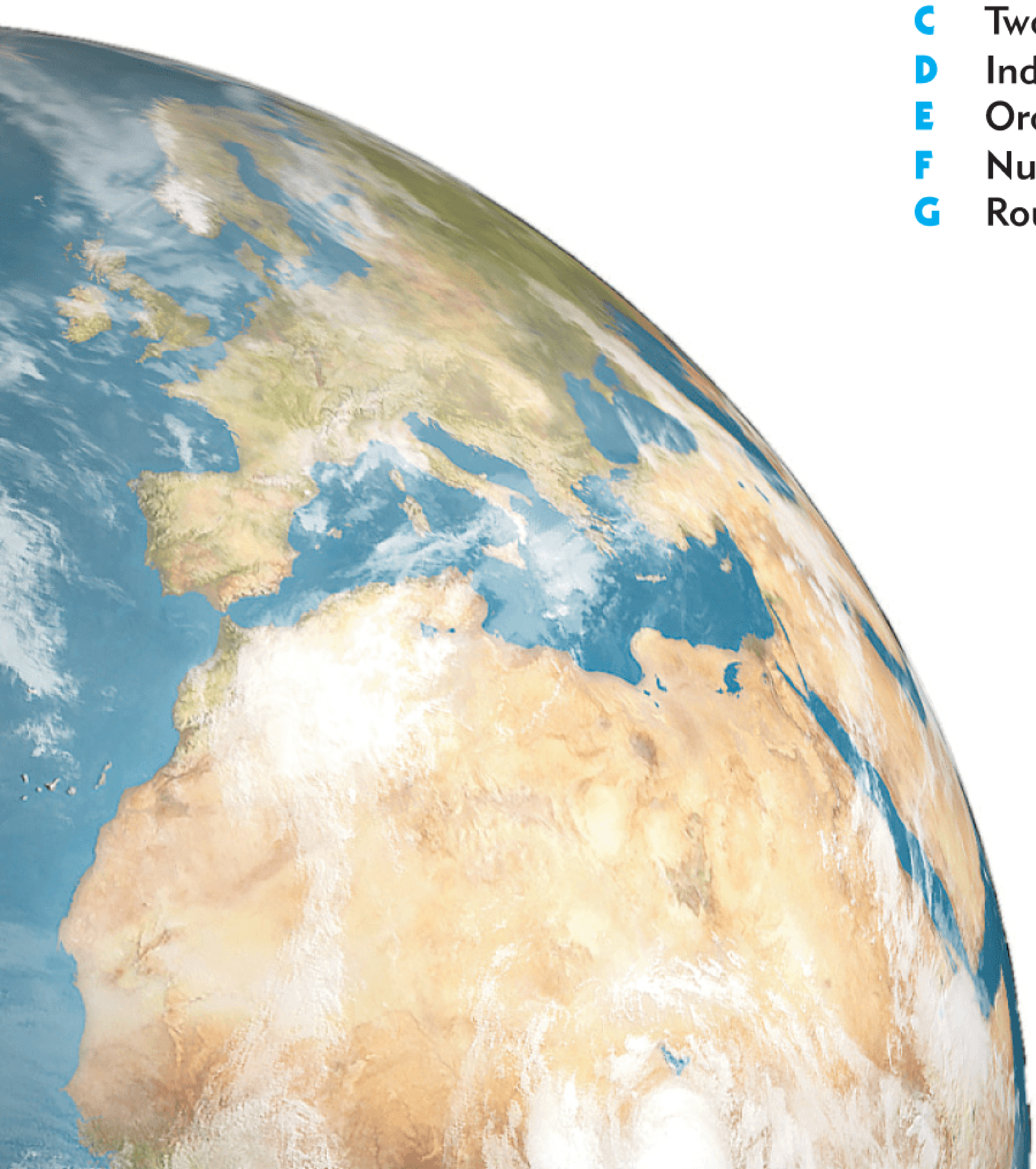
a 6 284 580 712 **b** 8 603 591 277 946

10 A total of one hundred and twenty nine million, eighty five thousand, four hundred and three formal votes were counted in the 2012 United States presidential election.
Write this number in numerals.

Chapter 2

Whole numbers

Contents:



OPENING PROBLEM

A concert hall holds 3000 people. A band called *The Angles* performed 5 concerts at the hall. There was a full audience each night, and each ticket cost $30.

Things to think about:

- **a** In total, how many people attended the concerts?
- **b** What was the total value of all the concert tickets?
- **c** The concert hall contains 20 sections, each of which holds the same number of people. How many people does each section hold?

In this chapter we will learn how to **operate** with whole numbers. We consider strategies for addition, subtraction, multiplication, and division.

A ADDITION AND SUBTRACTION

To find the **sum** of two or more numbers, we **add** them.

To find the **difference** between two numbers, we **subtract** the smaller number from the larger one.

Example 1 **Self Tutor**

Find:

- **a** the sum of 7, 8, and 11
- **b** the difference between 13 and 31.

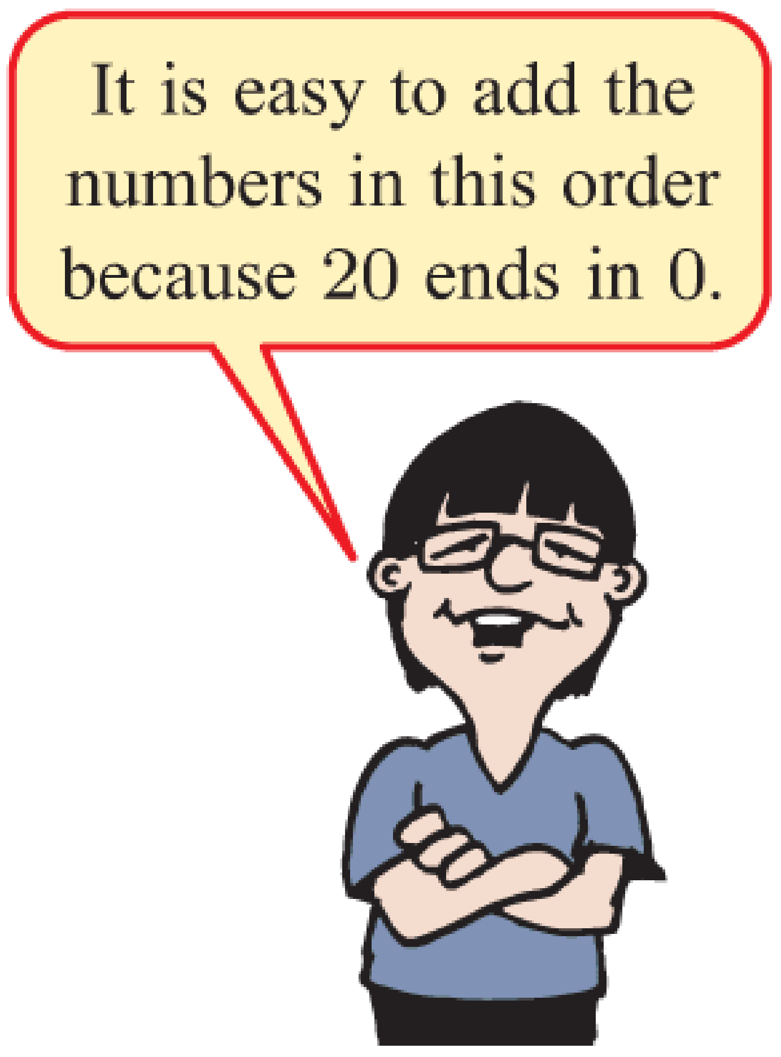
a The sum of 7, 8, and 11
$= 7 + 8 + 11$
$= 26$

b The difference between 13 and 31
$= 31 - 13$ {larger − smaller}
$= 18$

When we add three or more numbers together, we can rewrite them **in any convenient order** before we find the sum.

For example, in $8 + 39 + 12$, we notice that $8 + 12$ is 20.

$$\begin{aligned} \text{So,}\quad & 8 + 39 + 12 \\ &= \underbrace{8 + 12} + 39 \\ &= \quad 20 \quad + 39 \\ &= 59 \end{aligned}$$



Example 2 **Self Tutor**

Find:

a $74 + 23 + 7$

b $16 + 67 + 14$

a
$$\begin{aligned} & 74 + \underbrace{23 + 7} \\ &= 74 + \quad 30 \\ &= 104 \end{aligned}$$

b
$$\begin{aligned} & 16 + 67 + 14 \\ &= \underbrace{16 + 14} + 67 \\ &= \quad 30 \quad + 67 \\ &= 97 \end{aligned}$$

EXERCISE 2A.1

1 Find the sum of:

a 8 and 11 **b** 19 and 13 **c** 24 and 17 **d** 56, 14, and 28.

2 Find the difference between:

a 7 and 3 **b** 27 and 18 **c** 19 and 38 **d** 123 and 280.

3 Find:

a the sum of 4, 6, and 13

b the difference between 18 and 37

c by how much 83 is greater than 66

d the sum of the whole numbers from 2 to 6.

4 Find the following sums by adding them in the most convenient order:

a $3 + 6 + 7$ **b** $19 + 8 + 2$ **c** $3 + 6 + 7 + 4$

d $18 + 41 + 32$ **e** $21 + 98 + 19$ **f** $45 + 14 + 26$

g $98 + 57 + 102$ **h** $107 + 14 + 23$ **i** $28 + 13 + 12 + 37$

ADDING AND SUBTRACTING LARGER NUMBERS

For additions and subtractions which are too large to perform mentally, we write the numbers in columns with the place values lining up. We start from the units column, and work from right to left.

Example 3 **Self Tutor**

Find: $32 + 427 + 3274$

$$\begin{array}{r} 3\,2 \\ 4\,2\,7 \\ +\quad 3\,2\,7\,4 \\ {\scriptstyle 1\,1\;\;} \\ \hline 3\,7\,3\,3 \end{array}$$

EXERCISE 2A.2

1 Do these additions:

a $\begin{array}{r} 3\,4\,2 \\ +\ \ 4\,1\,5 \\ \hline \end{array}$ **b** $\begin{array}{r} 6\,0\,4 \\ +\ \ 3\,2\,9 \\ \hline \end{array}$ **c** $\begin{array}{r} 1\,9\,1\,7 \\ +\ \ 2\,0\,7\,8 \\ \hline \end{array}$ **d** $\begin{array}{r} 2\,7\,2\,5 \\ +\ \ 1\,9\,7\,2 \\ \hline \end{array}$

e $$\begin{array}{r} 913 \\ 24 \\ +\ 707 \\ \hline \end{array}$$

f $$\begin{array}{r} 217 \\ 106 \\ +\ 1274 \\ \hline \end{array}$$

g $$\begin{array}{r} 9004 \\ 216 \\ 23 \\ +\ 3816 \\ \hline \end{array}$$

h $$\begin{array}{r} 216 \\ 4519 \\ 76 \\ +\ 429 \\ \hline \end{array}$$

2 Find:

a $42+37$ **b** $72+35$ **c** $421+327$

d $624+72$ **e** $921+1234$ **f** $6214+324+27$

g $90+724$ **h** $32+627+4296$ **i** $912+6+427+3274$

Example 4

Self Tutor

Find:

a $87-53$ **b** $519-345$ **c** $4200-326$

a $$\begin{array}{r} 87 \\ -\ 53 \\ \hline 34 \end{array}$$

b $$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{11}{\cancel{1}}\,9 \\ -\ 3\,4\,5 \\ \hline 1\,7\,4 \end{array}$$

c $$\begin{array}{r} \overset{3}{\cancel{4}}\,\overset{11}{\cancel{2}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\ 3\,2\,6 \\ \hline 3\,8\,7\,4 \end{array}$$

3 Do these subtractions:

a $$\begin{array}{r} 97 \\ -\ 15 \\ \hline \end{array}$$

b $$\begin{array}{r} 63 \\ -\ 19 \\ \hline \end{array}$$

c $$\begin{array}{r} 648 \\ -\ 333 \\ \hline \end{array}$$

d $$\begin{array}{r} 247 \\ -\ 138 \\ \hline \end{array}$$

e $$\begin{array}{r} 713 \\ -\ 48 \\ \hline \end{array}$$

f $$\begin{array}{r} 602 \\ -\ 149 \\ \hline \end{array}$$

g $$\begin{array}{r} 6915 \\ -\ 1732 \\ \hline \end{array}$$

h $$\begin{array}{r} 6005 \\ -\ 2349 \\ \hline \end{array}$$

4 Find:

a $47-13$ **b** $62-14$ **c** $93-27$

d $40-18$ **e** $214-32$ **f** $623-147$

g $503-127$ **h** $5003-1236$ **i** $3000-583$

5 Five of these eight calculations have incorrect answers. Identify the errors, and rewrite the calculations so they have correct answers.

a $$\begin{array}{r} 239 \\ +\ 478 \\ \hline 707 \end{array}$$

b $$\begin{array}{r} 563 \\ -\ 281 \\ \hline 342 \end{array}$$

c $$\begin{array}{r} 702 \\ 87 \\ +\ 101 \\ \hline 890 \end{array}$$

d $$\begin{array}{r} 5900 \\ -\ 3814 \\ \hline 2186 \end{array}$$

e $$\begin{array}{r} 311 \\ 197 \\ +\ 648 \\ \hline 1155 \end{array}$$

f $$\begin{array}{r} 6913 \\ -\ 587 \\ \hline 6326 \end{array}$$

g $$\begin{array}{r} 5555 \\ +\ 6767 \\ \hline 12322 \end{array}$$

h $$\begin{array}{r} 3215 \\ -\ 3186 \\ \hline 39 \end{array}$$

GAME **SNAKES AND ADDERS**

Click on the icon to play a game of Snakes and Adders. You must add and subtract your way to the finish without being bitten by a snake!

SNAKES AND ADDERS

WORD PROBLEMS

We will now consider some **word problems** whose solution involves **addition** or **subtraction**.

To solve problems like these, we need to look for key words or phrases that tell us when to add or subtract. For example:

- "total" and "altogether" suggest that we need to add
- "have left", "more than", and "less than" suggest that we need to subtract.

To finish our solution, we write a **mathematical sentence** involving numbers that specifically answers the question.

Example 5 **Self Tutor**

Clive bought 450 g of potatoes, 175 g of carrots, and 340 g of onions. What was the total weight of Clive's vegetables?

To find the total weight, we need to add:

$$\begin{array}{rr} & 450 \\ & 175 \\ + & 340 \\ & {\scriptstyle 1} \\ \hline & 965 \end{array}$$

The total weight of Clive's vegetables was 965 g.

EXERCISE 2A.3

1 Jack bought three separate lengths of timber. Their lengths were 280 cm, 150 cm, and 65 cm. What was the total length of timber that Jack bought?

2 Xuen bought a game console for $255. She also purchased an extra controller for $50, a game for $95, and a bag to store these in for $32. How much did Xuen pay altogether?

3 Erika had 65 minutes of call time left on her prepaid cellphone. She made a 26 minute call to her boyfriend Marino. How many minutes did she have left after making this call?

4 At a tennis tournament, the first prize was £175 000 and second prize was £115 000. Find the difference between the prizes.

5 Herb's bank balance is €87. He deposits €246 and then €113. How much does he have in his account now?

6 An ice cream store sold 78 ice creams on Friday, 154 ice creams on Saturday, and 129 ice creams on Sunday. How many ice creams did the store sell in total?

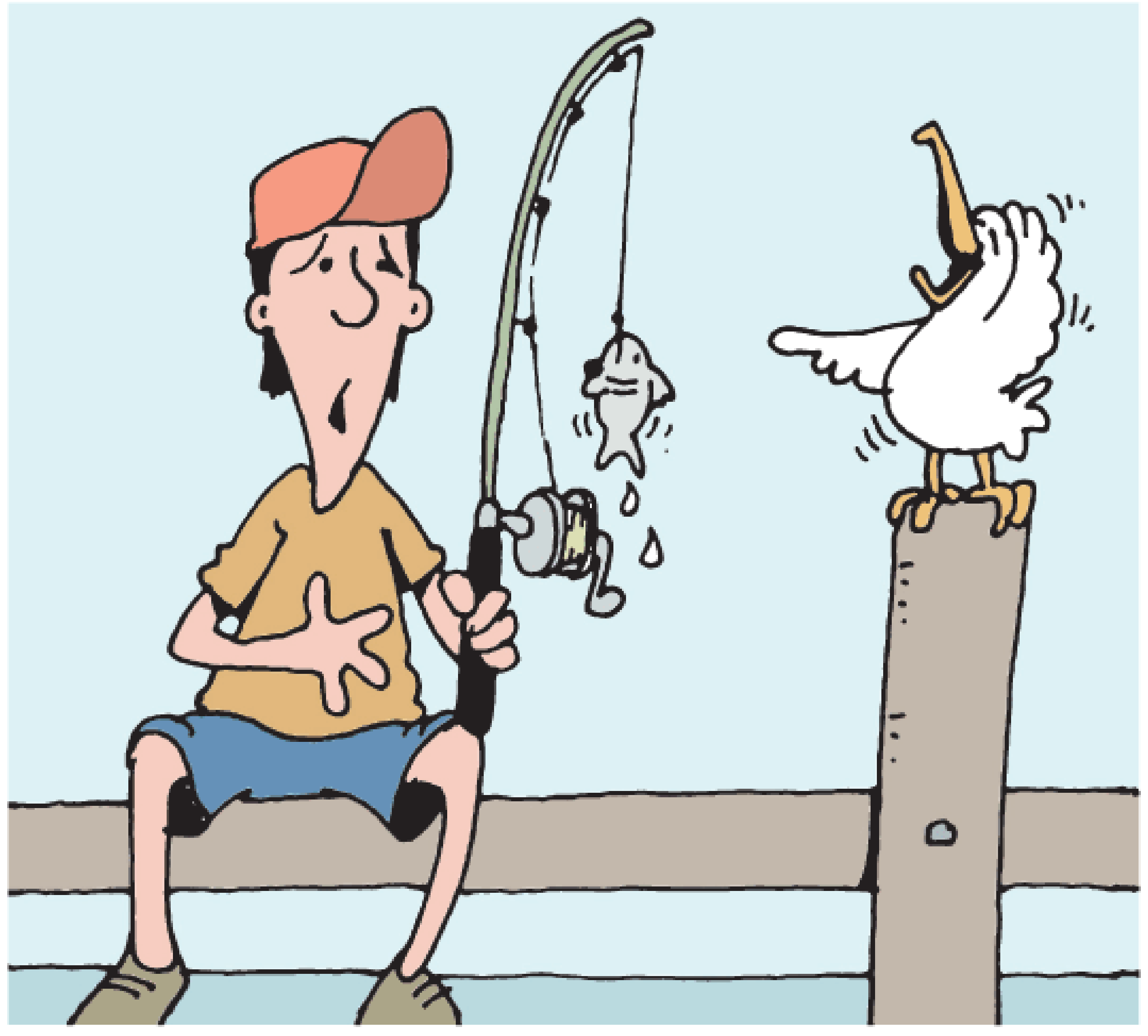
7 Justyn bought a jumper for £32. Sylvia bought a shirt for £16. How much more did Justyn spend than Sylvia?

8 Rima went on an overseas trip that required three plane flights. The first flight was 2142 km, the second was 732 km, and the third was 1049 km. What was the total distance that Rima flew?

9 During a 365 day year, it rained in New York on 192 days. On how many days did it *not* rain?

10 Darryl likes fishing. He caught 26 fish during June, 57 fish during July, and 34 fish during August.

- **a** Find the difference between the numbers of fish caught during:
  - **i** June and August
  - **ii** July and August.
- **b** Find the total number of fish caught by Darryl during this period.



PUZZLE **NUMBER PUZZLES**

In these number puzzles each letter stands for a different digit from 0 to 9. There are several solutions to each puzzle. Can you find one of them? Can you find all of them?

a

$$\begin{array}{ccccc} & & \mathrm{D} & \mathrm{O} & \mathrm{G} \\ + & & \mathrm{C} & \mathrm{A} & \mathrm{T} \\ \hline & \mathrm{H} & \mathrm{A} & \mathrm{T} & \mathrm{E} \end{array}$$

b

$$\begin{array}{ccccc} & \mathrm{S} & \mathrm{U} & \mathrm{R} & \mathrm{F} \\ - & \mathrm{S} & \mathrm{A} & \mathrm{N} & \mathrm{D} \\ \hline & & \mathrm{S} & \mathrm{E} & \mathrm{A} \end{array}$$

B MULTIPLICATION AND DIVISION

To find the **product** of two or more numbers, we **multiply** them.

For example, the product of 6 and 7 is 6×7, which is 42.

To find the **quotient** of two numbers, we divide the first number by the second number.

The number being divided is the **dividend**, and the number we are dividing by is called the **divisor**.

For example, the quotient of 63 and 9 is $63 \div 9$, which is 7.

$$\underset{\substack{\uparrow \\ \text{dividend}}}{63} \div \underset{\substack{\uparrow \\ \text{divisor}}}{9} = \underset{\substack{\uparrow \\ \text{quotient}}}{7}$$

Example 6 **Self Tutor**

Find:

a the product of 7 and 12

b the quotient of 56 and 7.

a The product of 7 and 12
$= 7 \times 12$
$= 84$

b The quotient of 56 and 7
$= 56 \div 7$
$= 8$

MULTIPLYING AND DIVIDING BY 10, 100, AND 1000

- When we multiply a whole number by 10, we make the number 10 times larger. We write one extra zero on the end of the number.
- When we multiply a whole number by 100, we make the number 100 times larger. We write two extra zeros on the end of the number.
- When we multiply a whole number by 1000, we make the number 1000 times larger. We write three extra zeros on the end of the number.

Example 7 **Self Tutor**

Find:

a 23×10 **b** 89×100 **c** 381×1000

a 23×10
$= 230$

b 89×100
$= 8900$

c 381×1000
$= 381\,000$

We do the reverse when we divide by 10, 100, or 1000.

- When we divide a whole number by 10, we make the number 10 times smaller. We remove one zero from the end of the number.
- When we divide a whole number by 100, we make the number 100 times smaller. We remove two zeros from the end of the number.
- When we divide a whole number by 1000, we make the number 1000 times smaller. We remove three zeros from the end of the number.

Example 8 **Self Tutor**

Find:

a $34\,000 \div 10$ **b** $34\,000 \div 100$ **c** $34\,000 \div 1000$

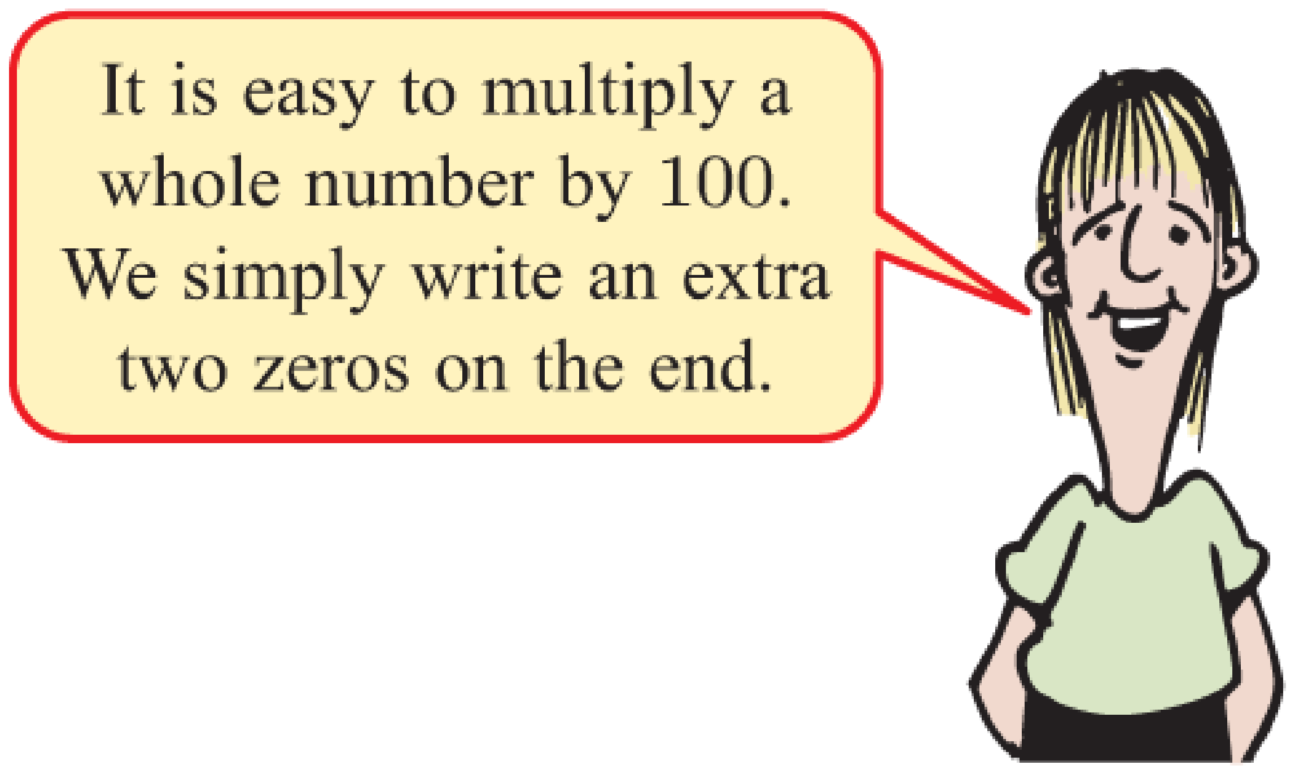
a $34\,000 \div 10$
$= 3400$

b $34\,000 \div 100$
$= 340$

c $34\,000 \div 1000$
$= 34$

When we multiply three or more numbers together, we can rearrange their order to make the multiplication easier.

For example, $4 \times 47 \times 25$
$= 4 \times 25 \times 47$
$= 100 \times 47$
$= 4700$



Example 9 **Self Tutor**

Find:

a $5 \times 19 \times 2$ **b** $16 \times 125 \times 8$

a
$$\begin{aligned} & 5 \times 19 \times 2 \\ &= 5 \times 2 \times 19 \\ &= 10 \times 19 \\ &= 190 \end{aligned}$$

b
$$\begin{aligned} & 16 \times 125 \times 8 \\ &= 16 \times 1000 \\ &= 16\,000 \end{aligned}$$

EXERCISE 2B.1

1 Find the product of:

a 6 and 9 **b** 11 and 13 **c** 2, 5, and 7 **d** 3, 8, and 10.

2 Find the quotient of:

a 12 and 3 **b** 28 and 7 **c** 99 and 9 **d** 165 and 11.

3 Find:

a 50×10 **b** 50×100 **c** 50×1000 **d** 69×100

e 69×1000 **f** $69 \times 10\,000$ **g** 123×100 **h** 246×1000

i 960×100 **j** $49 \times 10\,000$ **k** 490×100 **l** 4900×100

4 Find:

a $2000 \div 10$ **b** $2000 \div 100$ **c** $2000 \div 1000$

d $57\,000 \div 10$ **e** $57\,000 \div 100$ **f** $57\,000 \div 1000$

g $243\,000 \div 10$ **h** $243\,000 \div 100$ **i** $243\,000 \div 1000$

j $45\,000 \div 10$ **k** $45\,000 \div 100$ **l** $45\,000 \div 1000$

m $720\,000 \div 10$ **n** $720\,000 \div 100$ **o** $720\,000 \div 1000$

p $6\,000\,000 \div 10$ **q** $6\,000\,000 \div 100$ **r** $6\,000\,000 \div 1000$

5 Find the following products by rearranging their order:

a $5 \times 13 \times 2$ **b** $25 \times 19 \times 4$ **c** $50 \times 21 \times 2$ **d** $125 \times 19 \times 8$

e $4 \times 21 \times 25$ **f** $200 \times 97 \times 5$ **g** $500 \times 27 \times 2$ **h** $12 \times 125 \times 8$

Example 10 **Self Tutor**

Find:

a 3×4 **b** 30×4 **c** 30×400

a
$$\begin{aligned} & 3 \times 4 \\ &= 12 \end{aligned}$$

b
$$\begin{aligned} & 30 \times 4 \\ &= 3 \times 10 \times 4 \\ &= 12 \times 10 \\ &= 120 \end{aligned}$$

c
$$\begin{aligned} & 30 \times 400 \\ &= 3 \times 10 \times 4 \times 100 \\ &= 12 \times 1000 \\ &= 12\,000 \end{aligned}$$

6 Find these products:

a 3×2
b 30×2
c 30×20
d 300×20
e 5×7
f 5×70
g 50×70
h 50×700
i 3×11
j 30×11
k 300×11
l 300×1100

Example 11 **Self Tutor**

Find:

a $18 \div 3$
b $180 \div 3$
c $1800 \div 3$

a
$$\begin{aligned} & 18 \div 3 \\ &= 6 \end{aligned}$$

b
$$\begin{aligned} & 180 \div 3 \\ &= 18 \times 10 \div 3 \\ &= 18 \div 3 \times 10 \\ &= 6 \times 10 \\ &= 60 \end{aligned}$$

c
$$\begin{aligned} & 1800 \div 3 \\ &= 18 \times 100 \div 3 \\ &= 18 \div 3 \times 100 \\ &= 6 \times 100 \\ &= 600 \end{aligned}$$

7 Find these quotients:

a $6 \div 2$
b $60 \div 2$
c $600 \div 2$
d $6000 \div 2$
e $35 \div 7$
f $350 \div 7$
g $3500 \div 7$
h $35\,000 \div 7$
i $12 \div 3$
j $120 \div 3$
k $1200 \div 3$
l $12\,000 \div 3$

MULTIPLYING BY WHOLE NUMBERS

As with addition and subtraction, when we multiply whole numbers, we write the numbers in columns so that the place values line up.

Example 12 **Self Tutor**

Find: **a** 67×4 **b** 53×16

a
$$\begin{array}{r} 6\ 7 \\ \times\quad 4 \\ {}_{2}\ \ \\ \hline 2\ 6\ 8 \end{array}$$

b
$$\begin{array}{rl} 5\ 3 & \\ \times\quad 1\ 6 & \\ {}_{1}\ \ & \\ \hline 3\ 1\ 8 & \{\text{multiplying 53 by 6}\} \\ +\quad 5\ 3\ 0 & \{\text{multiplying 53 by 10}\} \\ \hline 8\ 4\ 8 & \{\text{adding}\} \end{array}$$

EXERCISE 2B.2

1 Do these multiplications:

a
$$\begin{array}{r} 7\ 2 \\ \times\quad 3 \\ \hline \end{array}$$

b
$$\begin{array}{r} 5\ 9 \\ \times\quad 4 \\ \hline \end{array}$$

c
$$\begin{array}{r} 1\ 2\ 5 \\ \times\quad 7 \\ \hline \end{array}$$

d
$$\begin{array}{r} 2\ 8 \\ \times\quad 1\ 2 \\ \hline \end{array}$$

e
$$\begin{array}{r} 3\ 1 \\ \times\quad 2\ 2 \\ \hline \end{array}$$

f
$$\begin{array}{r} 4\ 2 \\ \times\quad 1\ 3 \\ \hline \end{array}$$

g
$$\begin{array}{r} 4\ 3 \\ \times\quad 1\ 4 \\ \hline \end{array}$$

h
$$\begin{array}{r} 1\ 5\ 2 \\ \times\quad 2\ 3 \\ \hline \end{array}$$

2 Find:

a 24×5 **b** 37×4 **c** 62×8 **d** 53×24

e 27×15 **f** 56×49 **g** 324×45 **h** 642×36

DIVIDING BY WHOLE NUMBERS

When we divide whole numbers, we work from left to right. Make sure you write each digit of the answer in the correct place value column.

Example 13 **Self Tutor**

Find:

a $256 \div 4$ **b** $2502 \div 6$

a

$$\begin{array}{r} 6\ \ 4 \\ 4\,\overline{)\,2\ \ 5\ {}^{1}6} \end{array}$$

$\therefore\ \ 256 \div 4 = 64$

b

$$\begin{array}{r} 4\ \ 1\ \ 7 \\ 6\,\overline{)\,2\ \ 5\ {}^{1}0\ {}^{4}2} \end{array}$$

$\therefore\ \ 2502 \div 6 = 417$

When one whole number is divided by another, the result is not always a whole number. Sometimes we are left with a **remainder**.

Example 14 **Self Tutor**

Find $1851 \div 7$.

$$\begin{array}{r} 2\ \ 6\ \ 4 \\ 7\,\overline{)\,1\ \ 8\ {}^{4}5\ {}^{3}1\ {}^{3}} \end{array}$$

$\therefore\ \ 1851 \div 7 = 264$ with remainder 3.

EXERCISE 2B.3

1 Do these divisions:

a $3\,\overline{)\,4\ 2}$ **b** $4\,\overline{)\,2\ 1\ 6}$ **c** $8\,\overline{)\,1\ 6\ 8}$ **d** $6\,\overline{)\,5\ 0\ 4}$

e $5\,\overline{)\,3\ 7\ 5}$ **f** $4\,\overline{)\,1\ 1\ 6\ 4}$ **g** $7\,\overline{)\,6\ 3\ 0\ 7}$ **h** $11\,\overline{)\,6\ 8\ 0\ 9}$

2 Find:

a $48 \div 4$ **b** $125 \div 5$ **c** $312 \div 6$ **d** $240 \div 5$

e $203 \div 7$ **f** $624 \div 3$ **g** $328 \div 8$ **h** $7353 \div 9$

3 Find:

a $86 \div 3$ **b** $193 \div 5$ **c** $974 \div 6$ **d** $3948 \div 9$

WORD PROBLEMS

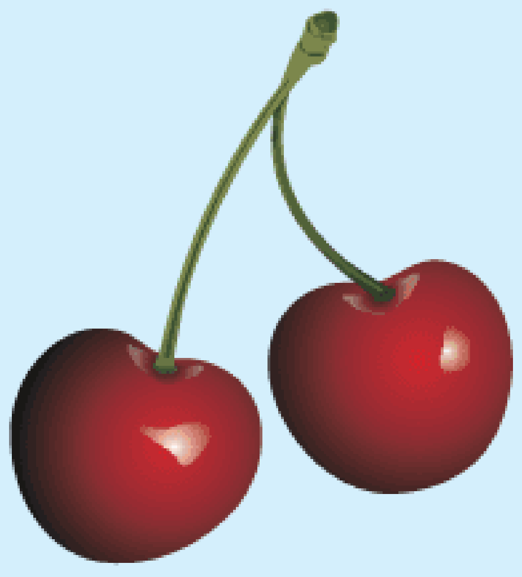
Example 15 **Self Tutor**

Jason works for a supermarket chain. He arranges to buy 217 baskets of fresh cherries at a price of $38 for each basket.

How much does the supermarket have to pay?

Total cost = 217 × $38

$$\begin{array}{r} 2\,1\,7 \\ \times \quad 3\,8 \\ {\scriptstyle 1\ 5} \\ {\scriptstyle 2\quad} \\ \hline 1\,7\,3\,6 \\ +\quad 6\,5\,1\,0 \\ {\scriptstyle 1\qquad} \\ \hline 8\,2\,4\,6 \end{array}$$

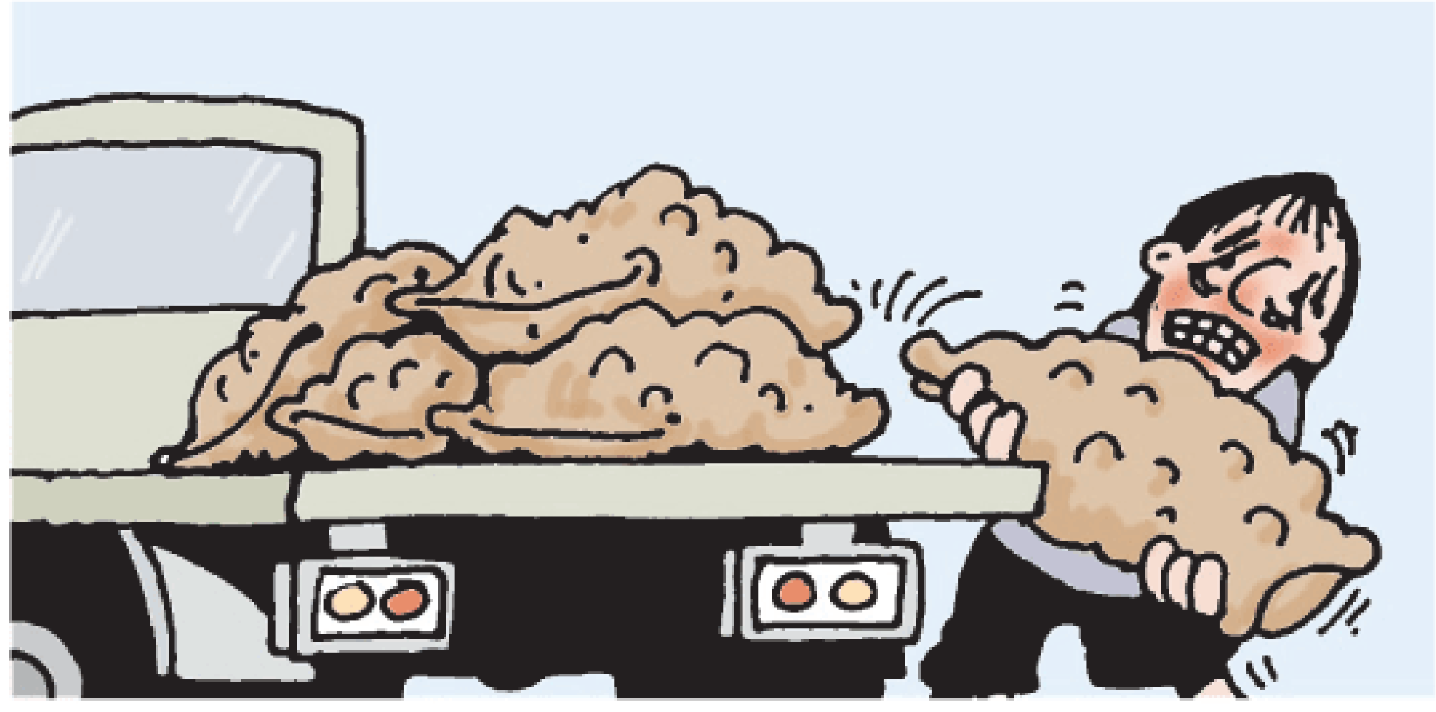


The supermarket must pay $8246.

EXERCISE 2B.4

1 60 rows of pine trees were planted. Each row contained 80 trees. How many pine trees were planted altogether?

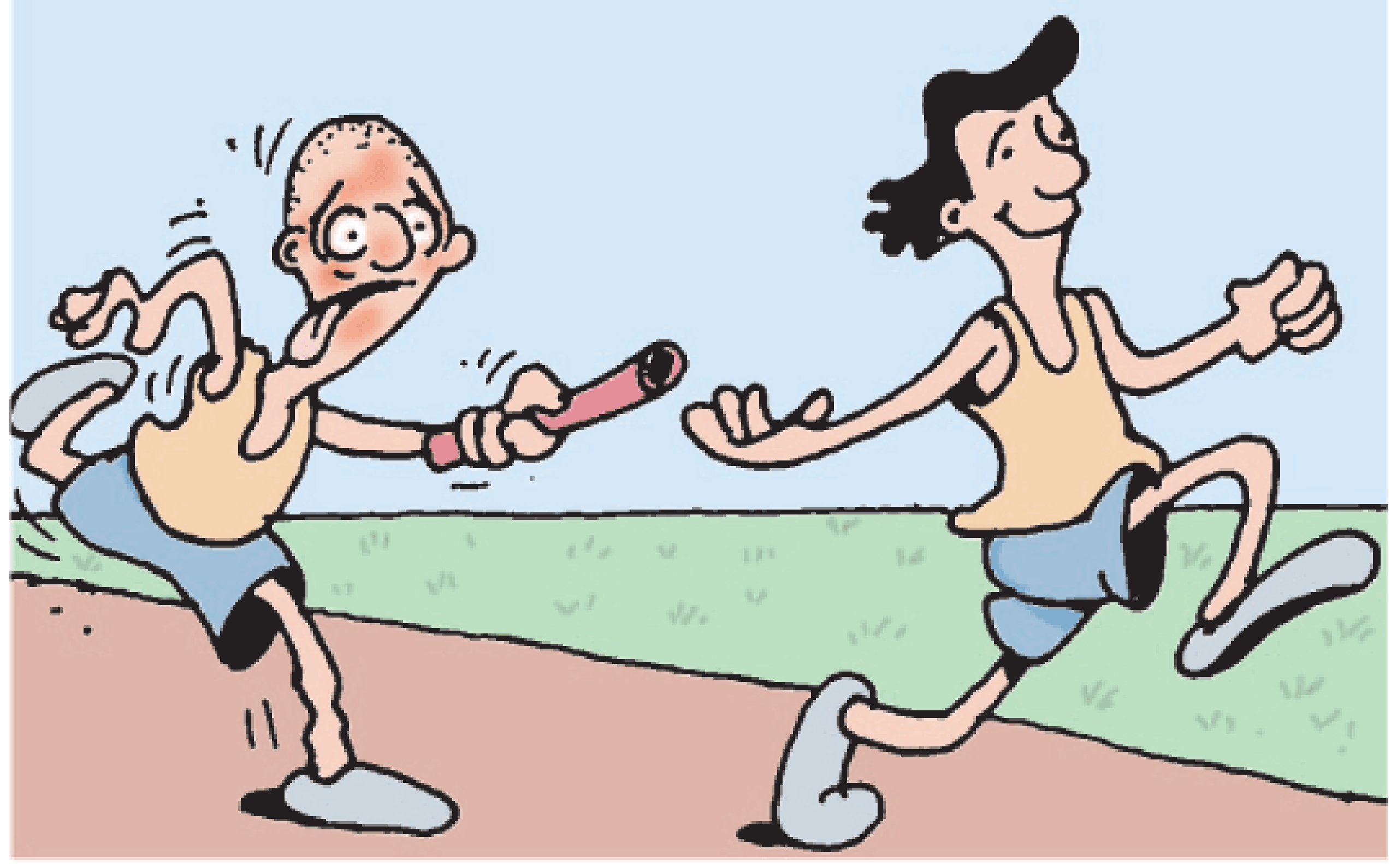
2 Carlos lifted five 18 kg bags of potatoes onto a truck. How many kilograms of potatoes did he lift altogether?



3 My three brothers and I received a gift of $320. If we share the money equally between us, how much will each person get?

4 A flute teacher charges $100 per lesson. She gives 17 lessons in one week. How much money does she earn?

5 A relay team of nine people took 738 minutes to complete a charity relay. If each team member ran for the same amount of time, for how long did each person run?



6 A function room contains 18 tables. Each table can seat 12 people. How many people can the function room hold?

7 I write at the rate of 8 words per minute. How long will it take me to write 648 words?

8 A London hotel has 6 floors, each with 50 rooms. The hotel is fully occupied, and the rooms cost £150 per night.

- **a** How many rooms does the hotel have?
- **b** How much income does the hotel receive for the night?
- **c** Find the total income for the hotel over a 20 day period in which it is fully occupied.

9 While training for half marathons, Paulo ran 42 000 m during one week. How far did he run each day if he ran:

  a the same distance on each of all 7 days
  b the same distance on each of 6 days and rested on the seventh
  c the same distance on each of 3 days and rested on the other four?

C TWO STEP PROBLEM SOLVING

Sometimes we need to perform more than one operation to solve a problem. In these situations it may be easier to solve the problem in two steps.

Example 16 **Self Tutor**

Each week Clancy is paid $350, plus $65 for each vacuum cleaner he sells. How much does Clancy earn if he sells 13 vacuum cleaners in a week?

$$\begin{aligned} \text{Money from sales} &= \$65 \times 13 \\ &= \$845 \\ \text{In total, Clancy earns} \quad & \$845 + \$350 \\ &= \$1195 \end{aligned}$$

$$\begin{array}{r} 6\,5 \\ \times\quad 1\,3 \\ {\scriptstyle 1} \\ \hline 1\,9\,5 \\ +\quad 6\,5\,0 \\ {\scriptstyle 1}\quad \\ \hline 8\,4\,5 \end{array} \qquad \begin{array}{r} 8\,4\,5 \\ +\quad 3\,5\,0 \\ \hline 1\,1\,9\,5 \end{array}$$

EXERCISE 2C

1 Deloris bought a shirt costing €29 and a pair of jeans costing €45. How much change did she get from €100?

2 Anne and Alan have a herd of 183 goats. Anne puts 75 goats into their largest paddock, then Alan divides the rest equally between two smaller paddocks. How many goats did Alan put in each smaller paddock?

3 The cost of placing a notice in a newspaper is $38, plus $13 for each line of type. If my notice takes 10 lines, how much will I pay?

4 Yesterday, June bought 3 buns for 84 cents. Today she bought 8 buns. How much will June pay for the buns today?

5 Marcia saved €620 during the year, and her sister saved twice that amount.

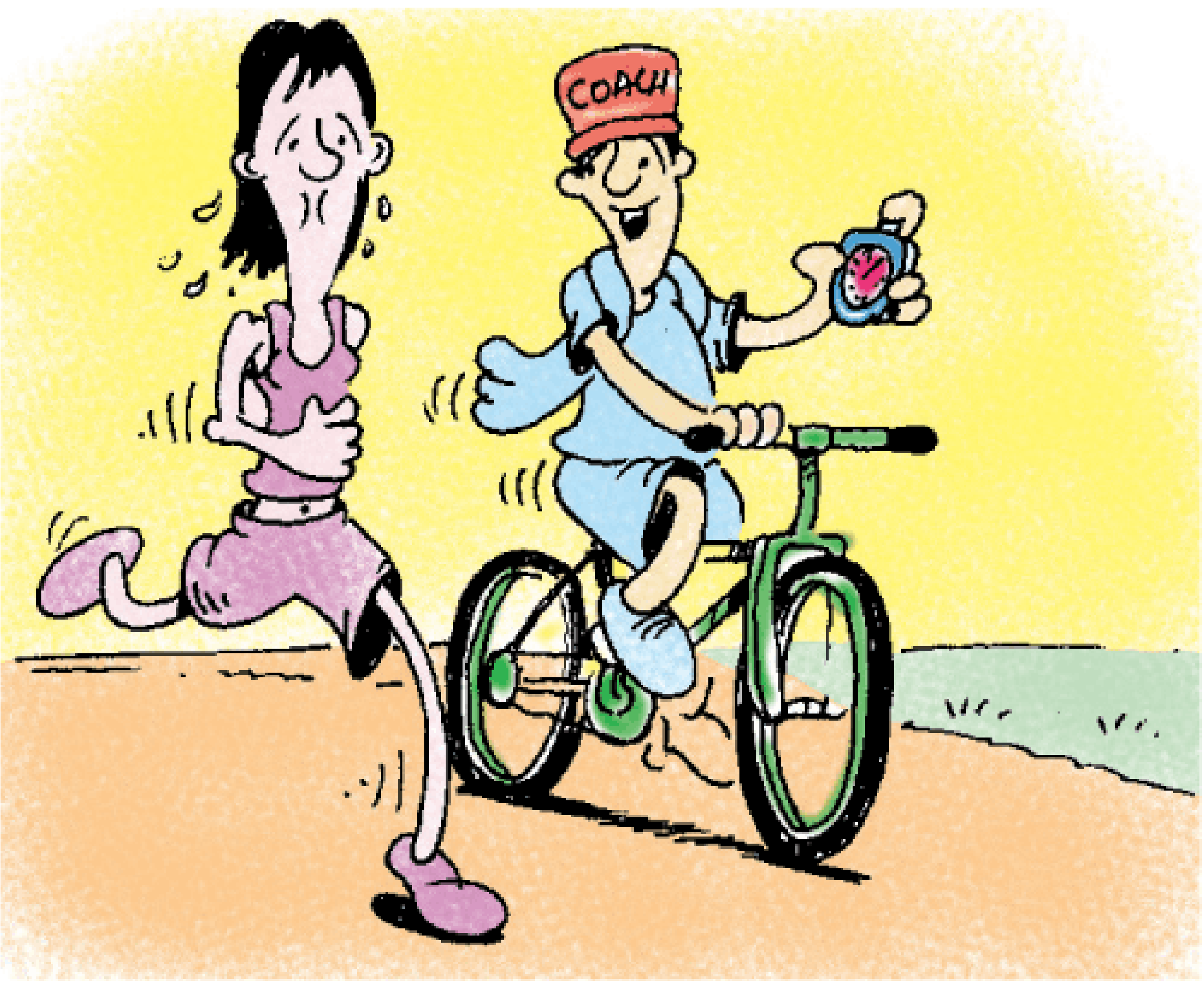
  a How much money did Marcia's sister save?
  b How much money did the sisters save in total?

6 Yuan worked 45 hours at one job for $24 per hour, then 35 hours at another job for $26 per hour.

  a In total, how much did Yuan earn over this period?
  b Yuan had hoped to earn $2000. Did he succeed or fail, and by how much?

7 Alicia ran 6 km each day during March, and 8 km each day during April. How far did she run in total over the two months?

8 A plastic crate contains 100 boxes of pens. Each box of pens weighs 86 grams. The total mass of the crate and pens is 9200 grams. Find the mass of the crate.



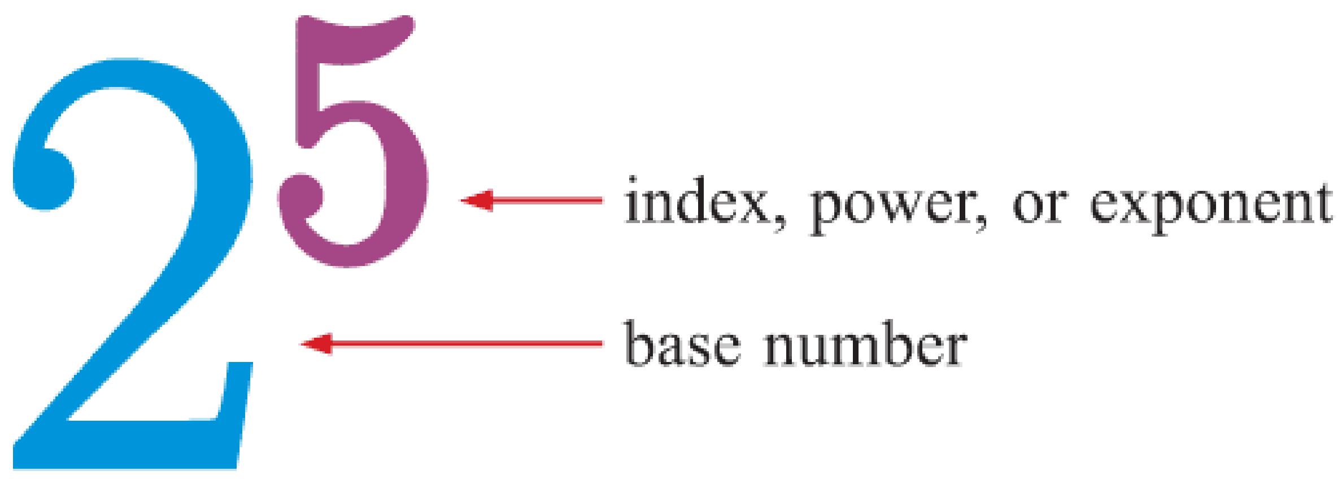
D INDEX NOTATION

We sometimes need to multiply the same number together many times.

To avoid writing long lists of identical numbers multiplied together, we can instead write the number using **index notation**.

For example, we can write $2\times 2\times 2\times 2\times 2$ as 2^5.

In this notation, the 2 is called the **base number** and the 5 is the **index**, **power**, or **exponent**. The index is the number of times the base number appears in the product.



The following table demonstrates correct language when talking about index notation.

Natural number	*Product form*	*Index form*	*Spoken form*
2	2	2^1	two
4	2×2	2^2	two squared
8	$2\times 2\times 2$	2^3	two cubed
16	$2\times 2\times 2\times 2$	2^4	two to the fourth
32	$2\times 2\times 2\times 2\times 2$	2^5	two to the fifth

Example 17 **Self Tutor**

Write in index form:

a $5\times 5\times 5$ **b** $2\times 2\times 3\times 3\times 3\times 3$

a
$$\begin{aligned} &5\times 5\times 5 \\ &= 5^3 \end{aligned}$$

b
$$\begin{aligned} &\underbrace{2\times 2}_{}\times\underbrace{3\times 3\times 3\times 3}_{} \\ &= \quad 2^2 \quad \times \quad 3^4 \\ &= 2^2\times 3^4 \end{aligned}$$

EXERCISE 2D

1 Write in index form:

a $2\times 2\times 2$

b 7×7

c $9\times 9\times 9\times 9$

d 13×13

e $3\times 3\times 3\times 3\times 3$

f $4\times 4\times 4\times 4\times 4\times 4\times 4$

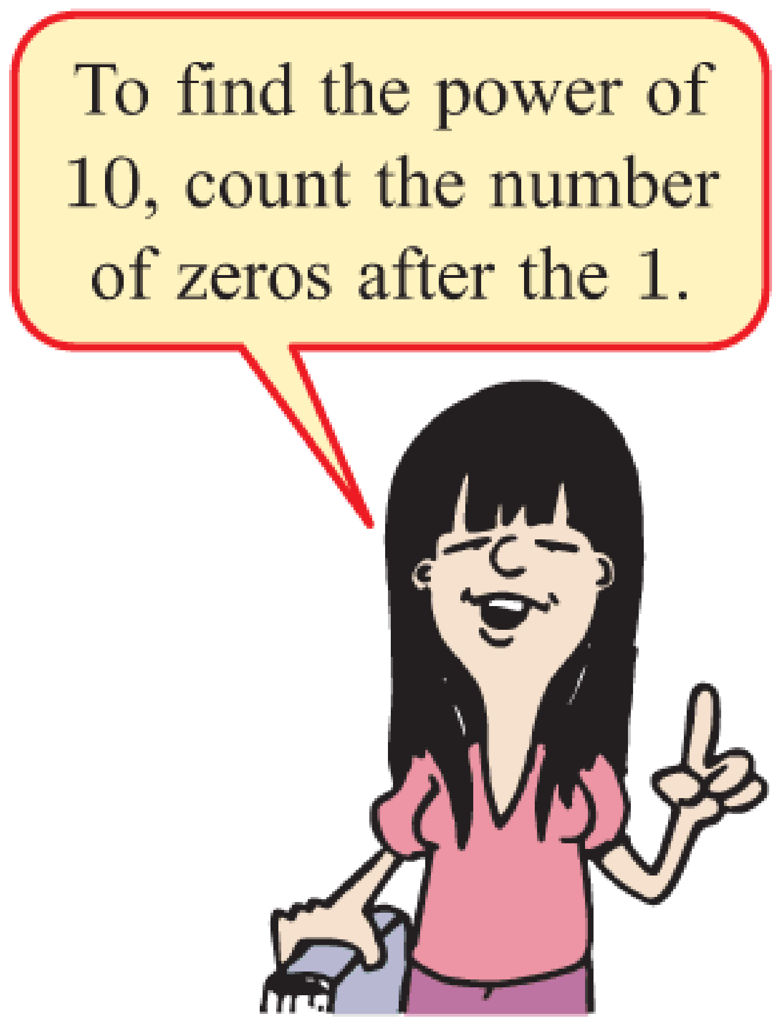
2 Write using index notation:

a $2 \times 2 \times 2 \times 3 \times 3$
b $4 \times 4 \times 7 \times 7 \times 7 \times 7$
c $5 \times 5 \times 5 - 6 \times 6 \times 6$
d $9 \times 9 \times 9 + 8 \times 8 \times 8 \times 8$
e $3 \times 3 \times 3 \times 5 \times 7 \times 7$
f $13 \times 13 - 2 \times 2 \times 2 \times 2 + 5 \times 5$

Example 18 **Self Tutor**

Write 1000 as a power of 10.

$$\begin{aligned}1000 &= 10 \times 10 \times 10\\ &= 10^3\end{aligned}$$



3 Write as a power of 10:

a 100
b 10 000
c 100 000
d one million
e one billion
f one trillion

Example 19 **Self Tutor**

Write as a natural number:

a 3^4
b $2^3 \times 4^2$

a $$\begin{aligned}&3^4\\ &= 3 \times 3 \times 3 \times 3\\ &= 81\end{aligned}$$

b $$\begin{aligned}&2^3 \times 4^2\\ &= 2 \times 2 \times 2 \times 4 \times 4\\ &= 8 \times 16\\ &= 128\end{aligned}$$

4 Write as a natural number:

a 3^2
b 2^3
c 4^2
d 3^3
e 4^3
f 2^4
g 5^3
h 9^2
i $6^2 \times 2^3$
j $3^3 \times 2^2$
k $5^2 \times 3^3$
l $8^2 \times 10^3$

5 Determine which of the following is larger:

a 2^3 or 3^2
b 2^4 or 4^2
c 5^2 or 2^5

E ORDER OF OPERATIONS

DISCUSSION

What do you think is the value of $20 + 8 \div 4$?

Share your answer with your classmates. Did everybody in the class get the same answer?

From the discussion with your class, you should have decided that the value of $20 + 8 \div 4$ depends on the order in which the operations are performed.

If we do the addition first and then the division, we get

$$\begin{aligned} & 20 + 8 \div 4 \\ = {} & 28 \div 4 \\ = {} & 7 \end{aligned}$$

If we do the division first and then the addition, we get

$$\begin{aligned} & 20 + 8 \div 4 \\ = {} & 20 + 2 \\ = {} & 22 \end{aligned}$$

To avoid confusion, we use a set of rules which state the order in which operations should be performed.

RULES FOR ORDER OF OPERATIONS

- Perform operations within **B**rackets first.
- Then, calculate any part involving **E**xponents.
- Then, working from the left, perform all **D**ivisions and **M**ultiplications.
- Finally, working from the left, perform all **A**dditions and **S**ubtractions.

We also note that:

- If an expression contains only $+$ and $-$ operations, we work from left to right.
- If an expression contains only $\times$ and $\div$ operations, we work from left to right.

Example 20 **Self Tutor**

Find the value of:

a $11 - 6 + 8$ **b** $18 - 8 \div 2$

a
$$\begin{aligned} & 11 - 6 + 8 && \{\text{perform subtraction and addition, working from the left}\} \\ = {} & 5 + 8 \\ = {} & 13 \end{aligned}$$

b
$$\begin{aligned} & 18 - 8 \div 2 && \{\text{perform division first}\} \\ = {} & 18 - 4 && \{\text{then perform subtraction}\} \\ = {} & 14 \end{aligned}$$

Example 21 **Self Tutor**

Find the value of:

a $7 + 3 \times 2 - 4$ **b** $9 \div 3 + 7 \times 2$

a
$$\begin{aligned} & 7 + 3 \times 2 - 4 && \{\text{perform multiplication first}\} \\ = {} & 7 + 6 - 4 && \{\text{then perform addition and subtraction, working from the left}\} \\ = {} & 13 - 4 \\ = {} & 9 \end{aligned}$$

b
$$\begin{aligned} & 9 \div 3 + 7 \times 2 && \{\text{perform division and multiplication first}\} \\ = {} & 3 + 14 && \{\text{then perform addition}\} \\ = {} & 17 \end{aligned}$$

EXERCISE 2E

1 Find the value of:

- **a** $12 - 6 + 2$
- **b** $12 + 6 - 8$
- **c** $12 \div 6 + 8$
- **d** $6 \times 2 \div 3$
- **e** $12 \div 3 + 2$
- **f** $15 - 6 - 4$
- **g** $6 \times 6 \div 2$
- **h** $12 + 6 \div 3$
- **i** $17 - 7 \times 2$
- **j** $30 - 10 \div 2$
- **k** $32 \div 4 \times 2$
- **l** $36 \div 6 \div 2$

2 Find the value of:

- **a** $7 + 6 - 5 + 2$
- **b** $18 \div 2 \times 3 - 1$
- **c** $18 \div 3 + 10 \times 3$
- **d** $8 \times 3 - 4 \times 5$
- **e** $30 - 3 \times 5 + 1$
- **f** $5 + 7 - 3 \times 4$
- **g** $22 \div 2 + 5 \times 4$
- **h** $60 - 24 \div 3 \times 2$
- **i** $20 \div 2 + 8 \div 4$

Example 22 **Self Tutor**

Find the value of: $23 - (17 - 2)$

$$\begin{aligned} & 23 - (17 - 2) && \{\text{perform operations within brackets first}\} \\ = \; & 23 - 15 \\ = \; & 8 \end{aligned}$$

3 Find the value of:

- **a** $(8 - 4) \div 2$
- **b** $11 - (2 + 3)$
- **c** $5 \times (6 + 1)$
- **d** $14 \div (10 - 3)$
- **e** $(11 + 19) \div 5$
- **f** $60 \div (3 \times 4)$
- **g** $(5 + 3) \times 4 - 1$
- **h** $5 + (3 \times 4) - 1$
- **i** $5 + 3 \times (4 - 1)$
- **j** $(6 + 19 - 3) \div 2$
- **k** $(7 + 17) \div (40 \div 5)$
- **l** $11 - 6 \div (3 - 1)$

Example 23 **Self Tutor**

Find the value of: $45 \div 3^2$

$$\begin{aligned} & 45 \div 3^2 && \{\text{exponent first}\} \\ = \; & 45 \div 9 && \{\text{then perform division}\} \\ = \; & 5 \end{aligned}$$

4 Find the value of:

- **a** $10 - 3^2$
- **b** $28 \div 2^2$
- **c** $8^2 \div 4$
- **d** $21 - 2^4 + 6$
- **e** $2 + 5^2$
- **f** $(2 + 5)^2$
- **g** 2×3^2
- **h** $(2 \times 3)^2$
- **i** $(4 - 2)^3 \div 8$
- **j** $(30 \div 6)^3$
- **k** $5 + (4 + 5)^2$
- **l** $3^4 - (3 \times 2)^2$

5 Replace $*$ by either $+$, $-$, $\times$, or $\div$ to make a correct statement:

- **a** $4 + 18 * 3 = 10$
- **b** $6 * 7 - 12 = 30$
- **c** $(17 * 3) \div 5 = 4$
- **d** $(18 - 2) * 8 = 2$
- **e** $3^3 * 2^2 = 23$
- **f** $4 + (21 * 7) = 7$

6 To find $15 - 7 + 3$, Derrick performed these steps:

$15 - 7 + 3$ {perform addition first}
$= 15 - 10$ {then perform subtraction}
$= 5$

a Explain the error in Derrick's working.

b Find the correct value of $15 - 7 + 3$.

PUZZLE

Click on the icon to obtain a printable version of this puzzle.

PUZZLE

1		2	■	3	4
	■	5	6		
7	8	■	9		■
■	10	11	■	12	13
14			15	■	
16		■	17		

Across

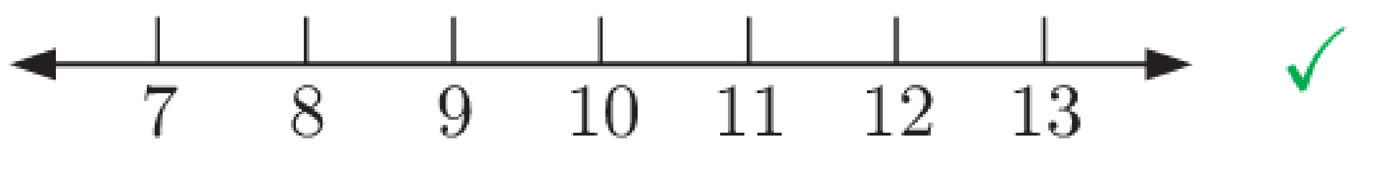
1 $40 \times 5 - 17$
3 $100 - (7 - 1)$
5 $(1 + 5 \times 50) \times 25$
7 $3 \times (3 + 20)$
9 $8 \times 11 - 7$
10 $100 - 9 \times 2$
12 $5 \times (6 + 7)$
14 $153 \div 3 + 3 \times 1000$
16 $90 - 4 \times 4$
17 $9 \times 100 + 8 \times 5$

Down

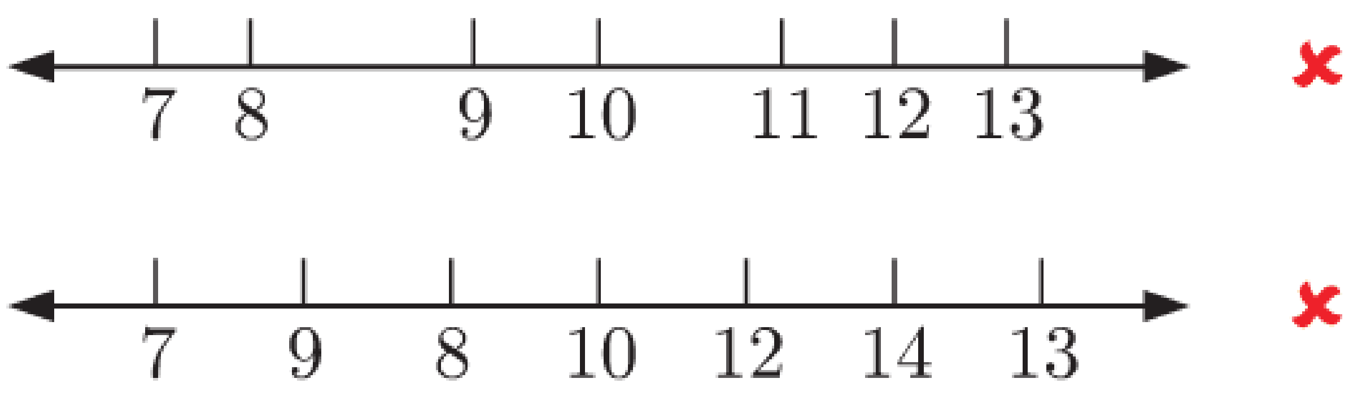
1 $100 + 24 \div 4$
2 $10 \times 4 - 20 \div 5$
3 $10\,000 - 3 \times 100 + 2 \times 8$
4 $7 \times 7 - 2 \times 2$
6 $(7 - 3) \times (6 + 1)$
8 $100 \times 100 - 14 \times 14$
11 $625 \div (20 + 5)$
13 $10 \times (9 \times 6)$
14 $70 - 3 \times 11$
15 $2 \times 5 + 3 \times 3$

F NUMBER LINES

A **number line** has equally spaced points marked with numbers in order and in the correct position relative to one another. Arrowheads are used to show that the line can continue indefinitely.

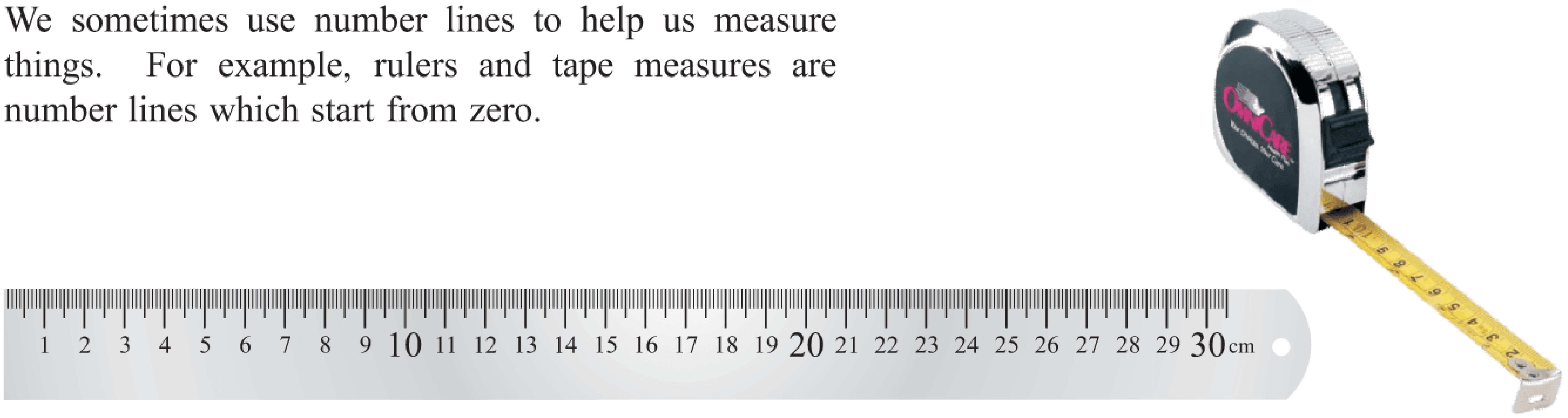


correct number line



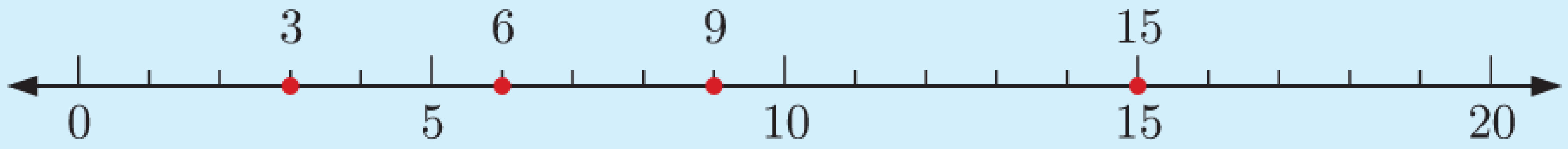
not number lines

We sometimes use number lines to help us measure things. For example, rulers and tape measures are number lines which start from zero.



Example 24 **Self Tutor**

Show the numbers 9, 15, 3, and 6 with dots on a number line.



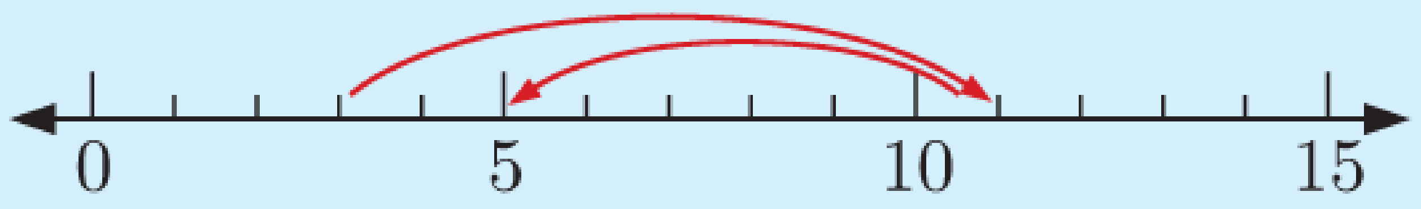
Number lines can also be used to show the four basic **operations** of addition, subtraction, multiplication, and division with whole numbers.

Example 25 **Self Tutor**

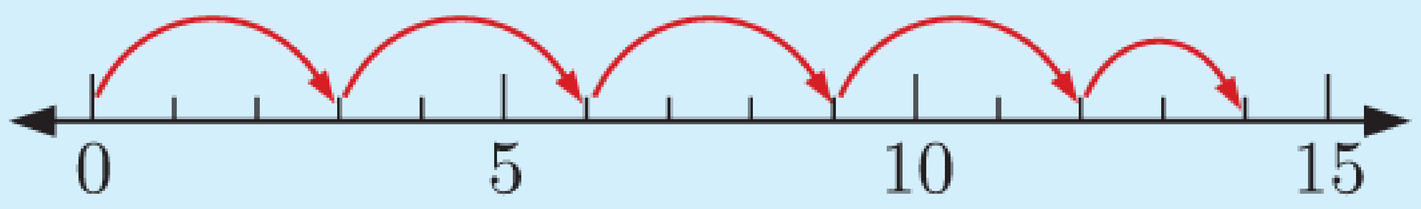
Perform the following operations on a number line:

a $3+8-6$ **b** $4\times 3+2$ **c** $23\div 5$

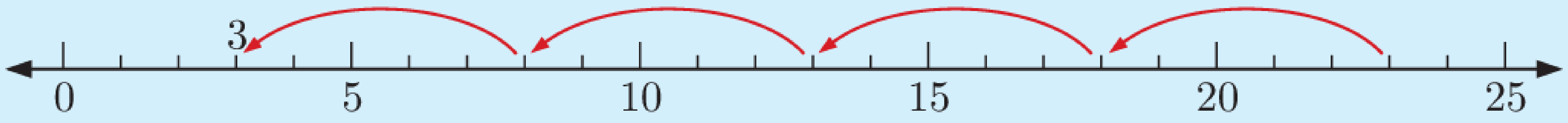
a Starting from 3, we jump 8 units to the right, then jump 6 units to the left.
$\therefore\ 3+8-6=5$



b Starting from 0, we jump 3 units to the right 4 times, then jump 2 units more to the right.
$\therefore\ 4\times 3+2=14$



c Starting from 23, we jump towards 0 in groups of 5 units.



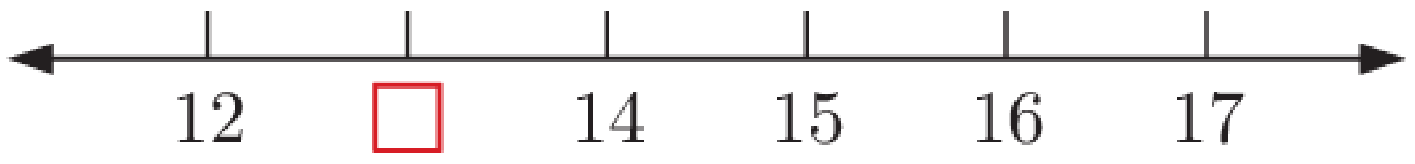
There are 4 complete jumps, with 3 left over, so $23\div 5=4$ with remainder 3.

EXERCISE 2F

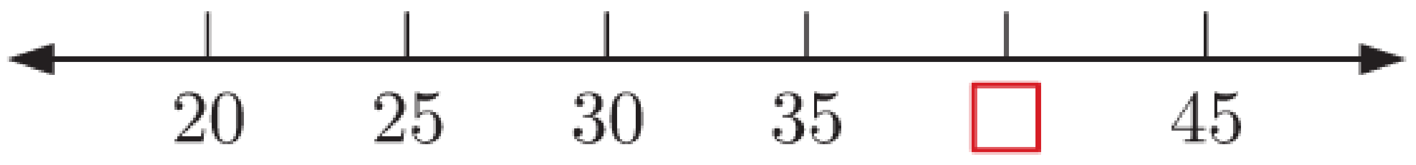
1 Find the missing values in the following number lines:

a

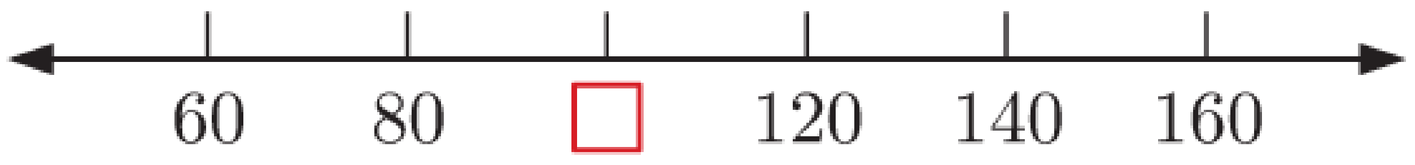
b



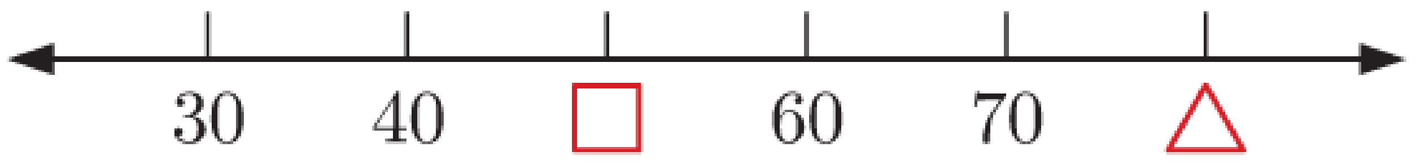
c



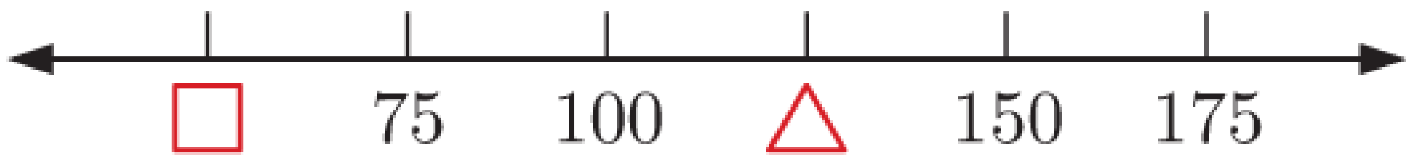
d



e



f



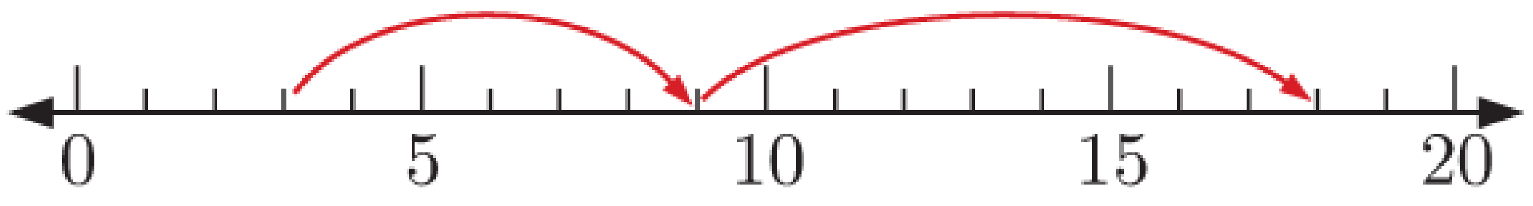
2 Show the following numbers on a number line:

a 9, 4, 8, 2, 7 **b** 14, 19, 16, 18, 13 **c** 28, 31, 23, 30, 34

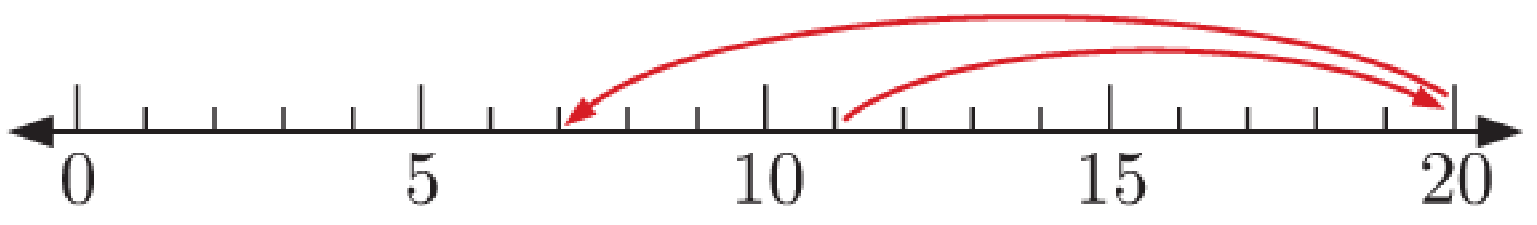
d 70, 30, 60, 90, 40 **e** 250, 75, 200, 25, 125 **f** 4000, 3000, 500, 2500, 1500

3 What operations do the following number lines show? Include the final answer in each case.

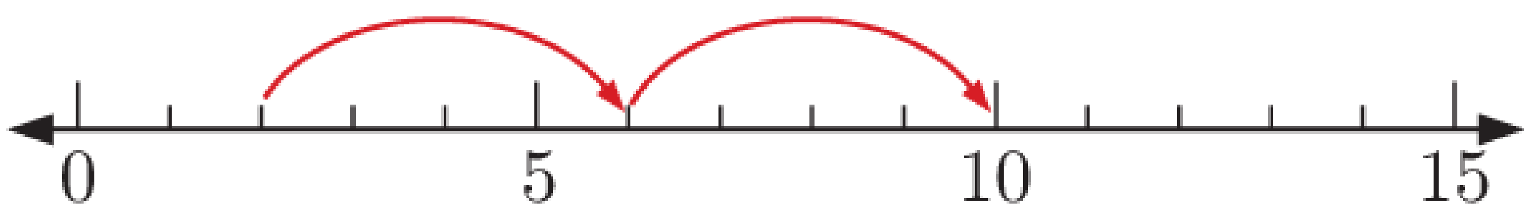
a



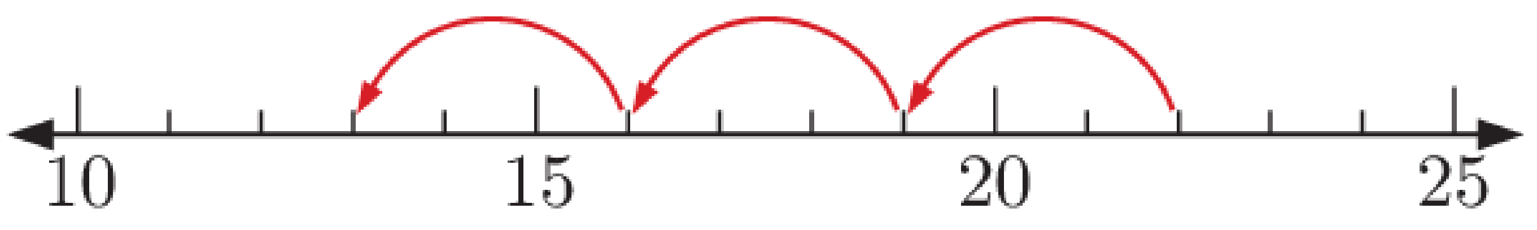
b



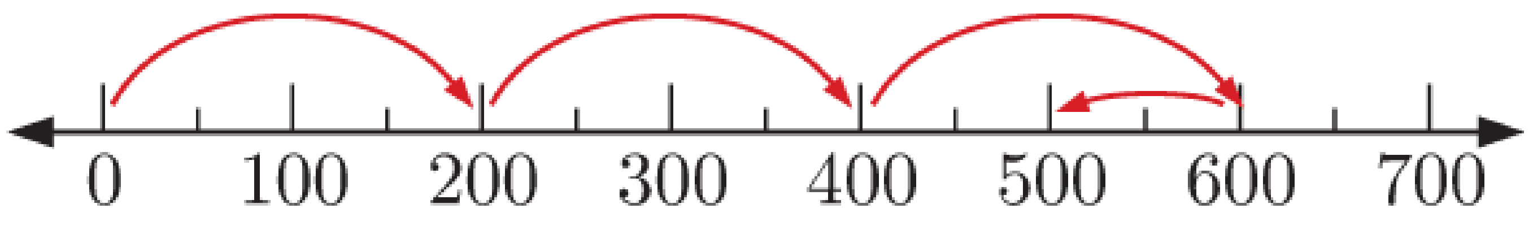
c



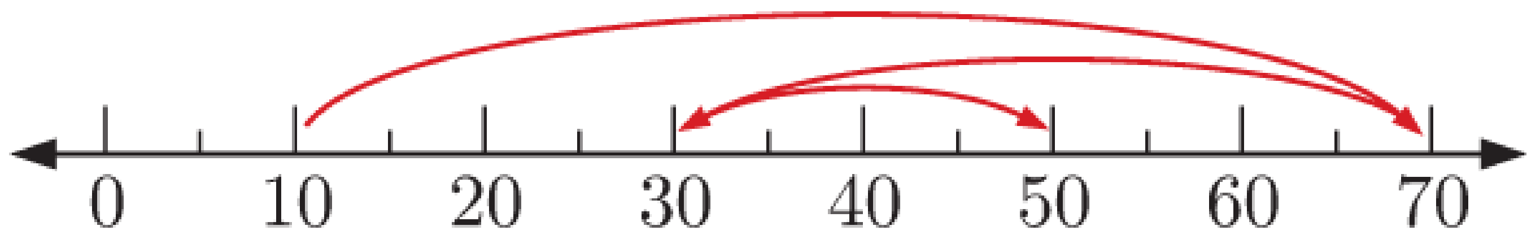
d



e



f



4 Draw a number line and show the following operations. Include the final answer in each case.

a $9 + 8 - 6$

b $2 + 4 + 8 - 2$

c $2 \times 7 + 9$

d $3 \times 6 - 8$

e $18 \div 3$

f $16 \div 7$

G ROUNDING NUMBERS

We often **round** numbers when we want an idea of how big a quantity is, but we do not need to know the exact number.

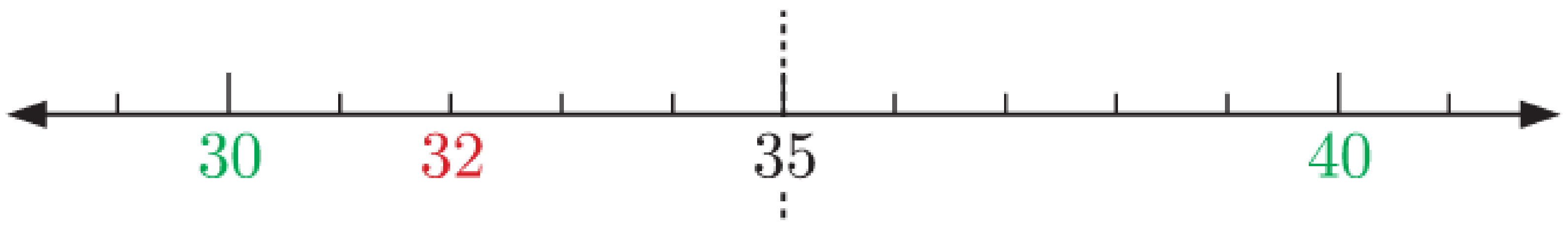
For example, suppose there were 306 competitors at an athletics carnival. We might say "there were about 300 competitors", since 300 is a good approximation for 306. In this case 306 has been **rounded** to the nearest hundred.

It is common to round off whole numbers to the **nearest ten**, **nearest hundred**, or **nearest thousand**.

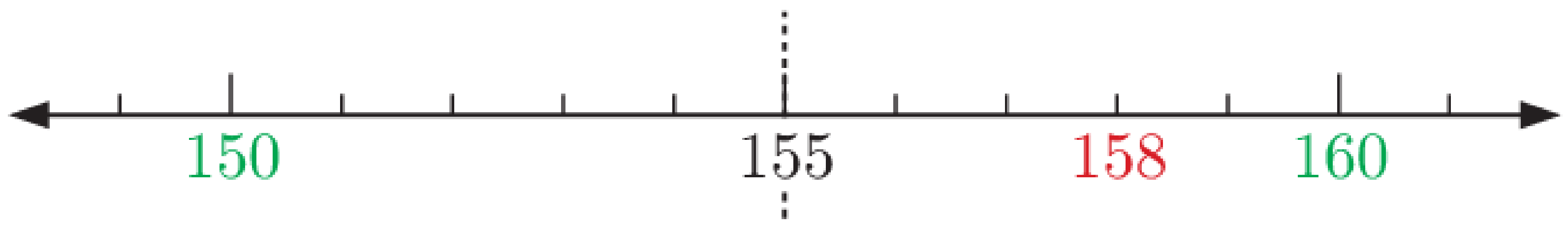
To learn how to round numbers, we start by looking at their position on a number line. For example, suppose we are rounding to the nearest 10:

In order to round a number to the nearest ten, we start by finding the multiples of ten on each side of the number.

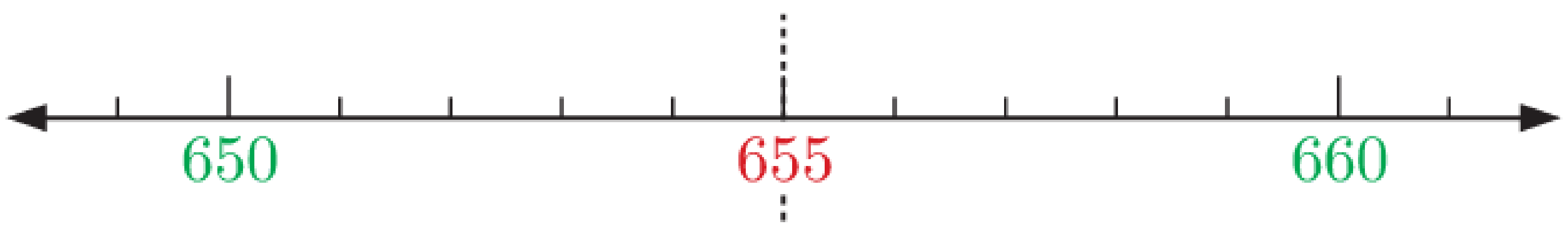
- 32 lies between 30 and 40.
  It is nearer to 30 than to 40, so 32 is **rounded down** to 30.



- 158 lies between 150 and 160.
  It is nearer to 160 than to 150, so 158 is **rounded up** to 160.



- 655 lies between 650 and 660.
  In fact, it is halfway between 650 and 660.
  In this case we agree to round up.
  So, 655 is **rounded up** to 660.



The rules for rounding off are:

- If the digit **after** the one being rounded off is **less than 5** (0, 1, 2, 3, or 4), then we round **down**.
- If the digit **after** the one being rounded off is **5 or more** (5, 6, 7, 8, or 9), then we round **up**.

Example 26 Self Tutor

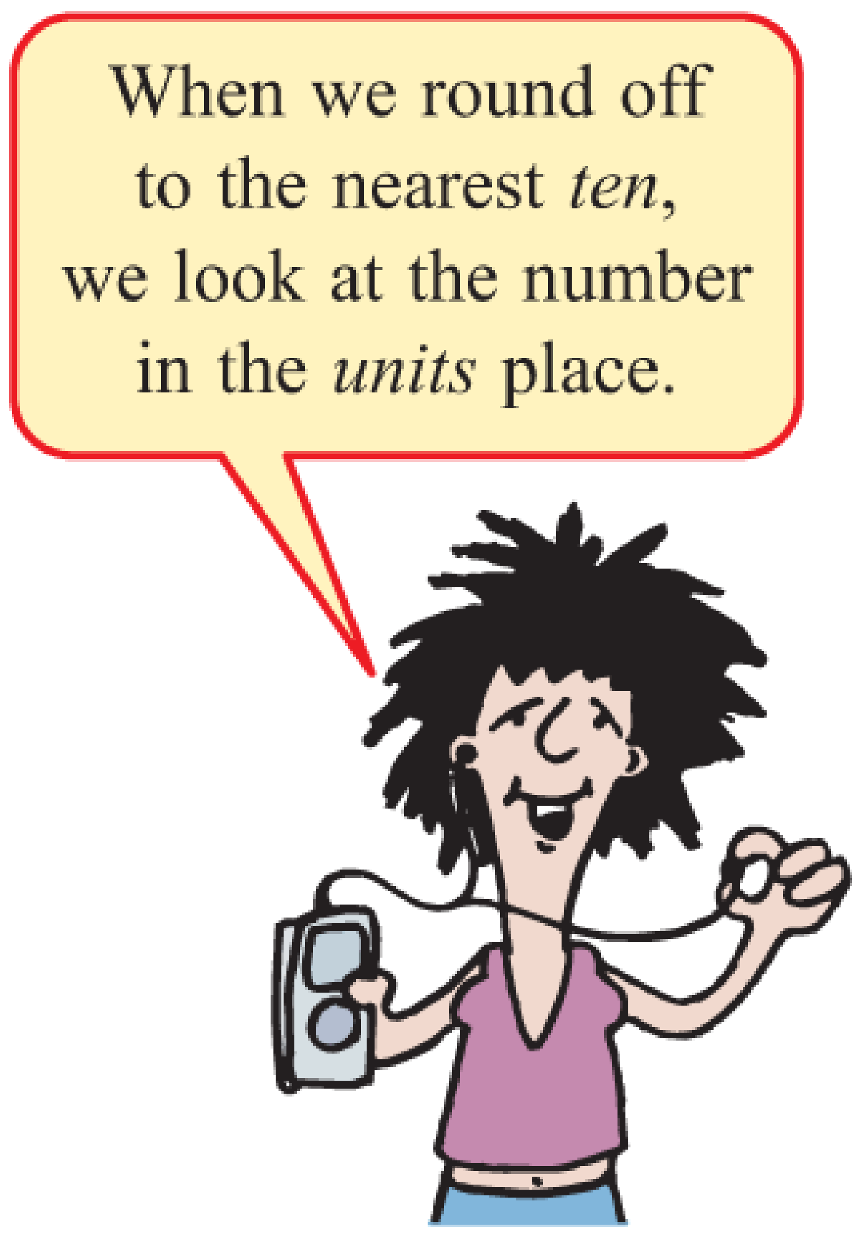
Round off to the nearest 10:

a 63 **b** 475 **c** 3029

a 63 lies between 60 and 70.
It is nearer to 60, so we round down.
$\therefore$ 63 is rounded to 60.

b 475 lies between 470 and 480.
It lies halfway between these numbers, so we round up.
$\therefore$ 475 is rounded to 480.

c 3029 lies between 3020 and 3030.
It is nearer to 3030, so we round up.
$\therefore$ 3029 is rounded to 3030.



EXERCISE 2G

1 Which of the two outer numbers is nearer to the number in bold, or are they the same distance away?

a 30, **38**, 40
b 70, **71**, 80
c 90, **95**, 100
d 130, **132**, 140
e 450, **457**, 460
f 730, **735**, 740
g 810, **818**, 820
h 1220, **1225**, 1230
i 6740, **6743**, 6750

2 Round off to the nearest 10:

a 17
b 53
c 35
d 71
e 97
f 206
g 311
h 502
i 888
j 3659
k 7444
l 8705
m 9606
n 14 075
o 30 122
p 47 777
q 69 569
r 70 099

Example 27 Self Tutor

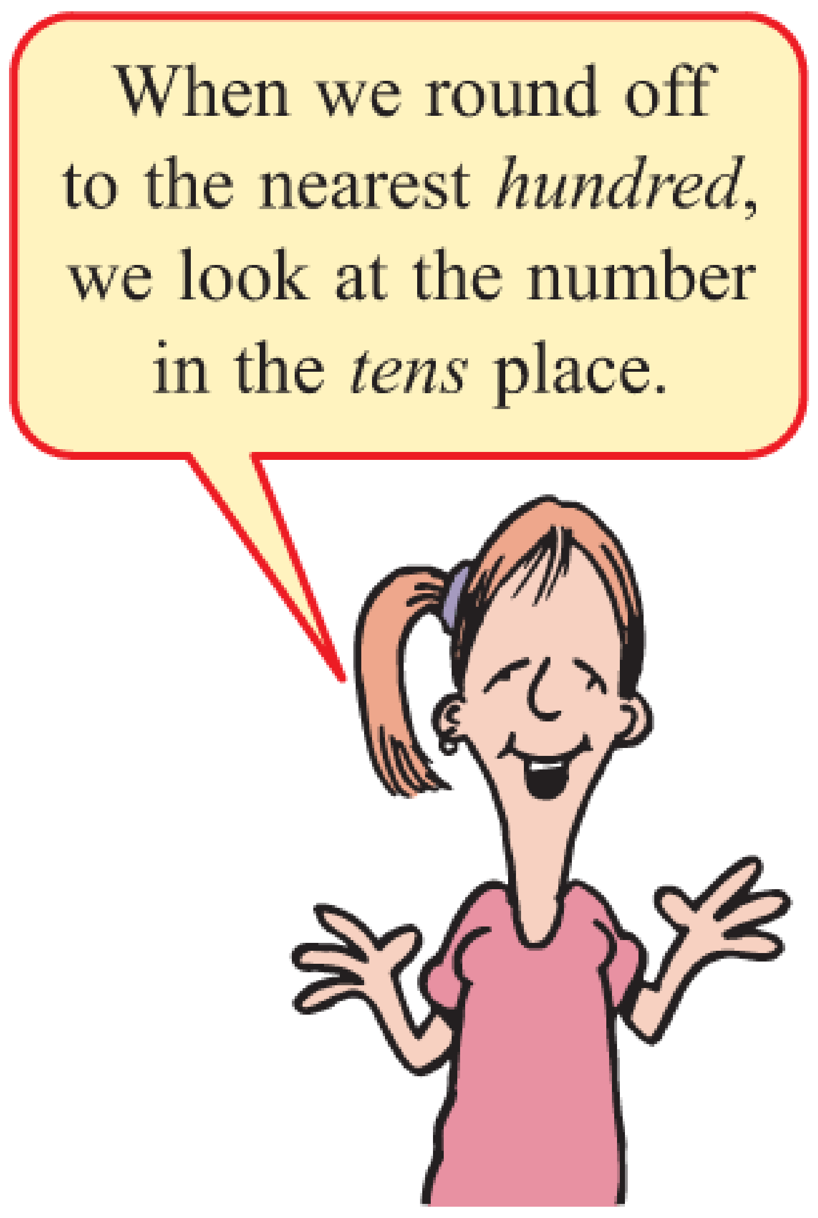
Round off to the nearest 100:

a 63 **b** 249 **c** 1655

a 63 lies between 0 and 100.
It is nearer to 100, so we round up.
$\therefore$ 63 is rounded to 100.

b 249 lies between 200 and 300.
It is nearer to 200, so we round down.
$\therefore$ 249 is rounded to 200.

c 1655 lies between 1600 and 1700.
It is nearer to 1700, so we round up.
$\therefore$ 1655 is rounded to 1700.



3 Which of the outer numbers is nearer to the number in bold?

a 500, **547**, 600 **b** 7600, **7631**, 7700 **c** 2900, **2985**, 3000

4 Round off to the nearest 100:

a 75 **b** 211 **c** 572 **d** 793 **e** 1050 **f** 2684
g 6998 **h** 13 208 **i** 27 660 **j** 38 457 **k** 55 443 **l** 85 074

Example 28 Self Tutor

Round off to the nearest 1000:

a 932 **b** 4500 **c** 44 482

a 932 lies between 0 and 1000.
It is nearer to 1000, so we round up.
$\therefore$ 932 is rounded to 1000.

b 4500 lies between 4000 and 5000.
It is halfway between these numbers, so we round up.
$\therefore$ 4500 is rounded to 5000.

c 44 482 lies between 44 000 and 45 000.
It is nearer to 44 000, so we round down.
$\therefore$ 44 482 is rounded to 44 000.

When we round to the nearest *thousand*, we look at the number in the *hundreds* place.

5 Round off to the nearest 1000:

a 834 **b** 495 **c** 1089 **d** 5485
e 7800 **f** 6500 **g** 9990 **h** 9399
i 13 095 **j** 7543 **k** 246 088 **l** 499 859

Example 29 Self Tutor

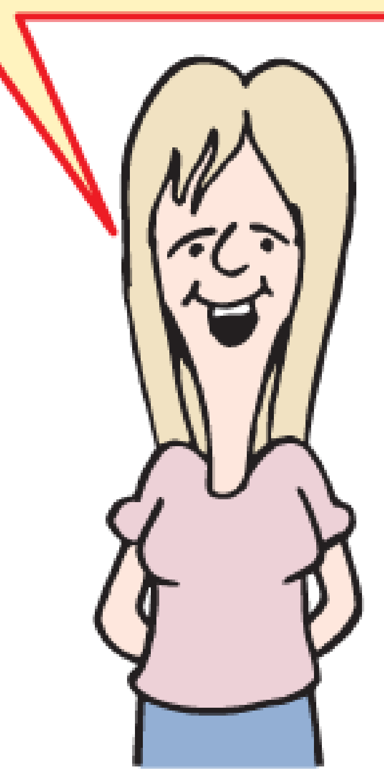
Round off to the nearest 10 000:

a 42 635 **b** 99 981

a 42 635 lies between 40 000 and 50 000.
It is nearer to 40 000, so we round down.
$\therefore$ 42 635 is rounded to 40 000.

b 99 981 lies between 90 000 and 100 000.
It is nearer to 100 000, so we round up.
$\therefore$ 99 981 is rounded to 100 000.

To round a number to the nearest 10 000, look at the number in the *thousands* place.



6 Round off to the nearest 10 000:

a 18 124 **b** 47 600 **c** 54 500 **d** 75 850
e 89 888 **f** 52 749 **g** 90 555 **h** 99 776

7 Round off to the nearest 100 000:

a 181 000 **b** 342 000 **c** 654 000 **d** 709 850
e 139 888 **f** 450 749 **g** 290 555 **h** 89 512

8 Round off to the accuracy given:

a 37 musicians in an orchestra (to the nearest 10)
b 55 singers in a youth choir (to the nearest 10)
c a payment of £582 (to the nearest £10)
d a tax bill of €4095 (to the nearest €10)
e a load of bricks weighs 687 kg (to the nearest 100 kg)
f a car costs $24 995 (to the nearest $100)
g the journey was 35 621 km (to the nearest 100 km)
h the circumference of the Earth is 40 008 km (to the nearest 10 000 km)
i the cost of a house is £463 590 (to the nearest £10 000)
j the population of Berlin is 3 450 889 (to the nearest 100 000)

PUZZLE ROUNDING WHOLE NUMBERS

Click on the icon to obtain a printable crossword puzzle for rounding numbers.

PRINTABLE WORKSHEET

Global context

click here

Family trees

Statement of inquiry:	Drawing diagrams can help us to understand our relationships with those around us.
Global context:	Identities and relationships
Key concept:	Relationships
Related concepts:	Representation, System
Objective:	Knowing and understanding
Approaches to learning:	Thinking, Research

KEY WORDS USED IN THIS CHAPTER

- addition
- dividend
- index
- power
- rounding
- base
- division
- index notation
- product
- subtraction
- brackets
- divisor
- multiplication
- quotient
- sum
- difference
- exponent
- number line
- remainder

REVIEW SET 2A

1 Find:

a the difference between 17 and 35
b the sum of 13, 27, and 38
c the product of 13 and 8
d the quotient of 115 and 5.

2 Do these additions:

a
$$\begin{array}{r} 217 \\ +\ 541 \\ \hline \end{array}$$

b
$$\begin{array}{r} 3576 \\ +\ 4385 \\ \hline \end{array}$$

c
$$\begin{array}{r} 178 \\ 2307 \\ +\ 765 \\ \hline \end{array}$$

3 Simplify:

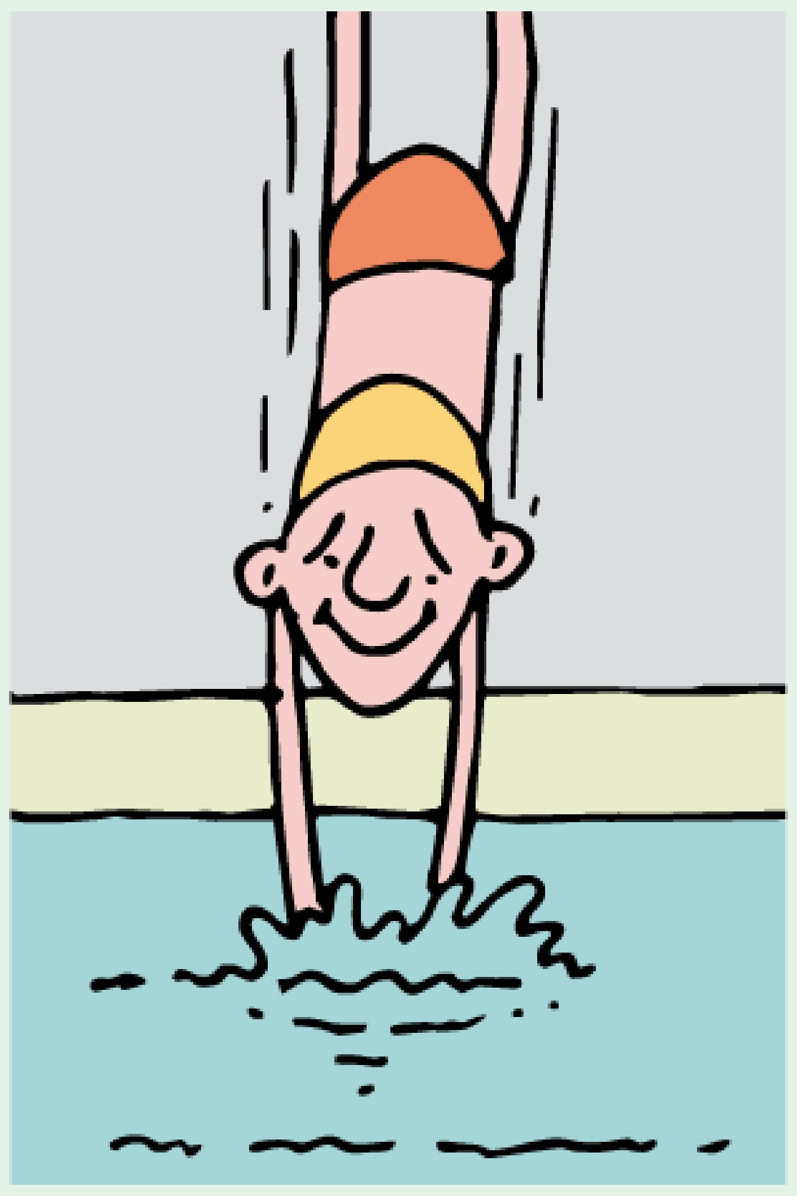
a $18 + 17 + 32$ **b** $108 - 29$ **c** $108 + 16 + 84$

d $2 \times 27 \times 5$ **e** $25 \times 17 \times 4$ **f** $23 \times 40 \times 5$

4 Find:

a $46 + 178$ **b** $311 - 39$ **c** 29×18 **d** $768 \div 6$

5 David scored 570 points in a diving competition. Victor scored 486 points. Find the difference between their scores.



6 Find:

a 34×100 **b** $59\,000 \div 1000$

7 Do these multiplications:

a
$$\begin{array}{r} 56 \\ \times\ 6 \\ \hline \end{array}$$

b
$$\begin{array}{r} 127 \\ \times\ 4 \\ \hline \end{array}$$

c
$$\begin{array}{r} 47 \\ \times\ 13 \\ \hline \end{array}$$

8 Write in index form:

a $6 \times 6 \times 6 \times 6$ **b** $2 \times 2 \times 2 \times 7 \times 7 \times 7 \times 7 \times 7$

9 Damien bought some shorts for £39, and a polo shirt for £32. How much change did he get from £100?

10 Find the total cost of 24 opera tickets at €112 each.

11 Round off to the nearest 100:

a 536 **b** 769

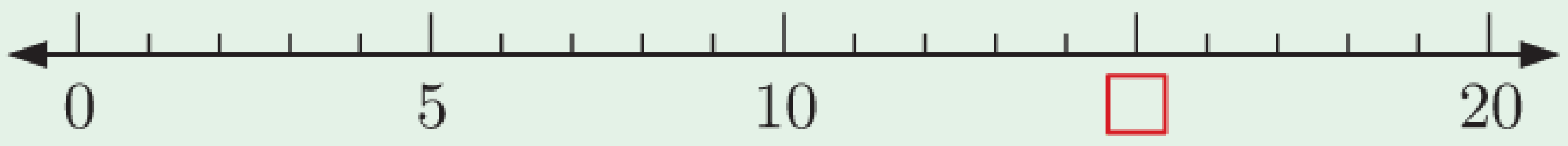
c 2309 **d** 49 471

12 Find:

a $19 - 8 \times 2$ **b** $10 + 6 \div 2$ **c** $24 \div (6 - 4)$

d $48 \div 4 \times 2$ **e** $72 \div 3^2$ **f** $36 \div (12 \div 6)^2$

13 Find the missing value in the number line alongside.



14 Show the following numbers on a number line:

a 5, 2, 7, 9 **b** 20, 80, 40, 30, 10

15 Round:

a 35 to the nearest 10 **b** 4384 to the nearest 1000

c 463 994 to the nearest 10 000 **d** 853 941 to the nearest 100 000.

REVIEW SET 2B

1 Do these subtractions:

a $\begin{array}{r} 85 \\ -\ 32 \\ \hline \end{array}$ **b** $\begin{array}{r} 629 \\ -\ 166 \\ \hline \end{array}$ **c** $\begin{array}{r} 2306 \\ -\ 512 \\ \hline \end{array}$

2 Find:

a $206 + 47 + 195$ **b** $3040 - 197$

3 Do these divisions:

a $4\overline{)1\ 3\ 6}$ **b** $5\overline{)3\ 8\ 5}$ **c** $7\overline{)3\ 9\ 2}$ **d** $9\overline{)5\ 5\ 3}$

4 50 000 tickets were sold in a charity home lottery. The cost of each ticket was \$20. How much money was raised?

5 Find:

a 23×39 **b** $408 \div 8$ **c** $3632 \div 7$

6 Julian has saved \$500, and wants to take a short vacation. The flights cost \$378, and the hotel costs \$147. Does Julian have enough money?

7 Derek takes 6 minutes to construct one section of fence. How many sections can he construct in 90 minutes?

8 Kathryn was paid €608 wages for the week. She also earned €24 per hour for 5 hours overtime. How much did Kathryn earn in total?

9 Draw a number line to show the operation $5 + 7 - 3$. Include the final answer.

10 Write using index notation:

a $2 \times 2 + 7 \times 7 \times 7$ **b** $11 \times 11 \times 11 - 3 \times 3 \times 3 \times 3$

11 Round off to the nearest 1000:

a 3487 **b** 9684 **c** 24 107 **d** 312 689

12 Find:

a $18 - 12 \div (1 + 5)$ **b** $9 \times 3 - 4 \times 6$ **c** $7 + (20 \div 4)^2$

13 At a sports store, Mildred bought 4 footballs for a total cost of \$68.

a How much did each football cost?

b Julie bought 3 footballs and 2 baseballs. Given that each baseball costs \$15, how much did Julie pay in total?

14 Replace $*$ by either $+$, $-$, $\times$, or $\div$ to make $2 \times 8 * 4 + 2 = 6$ a correct statement.

15 Round off to the accuracy given:

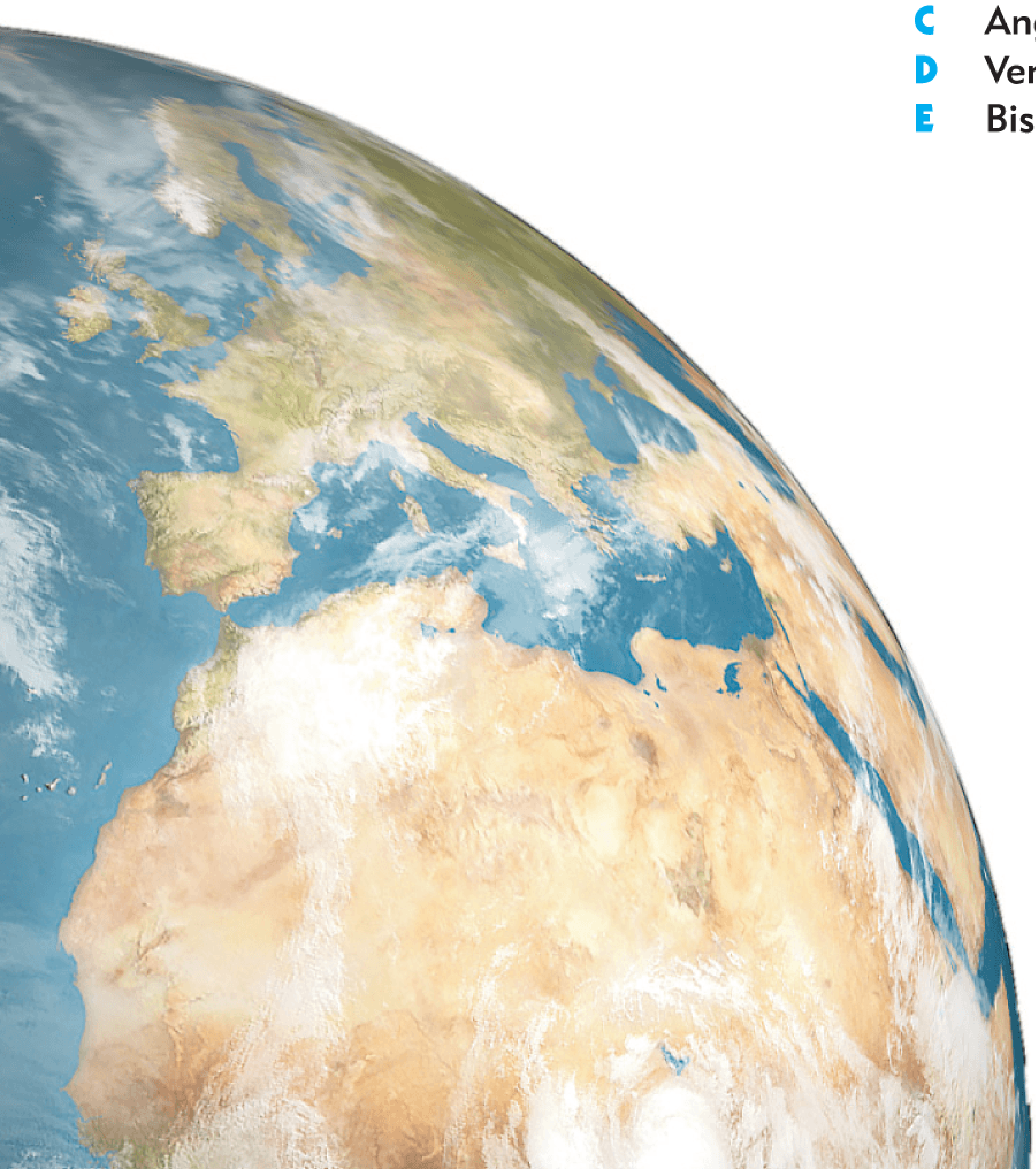
a a phone bill for £82 (to the nearest £10)

b the number of people in a sporting ground is 16 610 (to the nearest 1000)

Chapter 3

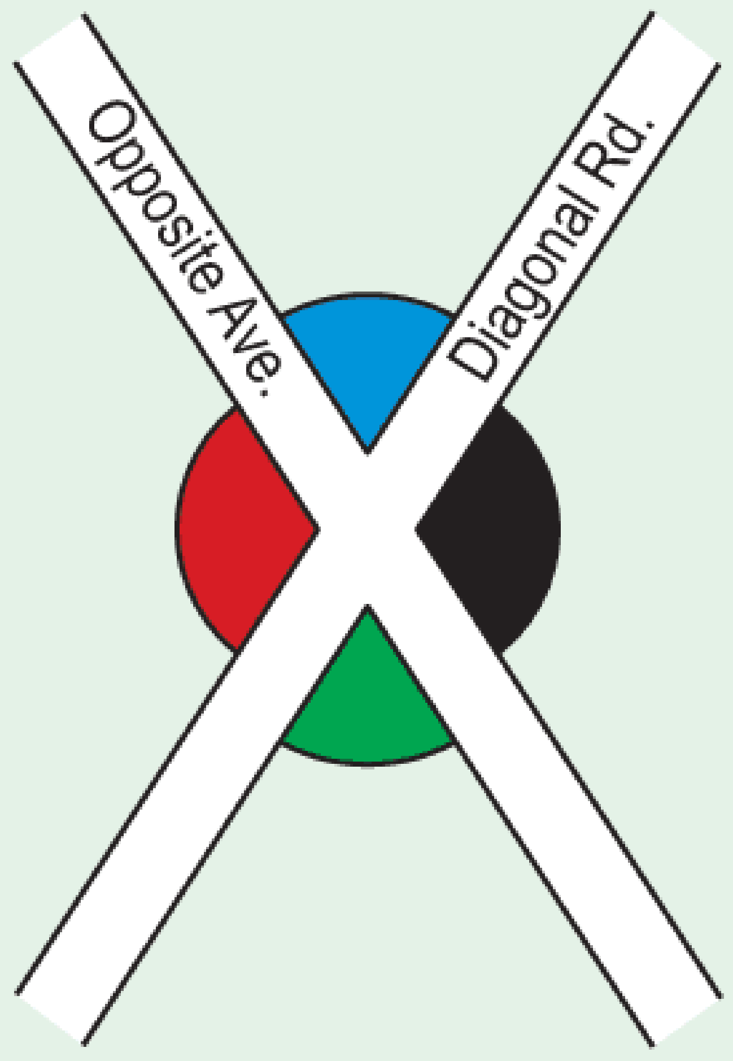
Points, lines, and angles

Contents:



OPENING PROBLEM

Henry was studying two straight roads on a map. He observed that at their point of intersection, there were four angles made. They are marked red, blue, black, and green.



Things to think about:

- **a** Do you think the blue angle is larger or smaller than the red angle?
- **b** How could you measure the sizes of the angles?
- **c** Are any of the angles equal in size?

We see points, lines, and angles all around us. Look at the picture alongside, and see how many points, lines, and angles you can find.

A POINTS AND LINES

POINTS

DISCUSSION WHAT IS A POINT?

In groups of 4 or 5, discuss the following:

- What is meant by a *point*?
- Give examples of points in your classroom.
- Give examples of things which could be used to *represent* a point.
- How small can a point be?

A **point** is used to mark a position or location.

Examples of points are:

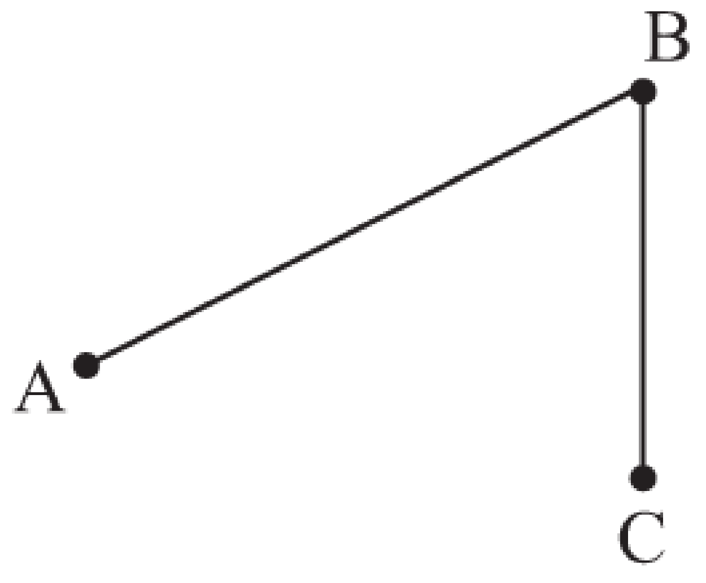
- the corner of your room where two walls and the ceiling all meet
- the tip of the mouse cursor on your computer
- a speck of dust.

A point does not have any size. However, we use small dots to represent points so that we can see them.

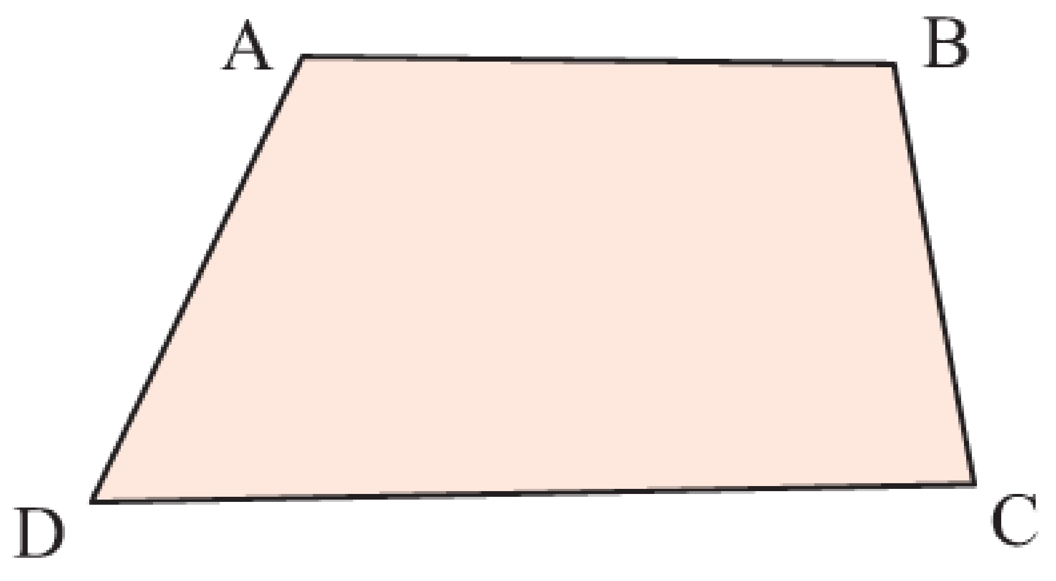
To help identify points, we label them with a capital letter.

We can then make statements like:

- "the distance from A to B is ...."
- "the angle at B measures ...."



VERTICES



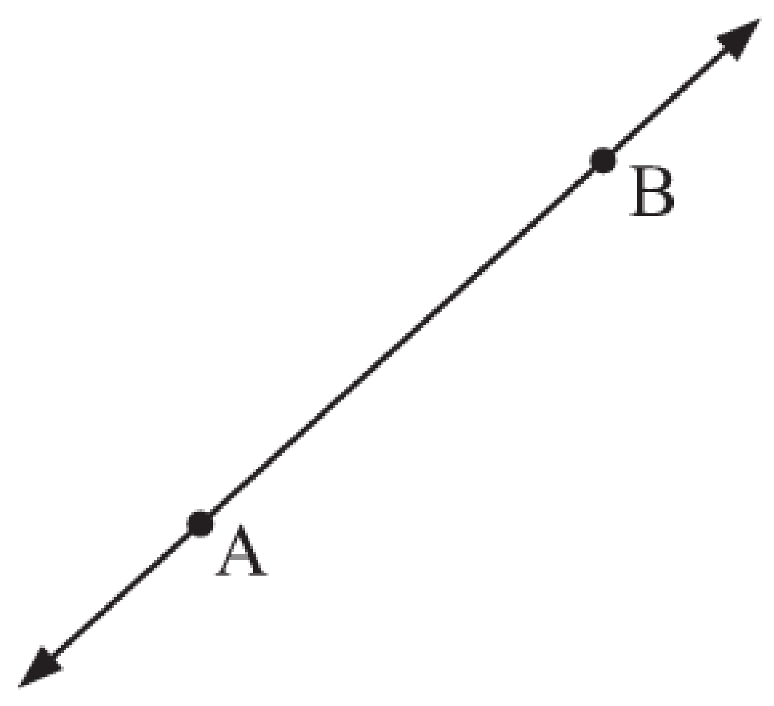
The figure alongside contains four points labelled A, B, C, and D. They are the corner points or **vertices** of the figure.

Vertices is the plural of **vertex**. We say that point B is a vertex of the figure.

LINES

A **straight line**, usually just called a **line**, is a continuous infinite collection of points in a particular direction. A line has no beginning and no end.

The line alongside passes through points A and B. We use arrowheads to show that the line continues endlessly in both directions. We can call the line "line AB" or "line BA".



DISCUSSION LINES

- How many different straight lines could be drawn through the single point A?
- Suppose A and B are two separate points. How many straight lines can be drawn which pass through both A and B?
- Suppose P, Q, and R are three different points. How many straight lines could be drawn which pass through P, Q, and R? Explain your answer.

We use the following notation to describe lines and parts of lines:

(AB) is the **line** which passes through A and B and continues endlessly in both directions.

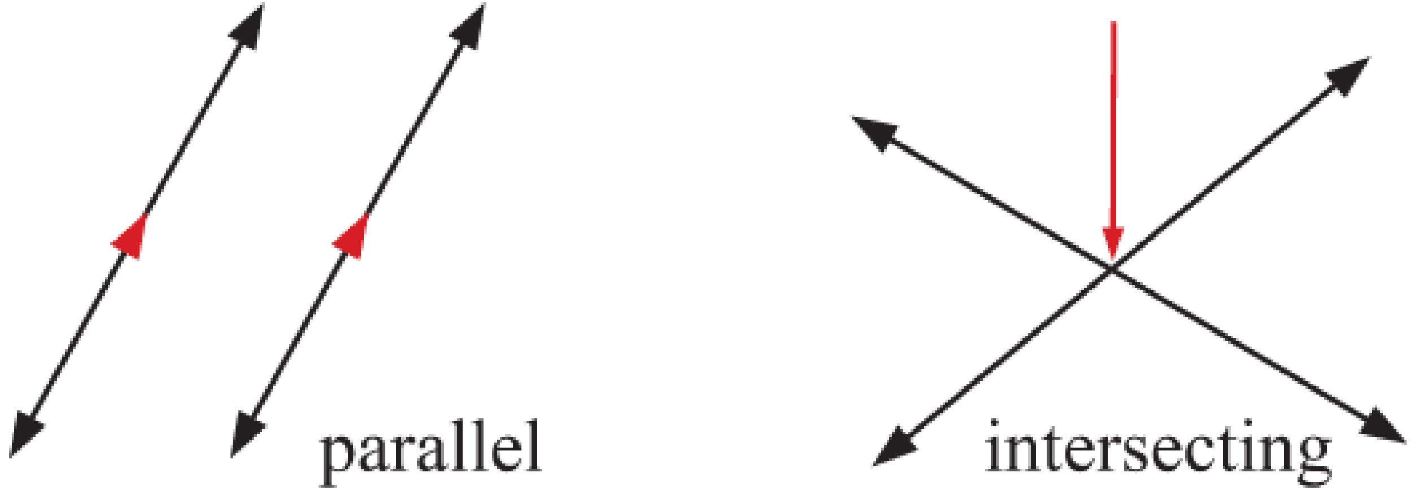
[AB] is the **line segment** which joins the two points A and B. It is only a part of the line (AB).

[AB) is the **ray** which starts at A, passes through point B, and continues on endlessly.

PARALLEL AND INTERSECTING LINES

In mathematics, a **plane** is a flat surface like a table top or a sheet of paper.

Two straight lines on a plane are either **parallel** or **intersecting**.

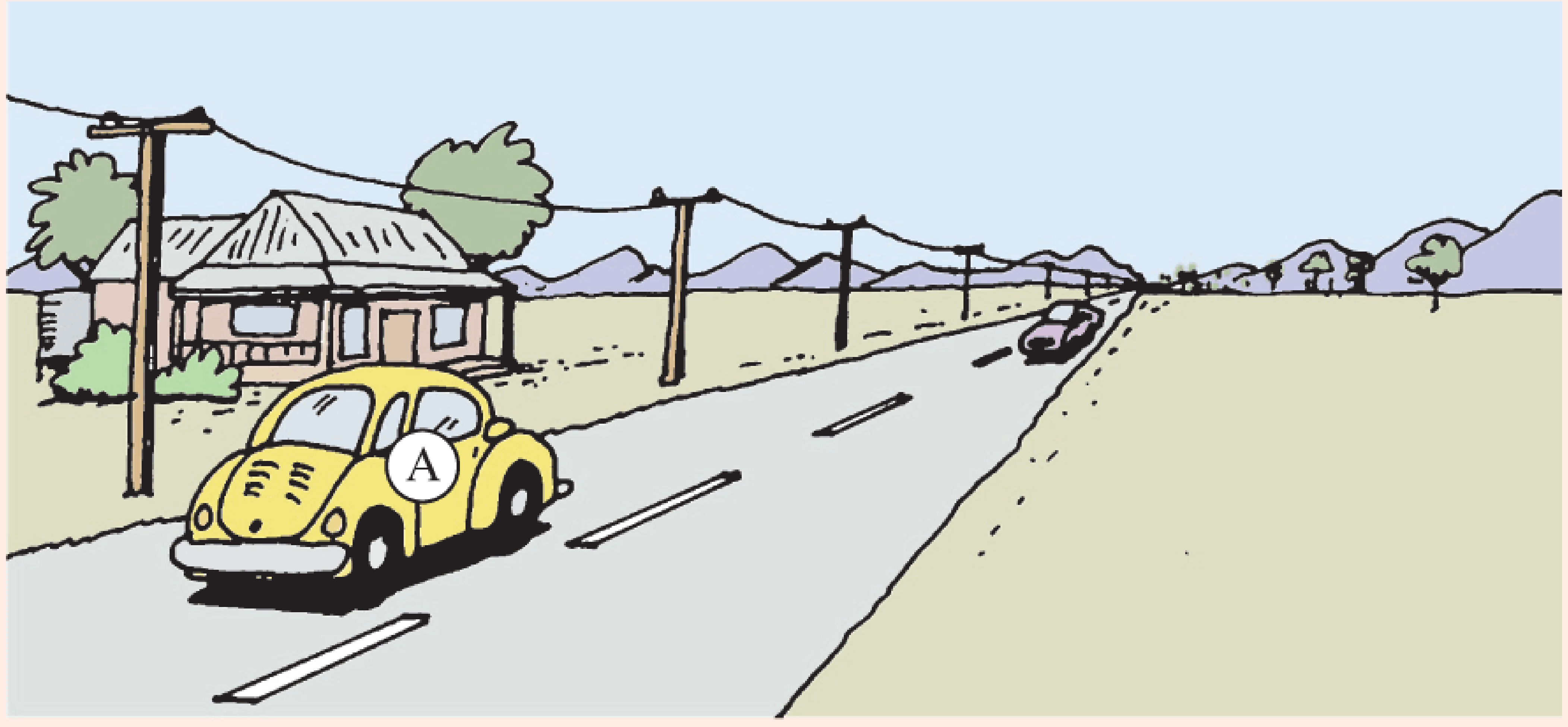


Parallel lines are lines which are always a fixed distance apart and never meet.

We draw arrowheads at the middle of parallel lines to indicate that they are parallel.

DISCUSSION

The edges of a long straight road are parallel lines. To the people in car A, the parallel lines appear to meet in the distance. Discuss the picture. Do the parallel lines really meet?



EXERCISE 3A

1 Give two examples in the classroom of:

a a point **b** a line.

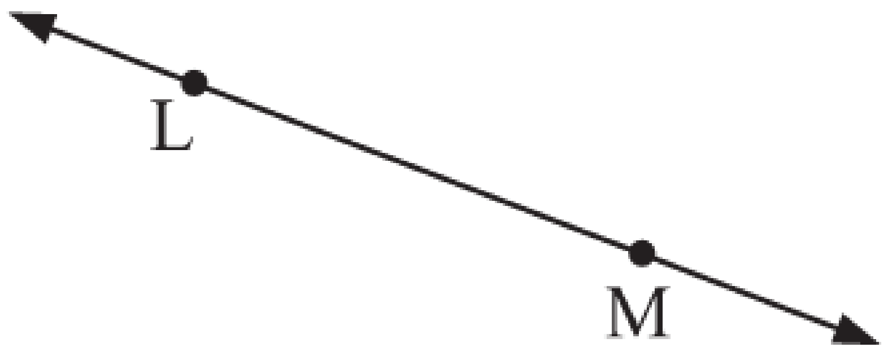
2 In geometry, what is meant by:

a a vertex **b** a point of intersection **c** parallel lines?

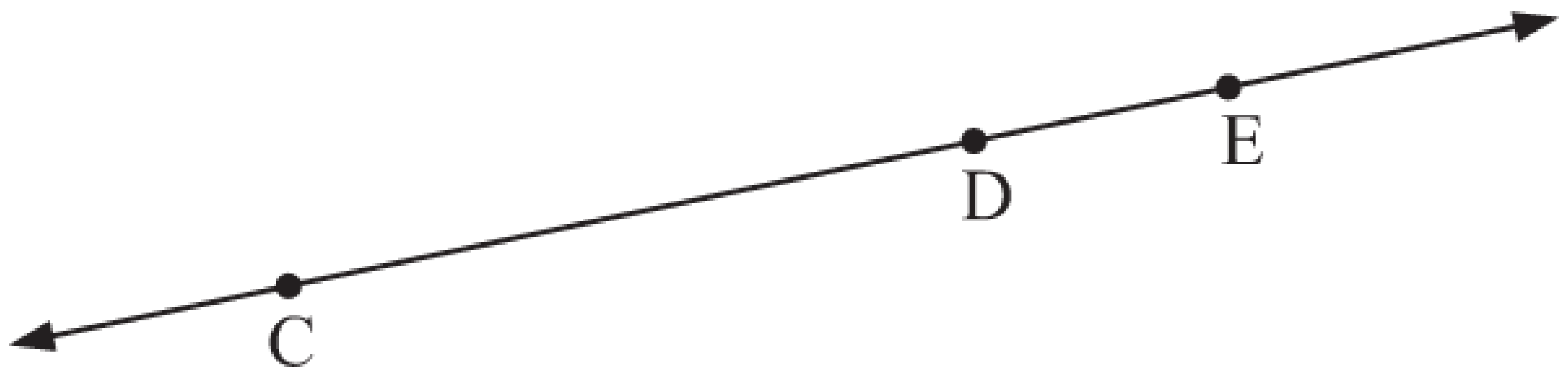
Draw diagrams to illustrate each.

3 Give *all* ways of naming these straight lines:

a

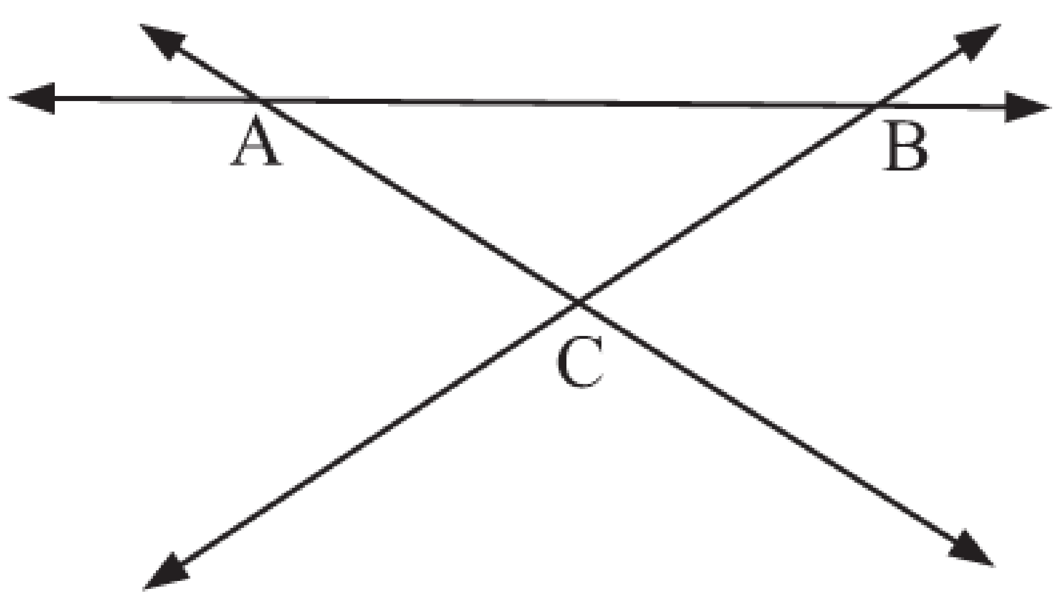


b



Hint: In **b** there are 6 answers.

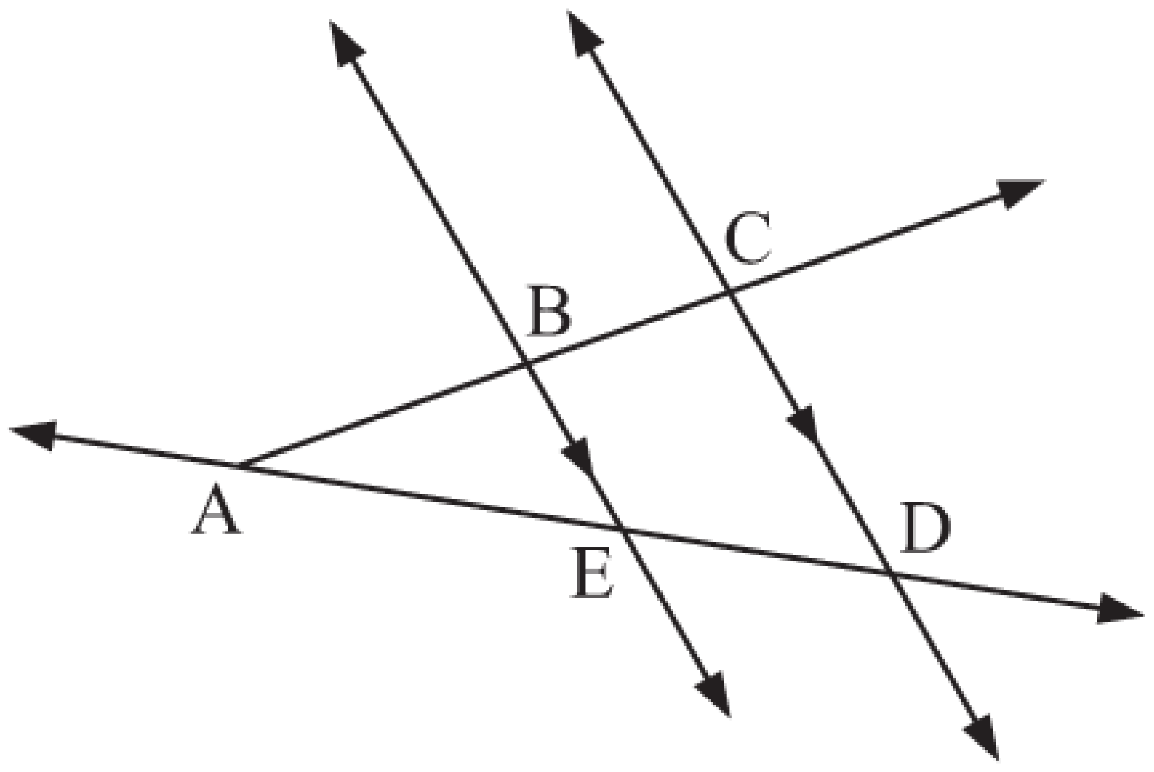
4 Find the intersection of:



- **a** lines (AB) and (BC)
- **b** lines (CB) and (CA).

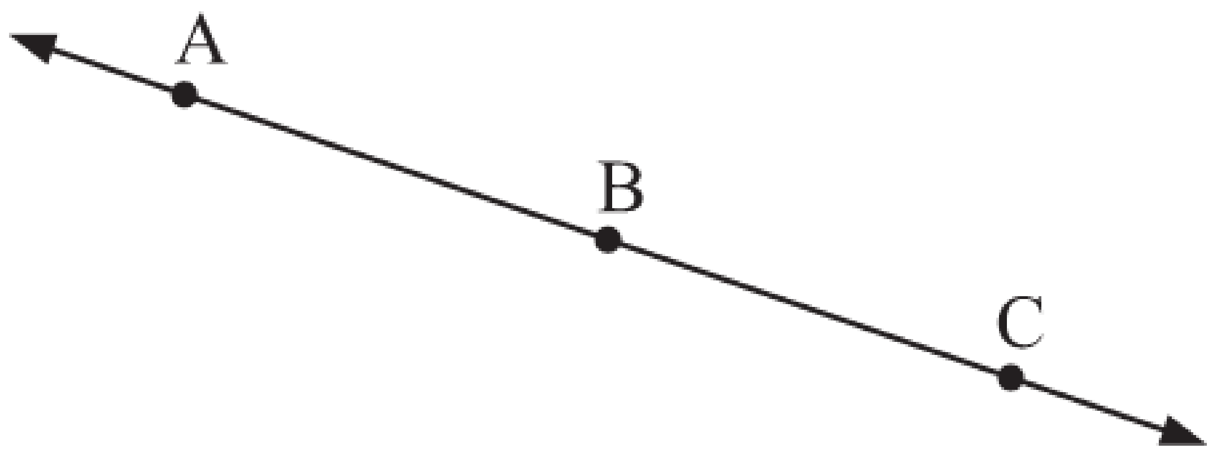
5 **a** At which point does the ray [AB) intersect the line (CD)?

- **b** Which points do *not* lie on the line (AD)?
- **c** Which two lines are parallel?



6 Find the intersection of:

- **a** [AB] and [BC]
- **b** [AB) and [BC]
- **c** [AB] and [AC].



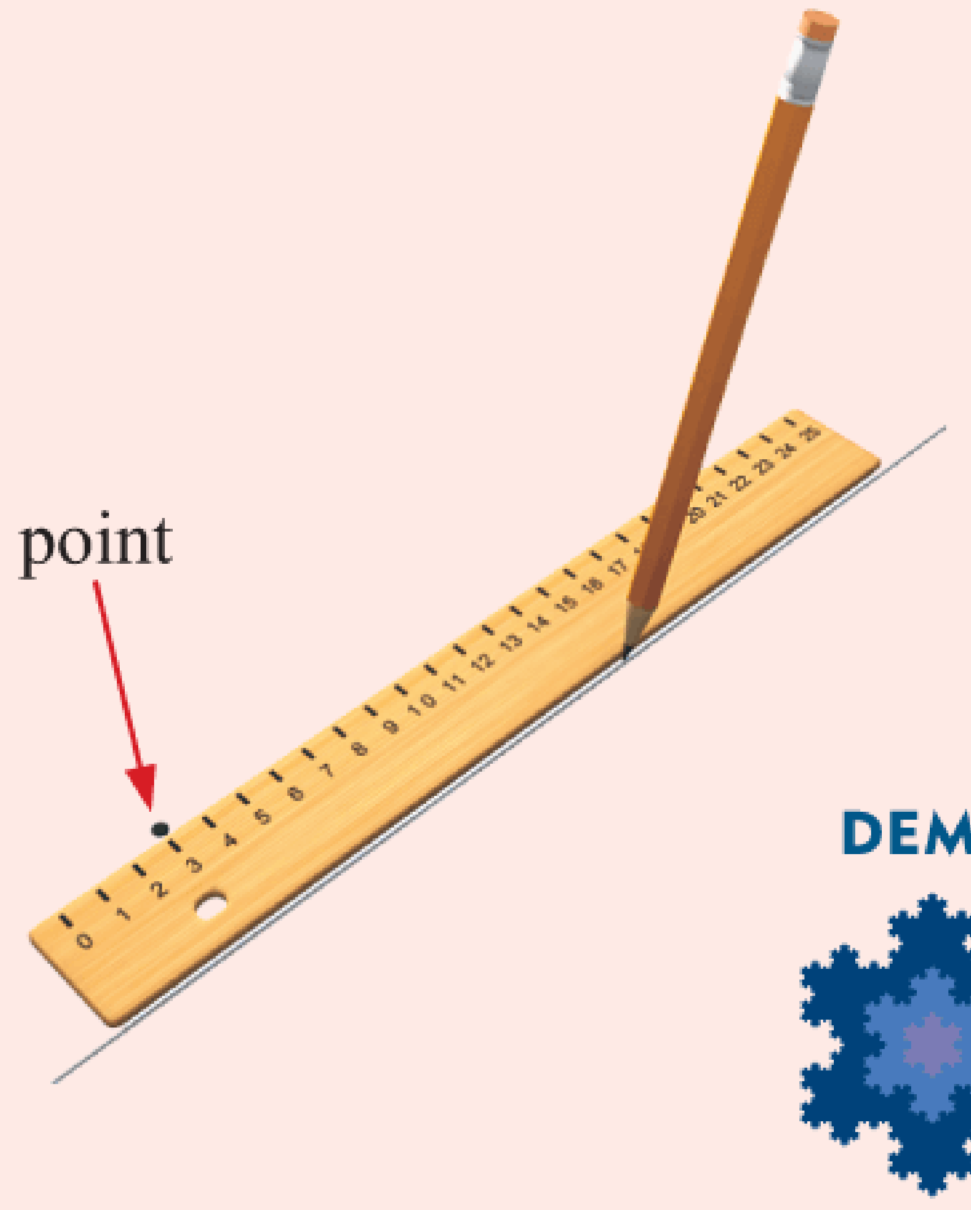
ACTIVITY STRAIGHT LINE SURPRISES

Part 1

You will need: a sheet of blank paper, a ruler, and a sharp pencil.

What to do:

1. Mark a point somewhere near the centre of the paper.
2. Line up one edge of your ruler so it passes through the point. Draw a line along the *other* edge of the ruler across the paper.
3. Change the position of the ruler, keeping one edge passing through the point. Draw a second line so that it intersects with your first line.



DEMO

4. Change the position of the ruler again, keeping one edge passing through the point. The other edge will allow you to form a triangle. Draw the third line.
5. Rule lots more lines like this.
6. Describe the shape formed by the intersecting lines as more lines are drawn. Can you explain why this shape is forming?

Part 2

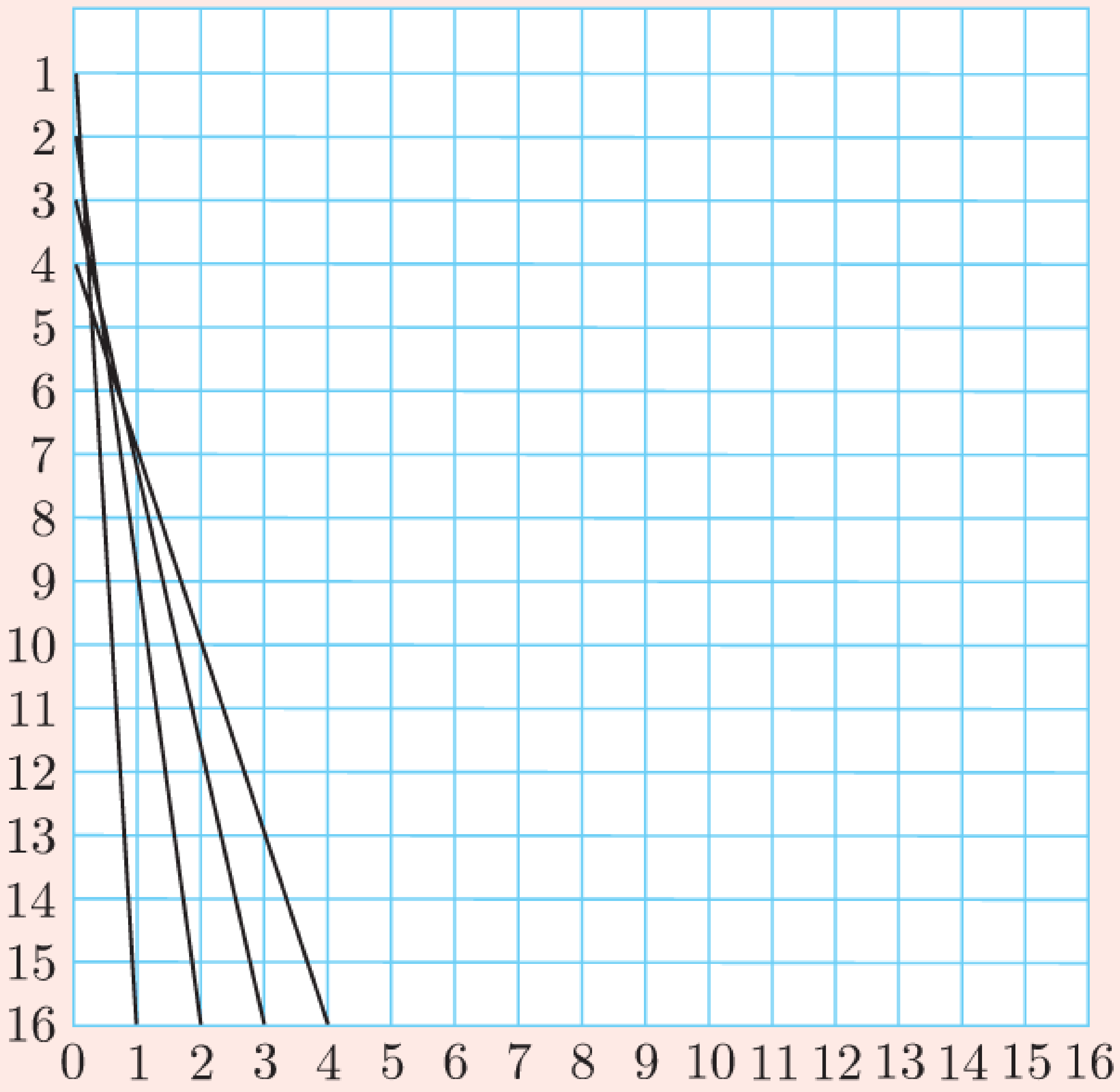
You will need:

a sheet of 5 mm graph paper, a ruler, and a sharp pencil.

PRINTABLE GRAPH PAPER

What to do:

1. On the graph paper draw a horizontal base line. Mark the numbers from 0 to 16 on it as shown.
2. Draw a vertical line at 0. Mark the numbers from 1 to 16 on it as shown.
3. Rule a straight line from 1 to 1, from 2 to 2, and so on until all of the pairs of points have been joined.



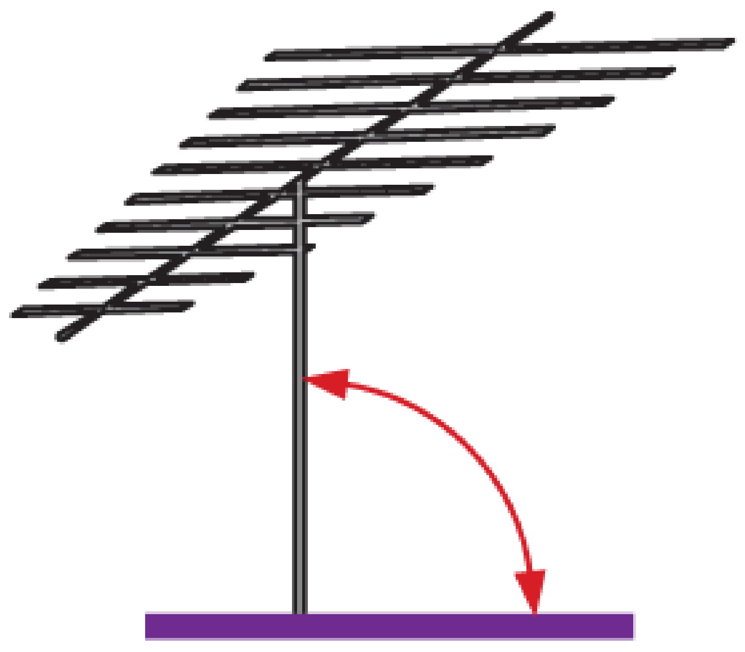
4. Now draw a vertical line at 16 on the base line, and repeat the pattern on the right hand side.
5. Turn the page upside down, and repeat the pattern so that you have drawn four sets of straight lines.

DEMO

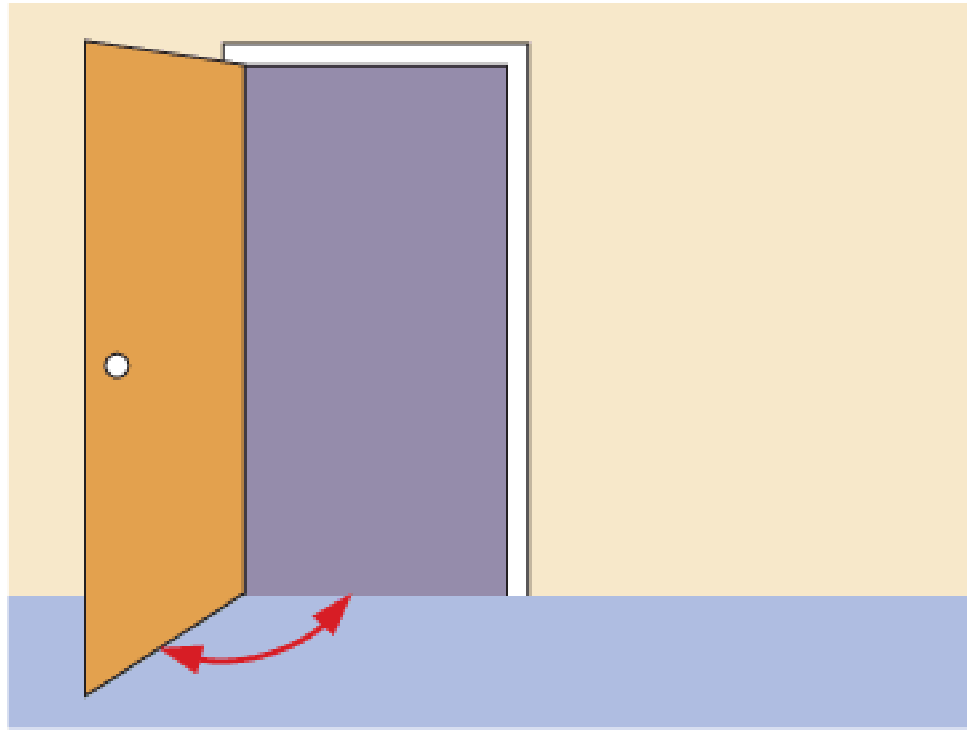
B ANGLES

Whenever two lines or edges meet, an **angle** is formed between them.

The angle between the pole and the ground.



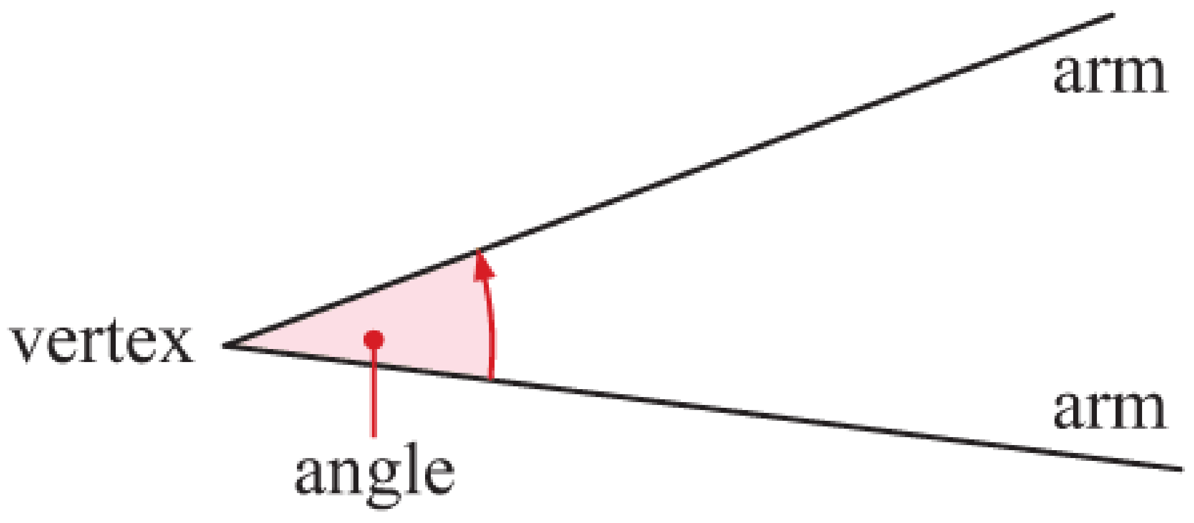
The angle between the wall and the door.



The angle between the hands of a clock.

An **angle** is made up of two arms which meet at a point called the **vertex**.

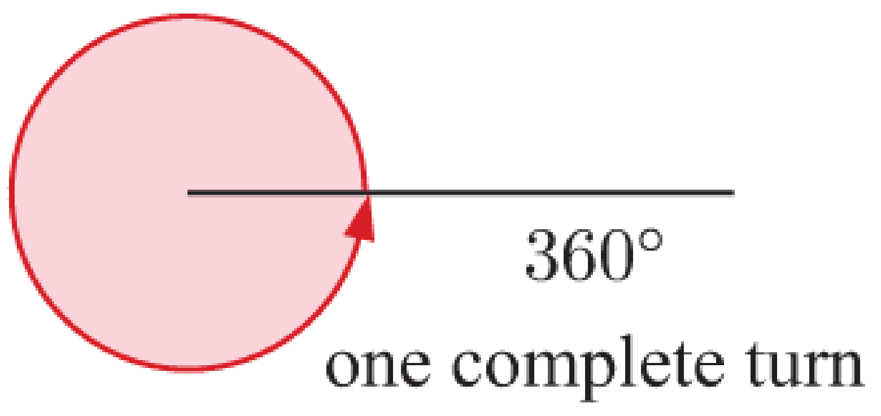
The **size** of the angle is measured by the amount of turning or rotation from one arm to the other.



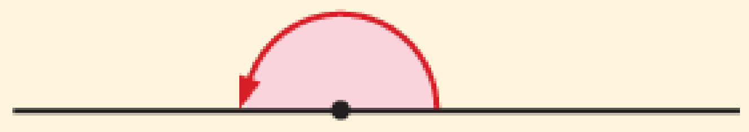
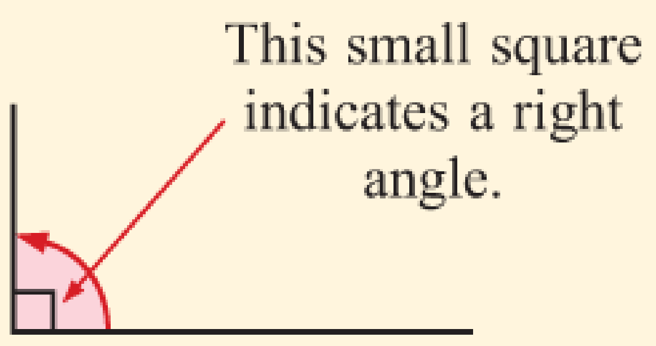
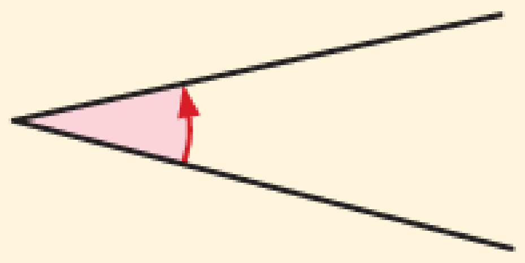
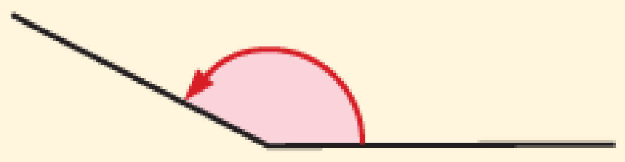
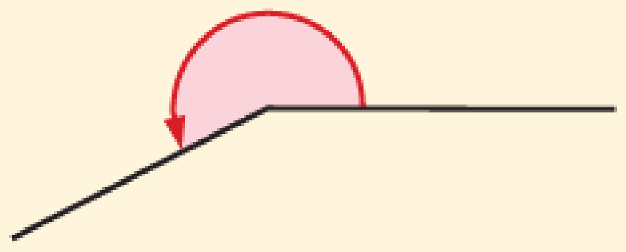
MEASURING ANGLES

In order to accurately find the size or measure of an angle, we need a unit of measurement. The unit we will use is the **degree**.

Ancient Babylonian astronomers decided that there would be 360 degrees in a **complete turn** or **revolution**. We write this as 360°. 360 was probably chosen because it can be divided by 2, 3, 4, 5, 6, 8, 9, 10, 12, and 15, to give whole number answers.



We can measure other angles by comparing their size with a complete turn.

Revolution	**Straight Angle**	**Right Angle**
 One complete turn. One revolution = 360°.	 $\frac{1}{2}$ turn A straight angle = 180°.	 $\frac{1}{4}$ turn A right angle = 90°.
Acute Angle	**Obtuse Angle**	**Reflex Angle**
 Less than a $\frac{1}{4}$ turn. An acute angle has size between 0° and 90°.	 Between $\frac{1}{4}$ turn and $\frac{1}{2}$ turn. An obtuse angle has size between 90° and 180°.	 Between $\frac{1}{2}$ turn and 1 turn. A reflex angle has size between 180° and 360°.

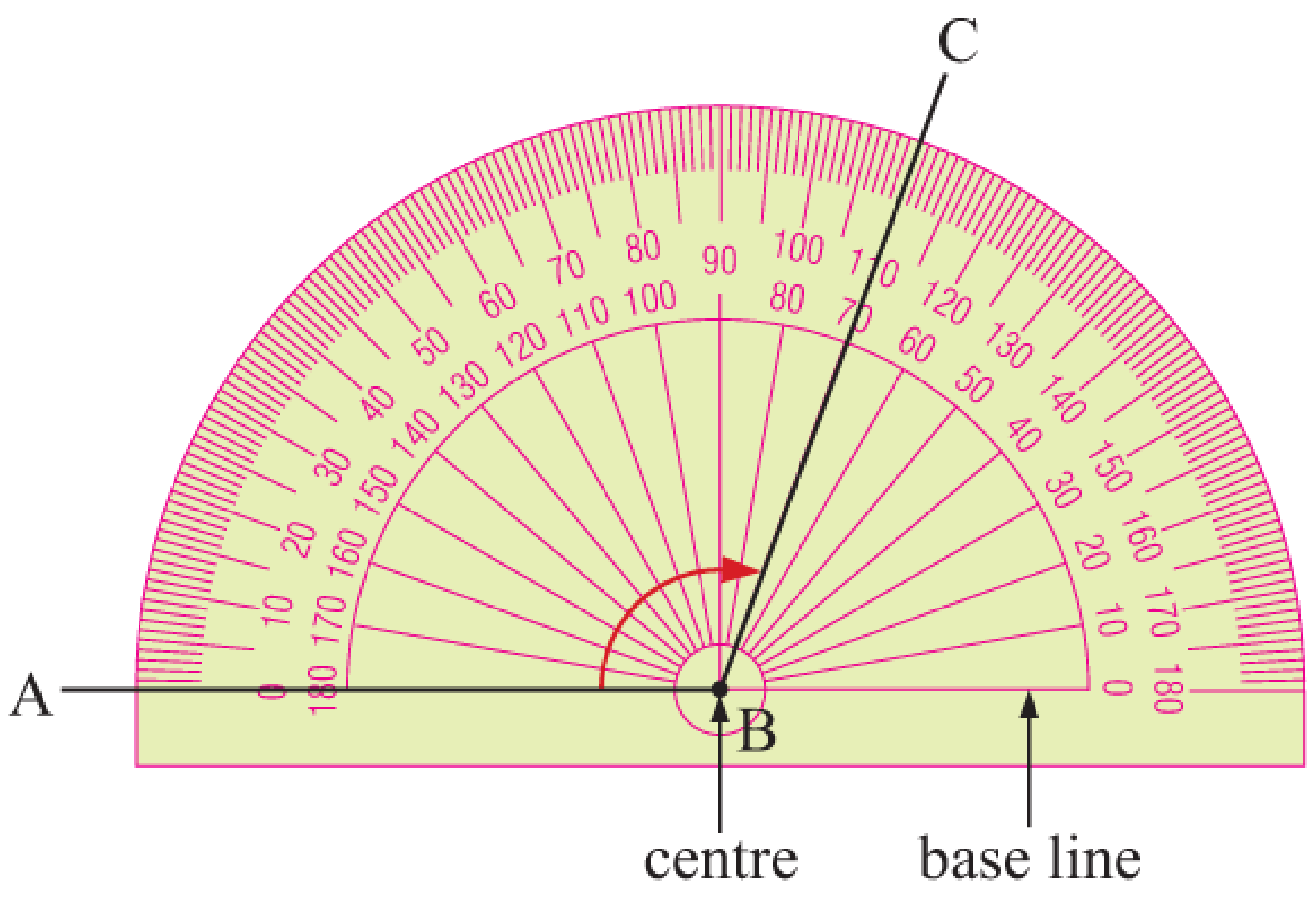
THE PROTRACTOR

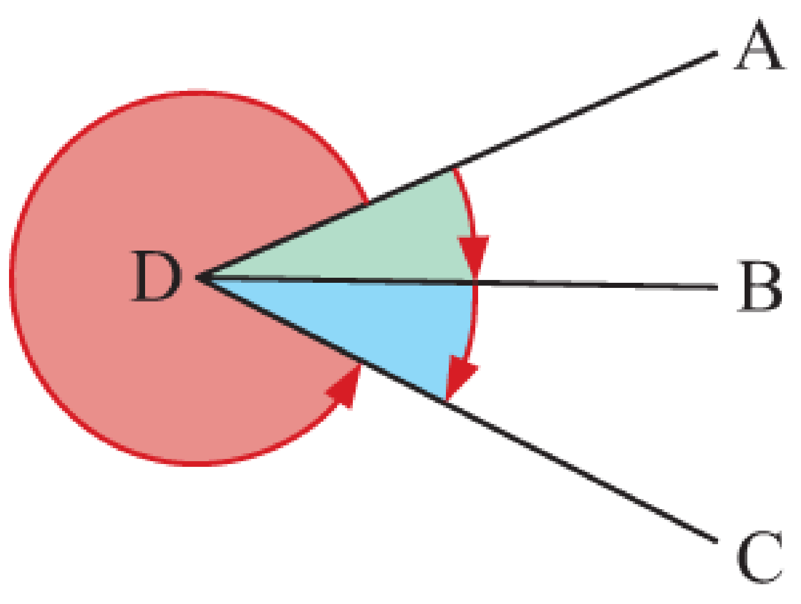
In order to measure angles, we use a **protractor** with 1° markings.

Alongside is a protractor placed with its centre at B and its base line on [AB]. The amount of turning from [AB] to [BC] is 110 degrees.

We write $A\widehat{B}C = 110°$ which reads 'the angle ABC measures 110 degrees'.

$A\widehat{B}C$ is called **three point notation**. We use it to make it clear which angle we are referring to.





For example:

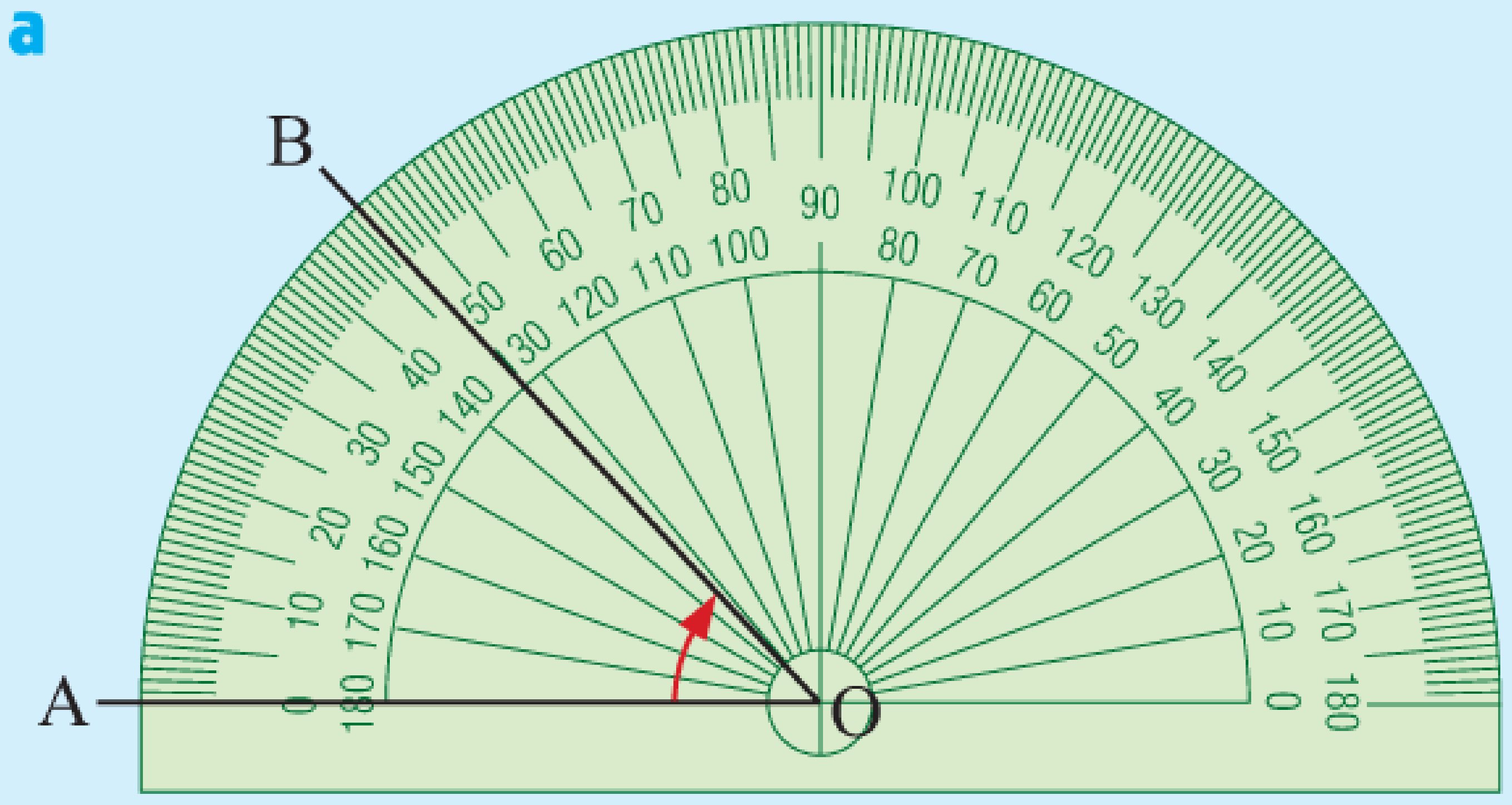
- the green angle is $A\widehat{D}B$ or $B\widehat{D}A$
- the blue angle is $B\widehat{D}C$ or $C\widehat{D}B$
- $A\widehat{D}C$ is made up of the green angle and the blue angle
- the red angle is *reflex* $A\widehat{D}C$, since its size is more than 180°.

Example 1

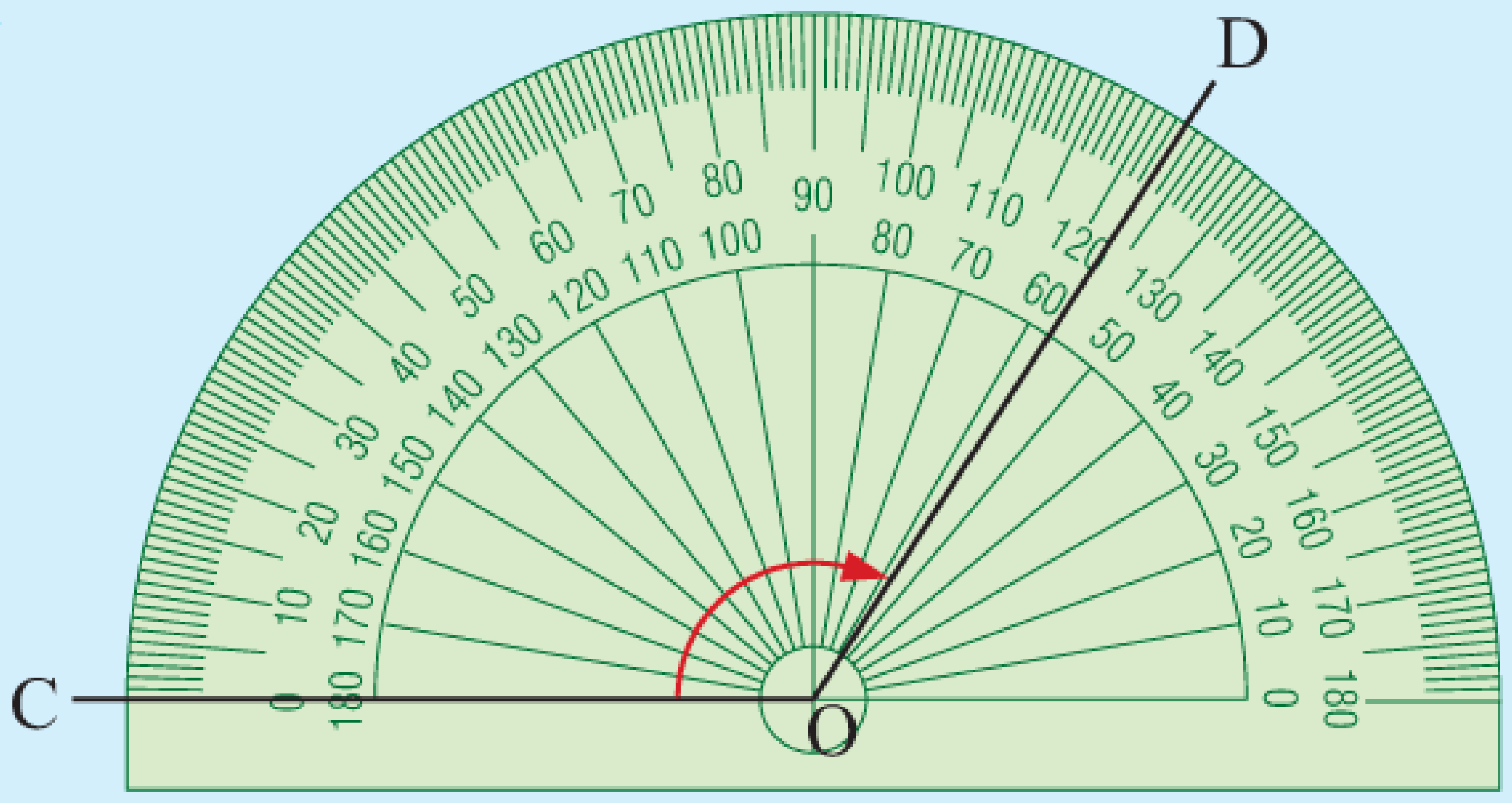
Self Tutor

Measure these angles:

a



b



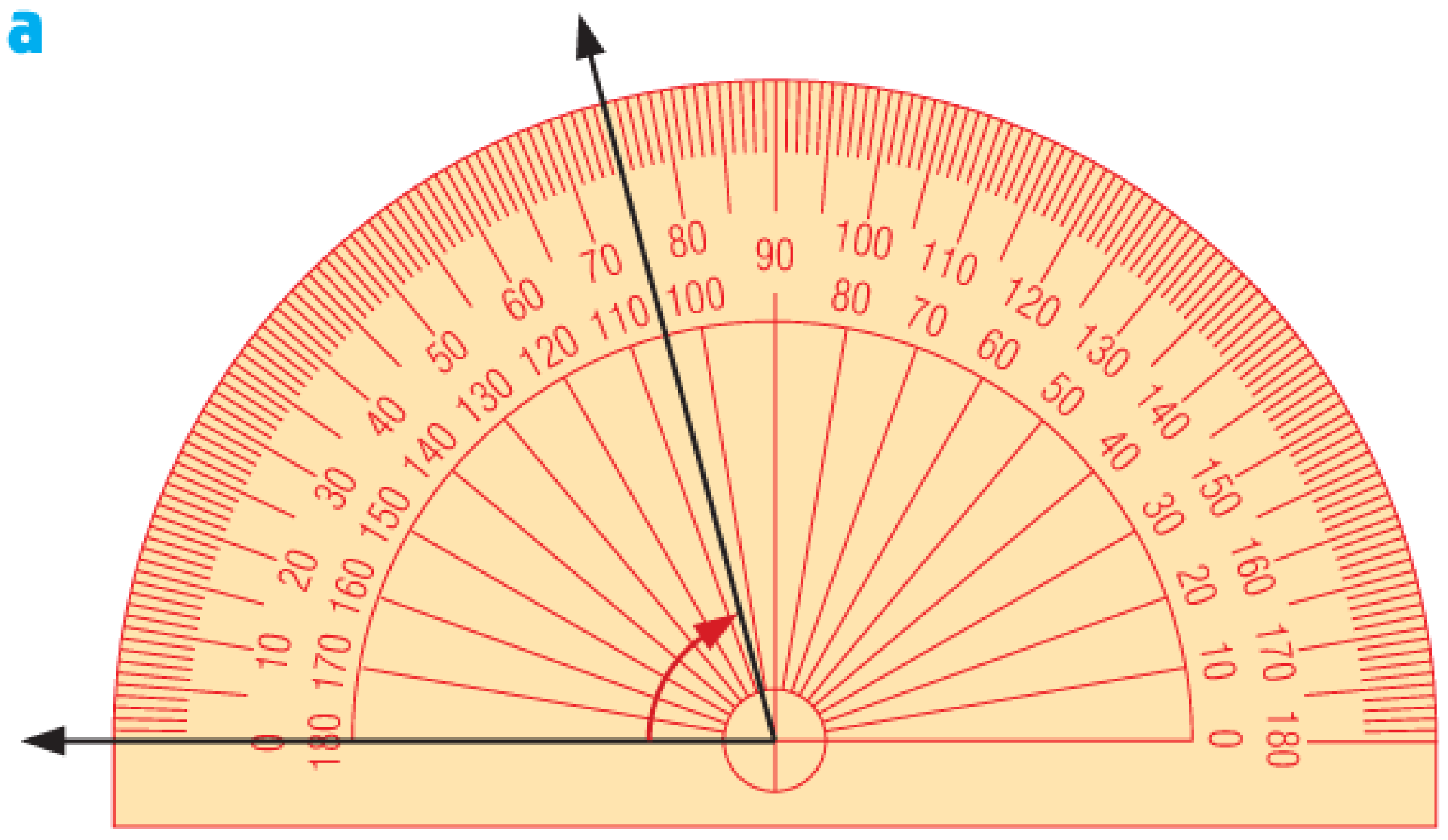
a $A\widehat{O}B$ has size 47°.

b $C\widehat{O}D$ has size 123°.

EXERCISE 3B

1 Measure these angles:

a



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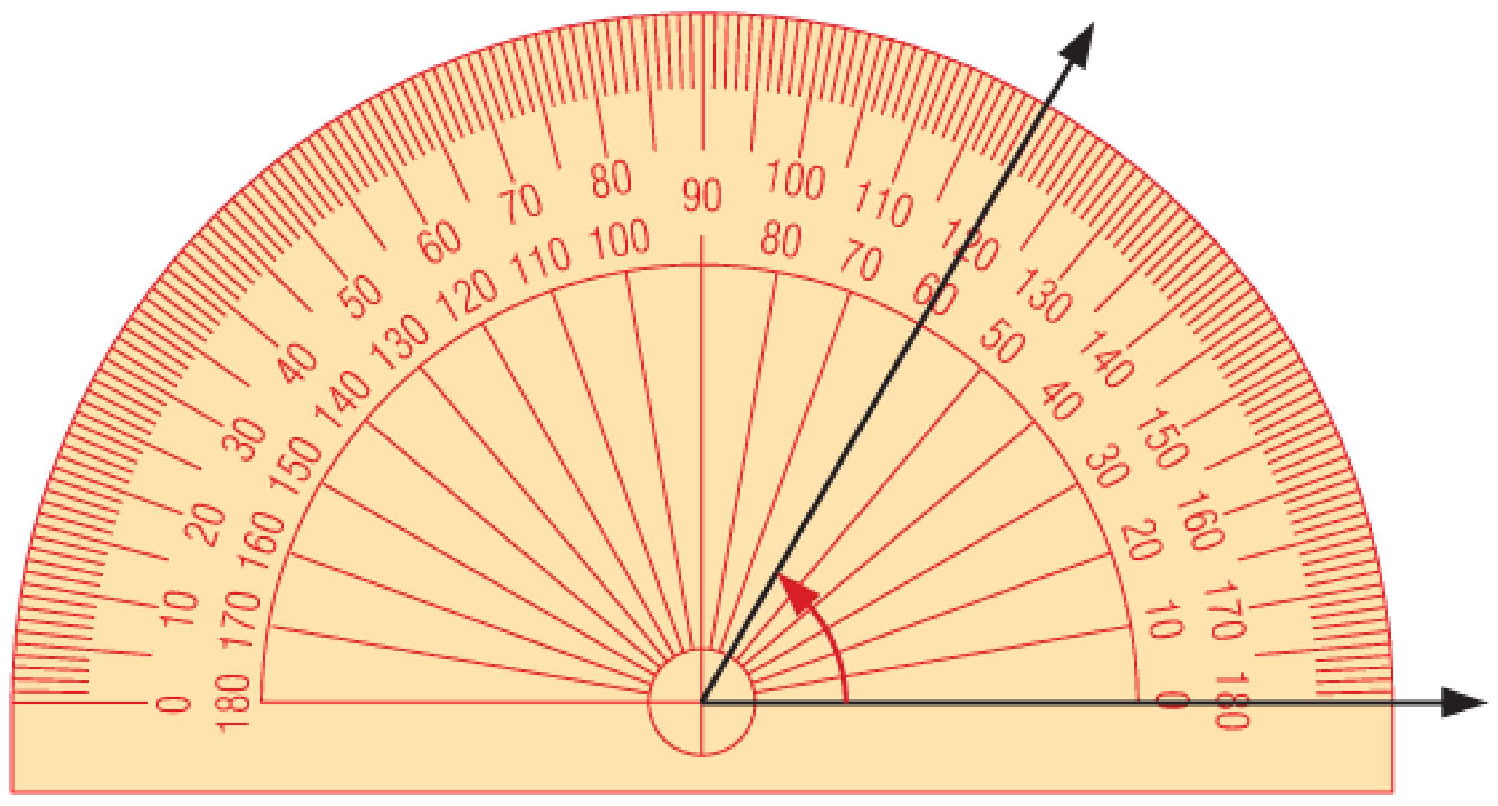

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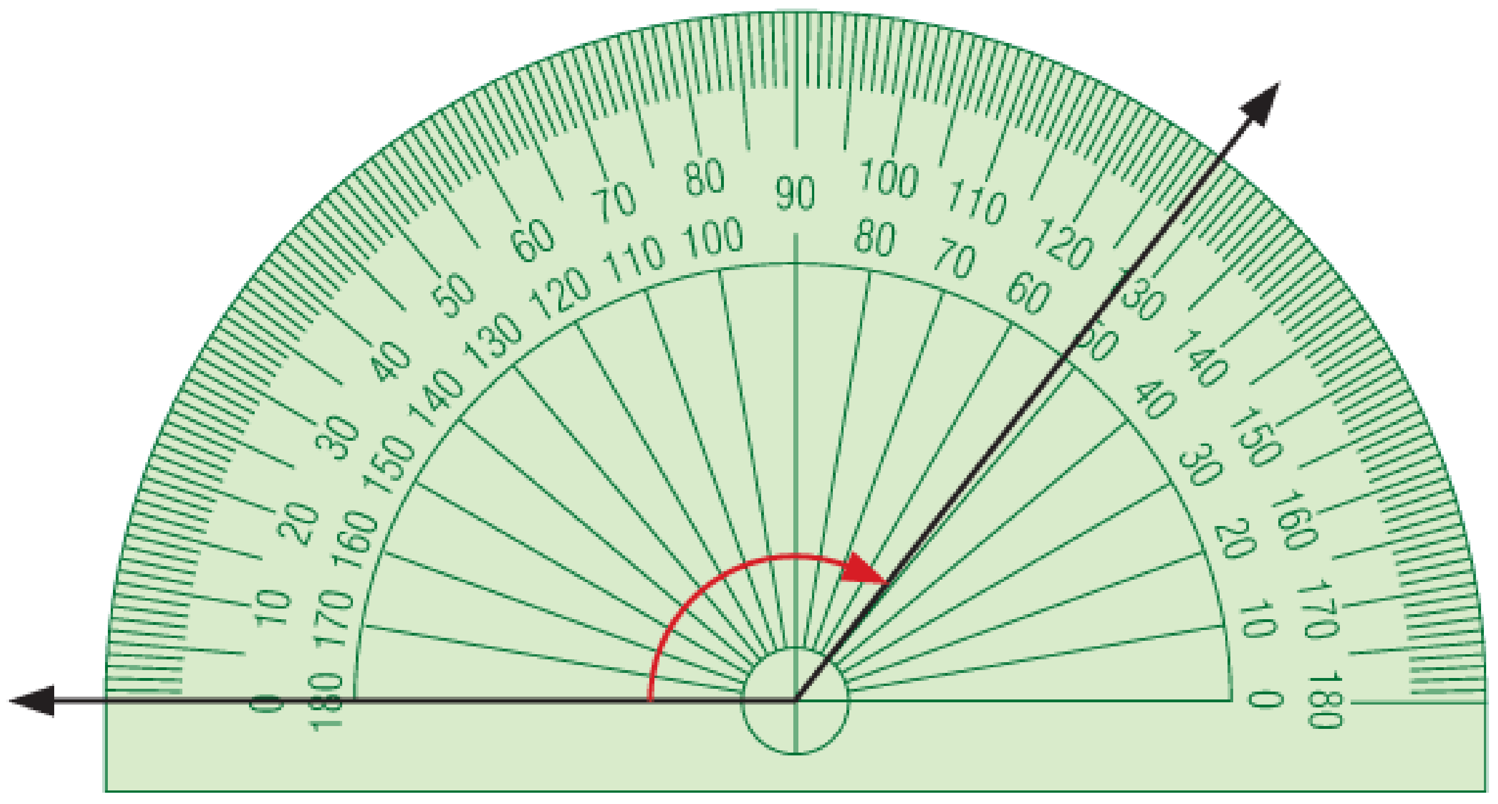

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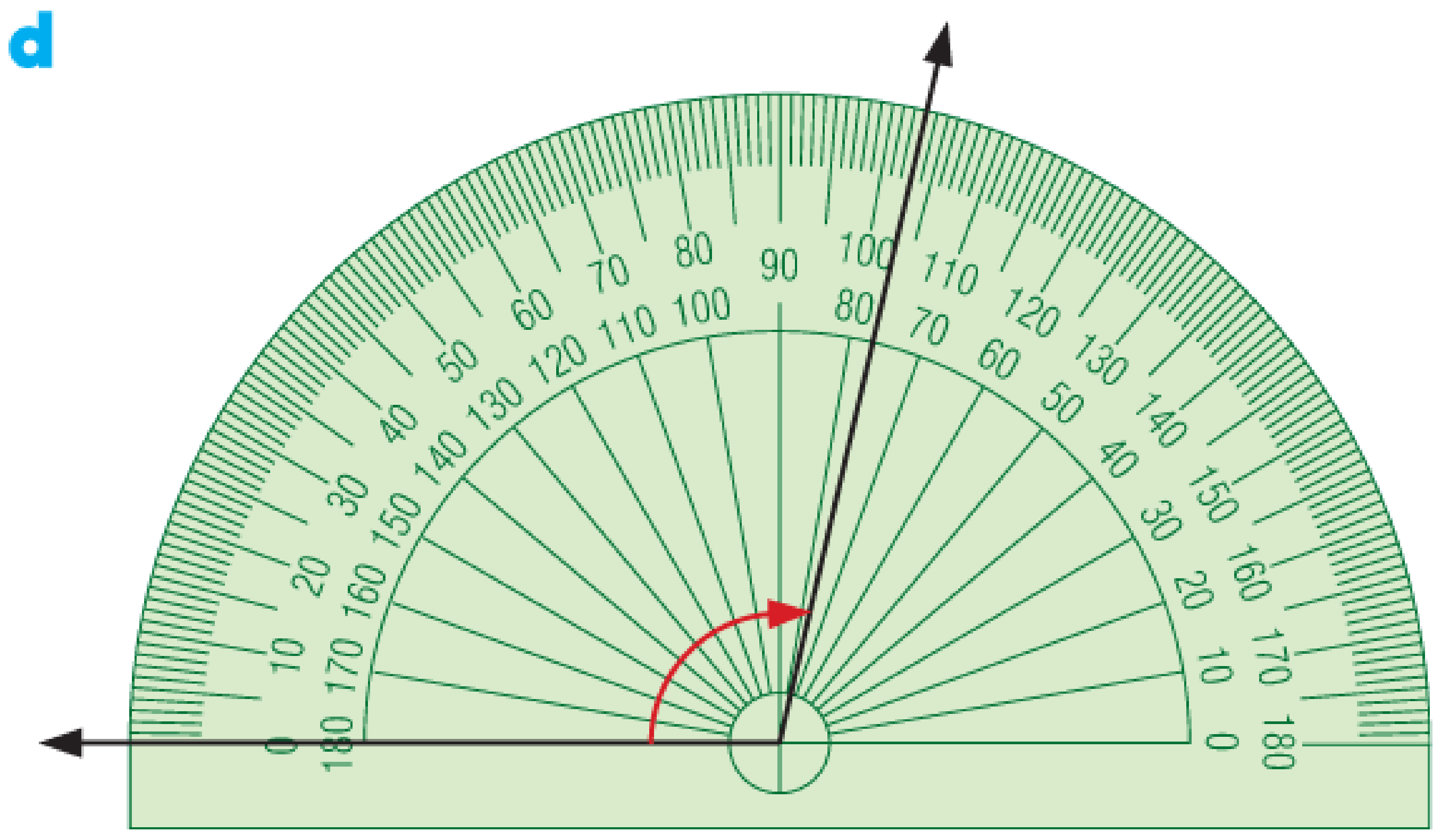

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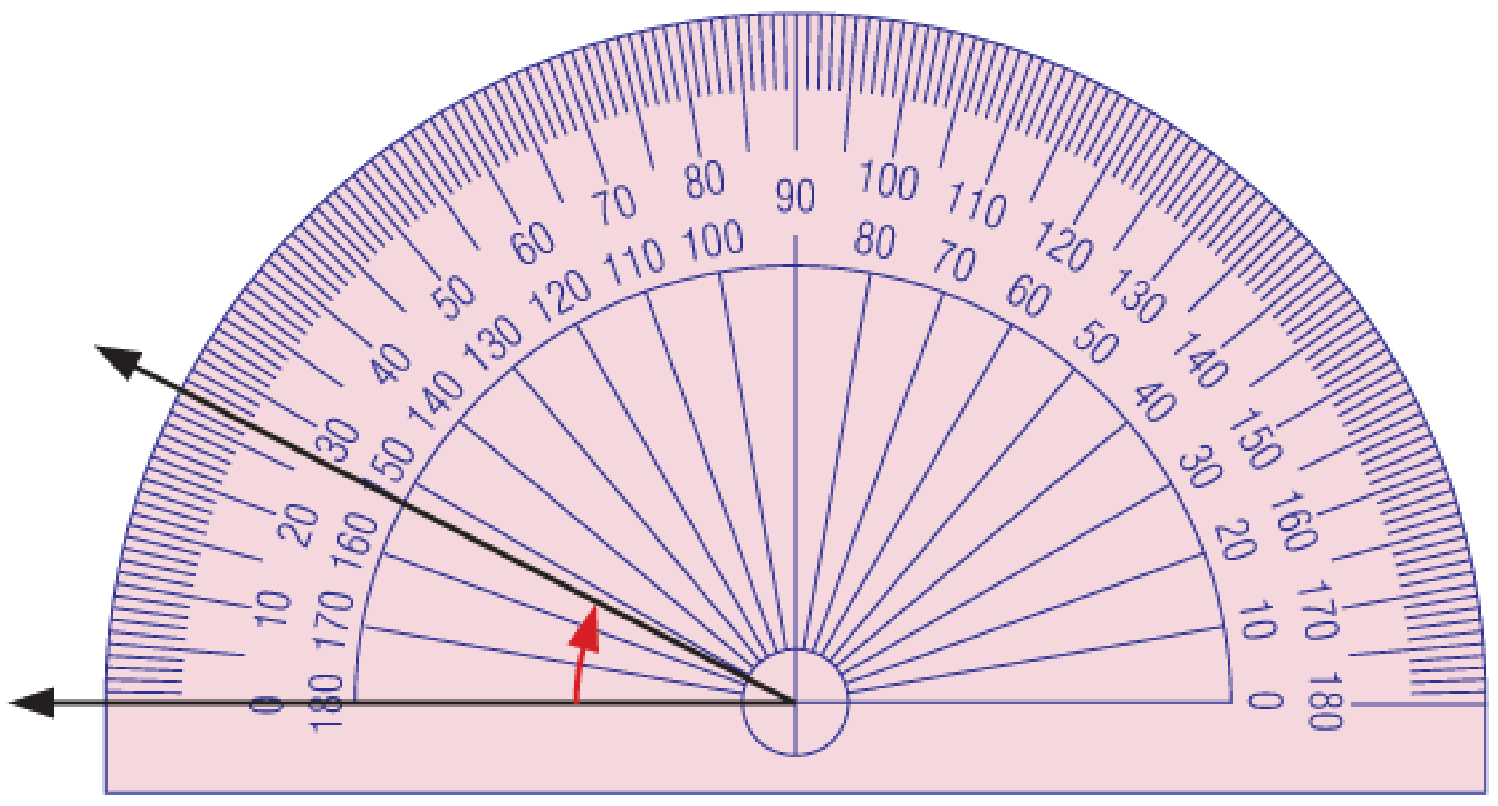

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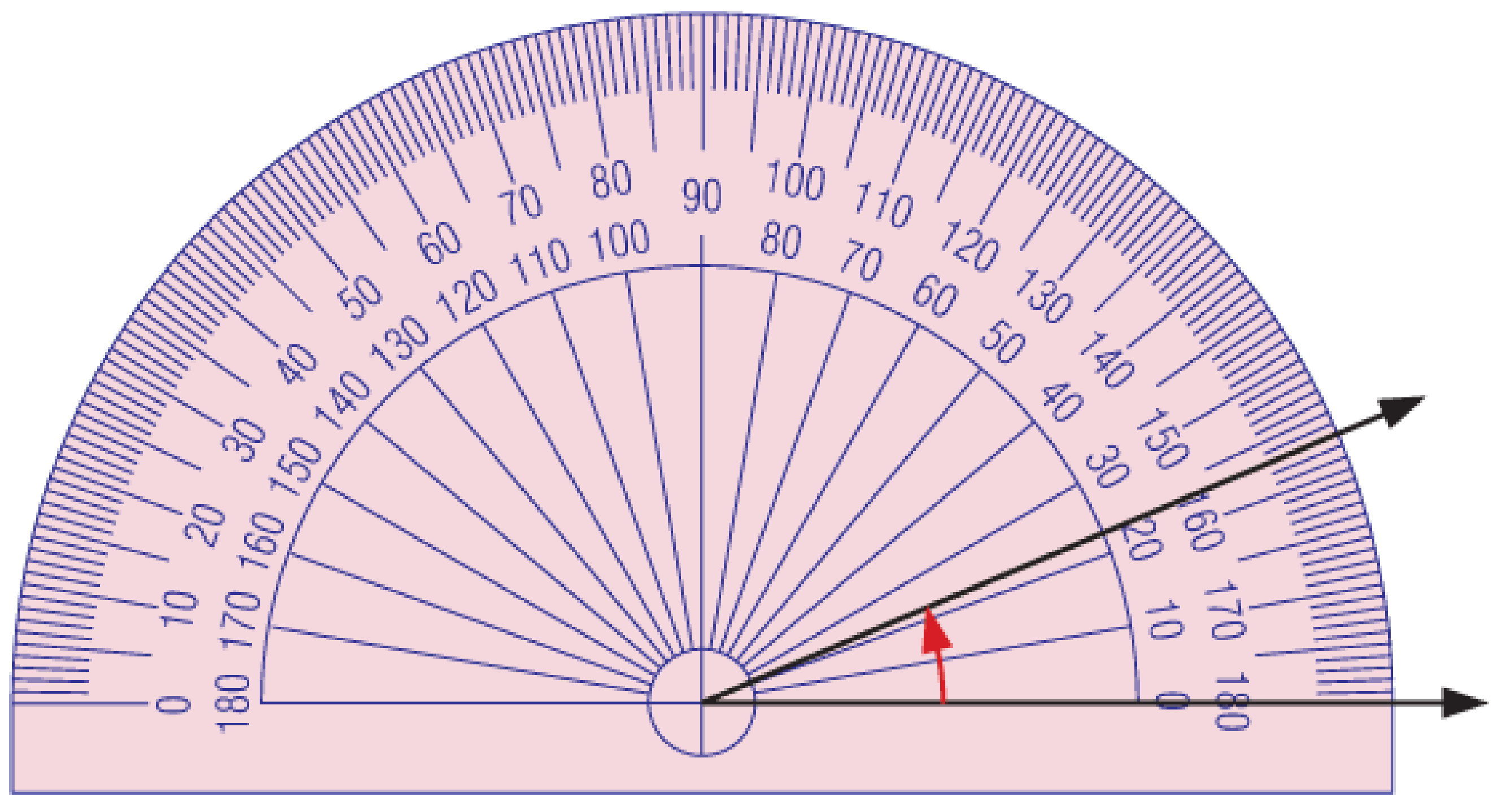

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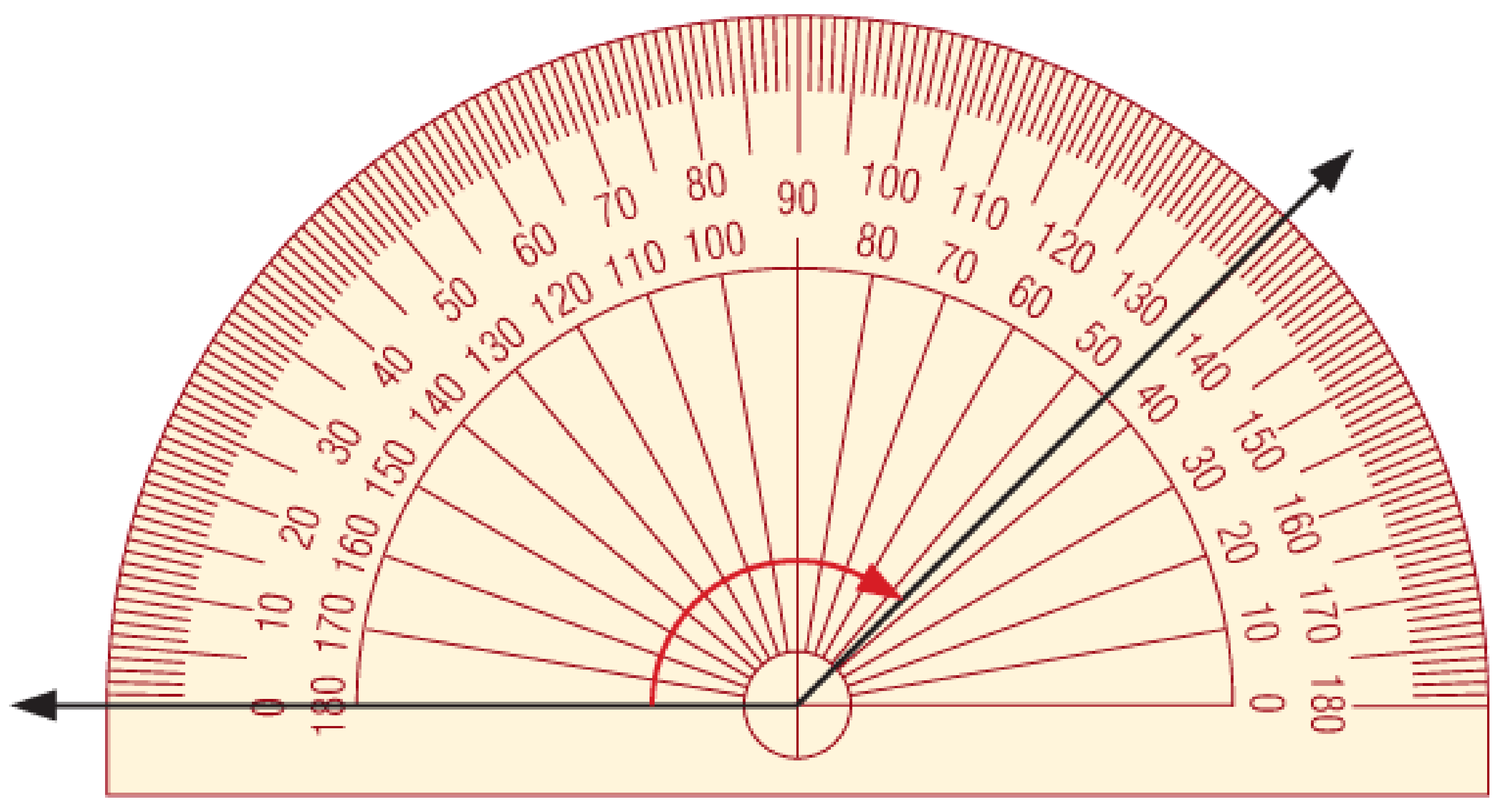

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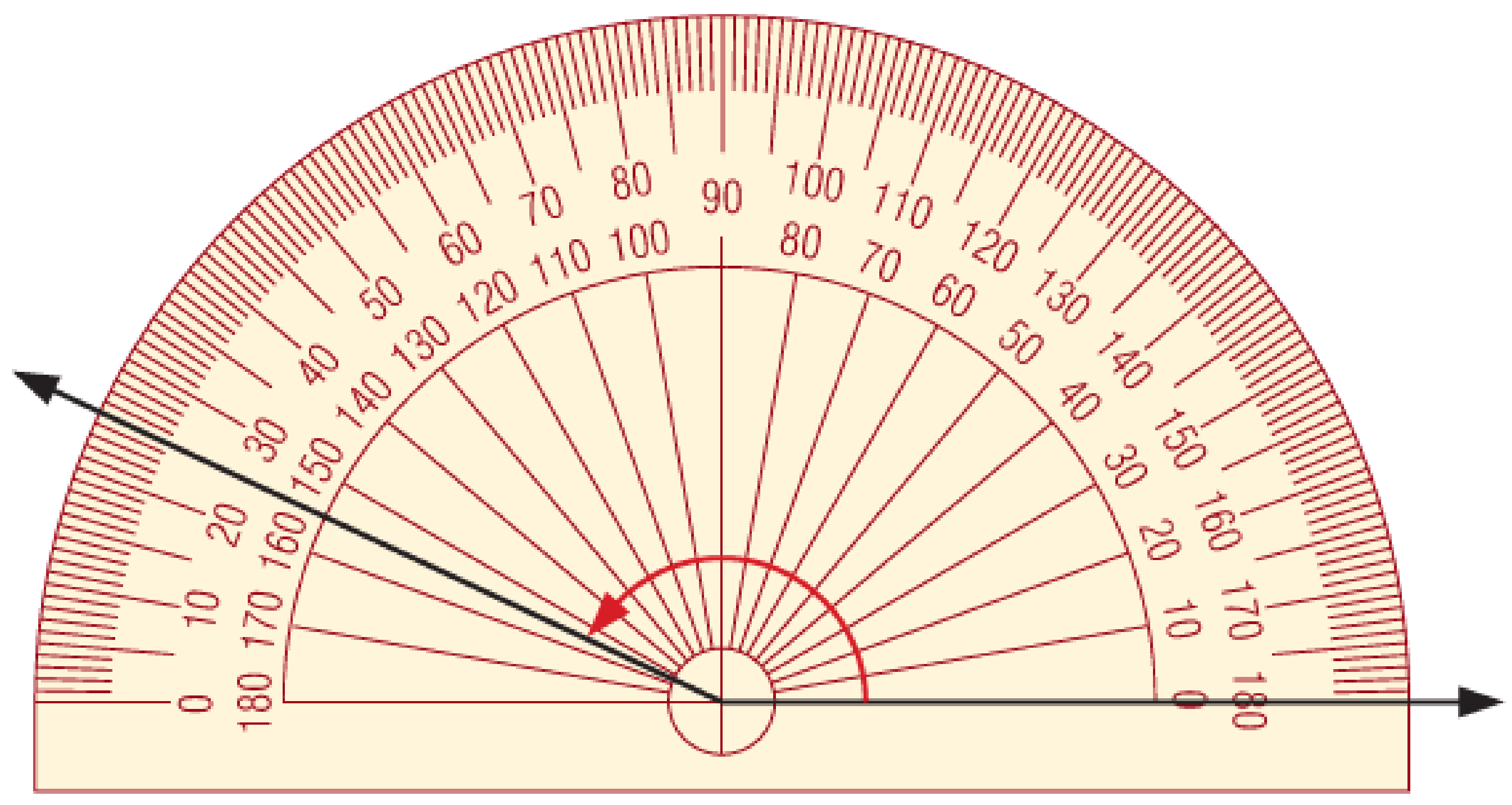

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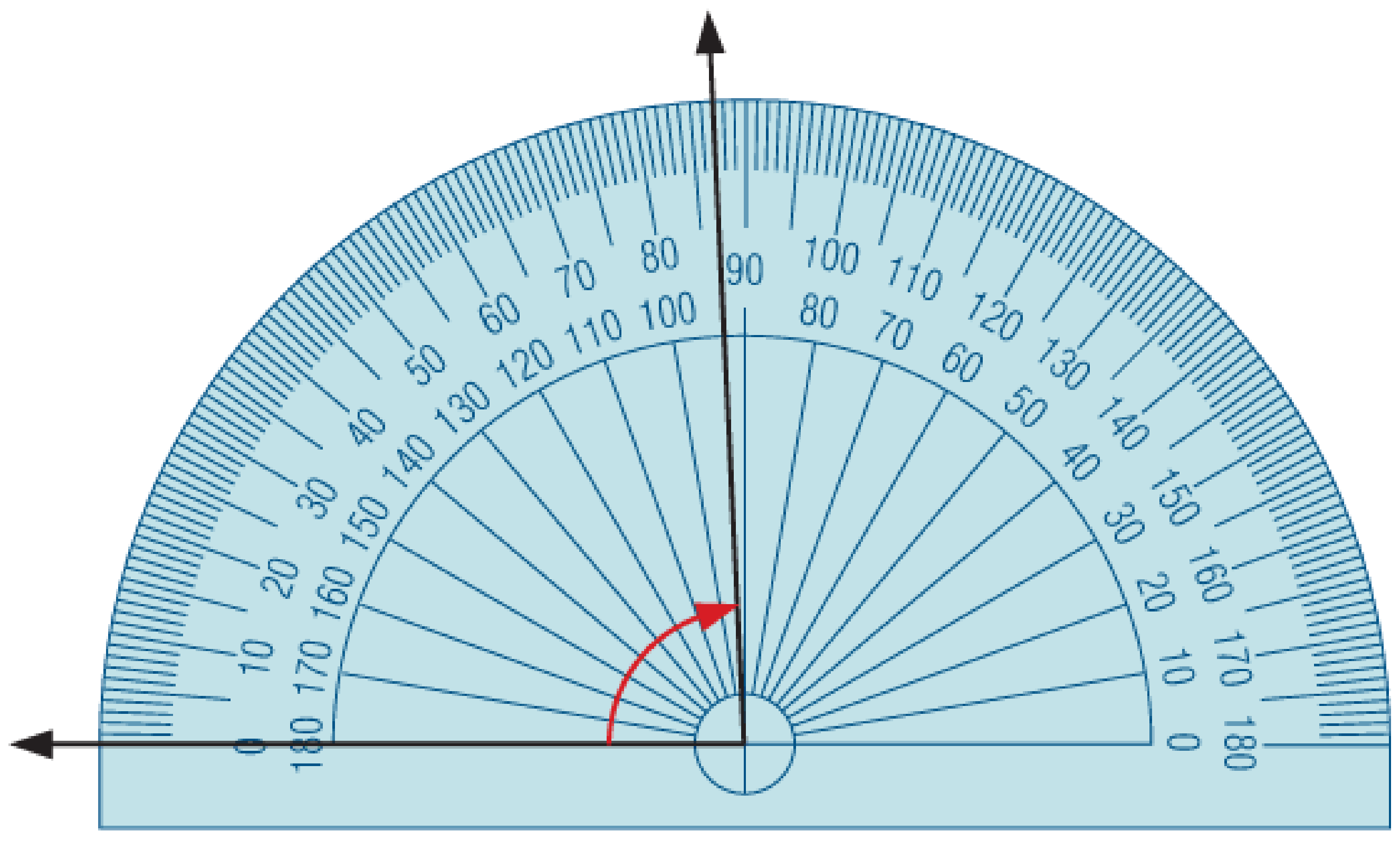

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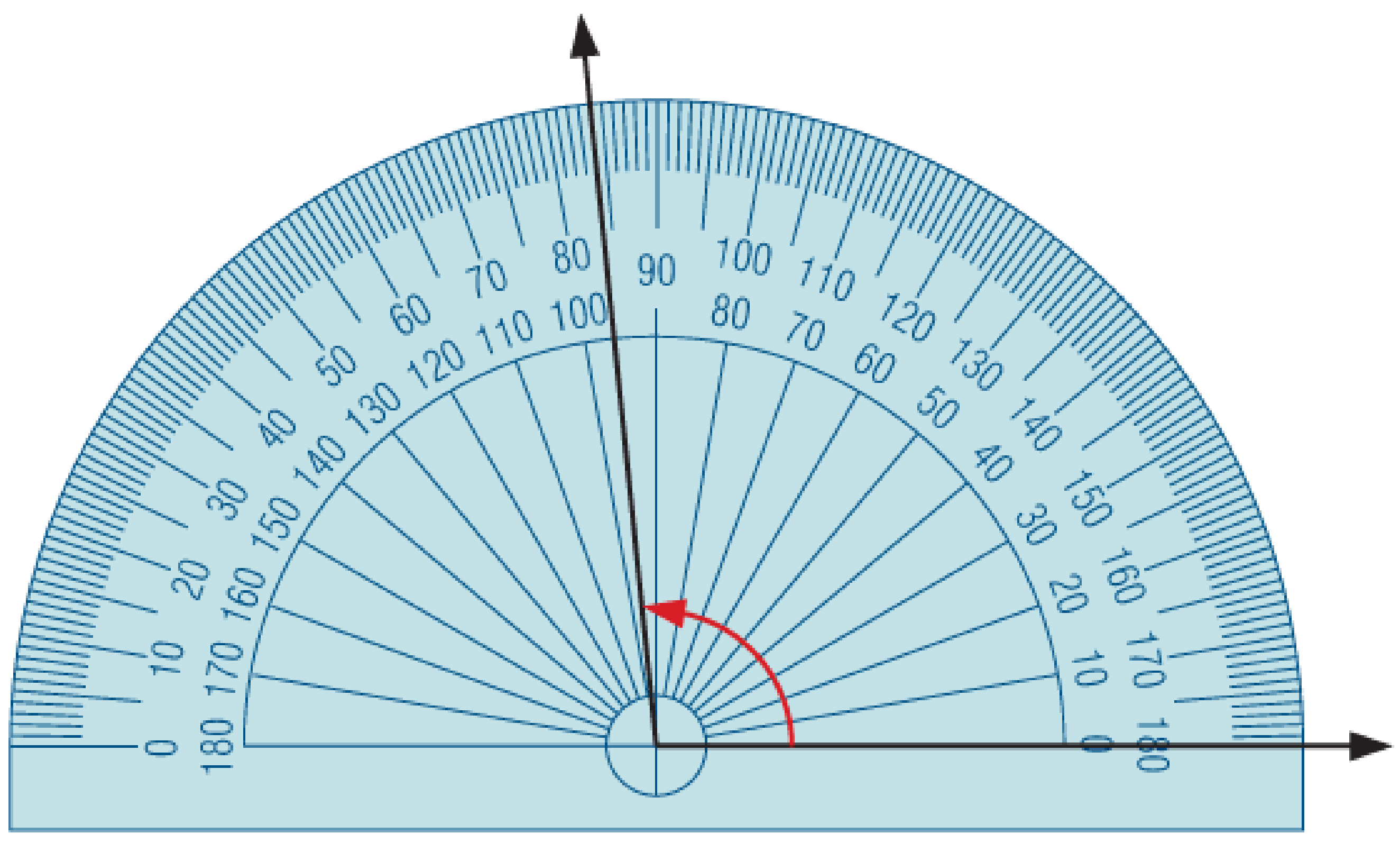
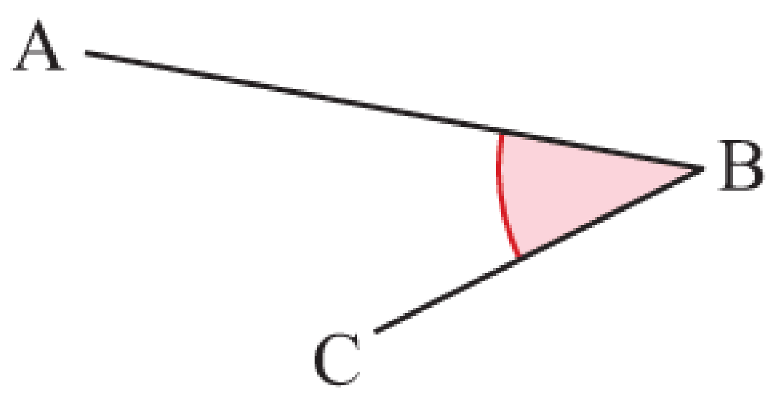


2 Draw a diagram to illustrate:

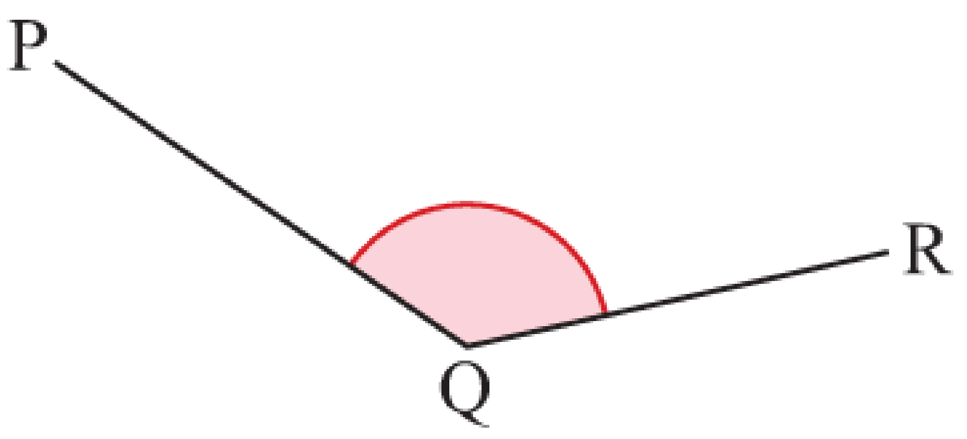
a a $\frac{1}{2}$ turn
b a $\frac{1}{4}$ turn
c a revolution
d an obtuse angle
e a straight angle
f a right angle.

3 Name the shaded angles in three point notation. State the type of angle in each case.

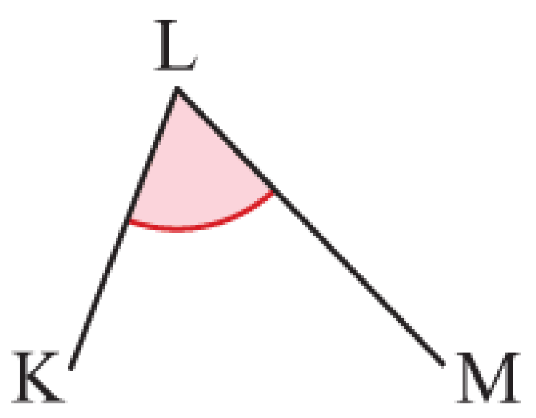
a



b



c



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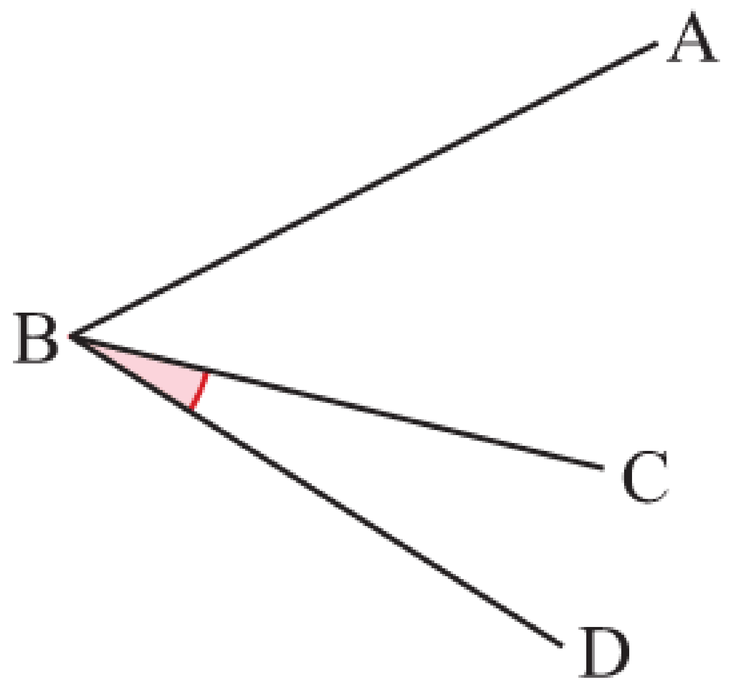

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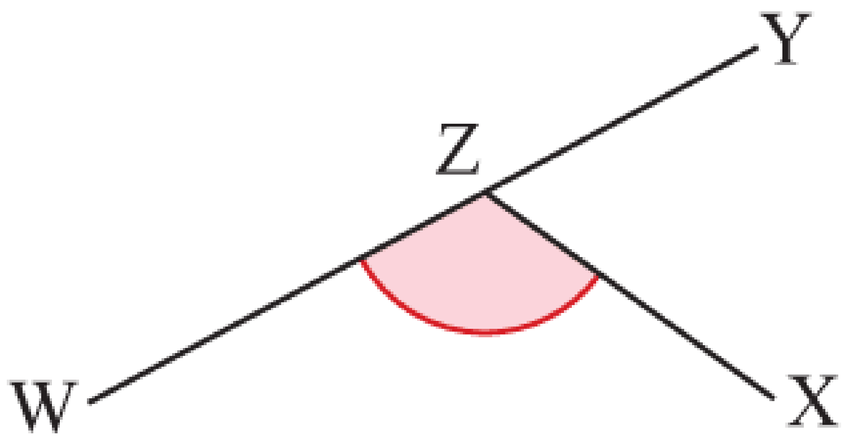

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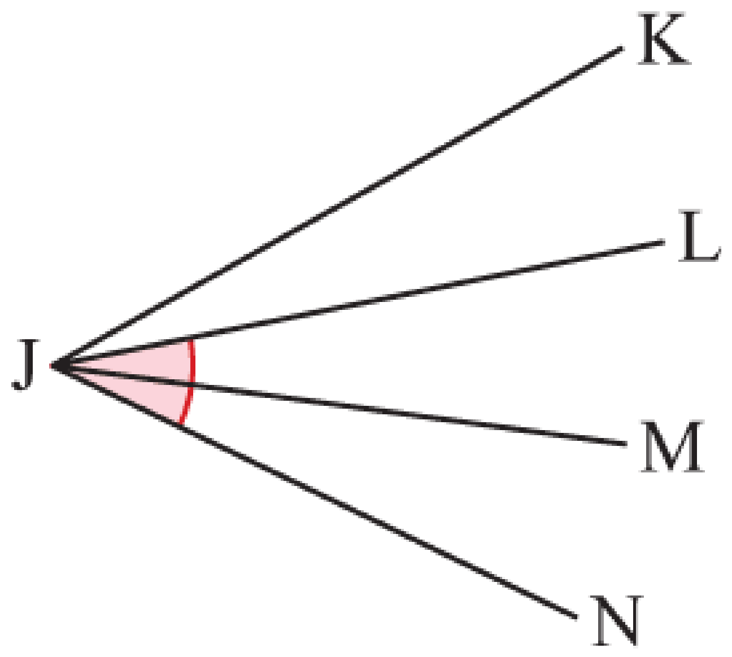
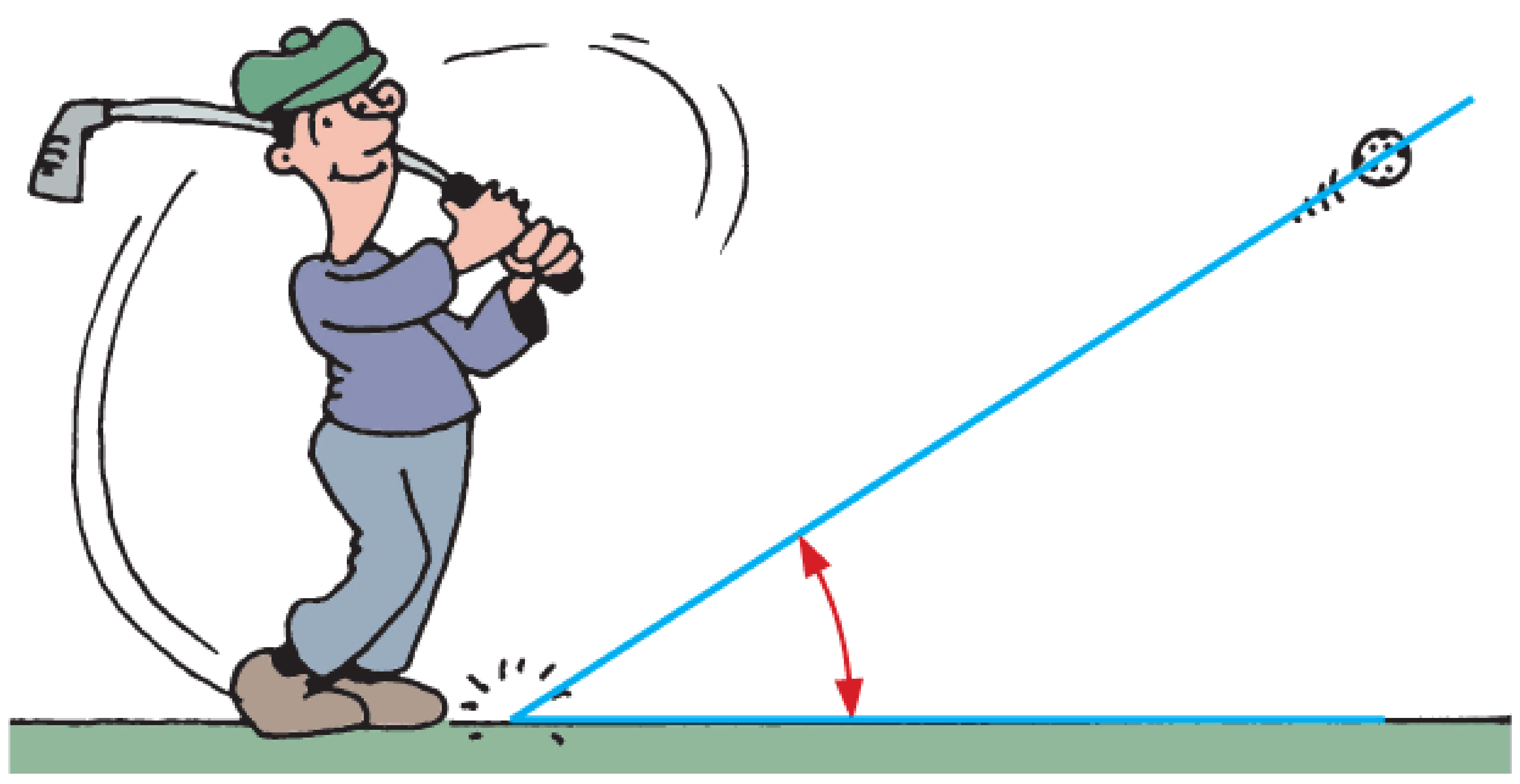


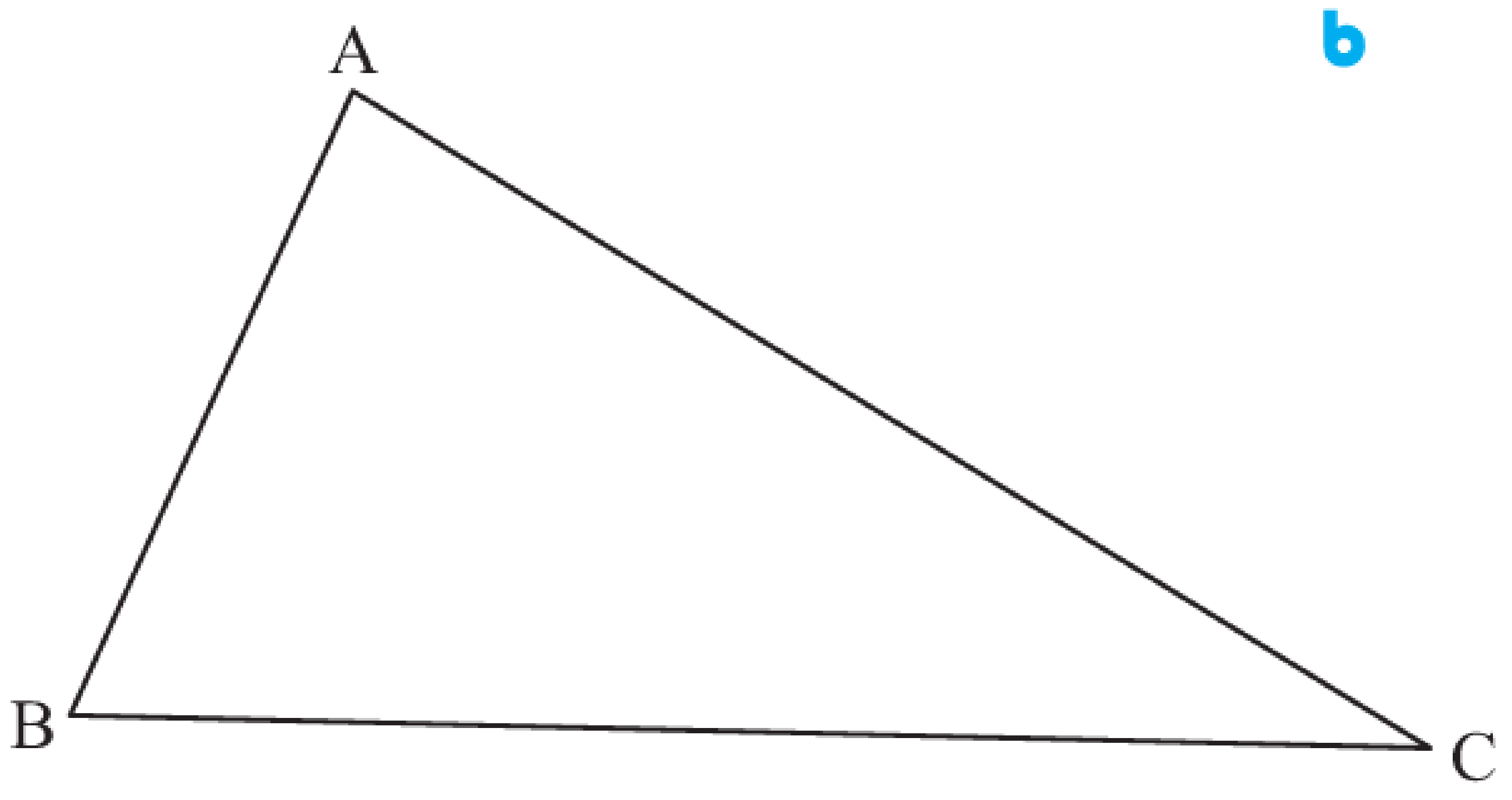
4 Use your protractor to find the angle between the ground and the direction of the ball.



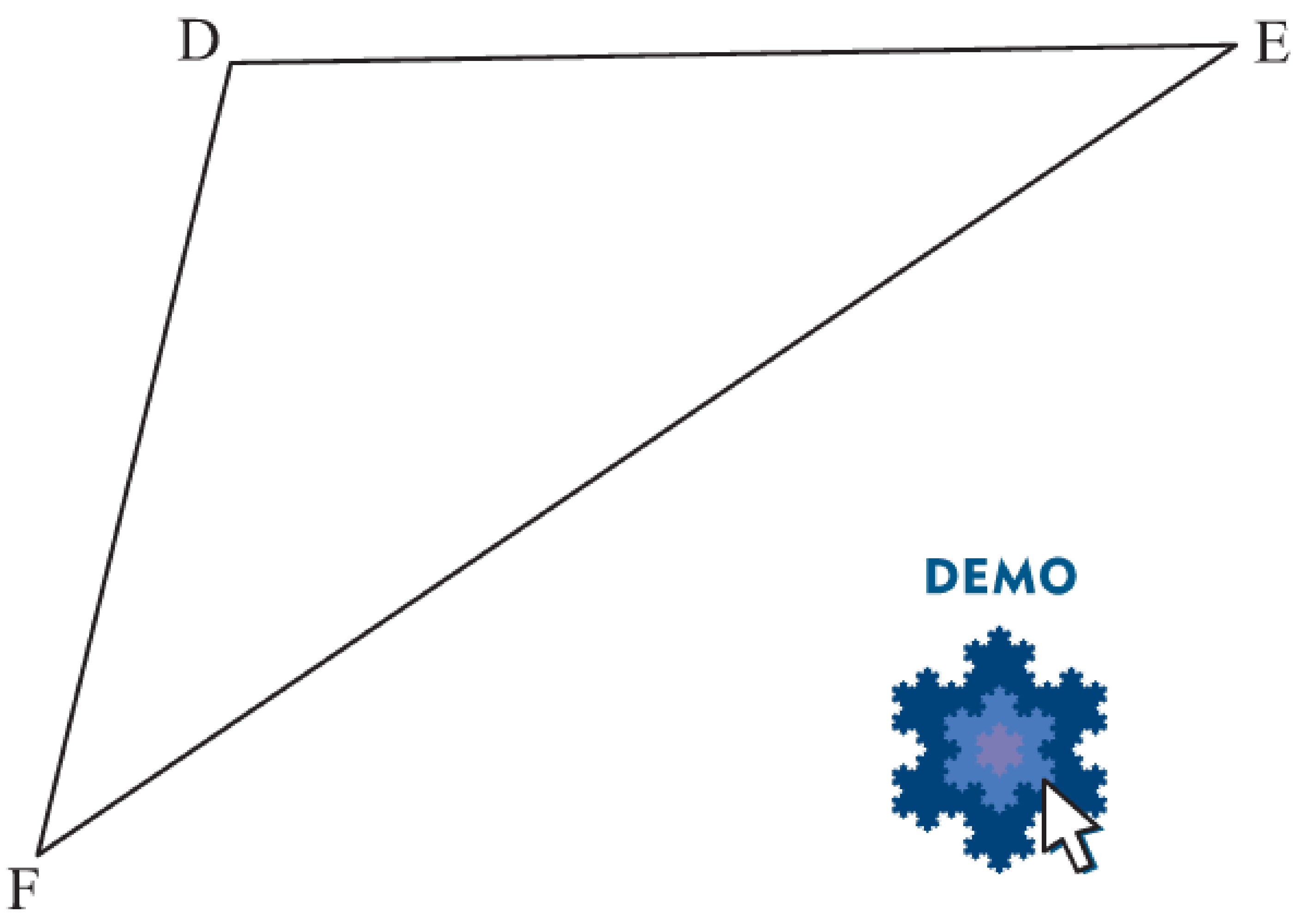
PRINTABLE DIAGRAMS

5 Measure all angles of the following figures. Use three point notation to write down your answers.

a



b



6 Use your ruler and protractor to draw the following angles:

a $A\widehat{B}C = 60°$
b $P\widehat{Q}R = 115°$
c $L\widehat{M}N = 48°$
d $D\widehat{E}F = 151°$
e $J\widehat{K}L = 137°$
f $X\widehat{Y}Z = 17°$

Ask a friend to check your answers.

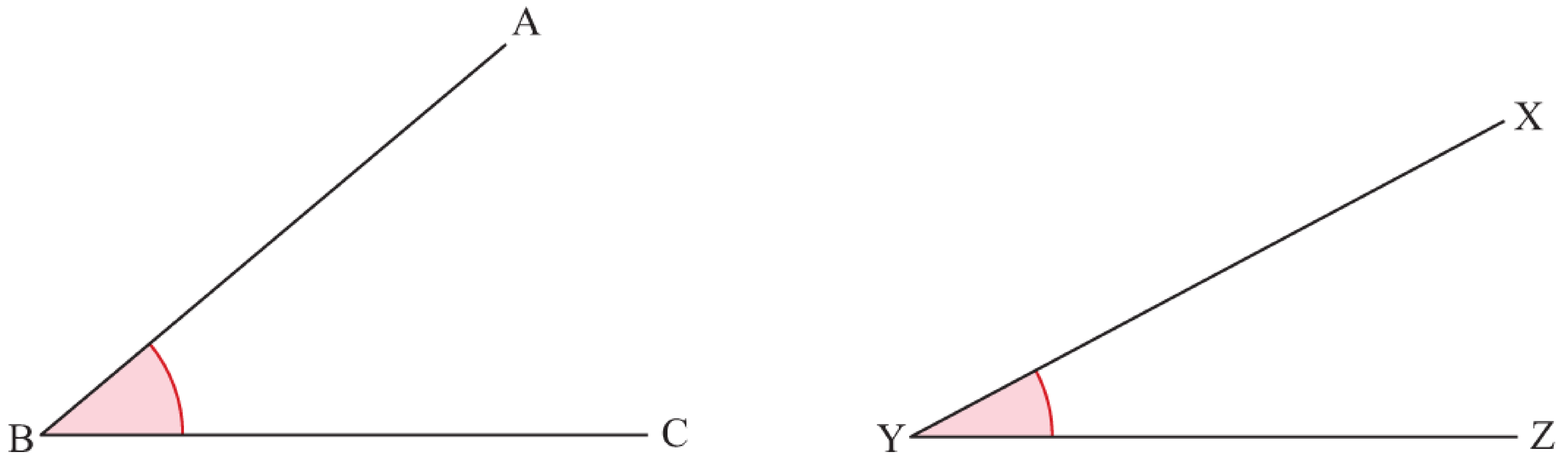
7 For the following angle sizes, state whether the angle is acute, right, obtuse, straight, or reflex:

a 73°
b 194°
c 90°
d 114°
e 13°
f 180°
g 277°
h 93°

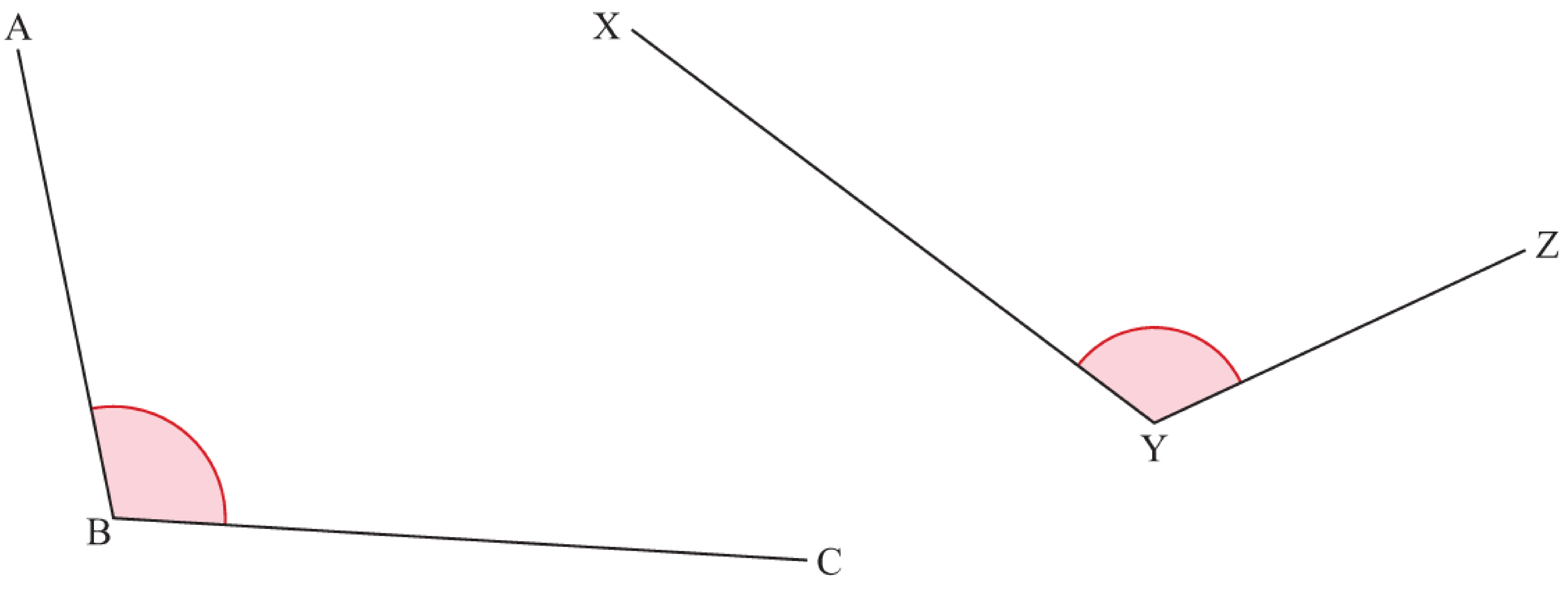
8 For each pair of angles:

- **i** Measure the sizes of $A\widehat{B}C$ and $X\widehat{Y}Z$.
- **ii** Determine whether $A\widehat{B}C$ or $X\widehat{Y}Z$ is larger.

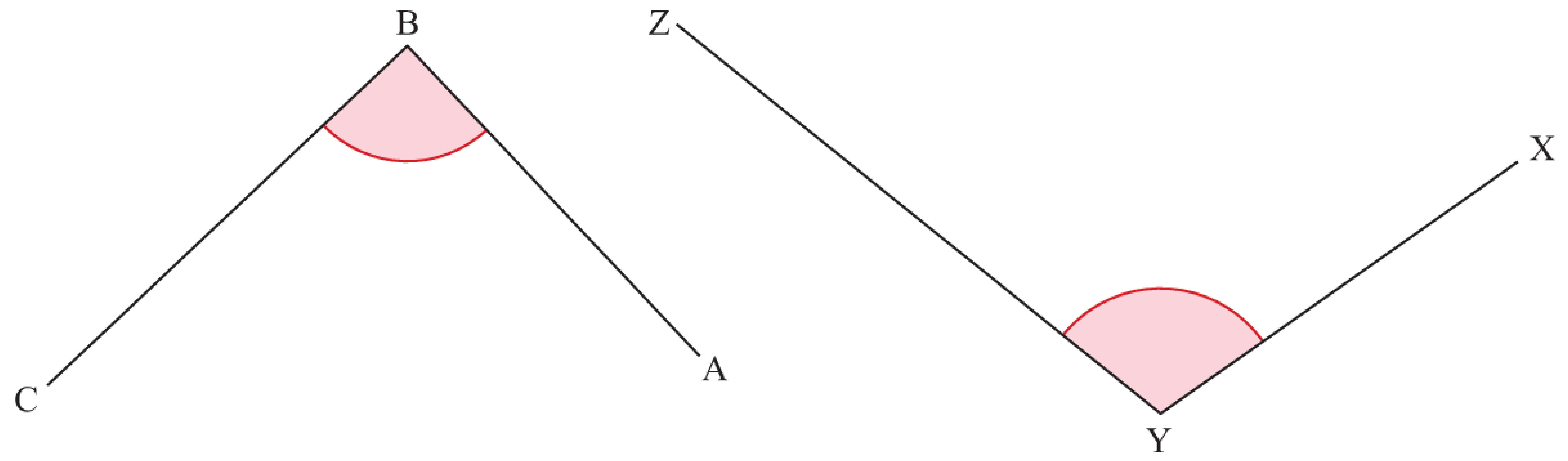
a



b



c

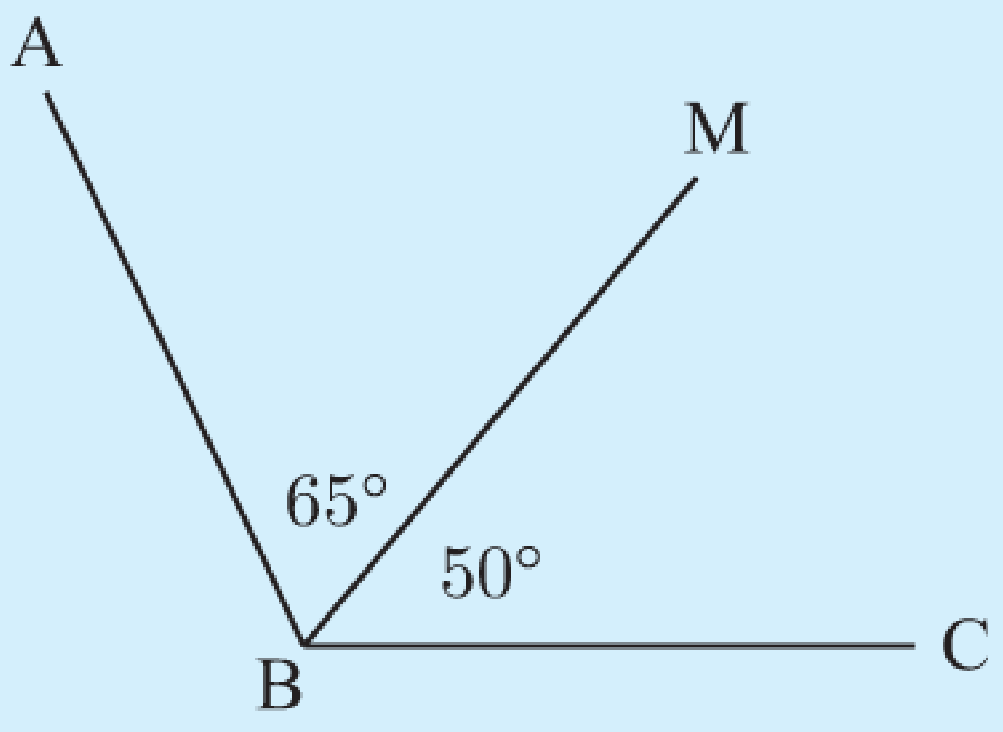


Example 2 **Self Tutor**

In each diagram:

- **i** Find the size of $A\widehat{B}C$ without using your protractor.
- **ii** State the type of $A\widehat{B}C$.

a



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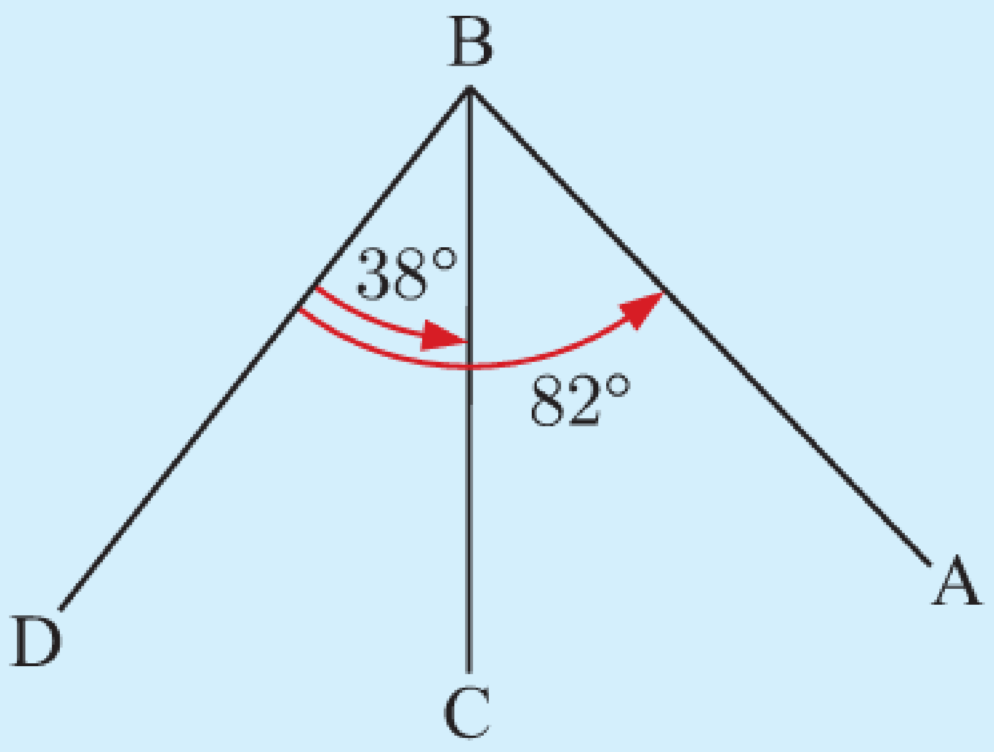


a **i** $A\widehat{B}C = 65° + 50°$
$= 115°$

ii $A\widehat{B}C$ is an obtuse angle.

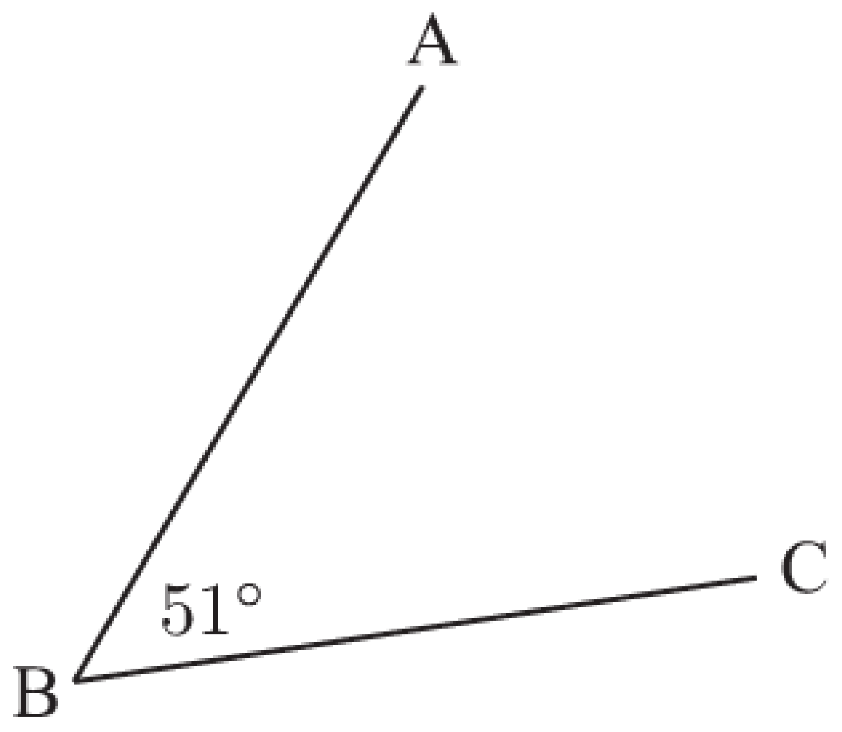
b **i** $A\widehat{B}C = 82° - 38°$
$= 44°$

ii $A\widehat{B}C$ is an acute angle.

9 In each diagram:

- **i** Find the size of $A\widehat{B}C$ without using your protractor.
- **ii** State the type of $A\widehat{B}C$.

a



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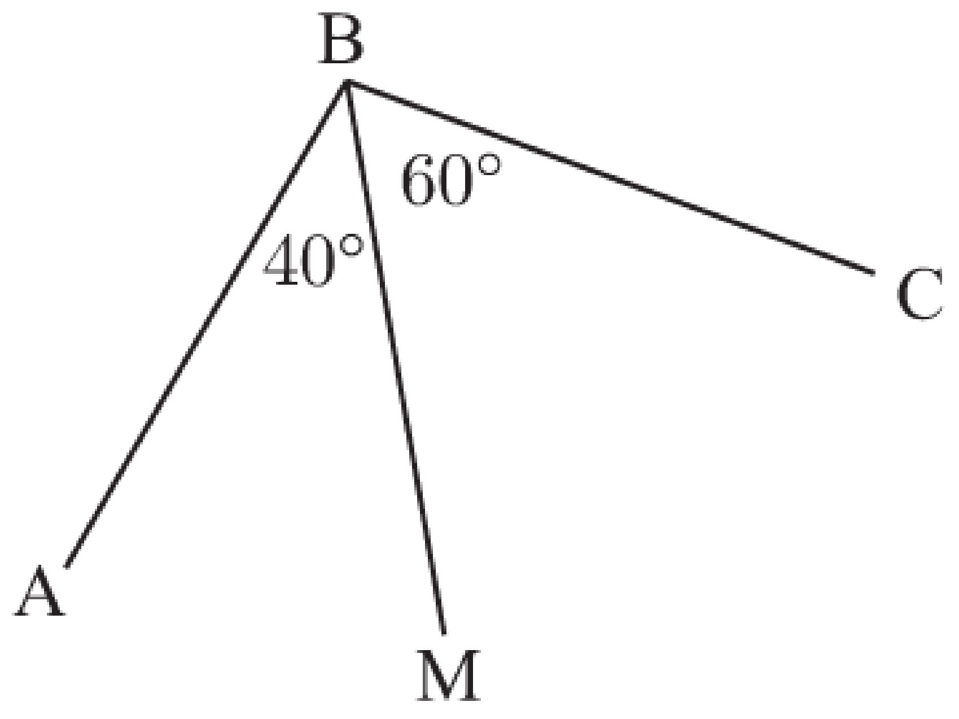

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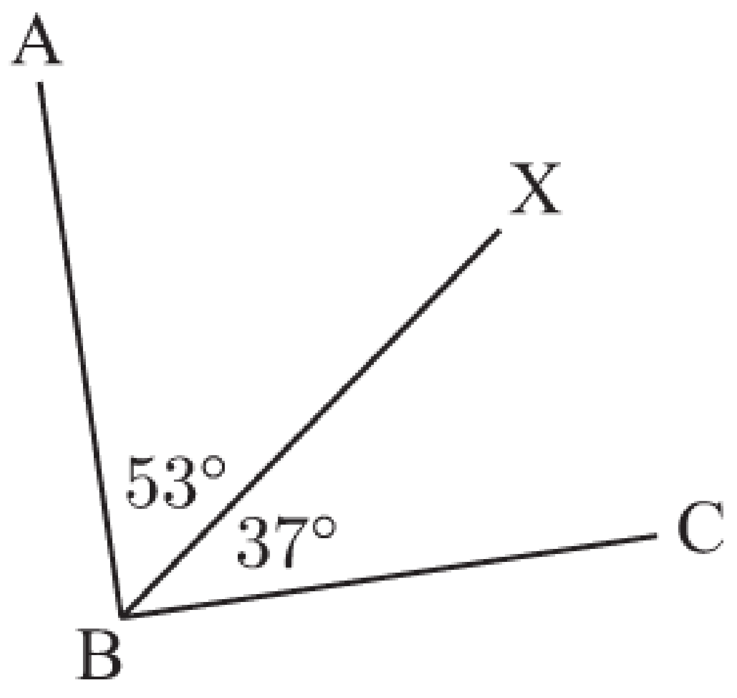

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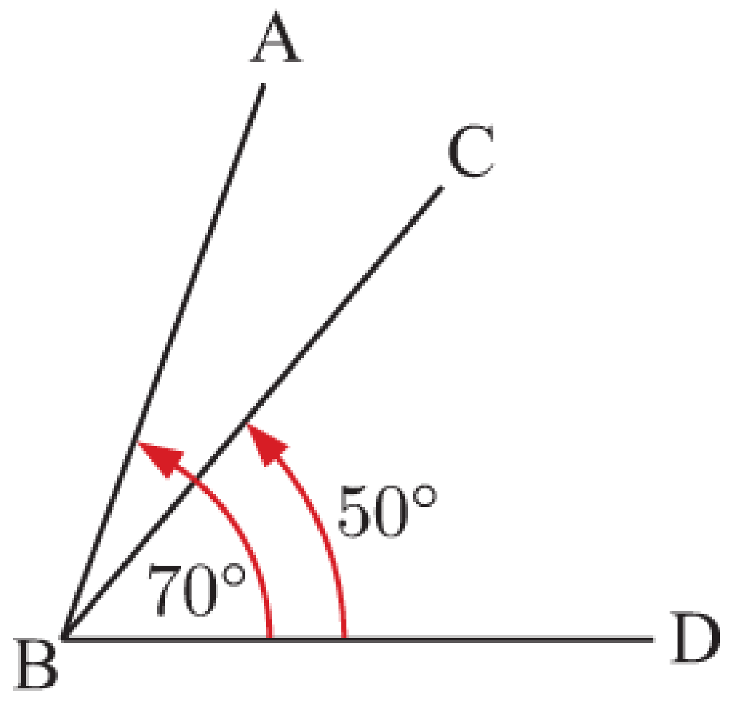

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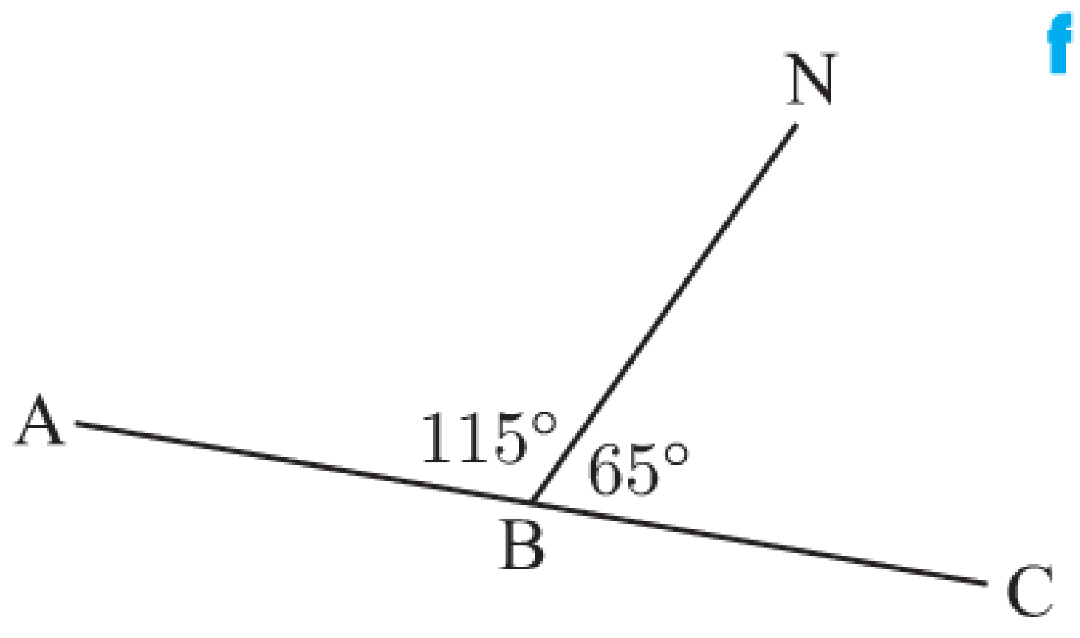

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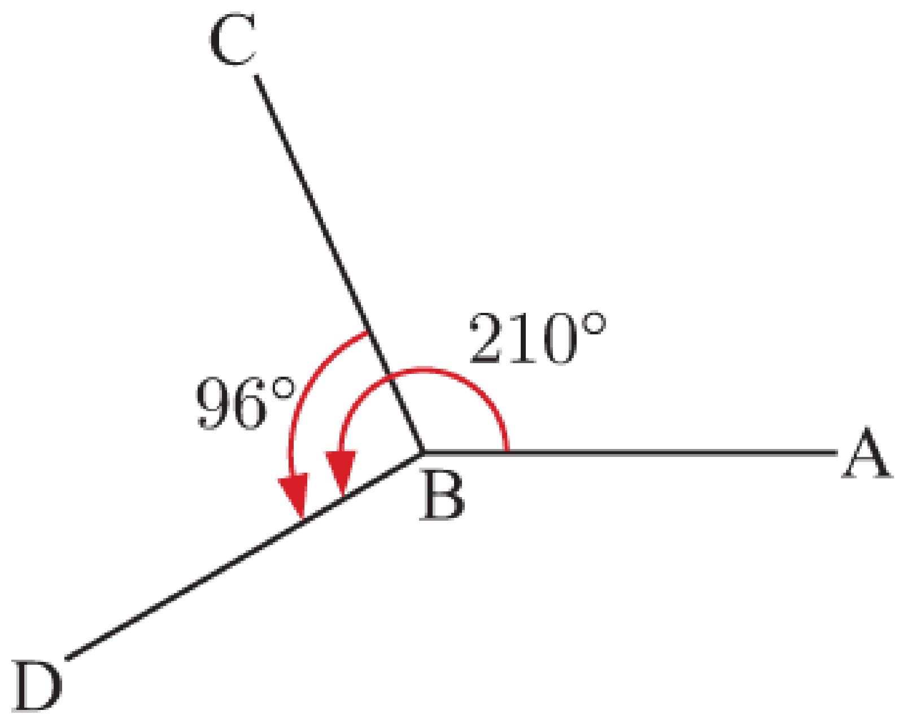
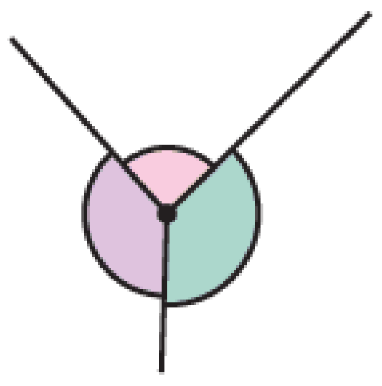


GAME **ANGLE GUESSING GAME**

Click on the icon to play an angle guessing game.

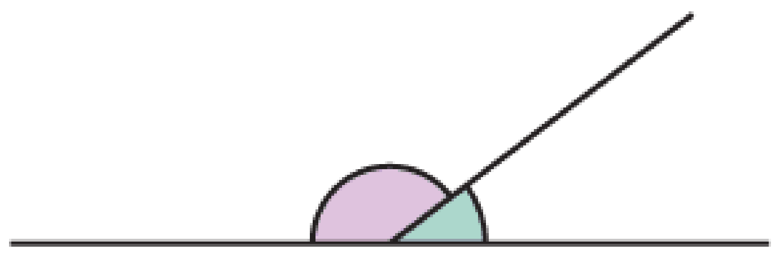
GAME

C ANGLES AT A POINT OR ON A LINE



These angles are **angles at a point**.

Angles at a point add to 360°.



These angles are **angles on a line**.

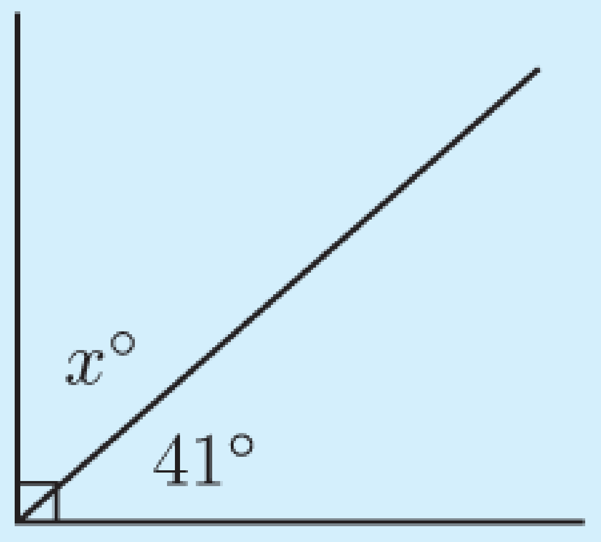
Angles on a line add to 180°.

We can use these facts to find unknown angles in figures. We often indicate unknown angles using letters such as x.

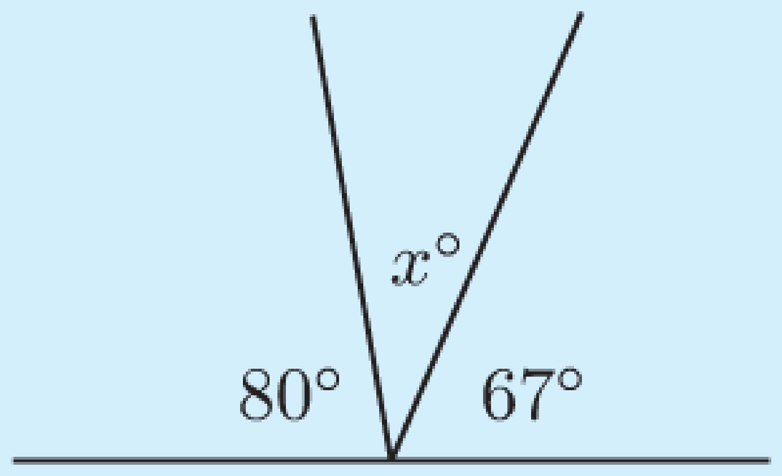
Example 3 **Self Tutor**

Find the value of x without using a protractor:

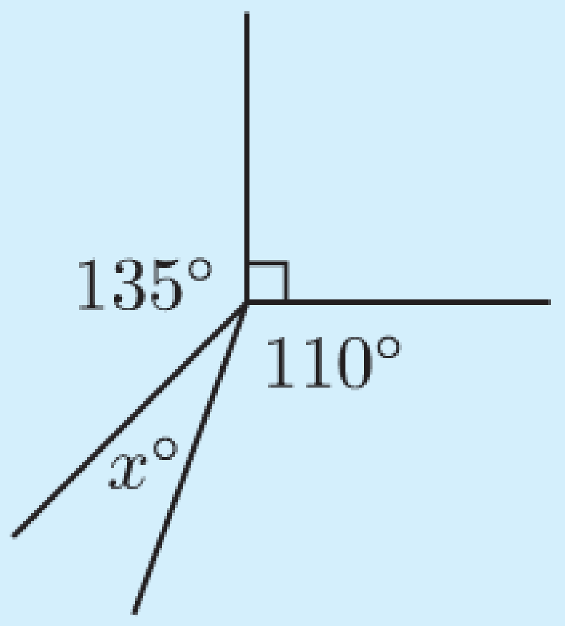
a



b



c



a The angles add to 90°

so $x = 90 - 41$

$= 49$

b The angles add to 180°

so $x = 180 - 80 - 67$

$= 33$

c The angles add to 360°

so

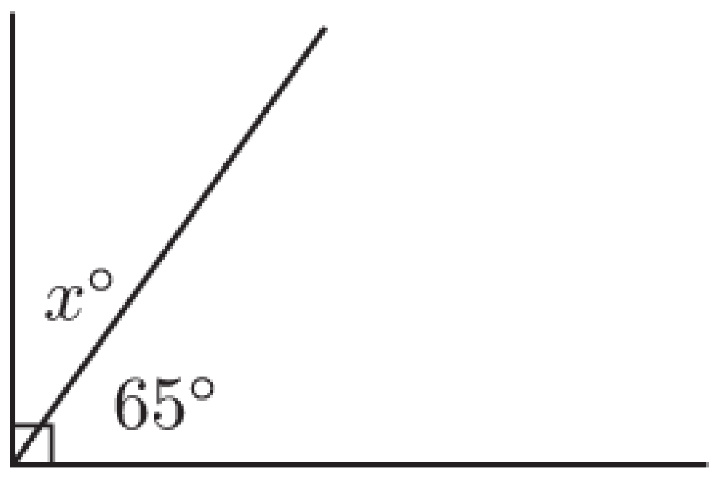
$x = 360 - 135 - 90 - 110$

$= 25$

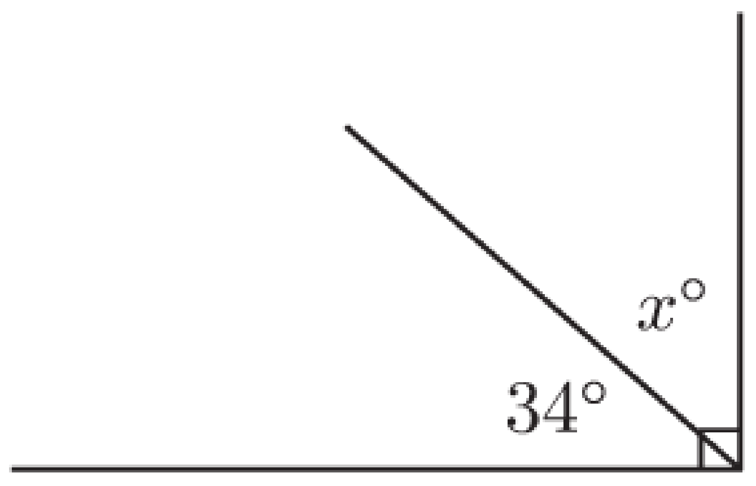
EXERCISE 3C

1 Find the value of the unknown:

a



b



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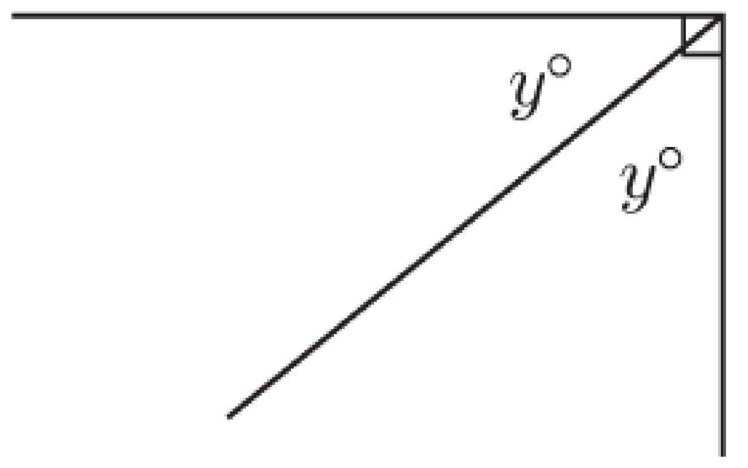

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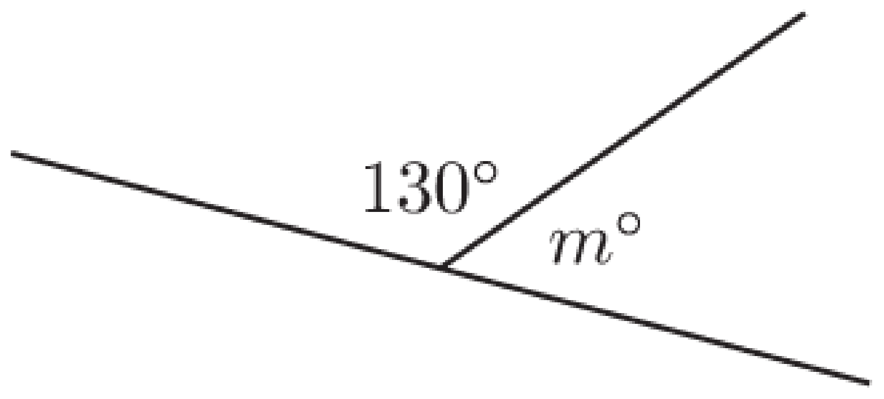
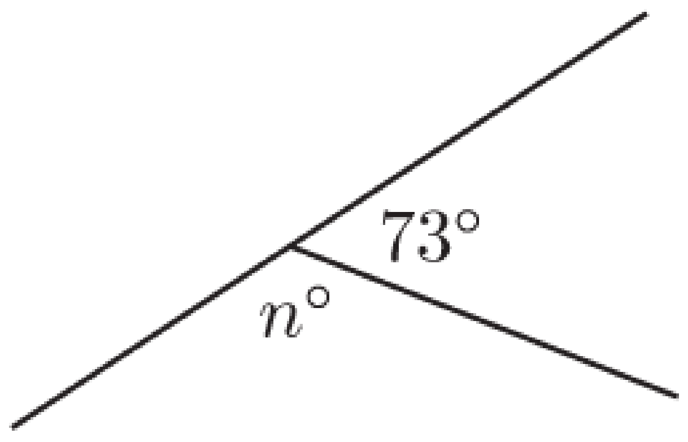


e



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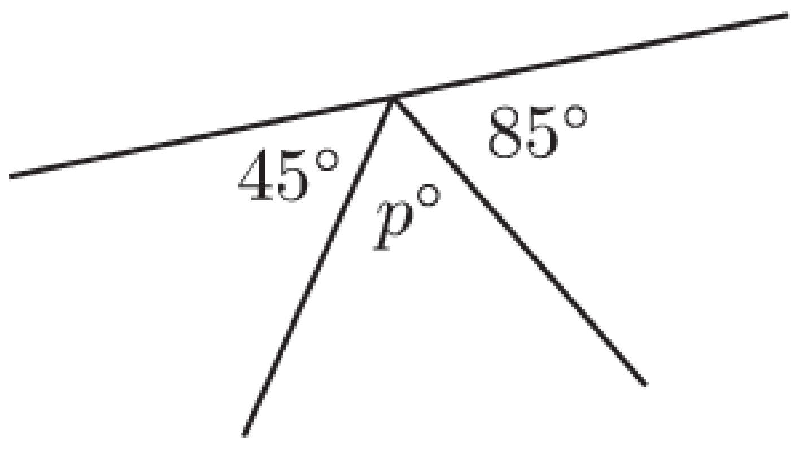
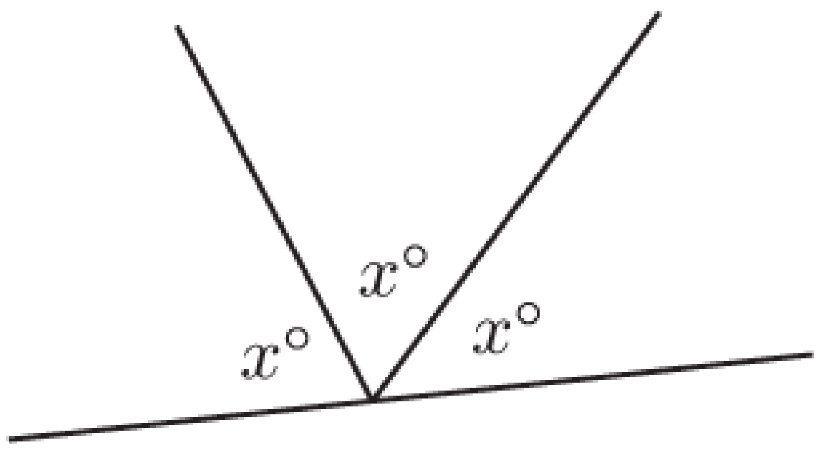
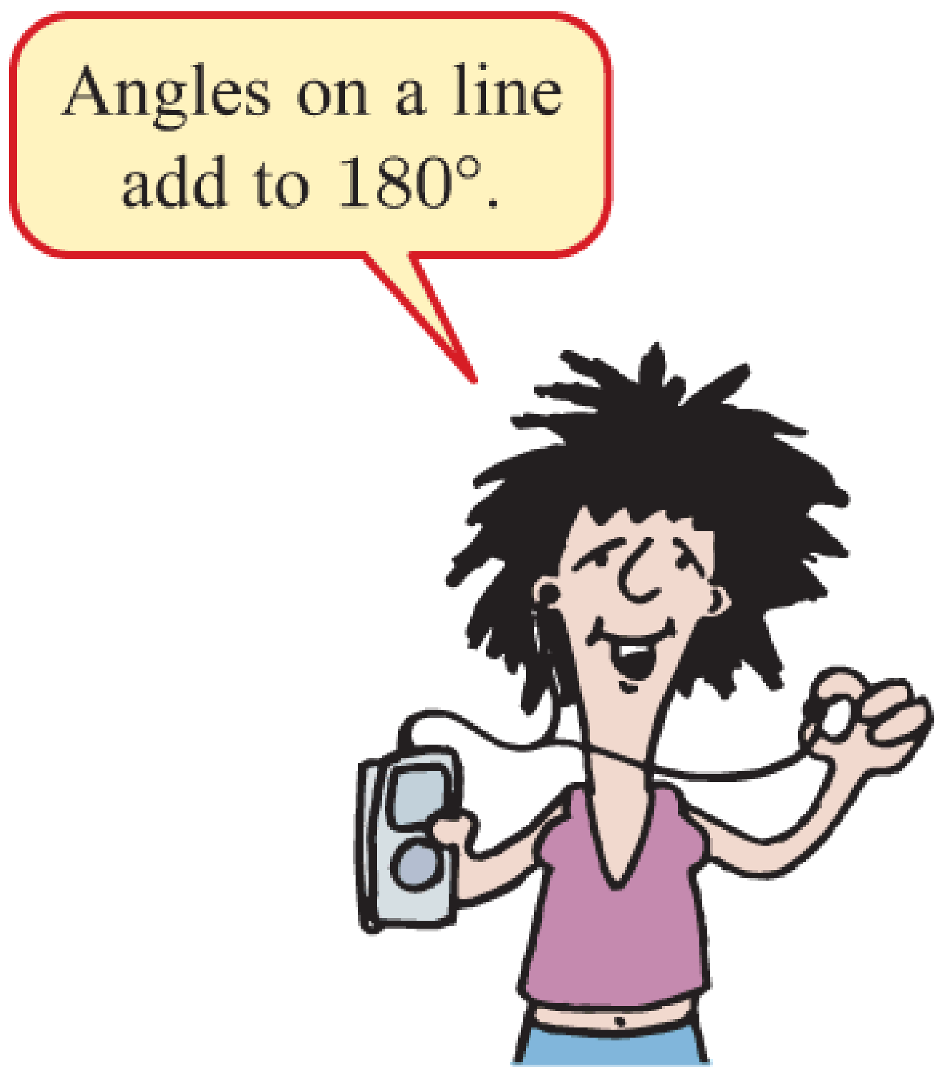


g





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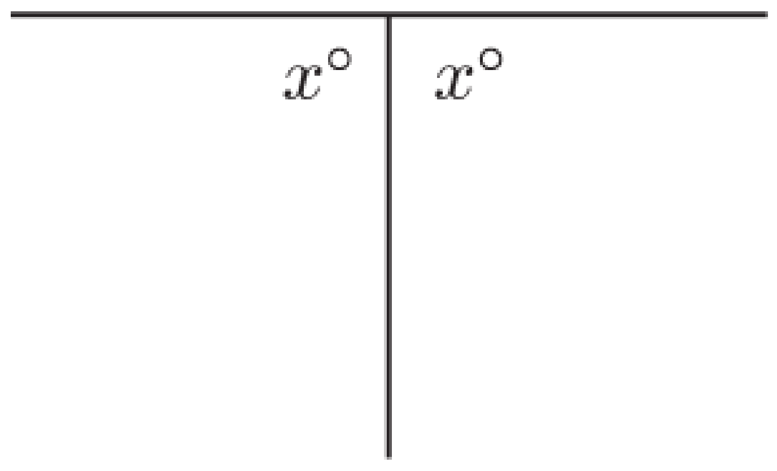
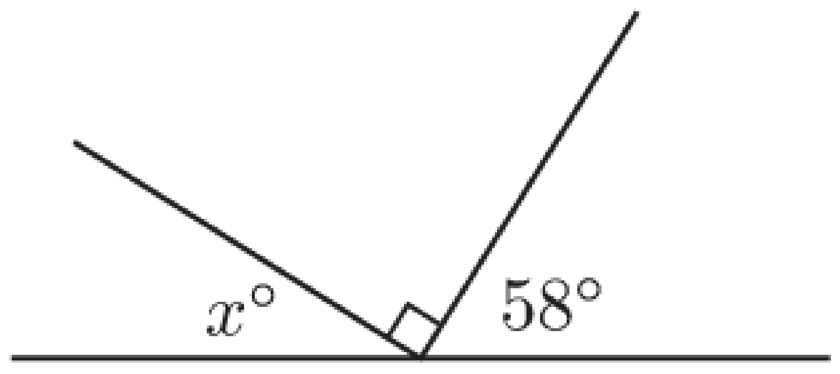
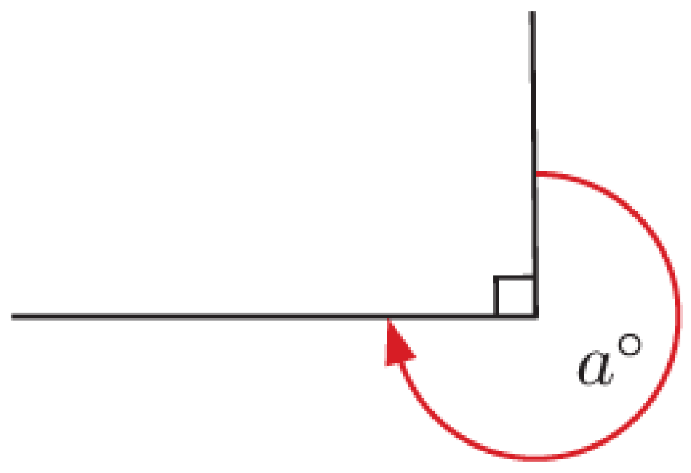


i



2 Find the value of the unknown:

a



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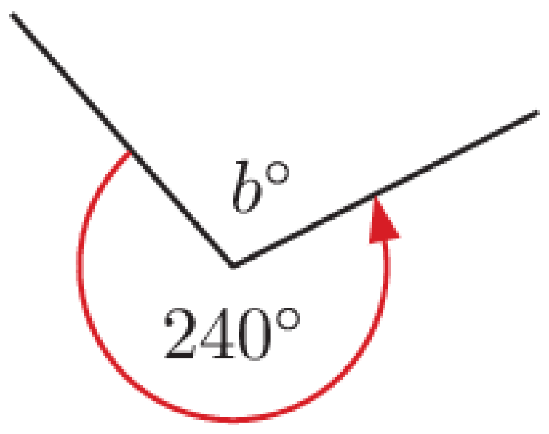

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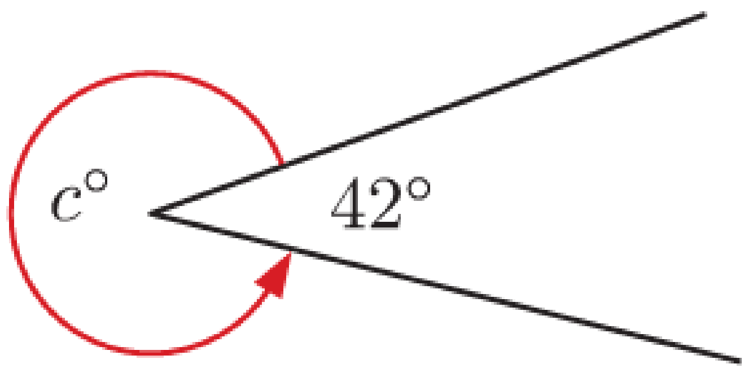

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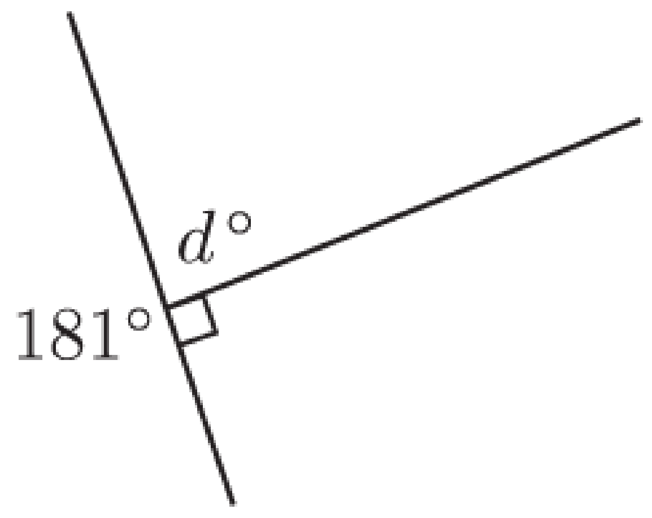

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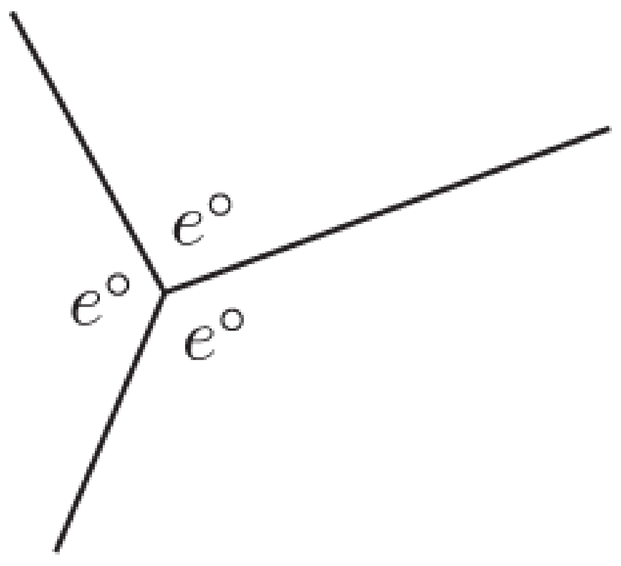

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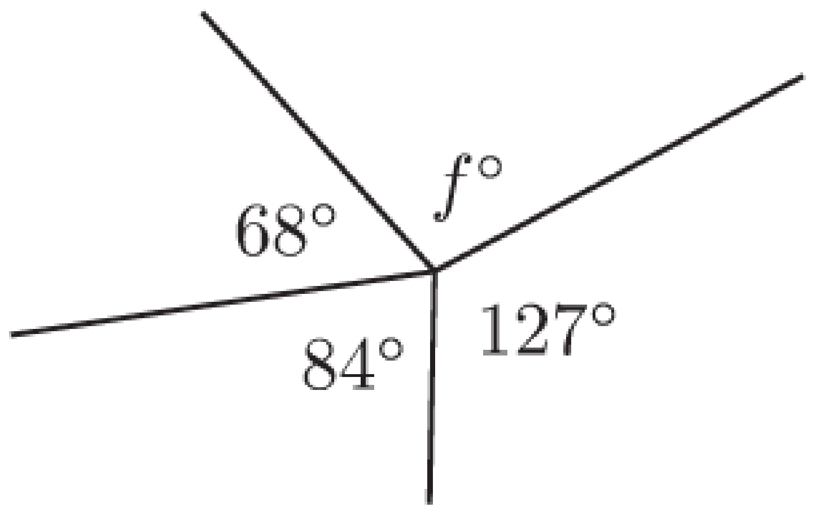

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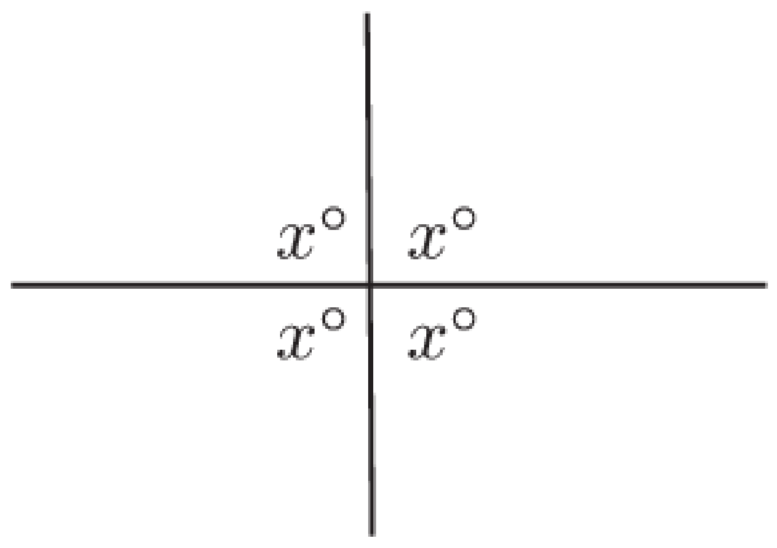

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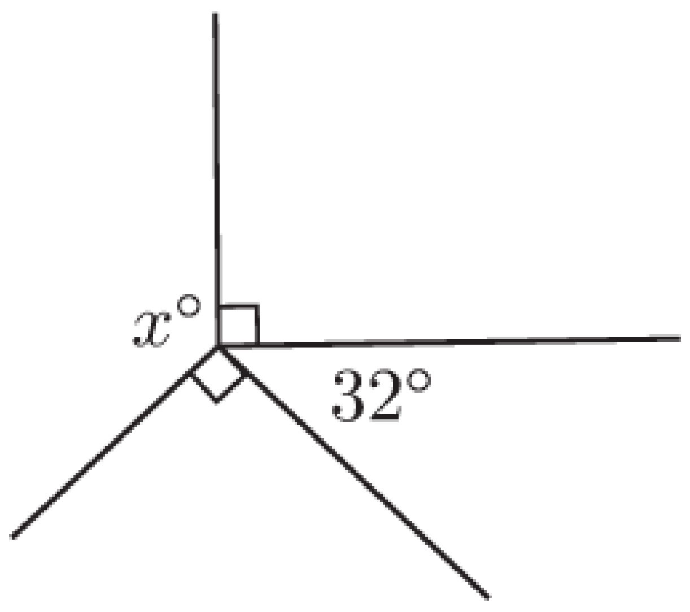

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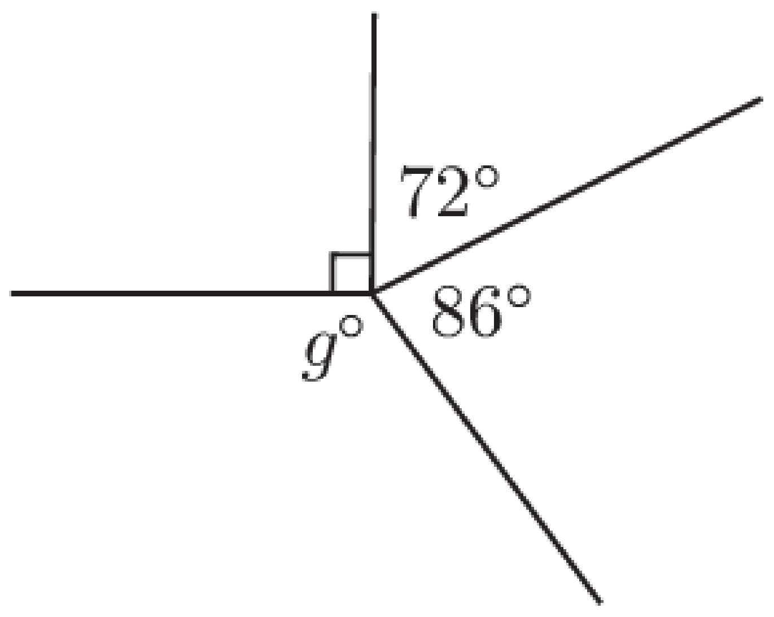
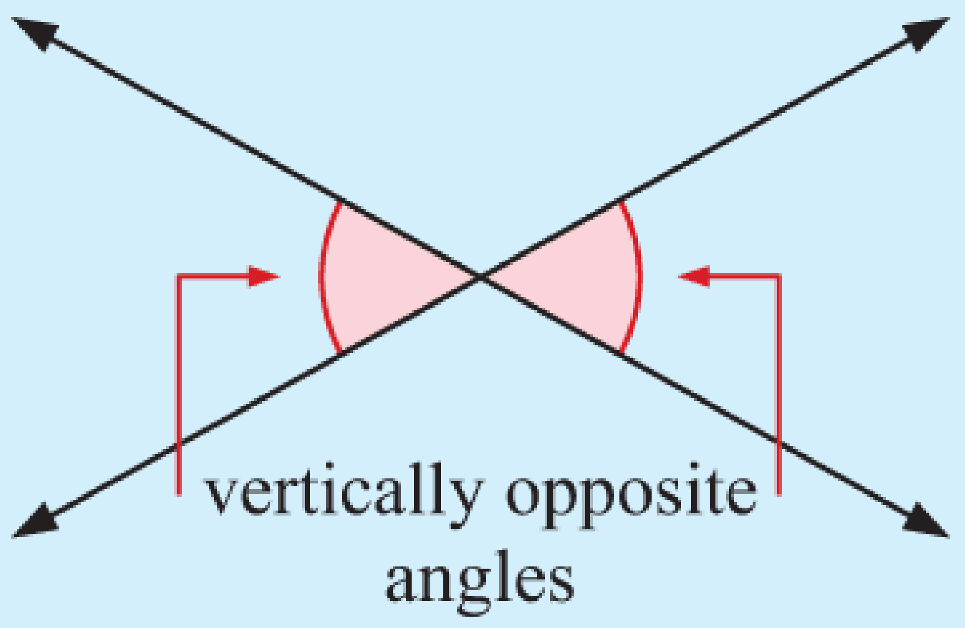


D VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are formed when two straight lines intersect.

The two angles are directly opposite each other through the vertex.



INVESTIGATION VERTICALLY OPPOSITE ANGLES

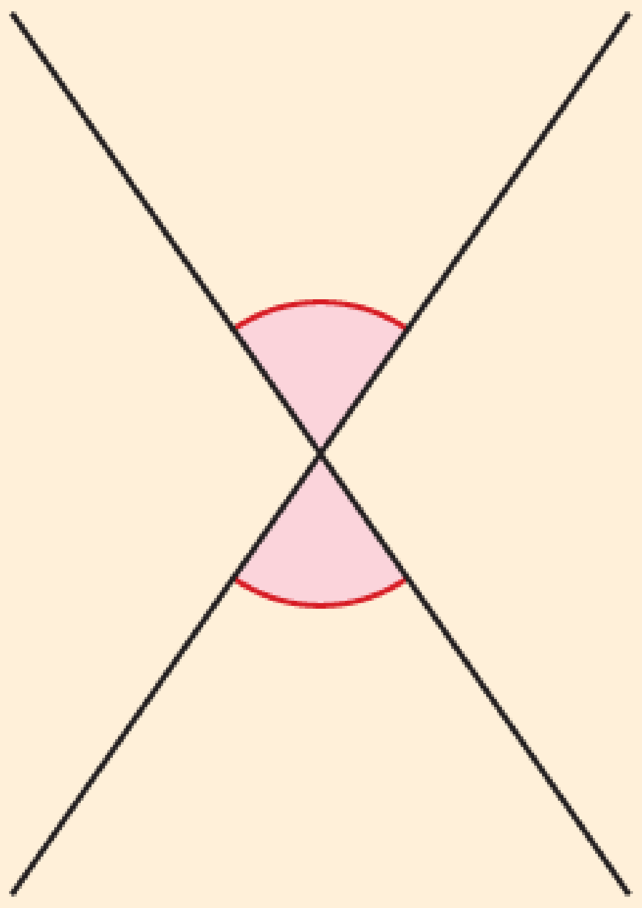
In this Investigation we will discover the relationship between the sizes of vertically opposite angles.

PRINTABLE DIAGRAMS

What to do:

1 For each set of intersecting line segments, use a protractor to measure each of the vertically opposite angles.

a



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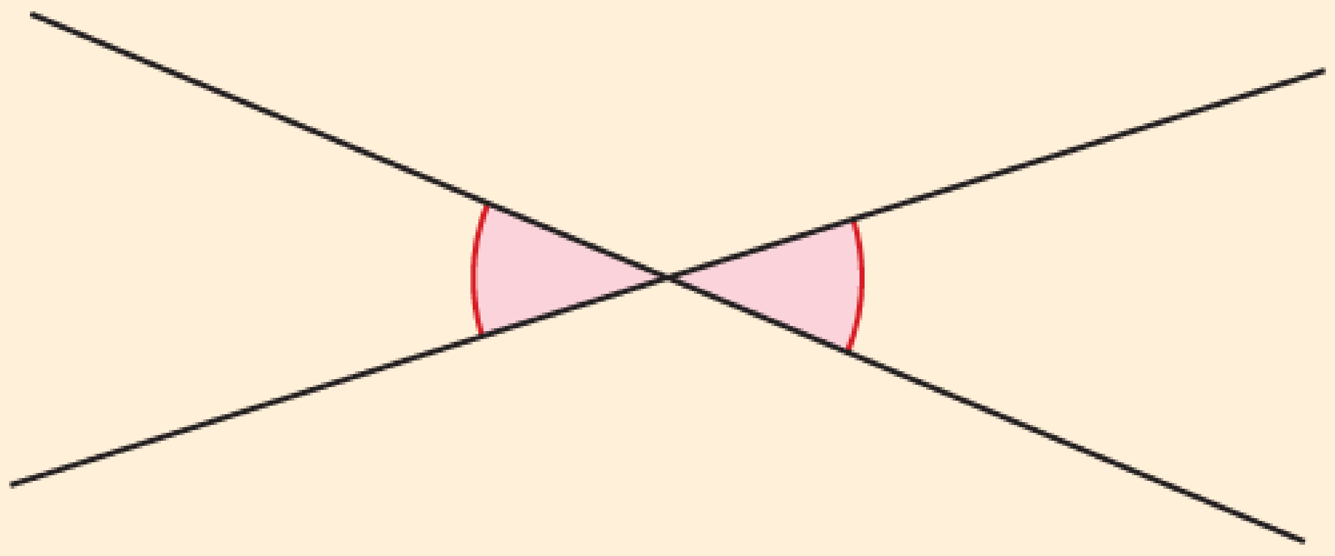
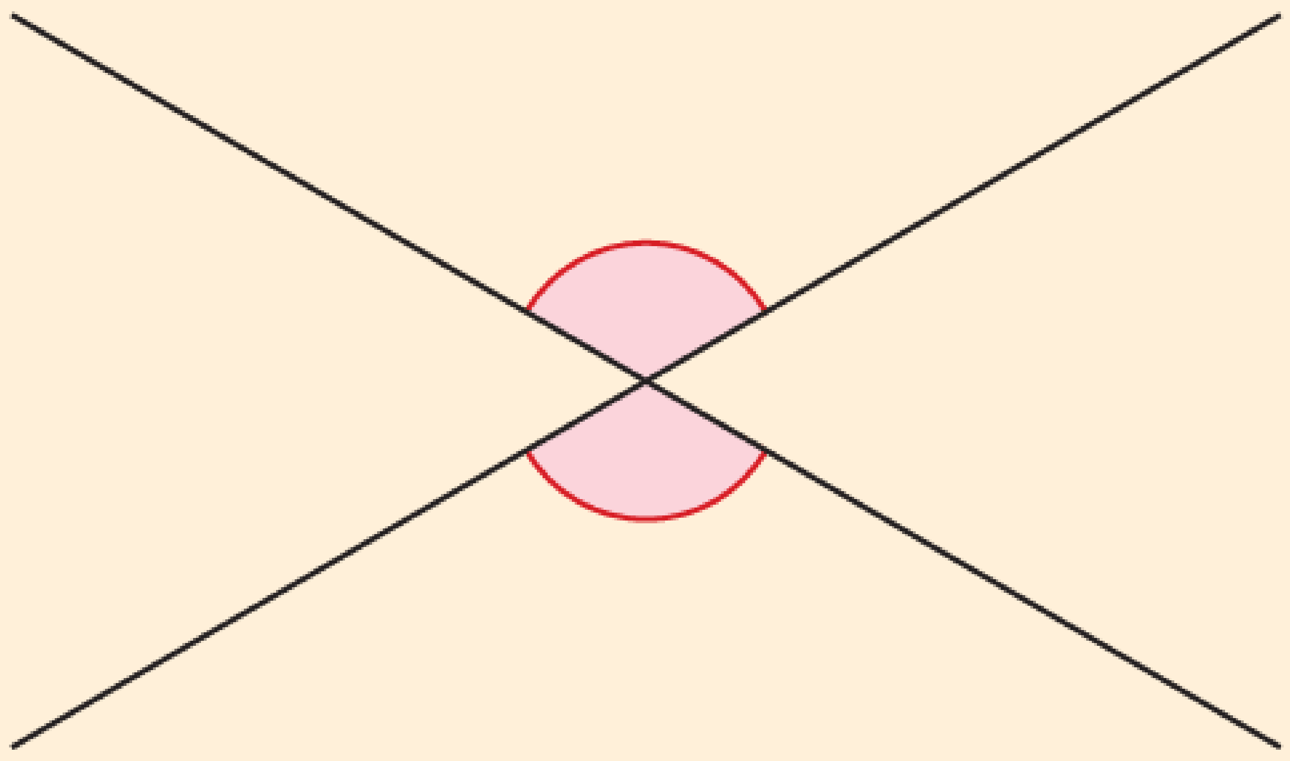


c



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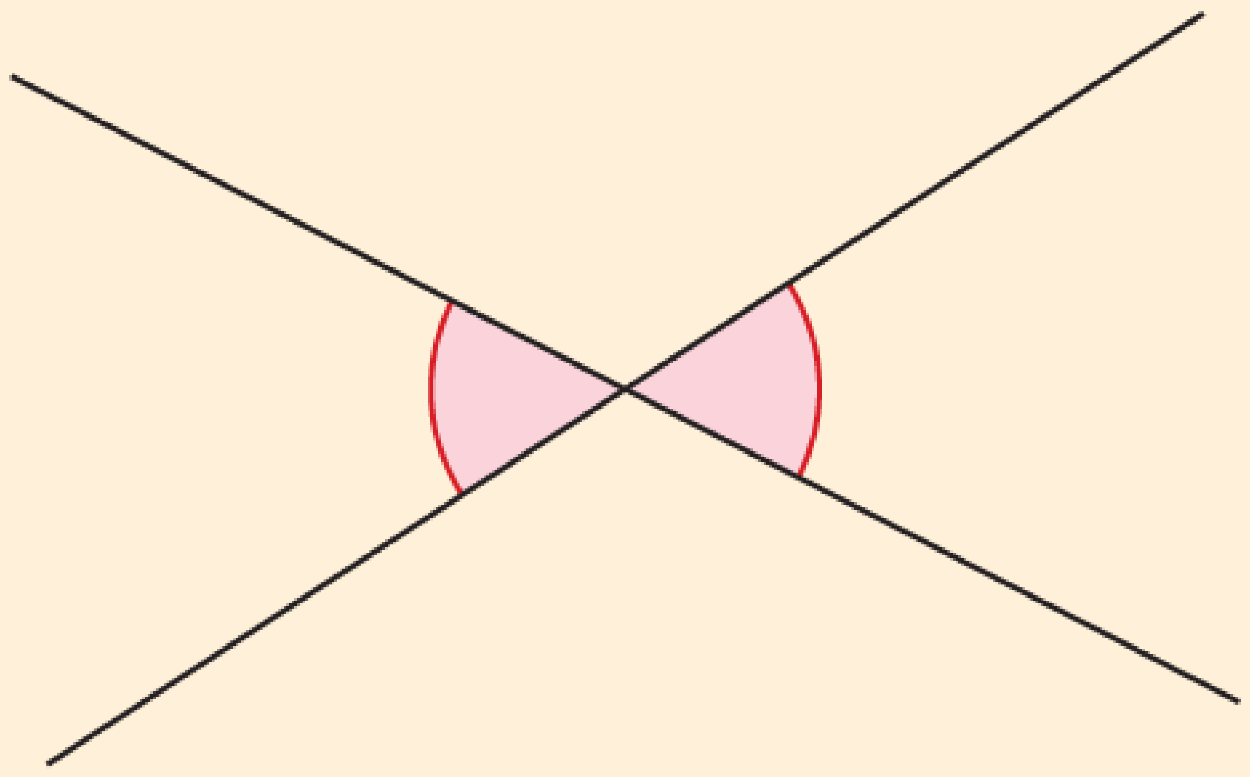

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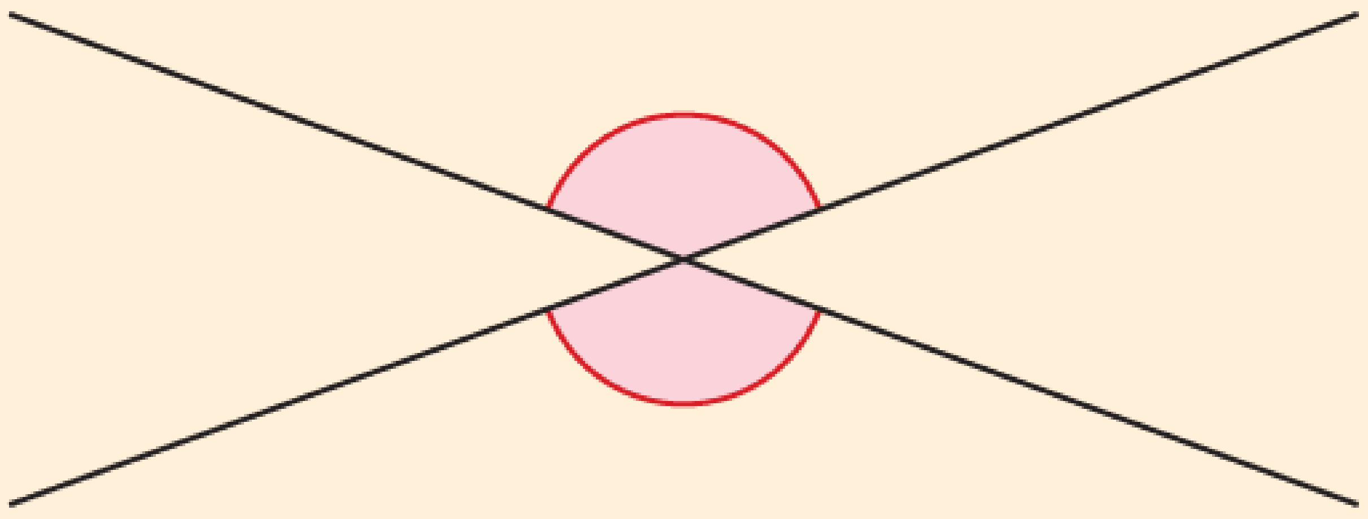

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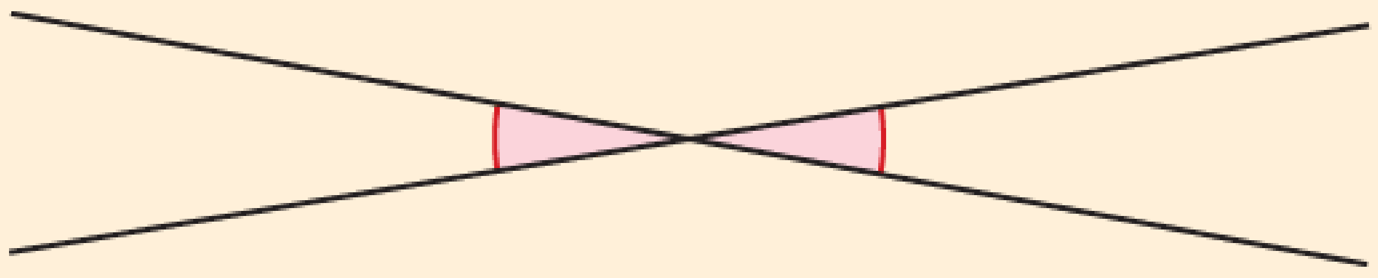
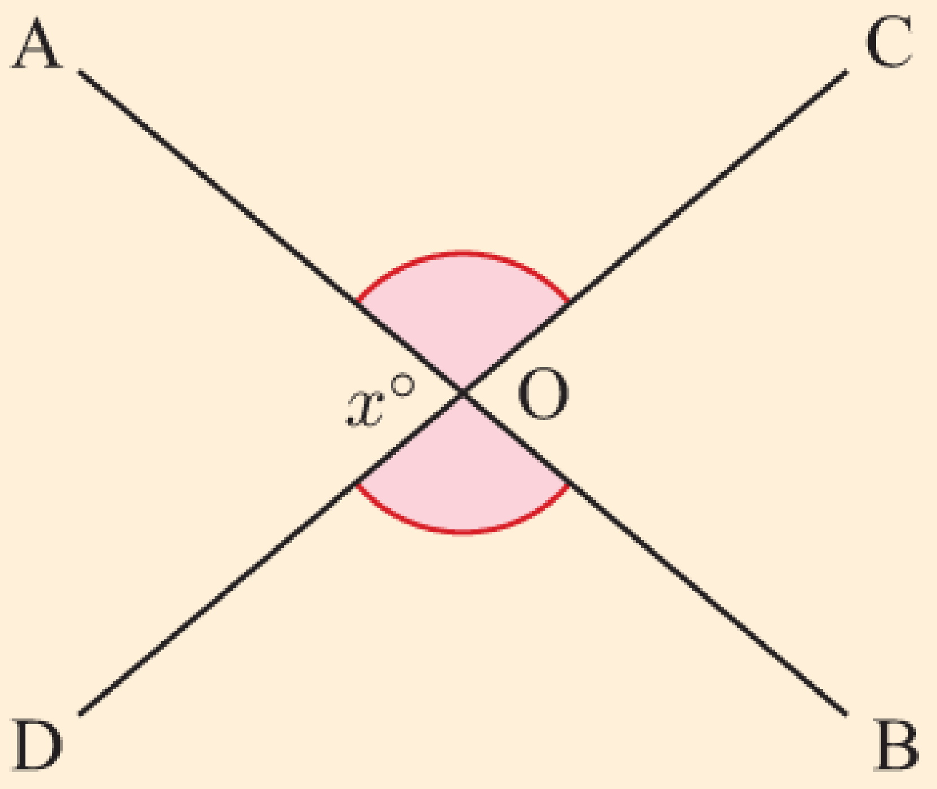


2 What do you suspect from **1**?

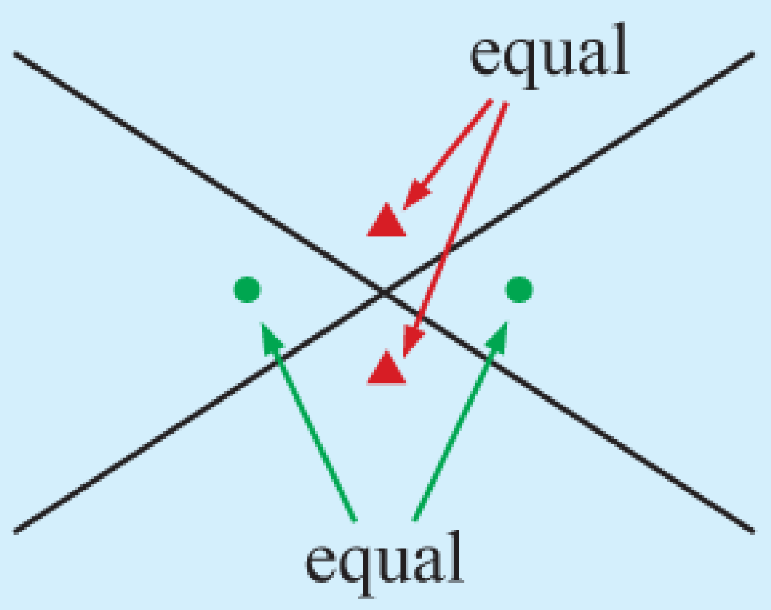
3 In the diagram alongside, $A\widehat{O}D = x^\circ$.
$A\widehat{O}C$ and $B\widehat{O}D$ are vertically opposite angles.

- **a** Explain why $A\widehat{O}C = (180 - x)^\circ$.
- **b** Explain why $B\widehat{O}D = (180 - x)^\circ$.
- **c** What can you conclude?



From the **Investigation** you should have discovered that:

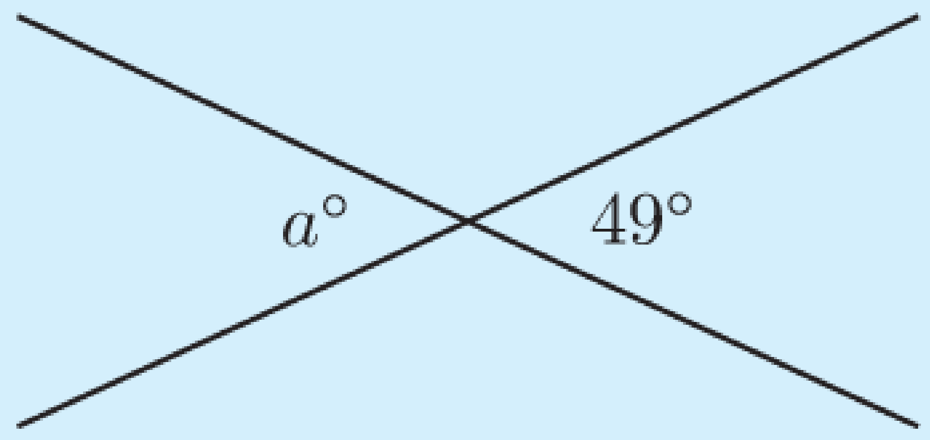
Vertically opposite angles are **equal in size**.



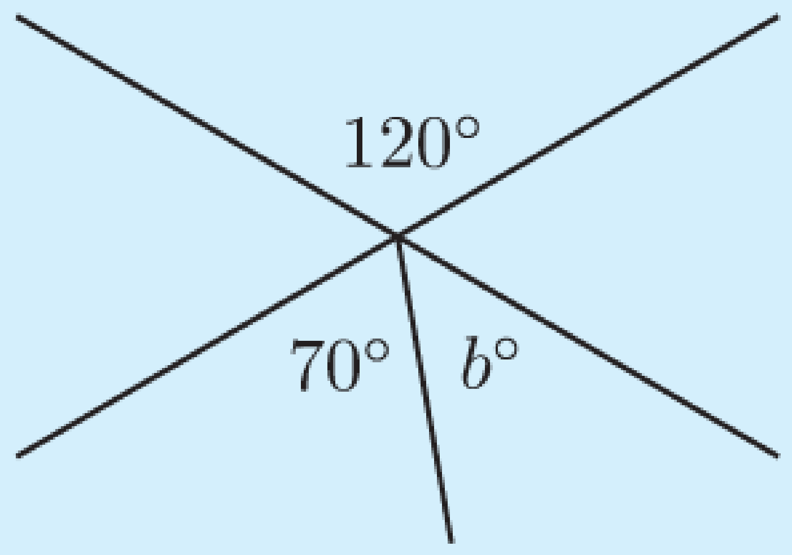
Example 4 **Self Tutor**

Find the value of the unknown:

a



b



a $a = 49$ {vertically opposite angles}

b $b + 70 = 120$

{vertically opposite angles}

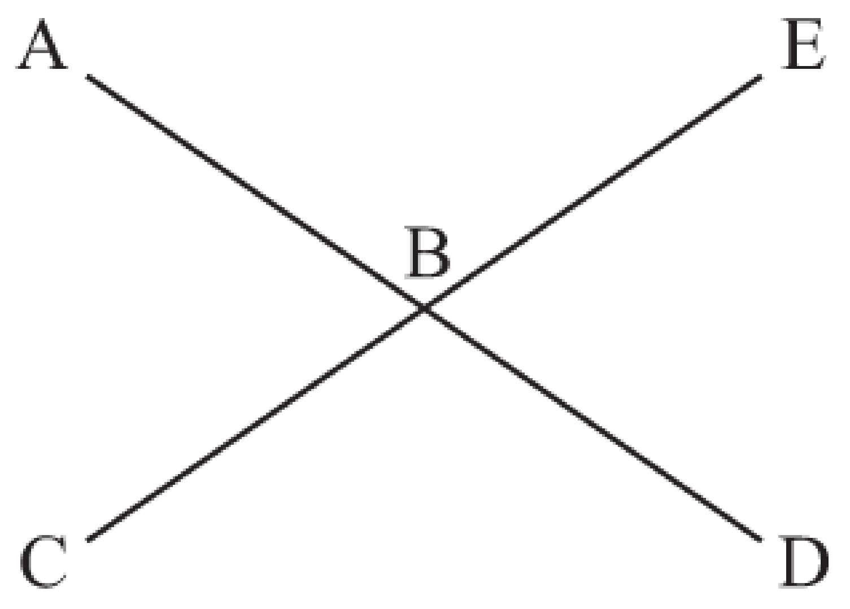
But $50 + 70 = 120$

$\therefore\ b = 50$

EXERCISE 3D

1 Name the angle which is vertically opposite $A\widehat{B}C$:

a



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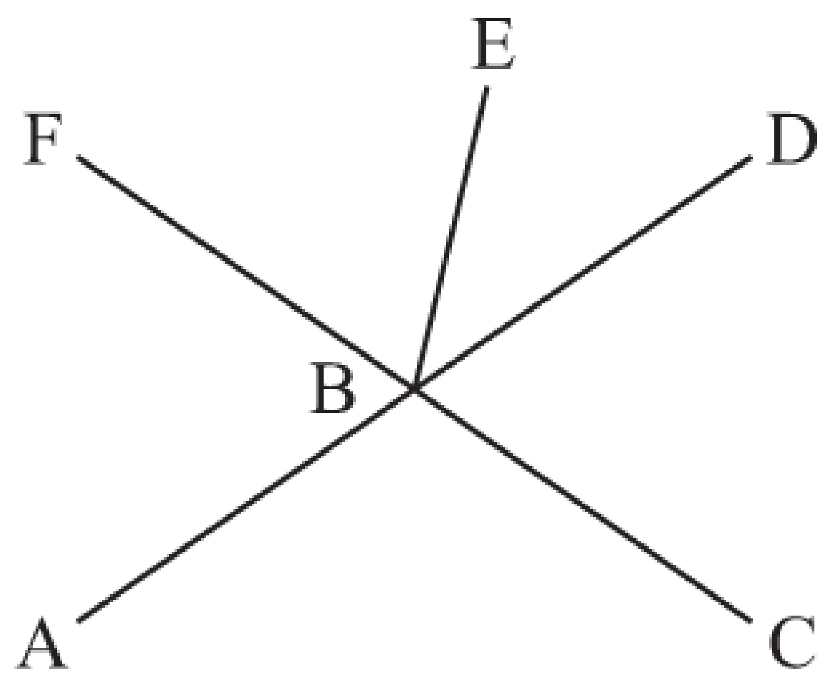

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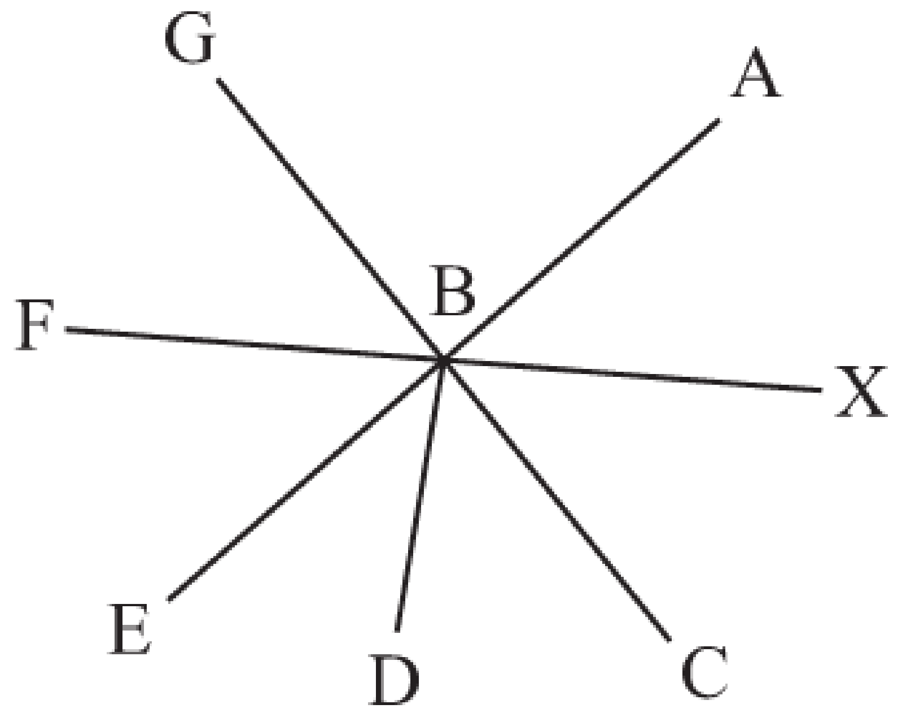
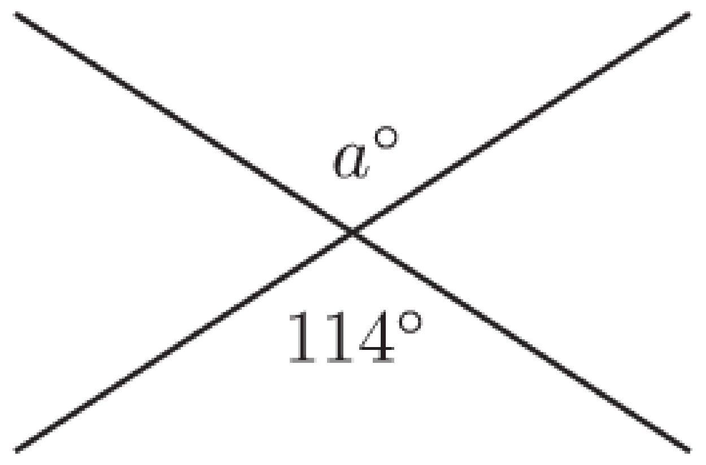


2 Find the value of the unknown:

a



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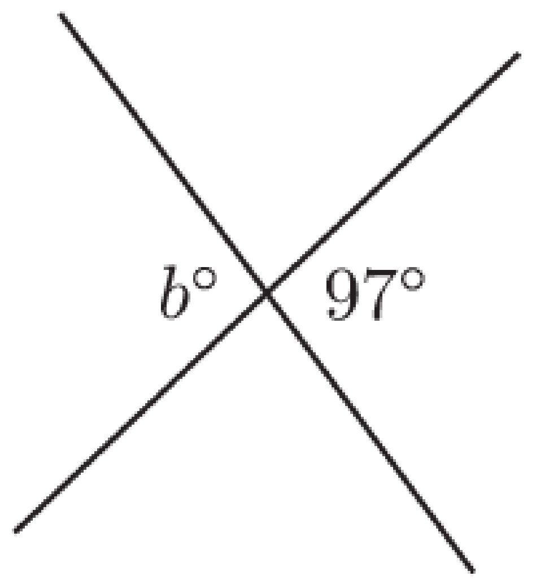

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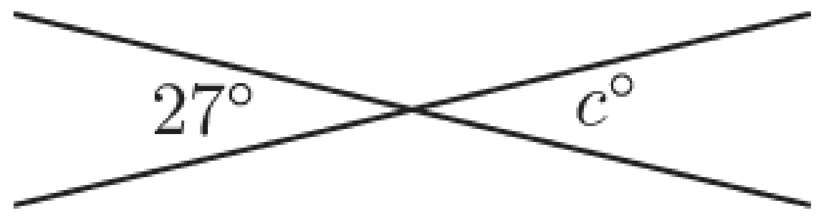

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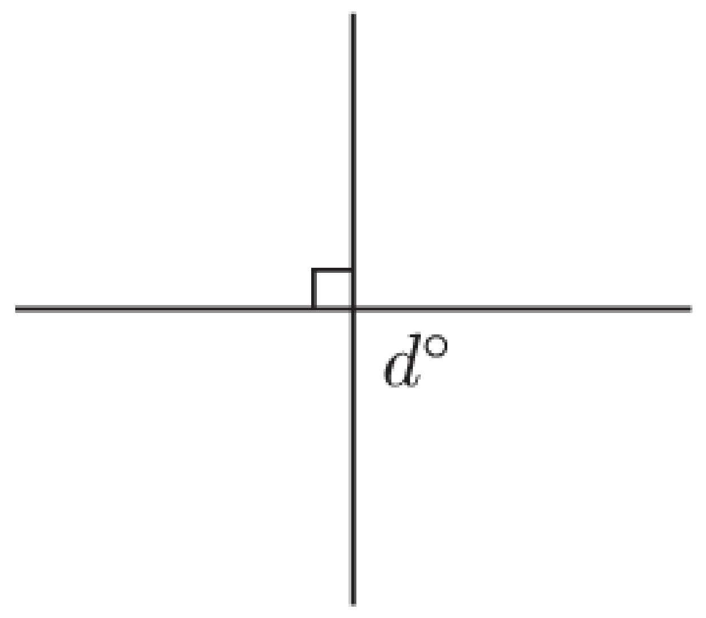

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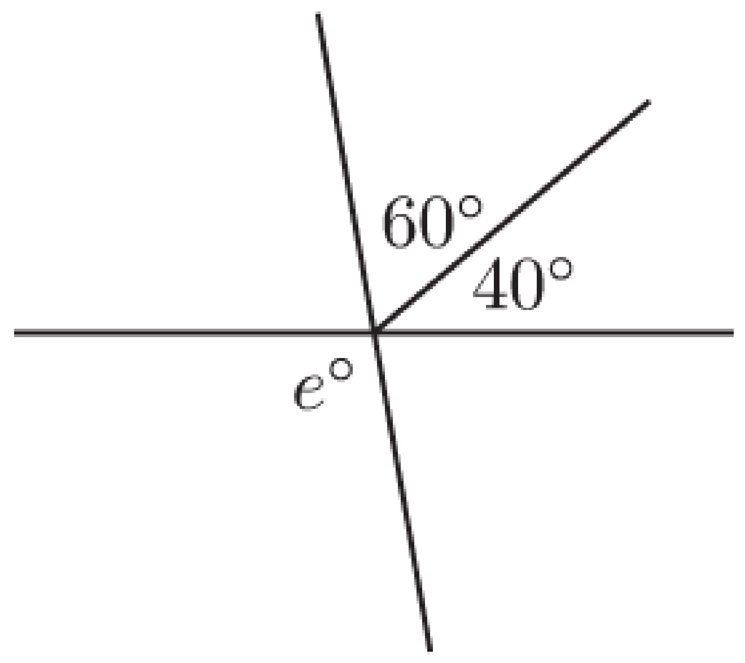

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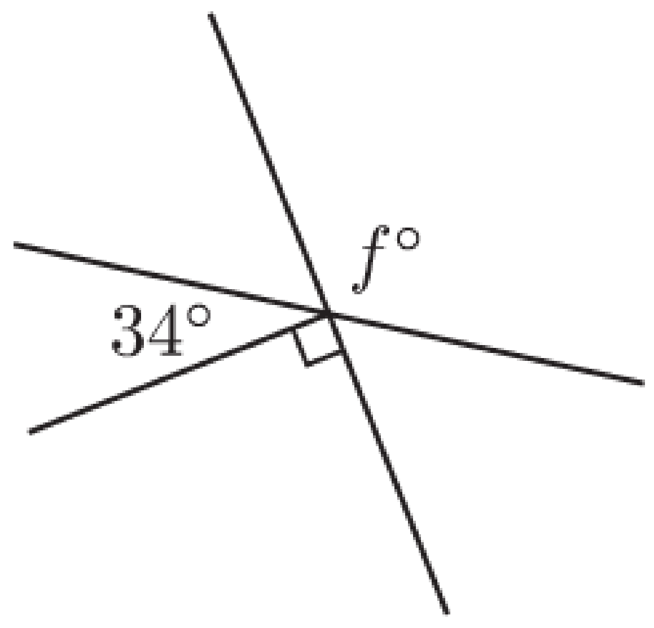

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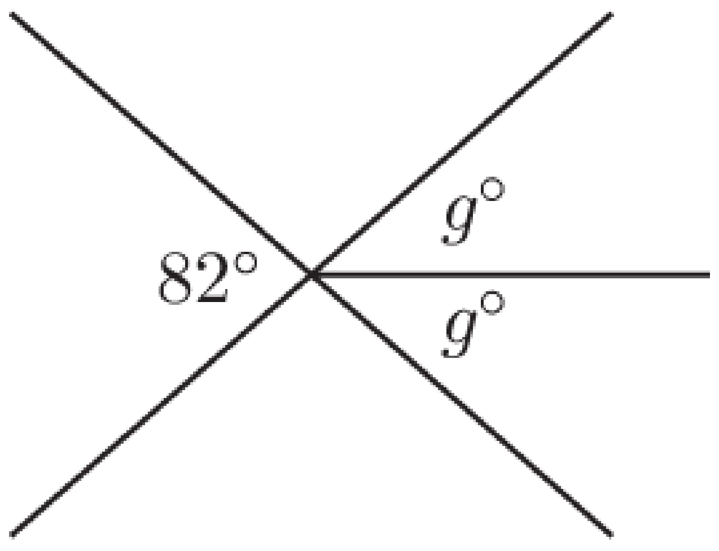

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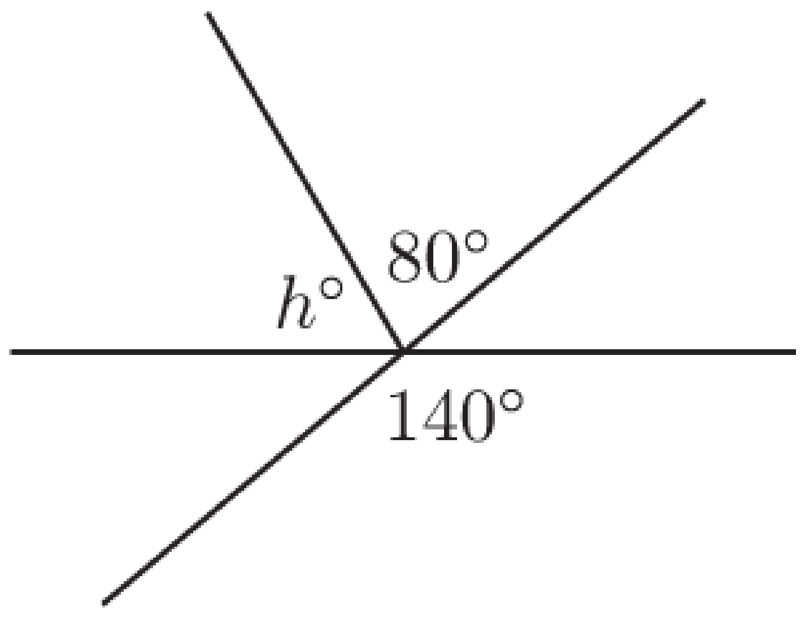

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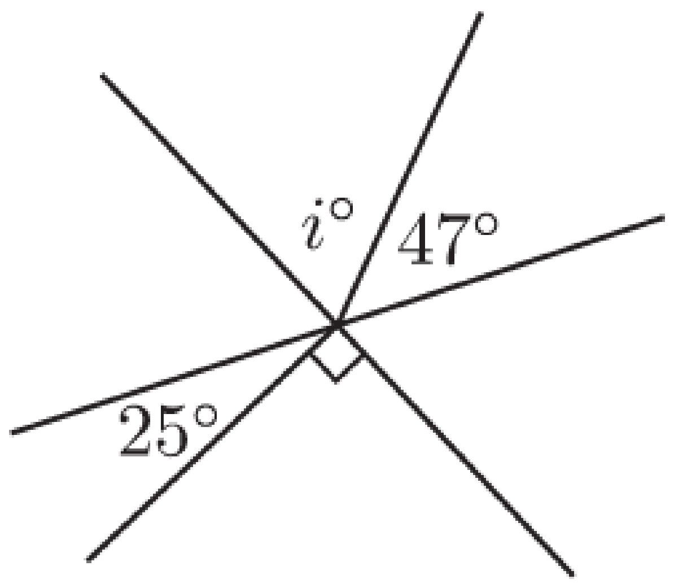

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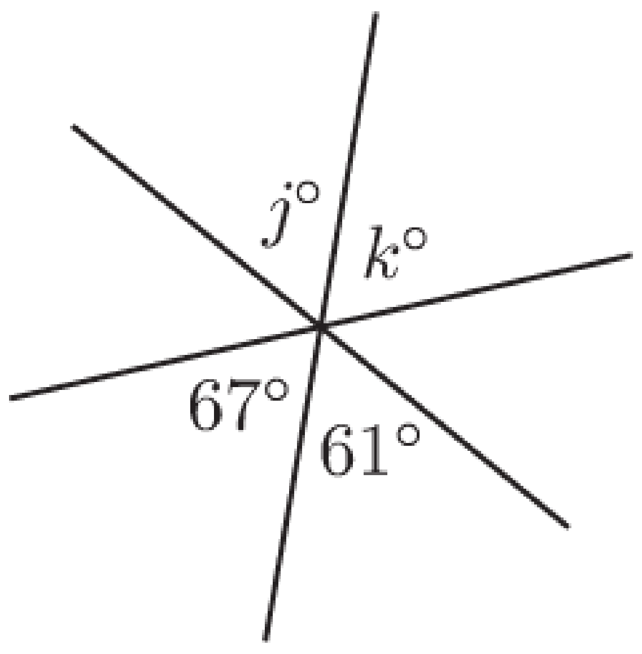

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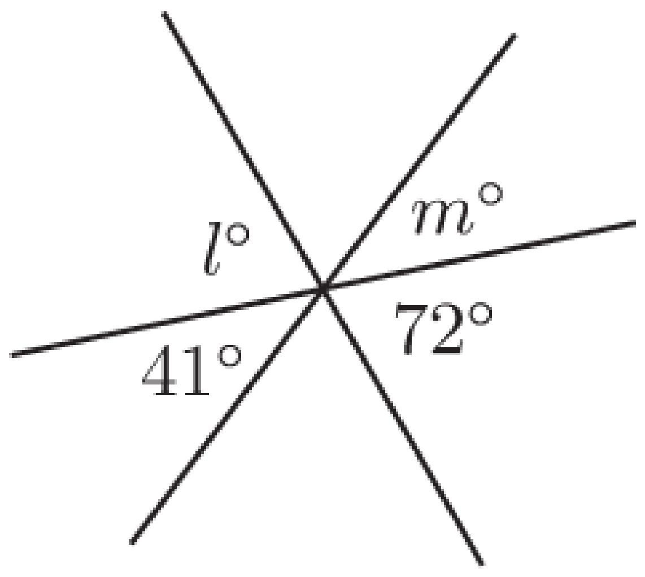

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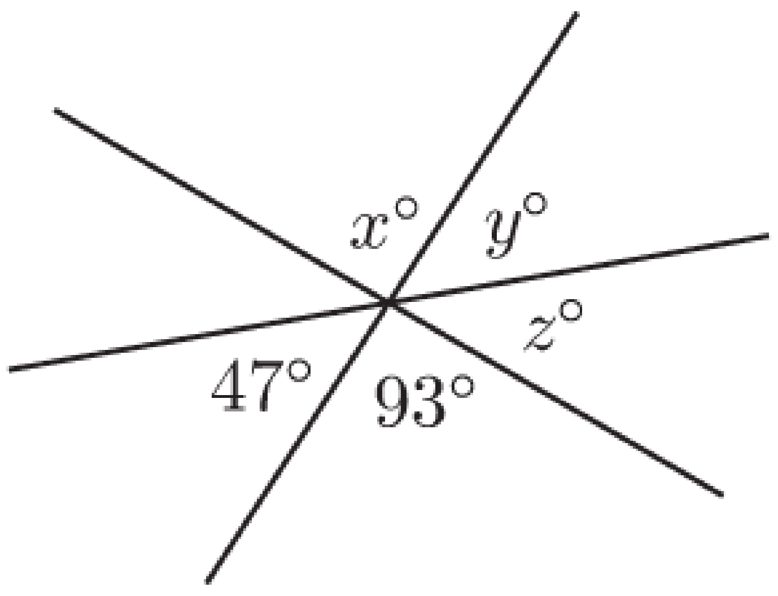


E BISECTING ANGLES

When we **bisect** an angle with a straight line, we divide it into two angles of equal size. In the following **Example** we show how to bisect an angle using a *compass and ruler only*.

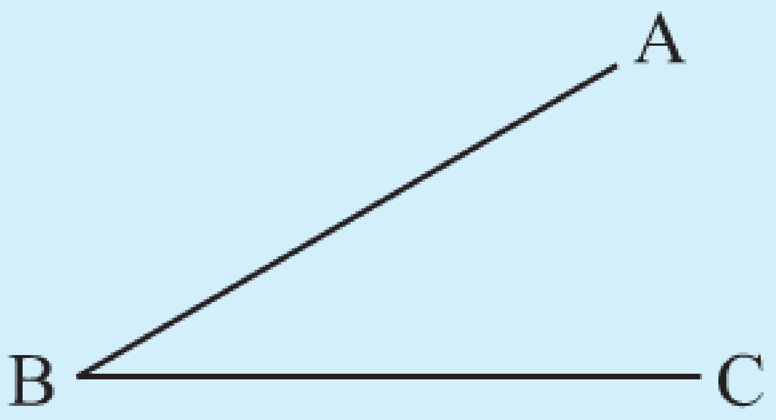
The **radius** of a compass is the distance from the sharp point to the tip of your pencil.

A diagram drawn using a compass and ruler is called a **construction**.

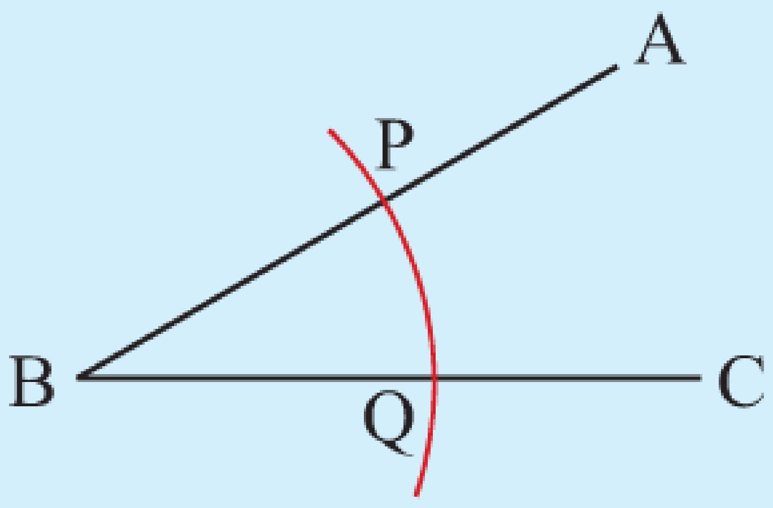
Example 5

Self Tutor

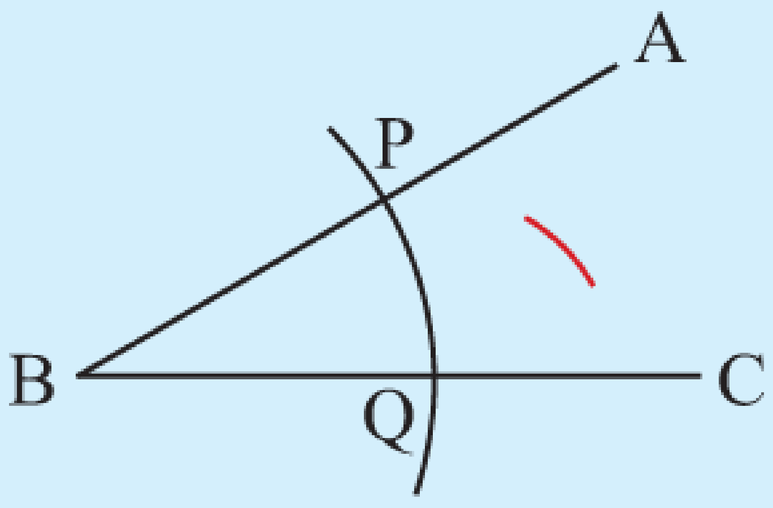
Bisect $A\widehat{B}C$.



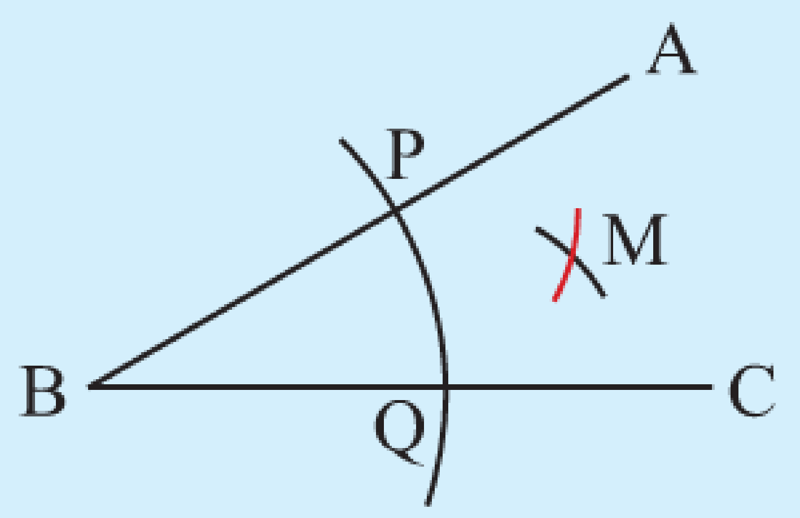
Step 1: With centre B, draw an arc which cuts [BA] and [BC] at P and Q respectively.



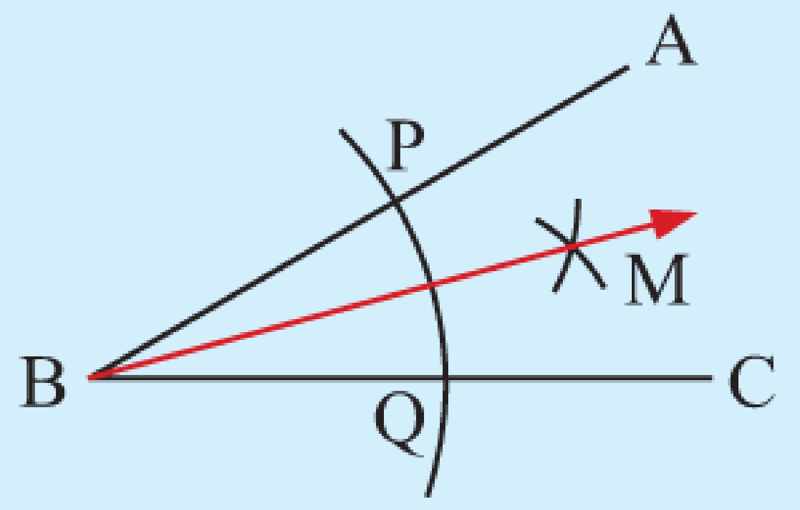
Step 2: With centre Q and radius [PQ], draw an arc within the angle ABC.



Step 3: With centre P and the same radius [PQ], draw another arc to intersect the previous one at M.



Step 4: Draw [BM].
[BM) bisects $A\widehat{B}C$,
so $A\widehat{B}M = C\widehat{B}M$.



EXERCISE 3E

1 **a** Use your protractor to draw $A\widehat{B}C$ of size 80°. Bisect $A\widehat{B}C$ using a compass and ruler only.
b Use your protractor to find the size of each of the two angles you constructed.

DEMO

2 Draw acute $X\widehat{Y}Z$ of your own choice. Bisect the angle using a compass and ruler only. Check your construction using your protractor.

3 Draw obtuse $A\widehat{B}C$ of your own choice. Bisect the angle using a compass and ruler only. Check your construction using your protractor.

4 **a** Draw any triangle ABC and carefully bisect its three angles.
b Repeat with another triangle DEF of different shape.
c Check with other students in your class for any observations about the three angle bisectors.
d Copy and complete: "The three angle bisectors of a triangle ....".

DEMO

5 **a** Use your protractor to draw an angle of size 140°.
b *Hence* use a compass and ruler to construct an angle of size 35°.

KEY WORDS USED IN THIS CHAPTER

- acute angle
- angle
- bisect
- compass
- degree
- intersecting
- line
- line segment
- obtuse angle
- parallel
- point
- ray
- reflex angle
- revolution
- right angle
- straight angle
- three point notation
- vertex
- vertically opposite angles

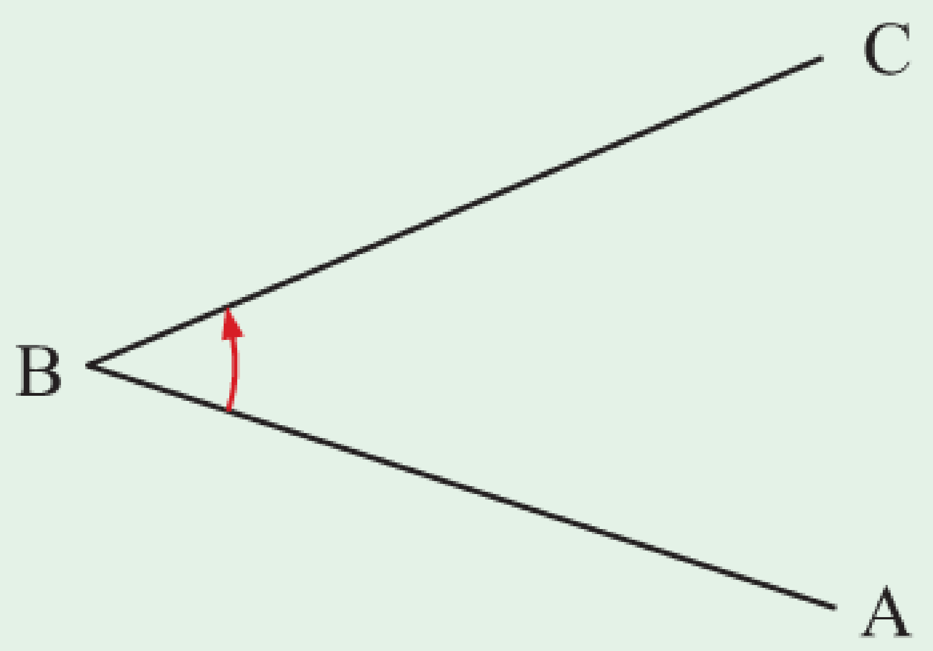
REVIEW SET 3A

1 Draw a diagram to illustrate:

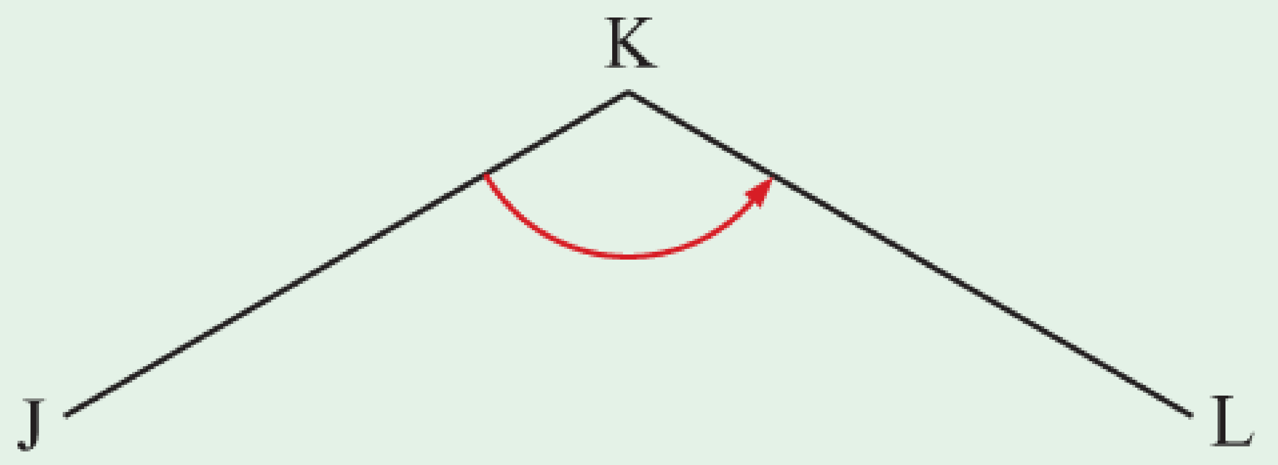
a a $\frac{3}{4}$ turn **b** an acute angle.

2 Use three point notation to name the following angles. State the type of angle in each case.

a



b

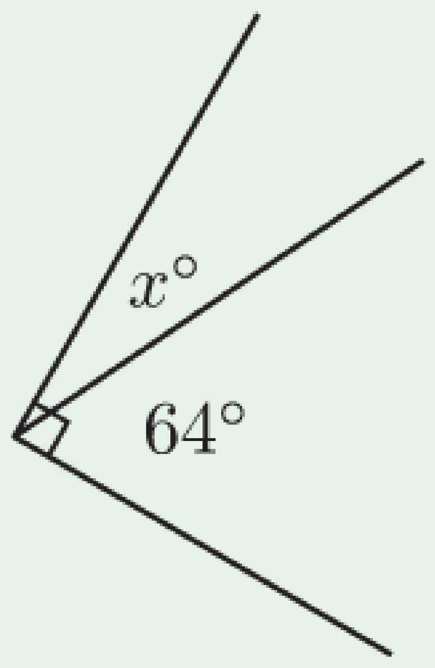


3 For the following angle sizes, state whether the angle is acute, right, obtuse, straight, or reflex:

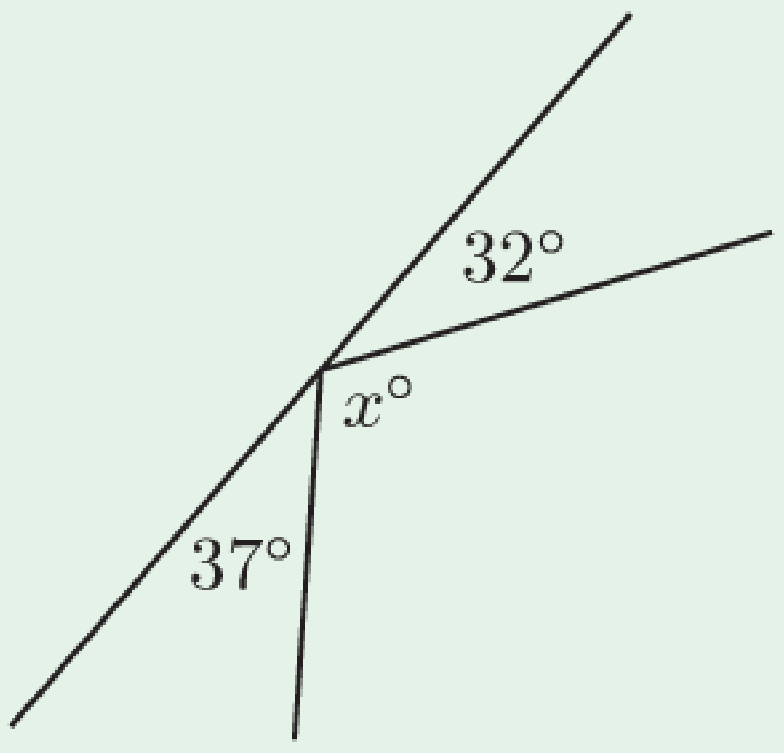
a 139° **b** 90° **c** 187° **d** 24°

4 Find the value of x:

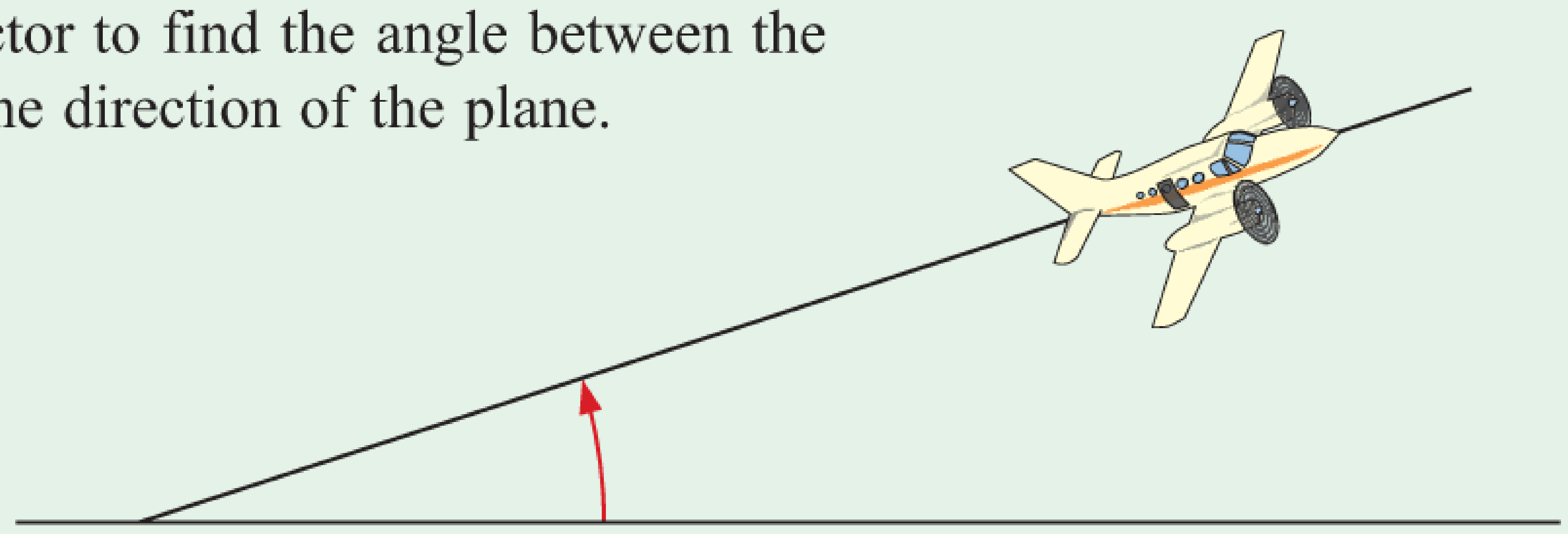
a



b



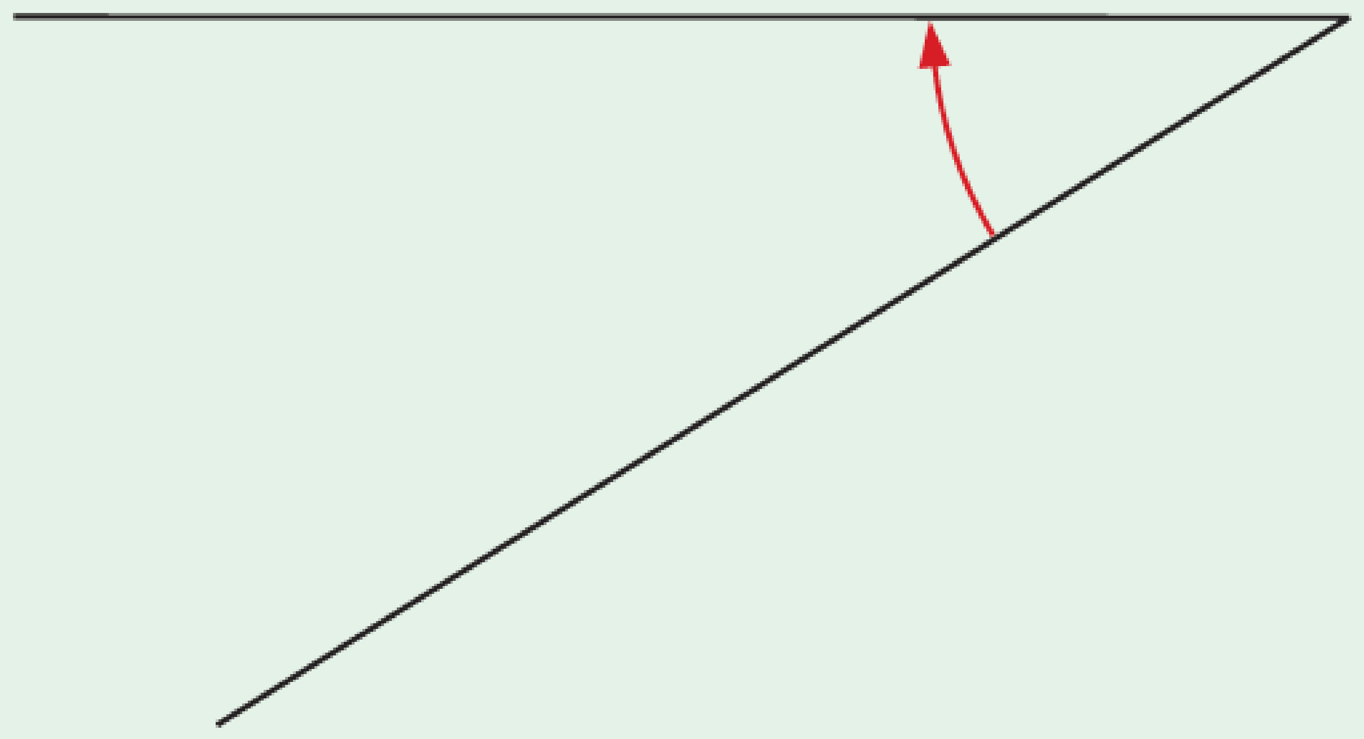
5 Use a protractor to find the angle between the ground and the direction of the plane.



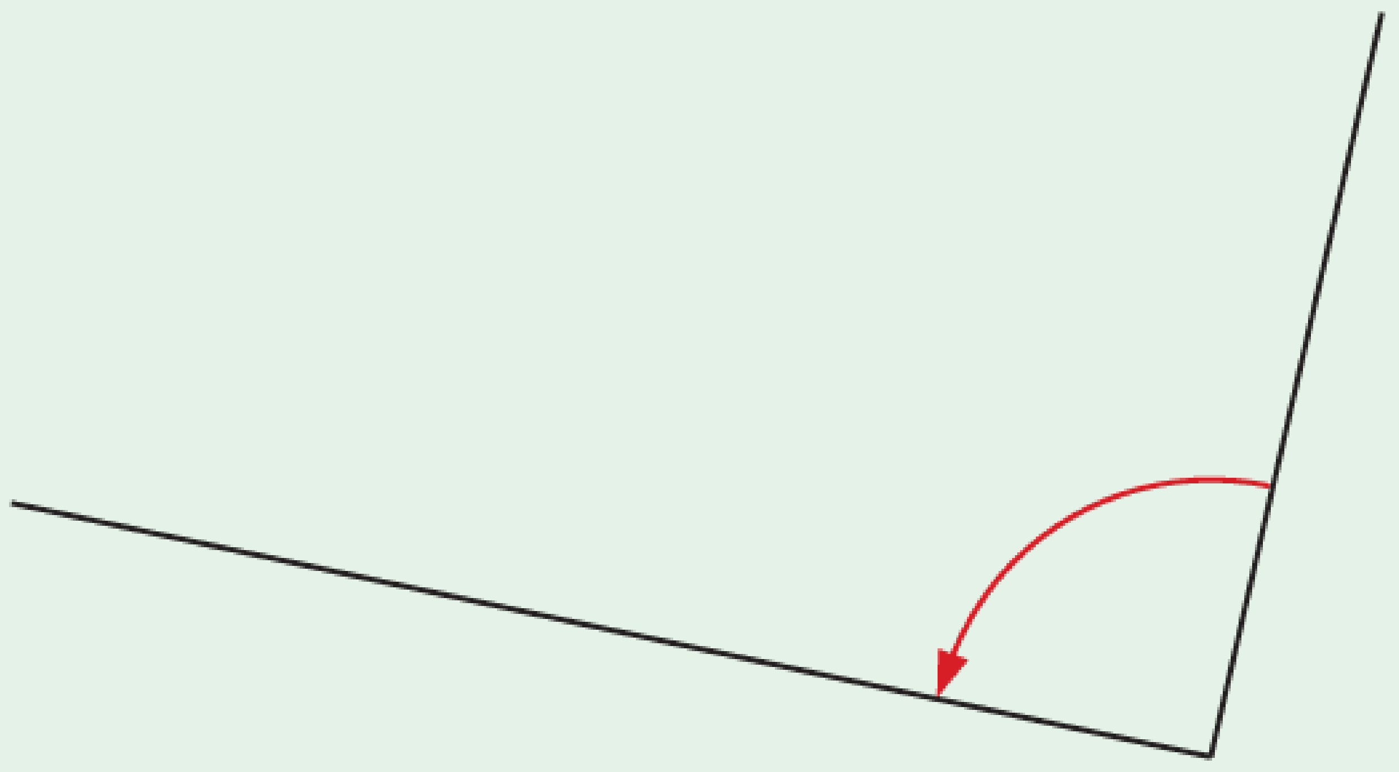
PRINTABLE DIAGRAMS

6 Use a protractor to measure the following angles. State the type of angle in each case.

a

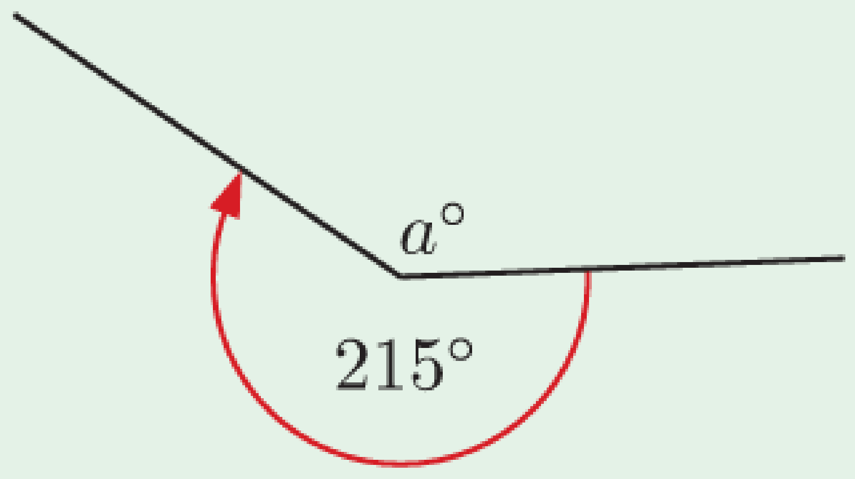


b

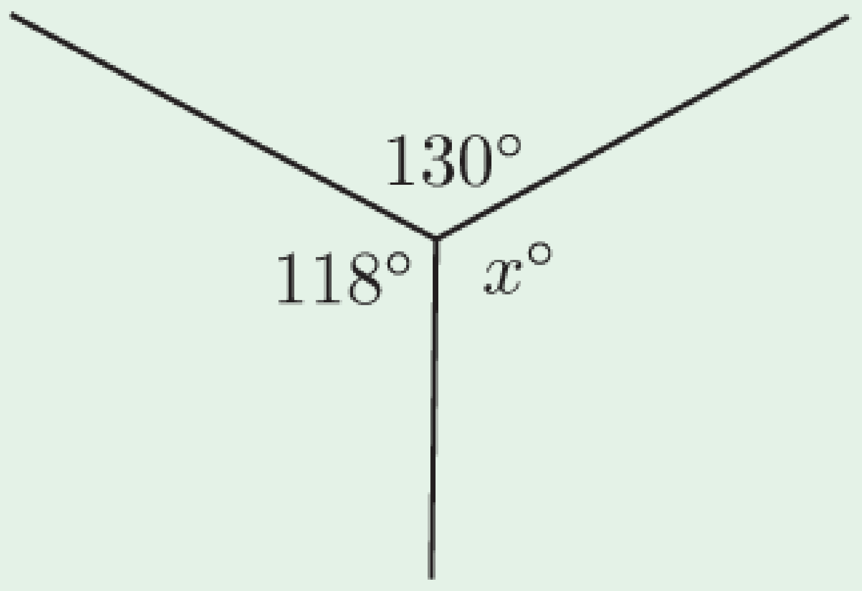


7 Find the value of the unknown:

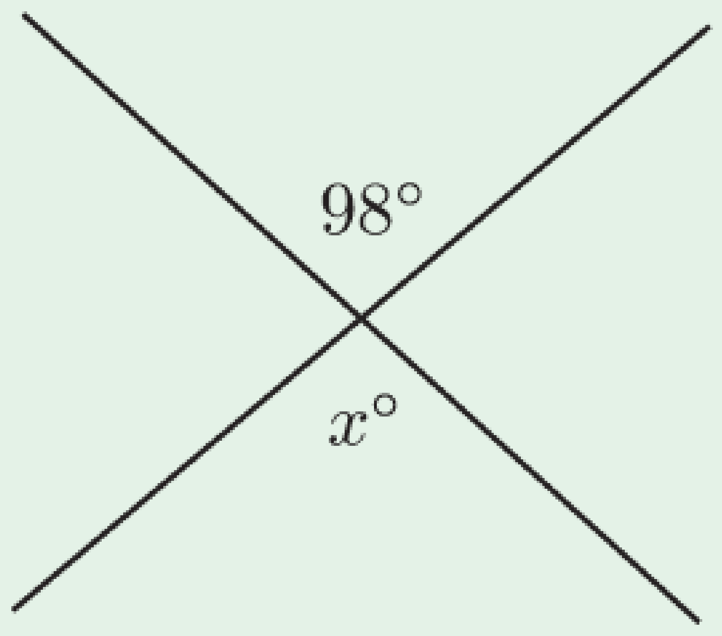
a



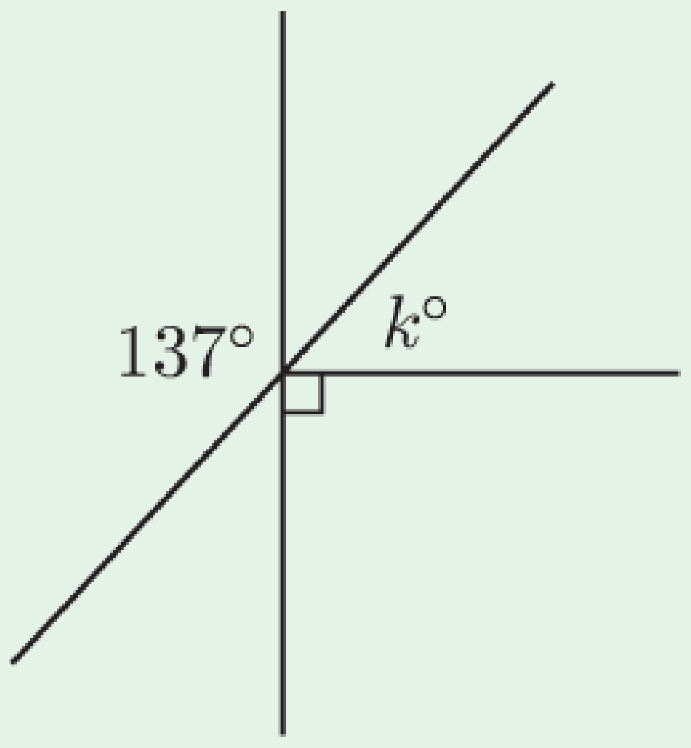
b



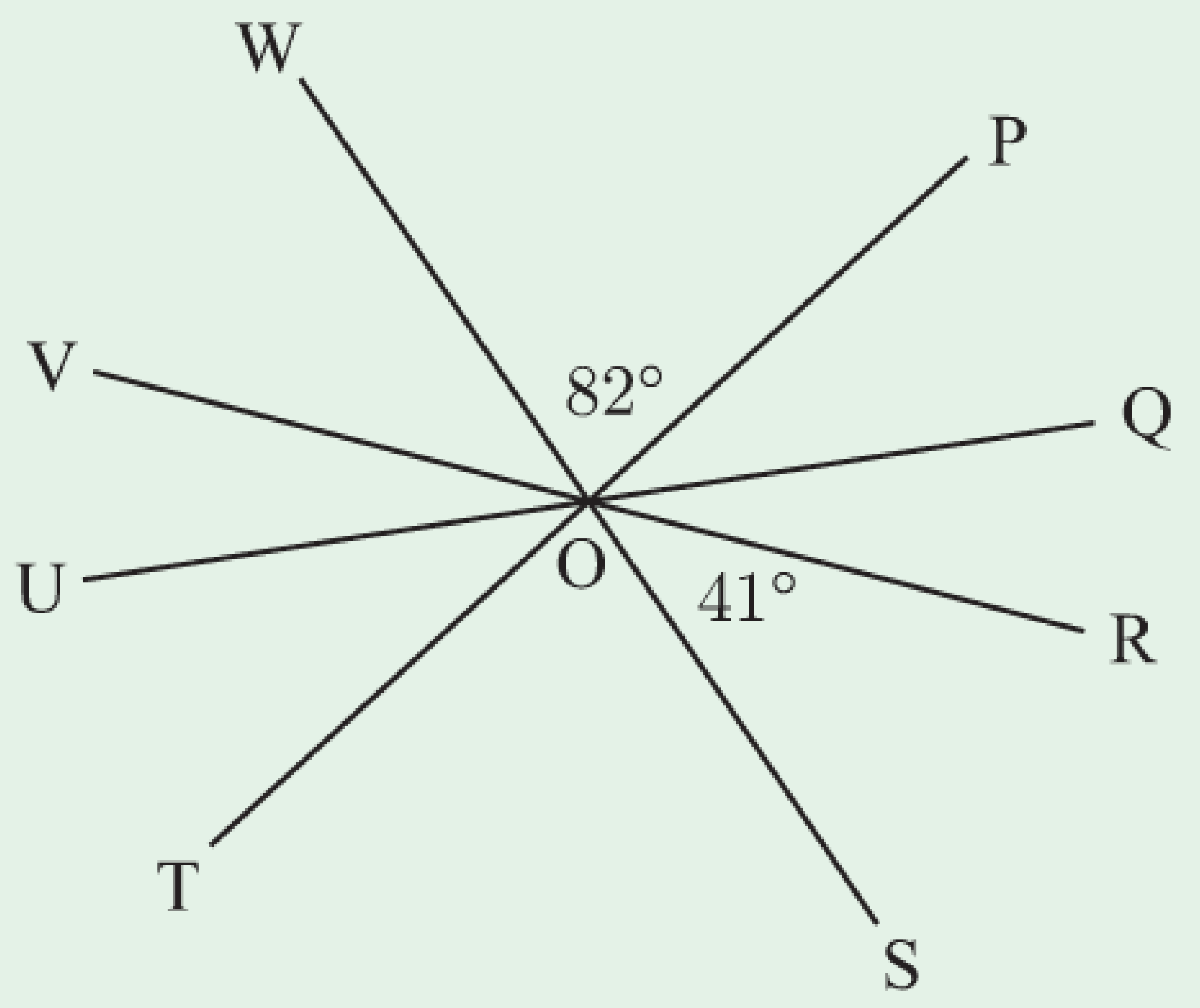
c



d



8



a Name the angle which is vertically opposite:

i $P\widehat{O}Q$ **ii** $W\widehat{O}V$

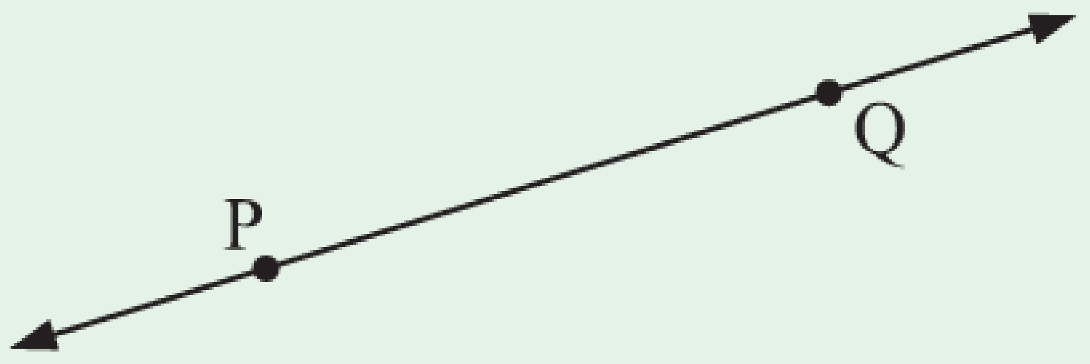
b Find the size of:

i $S\widehat{O}T$ **ii** $P\widehat{O}V$

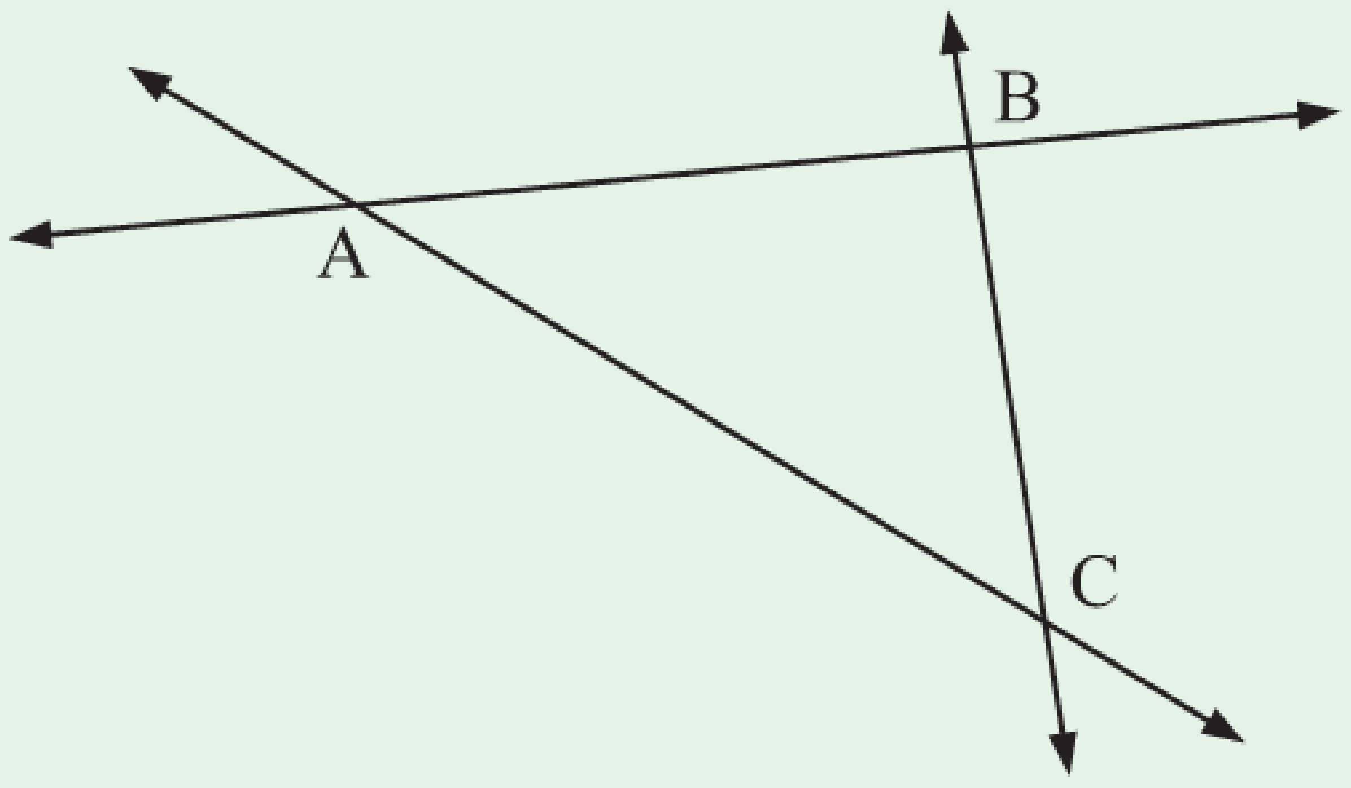
9 Use your protractor to draw an angle PQR of size 120°. Bisect this angle using a compass and ruler only. Check your construction using a protractor.

REVIEW SET 3B

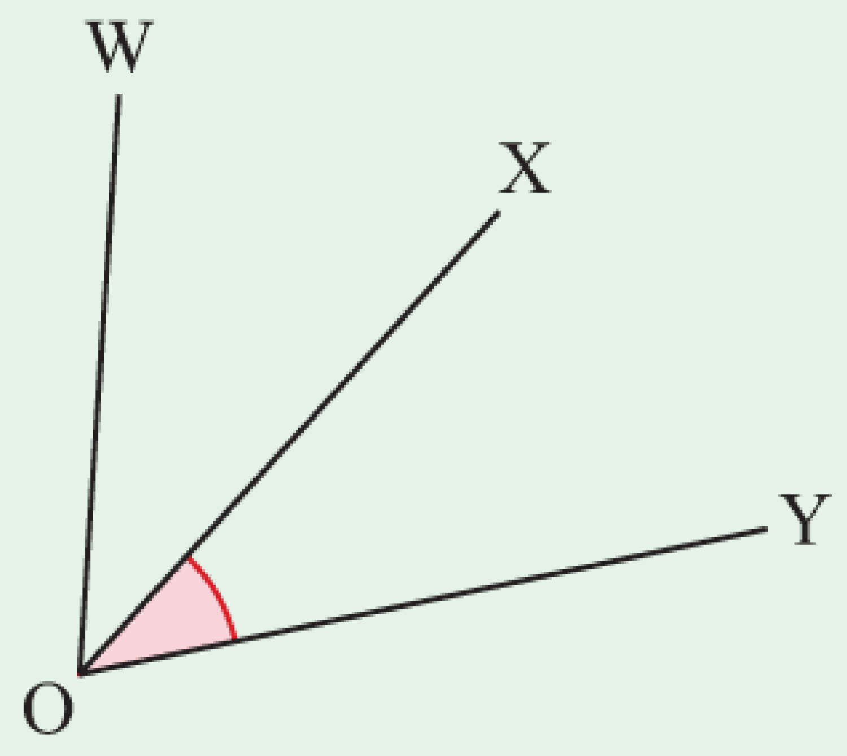
1 Give all the ways of naming this straight line.



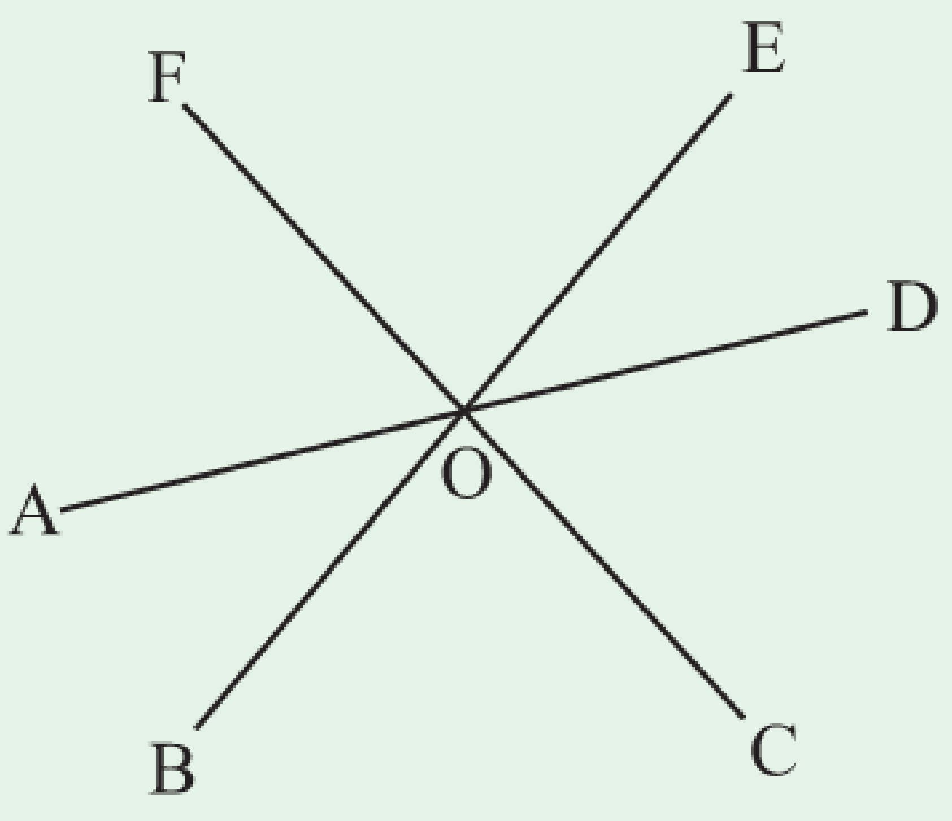
2 Name the intersection of (AB) and (AC).



3 Name the shaded angle in three point notation.



4

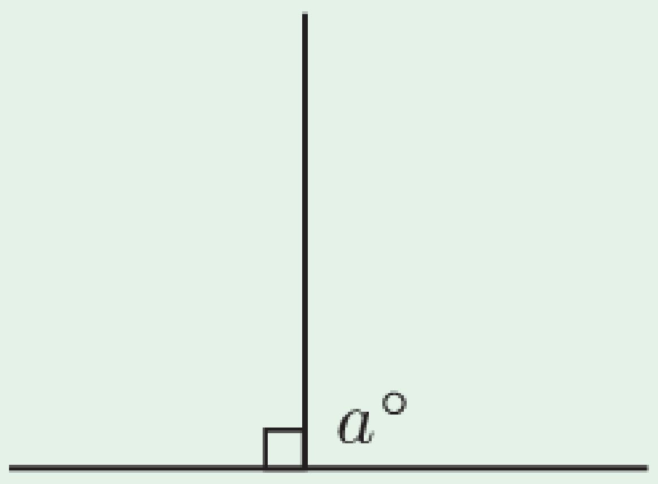


a Name the angle vertically opposite $A\widehat{O}B$.

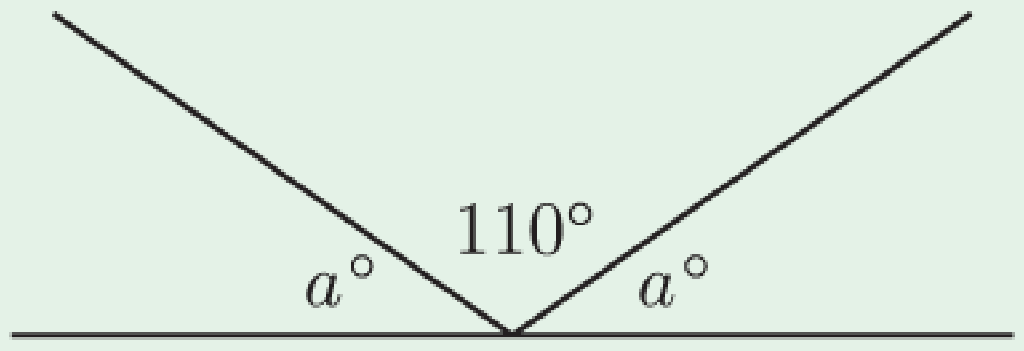
b Name *three* pairs of equal acute angles in the figure.

5 Find the value of a:

a

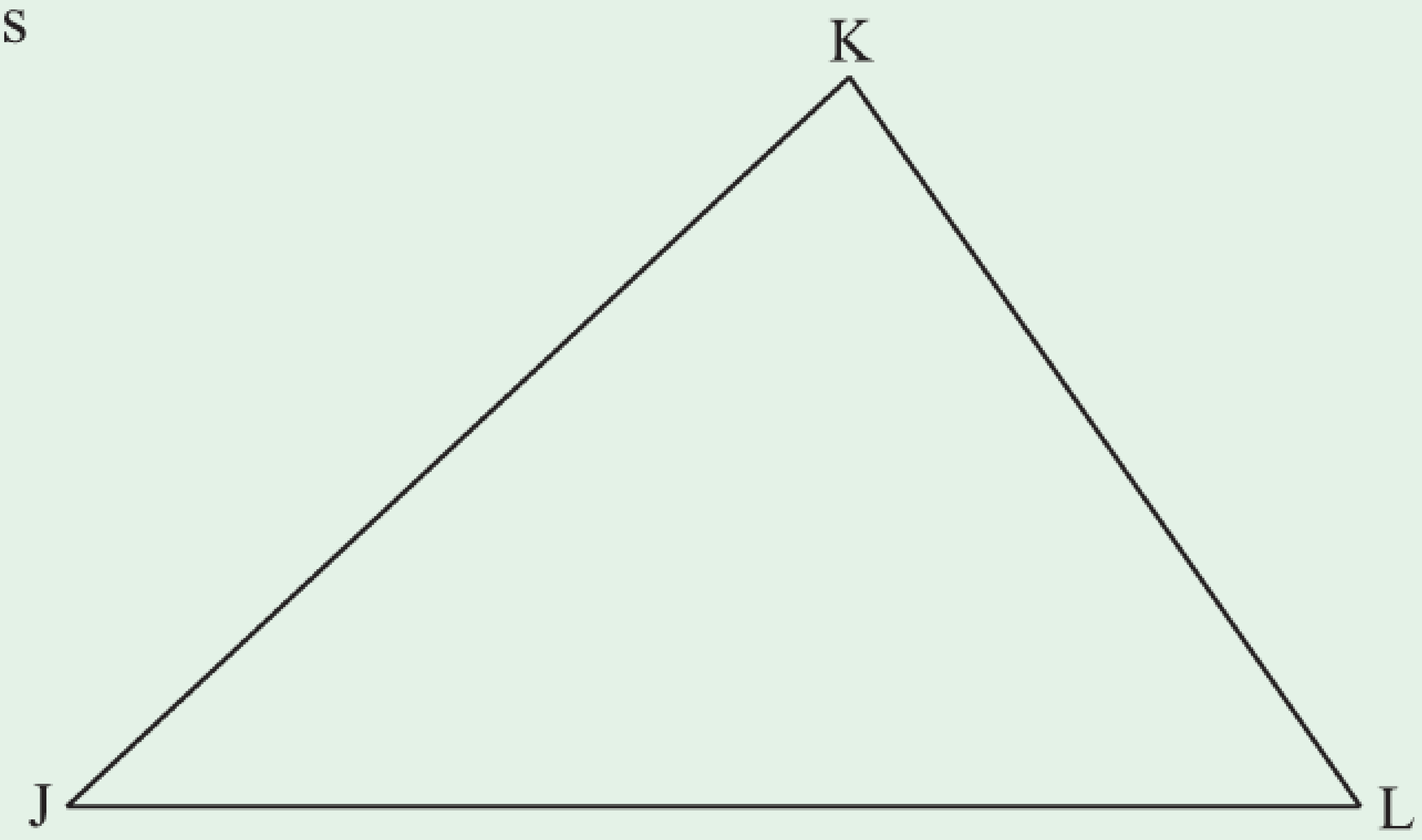


b

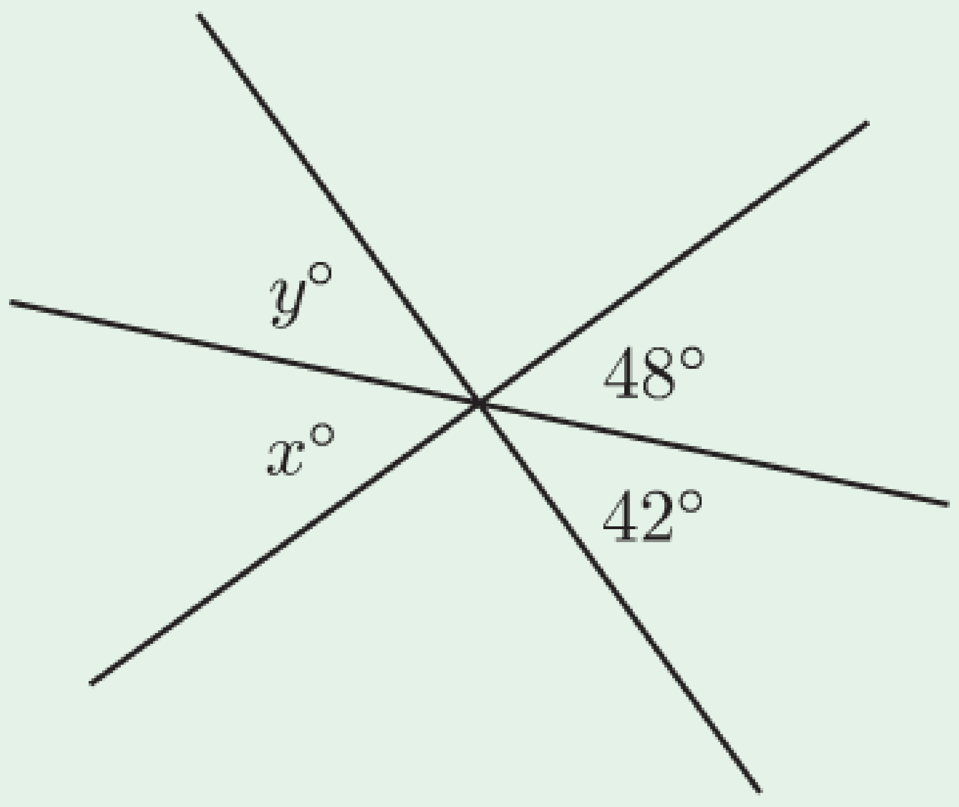


6 Use a protractor to measure each of the angles in this triangle.

PRINTABLE DIAGRAMS



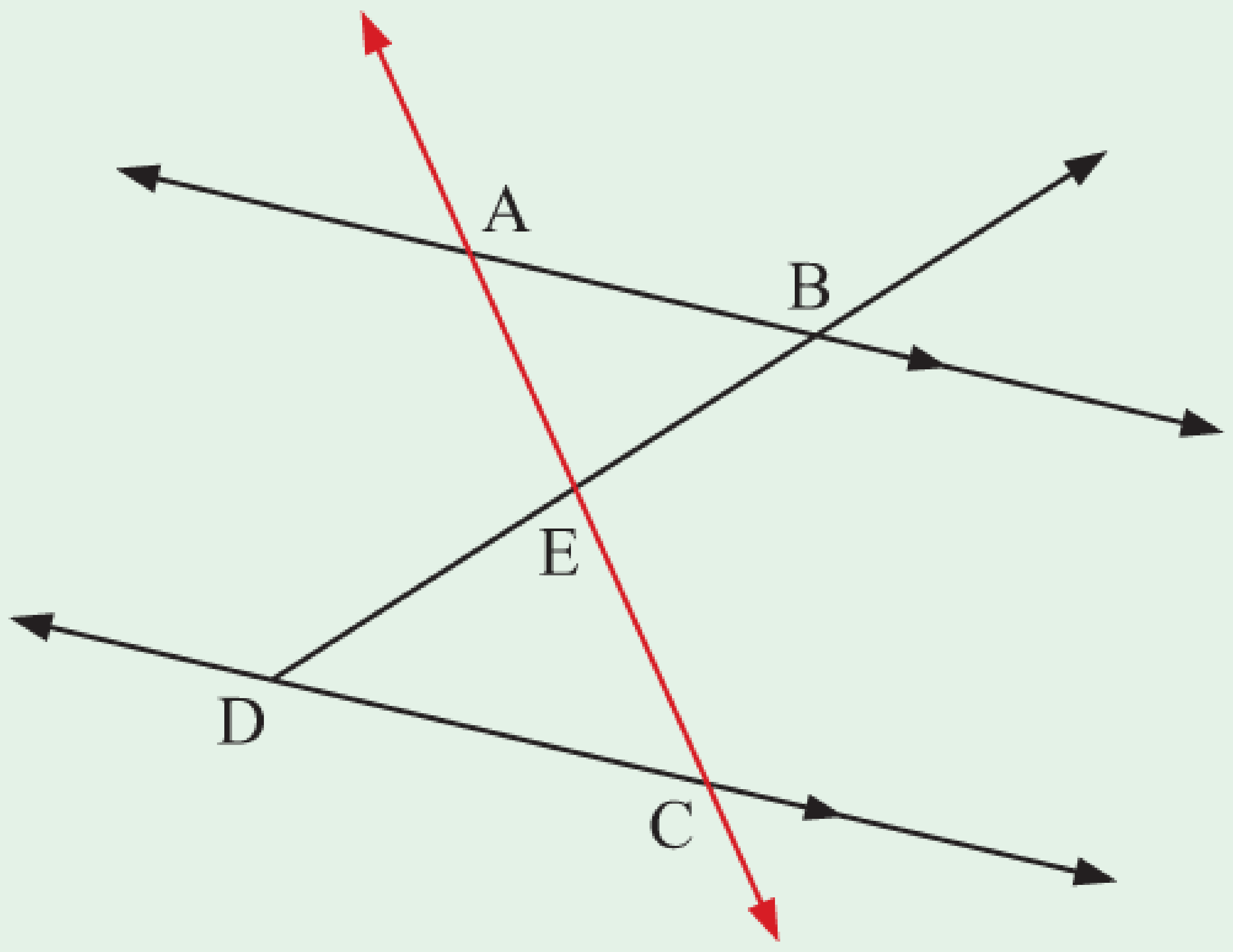
7 Find the values of the unknowns:



8 **a** Name the red line.

b Which two lines are parallel?

c Which points do not lie on the ray [DB)?

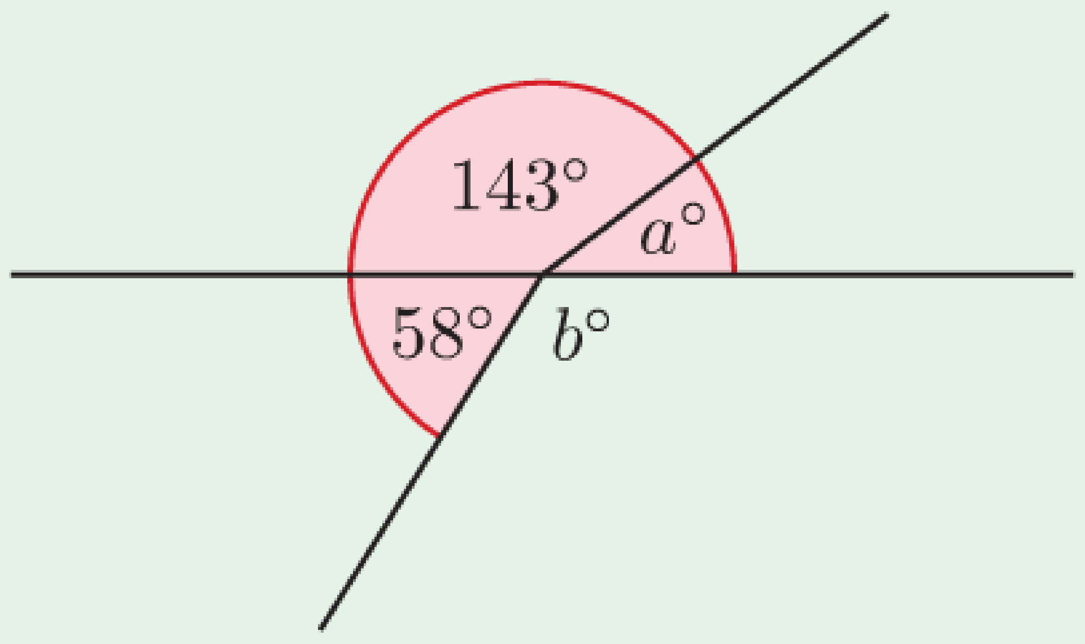


9 **a** Find the value of:

i a **ii** b

b Find the size of the shaded angle.

c State the type of the shaded angle.



10 **a** Use your protractor to draw an angle ABC of size 100°.

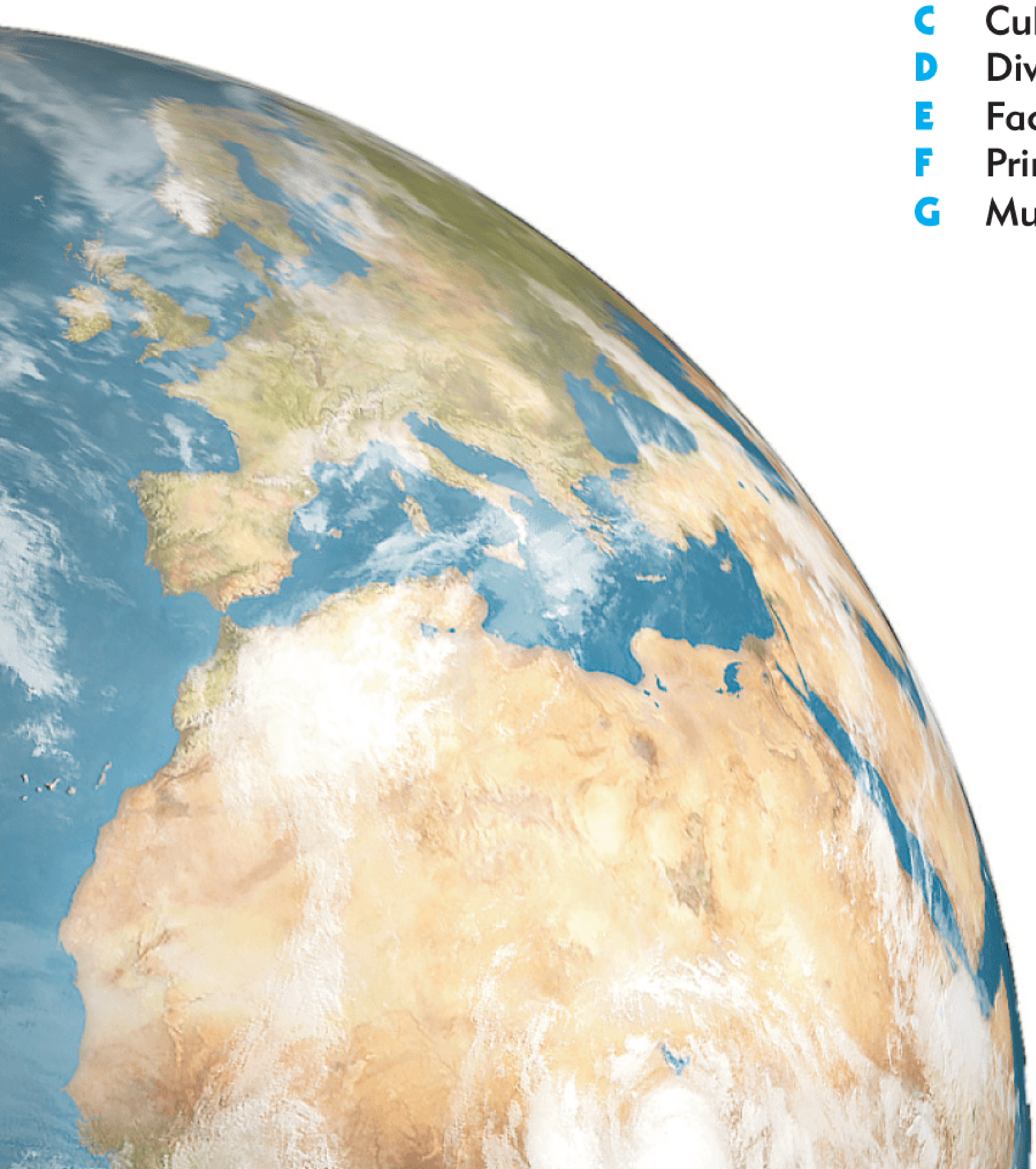
b Use your compass and ruler to draw a line [BM] which bisects $A\hat{B}C$.

c Use your compass and ruler to bisect $A\hat{B}M$.

Chapter 4

Number properties

Contents:



OPENING PROBLEM

There are 24 students in a class. The teacher wants to divide the students into smaller groups of equal size to work on a project.

Things to think about:

a Can the students split into 6 equal groups? How many students would be in each group?

b Can the students split into 5 equal groups?

c What other size groups can the students be split into equally?

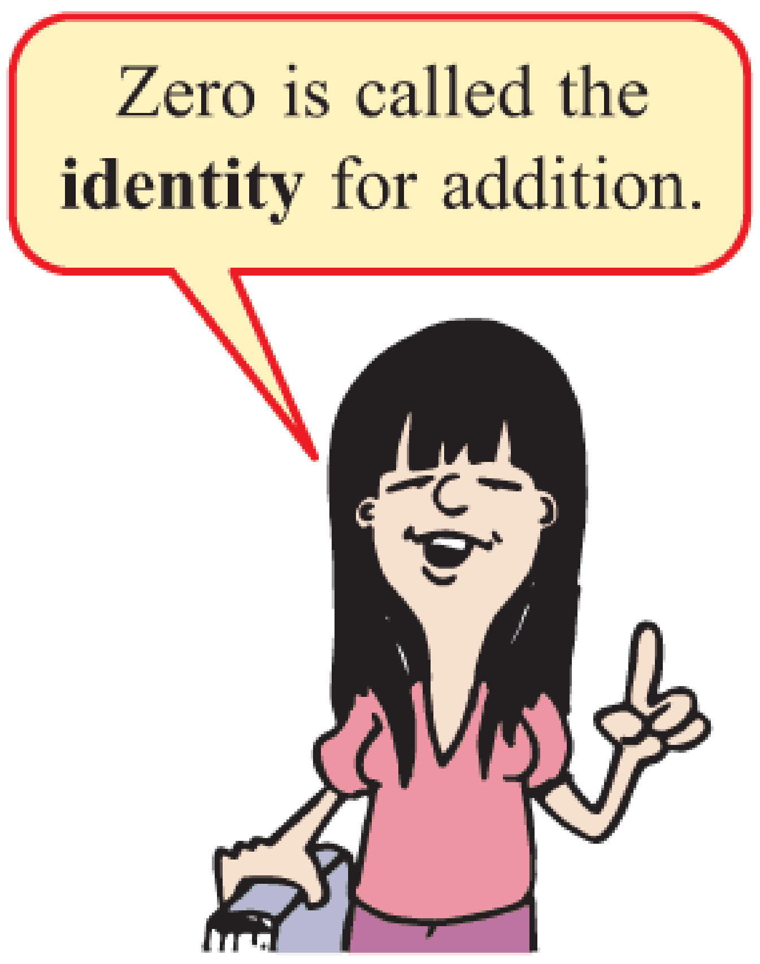
d Suppose one student has gone home sick, so there are only 23 students left in the class. Is it now possible to split the class into smaller groups of equal size?

In this chapter, we will explore some properties of numbers.

A ZERO AND ONE

Zero (0) and one (1) are very special numbers which have important properties.

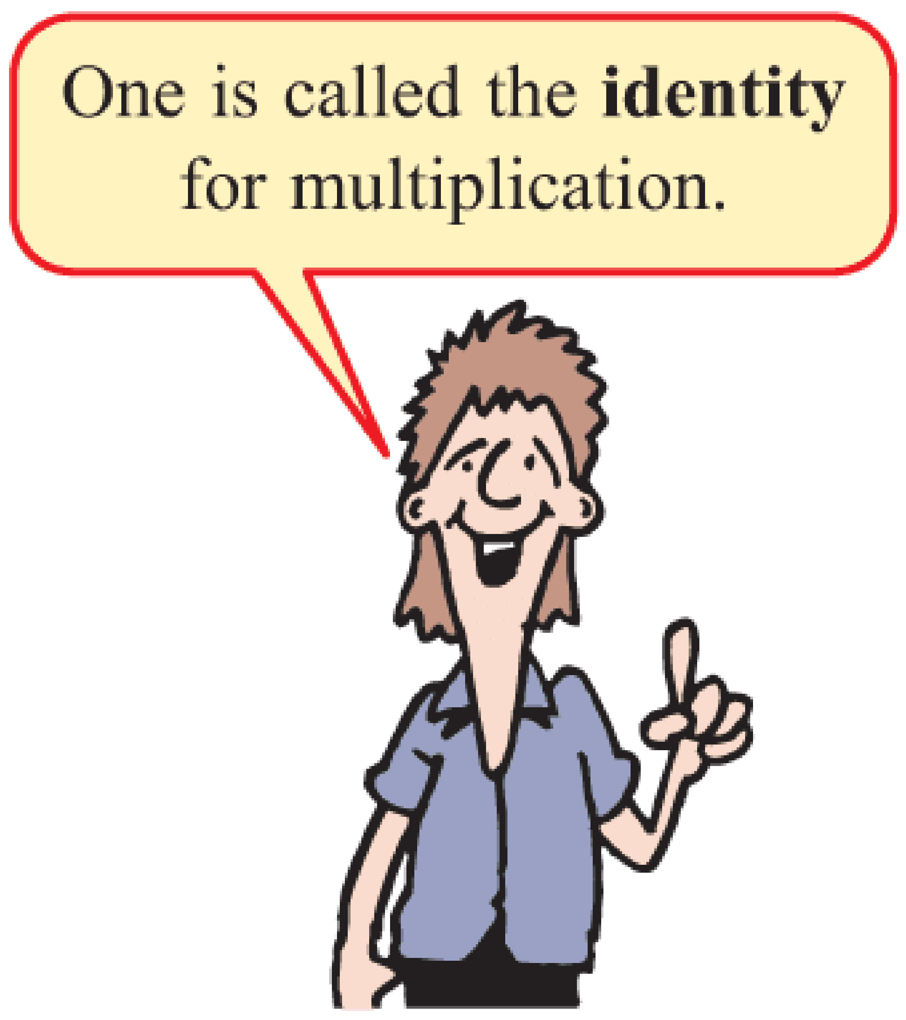
ZERO



- When 0 is added to a number, the number remains the same.
- When 0 is subtracted from a number, the number remains the same.
- When a number is multiplied by 0, the result is 0.
- It is meaningless to divide by 0, so the result is **undefined**.
- When 0 is divided by a non-zero number, the result is 0.

For example: $12 + 0 = 12$, $12 - 0 = 12$, $12 \times 0 = 0$, $12 \div 0$ is undefined, $0 \div 12 = 0$.

ONE



If we multiply or divide a number by 1, it remains the same.

For example: $5 \times 1 = 1 \times 5 = 5$, $5 \div 1 = 5$.

EXERCISE 4A

1 Find, if possible:

a $7+0$ **b** $7-0$ **c** 7×0 **d** $7\div 0$

e $18-0$ **f** $15+0-8$ **g** $18\div 0$ **h** $0\div 7$

i $8+7-0$ **j** $23-0-0$ **k** $6+0\times 5$ **l** $(6+0)\times 5$

2 Simplify, if possible:

a $0+73$ **b** $0\div 12$ **c** $12\div 0$ **d** $0\div 30$

e $30\div 0$ **f** 11×0 **g** 3×1 **h** 1×125

i $0\div 8$ **j** $45\div 1$ **k** 0×4 **l** 1×0

m 0×0 **n** $0\div 1$ **o** $235\div 1$ **p** $0\div 0$

B SQUARE NUMBERS

The product of two identical whole numbers is a **square number**.

We call it a square number because it can be represented by a square array of dots.

For example:

$1\times 1=1$ $\therefore\ 1^2=1$

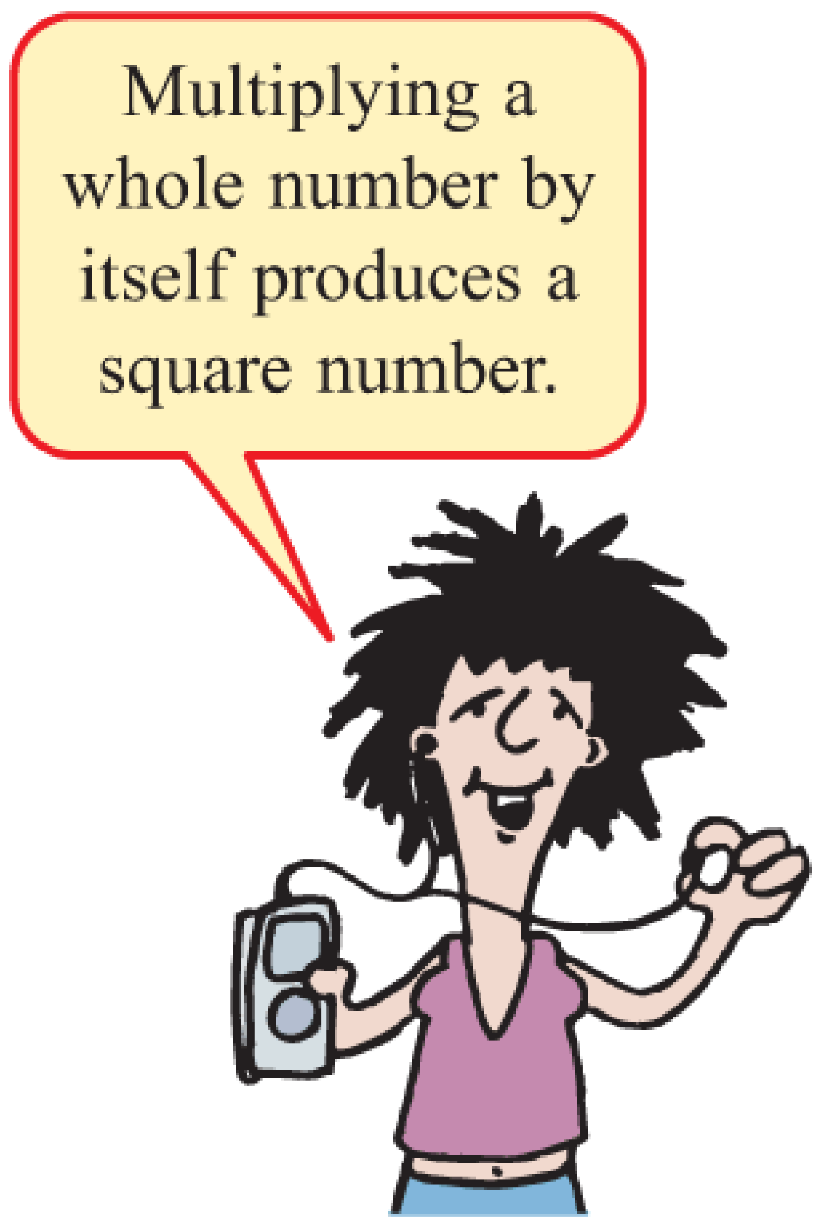
$2\times 2=4$ $\therefore\ 2^2=4$

$3\times 3=9$ $\therefore\ 3^2=9$

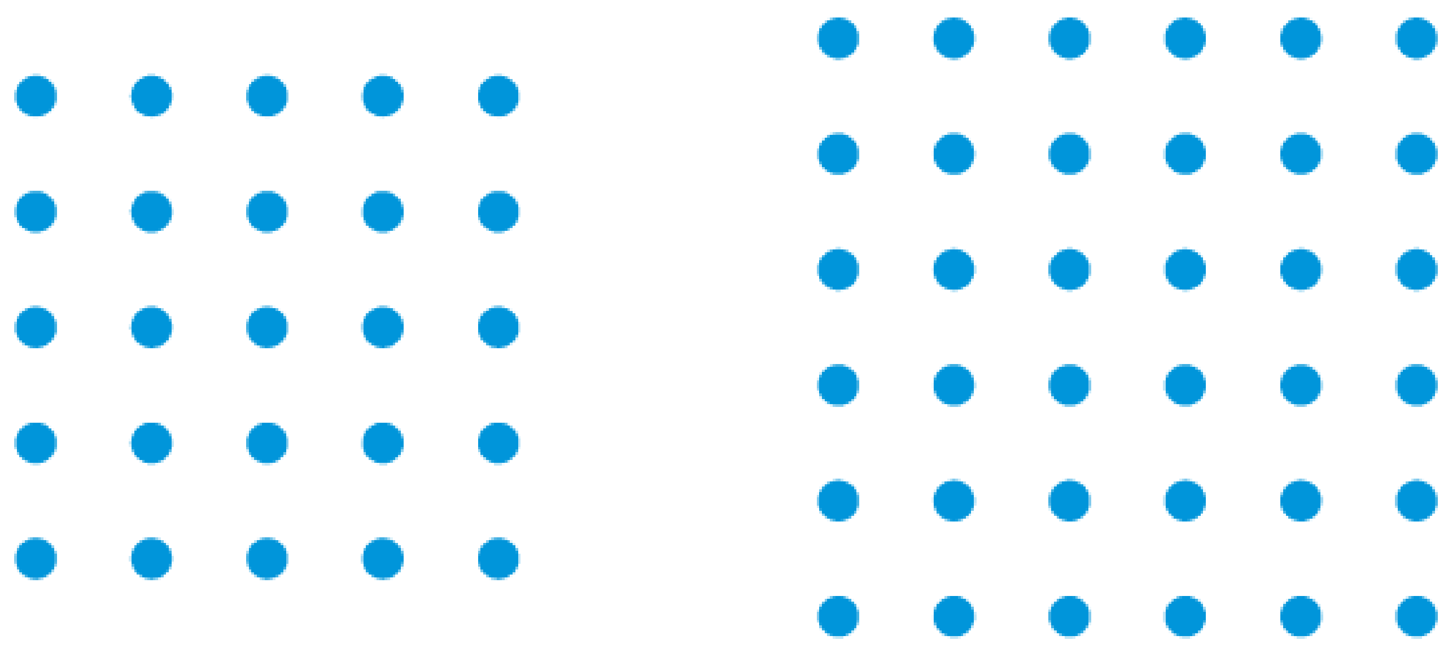
$4\times 4=16$ $\therefore\ 4^2=16$

So, 1, 4, 9, and 16 are all square numbers.

EXERCISE 4B



1 By counting the number of dots in these arrangements, find the fifth and sixth square numbers.



2 List the first ten square numbers.

3 Find the: **a** 12th **b** 15th **c** 22nd square number.

4 Find two square numbers whose sum is another square number.

5 **a** Find 0^2.

b Which numbers are equal to their own squares?

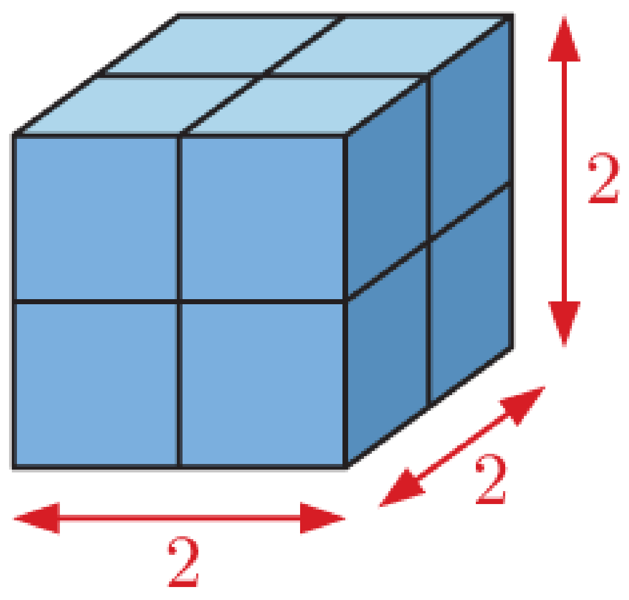
C CUBIC NUMBERS

The product of three identical numbers is a **cubic number**.

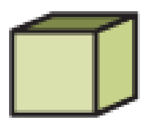
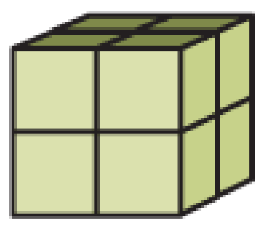
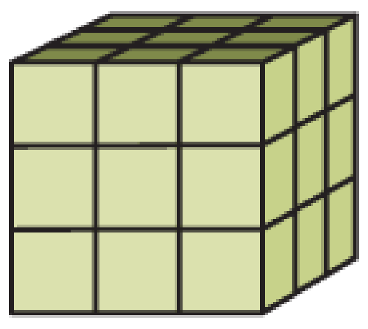
We call it a cubic number because it can be represented by a cubic array of blocks.

For example, the cube alongside contains
$2 \times 2 \times 2 = 2^3 = 8$ blocks.

So, 8 is a cubic number.



EXERCISE 4C

1  ,  ,  represent the first three cubic numbers.

- **a** Draw a sketch which represents the fourth cubic number.
- **b** Find the fourth cubic number.

2 List the first ten cubic numbers.

3 Find the difference between the sixth cubic number and the sixth square number.

4 Find two cubic numbers whose sum is a square number.

5 Find two square numbers whose sum is a cubic number.

6
- **a** Find 0^3.
- **b** Which natural numbers are equal to their own cubes?

D DIVISIBILITY

One number is **divisible** by another if, when we divide, the quotient is a whole number.

For example: 12 is divisible by 4 because $12 \div 4 = 3$.
12 is not divisible by 5 because $12 \div 5 = 2$ with remainder 2.

EVEN AND ODD NUMBERS

A natural number is **even** if it is divisible by 2.
A natural number is **odd** if it is not divisible by 2.

For example: 14 is even because $14 \div 2 = 7$.
19 is odd because $19 \div 2 = 9$ with remainder 1.

EXERCISE 4D.1

1 Determine whether:

a 18 is divisible by 3 **b** 20 is divisible by 7 **c** 40 is divisible by 10
d 67 is divisible by 10 **e** 27 is divisible by 4 **f** 35 is divisible by 5
g 31 is divisible by 9 **h** 88 is divisible by 11.

2 Are the following numbers even or odd?

a 12 **b** 27 **c** 39 **d** 34 **e** 60 **f** 53 **g** 79 **h** 104

3 Find the number between 50 and 60 which is divisible by 7.

4 Is zero odd or even? Explain your answer.

5 What is the smallest non-zero number which is divisible by 6 *and* by 8?

6 **a** Beginning with 8, write three consecutive even numbers.
b Beginning with 17, write five consecutive odd numbers.

7 **a** Write two even numbers which are *not* consecutive, and which add to 10.
b Write all the sets of two non-consecutive odd numbers which add to 20.
c Write all the sets of three different even numbers which add to 20.

8 Use the words "even" and "odd" to complete these sentences:

a The sum of two even numbers is always ......
b The sum of two odd numbers is always ......
c The sum of an odd number and an even number is always ......
d When an even number is subtracted from an odd number, the result is ......
e When an odd number is subtracted from an odd number, the result is ......
f The product of two odd numbers is always ......
g The product of an even number and an odd number is always ......

DISCUSSION

Are square and cubic numbers always even? Are they always odd?

By studying lists of square and cubic numbers, identify a pattern for which are odd and which are even.

DIVISIBILITY TESTS

There are some simple tests we can follow to determine whether one number is divisible by another, without actually doing the division!

Number	Divisibility Test
2	If the last digit is even, then the number is divisible by 2.
3	If the sum of the digits is divisible by 3, then the number is divisible by 3.
4	If the number formed by the last *two* digits is divisible by 4, then the original number is divisible by 4.
5	If the last digit is 0 or 5, then the number is divisible by 5.
6	If the number is divisible by both 2 and 3, then it is divisible by 6.
10	If the last digit is 0, then the number is divisible by 10.

Example 1 **Self Tutor**

Determine whether 768 is divisible by:

a 2 **b** 3 **c** 6.

a 768 ends in 8, which is even
$\therefore$ 768 is divisible by 2.

b The sum of the digits $= 7 + 6 + 8 = 21$.
Now 21 is divisible by 3 $\{$as $21 \div 3 = 7\}$
$\therefore$ 768 is divisible by 3.

c Since 768 is divisible by both 2 and 3, 768 is divisible by 6.

EXERCISE 4D.2

1 Determine whether the following numbers are divisible by 2:
a 216 **b** 3184 **c** 827 **d** 4770 **e** 123 456

2 Determine whether the following numbers are divisible by 10:
a 341 **b** 520 **c** 4313 **d** 87 600 **e** 211 003

3 Determine whether the following numbers are divisible by 3:
a 84 **b** 123 **c** 437 **d** 111 114 **e** 707 052

4 Determine whether the following numbers are divisible by 5:
a 400 **b** 628 **c** 735 **d** 21 063 **e** 384 005

5 Determine whether the following numbers are divisible by 4:
a 482 **b** 2556 **c** 8762 **d** 12 368 **e** 213 186

6 Determine whether the following numbers are divisible by 6:
a 162 **b** 381 **c** 1602 **d** 2156 **e** 5364

7 Consider the numbers of the form 37☐. Which digits could be put in place of ☐ so that the number is:
a even **b** divisible by 3 **c** divisible by 4 **d** divisible by 5?

8 What digits could replace ☐ so that these numbers are divisible by 3?
a 3☐2 **b** 8☐5 **c** 3☐14 **d** ☐229

E FACTORS

The **factors** of a natural number are the natural numbers which divide exactly into it.

For example: $4 \div 1 = 4$
$4 \div 2 = 2$
$4 \div 3 = 1$ with remainder 1
$4 \div 4 = 1$
So, the factors of 4 are 1, 2, and 4.

All natural numbers can be split into **factor pairs**.

For example: $4 = 1 \times 4$ or 2×2
$10 = 1 \times 10$ or 2×5.

EXERCISE 4E

1 **a** Is 5 a factor of 30? **b** Is 4 a factor of 18?
c Is 8 a factor of 26? **d** Is 7 a factor of 35?

2 List the factors of:
a 5 **b** 6 **c** 7 **d** 8
e 9 **f** 10 **g** 11 **h** 12

3 Which of these numbers are factors of 40?
a 1 **b** 3 **c** 4 **d** 6
e 8 **f** 12 **g** 15 **h** 40

4 Copy and complete these factor pairs:
a $22 = 2 \times$ ...... **b** $45 = 9 \times$ ...... **c** $30 = 3 \times$ ......
d $49 = 7 \times$ ...... **e** $72 = 12 \times$ ...... **f** $85 = 5 \times$ ......

5 List the factors of:
a 15 **b** 18 **c** 23 **d** 24
e 45 **f** 64 **g** 72 **h** 100

6 How many factors do the following numbers have?
a 14 **b** 19 **c** 28 **d** 36
e 43 **f** 54 **g** 60 **h** 66

7 Explain why every natural number greater than 1 has at least two factors.

8 **a** List the factors of 32 and 48.
b What factors do 32 and 48 have in common?
c What is the *highest* common factor of 32 and 48?

F PRIME AND COMPOSITE NUMBERS

When we split a number into **factor pairs**, we usually write the number as the product of two smaller numbers.

For example, 15 can be split into 5×3.

However, there are some numbers which cannot be split into smaller numbers. We call these **prime numbers**.

For example, we can only write 7 as 1×7. We cannot write 7 as the product of smaller numbers.

> A **prime** number is a natural number which has exactly two different factors, 1 and itself.
>
> A **composite** number is a natural number which has more than two factors.

For example, 5 is a prime number, because its only factors are 1 and 5.

6 is a composite number, because it has four factors: 1, 2, 3, and 6.

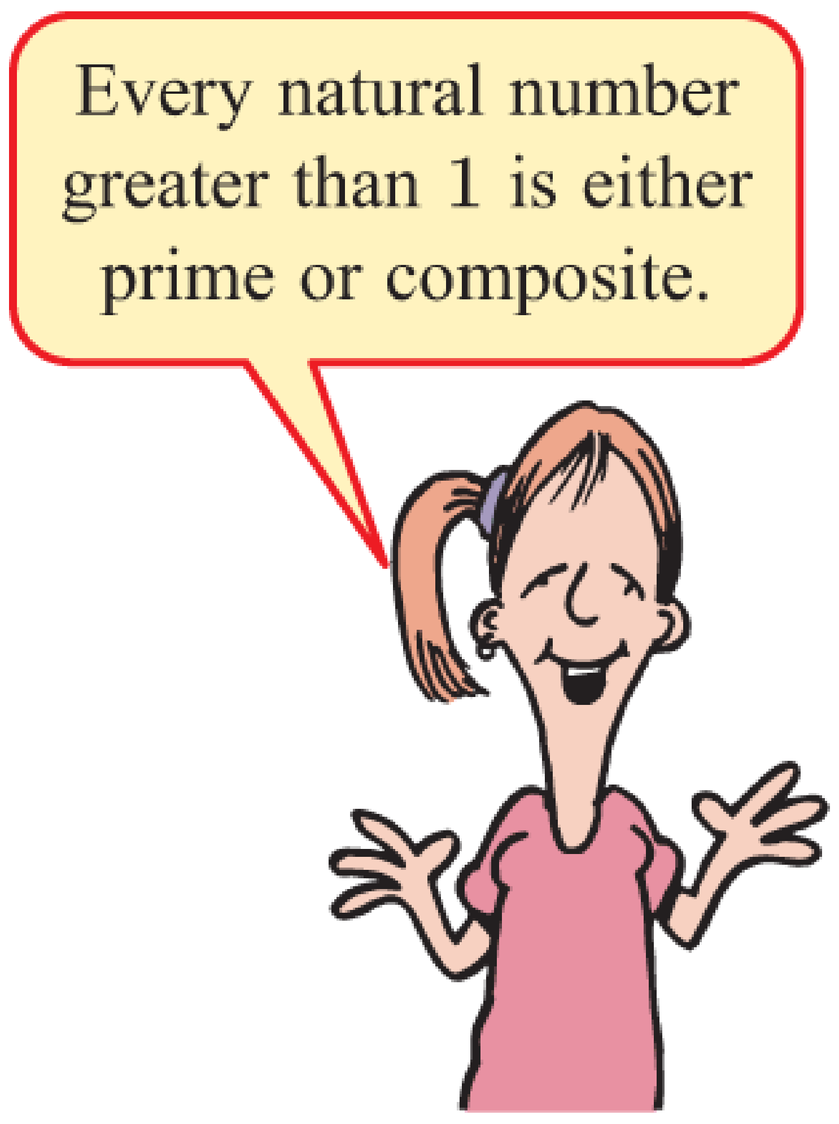
The first few prime numbers are:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ \ldots.$$

There are infinitely many prime numbers, so this list extends forever.

The number 1 is very special, since it has only one factor, itself!

The number **1** is neither prime nor composite.



EXERCISE 4F

1 Copy and complete the table below, describing each number as ‘prime’ or ‘composite’:

1	neither	11		21		31	
2	prime	12		22		32	
3	prime	13		23		33	
4	composite	14		24		34	
5		15		25		35	
6		16		26		36	
7		17		27		37	
8		18		28		38	
9		19		29		39	
10		20		30		40	

2 How many even prime numbers are there?

3 List the prime numbers between:

a 50 and 60 **b** 60 and 70 **c** 90 and 110.

4 Explain why the number 57 932 560 195 is composite.

5 Find the smallest two consecutive odd numbers which are both composite.

6 **a** Find two composite numbers whose *sum* is a prime number.

b Is it possible to find two composite numbers whose *product* is a prime number? Explain your answer.

Example 2 **Self Tutor**

a List the factors of 18. **b** List the *prime* factors of 18.

c Write 18 as the product of prime factors.

a The factors of 18 are 1, 2, 3, 6, 9, and 18. **b** The prime factors of 18 are 2 and 3.

c $18 = 2 \times 9$

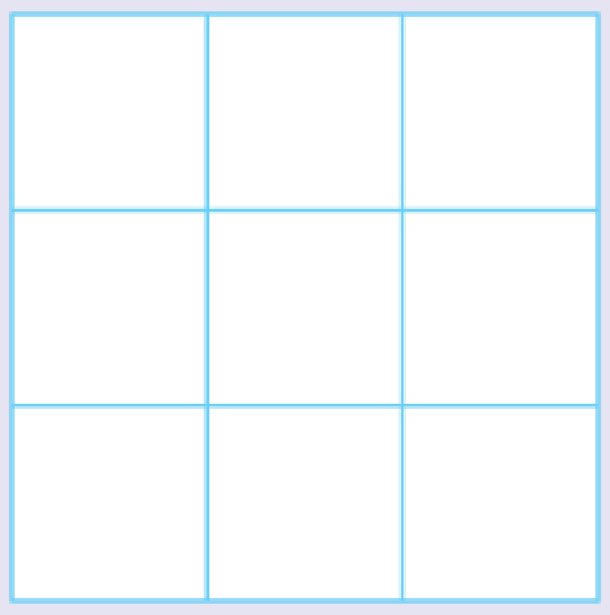
$\therefore\ 18 = 2 \times 3 \times 3$

7 **a** List the factors of 20. **b** List the *prime* factors of 20.
c Write 20 as the product of prime factors.

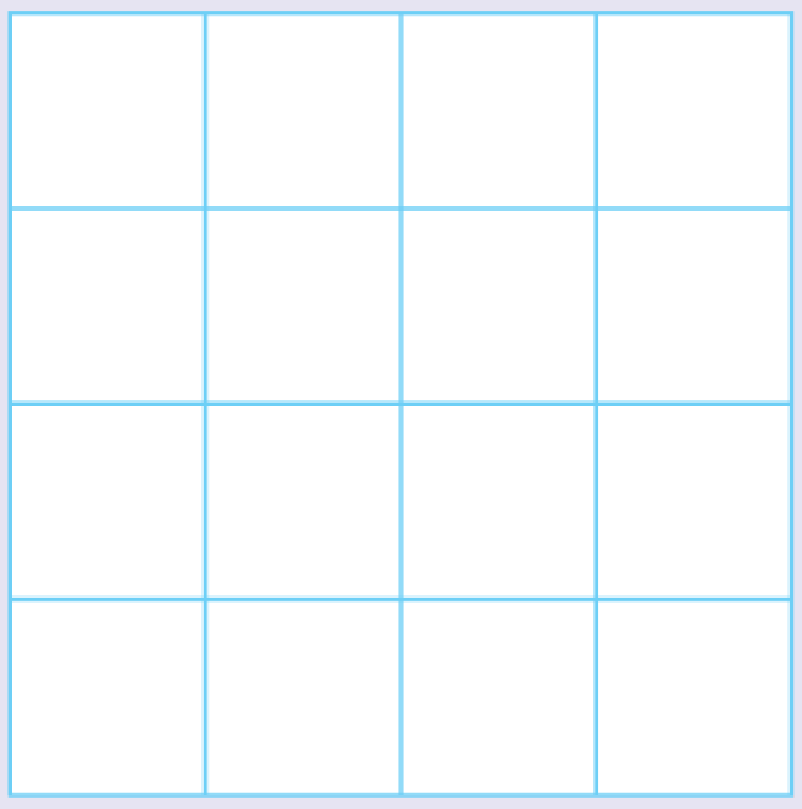
8 **a** List the factors of 27. **b** List the *prime* factors of 27.
c Write 27 as the product of prime factors.

PUZZLE

1 Fill the 3×3 square with the numbers 1 to 9, so that the sum of each row and column is a prime number.



2 Fill the 4×4 square with the numbers 1 to 16, so that the sum of each row and column is a prime number.



G MULTIPLES

The **multiples** of any whole number have that number as a factor. They are obtained by multiplying it by 1, then 2, then 3, then 4, and so on.

For example, the multiples of 7 are:

7,	14,	21,	28,	35,	....
↑	↑	↑	↑	↑	
7×1	7×2	7×3	7×4	7×5	

Example 3

Self Tutor

List the multiples of 6 which are between 40 and 50.

The multiples of 6 are	6,	12,	18,	24,	30,	36,	42,	48,	56,	....
	↑	↑	↑	↑	↑	↑	↑	↑	↑	
	6×1	6×2	6×3	6×4	6×5	6×6	6×7	6×8	6×9	

$\therefore$ the required multiples of 6 are 42 and 48.

EXERCISE 4G

1 List the first ten multiples of:
a 4 **b** 9 **c** 11

2 List the multiples of
a 3 which are between 10 and 20 **b** 12 which are between 40 and 70.

3 **a** List the first twelve multiples of 7.

b Use your list to determine which of these numbers are multiples of 7:

i 12 **ii** 21 **iii** 30 **iv** 47 **v** 63

4 **a** List the first twelve multiples of 8. **b** List the first ten multiples of 10.

c Write down the numbers less than 100 which are multiples of both 8 and 10.

d What is the *lowest* common multiple of 8 and 10?

5 Find the largest multiple of 11 which is less than 200.

PUZZLE

WHICH NUMBER AM I?

I am one of the numbers shown alongside.

The number to the left of me is a multiple of 6.

The number to the right of me is a multiple of 4.

The number above me is a multiple of 3.

The number below me is a multiple of 7.

Which number am I?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

INVESTIGATION

SQUARES

The perfect squares have some interesting properties. We will discover some patterns they form in this Investigation.

What to do:

1 List the perfect squares from 0^2 to 12^2.

2 Copy and complete:

a

$1^2 - 0^2 = ...$

$2^2 - 1^2 = ...$

$3^2 - 2^2 = ...$

$4^2 - 3^2 = ...$

$\vdots$

$12^2 - 11^2 = ...$

b The differences between consecutive squares are the ...... numbers.

3 Copy and complete:

a

$2^2 - 0^2 = ...$

$3^2 - 1^2 = ...$

$4^2 - 2^2 = ...$

$\vdots$

$12^2 - 10^2 = ...$

b The differences between every second square form the ......

4 **a** Copy and complete:

$$3^2 - 0^2 = \text{......}$$
$$4^2 - 1^2 = \text{......}$$
$$5^2 - 2^2 = \text{......}$$
$$\vdots$$
$$12^2 - 9^2 = \text{......}$$

b What do you notice about these differences?

Global context

click here

Cicadas

Statement of inquiry:	Mathematics can be used to explain occurrences in nature.
Global context:	Orientation in space and time
Key concept:	Form
Related concepts:	Generalisation, Pattern
Objectives:	Investigating patterns, Applying mathematics in real-life contexts
Approaches to learning:	Communication, Research

KEY WORDS USED IN THIS CHAPTER

- composite
- even
- odd
- undefined
- cubic number
- factor
- prime
- divisible
- multiple
- square number

REVIEW SET 4A

1 Simplify:

a $13 - 0 + 19$ **b** 23×0 **c** 14×1 **d** $0 \div 18$

2 Determine whether:

a 43 is divisible by 9 **b** 132 is divisible by 12

3 What digits could □ be if 2□8 is divisible by:

a 3 **b** 4?

4 **a** Draw a diagram to represent the 12th square number.

b Find the 12th square number.

5 Determine whether the following numbers are prime or composite:

a 44 **b** 51 **c** 83 **d** 87

6 **a** Find the sum of the 1st and the 8th square numbers.

b Find two other square numbers which have this sum.

7 **a** Find the sum of the first five prime numbers.

b Is this number prime or composite?

c List the factors of the number.

8 **a** List the factors of 16 and 40.
b What factors do 16 and 40 have in common?
c What is the *highest* common factor of 16 and 40?

9 How many cubic numbers are less than 500?

10 List the multiples of 6 which are between 50 and 70.

REVIEW SET 4B

1 If possible, find:
a 243×1 **b** 243×0 **c** $243 \div 1$ **d** $243 \div 0$

2 Find the sum of the first three cubic numbers.

3 Write down three different even numbers which sum to 30.

4 List the factors of 54.

5 How many natural numbers smaller than 30 are composite?

6 Decide whether the following numbers are divisible by 3:
a 97 **b** 252 **c** 17 206 **d** 519 462

7 Find the largest multiple of 9 which is less than 150.

8 **a** List the factors of 30.
b List the *prime* factors of 30.
c Write 30 as the product of prime factors.

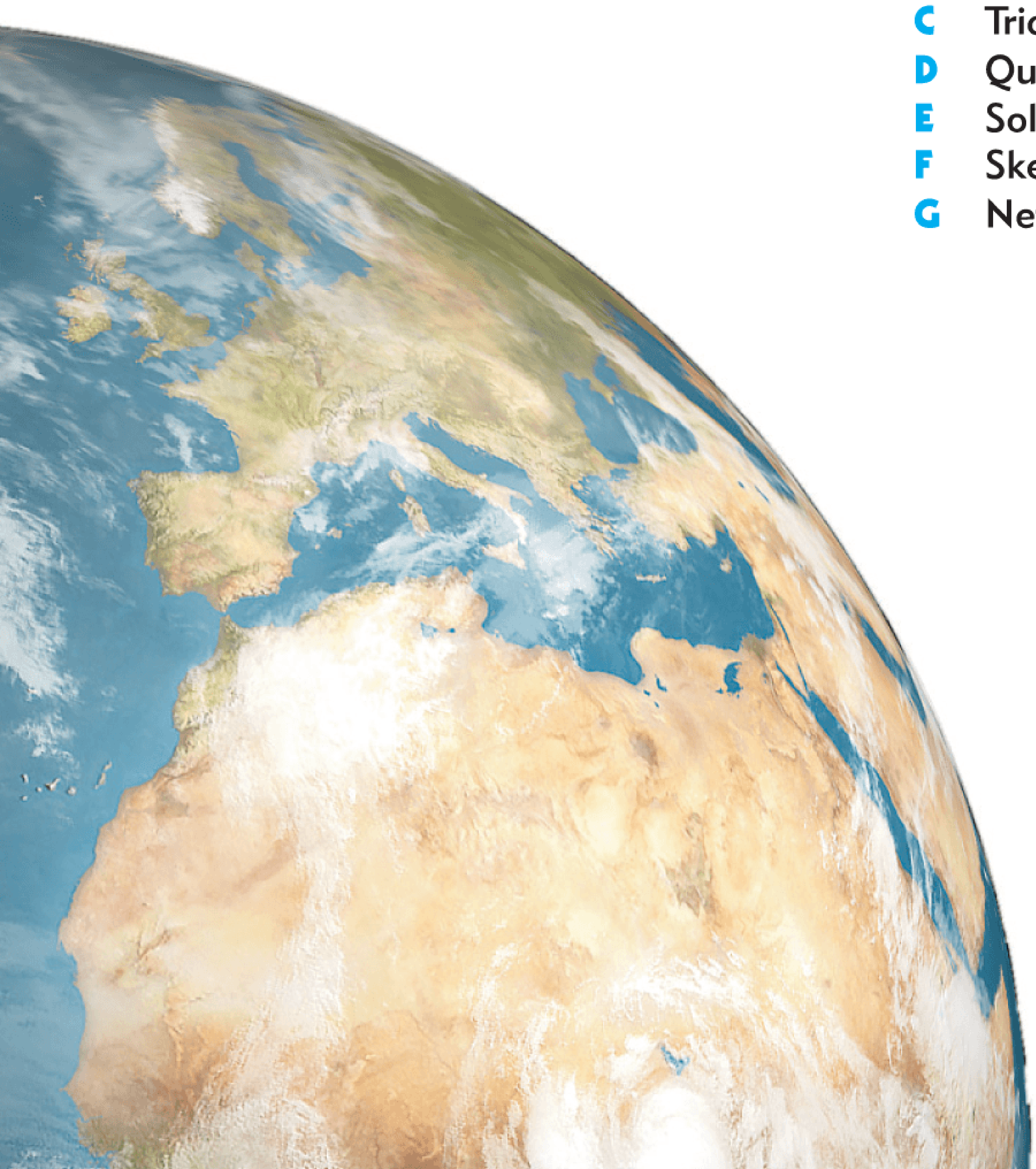
9 **a** Find:
i $7+9$ **ii** $21+23$ **iii** $29+31$ **iv** $37+39$
b Copy and complete: When two consecutive odd numbers are added together, the result is always divisible by ......

10 **a** List the first thirteen multiples of 6.
b List the first ten multiples of 8.
c Write down the numbers less than 80 which are multiples of both 6 and 8.
d What is the *lowest* common multiple of 6 and 8?

Chapter 5

Geometric shapes

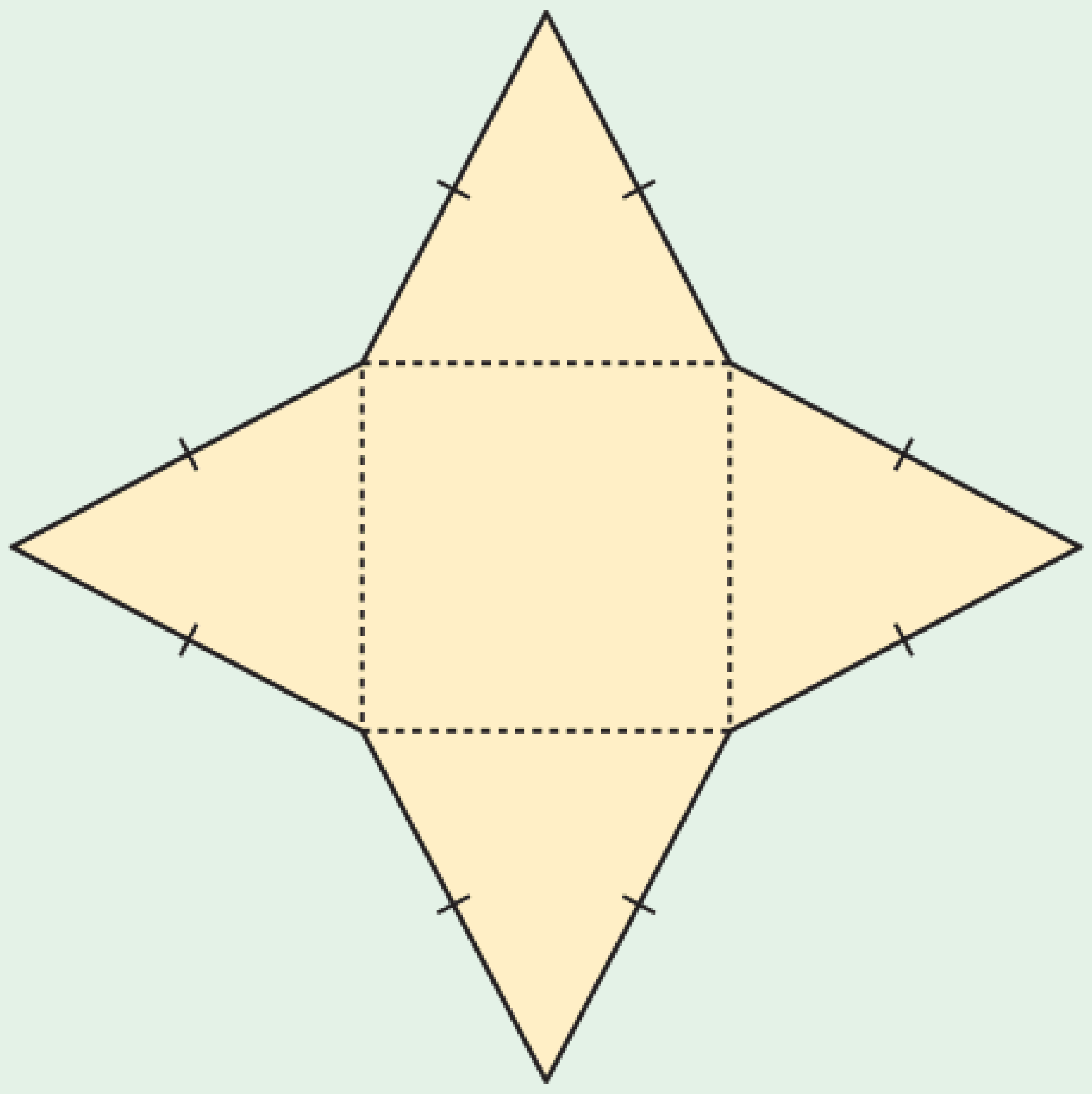
Contents:



OPENING PROBLEM

Things to think about:

- **a** What different shapes can you see in this diagram?
- **b** The triangles each have 2 sides of equal length. What name do we give to triangles like this?
- **c** If we fold the shape along the dotted lines, what type of solid can we construct?

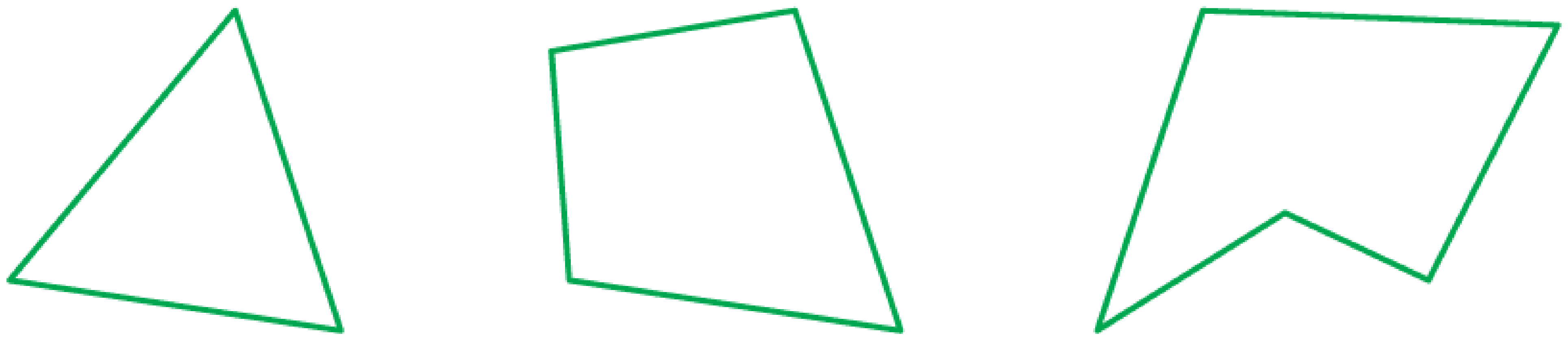


In this chapter we will explore two-dimensional **polygons** and **circles**, and three-dimensional **solids**.

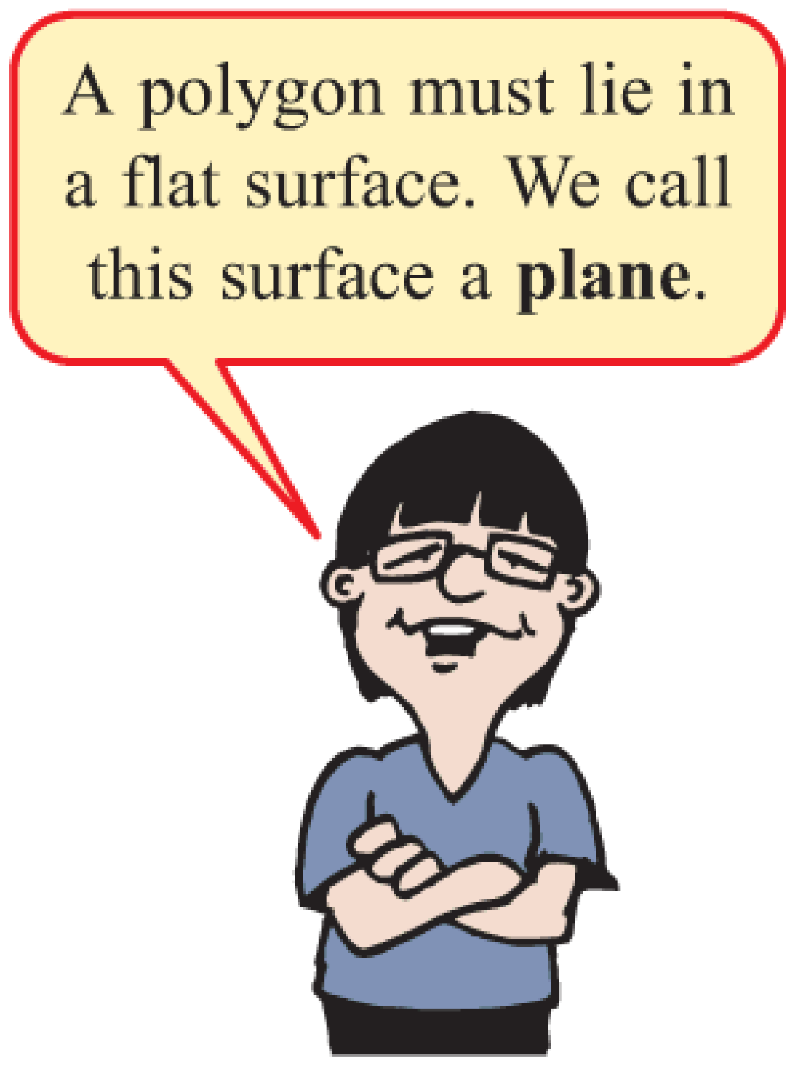
A POLYGONS

A **polygon** is a closed figure which has only straight line sides and which does not cross itself.

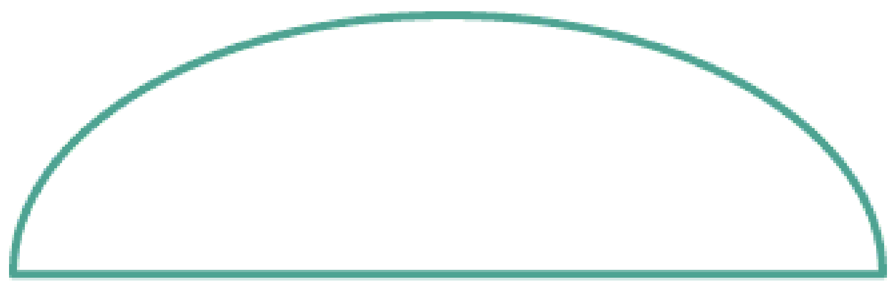
These figures are polygons:



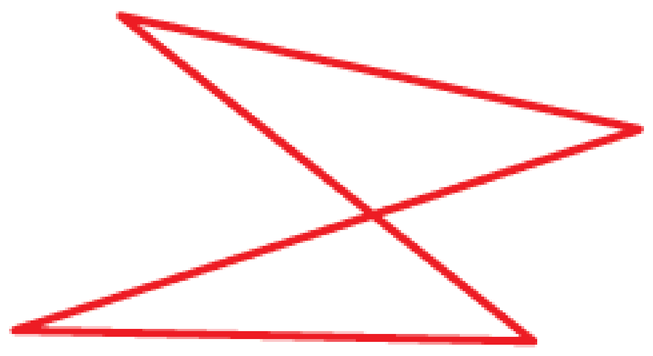
Here are some examples of polygons that we often see around us:



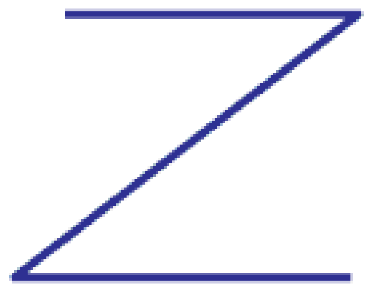
These figures are **not** polygons:



curved side



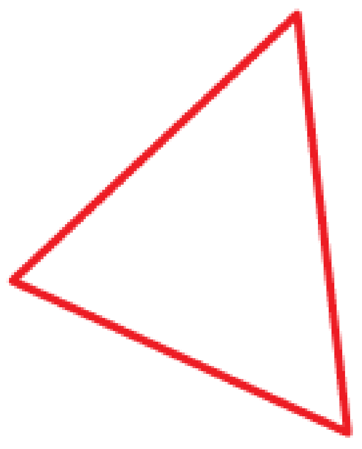
crosses itself



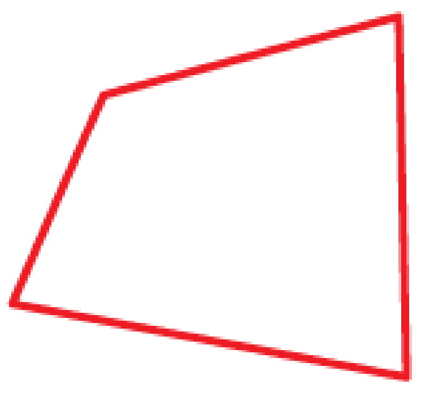
not closed

NAMING POLYGONS

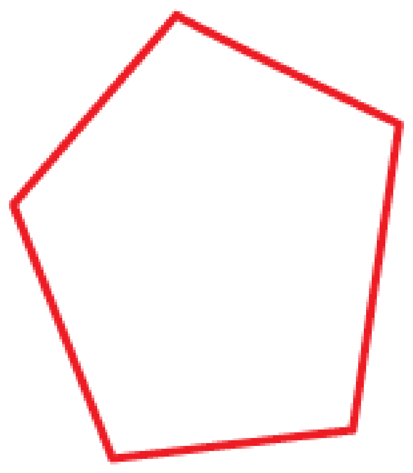
We name polygons according to how many sides they have:



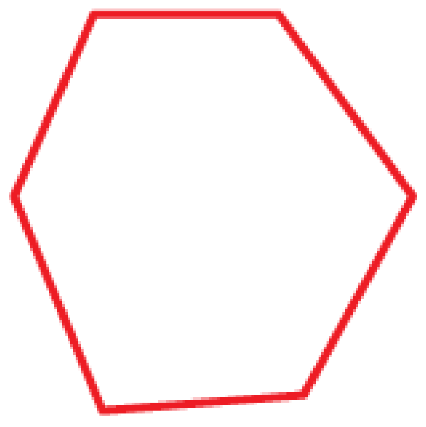
triangle
3 sides



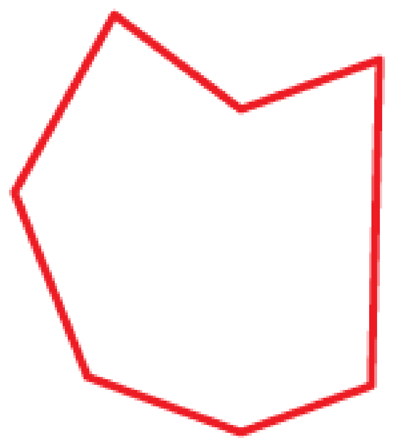
quadrilateral
4 sides



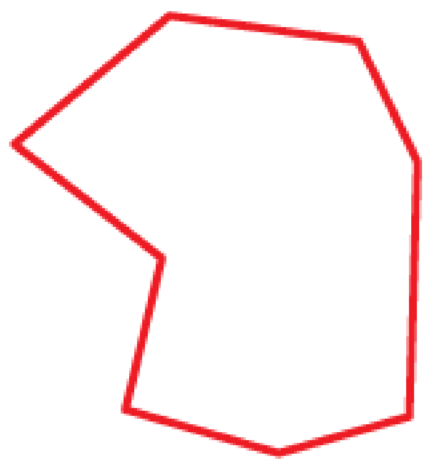
pentagon
5 sides



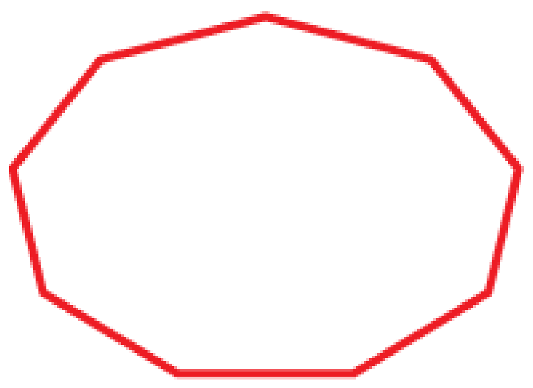
hexagon
6 sides



heptagon
7 sides

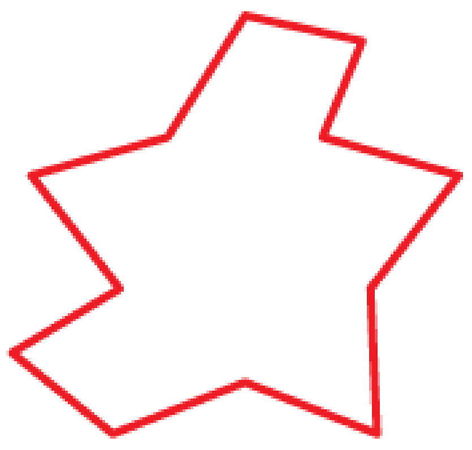


octagon
8 sides



nonagon
9 sides

decagon
10 sides

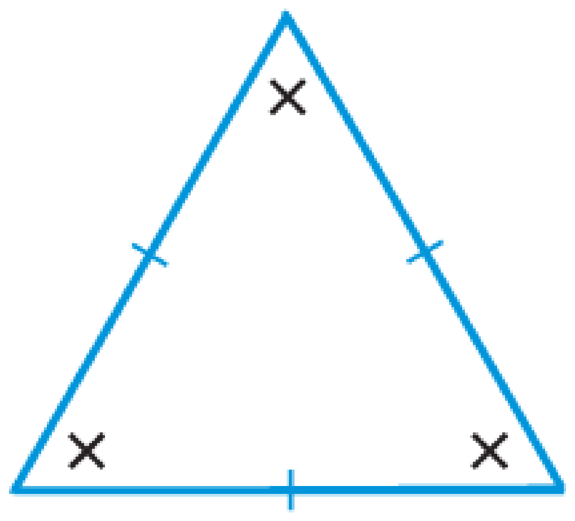


dodecagon
12 sides

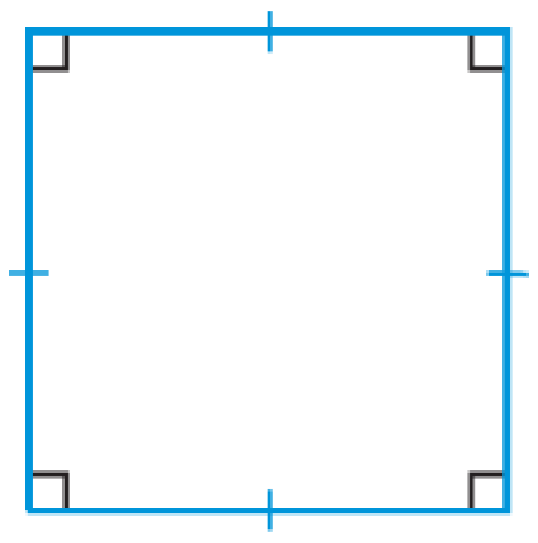
REGULAR POLYGONS

A **regular polygon** is a polygon with all sides the same length and all angles the same size.

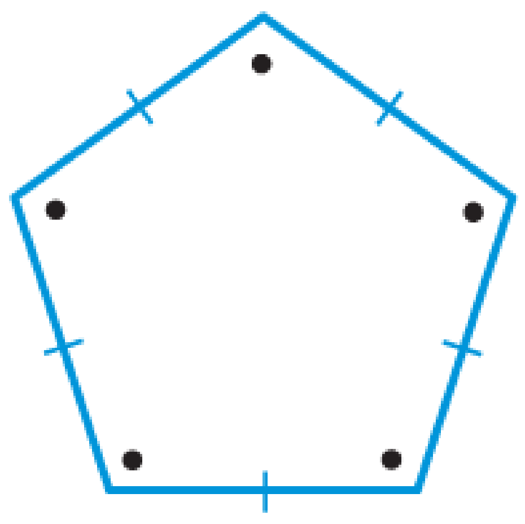
The polygons below are marked to show that they are regular:



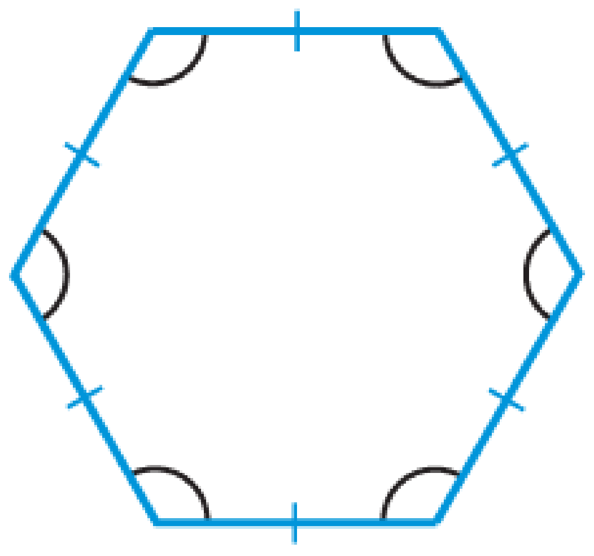
equilateral
triangle
3 equal sides
3 equal angles



square
4 equal sides
4 equal angles



regular
pentagon
5 equal sides
5 equal angles



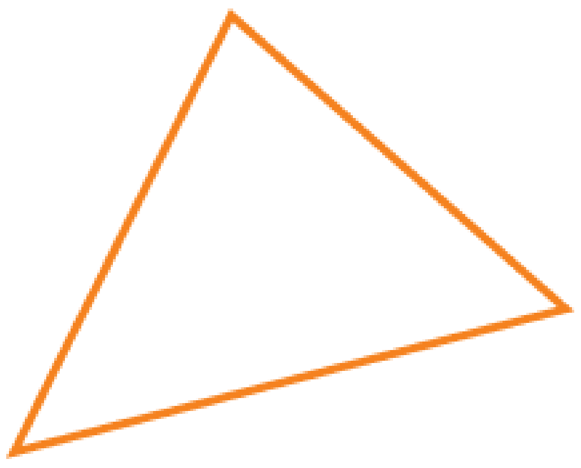
regular
hexagon
6 equal sides
6 equal angles

Equal sides are shown by small markings.
Equal angles are shown by using the same symbols.

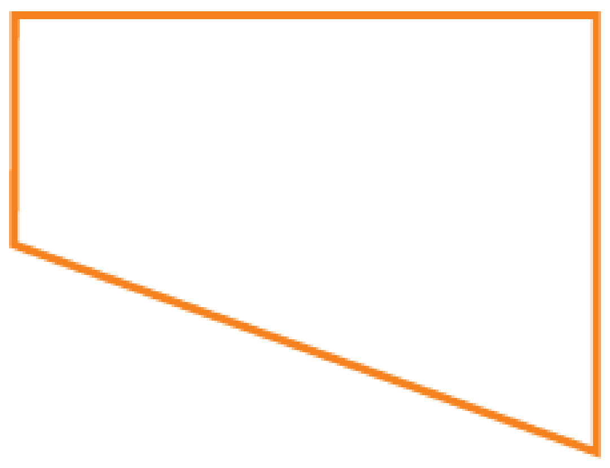
EXERCISE 5A

1 Name these polygons:

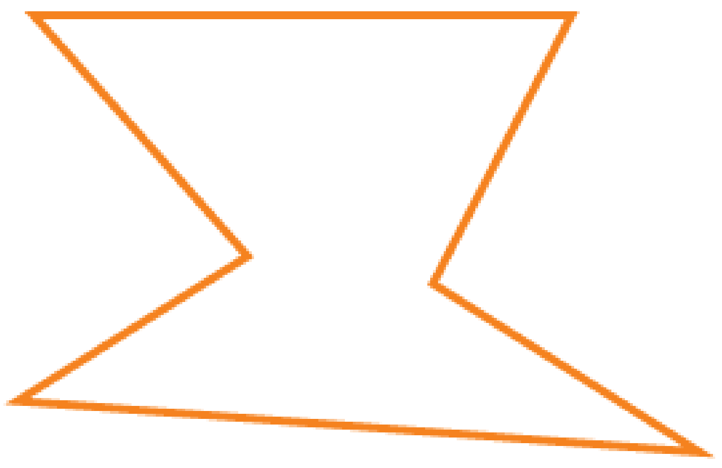
a



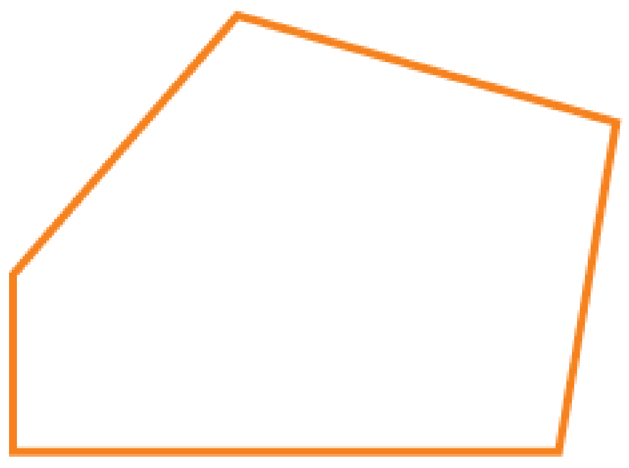
b



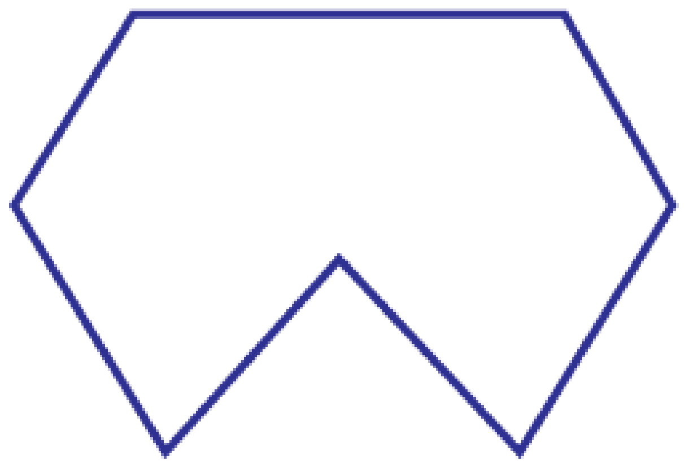
c



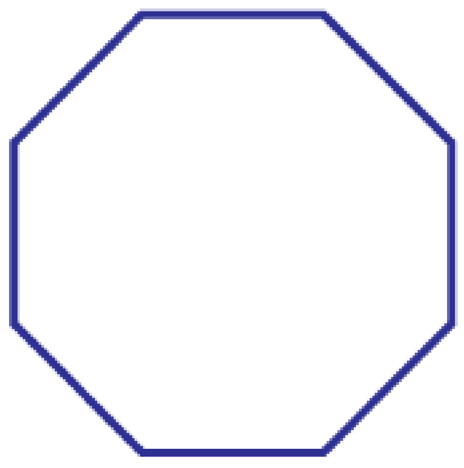
d



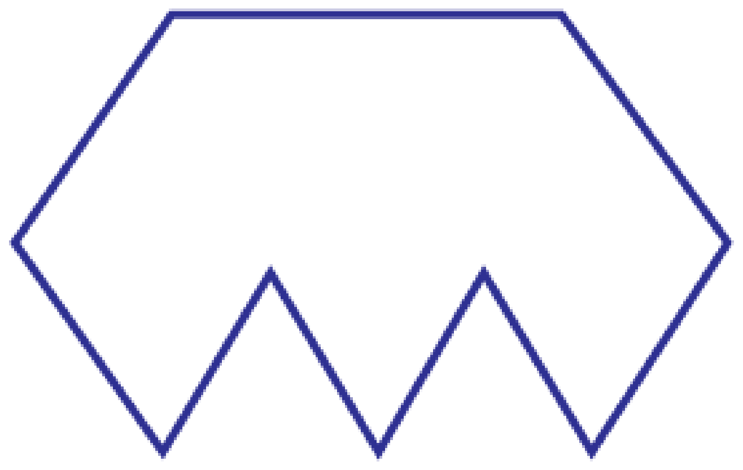
e



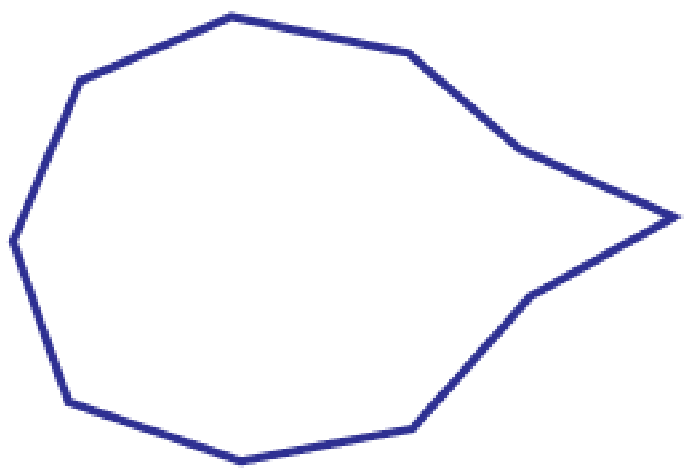
f



g

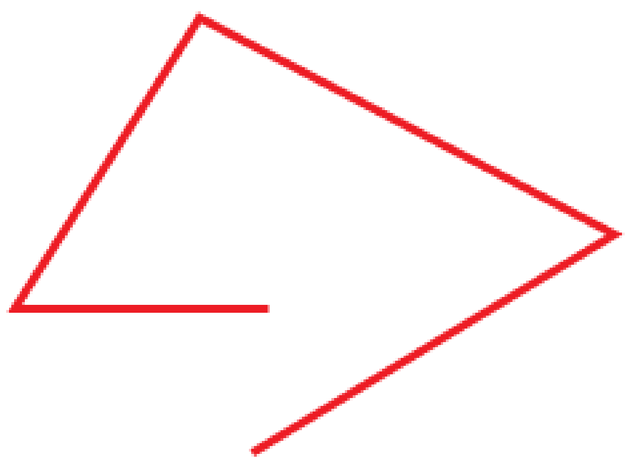


h

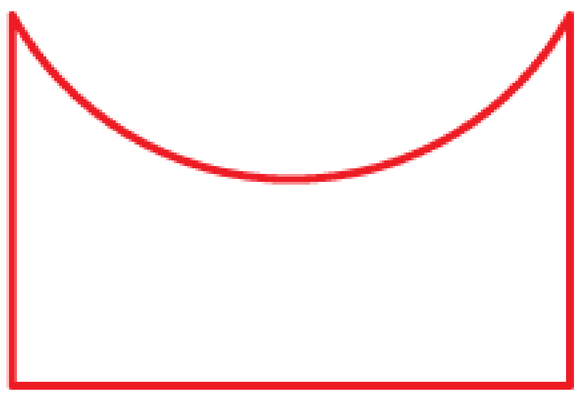


2 Explain why these shapes are not polygons:

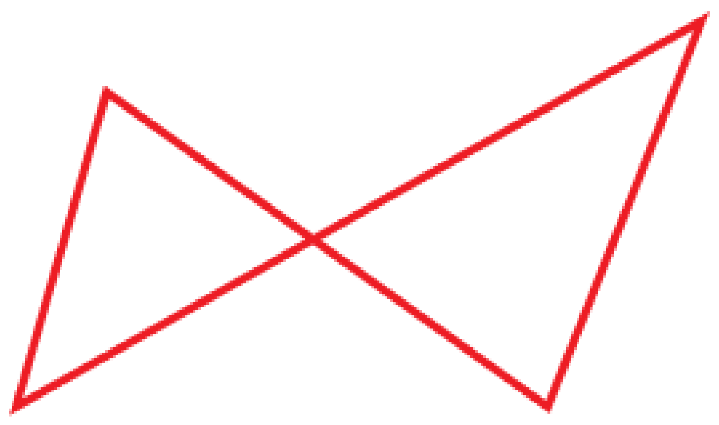
a



b



c



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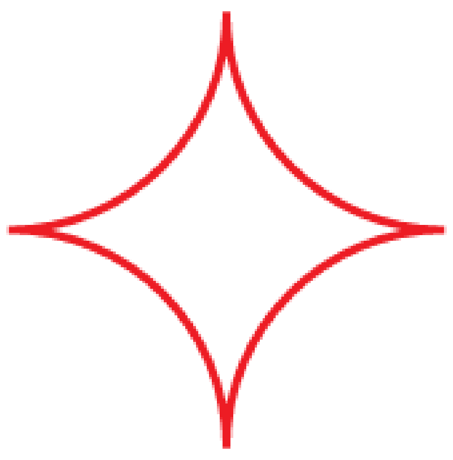
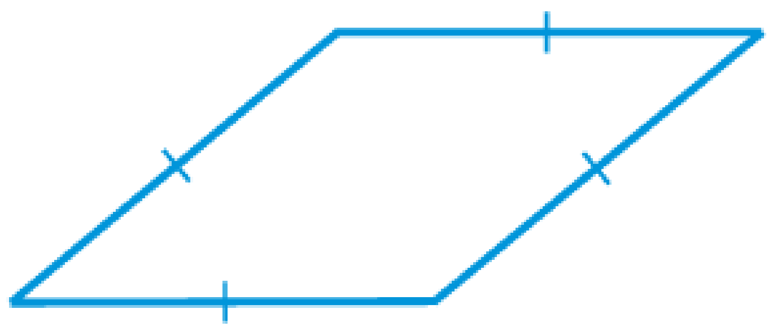
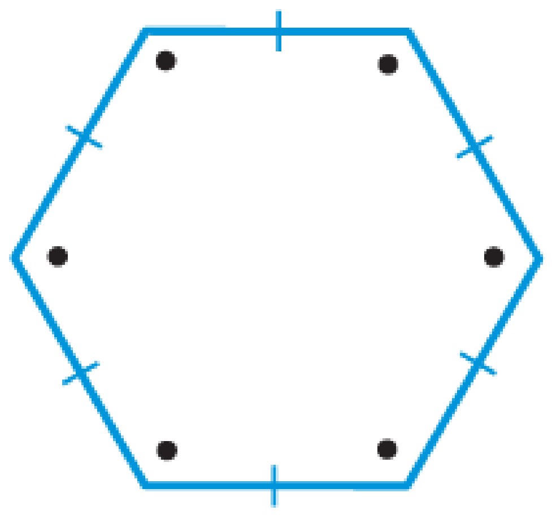


3 Which of the following are regular polygons?

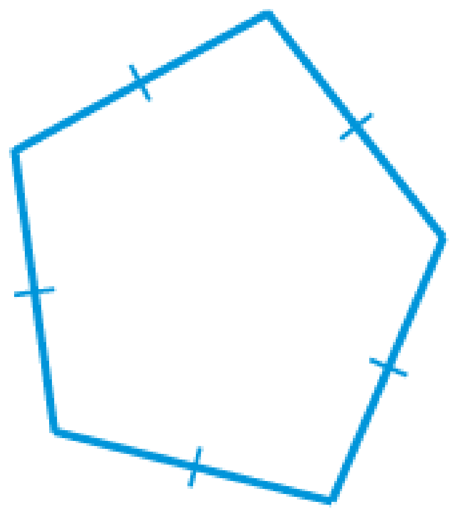
a



b



c



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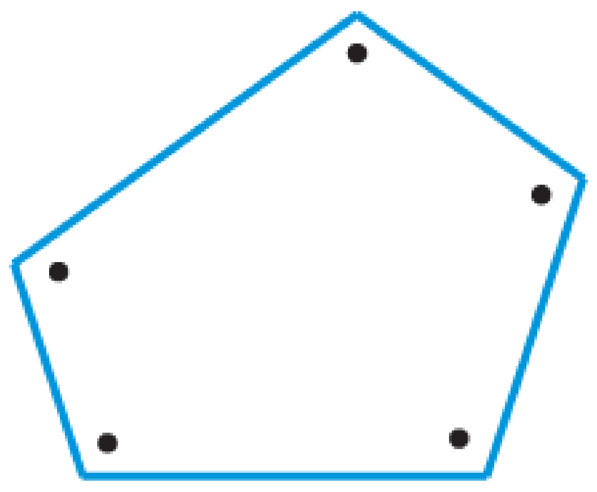
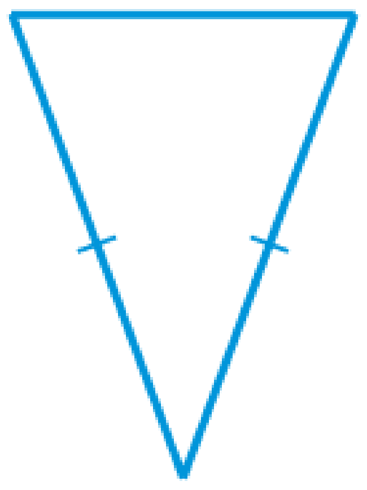


e



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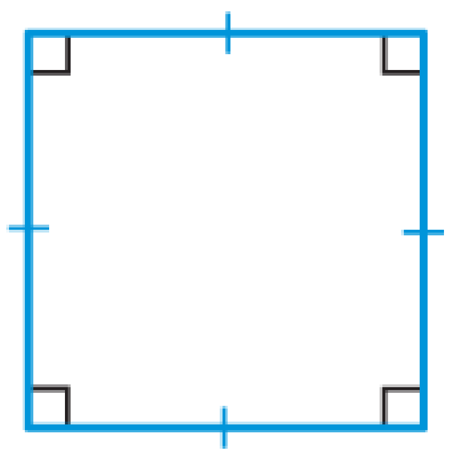
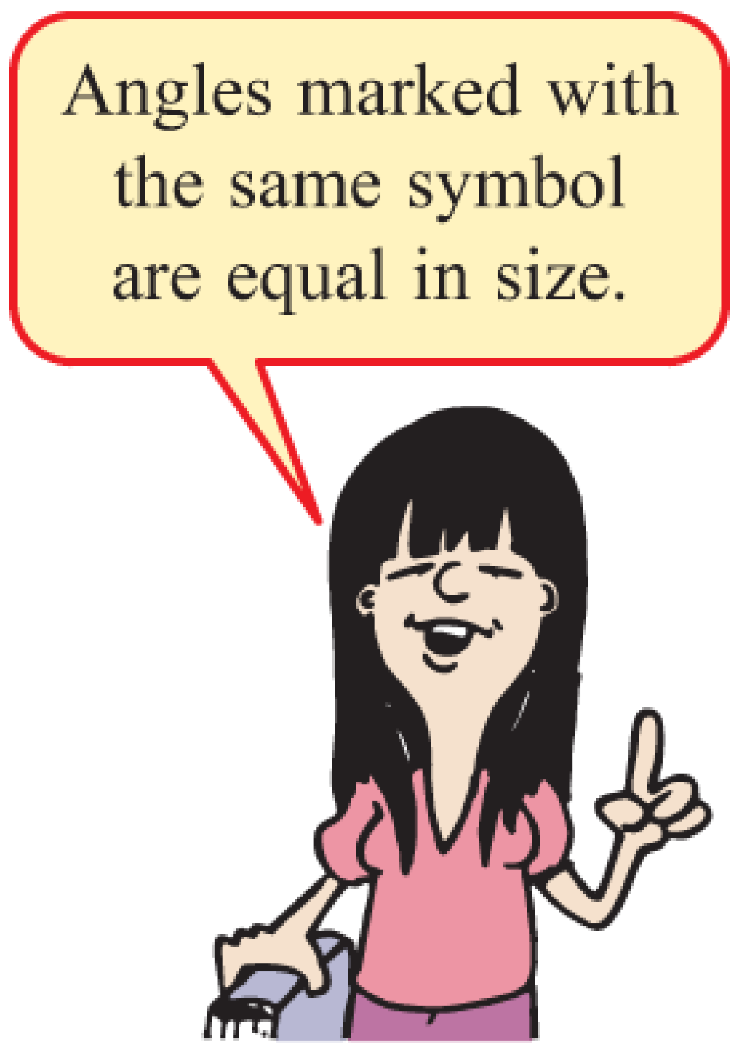




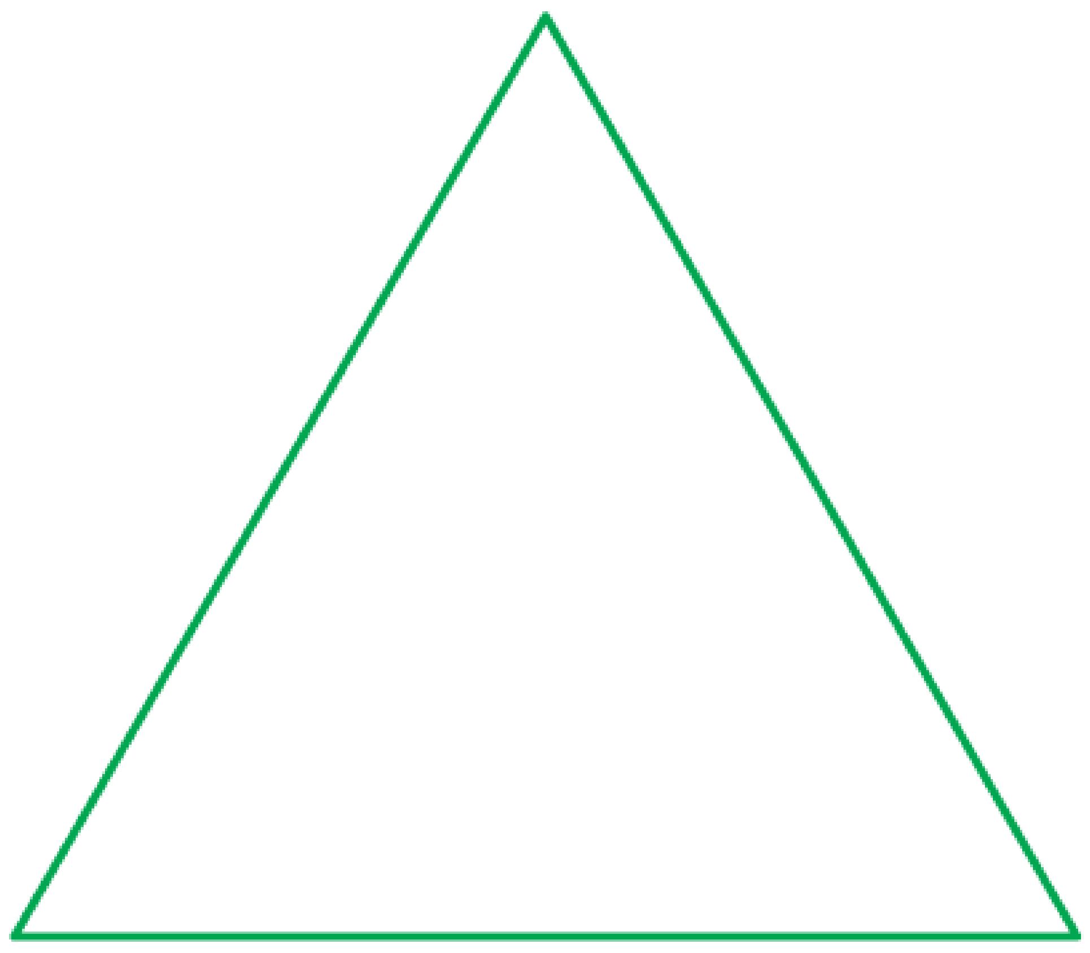
4 Draw the following polygons:

- **a** a quadrilateral with 3 equal sides
- **b** an octagon with equal sides, but with unequal angles
- **c** a hexagon with 3 right angles.

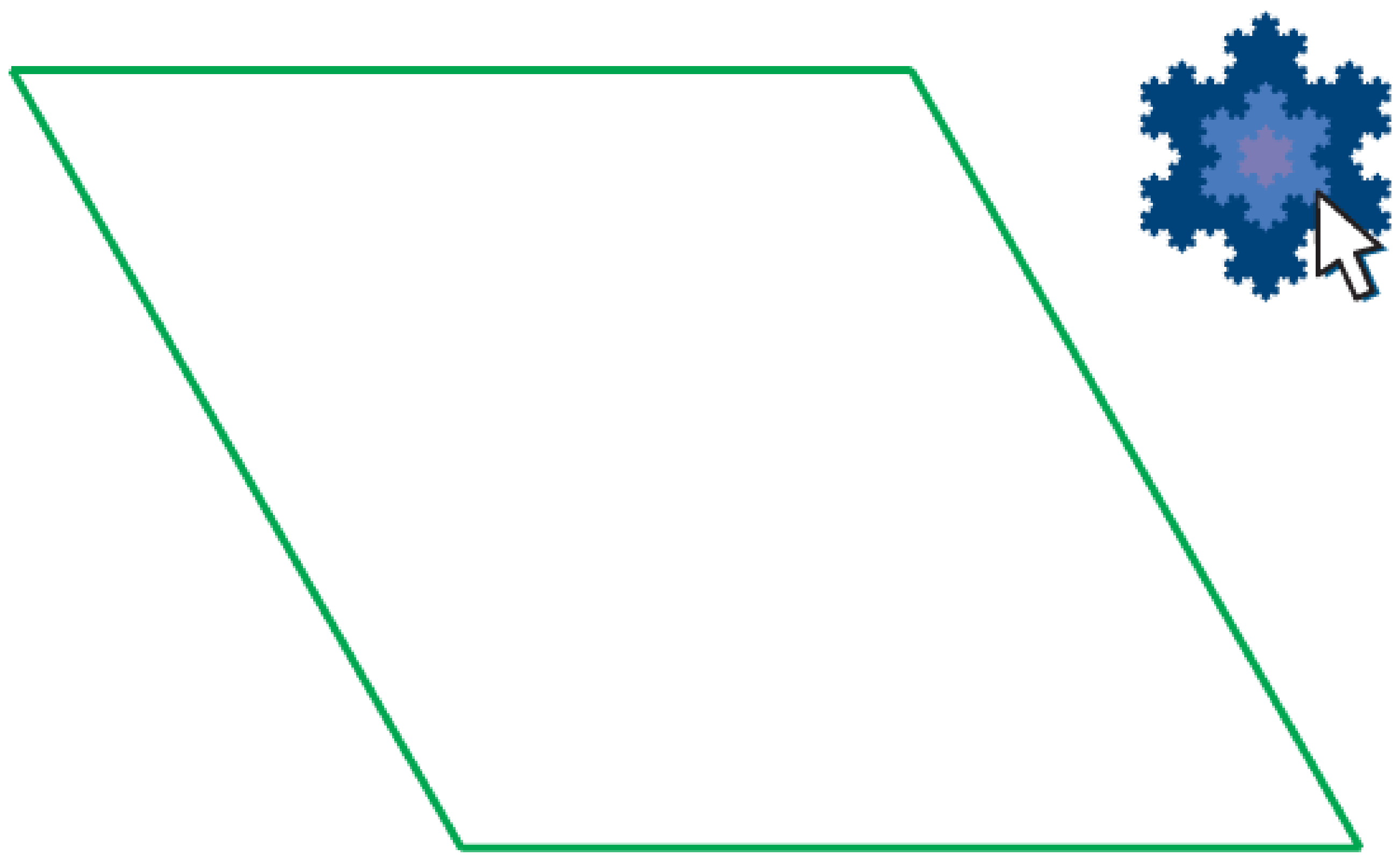
5 Use a ruler and protractor to determine whether the following polygons are regular:

PRINTABLE DIAGRAMS

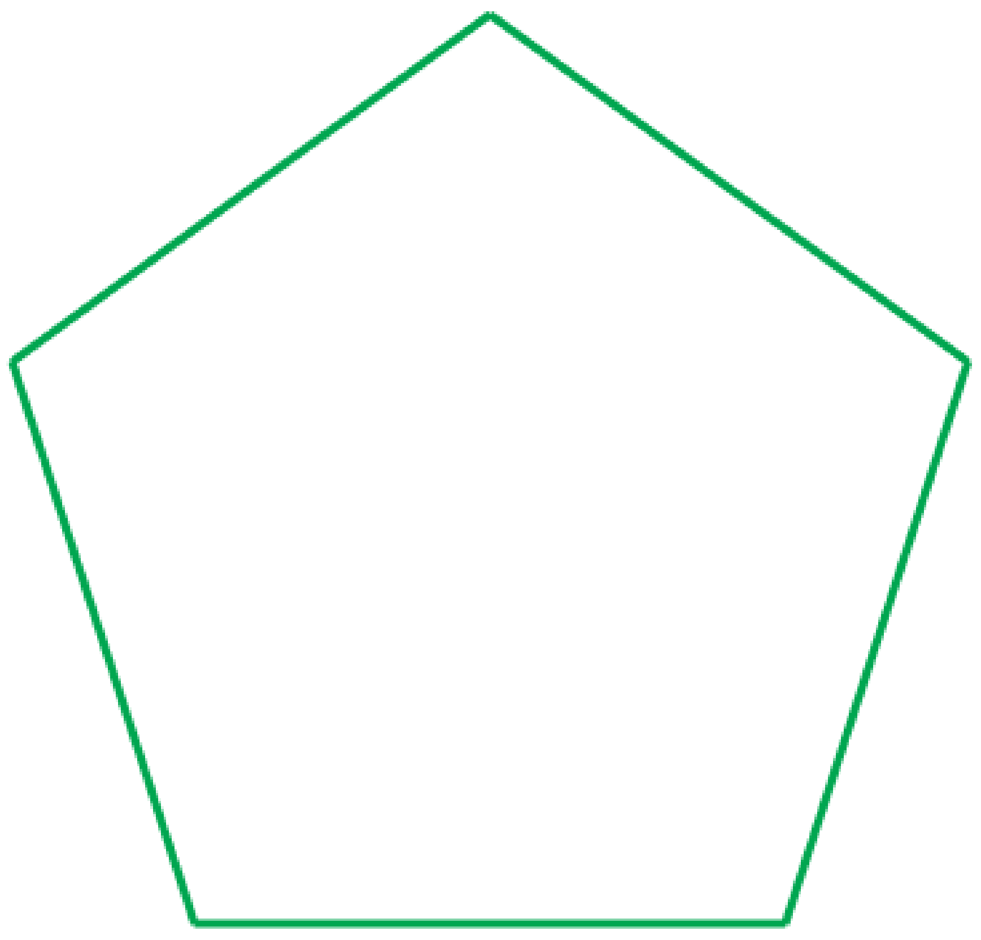
a



b



c

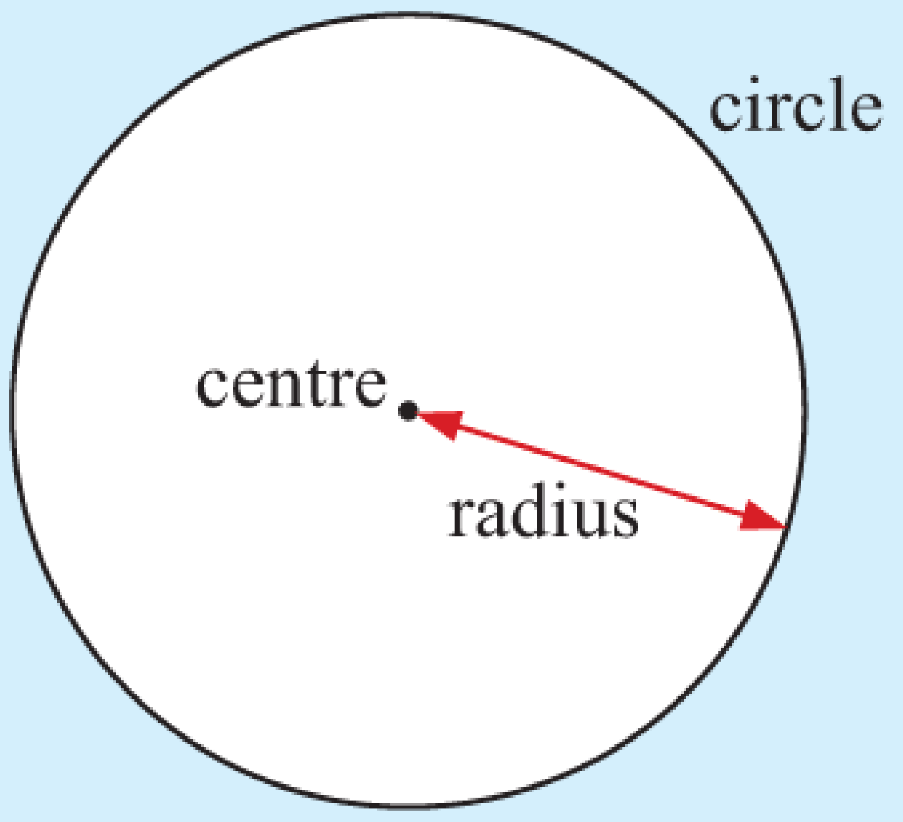


d

B CIRCLES

Every point on a **circle** is the same distance from a fixed point called its **centre**. A circle is a closed curve.

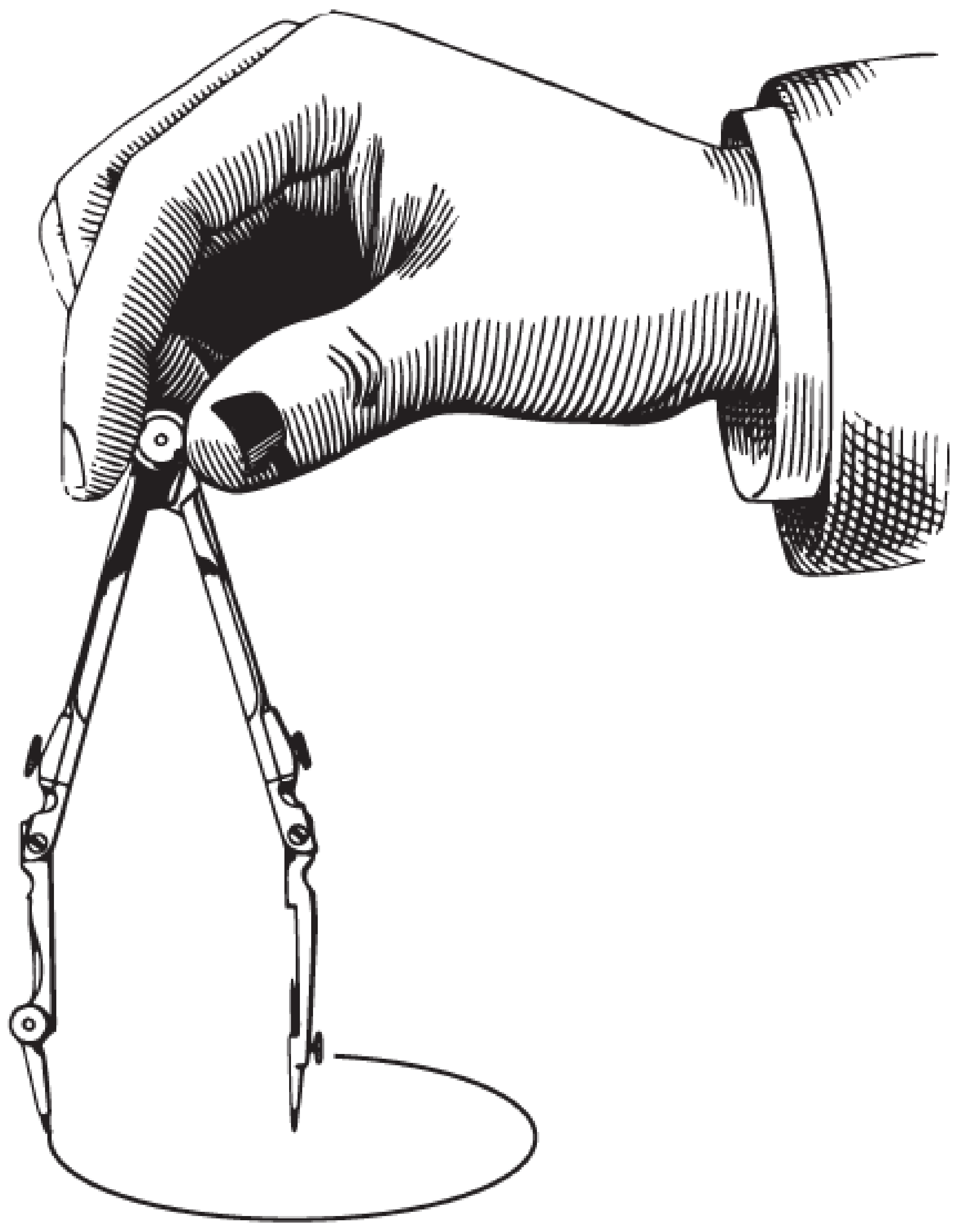
The distance from the centre to the circle is called the **radius** of the circle.



You can use a **compass** to draw a circle with a particular radius.

The point of the compass is placed at the centre of the circle, and the arms set so the distance from the needle to the tip of your pencil is the radius that you have chosen.

VIDEO DEMO

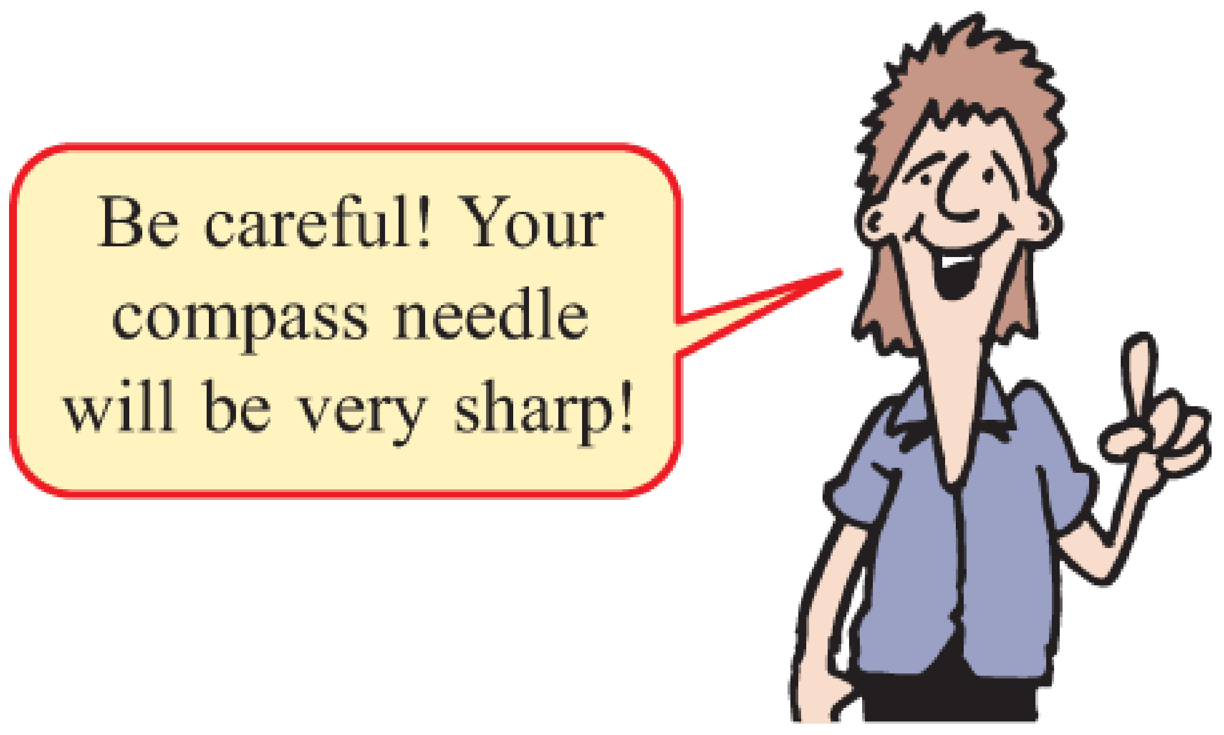


EXERCISE 5B

1 Explain why a circle is not a polygon.

2 Construct a circle with radius:

a 1 cm **b** 2 cm **c** 3 cm

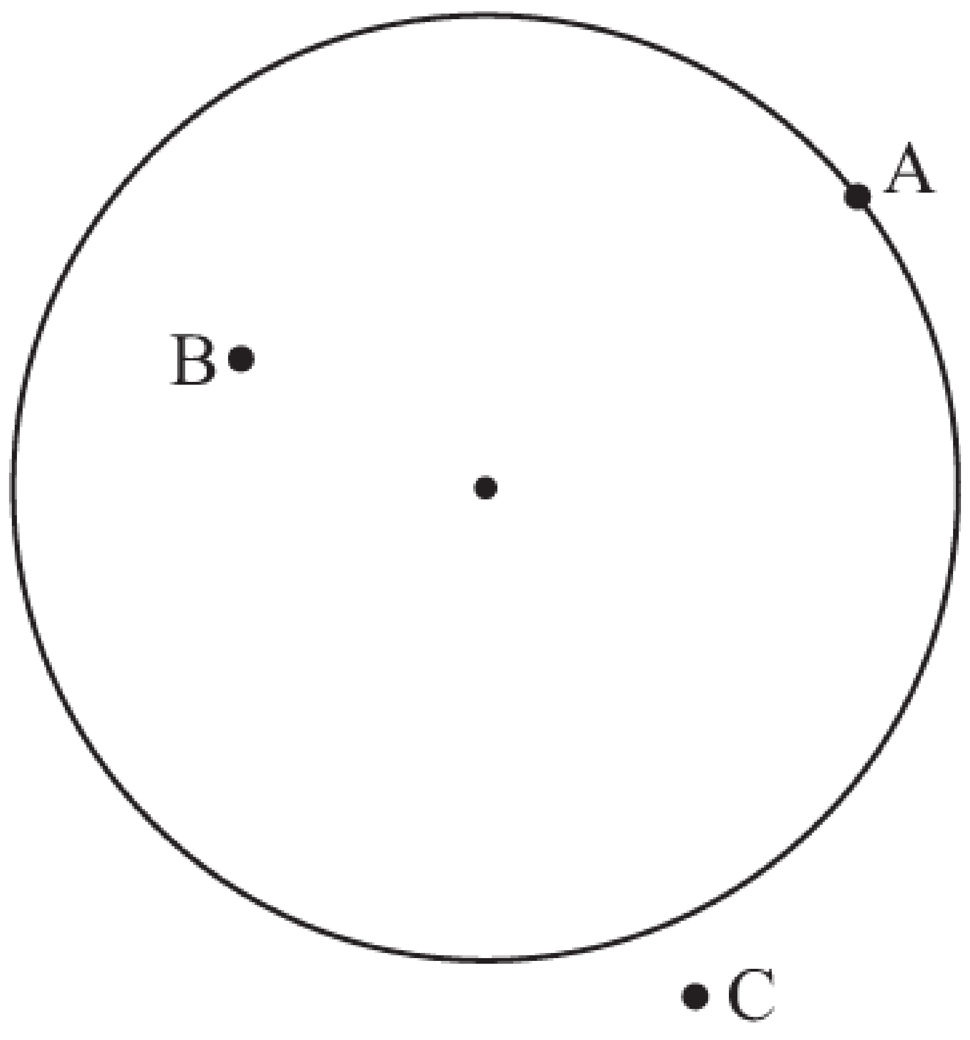


3 This circle has radius 2 cm.
What can we say about the distance between the centre of the circle and:

a point A

b point B

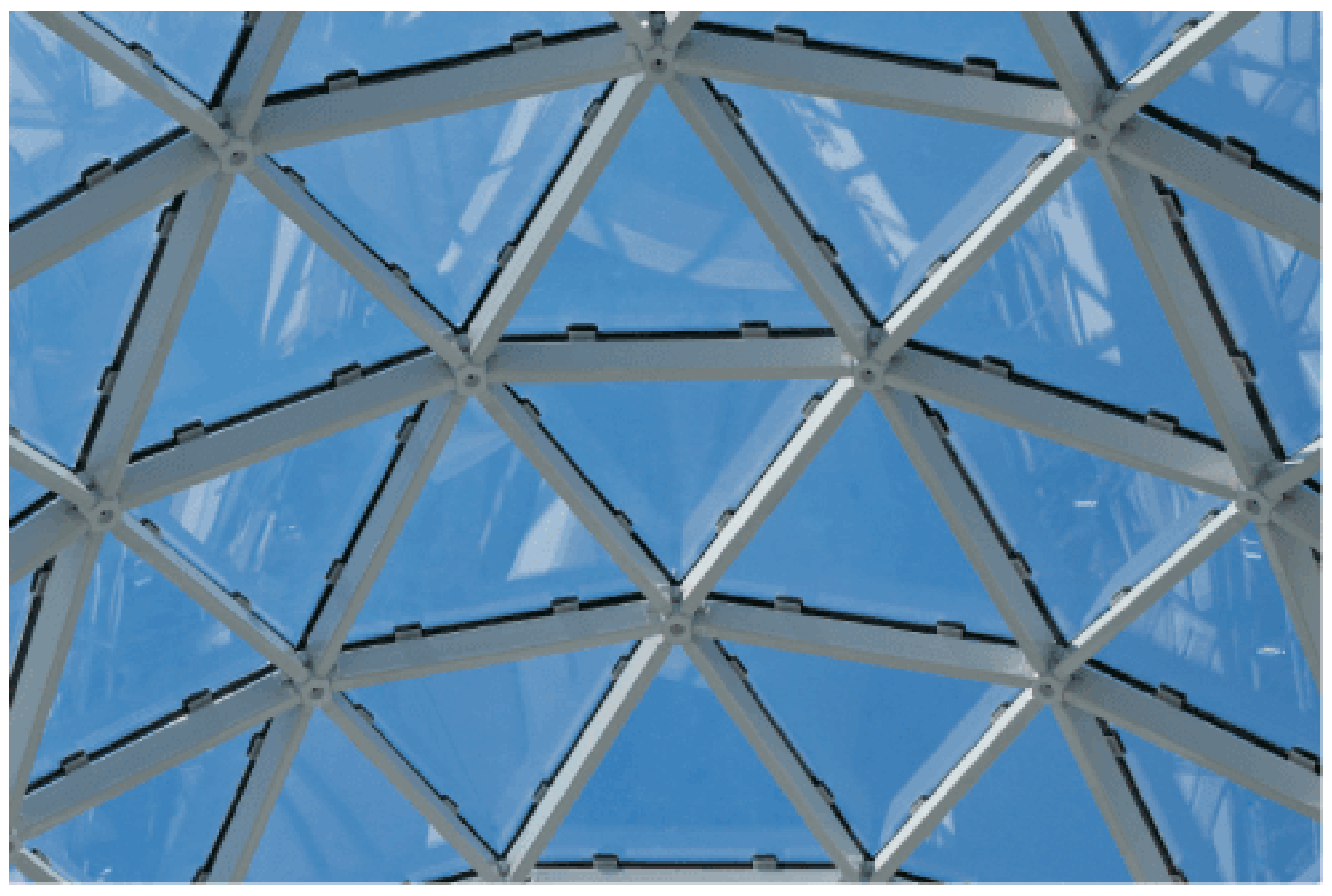
c point C?



C TRIANGLES

A **triangle** is a polygon with three sides.

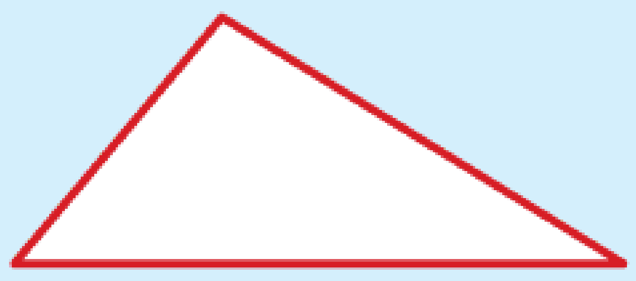
We often see triangles in buildings and bridges because they provide strength and stability.



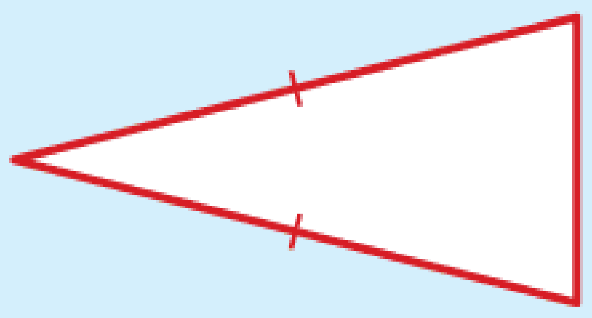
We can classify triangles according to the number of sides which are equal in length.

A triangle is:

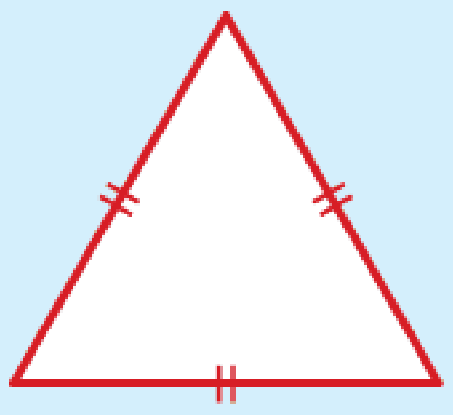
- **scalene** if the three sides all have different lengths



- **isosceles** if at least two sides have the same length



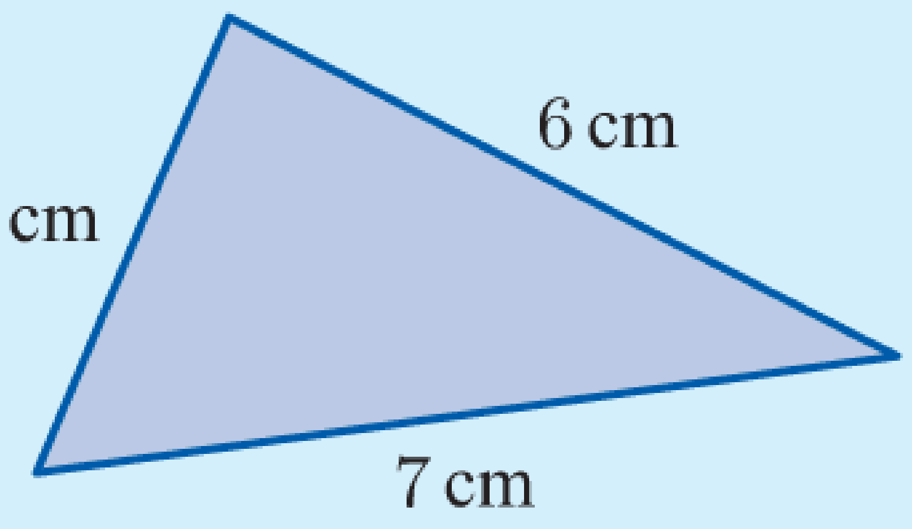
- **equilateral** if all three sides have the same length.



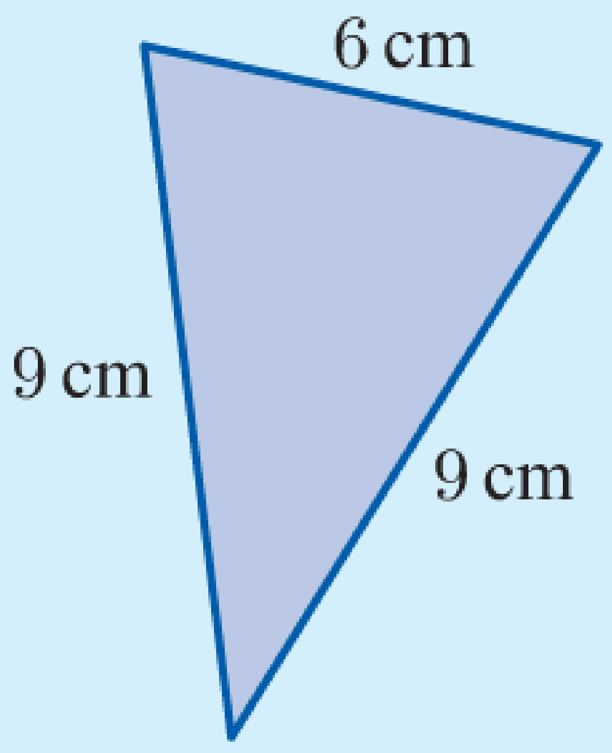
Example 1 **Self Tutor**

Classify the following triangles:

a



b



a The three sides have different lengths, so the triangle is scalene.

b Two of the sides have the same length, so the triangle is isosceles.

CONSTRUCTING A TRIANGLE

If we are given the side lengths, we can use a **compass and ruler** to construct a triangle.

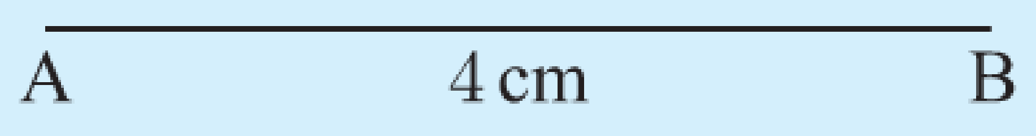
Example 2

Self Tutor

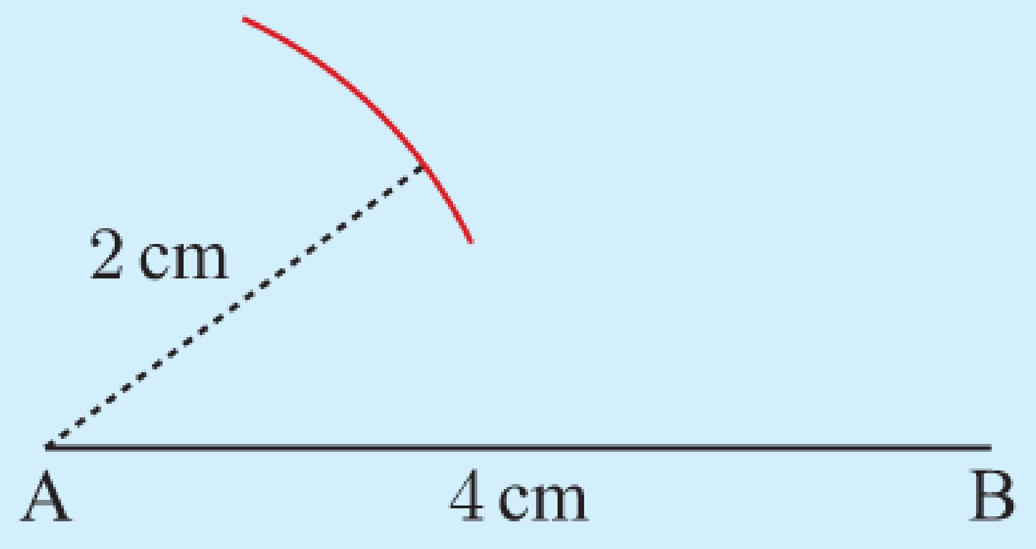
Construct a triangle ABC with sides 4 cm, 3 cm, and 2 cm long.

VIDEO CLIP

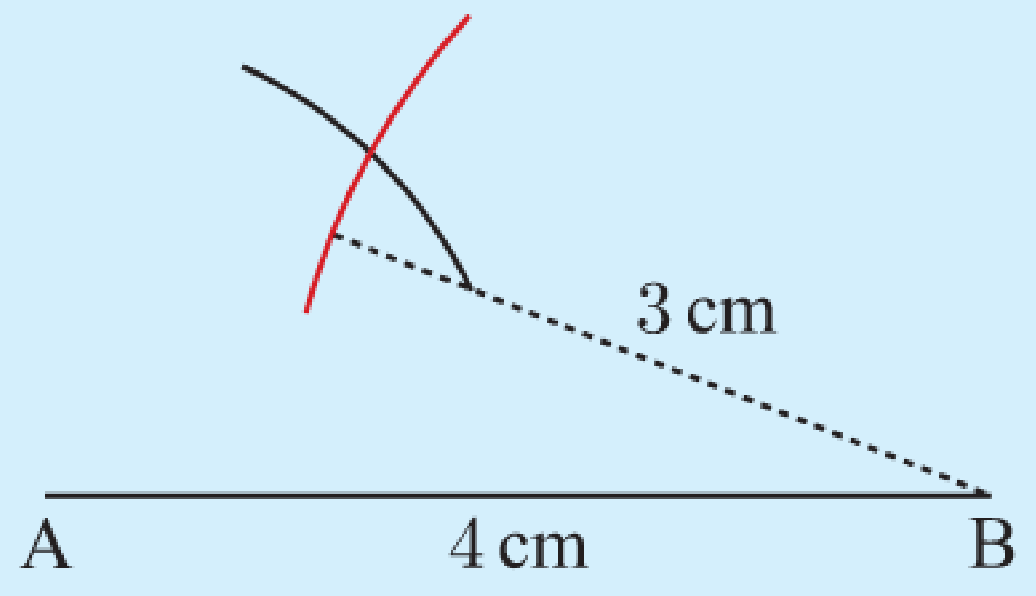
Step 1: Draw a line segment of length 4 cm. We will call this line segment [AB], and use it as the base of the triangle.



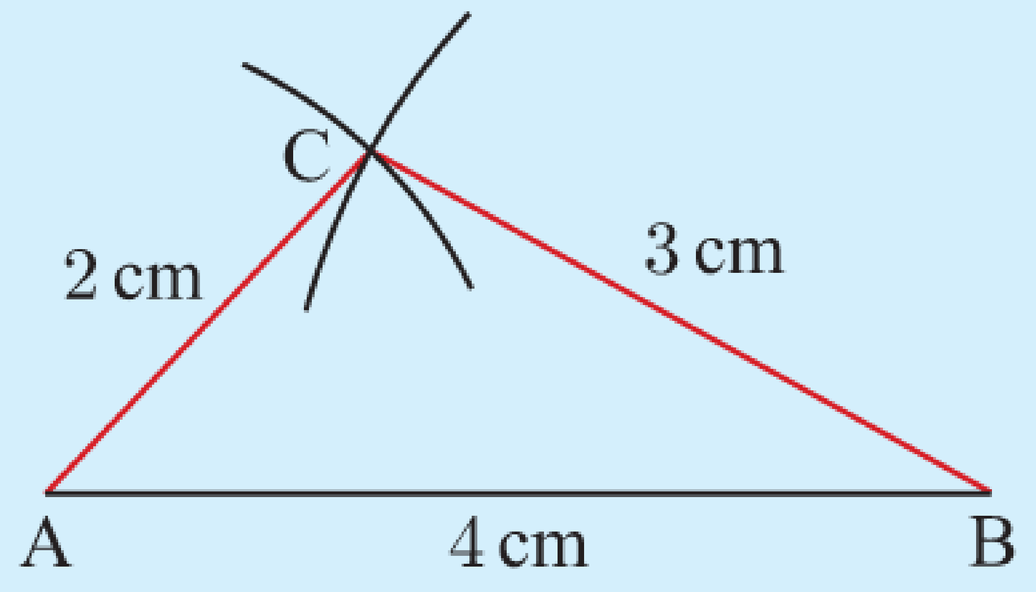
Step 2: Open your compass to a radius of 2 cm. Using this radius, draw an arc from A.



Step 3: Now open the compass to a radius of 3 cm. Draw an arc from B to intersect the first arc.



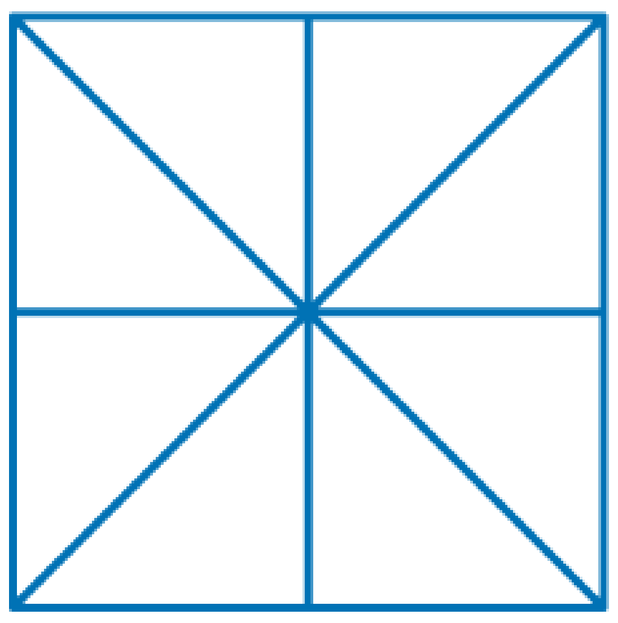
Step 4: The point of intersection of the two arcs is the third vertex C of the triangle ABC. Draw line segments [AC] and [BC] to complete the triangle.



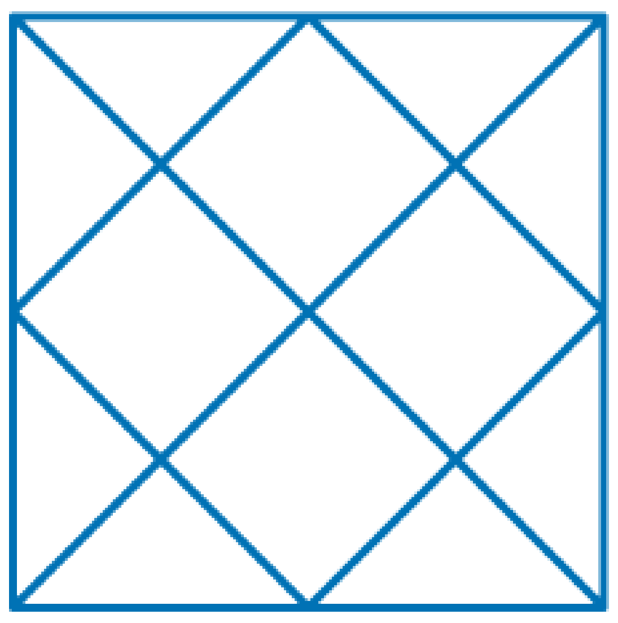
EXERCISE 5C

1 How many triangles can you find in the given figures?

a

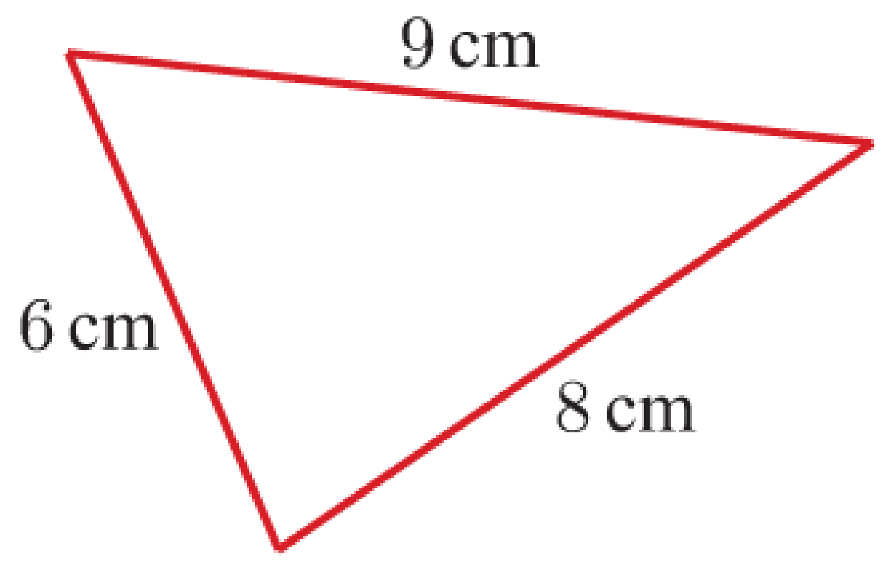


b

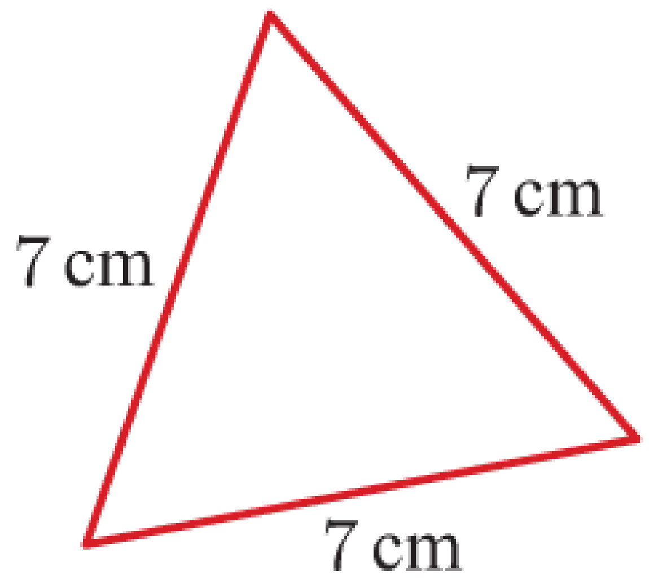


2 Classify the following triangles:

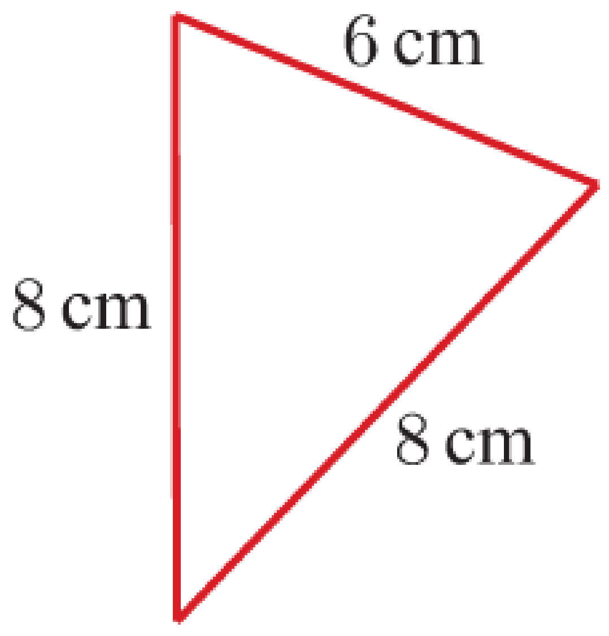
a



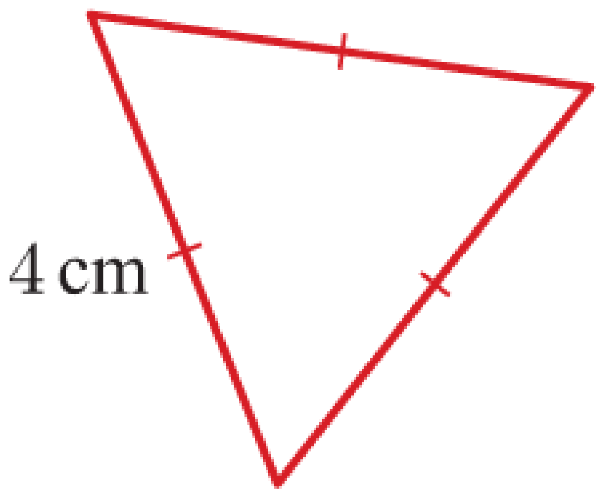
b



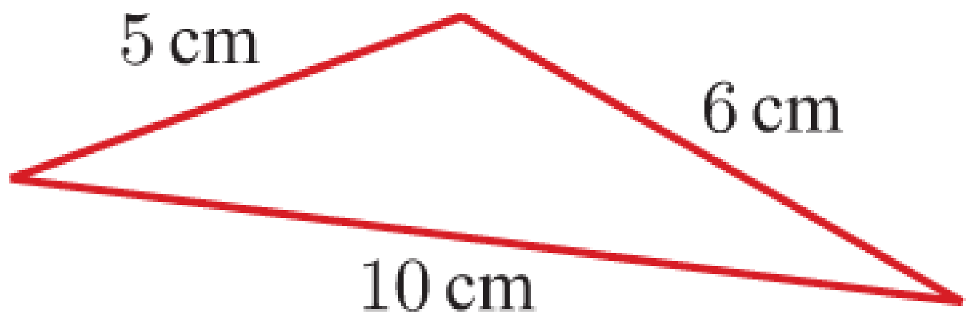
c



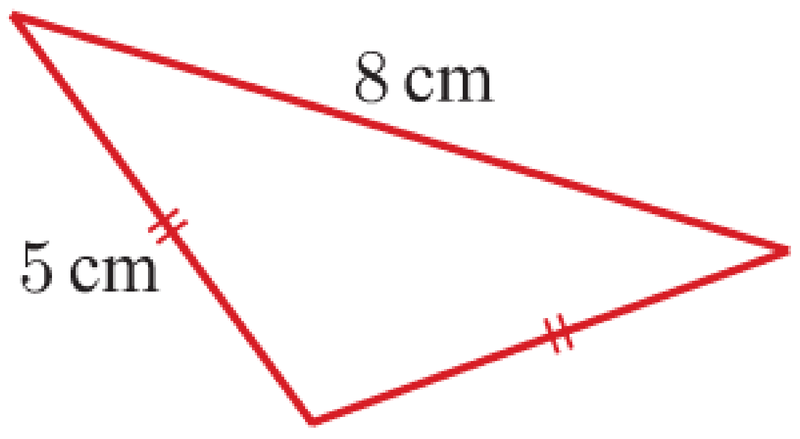
d



e

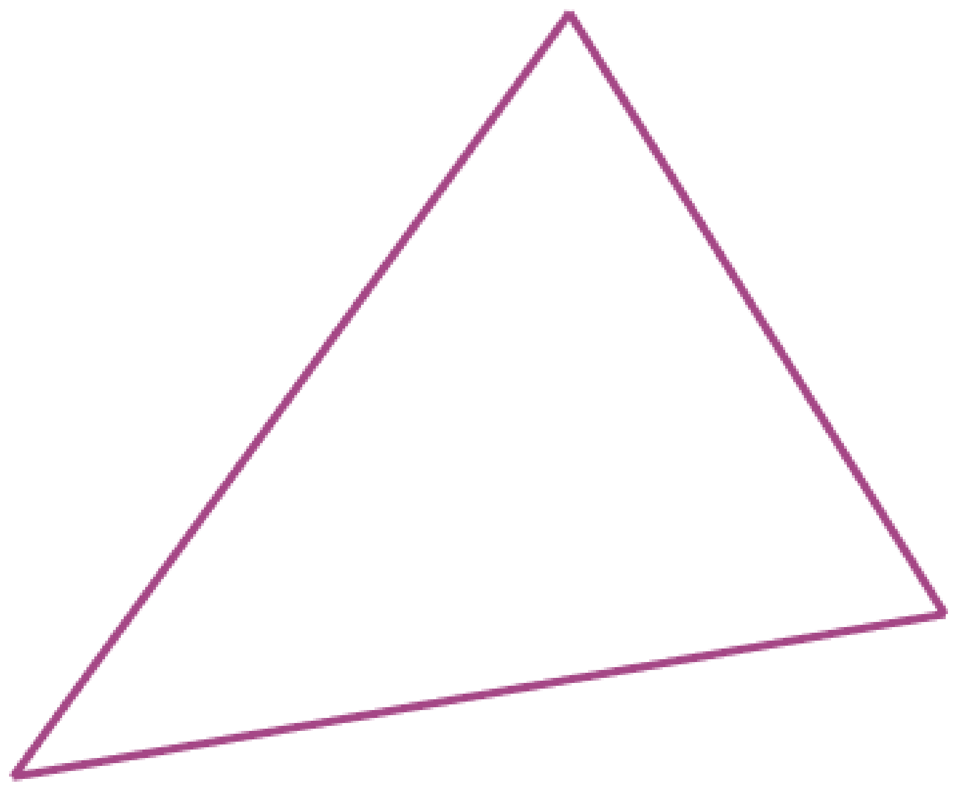


f

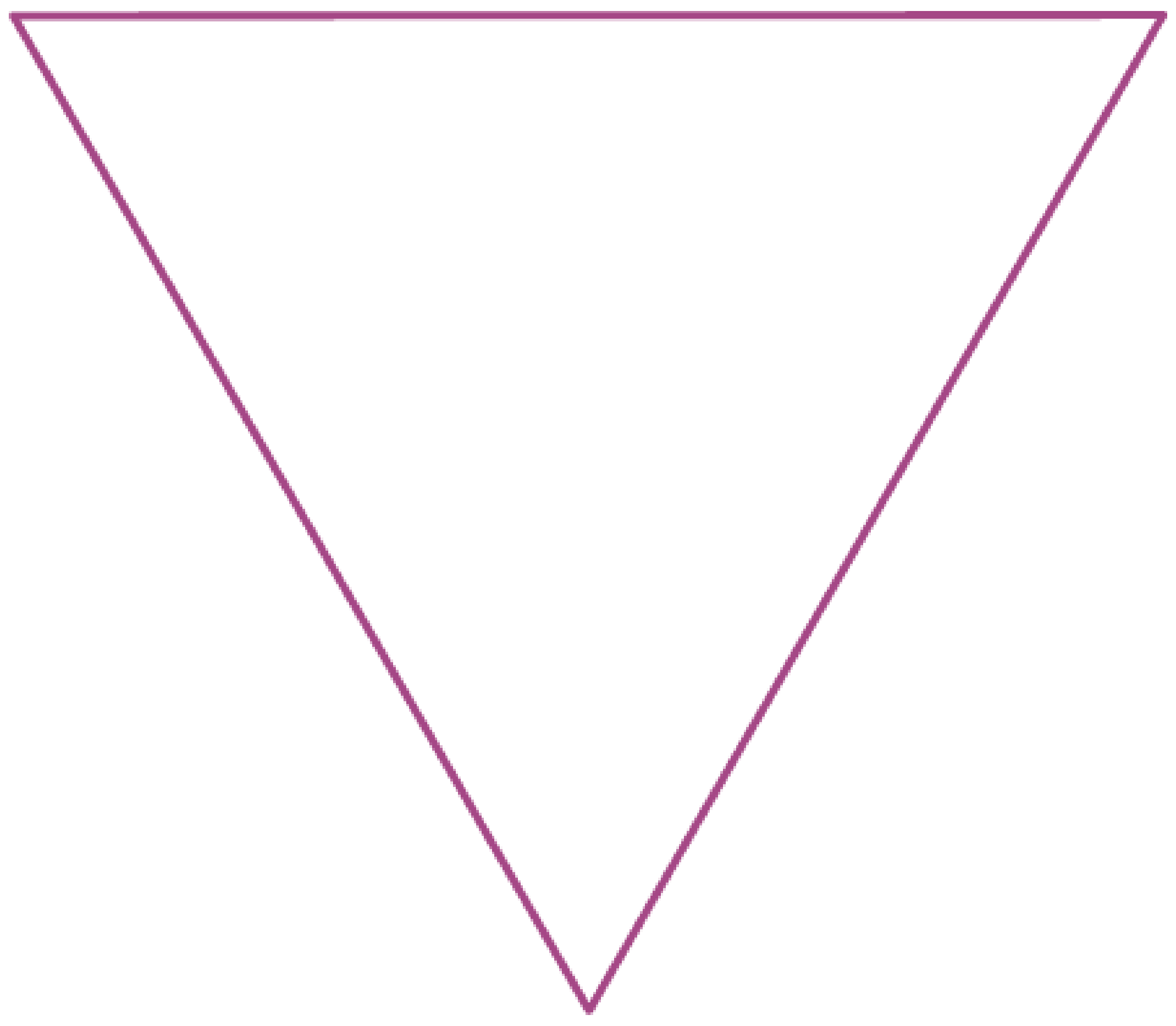


3 Use a ruler to measure each side of these triangles. Hence classify each triangle.

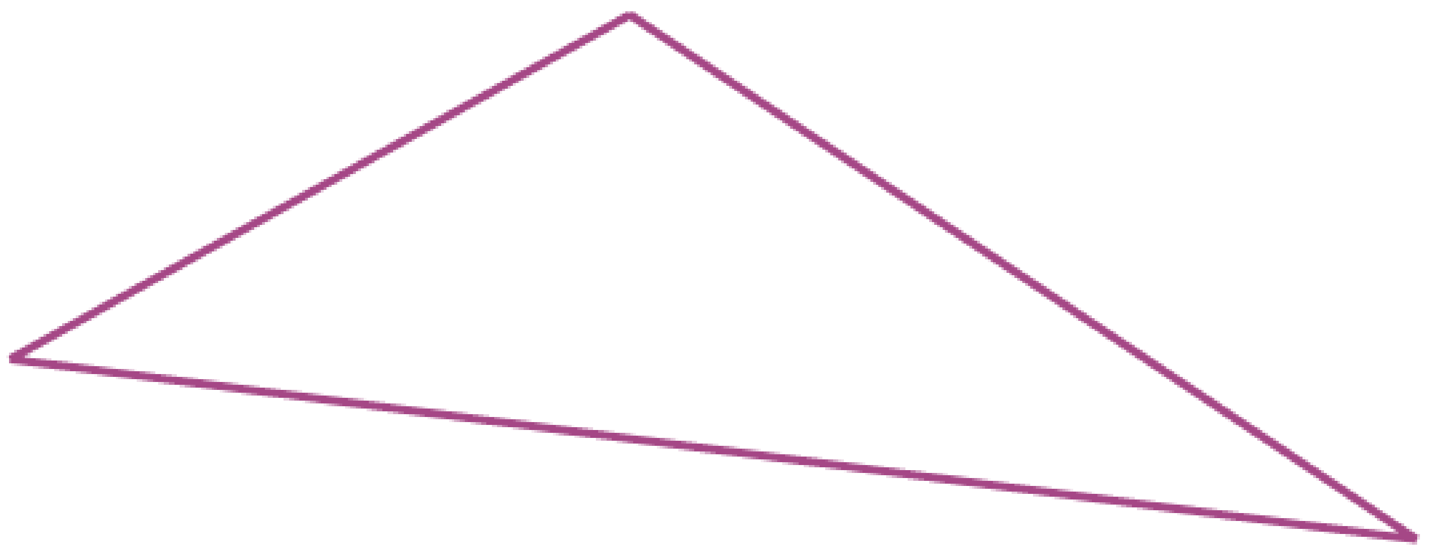
a



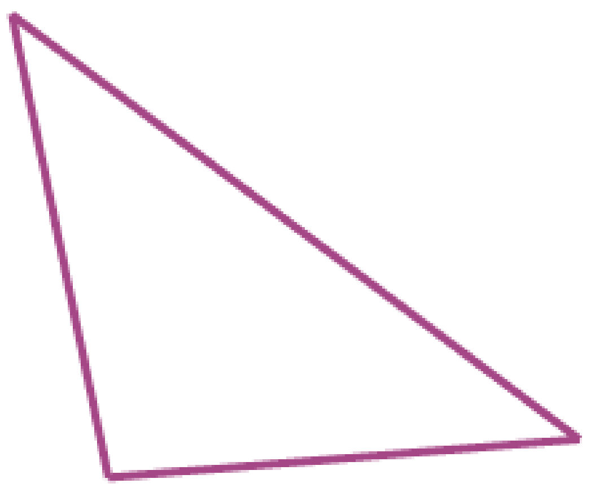
b



c



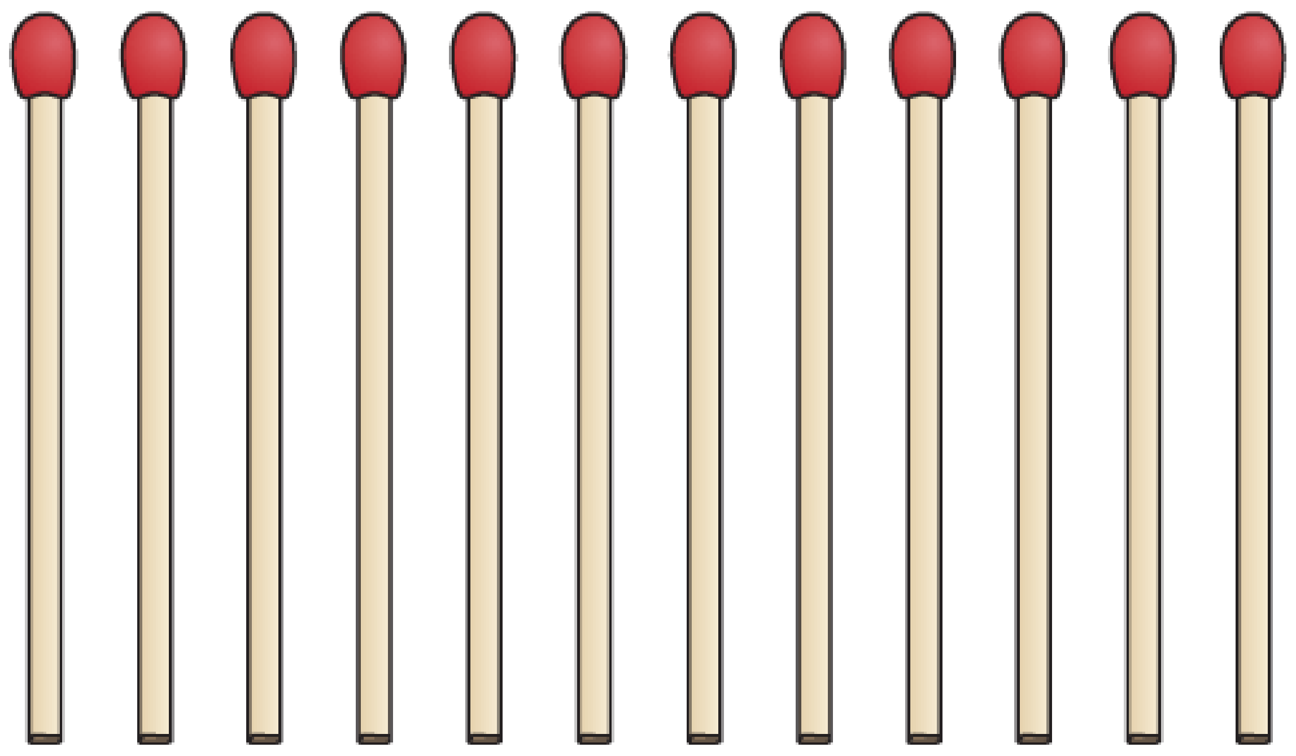
d



PRINTABLE DIAGRAMS

4 Show how you can arrange 12 matchsticks of equal size to form:

- **a** an equilateral triangle
- **b** an isosceles triangle
- **c** a scalene triangle.



5 Accurately construct a triangle with sides:

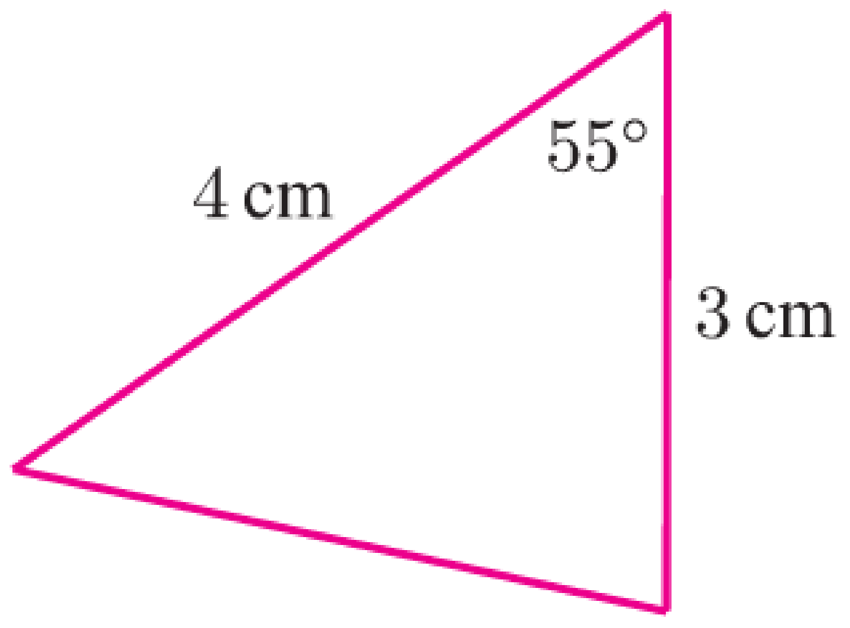
a 4 cm, 5 cm, and 6 cm

b 3 cm, 6 cm, and 7 cm.

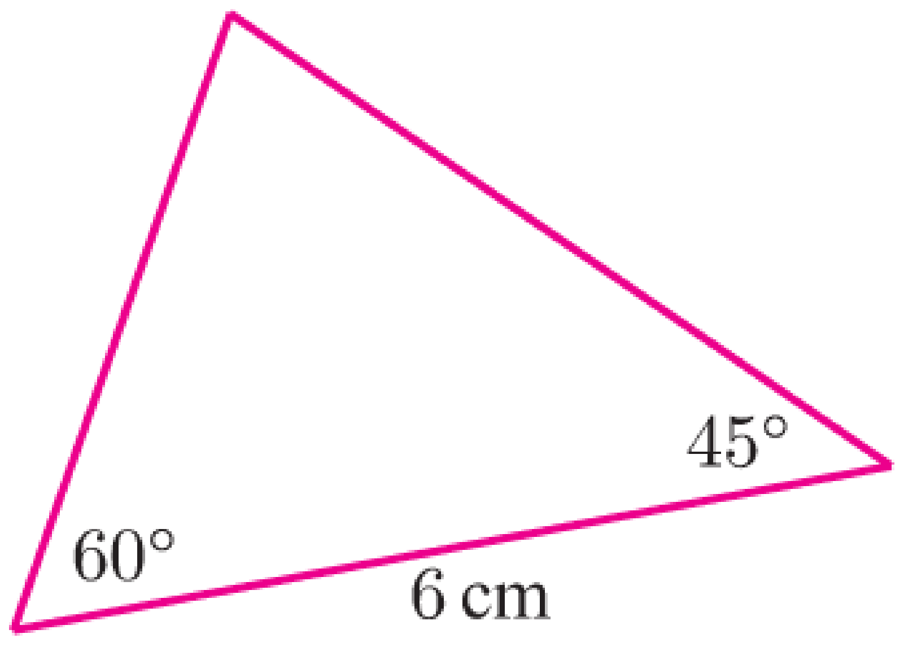
6 Is it possible to construct a triangle with sides 3 cm, 4 cm, and 9 cm? Explain your answer.

7 Use a protractor and ruler to accurately construct these triangles:

a



b

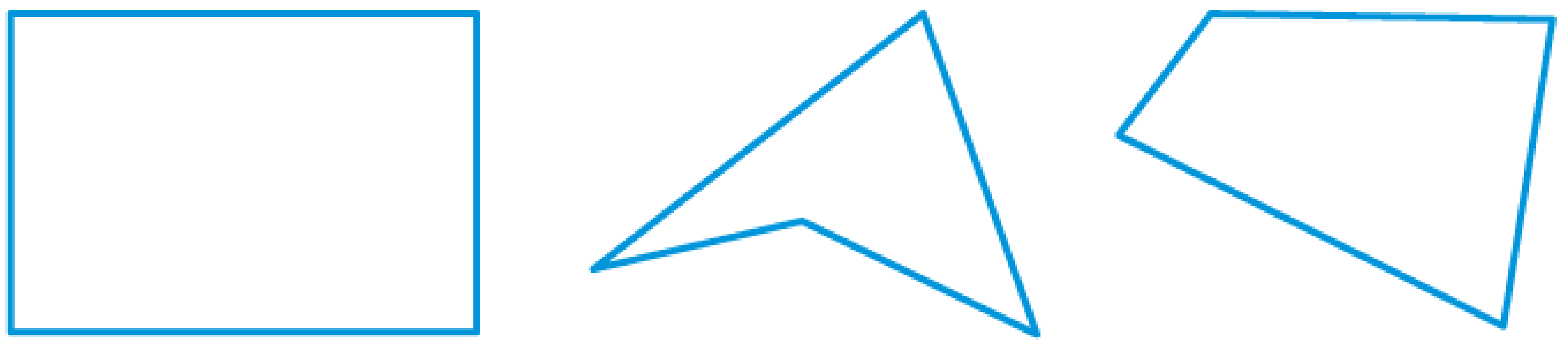


8 **a** Construct a triangle ABC whose side lengths are all 6 cm.
b What type of triangle is ABC?
c Measure the angles of the triangle using a protractor.
d Copy and complete: All angles of an equilateral triangle measure ......°.

D QUADRILATERALS

A **quadrilateral** is a polygon with four sides.

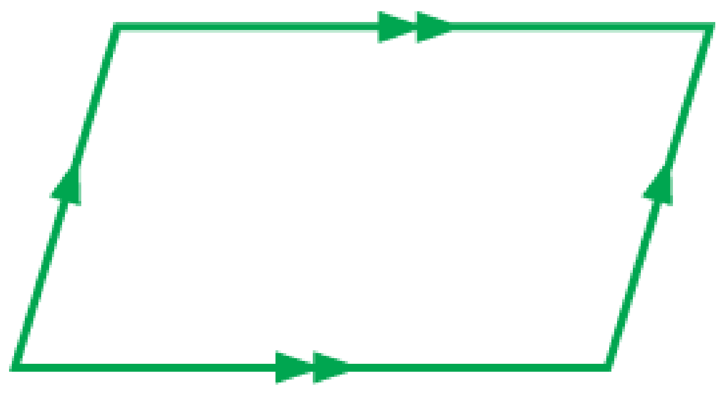
The shapes alongside are all quadrilaterals.



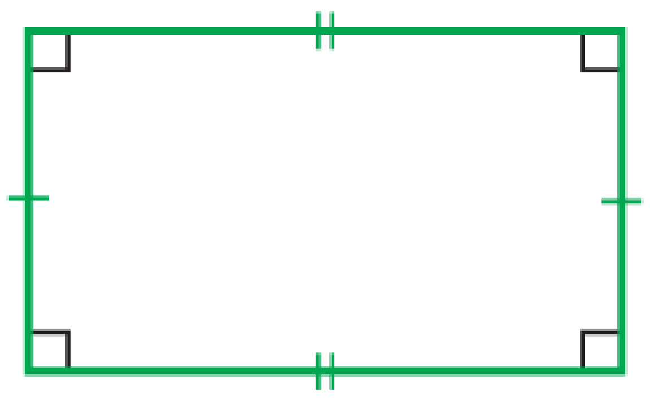
To classify quadrilaterals, we need to consider side lengths, angles, and also whether opposite sides are parallel.

There are six special quadrilaterals:

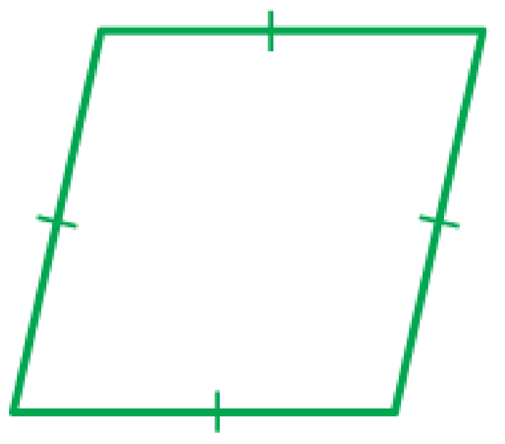
- A **parallelogram** has both pairs of opposite sides parallel.
  The opposite sides of a parallelogram are equal in length.



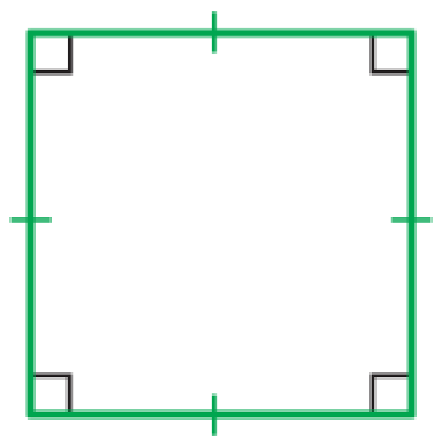
- A **rectangle** is a parallelogram with right angled corners.
  The opposite sides of a rectangle are equal in length.



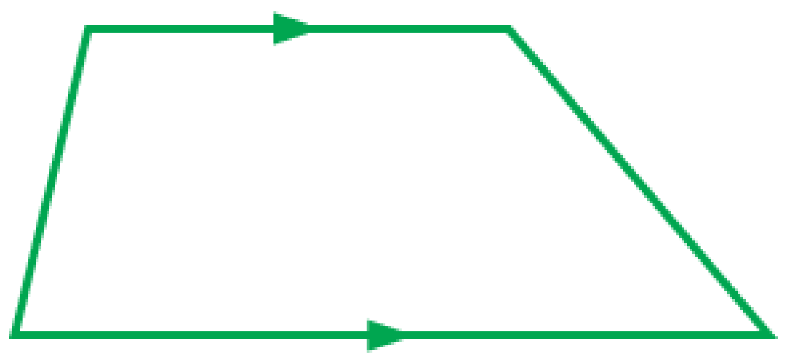
- A **rhombus** is a parallelogram with all four sides equal in length.



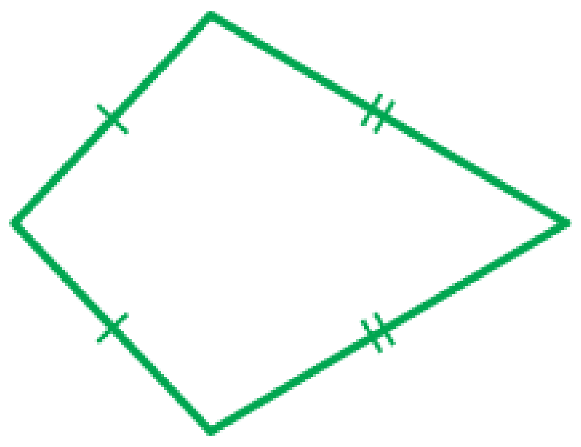
- A **square** is a rectangle with all sides equal in length.
  Both pairs of opposite sides of a square are parallel.



- A **trapezium** has one pair of opposite sides which are parallel.



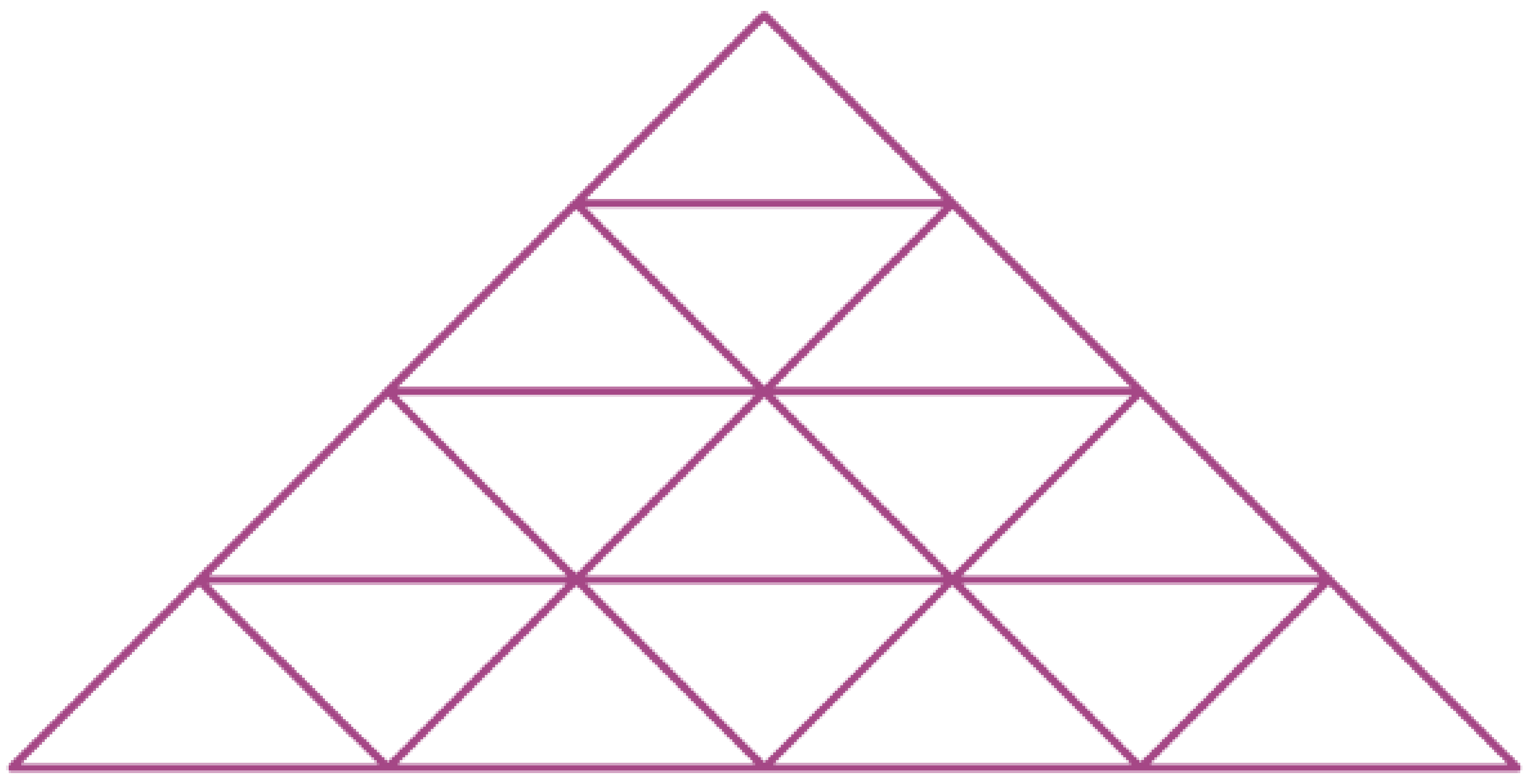
- A **kite** has two pairs of adjacent sides which are equal in length.



EXERCISE 5D

1 In the diagram alongside, identify a:

- **a** square
- **b** rectangle
- **c** parallelogram
- **d** trapezium.

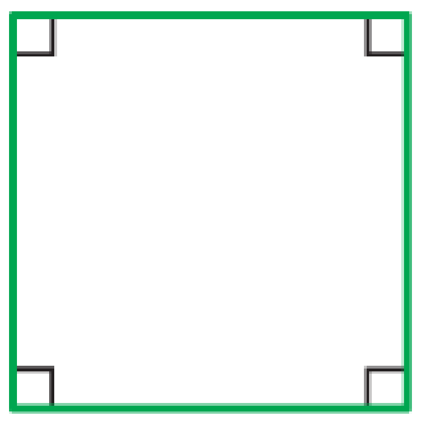


2 Draw an example of a:

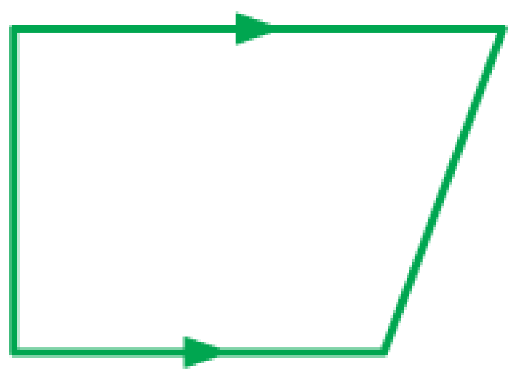
- **a** rhombus
- **b** rectangle
- **c** trapezium
- **d** kite.

3 Name the following quadrilaterals. If necessary, use a ruler to measure the sides.

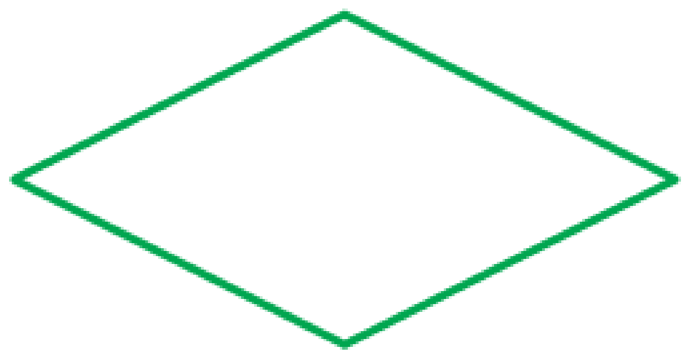
a



b

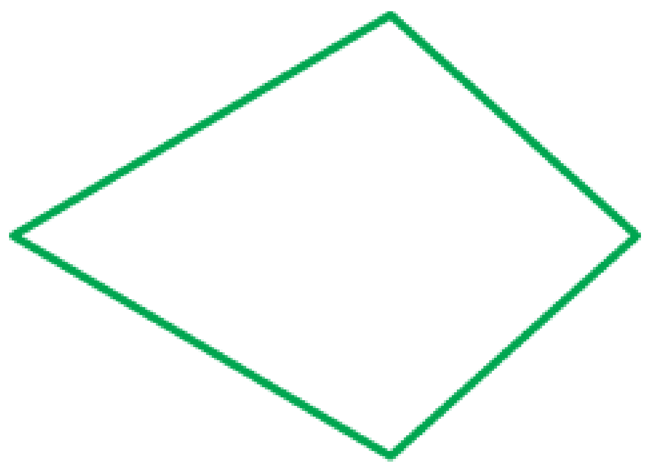


c

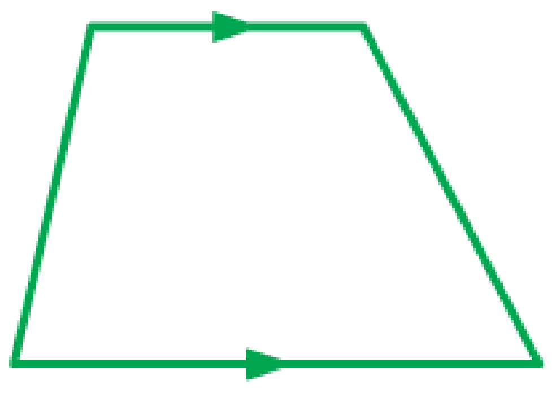


d

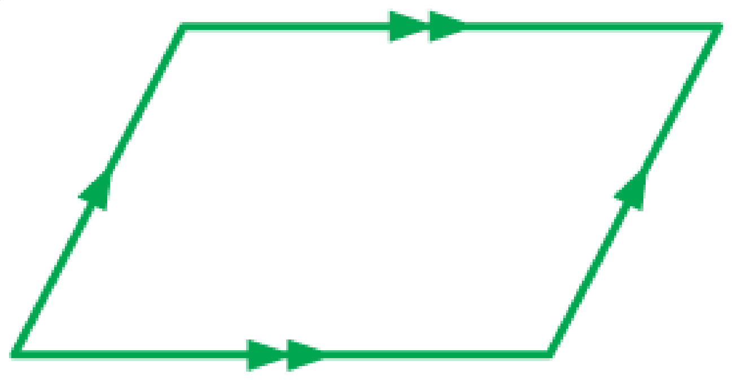
e



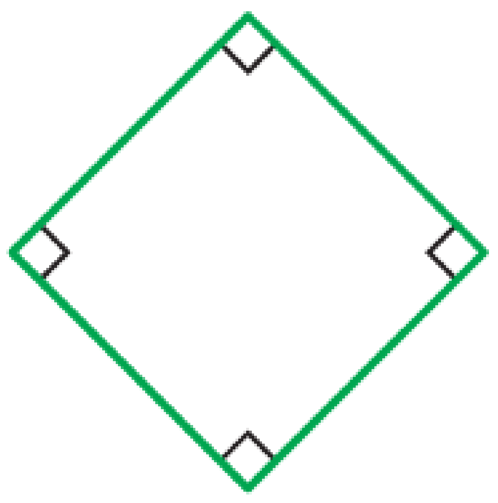
f



g



h



4 Show how:

- **a** two squares can be combined to form a rectangle
- **b** two rectangles can be combined to form a square
- **c** two trapezia can be combined to form a parallelogram
- **d** two equilateral triangles can be combined to form a rhombus
- **e** two isosceles triangles can be combined to form a kite.

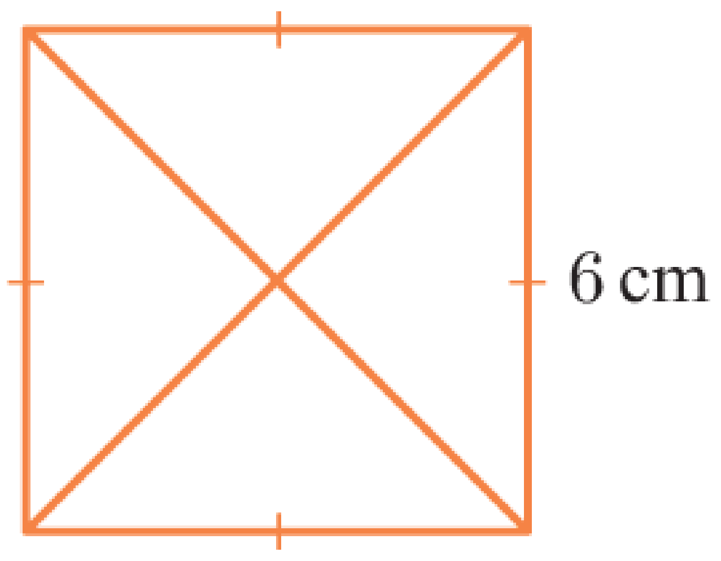
PRINTABLE SHAPES

5 True or false?

- **a** A square is a special type of rhombus.
- **b** A rectangle is a special type of square.
- **c** A square is a special type of parallelogram.
- **d** A rectangle is a special type of parallelogram.

6 **a** Use a ruler and protractor to draw a square with side length 6 cm.

- **b** Draw the diagonals of the square.
- **c** Measure the lengths of the diagonals. What do you notice?

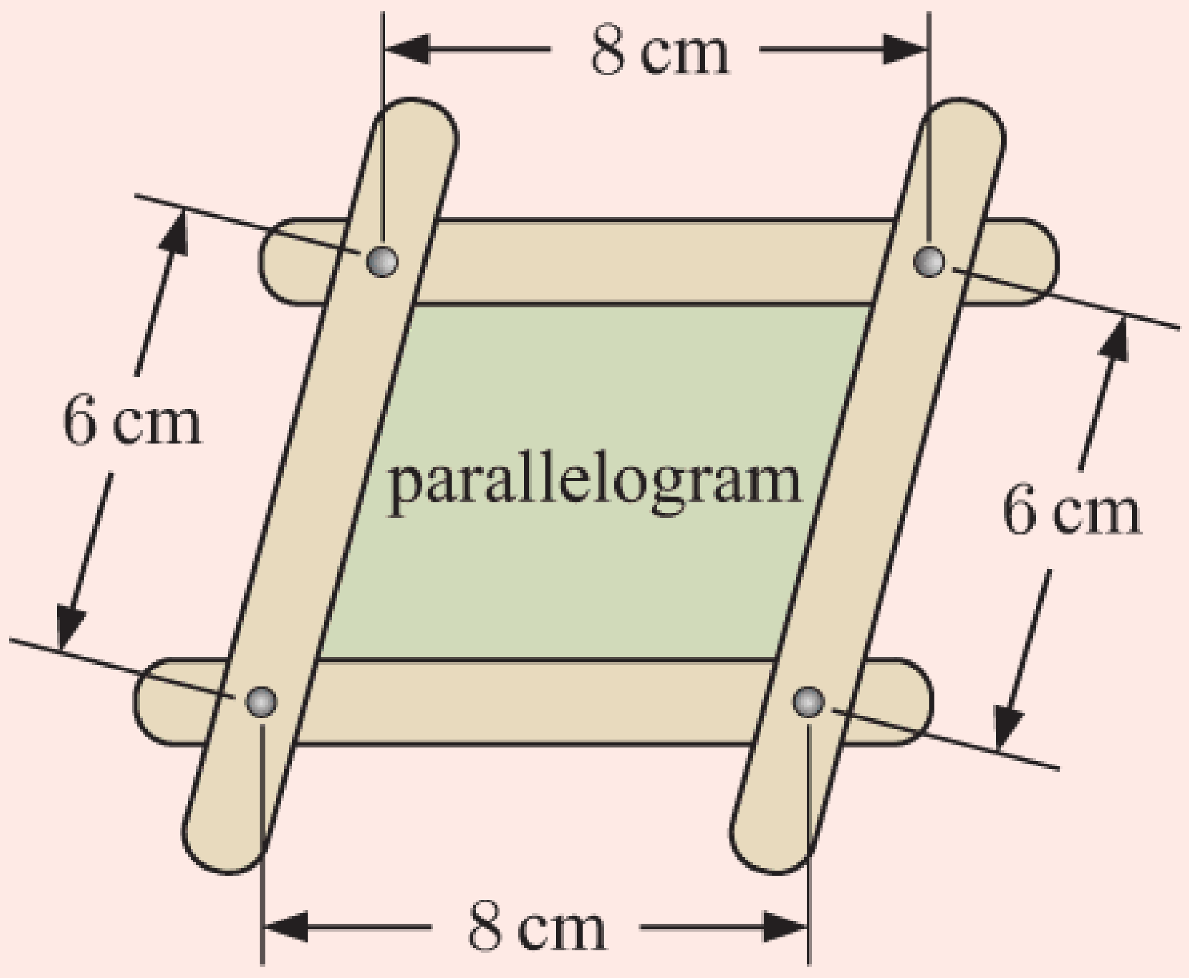


ACTIVITY 1 MAKING PARALLELOGRAMS

To make parallelograms with different angles you could use ice-block sticks.

What to do:

1. Join the ice-block sticks with four small bolts and nuts. You do not need to tighten the bolts.
2. Use a pencil to draw the parallelogram inside the frame.
3. The parallelogram may be changed by moving the wooden frame.
   Use this technique to draw *five* different parallelograms.

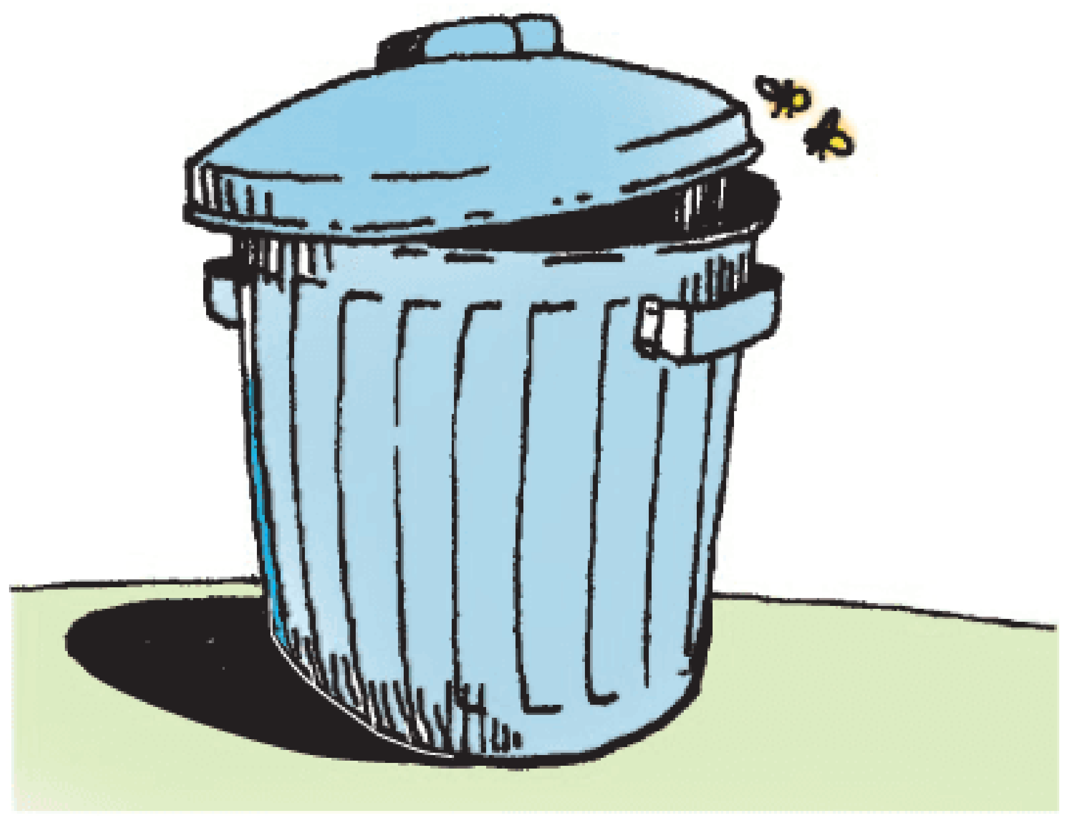


E SOLIDS

Solids are objects which occupy space.

A solid needs to be fully enclosed. However, unlike the name suggests, it may be hollow.

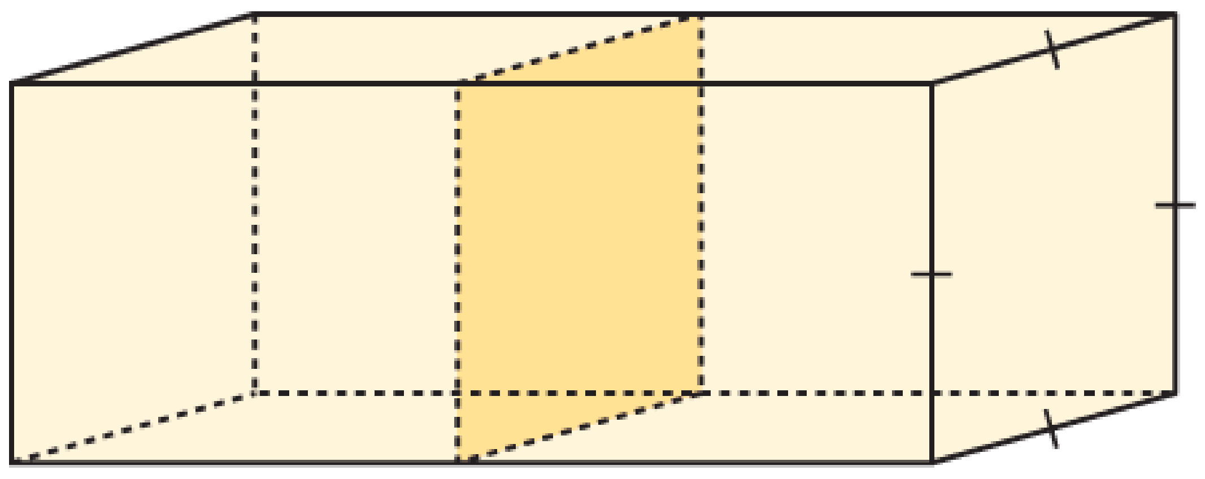
A piece of timber is a solid, and so is a rubbish bin.



CROSS-SECTIONS OF SOLIDS

A **cross-section** of a solid is the shape of a slice through it.

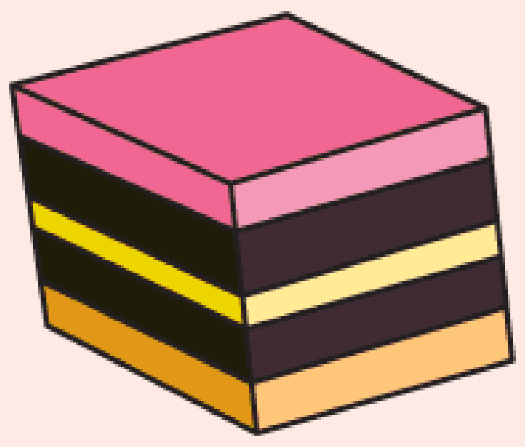
If we slice this box vertically, the cross-section is a square.



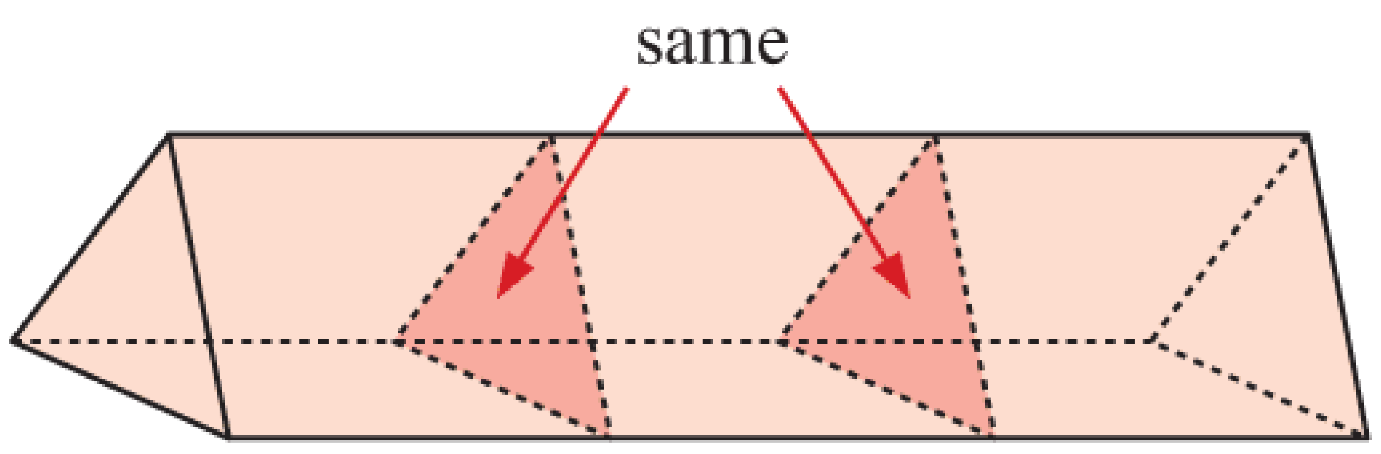
ACTIVITY 2 CROSS-SECTIONS OF SOLIDS

Draw cross-sections of:

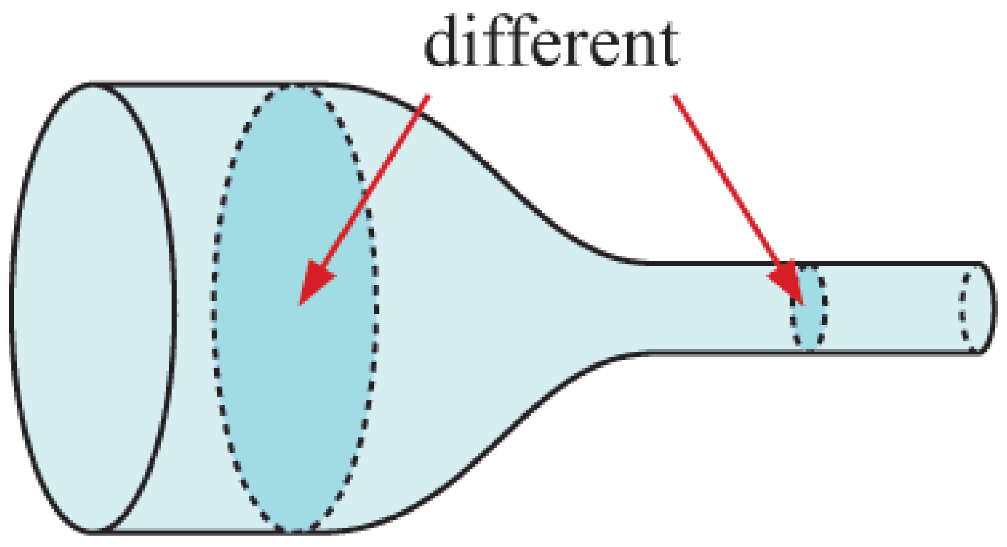
- a loaf of bread
- a Swiss roll
- a licorice allsort
- an empty match box.



For some solids, the cross-section is the same no matter where the slice is made. These solids are called **solids of uniform cross-section**.



uniform cross-section

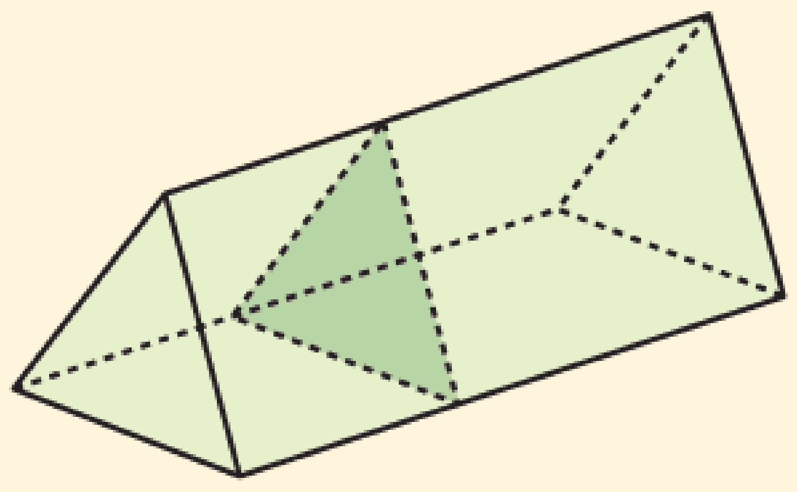
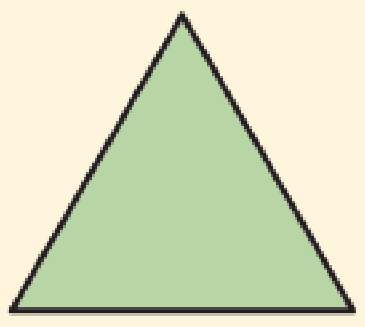
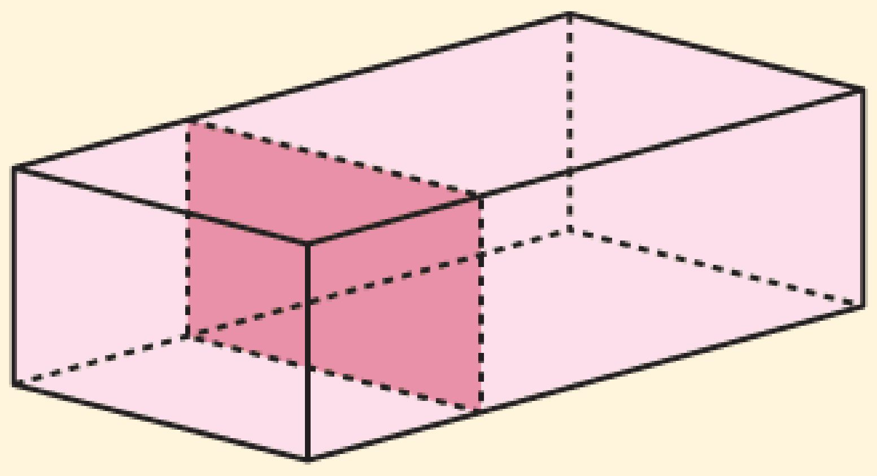
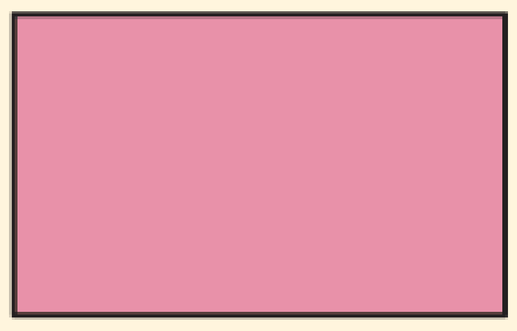
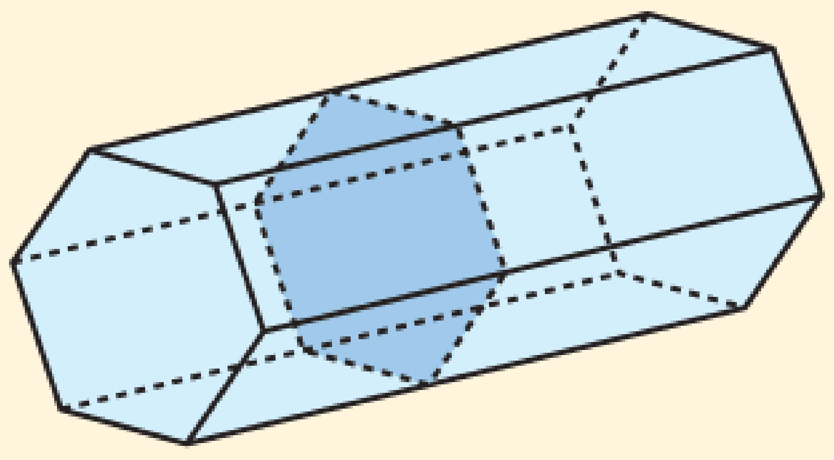
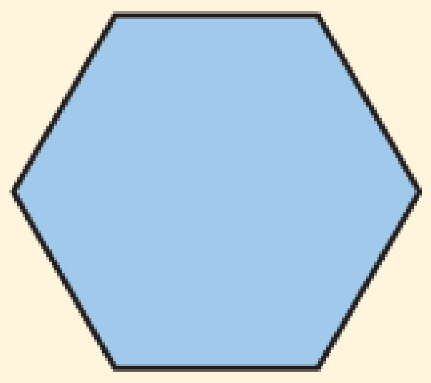


not uniform cross-section

PRISMS

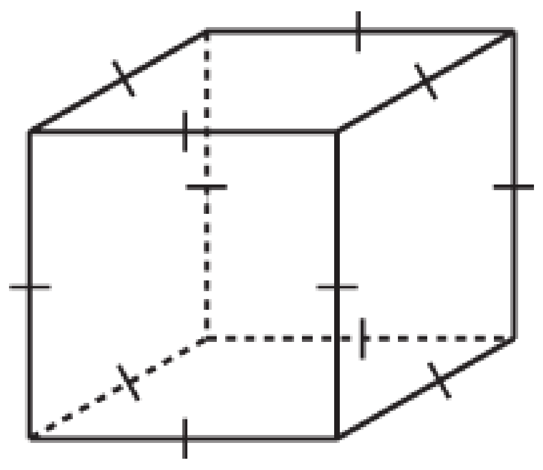
A **prism** is a solid with a uniform cross-section that is a polygon.

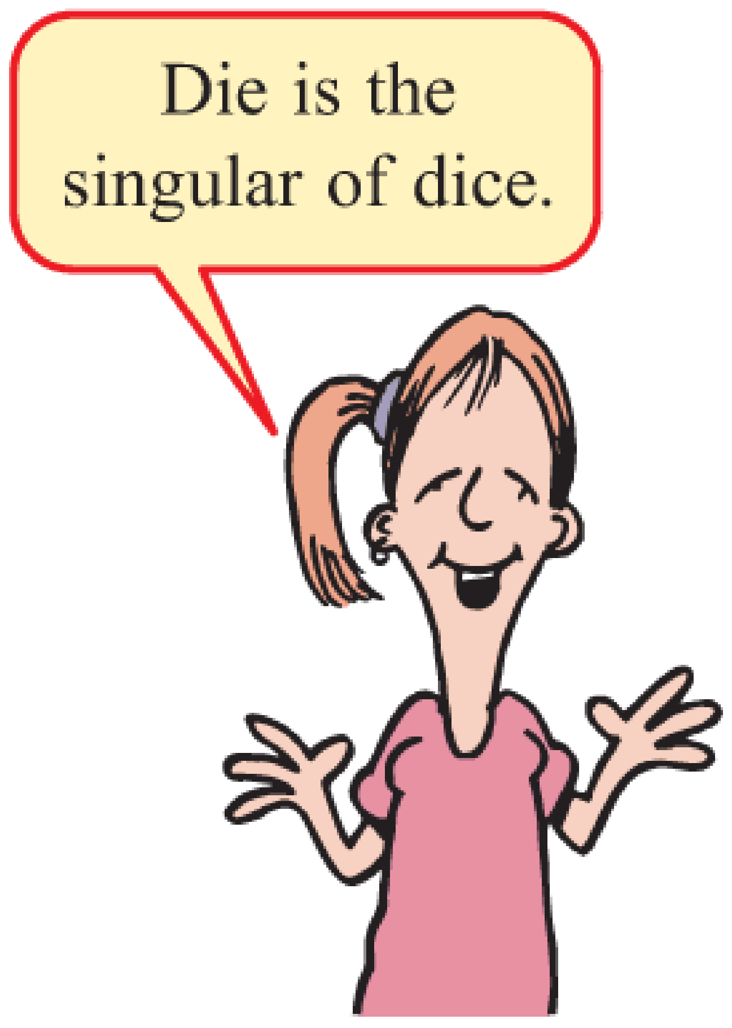
Prisms are named according to the shape of the cross-section.

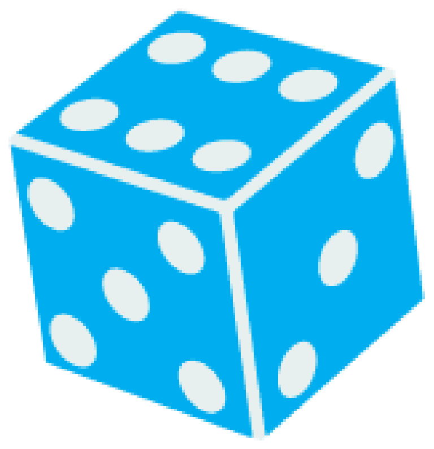
Name	*Figure*	*Cross-section*
Triangular prism		
Rectangular prism		
Hexagonal prism		

CUBES

A **cube** is a rectangular prism whose edges are all the same length.





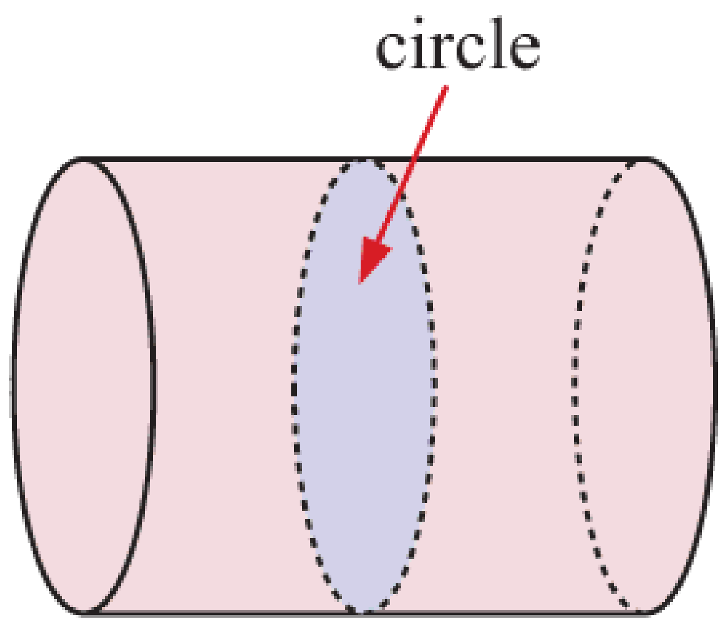


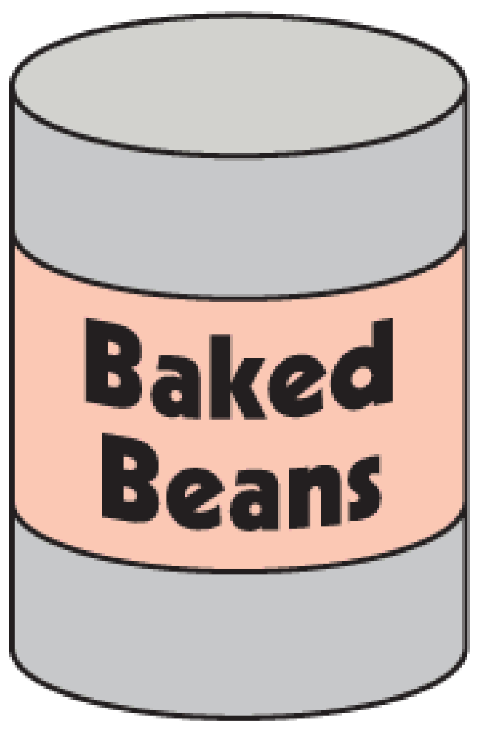
A die is an example of a cube.

CYLINDERS

A **cylinder** is a solid with a uniform cross-section that is a circle.

An aluminium can is an example of a cylinder.

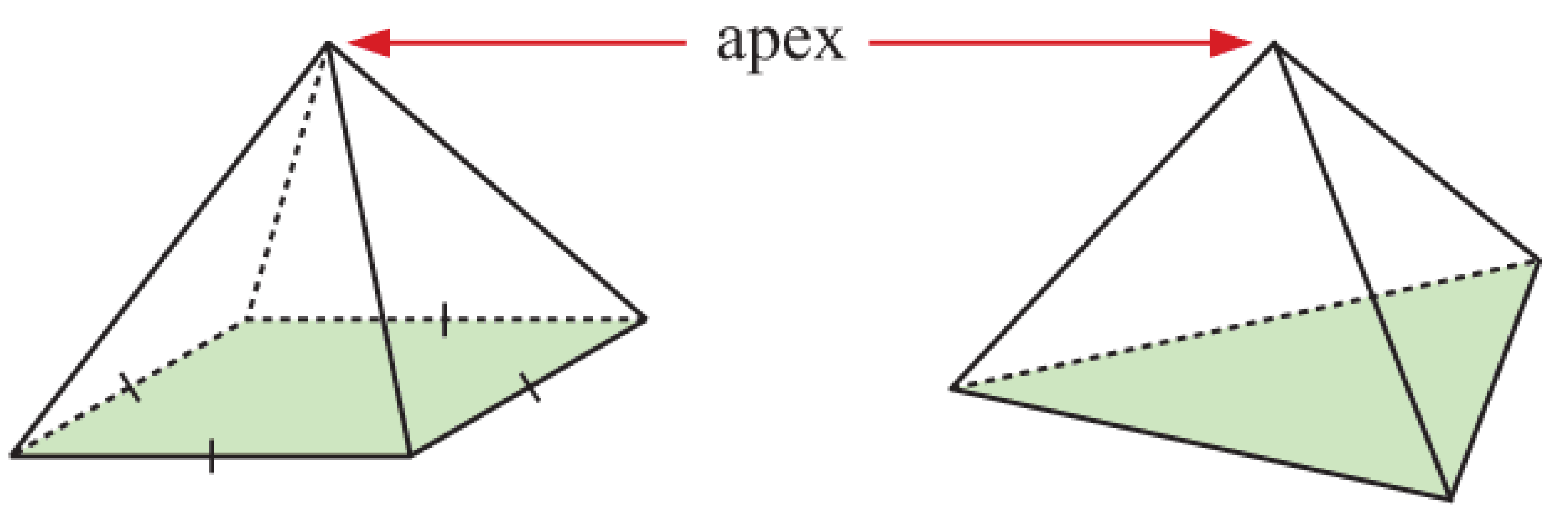




PYRAMIDS

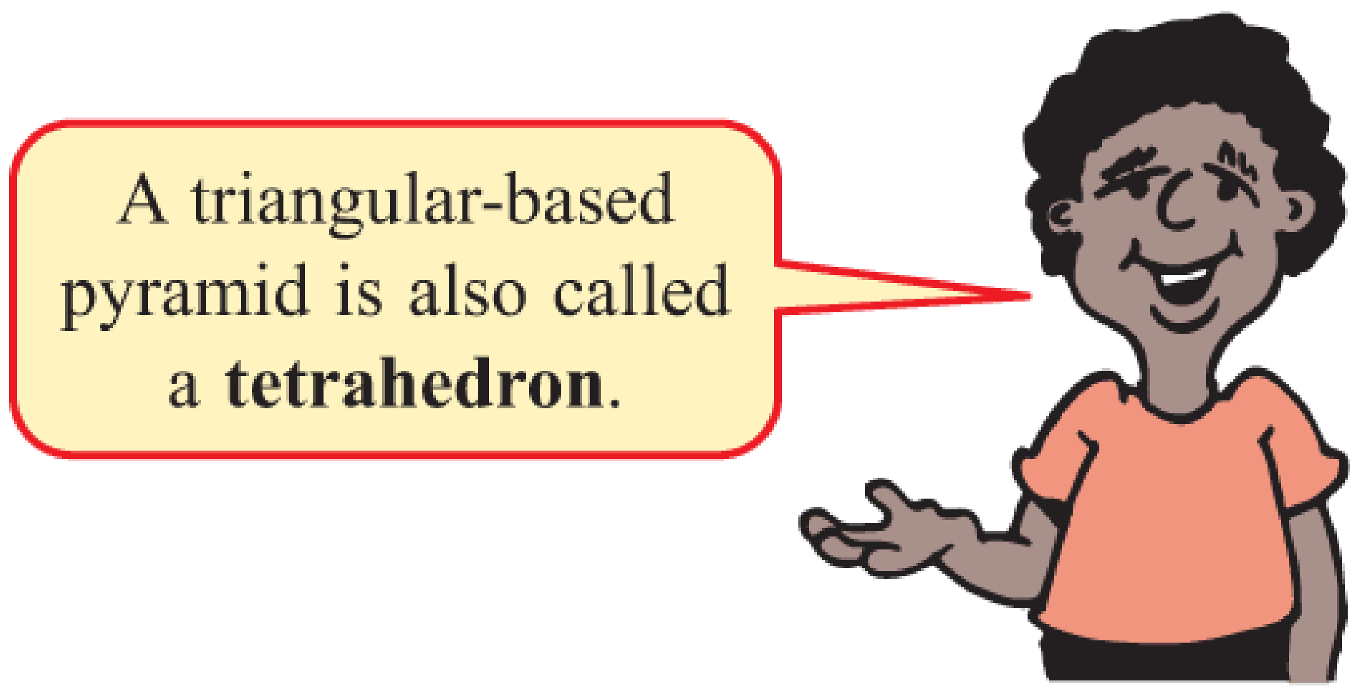
A **pyramid** is a solid with a polygon base. It has triangular faces which come from its base to meet at a point called the **apex**.

Pyramids are named according to the shape of their base.



square-based pyramid

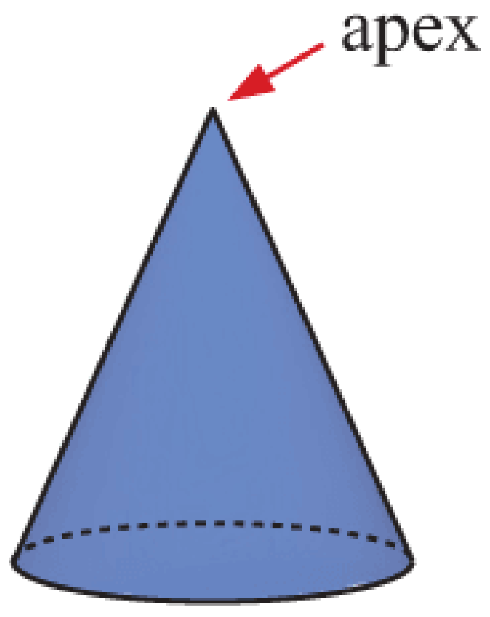
triangular-based pyramid



DEMO

CONES

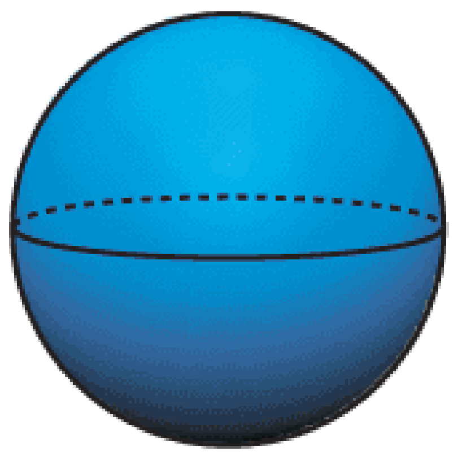
A **cone** is a solid with a circular base and a curved surface from the base to the apex.



cone

SPHERES

A **sphere** is a ball-shaped solid.

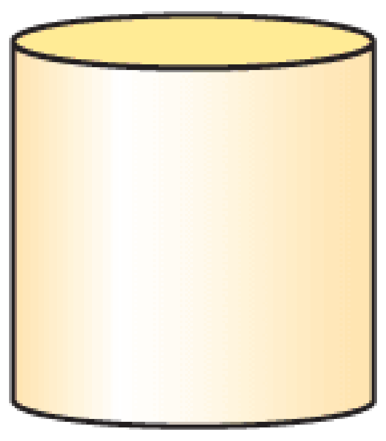


sphere

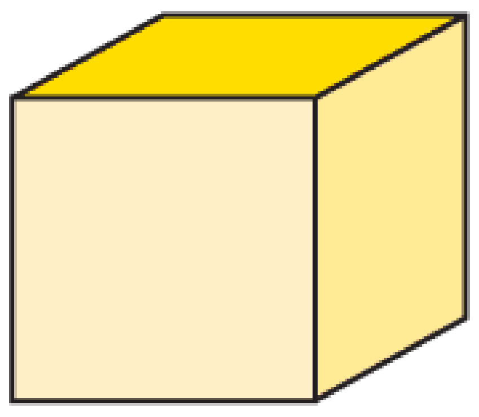
EXERCISE 5E

1 Name these solids:

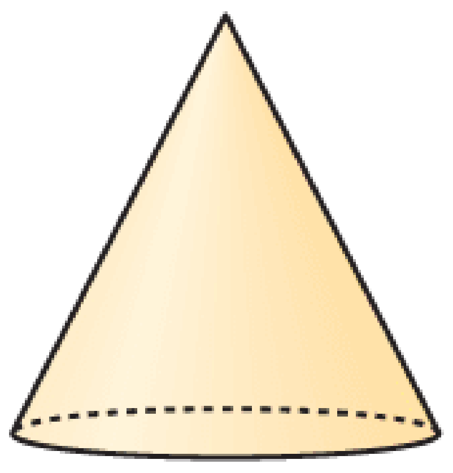
a



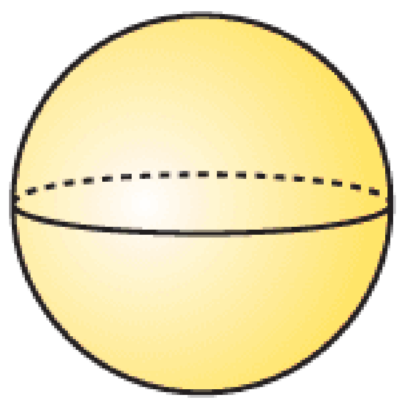
b



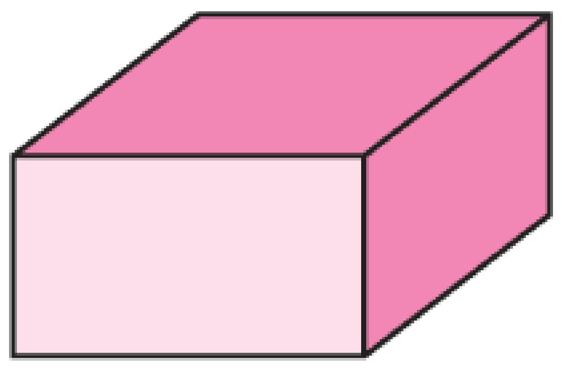
c



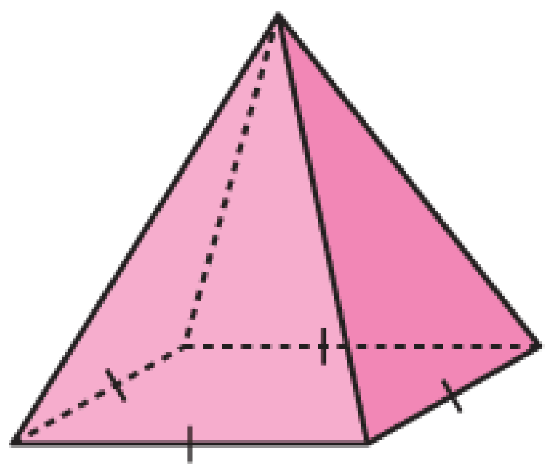
d



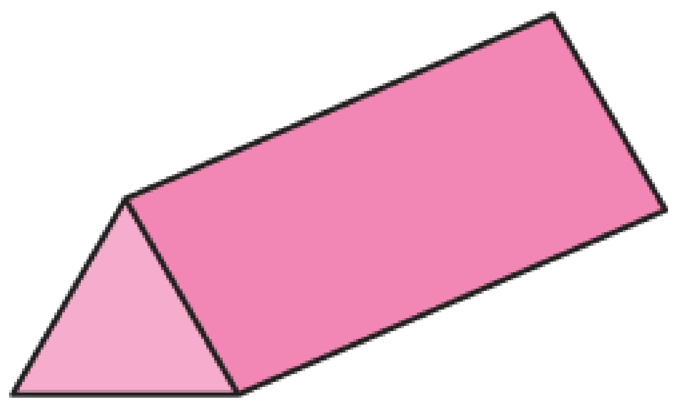
e



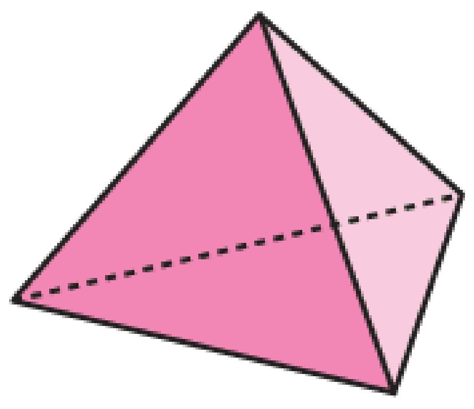
f



g



h



2 Which solid would best describe the shape of:

a a refrigerator

b a battery

c this tent?

3 State whether the following solids have:

A only flat surfaces
B only curved surfaces
C both flat and curved surfaces.

a triangular prism **b** sphere **c** square-based pyramid **d** cylinder

4 Name a solid which has:

a only rectangular surfaces **b** only triangular surfaces.

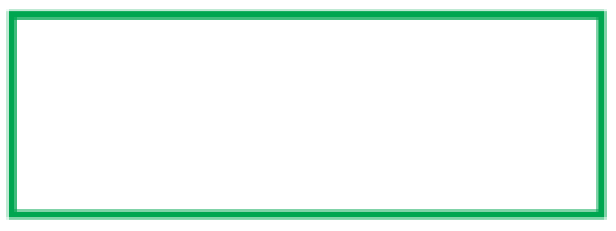
F SKETCHING SOLIDS

To sketch a solid, we need to represent a three-dimensional object on a two-dimensional page.

This task is not easy, but the following step-by-step instructions will help you.

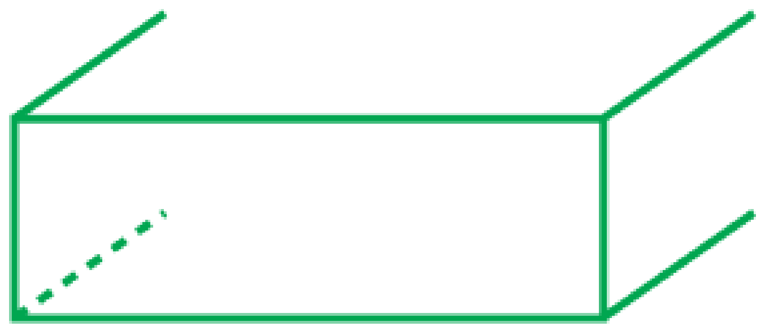
RECTANGULAR PRISM

Step 1:



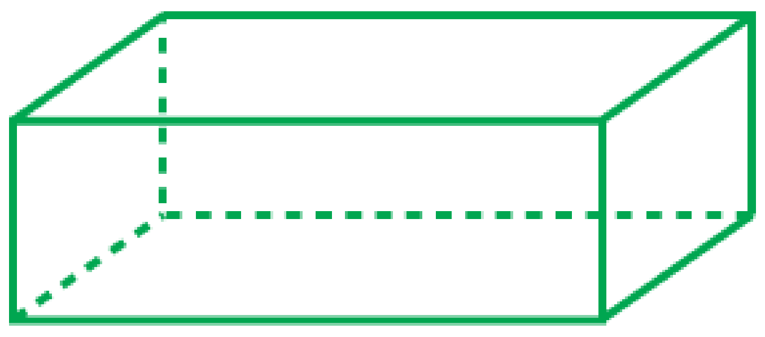
Draw a rectangle for the **front face**.

Step 2:



From each of the vertices draw an angled line back. Use dashes for the edge that is hidden. The lengths are drawn slightly shorter than their real length to give perspective.

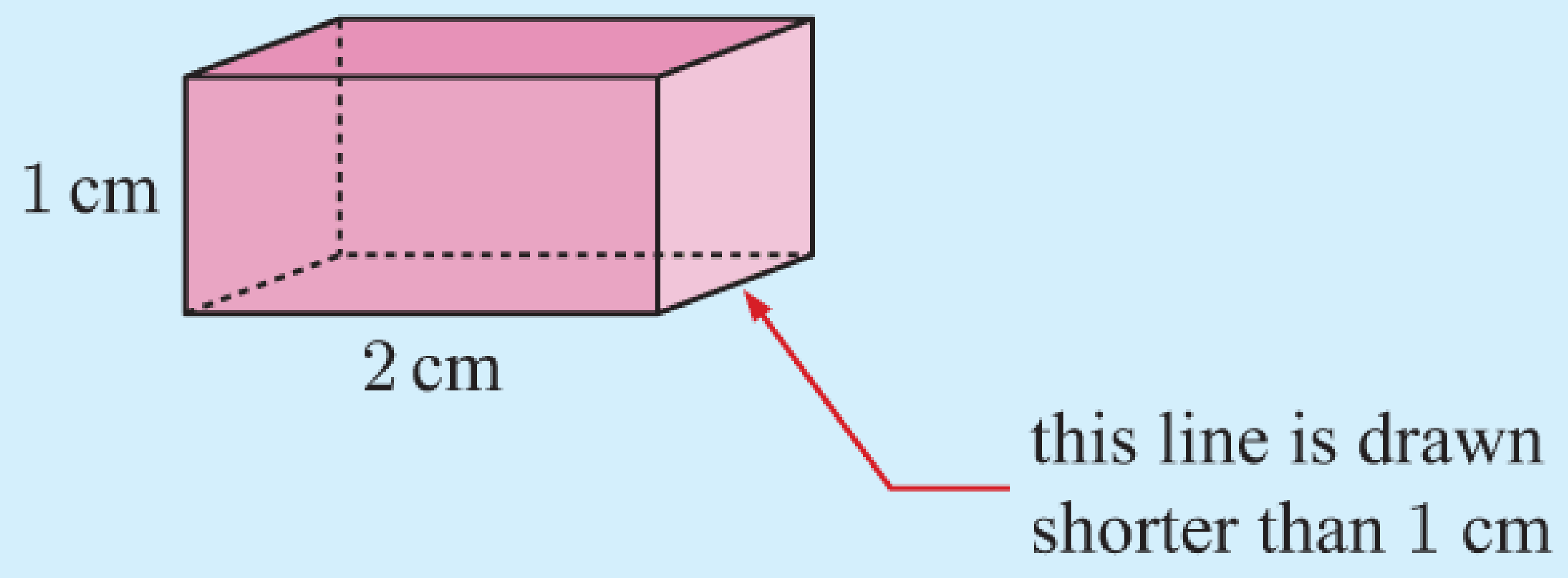
Step 3:

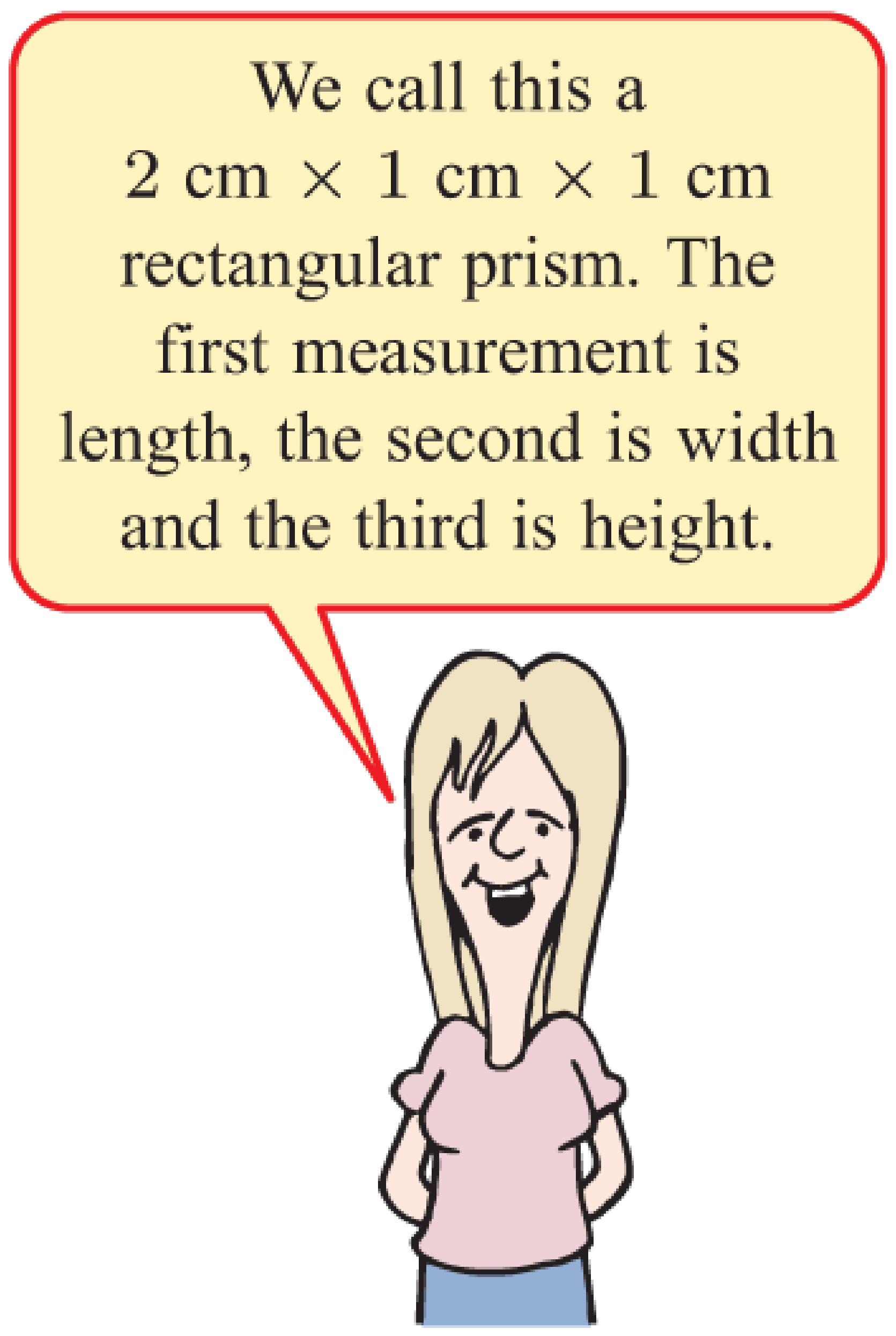


Complete the drawing by joining the appropriate vertices with another rectangle. Use dotted lines for the hidden edges.

Example 3 🔊 **Self Tutor**

Draw a rectangular prism 2 cm long by 1 cm wide by 1 cm high.





PYRAMIDS

In the picture of the pyramid alongside, only five edges, four vertices, and two faces can be seen.

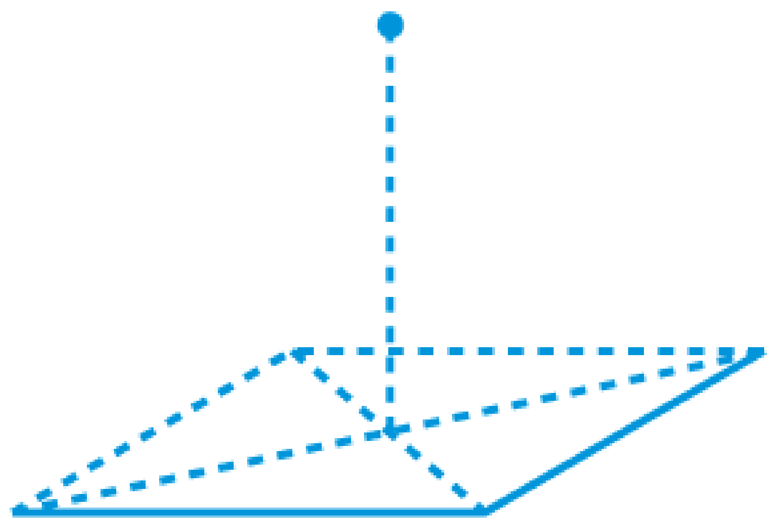
In fact, this pyramid has a square base and four triangular faces.

To draw a square-based pyramid we use these steps:

Step 1:

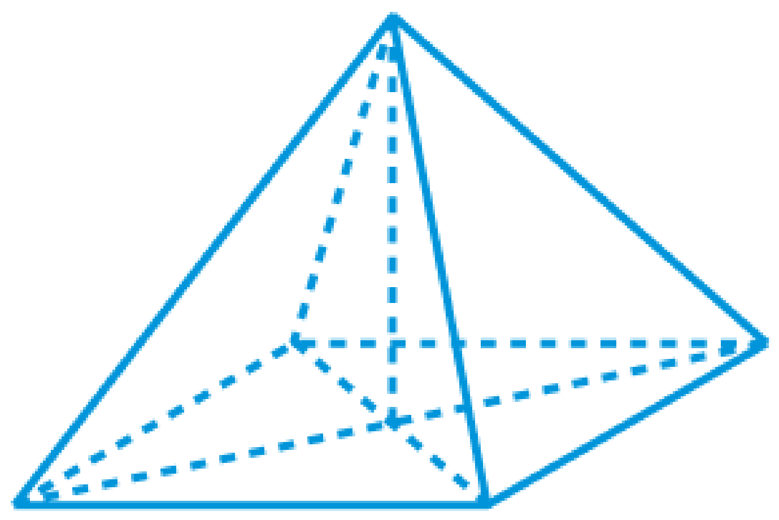
Draw a parallelogram to represent the base. Use dashed lines for the edges that will be hidden.

Step 2:



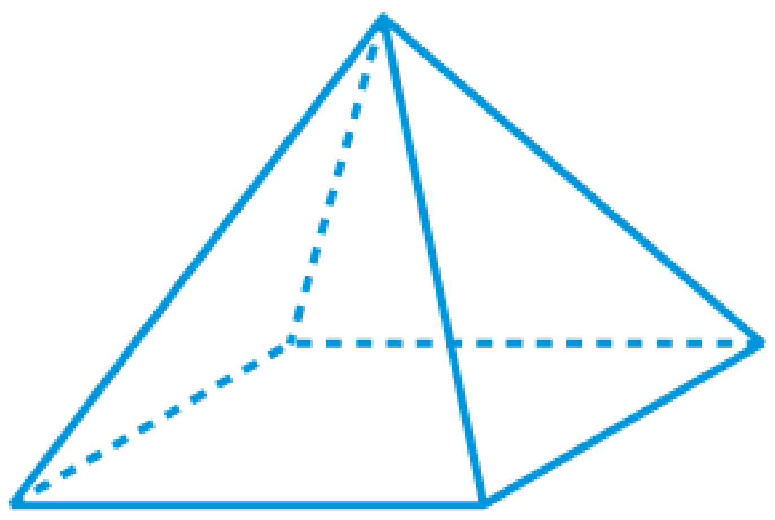
To find the centre of the parallelogram, draw its diagonals and find their intersection. Draw a point above the centre to represent the apex of the pyramid.

Step 3:



Join each vertex of the base to the apex to complete the pyramid. Use a dashed line for the hidden edge.

Step 4:



Remove the construction lines.

CYLINDERS

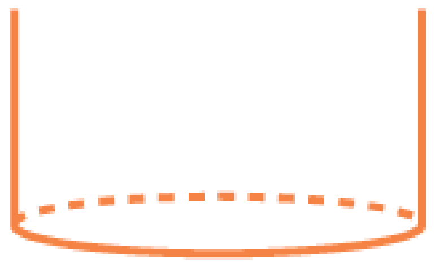
You are probably familiar with cylinders such as tin cans. Their top and bottom are both circles, but when we look at them on an angle they will *appear* to be **ellipses** or ovals.

To draw a cylinder we use these steps:

Step 1:

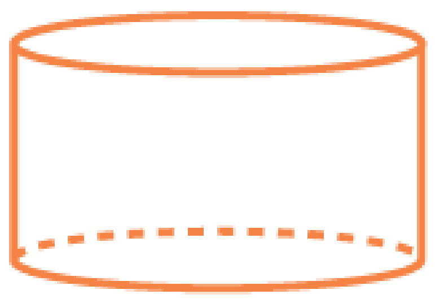
Draw an ellipse to represent the base. Use dashes for the top half.

Step 2:



Draw the sides of the cylinder from the "ends" of the ellipse.

Step 3:



Complete the cylinder by drawing another ellipse on the top.

CONES

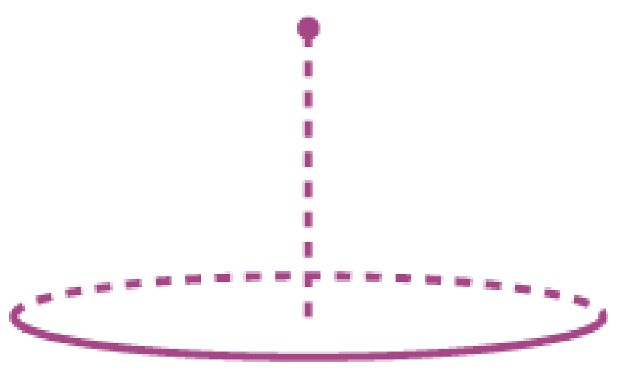
Just like a cylinder, we represent the circular base of a cone using an ellipse.

To draw a cone we use these steps:

Step 1:

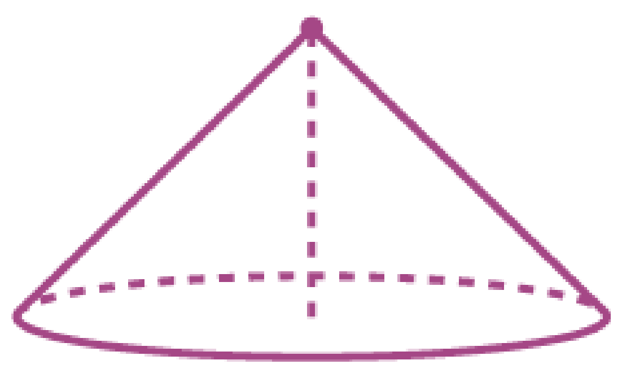
Draw an ellipse to represent the base. Use dashes for the top half.

Step 2:



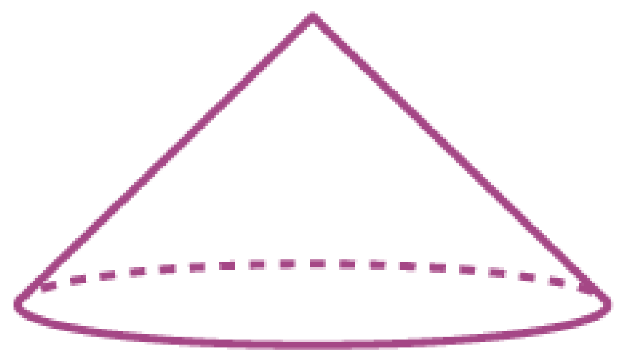
Mark a point directly above the centre of the ellipse. This will be the apex of the cone.

Step 3:



Join the "ends" of the ellipse to the apex to complete the cone.

Step 4:



Remove the construction line.

EXERCISE 5F

1 Draw a rectangular prism that is:

a 1 cm × 1 cm × 2 cm **b** 3 cm × 2 cm × 1 cm **c** 4 cm × 2 cm × 3 cm

2 Sketch:

- **a** a square-based pyramid
- **b** a hexagonal prism
- **c** a triangular-based pyramid
- **d** a hexagonal-based pyramid.

3 Draw:

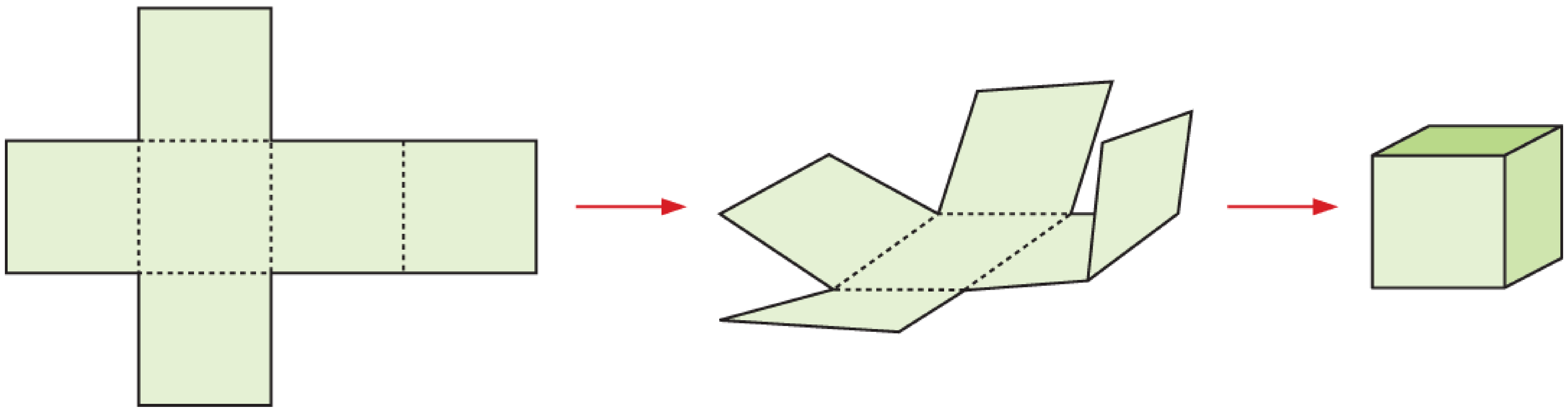
- **a** a cylinder which is 3 cm high and whose base is 2 cm wide
- **b** a cone which is 4 cm high and whose base is 3 cm wide.

4 Sketch a sphere. Use shading to show how it curves.

G NETS OF SOLIDS

Nets are patterns which can be folded along certain lines to form three-dimensional models of solids.

For example, when this net is cut out and folded along the dashed lines, we form a **cube**.



DEMO

ACTIVITY 3 NETS

Click on the icon to obtain these printable nets.

PRINTABLE NETS

Print them onto light card, and use them to construct a:

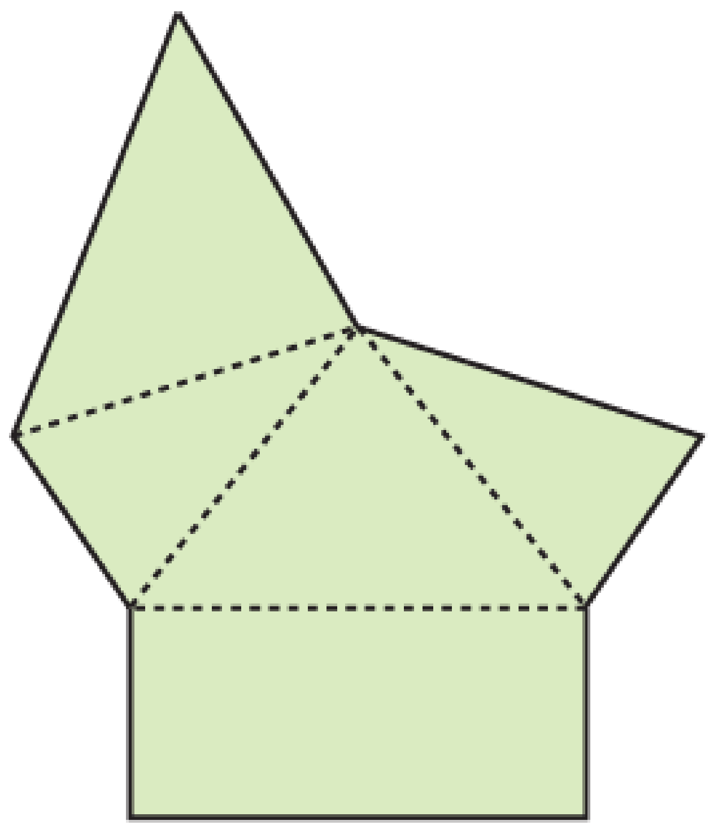
- cube
- square-based pyramid
- triangular prism.

EXERCISE 5G

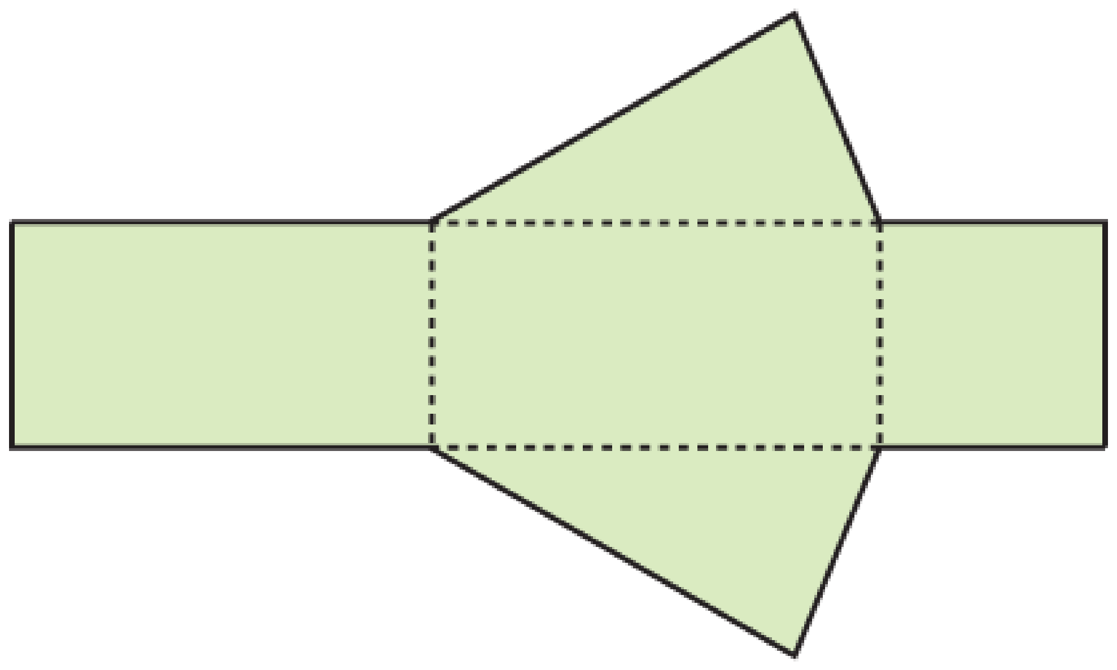
1 Draw and name the solids which would be formed from these nets.

PRINTABLE NETS

a



b



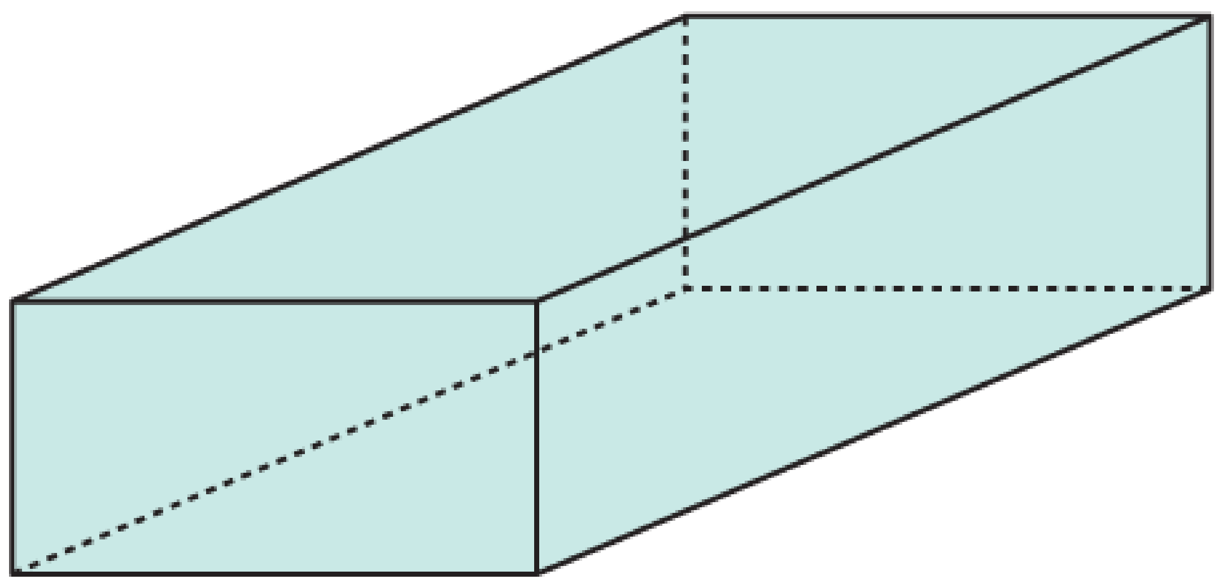
2 Draw a net for a:

a triangular-based pyramid **b** hexagonal prism.

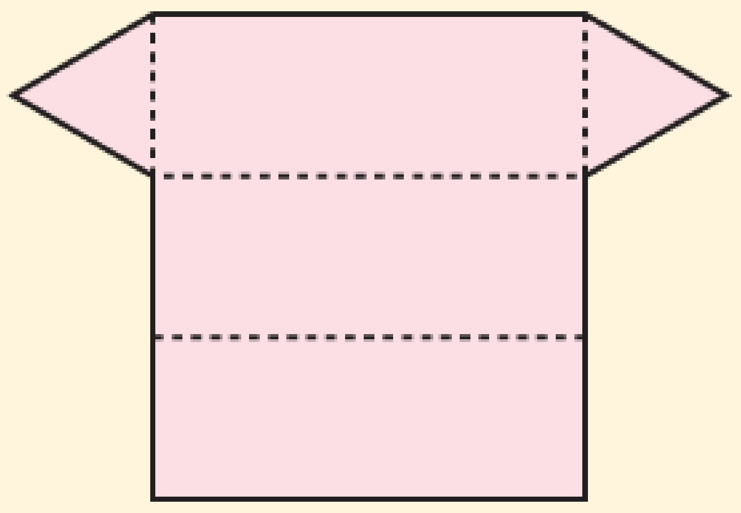
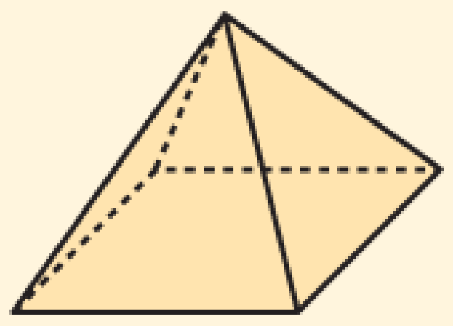
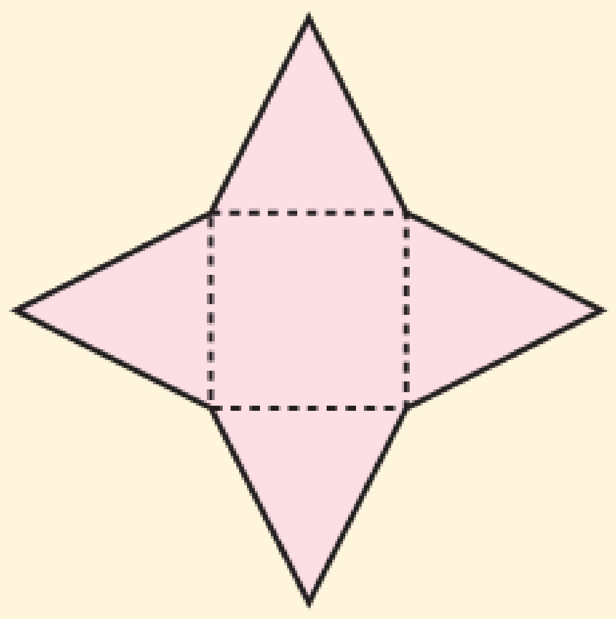
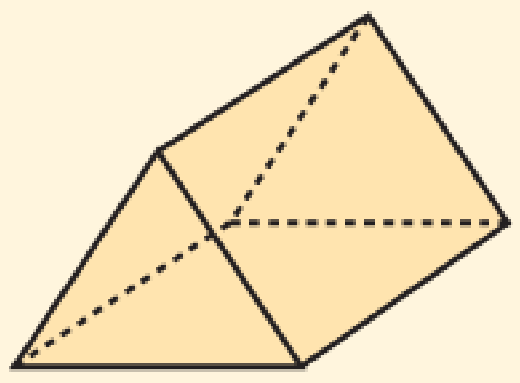
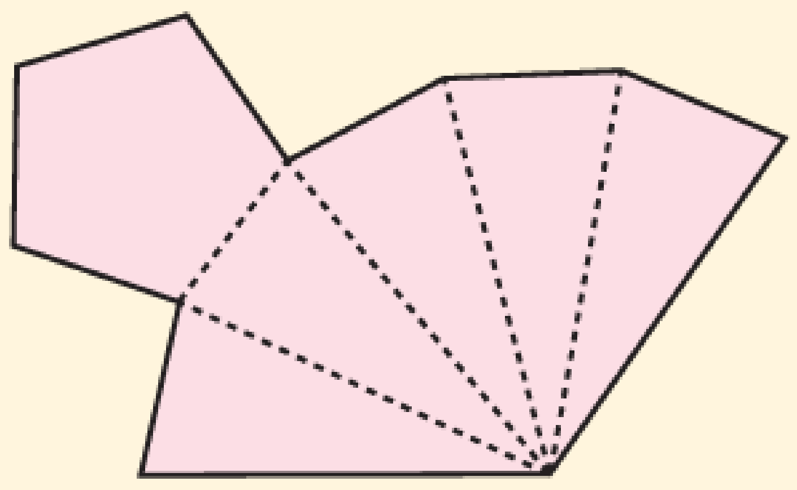
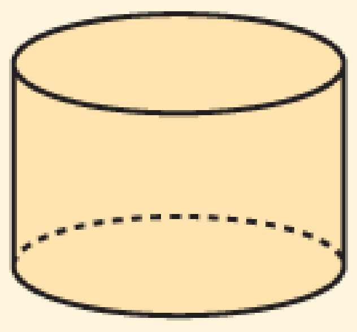
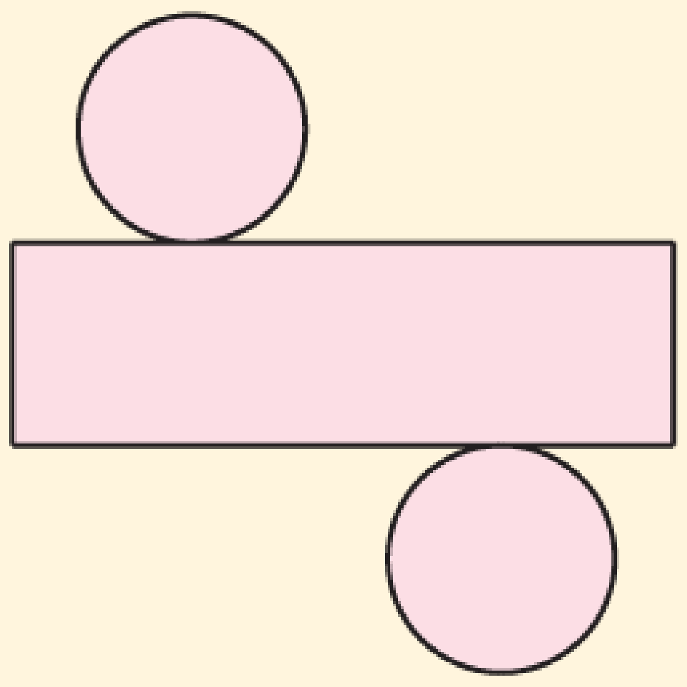
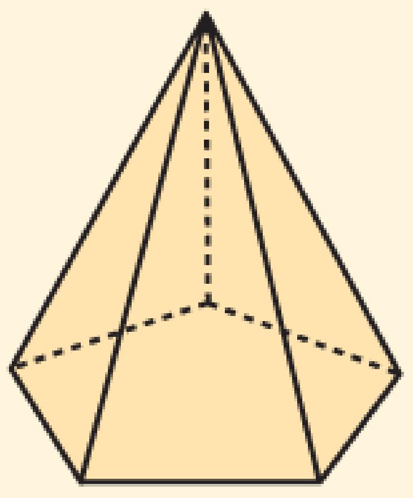
3 **a** Draw the net which could be used to construct a box like this one.

b How would you change this net so that the box is open at the top?

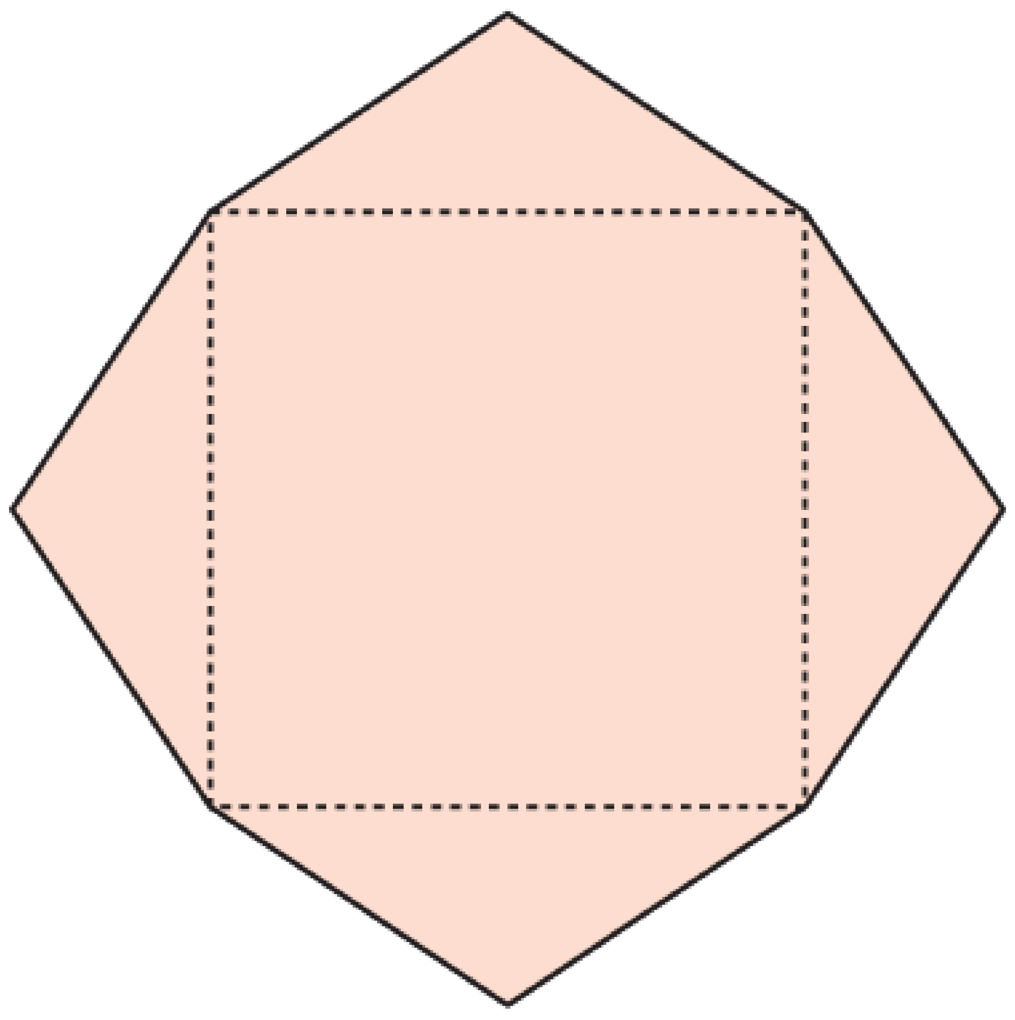
c How would you sketch the box to show it is open at the top?



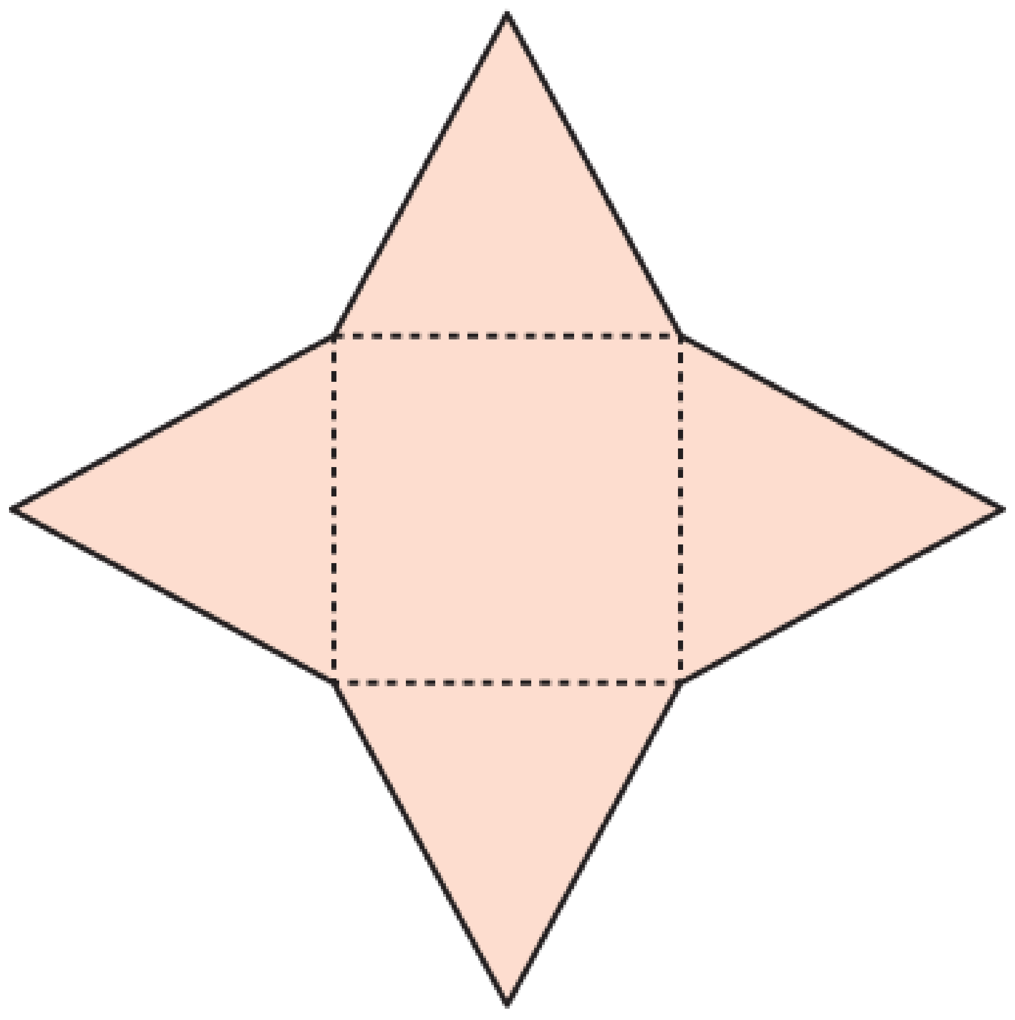
4 Match the net given in the first column with the correct solid and the correct name.

Net	*Solid*	*Name*
a 	**A** 	**1** pentagonal-based pyramid
b 	**B** 	**2** cylinder
c 	**C** 	**3** triangular prism
d 	**D** 	**4** square-based pyramid

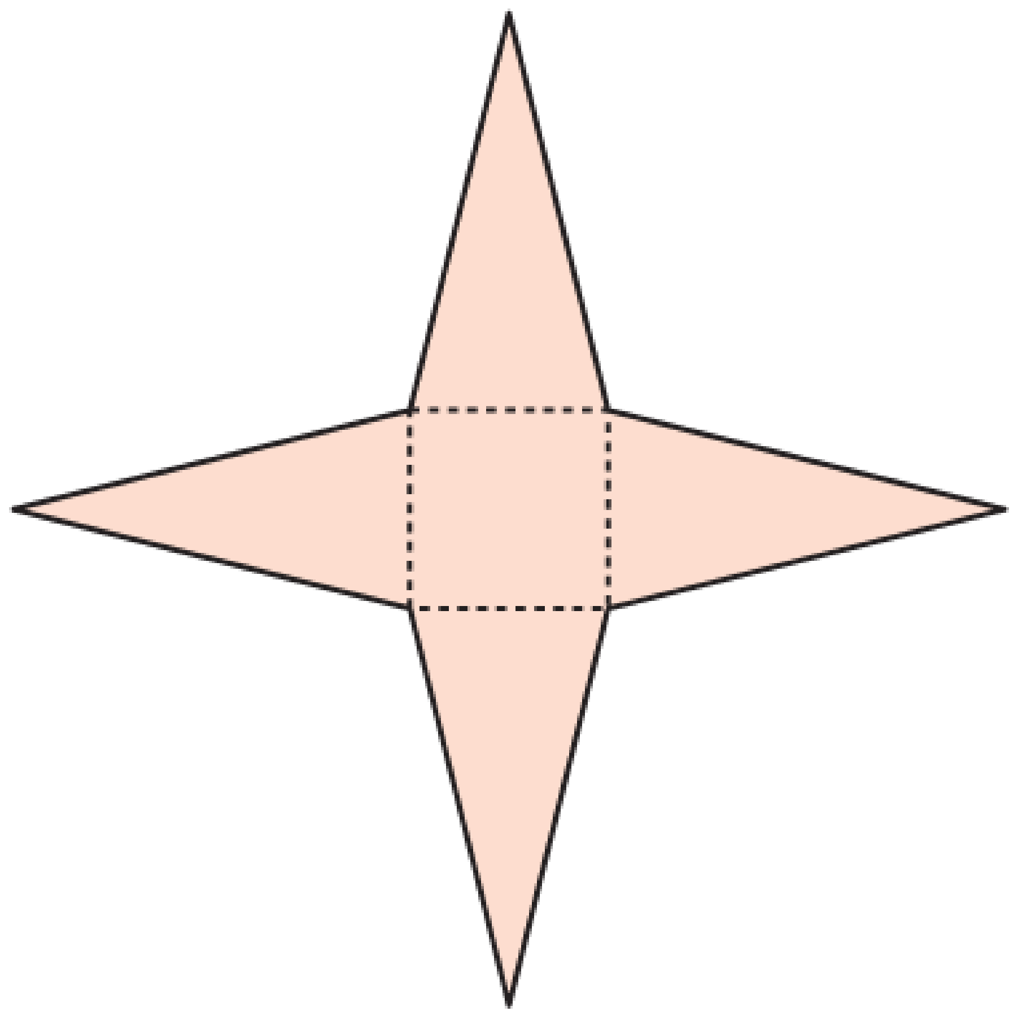
5 Three students were asked to draw a net for a square-based pyramid. The nets they drew are shown below:



Claire



Derek



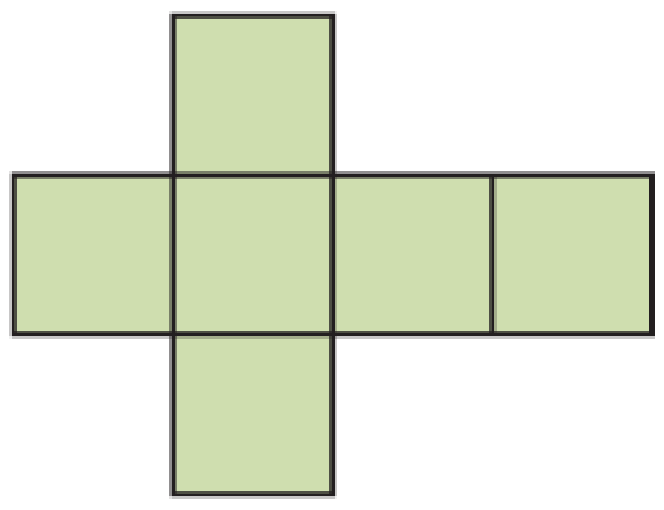
Eric

a Explain why it is not possible to construct a pyramid from Claire's net.

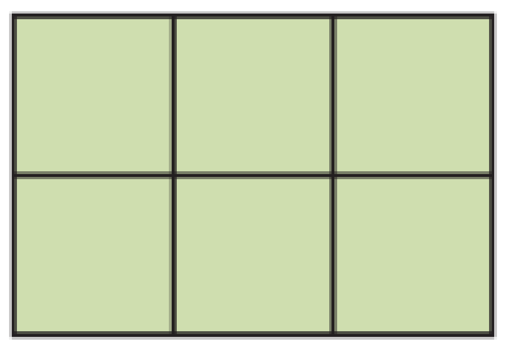
b Which of the remaining nets will produce a higher pyramid? Explain your answer.

6 Which of these nets can be used to make a cube?

a

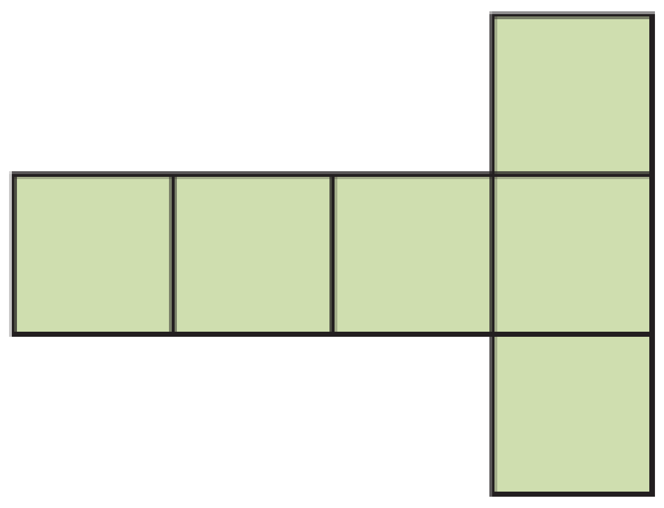


b

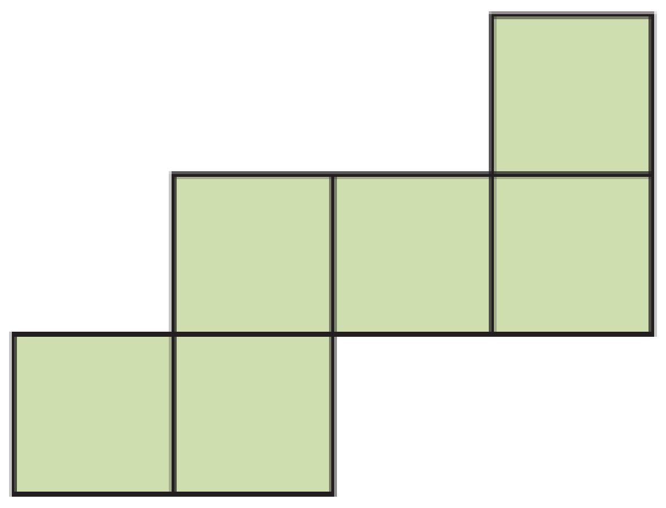


DEMO

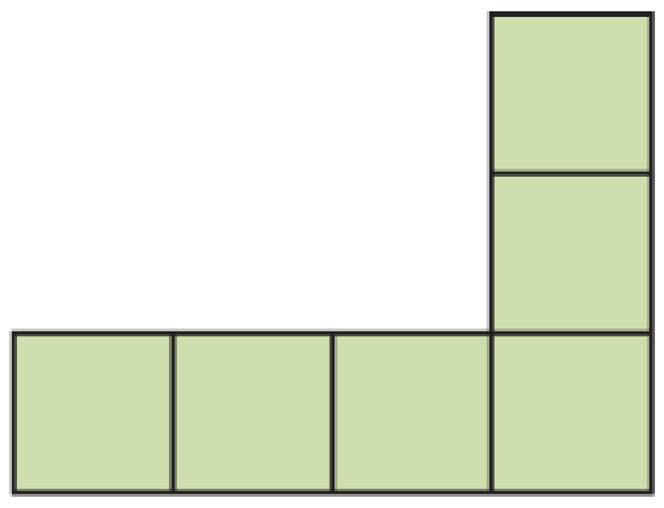
c



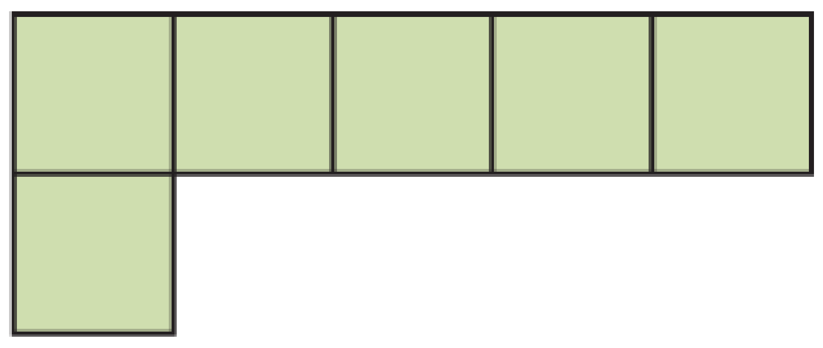
d



e

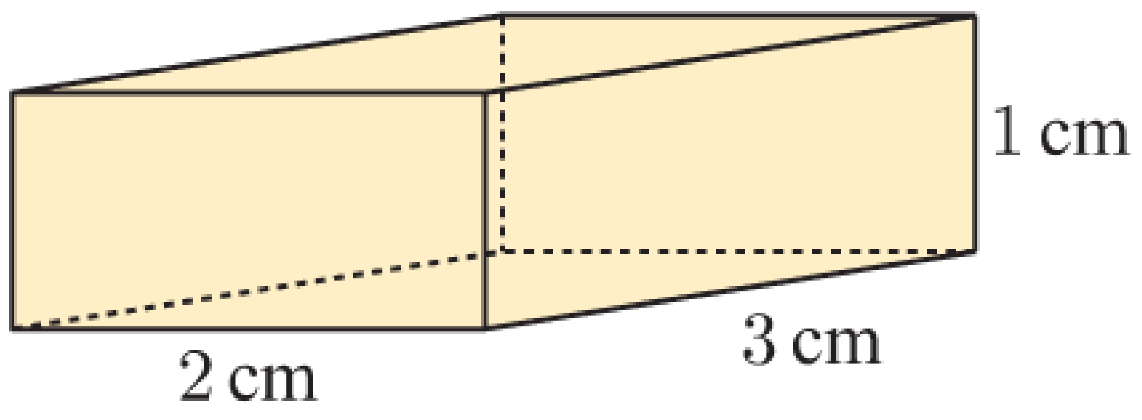


f

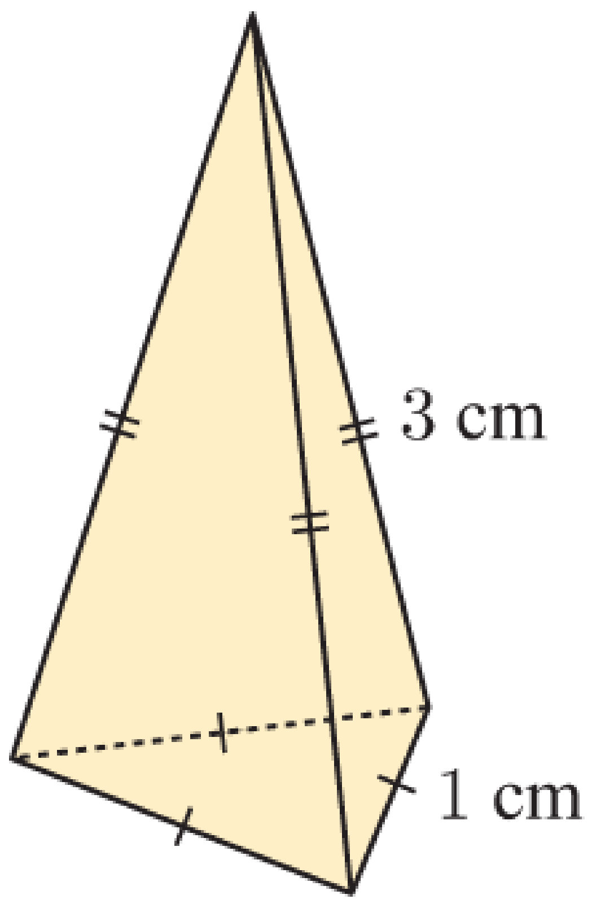


7 Draw an exact net which could be used to construct:

a



b



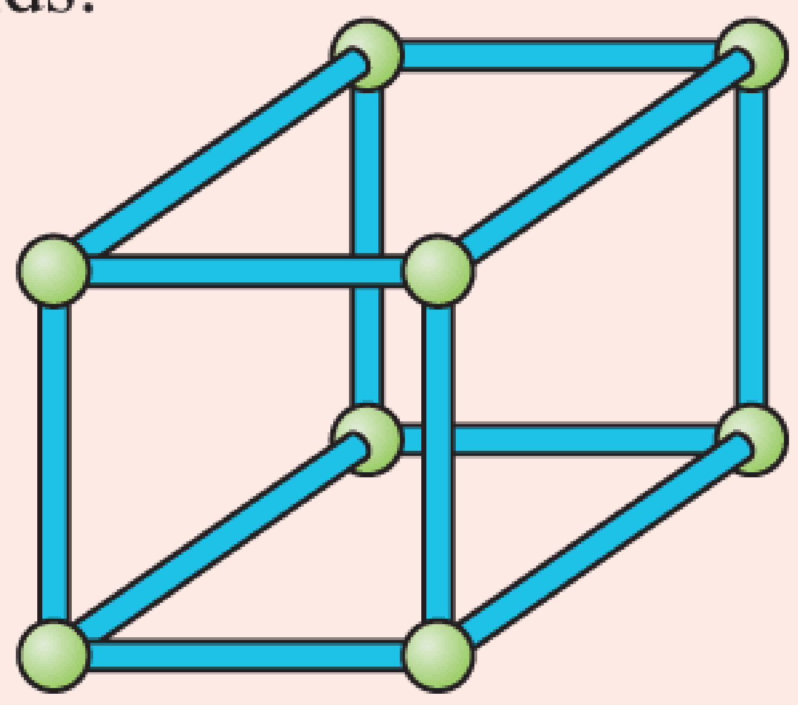
ACTIVITY 4 MODELS OF SOLIDS

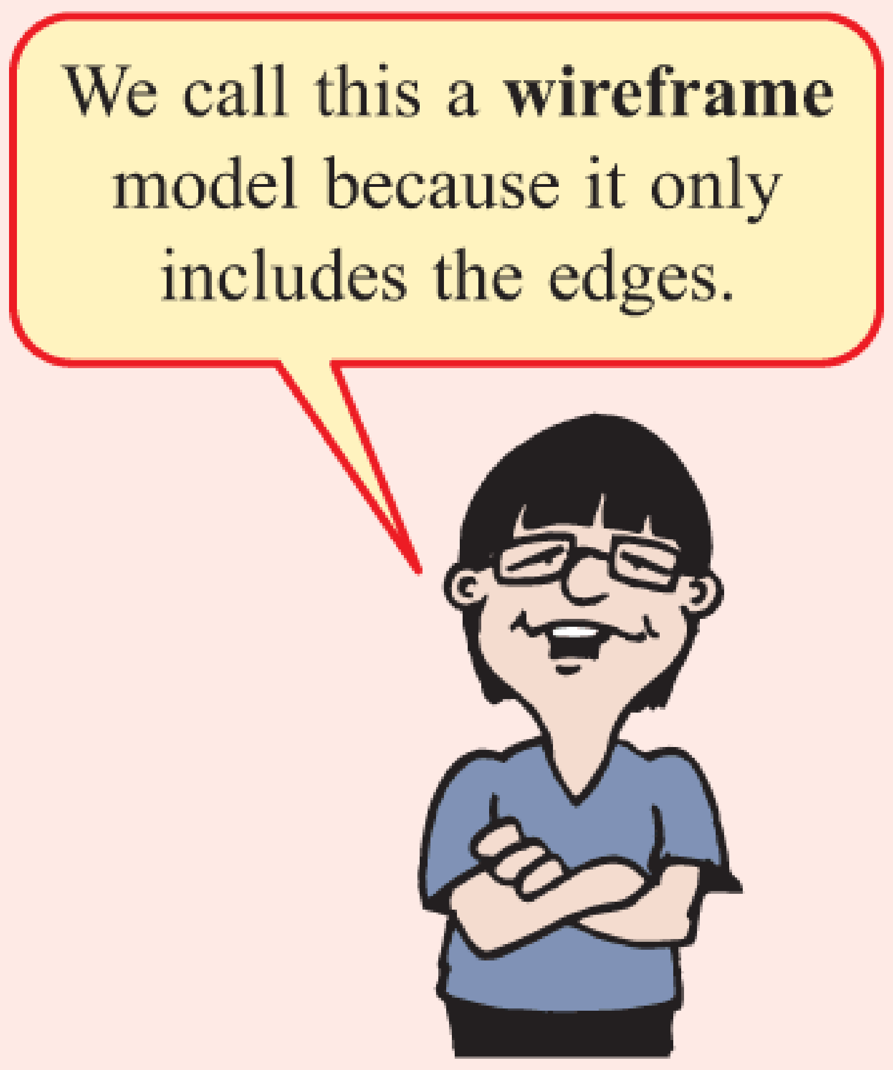
You will need: plastic straws, modelling clay

What to do:

Using straws as the edges, and modelling clay to hold the edges together, create the following solids:

- cube
- rectangular prism
- triangular prism
- square-based pyramid
- triangular-based pyramid



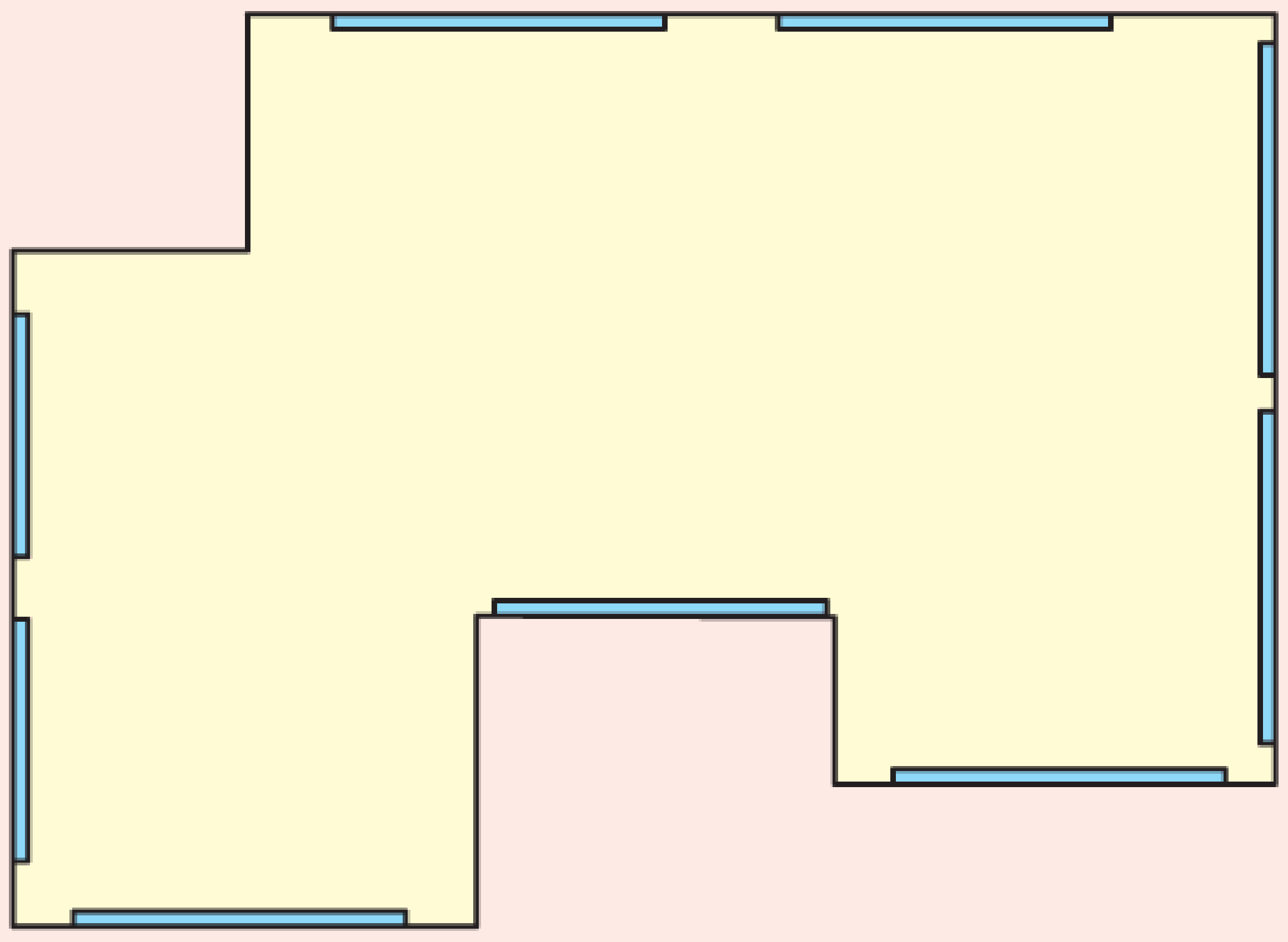


Experiment using straws of different lengths. For each solid, determine which edges must be the same length, and which edges can be different lengths.

ACTIVITY 5 ART GALLERIES

Suppose you are the manager of an art gallery. A room containing important paintings has the shape of a polygon with right angled corners.

Security guards must be positioned in the corners to keep watch over the paintings. How many security guards are needed to view the entire gallery?



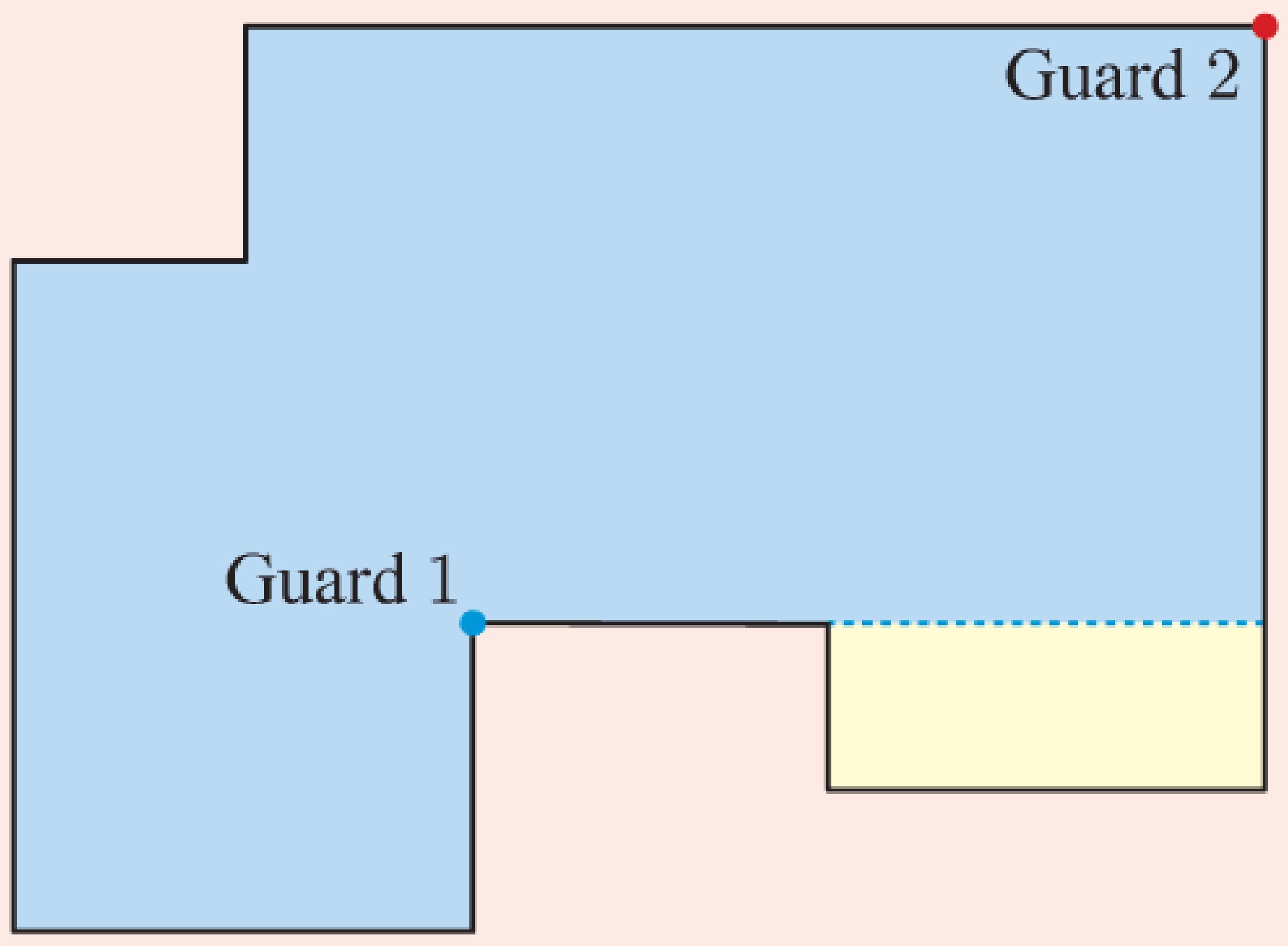
This problem can be solved by performing these steps:

1. Count the number of corners in the gallery.
2. Divide this number by 4, ignoring any remainder. This is the number of guards needed.

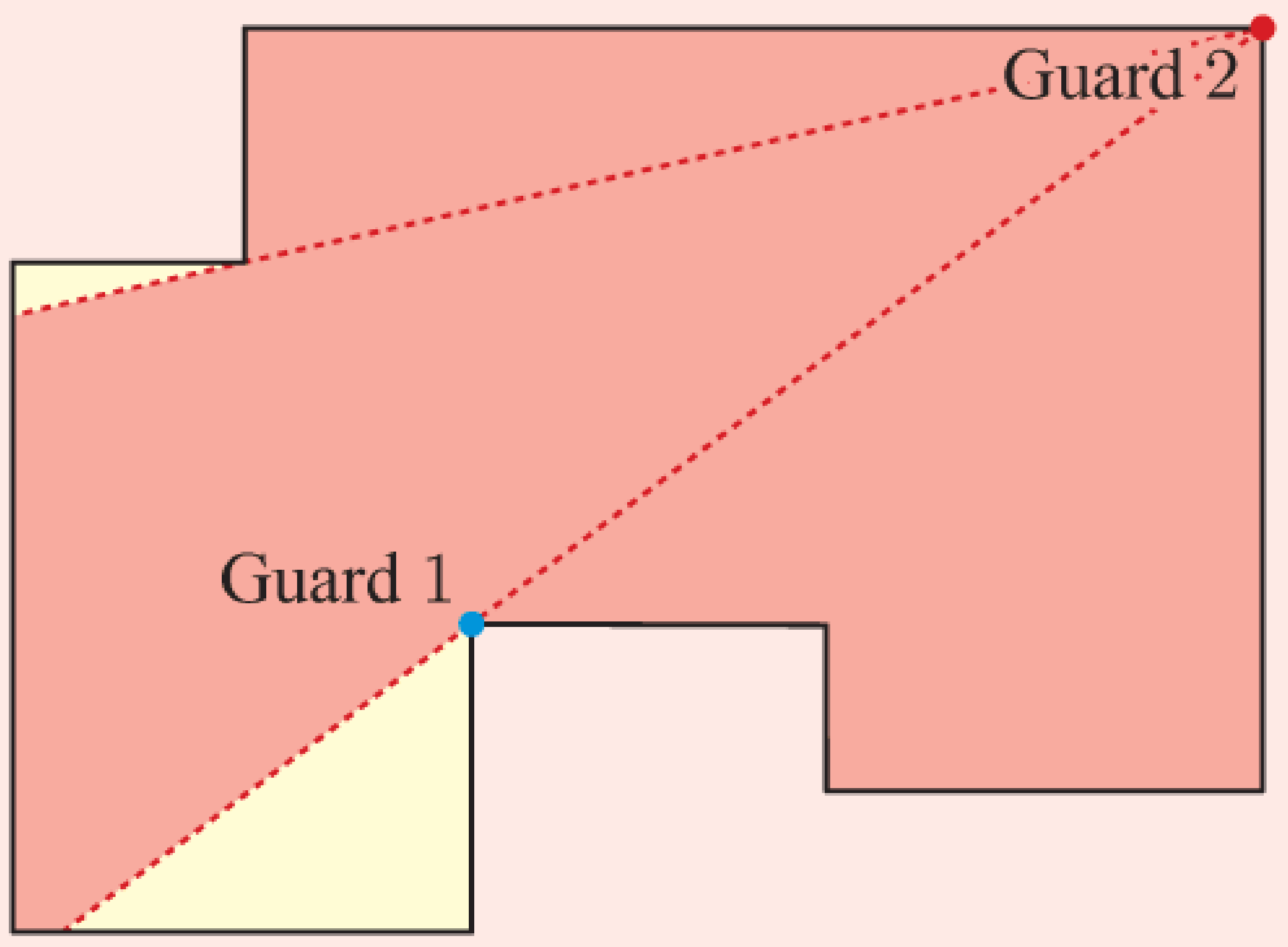
The gallery has 10 corners, and $10 \div 4 = 2$ remainder 2, so 2 guards are needed.

For example, for the gallery given, we could position the two security guards as shown below.

Guard 1 can see the blue shaded area.



Guard 2 can see the red shaded area.

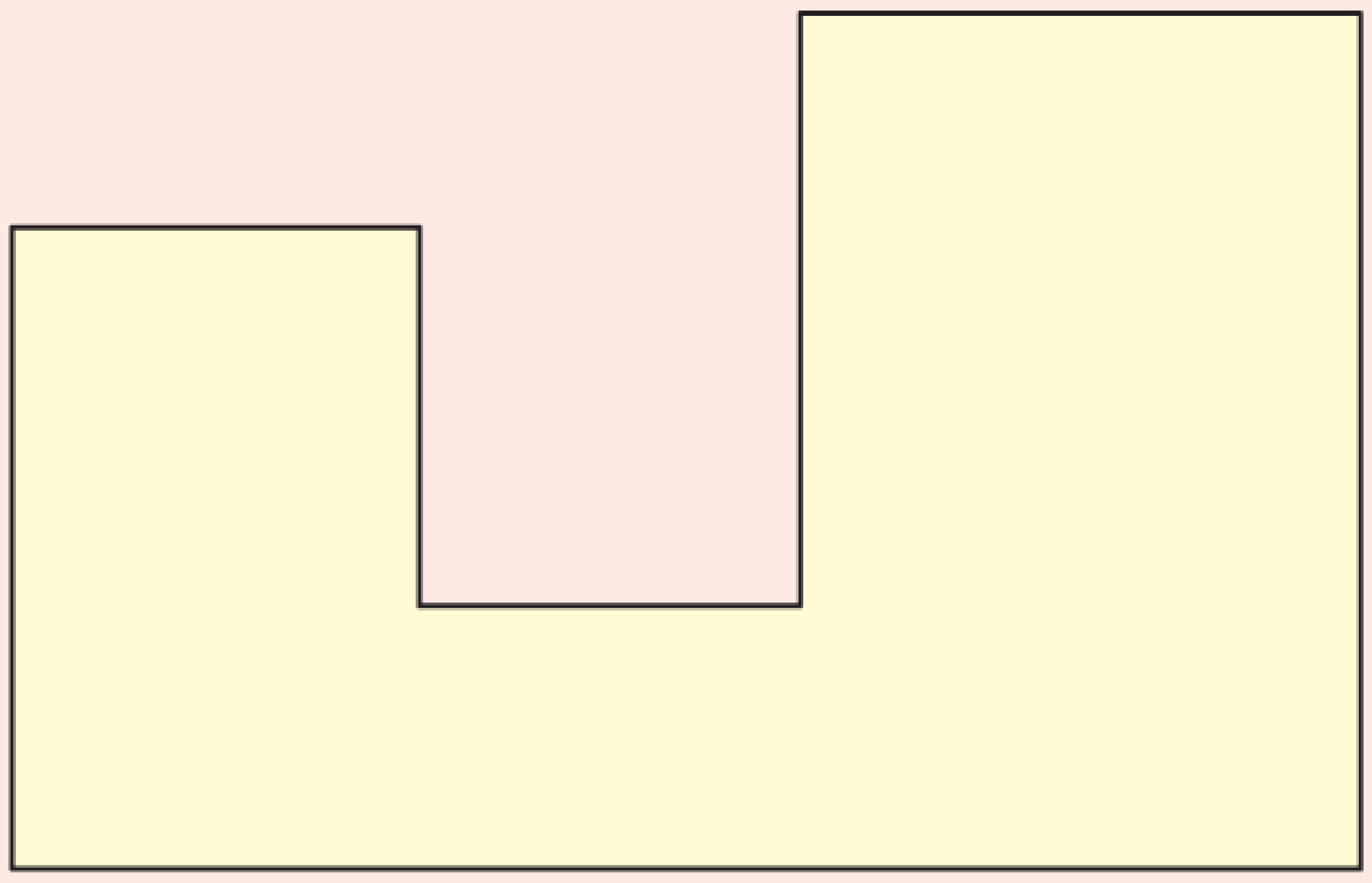


Together, they can see the whole gallery.

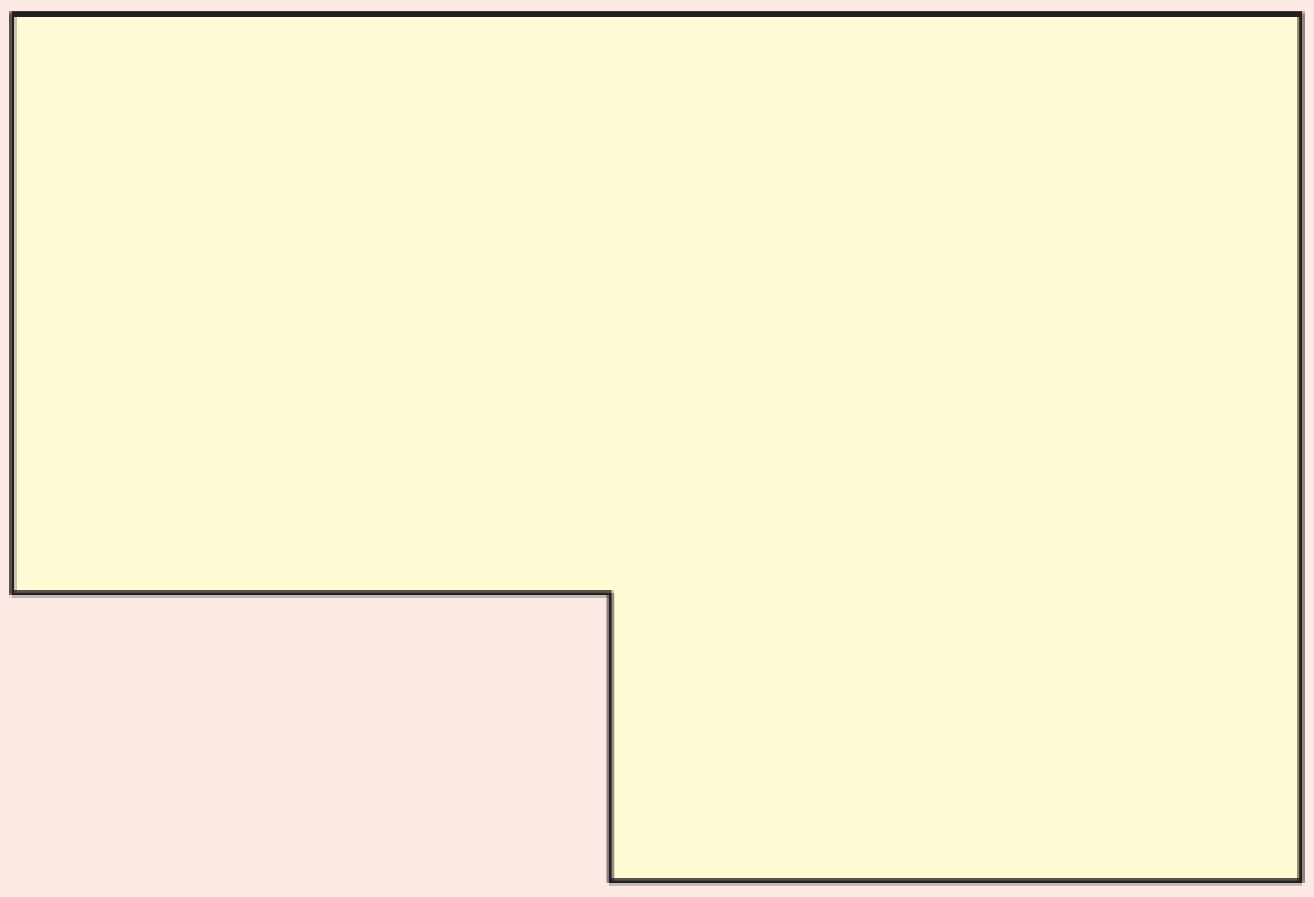
What to do:

Determine how many guards are needed to view each of these galleries, and decide where the guards should be positioned:

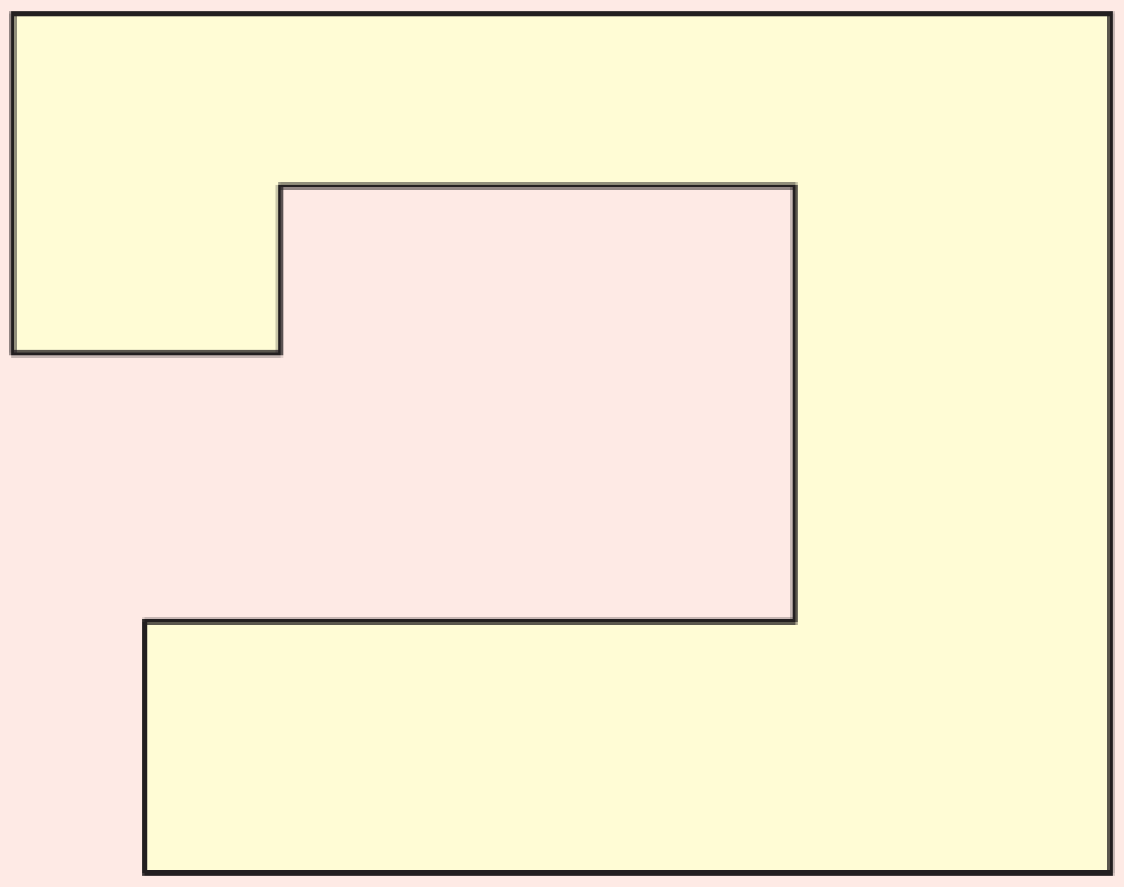
1



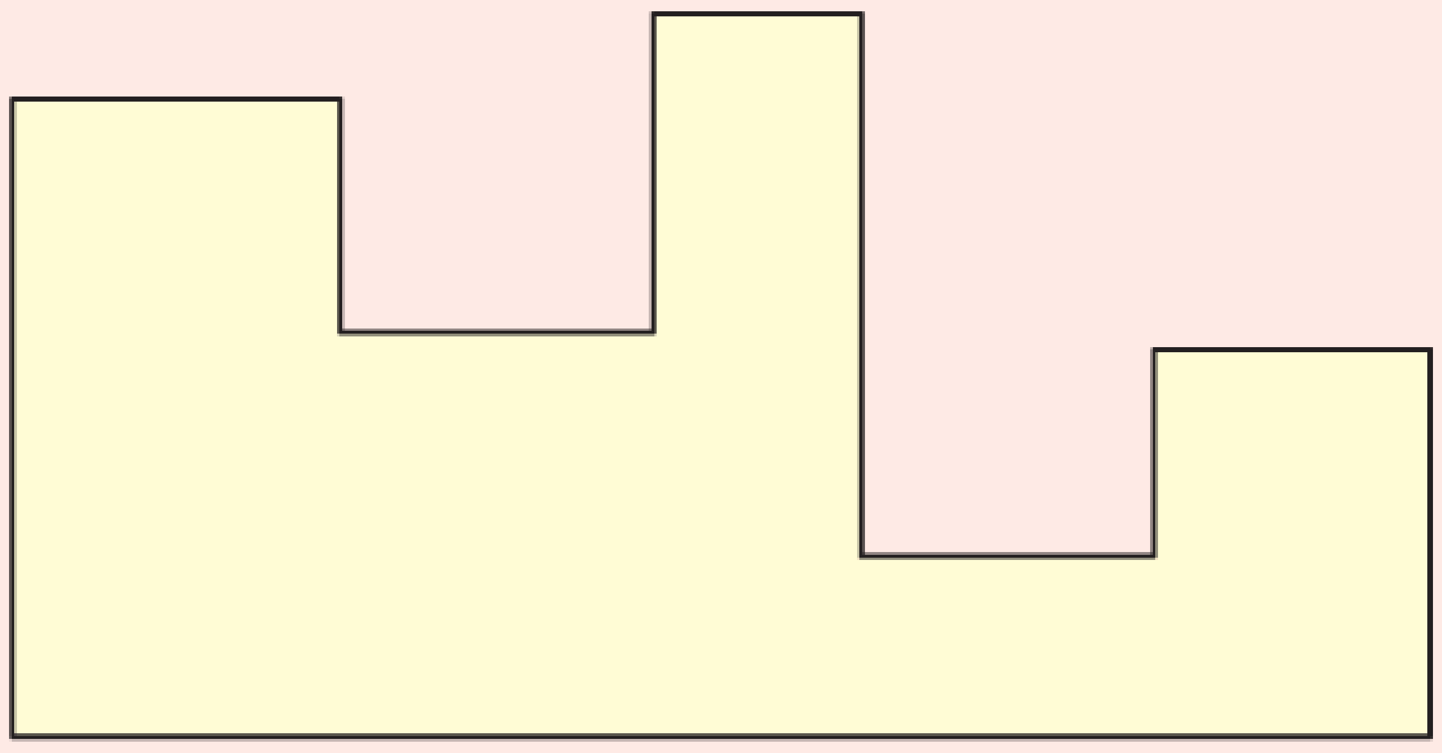
2



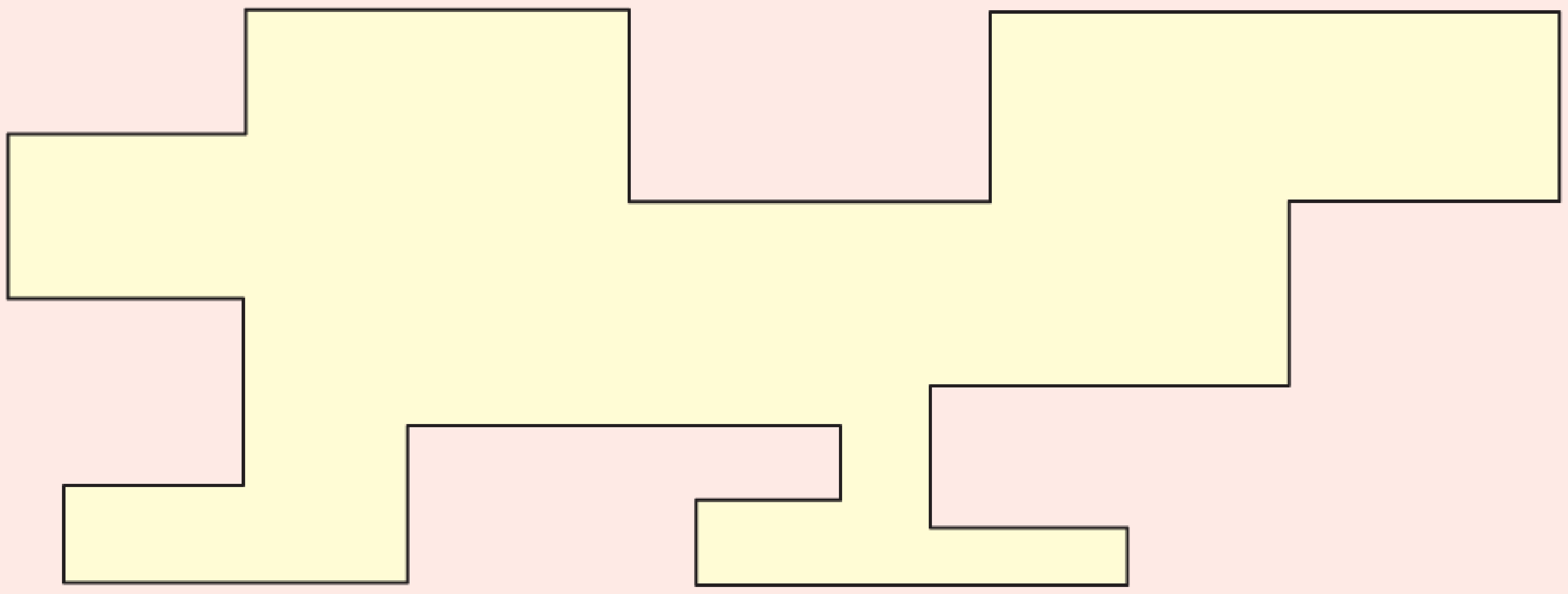
3



4



5



PRINTABLE ACTIVITY

Global context

click here

Platonic solids

Statement of inquiry:	Solids can be classified according to their properties.
Global context:	Scientific and technical innovation
Key concept:	Form
Related concepts:	Representation, Space
Objective:	Communicating
Approaches to learning:	Thinking, Research

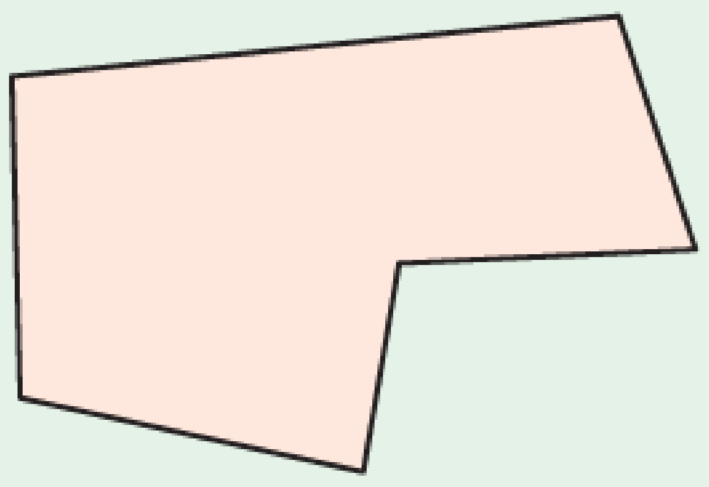
KEY WORDS USED IN THIS CHAPTER

- apex
- cube
- kite
- polygon
- radius
- scalene
- trapezium
- circle
- cylinder
- net
- prism
- rectangle
- solid
- triangle
- cone
- equilateral
- parallelogram
- pyramid
- regular polygon
- sphere
- cross-section
- isosceles
- plane
- quadrilateral
- rhombus
- square

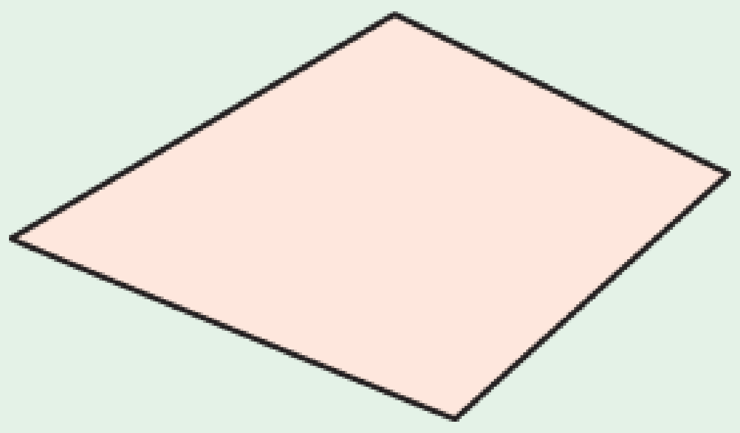
REVIEW SET 5A

1 Name these polygons:

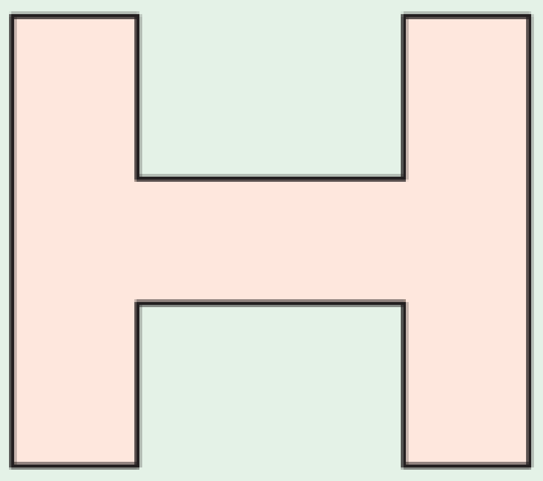
a



b



c

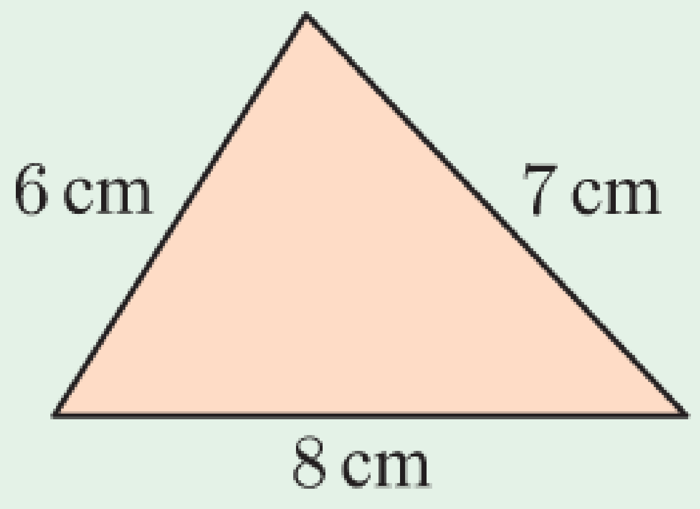


2 Draw these polygons:

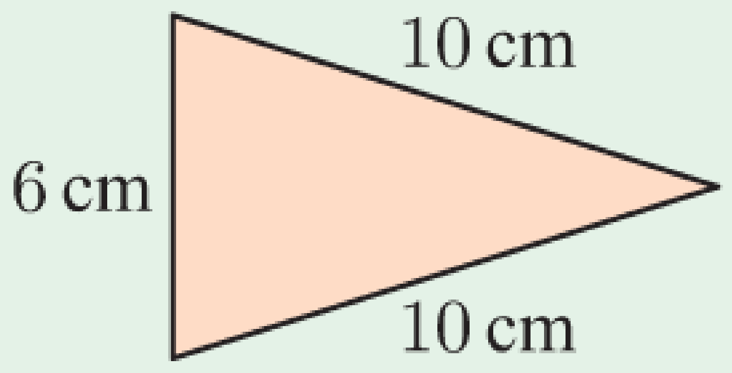
a isosceles triangle **b** regular hexagon **c** rhombus

3 Classify these triangles:

a



b



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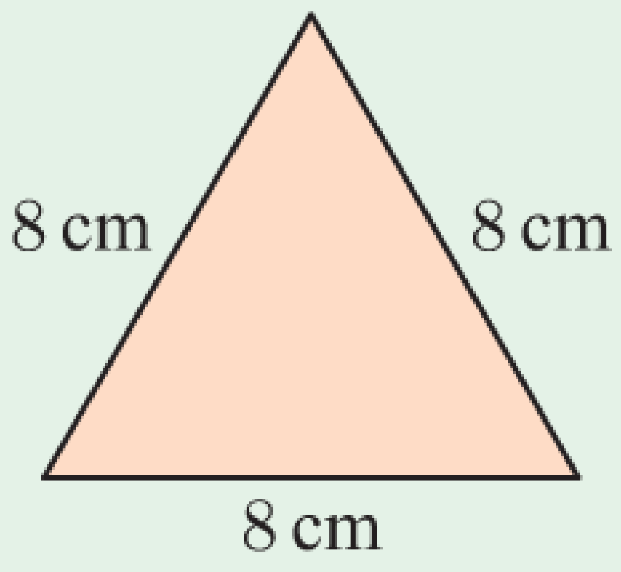


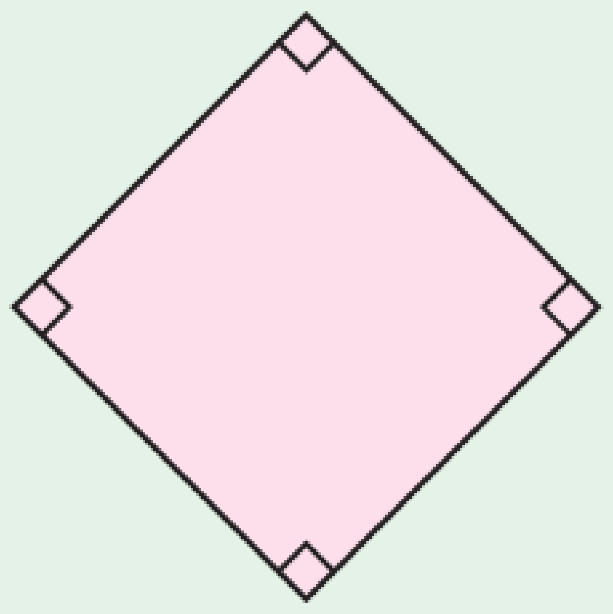
4 Construct a circle with radius 4 cm.

5 Using a compass and ruler only, construct an isosceles triangle with base length 5 cm and equal sides of length 4 cm.

6 Name these quadrilaterals. If necessary, use a ruler to measure the sides.

PRINTABLE DIAGRAMS

a



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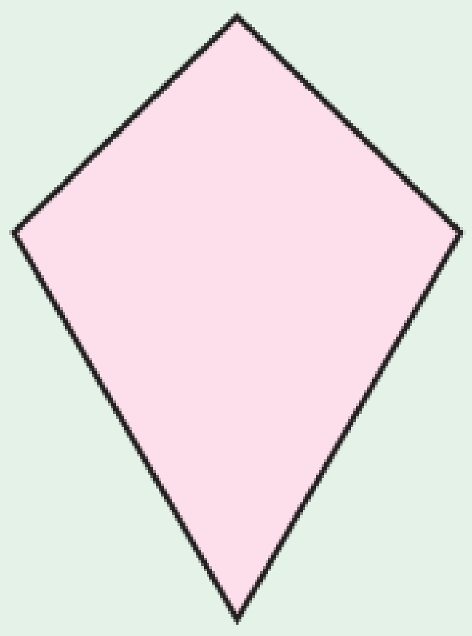

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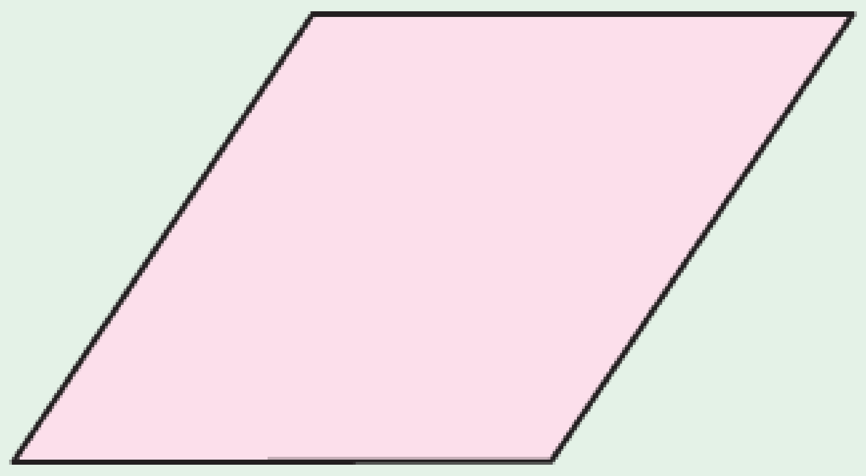

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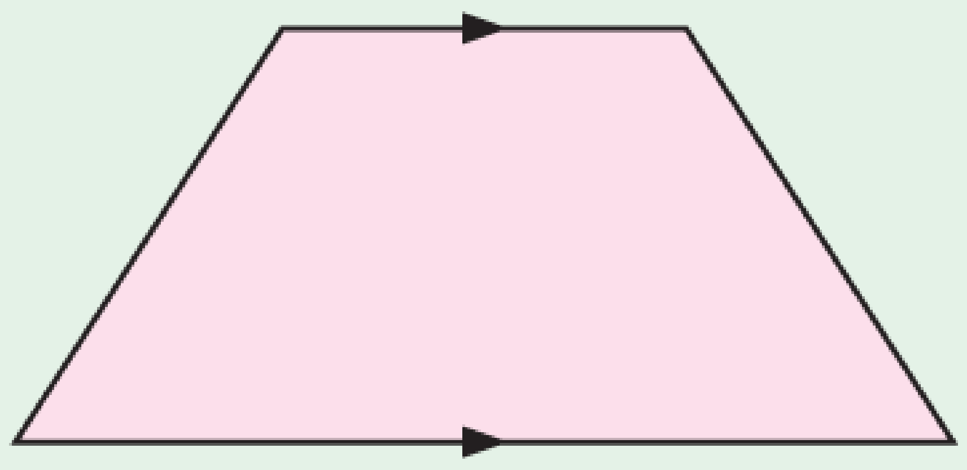
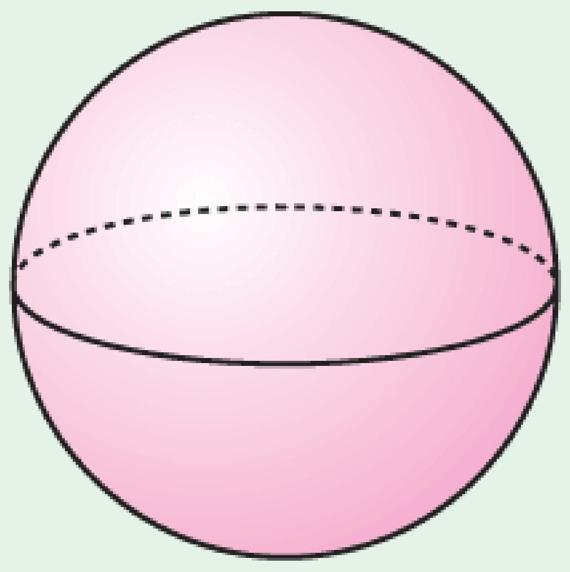


7 Name these solids:

a



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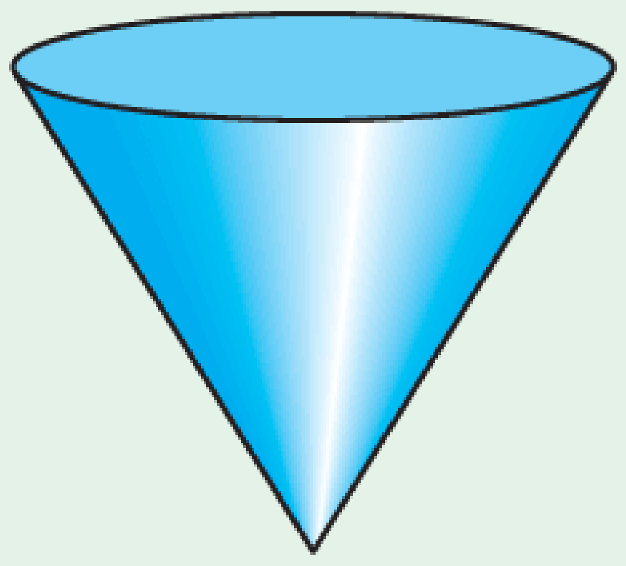

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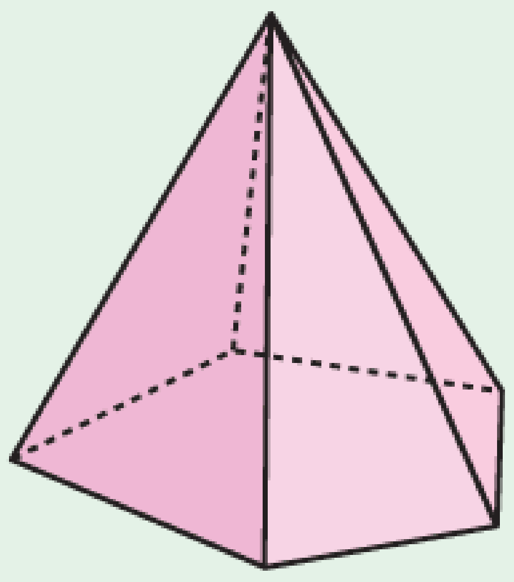


8 Explain whether:

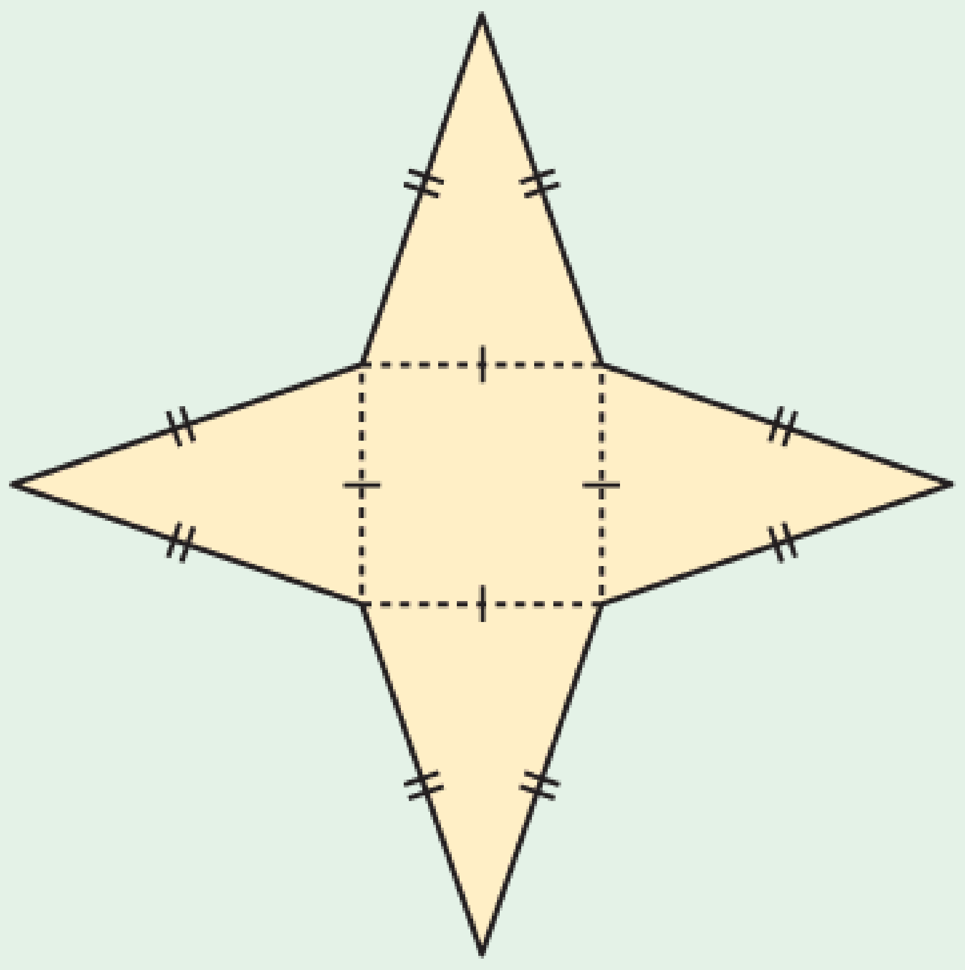
a an equilateral triangle is also isosceles

b a square is a special type of kite.

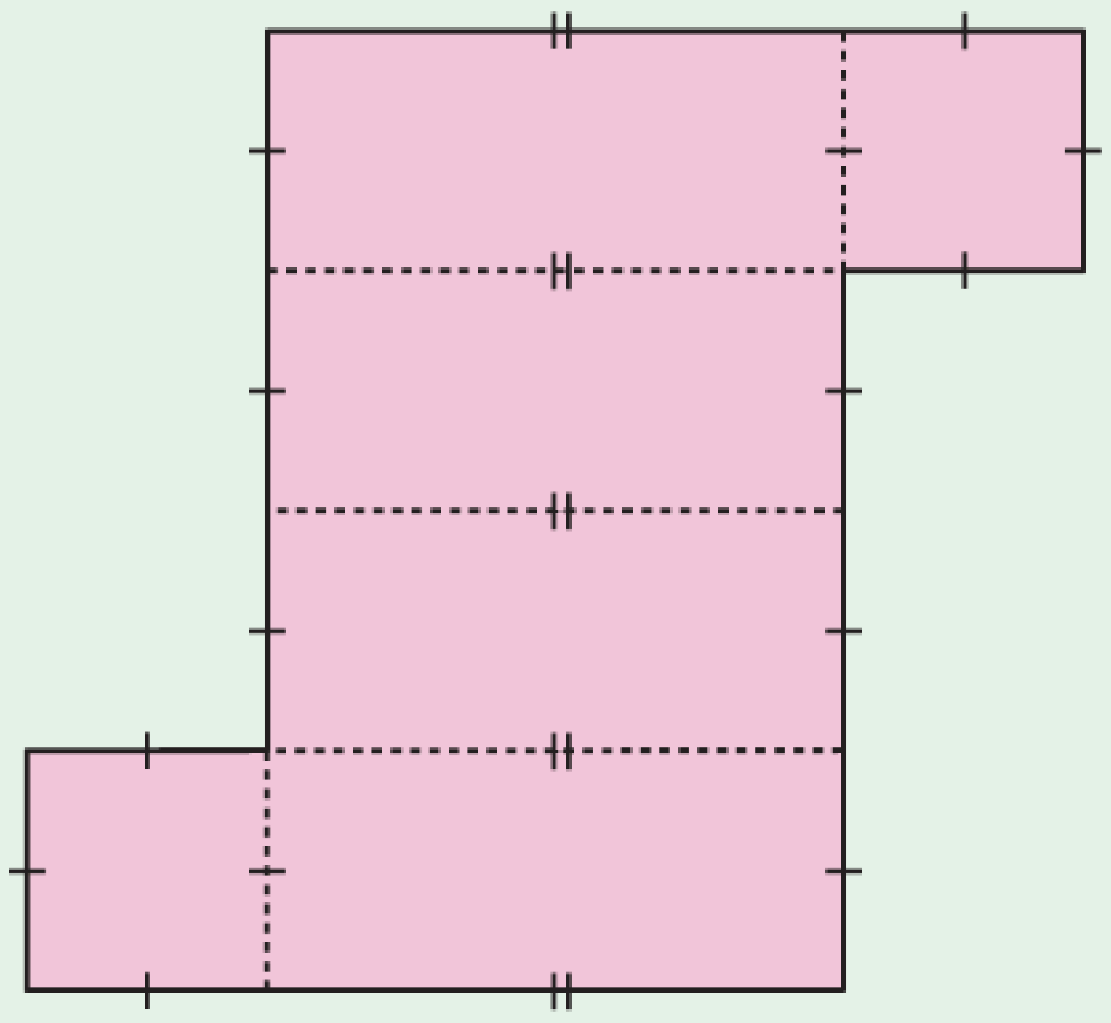
9 Draw a cylinder which is 5 cm high and has a base 3 cm wide.

10 Draw and name the solids which would be formed from the following nets:

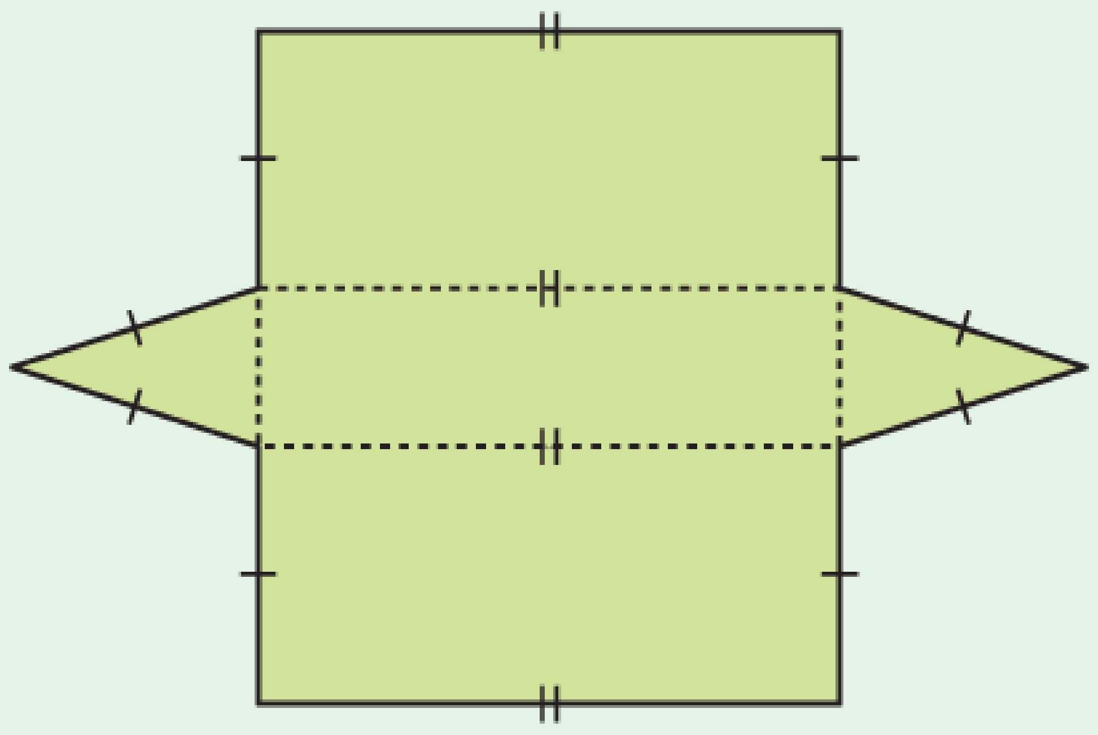
a



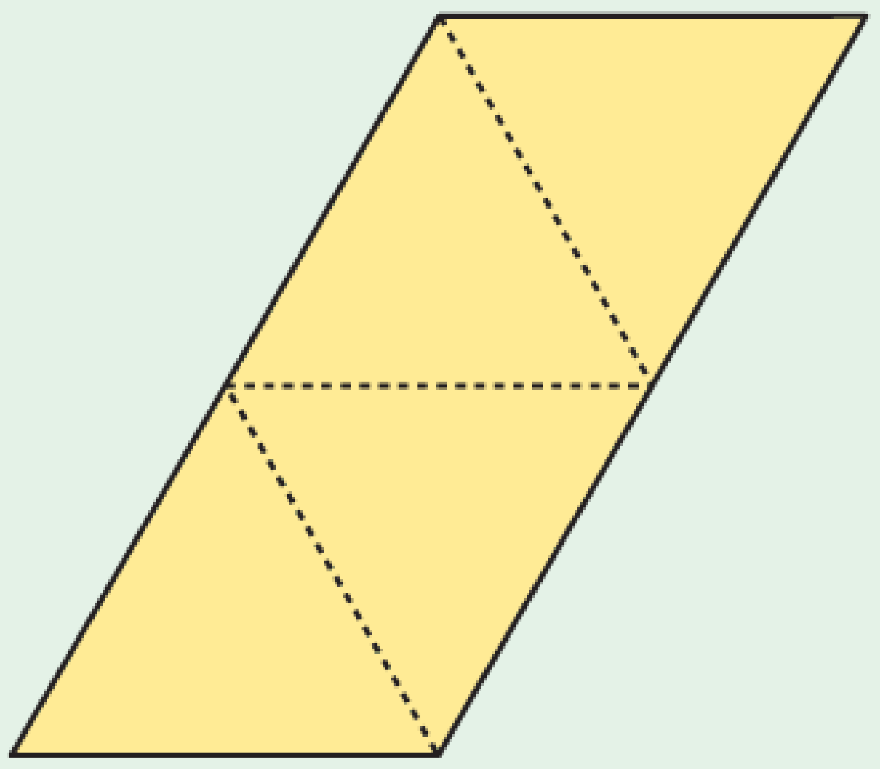
b



c



d

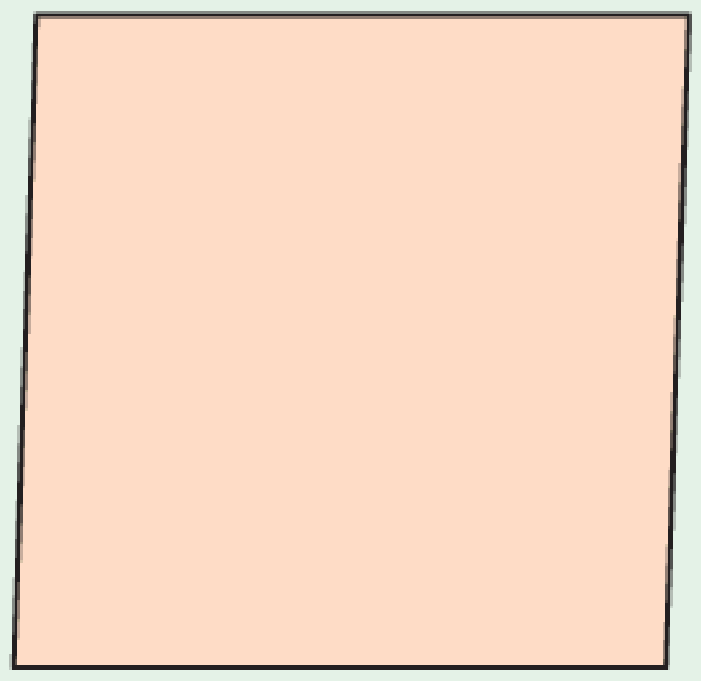


REVIEW SET 5B

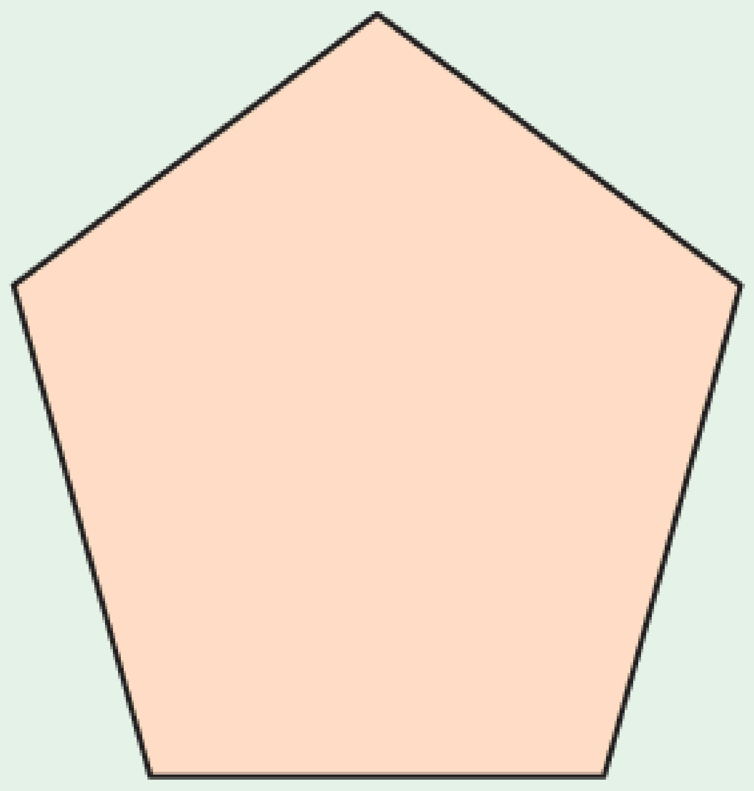
1 Using a ruler and protractor, determine whether these polygons are regular:

PRINTABLE DIAGRAMS

a

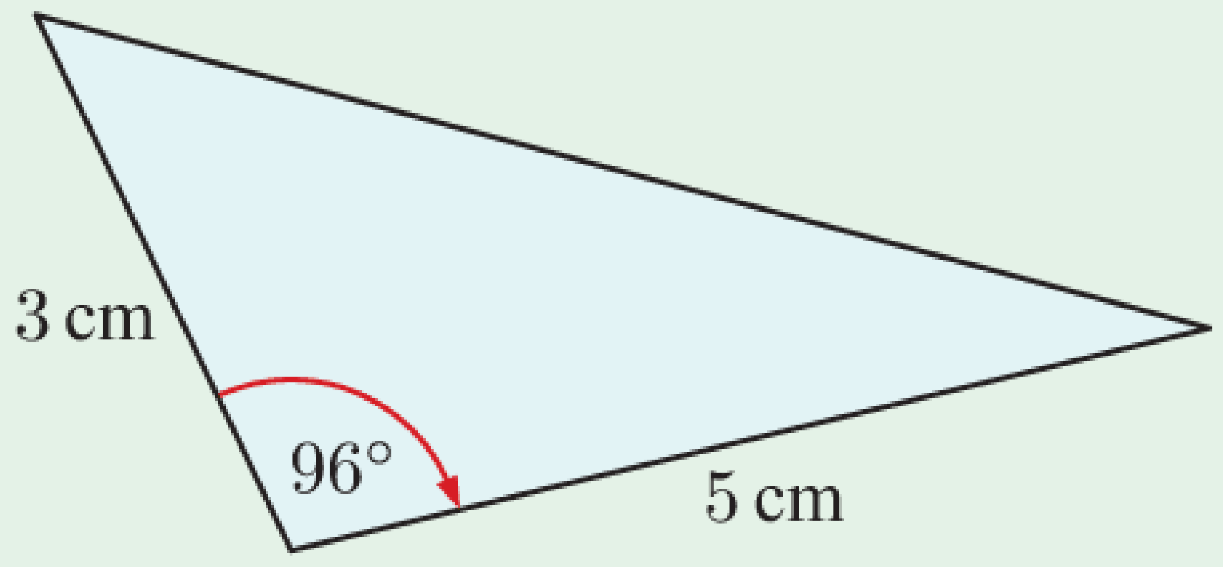


b



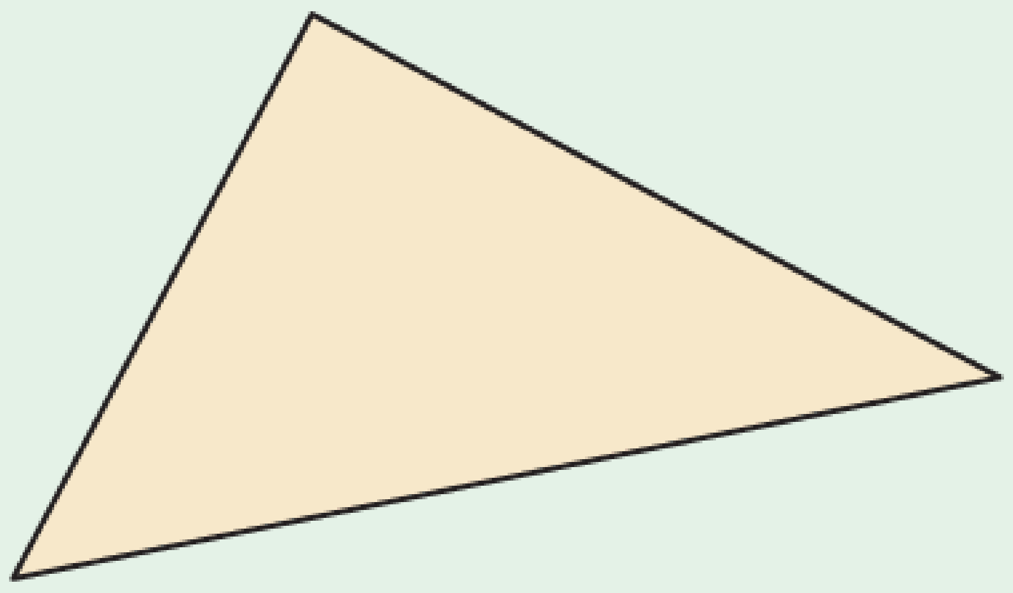
2 Using a compass and ruler only, construct a triangle with sides of length 3 cm, 4 cm, and 6 cm.

3 Use a protractor and ruler to construct a triangle with the measurements shown.



4 Answer the **Opening Problem** on page **84**.

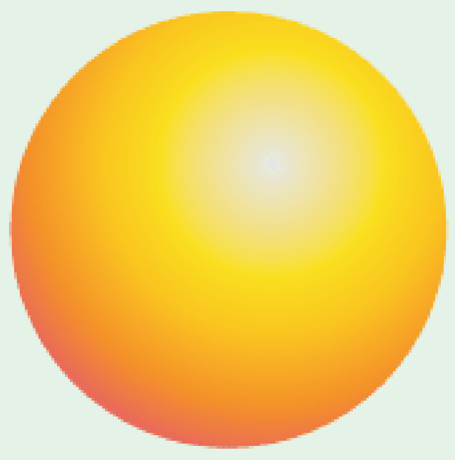
5 Classify this triangle by measuring its sides:



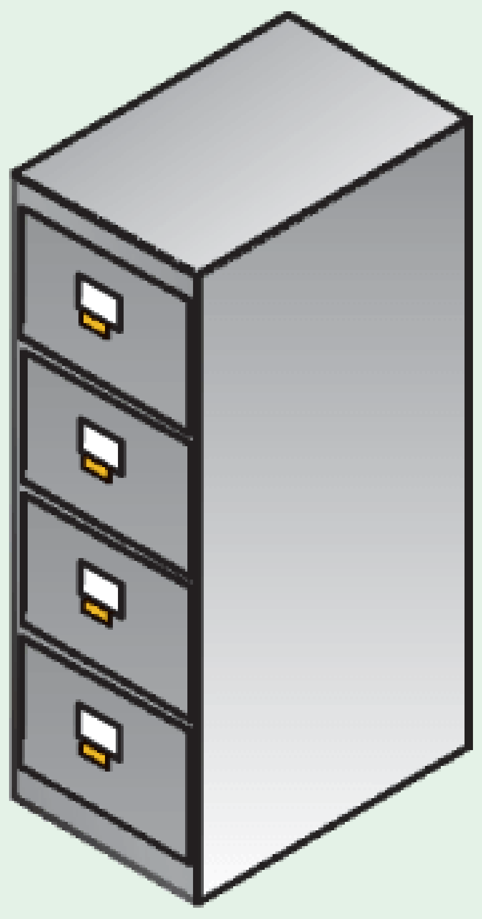
6 Draw a quadrilateral which has 3 obtuse angles.

7 What solid would best describe the shape of:

a a marble



b a filing cabinet?



8 Point C is the centre of both of these circles. The larger circle has radius 3 cm, and the smaller circle has radius 1 cm. Which of the labelled points is:

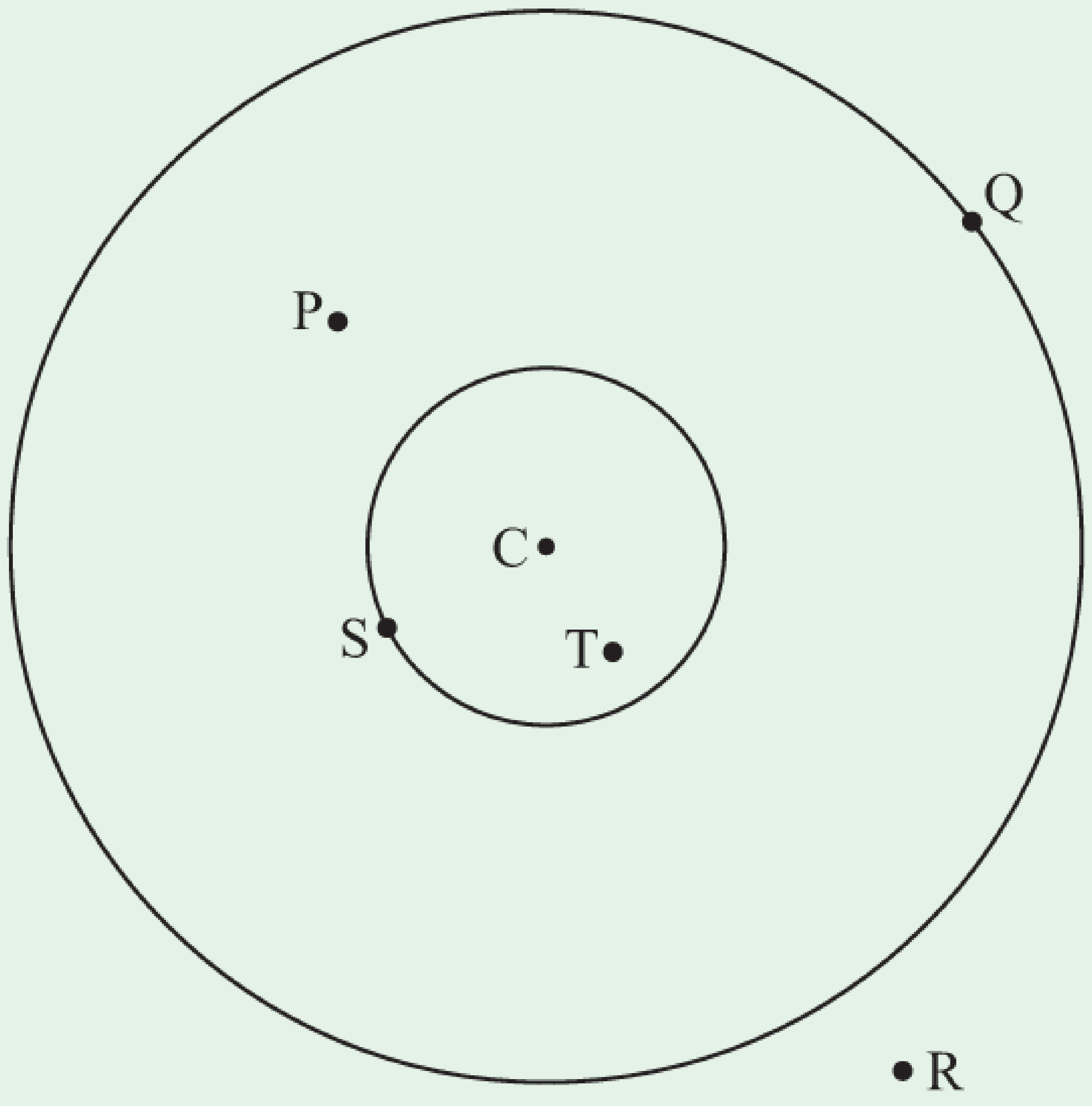
a 3 cm from C

b less than 1 cm from C

c 1 cm from C

d more than 3 cm from C

e between 1 cm and 3 cm from C?



9 Use your ruler to draw:

a a cube

b a 2 cm × 1 cm × 3 cm rectangular prism.

10 Draw the net for a hexagonal-based pyramid.

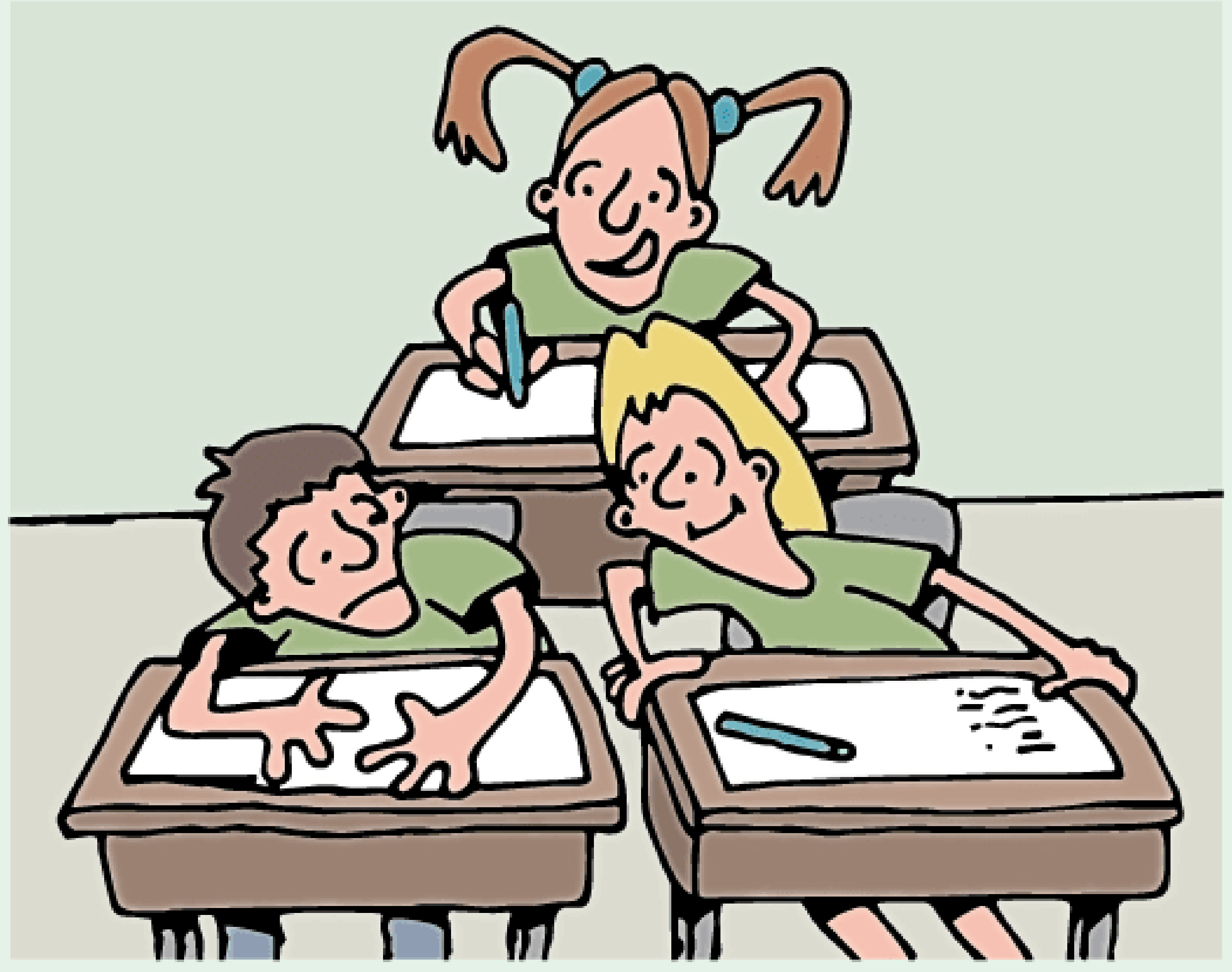
Chapter 6

Fractions

Contents:

OPENING PROBLEM

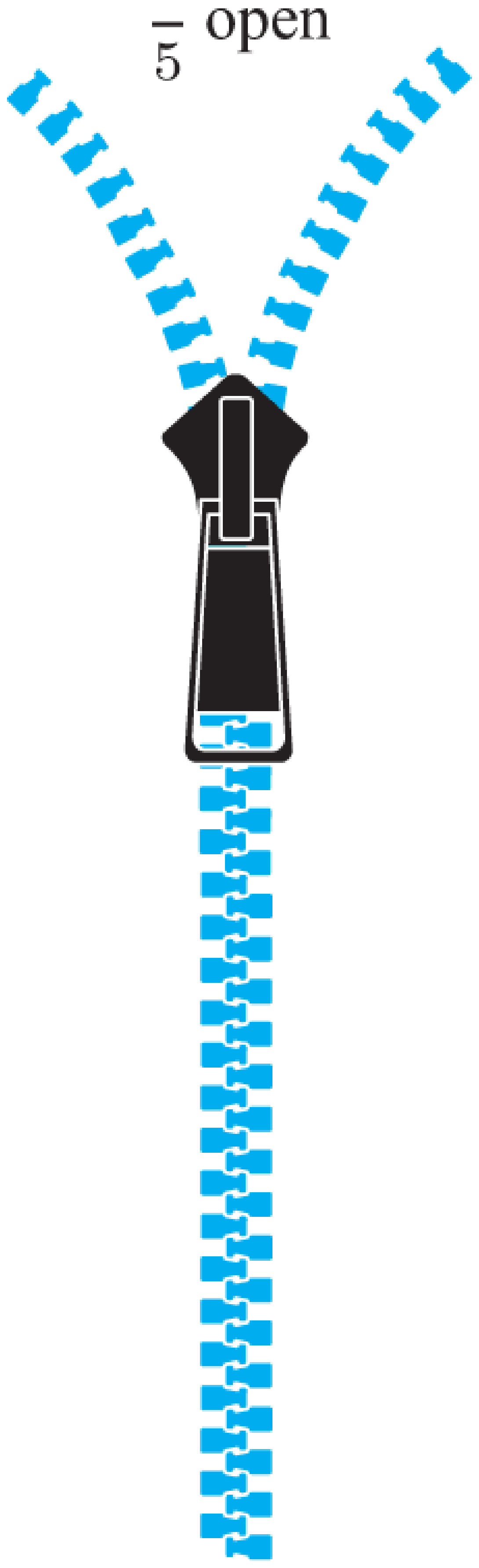
The students in Amelia's class have all been given a week to do a project. So far, Amelia has done $\frac{3}{8}$, Charlie has done $\frac{5}{8}$, and Matilda has done $\frac{1}{2}$.

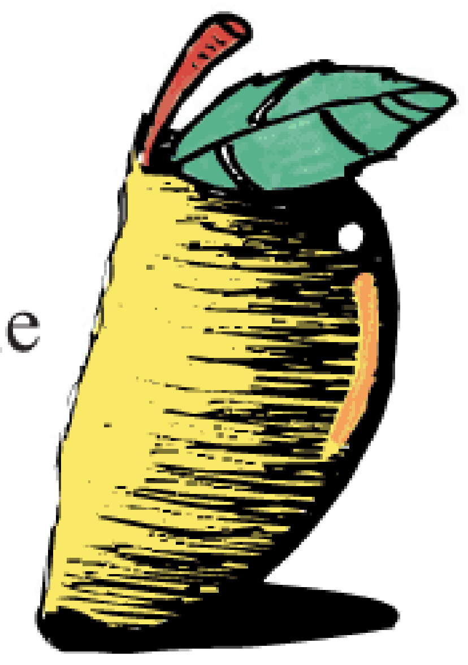


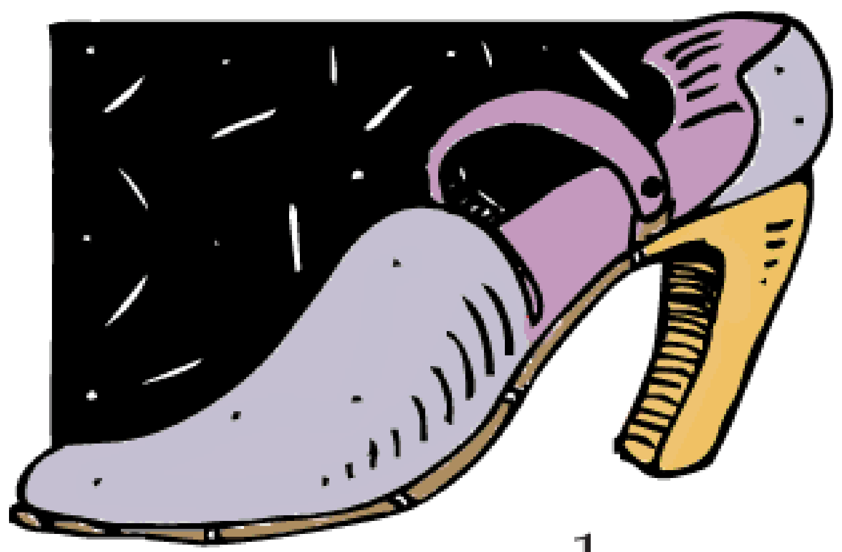
Things to think about:

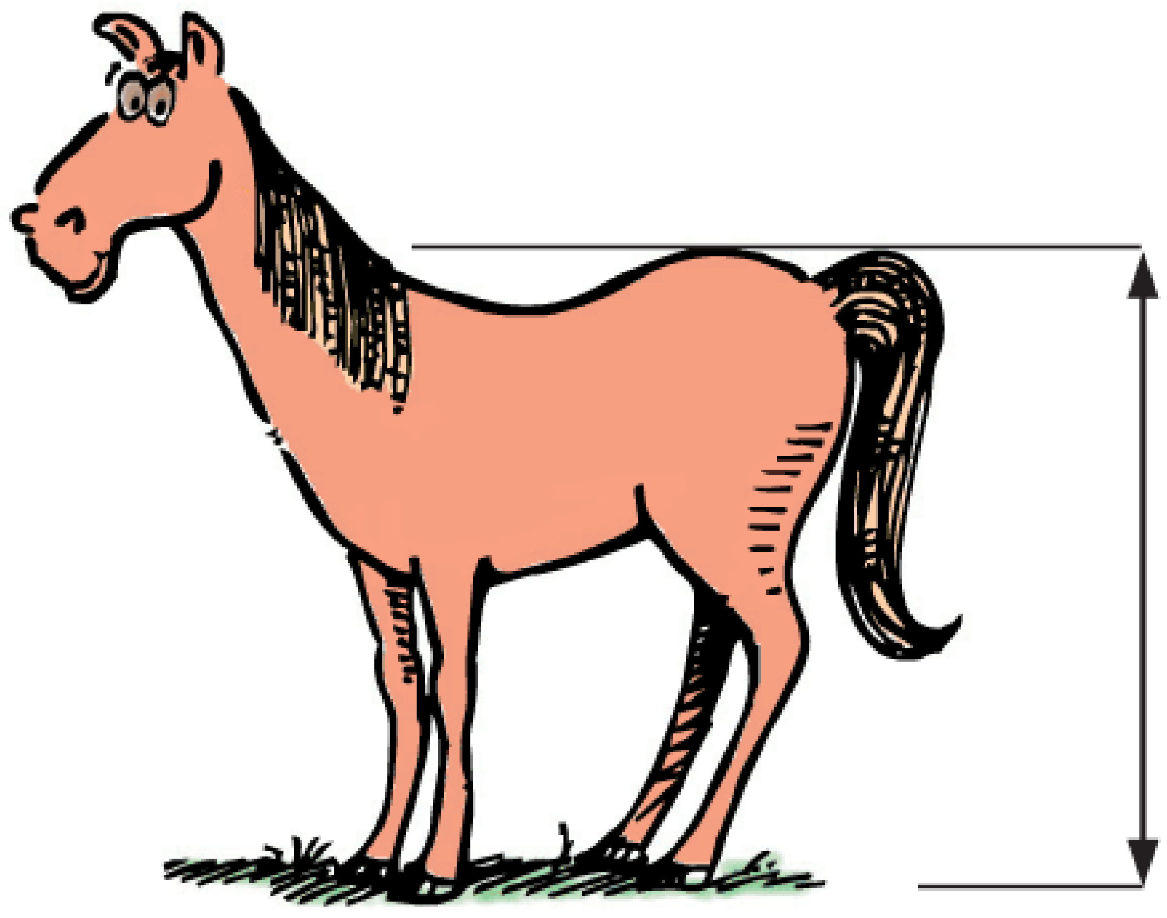
- **a** Has Amelia completed more of the project than Charlie?
- **b** Has Amelia completed more of the project than Matilda?
- **c** Which of the problems **a** or **b** was easier to solve? What made the other one harder?
- **d** The next night, Amelia completed another $\frac{1}{2}$ of her project. What total fraction has she completed now?

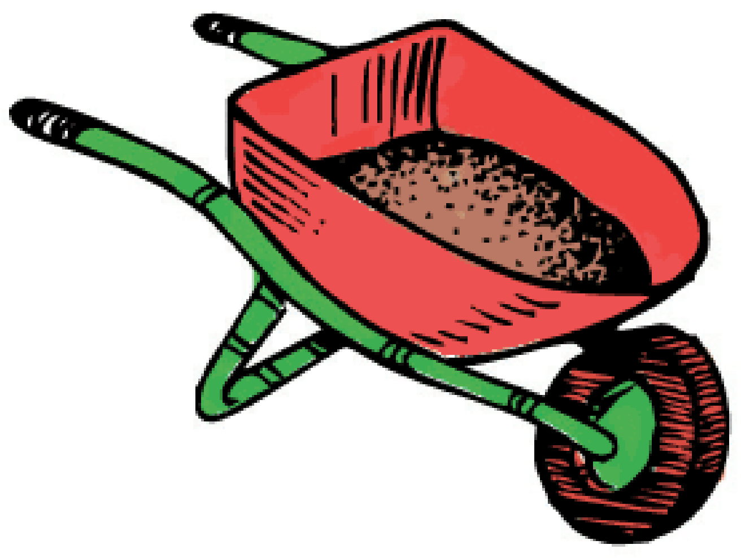
Every day we see quantities which can be expressed as fractions. It is therefore important that we can understand, compare, add, and subtract fractions.







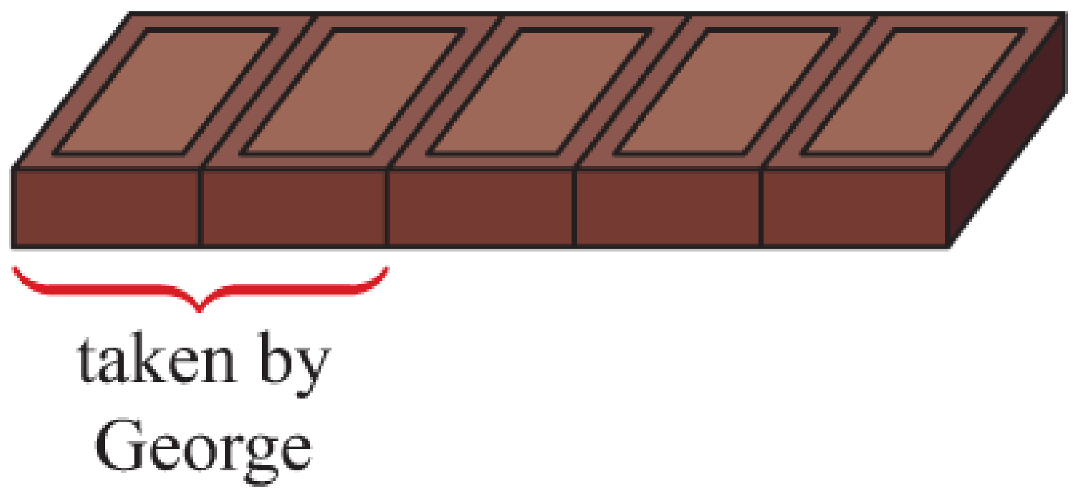




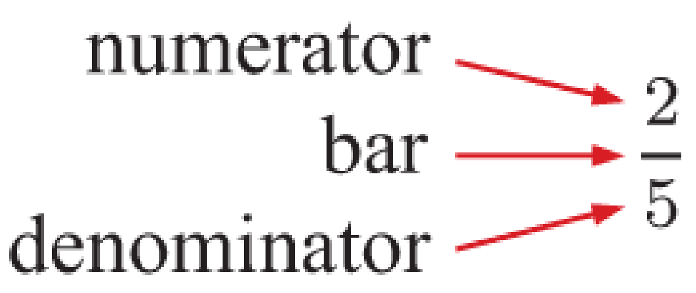
A FRACTIONS

A **fraction** is a part of any quantity.

For example, a chocolate bar is divided into 5 equal parts. If George takes 2 of the parts, we say that George has taken $\frac{2}{5}$ of the chocolate bar.



$\frac{2}{5}$ is a fraction which shows that we had a whole, we divided it into 5 **equal parts**, and we are looking at 2 of them.

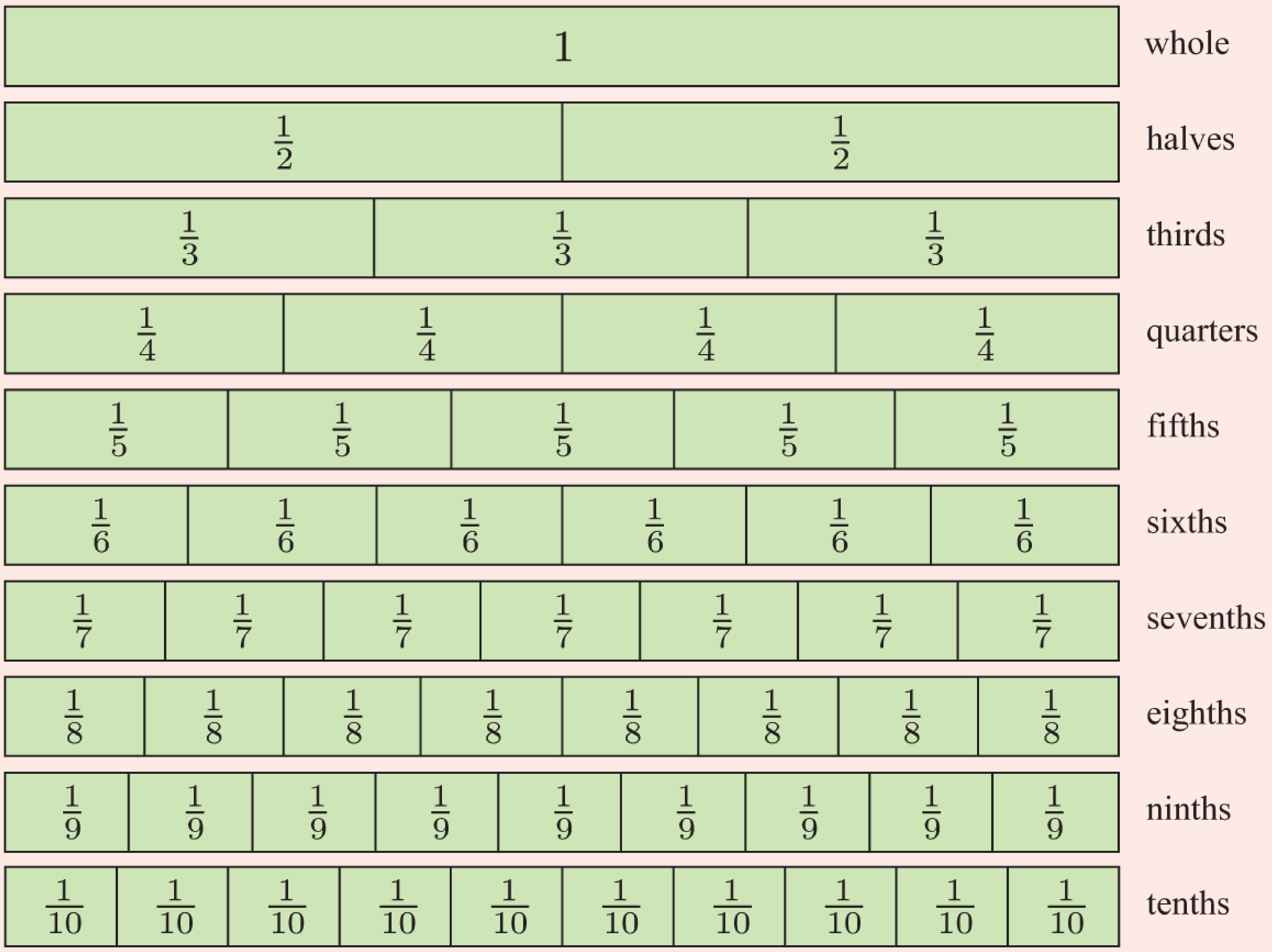


The **numerator** shows how many parts we are looking at.

The **denominator** shows how many equal parts there are altogether.

ACTIVITY FRACTION WALLS

The fraction wall below shows some of the different ways of dividing a whole into equal parts.



What to do:

Use the fraction wall to complete this table:

PRINTABLE TABLE

	Number of equal parts	*One part as a fraction*	*Fraction in words*	*All parts form the fraction*
	1	$\frac{1}{1}$	one whole	$\frac{1}{1}$
a	2		one half	
b		$\frac{1}{3}$		
c			one quarter	
d			one fifth	
e		$\frac{1}{6}$		$\frac{6}{6}$
f	7			
g			one eighth	
h		$\frac{1}{9}$		$\frac{9}{9}$
i	10			

EXERCISE 6A

1 Write a fraction to show:

- **a** three quarters
- **b** two thirds
- **c** two fifths
- **d** four fifths
- **e** three eighths
- **f** five eighths
- **g** two sevenths
- **h** three tenths
- **i** one hundredth

2 Write in words:

- **a** $\frac{1}{3}$
- **b** $\frac{2}{9}$
- **c** $\frac{3}{5}$
- **d** $\frac{7}{8}$
- **e** $\frac{4}{9}$
- **f** $\frac{5}{7}$
- **g** $\frac{5}{12}$
- **h** $\frac{17}{20}$
- **i** $\frac{11}{30}$
- **j** $\frac{4}{25}$
- **k** $\frac{3}{100}$
- **l** $\frac{97}{100}$

3 For each fraction, state the numerator:

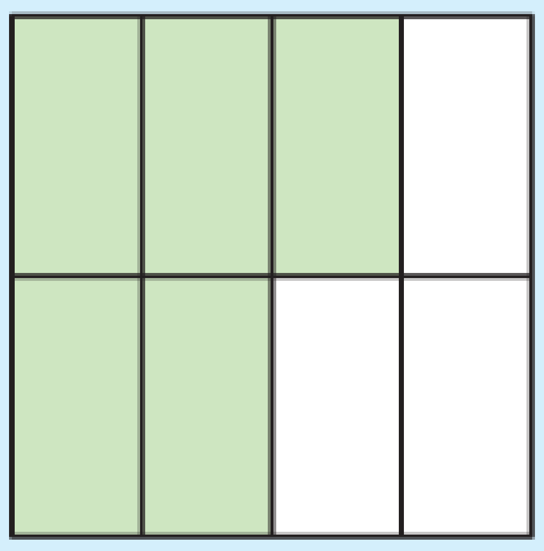
- **a** $\frac{2}{3}$
- **b** $\frac{4}{5}$
- **c** $\frac{3}{7}$
- **d** $\frac{1}{8}$

4 For each fraction, state the denominator:

- **a** $\frac{2}{3}$
- **b** $\frac{4}{5}$
- **c** $\frac{3}{7}$
- **d** $\frac{1}{8}$

Example 1 **Self Tutor**

a What fraction of the square is shaded?

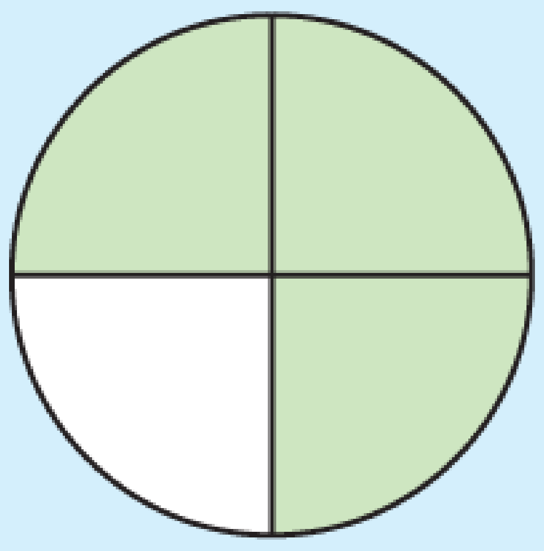


b Draw a diagram to represent the fraction $\frac{3}{4}$.

a The square is divided into 8 equal parts, and 5 of them are shaded.

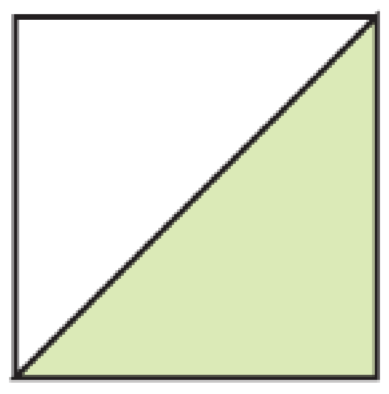
$\therefore$ the fraction shaded is $\frac{5}{8}$.

b We divide a circle into 4 equal parts, then shade 3 of them.

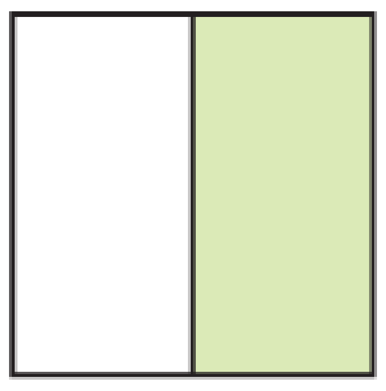


5 What fraction of the square is shaded?

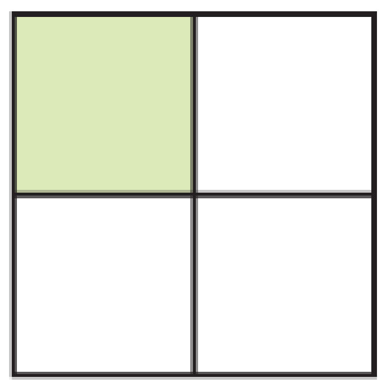
a



b



c



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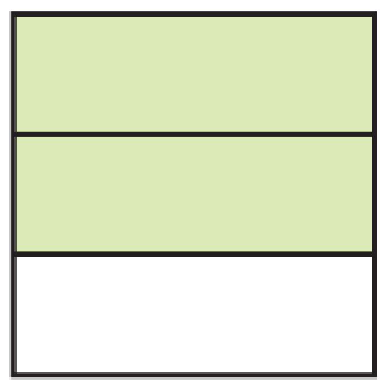

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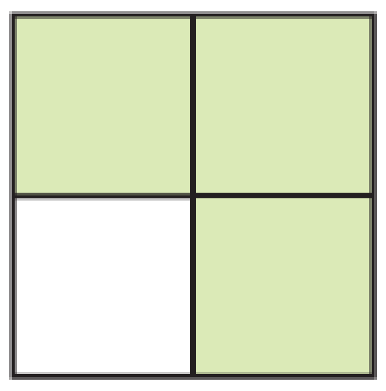

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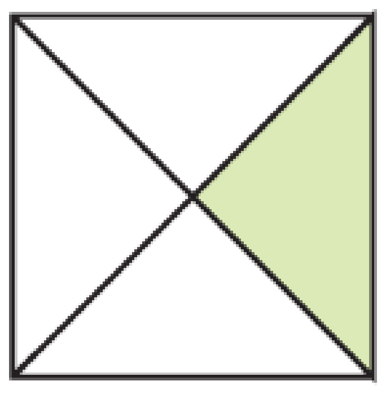
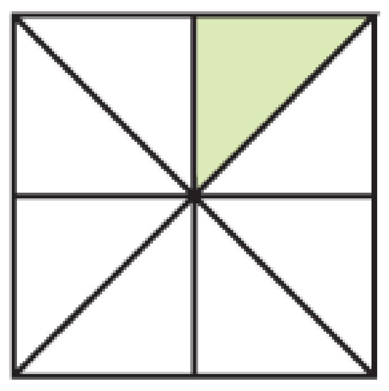


g



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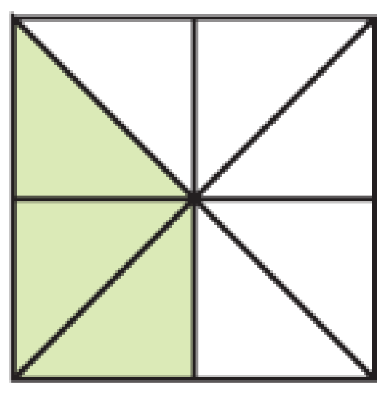

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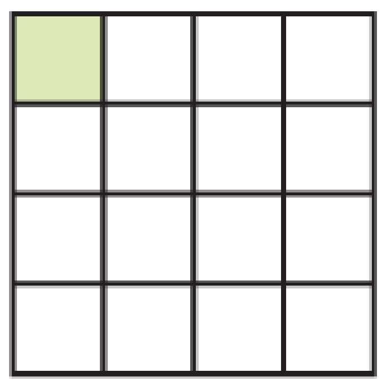

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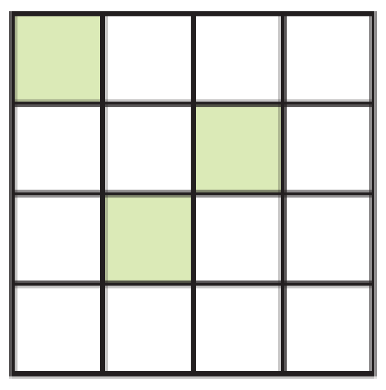

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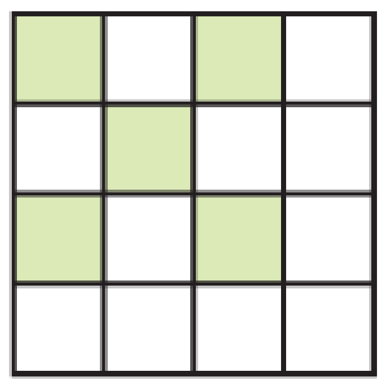

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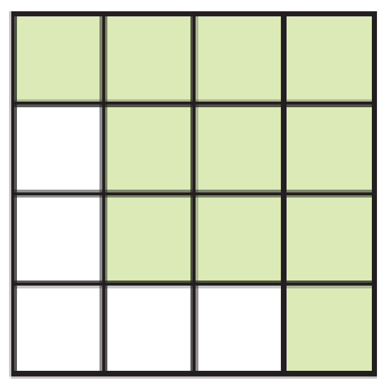
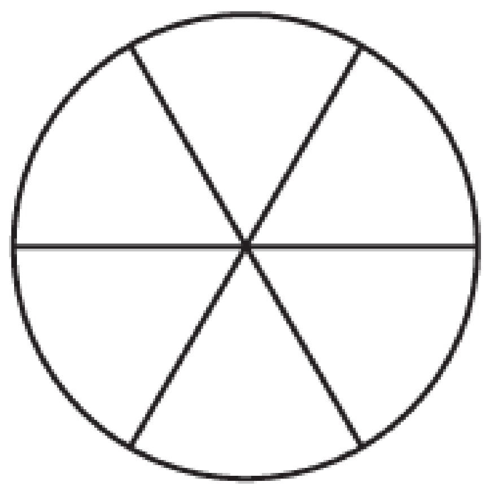
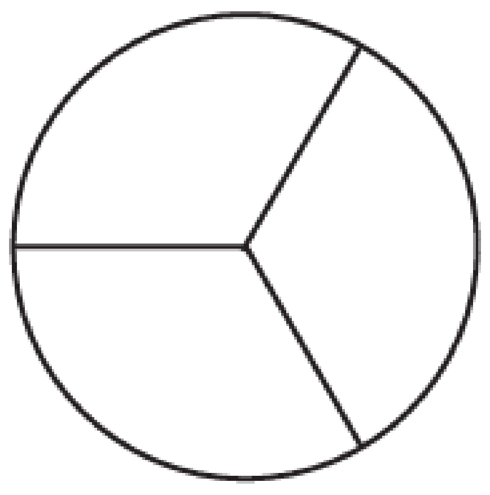


6 Copy the circles and shade the fractions given:

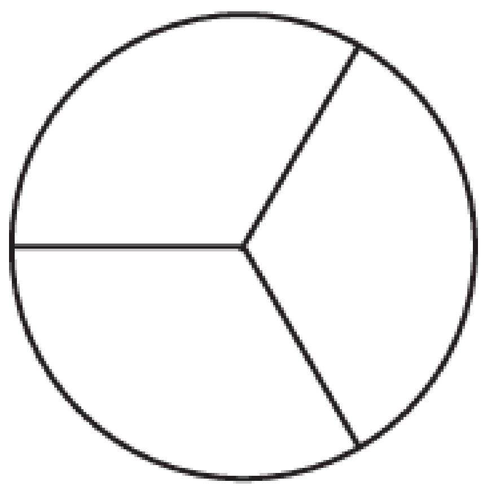
a $\frac{1}{6}$



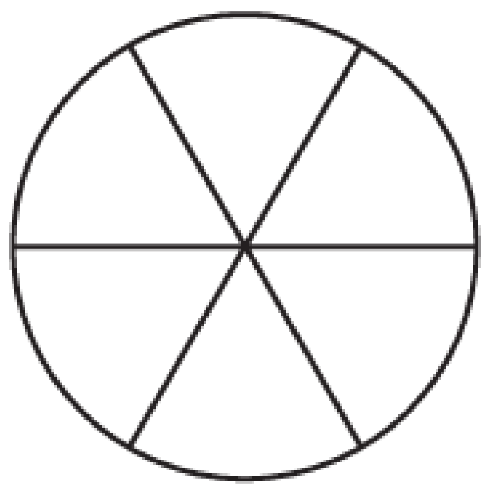
b $\frac{1}{3}$



c $\frac{2}{3}$



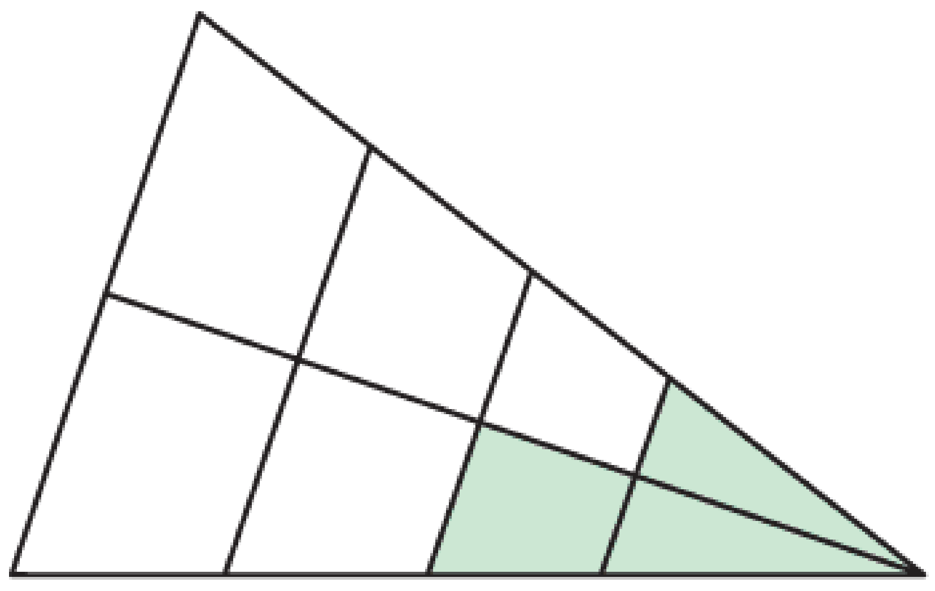
d $\frac{5}{6}$



7 Draw a diagram to represent:

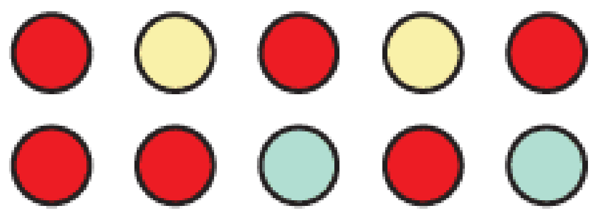
a $\frac{2}{4}$ **b** $\frac{5}{8}$ **c** $\frac{1}{9}$ **d** $\frac{4}{12}$

8 Is $\frac{3}{8}$ of this triangle shaded? Explain your answer.

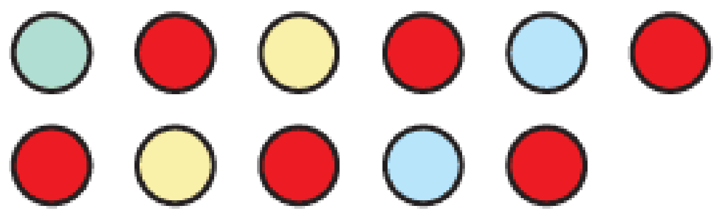


9 What fraction of the dots are red?

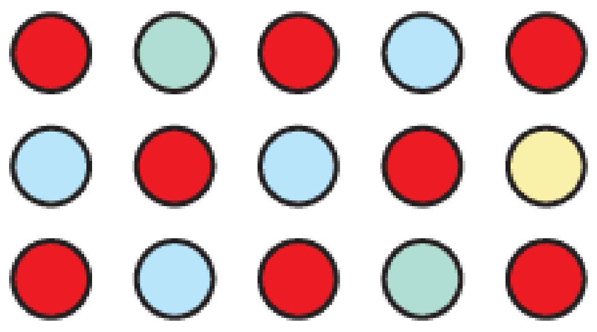
a



b



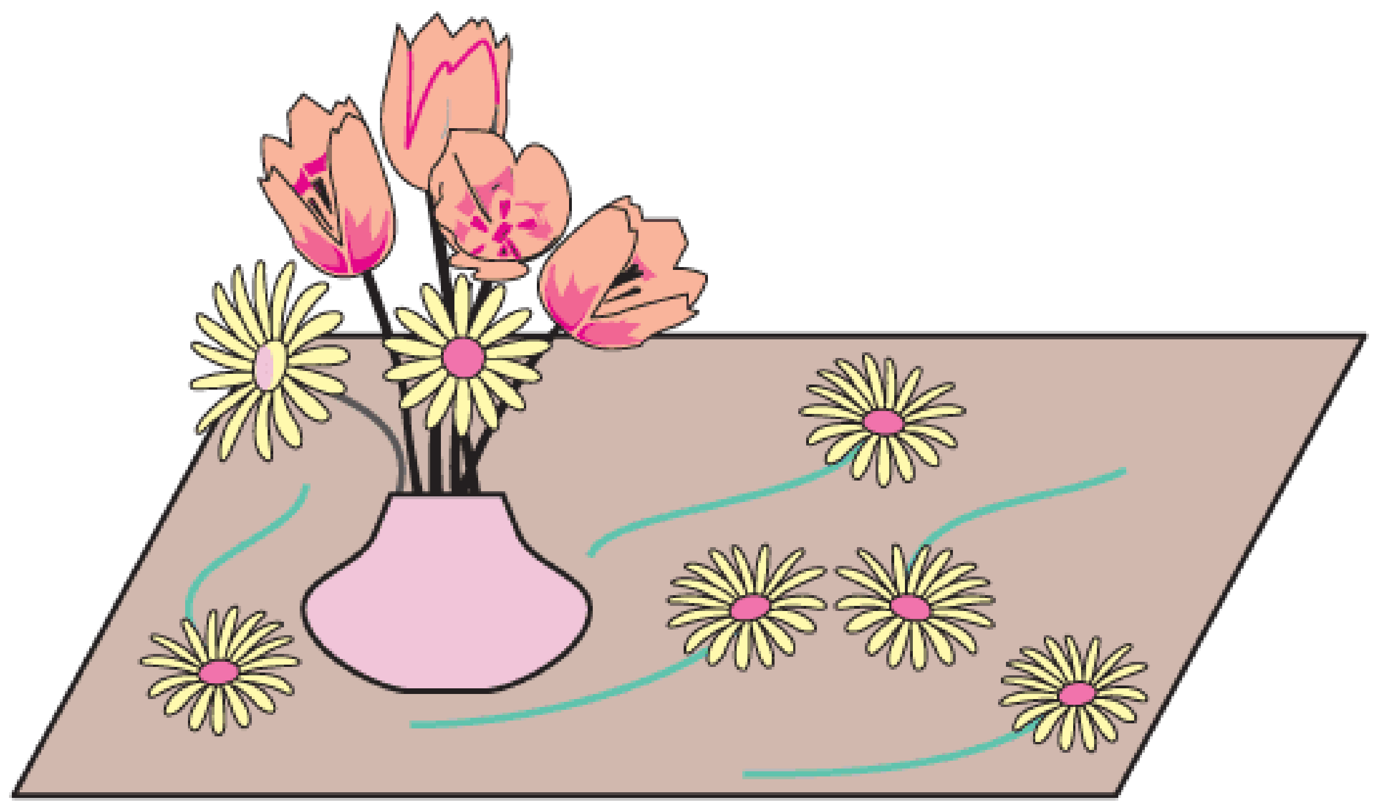
c



10

What fraction of the cats are: **a** black **b** white?

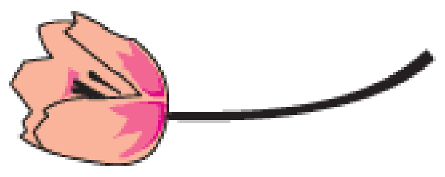
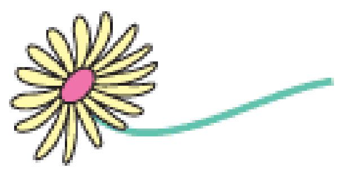
11



a What fraction of the flowers are:

i in the vase **ii** lying on the table?

b What fraction of the flowers are:

i tulips  **ii** daisies? 

12

a What fraction of the children are:

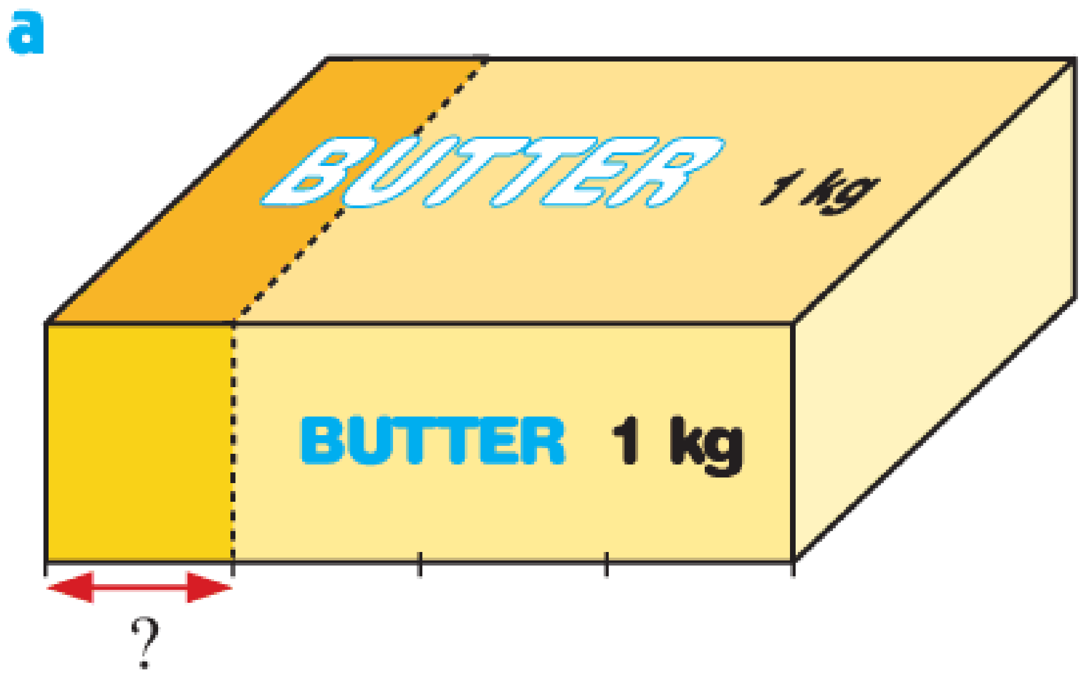
i wearing hats **ii** not wearing hats?

b What fraction of the children are:

i boys **ii** girls?

13 Give the fraction shown in each diagram:

a



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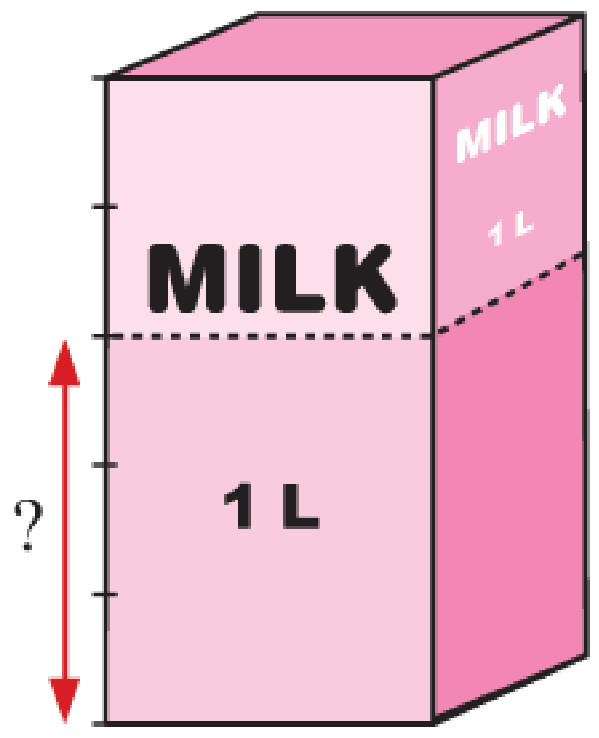
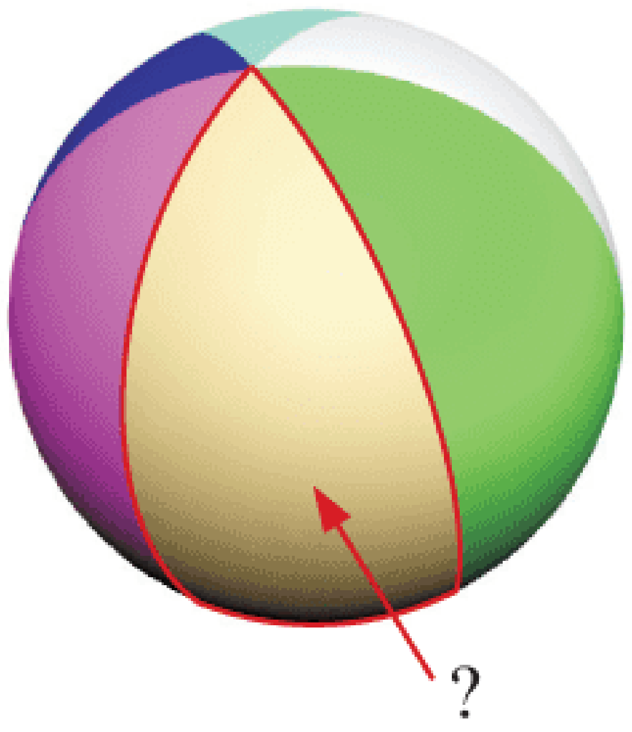
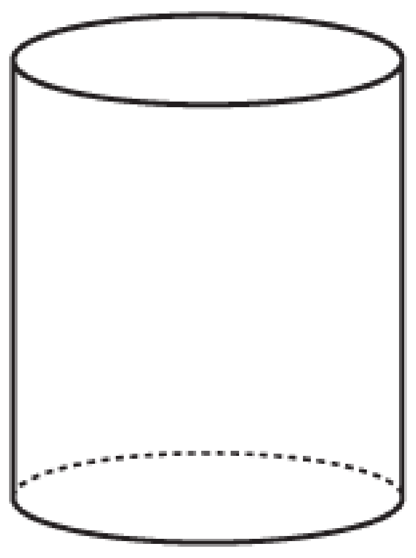


c



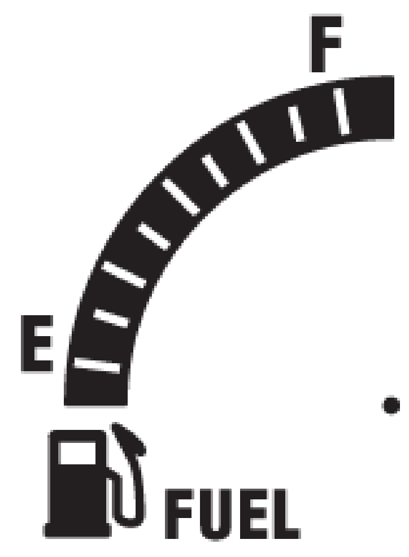
14 Copy and complete the following sketches to show:

a



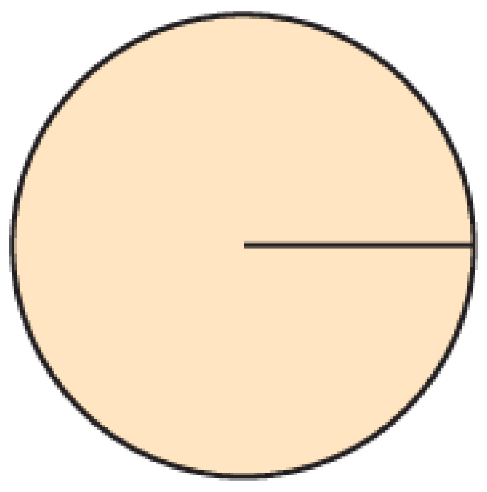
a glass which is half full of water

b



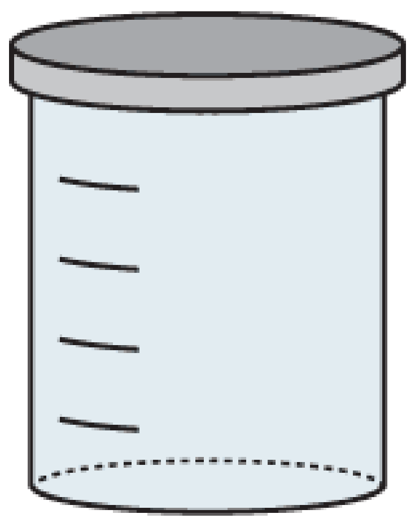
a petrol gauge showing the tank is three quarters full

c



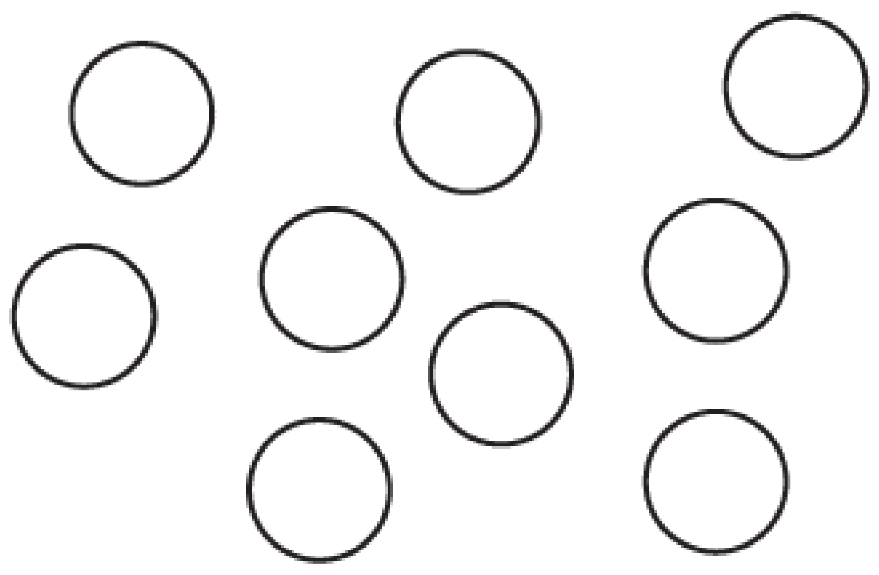
a pizza with one third missing

d



a container which is $\frac{3}{5}$ full

e



five ninths of the balls are blue.

PRINTABLE DIAGRAMS

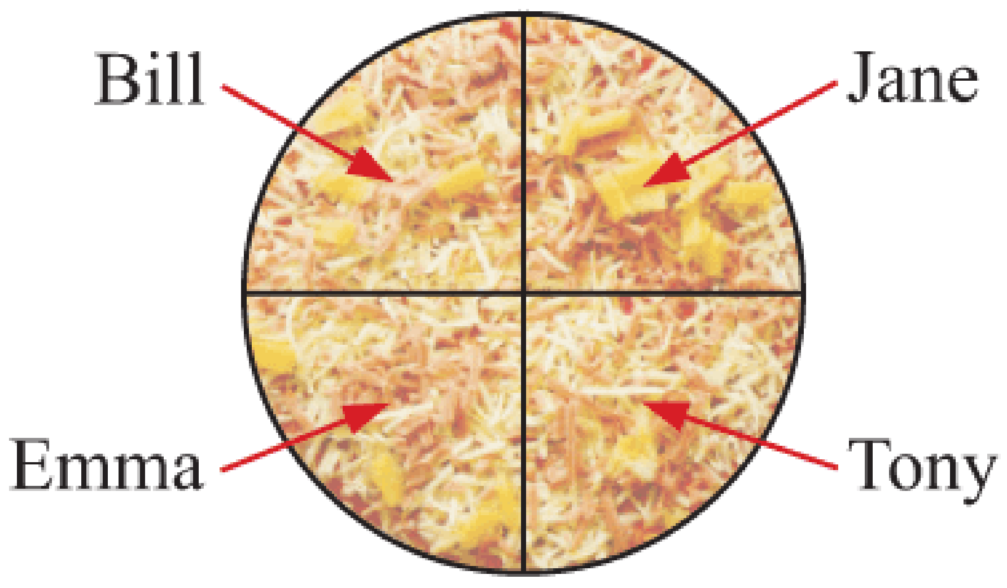
B FRACTIONS AS DIVISION

When we write a fraction such as $\frac{1}{4}$, the bar indicates division.

The fraction $\frac{1}{4}$ is equivalent to the division $1 \div 4$.

We can see this by dividing a pizza into four equal portions. Each person will get one quarter of the pizza.

$$\underset{\text{pizza}}{1} \quad \div \quad \underset{\text{people}}{4} \quad = \quad \frac{1}{4} \text{ of a pizza each}$$



Example 2 **Self Tutor**

Write as a fraction:

a $2 \div 7$ **b** $3 \div 8$

a $2 \div 7 = \frac{2}{7}$ **b** $3 \div 8 = \frac{3}{8}$

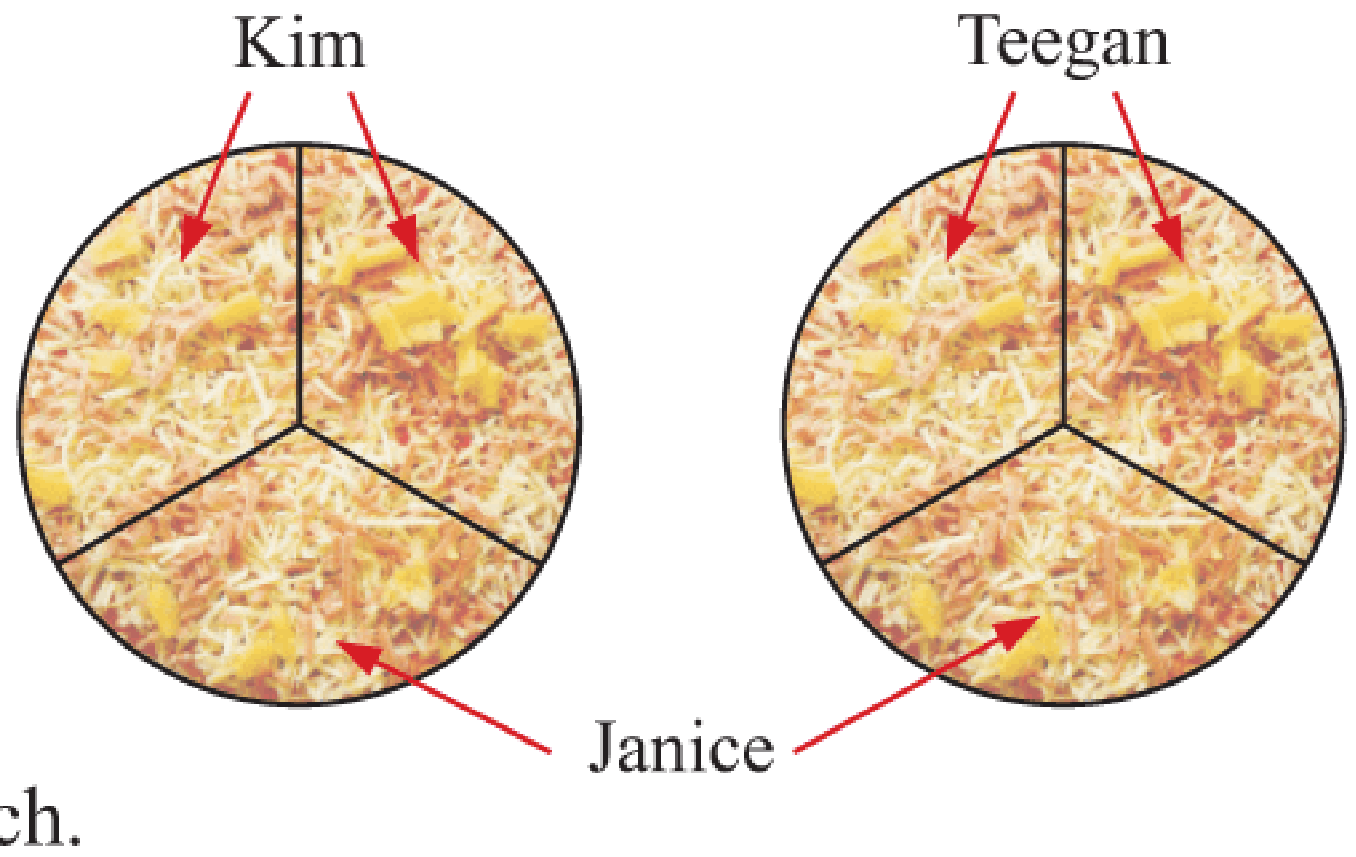
EXERCISE 6B

1 Write as a fraction:

a $4 \div 5$ **b** $1 \div 7$ **c** $3 \div 10$

d $8 \div 9$ **e** $2 \div 11$ **f** $12 \div 13$

2 Suppose 2 pizzas are shared equally between 3 people.



- **a** Look at what Kim gets. What fraction of a pizza is this?
- **b** Check that the other two people each get the same amount as Kim.
- **c** Copy and complete:
  ...... pizzas ÷ ...... people = ... of a pizza each.

Example 3 **Self Tutor**

Write as a division:

a $\frac{1}{6}$ **b** $\frac{4}{9}$

a $\frac{1}{6} = 1 \div 6$ **b** $\frac{4}{9} = 4 \div 9$

3 Write as a division:

a $\frac{1}{3}$	**b** $\frac{2}{5}$	**c** $\frac{7}{8}$
d $\frac{3}{4}$	**e** $\frac{8}{13}$	**f** $\frac{11}{20}$

Example 4 **Self Tutor**

Write as a division, and hence as a whole number:

a $\frac{12}{4}$ **b** $\frac{42}{6}$

a $\frac{12}{4} = 12 \div 4$
$= 3$

b $\frac{42}{6} = 42 \div 6$
$= 7$

4 Write as a division, and hence as a whole number:

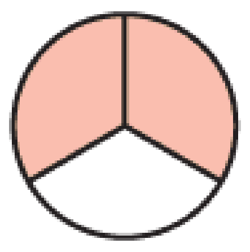
a $\frac{20}{5}$	**b** $\frac{27}{3}$	**c** $\frac{55}{11}$	**d** $\frac{7}{7}$
e $\frac{24}{12}$	**f** $\frac{19}{19}$	**g** $\frac{0}{8}$	**h** $\frac{108}{9}$

C PROPER AND IMPROPER FRACTIONS

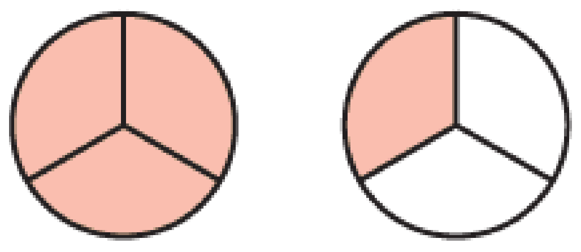
A fraction which has numerator **less** than its denominator is called a **proper fraction**.

A fraction which has numerator **greater** than its denominator is called an **improper fraction**.

For example: $\frac{2}{3}$ is a proper fraction.

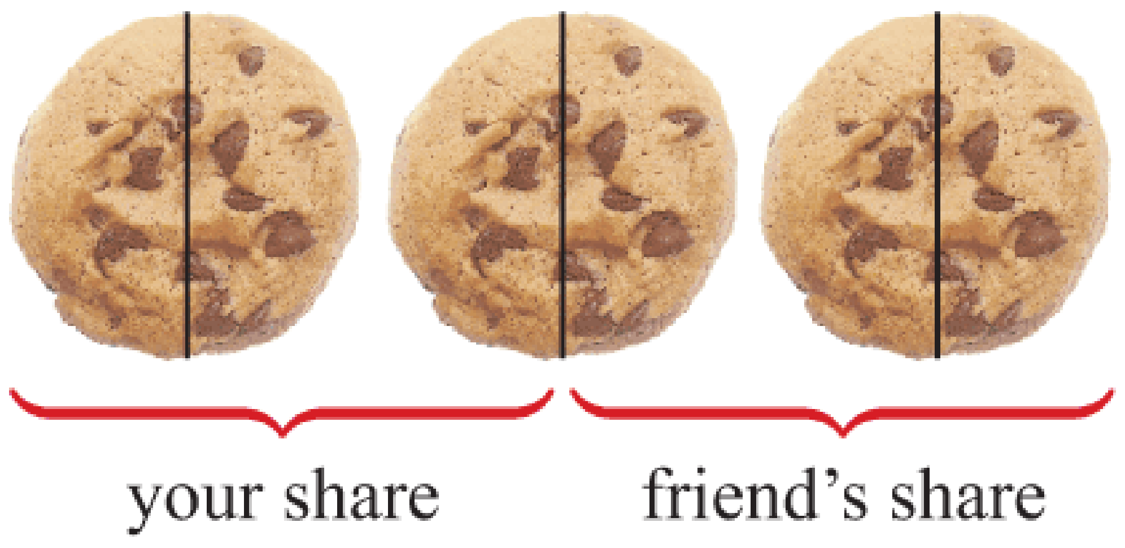


$\frac{4}{3}$ is an improper fraction.



$$\frac{4}{3} = \frac{3}{3} + \frac{1}{3} = 1 + \frac{1}{3}$$

To see how improper fractions occur, suppose you and a friend share three cookies. Each person receives three halves of a cookie, which is $\frac{3}{2}$ cookies.

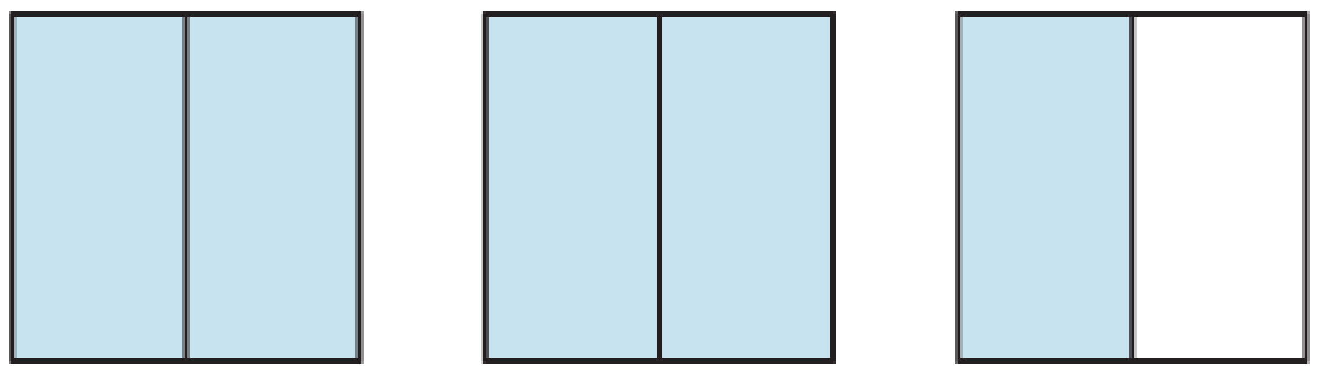


We can also see that $\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2}$.

So each person receives one and a half cookies. We can write this as $1\frac{1}{2}$ cookies.

When an improper fraction is written as a whole number and a proper fraction, it is called a **mixed number**.

For example, $2\frac{1}{2}$ is a mixed number. It means two wholes and one half.

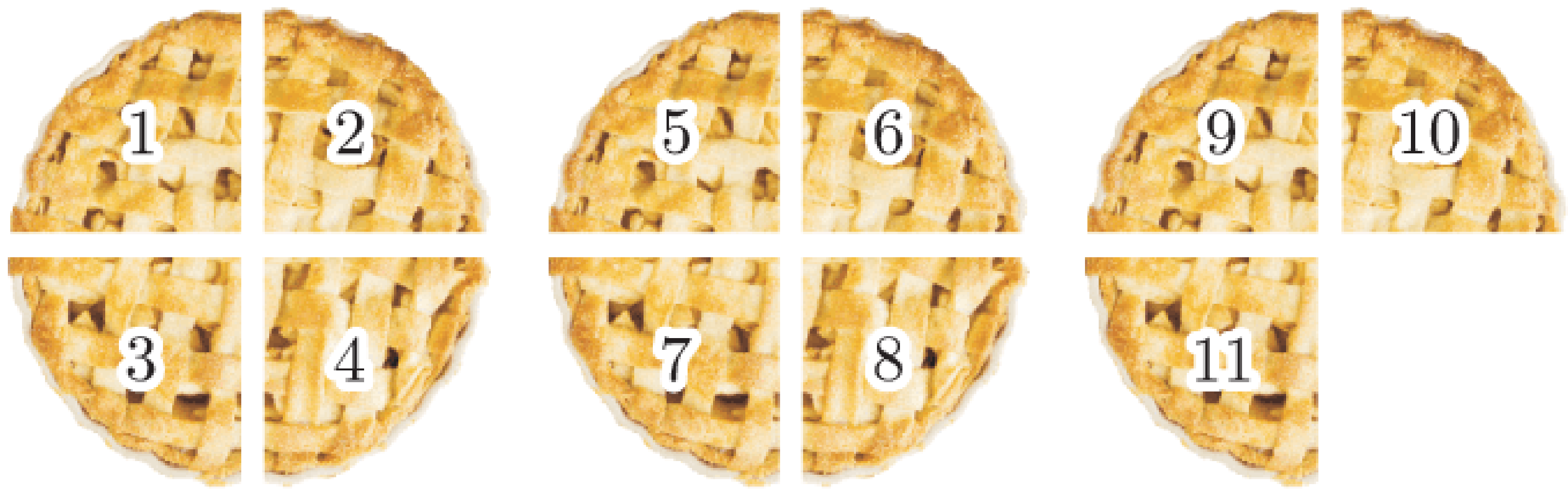


We can write mixed numbers as improper fractions, and vice versa.

For example, at a class picnic there were 3 apple pies, each cut into quarters.

Sam ate one quarter of a pie.

We see there are now $2\frac{3}{4}$ pies remaining.



Each whole pie has 4 quarters, and we have 3 quarters of the third pie, so we have $2 \times 4 + 3 = 11$ quarters.

So, $2\frac{3}{4} = \frac{11}{4}$.

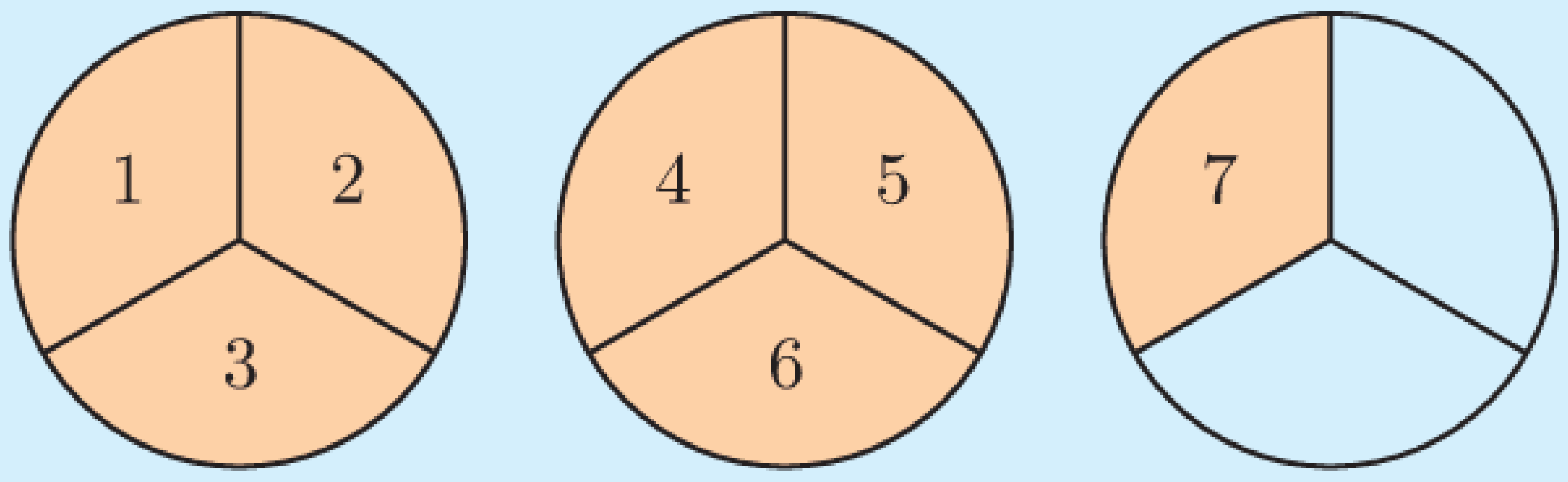
Example 5 **Self Tutor**

Write $2\frac{1}{3}$ as an improper fraction.

$2\frac{1}{3}$ is 2 wholes and one third.

Each whole has 3 thirds, so there are $2 \times 3 + 1 = 7$ thirds.

$\therefore \quad 2\frac{1}{3} = \frac{7}{3}$



EXERCISE 6C

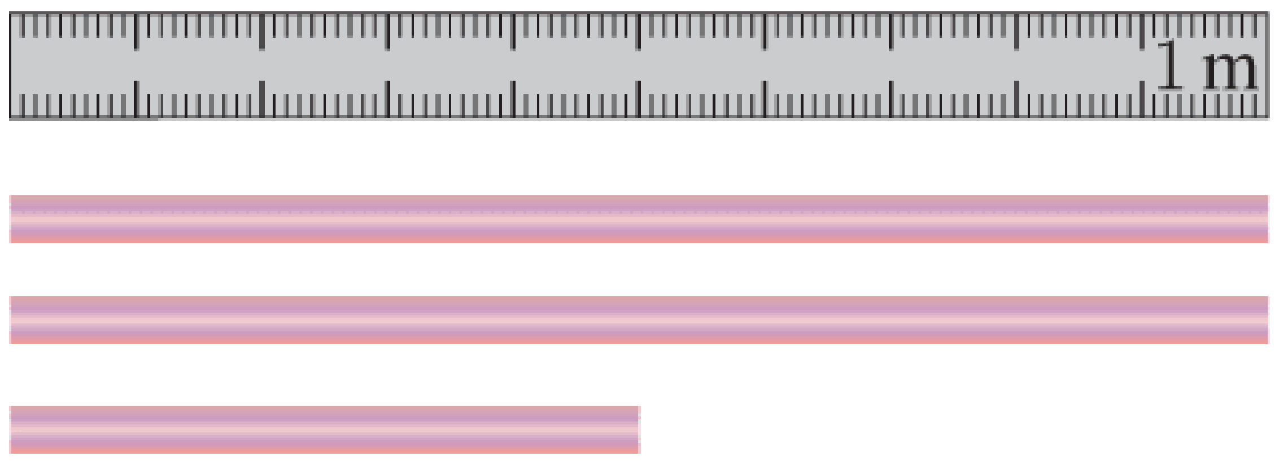
1 Determine whether each of the following is a proper fraction, an improper fraction, or a mixed number:

a $\frac{3}{5}$ **b** $\frac{7}{6}$ **c** $\frac{1}{9}$ **d** $3\frac{1}{3}$

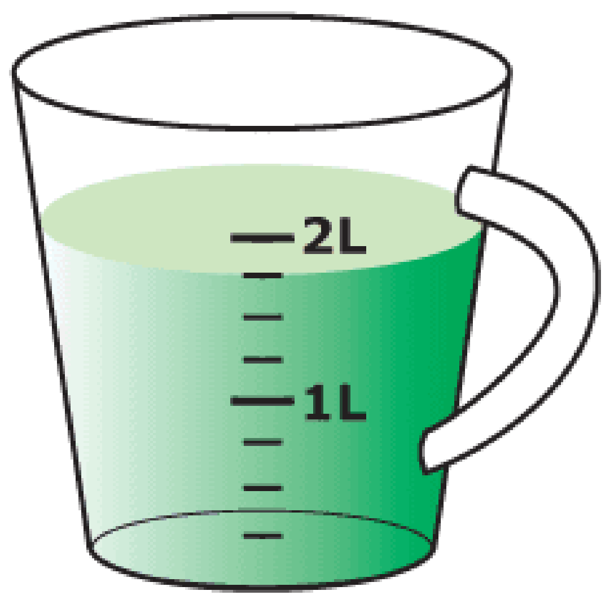
e $\frac{11}{8}$ **f** $\frac{8}{11}$ **g** $4\frac{2}{5}$ **h** $\frac{40}{7}$

2 Write down the mixed number shown in the diagram:

a

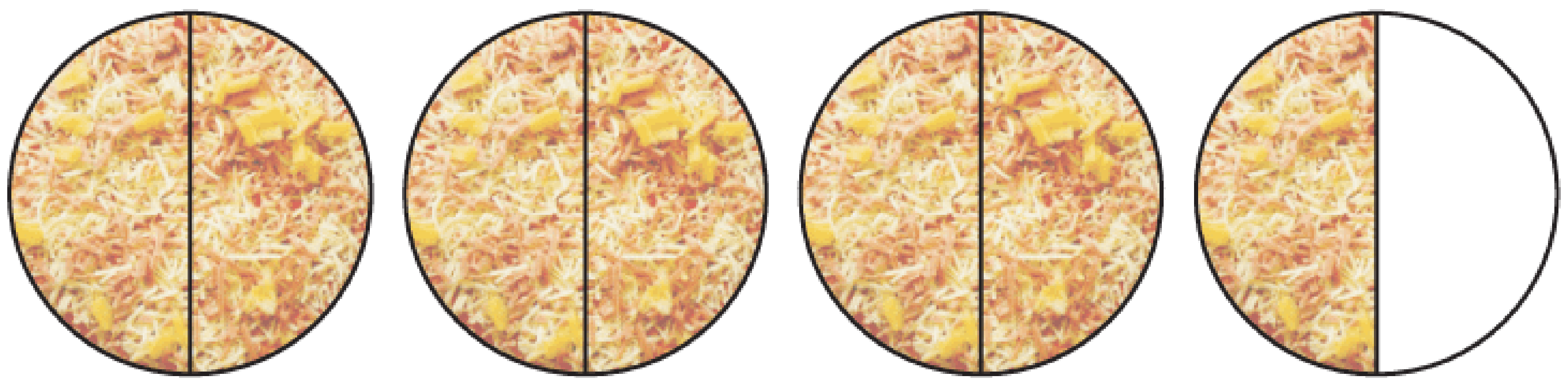


b



c

3

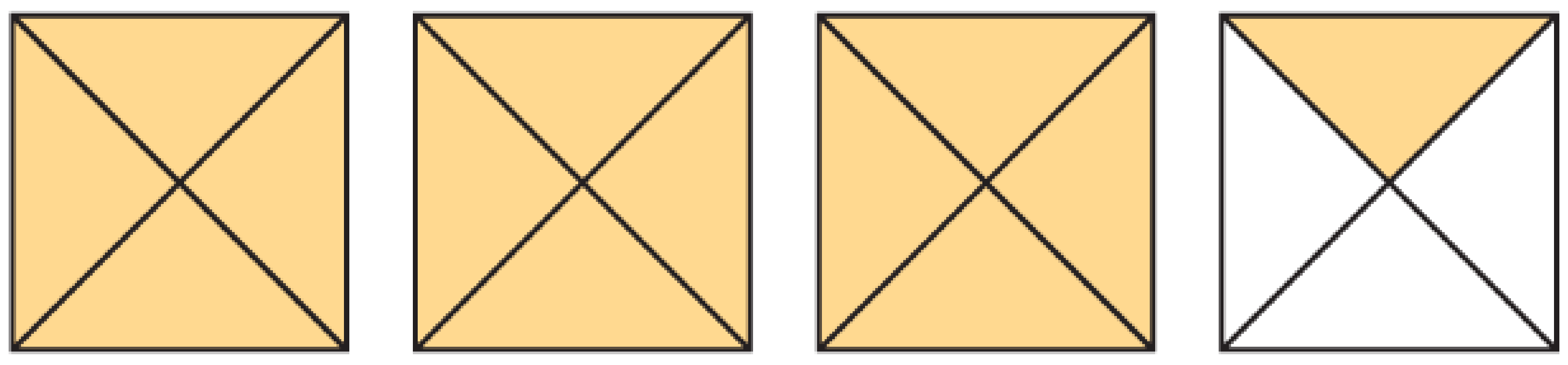


This diagram shows $3\frac{1}{2}$ pizzas.

a How many halves are there in $3\frac{1}{2}$ pizzas?

b Copy and complete: $3\frac{1}{2} = \frac{....}{2}$

4



a What mixed number is represented by this diagram?

b How many quarters are shaded?

c Copy and complete: ...... $= \frac{....}{4}$

5 Write as an improper fraction:

a $1\frac{1}{4}$ **b** $2\frac{1}{2}$ **c** $3\frac{2}{3}$ **d** $2\frac{5}{6}$ **e** $1\frac{3}{5}$

f $5\frac{1}{3}$ **g** $6\frac{1}{2}$ **h** $2\frac{3}{8}$ **i** $4\frac{1}{6}$ **j** $3\frac{7}{10}$

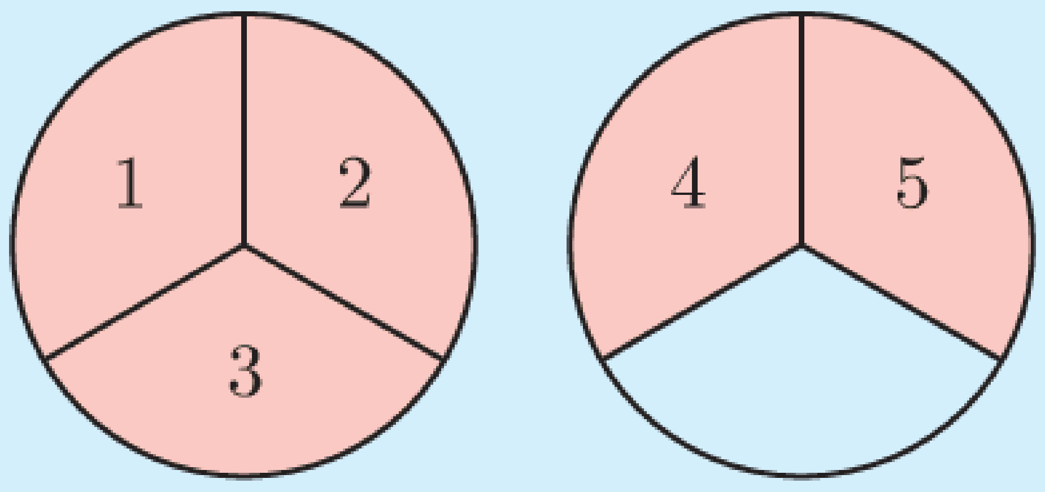
Example 6 **Self Tutor**

Write $\frac{5}{3}$ as a mixed number.

$\frac{5}{3}$ is 5 thirds.

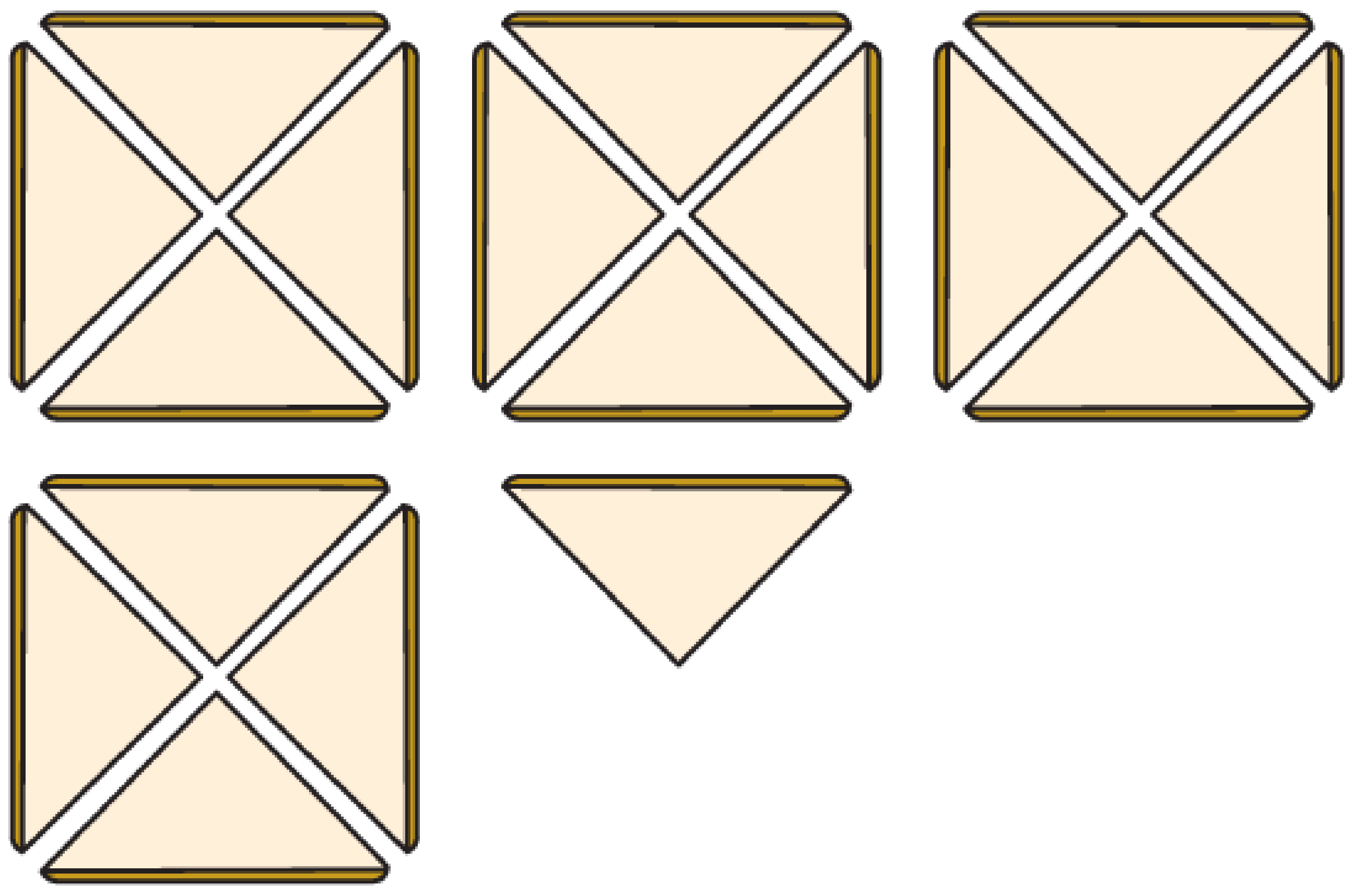
This is 1 whole, and 2 thirds left over.

So, $\frac{5}{3} = 1\frac{2}{3}$



6 After the school picnic there were 17 quarter sandwiches left over.

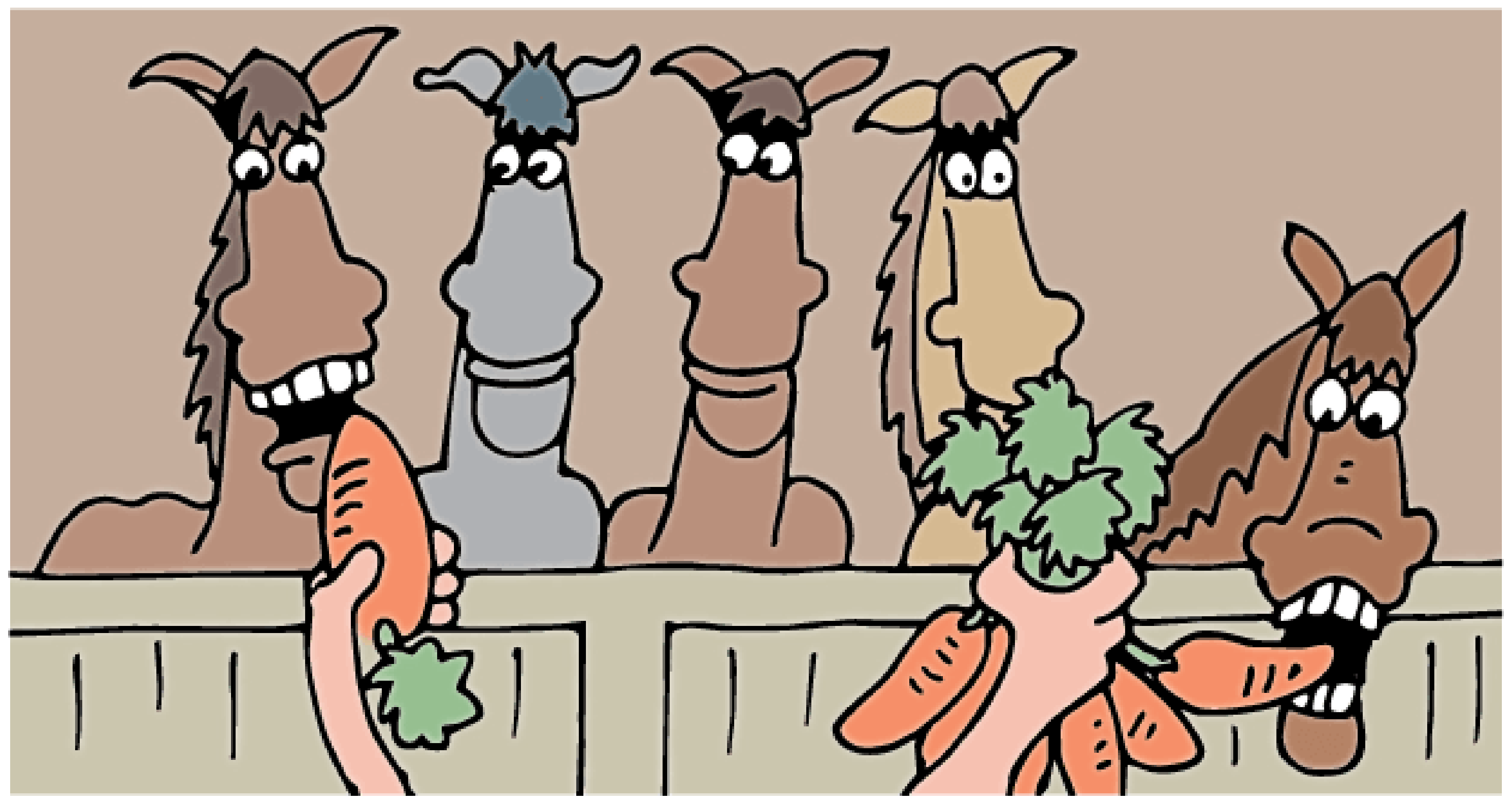
- **a** How many whole sandwiches can be formed from the quarters?
- **b** Once the whole sandwiches have been formed, how many quarters are left over?
- **c** Copy and complete: $\frac{17}{4} = ...$



7 Write as a mixed number:

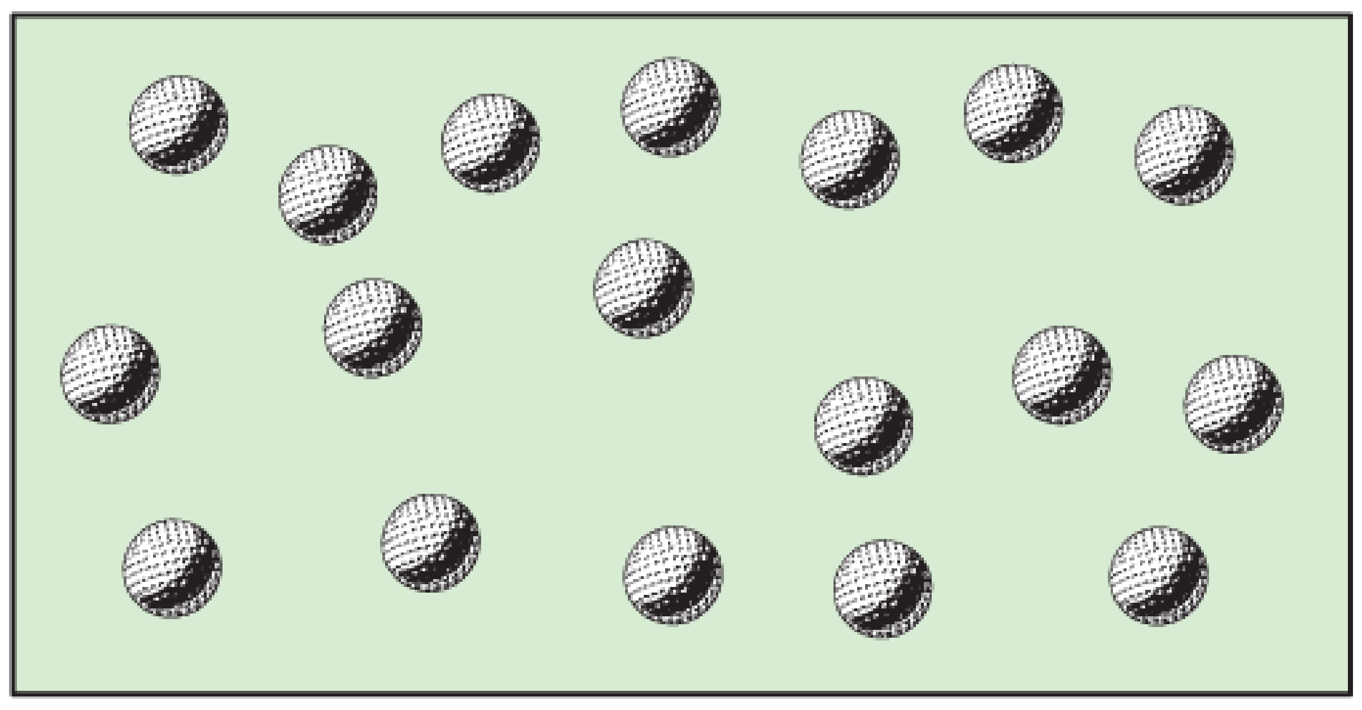
a $\frac{4}{3}$	**b** $\frac{9}{4}$	**c** $\frac{11}{6}$	**d** $\frac{16}{5}$	**e** $\frac{19}{4}$
f $\frac{15}{2}$	**g** $\frac{14}{3}$	**h** $\frac{17}{7}$	**i** $\frac{33}{10}$	**j** $\frac{35}{8}$

8 19 carrots are shared equally between 5 horses. How many carrots does each horse receive? Give your answer as a mixed number.



D FRACTIONS OF QUANTITIES

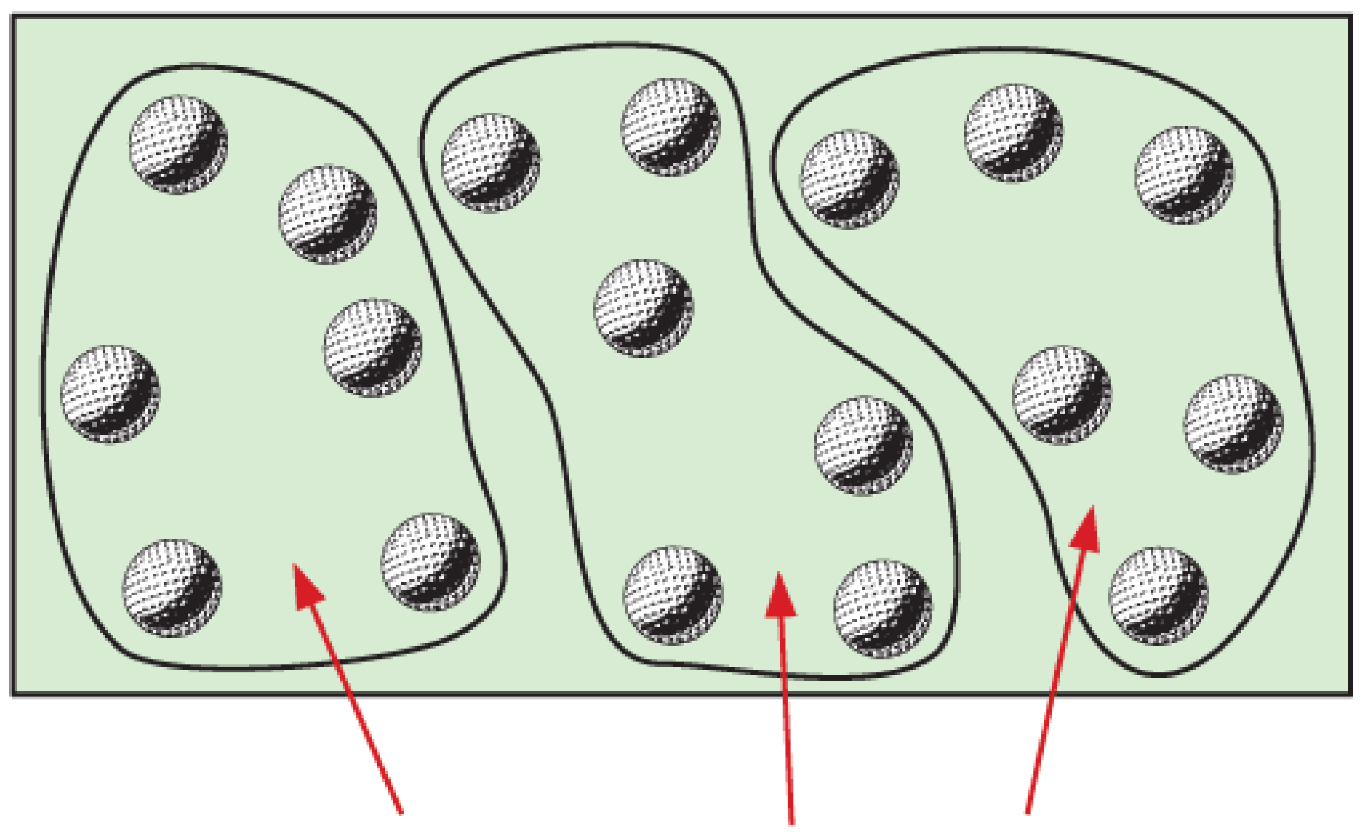
Chris has 18 golf balls. He gives one third of them to his brother Josh. How many golf balls does Josh receive?



To find out, we can divide the golf balls into 3 equal groups.

We see that $\frac{1}{3}$ of 18 balls is 6 balls.

We also notice that $18 \div 3 = 6$.



Each group contains $\frac{1}{3}$ of the golf balls.

So, to find $\frac{1}{3}$ of a number, we divide the number by 3.

Example 7 **Self Tutor**

Find $\frac{1}{4}$ of 24.

$$\begin{aligned}\tfrac{1}{4} \text{ of } 24 &= 24 \div 4 \\ &= 6\end{aligned}$$

EXERCISE 6D

1 Find:

a $\frac{1}{2}$ of 10 **b** $\frac{1}{2}$ of 36 **c** $\frac{1}{3}$ of 12 **d** $\frac{1}{4}$ of 20

e $\frac{1}{4}$ of 44 **f** $\frac{1}{5}$ of 30 **g** $\frac{1}{6}$ of 30 **h** $\frac{1}{8}$ of 48

i $\frac{1}{10}$ of 70 **j** $\frac{1}{3}$ of 45 **k** $\frac{1}{5}$ of 120 **l** $\frac{1}{12}$ of 600

2 Find:

a $\frac{1}{3}$ of 30 people **b** $\frac{1}{4}$ of 20 lollies **c** $\frac{1}{5}$ of 35 drinks

d $\frac{1}{10}$ of 650 g **e** $\frac{1}{2}$ of €38 **f** $\frac{1}{4}$ of 60 minutes

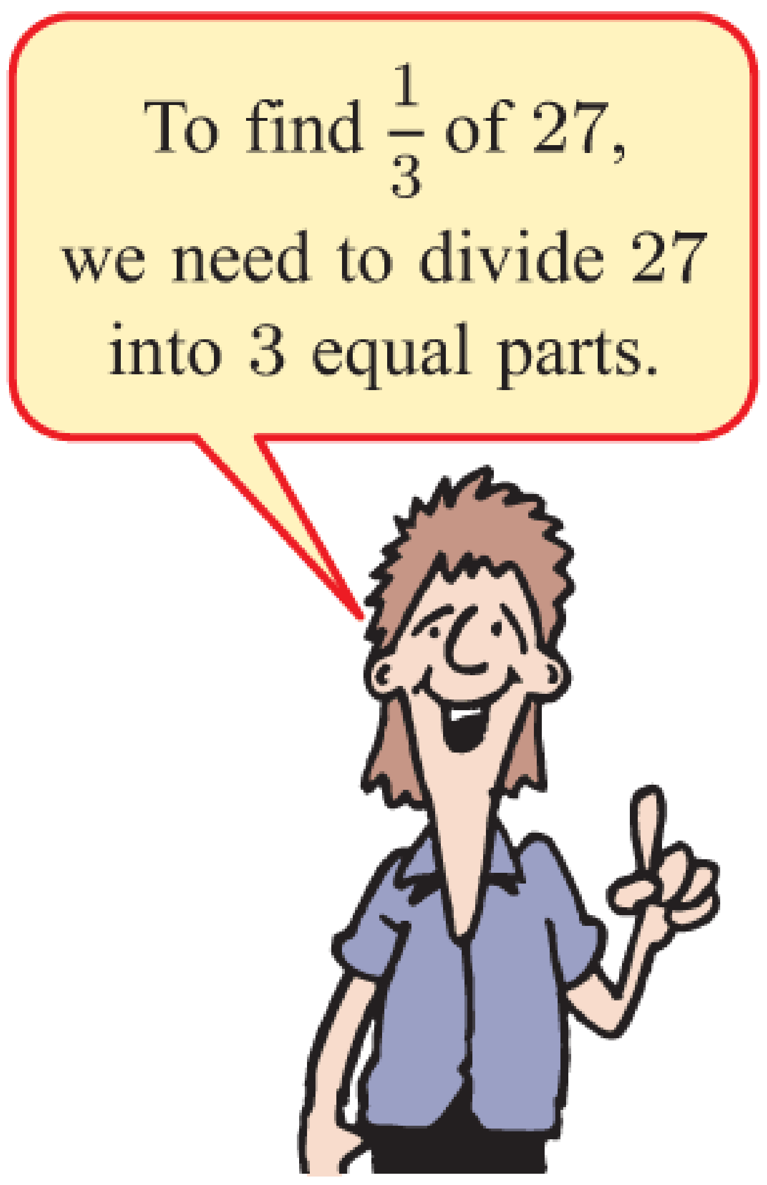
Example 8 **Self Tutor**

On the first day of school this year, there were 27 Year 6 students in a class. $\frac{1}{3}$ of the students were aged 12 years or older. How many students were 12 years or older?

The full class is 27 students.

So, $\frac{1}{3}$ of 27 is $27 \div 3 = 9$ students.

There were 9 students aged 12 years or older.

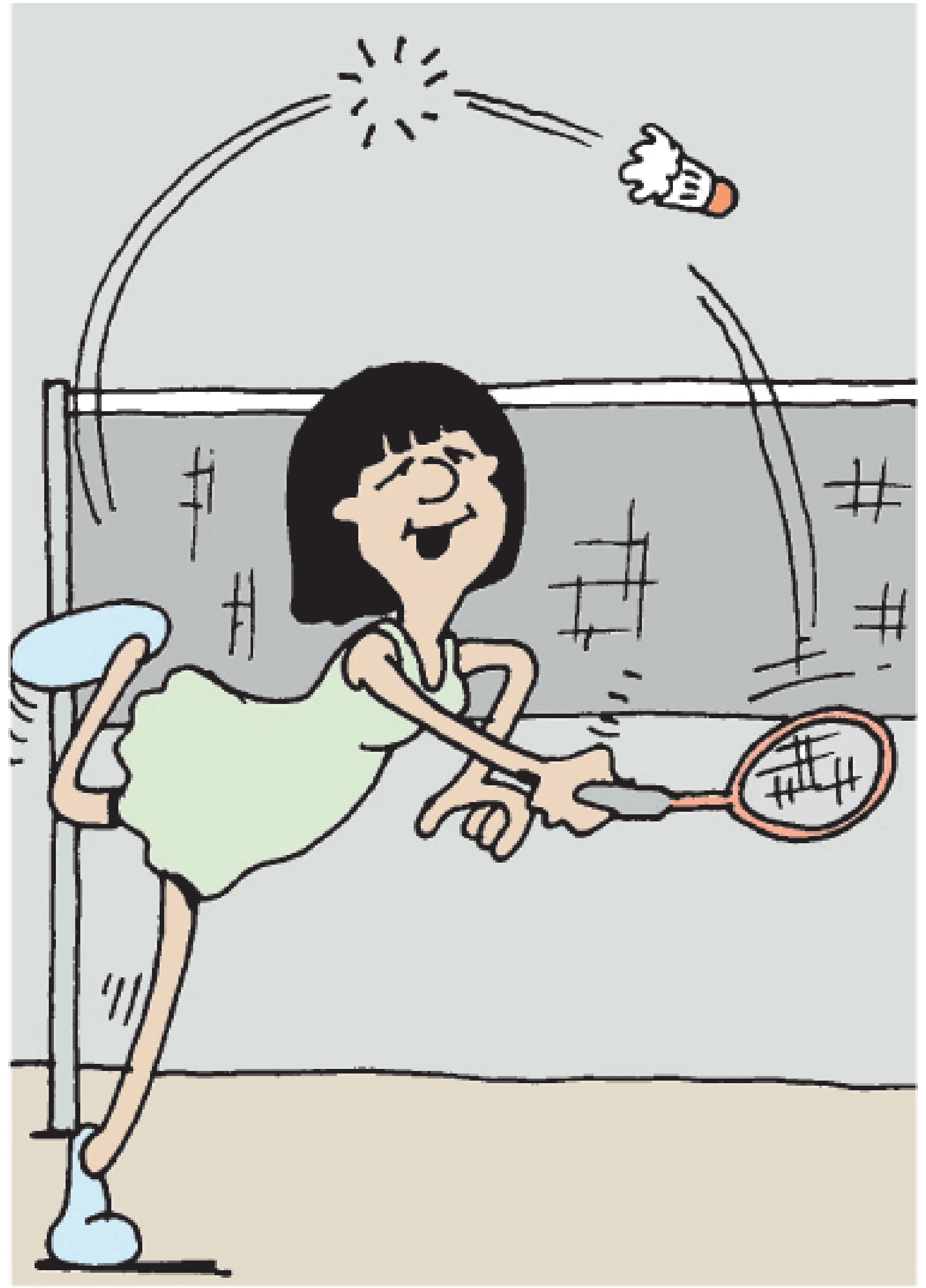


3 Viktor played 15 games of tennis for his school team. He won one third of them. How many games did Viktor win?

4 Of the 250 students at a school, one fifth were absent with chicken pox. How many students were absent?

5 One sixth of the cars from an assembly line were painted white. If 480 cars came from the assembly line, how many were painted white?

6 Ling had $900 in her bank account. She spent one fifth of her money on a new badminton racquet. How much did the racquet cost?



7 While Evan was on holidays, one eighth of the tomato plants in his greenhouse died. If he had 96 plants alive when he went away, how many plants:

a died **b** were still alive?

Example 9 Self Tutor

Find $\frac{2}{5}$ of 30.

$\frac{1}{5}$ of 30 is $30 \div 5 = 6$

$\therefore$ $\frac{2}{5}$ of 30 is $6 \times 2 = 12$

8 Find:

a $\frac{2}{3}$ of 9 **b** $\frac{3}{4}$ of 24 **c** $\frac{2}{5}$ of 45

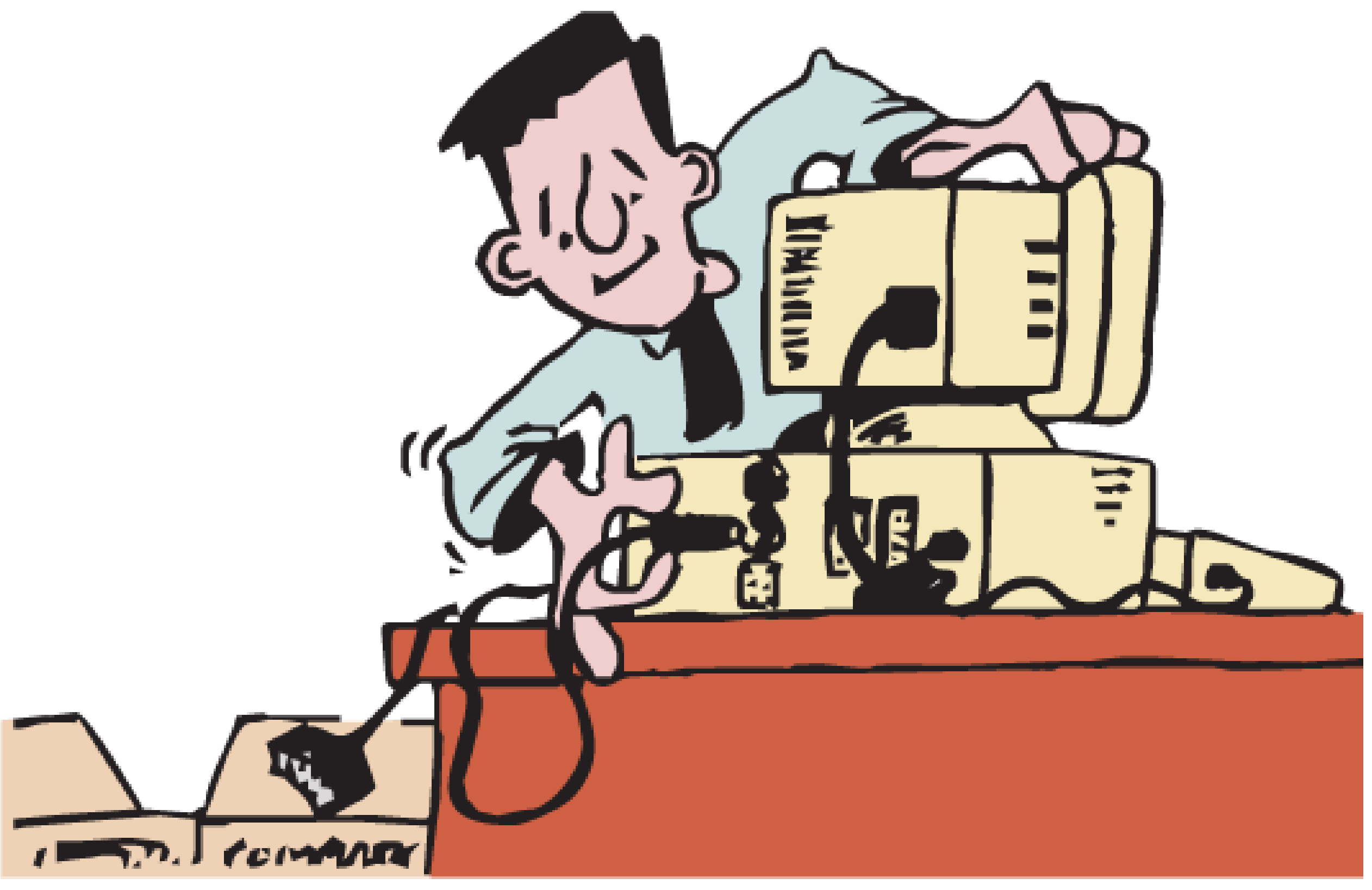
d $\frac{3}{5}$ of 35 **e** $\frac{4}{7}$ of 21 **f** $\frac{5}{6}$ of 54

g $\frac{7}{10}$ of 120 **h** $\frac{8}{9}$ of 72 **i** $\frac{13}{20}$ of 400

9 55 passengers were on the bus one morning. Two fifths of the passengers were school children. How many school children were on the bus?

10 Richard spent three quarters of his working day installing computers, and the remainder of the time travelling between jobs. If his working day was 8 hours, how much time did Richard spend installing computers?

11 Sasha shot for goal 16 times during a netball match. She scored a goal with seven eighths of her shots. How many goals did Sasha score?



12

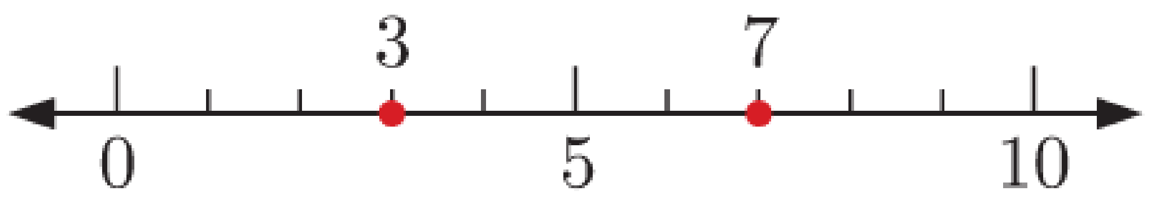
A business hired a truck to transport boxes of equipment. The total weight of the equipment was 3000 kg, but the truck could only carry $\frac{5}{8}$ of the boxes in one load.

a What weight did the truck carry in the first load?

b If there were 80 boxes of equal weight, how many boxes did the truck carry in the *second* load?

E FRACTIONS ON A NUMBER LINE

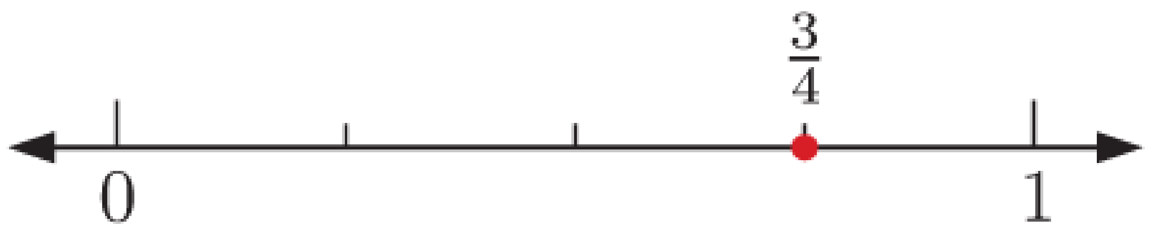
In **Chapter 2**, we placed natural numbers on a number line.



We can do the same thing with fractions.

For example, to place the fraction $\frac{3}{4}$ on a number line, we divide the interval between 0 and 1 into 4 equal parts. Each of the small intervals has length $\frac{1}{4}$.

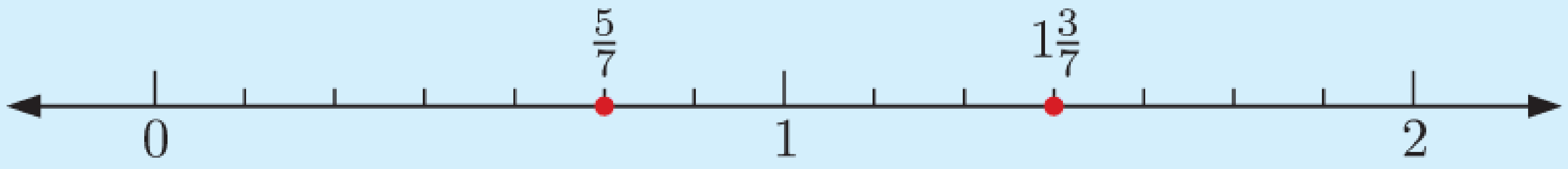
We count 3 intervals from 0, and mark $\frac{3}{4}$ with a dot.



Example 10 **Self Tutor**

Place the fractions $\frac{5}{7}$ and $1\frac{3}{7}$ on a number line.

Since these fractions both involve sevenths, we divide the number line into intervals of length $\frac{1}{7}$.



EXERCISE 6E

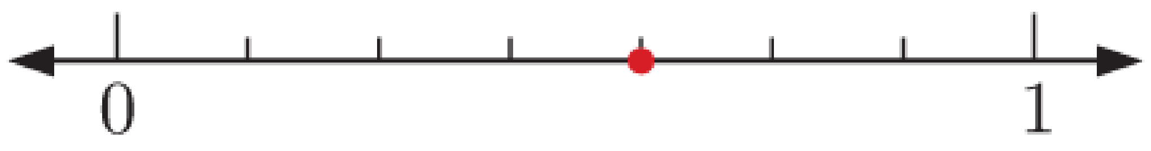
1 Place the following fractions on a number line:

- **a** $\frac{2}{5}$ and $\frac{4}{5}$
- **b** $\frac{3}{6}$ and $\frac{5}{6}$
- **c** $\frac{1}{4}$ and $\frac{7}{4}$
- **d** $\frac{2}{3}$ and $2\frac{1}{3}$
- **e** $\frac{6}{5}$ and $\frac{13}{5}$
- **f** $\frac{5}{8}$ and $1\frac{7}{8}$
- **g** $\frac{2}{10}$, $\frac{5}{10}$, and $\frac{9}{10}$
- **h** $\frac{1}{6}$, $\frac{11}{6}$, and $1\frac{1}{6}$
- **i** $\frac{6}{7}$, $\frac{12}{7}$, and $2\frac{2}{7}$

2 Identify the value indicated by the red dot:

a

b



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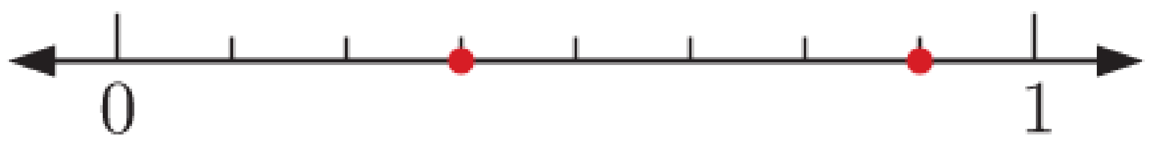

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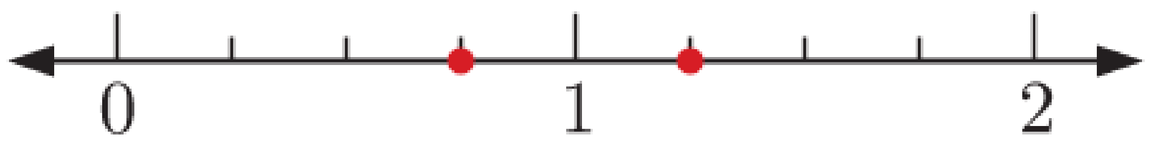

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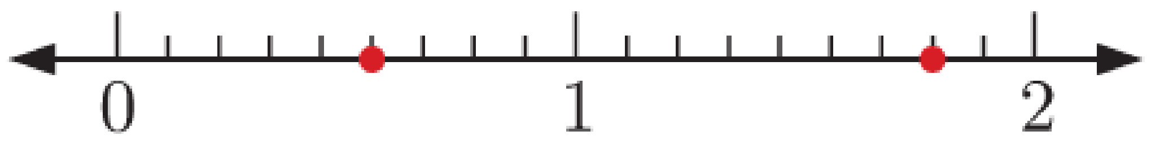

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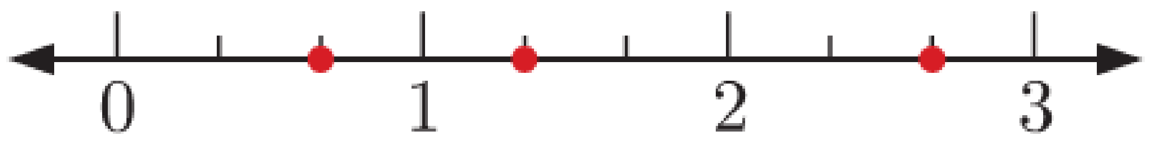

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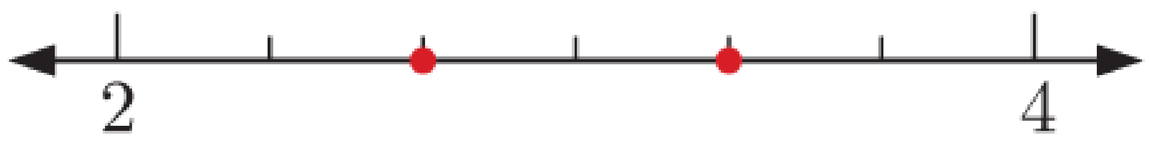
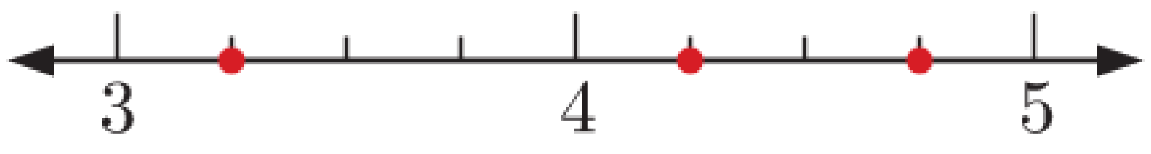


h



3 **a** Place the fraction $\frac{1}{4}$ on a number line.

b On the same number line, place the fraction $\frac{3}{8}$.

c Which value is larger, $\frac{1}{4}$ or $\frac{3}{8}$? Explain your answer.

4 **a** Place the fractions $\frac{2}{3}$ and $\frac{4}{6}$ on the same number line.

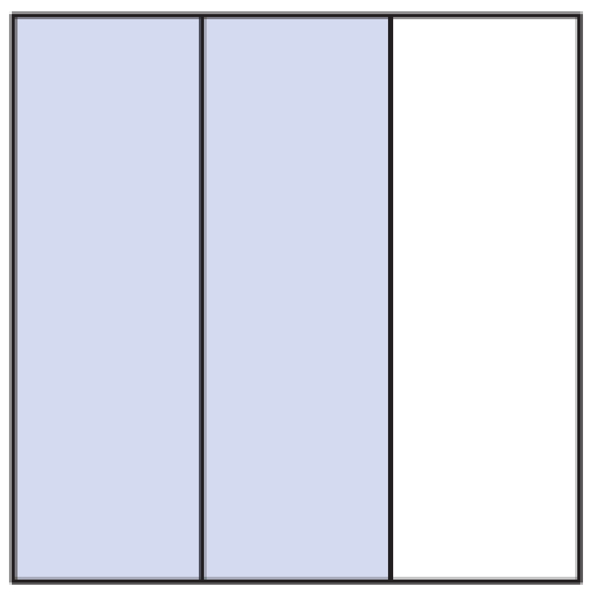
b What do you notice about these fractions?

F EQUAL FRACTIONS

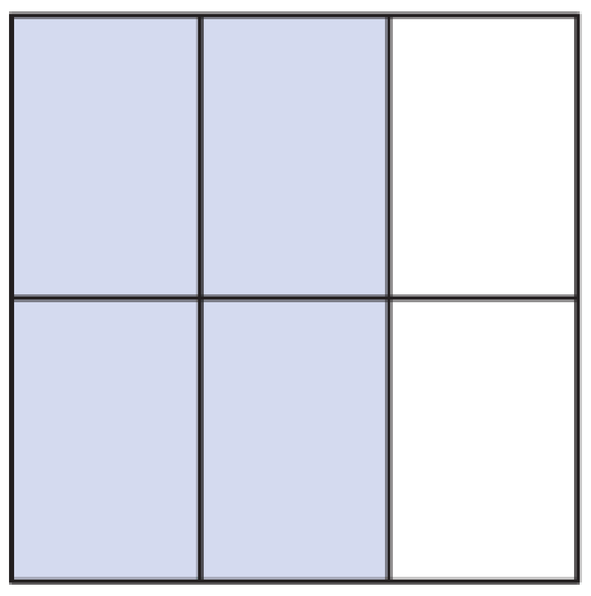
Two fractions are **equal** if they describe the same amount. They lie at the same place on the number line.

For example, we can represent the fractions $\frac{2}{3}$ and $\frac{4}{6}$ by shading diagrams.

We see the same amount is shaded in each diagram, so $\frac{2}{3} = \frac{4}{6}$.



$\frac{2}{3}$ is shaded



$\frac{4}{6}$ is shaded

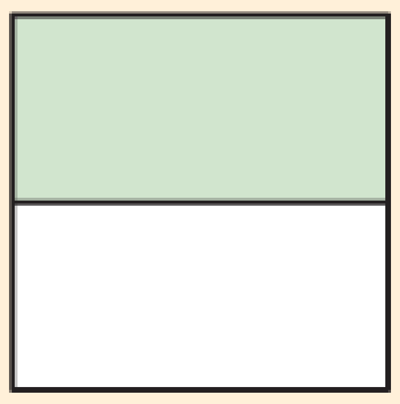
DEMO

INVESTIGATION EQUAL FRACTIONS

What to do:

1 **a** Use grid paper to construct three identical squares with sides 6 cm long, or click on the icon to obtain a template.

TEMPLATE

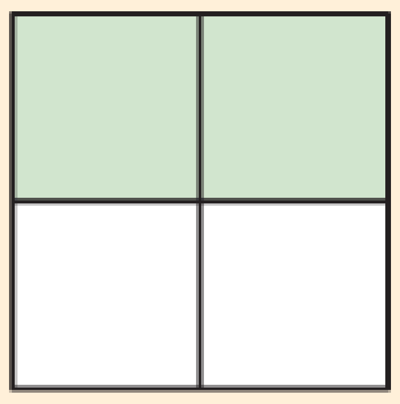


b Divide the first square into two equal halves.

Shade one of the halves, so that $\frac{1}{2}$ of the square is shaded.

c Divide the second square into four equal quarters.

Shade two of the quarters, so that $\frac{2}{4}$ of the square is shaded.

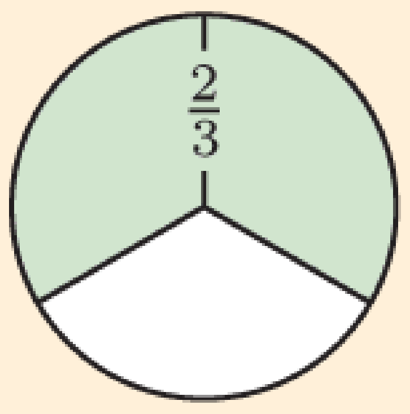


d Do you think that the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are equal? Check your answer by placing them on a number line.

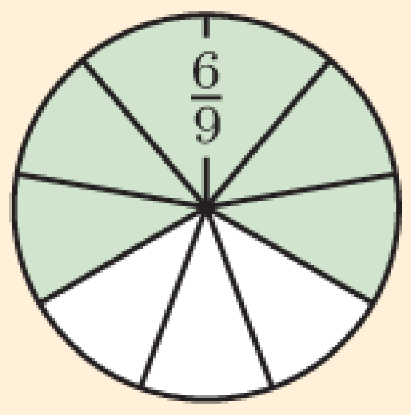
e Divide the third square into six equal sixths. How many sixths do you need to shade, to make a fraction equal to $\frac{1}{2}$ and $\frac{2}{4}$?

2 **a** Draw two circles with radius 3 cm, or print them using the template.

b From the centre of the first circle, measure and rule 3 lines that are 120° apart. Since $3 \times 120° = 360°$, you have divided the circle into thirds. Shade $\frac{2}{3}$ of the circle.



c From the centre of the second circle, measure and rule 9 lines that are 40° apart. Since $9 \times 40° = 360°$, you have divided the circle into ninths. Shade $\frac{6}{9}$ of the circle.



d Do you think that the fractions $\frac{2}{3}$ and $\frac{6}{9}$ are equal? Check your answer by placing them on a number line.

In the **Investigation**, you should have found that $\frac{1}{2} = \frac{2}{4}$, and $\frac{2}{3} = \frac{6}{9}$.

Notice how these numbers are related:

$$\frac{1}{2} \underset{\overset{\curvearrowright}{\times 2}}{\overset{\overset{\times 2}{\curvearrowright}}{=}} \frac{2}{4} \quad \text{or} \quad \frac{1}{2} \underset{\overset{\curvearrowleft}{\div 2}}{\overset{\overset{\div 2}{\curvearrowleft}}{=}} \frac{2}{4} \qquad\qquad \frac{2}{3} \underset{\overset{\curvearrowright}{\times 3}}{\overset{\overset{\times 3}{\curvearrowright}}{=}} \frac{6}{9} \quad \text{or} \quad \frac{2}{3} \underset{\overset{\curvearrowleft}{\div 3}}{\overset{\overset{\div 3}{\curvearrowleft}}{=}} \frac{6}{9}$$

This suggests that:

Multiplying or dividing both the numerator and the denominator by the same non-zero number produces an equal fraction.

This rule allows us to write a given fraction with a different numerator or with a different denominator, without changing the fraction's value.

Example 11 **Self Tutor**

Copy and complete these equal fraction statements:

a $\frac{5}{6} = \frac{....}{12}$ **b** $\frac{8}{20} = \frac{2}{....}$

a The denominator has been multiplied by 2, so we must also multiply the numerator by 2.

$$\frac{5}{6} \underset{\overset{\curvearrowright}{\times 2}}{\overset{\overset{\times 2}{\curvearrowright}}{=}} \frac{10}{12}$$

b The numerator has been divided by 4, so we must also divide the denominator by 4.

$$\frac{8}{20} \underset{\overset{\curvearrowright}{\div 4}}{\overset{\overset{\div 4}{\curvearrowright}}{=}} \frac{2}{5}$$

EXERCISE 6F.1

1 Write a fraction equal to $\frac{6}{10}$ by:

- **a** multiplying both the numerator and denominator by 3
- **b** dividing both the numerator and denominator by 2.

2 Write a fraction equal to $\frac{4}{12}$ by:

- **a** multiplying both the numerator and denominator by 5
- **b** dividing both the numerator and denominator by 4.

3 Copy and complete these equal fraction statements:

a $\frac{1}{4} = \frac{....}{8}$ **b** $\frac{5}{8} = \frac{15}{....}$ **c** $\frac{16}{22} = \frac{....}{11}$

d $\frac{24}{30} = \frac{....}{10}$ **e** $\frac{2}{7} = \frac{10}{....}$ **f** $\frac{6}{7} = \frac{....}{42}$

Example 12 **Self Tutor**

Write with denominator 18:

a $\frac{7}{9}$ **b** $\frac{5}{6}$ **c** $\frac{22}{36}$

a
$$\begin{aligned} &\frac{7}{9} \\ &= \frac{7 \times 2}{9 \times 2} \quad \{9 \times 2 = 18\} \\ &= \frac{14}{18} \end{aligned}$$

b
$$\begin{aligned} &\frac{5}{6} \\ &= \frac{5 \times 3}{6 \times 3} \quad \{6 \times 3 = 18\} \\ &= \frac{15}{18} \end{aligned}$$

c
$$\begin{aligned} &\frac{22}{36} \\ &= \frac{22 \div 2}{36 \div 2} \quad \{36 \div 2 = 18\} \\ &= \frac{11}{18} \end{aligned}$$

4 Write $\frac{3}{4}$ with denominator:

a 8 **b** 12 **c** 16 **d** 20

5 Write $\frac{4}{10}$ with denominator:

a 20 **b** 30 **c** 50 **d** 5

6 Write with denominator 8:

a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** 1 **e** $\frac{10}{16}$

7 Write with denominator 30:

a $\frac{1}{2}$ **b** $\frac{4}{5}$ **c** $\frac{5}{6}$ **d** $\frac{3}{10}$ **e** $\frac{1}{5}$

f $\frac{2}{3}$ **g** 1 **h** $\frac{3}{5}$ **i** $\frac{14}{60}$ **j** $\frac{13}{10}$

8 Write in hundredths:

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{9}{10}$ **e** $\frac{7}{25}$

f $\frac{13}{50}$ **g** 1 **h** $\frac{17}{20}$ **i** $\frac{34}{200}$ **j** $\frac{61}{50}$

GAME EQUAL FRACTIONS

Click on the icon to play a game where you must find equal fractions.

GAME

SIMPLEST FORM

A fraction is written in **simplest form** if it is written with the smallest possible whole number numerator and denominator.

For example, the fraction $\frac{9}{12}$ is not written in simplest form, because we can write $\frac{9}{12}$ as $\frac{3}{4}$.

$$\underset{\div 3}{\overset{\div 3}{\frac{9}{12} = \frac{3}{4}}}$$

To write a fraction in simplest form, we must find the **largest** number that is a factor of both the numerator and the denominator. We then divide the numerator and denominator by this value.

Example 13 **Self Tutor**

Write in simplest form:

a $\frac{5}{20}$ **b** $\frac{8}{12}$

a $\frac{5}{20}$

$= \frac{5 \div 5}{20 \div 5}$ {5 is a factor of both 5 and 20}

$= \frac{1}{4}$

b $\frac{8}{12}$

$= \frac{8 \div 4}{12 \div 4}$ {4 is the largest factor of both 8 and 12}

$= \frac{2}{3}$

A fraction is in simplest form when its numerator and denominator do not have any factors in common, except 1.

EXERCISE 6F.2

1 Write in simplest form:

a $\frac{2}{4}$ **b** $\frac{4}{8}$ **c** $\frac{3}{9}$ **d** $\frac{2}{10}$ **e** $\frac{5}{15}$

f $\frac{4}{24}$ **g** $\frac{6}{10}$ **h** $\frac{20}{30}$ **i** $\frac{18}{21}$ **j** $\frac{24}{32}$

2 Which of these fractions is written in simplest form?

A $\frac{6}{8}$ **B** $\frac{3}{12}$ **C** $\frac{10}{21}$ **D** $\frac{14}{20}$ **E** $\frac{7}{28}$

G COMPARING FRACTIONS

We often wish to compare the size of two fractions.

For example, if you were offered $\frac{3}{5}$ or $\frac{7}{10}$ of a block of chocolate, which would you choose?

If two fractions are written with the same denominator, we can simply compare the sizes of the numerators.

Example 14 **Self Tutor**

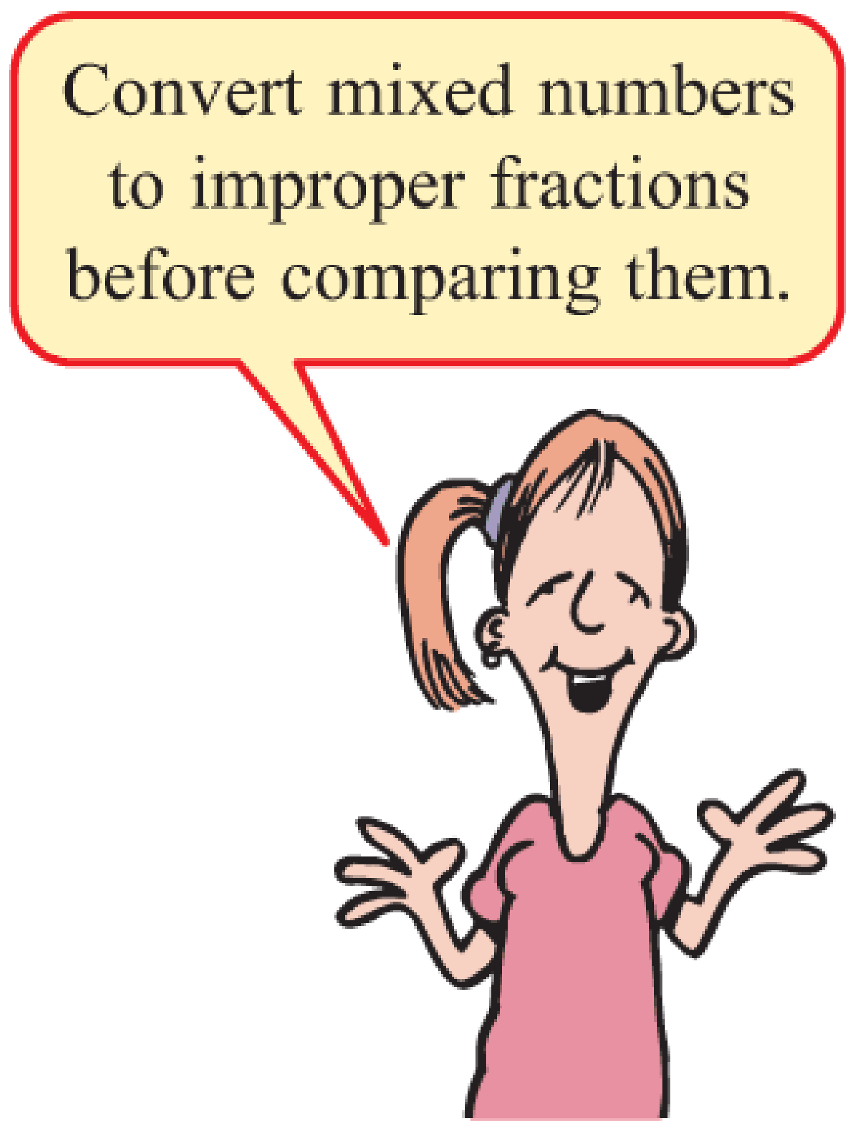
Which is larger:

a $\frac{4}{7}$ or $\frac{6}{7}$ **b** $\frac{13}{5}$ or $2\frac{1}{5}$?

a 6 is larger than 4, so $\frac{6}{7}$ is larger than $\frac{4}{7}$.

b $2\frac{1}{5}$ as an improper fraction is $\frac{11}{5}$.

13 is larger than 11, so $\frac{13}{5}$ is larger than $2\frac{1}{5}$.



If two fractions do *not* have the same denominator, we write one of them as an equal fraction which has the same denominator as the fraction we are comparing with. We can then compare the numerators.

Example 15 **Self Tutor**

Which is larger: $\frac{3}{5}$ or $\frac{7}{10}$?

We multiply the numerator and denominator of $\frac{3}{5}$ by 2, so that both fractions have denominator 10.

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

$\frac{7}{10}$ is greater than $\frac{6}{10}$, so $\frac{7}{10}$ is greater than $\frac{3}{5}$.

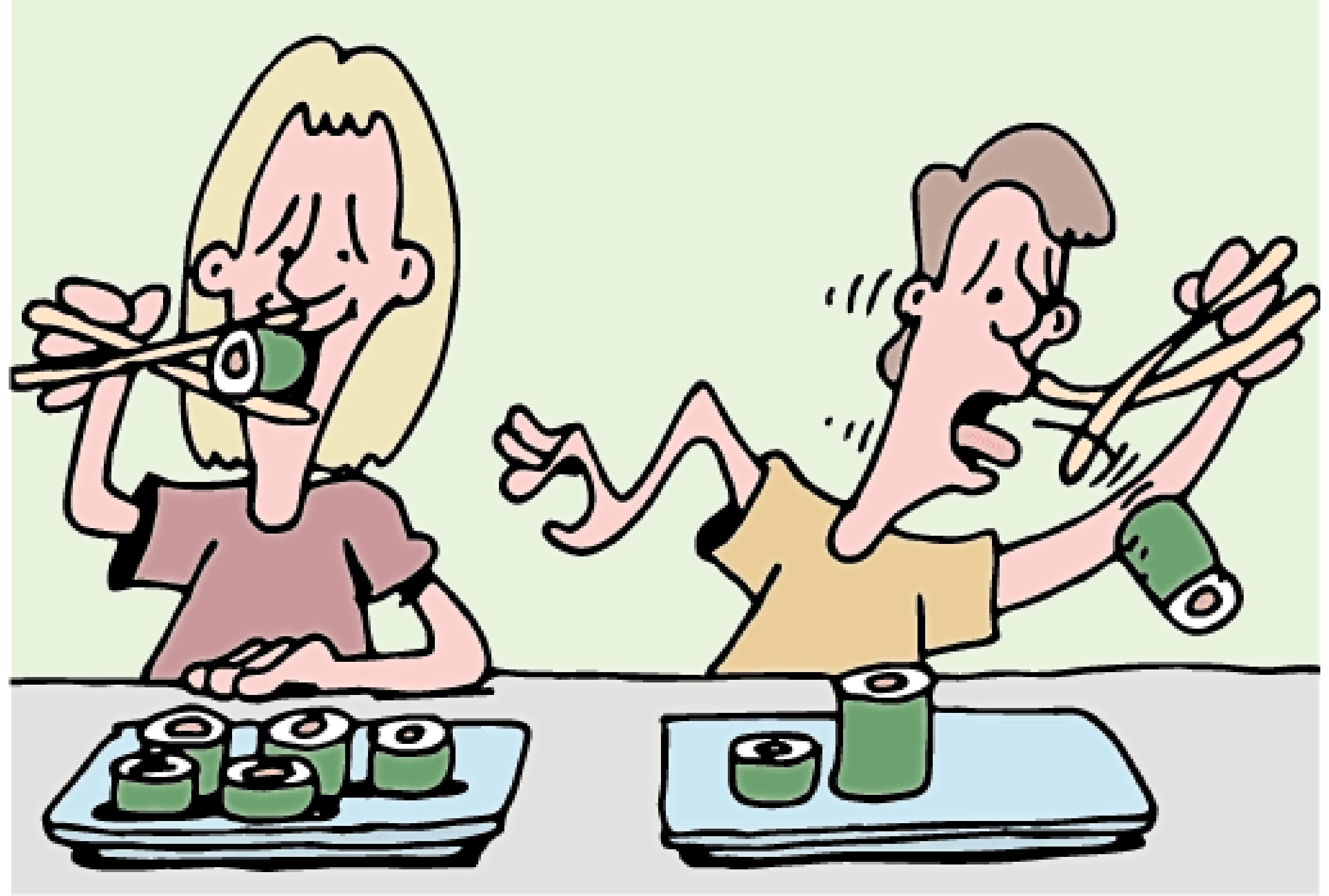
EXERCISE 6G

1 Which is larger:

a $\frac{5}{12}$ or $\frac{7}{12}$ **b** $\frac{4}{5}$ or $\frac{3}{5}$ **c** $\frac{8}{9}$ or $\frac{13}{9}$

d $\frac{11}{7}$ or $1\frac{3}{7}$ **e** $\frac{19}{4}$ or $5\frac{1}{4}$ **f** $\frac{28}{6}$ or $4\frac{5}{6}$?

2 Keith and Caroline ate sushi for dinner. Keith had $3\frac{1}{3}$ pieces of sushi. Caroline cut her sushi pieces into thirds, and ate 8 of the thirds.
Who had more sushi for dinner?

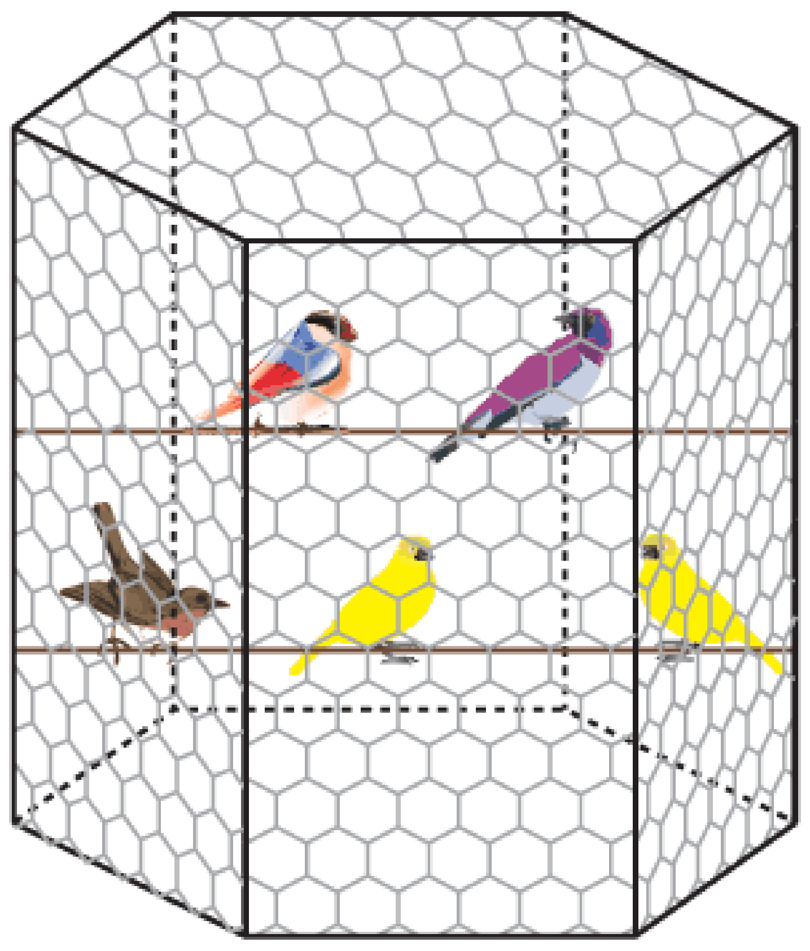


3 Which is larger:

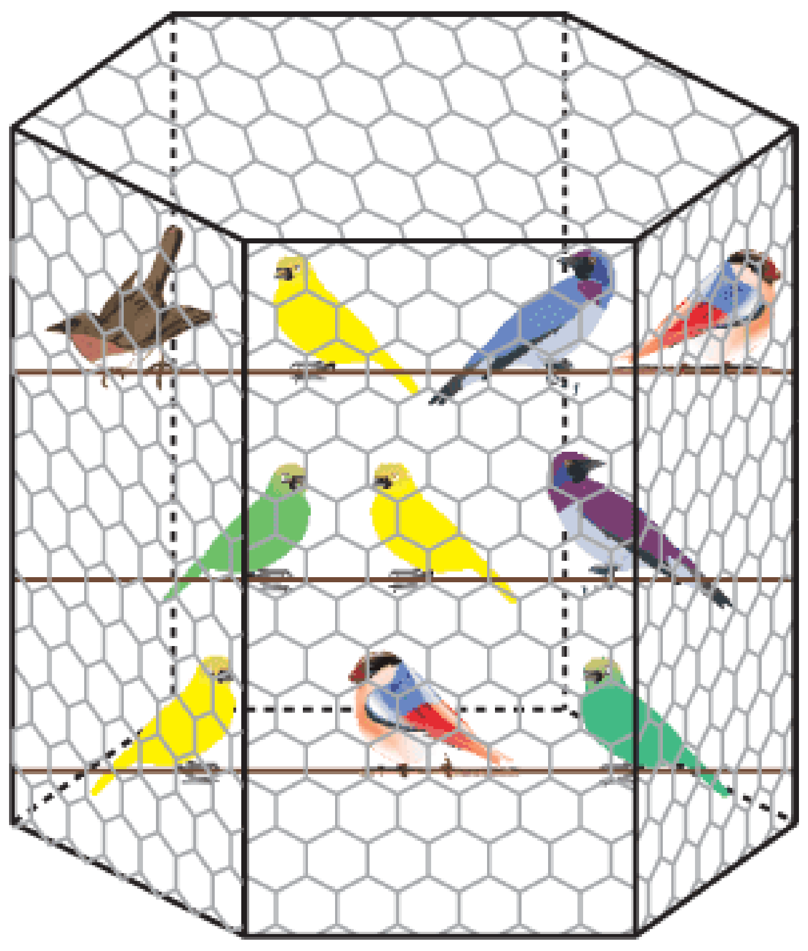
a $\frac{1}{2}$ or $\frac{3}{4}$
b $\frac{1}{3}$ or $\frac{3}{6}$
c $\frac{3}{4}$ or $\frac{7}{8}$
d $\frac{5}{8}$ or $\frac{1}{2}$
e $\frac{2}{3}$ or $\frac{5}{9}$
f $\frac{1}{4}$ or $\frac{5}{20}$
g $\frac{4}{3}$ or $\frac{5}{6}$
h $\frac{13}{15}$ or $\frac{6}{5}$
i $\frac{15}{4}$ or $\frac{7}{2}$?

4 Arnold spends $\frac{1}{3}$ of his income on rent, and $\frac{2}{9}$ of his income on groceries. Does he spend more on rent or on groceries?

5 Trent and Meredith each own a cage of birds.



Trent's cage



Meredith's cage

a What fraction of Trent's birds are yellow?
b What fraction of Meredith's birds are yellow?
c In which cage is there a greater fraction of yellow birds?

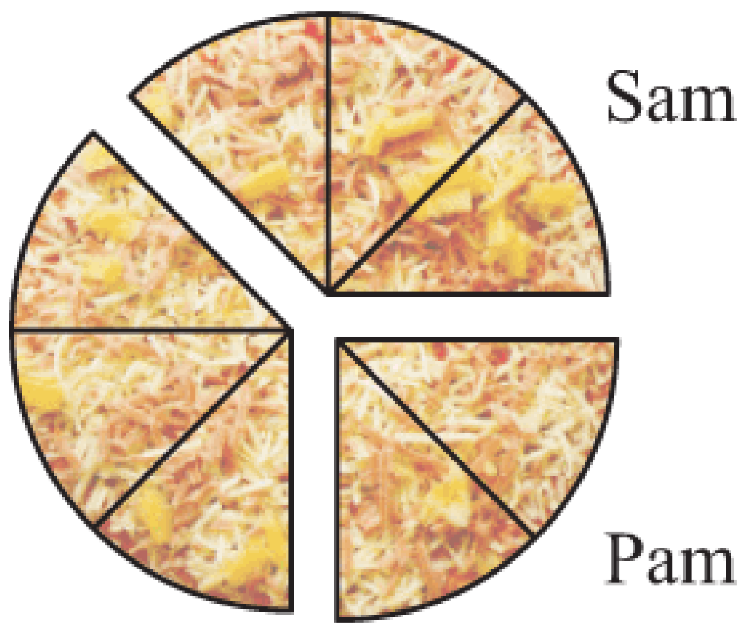
DISCUSSION

Are improper fractions always larger in size than proper fractions?

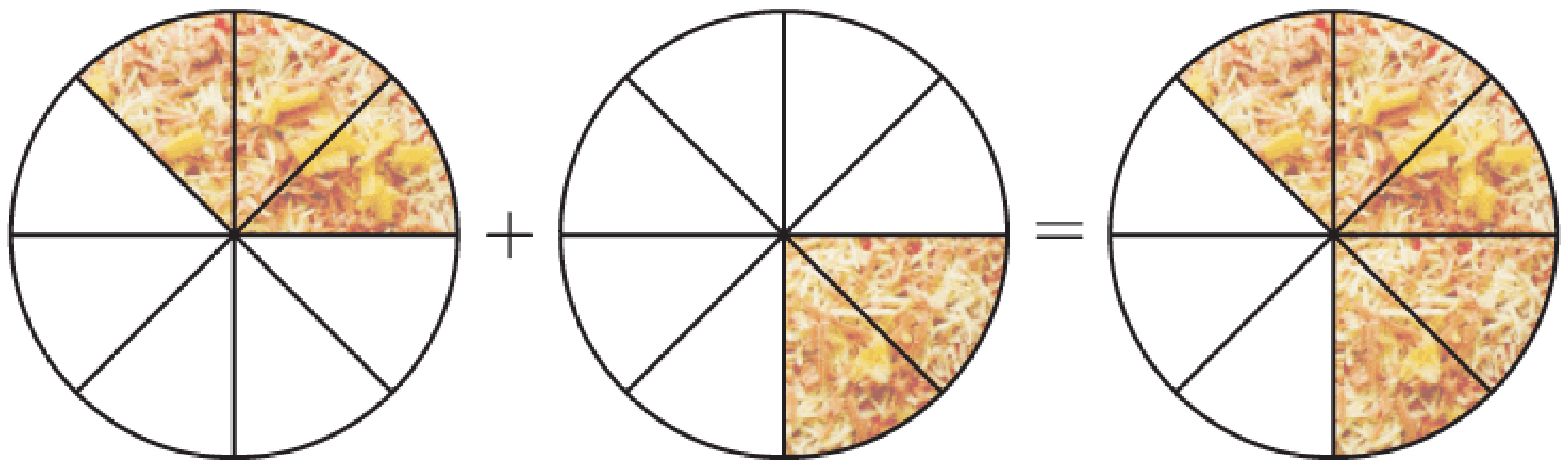
H ADDING AND SUBTRACTING FRACTIONS

A pizza is divided into 8 equal pieces. Sam takes 3 pieces and Pam takes 2 pieces, so together they have taken a total of 5 pieces.

Notice that Sam has taken $\frac{3}{8}$ of the pizza, Pam has taken $\frac{2}{8}$, and together they have taken $\frac{5}{8}$.



So,



$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

Sam eats 1 of his pieces of pizza, so he has 2 pieces remaining. We can also say that Sam took $\frac{3}{8}$ of the pizza, he ate $\frac{1}{8}$ of the pizza, and he has $\frac{2}{8}$ of the pizza remaining.

So,

$$\frac{3}{8} \quad - \quad \frac{1}{8} \quad = \quad \frac{2}{8}$$

To add or subtract fractions with the same denominator, we add or subtract the numerators. The denominator stays the same.

Example 16 **Self Tutor**

Find: **a** $\frac{2}{5} - \frac{1}{5}$ **b** $\frac{4}{9} + \frac{7}{9}$

a
$$\begin{aligned} &\frac{2}{5} - \frac{1}{5} \\ &= \frac{2-1}{5} \\ &= \frac{1}{5} \end{aligned}$$

b
$$\begin{aligned} &\frac{4}{9} + \frac{7}{9} \\ &= \frac{4+7}{9} \\ &= \frac{11}{9} \\ &= 1\frac{2}{9} \end{aligned}$$

EXERCISE 6H.1

1 Find:

a $\frac{1}{4} + \frac{2}{4}$ **b** $\frac{2}{3} - \frac{1}{3}$ **c** $\frac{5}{4} - \frac{2}{4}$

d $\frac{3}{8} + \frac{4}{8}$ **e** $\frac{1}{5} + \frac{3}{5}$ **f** $\frac{5}{7} - \frac{3}{7}$

g $\frac{6}{11} - \frac{2}{11}$ **h** $\frac{9}{20} + \frac{8}{20}$ **i** $\frac{21}{25} - \frac{13}{25} + \frac{1}{25}$

2 Find:

a $\frac{4}{5} + \frac{2}{5}$ **b** $\frac{7}{10} + \frac{6}{10}$ **c** $\frac{6}{7} + \frac{5}{7}$

d $\frac{11}{15} + \frac{8}{15}$ **e** $\frac{10}{13} + \frac{8}{13} + \frac{11}{13}$ **f** $\frac{11}{14} + \frac{13}{14} - \frac{1}{14}$

Example 17 Self Tutor

Find: $2+\frac{4}{7}+\frac{6}{7}$

$$\begin{aligned}&2+\frac{4}{7}+\frac{6}{7}\\=\ &2+\frac{4+6}{7}\\=\ &2+\frac{10}{7}\\=\ &2+1\frac{3}{7}\\=\ &3\frac{3}{7}\end{aligned}$$

3 Find:

a $3+\frac{1}{9}+\frac{4}{9}$ **b** $2+\frac{3}{10}+\frac{4}{10}$ **c** $5+\frac{6}{7}-\frac{4}{7}$

d $1+\frac{5}{6}+\frac{2}{6}$ **e** $4+\frac{13}{15}+\frac{4}{15}$ **f** $7+\frac{12}{17}+\frac{10}{17}$

Example 18 Self Tutor

Find $\frac{3}{8}-\frac{2}{8}+\frac{5}{8}$, giving your answer in simplest form.

$$\begin{aligned}&\frac{3}{8}-\frac{2}{8}+\frac{5}{8}\\=\ &\frac{3-2+5}{8}\\=\ &\frac{6}{8}\\=\ &\frac{6\div 2}{8\div 2}\qquad \{2 \text{ is a factor of both 6 and 8}\}\\=\ &\frac{3}{4}\end{aligned}$$

4 Find, giving your answer in simplest form:

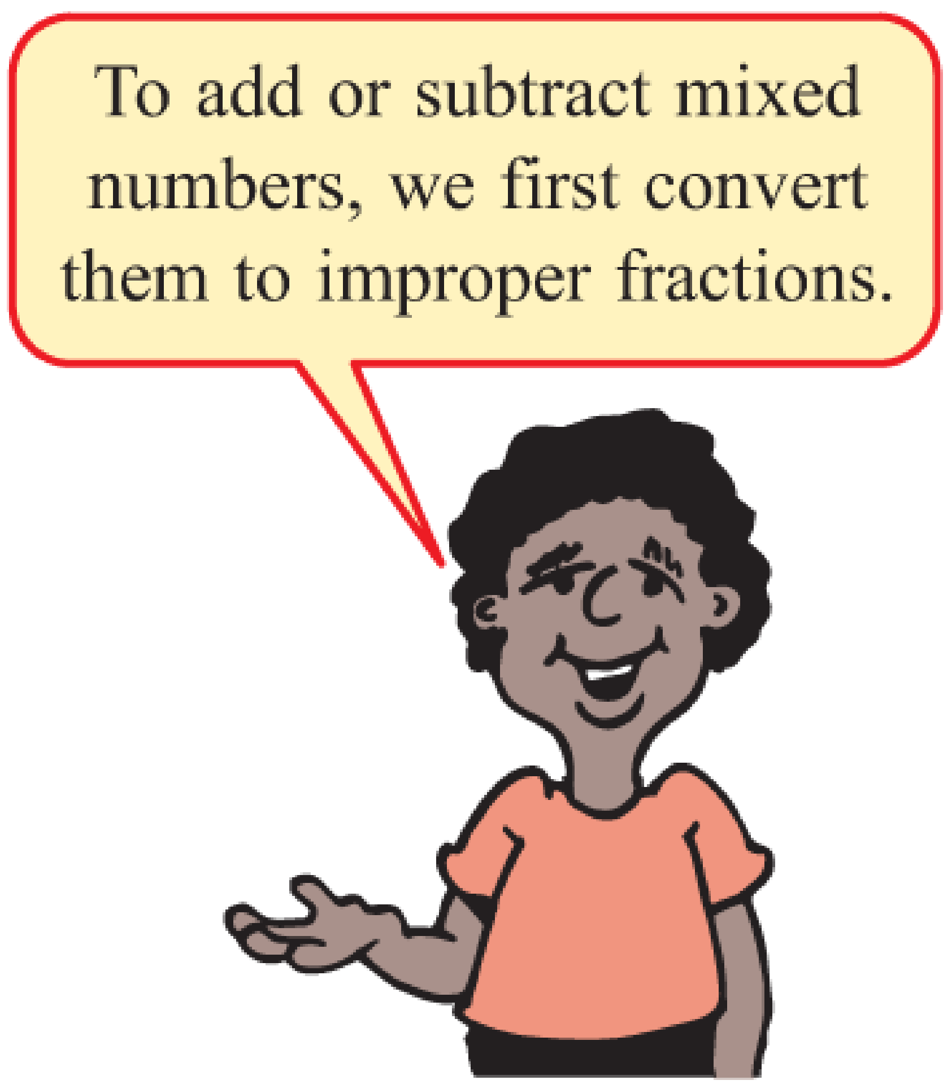
a $\frac{3}{4}-\frac{1}{4}$ **b** $\frac{2}{9}+\frac{1}{9}$ **c** $\frac{7}{6}-\frac{3}{6}$

d $\frac{1}{8}+\frac{2}{8}+\frac{3}{8}$ **e** $\frac{8}{4}-\frac{3}{4}-\frac{3}{4}$ **f** $\frac{3}{10}+\frac{7}{10}-\frac{2}{10}$

Example 19 Self Tutor

Find $2\frac{3}{5}+8\frac{4}{5}$.

$$\begin{aligned}&2\frac{3}{5}+8\frac{4}{5}\\=\ &\frac{13}{5}+\frac{44}{5}\\=\ &\frac{13+44}{5}\\=\ &\frac{57}{5}\\=\ &11\frac{2}{5}\end{aligned}$$



5 Find:

a $2\frac{2}{3} + 1\frac{2}{3}$

b $4\frac{3}{5} - 2\frac{1}{5}$

c $3\frac{2}{7} + 5\frac{3}{7}$

d $6\frac{2}{8} - 3\frac{5}{8}$

e $1\frac{5}{9} + 3\frac{2}{9} + 4\frac{7}{9}$

f $8\frac{1}{10} - 5\frac{7}{10} + 6\frac{3}{10}$

6 Simon and Shane went hiking. On the first day they walked $\frac{5}{9}$ of the total distance. They had a steep climb on the second day and only walked $\frac{2}{9}$ of the total distance.
What fraction of the total distance was completed after two days?

7 Leah wrote $1\frac{1}{4}$ pages of a story before dinner, and another $2\frac{1}{4}$ pages after dinner. How many pages had she completed?

8 Spiros had $\frac{9}{10}$ of a bag of fertiliser. He used $\frac{6}{10}$ of a bag for his tomatoes.

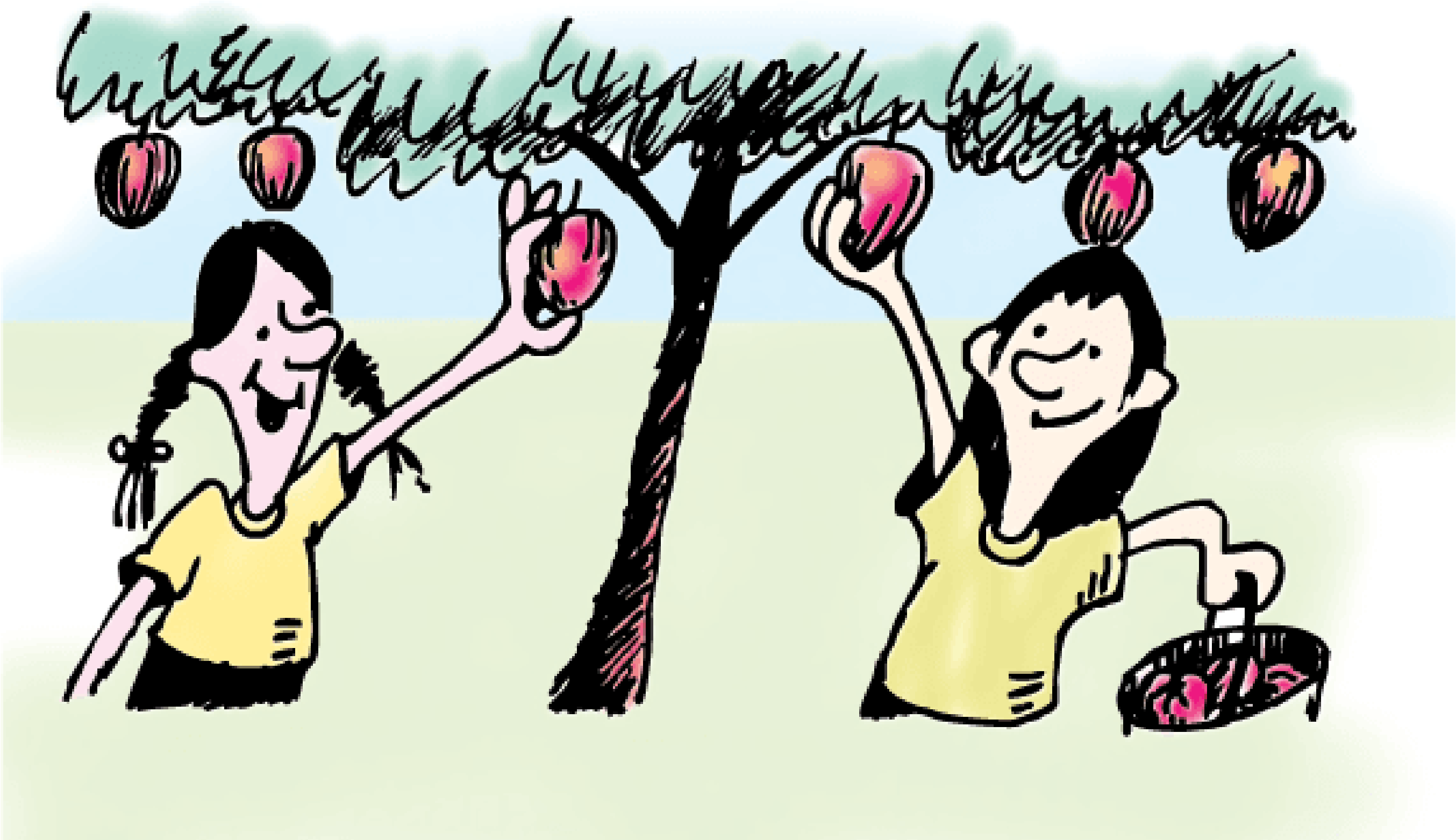
a What fraction of the bag of fertiliser was left?

b The full bag of fertiliser contained 20 kg. How many kilograms of fertiliser does Spiros have left?

9 Sarah and Jane went apple picking.
Sarah picked $1\frac{3}{5}$ bags and Jane picked $2\frac{4}{5}$ bags.

a How many bags of apples did they pick altogether?

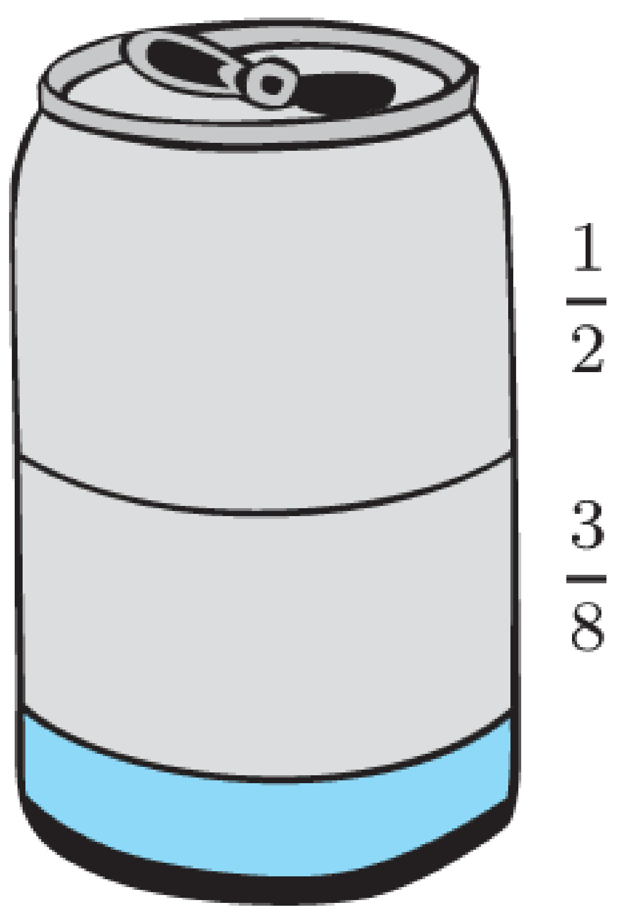
b How many more bags did Jane pick than Sarah?



ADDING AND SUBTRACTING FRACTIONS WITH UNEQUAL DENOMINATORS

Sometimes the fractions we want to add or subtract do not have the same denominator.

For example, suppose Anita drinks $\frac{1}{2}$ of a can of soft drink, and Melissa drinks $\frac{3}{8}$ of the can. Altogether, what fraction of the can have they drunk?

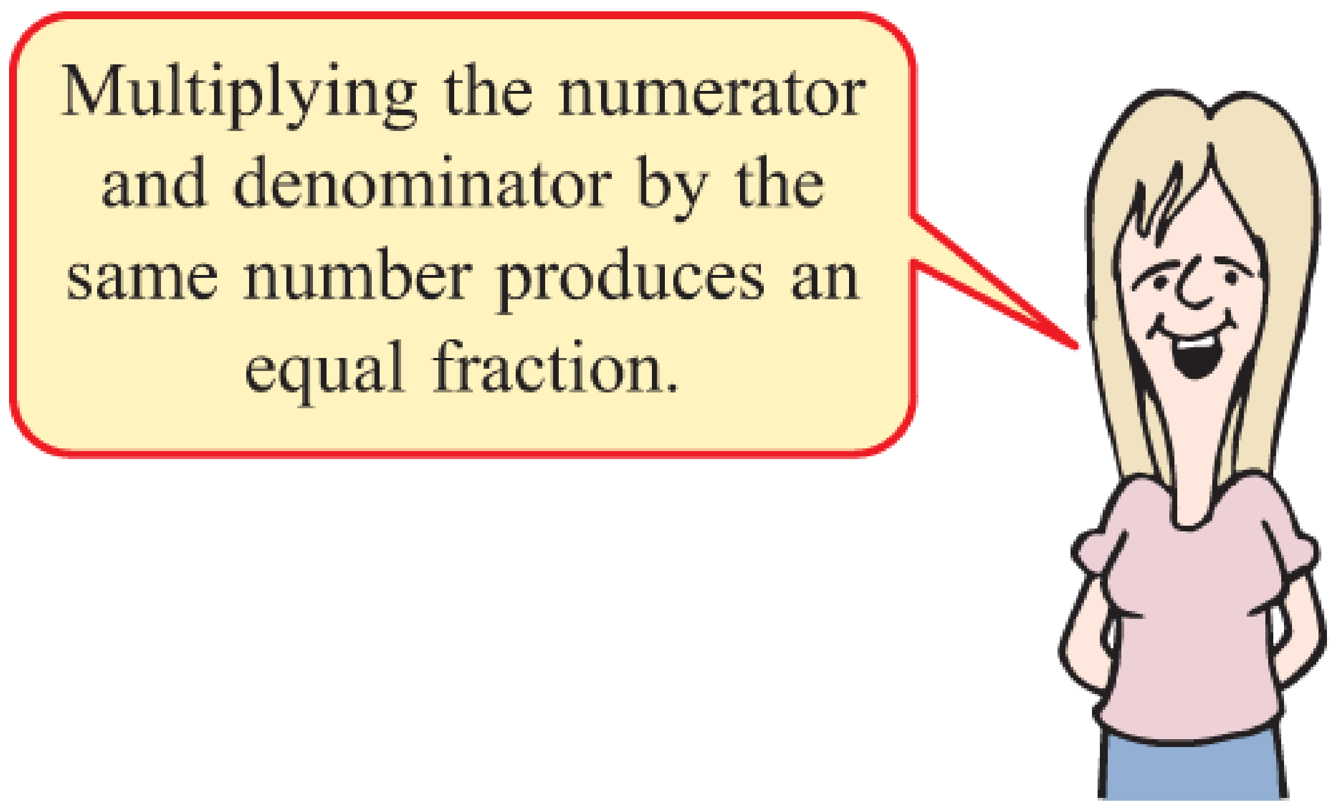


In the same way that it is easier to compare two fractions if they have the same denominator, it is also easier to add or subtract fractions if they have the same denominator.

In the situation above, we need to find $\frac{1}{2}+\frac{3}{8}$.

We can write $\frac{1}{2}$ with denominator 8 by multiplying both the numerator and denominator by 4.

$$\begin{aligned}&\frac{1}{2}+\frac{3}{8}\\ &=\frac{1\times 4}{2\times 4}+\frac{3}{8}\\ &=\frac{4}{8}+\frac{3}{8}\\ &=\frac{7}{8}\end{aligned}$$



So, Anita and Melissa drank $\frac{7}{8}$ of the can of soft drink.

Example 20 **Self Tutor**

Find:

a $\frac{5}{9}-\frac{1}{3}$

b $\frac{3}{5}+\frac{7}{10}$

a
$$\begin{aligned}&\frac{5}{9}-\frac{1}{3}\\ &=\frac{5}{9}-\frac{1\times 3}{3\times 3} \quad \{\text{converting to 9ths}\}\\ &=\frac{5}{9}-\frac{3}{9}\\ &=\frac{2}{9}\end{aligned}$$

b
$$\begin{aligned}&\frac{3}{5}+\frac{7}{10}\\ &=\frac{3\times 2}{5\times 2}+\frac{7}{10} \quad \{\text{converting to 10ths}\}\\ &=\frac{6}{10}+\frac{7}{10}\\ &=\frac{13}{10}\\ &=1\frac{3}{10}\end{aligned}$$

EXERCISE 6H.2

1 Find:

a $\frac{1}{2}-\frac{1}{4}$ **b** $\frac{1}{6}+\frac{2}{3}$ **c** $\frac{5}{8}-\frac{1}{4}$

d $\frac{1}{3}+\frac{1}{12}$ **e** $\frac{19}{30}-\frac{2}{5}$ **f** $\frac{2}{7}+\frac{30}{49}$

2 Find:

a $\frac{4}{5}+\frac{3}{10}$ **b** $\frac{10}{12}+\frac{3}{4}$ **c** $\frac{7}{9}+\frac{21}{45}$ **d** $\frac{23}{25}+\frac{51}{100}$

3 Find:

a $2+\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{6}+\frac{1}{3}+\frac{1}{6}$ **c** $\frac{1}{2}+\frac{3}{4}+\frac{1}{8}$

Example 21 **Self Tutor**

Find:

a $1\frac{1}{4}+2\frac{1}{2}$ **b** $4-1\frac{2}{3}$

a $1\frac{1}{4}+2\frac{1}{2}$

$=\frac{5}{4}+\frac{5}{2}$ {converting to improper fractions}

$=\frac{5}{4}+\frac{5\times 2}{2\times 2}$ {converting to quarters}

$=\frac{5}{4}+\frac{10}{4}$

$=\frac{15}{4}$

$=3\frac{3}{4}$

b $4-1\frac{2}{3}$

$=\frac{4}{1}-\frac{5}{3}$ {converting to improper fractions}

$=\frac{4\times 3}{1\times 3}-\frac{5}{3}$ {converting to thirds}

$=\frac{12}{3}-\frac{5}{3}$

$=\frac{7}{3}$

$=2\frac{1}{3}$

4 Find:

a $1\frac{1}{2}+2\frac{3}{8}$ **b** $3\frac{1}{3}-2\frac{1}{6}$ **c** $1\frac{2}{5}+1\frac{9}{10}$

d $7\frac{1}{2}-5\frac{3}{4}$ **e** $2\frac{2}{7}+1\frac{10}{21}$ **f** $6\frac{2}{3}-3\frac{2}{15}$

5 Find:

a $1-\frac{4}{9}$ **b** $3-1\frac{5}{8}$ **c** $7-4\frac{2}{7}$ **d** $9-6\frac{5}{12}$

6 Joshua baked a cake to share with friends. Lisa ate $\frac{2}{9}$, and Rebecca ate $\frac{1}{3}$ of the cake. What fraction of the cake did the girls eat between them?

7 Rob is planting flowers in his garden. He planted $\frac{1}{4}$ of the flowers on the first day, and $\frac{3}{8}$ of the flowers on the second day. What fraction of the flowers has he planted so far?

8 Every day, Angus feeds his chickens $\frac{1}{5}$ of a large tub of feed. If Angus' tub is $\frac{9}{10}$ full at the start of the day, how much is left after he has fed his chickens?

9 Samantha is an artist. She spends $3\frac{1}{2}$ hours on Saturday painting a portrait, and a further $2\frac{1}{4}$ hours finishing it on Sunday.
In total, how long did it take her to paint the portrait?

10 

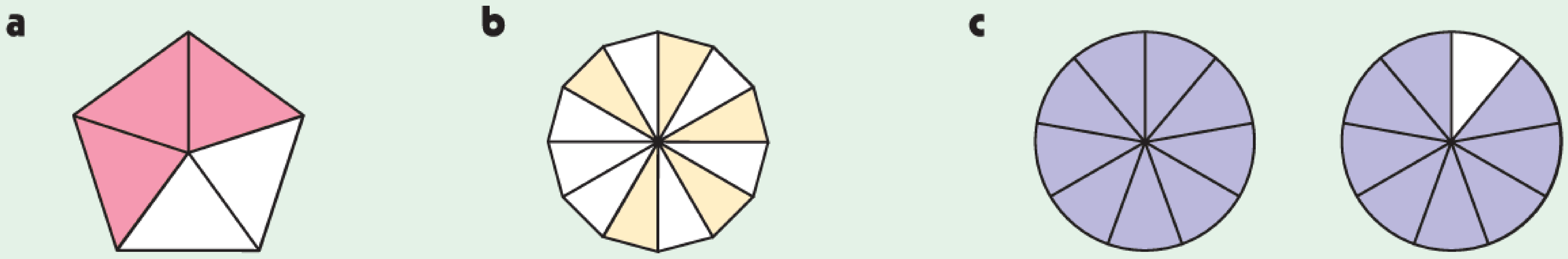
$3\frac{1}{8}$ tonnes of earth must be removed to level a housing block. A truck moves $1\frac{1}{2}$ tonnes in the first load. How much earth still needs to be moved?

KEY WORDS USED IN THIS CHAPTER

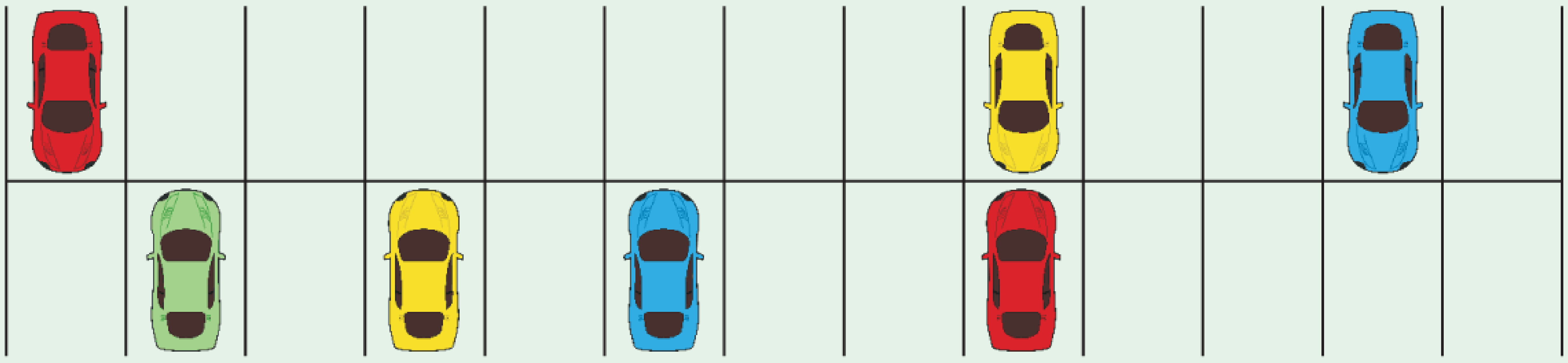
- bar
- fraction
- number line
- simplest form
- denominator
- improper fraction
- numerator
- equal fractions
- mixed number
- proper fraction

REVIEW SET 6A

1 Write down the fraction represented by:



2 What fraction of the cars in this car park are blue?



3 Write as a fraction:

a $6 \div 11$ **b** $15 \div 19$

4 Write as a mixed number:

a $\frac{9}{5}$ **b** $\frac{13}{3}$ **c** $\frac{35}{6}$

5 Copy and complete these equal fraction statements:

a $\frac{3}{8} = \frac{....}{32}$ **b** $\frac{18}{21} = \frac{6}{....}$ **c** $\frac{16}{72} = \frac{....}{9}$

6 Find:

a $\frac{1}{4}$ of £200 **b** $\frac{2}{5}$ of 100 g **c** $\frac{3}{8}$ of 56 cm

7 Place the following fractions on a number line:

a $\frac{1}{6}$ and $\frac{4}{6}$ **b** $\frac{4}{8}$ and $\frac{11}{8}$ **c** $\frac{6}{7}$ and $1\frac{4}{7}$

8 An athlete runs $\frac{2}{5}$ of a race in the first hour and $\frac{3}{10}$ in the second hour.

What fraction of the race has he completed?

9 Which is larger:

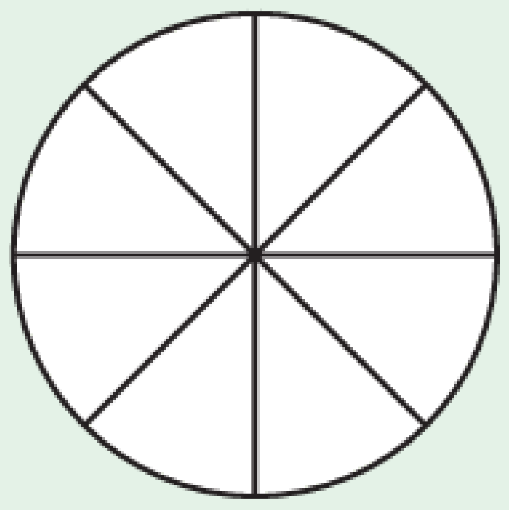
a $\frac{6}{10}$ or $\frac{3}{10}$ **b** $\frac{19}{7}$ or $2\frac{3}{7}$ **c** $\frac{4}{5}$ or $\frac{22}{25}$ **d** $\frac{17}{3}$ or $\frac{31}{6}$?

10 Find:

a $\frac{12}{7}-\frac{8}{7}$ **b** $\frac{8}{11}+\frac{9}{11}$ **c** $\frac{3}{8}+\frac{1}{4}$ **d** $5\frac{1}{3}-1\frac{1}{9}$

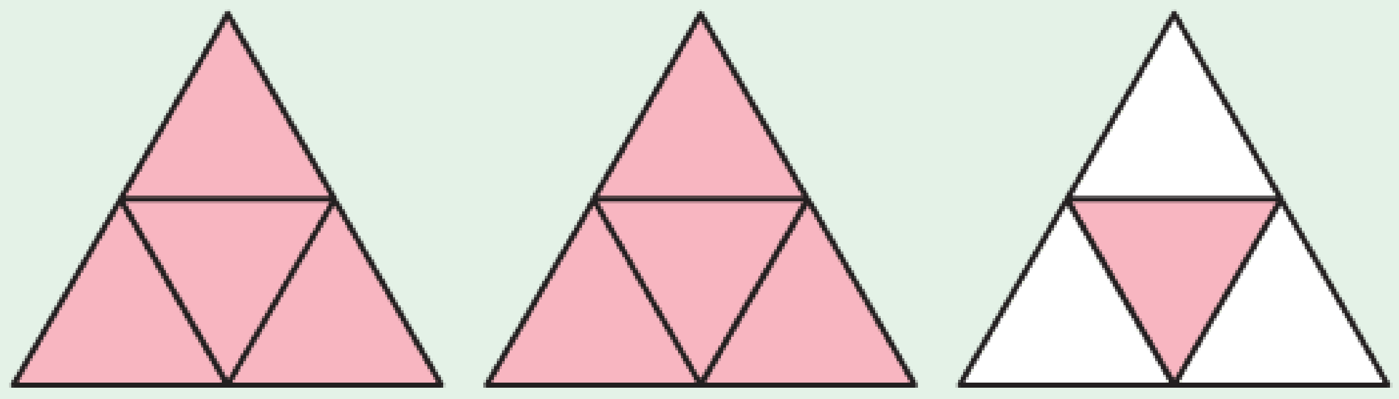
REVIEW SET 6B

1 Copy this circle and shade $\frac{5}{8}$ of it.



2 **a** What mixed number is represented by this diagram?

b Write the mixed number as an improper fraction.



3 Write as a division, and hence as a whole number:

a $\frac{40}{8}$ **b** $\frac{72}{9}$ **c** $\frac{99}{11}$

4 Write as an improper fraction:

a $3\frac{5}{6}$ **b** $4\frac{3}{7}$ **c** $5\frac{2}{5}$

5 Write in simplest form:

a $\frac{2}{16}$ **b** $\frac{25}{45}$ **c** $\frac{32}{40}$

6 Write $\frac{5}{6}$ with denominator:

a 12 **b** 30 **c** 48

7 Sarah went on a holiday for 20 days. It rained on one quarter of the days. On how many days did it rain?

8 Which is greater, $\frac{3}{7}$ or $\frac{10}{21}$?

9 Find:

a $3 + \frac{3}{5} + \frac{4}{5}$

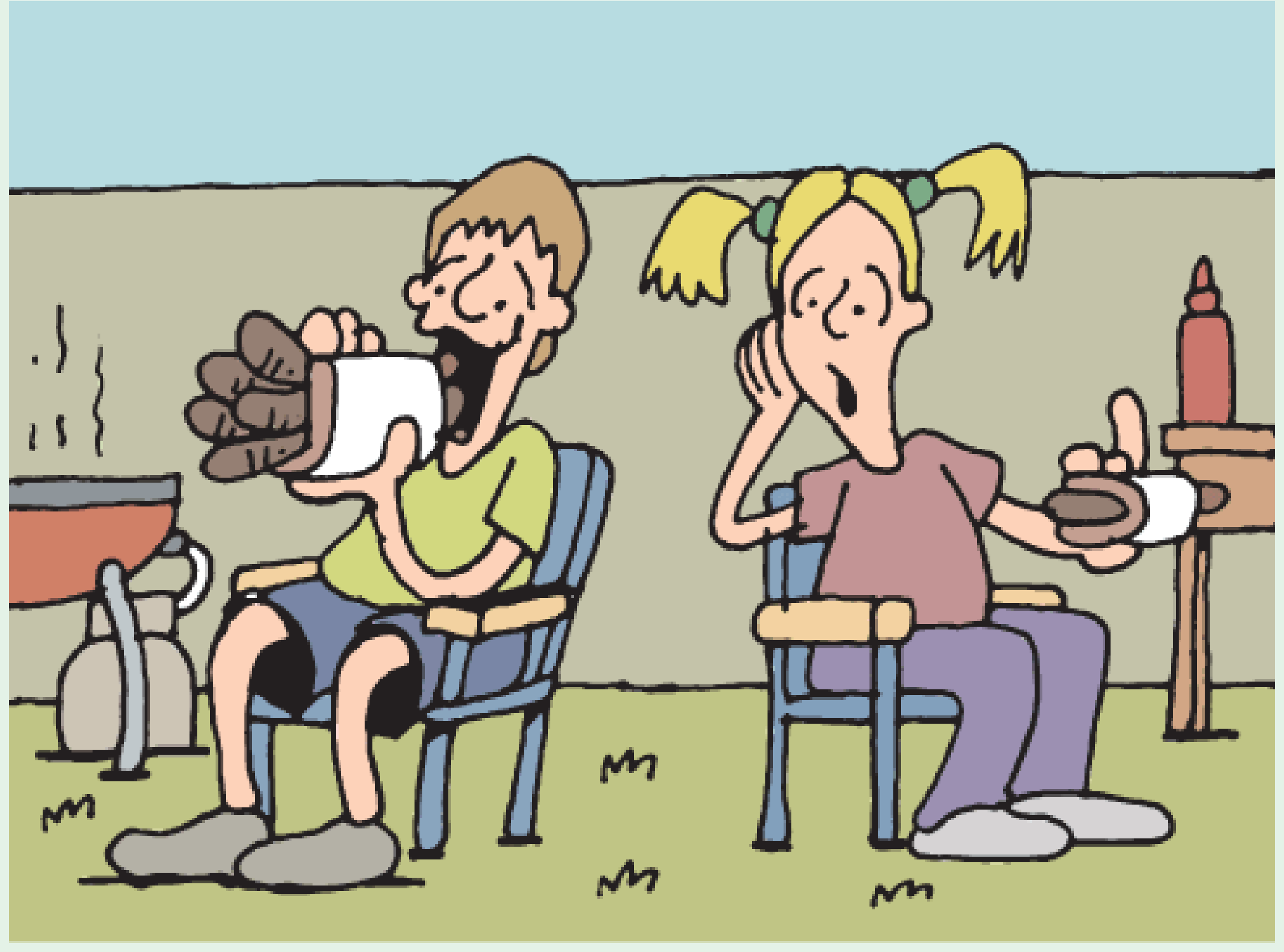
b $4\frac{1}{10} - 2\frac{4}{10}$

c $\frac{5}{6} + \frac{14}{18}$

10 At a barbecue, Adam ate $5\frac{1}{3}$ sausages, and Jill ate $2\frac{1}{2}$ sausages.

a How many sausages did they eat in total?

b How many more sausages did Adam eat than Jill?



Chapter 7

Decimal numbers

Contents:

OPENING PROBLEM

Petrol stations often use signs to display the price of petrol.

In some countries, the price is displayed as a mixed number:

In other countries, we see prices like this one:

Things to think about:

- **a** What type of number is 141.7?
- **b** What does the dot indicate?
- **c** Between which two whole numbers is the value 141.7?
- **d** How would you write 141.7 as a mixed number?

In **Chapter 6**, we saw how fractions can be used to describe values between the whole numbers. Another way to describe these values is by using **decimal numbers**.

In this chapter we will explore the relationship between fractions and decimal numbers. We will also learn how to add, subtract, multiply, and divide with decimals.

A CONSTRUCTING DECIMAL NUMBERS

We have seen previously how whole numbers are written with digits in different **place values**.

For example, 84 has the place value table since $84 = 8 \times 10 + 4 \times 1$.

8	4
tens	*units*

By extending this table to the right, we can construct values between the whole numbers 84 and 85.

After the units place we write a **decimal point**. This separates the whole number part to the left of the dot, from the fractional part to the right of the dot. The next place value will be for tenths or $\frac{1}{10}$ of a whole. After tenths come hundredths, thousandths, and so on.

For example:

8	4	.	7	3
tens	*units*		*tenths*	*hundredths*

The number 84.73 has value 84, 7 tenths, and 3 hundredths.

If the whole number part is zero, we write a zero in front of the decimal point. For example, we write 0.37 instead of .37.

DISCUSSION

- Why do you think we write 0.37 instead of .37?
- Are the numbers 1.5 and 1.05 the same?
- Are the numbers 1.5 and 1.50 the same?
- In what situations would we write 1.50 instead of 1.5?

Example 1

Self Tutor

Express in oral form:

a 0.9 **b** 3.06 **c** 11.407

a 0.9 is ‘zero point nine’.
b 3.06 is ‘three point zero six’.
c 11.407 is ‘eleven point four zero seven’.

Oral form means how you would say it.

EXERCISE 7A

1 Express in oral form:

a 0.6 **b** 0.45 **c** 0.908 **d** 8.3 **e** 6.08
f 96.02 **g** 5.864 **h** 34.003 **i** 7.581 **j** 60.264

2 Write in decimal form:

a eight point three seven **b** twenty one point zero five
c nine point zero zero four **d** thirty eight point two zero six

3 Between which two whole numbers do the following decimal numbers lie?

a 5.7 **b** 13.4 **c** 9.8 **d** 6.27
e 19.76 **f** 32.09 **g** 0.46 **h** 8.506

4 State the number of decimal places in these decimal numbers:

a 9.1 **b** 3.26 **c** 17.2
d 47.94 **e** 2.507 **f** 57.813
g 34.0 **h** 13.80 **i** 23.006

The “number of decimal places” is the number of digits after the decimal point.

Example 2 **Self Tutor**

Write in a place value table and then as a decimal number:

a 7 hundredths **b** $23 + \frac{4}{10} + \frac{9}{1000}$

	Number	*tens*	*units*		*tenths*	*hundredths*	*thousandths*	*Decimal number*
a	7 hundredths			.	0	7		0.07
b	$23 + \frac{4}{10} + \frac{9}{1000}$	2	3	.	4	0	9	23.409

5 Write in a place value table and then as a decimal number:

PRINTABLE WORKSHEET

a $\frac{8}{10} + \frac{3}{100}$ **b** $4 + \frac{1}{10} + \frac{2}{100} + \frac{8}{1000}$

c $9 + \frac{4}{1000}$

d $28 + \frac{6}{10} + \frac{9}{100} + \frac{9}{1000}$

e $\frac{5}{100} + \frac{6}{1000}$

f $139 + \frac{7}{100} + \frac{7}{1000}$

Number	*thousands*	*hundreds*	*tens*	*units*		*tenths*	*hundredths*	*thousandths*	*Decimal number*
					.				

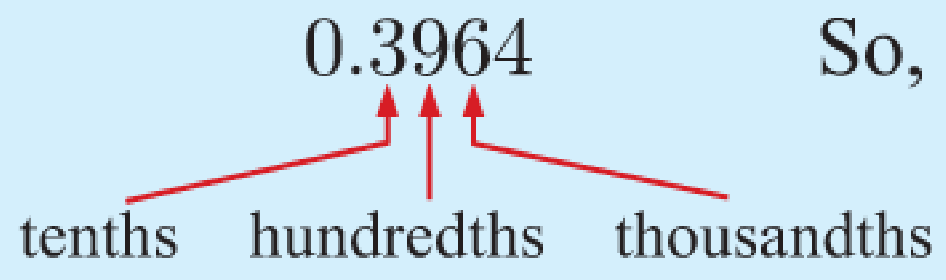
6 Write in a place value table and then as a decimal number:

a 8 tenths
b 4 hundredths
c 3 thousandths
d 7 tens and 8 tenths
e 5 units and 6 hundredths
f 9 thousands and 2 thousandths
g 2 hundreds, 9 units, and 4 hundredths
h 8 thousands, 4 tenths, and 2 thousandths
i 6 tens, 8 tenths, and 9 hundredths

If a word for a digit ends in **ths**, it indicates *part* of a whole. The digit will follow the decimal point.

Example 3 **Self Tutor**

State the value of the digit 6 in 0.3964



So, the 6 stands for 6 thousandths, or $\frac{6}{1000}$.

7 State the value of the digit 3 in:

a 4325.9
b 6.374
c 32.098
d 150.953
e 43.4444
f 82.7384
g 3874.941
h 24.8403

8 State the value of the digit 5 in:

a 18.945
b 596.08
c 4.5972
d 94.8573
e 75 948.264
f 275.183
g 358 946.843
h 0.0005

Example 4

Self Tutor

Express 5.706 in expanded form.

$$5.706 = 5 + \frac{7}{10} + \frac{0}{100} + \frac{6}{1000}$$
$$= 5 + \frac{7}{10} + \frac{6}{1000}$$

9 Express in expanded form:

a 5.4
b 14.9
c 2.03
d 32.86
e 2.264
f 1.308
g 3.002
h 0.952
i 4.024
j 2.973
k 20.816
l 7.777
m 9.008
n 154.451
o 808.808
p 0.064

10 Write in decimal form:

a $\frac{6}{10}$
b $\frac{2}{10}$
c $\frac{4}{10} + \frac{3}{100}$
d $\frac{7}{10} + \frac{1}{100}$
e $\frac{8}{10} + \frac{9}{1000}$
f $\frac{9}{100}$
g $\frac{7}{1000}$
h $\frac{5}{100} + \frac{2}{1000}$
i $\frac{5}{10} + \frac{6}{100} + \frac{8}{1000}$
j $\frac{2}{1000} + \frac{3}{10\,000}$
k $4 + \frac{3}{10} + \frac{8}{100} + \frac{7}{1000}$
l $\frac{3}{100} + \frac{8}{10\,000}$
m $\frac{3}{10} + \frac{3}{1000} + \frac{3}{10\,000}$
n $5 + \frac{5}{10} + \frac{5}{100} + \frac{5}{1000}$
o $\frac{2}{10} + \frac{5}{100\,000}$

Example 5

Self Tutor

Write $\frac{39}{1000}$ in decimal form.

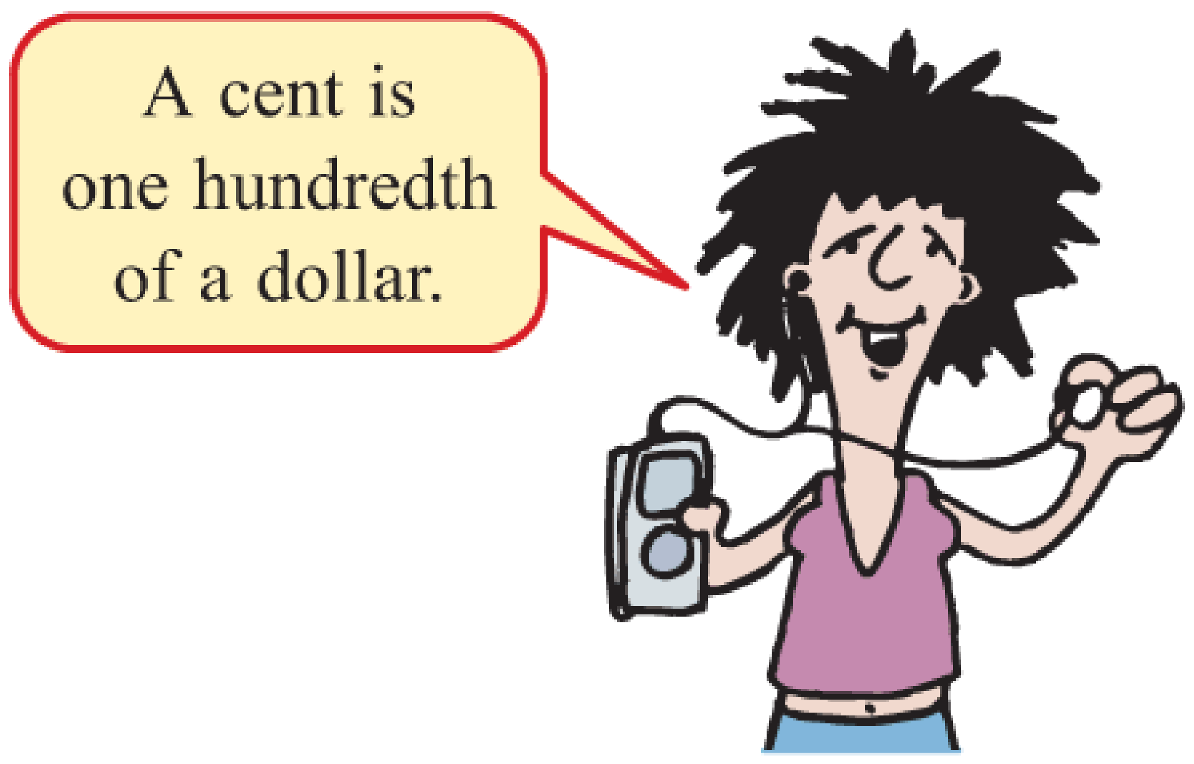
$$\frac{39}{1000} = \frac{3\cancel{0}}{100\cancel{0}} + \frac{9}{1000}$$
$$= \frac{3}{100} + \frac{9}{1000}$$
$$= 0.039$$

11 Write in decimal form:

a $\frac{23}{100}$
b $\frac{79}{100}$
c $\frac{307}{1000}$
d $\frac{117}{1000}$
e $\frac{600}{1000}$
f $\frac{703}{1000}$
g $\frac{469}{100}$
h $\frac{540}{1000}$
i $\frac{4672}{10\,000}$
j $\frac{3600}{10\,000}$

12 Write the following values in dollars using a decimal point:

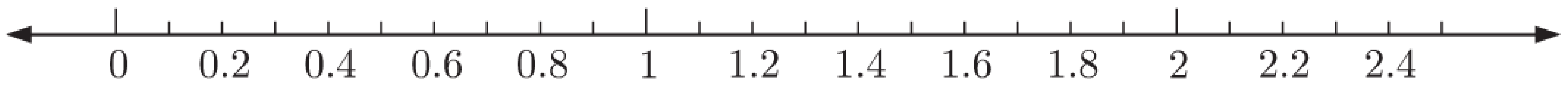
- **a** one dollar thirty cents
- **b** 10 dollars 95 cents
- **c** 45 cents
- **d** thirty seven dollars eight cents



B DECIMALS ON A NUMBER LINE

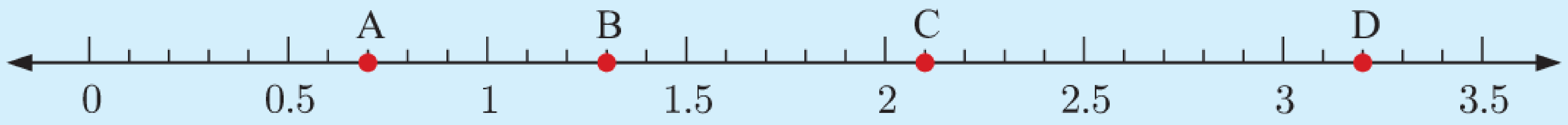
Just as whole numbers and fractions can be marked on a number line, we can do the same with decimal numbers.

Consider the following number line where each whole number shown has ten equal divisions. Each division on the number line represents $\frac{1}{10}$ or 0.1



Example 6 **Self Tutor**

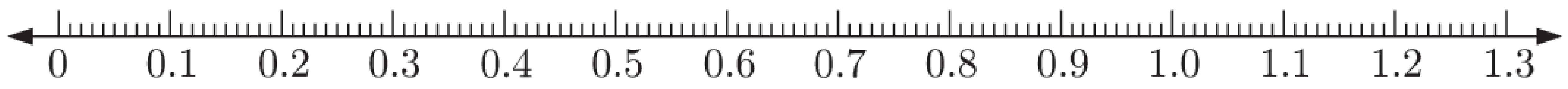
Write down the decimal values of A, B, C, and D marked on this number line:



Each division on the number line represents 0.1

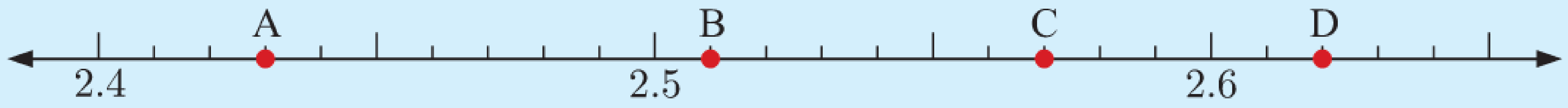
∴ A is 0.7, B is 1.3, C is 2.1, and D is 3.2

Suppose we divide each of the parts which represent $\frac{1}{10}$ into 10 equal parts. Each unit is now divided into 100 equal parts, and each division is $\frac{1}{100}$ or 0.01 of the unit.



Example 7 **Self Tutor**

Write down the decimal values of A, B, C, and D marked on this number line:



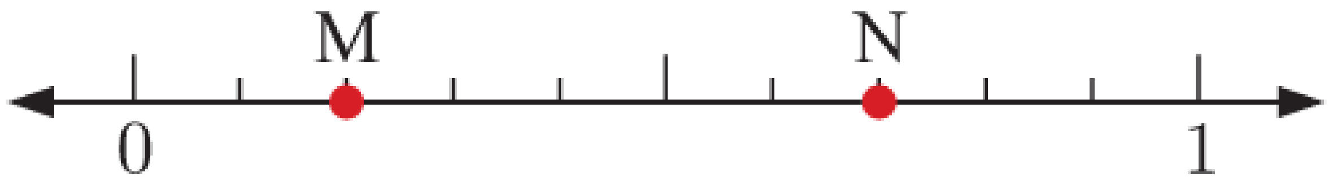
Each division on the number line represents 0.01

∴ A is 2.43, B is 2.51, C is 2.57, and D is 2.62

EXERCISE 7B

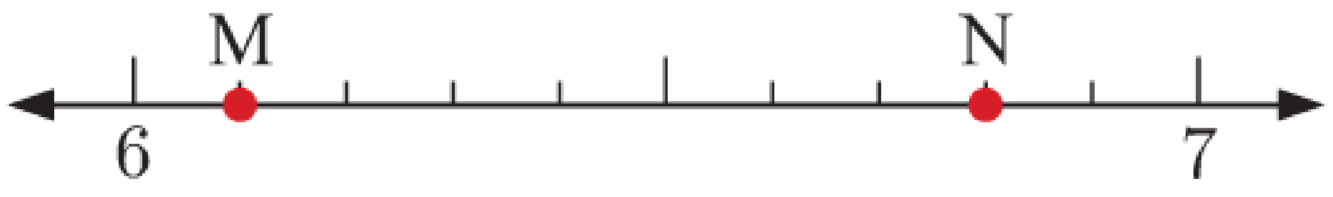
1 Write down the decimal values of M and N:

a

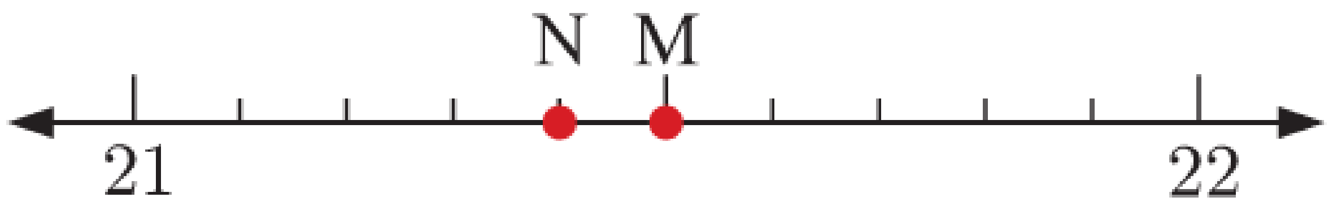


b

c

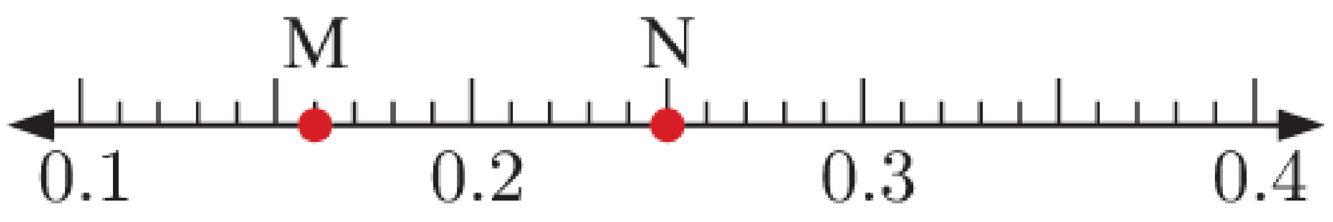


d

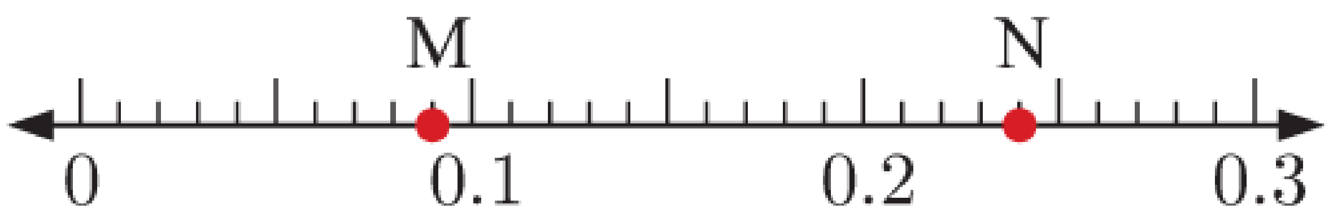


2 Write down the decimal values of M and N:

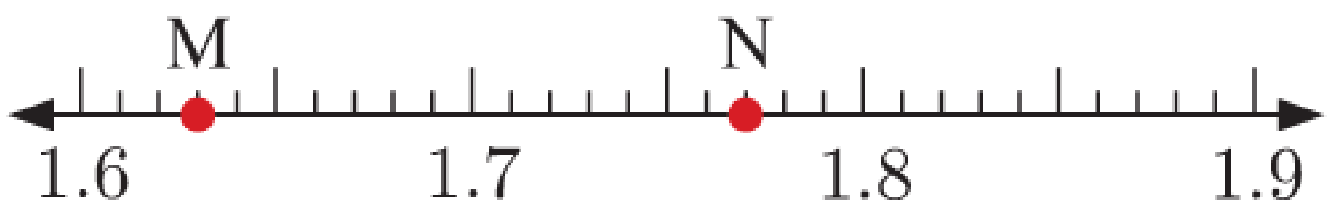
a



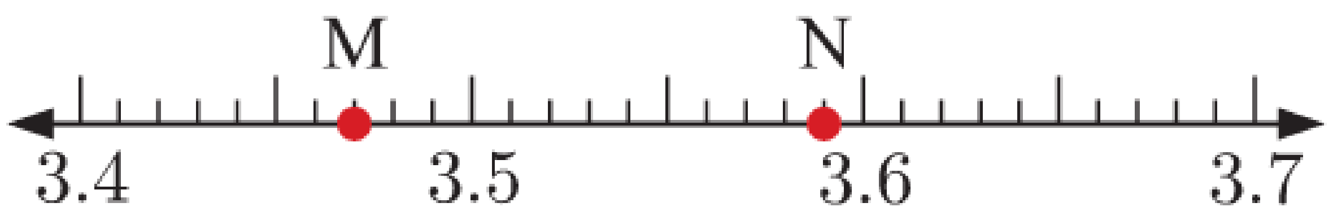
b



c

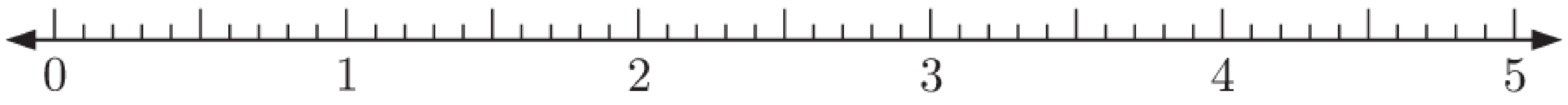


d



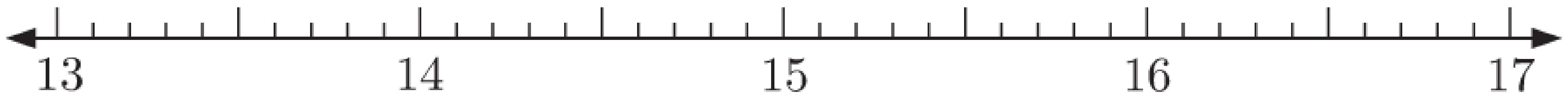
3 Copy the number lines and mark the given numbers on them:

a A = 1.6, B = 2.5, C = 2.9, D = 4.1

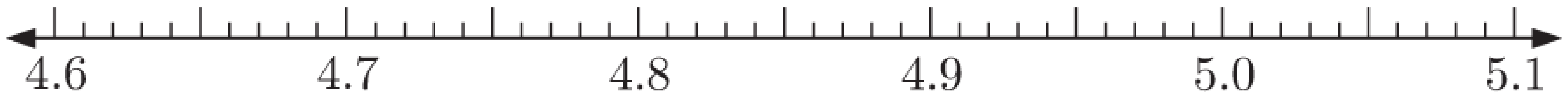


PRINTABLE NUMBER LINES

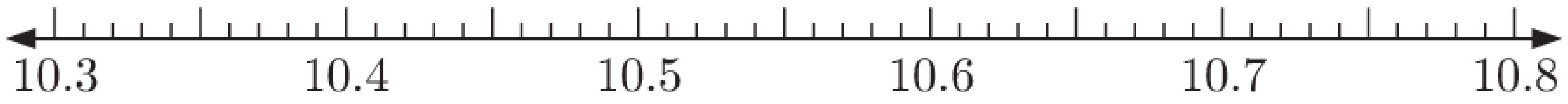
b E = 13.7, F = 14.2, G = 15.3, H = 16.5



c A = 4.61, B = 4.78, C = 4.83, D = 4.97



d E = 10.35, F = 10.46, G = 10.62, H = 10.79

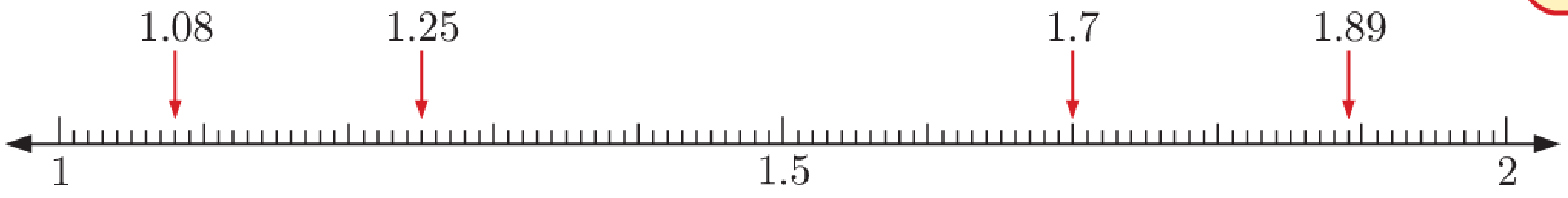


C COMPARING DECIMAL NUMBERS

We can use a number line to help compare the sizes of decimal numbers.

< means "is less than".
> means "is greater than".

For example, consider the following number line:



As we look from left to right, the numbers are increasing.

So, $1.08 < 1.25 < 1.7 < 1.89$

To compare decimal numbers without having to construct a number line, we place zeros on the end so each number has the same number of decimal places.

We can do this because adding zeros on the end after the decimal point does not affect the place values of the other digits.

Example 8 **Self Tutor**

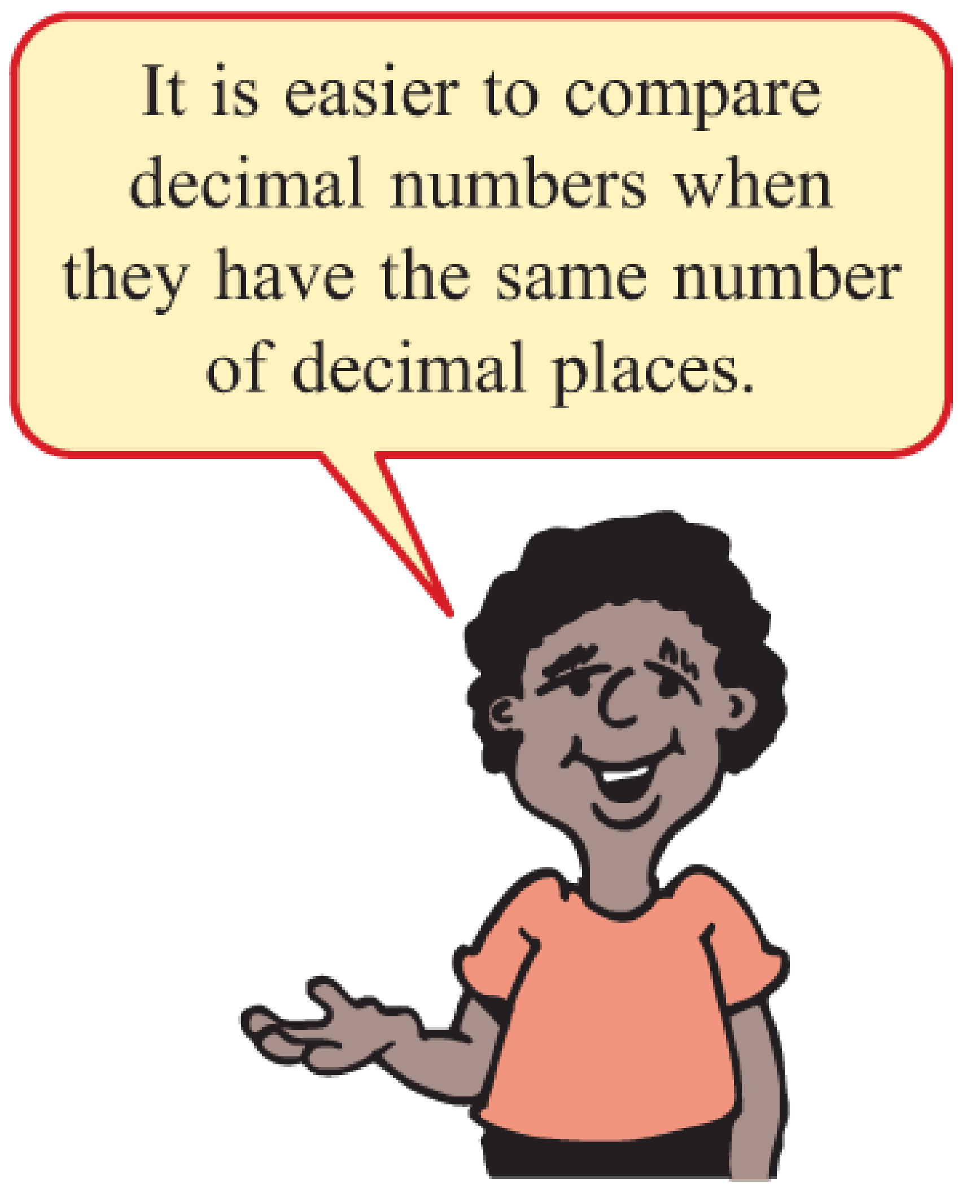
Insert $>$, $<$, or $=$ in the box to make the statement true:

a 0.305 ☐ 0.35 **b** 0.88 ☐ 0.808

We start by writing the numbers with the same number of decimal places.

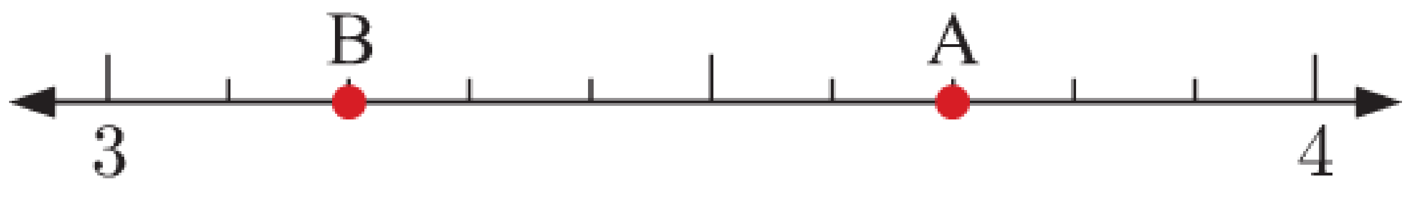
a 0.305 ☐ 0.350
So, $0.305 < 0.35$

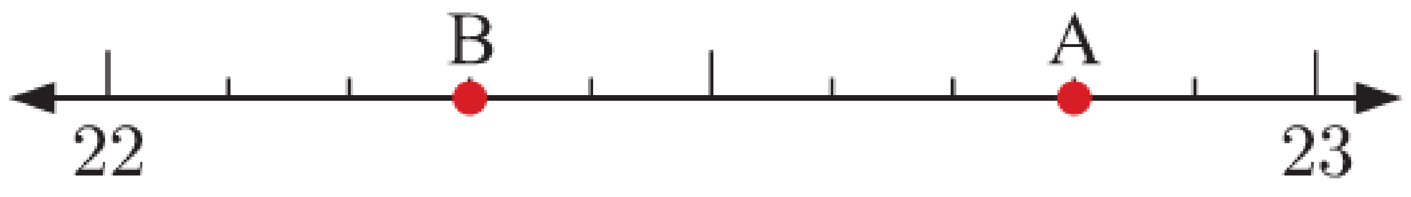
b 0.880 ☐ 0.808
So, $0.88 > 0.808$

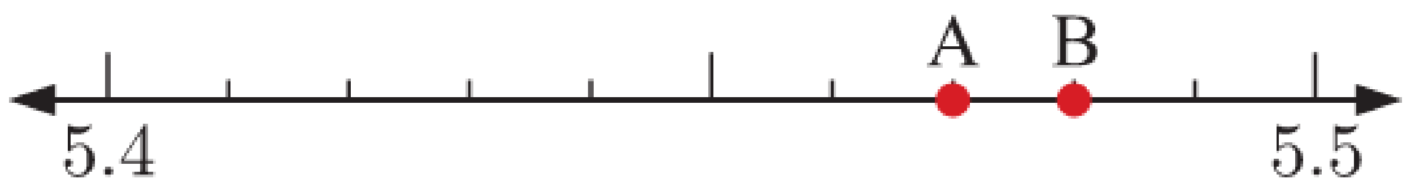


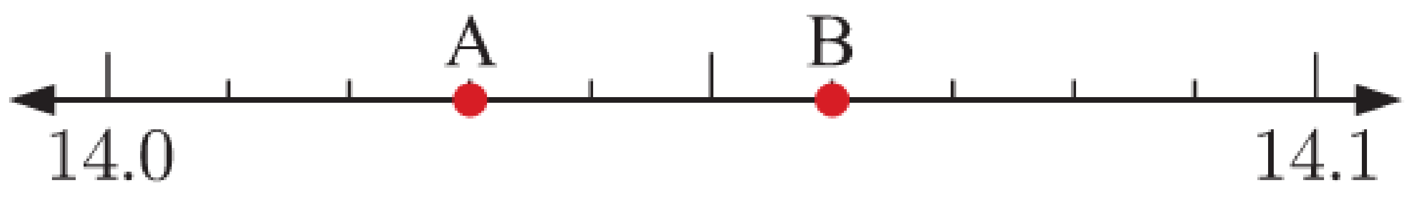
EXERCISE 7C

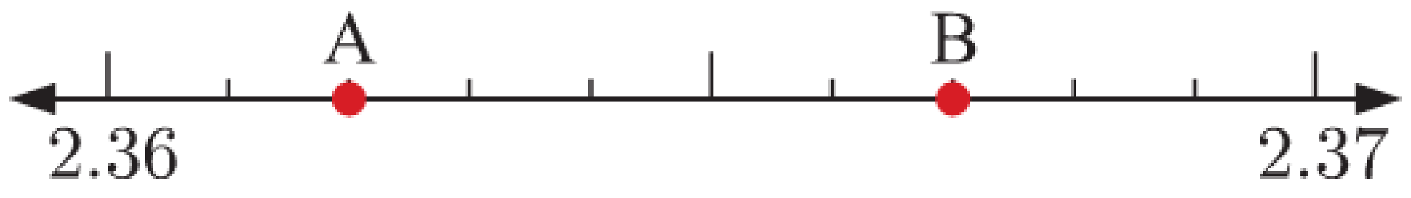
1 Write down the values of A and B on each number line, and determine whether $A > B$ or $A < B$:

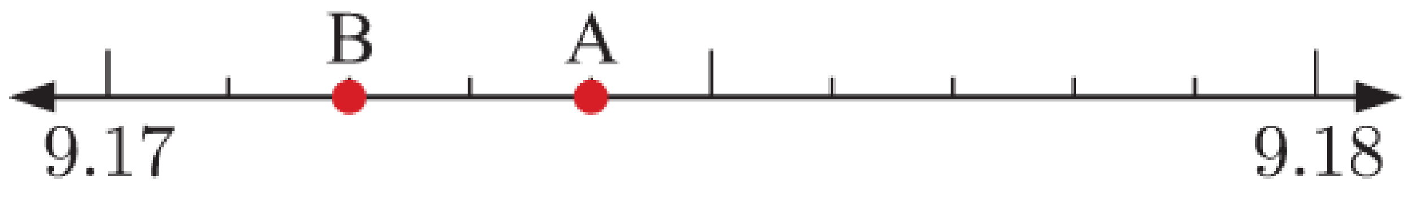
a


b


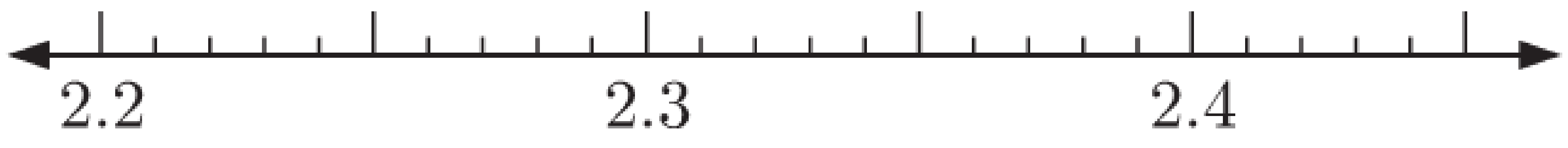
c


d


e


f


2 **a** Mark the numbers 2.34, 2.4, 2.26, and 2.3 on this number line.



b Write the numbers in **a** in order from smallest to largest.

3 Insert $>$, $<$, or $=$ to make the statement true:

a 0.7 ☐ 0.8
b 0.06 ☐ 0.05
c 0.2 ☐ 0.19
d 4.01 ☐ 4.1
e 0.81 ☐ 0.803
f 2.5 ☐ 2.50
g 0.304 ☐ 0.34
h 0.03 ☐ 0.2
i 6.05 ☐ 60.50
j 0.29 ☐ 0.290
k 5.01 ☐ 5.016
l 1.15 ☐ 1.035
m 21.021 ☐ 21.210
n 8.09 ☐ 8.090
o 0.904 ☐ 0.94

4 Write these numbers in order from smallest to largest:

a 0.8, 0.4, 0.6
b 0.4, 0.1, 0.9
c 0.14, 0.09, 0.06
d 0.46, 0.5, 0.51
e 1.06, 1.59, 1.61
f 2.6, 2.06, 0.206
g 0.095, 0.905, 0.0905
h 15.5, 15.05, 15.55

5 Write these numbers in order from largest to smallest:

a 0.9, 0.4, 0.3, 0.8

b 0.51, 0.49, 0.5, 0.47

c 0.6, 0.596, 0.61, 0.609

d 0.02, 0.04, 0.42, 0.24

e 6.27, 6.271, 6.027, 6.277

f 0.31, 0.031, 0.301, 0.311

g 8.088, 8.008, 8.080, 8.880

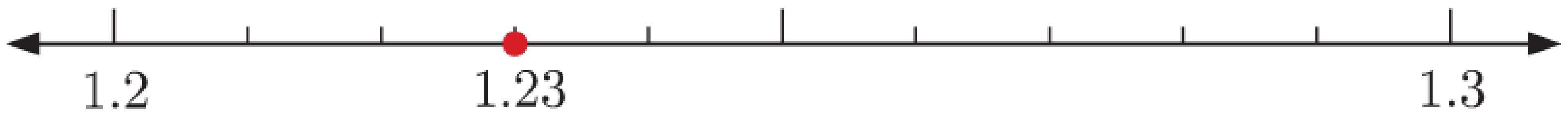
h 7.61, 7.061, 7.01, 7.06

D ROUNDING DECIMAL NUMBERS

We are often given measurements as decimal numbers. For example, my digital bathroom scales tell me I weigh 59.4 kg. In reality I do not weigh *exactly* 59.4 kg, but this is an *approximation* of my actual weight. It is not necessary to measure my weight to greater accuracy, so my weight has been *rounded* to one decimal place.

We round off decimal numbers in the same way we do whole numbers. We look at values on the number line either side of our number, and work out which is closer.

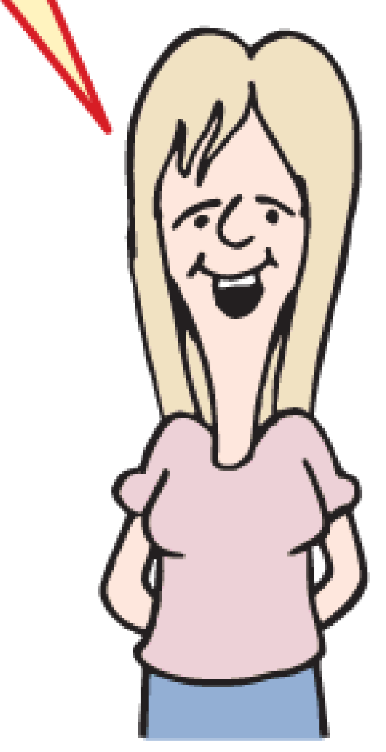
For example, consider the decimal number 1.23 on this number line.



1.23 is closer to 1.2 than it is to 1.3, so we round down.

We write $1.23 \approx 1.2$

$\approx$ means "is approximately equal to".



RULES FOR ROUNDING OFF DECIMAL NUMBERS

The rules for rounding decimal numbers are the same as those for rounding whole numbers.

- If the digit after the one being rounded is **less than 5** (0, 1, 2, 3, or 4) then we round **down**.
- If the digit after the one being rounded is **5 or more** (5, 6, 7, 8, or 9) then we round **up**.

Example 9 **Self Tutor**

Round:

a 3.26 to 1 decimal place

b 5.273 to 2 decimal places

c 4.9851 to 2 decimal places

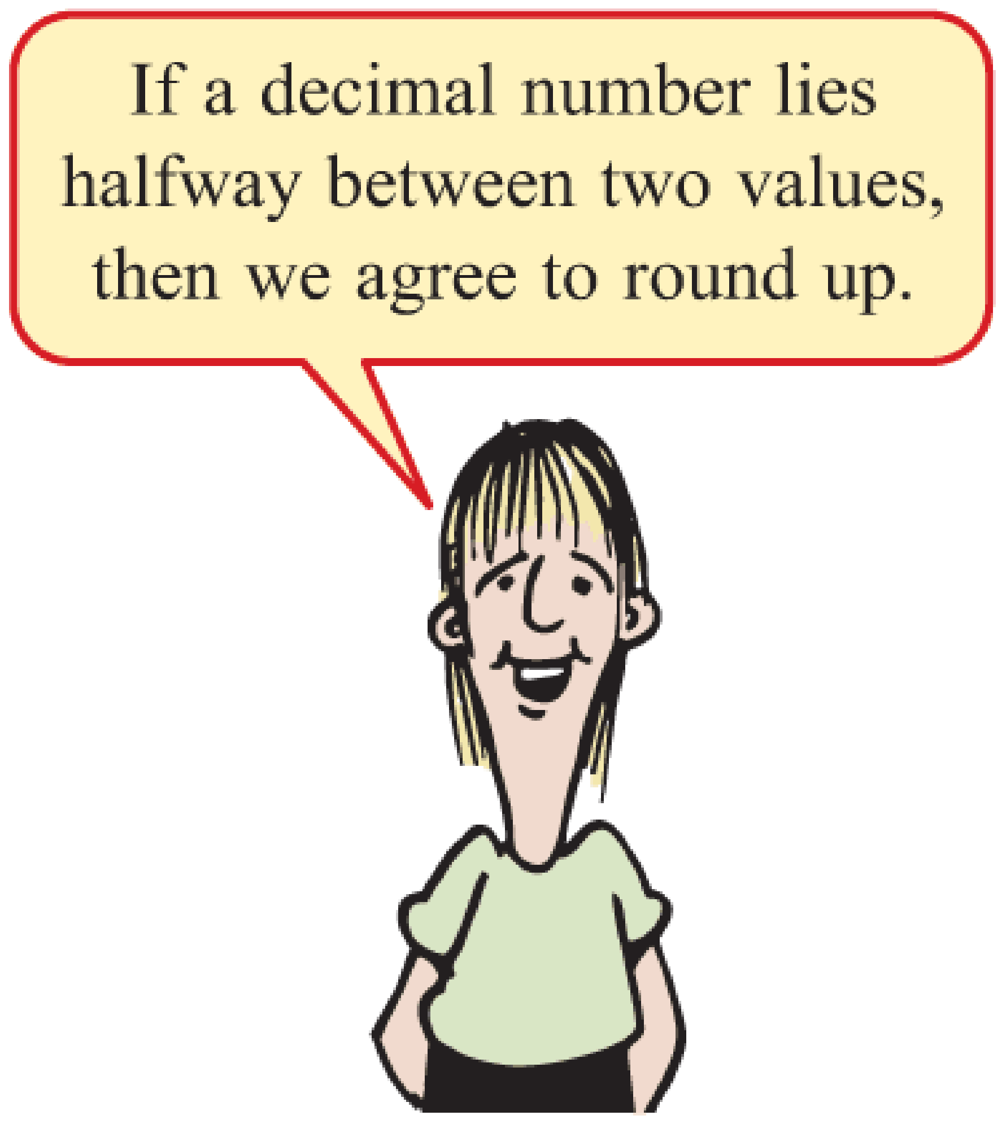
a 3.26 is closer to 3.3 than to 3.2, so we round up.
So, $3.26 \approx 3.3$

b 5.273 is closer to 5.27 than to 5.28, so we round down.
So, $5.273 \approx 5.27$

c 4.9851 is closer to 4.99 than to 4.98, so we round up.
So, $4.9851 \approx 4.99$

"Rounding to 1 decimal place" means that there should be 1 digit after the decimal point.

EXERCISE 7D



1 Round to 1 decimal place:

a 2.43 **b** 3.57 **c** 4.92
d 7.75 **e** 0.639 **f** 4.275

2 Round to 2 decimal places:

a 4.236 **b** 2.731 **c** 5.625
d 4.377 **e** 6.5237 **f** 1.0871

3 Round to the nearest whole number:

a 3.7 **b** 6.1 **c** 7.48
d 12.63 **e** 21.082

4 Round 0.486 to:

a 1 decimal place **b** 2 decimal places.

5 Round 3.789 to:

a 1 decimal place **b** 2 decimal places.

6 Round 5.183 75 to:

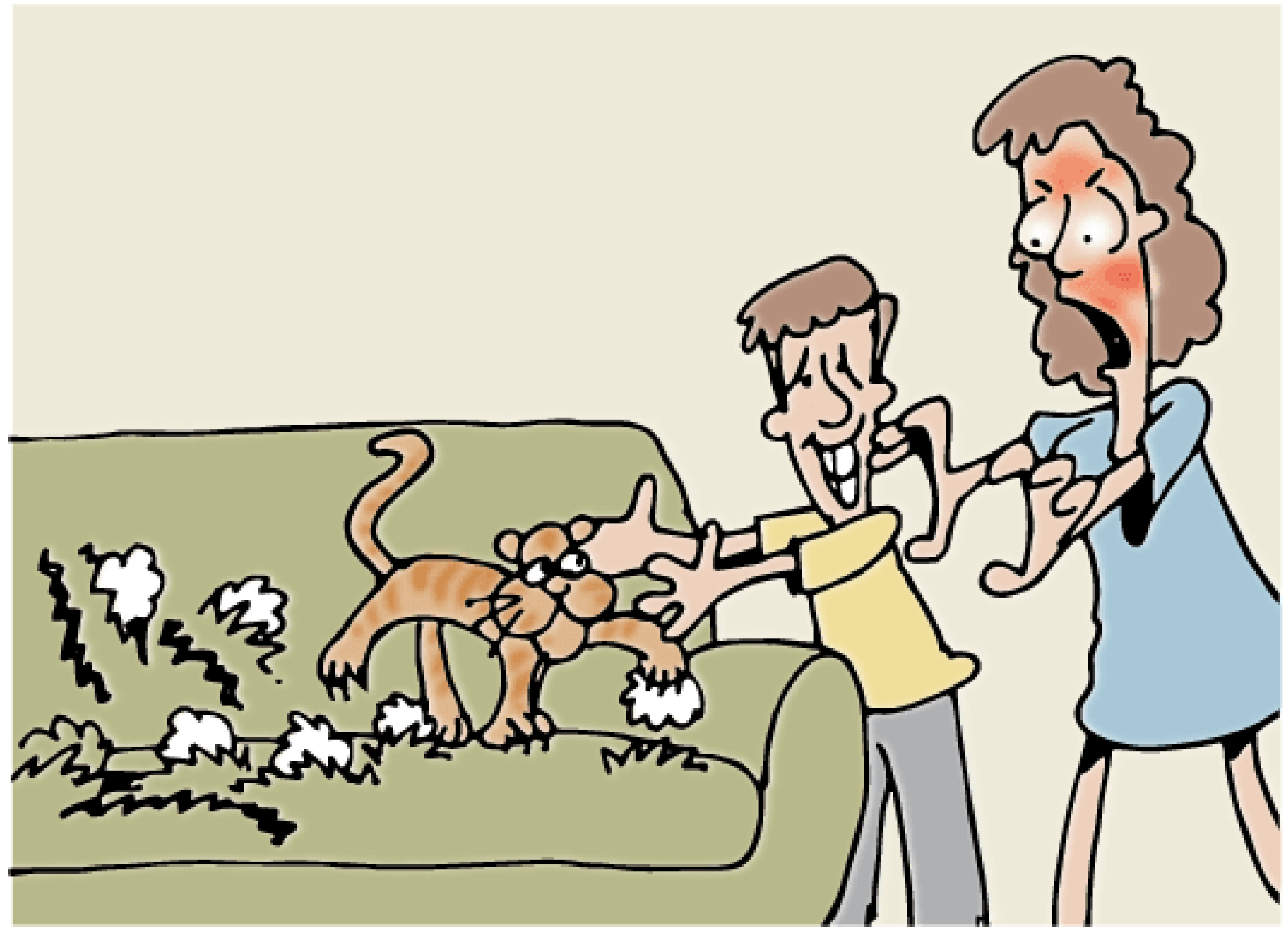
a the nearest whole number **b** 1 decimal place
c 2 decimal places **d** 3 decimal places.

7 Round:

a 3.87 to the nearest tenth **b** 4.3 to the nearest whole number
c 6.09 to 1 decimal place **d** 0.461 72 to 3 decimal places
e 2.9467 to 2 decimal places **f** 0.175 61 to 3 decimal places.

8 Round the following values to the accuracy given:

a Frank's cat weighs approximately 4.327 kg. {1 decimal place}
b The maximum temperature reached today was approximately 32.694°C. {1 decimal place}
c The tree in Tina's backyard is approximately 2.9381 m high. {2 decimal places}
d Nick swam 100 m in approximately 86.825 seconds. {whole number}



E CONVERTING BETWEEN DECIMALS AND FRACTIONS

Decimals and fractions can both be used to describe values between whole numbers. In this Section we will learn how to convert between decimals and fractions.

CONVERTING DECIMALS TO FRACTIONS

Decimal numbers can be easily written as fractions with powers of 10 as their denominators.

Example 10 Self Tutor

Write as a fraction or as a mixed number:

a 0.7 **b** 0.79 **c** 2.013

a $0.7 = \frac{7}{10}$

b $0.79 = \frac{79}{100}$

c $2.013 = 2 + \frac{13}{1000} = 2\frac{13}{1000}$

Sometimes the fraction can be simplified so the denominator is not a power of 10.

Example 11 Self Tutor

Write as a fraction in simplest form:

a 0.4 **b** 0.72 **c** 0.275

a $0.4 = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

b $0.72 = \frac{72}{100} = \frac{72 \div 4}{100 \div 4} = \frac{18}{25}$

c $0.275 = \frac{275}{1000} = \frac{275 \div 25}{1000 \div 25} = \frac{11}{40}$

EXERCISE 7E.1

1 Write as a fraction or as a mixed number:

a 0.1 **b** 0.9 **c** 0.19 **d** 0.67 **e** 0.07 **f** 0.191
g 0.523 **h** 0.049 **i** 4.3 **j** 0.87 **k** 0.01 **l** 5.271

2 Write as a fraction or mixed number in simplest form:

a 0.8 **b** 0.5 **c** 0.26 **d** 0.35 **e** 0.25 **f** 0.106
g 0.015 **h** 0.175 **i** 0.625 **j** 7.6 **k** 4.56 **l** 3.95

CONVERTING FRACTIONS TO DECIMALS

We have already seen how to convert fractions with denominators 10, 100, and 1000 into decimal numbers.

Example 12 Self Tutor

Write as a decimal:

a $\frac{4}{10}$ **b** $\frac{57}{100}$ **c** $\frac{129}{1000}$

a $\frac{4}{10} = 0.4$

b $\frac{57}{100} = 0.57$

c $\frac{129}{1000} = 0.129$

If the denominator is not a power of 10, we can sometimes make it a power of 10 by multiplying both the numerator and denominator by a particular number.

Example 13 **Self Tutor**

Write as a decimal:

a $\frac{3}{4}$ **b** $\frac{7}{20}$ **c** $\frac{23}{125}$

a $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$

b $\frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 0.35$

c $\frac{23}{125} = \frac{23 \times 8}{125 \times 8} = \frac{184}{1000} = 0.184$


When we multiply the numerator and denominator by the same amount, we do not change the value of the fraction.

EXERCISE 7E.2

1 Write as a decimal:

a $\frac{3}{10}$ **b** $\frac{21}{100}$ **c** $\frac{77}{100}$ **d** $\frac{319}{1000}$ **e** $4\frac{91}{100}$ **f** $2\frac{137}{1000}$

2 By what whole number would you multiply the following, to obtain a power of 10?

a 2 **b** 5 **c** 4 **d** 8 **e** 20

f 25 **g** 50 **h** 125 **i** 40 **j** 250

3 Write as a decimal:

a $\frac{1}{5}$ **b** $\frac{3}{20}$ **c** $\frac{17}{20}$ **d** $\frac{9}{25}$ **e** $\frac{21}{25}$ **f** $\frac{13}{50}$

g $\frac{31}{50}$ **h** $\frac{138}{500}$ **i** $\frac{6}{250}$ **j** $\frac{91}{250}$ **k** $\frac{11}{500}$ **l** $\frac{9}{125}$

m $\frac{68}{125}$ **n** $\frac{9}{40}$ **o** $\frac{3}{8}$ **p** $5\frac{3}{4}$ **q** $7\frac{11}{20}$ **r** $3\frac{21}{125}$

4 Copy and complete these conversions to decimals:

a $\frac{1}{2} =$ ......

b $\frac{1}{5} =$ ......, $\frac{2}{5} =$ ......, $\frac{3}{5} =$ ......, $\frac{4}{5} =$ ......

c $\frac{1}{4} =$ ......, $\frac{2}{4} =$ ......, $\frac{3}{4} =$ ......

d $\frac{1}{8} =$ ......, $\frac{2}{8} =$ ......, $\frac{3}{8} =$ ......, $\frac{4}{8} =$ ......,
$\frac{5}{8} =$ ......, $\frac{6}{8} =$ ......, $\frac{7}{8} =$ ......

You should remember the decimal values of these fractions.

5 By first converting the fractions into decimals, write the numbers $\frac{7}{10}$, 0.63, $\frac{13}{20}$, 0.74, and $\frac{18}{25}$ in order from smallest to largest.

Example 14 **Self Tutor**

Write as a decimal:

a $\frac{9}{20}$ kg **b** $3\frac{2}{5}$ m

a
$$\begin{aligned} &\frac{9}{20}\text{ kg} \\ &= \frac{9 \times 5}{20 \times 5}\text{ kg} \\ &= \frac{45}{100}\text{ kg} \\ &= 0.45\text{ kg} \end{aligned}$$

b
$$\begin{aligned} &3\frac{2}{5}\text{ m} \\ &= 3 + \frac{2 \times 2}{5 \times 2}\text{ m} \\ &= 3 + \frac{4}{10}\text{ m} \\ &= 3.4\text{ m} \end{aligned}$$

6 Write as decimals:

a $\frac{1}{2}$ g **b** $\frac{1}{5}$ kg **c** $\frac{7}{20}$ m **d** $\frac{23}{50}$ cm

e $4\frac{3}{5}$ seconds **f** $1\frac{19}{20}$ dollars **g** $4\frac{23}{25}$ tonnes **h** $7\frac{7}{8}$ L

F ADDING AND SUBTRACTING DECIMALS

To **add** or **subtract** decimal numbers, we write the numbers so the decimal points are vertically underneath each other.

When this is done, the digits in each place value will also lie under one another. We then add or subtract as for whole numbers.

Example 15 **Self Tutor**

Find 3.84 + 0.372

$$\begin{array}{r} 3\,.\,8\,4\,0 \\ +\quad 0\,.\,3\,7\,2 \\ {\scriptstyle 1\ \ \ 1}\quad \\ \hline 4\,.\,2\,1\,2 \end{array}$$

We write a 0 on the end of 3.84 so that both numbers have the same number of decimal places.

EXERCISE 7F

1 Find:

a 0.4 + 0.5 **b** 0.6 + 2.7 **c** 0.9 + 0.23

d 0.94 + 0.09 **e** 0.57 + 0.96 **f** 23.04 + 4.78

g 15.79 + 2.64 **h** 0.4 + 0.3 + 4 **i** 0.009 + 0.435

j 0.007 + 2.948 **k** 0.0036 + 0.697 **l** 0.3 + 0.9 + 0.6

m 0.41 + 0.12 + 0.6 **n** 7.1 + 2.4 + 3.8 **o** 0.95 + 1.23 + 8.74

Example 16 **Self Tutor**

Find: **a** 26.9 – 9.5 **b** 5 – 2.32

a
$$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{16}{\cancel{6}}\,.\,9 \\ -\quad 9\,.\,5 \\ \hline 1\,7\,.\,4 \end{array}$$

b
$$\begin{array}{r} \overset{4}{\cancel{5}}\,.\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\quad 2\,.\,3\,2 \\ \hline 2\,.\,6\,8 \end{array}$$

2 Find:

a 0.86 – 0.34
b 1.75 – 0.41
c 6.42 – 3.3
d 1.7 – 0.9
e 2.3 – 0.8
f 4.2 – 3.8
g 2 – 0.6
h 4 – 1.7
i 3 – 0.74
j 4.5 – 1.83
k 1 – 0.99
l 10 – 0.98

3 Find the sum of:

a 31.1, 8.4, and 4.7
b 3.56, 1.12, and 9.7
c 1.01, 0.101, and 10.1
d 4, 4.004, 0.044, and 0.404

4 Subtract:

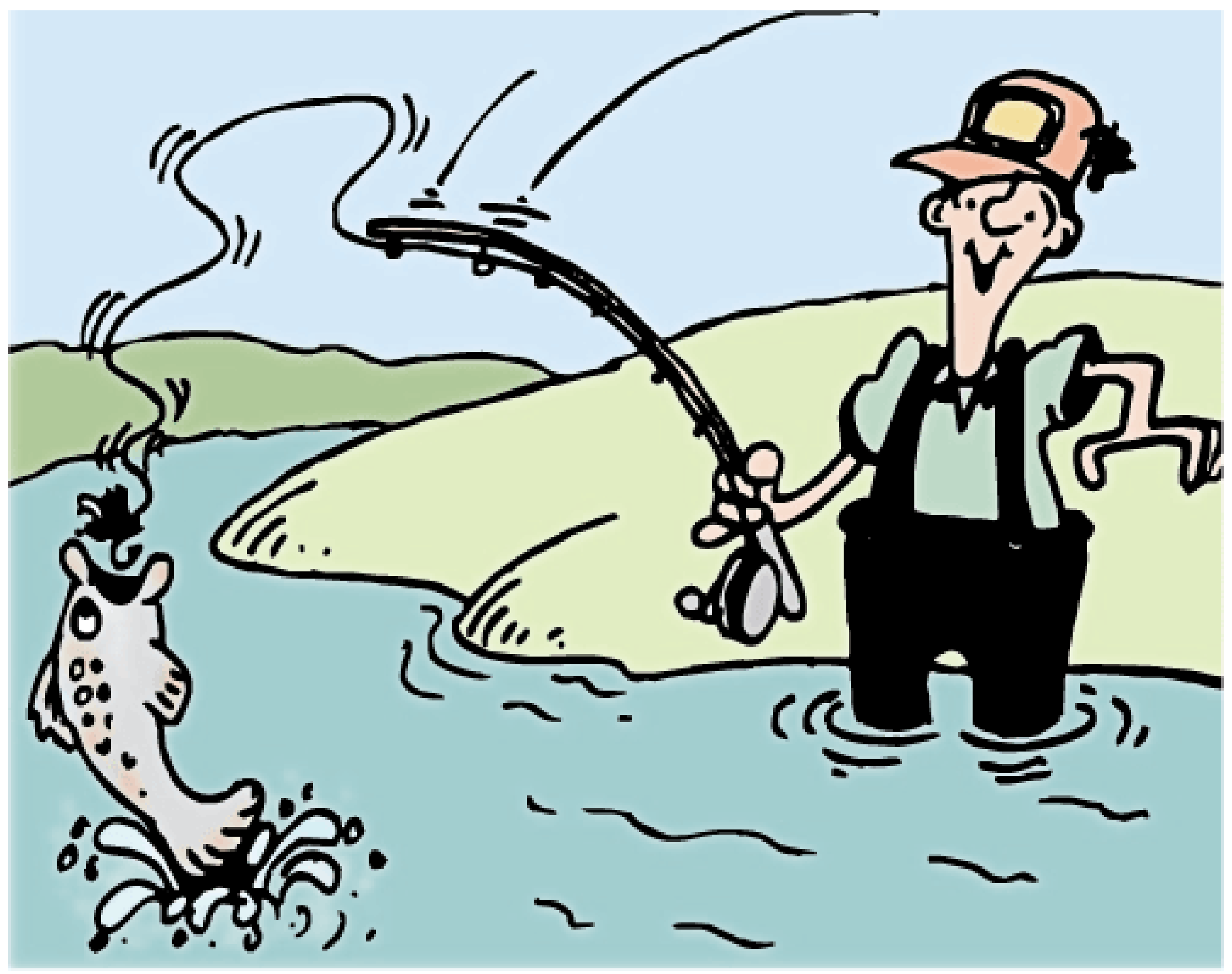
a 29.5 from 35.6
b 1.3 from 23.48
c 6.08 from 7.1
d 3.7 from 171.2
e 9.67 from 68.3
f 8.096 from 10.11

5 John gets $5.40 pocket money, Pat gets $3.85, and Jill gets $7.85. How much pocket money do they get altogether?

6 Helena is 1.75 m tall and Fred is 1.38 m tall. How much taller is Helena than Fred?

7 At a golf tournament, two players hit the same ball, one after the other. First Jeff hit the ball 132.6 m. Janet then hit the ball 104.8 m. How far did the ball travel altogether?

8 Our class went trout fishing and caught five fish. Their weights were 10.6 kg, 3.45 kg, 6.23 kg, 1.83 kg, and 5.84 kg. What was the total weight of all five fish?



9 Three pieces of timber have lengths 2.755 m, 3.084 m, and 7.24 m. Find the total length of these timber pieces.

10 Elizabeth bought items costing €10.85, €37.65, €19.05, and €24.35. How much change will she receive from €100?

G MULTIPLYING BY POWERS OF 10

Consider the multiplication $8 \times 10 = 80$.

If we write 8 as 8.0, we see that multiplication by 10 shifts the decimal point one place to the right.

$$8.\overset{\curvearrowright}{0} \times 10 = 80$$

Now consider the multiplication $8 \times 100 = 800$.

If we write 8 as 8.00, we see that multiplication by 100 shifts the decimal point two places to the right.

$$8.\overset{\curvearrowright}{0}\overset{\curvearrowright}{0} \times 100 = 800$$

Similarly, multiplication by 1000 shifts the decimal point three places to the right.

$$8.\overset{\curvearrowright}{0}\overset{\curvearrowright}{0}\overset{\curvearrowright}{0} \times 1000 = 8000$$

- When multiplying by 10, we move the decimal point one place to the right.
- When multiplying by 100, we move the decimal point two places to the right.
- When multiplying by 1000, we move the decimal point three places to the right.

Example 17 Self Tutor

Multiply 0.14 by:

a 10 **b** 100 **c** 1000

a 0.14×10
$= 0.\overset{\curvearrowright}{1}4 \times 10$
$= 1.4$

b 0.14×100
$= 0.\overset{\curvearrowright}{1}\overset{\curvearrowright}{4} \times 100$
$= 14$

c 0.14×1000
$= 0.\overset{\curvearrowright}{1}\overset{\curvearrowright}{4}\overset{\curvearrowright}{0} \times 1000$
$= 140$

EXERCISE 7G

1 Multiply the following numbers:

a 2×10
b 6.3×10
c 0.2×10
d 0.01×10
e 0.238×10
f 60.6×10
g 6×100
h 9.2×100
i 0.7×100
j 0.54×100
k 70.4×100
l $0.057\,98 \times 100$
m 7×1000
n 6.2×1000
o 0.7×1000
p 0.38×1000
q 6.75×1000
r 0.0824×1000

2 Multiply the numbers to complete the table:

	Number	× 10	× 100	× 1000
a	0.009			
b	0.12			
c	0.5			
d	4.6			
e	19.07			

3 Write the multiplier to complete these products:

a $9 \times \square = 900$ **b** $33 \times \square = 330$ **c** $3.4 \times \square = 34$

d $0.02 \times \square = 2$ **e** $0.003 \times \square = 0.03$ **f** $5.64 \times \square = 5640$

4 An ice cream costs $1.80. Find the cost of:

a 10 ice creams **b** 100 ice creams **c** 1000 ice creams.

5 A bicycle helmet costs €45.60. Find the cost of:

a 10 helmets **b** 100 helmets **c** 1000 helmets.

H DIVIDING BY POWERS OF 10

Consider the division $8 \div 10 = \frac{8}{10} = 0.8$

If we write 8 as 8.0, we see that division by 10 shifts the decimal point one place to the left.

$$\overset{\curvearrowleft}{8}.0 \div 10 = 0.8$$

Similarly, division by 100 shifts the decimal point two places to the left:

$$\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{8}.0 \div 100 = \frac{8}{100} = 0.08$$

- When dividing by 10, we move the decimal point one place to the left.
- When dividing by 100, we move the decimal point two places to the left.
- When dividing by 1000, we move the decimal point three places to the left.

Example 18 **Self Tutor**

Divide 60.9 by:

a 10 **b** 100 **c** 1000

a $60.9 \div 10$
$= 6\overset{\curvearrowleft}{0}.9 \div 10$
$= 6.09$

b $60.9 \div 100$
$= \overset{\curvearrowleft}{6}\overset{\curvearrowleft}{0}.9 \div 100$
$= 0.609$

c $60.9 \div 1000$
$= \overset{\curvearrowleft}{0}\overset{\curvearrowleft}{6}\overset{\curvearrowleft}{0}.9 \div 1000$
$= 0.0609$

EXERCISE 7H

1 Find:

a $2 \div 10$ **b** $6.3 \div 10$ **c** $0.2 \div 10$

d $0.01 \div 10$ **e** $54.02 \div 10$ **f** $606 \div 10$

g $6 \div 100$ **h** $9.2 \div 100$ **i** $0.7 \div 100$

j $50 \div 100$ **k** $166 \div 100$ **l** $300.7 \div 100$

2 Find:

a $7 \div 1000$ **b** $6.2 \div 1000$ **c** $56.1 \div 1000$

d $499 \div 1000$ **e** $701 \div 1000$ **f** $6854.9 \div 1000$

3 Divide the numbers to complete the table:

	Number	$\div 10$	$\div 100$	$\div 1000$
a	8			
b	4.6			
c	50			
d	19.07			
e	231.4			

4 Write the divisor to complete these divisions:

a $6 \div \square = 0.6$ **b** $33 \div \square = 0.33$ **c** $3.4 \div \square = 0.34$

d $0.2 \div \square = 0.002$ **e** $49 \div \square = 0.49$ **f** $634.1 \div \square = 0.6341$

5 A prize of \$27 565 is divided equally between 100 people.
How much money does each person receive?

6 A supermarket pays £1820 for 1000 two-litre bottles of milk. The bottles arrive in plastic crates containing 10 bottles each. Find the amount the supermarket pays for:

a each bottle of milk **b** a crate of 10 bottles.

I MULTIPLYING DECIMALS BY WHOLE NUMBERS

We know that 2×3 means "2 lots of 3", which is $3 + 3 = 6$.

In the same way, 2×1.4 means "2 lots of 1.4", which is $1.4 + 1.4 = 2.8$

$$\begin{array}{r} 1.4 \\ +\ 1.4 \\ \hline 2.8 \end{array}$$

We observe that $2 \times 14 = 28$
and $2 \times 1.4 = 2.8$

INVESTIGATION MULTIPLICATION OF DECIMALS

What to do:

1 Find the following products without using technology:

a 2×8 and 2×0.8 **b** 3×12 and 3×1.2

c 2×34 and 2×3.4 **d** 3×15 and 3×0.15

2 Use your calculator to find the following products:

a 9×6 and 9×0.6 **b** 6×25 and 6×0.25

3 Discuss your findings with your classmates.

To multiply a decimal by a whole number, we follow these steps:

Step 1: Multiply the numbers as though they were whole numbers.

Step 2: Insert the decimal point so that the number of decimal places in the answer equals the number of decimal places in the question.

Step 3: Remove any unnecessary zeros from the end of the answer.

Example 19 **Self Tutor**

Find:

a 9×0.7 **b** 0.11×8 **c** 6×0.05

a $9 \times 7 = 63$

$\therefore\ 9 \times 0.7 = 6.3$ {1 decimal place in the question and answer}

b $11 \times 8 = 88$

$\therefore\ 0.11 \times 8 = 0.88$ {2 decimal places in the question and answer}

c $6 \times 5 = 30$

$\therefore\ 6 \times 0.05 = 0.30$ {2 decimal places in the question and answer}

$= 0.3$ {removing the unnecessary zero}

EXERCISE 7I

1 Find:

a 3×0.7 **b** 0.8×4 **c** 5×0.5 **d** 9×1.2

e 6×0.07 **f** 0.03×8 **g** 3×0.02 **h** 0.13×4

i 12×0.08 **j** 25×0.05 **k** 7×0.008 **l** 0.0012×3

2 Find:

a 8×0.5 **b** 5×0.6 **c** 4×0.15

d 0.04×5 **e** 2×0.05 **f** 15×0.04

g 50×0.008 **h** 6×0.025 **i** 12×0.005

Example 20 **Self Tutor**

Find:

a 21.3×4 **b** 428×0.25

a

$$\begin{array}{cccc} & 2 & 1 & 3 \\ \times & & & 4 \\ & & {\scriptstyle 1} & \\ \hline & 8 & 5 & 2 \end{array}$$

$213 \times 4 = 852$

$\therefore\ 21.3 \times 4 = 85.2$ {1 decimal place in question and answer}

b

$$\begin{array}{cccccc} & & & 4 & 2 & 8 \\ & \times & & & 2 & 5 \\ & & & {\scriptstyle 1} & {\scriptstyle 4} & \\ & & & {\scriptstyle 1} & & \\ \hline & & 2 & 1 & 4 & 0 \\ & & 8 & 5 & 6 & 0 \\ & {\scriptstyle 1} & & {\scriptstyle 1} & & \\ \hline & 1 & 0 & 7 & 0 & 0 \end{array}$$

$428 \times 25 = 10\,700$

$\therefore\ 428 \times 0.25 = 107.00$ {2 decimal places in question and answer}

$= 107$ {removing unnecessary zeros}

3 Find:

a 3.1×7 **b** 21×1.9 **c** 28.7×6

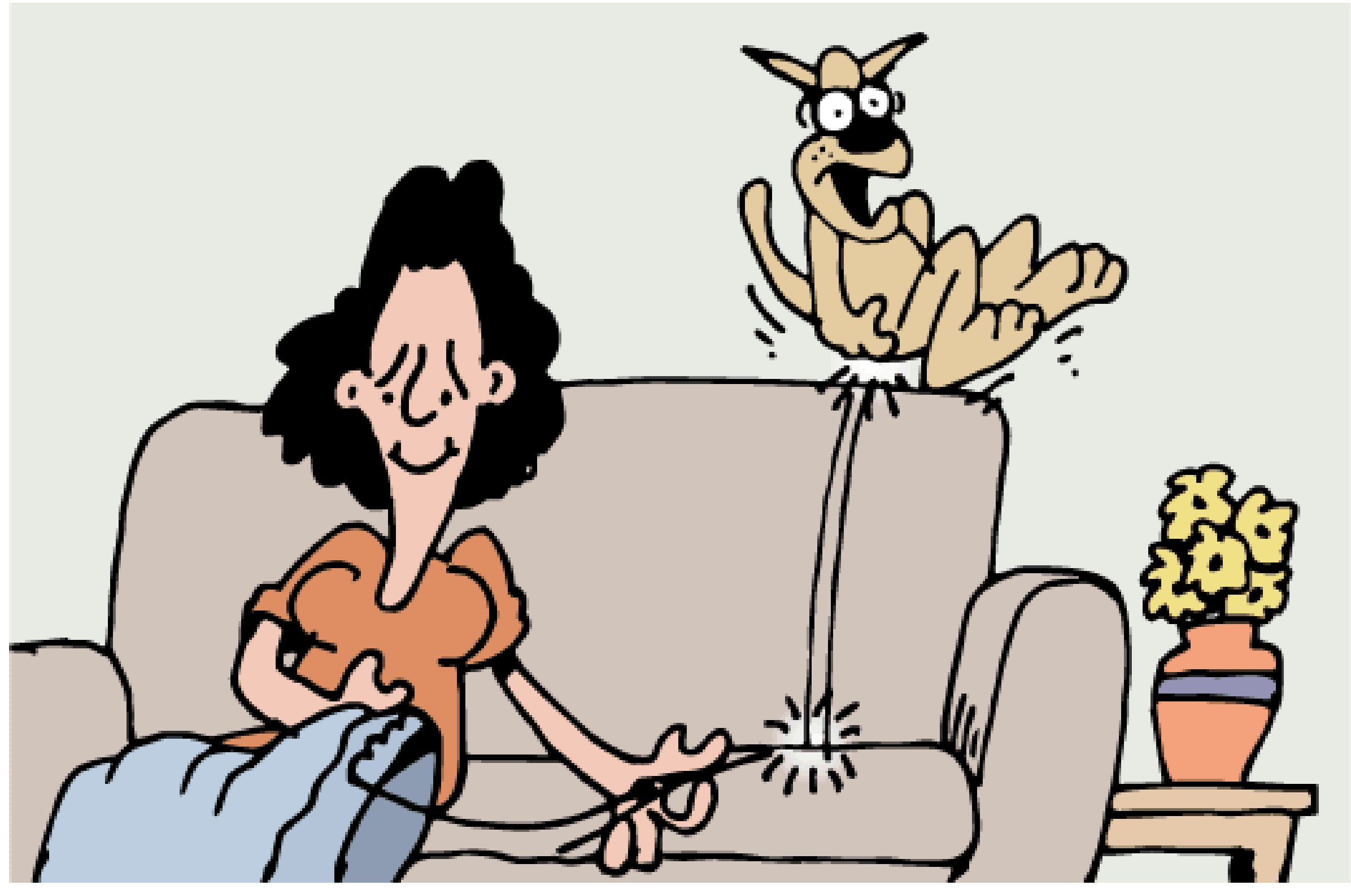
d 142×0.23 **e** 3.65×8 **f** 280×0.65

4 If one baseball bat weighs 2.3 kg, how much would seven similar baseball bats weigh in total?

5 Dawn is able to sew a skirt hem in 10.4 minutes. How long will it take her to sew nine skirt hems if she works at the same rate?

6 25 students went on an excursion. Their bus tickets cost €2.35 each. Find the total amount paid.

7 Find the total weight of 200 boxes of muesli bars if each box weighs 0.18 kg.



ACTIVITY ESTIMATING DECIMAL PRODUCTS

When we multiply decimals, it is important to think about what answer to expect. An **estimate** can warn us of an error we may have made.

We can estimate products by rounding the decimal to the nearest whole number.

For example, $12 \times 3.9 \approx 12 \times 4$, so we expect the value of 12×3.9 to be somewhere near 48.

What to do:

1 Estimate the following products, then use your estimate to choose the correct answer.

- **a** $4.387 \times 6 =$
  - **A** 263.22 **B** 26.322 **C** 2.6322 **D** 2632.2
- **b** $150 \times 2.8 =$
  - **A** 4.2 **B** 42 **C** 420 **D** 4200
- **c** $7.234 \times 11 =$
  - **A** 795.74 **B** 79.574 **C** 7.9574 **D** 0.795 74

2 **a** Estimate the product 17×3.1

b Will your estimate be more or less than the actual value? Explain your answer.

J DIVIDING DECIMALS BY WHOLE NUMBERS

To divide a decimal by a whole number, we perform the division as we would for whole numbers, except we place the decimal point in the answer directly above the decimal point in the question.

Example 21 Self Tutor

Find: **a** $4.64 \div 4$ **b** $5.28 \div 8$

a
$$\begin{array}{r} 1\,.\,1\;\;6 \\ 4\,\overline{)\,4\,.\,6\;{}^{2}4} \end{array}$$
So, $4.64 \div 4 = 1.16$

b
$$\begin{array}{r} 0\,.\,6\;\;6 \\ 8\,\overline{)\,5\,.\,2\;{}^{4}8} \end{array}$$
So, $5.28 \div 8 = 0.66$

EXERCISE 7J

1 Find:

a $3.2 \div 4$ **b** $7.5 \div 5$ **c** $1.26 \div 3$ **d** $3.57 \div 7$

e $24.16 \div 8$ **f** $2.46 \div 6$ **g** $0.72 \div 9$ **h** $81.6 \div 4$

Example 22 **Self Tutor**

A 6.4 m length of timber is cut into four equal lengths.
How long is each piece?

$$\begin{array}{r} 1\,.\,6 \\ 4\,\overline{\big)\,6\,.\,{}^{2}4} \end{array}$$

$6.4 \text{ m} \div 4 = 1.6 \text{ m}$

Each piece is 1.6 m long.

2 A 10.75 kg tub of ice cream is divided equally among 5 people. How much ice cream does each person receive?

3 Simone bought 4 kg of bananas for \$9.16. How much did each kilogram of bananas cost?

Example 23 **Self Tutor**

Find:

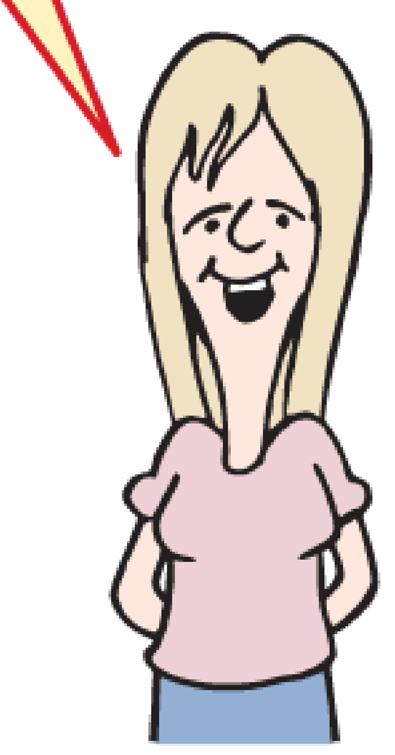
a $6.3 \div 5$ **b** $3.5 \div 4$

a
$$\begin{array}{r} 1\,.\,2\;\;6 \\ 5\,\overline{\big)\,6\,.\,{}^{1}3\;{}^{3}0} \end{array}$$

So, $6.3 \div 5 = 1.26$

b
$$\begin{array}{r} 0\,.\,8\;\;7\;\;5 \\ 4\,\overline{\big)\,3\,.\,5\;{}^{3}0\;{}^{2}0} \end{array}$$

So, $3.5 \div 4 = 0.875$



Sometimes we need to add extra zeros to the number we are dividing into.

4 Find:

a $5.3 \div 2$ **b** $6.1 \div 5$ **c** $3.4 \div 4$ **d** $3.4 \div 8$

e $6.5 \div 2$ **f** $5.9 \div 4$ **g** $2.41 \div 2$ **h** $6.32 \div 5$

5 Ingrid completed 8 laps of a bicycle circuit, which was a total distance of 26.8 km. Find the length of each lap.

KEY WORDS USED IN THIS CHAPTER

- decimal number
- place value
- decimal point
- tenth
- hundredth
- thousandth

REVIEW SET 7A

1 Between which two whole numbers do the following decimal numbers lie?

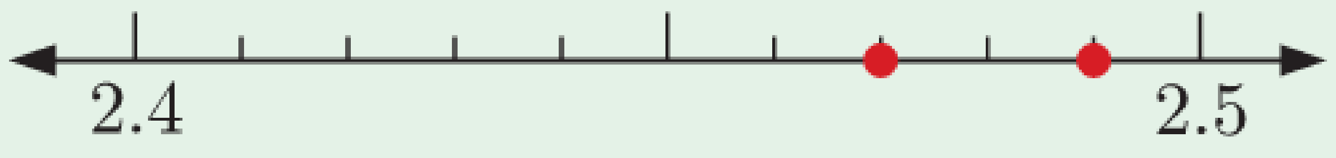
a 7.5 **b** 15.6 **c** 41.07 **d** 67.83

2 Write as a decimal number:

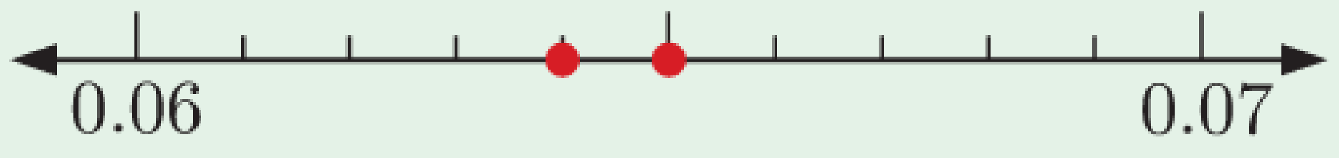
a $\frac{7}{10} + \frac{3}{100}$ **b** $\frac{1}{10} + \frac{7}{1000}$ **c** $5 + \frac{6}{100} + \frac{9}{1000}$

3 Write down the values of A and B on these number lines:

a



b



4 Round 3.8551 to:

a 1 decimal place

b 2 decimal places.

5 Insert $>$, $<$, or $=$ to make the statement true:

a 0.57 ☐ 0.41

b 0.09 ☐ 0.1

c 3.07 ☐ 3.7

6 Write as a fraction in simplest form:

a 0.23

b 0.2

c 0.059

d 0.68

7 Write as a decimal number:

a $\frac{71}{100}$

b $\frac{4}{5}$

c $\frac{19}{20}$

d $\frac{81}{500}$

8 Find:

a $0.41 + 0.27$

b $7.39 - 5.16$

c $16.5 + 3.74$

d $8 - 2.49$

9 In three seasons, a vineyard produced 638.17 tonnes, 582.35 tonnes, and 717.36 tonnes of grapes. What was the total harvest of grapes for the three years?

10 Find:

a 6.2×10

b 2.158×100

c $5.6 \div 10$

d $4.2 \div 100$

11 Find:

a 9×0.3

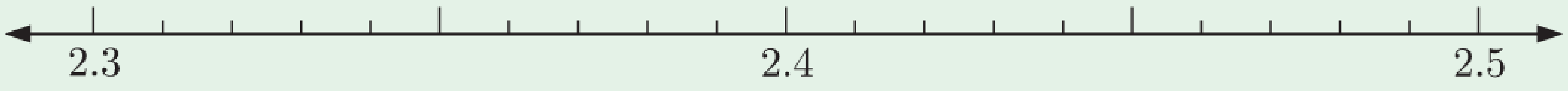
b 0.06×8

c $0.021 \div 7$

d $0.15 \div 4$

12 Determine the total cost of 14 showbags valued at \$7.85 each.

13 **a** Place the decimal numbers 2.39 and 2.46 on this number line.



b Find the sum of 2.39 and 2.46

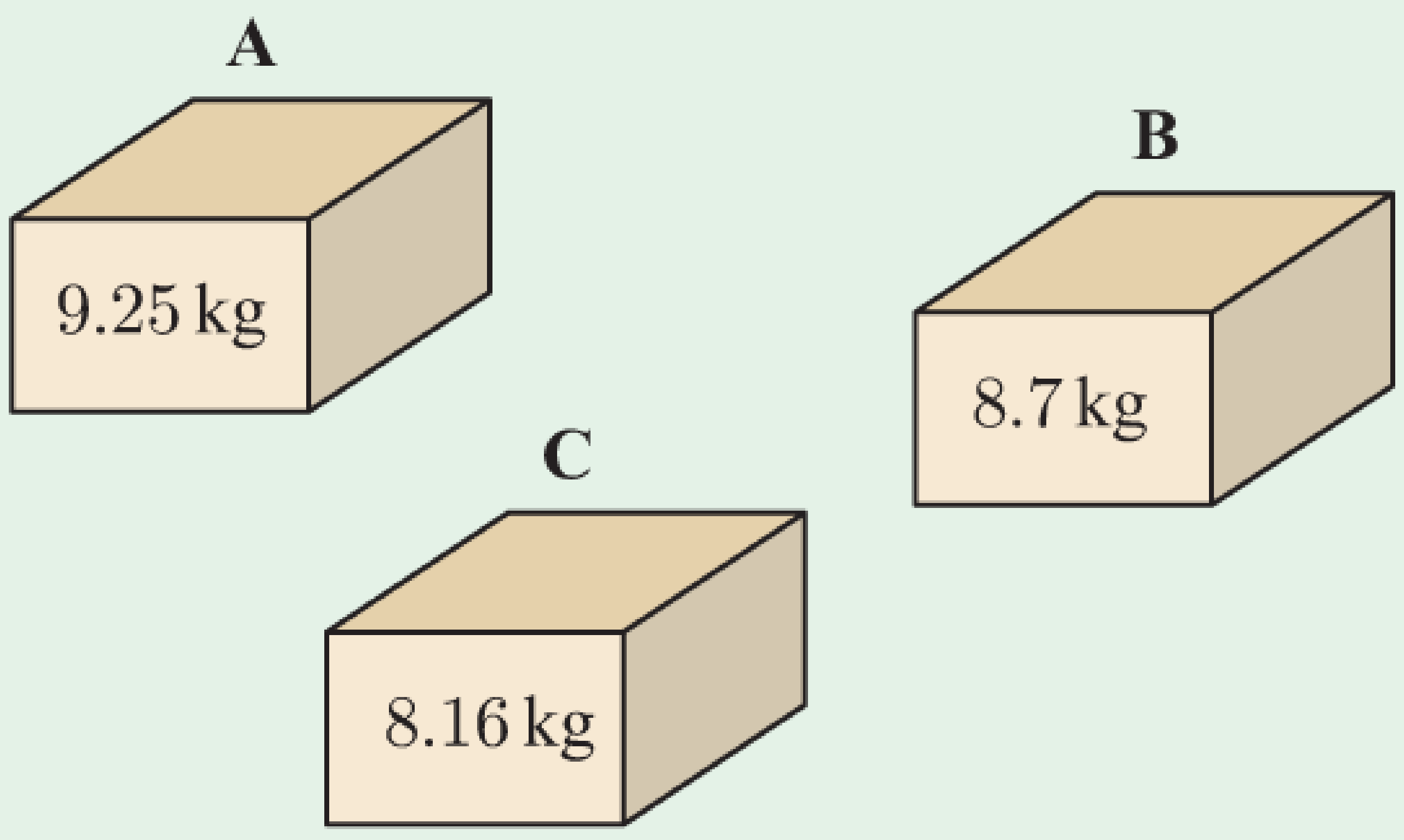
c Round the sum to 1 decimal place.

14 Kim sent some birthday presents to his cousins overseas using three parcels.

a Find the total weight of the parcels.

b How much heavier is parcel **A** than parcel **C**?

c The parcels cost £3 per kilogram to send.
Find the cost of sending parcel **B**.



REVIEW SET 7B

1 State the number of decimal places in these decimal numbers:

a 30.7 **b** 9.16 **c** 92.70 **d** 0.108

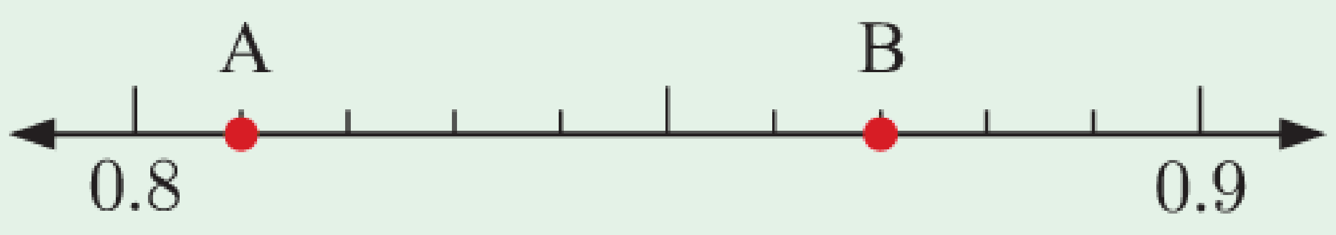
2 Write 'sixteen point five seven four' in decimal form.

3 Write $\frac{31}{1000}$ as a decimal number.

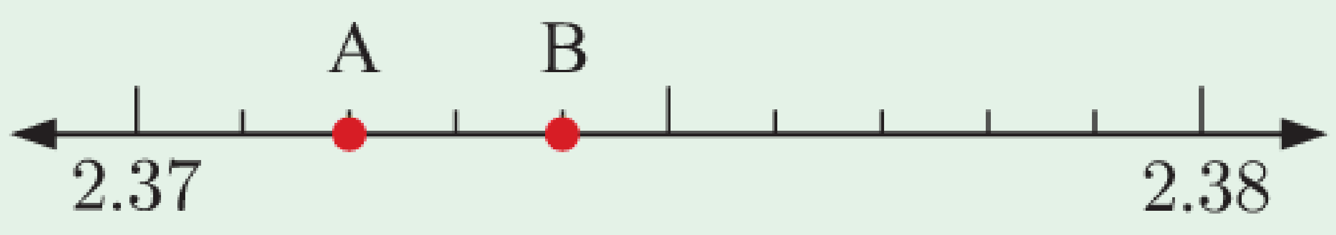
4 State the value of the digit 3 in 10.003

5 Write down the values of A and B on these number lines:

a



b



6 Write 2.049 in expanded form.

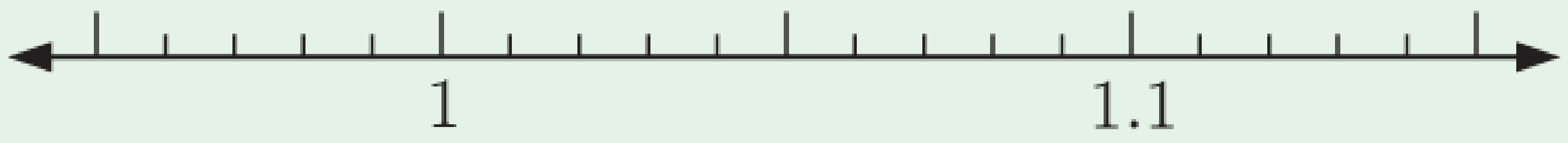
7 Complete the following statements:

a $203 \div \square = 2.03$ **b** $2.03 \times \square = 2030$ **c** $0.203 \div \square = 0.002\,03$

8 Find:

a $0.23 + 0.69$ **b** $6.81 - 4.25$ **c** $5.8 + 3.74$ **d** $9 - 1.83$

9 **a** Mark the numbers 1.11, 1.0, 1.1, and 1.01 on this number line.



b Write the numbers in **a** in order from largest to smallest.

10 Round to 2 decimal places:

a 0.523 **b** 1.0908 **c** 12.3671

11 Write the following amounts as decimals:

a $\frac{17}{20}$ km **b** $\frac{3}{25}$ L **c** $4\frac{21}{50}$ kg

12 Find:

a 0.07×6 **b** 23×1.6 **c** $8.7 \div 3$ **d** $7.9 \div 5$

13 The first horse in a 1000 metre sprint finished in 56.98 seconds. The second and third horses were 0.07 seconds and 0.23 seconds behind the winner.

a How long did the **i** second horse **ii** third horse take to finish the sprint?

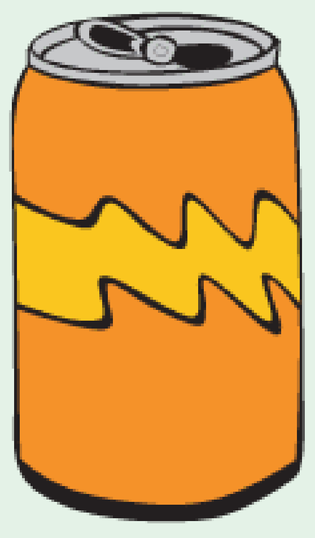
b Find the difference in time between the second horse and the third horse.

14 Anna decided to go to a movie. She spent $2.85 on a bus ticket, $15.50 on the movie ticket, $1.55 on a drink, and $1.85 on an ice cream.

a How much did Anna spend in total?

b How much change would she have from $50?

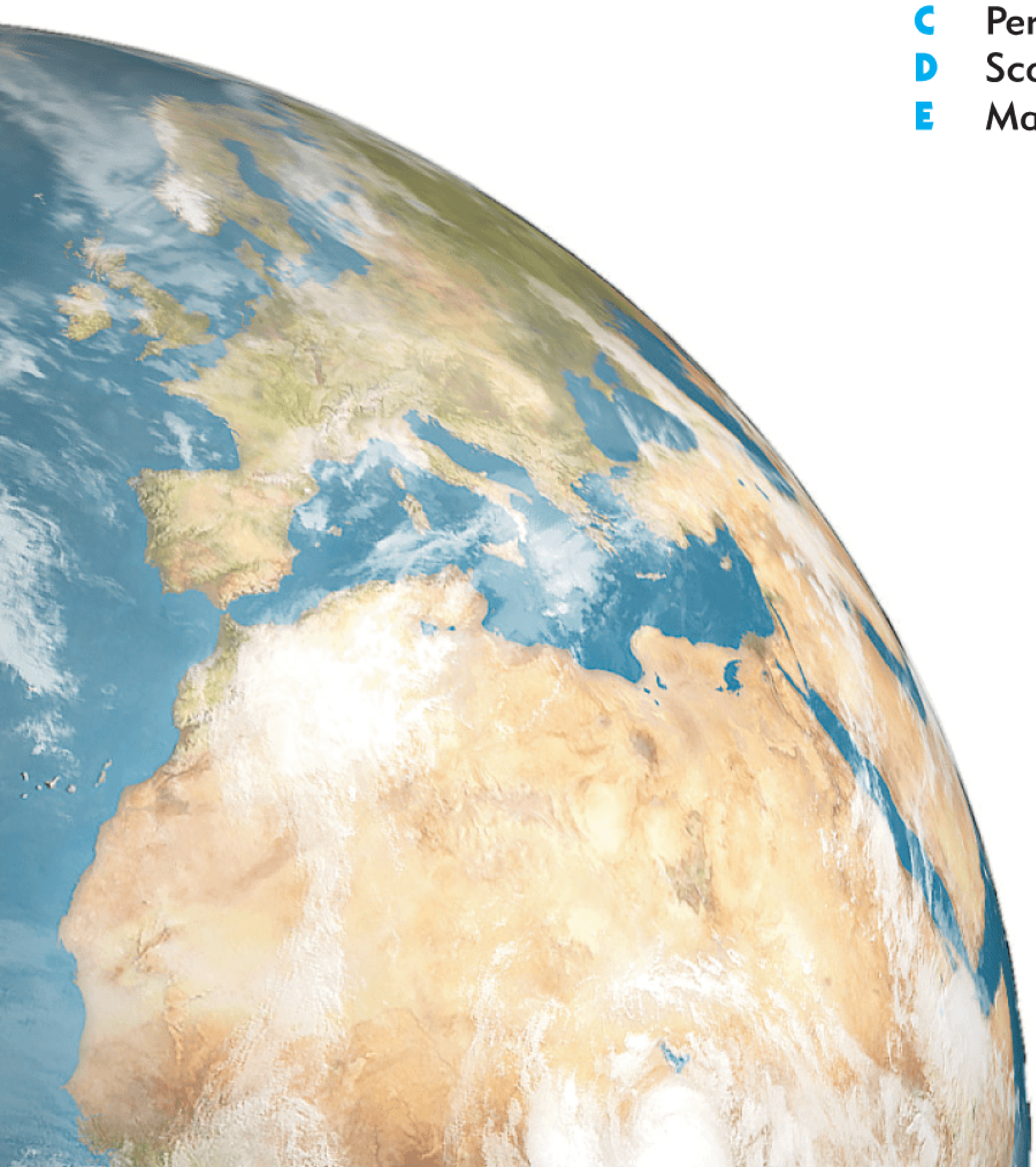
c Four of Anna's friends went to the movies with her, and they each spent the same amount as Anna. In total, how much was spent by the five children?



Chapter 8

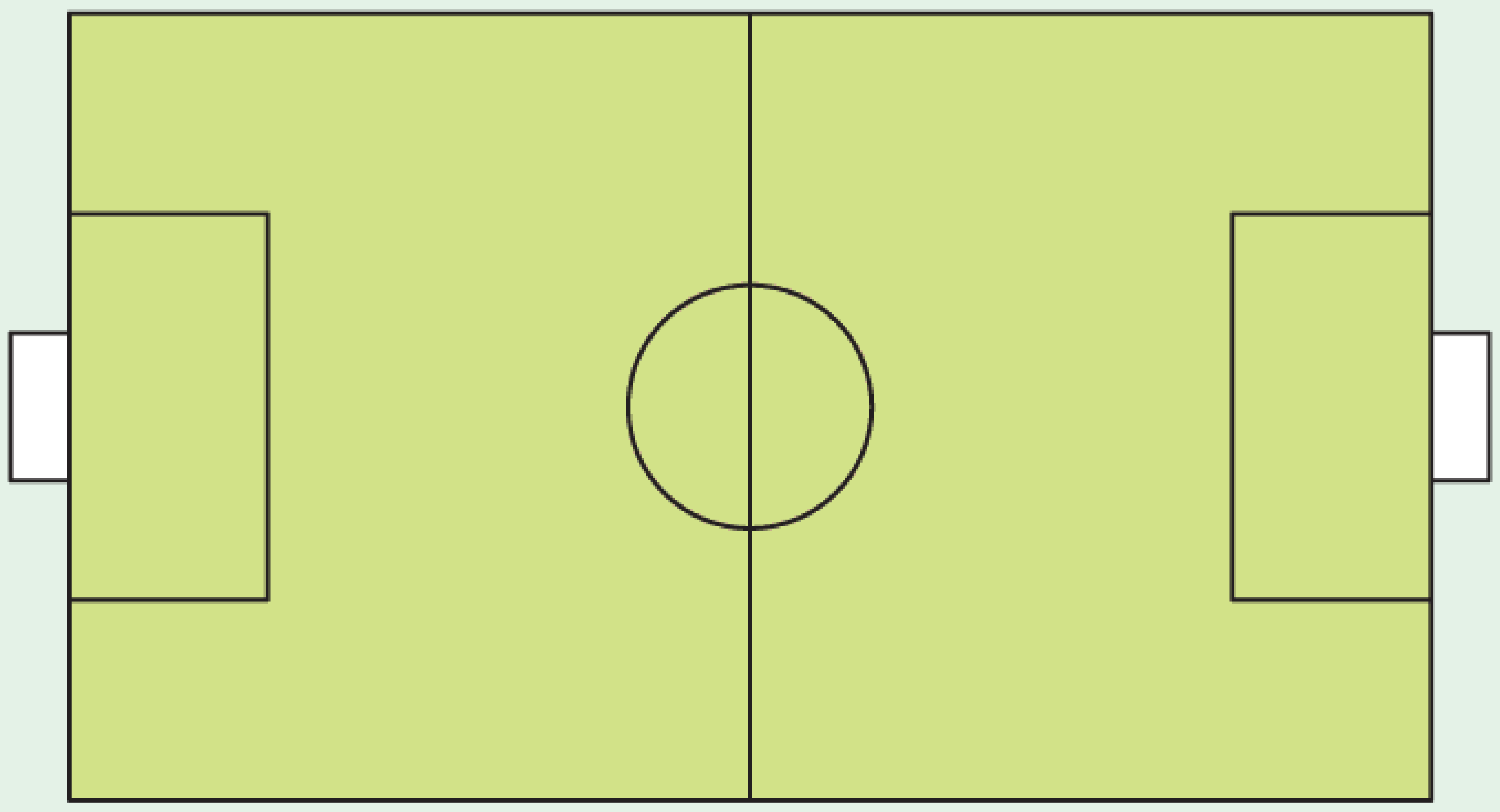
Measurement

Contents:



OPENING PROBLEM

Byron is about to play a game of football on a pitch like the one alongside. You may have a pitch like this at your school.



Things to think about:

- **a** How *long* do you think a football pitch is?
- **b** How *wide* do you think a football pitch is?
- **c** How could you calculate the total *distance* around the boundary of the pitch?

In our everyday life we **measure** many things. Measurement gives us a number value for the **size** of a quantity.

There are several common *types* of quantities we measure:

Measurement	*Example*
Distance or length	How far we have travelled.
Mass or weight	How heavy we are.
Time	How long a tennis match will last.
Temperature	How hot it is going to be tomorrow.
Area	The size of the block of land I want to buy.
Volume	How much concrete I need for the driveway.
Speed	How fast the car is travelling.

The different types of quantities require different **instruments** to measure them.

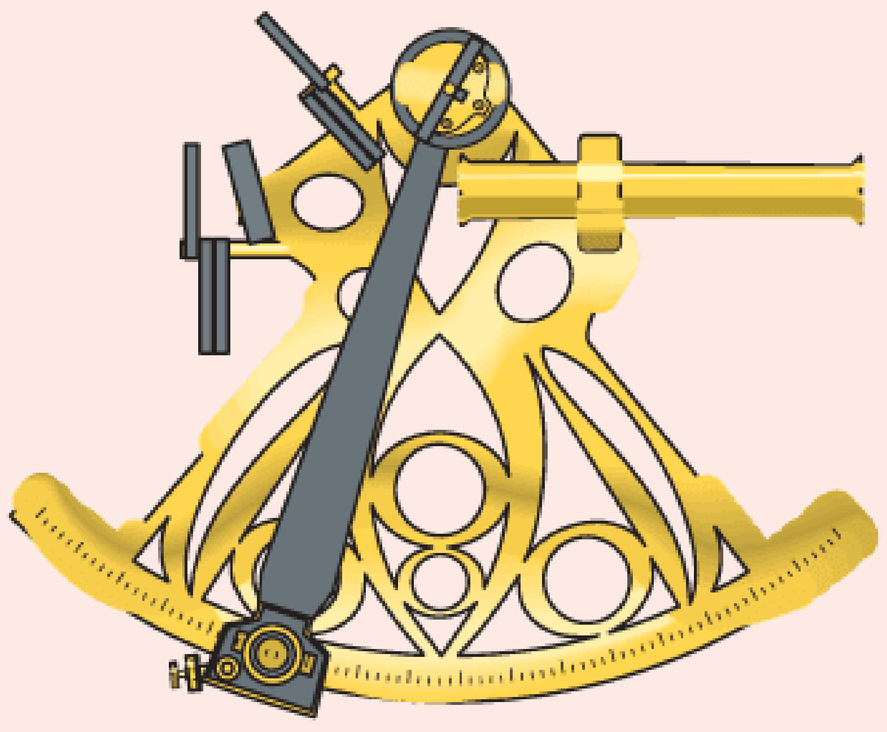
ACTIVITY 1 MEASURING INSTRUMENTS

What to do:

Match each instrument to its name. Find out what the instrument measures.

a

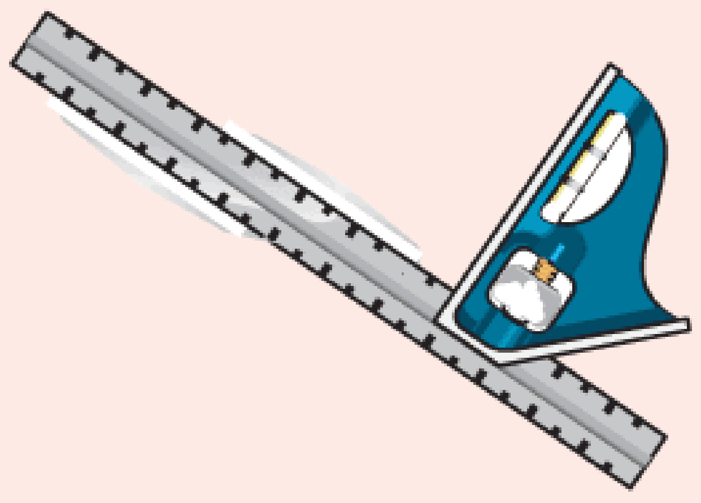
b



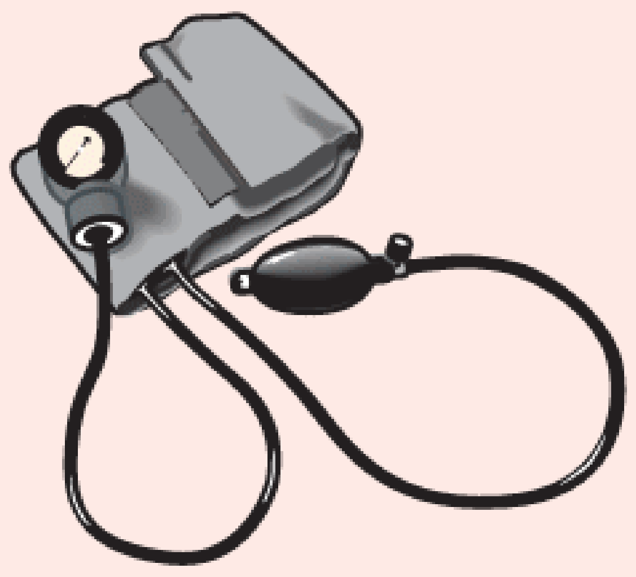
c

d

e



f



g

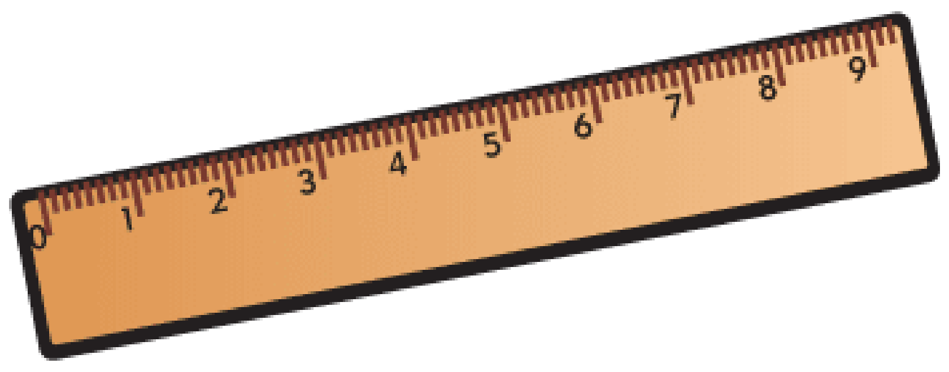
h

A pocket watch
B electricity meter
C builders' square
D sphygmomanometer
E thermometer
F micrometer
G fuel gauge
H sextant

A READING SCALES

Measuring instruments usually have a **scale** marked on them.

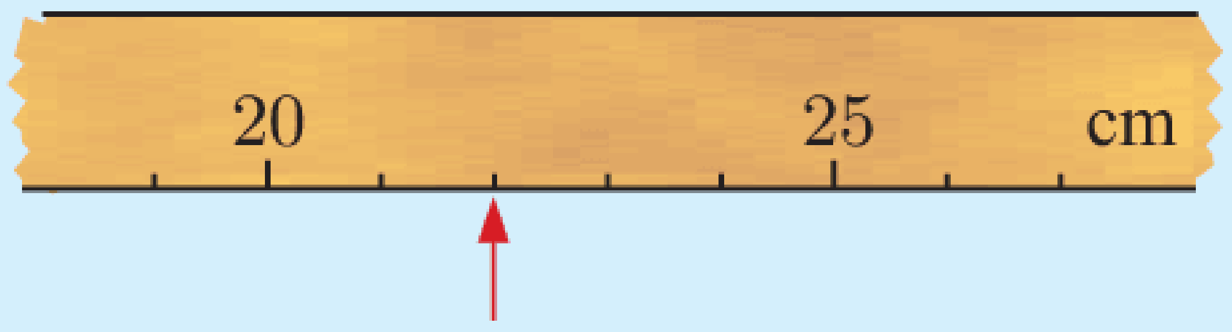
We are all familiar with a **ruler** for measuring lengths. Rulers usually have a scale marked in both millimetres and centimetres.



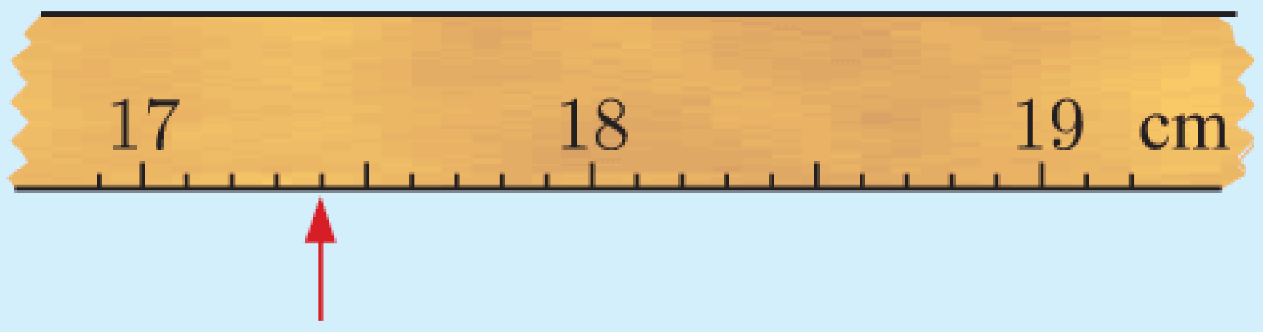
Example 1 **Self Tutor**

Read these ruler measurements:

a



b

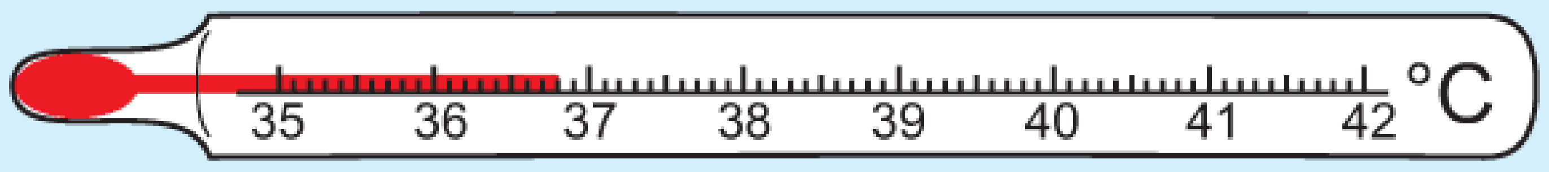


a There are 5 divisions between 20 and 25 cm, so each division is 1 cm.
The measurement is 22 cm.

b There are 10 divisions between 17 and 18 cm, so each division is $\frac{1}{10}$ of a cm.
The measurement is 17.4 cm.

Example 2 **Self Tutor**

Read the temperature on this thermometer:

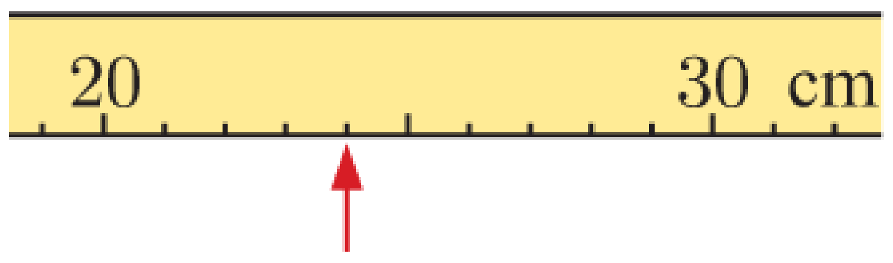


There are 10 divisions between 36°C and 37°C, so each division is 0.1°C.
The temperature is 36.8°C.

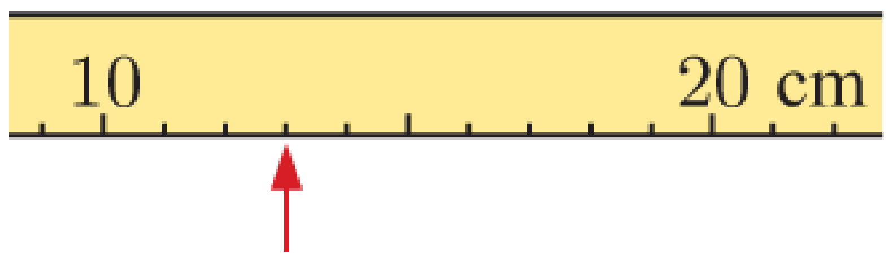
EXERCISE 8A

1 Read these ruler measurements:

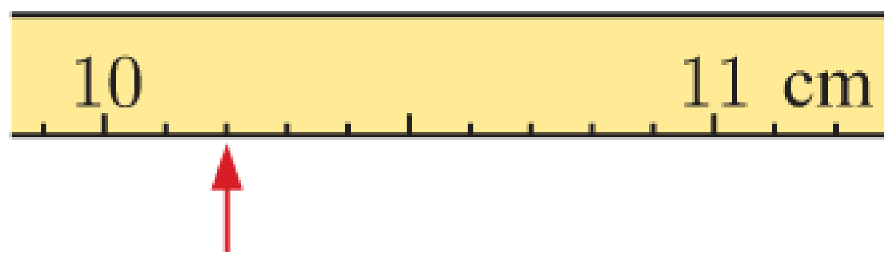
a



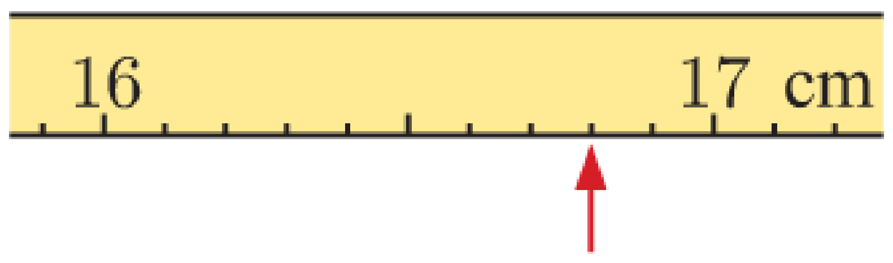
b



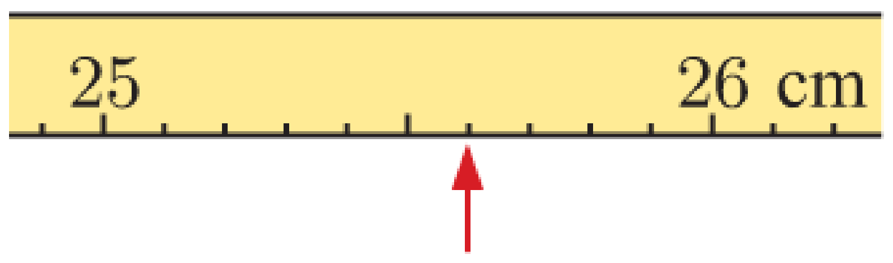
c



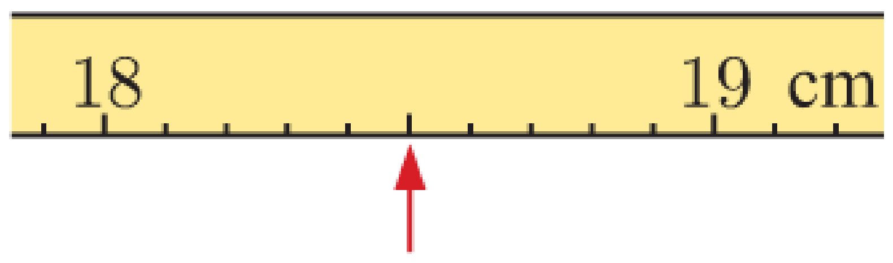
d



e

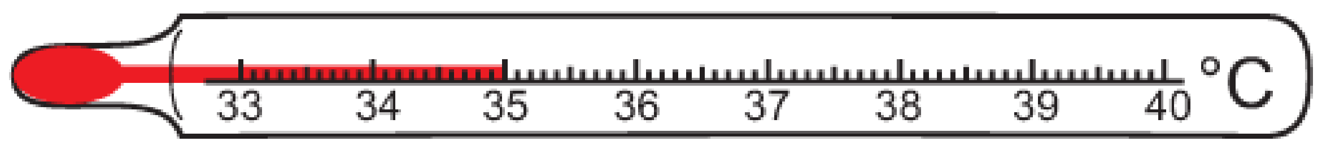


f

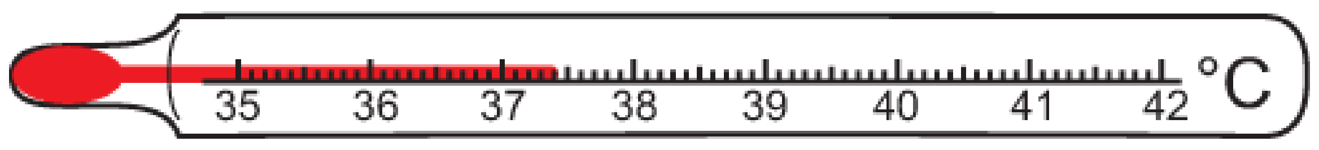


2 Read the temperature on each thermometer:

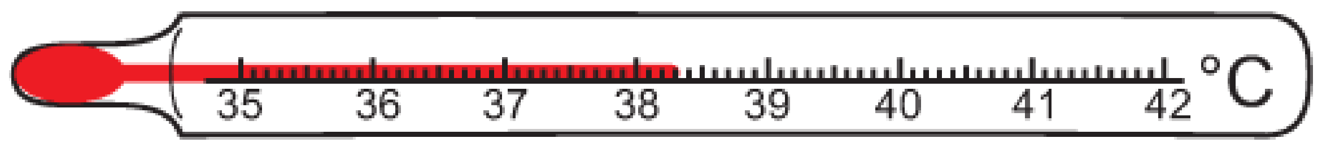
a



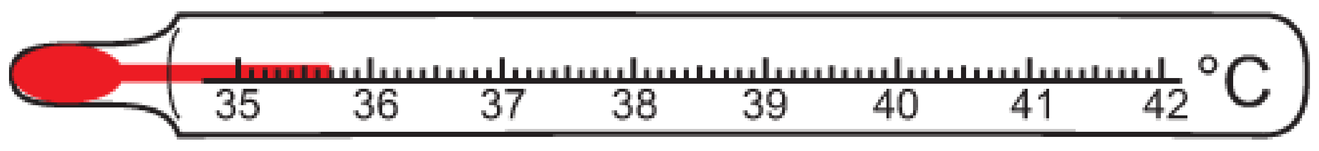
b



c



d

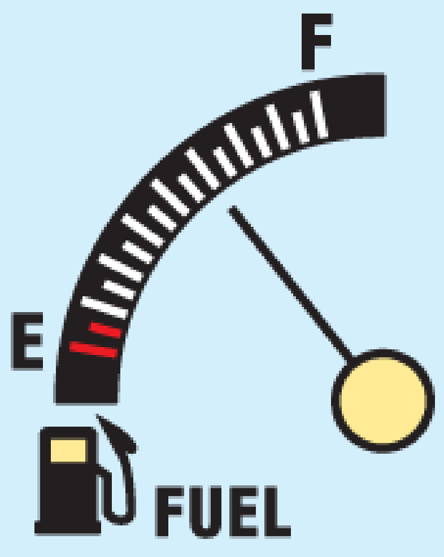


Example 3

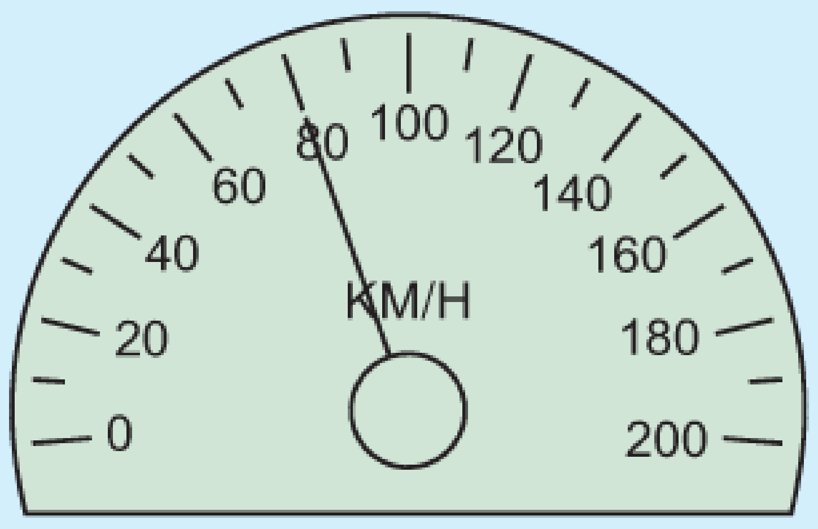
Self Tutor

Read as accurately as possible, the measurement on:

a the fuel gauge



b the speedometer.

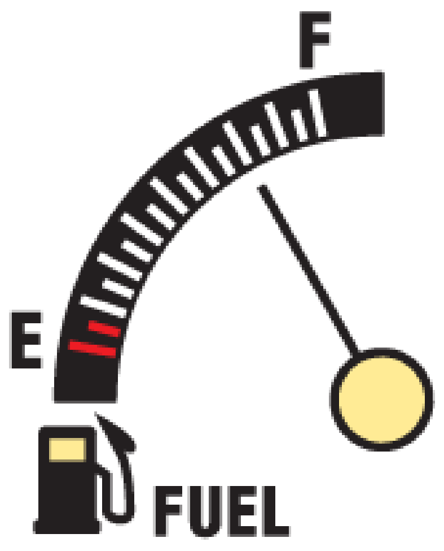


a There are eight main divisions from empty to full, so the fuel tank is $\frac{5}{8}$ full.

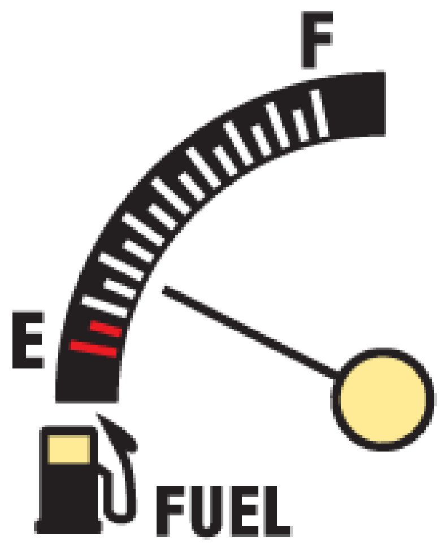
b The speed is 80 km/h.

3 Read these fuel gauges:

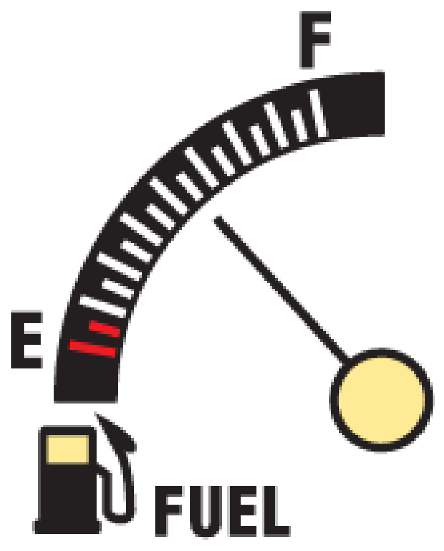
a



b

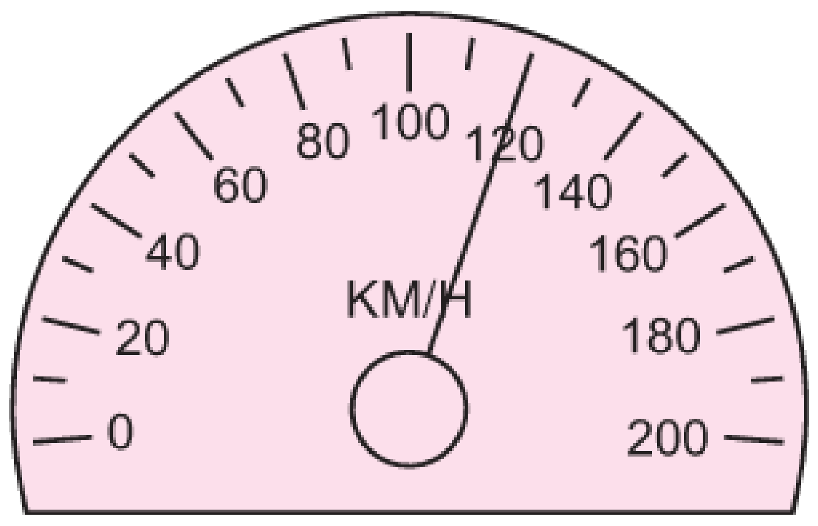


c

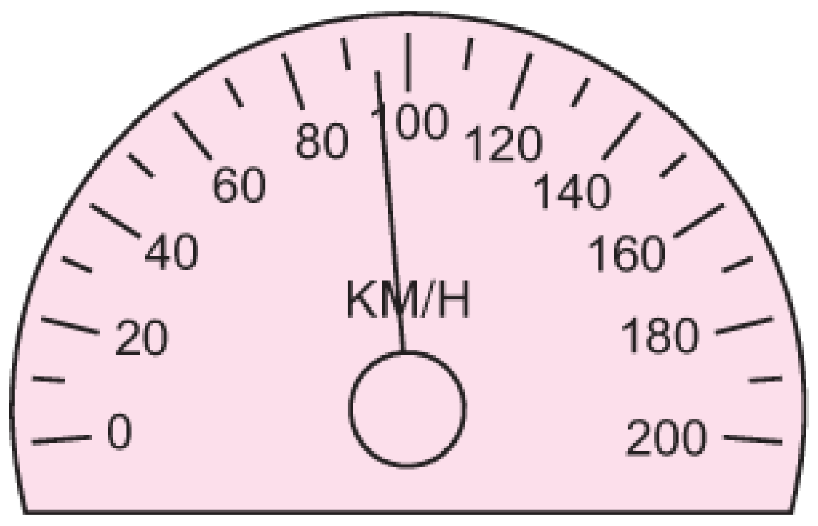


4 Read as accurately as possible, the speeds on these speedometers:

a



b



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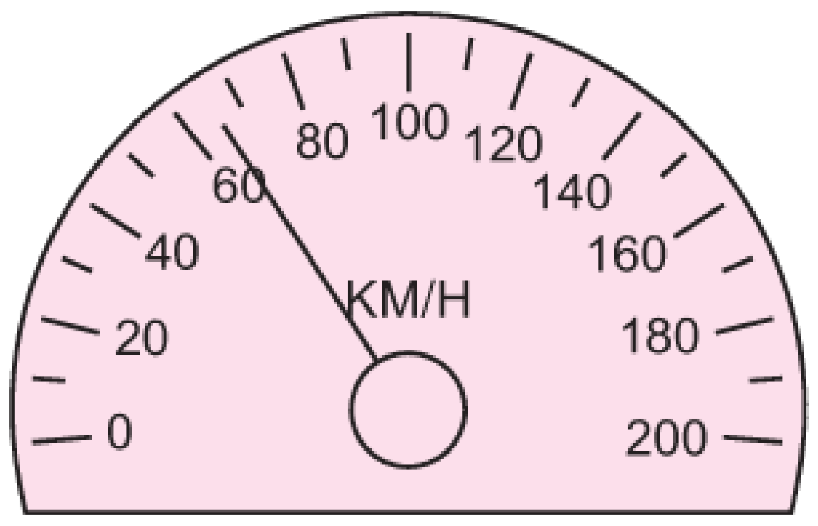
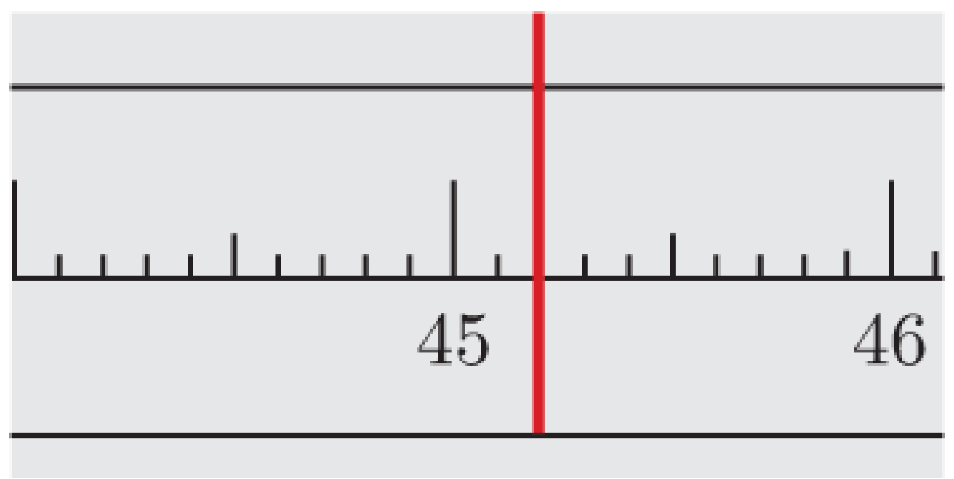
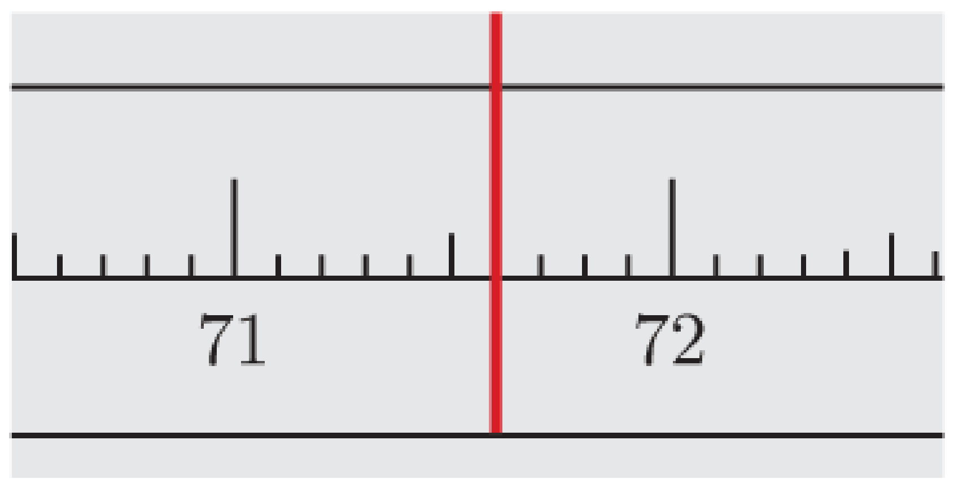


5 Find the weights, in kilograms, shown by these bathroom scales:

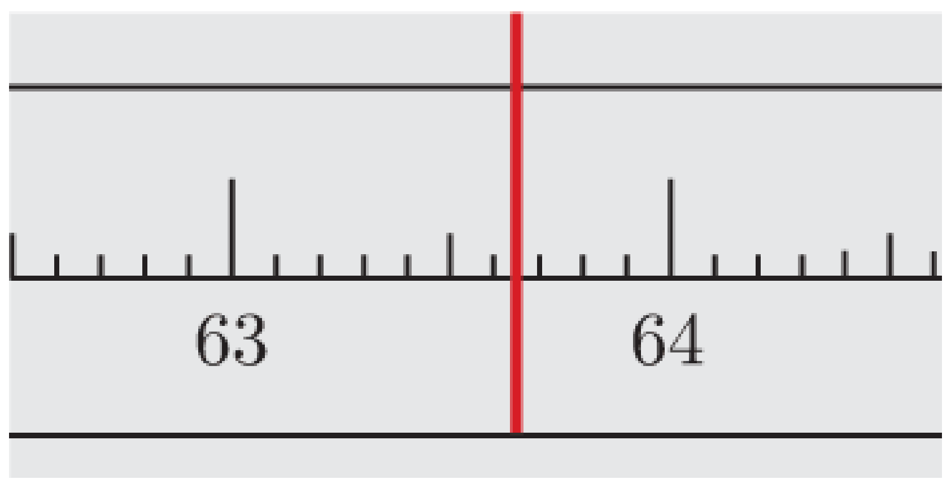
a



b

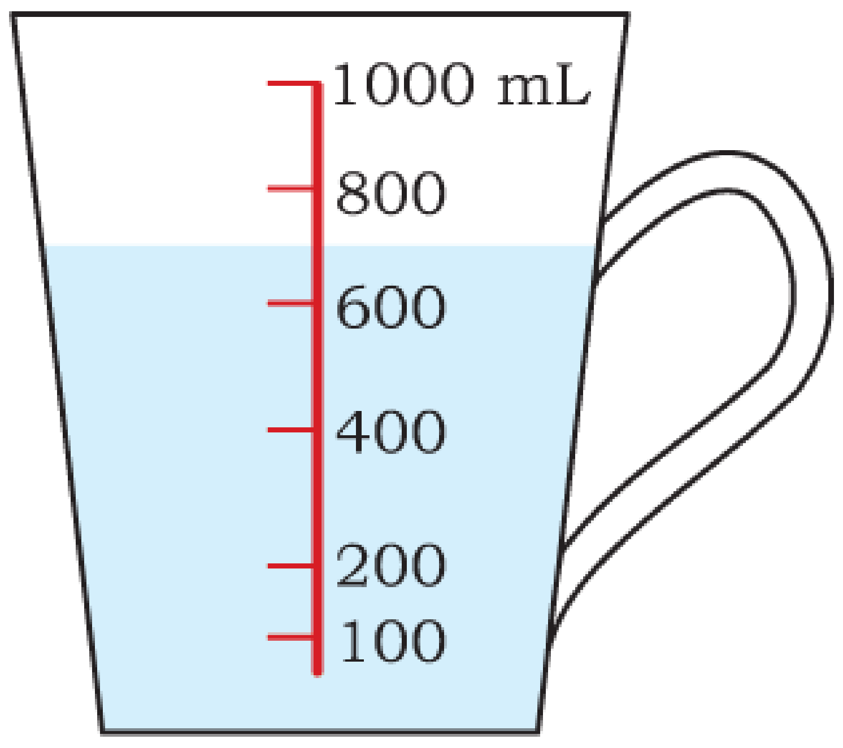


c

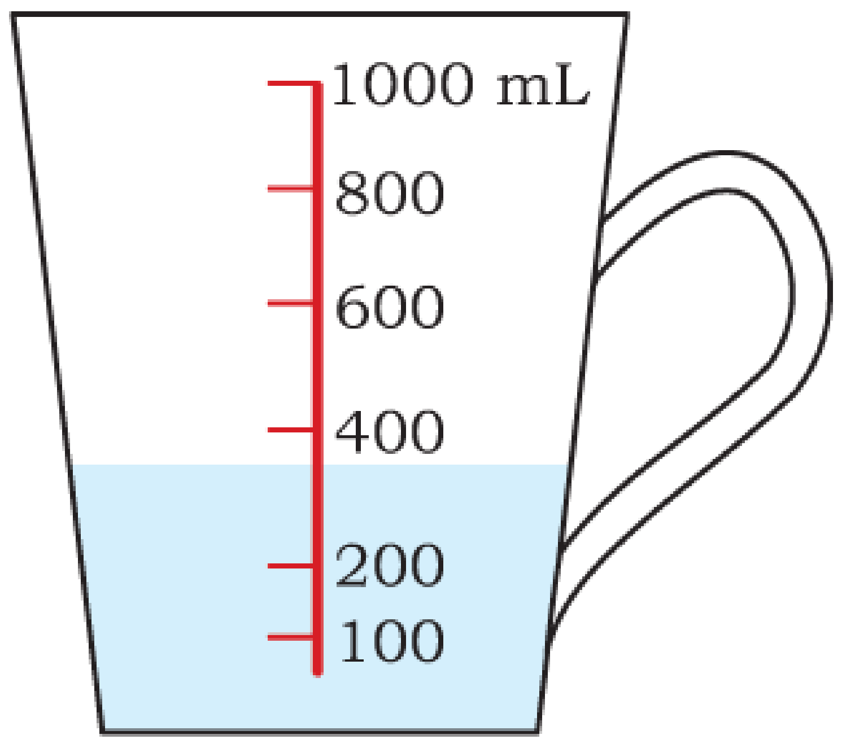


6 Find the quantity of fluid in each jug:

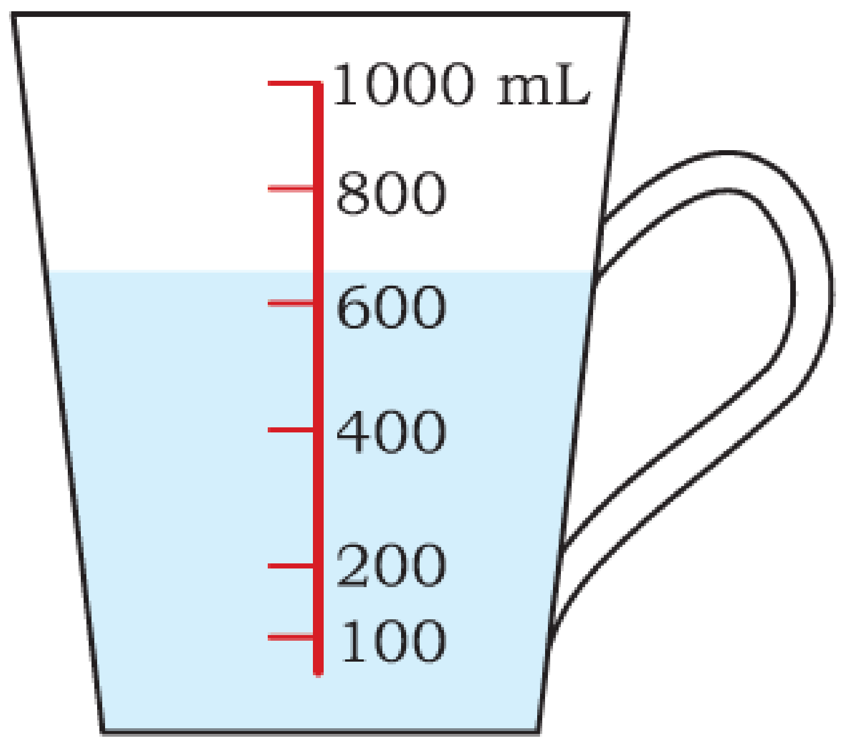
a



b



c



B LENGTH

A **length** is a measure of distance.

Lengths are used to express how far one object is from another. For example:

"I live 5 kilometres from school."

"His ball is 150 metres from the hole."

"Rule a margin 2 centimetres from the edge of the page."

UNITS OF LENGTH

In this course we use the **International System of Units**, called the **SI System**, from the French *Le Systéme international d'unités*. It is also commonly known as the **metric system**.

In the metric system, the base unit for length is the **metre**.

Other commonly used units of length based on the metre are the **millimetre**, **centimetre**, and **kilometre**.

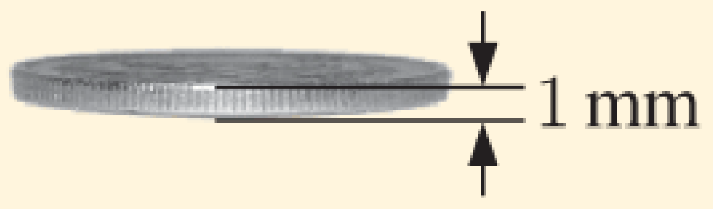
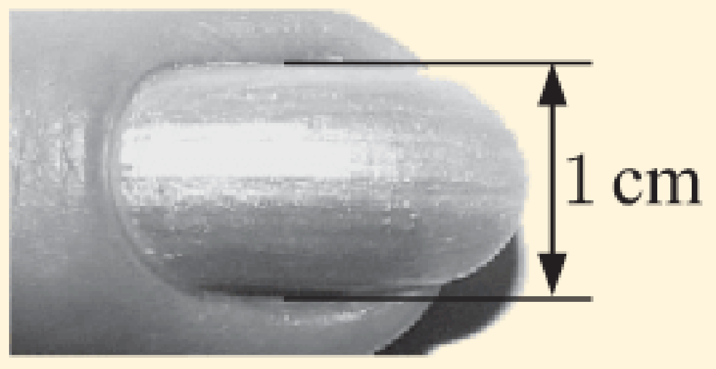
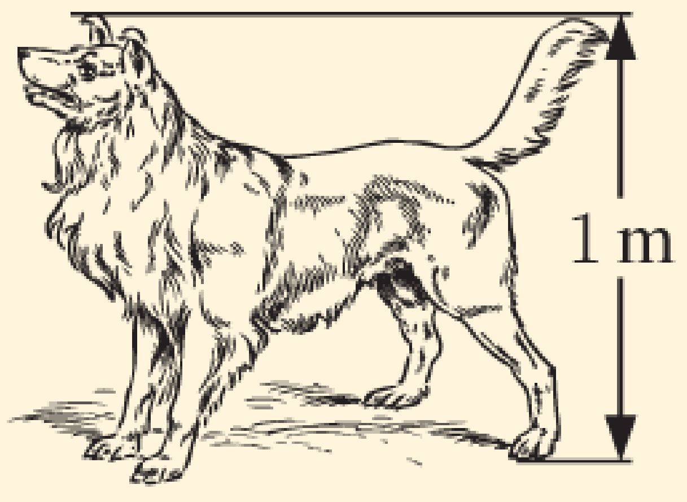
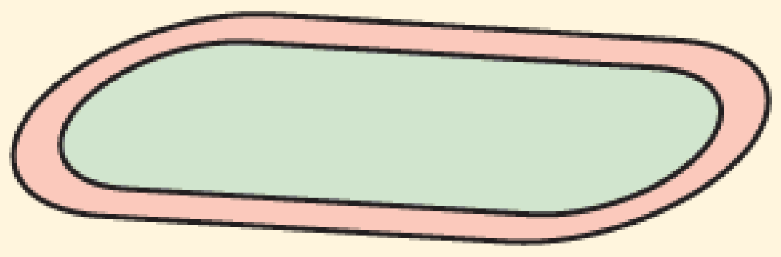
$$1 \text{ millimetre (mm)} = \frac{1}{1000} \text{ m} = 0.001 \text{ m}$$

$$1 \text{ centimetre (cm)} = \frac{1}{100} \text{ m} = 0.01 \text{ m}$$

$$1 \text{ kilometre (km)} = 1000 \text{ m}$$

milli means "thousandth".
centi means "hundredth".
kilo means "thousand".

These objects may help you visualise some of the units of length.

1 mm	the thickness of a coin	
1 cm	the width of a fingernail	
1 m	the height of a large dog	
1 km	$2\frac{1}{2}$ times around an athletics track	

LENGTH CONVERSIONS

When we convert from one unit to a **smaller** unit, there will be more smaller units. We therefore need to **multiply**.

When we convert from one unit to a **larger** unit, there will be less larger units. We therefore need to **divide**.

In the metric system, all of the conversions involve multiplication or division by powers of 10.

$$\begin{tikzcd}
\boxed{\text{km}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{m}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \boxed{\text{cm}} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \boxed{\text{mm}} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

Example 4 **Self Tutor**

Write the following in metres:

a 300 cm **b** 2.1 km **c** 4700 mm

a We are converting from a smaller unit to a larger one, so we divide.

$$\begin{aligned} & 300 \text{ cm} \\ &= (300 \div 100) \text{ m} \\ &= 3 \text{ m} \end{aligned}$$

b We are converting from a larger unit to a smaller one, so we multiply.

$$\begin{aligned} & 2.1 \text{ km} \\ &= (2.1 \times 1000) \text{ m} \\ &= 2100 \text{ m} \end{aligned}$$

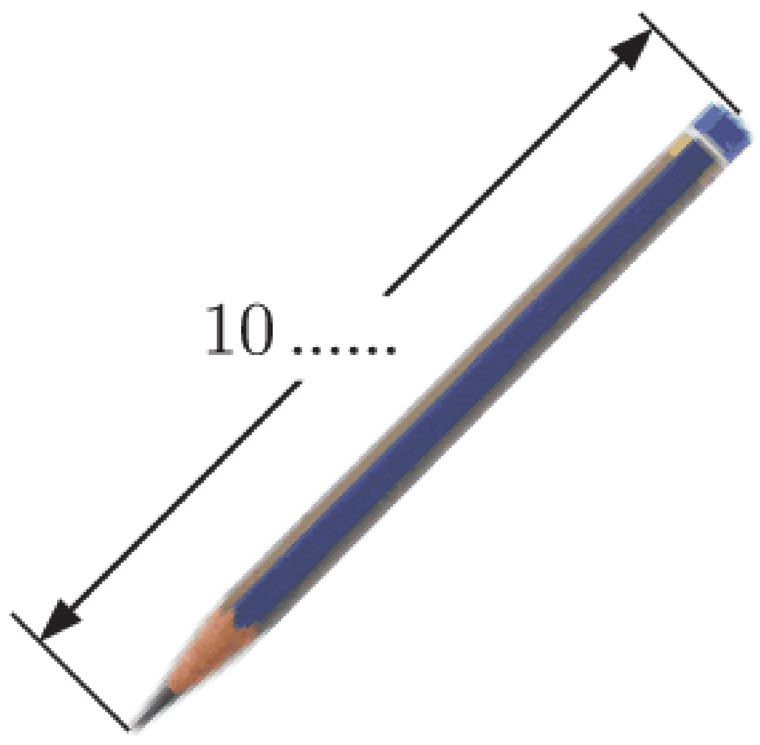
c We are converting from a smaller unit to a larger one, so we divide.

$$\begin{aligned} & 4700 \text{ mm} \\ &= (4700 \div 10) \text{ cm} \\ &= 470 \text{ cm} \\ &= (470 \div 100) \text{ m} \\ &= 4.7 \text{ m} \end{aligned}$$

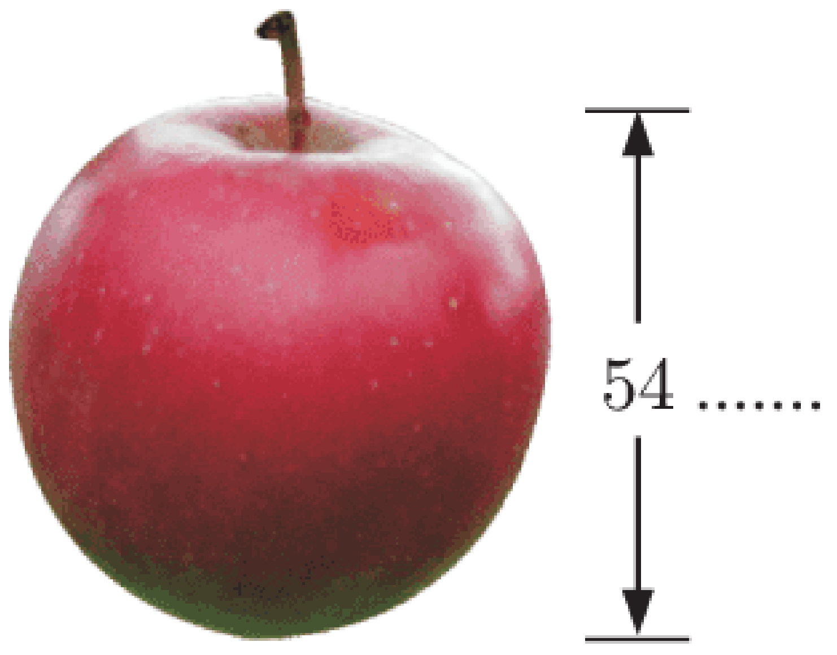
EXERCISE 8B

1 The pictures below are missing the units from their measurements. What should they be?

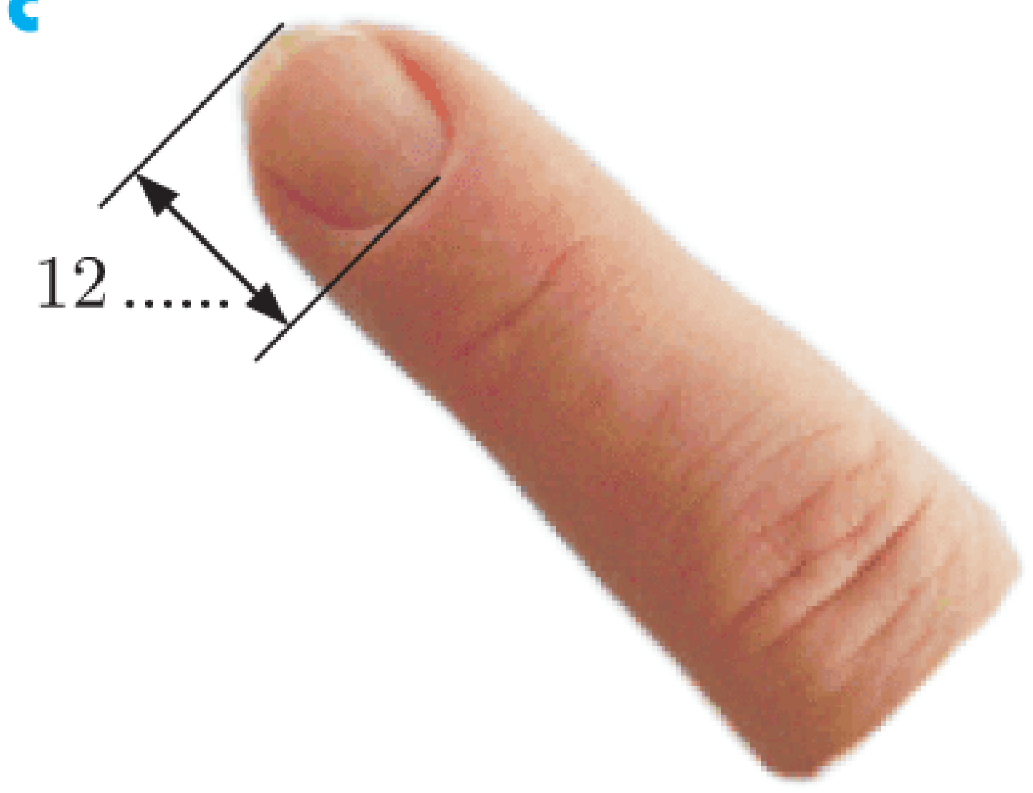
a



b

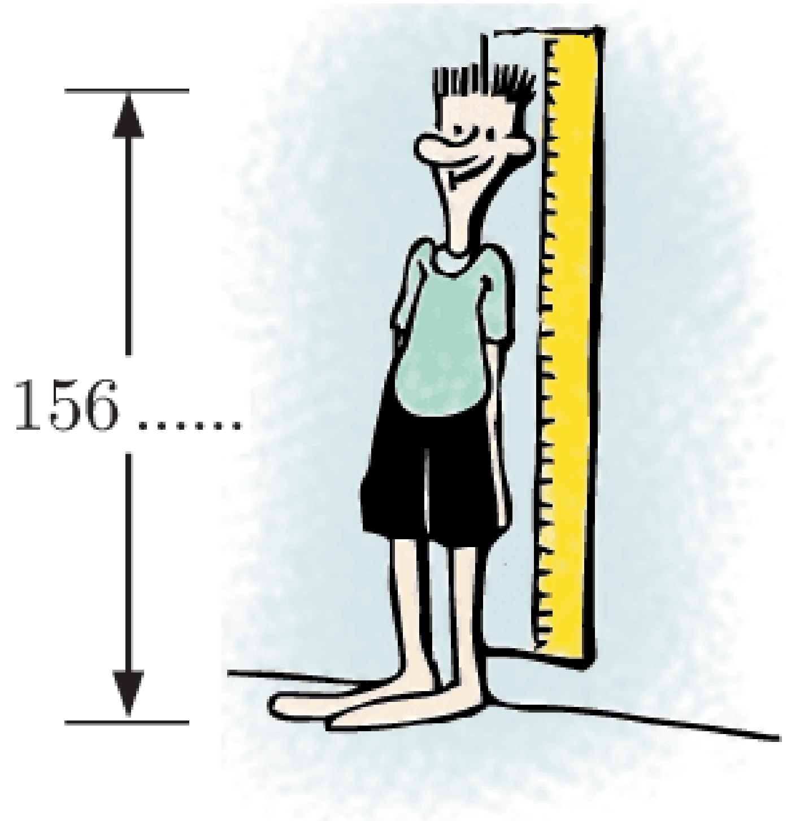


c

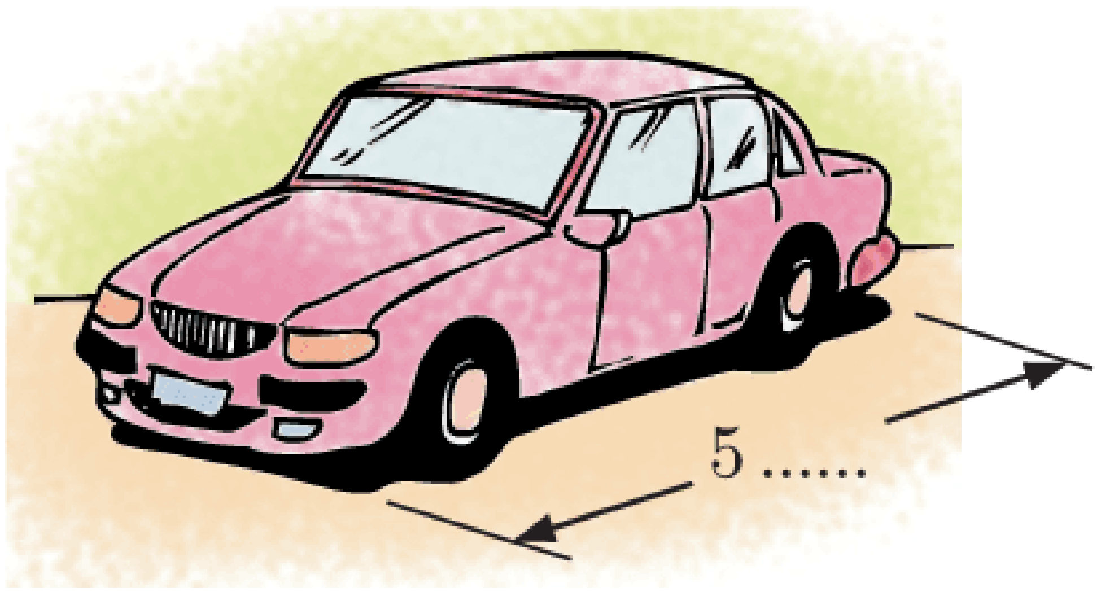


d

e



f



2 Choose the correct answer.

- **a** The length of a fly would be:
  - **A** 7 cm **B** 7 mm **C** 7 m **D** 7 km
- **b** The flight distance from Rome to Moscow would be:
  - **A** 23 800 m **B** 238 cm **C** 23.8 mm **D** 2380 km
- **c** The width of a public swimming pool would be:
  - **A** 200 m **B** 200 cm **C** 20 m **D** 20 km

3 Write in metres:

a 760 cm **b** 400 mm **c** 2 km **d** 25 cm
e 2763 mm **f** 25 000 cm **g** 4.7 km **h** 0.09 km

4 Write in centimetres:

a 550 m **b** 47 mm **c** 1.3 m **d** 435 mm
e 1377 mm **f** 8.7 km **g** 290 m **h** 1.96 km

5 Write in millimetres:

a 2.5 cm **b** 1.83 m **c** 49 cm **d** 0.92 m

6 Write in kilometres:

a 3371 m **b** 21 901 m **c** 267 000 cm **d** 38 800 cm

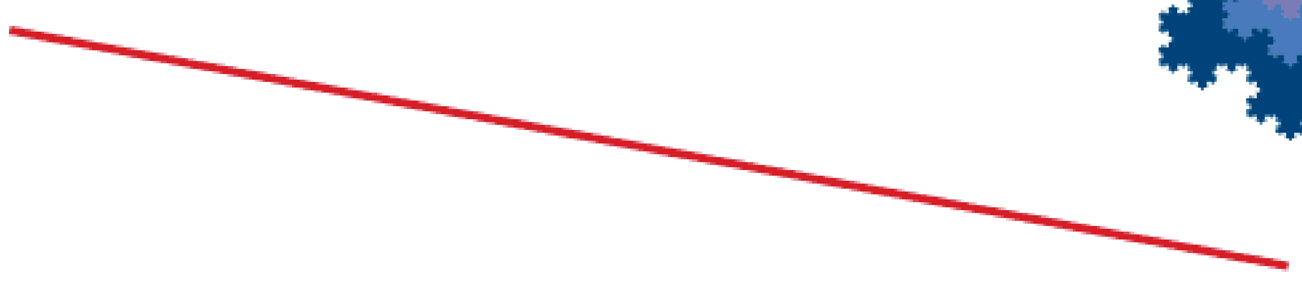
7 Use a ruler to find the total length of each line. Give your answer in:

i centimetres **ii** millimetres.

PRINTABLE DIAGRAMS

a

b



c

d

8 Peter's long jump distance was measured as 2.265 metres. Write the length of his jump in centimetres.

9 Jodie measured the length of a big black ant as 2.35 cm. Write the length of the ant in millimetres.

Example 5 **Self Tutor**

By writing each length in metres, find the sum of these lengths: 2 km, 240 m, 57 cm.

$$\begin{aligned} 2\text{ km} + 240\text{ m} + 57\text{ cm} &= 2000\text{ m} + 240\text{ m} + 0.57\text{ m} \\ &= 2240.57\text{ m} \end{aligned}$$

10 By writing each length in metres, find the sum of these lengths:

a 3 km, 580 m

b 1 km, 38 m

c 5 km, 674 m, 22 cm

d 48 m, 91 cm, 6 mm

11 To get to school, Katrina walked 600 m to the train station, then travelled 7 km by train, then travelled 4.1 km by bus. Find the total distance travelled by Katrina in:

a metres

b kilometres.

12 Write the following lengths in order of size from shortest to longest:

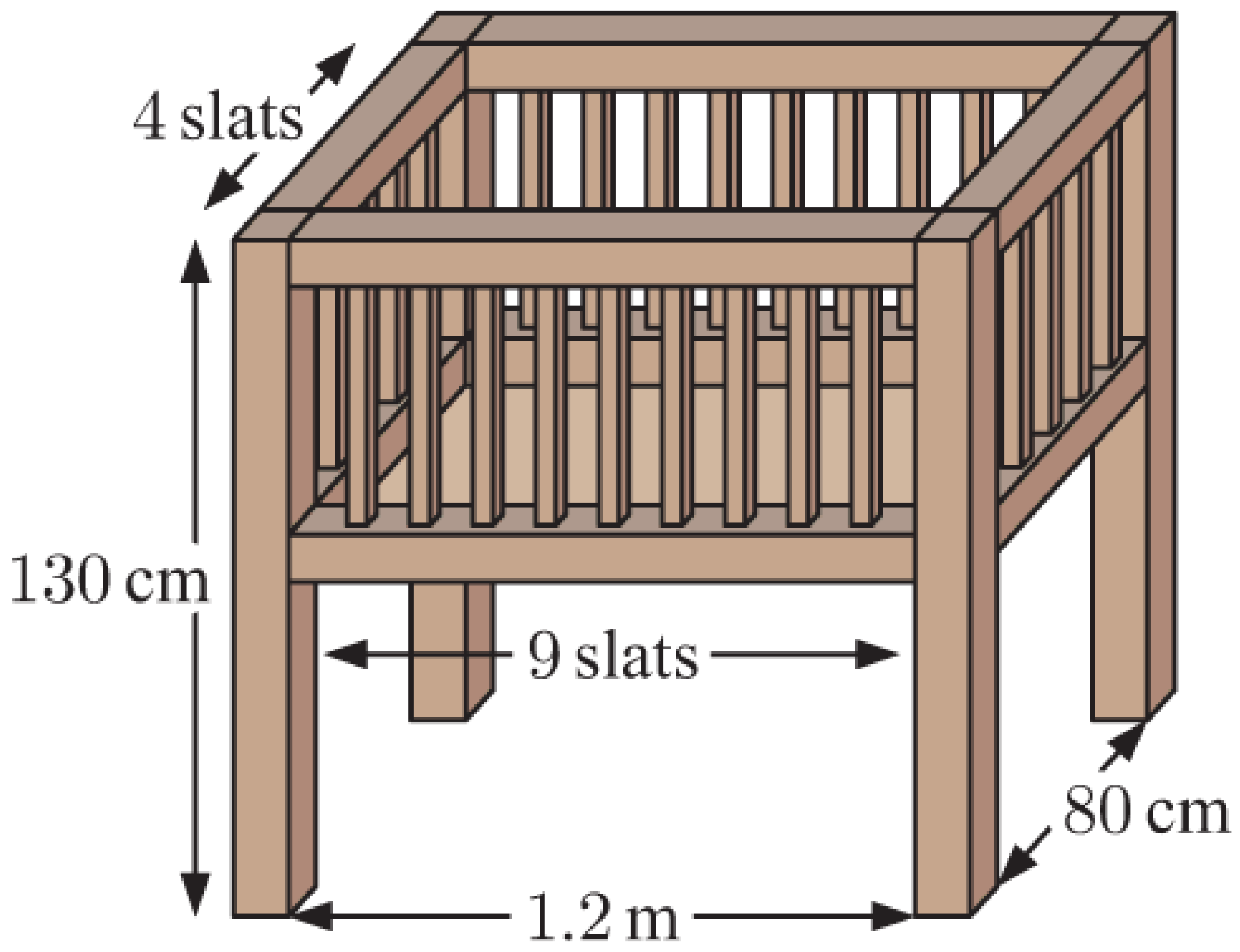
0.13 m, 7.8 cm, 95 mm, 29.2 cm.

13 Joe is building this wooden cot frame for his baby's nursery.

a **i** Find the total number of slats.

ii The slats are 60 cm long. Find the total length of timber required for the slats.

b Find the total length of timber required for the posts and horizontal beams.



ACTIVITY 2 STEP ESTIMATION

Some of us are short and others are tall. When we walk, our step lengths vary from one person to another.

By knowing your step length, you can estimate long distances with reasonable accuracy.

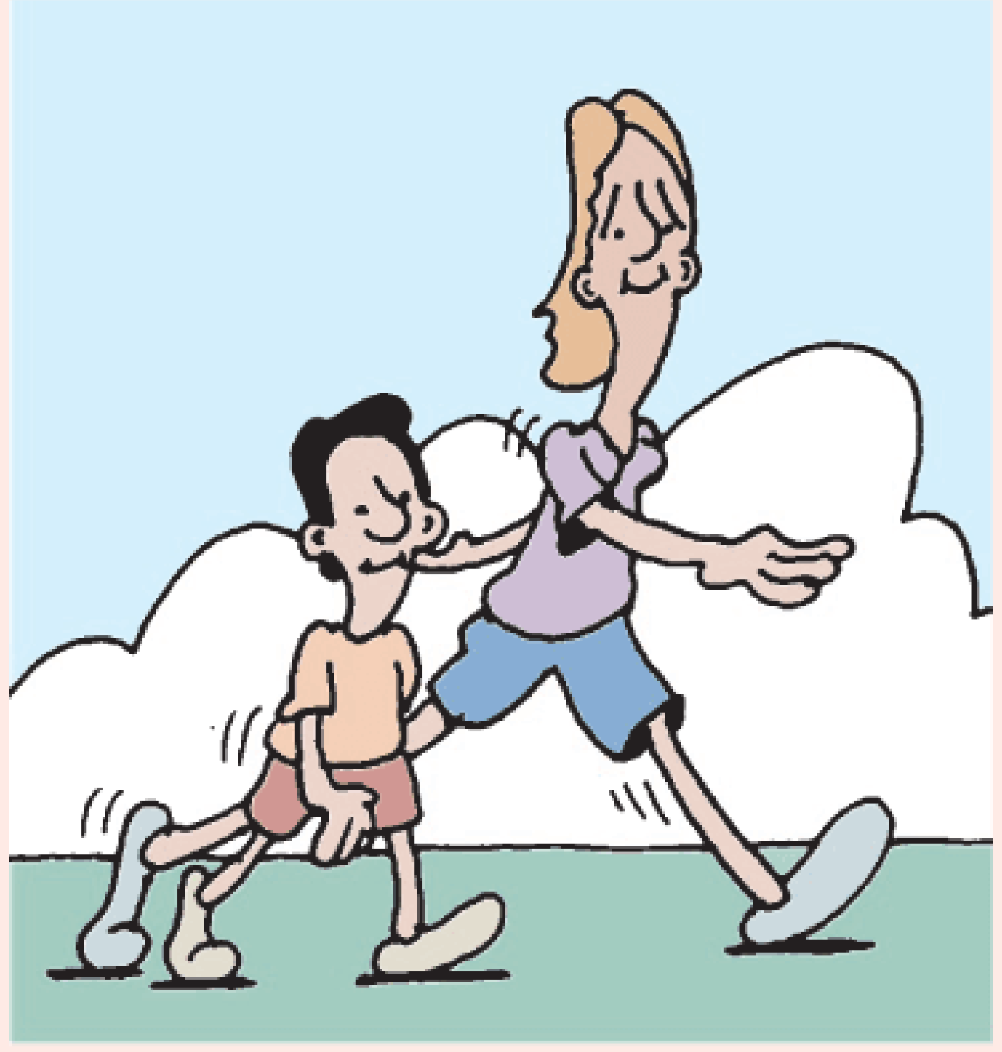
Consider Seani's experiment:

Seani set up two flags which she measured to be 100 metres apart. Using her usual walking step, she took $128\frac{1}{2}$ steps to walk between them.

$100 \text{ m} \div 128.5$ is about 0.78 m, so Seani calculated that her usual step length is about 0.78 m.

When Seani walked to school, she took 2186 steps.

Since $2186 \times 0.78 \approx 1705$, she estimated the distance to school to be 1705 metres.



What to do:

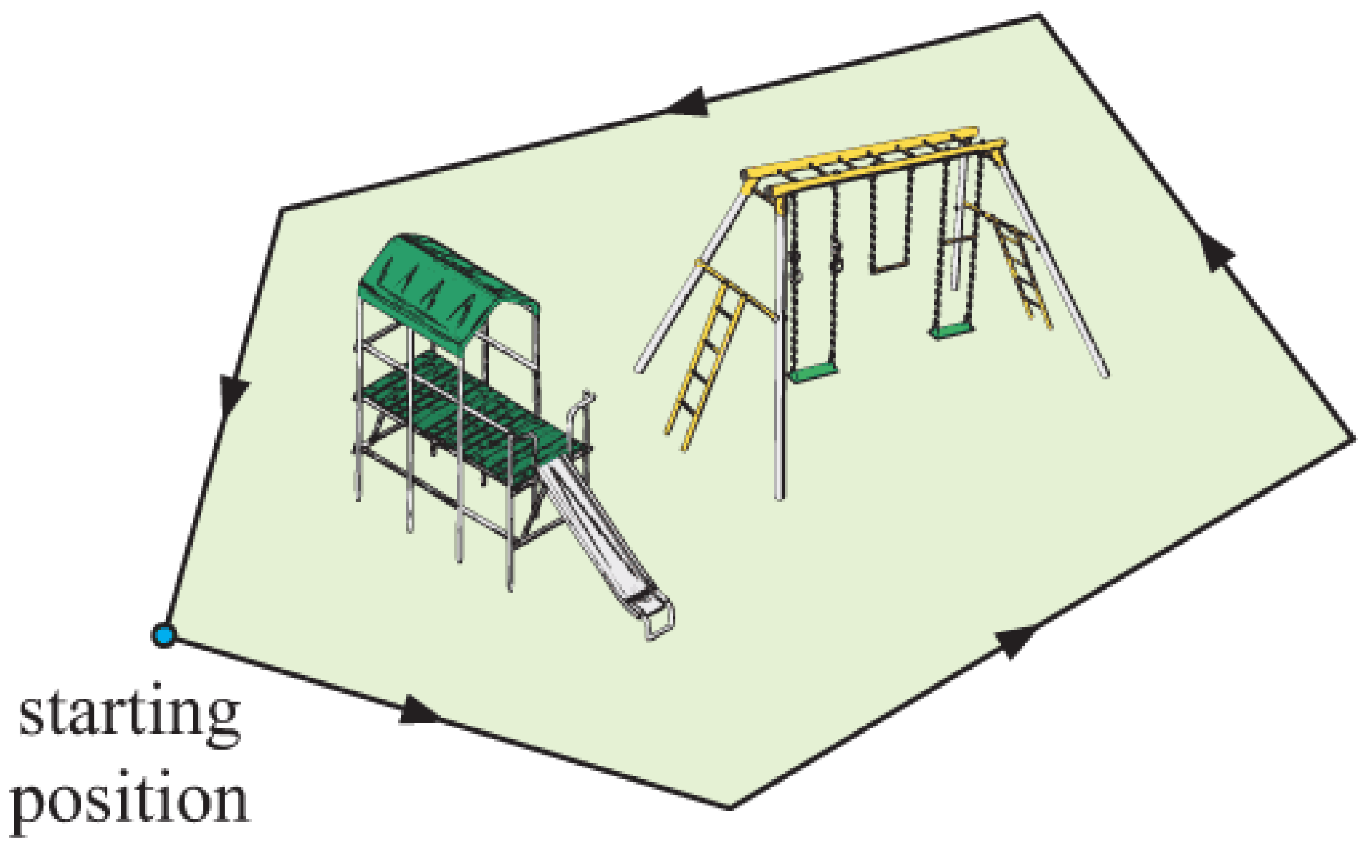
1. Use a long tape measure or trundle wheel to help place two flags exactly 100 metres apart.
2. Walk with your usual step from one flag to the other. Count the steps you take.
3. Using Seani's method, calculate your usual step length. Round your answer to 2 decimal places.
4. Choose *three* suitable distances around the school to estimate. Use Seani's method to estimate them.
5. Compare your estimates with other students. You could organise a competition to find the best distance estimator in your class.

C PERIMETER

The **perimeter** of a closed figure is a measurement of the distance around its boundary.

Suppose you stood at one corner of this playground area, then walked around the boundary until you returned to your starting point. The distance you have walked is the **perimeter** of the playground.

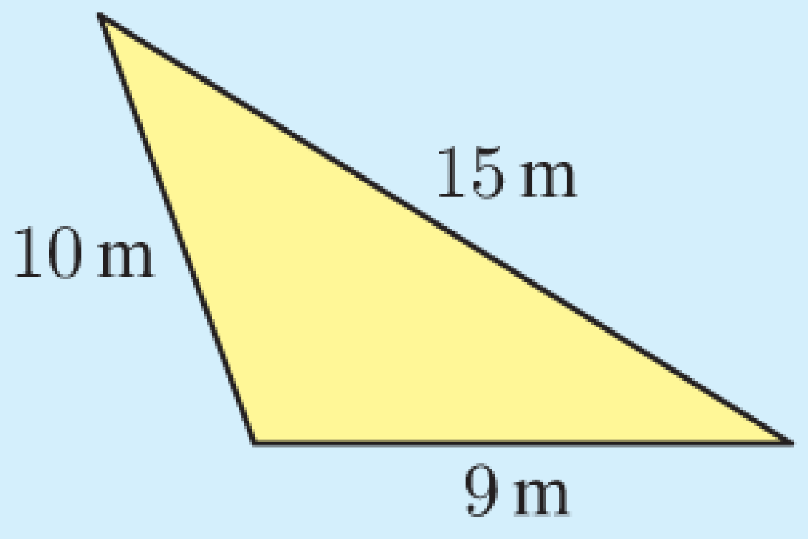
The perimeter of a polygon is found by adding the lengths of its sides.



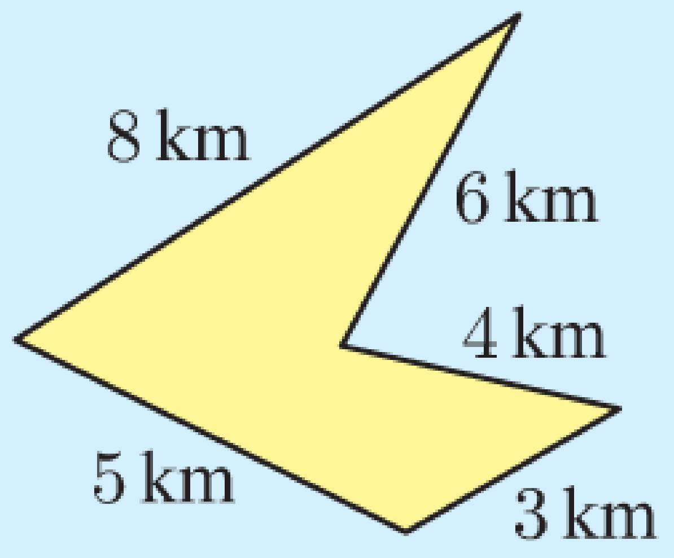
Example 6 **Self Tutor**

Find the perimeter of:

a



b



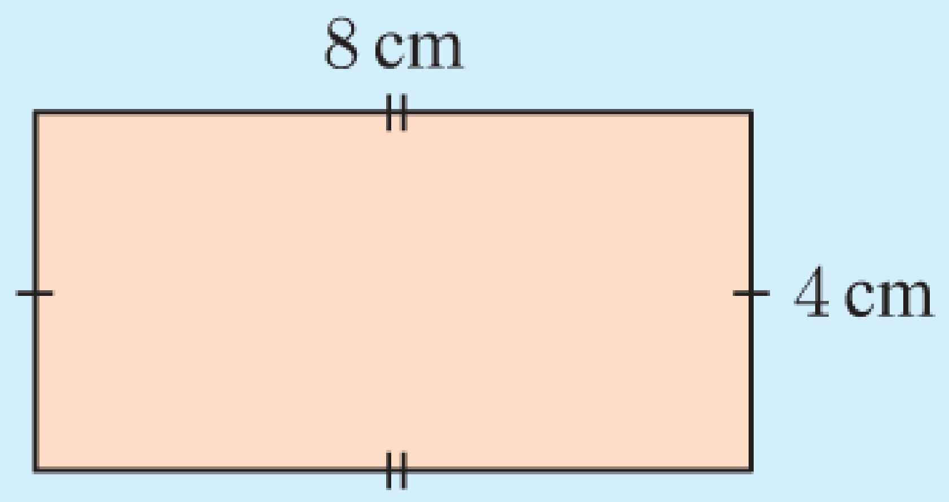
a Perimeter $= 10 + 9 + 15$ m
$= 34$ m

b Perimeter $= 8 + 5 + 3 + 4 + 6$ km
$= 26$ km

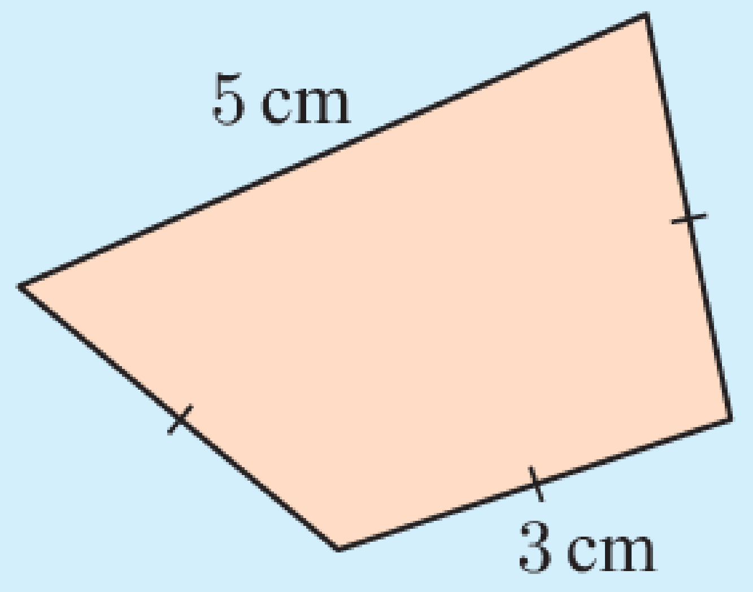
Example 7 **Self Tutor**

Find the perimeter of:

a



b



a Perimeter
$= 2 \times 4 \; + \; 2 \times 8$ cm
$= 8 + 16$ cm
$= 24$ cm

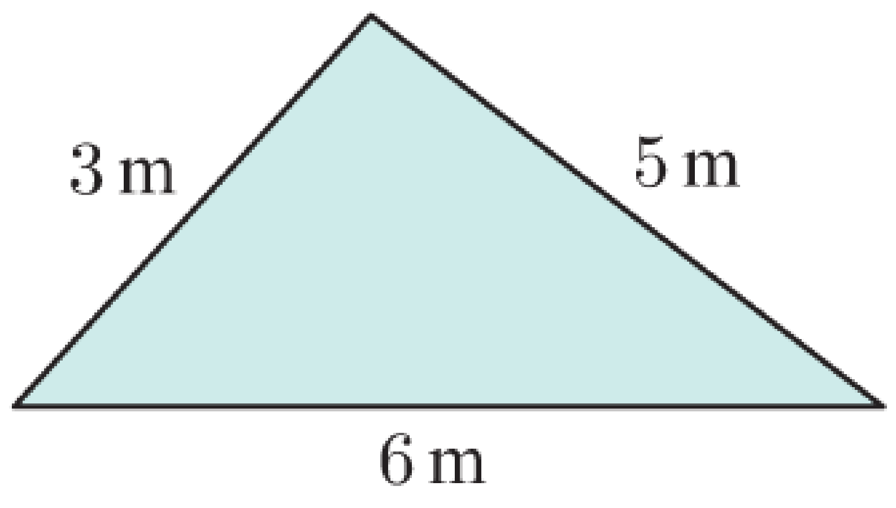
b Perimeter
$= 5 + 3 \times 3$ cm
$= 5 + 9$ cm
$= 14$ cm

The same markings on the sides of a polygon show that those sides have the same length.

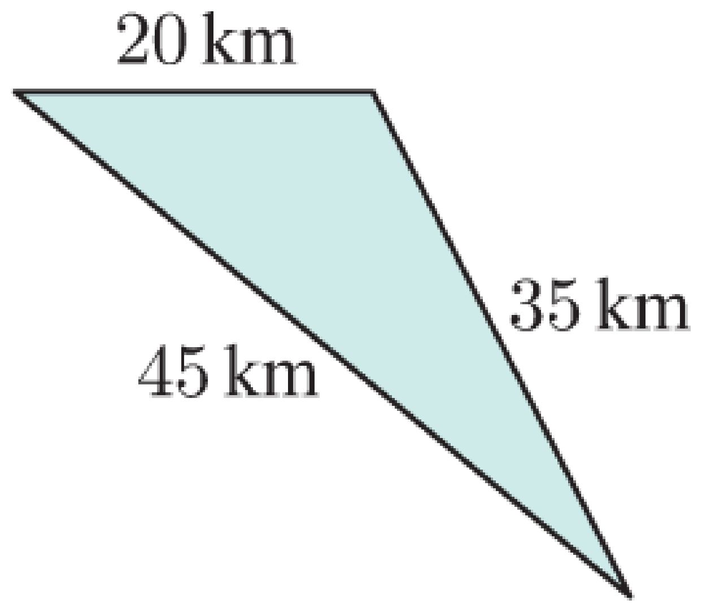
EXERCISE 8C

1 Find the perimeter of:

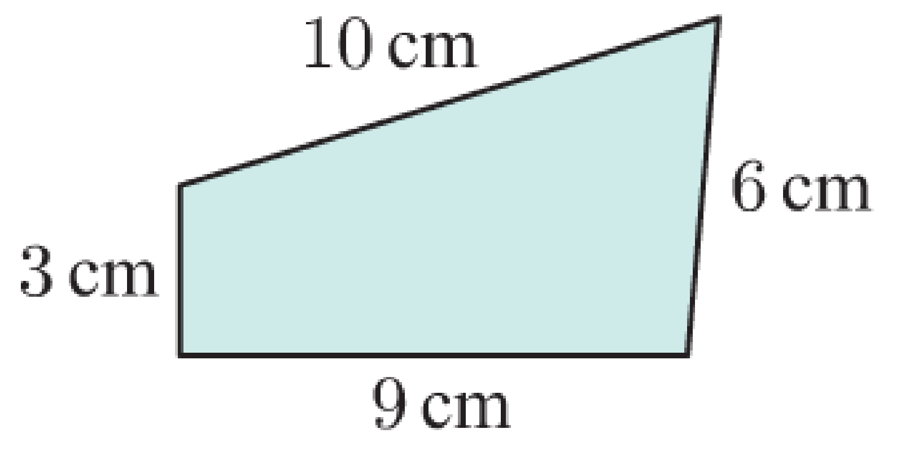
a



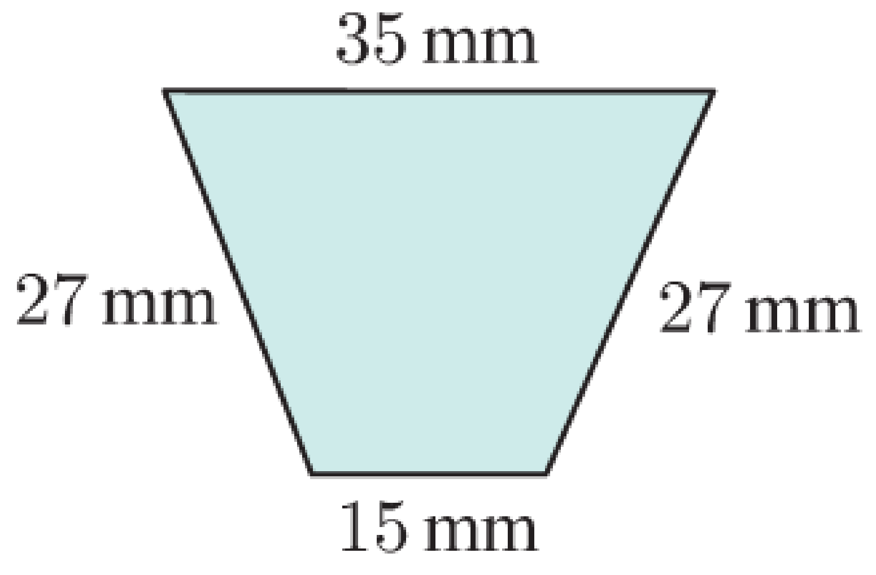
b



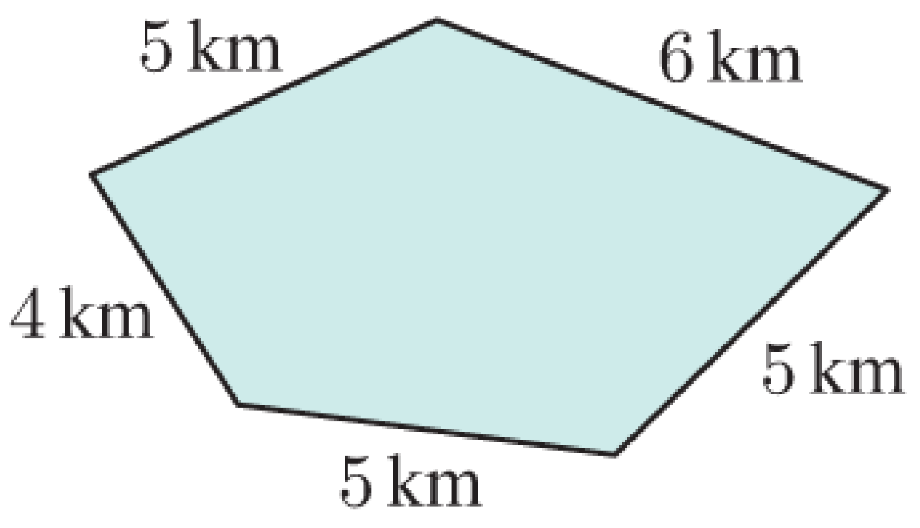
c



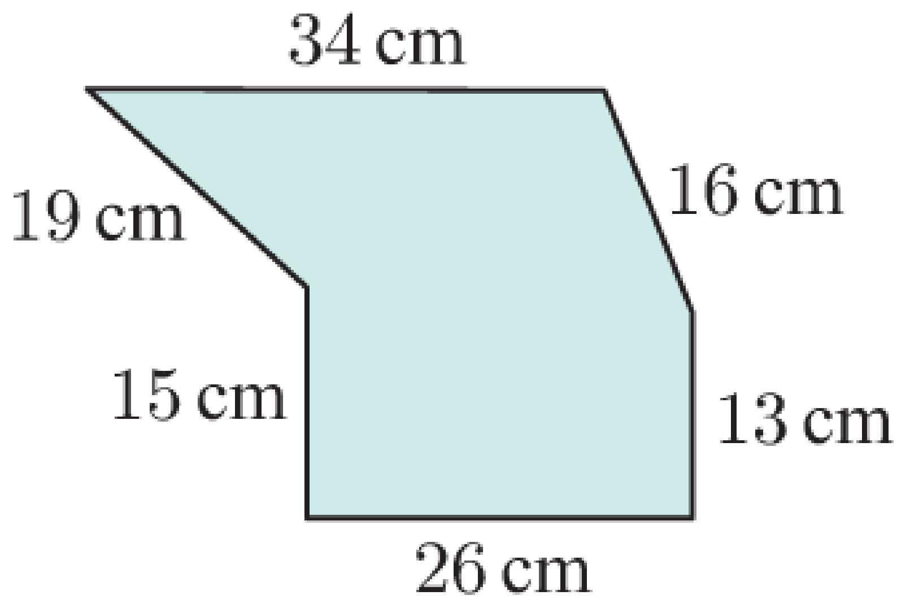
d



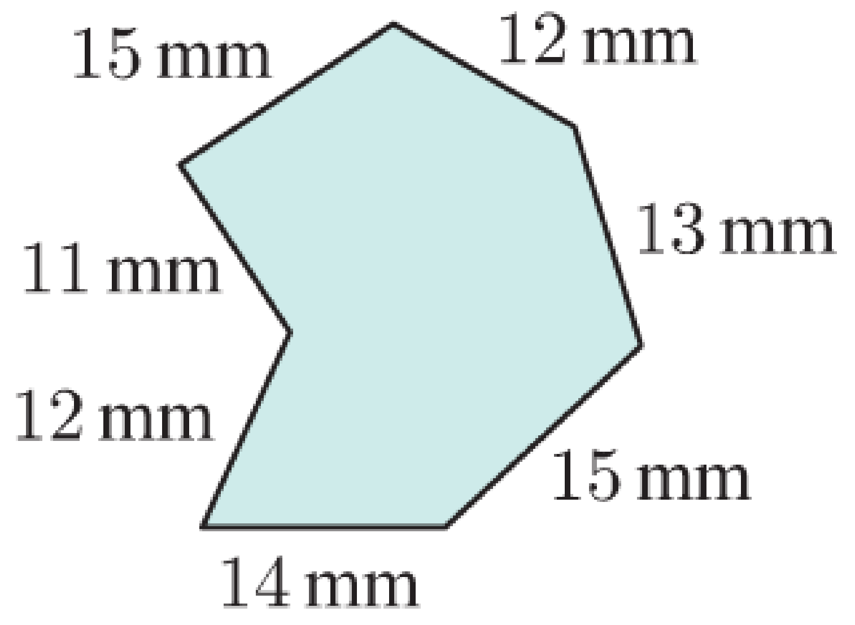
e



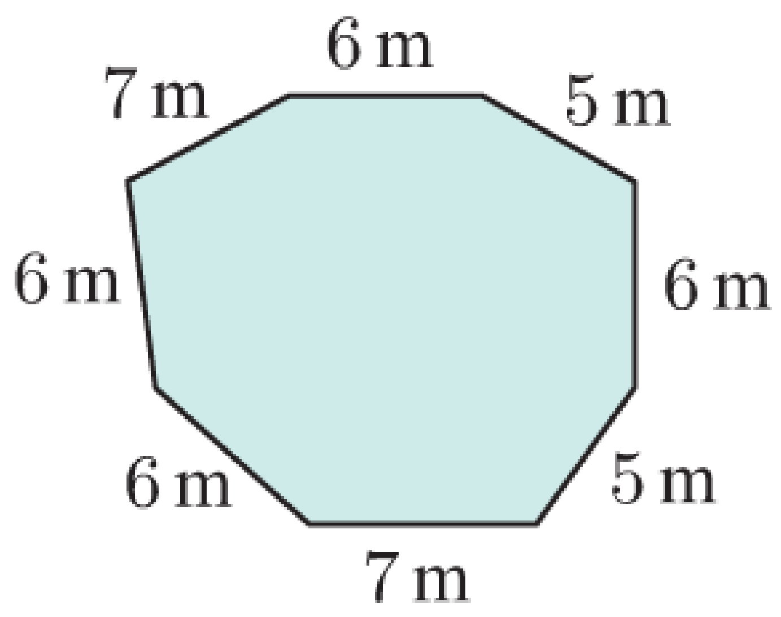
f



g

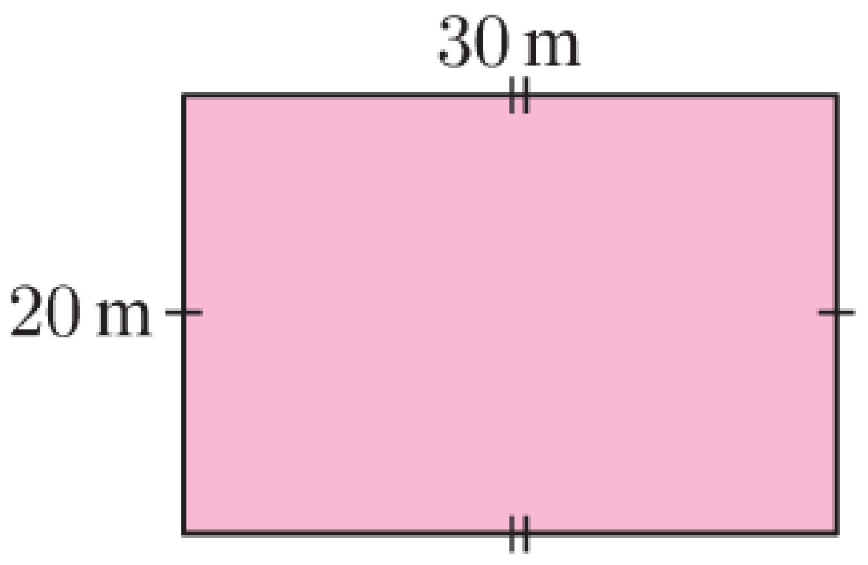


h

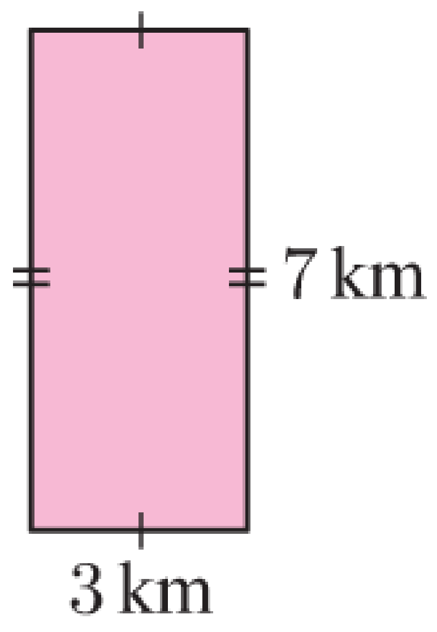


2 Find the perimeter of these rectangles:

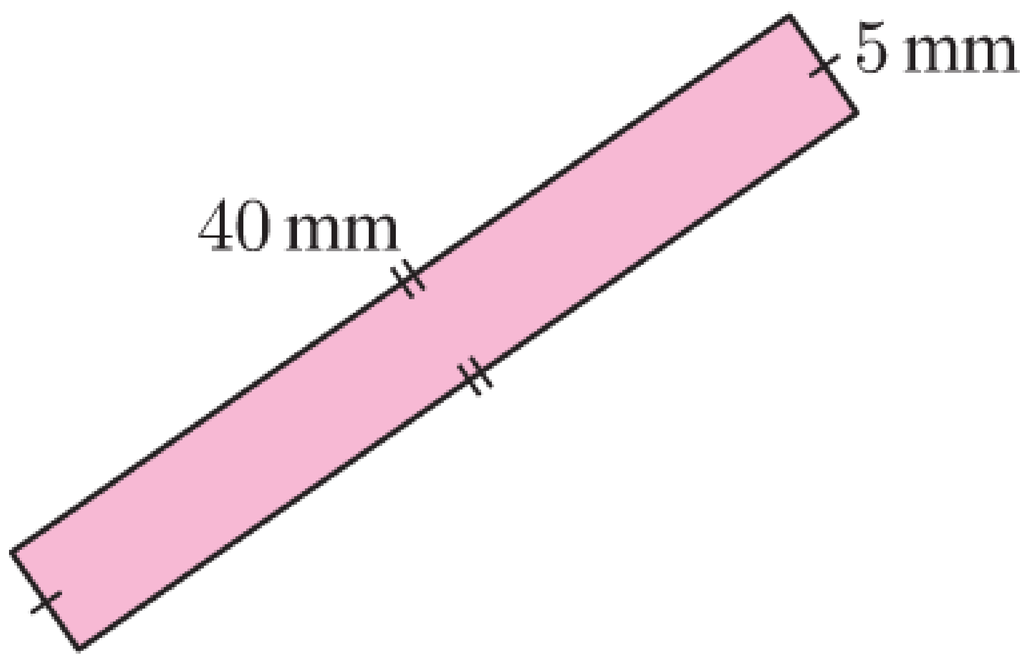
a



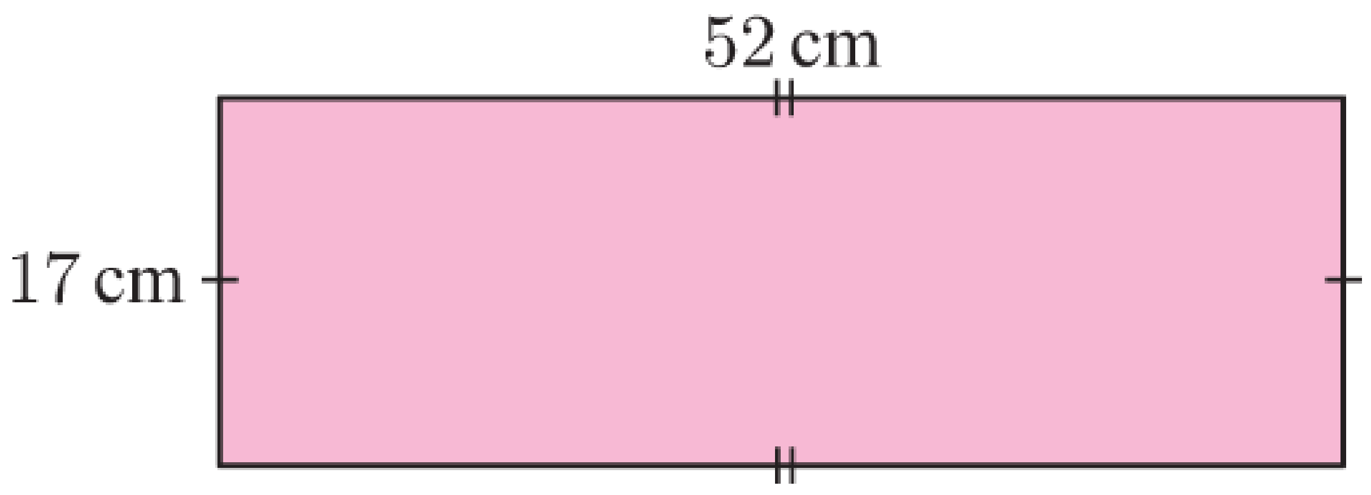
b



c

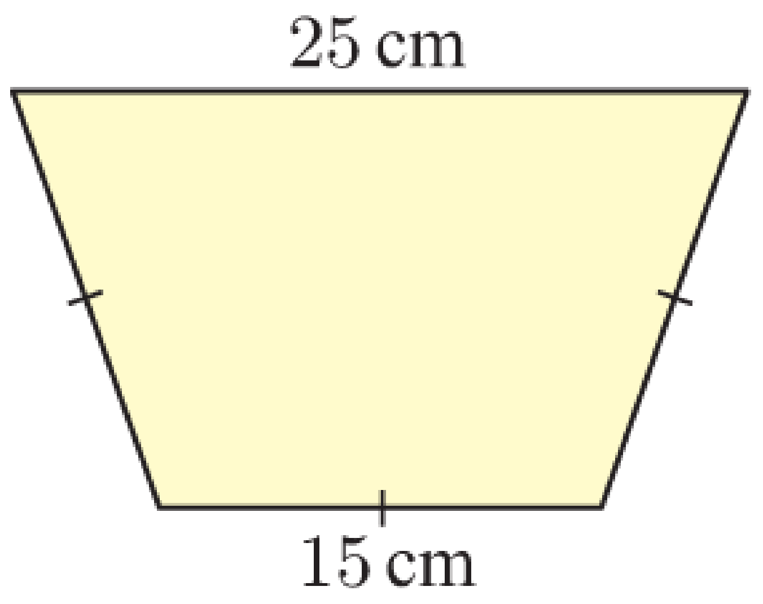


d

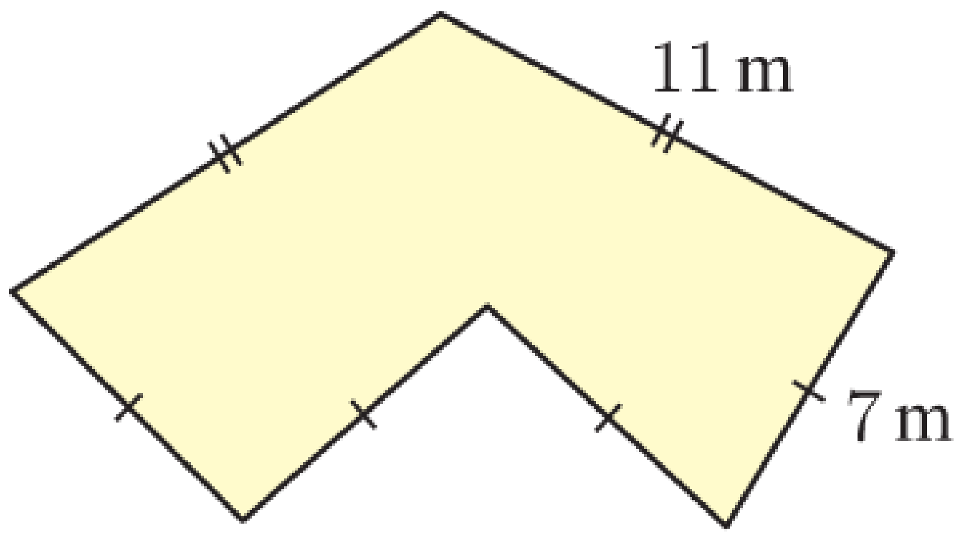


3 Find the perimeter of these figures:

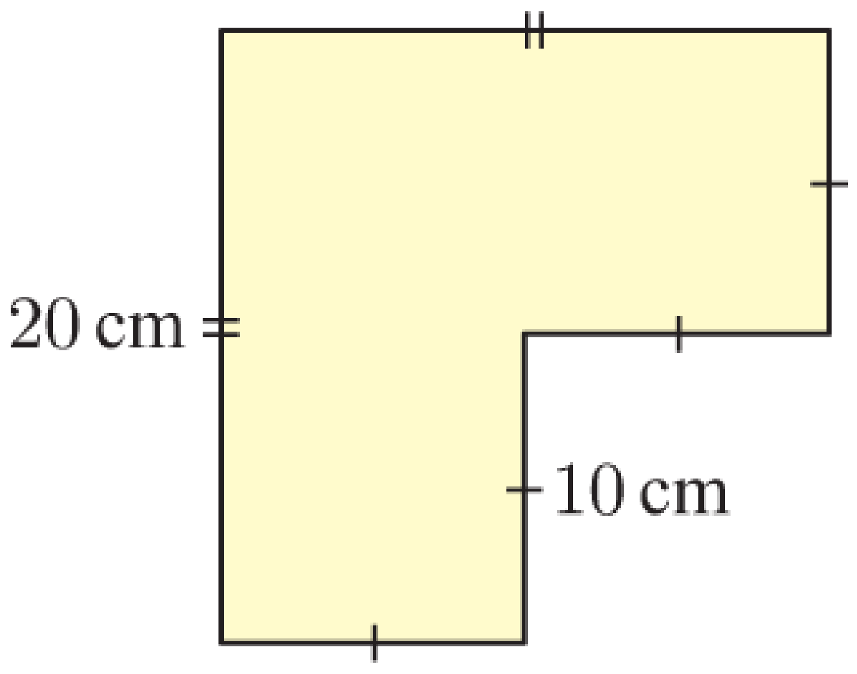
a



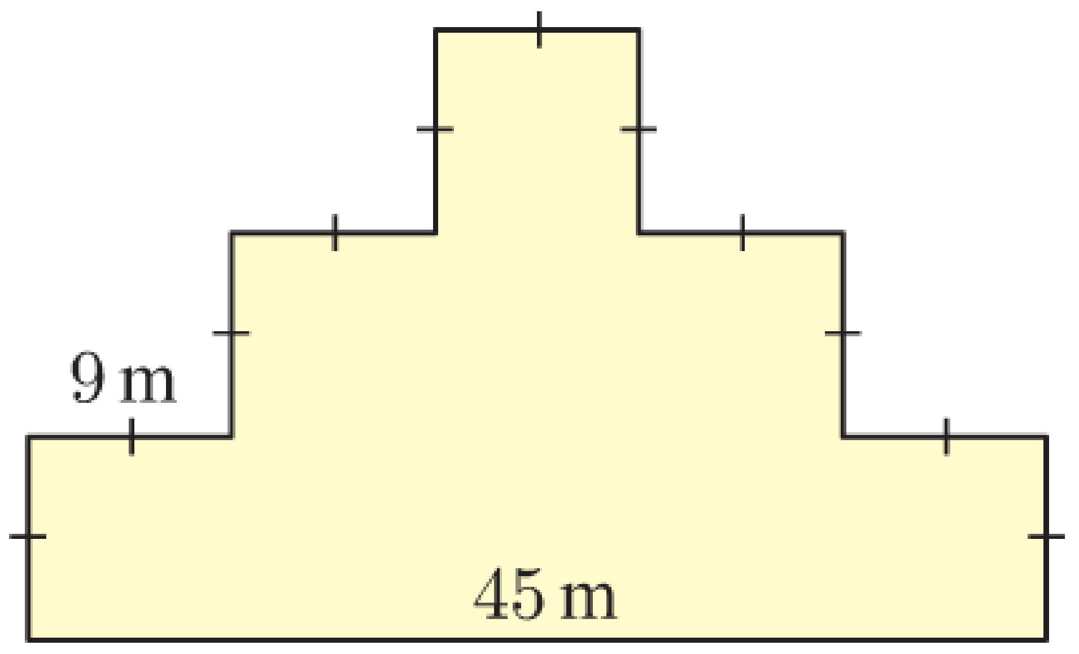
b



c

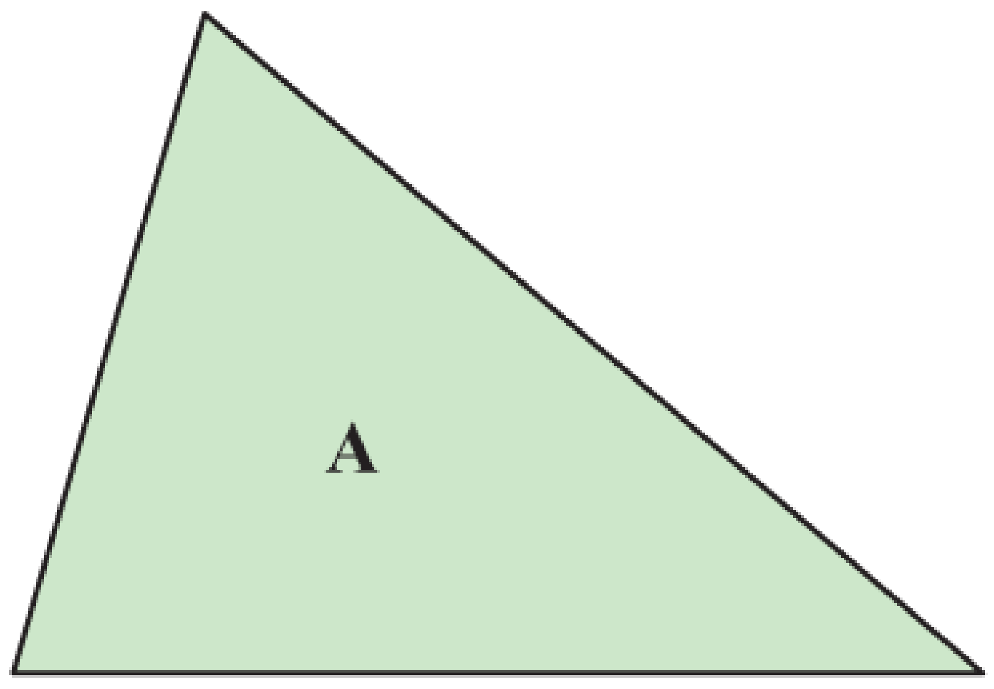


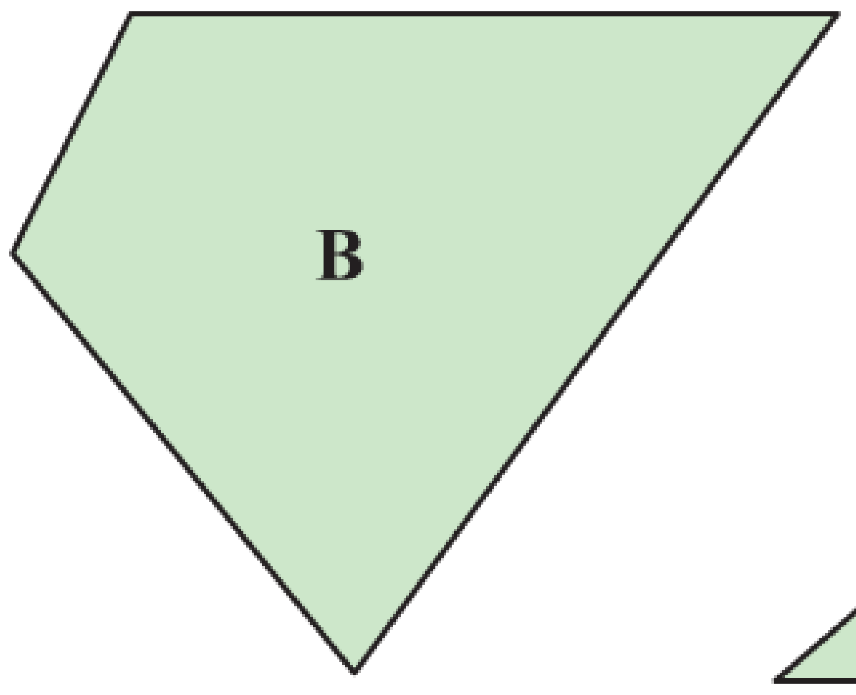
d

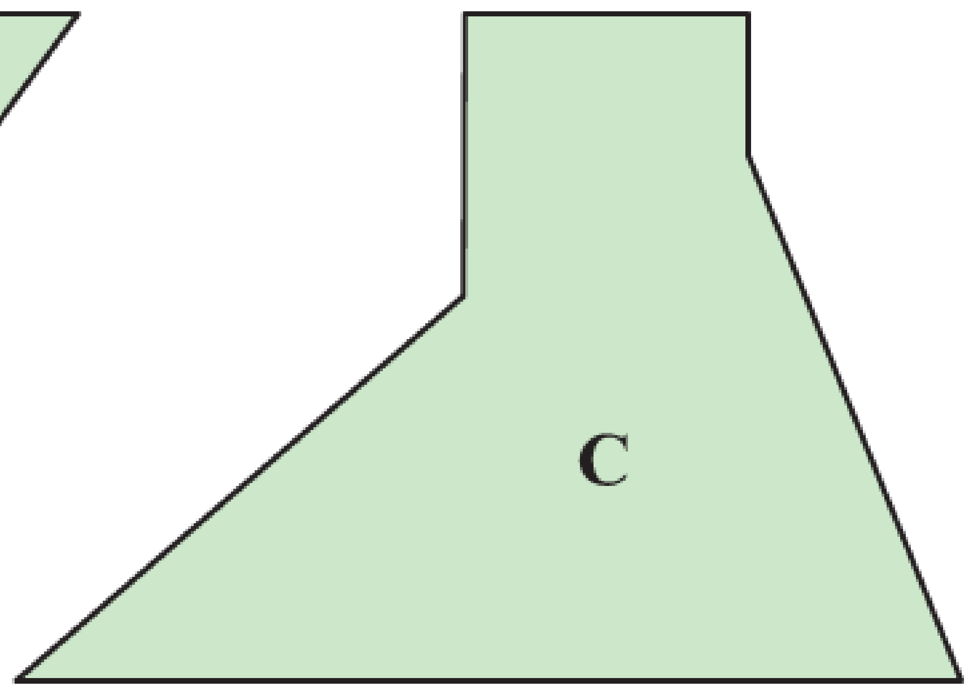


4 **a** Use a ruler to find the perimeter of each figure:

PRINTABLE DIAGRAMS





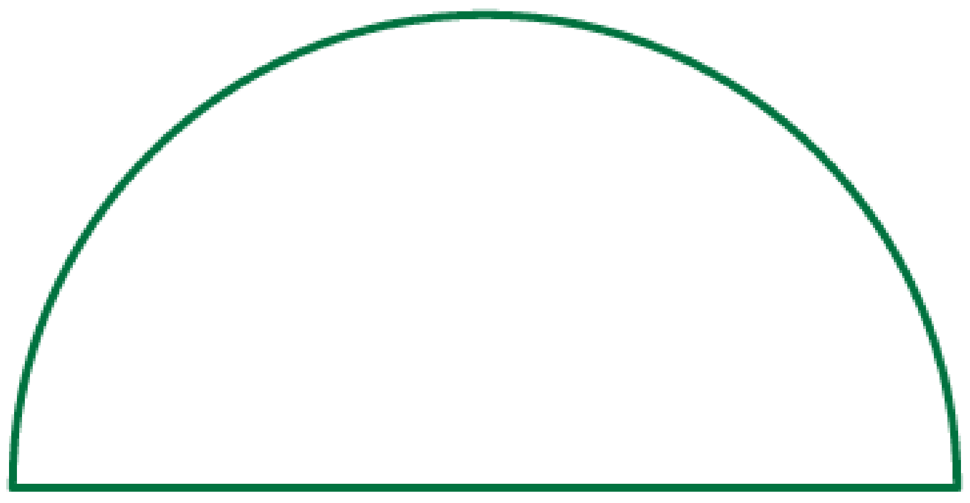


b Which of the figures has the longest perimeter?

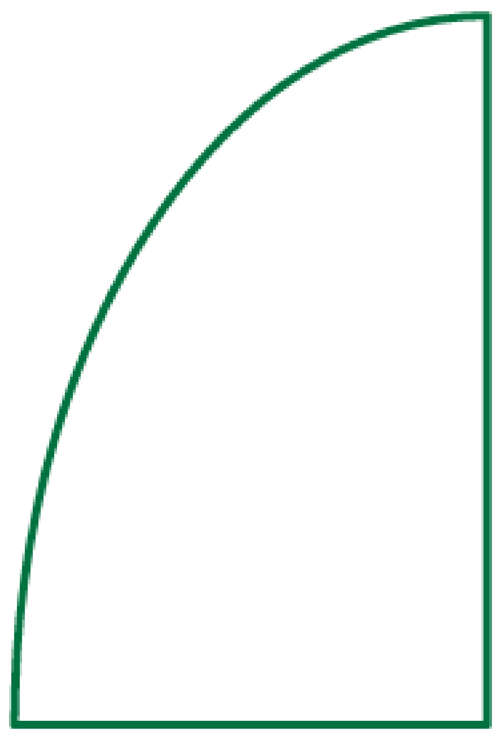
5 Use a piece of string to estimate, as accurately as possible, the perimeter of:

VIDEO DEMO

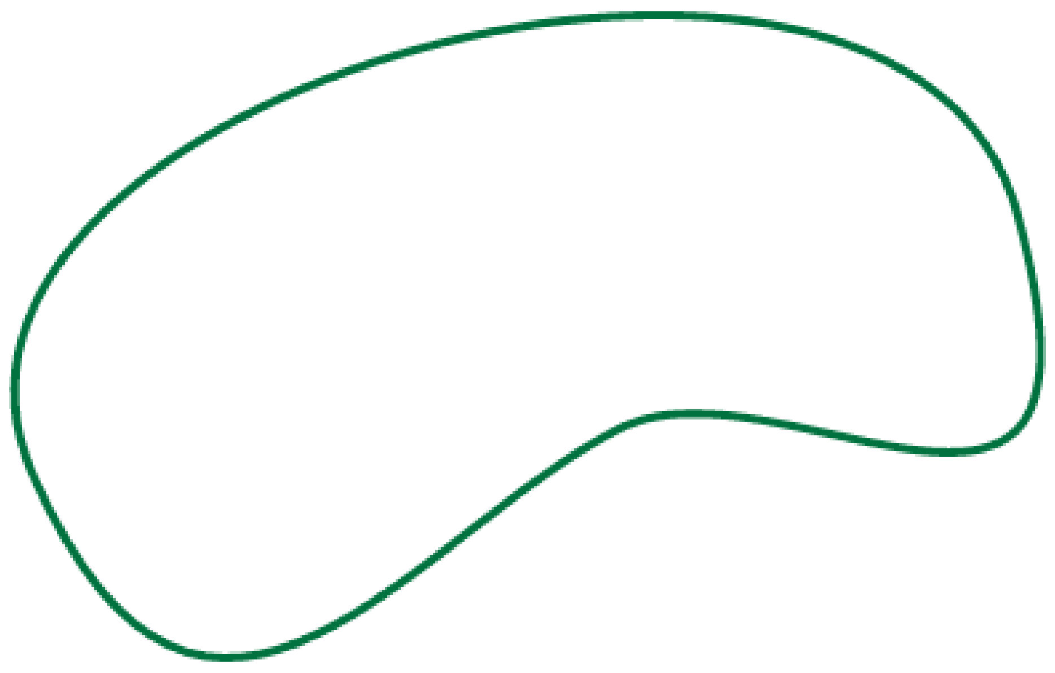
a



b



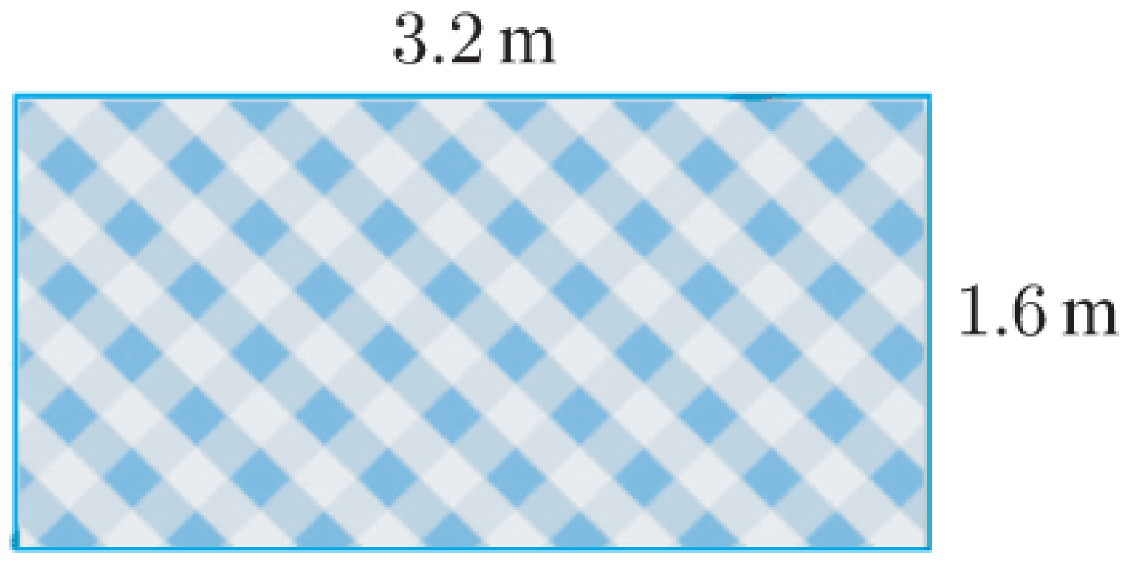
c



6 Find the length of the fence around a rectangular paddock which is 120 m by 260 m.

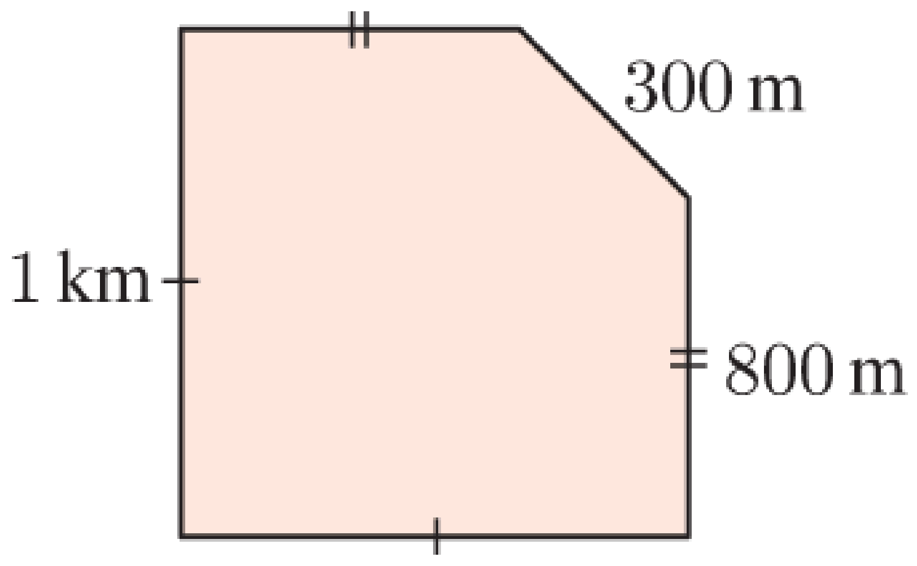
7 Martine has a rectangular tablecloth with the dimensions shown. She wants to sew lace trimming along its border.

- **a** Find the length of the lace required.
- **b** If the lace costs €4.65 per metre, find the total cost of the lace Martine needs.



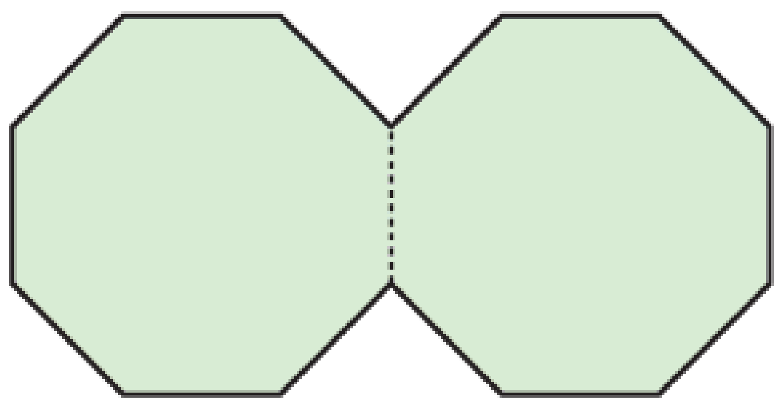
8 The jogging track in a city park is shaped as shown.

- **a** If I run one lap of the track, how far do I run?
- **b** Find the total distance that I would run in 5 laps.



9 An equilateral triangle has perimeter 27 cm. Find the length of each side of the triangle.

10 When two identical regular octagons are joined exactly on one side, the perimeter of the figure obtained is 98 cm. If the octagons are separated, what is the perimeter of each?



ACTIVITY 3 FINDING PERIMETERS

You will need: Tape measure or metre rule, trundle wheel.

What to do:

1 Make a copy of the table below.

PRINTABLE TABLE

Perimeter of:	*Unit of measure*	*Estimate*	*Actual*
front cover of this textbook			
a bank note of your choosing			
your desk			
largest window in your classroom			
playing card			
your school gymnasium			

2 Record the unit you will use to measure the perimeter of each item.

3 Estimate the perimeter of each item.

4 Measure each perimeter carefully, and record it in the table.

D SCALE DIAGRAMS

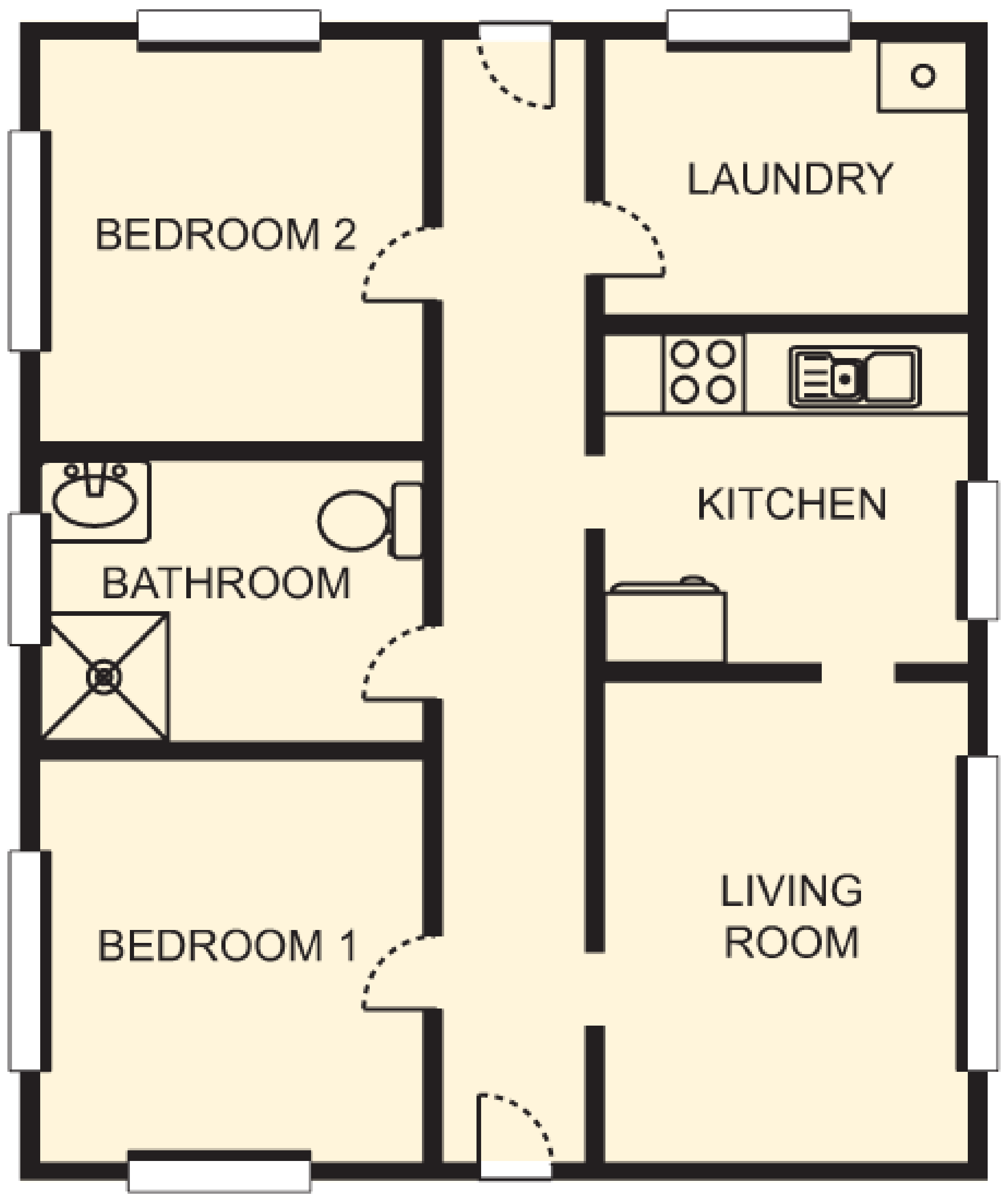
A **scale diagram** is a drawing which is either smaller or larger than the original, but with all sizes in the correct proportion.

It will have a **scale** which shows the connection between the lengths on the diagram compared with those for the real object.

The house plan shown is a scale diagram.

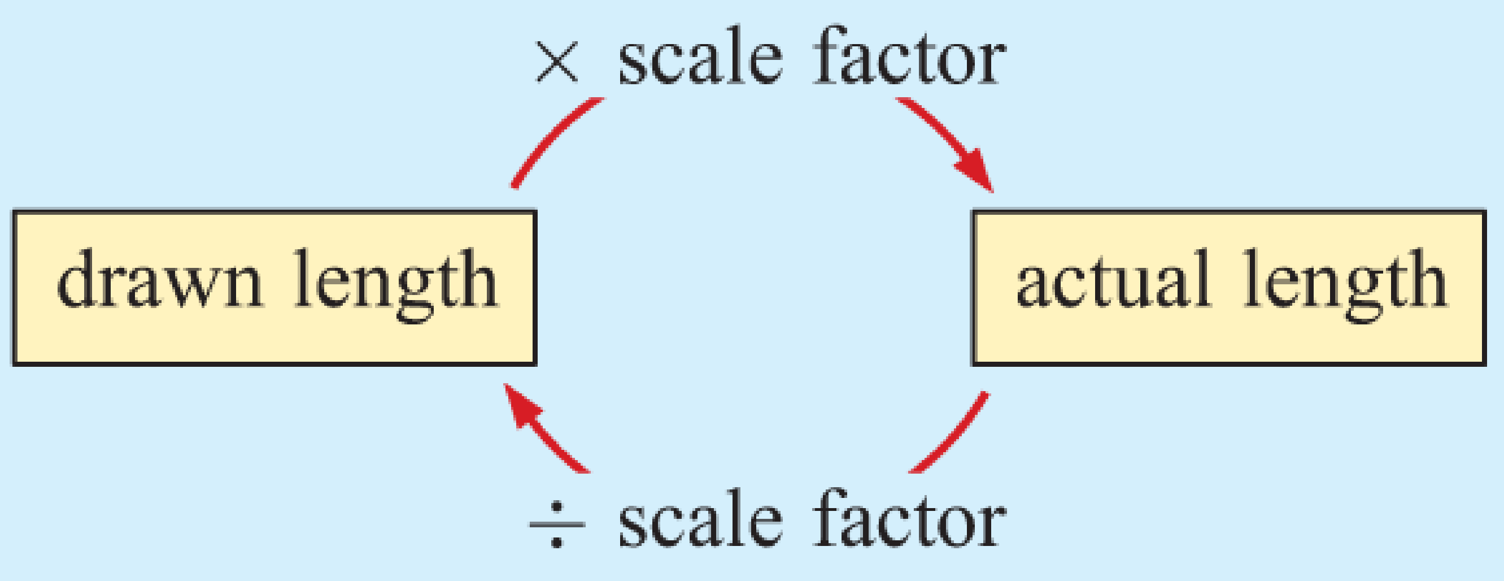
Its scale of "1 represents 200" means that lengths on the scale diagram are 200 times longer in reality. We say that the **scale factor** is 200.

This house plan is 5 cm wide. This tells us that the actual width of the house is $5 \times 200 = 1000$ cm, or 10 m.



Scale: 1 represents 200

- To find the actual length, we **multiply** the drawn length by the scale factor.
- To find the drawn length, we **divide** the actual length by the scale factor.



Example 8 **Self Tutor**

On a scale diagram, the scale is "1 represents 20". Find:

a the actual length if the drawn length is 3.4 cm

b the drawn length if the actual length is 2.4 m.

a actual length
$= $ drawn length $\times$ scale factor
$= 3.4 \text{ cm} \times 20$
$= 68 \text{ cm}$

b drawn length
$= $ actual length $\div$ scale factor
$= 2.4 \text{ m} \div 20$
$= 0.12 \text{ m}$
$= (0.12 \times 100) \text{ cm}$
$= 12 \text{ cm}$

EXERCISE 8D

1 The scale on a diagram is "1 represents 200".

a Find the actual length if the drawn length is:
i 3 cm **ii** 7 cm **iii** 8.2 cm **iv** 0.8 cm

b Find the drawn length if the actual length is:
i 200 m **ii** 18 m **iii** 5.6 m **iv** 12.2 m

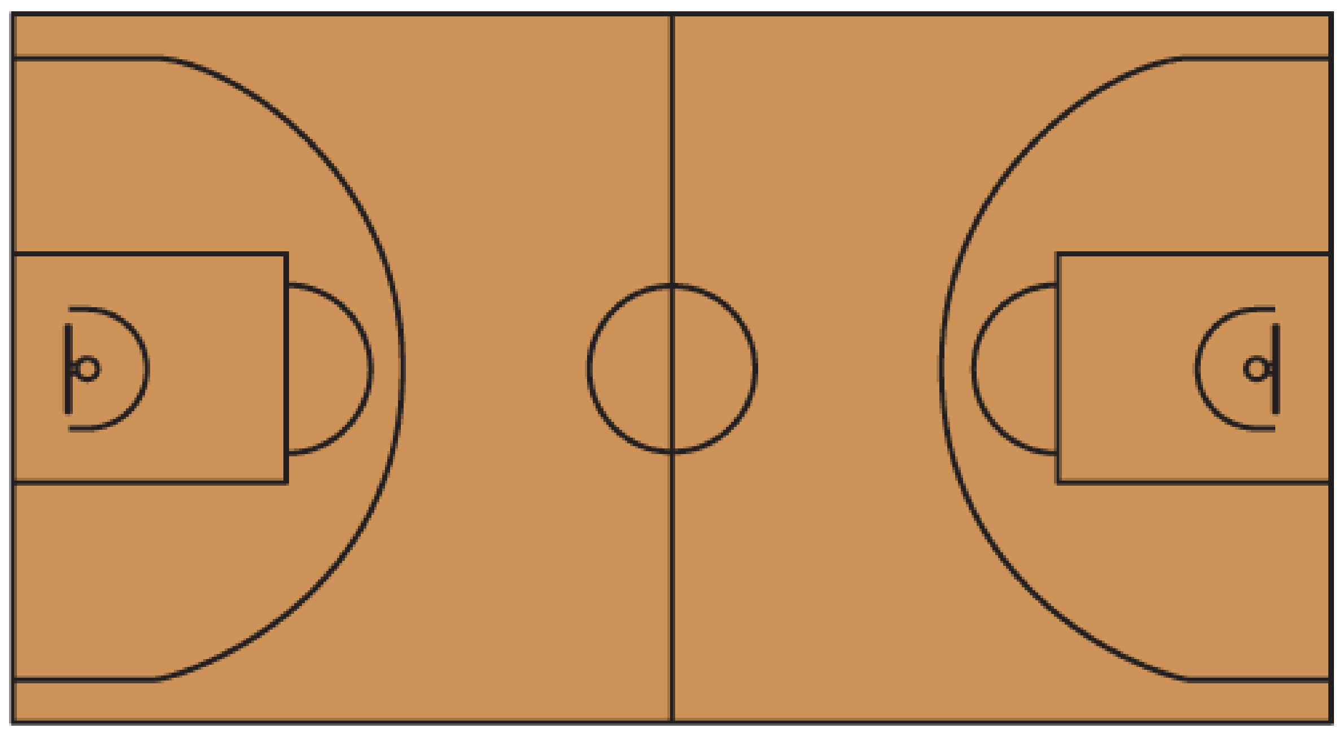
2 The scale on a diagram is "1 represents 5000".

a Find the actual length if the drawn length is:
i 4 cm **ii** 5.8 cm **iii** 2.4 cm **iv** 12.6 cm

b Find the drawn length if the actual length is:
i 500 m **ii** 175 m **iii** 20 m **iv** 108 m

3 Match each diagram with the correct scale:

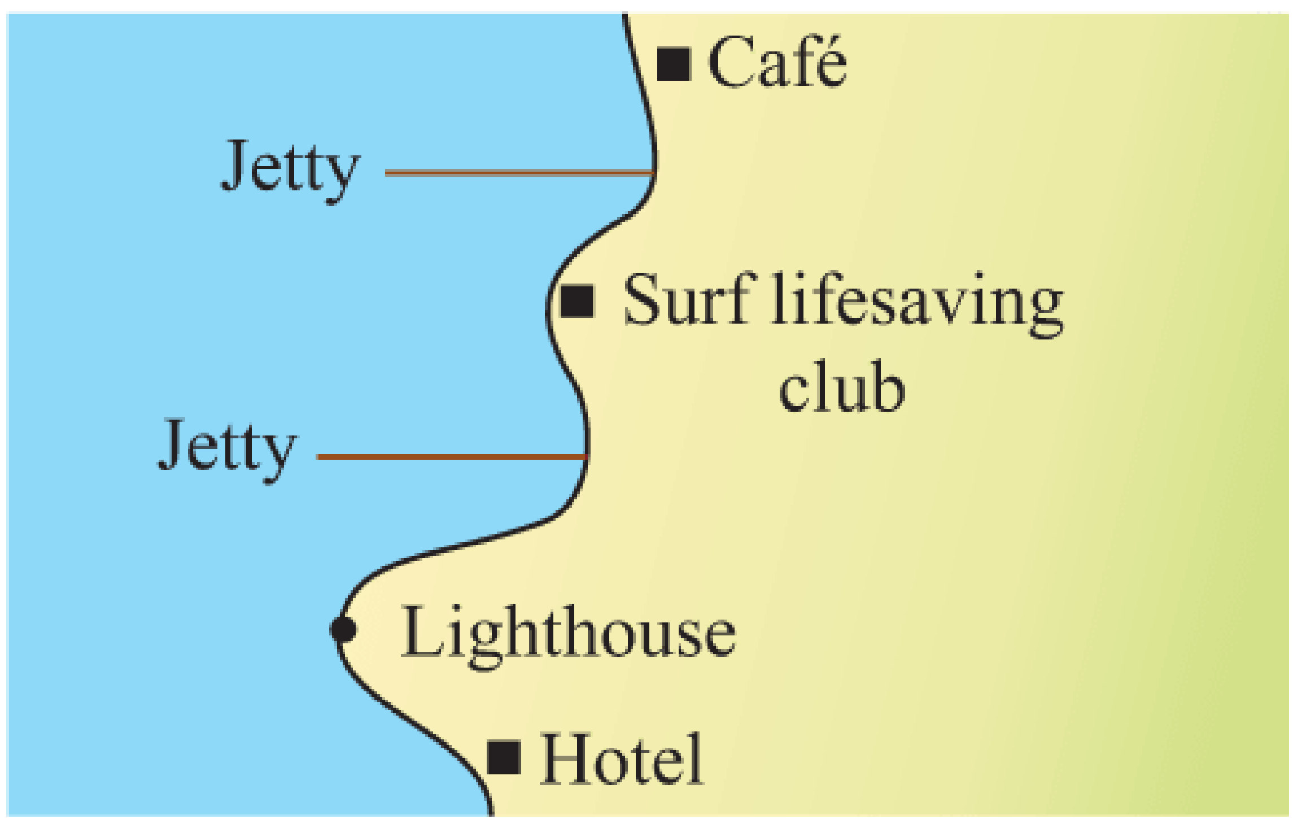
a



b

© OpenStreetMap contributors

c

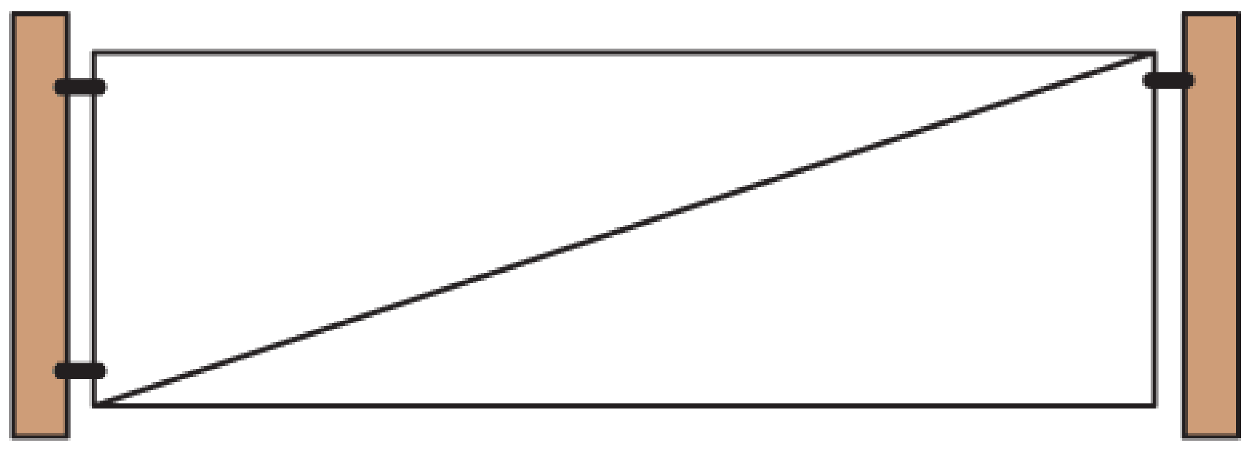


d

A "1 represents 30"
B "1 represents 400"
C "1 represents 125 000"
D "1 represents 30 000 000"

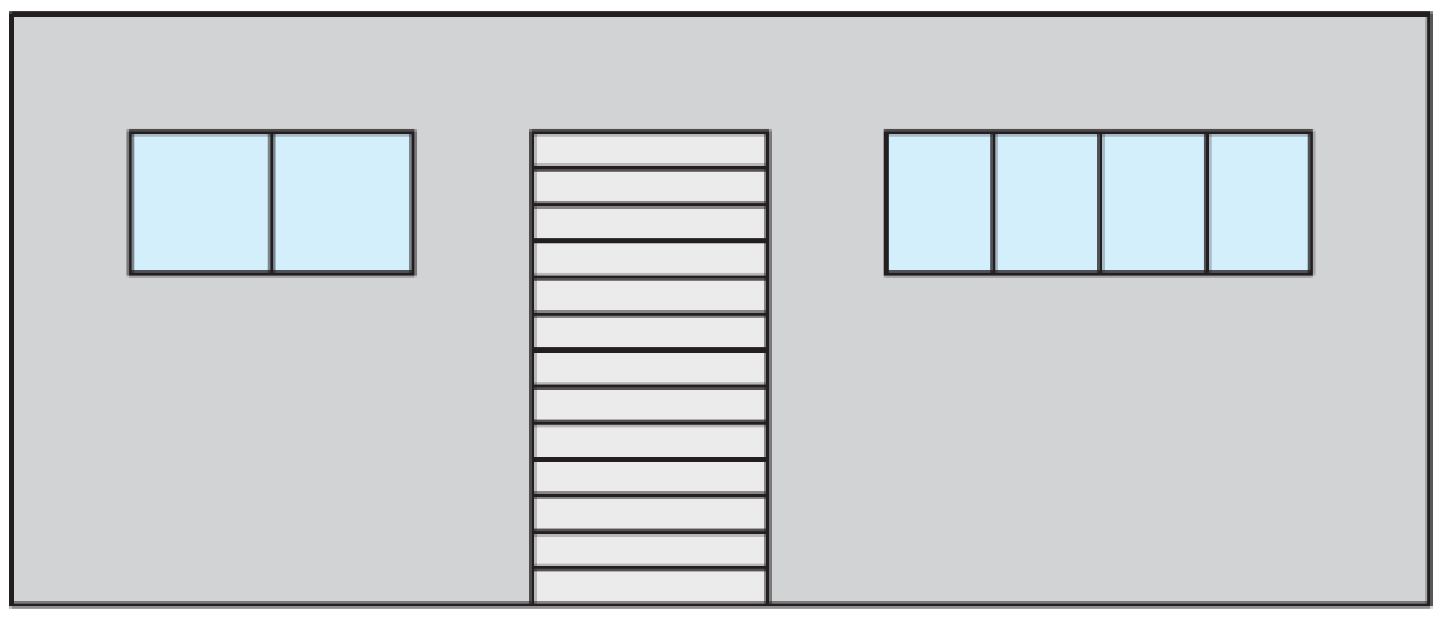
4 The drawing of a gate alongside has the scale “1 represents 100”. Find the actual:

- **a** width of the gate
- **b** height of the gate
- **c** length of the diagonal support.



PRINTABLE DIAGRAMS

5

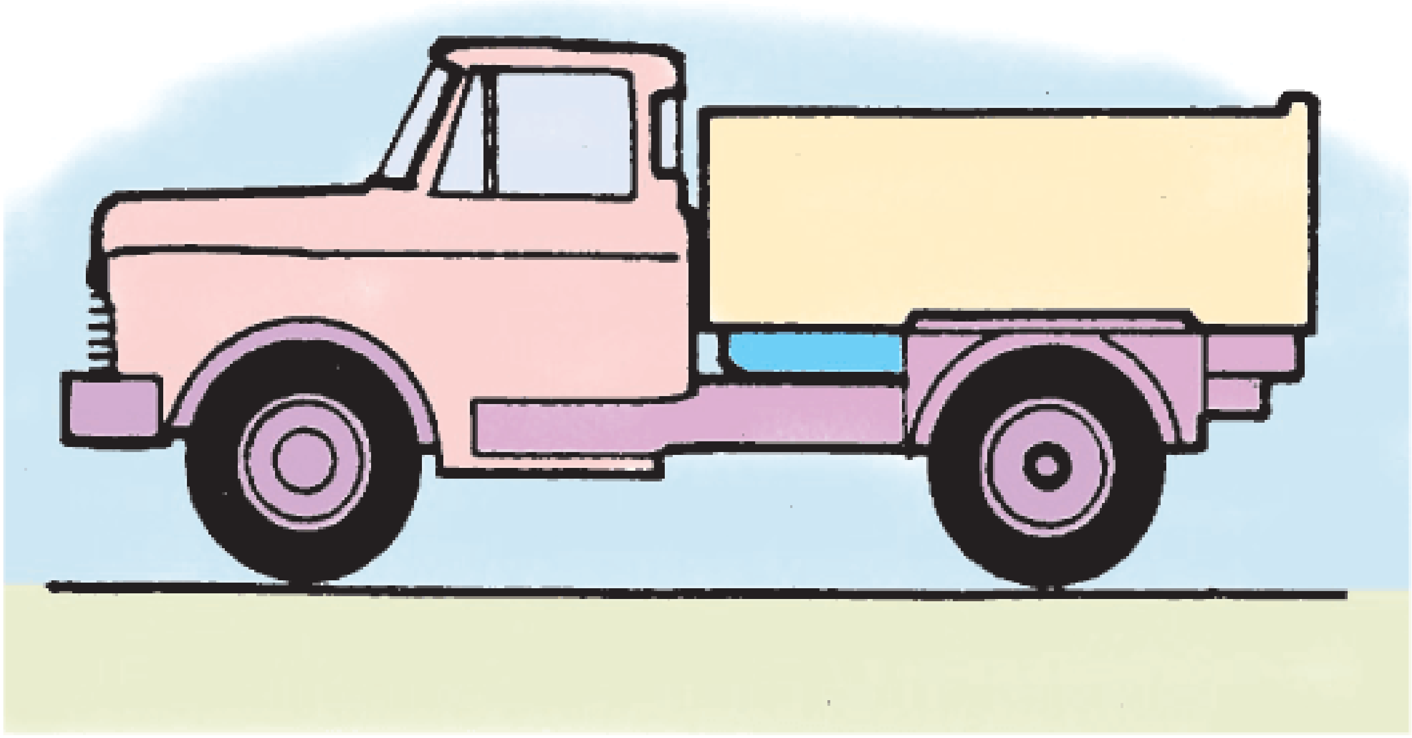


The plan of a factory wall has been drawn with the scale “1 represents 200”. Find the actual:

- **a** length of the wall
- **b** height of the wall
- **c** measurements of the door
- **d** measurements of the windows.

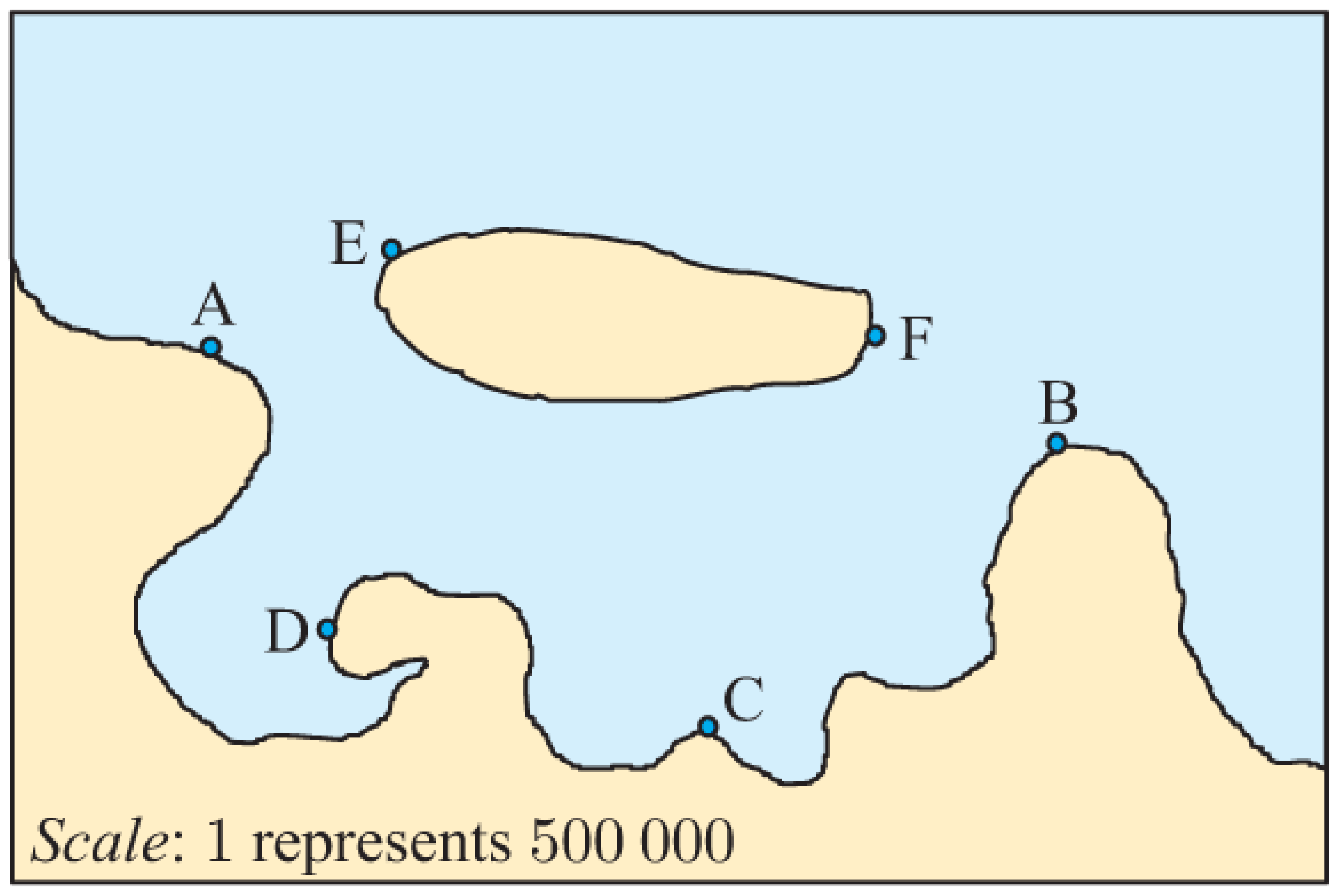
6 The drawing of the truck has the scale “1 represents 100”. Find:

- **a** the actual length of the truck
- **b** the maximum height of the truck.



7 Using the scale shown on the map, find:

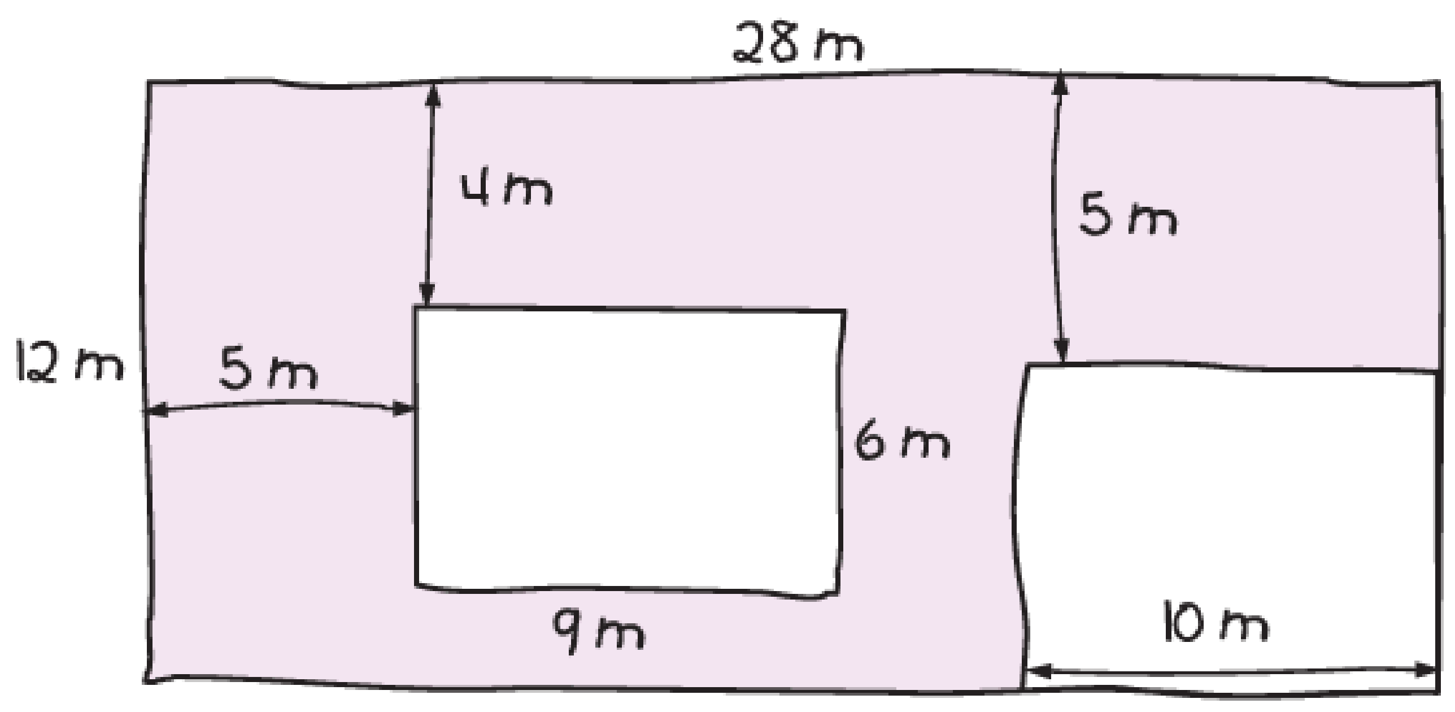
- **a** the actual distance represented by 1 cm on the map
- **b** the actual distance from
  - **i** A to B
  - **ii** D to E
  - **iii** C to F.



8 Draw a scale diagram of a circle with radius 1.5 m. Use the scale “1 represents 50”.

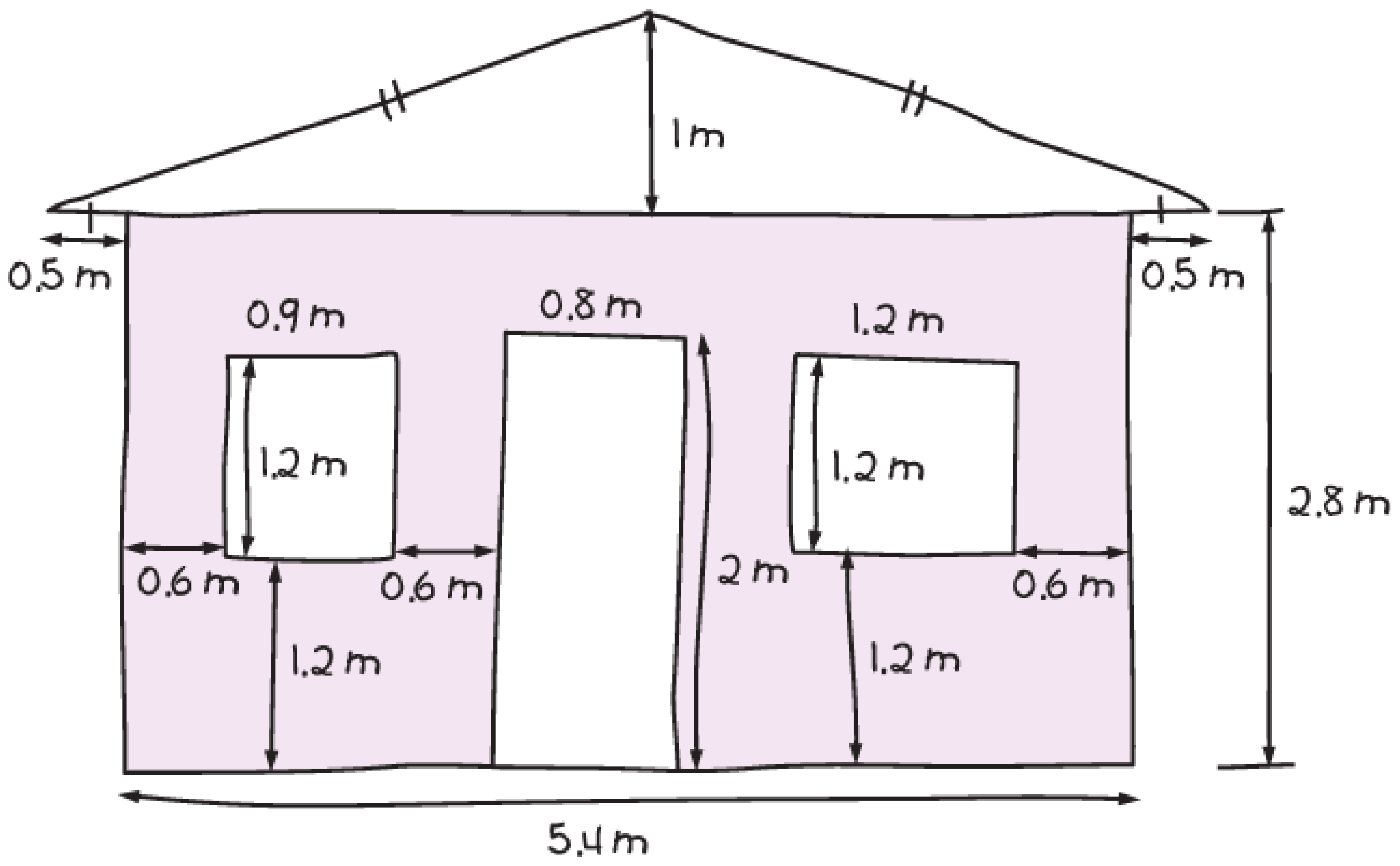
9 The rectangular park outside Kerri’s house is 80 m long and 60 m wide. Use a scale of “1 represents 2000” to draw a scale diagram of the park.

10



Use the measurements on this rough sketch to draw an accurate scale diagram. Use the scale “1 represents 400”.

11 The front view of a house is shown in this rough sketch. Using the scale "1 represents 100", draw an accurate scale diagram of this view.



12 Find the scale if:

a an aeroplane has wingspan 50 m and on its scale diagram it is 5 cm

b a truck is 15 m long and on its scale diagram it has length 12 cm.

13 Look at this map of Australia.

© OpenStreetMap contributors

The actual distance between Adelaide and Melbourne is 800 km.

a Measure the distance between Adelaide and Melbourne on the map.

b Find the scale for the map.

c Which of the cities shown is approximately 3000 km from Hobart?

d Estimate the distance between Brisbane and Cairns.

e Sue is travelling from Brisbane to Cairns by car. Do you think the distance she will travel will be greater or less than the distance in **d**? Explain your answer.

E MASS

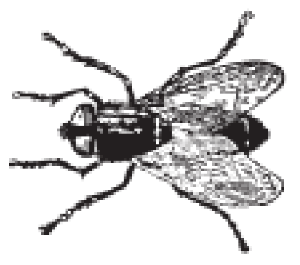
The **mass** of an object is a measure of how heavy the object is.

In the SI System or metric system, the **kilogram** (kg) is the base unit of mass. Other units of mass which are commonly used are the **milligram** (mg), **gram** (g), and **tonne** (t).

1 g = 1000 mg 1 kg = 1000 g 1 t = 1000 kg

You would use:

- **milligrams** to measure the mass of a fly

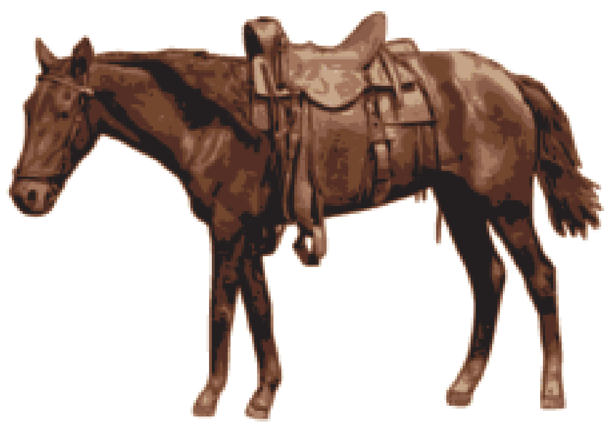


50 mg

- **grams** to measure the mass of a potato

400 g

- **kilograms** to measure the mass of a horse

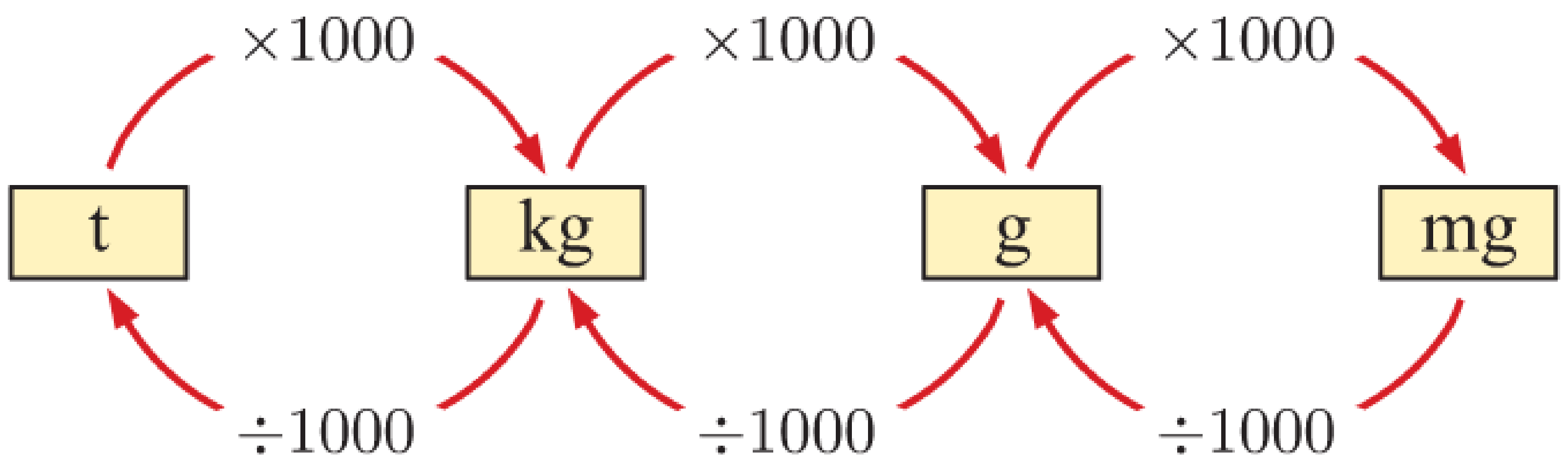


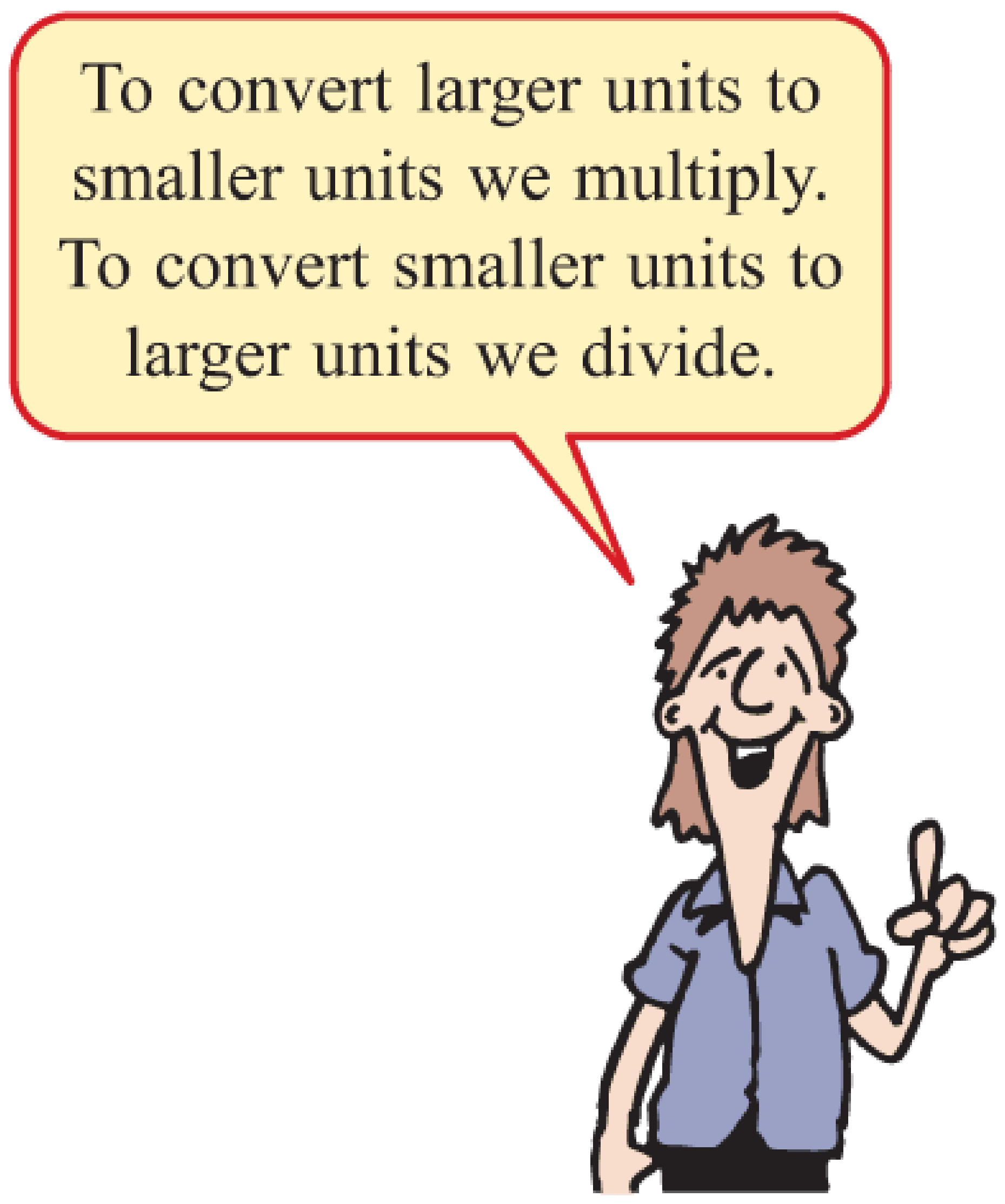
500 kg

- **tonnes** to measure the mass of a truck.

6 tonnes

MASS CONVERSIONS





Example 9 🔊 **Self Tutor**

Write in kilograms:

a 350 g **b** 8.5 t **c** 7 500 000 mg

a 350 g
= (350 ÷ 1000) kg
= 0.35 kg

b 8.5 t
= (8.5 × 1000) kg
= 8500 kg

c 7 500 000 mg
= (7 500 000 ÷ 1000) g
= 7500 g
= (7500 ÷ 1000) kg
= 7.5 kg

EXERCISE 8E

1 Give the units you would use to measure the mass of:

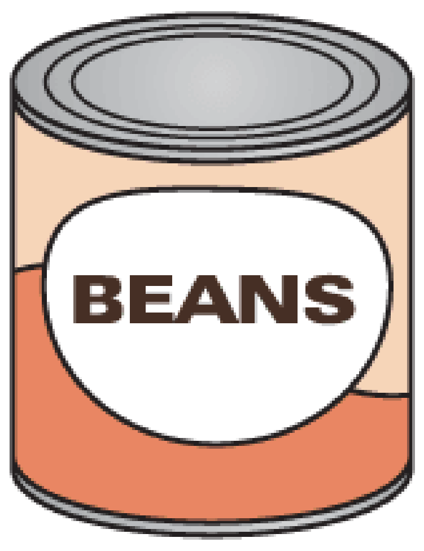
a a sheep **b** a ship **c** a book
d a banana **e** a lounge suite **f** a raindrop
g your school lunch **h** a baseball bat **i** a refrigerator
j a dinner plate **k** a school ruler **l** a bulldozer.

2 Match each object with the correct mass:

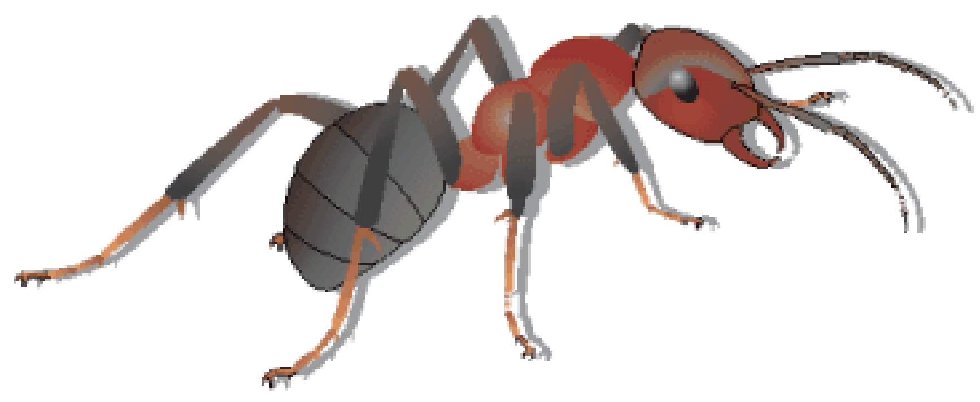
a

b

c



d



e

f

A 500 g **B** 400 t **C** 7 kg **D** 2 t **E** 5 mg **F** 70 kg

3 Write in grams:

a 8 kg **b** 3.2 kg **c** 14.2 kg **d** 380 mg
e 4250 mg **f** 75 420 mg **g** 6.8 t **h** 0.56 t

4 Write in kilograms:

a 13 870 g **b** 3.4 t **c** 786 g **d** 3496 mg

5 A block of chocolate has mass 120 g. Find the mass of 200 chocolate blocks, giving your answer in kilograms.

6 350 sandbags are used to stop floodwaters from reaching a school. Each sandbag weighs 20 kg. Find the total mass of the sandbags, giving your answer in tonnes.

7 There are 26 students in Jacqui's science class. Her teacher has placed a dish containing 3 g of a purple powder on the front bench. Each student requires 110 mg of the powder for an experiment. Will there be enough powder?

8 A bale of lucerne hay weighs 14 kg.

a A truck is loaded with 66 bales of lucerne. Find the mass of hay in the truck.

b Is the truck carrying more or less than a tonne of hay?

9 A tree trunk weighs 3.2 tonnes. If it is cut into 80 planks of equal size, what is the mass of each plank?

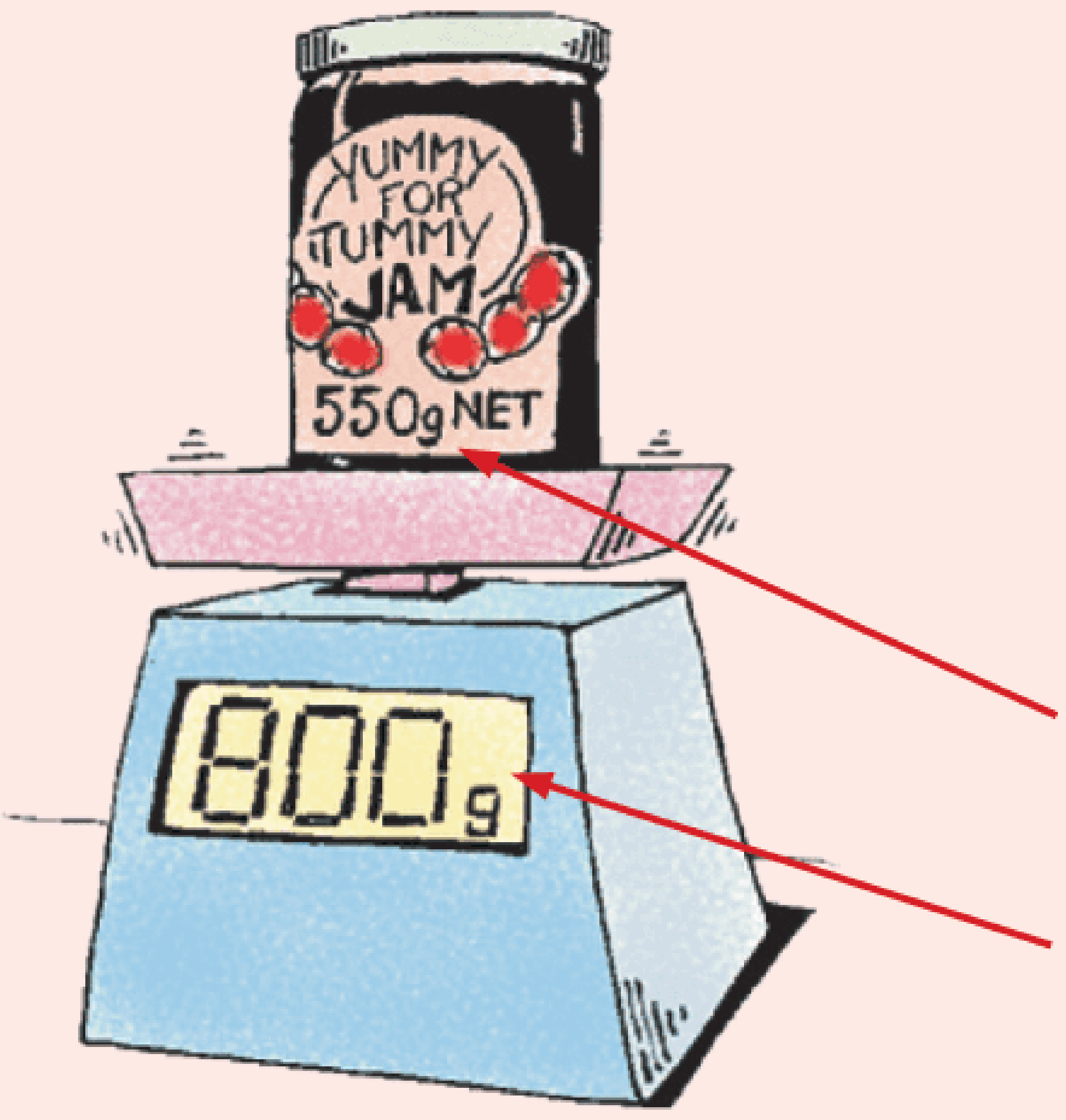
10 An aeroplane starting a long trip is initially carrying 108 tonnes of fuel. During the flight, the aeroplane burns 150 kg of fuel per minute.

- **a** In total, how much fuel does the plane burn during a 7 hour flight?
- **b** How many tonnes of fuel is the plane still carrying after 2 hours?

ACTIVITY 4 NET MASS

You may have noticed that many food packages use the word ‘net’ when describing the mass of their contents.

Net mass is the mass of contents *not* including the mass of the container. **Gross mass** is the mass of the contents *and* the container.



The jam weighs 550 g.
The **net** mass is 550 g.

The jam and the jar and lid together weigh 800 g.
The **gross** mass is 800 g.

The weight of the jar and lid is $800 \text{ g} - 550 \text{ g} = 250 \text{ g}$.

What to do:

1. Collect some food items which are packed in cardboard, plastic, or glass. Find the net weight stated on each container.
2. Weigh each item, and record the gross mass.
3. Use **1** and **2** to find the mass of each of the containers. Record your observations in a table like this.

Item	*Net mass*	*Gross mass*	*Mass of container*

4. Discuss why food packages have net masses printed on them.

Global context

click here

Calculating your carbon footprint

Statement of inquiry:	Examining the resources we use helps us to measure our impact on the environment.
Global context:	Globalisation and sustainability
Key concept:	Relationships
Related concepts:	Measurement, Quantity
Objectives:	Communicating, Applying mathematics in real-life contexts
Approaches to learning:	Communication, Research

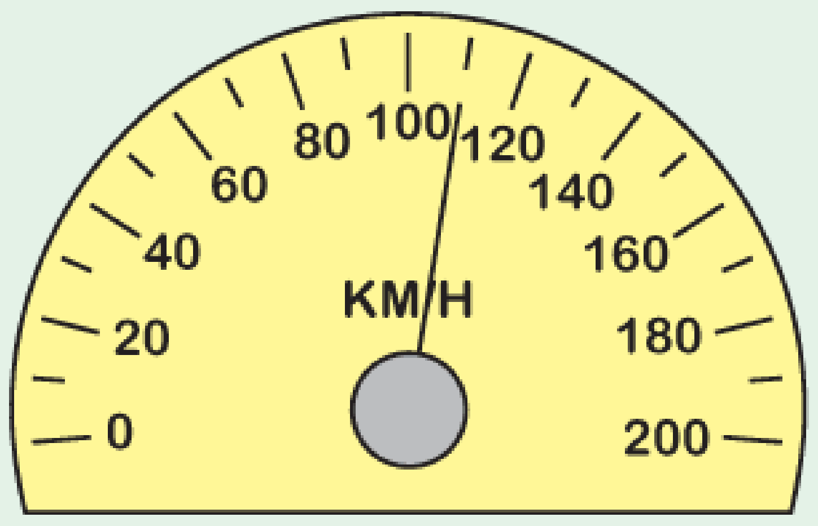
KEY WORDS USED IN THIS CHAPTER

- centimetre
- length
- millimetre
- scale factor
- gram
- mass
- perimeter
- tonne
- kilogram
- metre
- scale
- kilometre
- milligram
- scale diagram

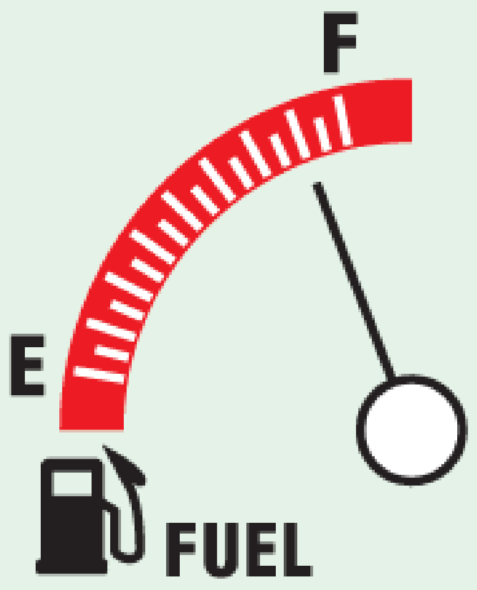
REVIEW SET 8A

1 Read these scales:

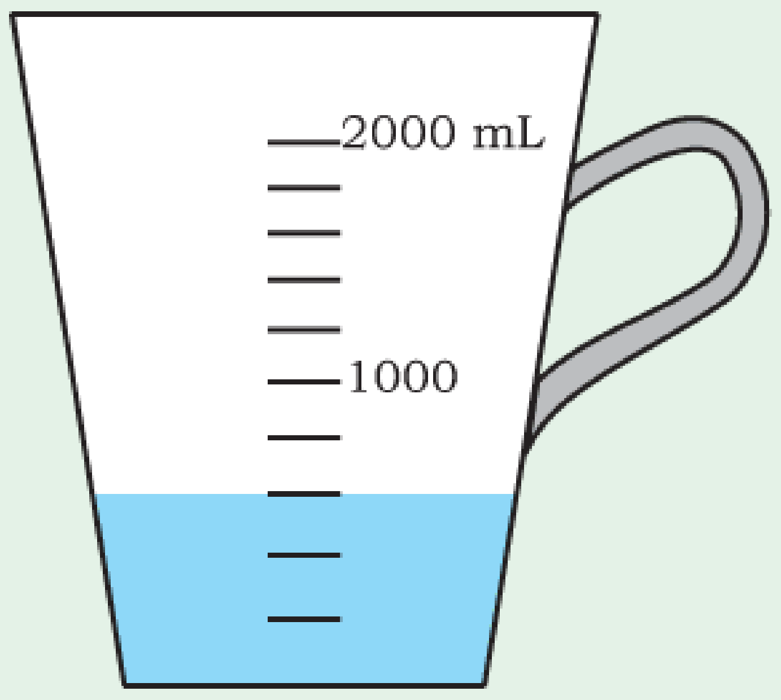
a



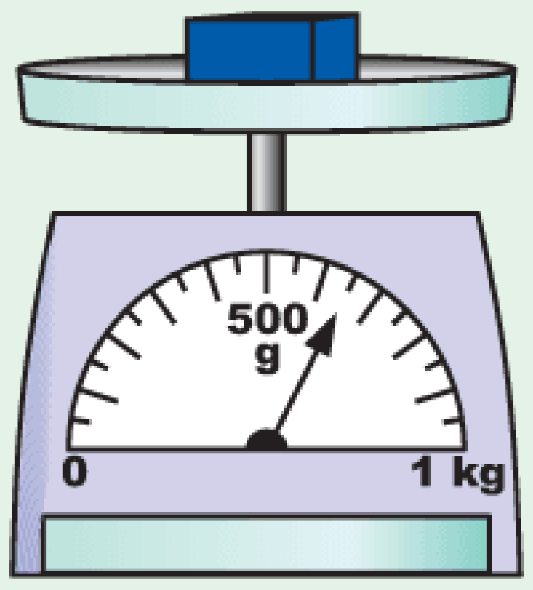
b



c



d

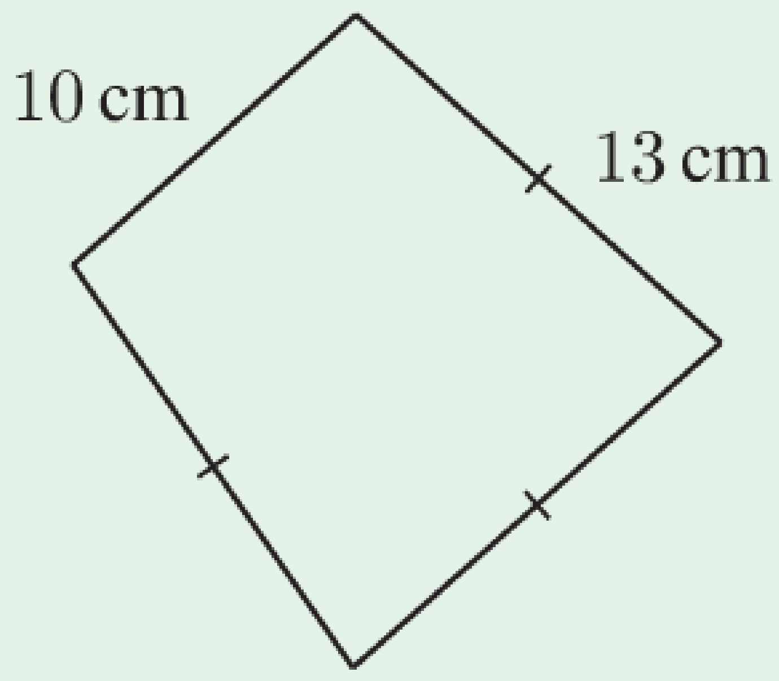


2 Convert:

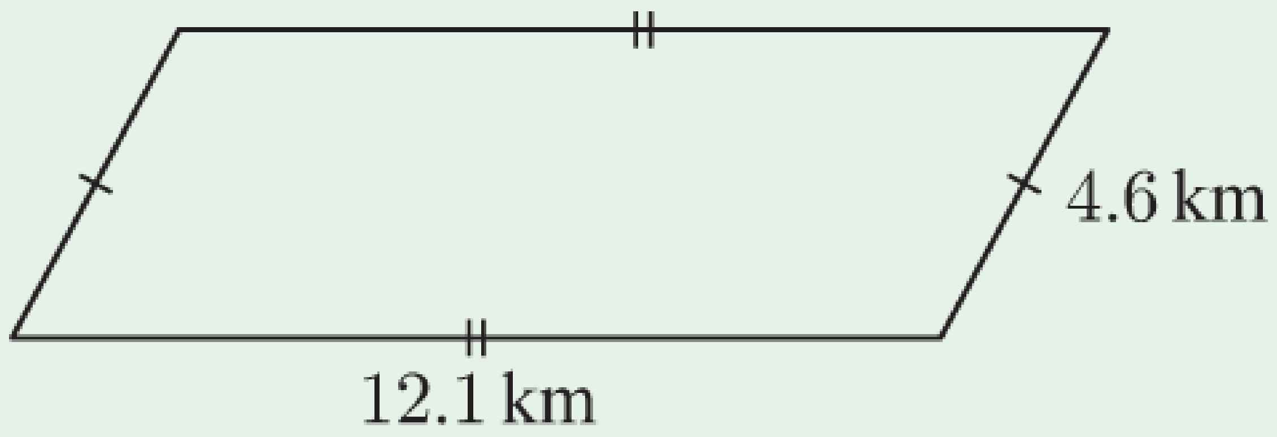
a 356 cm to m **b** 450 m to km **c** 7.63 m to mm.

3 Find the perimeter of:

a



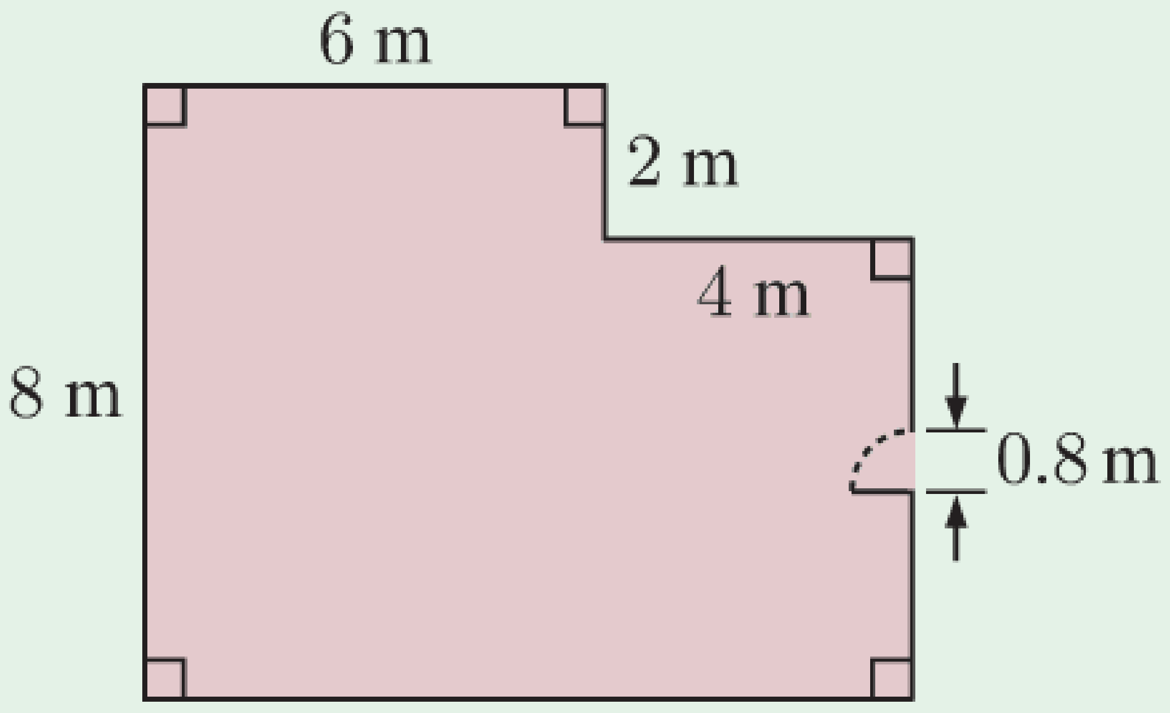
b



4 A room with this floorplan needs skirting board fitted to the bottom of each wall. The skirting board costs $16.50 per metre.

a Find the total length of skirting board required.

b What is the total cost of the skirting board?



5 By writing each length in metres, find the sum of these lengths:

a 6 km, 207 m **b** 9 m, 38 cm, 4 mm

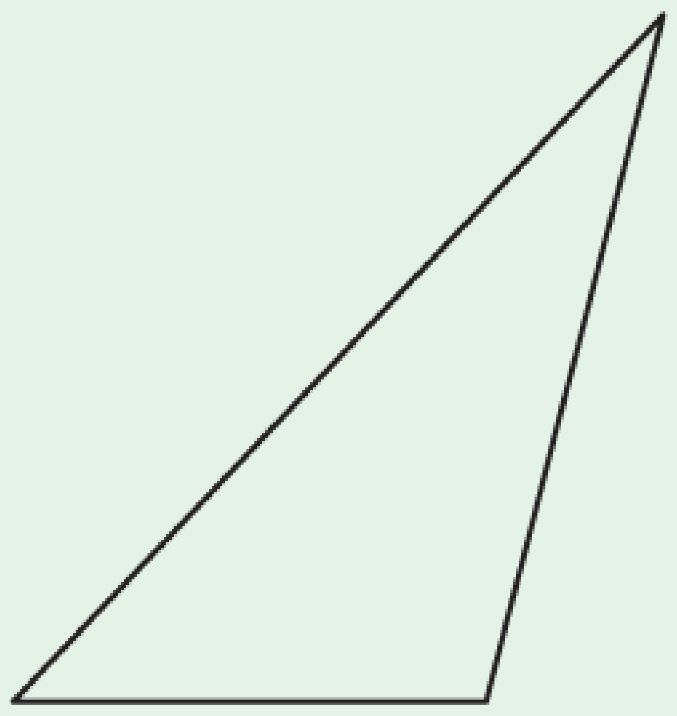
6 My suitcase weighs 21.2 kg. Write this mass in:

a grams **b** tonnes.

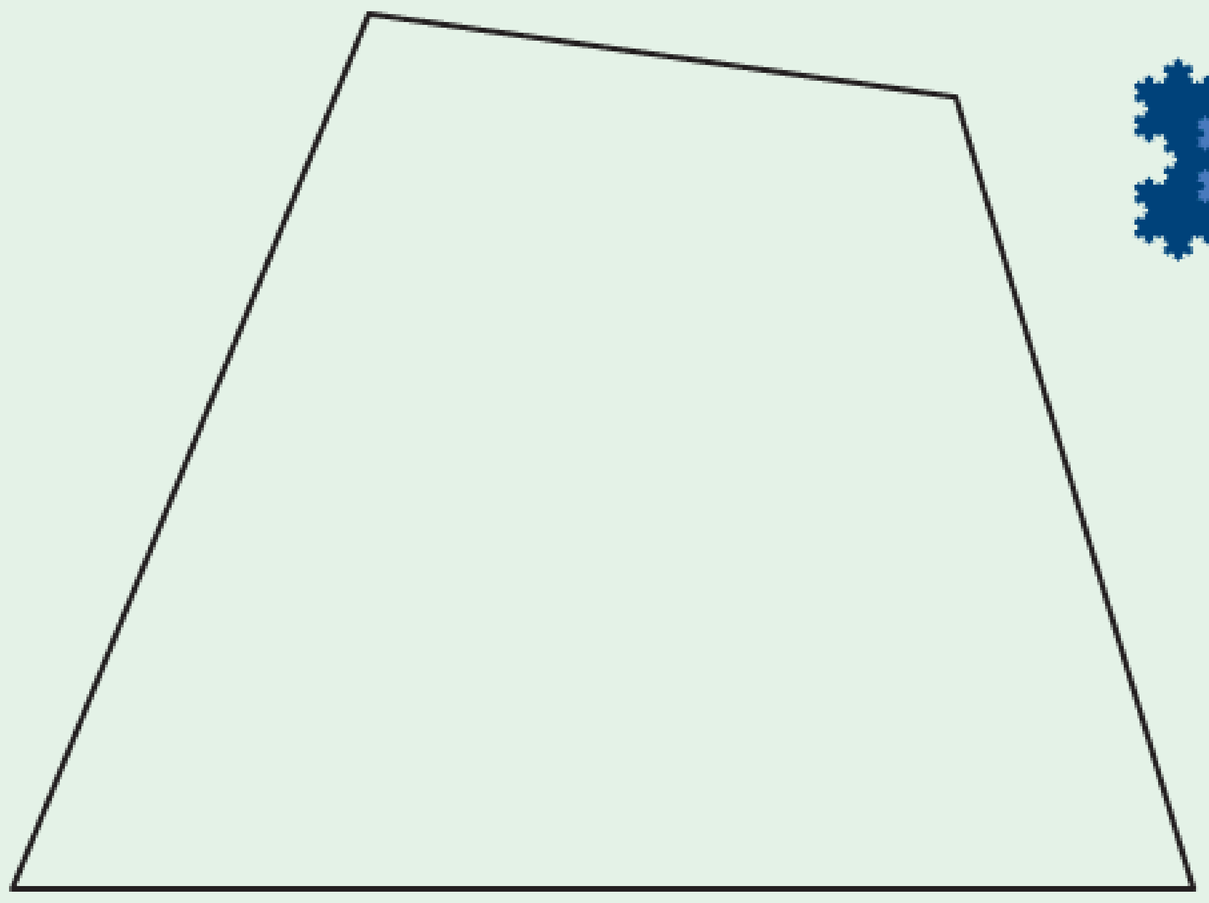
7 Use a ruler to find the perimeter of these figures:

PRINTABLE DIAGRAMS

a



b



8 A scale diagram has the scale "1 represents 500 000".

a Find the actual length if the drawn length is:

i 3.8 cm **ii** 6.4 cm **iii** 12.2 cm

b Find the drawn length if the actual length is:

i 50 km **ii** 22 km **iii** 130 km

9 Find the total mass of 1500 oranges if each orange has mass 180 g. Give your answer in kilograms.

10 This rectangle is drawn with the scale "1 represents 10".

a Find the actual dimensions of the rectangle.

b Which of the following could the rectangle represent?

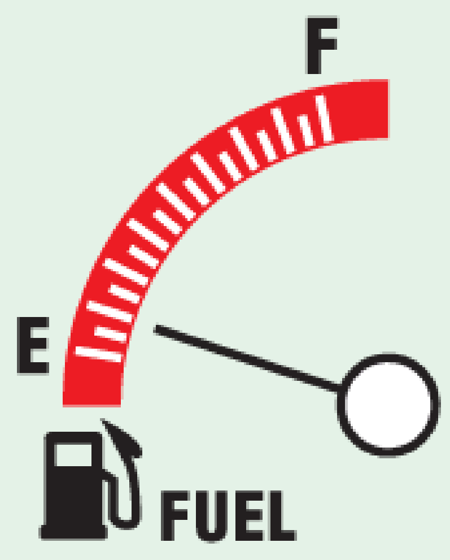
A a room **B** a stamp **C** a paver

Give reasons for your answer.

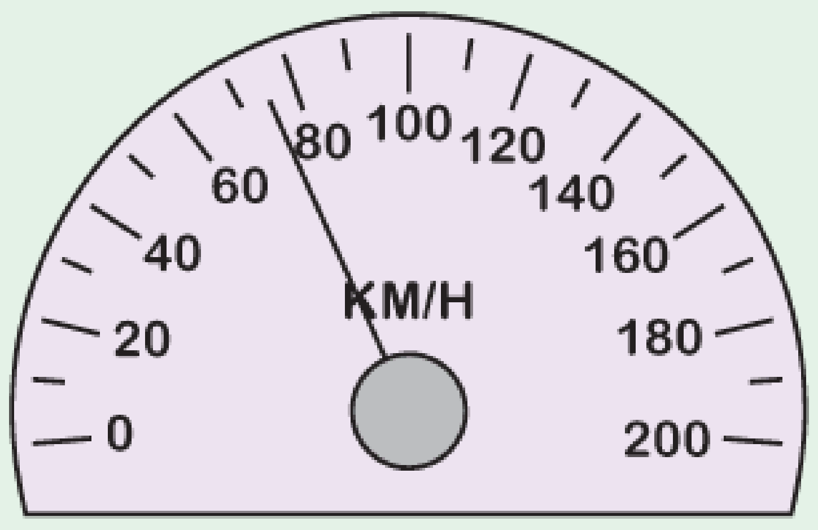
REVIEW SET 8B

1 Read these scales:

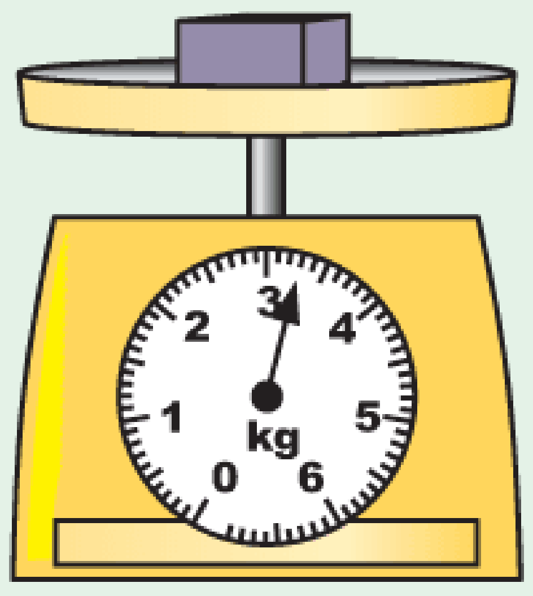
a



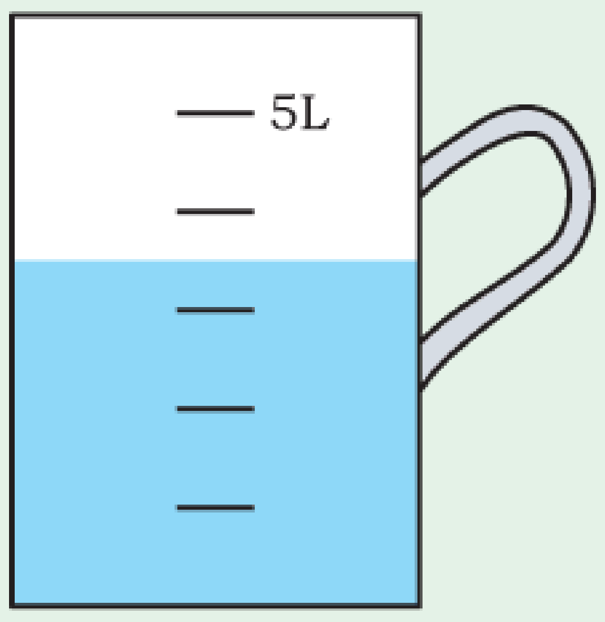
b



c

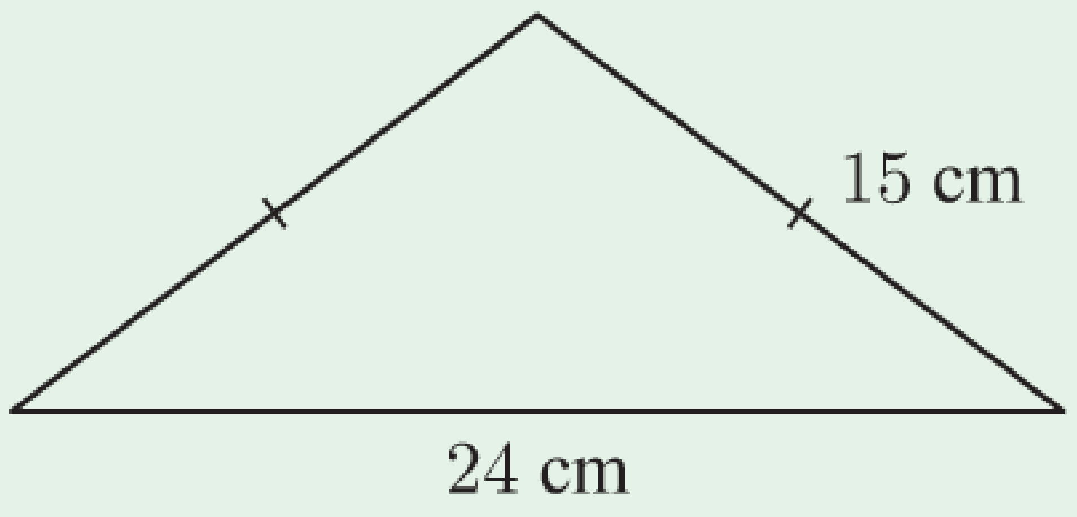


d

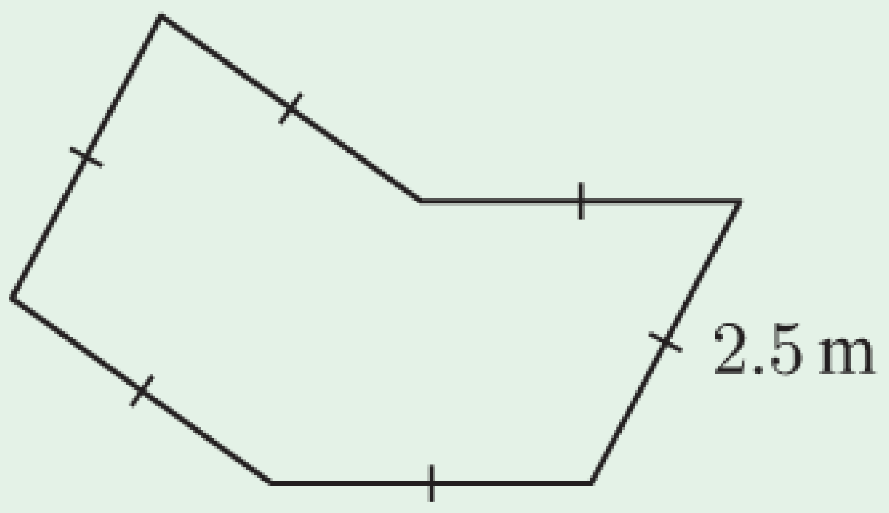


2 Find the perimeter of each figure:

a



b



3 Grant competed in the 100 m, 200 m, 400 m, and 1500 m freestyle events during the swimming tournament today. How many kilometres has he swum in total?

4 Write these measurements in order of size from smallest to largest:
423 mm, 21 cm, 0.35 m, 47.1 mm.

5 A 2.3 cm line on a scale diagram represents a real length of 460 m. What is the scale?

6 Use a ruler to find the length of these lines, giving your answer in:

i millimetres **ii** centimetres.

PRINTABLE DIAGRAMS

a

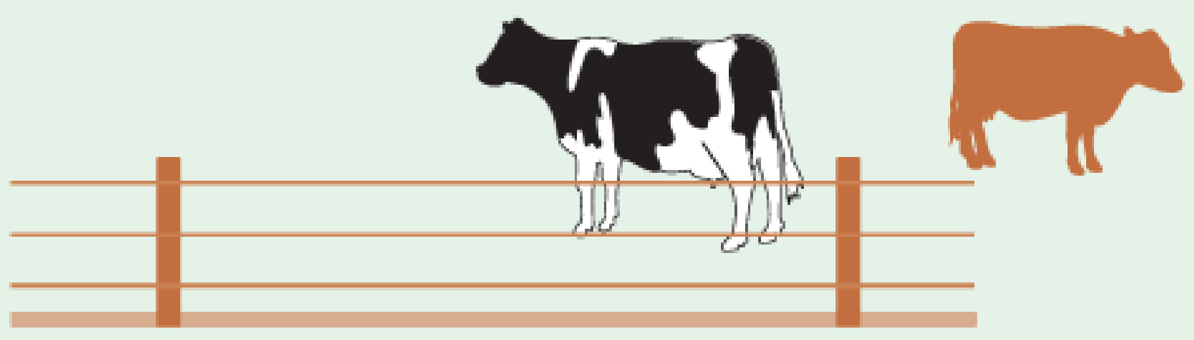
b

7 Convert:

a 3200 g to kg **b** 4.6 g to mg **c** 0.7 t to g.

8 A truck can carry 800 kg of soil. How many tonnes of soil can be carried in 25 truckloads?

9 A farmer fences a 250 m by 400 m rectangular paddock with a 3 strand wire fence.

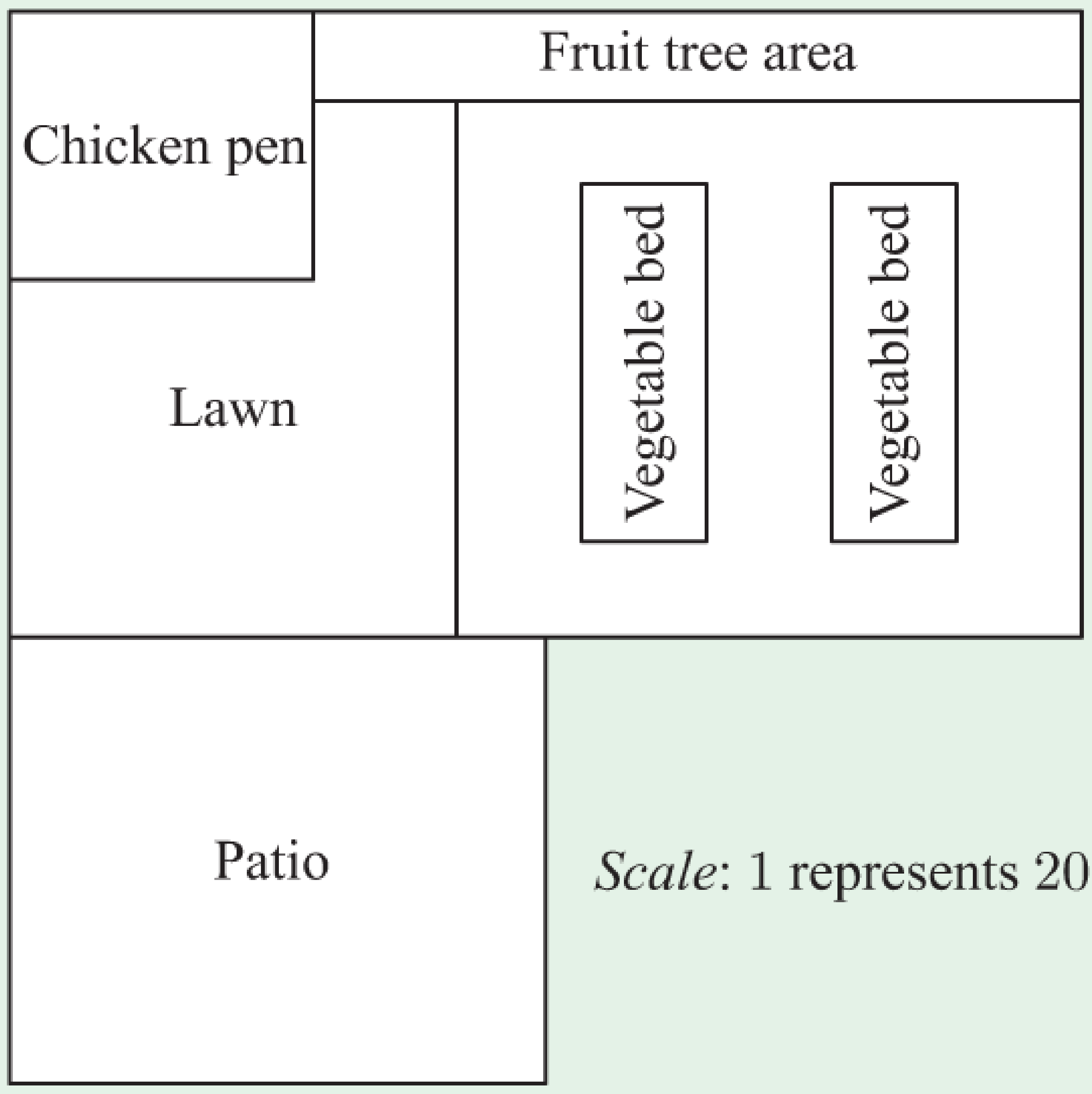


a Find the perimeter of the paddock.

b Find the total length of wire needed.

c The wire costs £2.40 per metre. Find the total cost of the wire.

10 Mr Brown is designing a garden. He draws the scale diagram shown.



Scale: 1 represents 200

a Find the actual measurements of:

i the chicken pen

ii the fruit tree area.

b Mr Brown wishes to add another vegetable bed between the two existing beds. He wants it to be 2 m wide by 5 m long.

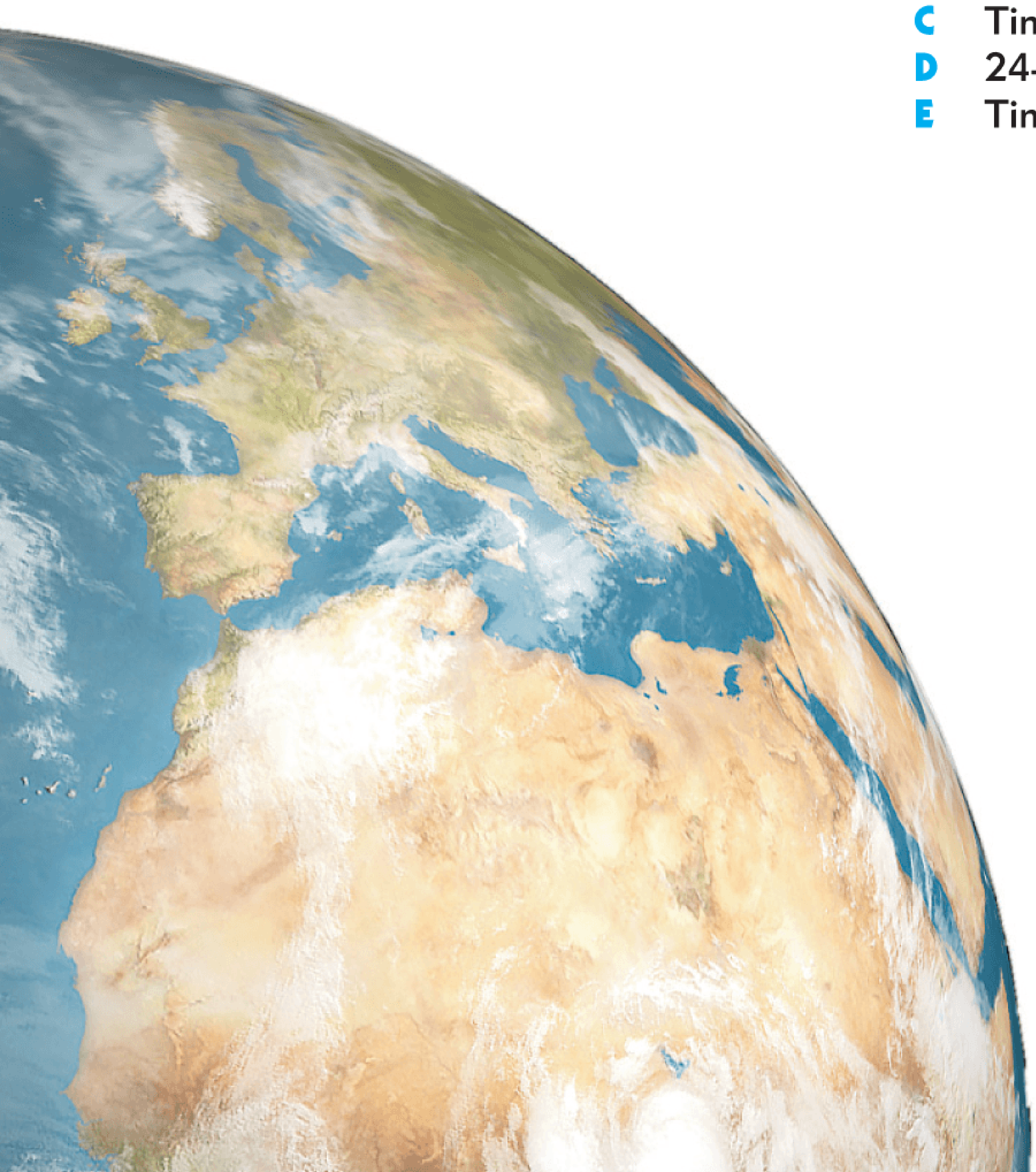
i How big would this new bed be on the diagram?

ii Will this new bed fit? Explain your answer.

Chapter 9

Time

Contents:



OPENING PROBLEM

Eight teams are competing in a junior soccer carnival.

This timetable shows the games which are to be played.

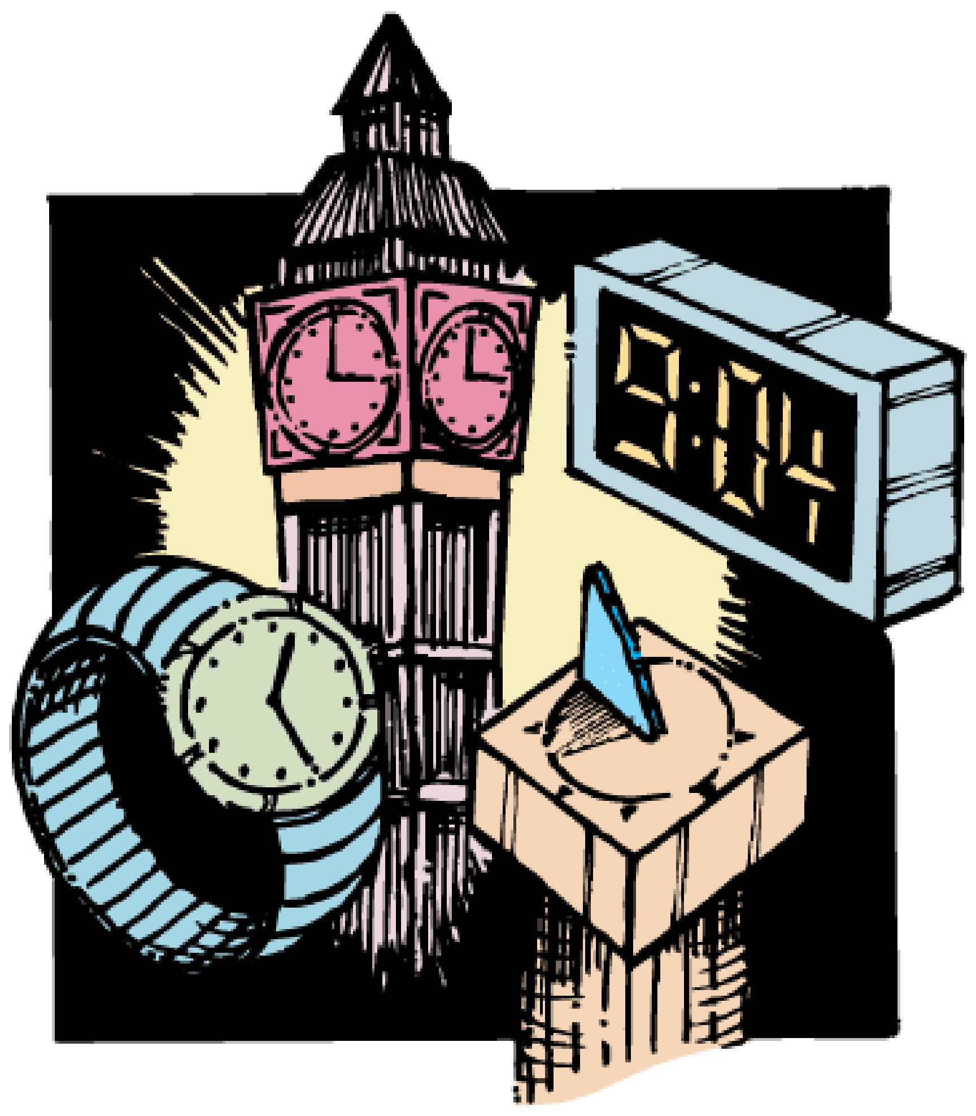
Time	*Field 1*	*Field 2*	*Field 3*
8:00	1 v 2	3 v 4	5 v 6
9:15	1 v 5	2 v 7	3 v 8
10:30	3 v 5	4 v 8	6 v 7
11:45	2 v 6	1 v 7	4 v 5
13:00	3 v 6	4 v 7	1 v 8
14:15	2 v 8		

Things to think about:

- **a** At what time does the carnival start?
- **b** At what time does Team 4 play Team 8?
- **c** What does the time 13:00 mean?
- **d** What is the time difference between Team 2 starting their first game, and starting their last game?

For most of us, it seems that time controls our lives. We are always asking questions such as:

- How long until the end of the lesson?
- At what time do you have to be at school?
- At what time shall we meet?
- When did the Second World War finish?
- For how long did Sachin Tendulkar play for the Indian cricket team?
- At what time does the football start?
- How long will it take us to travel to Tokyo?



ACTIVITY 1 TIME OUT

What to do:

Work in small groups to discuss the following topics:

1 List ways by which the following would be aware of time changes:

- **a** animals in the wild
- **b** domestic animals
- **c** human infants
- **d** farmers
- **e** sailors at sea 300 years ago.

2 Outline some of the problems people may have had in measuring time using:

- **a** candles
- **b** sundials
- **c** sand-glasses
- **d** pendulum clocks
- **e** mechanical clocks.

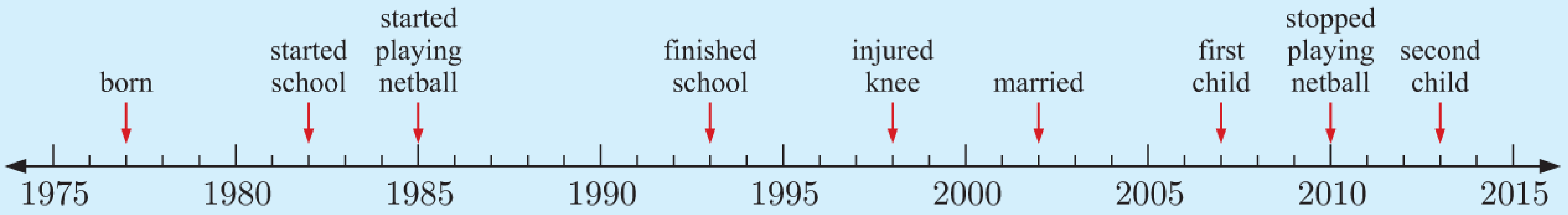
3 List 10 occupations where time or timing is very important, for example, musicians and restaurant chefs. For each of the occupations listed, write two consequences of wrong timing.

A TIME LINES

Time lines are simple graphs which display times or dates, and key events that correspond to these times.

Example 1 Self Tutor

The following time line shows some of the important dates in Sarah's life:



a Determine when Sarah:
  i was born **ii** was married **iii** finished school **iv** injured her knee.

b For how long did Sarah play netball?

c Find the age difference between Sarah's two children.

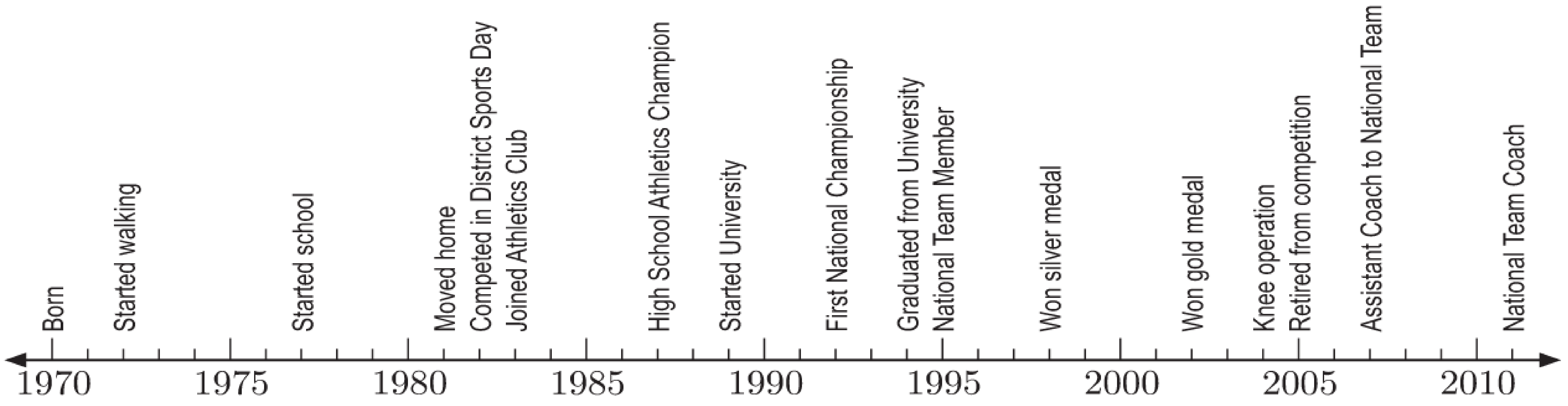
a **i** 1977 **ii** 2002 **iii** 1993 **iv** 1998

b She started in 1985 and stopped in 2010, so she played for 25 years.

c Her first child was born in 2007 and the second was born in 2013, so there is a difference of 6 years in age.

EXERCISE 9A

1 The following time line shows some of the important dates in Gavin's life:



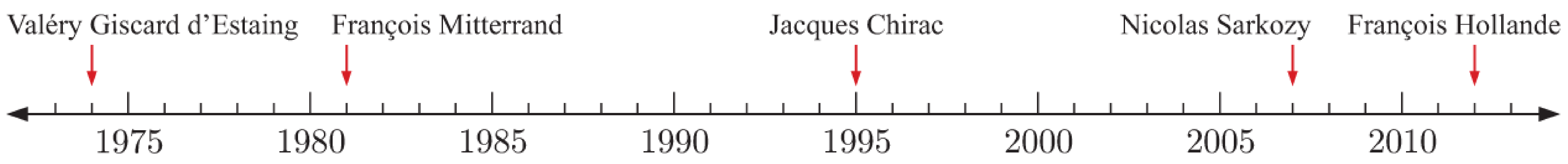
a When was Gavin born?

b When did Gavin:
  i join the athletics club **ii** start university **iii** win a gold medal?

c For how long did Gavin continue to compete after he was High School Athletics Champion?

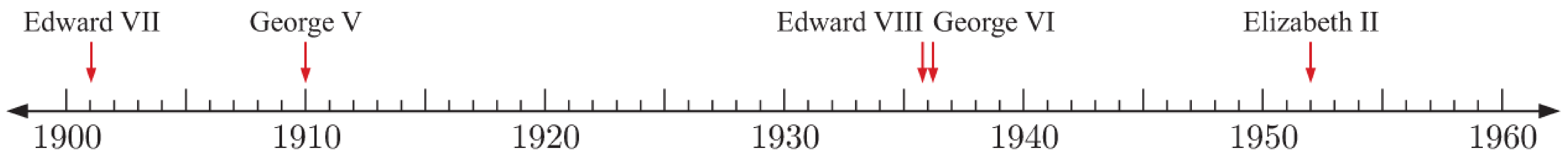
d How old was Gavin when he:
  i started school **ii** was appointed National Team coach?

2 This time line shows the French Presidents since 1975:



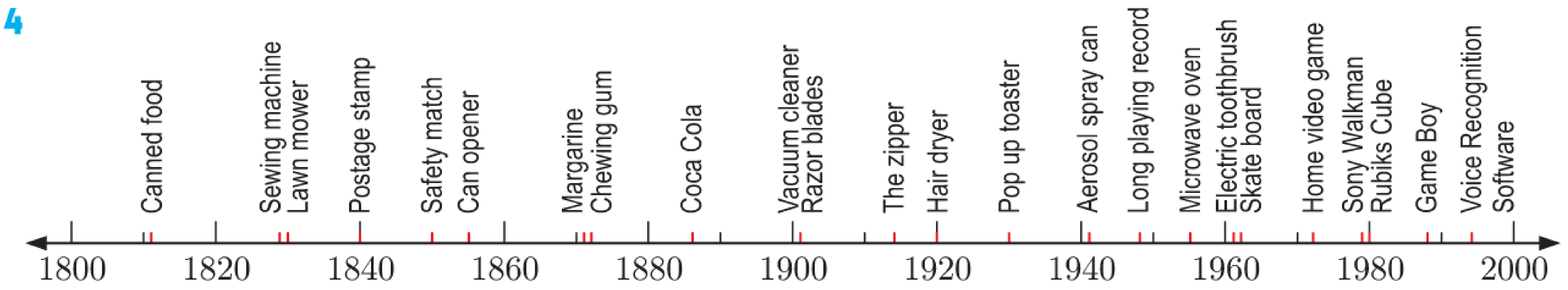
- **a** In which year did François Hollande become President?
- **b** Who was President in 1980?
- **c** For how long was Nicolas Sarkozy President?
- **d** Who was President for longer, François Mitterrand or Jacques Chirac?

3 The following time line shows the monarchs of England during the 20th century:



- **a** When did the reign of George V begin?
- **b** Which monarch reigned immediately after George VI?
- **c** How long did the reign of Edward VII last?

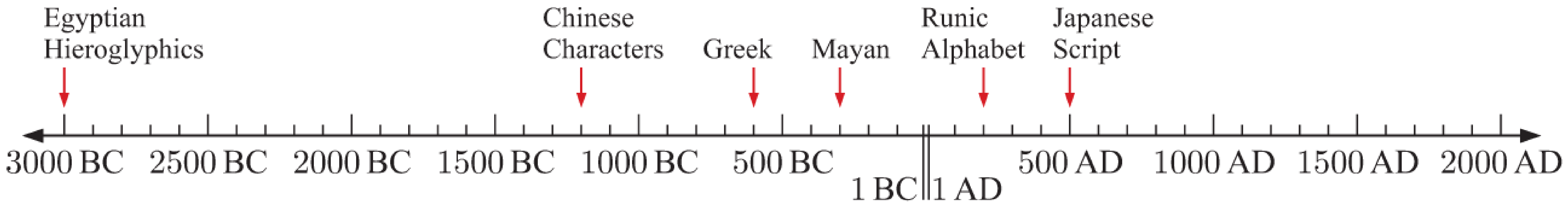
4



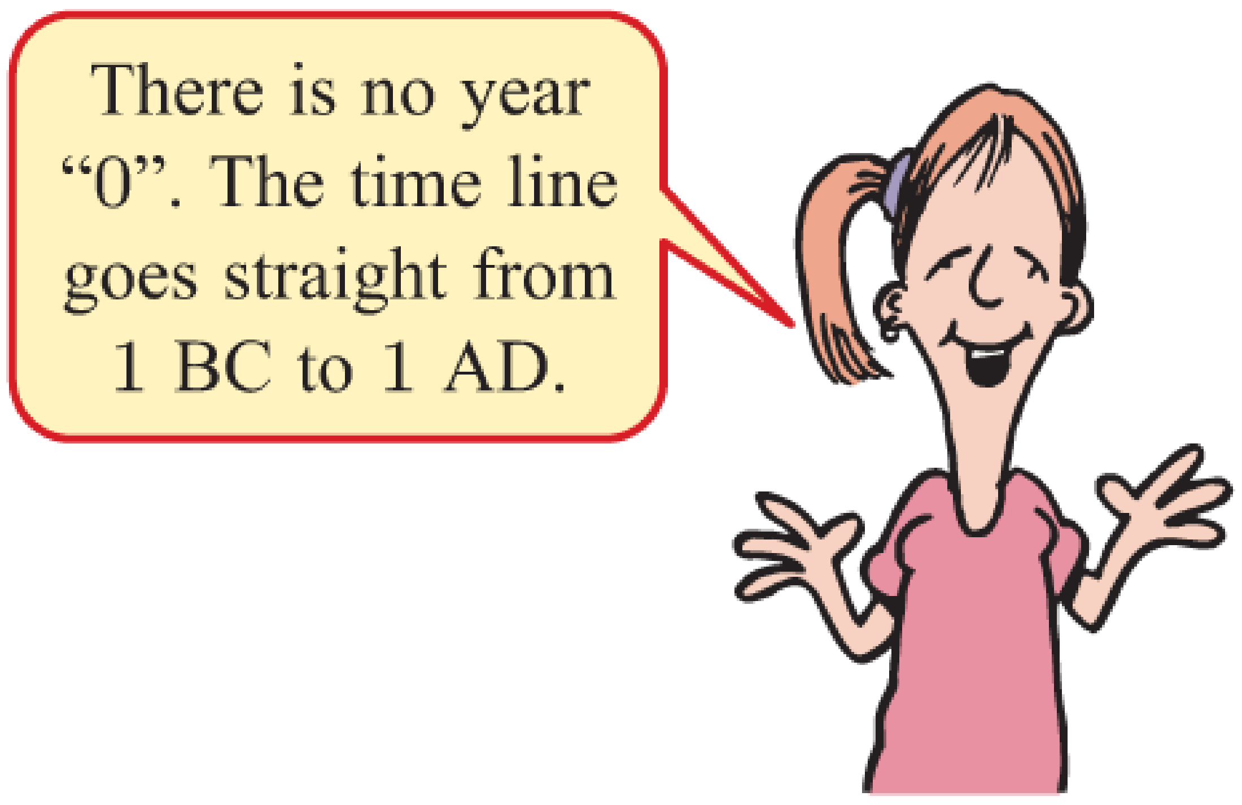
The time line above shows the years in which certain products first went on sale.

- **a** Which product was first sold in 1930?
- **b** Which product was available earlier, the safety match or the lawn mower?
- **c** In which year was the can opener first sold?
- **d** In which year was more than one of these products first sold?
- **e** How many years after chewing gum was first sold did the electric toothbrush become available?

5 The following time line shows when various methods of writing first appeared:



- **a** Estimate when the Runic Alphabet first appeared.
- **b** Estimate when Greek writing was first used.
- **c** About how long was there between the appearance of:
  - **i** Egyptian Hieroglyphics and Chinese Characters
  - **ii** Mayan writing and Japanese script?



6 Some important moments in history are listed below:

A	1947	A polaroid camera produces pictures in under one minute.
B	1450	Gutenberg builds a printing press.
C	700 BC	Coins are used in Turkey for buying and selling goods.
D	1890	An electric counting machine helps the American census.
E	999	The first mechanical clock is invented by a monk.
F	810	First description of Arabic numerals.
G	1636	An accurate pendulum clock is built.
H	3000 BC	An abacus, the first adding machine, is invented by Babylonians.
I	1569	Mercator shows a new way of drawing maps.
J	1938	The first ballpoint pen is introduced.
K	1642	A faster adding machine is designed by Pascal.
L	2800 BC	Egyptians devised the 12 month, 365 day calendar.
M	1614	Scottish mathematician John Napier invents logarithmic tables.
N	1946	The first electronic computer is demonstrated.
O	100	Paper is invented in China.
P	1500	The first watches are made.

a Draw a 20 cm line. Divide the line into 5 equal lengths as shown below.

b Using the matching letters of the alphabet, arrange the above events on your time line.

c Which of the events listed occurred most recently?

d How many of the events listed occurred before paper was invented?

e How many years before Pascal's adding machine was the Babylonian abacus used?

f How long after the first mechanical clock was invented, were the first watches made?

ACTIVITY 2 DEVELOPING TIME LINES

In this Activity you will draw at least one time line based on your own research.

What to do:

1 Decide on a suitable topic to research. Here are some suggestions, but you should research a topic which particularly interests you:

- Presidents of the USA
- Wars of the 20th century
- Significant dates in your family's history, such as birthdays, marriages, and special holidays.

2 Choose a suitable scale for your time line, and complete it with the relevant information.

3 Write down four questions based on your time line, and ask another student to answer them.

B UNITS OF TIME

The units for measuring time are based on the movement of the Earth and the Sun.

A **year** is the time it takes for the Earth to complete one orbit around the Sun.

DEMO

A **day** is the time it takes for the Earth to perform one complete rotation on its axis.

Other units we use include months, weeks, hours, minutes, and seconds.

1 year
= 12 months
= 52 weeks (approximately)
= 365 days (or 366 in a leap year)

1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

The number of days in a month varies:

January	31		May	31	September	30
February	28	(29 in a leap year)	June	30	October	31
March	31		July	31	November	30
April	30		August	31	December	31

A **leap year** occurs if the year is divisible by 4 but not by 100, except if the year is divisible by 400.

For example: 1996 was a leap year.
2000 was a leap year.
2100 will not be a leap year.

Example 2 **Self Tutor**

Convert:

a 5 days to hours **b** 120 minutes to hours **c** 3 hours to seconds.

a 5 days
$= 5 \times 24$ hours
$= 120$ hours

b 120 minutes
$= 120 \div 60$ hours
$= 2$ hours

c 3 hours
$= 3 \times 60$ minutes
$= 180$ minutes
$= 180 \times 60$ seconds
$= 10\,800$ seconds

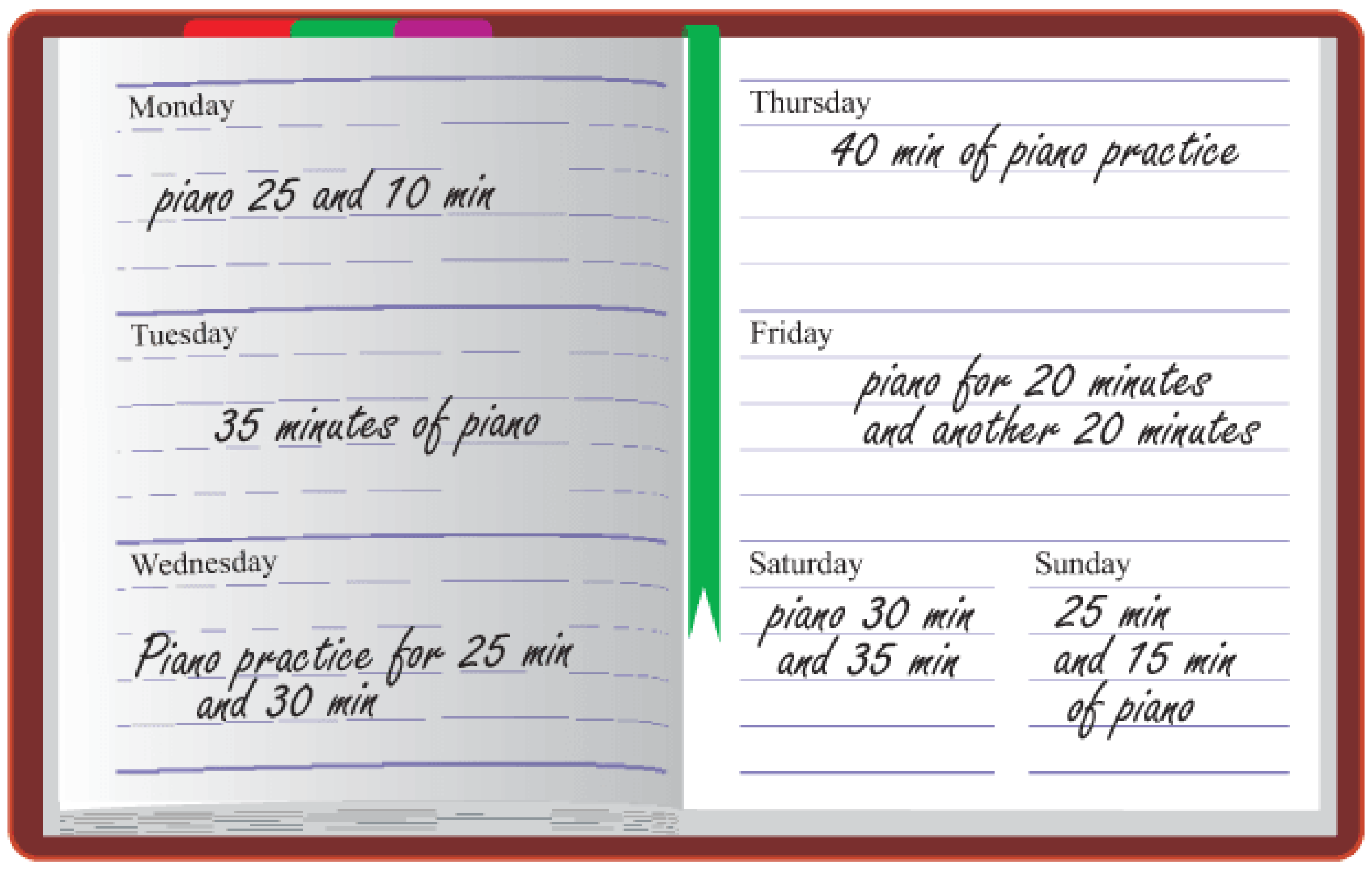
EXERCISE 9B

1 Convert:

a 10 minutes to seconds **b** 6 days to hours **c** 24 months to years
d 5 weeks to days **e** 240 seconds to minutes **f** 7 years to months
g 140 days to weeks **h** 4 hours to seconds **i** 3 weeks to hours.

2 Jill has 120 cans to put in her supermarket display. It takes her 4 seconds to position each can. How many minutes will it take Jill to complete the job?

3 Max has been told that he must practise the piano for 6 hours each week. His diary shows the times in minutes he has spent practising this week.
Did Max spend enough time practising this week?



Monday
piano 25 and 10 min

Tuesday
35 minutes of piano

Wednesday
Piano practice for 25 min and 30 min

Thursday
40 min of piano practice

Friday
piano for 20 minutes and another 20 minutes

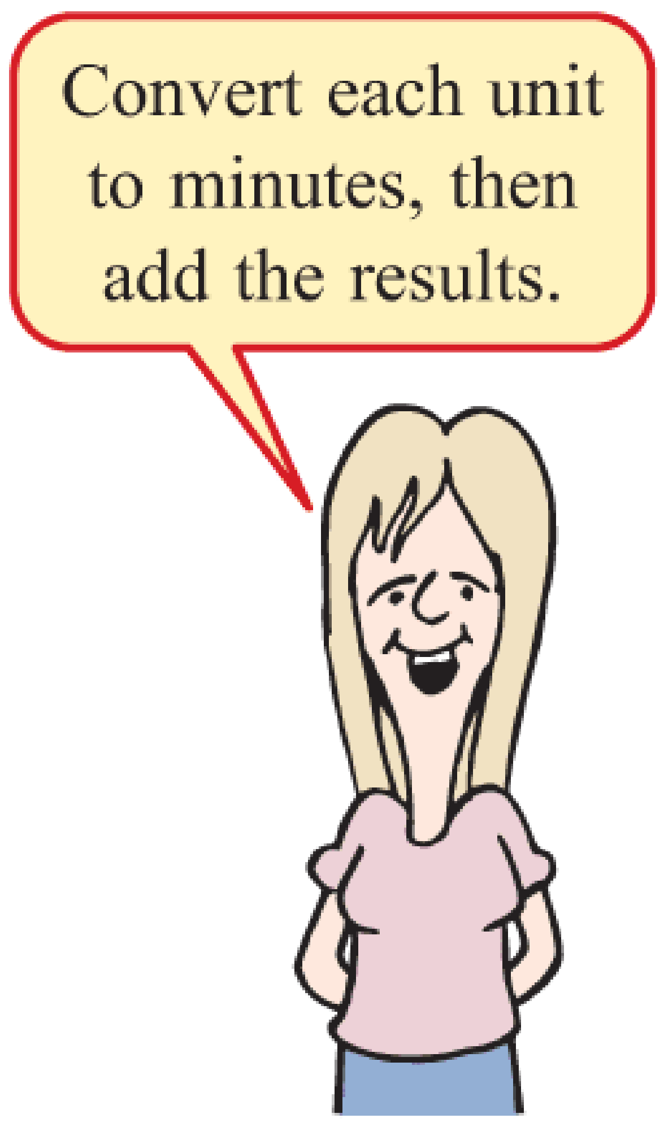
Saturday
piano 30 min and 35 min

Sunday
25 min and 15 min of piano

Example 3 **Self Tutor**

Convert 3 days, 9 hours, and 42 minutes to minutes.

3 days	9 hours		4320 min
$= 3 \times 24$ hours	$= 9 \times 60$ min		540 min
$= 3 \times 24 \times 60$ min	$= 540$ min	$+$	42 min
$= 4320$ min		Total	4902 min



4 Convert to minutes:

a 7 hours 24 min
b 3 days 5 hours 43 min
c 12 days 15 hours 36 min
d 2 weeks 3 days 8 hours 17 min

5 Convert to seconds:

a 40 min 38 s
b 3 h 35 min 27 s
c 14 h 12 min 43 s
d 22 h 52 min 11 s

6 Find the number of hours in:

a June
b March.

7 Find the number of:

a days in 2010
b hours in 2012
c minutes in 2013.

8 Consider a four year period which includes a leap year. Convert this period of time into:

a days
b hours
c minutes.

Example 4 **Self Tutor**

Write 134 seconds in minutes and seconds.

$$\begin{aligned} 120 \text{ seconds} &= 2 \text{ minutes} \\ \therefore \quad 134 \text{ seconds} &= 2 \text{ minutes and } 14 \text{ seconds.} \end{aligned}$$

9 Write:

- **a** 200 seconds in minutes and seconds
- **b** 50 hours in days and hours
- **c** 150 minutes in hours and minutes
- **d** 30 days in weeks and days
- **e** 40 months in years and months.

10 Jason ran for 25 minutes every day for two weeks. Find the total time he spent running, in hours and minutes.

ACTIVITY 3 HOW LONG DOES IT TAKE?

You will need: a stopwatch or digital watch with similar functions, and a partner to work with.

What to do:

1. Estimate the time it will take to complete each of the activities in the list below.
2. Complete each activity, using the stopwatch to measure the actual time it takes. Take turns with your partner to do the activity and operate the stopwatch.
3. Find the difference between the estimated time and the actual time for each activity.

PRINTABLE TABLE

Activity	*Estimated time*	*Actual time*	*Difference*
Count from 1 to 200			
Accurately write down your 8 and 9 times tables			
Carefully read aloud one page from a novel			
Walk 100 metres			
Roll a pair of sixes with 2 dice			
Bounce a basketball 100 times			

4. Are some times more easy to predict than others? Explain your answer.

GAME ESTIMATING TIME

Click on the icon to load a game for improving your time estimation skills.

GAME

C TIME CALCULATIONS

Time calculations are frequently made by travellers, credit card owners, and businesses.

Example 5 Self Tutor

How many days is it from April 24th to July 17th?

April has 30 days, so there are 30 − 24 = 6 days remaining in April.

	April	6
	May	31
	June	30
+	July	17
	Total	84 days

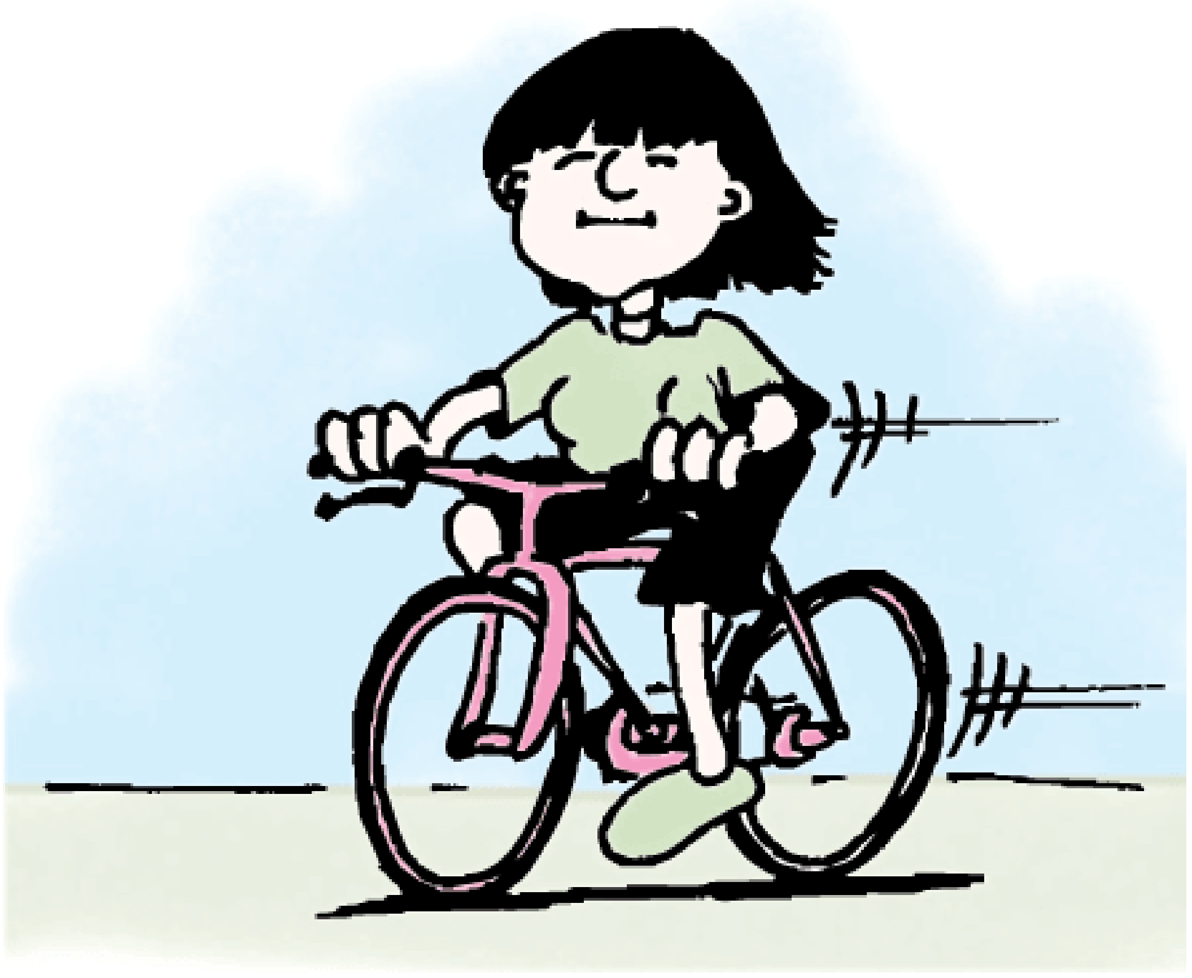
EXERCISE 9C

1 Find the number of days from:

- **a** March 11th to April 7th
- **b** May 11th to June 23rd
- **c** July 12th to November 6th
- **d** September 19th to January 8th
- **e** January 7th to March 16th in a non-leap year
- **f** February 6th to August 3rd in a leap year.

2 Lou Wong is saving money to buy a bicycle. Today is the 23rd of March, and the shop will hold the bicycle until May 7th at the price of $279.

- **a** How many days does Lou have to save for the bicycle?
- **b** How much does Lou need to save each day to reach the $279 target?



3 Candice must take her medication every second day, starting from April 11th. Will she take her medication on November 4th?

4 Sung Kim arrived in Australia on 17th February 2012. On what date did he celebrate being in Australia for 100 days?

Example 6 Self Tutor

What is the time:

- **a** 2 hours 15 minutes after 3:10 pm
- **b** 3 hours 40 minutes before 9:15 am?

a The time is

3:10 pm + 2 hours + 15 minutes
= 5:10 pm + 15 minutes
= 5:25 pm

b The time is

9:15 am − 3 hours − 40 minutes
= 6:15 am − 40 minutes
= 6:15 am − 15 minutes − 25 minutes
= 6:00 am − 25 minutes
= 5:35 am

5 What is the time:

- **a** 4 hours after 3:00 am
- **b** 5 hours before 8:00 pm
- **c** 34 minutes after 6:15 am
- **d** 45 minutes before 7:21 pm
- **e** 2 hours 13 minutes after 8:19 pm
- **f** 3 hours 27 minutes after 12:42 pm
- **g** 2 hours 55 minutes before 2 pm
- **h** 5 hours 18 minutes before noon
- **i** 1 hour 47 minutes after 1:30 pm
- **j** 3 hours 16 minutes before 2 am Mon?

6 High tide at the beach occurs at 1:25 am. The following low tide occurs 6 hours and 20 minutes later. At what time does the low tide occur?

7 David has agreed to meet some friends at his beach house at 12:30 pm. It will take David 1 hour and 40 minutes to drive there. At what time should David start the journey?

Example 7 **Self Tutor**

It is now 10:45 am. Ken is waiting for his plane to take off at 3:20 pm. How long is it before departure?

$$\begin{aligned} \text{10:45 am to 11:00 am} &= \phantom{4\text{ h }}15\text{ min} \\ \text{11:00 am to 3:00 pm} &= 4\text{ h} \\ \text{3:00 pm to 3:20 pm} &= \underline{\phantom{4\text{ h }}20\text{ min}} \\ \therefore\ \text{the time before departure} &= 4\text{ h }35\text{ min} \end{aligned}$$

8 Find the time difference between:

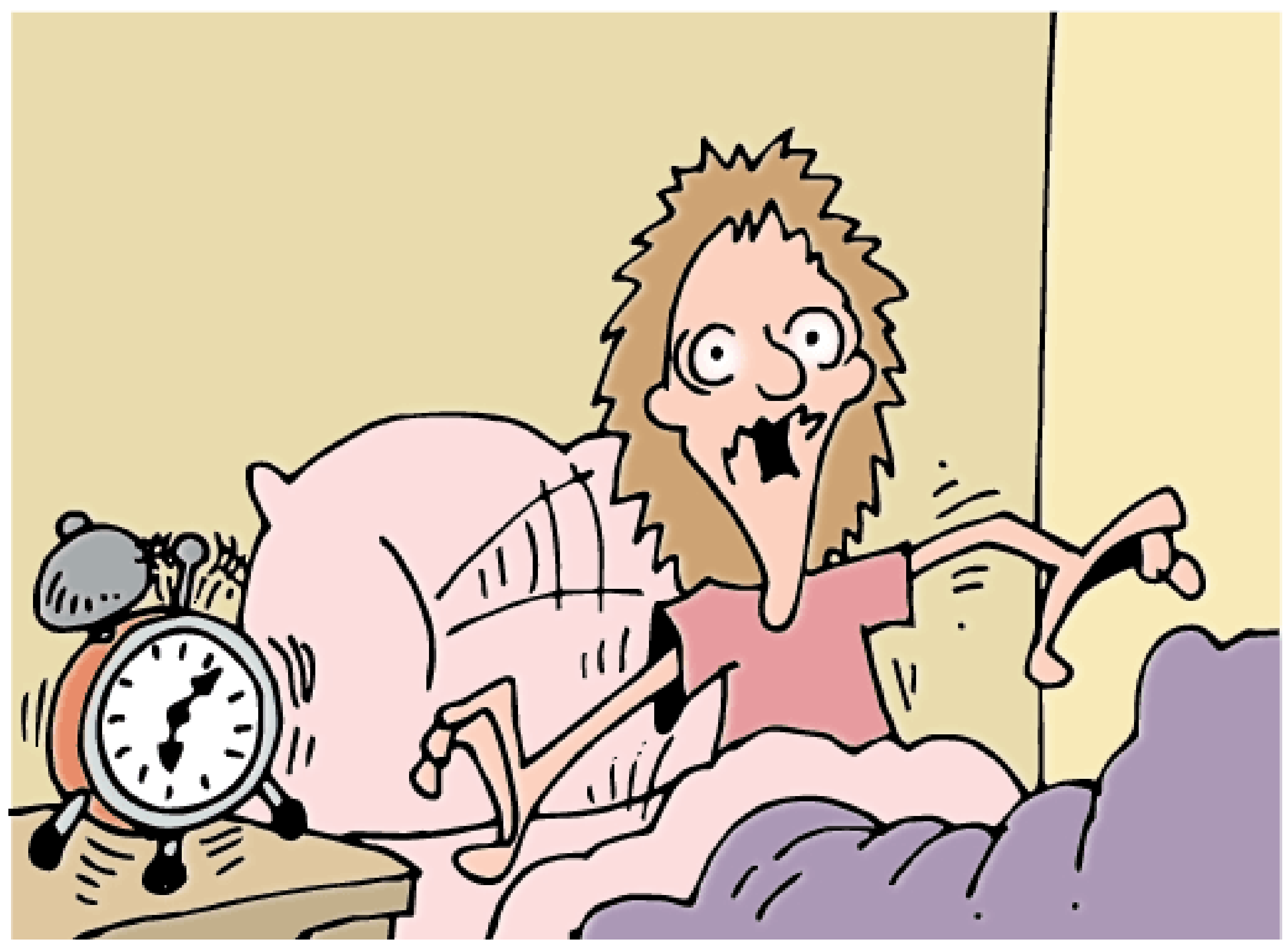
- **a** 7:50 am and 10:10 am
- **b** 11:45 am and 5:20 pm
- **c** 3:30 pm and 9:46 pm
- **d** 8:24 am and 7:51 pm.

9 A cyclist rides from 7:47 am until 11:31 am. For how long did the cyclist ride?

10 Jody sent a text message to her friend Monique at 6:27 pm. Monique replied to Jody at 9:09 pm. How long did Monique take to reply?

11 Shelley fell asleep at 10:26 pm. She woke up at 6:05 am the next day. For how long did she sleep?

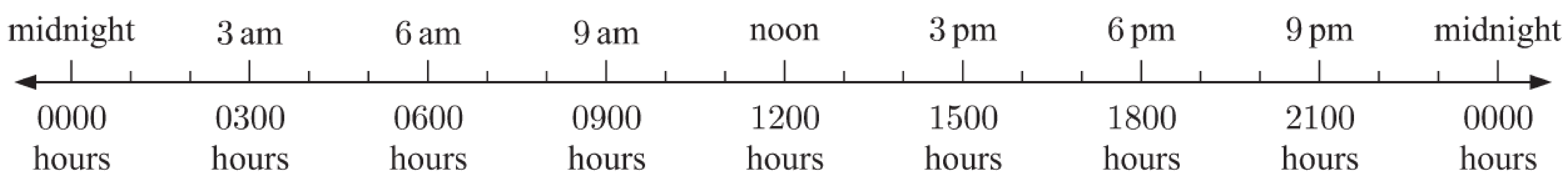
12 Mary's watch loses 3 seconds every hour. If it shows the correct time at 8 am on Wednesday, how slow will it be when the real time is 5 pm on Friday of the same week?



D 24-HOUR TIME

When a digital clock displays the time 6:20, this could mean 6:20 am or 6:20 pm. To avoid this confusion, we can use **24-hour time**.

24-hour time indicates the amount of time which has passed since midnight.



A 24-hour clock would display 6:20 for 6:20 am, and 18:20 for 6:20 pm.

FOUR DIGIT NOTATION

We use four digit notation when writing 24-hour time.

7:15 am appears as 7: 15, and is written as 0715 hours.

8:15 pm appears as 20: 15, and is written as 2015 hours.

In the table below we compare 12-hour time and 24-hour time:

12-hour time	*Digital display*	*24-hour time*
midnight	0:00	0000 hours
7:42 am	7:42	0742 hours
midday (noon)	12:00	1200 hours
11:29 pm	23:29	2329 hours


24-hour time always uses 4 digits.

Example 8 Self Tutor

Write in 24-hour time:

a 8:55 am **b** 9:25 pm

a 8:55 am is 0855 hours. **b** 9:25 pm is 2125 hours.

Example 9 Self Tutor

Write in 12-hour time:

a 0734 hours **b** 1630 hours

a 0734 hours is 7:34 am.

b 1630 hours
= 12 hours + 4 hours 30 minutes
= 4:30 pm

Times from 0000 hours to 1159 hours are am times.

Times from 1200 hours to 2359 hours are pm times.

EXERCISE 9D

1 Write in 24-hour time:

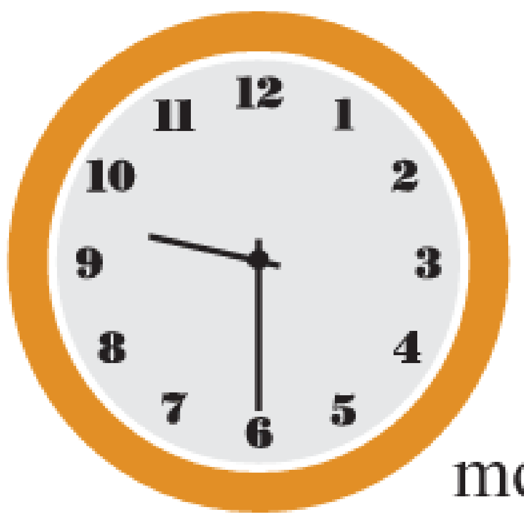
a 3:13 am **b** 11:17 am **c** midnight **d** 12:47 pm
e 5:41 pm **f** noon **g** 8:19 pm **h** 11:59 pm

2 Write in 12-hour time:

- **a** 0300 hours
- **b** 0630 hours
- **c** 1800 hours
- **d** 1200 hours
- **e** 0615 hours
- **f** 1545 hours
- **g** 2017 hours
- **h** 2348 hours

3 Write the following as 24-hour times:

a



morning

b

afternoon

c

evening

4 What, if anything, is wrong with the following 24-hour times?

- **a** 0862 hours
- **b** 0713 hours
- **c** 2541 hours

5 The following arrivals appear on a display at Singapore Changi Airport.

- **a** Which flight is due to arrive at 3:50 pm?
- **b** A thunderstorm delays the arrival of flight QF14 by 1 hour 35 minutes. At what time will the flight arrive? Write your answer using 24-hour time.

ARRIVALS		
Flight	*From*	*Arrival time*
JAL130	Tokyo	14:50
BA10	London	15:50
SQ71	Rome	16:25
QF14	Perth	16:45
EM16	Dubai	17:15

E TIMETABLES

Timetables are tables of information which tell us when events are to occur.

We can use timetables to find out when the next bus or train is coming, when our favourite television show will be on, and when the sun will rise tomorrow.

Example 10 **Self Tutor**

This timetable gives information about the phases of the moon, and the rising and setting of the planets of our solar system.

- **a** When is the next full moon?
- **b** At what time will:
  - **i** Mercury rise tomorrow
  - **ii** Saturn set tomorrow?

The Moon

New	First $\frac{1}{4}$	Full	Last $\frac{1}{4}$
●	◐	○	◑
Sep 21	Sep 29	Oct 6	Oct 12

The Sun and Planets

Tomorrow	Rise	Set
Sun	6:15 am	6:07 pm
Moon	2:29 am	1:00 pm
Mercury	5:59 am	5:20 pm
Venus	5:51 am	5:09 pm
Mars	4:43 am	3:11 pm
Jupiter	6:01 am	6:34 pm
Saturn	9:10 am	8:23 pm

a The next full moon is on October 6th.

b **i** Mercury rises at 5:59 am tomorrow. **ii** Saturn sets at 8:23 pm tomorrow.

EXERCISE 9E

1 This timetable shows the classes scheduled for a school day.

Spelling	9:00 - 9:45
Music	9:45 - 10:30
Recess	10:30 - 11:00
Mathematics	11:00 - 11:50
Science	11:50 - 12:40
Lunch	12:40 - 1:40
Sport	1:40 - 2:25
Language	2:25 - 3:10

- **a** At what time does the Science class start?
- **b** At what time does the Spelling class end?
- **c** What class is being taken at 2 pm?
- **d** How long is the:
  - **i** Music class
  - **ii** Mathematics class?
- **e** How long is the school day?

2 The timetable of events at a school camp is shown alongside.

- **a** At what time does the kayaking start?
- **b** Which activity begins at 3:30 pm?
- **c** For how long do the students sing campfire songs?
- **d** How much free time do the students have during the day?

Breakfast	8:30 am - 9:00 am
Kayaking	9:00 am - 10:45 am
Free time	10:45 am - 12:30 pm
Lunch	12:30 pm - 1:30 pm
Nature hike	1:30 pm - 3:00 pm
Free time	3:00 pm - 3:30 pm
Volleyball	3:30 pm - 5:00 pm
Journal writing	5:00 pm - 6:00 pm
Dinner	6:00 pm - 7:00 pm
Chores	7:00 pm - 7:30 pm
Campfire songs	7:30 pm - 9:00 pm
Bedtime	9:00 pm

3 This timetable shows the tide times on a particular day.

Tide times	*High Tide*		*Low Tide*	
Port Xenon	3:00 am	1.2 m	9:56 am	0.4 m
	3:07 pm	0.7 m	7:21 pm	0.3 m
Port Dowell	3:54 am	1.5 m	10:37 am	0.2 m
	4:37 pm	1.0 m	10:09 pm	0.4 m
Windcok	2:05 am	1.9 m	8:53 am	0.6 m
	3:52 pm	1.4 m	9:02 pm	0.3 m
Joseph's Bay	3:07 am	1.6 m	8:58 am	0.5 m
	3:05 pm	1.4 m	8:52 pm	0.3 m
Paradise Point	4:35 am	2.5 m	10:27 am	0.9 m
	4:16 pm	2.2 m	10:27 pm	0.5 m
Sunny Inlet	2:44 am	1.1 m	9:27 am	0.4 m
	2:26 pm	0.7 m	8:06 pm	0.3 m

- **a** When is the tide highest in the morning at Port Xenon?
- **b** When is the tide lowest in the evening at Paradise Point?
- **c** What is the lowest tide at Joseph's Bay in the morning? At what time does it occur?
- **d** What is the highest tide at Port Dowell in the afternoon? At what time does it occur?

4

Channel 4

05:30	Weather Watch	14:30	Movie Matinee
06:30	Roger Robot	16:45	Cartoon Capers
07:00	Cartoon Collection	17:30	Pick a Prize
08:00	Dazzlers	18:00	News and Weather
08:30	Kids Korner	19:00	Animal Antics
09:30	Hot Hits	19:30	North Park
11:00	Growing Gardens	20:30	Saturday Special
11:30	Football Flashbacks	23:15	Sports Roundup
12:00	Spectator Sports	00:10	Temporary close

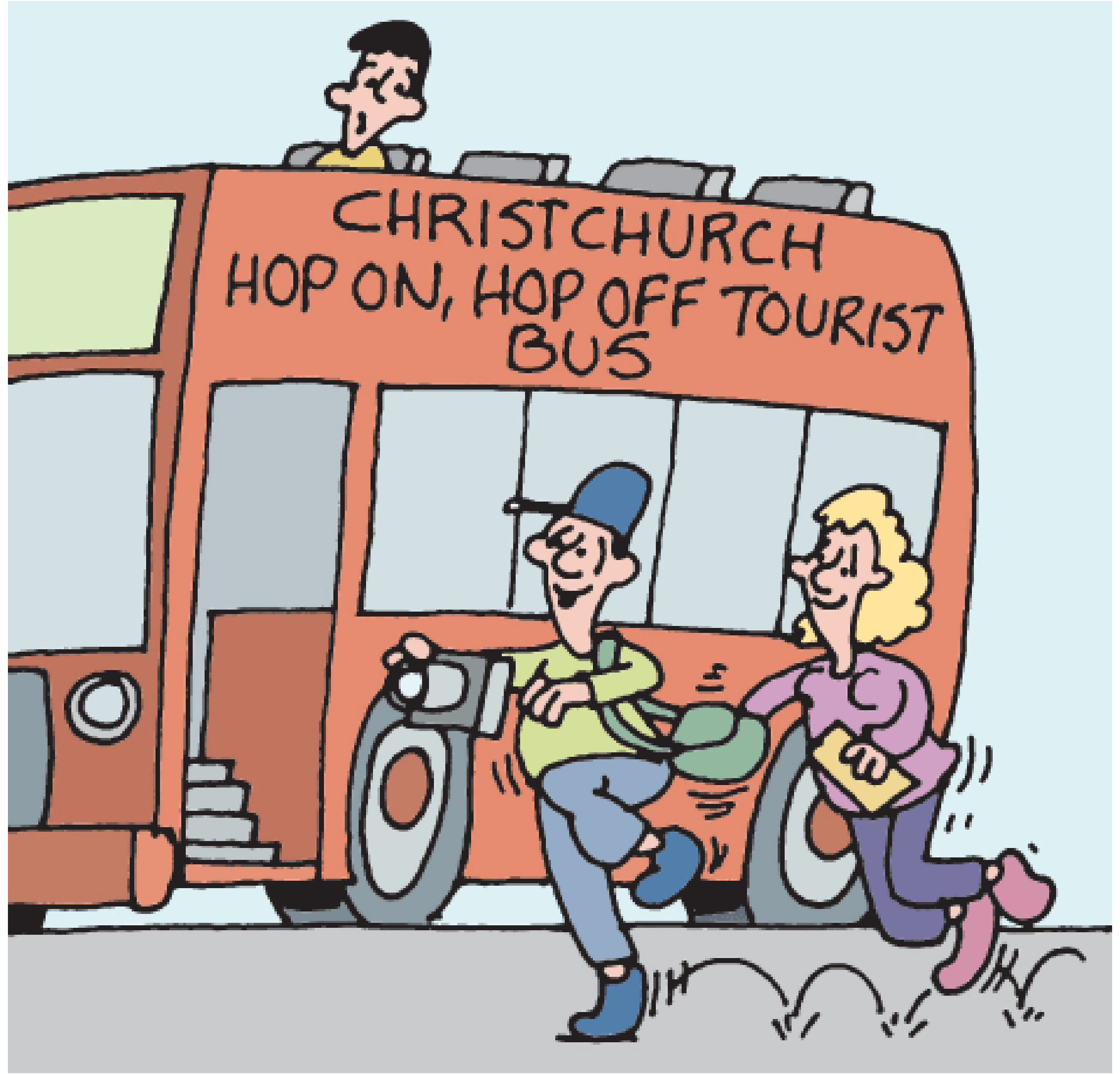
Use the television guide to answer the following questions:

- **a** At what time does Kids Korner start?
- **b** Which program starts at 7:30 pm?
- **c** For how many minutes is the News and Weather shown?
- **d** In total, how much time is spent showing sport?
- **e** How much time is there between the end of Cartoon Collection and the start of Cartoon Capers?

5 Below is the summer timetable for a tourist bus service in Christchurch, New Zealand.

Departure Times	*Bus A*	*Bus B*	*Bus C*	*Bus D*	*Bus E*	*Bus F*
Central Station	9:00	9:15	9:30	9:45	10:00	10:15
Canterbury Museum	9:30	9:45	10:00	10:15	10:30	10:45
Christchurch Gondola	10:50	11:05	11:20	11:35	11:50	12:05
Lyttelton Harbour	11:20	11:35	11:50	12:05	12:20	12:35
Akaroa	1:00	1:15	1:30	1:45	2:00	2:15
Airforce World	2:30	2:45	3:00	3:15	3:30	3:45
Yaldhurst Transport Museum	3:15	3:30	3:45	4:00	4:15	4:30
International Antarctic Centre	4:30	4:45	5:00	5:15	5:30	5:45
Arrive at Central Station	5:30	5:45	6:00	6:15	6:30	6:45

- **a** How many bus services are available?
- **b** What is the latest departure time?
- **c** What is the earliest arrival time back at Central Station?
- **d** How long does it take to get from:
  - **i** Lyttelton Harbour to Airforce World
  - **ii** the Gondola to the International Antarctic Centre?
- **e** How long does a complete trip last?
- **f** If you wanted to arrive at Akaroa at 2:00, which bus should you take?
- **g** If you are to meet a friend at Yaldhurst Transport Museum at 3:25, which bus is it best to travel on?



6 Consider the train timetable for the Carlingford to Wynyard line.

a What does it mean by:

i arr **ii** dep?

b If I catch the 4:17 pm train at Rydalmere, at what time will I arrive at Central?

c At what time should I catch the train from Dundas in order to arrive at Lidcombe by 6:00 pm?

d If I miss the 5:00 pm train from Clyde, what is the earliest time I can now arrive at Wynyard?

e **i** If I come out of the cinema at 3:45 pm at Carlingford, when is the first train that I can catch to Strathfield?

ii At what time will this train reach Strathfield?

CARLINGFORD-WYNYARD TRAIN TIMETABLE

	pm	pm	pm	pm	pm	pm	pm
Carlingford	3:32	4:11	4:45	5:23	5:55	6:26	6:52
Telopea	3:34	4:13	4:47	5:25	5:57	6:28	6:54
Dundas	3:36	4:15	4:49	5:27	5:59	6:30	6:56
Rydalmere	3:38	4:17	4:51	5:29	6:01	6:32	6:58
Camellia	3:40	4:19	4:53	5:31	6:03	6:34	7:00
Rosehill UA	3:42	4:21	4:55	5:33	6:05	6:36	7:02
Clyde ....**arr**	3:45X	4:24X	4:58X	5:36X	6:08X	6:39X	7:05
dep	3:51	4:26	5:00	5:48	6:18	6:48	7:06
Lidcombe.....**arr**	3:55	4:29	5:04	5:52	6:22	6:52	7:10
dep	3:57	4:31	5:06	5:54	6:24	6:54	7:12
Strathfield....**arr**	4:02	4:36	5:11	5:59	6:29	6:59	7:18X
dep	4:03	4:37	5:12	6:00	6:30	7:00	7:23
Central........**arr**	4:17	4:50	5:26	6:14	6:44	7:14	7:36
dep	4:18	4:51	5:27	6:15	6:45	7:15	7:37
Townhall	4:21	4:54	5:30	6:18	6:48	7:18	7:40
Wynyard	4:24	4:57	5:33	6:20	6:50	7:20	7:42

PUZZLE CINEMA SCHEDULING

Imagine you operate a cinema complex which contains 3 cinemas. During the day, you want to show:

- 5 screenings of film A, which is 160 minutes long
- 5 screenings of film B, which is 70 minutes long
- 3 screenings of film C, which is 100 minutes long
- 3 screenings of film D, which is 120 minutes long.

In addition, these rules must be followed:

- No film must start earlier than 10 am.
- No film must start later than 9 pm.
- No film must finish later than 11 pm.
- There must be at least 30 minutes between films in the same cinema.
- There must be at least 10 minutes between the starting times of films in different cinemas.
- There must be at least 60 minutes between the starting times of the *same film* in different cinemas.

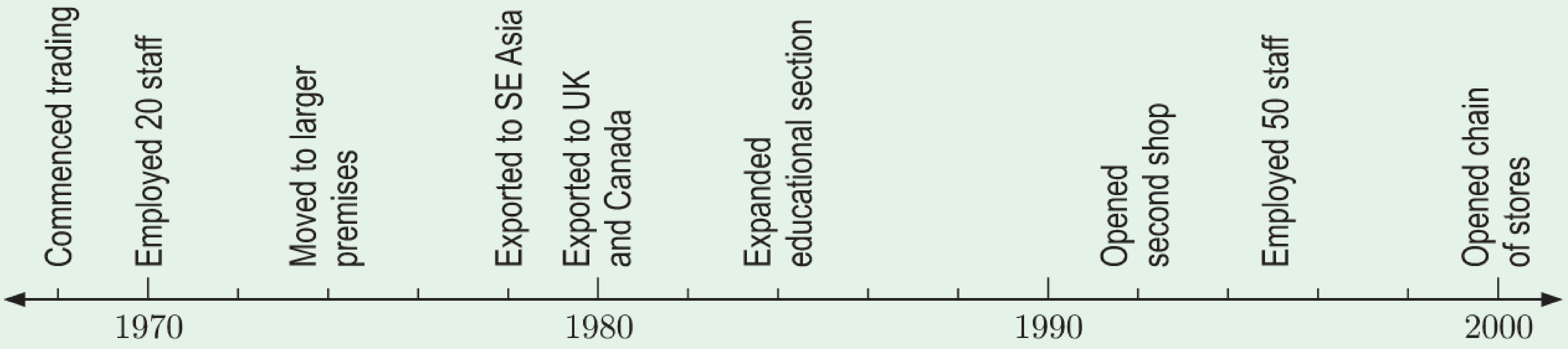
Can you create a schedule of films which satisfies all of these requirements?

KEY WORDS USED IN THIS CHAPTER

- 24-hour time
- leap year
- second
- week
- day
- minute
- time line
- year
- hour
- month
- timetable

REVIEW SET 9A

1 The following time line shows some important dates in the history of The Book Company.



a When did The Book Company:

i start trading **ii** employ 20 staff

iii export to SE Asia **iv** expand their educational section?

b How many years was it between when The Book Company:

i employed 20 staff and employed 50 staff

ii opened a second shop and opened a chain of stores?

2 Convert:

a 12 hours to minutes **b** 1 week to hours **c** 72 hours to days.

3 Write 168 minutes in hours and minutes.

4

TIMETABLE - Melbourne to Sydney		
Melbourne	depart	7:30 am
Albury	arrive	11:10 am
	depart	11:50 am
Canberra	arrive	4:05 pm
	depart	4:45 pm
Goulburn	arrive	5:55 pm
	depart	6:25 pm
Sydney	arrive	9:30 pm

The Happy Travellers Bus Service has a regular daily run between Melbourne and Sydney.

a Find the time taken to go from:

i Melbourne to Canberra **ii** Canberra to Sydney **iii** Melbourne to Sydney.

b For how long does the bus stop in:

i Albury **ii** Canberra **iii** Goulburn?

c If the bus was 15 minutes late departing Goulburn, at what time did it actually depart?

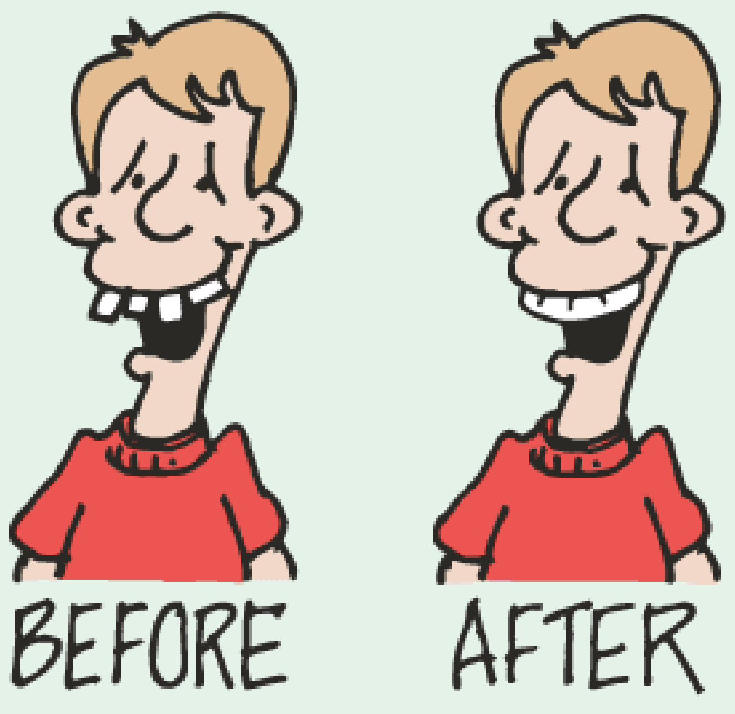
5 Mitchell is spending the day at the beach. He applies sunscreen at 9:40 am, when he arrives. The sunscreen lasts for 2 hours and 30 minutes. At what time should Mitchell reapply sunscreen?

6 Josh began saving £15 a day from the 4th of April. He needs £3000 to have his teeth straightened on September 27th.

a How many days does he have available to save?

b What is the total he will have saved by then?

c How much will he still owe the orthodontist?



7 Write:

a 3 hours 18 minutes in minutes **b** 9 minutes 12 seconds in seconds.

8 Write in 24-hour time:

a 12:32 am **b** 10:15 am **c** 5:49 pm

9 Answer the **Opening Problem** on page **180**.

10 Daphne is going to a concert which starts at 8:20 pm. When she arrives at the concert hall, the clock reads 19:57.

a Write this time in 12-hour time.

b How long will it be before the concert begins?

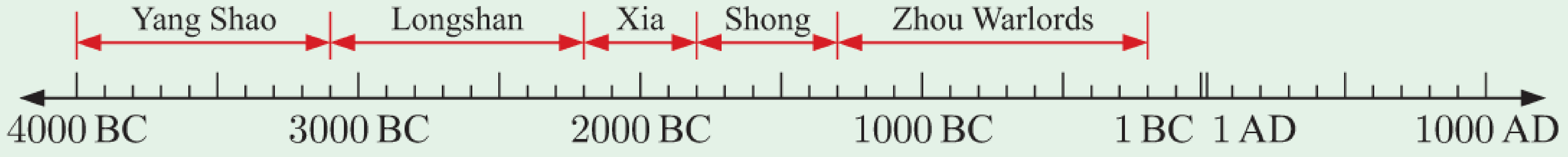
c The concert lasts for 135 minutes.

  i Write 135 minutes in hours and minutes.

  ii At what time will the concert finish?

REVIEW SET 9B

1 The following time line shows the periods of various Chinese civilisations:



a In which year did the Longshan dynasty begin?

b For how long did the Xia dynasty last?

2 What is the time:

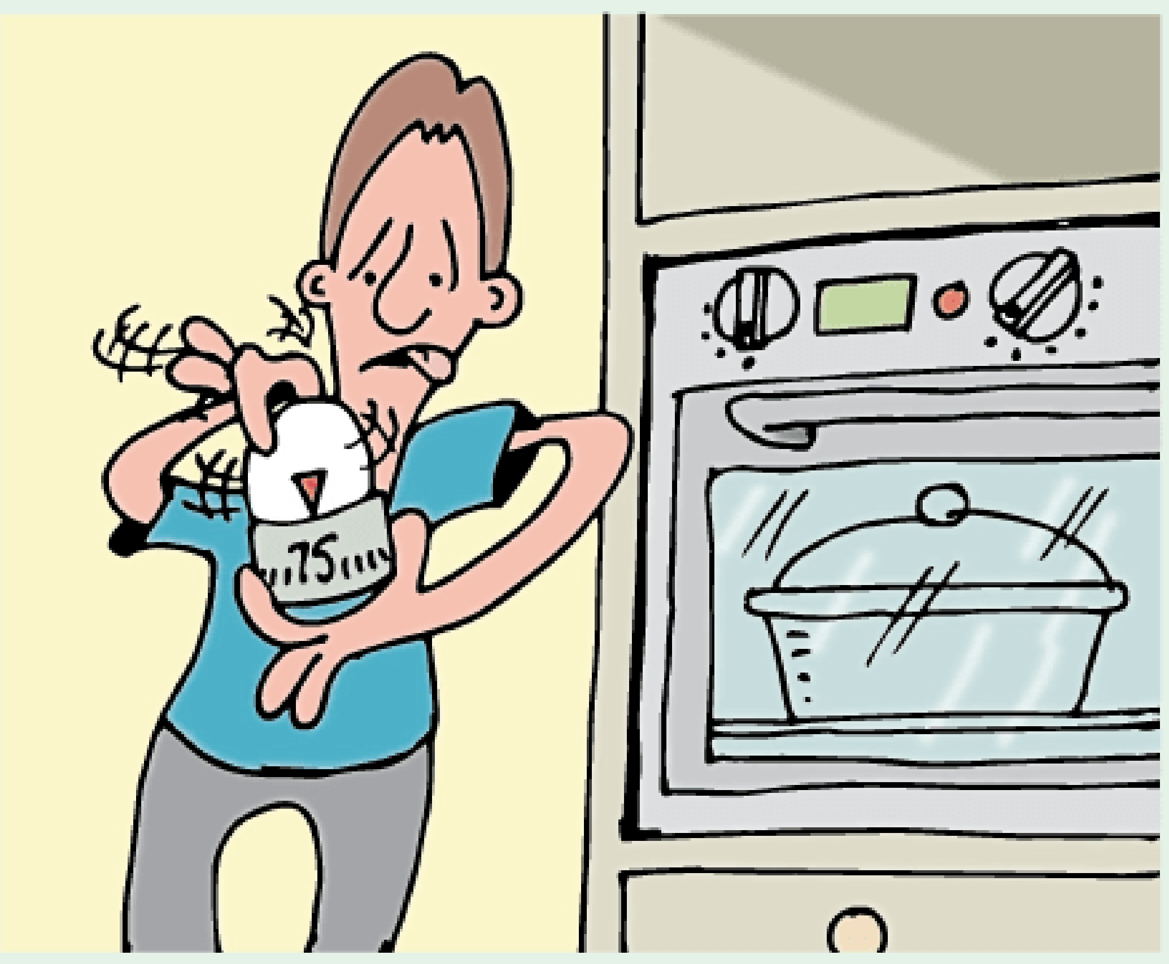
a 6 hours after 10 am **b** 3 hours 21 min before 8 pm?

3 Find the time difference between 11:17 am and 2:56 pm.

4 Find the number of:

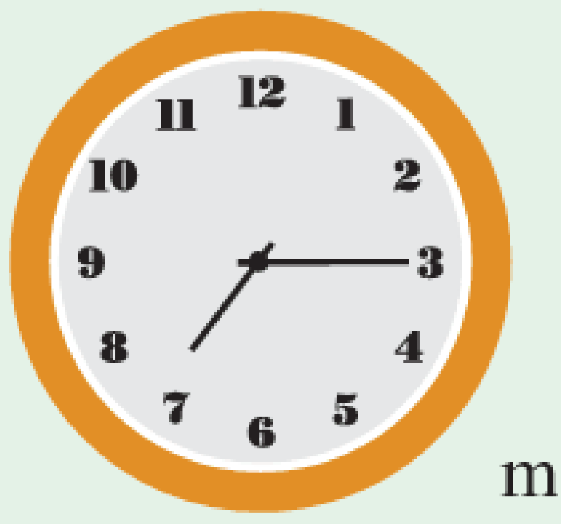
a hours in September **b** days in 2016.

5 Roger needs to cook a casserole in the oven for 75 minutes. If he put the casserole in the oven at 3:57 pm, at what time should he take it out?



6 Write in 24-hour time:

a



b

7 Sean can save $18 each day. Today is May 9th, and on November 20th he wishes to travel to Fiji on a package deal costing $5449. If he does not reach the target of $5449, he will have to borrow the remainder from a bank.

- **a** How many days does Sean have available to save money for the trip?
- **b** How much money will he save in this time?
- **c** Will Sean need to borrow money? If so, how much?

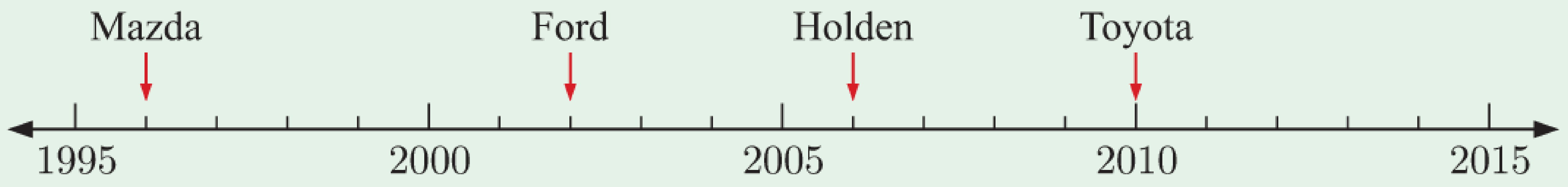
FR. $5449 p.p

BUSINESS CLASS FIJI

• Return business class airfares • 7 days FREE car hire and 5 nights luxury Hilton accommodation.

TRAVELAND HOLIDAYS

8 The time line below shows the cars that Gordon has owned:



- **a** In which year did Gordon buy the Holden?
- **b** Which car did he buy most recently?
- **c** Which car was Gordon driving in 2000?
- **d** For how long did he own the Ford?

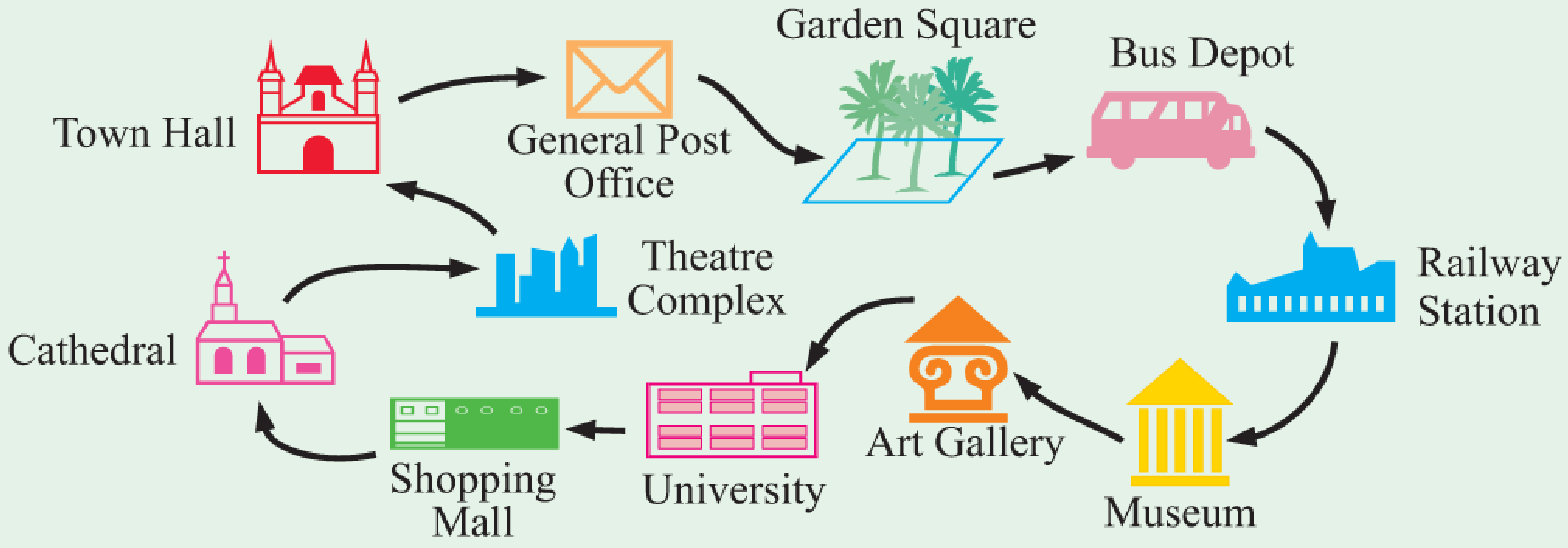
9 Write these 24-hour times in 12-hour time:

- **a** 0415 hours
- **b** 1300 hours
- **c** 2335 hours

10 A City Circuit bus leaves the Town Hall every 10 minutes. Use the timetable given for the 10 am bus to answer the following questions:

Timetable	
Town Hall	10:00 am
General Post Office	10:03 am
Garden Square	10:05 am
Bus Depot	10:08 am
Railway Station	10:10 am
Museum	10:12 am
Art Gallery	10:14 am
University	10:19 am
Shopping Mall	10:23 am
Cathedral	10:25 am
Theatre Complex	10:27 am
Town Hall	10:30 am

- **a** How long does it take the bus to complete one circuit?
- **b** Find how long it takes to go from:
  - **i** the Town Hall to the Railway Station
  - **ii** the Bus Depot to the Shopping Mall
  - **iii** Garden Square to the Cathedral.
- **c** Write the times that the *next* bus will depart from the:
  - **i** Town Hall
  - **ii** Museum.
- **d** How many other buses will leave the Town Hall before this bus returns?



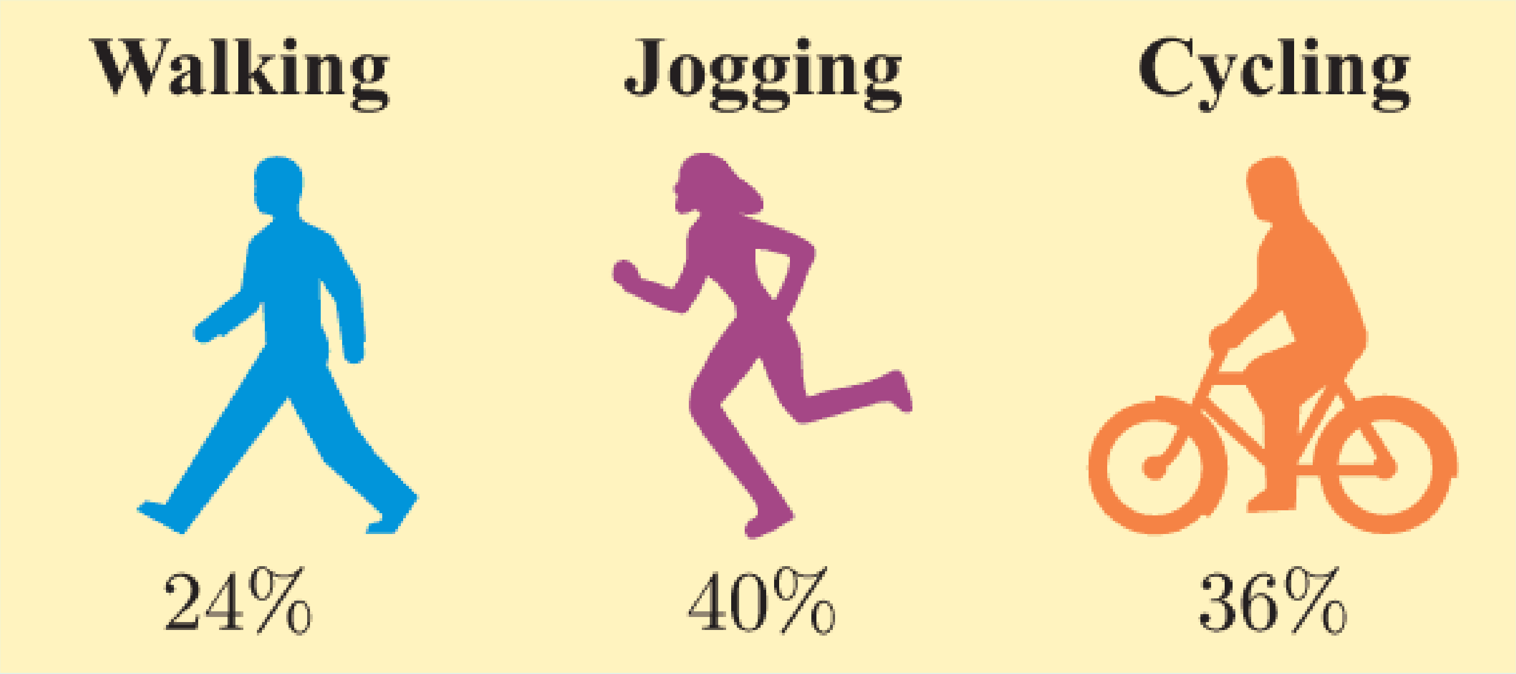
Chapter 10

Percentage

Contents:

OPENING PROBLEM

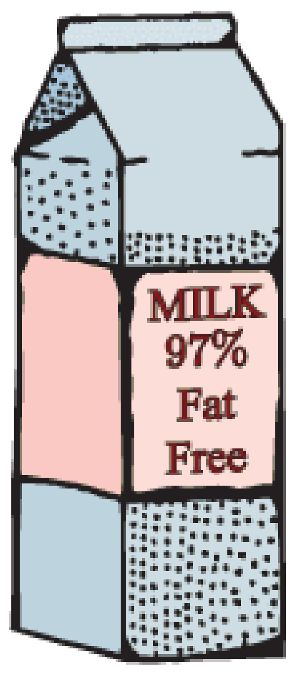
In order to predict maintenance costs, a survey was conducted to determine how the trails in a park are used. Each person was asked what they mainly used the trails for. The results are shown alongside.



Things to think about:

- **a** What does 24% mean?
- **b** Find the sum of the percentages. What do you notice?
- **c** What *fraction* of people use the trails for jogging?
- **d** Suppose 300 people use the trails on one day. About how many people would you expect to use the trails for cycling?

You have probably seen the symbol % many times:



CHEAP LOANS

ONLY 3.95%
INTEREST!

Values such as these are called **percentages**. They are used to compare a portion with a whole amount.

A PERCENTAGES

In **Chapter 6** we used **fractions** to write portions of a whole.

For example, the fraction $\frac{3}{5}$ means we divide a whole into 5 equal parts, and consider 3 of the parts.

A **percentage** is another way to compare a portion with a whole.

Suppose we divide a whole into 100 equal parts. We say that each part is 1% or "one per cent" of the whole.

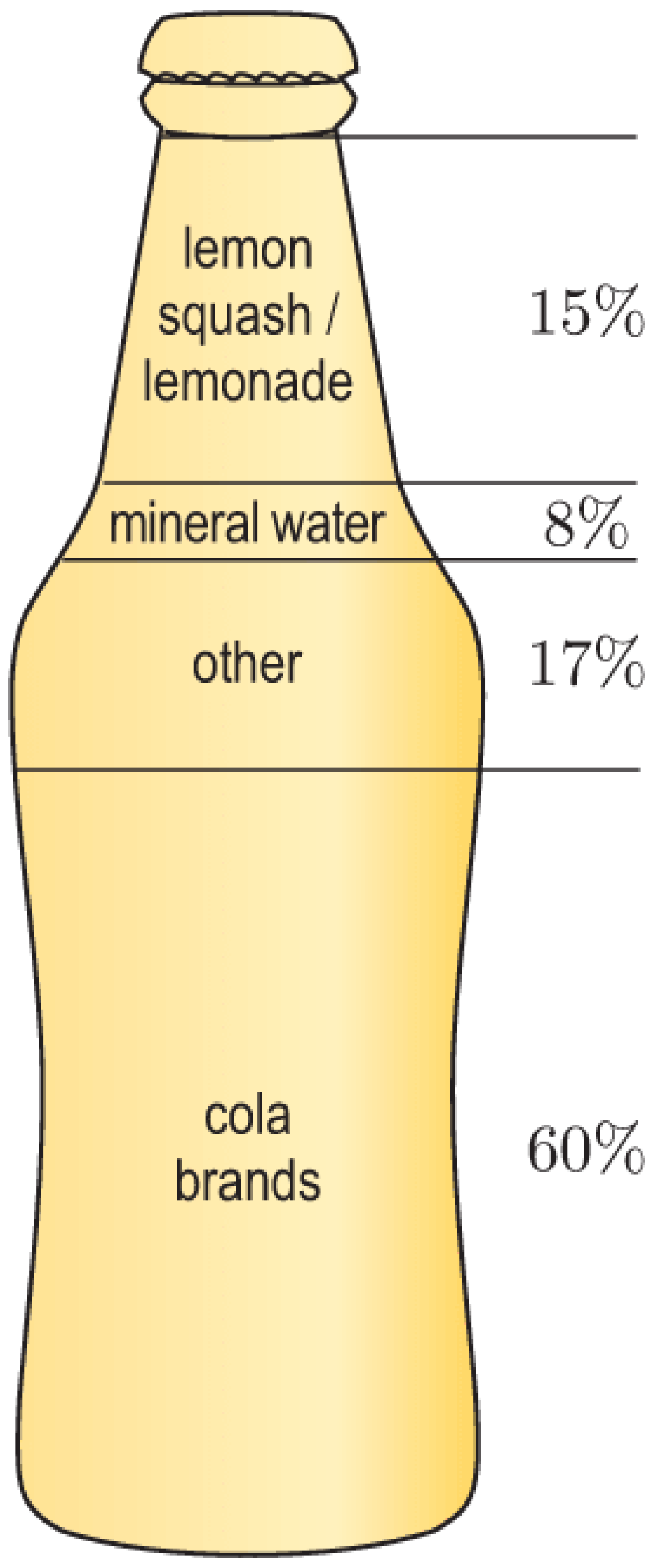
Percentages are like fractions which have denominator 100.

For example, $12\% = \frac{12}{100}$ and means "12 out of every 100".

The diagram alongside shows the sales of different types of soft drink. Notice that:

- we cannot see *how many* of each soft drink type were sold, but we can *compare* the sales of the different types
- the percentages add up to 100%, which represents all soft drink sales.

> A **percentage** is a comparison with a whole, which is 100%.



Sales of all carbonated soft drinks

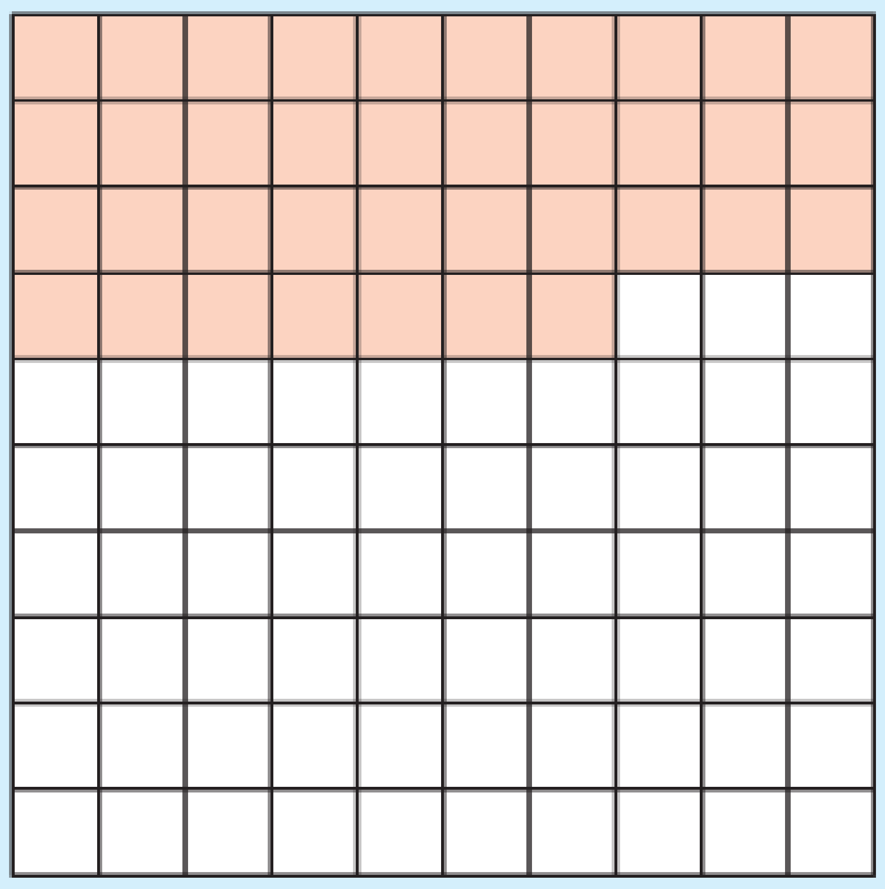
Example 1

Self Tutor

There are 100 tiles in this pattern.

Write the portion of tiles which are coloured as a:

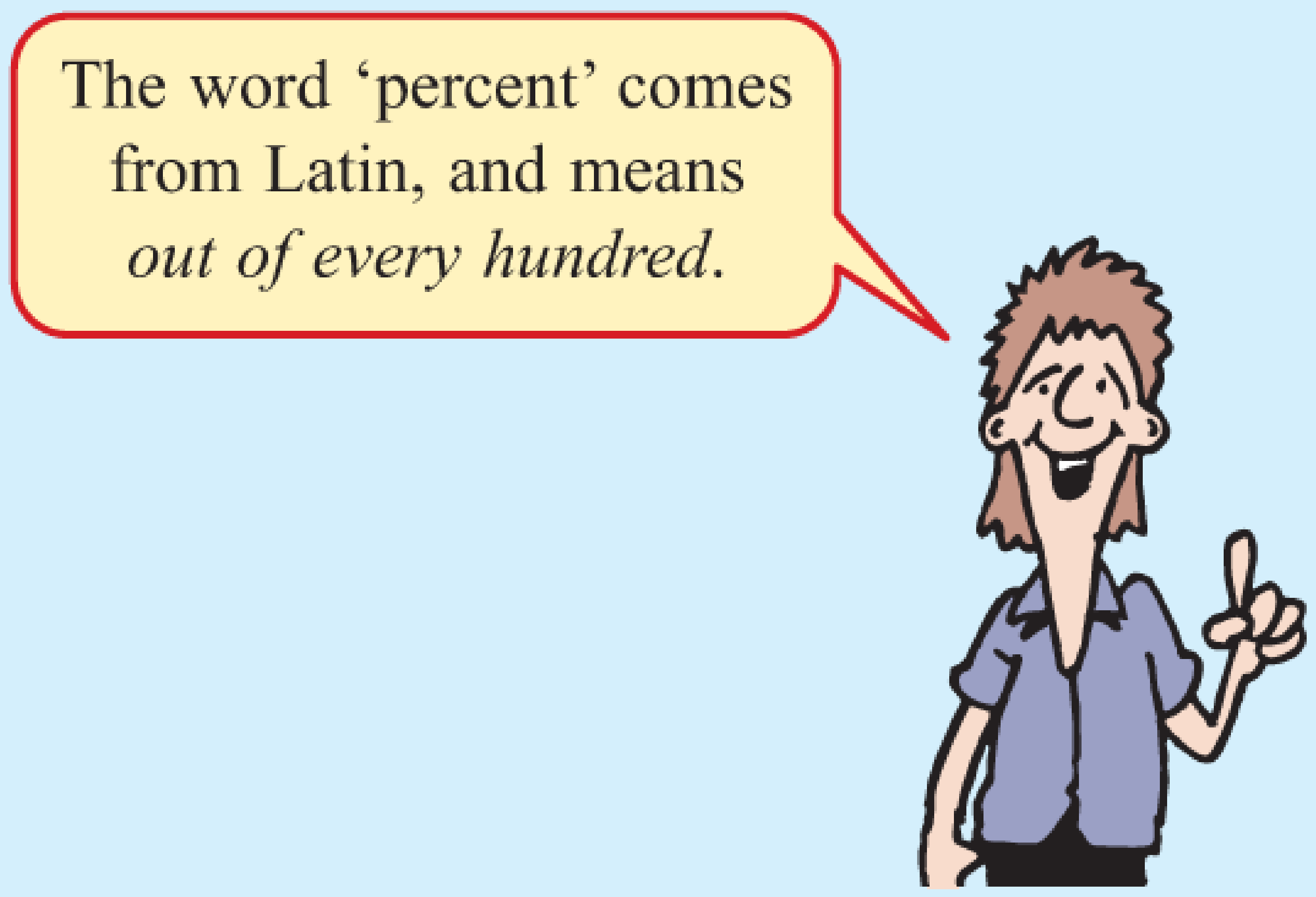
a fraction **b** percentage.



Of the 100 tiles, 37 are coloured.

a $\frac{37}{100}$ of the tiles are coloured.

b 37% of the tiles are coloured.

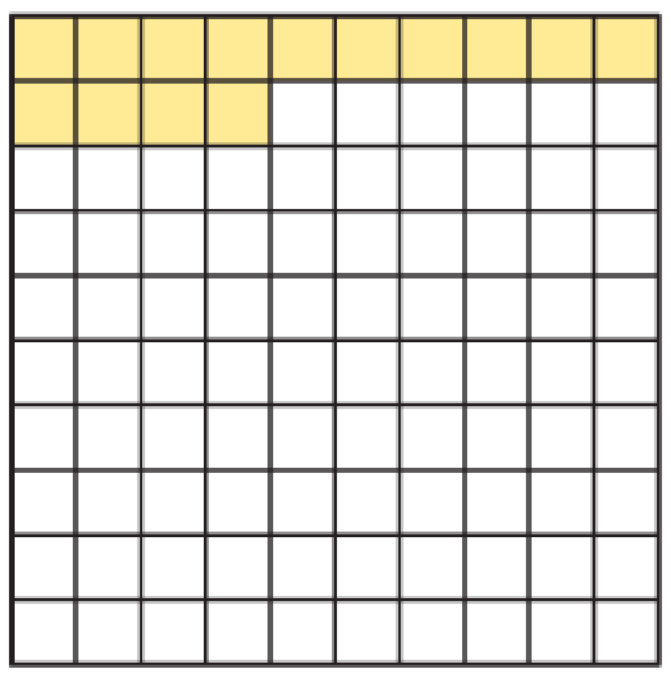


EXERCISE 10A

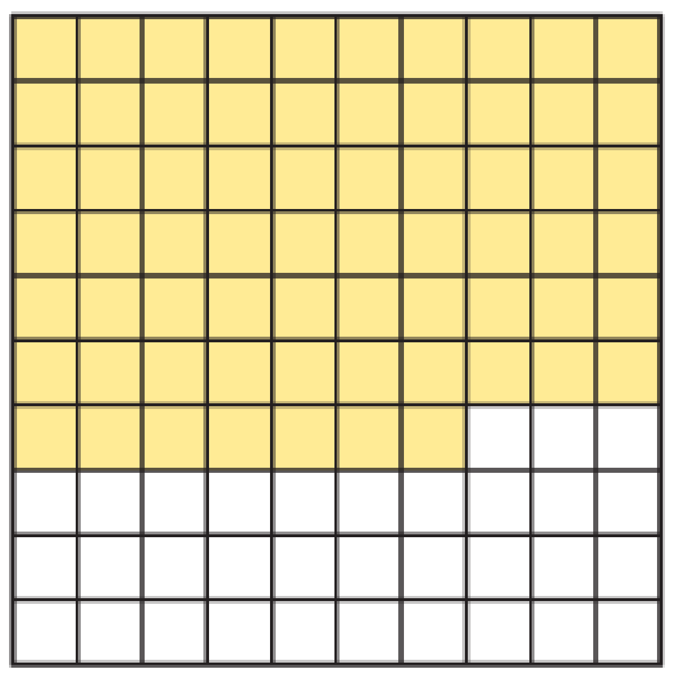
1 There are 100 tiles in each of the following patterns. For each pattern:

- **i** Write the fraction of tiles which are coloured, leaving your answer with the denominator 100.
- **ii** Write the percentage of tiles which are coloured.

a



b



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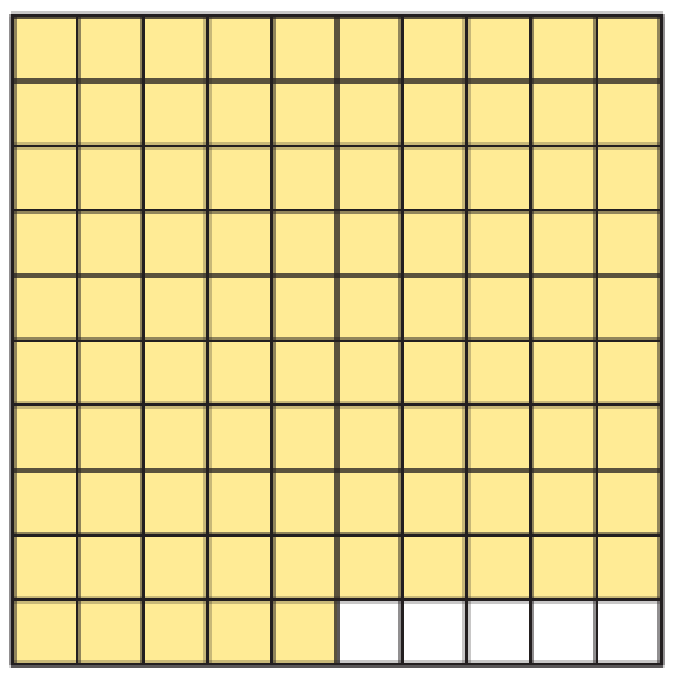

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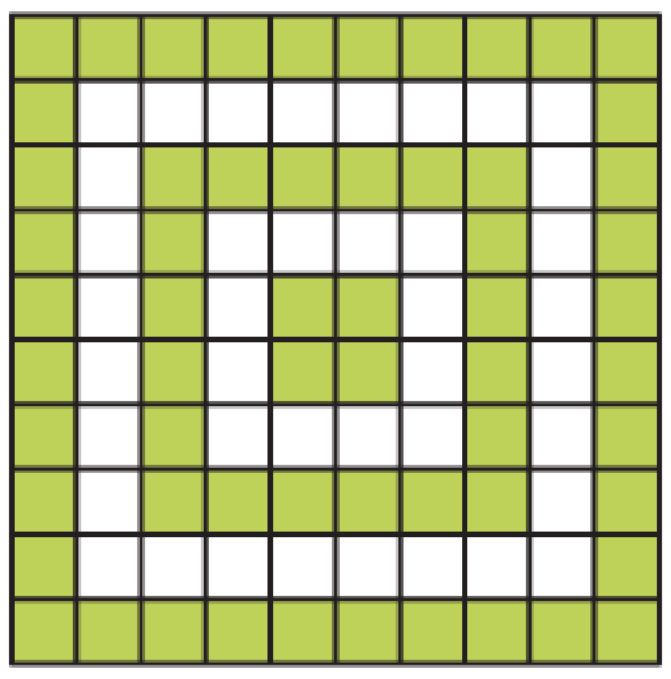

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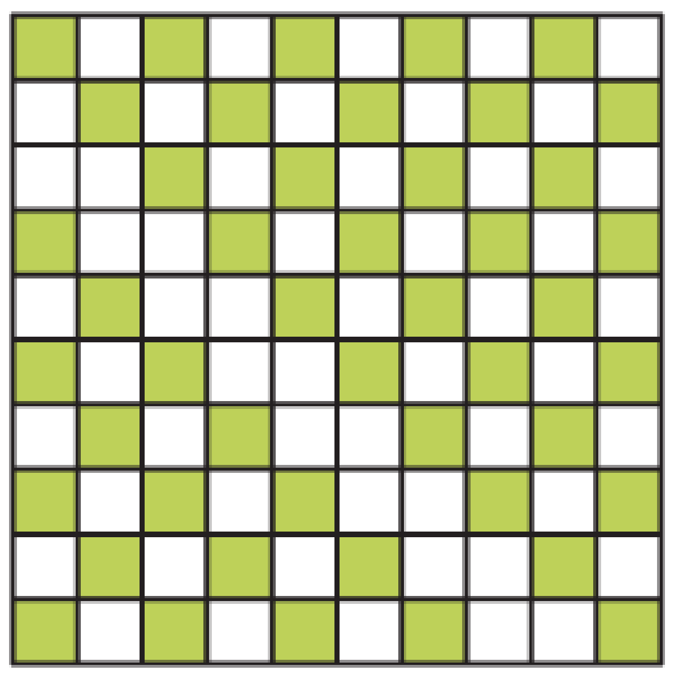

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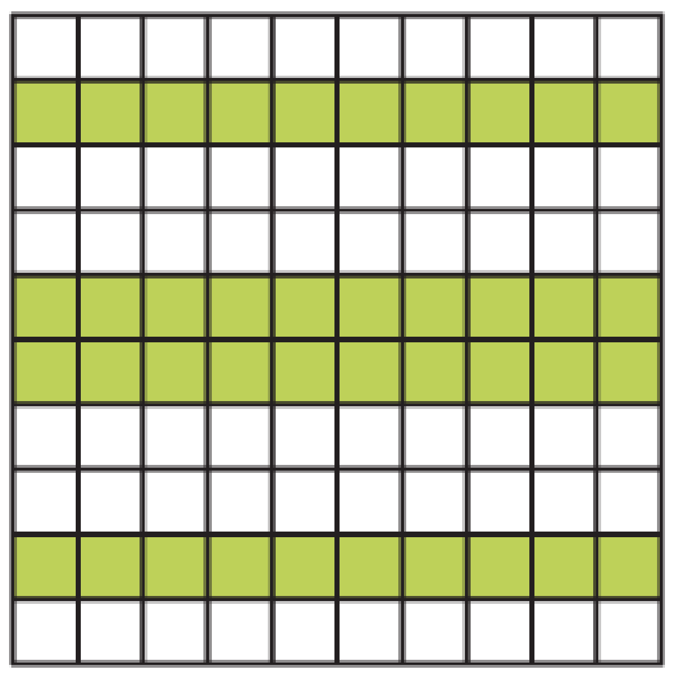
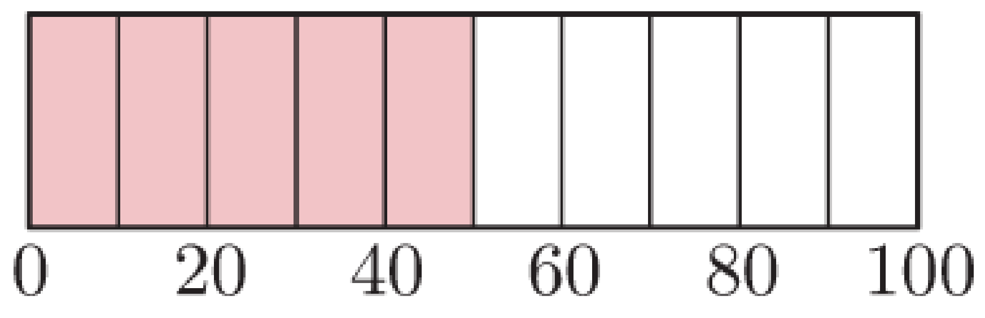


2 Estimate the percentage of each diagram which is shaded:

a



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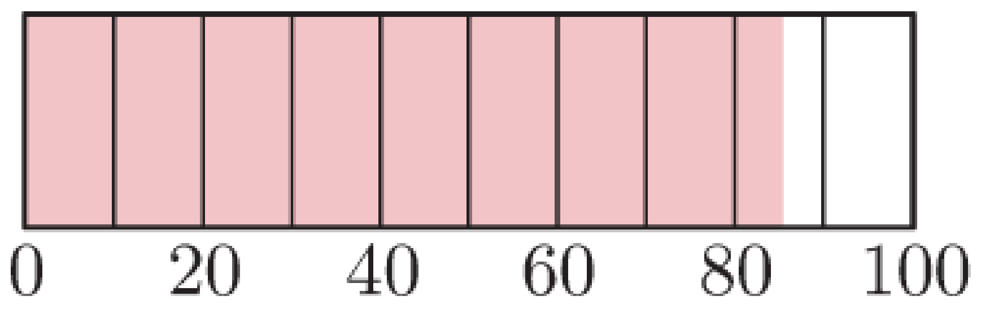
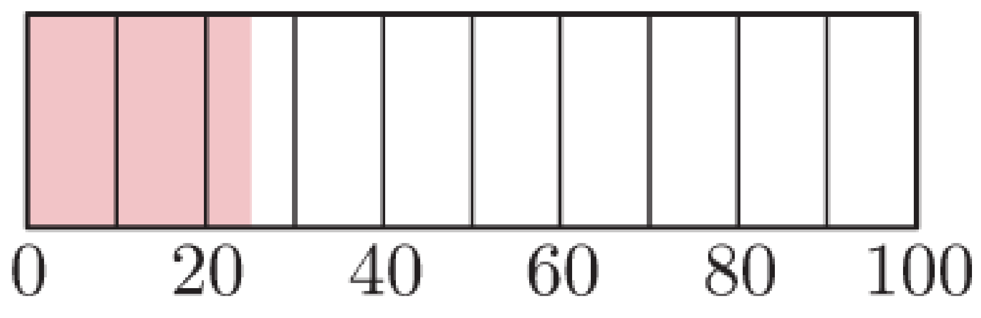
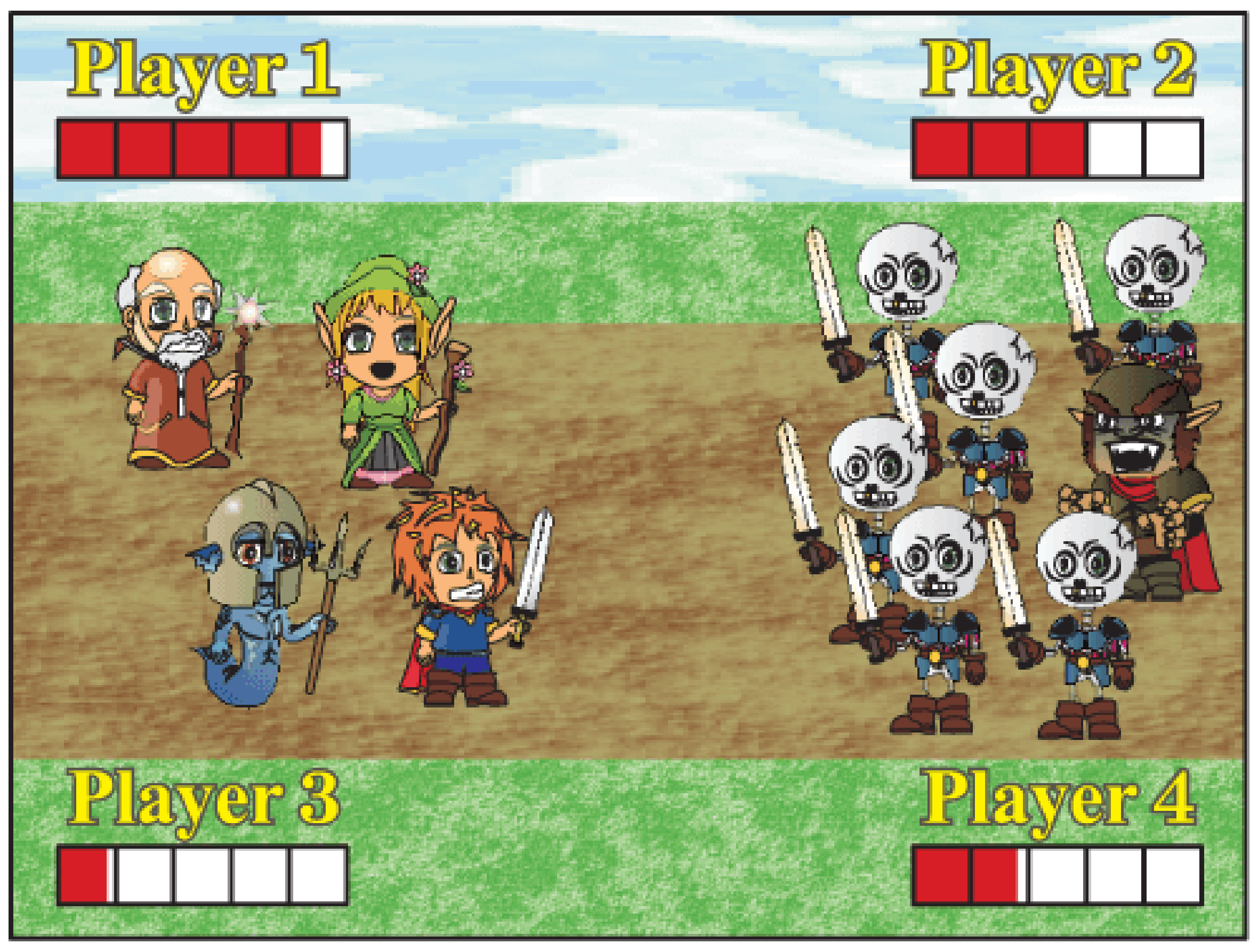


c



3 In a 4 player video game, the red bars show how healthy each player is.
Which player has a health level of:

a 60%
b 36%
c 91%
d 17%?

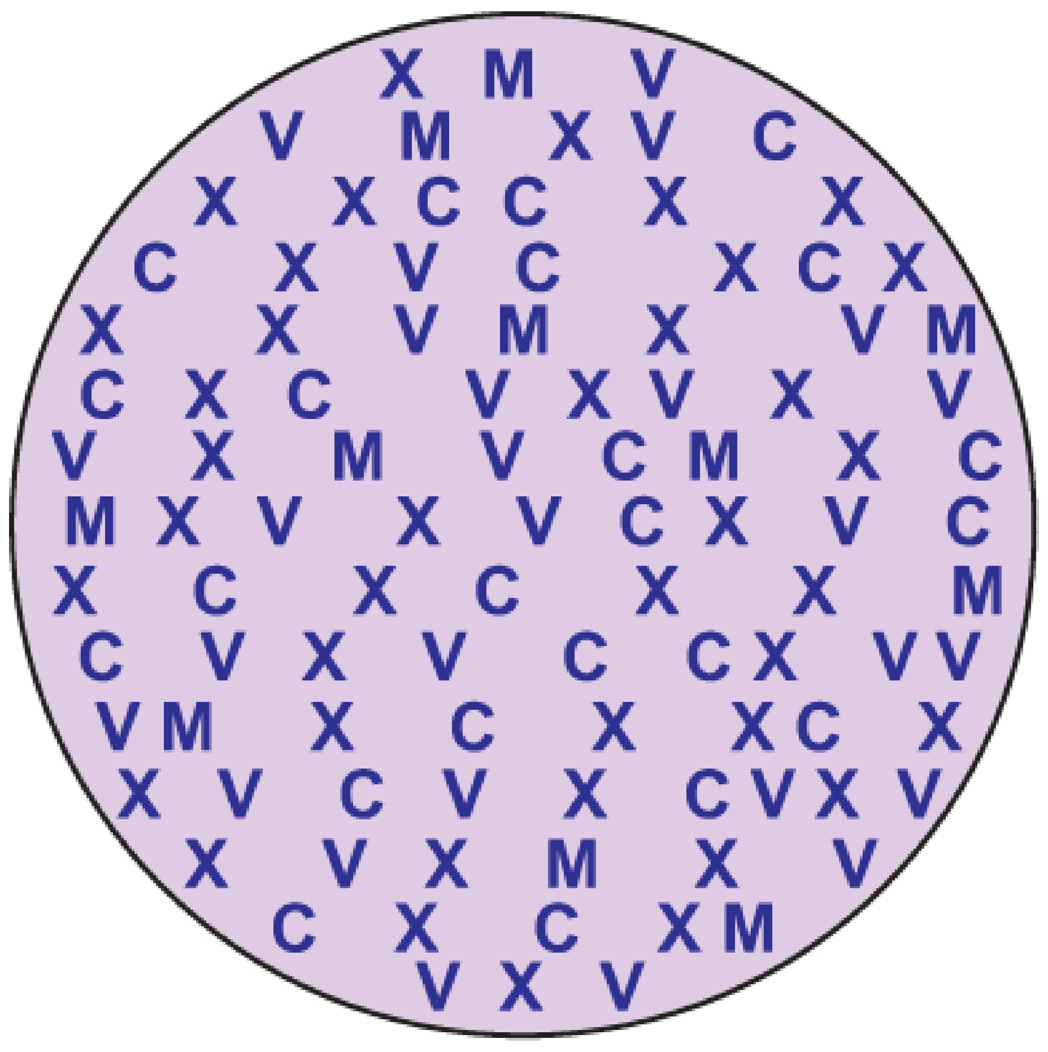


4 There are 100 symbols in the circle.

a What fraction of the symbols are:
i M **ii** C **iii** X **iv** V?

b What percentage of the symbols are:
i M **ii** C **iii** X **iv** V?

c Find the sum of the percentages in **b**. Explain this result.

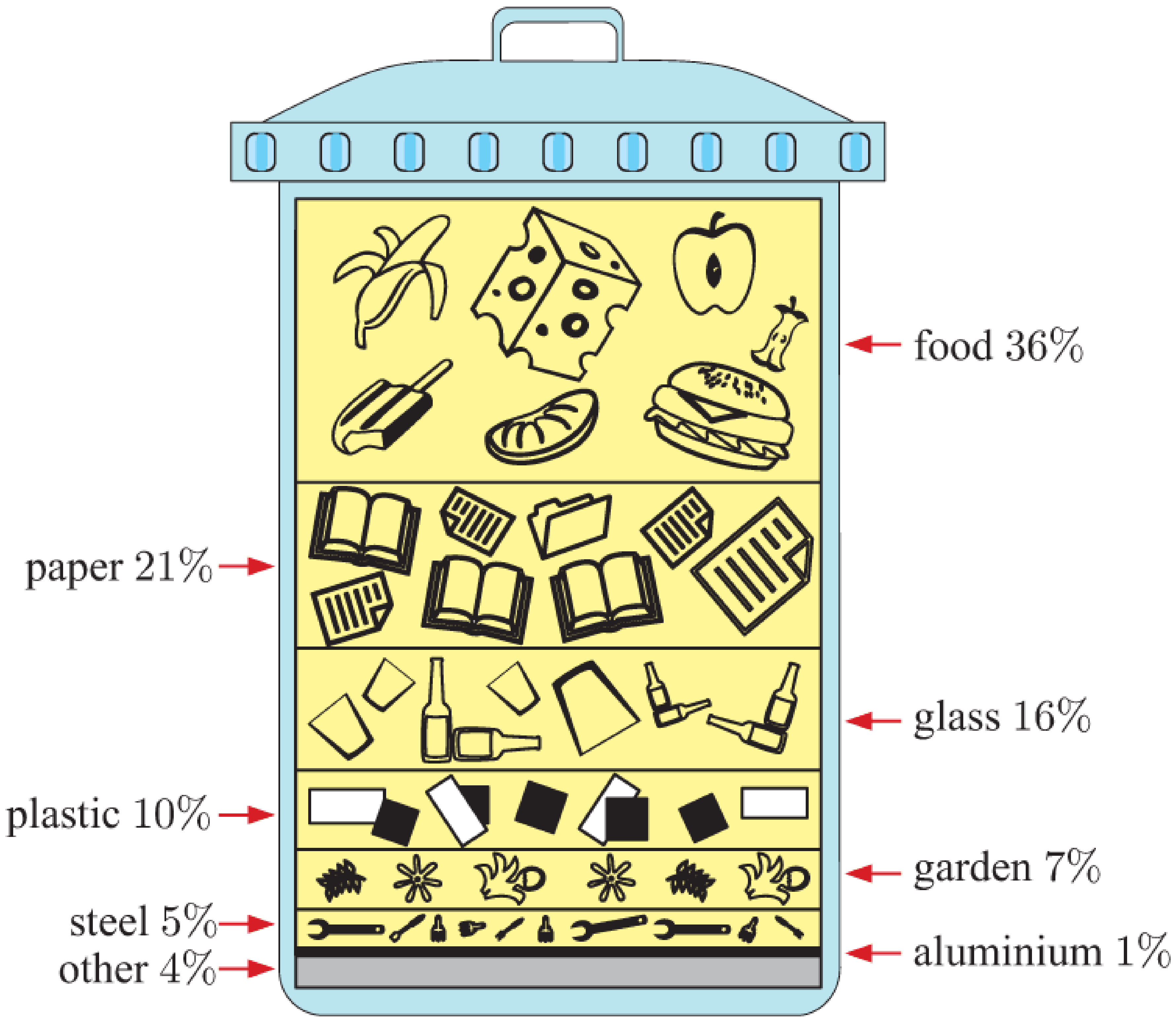


5 The diagram illustrates the different things which we throw away.

a Check that the percentages add to 100%.

b Does paper or glass make up the greater percentage of waste?

c What percentage of waste is:
i food **ii** steel
iii either plastic or aluminium?



B CONVERTING BETWEEN PERCENTAGES AND FRACTIONS

CONVERTING PERCENTAGES TO FRACTIONS

To convert a percentage into a fraction, we first write the percentage as a fraction with denominator 100. We then express the fraction in simplest form.

Example 2 **Self Tutor**

Write as a fraction:

a 29% **b** 85%

a $$29\% = \frac{29}{100}$$

b $$85\% = \frac{85}{100} = \frac{85 \div 5}{100 \div 5} = \frac{17}{20}$$

Convert to a fraction with denominator 100, then write in simplest form.

EXERCISE 10B.1

1 Write as a fraction:

a 59% **b** 13% **c** 3% **d** 97%

2 Write as a fraction in simplest form:

a 10% **b** 50% **c** 90% **d** 5%
e 22% **f** 74% **g** 15% **h** 65%
i 25% **j** 80% **k** 35% **l** 75%
m 4% **n** 48% **o** 56% **p** 64%

CONVERTING FRACTIONS TO PERCENTAGES

Many fractions can be converted to percentage form by first writing the fraction with denominator 100.

Example 3 **Self Tutor**

Write as a percentage:

a $\frac{19}{100}$ **b** $\frac{2}{5}$ **c** $\frac{557}{1000}$

a $$\frac{19}{100} = 19\%$$

b $$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%$$

c $$\frac{557}{1000} = \frac{557 \div 10}{1000 \div 10} = \frac{55.7}{100} = 55.7\%$$

EXERCISE 10B.2

1 Write as a percentage:

a $\frac{21}{100}$ **b** $\frac{53}{100}$ **c** $\frac{91}{100}$ **d** $\frac{8}{100}$
e $\frac{3}{10}$ **f** $\frac{7}{10}$ **g** $\frac{0}{10}$ **h** $\frac{10}{10}$

2 Write as a percentage:

a $\frac{1}{2}$ **b** $\frac{13}{50}$ **c** $\frac{1}{5}$ **d** $\frac{41}{50}$ **e** $\frac{3}{20}$

f $\frac{3}{5}$ **g** $\frac{7}{25}$ **h** $\frac{19}{20}$ **i** $\frac{12}{25}$ **j** $\frac{19}{25}$

3 Write as a percentage:

a $\frac{29}{200}$ **b** $\frac{231}{1000}$ **c** $\frac{759}{1000}$ **d** $\frac{103}{500}$

4 **a** Write $\frac{200}{100}$ in simplest form. **b** Write 2 as a percentage.

5 Use the illustration to complete the table below.

PRINTABLE TABLE

	Students	*Number*	*Fraction*	*Fraction with denominator* 100	*Percentage*
a	wearing shorts				
b	with a ball				
c	not wearing a hat				
d	wearing shorts and with a ball				
e	wearing track pants, baseball cap, and a green top				
f	wearing shorts or long pants				
g	wearing shoes				

6 Copy and complete these patterns:

a
$\frac{1}{5} = 20\%$
$\frac{2}{5} =$ ......
$\frac{3}{5} =$ ......
$\frac{4}{5} =$ ......
$\frac{5}{5} =$ ......

b
$\frac{1}{4}$ is 25%
$\frac{2}{4}$ is ......
$\frac{3}{4}$ is ......
$\frac{4}{4}$ is ......

c
$\frac{1}{3}$ is $33\frac{1}{3}\%$
$\frac{2}{3}$ is ......
$\frac{3}{3}$ is ......

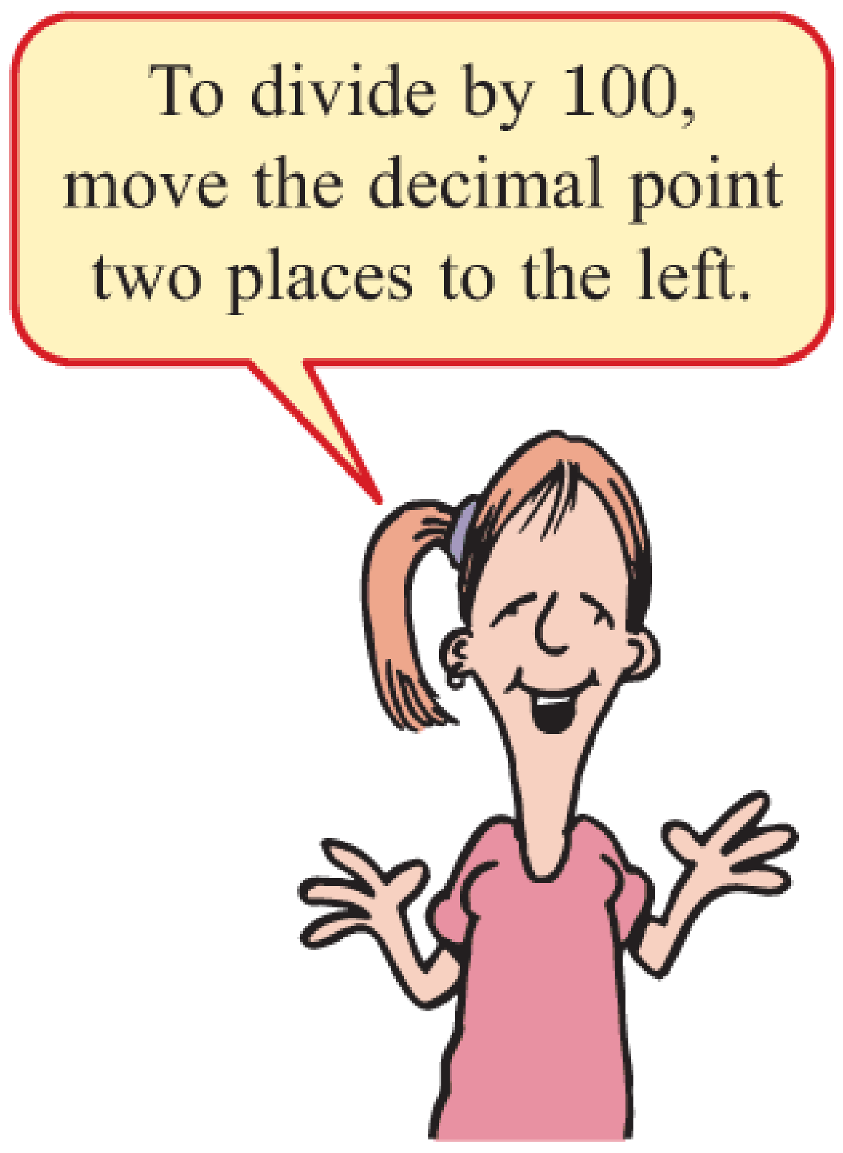
d
1 is 100%
$\frac{1}{2}$ is 50%
$\frac{1}{4}$ is ......
$\frac{1}{8}$ is ......
$\frac{1}{16}$ is ......

C CONVERTING BETWEEN PERCENTAGES AND DECIMALS

CONVERTING PERCENTAGES TO DECIMALS

To write a percentage as a decimal number, we **divide by 100%**.

Since $100\% = \frac{100}{100} = 1$, dividing by 100% is the same as dividing by 1. We therefore do not change the value of the number.



Example 4 Self Tutor

Write as a decimal:

a 21% **b** 6.7%

a 21%
$= 21. \div 100$
$= 0.21$

b 6.7%
$= 06.7 \div 100$
$= 0.067$

EXERCISE 10C.1

1 Write as a decimal:

a 10% **b** 50% **c** 25% **d** 5%
e 33% **f** 57% **g** 94% **h** 6%
i 40% **j** 11% **k** 1% **l** 90%

2 Write as a decimal:

a 17.5% **b** 81.6% **c** 60.7% **d** 9.4%
e 3.9% **f** 4.3% **g** 1.7% **h** 0.8%

3 Write each percentage as:

i a decimal **ii** a fraction in simplest form.

a 71% **b** 65% **c** 30% **d** 8%

CONVERTING DECIMALS TO PERCENTAGES

To write a decimal number as a percentage, we **multiply by 100%**.

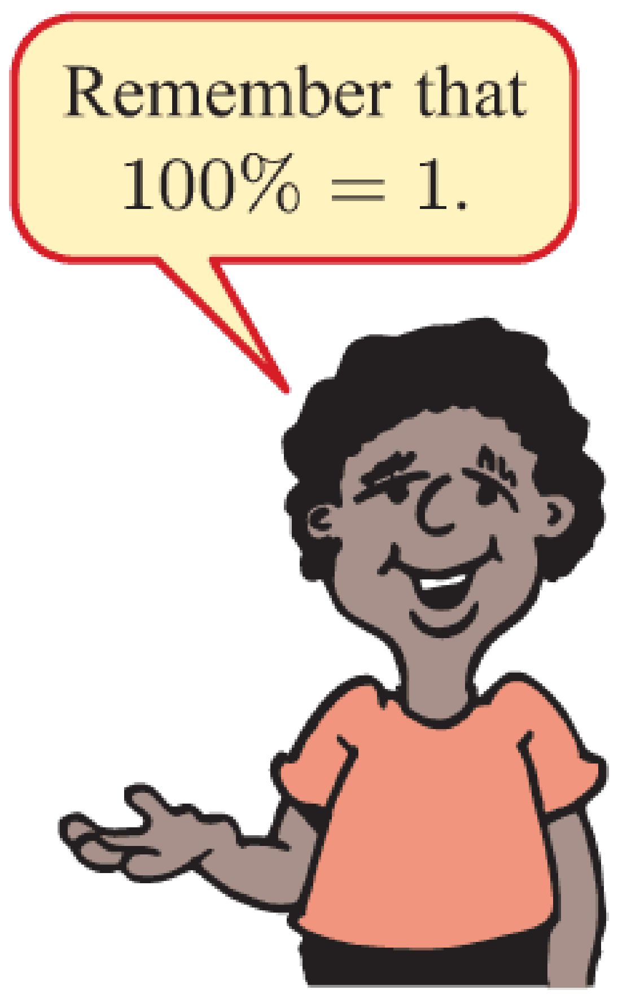
Example 5 **Self Tutor**

Write as a percentage:

a 0.27 **b** 0.055

a 0.27
$= 0.\overset{\frown}{2}\overset{\frown}{7} \times 100\%$
$= 27\%$

b 0.055
$= 0.\overset{\frown}{0}\overset{\frown}{5}5 \times 100\%$
$= 5.5\%$



EXERCISE 10C.2

1 Write as a percentage:

a 0.37 **b** 0.89 **c** 0.15 **d** 0.49
e 0.73 **f** 0.11 **g** 0.05 **h** 0.02

2 Write as a percentage:

a 0.2 **b** 0.7 **c** 0.9 **d** 0.4
e 0.074 **f** 0.739 **g** 0.086 **h** 0.001

3 Copy and complete:

	Percent	*Fraction*	*Decimal*
a	20%		0.2
b	40%	$\frac{2}{5}$	
c			0.5
d		$\frac{3}{4}$	
e			0.85

	Percent	*Fraction*	*Decimal*
f		$\frac{2}{25}$	
g			0.35
h	84%		
i	100%		
j		$\frac{3}{20}$	

4 Write:

a 28% as a fraction and as a decimal
b $\frac{4}{5}$ as a percentage and as a decimal
c 0.45 as a percentage and as a fraction
d 0.25 as a percentage and as a fraction.

DISCUSSION

- Does it make sense to talk about percentages greater than 100%, such as 110% or 250%?
- What would be meant by the statement "My business has grown by 300% over the last 2 years."?

D NUMBER LINES

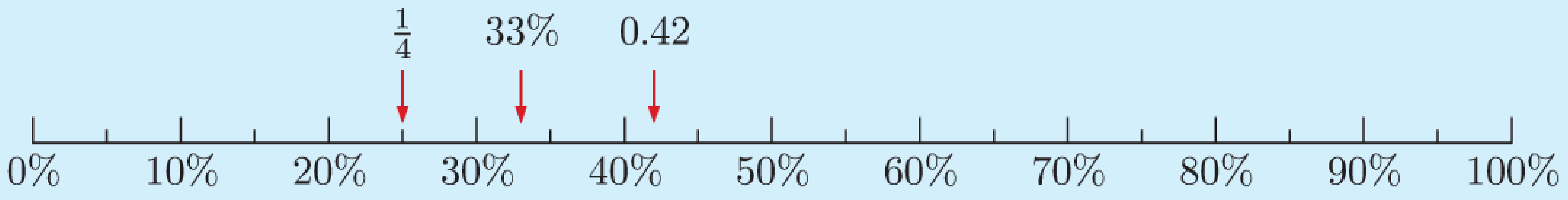
Plotting numbers on a number line can be difficult when the numbers are given as a mixture of fractions, decimals, and percentages. However, we can make the process easier by converting all fractions and decimals to percentages.

Example 6 **Self Tutor**

Convert $\frac{1}{4}$, 0.42, and 33% to percentages, and place them on a number line.

- $\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 25\%$
- $0.42 = 0.\overset{\curvearrowright\curvearrowright}{42} \times 100\% = 42\%$
- 33% is already a percentage

We use the percentages to place the numbers on a number line.



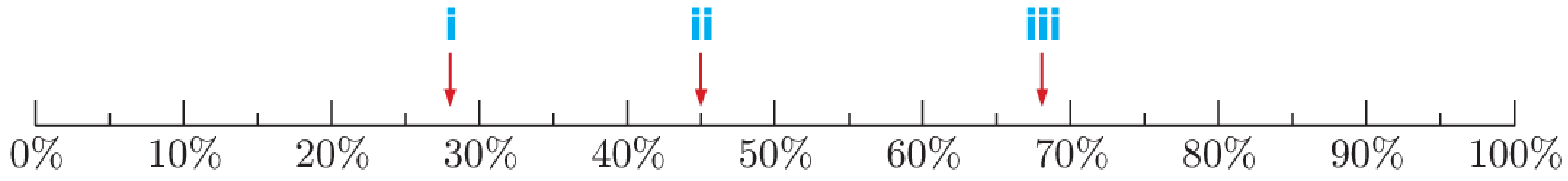
EXERCISE 10D

1 Convert each set of numbers to percentages, and place them on a number line:

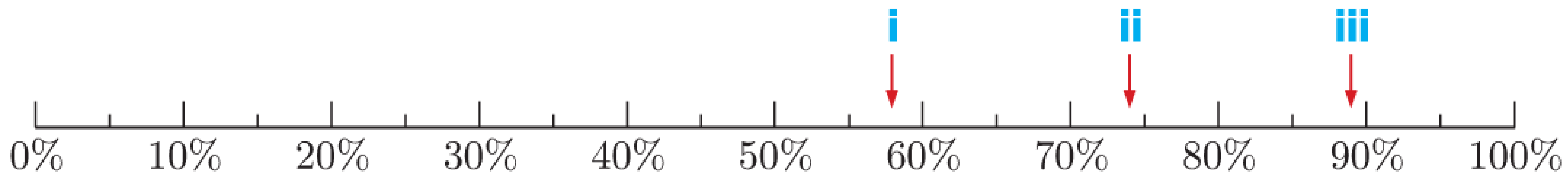
a $\frac{3}{5}$, 70%, 0.65
b 55%, $\frac{9}{20}$, 0.83
c 0.93, 79%, $\frac{17}{20}$
d 0.85, $\frac{3}{4}$, 92%
e $\frac{27}{50}$, 67%, 0.59
f 47%, 0.74, $\frac{7}{10}$
g $\frac{3}{4}$, 0.65, 42%
h 0.39, 58%, $\frac{7}{20}$, $\frac{2}{5}$
i $\frac{661}{1000}$, 73%, $\frac{13}{20}$, 0.47

2 Write each of the following number line positions as a fraction with denominator 100, as a decimal, and as a percentage:

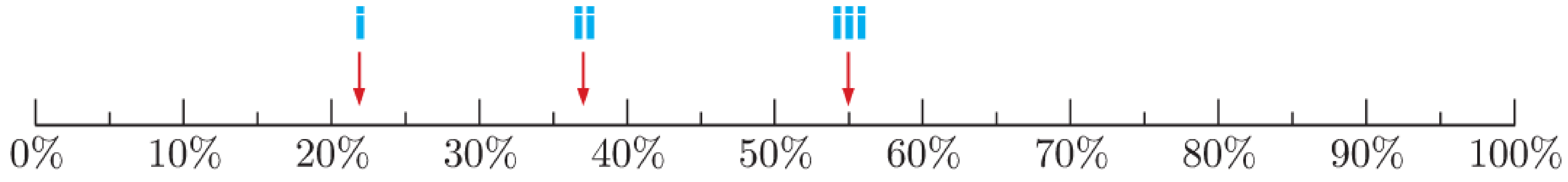
a



b



c



3 **a** Write each fraction or decimal as a percentage:

i $\frac{17}{25}$
ii 0.43
iii $\frac{39}{50}$
iv 0.627

b Place the values in **a** on a number line.

c Hence write the values in order from smallest to largest.

COMMON CONVERSIONS

Some percentages occur frequently in business calculations and advertisements, so it is useful to learn their values as fractions.

Copy this table of common conversions, and add others if you wish:

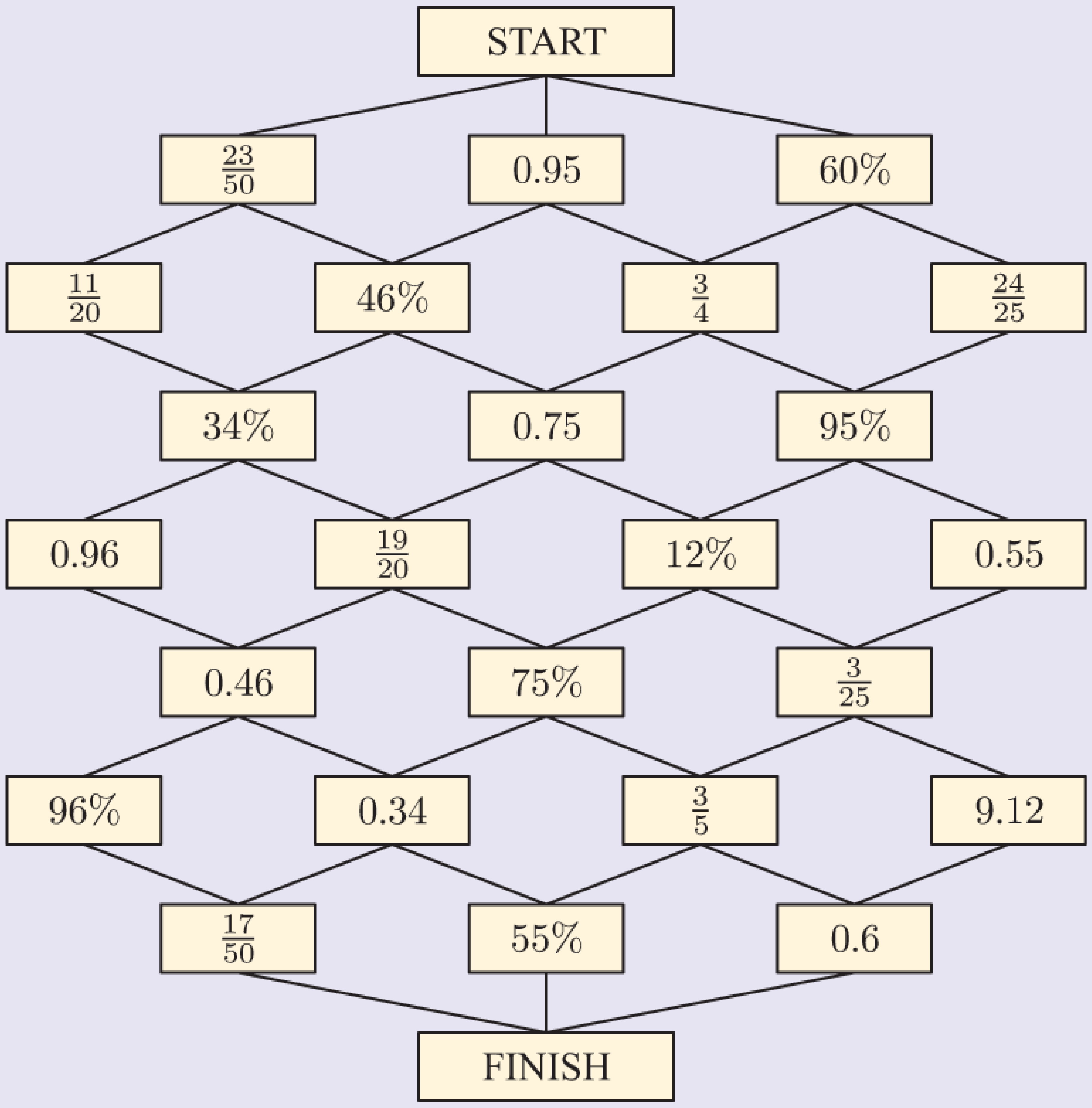
Percentage	*Common fraction*	*Decimal*
100%	1	1.0
75%	$\frac{3}{4}$	0.75
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
10%	$\frac{1}{10}$	0.1
5%	$\frac{1}{20}$	0.05

Percentage	*Common fraction*	*Decimal*
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.3333 ....
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.6666 ....
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125
$6\frac{1}{4}\%$	$\frac{1}{16}$	0.0625
$\frac{1}{2}\%$	$\frac{1}{200}$	0.005

PUZZLE

In this Puzzle, you must find a path from the Start to the Finish so that no two numbers on the path have the same value.

PRINTABLE PUZZLE



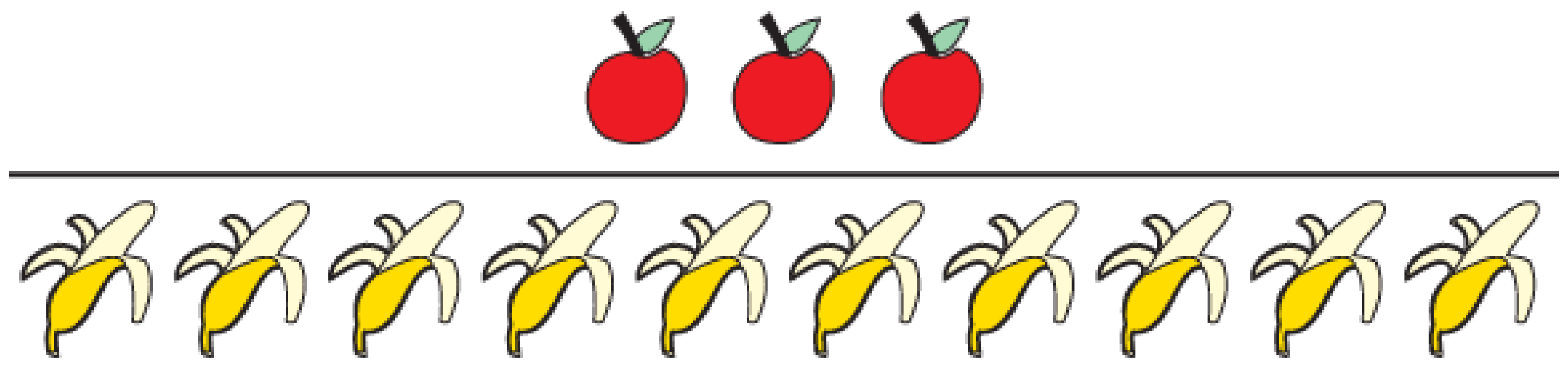
E ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Percentages are often used to compare quantities, so it is useful to express one quantity as a percentage of another.

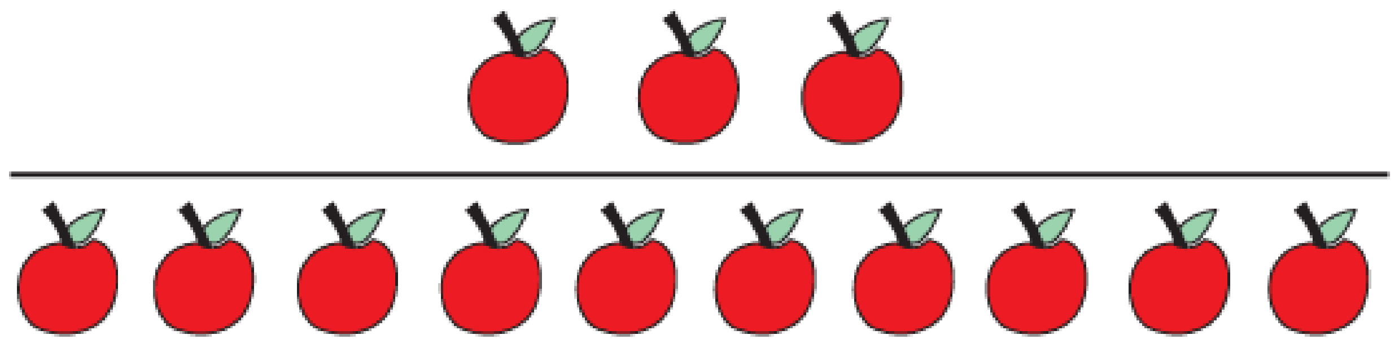
We must be careful to only compare **like with like**.

For example:

We cannot express 3 apples as a percentage of 10 bananas.



However, we *can* express 3 apples as a percentage of 10 apples.



We must also make sure that the quantities are compared in the same units.

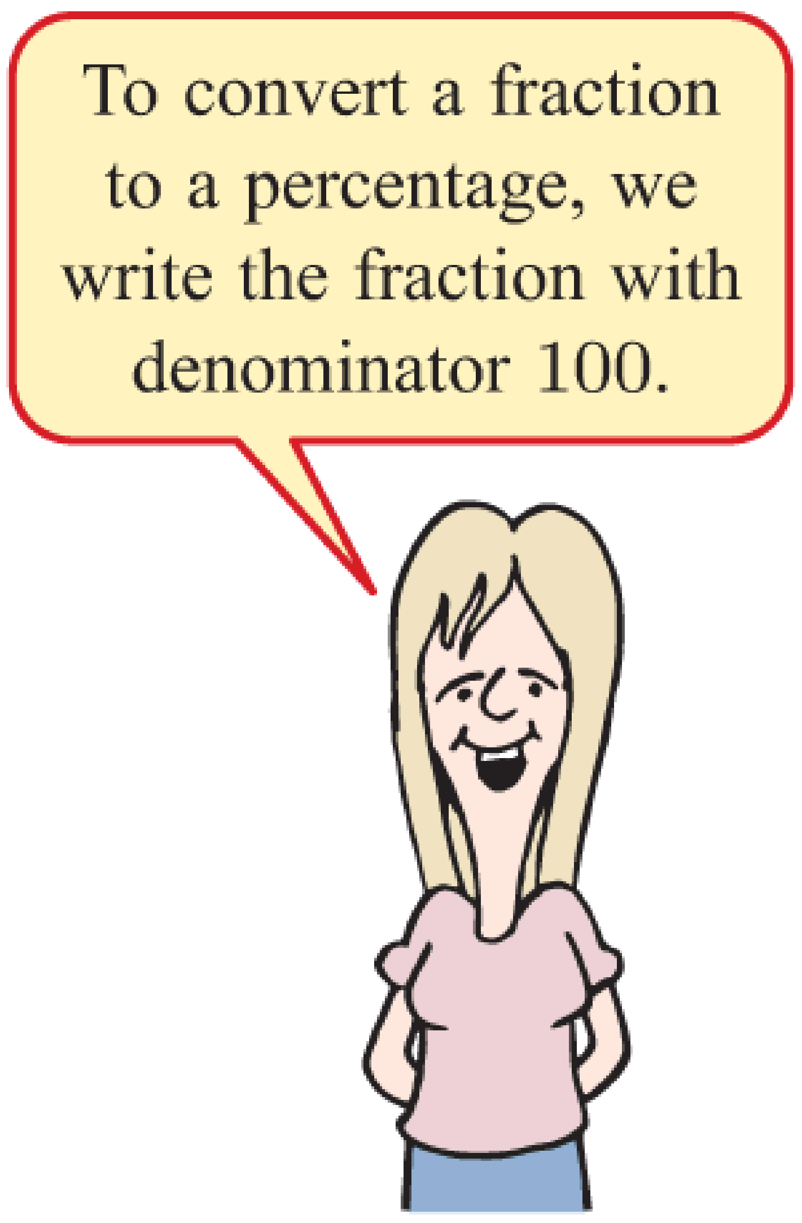
For example, if we are asked to express 35 cm as a percentage of 7 m, we would first convert the larger unit to the smaller one. In this case we would find 35 cm as a percentage of 700 cm.

To express one quantity as a percentage of another, we first write them as a fraction, and then convert the fraction to a percentage.

Example 7 **Self Tutor**

Express a mark of 22 out of 25 as a percentage.

$$\begin{aligned}\frac{22 \text{ marks}}{25 \text{ marks}} &= \frac{22}{25}\\ &= \frac{22 \times 4}{25 \times 4}\\ &= \frac{88}{100}\\ &= 88\%\end{aligned}$$



EXERCISE 10E

1 Express as a percentage:

a 17 marks out of 20

b 11 marks out of 25

c 37 marks out of 50

d 138 marks out of 200

Example 8 **Self Tutor**

Express the first quantity as a percentage of the second:

a 60 cm, 3 m **b** 160 g, 2 kg

a $\frac{60\text{ cm}}{3\text{ m}}$

$= \frac{60\text{ cm}}{300\text{ cm}}$

$= \frac{60 \div 3}{300 \div 3}$

$= \frac{20}{100}$

$= 20\%$

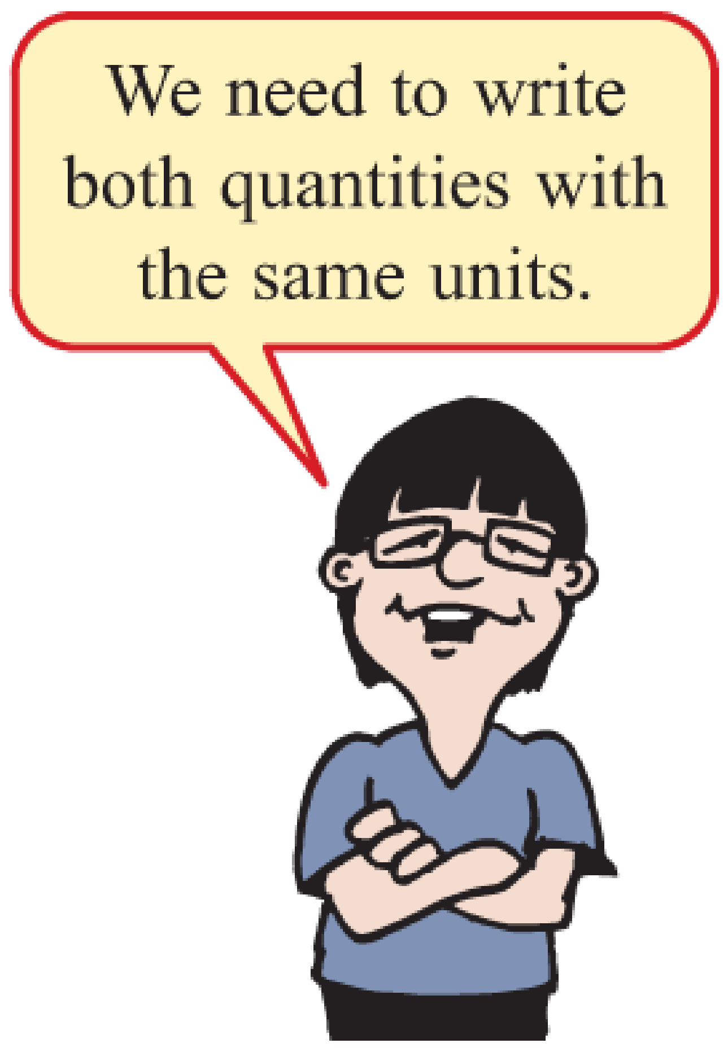
b $\frac{160\text{ g}}{2\text{ kg}}$

$= \frac{160\text{ g}}{2000\text{ g}}$

$= \frac{160 \div 20}{2000 \div 20}$

$= \frac{8}{100}$

$= 8\%$



2 Express the first quantity as a percentage of the second:

a 20 cm, 100 cm **b** 10 km, 50 km **c** 3 m, 4 m
d 7 mm, 2 cm **e** 50 g, 1 kg **f** 84 cm, 4 m
g 48 seconds, 5 minutes **h** 720 kg, 2 tonnes **i** 63 cents, 9 dollars
j 24 minutes, 10 hours **k** 50 cents, $25 **l** 1 mL, 1 litre

3 Express as a percentage:

a 72 diners in a restaurant that seats 200 diners
b 405 books sold out of 500 printed
c 660 square metres of lawn in a 2000 square metre garden
d 28 000 spectators in a 40 000 seat stadium
e a ten pin bowler scores 186 points out of a possible 300 points
f €60 off the price of a television marked at €400.

Example 9 **Self Tutor**

In a class of 25 students, 6 have black hair.
What percentage of the class have black hair?

The fraction with black hair $= \frac{6}{25}$

$= \frac{6 \times 4}{25 \times 4}$

$= \frac{24}{100}$

So, 24% of the class have black hair.

4 In a class of 25 students, 13 have blue eyes. What percentage of the class have blue eyes?

5 2 km of gas pipes need to be laid. So far, 480 m of pipes have been laid. What percentage of the pipes have been laid?

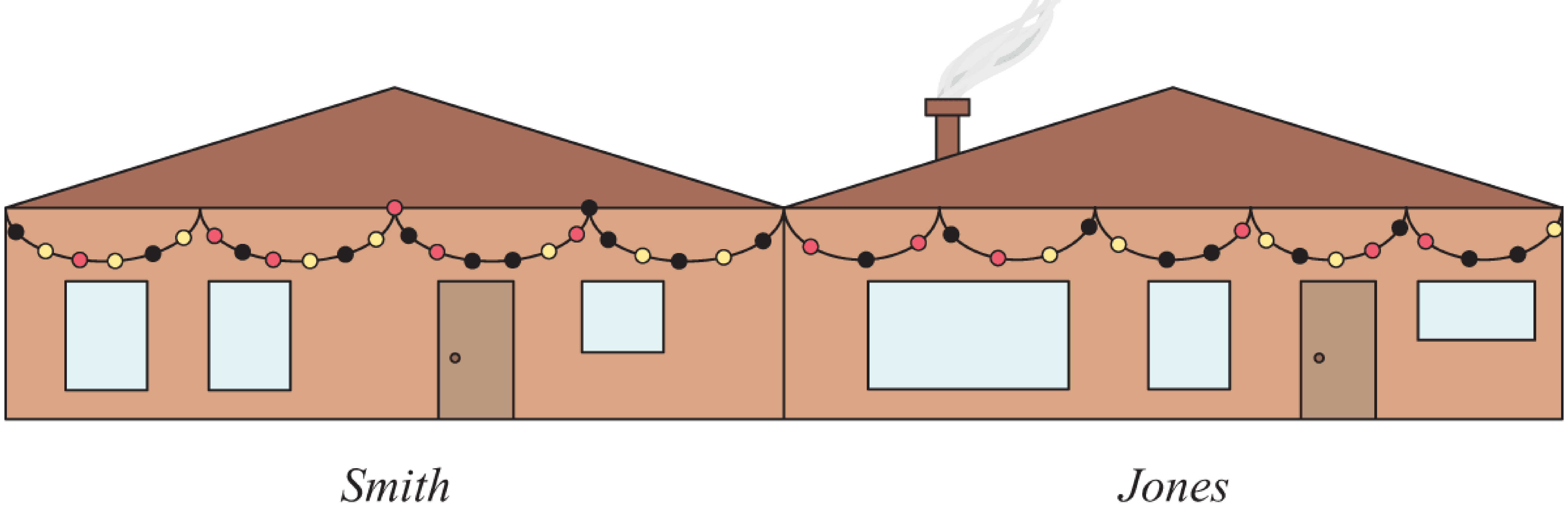
6 There are 50 singers in a school choir. 12 of them are in Grade 4, 17 are in Grade 5, and 21 are in Grade 6.

- **a** Find the percentage of singers who are in:
  - **i** Grade 4
  - **ii** Grade 5
  - **iii** Grade 6.
- **b** Check that the sum of the percentages in **a** is 100%.

7 Maria is taking a cooking course with 20 classes. To earn her certificate, she needs to attend at least 80% of the classes. Maria was unable to attend 3 classes.

- **a** What percentage of the classes did Maria attend?
- **b** Will Maria receive her certificate?

8 The Smith family and the Jones family each put up a display of coloured lights on their house during the holiday season.



- **a** Find the percentage of lights which are working on each house.
- **b** Which house has the higher percentage of working lights?
- **c** Of all the lights that are working, what percentage are:
  - **i** red
  - **ii** yellow?

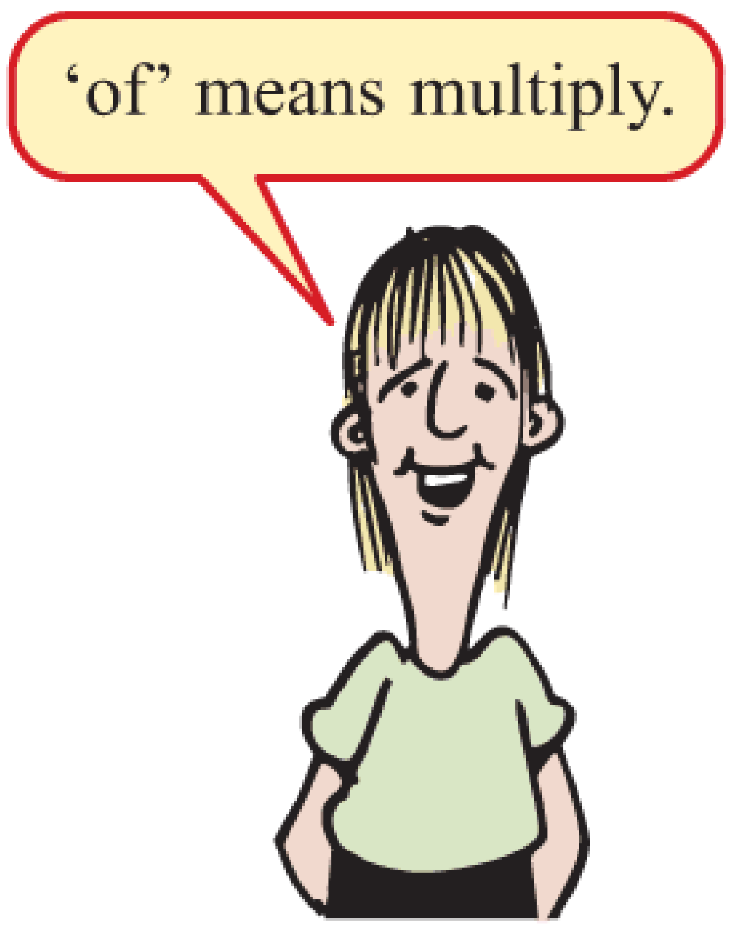
F FINDING PERCENTAGES OF QUANTITIES

To find a percentage of a quantity, we first convert the percentage to a decimal. We then multiply to find the answer.

Example 10 **Self Tutor**

Find 53% of 4000 people.

53% of 4000 people
$= 0.53 \times 4000$ people $\quad \{53\% = 0.53\}$
$= 2120$ people $\quad$ *Calculator*: $0.53 \boxed{\times} 4000 \boxed{=}$



EXERCISE 10F

1 Find:

- **a** 15% of \$200
- **b** 80% of 250 people
- **c** 27% of 30 kg
- **d** 75% of 320 litres
- **e** 7% of 70 cm
- **f** 45% of 35 seconds

Example 11 **Self Tutor**

Find 12% of 3 km, giving your answer in m.

$$\begin{aligned} &12\% \text{ of } 3 \text{ km} \\ &= 0.12 \times 3000 \text{ m} && \{12\% = 0.12, \quad 3 \text{ km} = 3000 \text{ m}\} \\ &= 360 \text{ m} && \textit{Calculator}: \quad 0.12 \ \boxed{\times} \ 3000 \ \boxed{=} \end{aligned}$$

2 Find the following, giving each answer in the units indicated:

- **a** 27% of \$1 (in cents)
- **b** 5% of 9 m (in cm)
- **c** 35% of 2 kg (in g)
- **d** 10% of 3 hours (in min)
- **e** 60% of 8 kL (in L)
- **f** 42% of 4 cm (in mm)
- **g** 22% of 5 days (in hours)
- **h** 7% of \$14 (in cents)

3 A school has 485 students. A teacher takes 20% of them on an excursion to the museum. How many students went to the museum?

4 A council collects 4500 tonnes of rubbish each year. 27% of the rubbish is recycled. How many tonnes of rubbish is this?

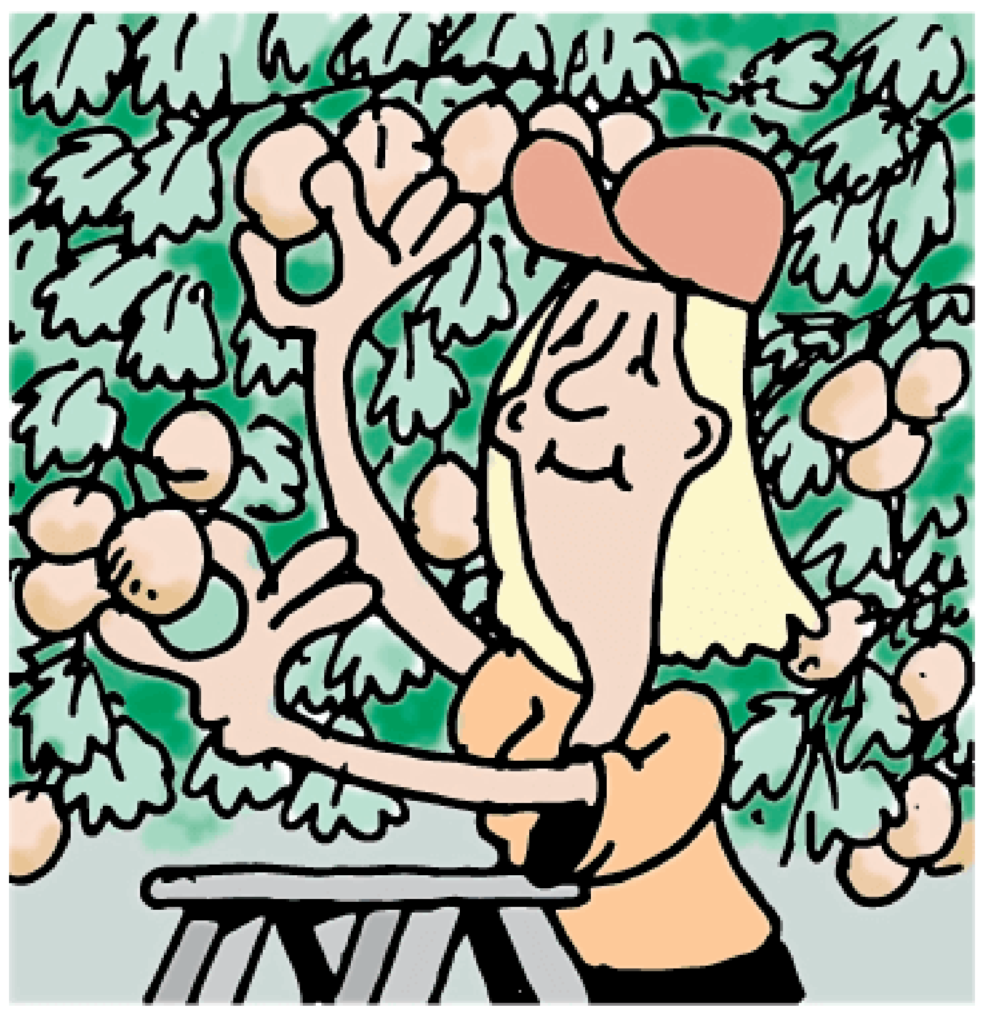
5 15% of an energy drink is sugar. How many grams of sugar are there in a 450 g can of energy drink?

6 

30% of a farmer's crop was barley, and the rest was wheat.

- **a** What percentage of the crop was wheat?
- **b** If the farmer planted 2400 acres in total, how many acres were planted with:
  - **i** barley
  - **ii** wheat?

7 An orchardist picks 2400 kg of apricots for drying. 85% of the weight is lost in the drying process. How many kilograms of dried apricots are produced?



8 A new company policy requires 5% of the workers in each office to have first aid training. How many workers need first aid training in:

- **a** a small office containing 20 workers
- **b** a large office containing 300 workers?

9 In a series of triathlon races, prize money is awarded to the top three competitors. The winner receives 50% of the prize money, second place receives 35%, and third place receives 15%.

The total prize money and results for the first two races are shown alongside.

Find the prize money won so far by:

a Shane **b** Daniel.

	Total prize money	1st	2nd	3rd
Race 1	$3000	Daniel	Matt	Shane
Race 2	$5000	Justin	Daniel	Trent

10 When a painting is sold at an art gallery, the art gallery receives a fixed percentage of the selling price, and the artist receives the rest.

- **a** When a painting was sold for $200, the art gallery received $66.
  - **i** What percentage of the price did the art gallery receive?
  - **ii** What percentage of the price did the artist receive?
- **b** A second painting is sold for $450. How much money will be received by the:
  - **i** art gallery **ii** artist?

PUZZLE

In this Puzzle your task is to move from the START to the FINISH.

You may move one square at a time, in any direction, including diagonally.

However, you may only move to a square if the row percentage of the column number is a whole number.

For example, you could move from the start to the top left square, since 50% of 40 is 20, which is a whole number.

PRINTABLE GRID

Column numbers

Row percentages

	40	55	25	24	70	50	65	60
	START							
50%								
25%								
8%								
40%								
$33\frac{1}{3}\%$								
75%								
150%								
5%								
	FINISH!							

KEY WORDS USED IN THIS CHAPTER

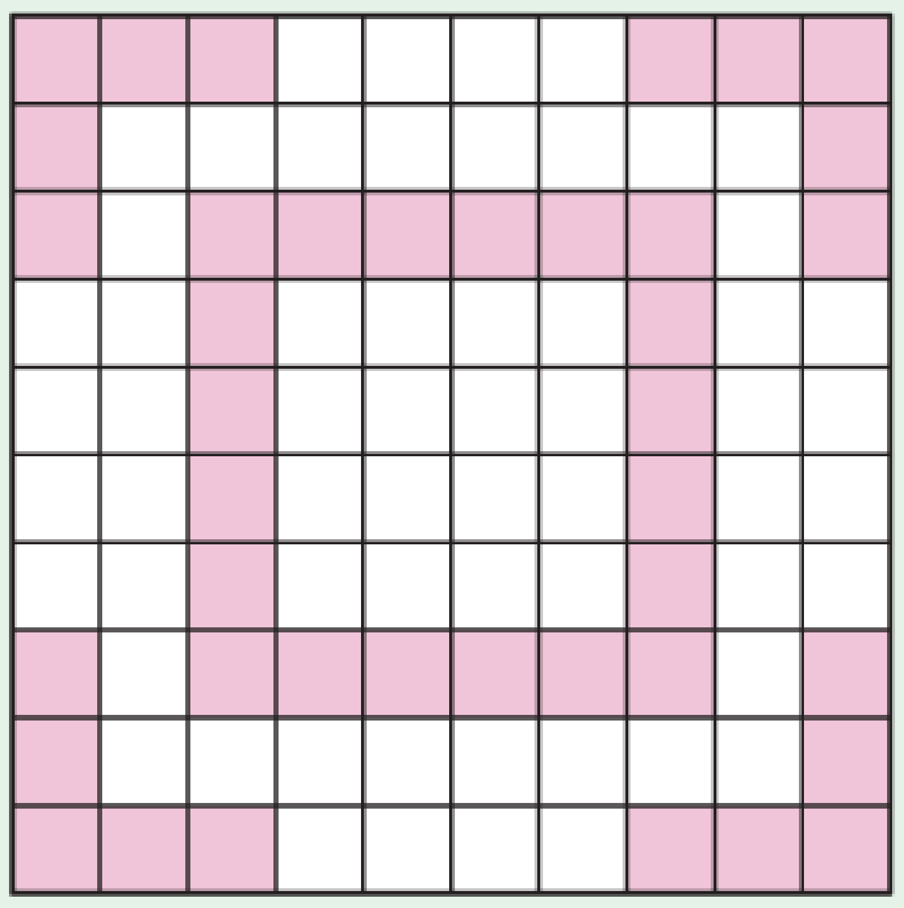
- decimal
- fraction
- percentage

REVIEW SET 10A

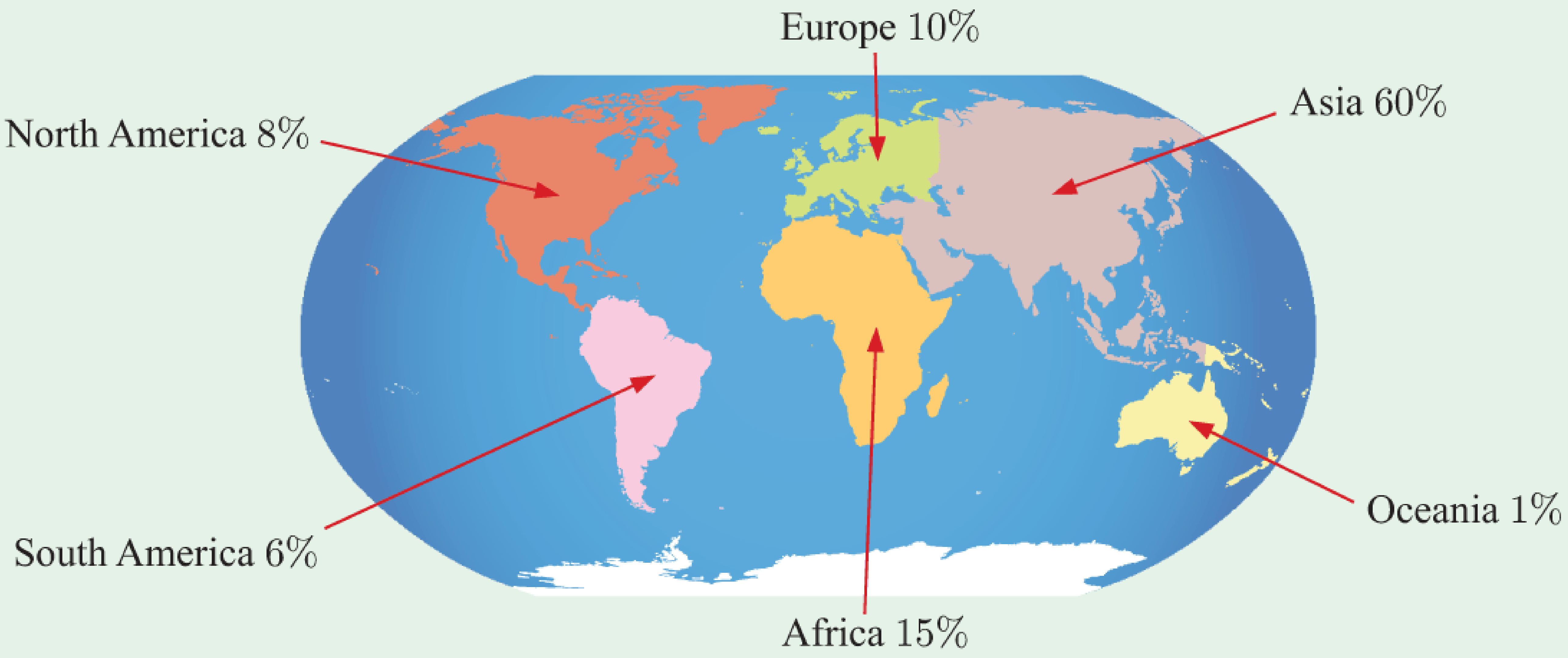
1 There are 100 tiles in the pattern shown.

a Write the fraction of tiles which are coloured, leaving your answer with denominator 100.

b Write the percentage of tiles which are coloured.



2 This map shows the percentage of people living in each continent.



a Find the sum of the percentages.

b What percentage of the world's population lives in:

i Asia **ii** North or South America?

c What *fraction* of the world's population lives in:

i Europe **ii** Africa?

3 Write as a percentage: **a** 0.47 **b** 0.306

4 Write as a fraction in simplest form:

a 31% **b** 16% **c** 94%

5 Find:

a 45% of £60 **b** 12% of 4 m (in cm)

6 50 students attended the annual quiz night. 27 of the students won at least one prize during the night. What percentage of the students won at least one prize?

7 Write $\frac{221}{1000}$ as a percentage.

8 Write as a decimal:

a 81% **b** 2% **c** 10.8%

9 Marcia has travelled 620 kilometres of a 2000 km journey. What percentage of the journey has she travelled?

10 Write $\frac{3}{4}$, 0.78, and 72% as percentages, and then place them on a number line.

11 A small country town has 280 households. 45% use a wood burning fire to warm their homes, 30% use electricity, 15% use gas, and 10% use oil or kerosene. How many households use:

a electricity **b** fire or gas?

12 Brenda makes her own mayonnaise. 40% of the mayonnaise is egg, 35% is oil, 20% is lemon juice, and the rest is mustard.

- **a** Does the mayonnaise contain more oil or lemon juice?
- **b** What percentage of the mayonnaise is mustard?
- **c** How much:
  - **i** lemon juice is there in 300 mL of mayonnaise
  - **ii** oil is there in 800 mL of mayonnaise?

REVIEW SET 10B

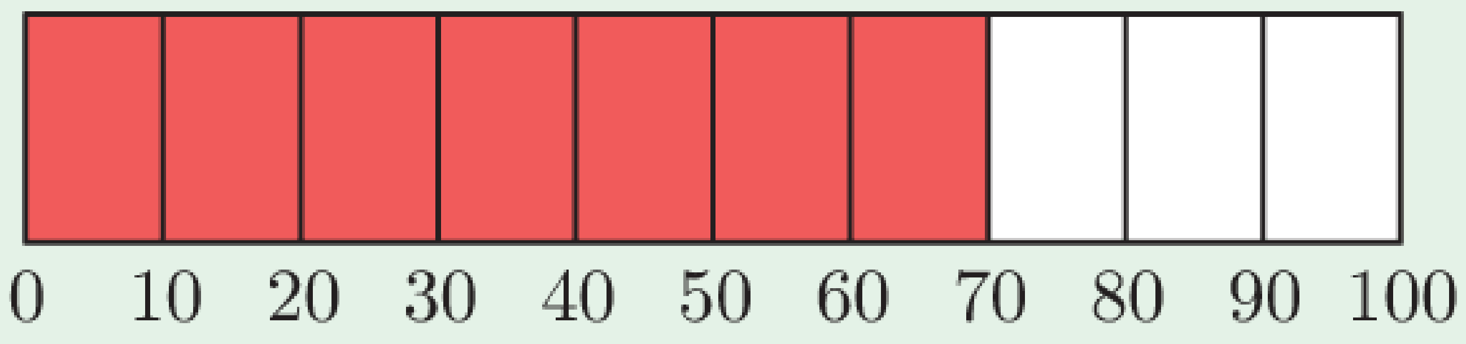
1 There are 100 symbols in the pattern shown.

- **a** Count the number of each type of symbol.
- **b** Write the number of each type as a fraction of the total.
- **c** Write the number of each type as a percentage of the total.
- **d** Check that your percentages in **c** sum to 100%.

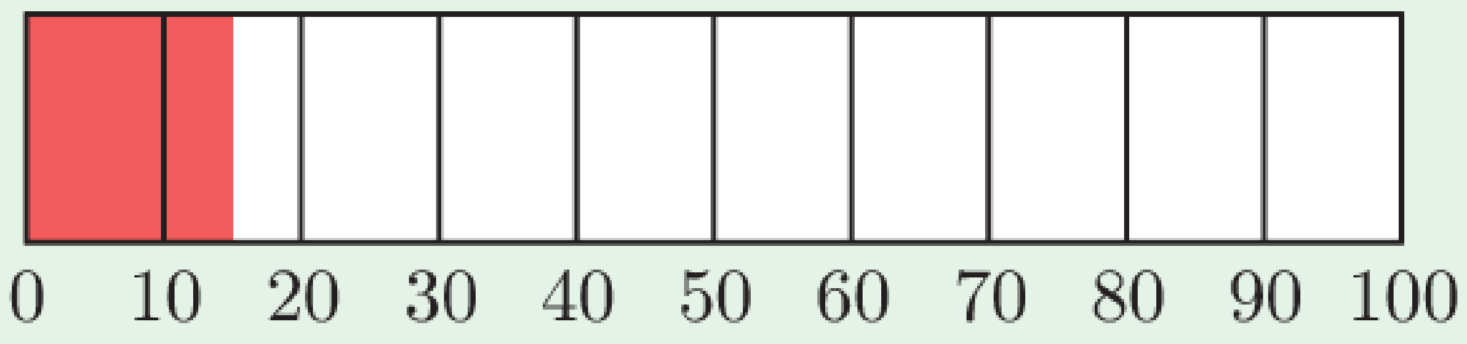
X	V	V	X	V	X	X	V	V	X
V	X	V	V	X	V	V	X	V	V
V	V	X	X	V	X	V	V	X	V
X	V	V	X	V	V	X	V	V	X
V	X	V	V	X	V	X	X	V	V
V	V	X	V	V	X	V	V	X	V
X	V	X	X	V	V	X	V	V	X
V	X	V	V	X	V	V	X	V	V
V	V	X	V	V	X	V	X	X	V
X	V	V	X	V	V	X	V	V	X

2 Estimate the percentage shaded in each diagram:

a



b



3 Write as a percentage:

a 0.09 **b** 0.136 **c** 0.702

4 In a group of 200 children, 34 are allergic to peanuts. What percentage of the children are allergic to peanuts?

5 Write 74% as a fraction and as a decimal.

6 8% of the students at a school are left-handed. The school has 375 students. How many students at the school are left-handed?

7 Write as a percentage:

a $\frac{27}{100}$ **b** $\frac{18}{25}$ **c** $\frac{13}{20}$

8 Express the first quantity as a percentage of the second:

a 13 goals from 25 shots **b** 58 cm of 2 m

9 Klaus spent €15 from the €50 he was given for his birthday. What percentage of his money did he spend?

10 A cordial mixture contains 15% cordial and 85% water. How much:

a cordial is in a 200 mL glass of cordial mixture

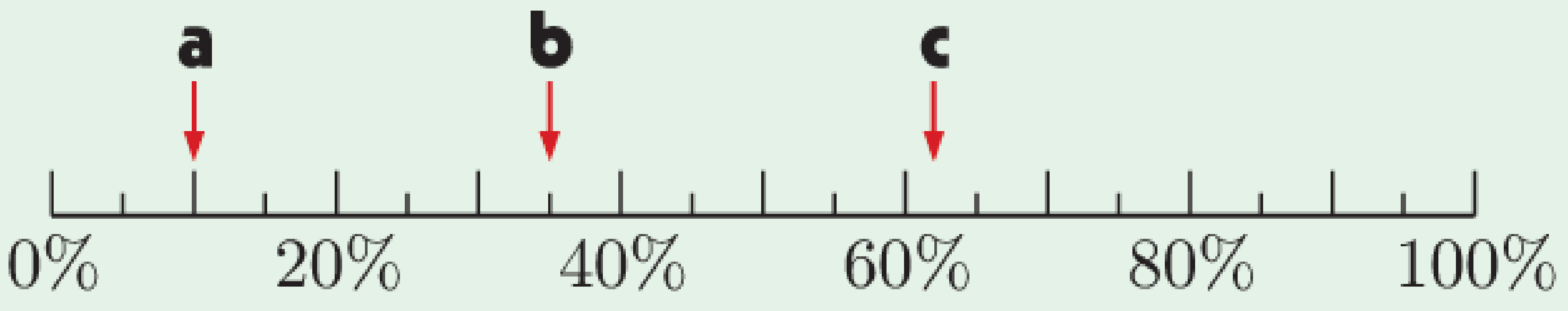
b water is in an 800 mL bottle of cordial mixture?

11 Write each number line position indicated as:

i a fraction with denominator 100

ii a decimal

iii a percentage.

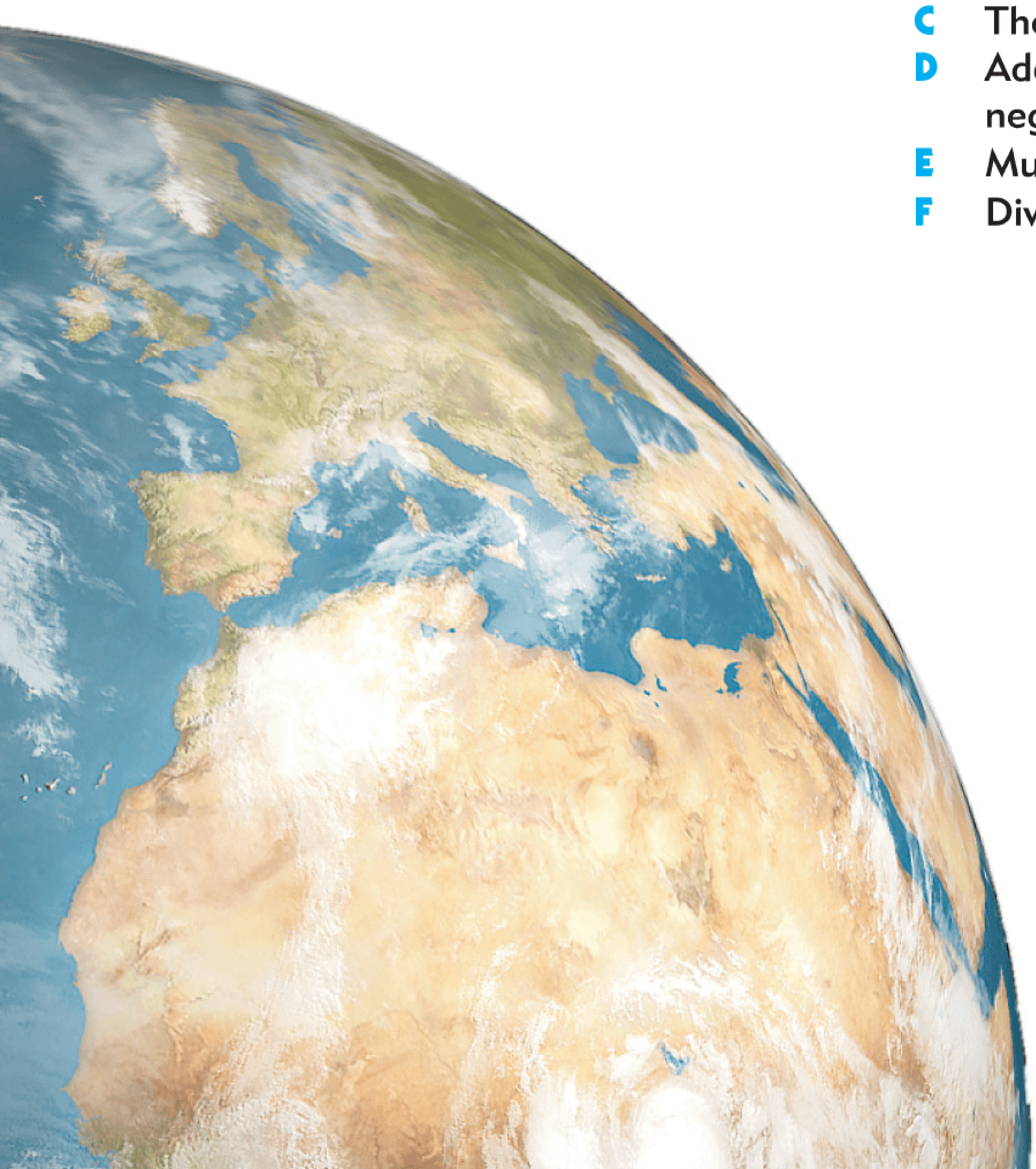


12 600 people applied to become a firefighter, but only 24 of them were accepted into the training program. What percentage of the people who applied were accepted?

Chapter 11

Positive and negative numbers

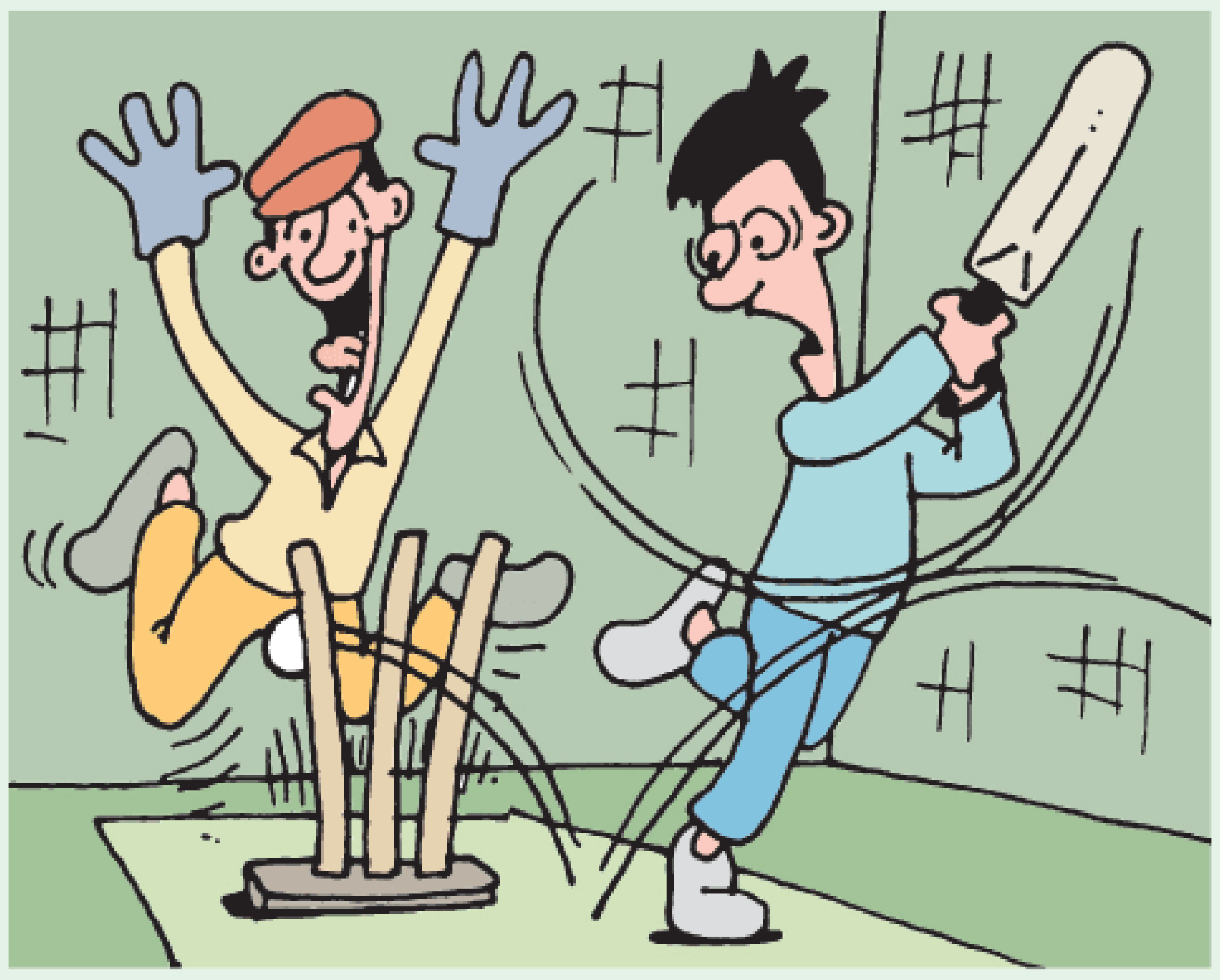
Contents:



OPENING PROBLEM

In indoor cricket, you are allowed to keep batting when your wicket is lost. However, 5 runs are subtracted from your score.

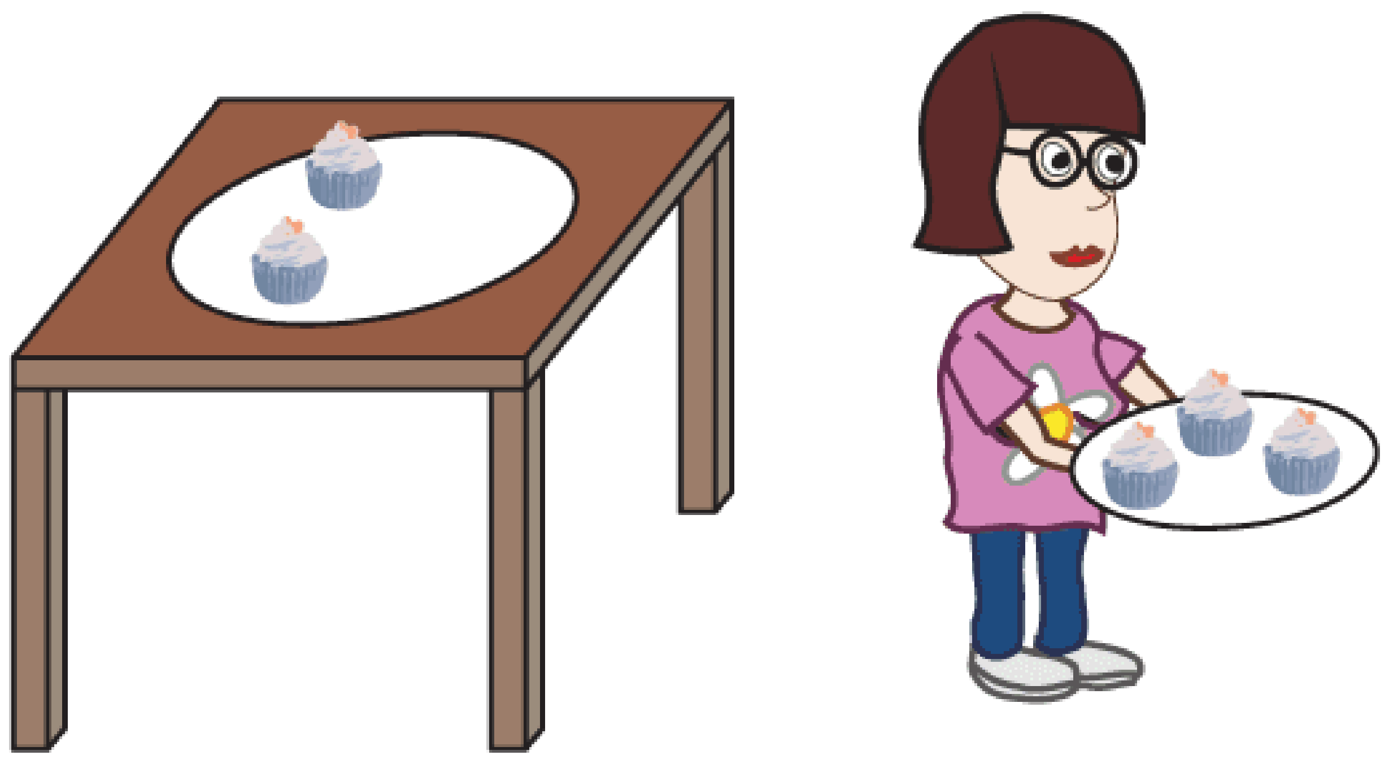
Blake had only scored 3 runs when he lost his wicket.

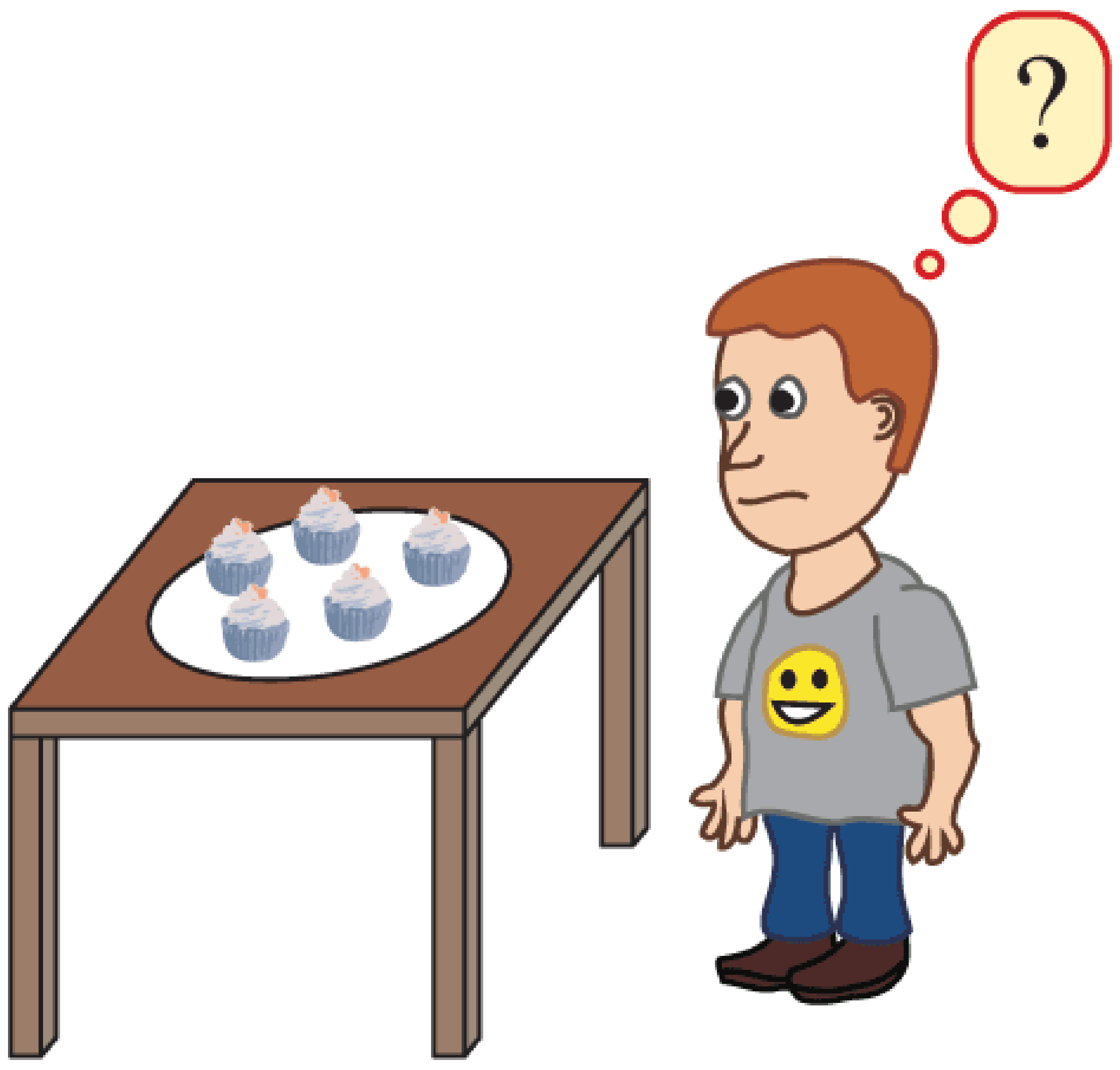


Things to think about:

- **a** What operation would you need to find Blake's score?
- **b** Is it possible to subtract 5 from 3?
- **c** What number can we use to display Blake's score?
- **d** Suppose Blake scored 6 runs on the next ball. What would his new score be?

Suppose Amanda has 5 cupcakes and eats 3 of them. The number of cupcakes remaining is $5 - 3 = 2$.





Now suppose Adam has 5 cupcakes, and he wants to eat 7. This is impossible, because once Adam has eaten 5 cupcakes, zero cupcakes remain, and there are no more left to eat. It does not make sense to say the number of cupcakes remaining is $5 - 7$, because we cannot have less than zero cupcakes.

However, there are many situations where it *does* make sense to talk about numbers less than zero.

For example, suppose the temperature in Oslo, Norway, is 5°C at 8 pm. Between 8 pm and midnight, the temperature falls by 7°C. The temperature at midnight is given by 5°C − 7°C. The answer is less than zero, so we call it a **negative number**.

A OPPOSITES

Many mathematical problems use **opposites**.

For example:

- *earning* money is the opposite of *spending* money
- going *up* is the opposite of going *down*
- getting *warmer* is the opposite of getting *cooler*.

Example 1 **Self Tutor**

Write the opposite of:

a winning by 7 points **b** travelling 10 km north.

a The opposite of winning by 7 points is losing by 7 points.

b The opposite of travelling 10 km north is travelling 10 km south.

Instead of using words to describe opposites, we can use **positive** and **negative** numbers.

POSITIVE NUMBERS

Positive numbers are numbers which are greater than zero.

The positive whole numbers are 1, 2, 3, 4, 5, ...

They can be written with a **positive sign** (+) before the number, but we normally see them without the positive sign and *assume* the number is positive.

NEGATIVE NUMBERS

Negative numbers are numbers which are less than zero.

They are written with a **negative sign** (−) before the number.

The negative whole numbers are −1, −2, −3, −4, −5, ...

WORDS INDICATING POSITIVE AND NEGATIVE

Some common uses of positive and negative signs are listed in the table alongside. Notice how the words are grouped in pairs of opposites.

For example, we could write "a profit of $5" as +5. The *opposite* statement "a loss of $5" would be written as −5.

Positive (+)	*Negative* (−)
above	below
over	under
increase	decrease
profit	loss
right	left
faster	slower
win	lose
north	south
east	west

EXERCISE 11A

1 Write the opposite of:

a winning €20
b 10 minutes early
c moving 2 steps to the right
d 6 m above sea level.

2 Copy and complete:

	Statement	*Number*	*Opposite of statement*	*Opposite number*
a	losing by 3 points	-3		
b	a clock is 5 minutes fast			
c	4 m above the water			
d	2°C below zero			
e	an increase of 6 kg			

3 If north is the positive direction, write these positions as positive or negative numbers:

a 7 km south
b 12 km north
c 15 km south

4 If temperatures above zero are positive, write these temperatures as positive or negative numbers:

a 7°C below zero
b 32°C above zero
c 40°C below zero

DISCUSSION

Why do you think we choose:

- north to be positive and south to be negative
- east to be positive and west to be negative?

B COMBINED EFFECTS

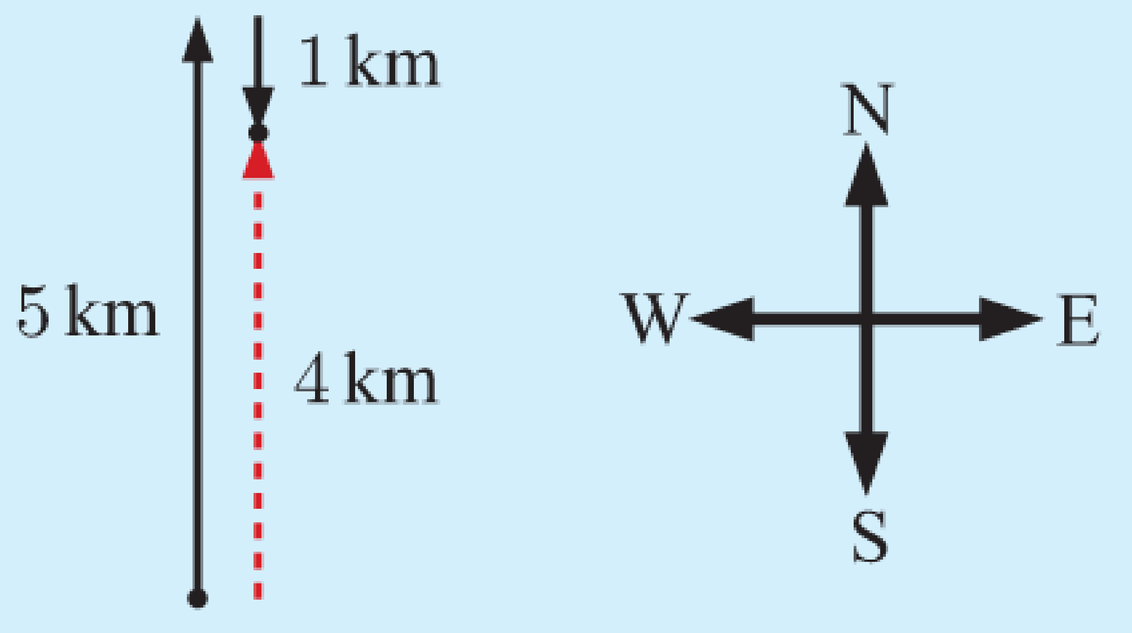
In situations where two things happen in opposite directions, we can use a diagram to help find the combined effect.

Example 2 **Self Tutor**

Find the combined effect of travelling:

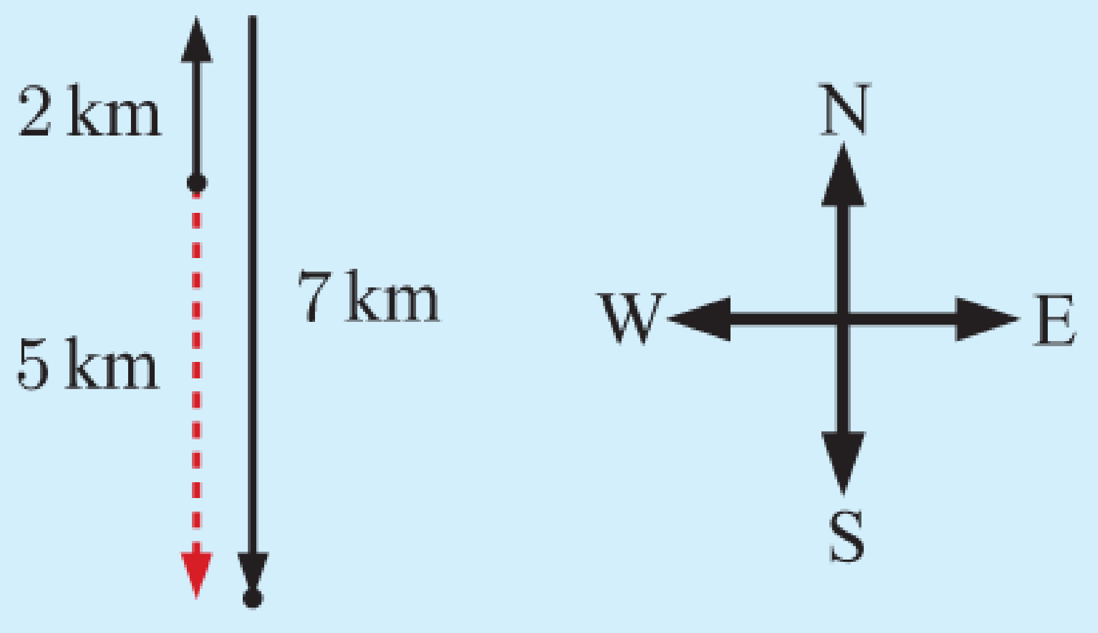
a 5 km north and then 1 km south
b 2 km north and then 7 km south.

a



The combined effect is travelling 4 km north.

b

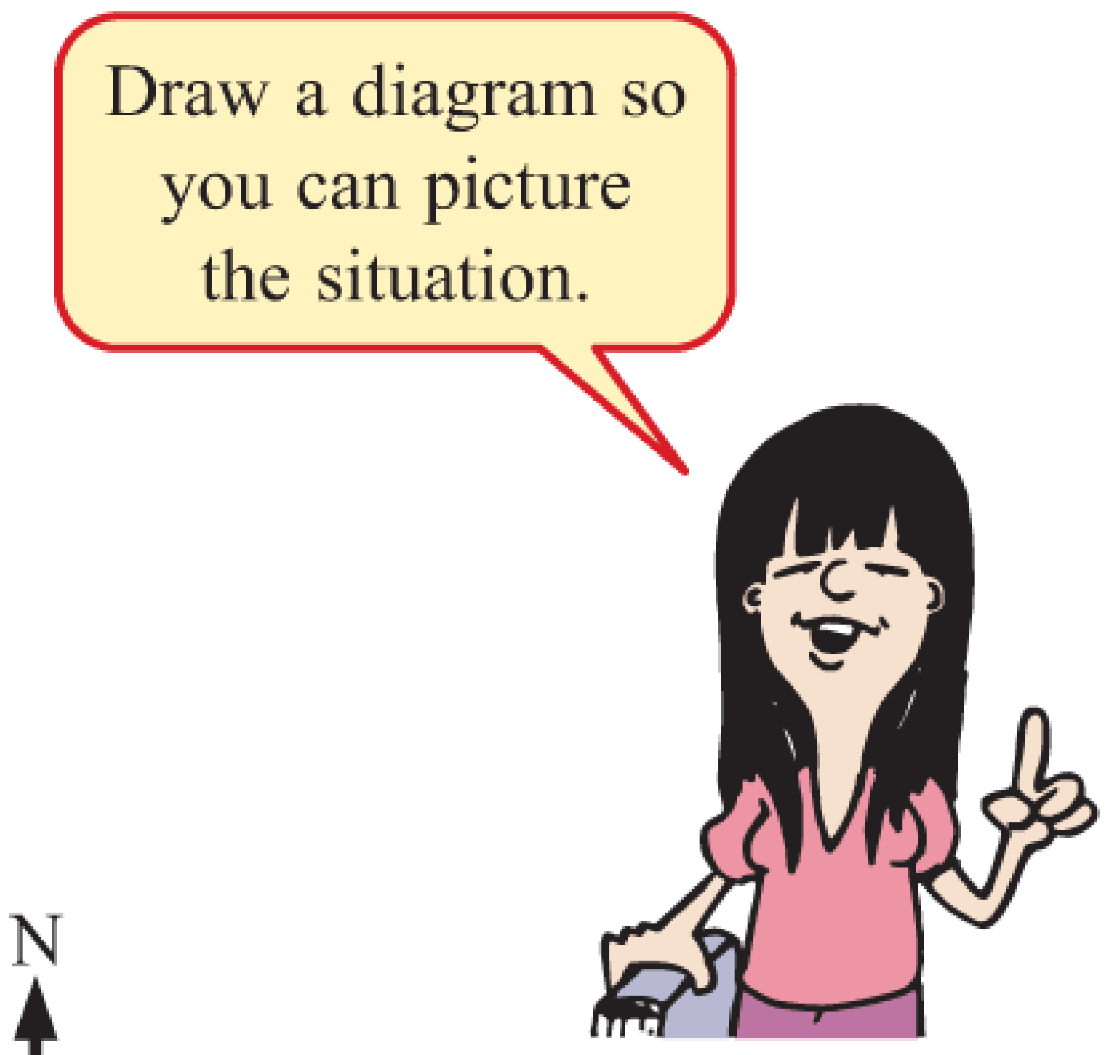


The combined effect is travelling 5 km south.

EXERCISE 11B

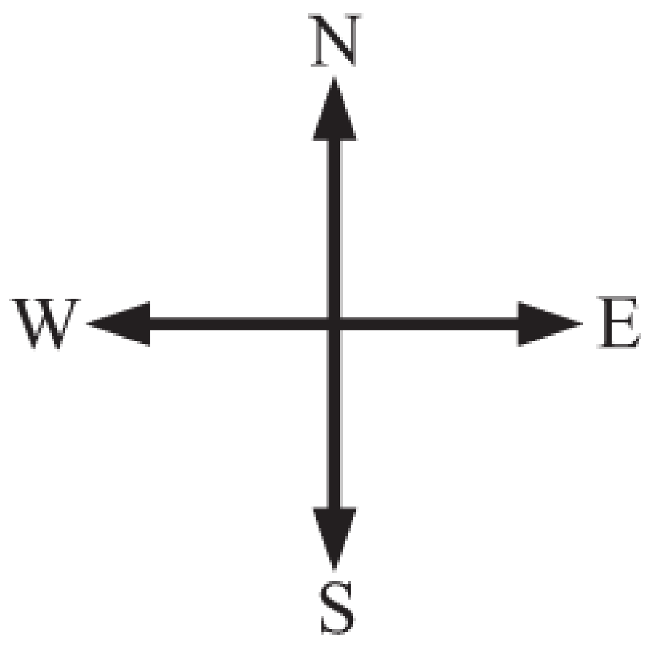
1 Henry is travelling in a lift. Find the combined effect of going:

- **a** up 3 floors and then down 1 floor
- **b** up 4 floors and then down 5 floors
- **c** down 2 floors and then up 6 floors
- **d** down 3 floors and then down 4 floors
- **e** down 8 floors and then up 5 floors.



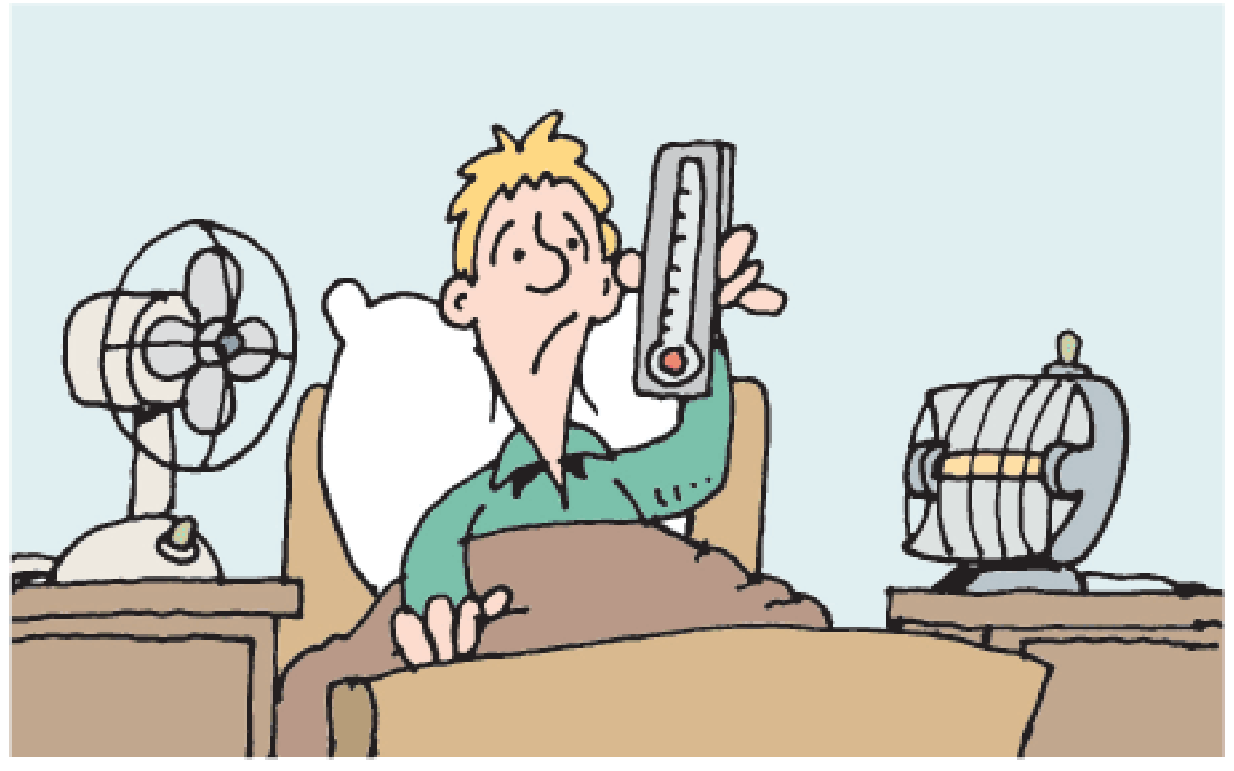
2 Find the combined effect of travelling:

- **a** 10 km east and then 6 km west
- **b** 7 km east and then 9 km west
- **c** 4 km west and then 5 km east
- **d** 11 km west and then 3 km east.



3 Boris checks the temperature in his bedroom on a regular basis. Find the combined effect if his bedroom gets:

- **a** 5°C warmer, then 2°C cooler
- **b** 1°C warmer, then 6°C cooler
- **c** 4°C cooler, then 11°C warmer
- **d** 8°C cooler, then 7°C warmer.

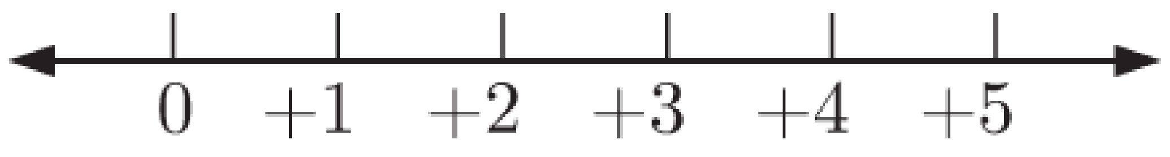


4 Find the combined effect of:

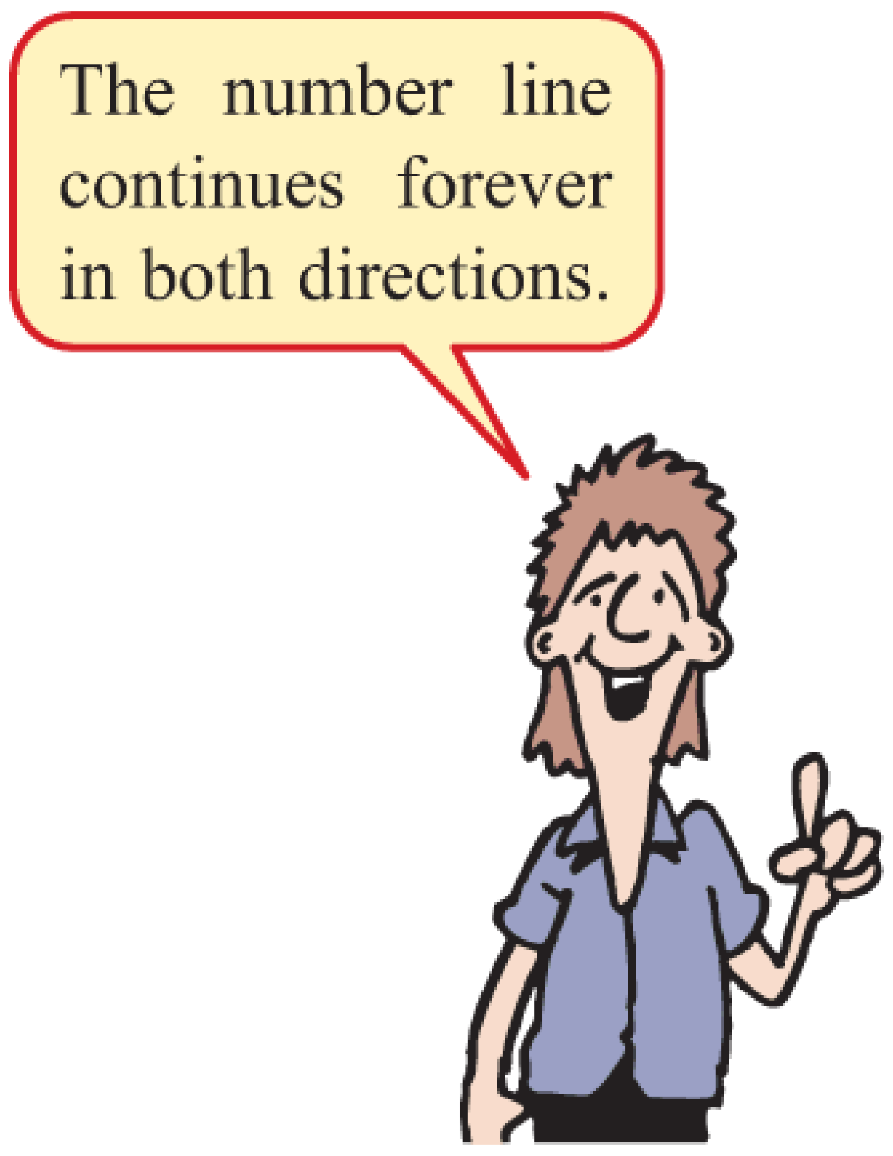
- **a** earning \$50 and then spending \$20
- **b** earning £5 and then spending £15
- **c** spending €10 and then earning €30
- **d** spending \$40 and then earning \$40.

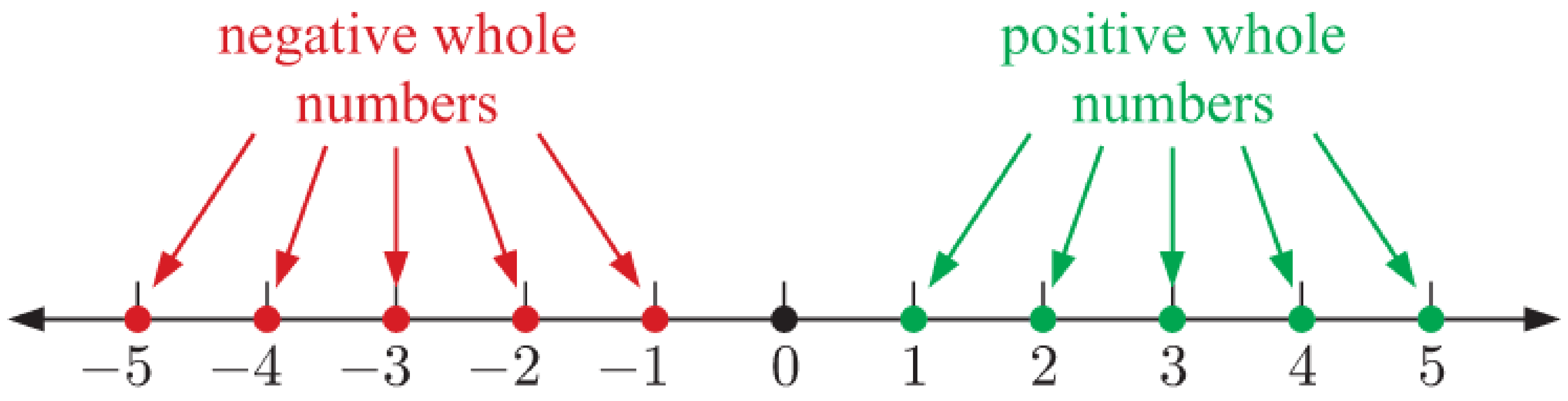
C THE NUMBER LINE

We have seen previously that we can place positive numbers on a number line.



We can also place negative numbers on a number line by extending the number line to the left:





Zero is neither positive nor negative.

The negative whole numbers, zero, and the positive whole numbers are together known as the **integers**.

In the same way that earning \$5 and spending \$5 are opposites, we say that +5 and −5 are **opposites**. They are the same distance from 0, but on opposite sides of 0.

Example 3 **Self Tutor**

Write the opposite of:

a +6 **b** −3

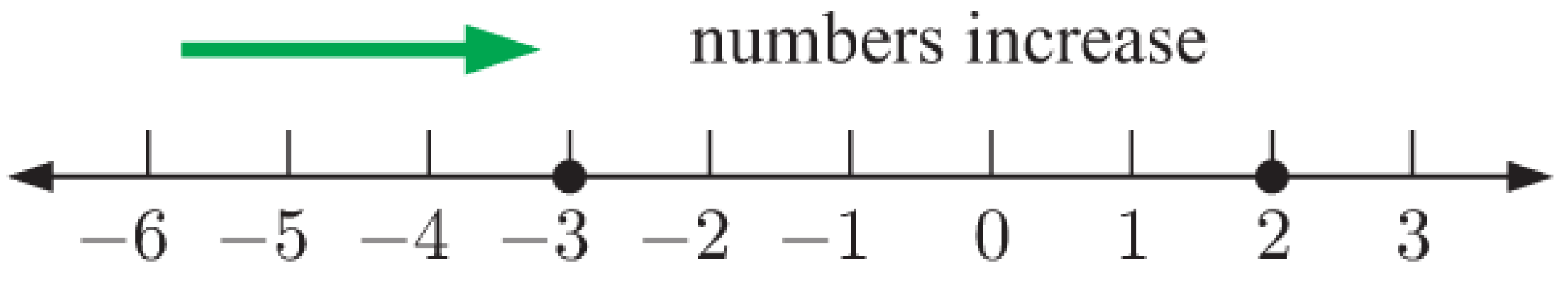
a The opposite of +6 is −6. **b** The opposite of −3 is +3.

DISCUSSION

Does zero have an opposite?

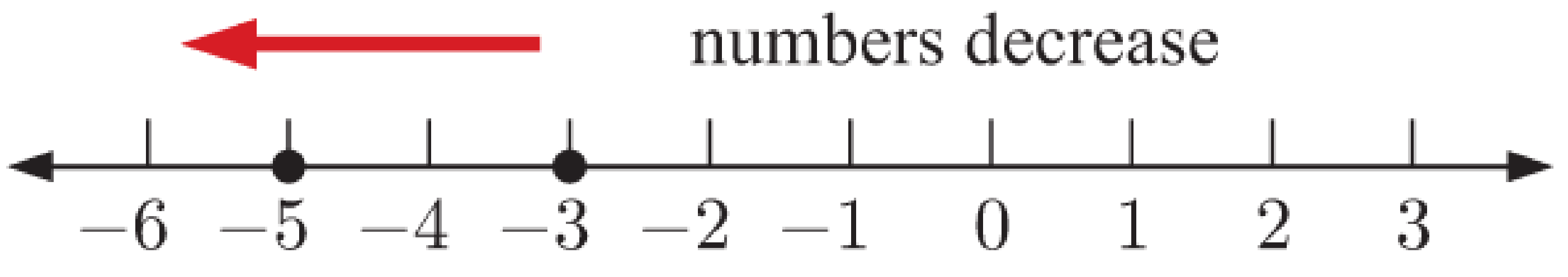
We can use a number line to compare the sizes of different numbers and arrange them in order.

As you move along the number line from *left* to *right*, the numbers increase. In a group of numbers, the number furthest to the right is the greatest number.



+2 is *greater* than −3 because it is further to the right on the number line.

As you move along the number line from *right* to *left*, the numbers decrease in size. In a group of numbers, the number furthest to the left is the least number.

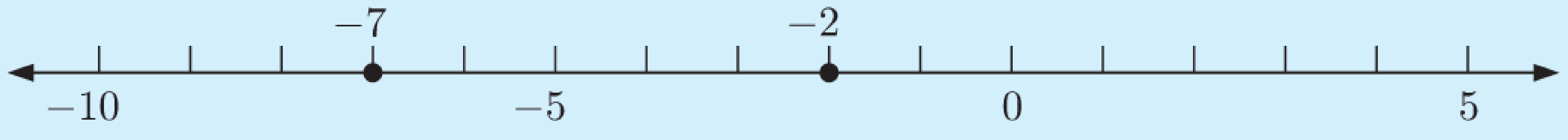


−5 is *less* than −3 because it is further to the left on the number line.

Example 4 **Self Tutor**

a Place −7 and −2 on a number line. **b** Which of these numbers is larger?

a



b −2 is further to the right on the number line, so −2 is larger than −7.

EXERCISE 11C

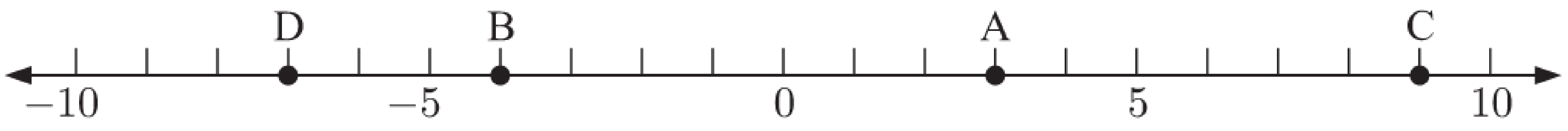
1 State whether these integers are positive, negative, or neither:

a 5 **b** −4 **c** −1 **d** +3
e −7 **f** 0 **g** 9 **h** −8

2 Write the opposite of:

a +7 **b** −1 **c** 2 **d** −4 **e** +11 **f** −13

3 Write the values of A, B, C, and D:



4 Place the following numbers on a number line. Use a different number line for each part.

- **a** $2,\ -5,\ 1,\ -3$
- **b** $-8,\ 4,\ -1,\ 6$
- **c** $-2,\ 0,\ 7,\ -4$
- **d** $-9,\ 3,\ -7,\ -6,\ 5$

5 For each pair of values:

- **i** place the values on a number line
- **ii** determine which value is larger.

- **a** 2 and -1
- **b** -6 and -3
- **c** -5 and 3
- **d** -5 and -8

6 Consider the numbers $4,\ -8,\ 0,\ -2,\ -5,$ and 1.

- **a** Place the numbers on a number line.
- **b** Use the number line to write the numbers in order from smallest to largest.

7 The table alongside shows the elevation above sea level of various cities.

- **a** Place these values on a number line.
- **b** Which of these cities are below sea level?
- **c** Which of these cities has the:
  - **i** highest elevation
  - **ii** lowest elevation?

Location	*Elevation*
Dublin, Ireland	8 m
Hachirogata, Japan	-4 m
Tripoli, Libya	5 m
New Orleans, USA	-2 m
Jakarta, Indonesia	3 m

D ADDITION AND SUBTRACTION WITH NEGATIVE NUMBERS

We can use a number line to perform additions and subtractions involving negative numbers.

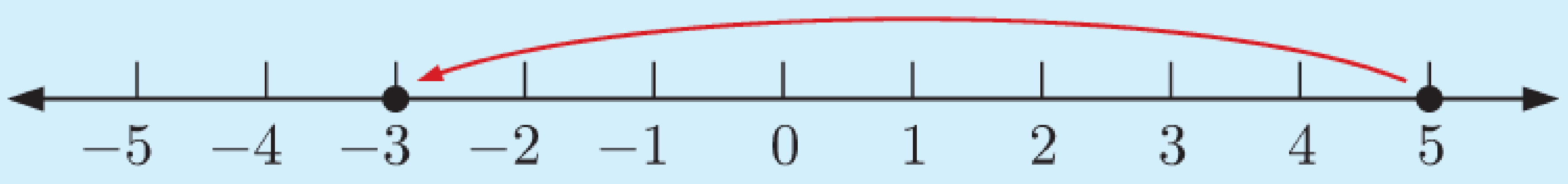
- To **increase** a number, we move **right** along the number line.
- To **decrease** a number, we move **left** along the number line.

Example 5 **Self Tutor**

Use a number line to find:

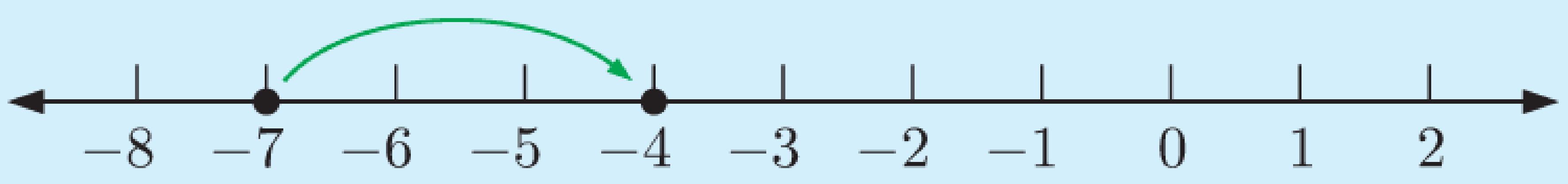
- **a** $5-8$
- **b** $-7+3$

a We start at 5, and move 8 units to the left.



So, $5-8=-3$

b We start at -7, and move 3 units to the right.



So, $-7+3=-4$

Subtracting 8 means we *decrease* the number by 8. We move to the *left*.

EXERCISE 11D.1

1 Use a number line to find:

a $3-5$ **b** $-1+4$ **c** $0-6$ **d** $-9+2$

e $-4-4$ **f** $7-12$ **g** $-3+0$ **h** $-8-5$

i $1-9$ **j** $-5+5$ **k** $-11+7$ **l** $-6-7$

2 Use a number line to find:

a $2+3-6$ **b** $-7+4-2$ **c** $-5+8+3$

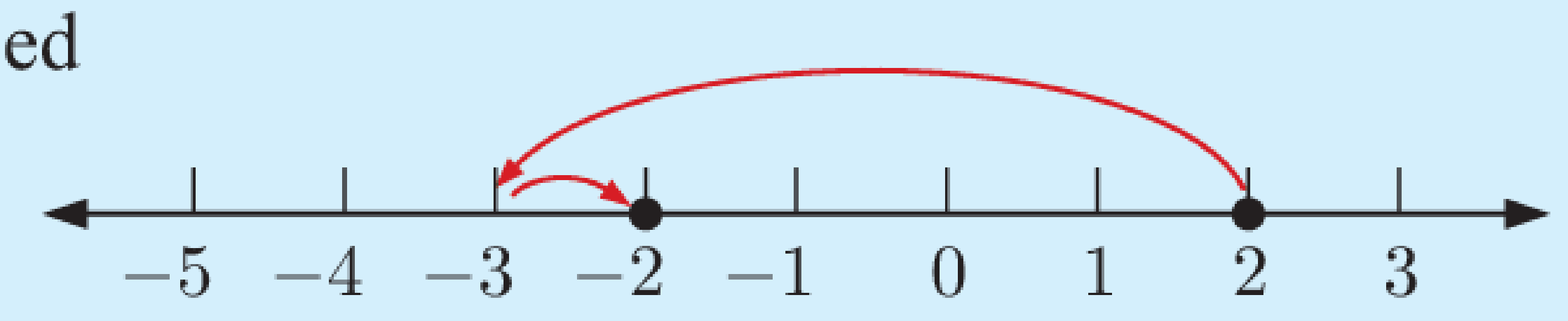
d $3-10+1$ **e** $-8+6-7$ **f** $-9-5-2$

Example 6 **Self Tutor**

A lift started 2 floors above ground level. It travelled down 5 floors, then up 1 floor.

How many floors above or below ground level is the lift now?

The lift starts at $+2$ or just 2. We therefore need to find $2-5+1$.



Using a number line, $2-5+1=-2$.

$\therefore$ the lift is now 2 floors *below* ground level.

3 A lift started 1 floor below ground level. It travelled 4 floors up, then 6 floors down.
How many floors above or below ground level is the lift now?

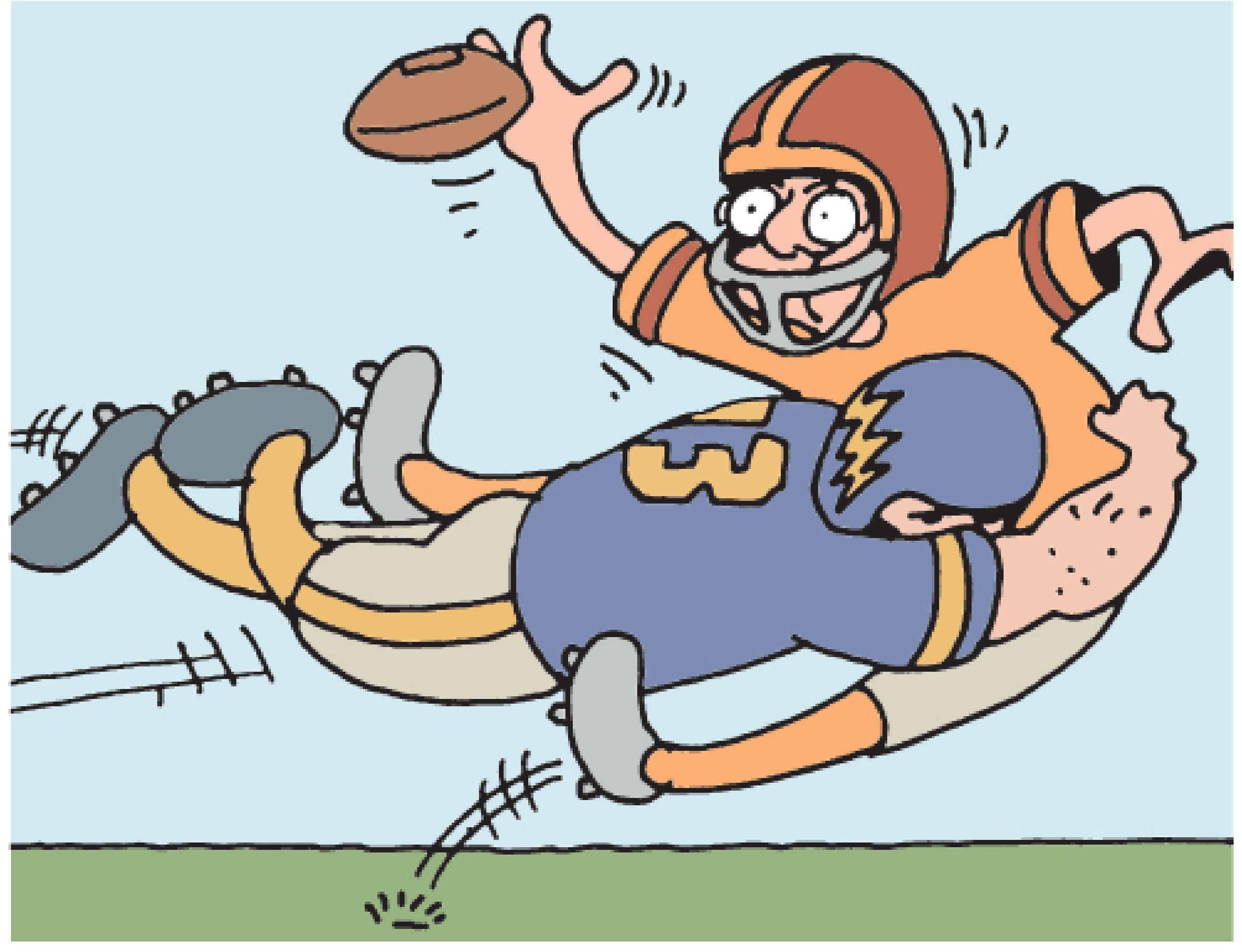
4 At midnight, the temperature in Chicago was -3°C. Between midnight and 9 am, the temperature dropped by 4°C, then rose by 6°C. What was the temperature at 9 am?

5 A gridiron team was 7 points ahead at quarter time. They lost the 2nd quarter by 15 points, won the 3rd quarter by 13 points, and lost the 4th quarter by 9 points.

a By how many points was the team winning or losing:

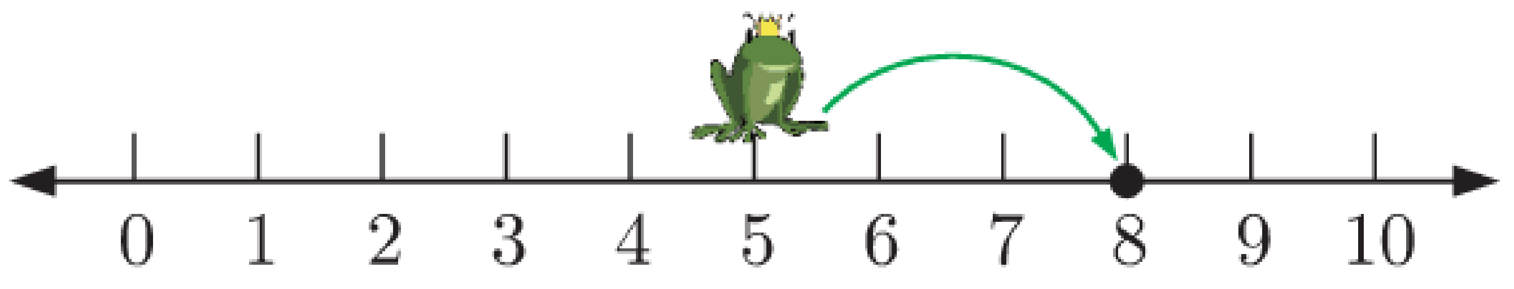
i at half time **ii** after three quarters?

b Did the team win or lose the game? By how much?



ADDING A NEGATIVE NUMBER

Freddy the frog is sitting at 5 on the number line. He is performing an addition, so he faces the right.



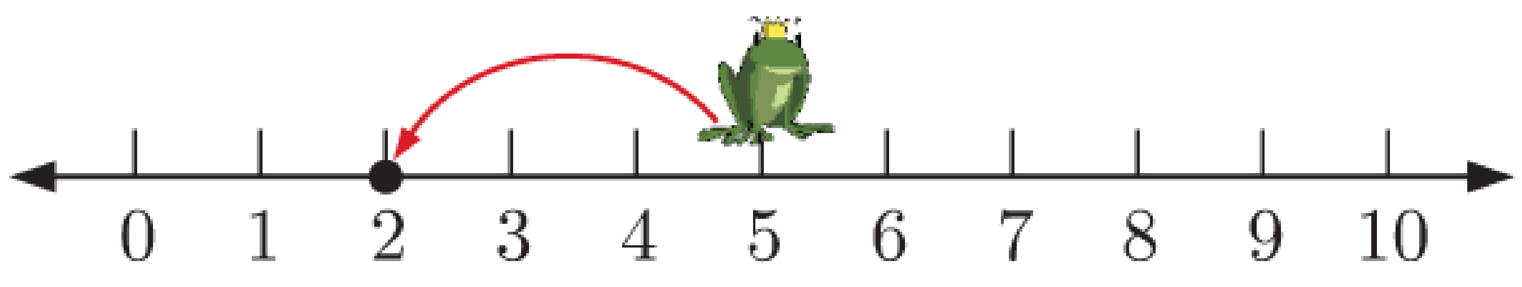
To find $5+3$, Freddy starts at 5 then hops 3 units forwards, to the right.

So, $5+3=8$.

DEMO

Freddy can use the same approach to find $5 + -3$.

He again starts at 5, facing the right. To add *negative* 3, he hops 3 units *backwards*. He ends up 3 units to the left of his starting point.



So, $5 + -3 = 2$.

Notice also that $5 - 3 = 2$.

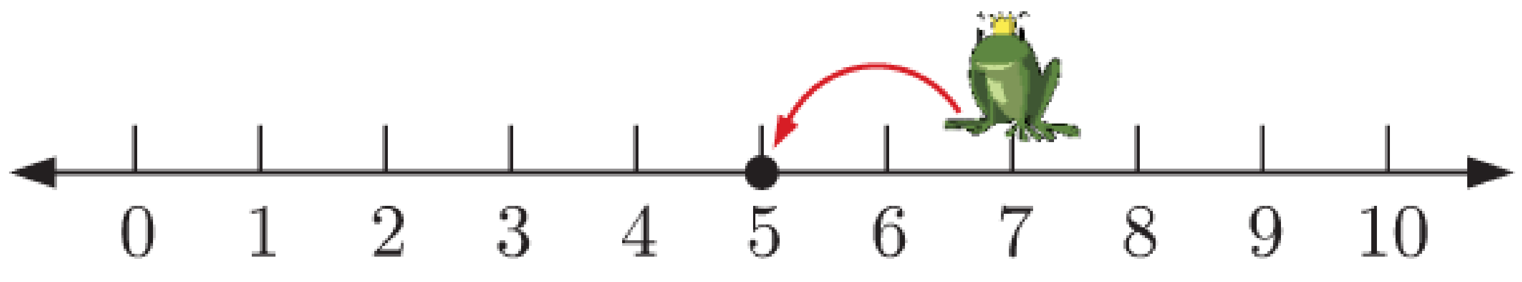
Adding a **negative** number is the same as **subtracting** its **opposite**.

For example, $1 + -4$ is the same as $1 - 4$.

SUBTRACTING A NEGATIVE NUMBER

To perform a subtraction, Freddy faces the left.

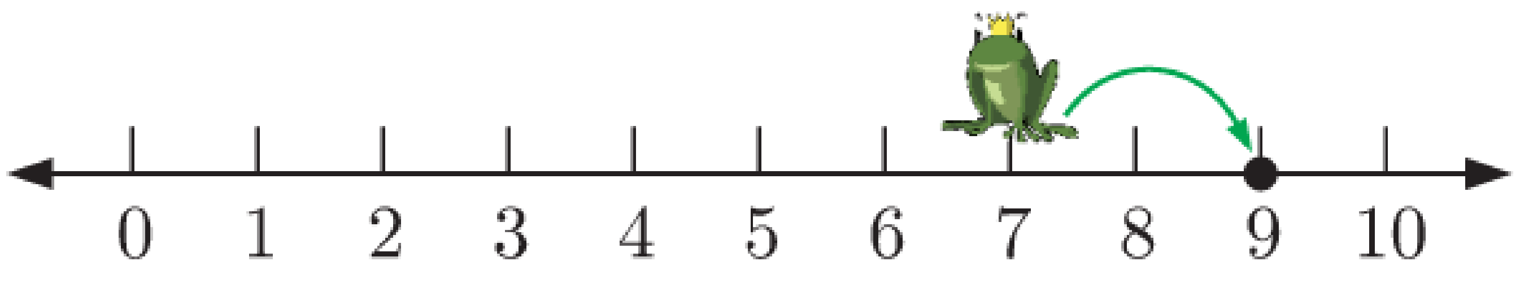
To find $7 - 2$, Freddy starts at 7 then hops 2 units forwards, to the left.



So, $7 - 2 = 5$.

DEMO

To find $7 - -2$, Freddy starts at 7, again facing the left. To subtract *negative* 2, he hops 2 units *backwards*. He ends up 2 units to the right of his starting point.



So, $7 - -2 = 9$.

Notice also that $7 + 2 = 9$.

Subtracting a **negative** number is the same as **adding** its **opposite**.

For example, $6 - -3$ is the same as $6 + 3$.

Example 7 **Self Tutor**

Simplify and then find:

a $3 + -7$ **b** $3 - -7$ **c** $-3 + -7$ **d** $-3 - -7$

a $3 + -7$
$= 3 - 7$
$= -4$

b $3 - -7$
$= 3 + 7$
$= 10$

c $-3 + -7$
$= -3 - 7$
$= -10$

d $-3 - -7$
$= -3 + 7$
$= 4$

EXERCISE 11D.2

1 Simplify and then find:

a $6 + -2$ **b** $6 - -2$ **c** $-6 + -2$ **d** $-6 - -2$
e $5 + -9$ **f** $5 - -9$ **g** $-5 + -9$ **h** $-5 - -9$

2 Simplify and then find:

a $10 + -4$
b $1 - -7$
c $-3 + -5$
d $-8 - -14$
e $-9 - -6$
f $-4 + -9$
g $0 - -5$
h $-11 - -18$
i $-4 + -5$
j $2 - -11$
k $-3 + -4$
l $-12 - -4$

3 Simplify and then find:

a $2 + 7 + -3$
b $8 - 5 + -4$
c $10 - -6 - 8$
d $-3 + -8 - -5$
e $7 + -11 + -1$
f $-5 + 14 + -9$

Example 8 **Self Tutor**

Find the difference between:

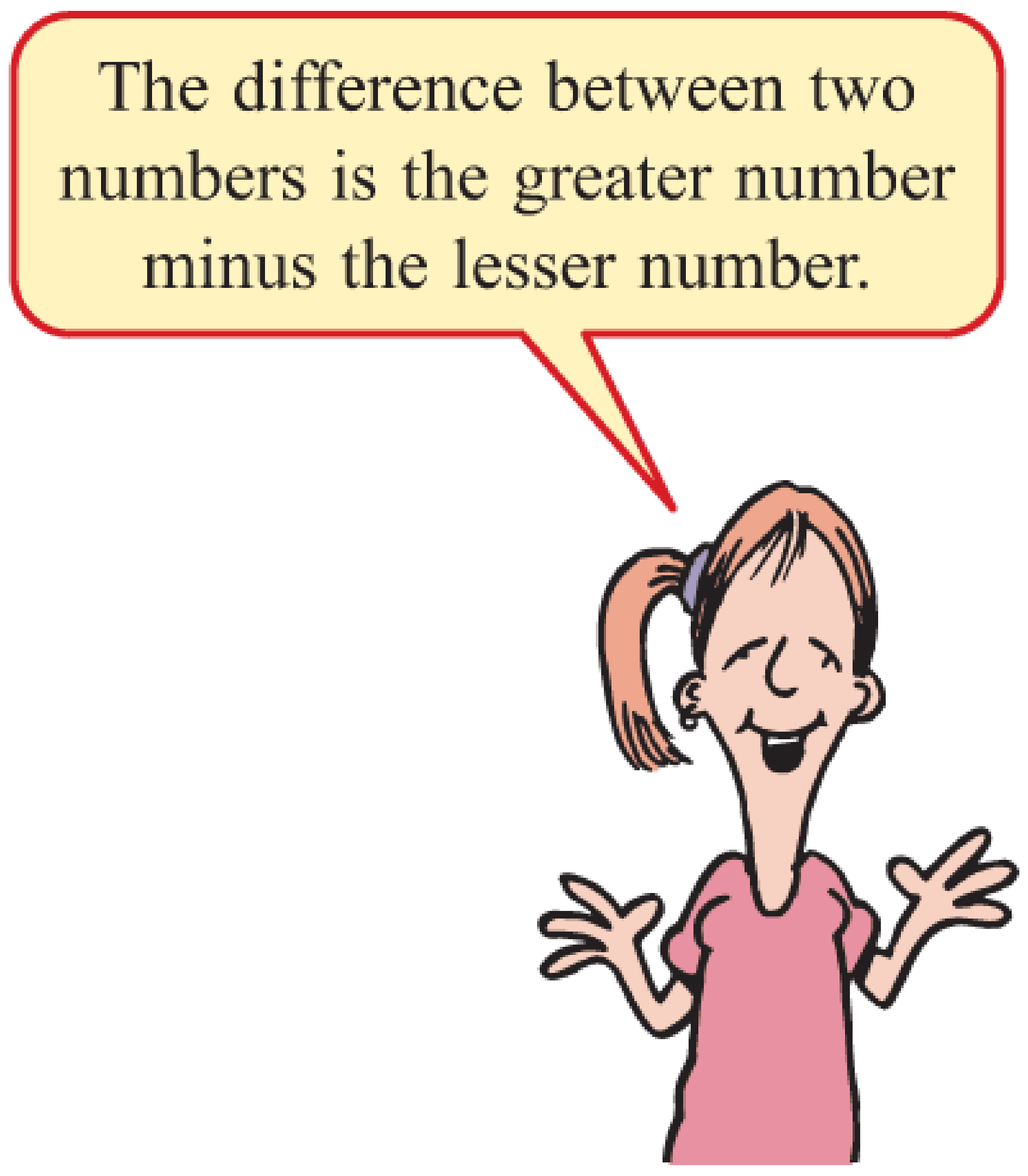
a 2 and -6
b -11 and -1

a 2 is to the right of -6 on the number line, so 2 is greater than -6.

$$\begin{aligned}\text{The difference between } 2 \text{ and } -6 &= 2 - -6\\ &= 2 + 6\\ &= 8\end{aligned}$$

b -1 is to the right of -11 on the number line, so -1 is greater than -11.

$$\begin{aligned}\text{The difference between } -11 \text{ and } -1 &= -1 - -11\\ &= -1 + 11\\ &= 10\end{aligned}$$



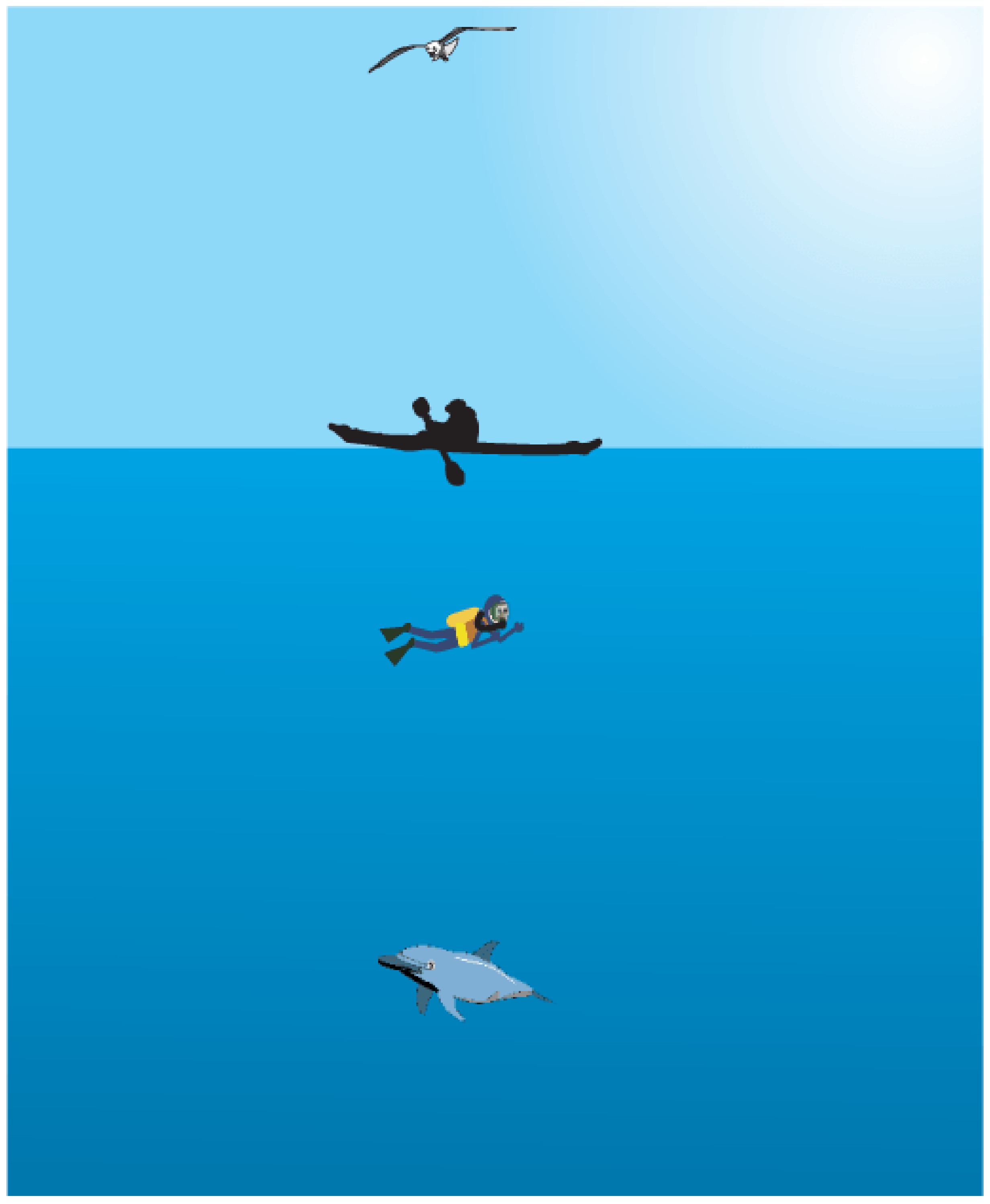
4 Find the difference between:

a 3 and -4
b -7 and 2
c -4 and -8
d 0 and -6
e 11 and -8
f -13 and -9
g -4 and 10
h -7 and -5
i 5 and -7

5 A seagull is flying 5 metres above sea level, a kayaker is paddling at sea level, a diver is swimming 2 metres below sea level, and a dolphin is 6 metres below sea level.

a Write an integer to describe the height above sea level of each person or animal.

b Find the distance between the:
  i kayaker and the diver
  ii seagull and the dolphin
  iii dolphin and the kayaker
  iv diver and the dolphin.



6 Write the following statements using integers, and calculate the resulting quantity:

- **a** spending \$50 then spending \$40
- **b** a fall in temperature of 20°C followed by a rise of 13°C
- **c** a profit of £310 followed by a loss of £97
- **d** a fall in temperature of 5°C followed by a fall of 8°C
- **e** a journey 21 km north followed by a journey 29 km south
- **f** winning by 13 points in the 1st half then losing by 17 points in the 2nd half
- **g** a loss of 3 kg followed by a loss of 2 kg.

7 Team A is playing Team B in the card game canasta. This table shows the number of points scored by each team for the first three hands.

	Team A	*Team B*
Hand 1	-50	810
Hand 2	400	140
Hand 3	900	-300

- **a** Find the difference in points scored by the teams in:
  - **i** hand 1
  - **ii** hand 2
  - **iii** hand 3.
- **b** Find the total number of points scored by each team so far.

8 This table shows the minimum and maximum temperatures in Juneau, Alaska, during a week in February 2014:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum temperature	-4°C	-6°C	-2°C	-3°C	-5°C	-12°C	-13°C
Maximum temperature	1°C	3°C	1°C	0°C	0°C	-3°C	-2°C

- **a** Which day had the:
  - **i** warmest maximum temperature
  - **ii** coolest maximum temperature
  - **iii** warmest minimum temperature
  - **iv** coolest minimum temperature?
- **b** Find the difference between the minimum and maximum temperatures on:
  - **i** Monday
  - **ii** Thursday
  - **iii** Saturday.

9 Steve, Paul, Peter, and William played together in the first round of a golf tournament. Each player's score shows how many shots above or below the 'par' score they took. In golf, a lower score is better.

Player	*Score*
Steve	$+2$
Paul	-3
Peter	-1
William	$+5$

- **a** Rank these scores in order, from best to worst.
- **b** Find the difference between:
  - **i** Steve's score and Paul's score
  - **ii** Paul's score and Peter's score.
- **c** In the next three rounds, William scores -4, -3, and $+1$. Find William's combined score for the tournament.

10 The children at a party are trying to guess how many lollies are in a jar. The results of the game are shown below. A positive number means the child guessed too high, and a negative number means the child guessed too low.

Child	*Result*
Max	$+30$
Xavier	-12
Lauren	-4
Molly	$+21$
Claire	$+3$
Jack	-24
Gabrielle	$+9$
Luis	-13
Kevin	-7

- **a** How many children guessed too low?
- **b** How many children were within 10 of the correct answer?
- **c** Which child's guess was the closest?
- **d** Find the difference between:
  - **i** Xavier's guess and Kevin's guess
  - **ii** Gabrielle's guess and Luis' guess.
- **e** Jack's guess was 50 lollies.
  - **i** How many lollies were in the jar?
  - **ii** What was Molly's guess?

GAME

This game can be played by 2 or more people.

You will need: Two sets of cards with the numbers -15 to 15 written on them (62 cards in total). You can make your own cards, or print them off by clicking on the icon.

PRINTABLE CARDS

$\boxed{-15}\ \boxed{-14}\ ....\ \boxed{-1}\ \boxed{0}\ \boxed{1}\ ....\ \boxed{14}\ \boxed{15}$

$\boxed{-15}\ \boxed{-14}\ ....\ \boxed{-1}\ \boxed{0}\ \boxed{1}\ ....\ \boxed{14}\ \boxed{15}$

What to do:

1. Shuffle the cards, then deal 6 cards to each player. Place the remaining cards face down in a pile.
2. Select a player to go first. The player takes a card from the top of the pile. The player then looks at his or her cards, and tries to form a set of 3 cards which sum to zero.

   Examples of sets include $\boxed{-8}\ \boxed{12}\ \boxed{-4}$ and $\boxed{7}\ \boxed{-7}\ \boxed{0}$.
3. If the player can form a set, the player puts the cards to one side, and then takes another turn. If not, play passes to the next player.
4. Play continues until all of the cards in the pile have been taken. The winner is the player with the most sets.

E MULTIPLYING NEGATIVE NUMBERS

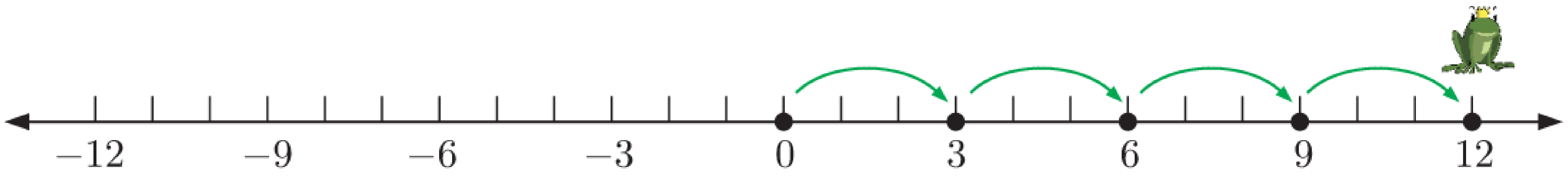
We have already seen how to add and subtract negative numbers. In this Section we look for rules for their **multiplication**.

For example, we know that $4 \times 3 = 12$, but we also need to be able to calculate:

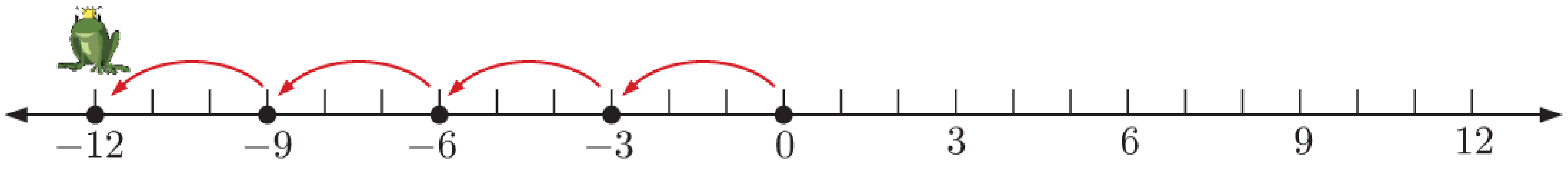
- 4×-3
- -4×3
- -4×-3

You have probably seen previously how we can think of 4×3 as "4 lots of three".

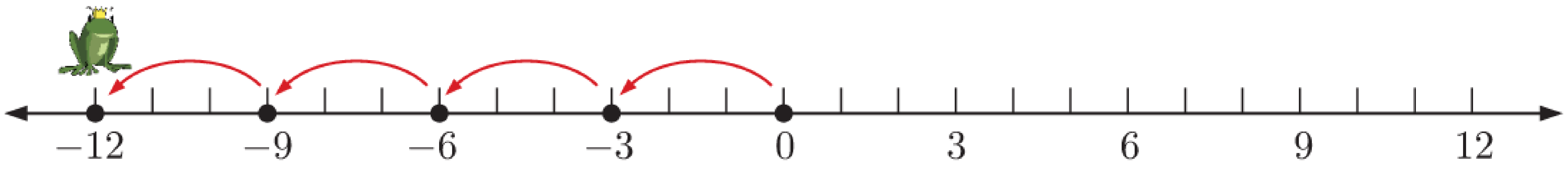
On the number line below, Freddy the Frog makes four jumps of positive 3. Each jump is positive, so he faces the right. We see that $4 \times 3 = 12$.



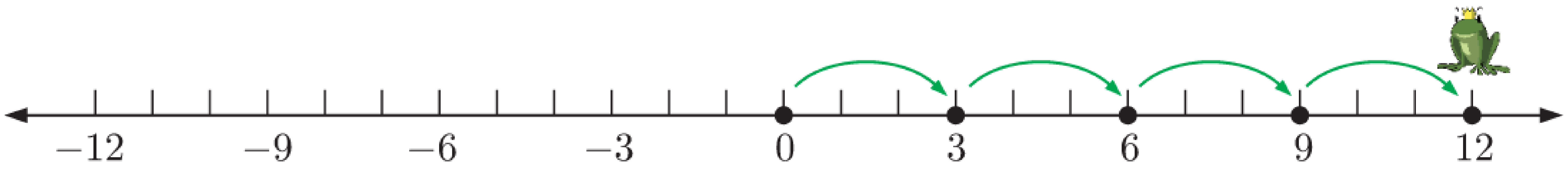
Now we consider 4×-3. Freddy now makes 4 jumps of negative 3. Each jump is negative, so he faces the left. We see that $4 \times -3 = -12$.



For the case -4×3, Freddy is again jumping 3 units each time, and he is facing the right. However, he now jumps four times *backwards*. We see that $-4 \times 3 = -12$.



Finally, we have -4×-3. Freddy is jumping three units each time, and is facing the left. He jumps four times *backwards*. We see that $-4 \times -3 = 12$.

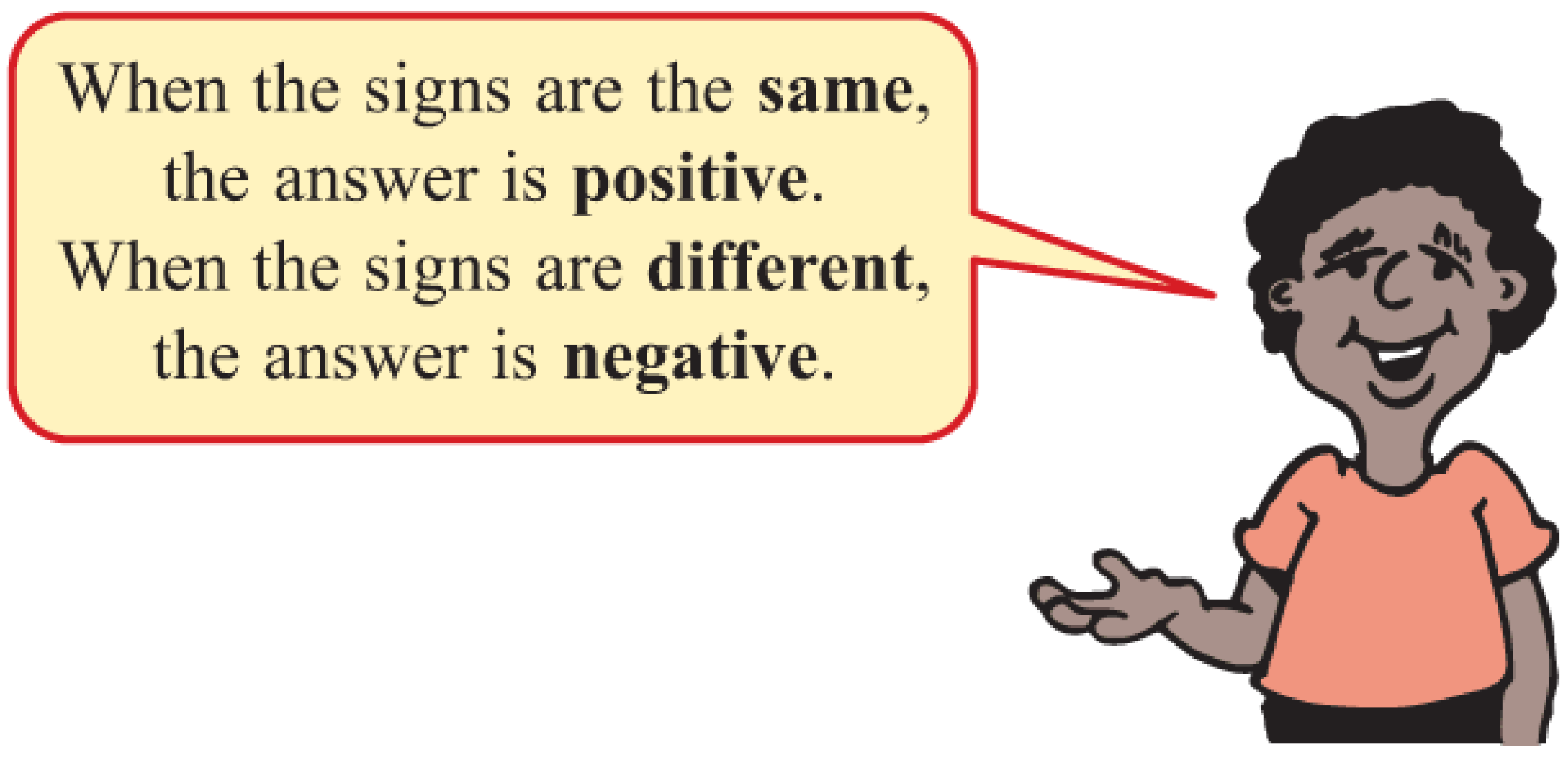


You can click on the icon for a demonstration.

DEMO

RULES FOR MULTIPLICATION

- **(positive) × (positive) = (positive)**
- **(positive) × (negative) = (negative)**
- **(negative) × (positive) = (negative)**
- **(negative) × (negative) = (positive)**



Example 9 Self Tutor

Find:

a 3×2 **b** 3×-2 **c** -3×2 **d** -3×-2

a $3 \times 2 = 6$ **b** $3 \times -2 = -6$ **c** $-3 \times 2 = -6$ **d** $-3 \times -2 = 6$

EXERCISE 11E

1 Find:

a 2×5 **b** 2×-5 **c** -2×5 **d** -2×-5
e 5×-2 **f** -5×2 **g** 5×2 **h** -5×-2

2 Find:

a 4×-2 **b** -5×3 **c** 7×-4 **d** 9×-1
e -3×-6 **f** -8×5 **g** -1×-7 **h** -4×11
i 5×-5 **j** -8×-8 **k** -9×7 **l** -12×10
m -8×7 **n** -6×-11 **o** -5×9 **p** -7×-12

3 In a multiple choice test containing 12 questions, students gain 5 marks for each question answered correctly, and they lose 3 marks for each question answered incorrectly.
What is the:

a highest possible score **b** lowest possible score?

4 List all the ways of writing -30 as the product of two whole numbers.

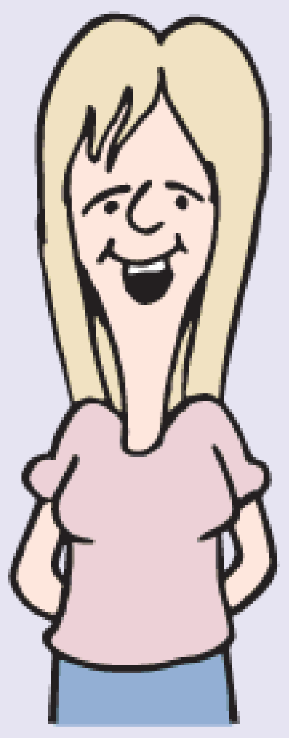
PUZZLE

Place these 10 numbers in a row, so that when any two adjacent numbers are multiplied together, the result is between -20 and -1 inclusive.

Adjacent numbers are numbers which are next to each other.

$5 \quad -4 \quad 3 \quad -3 \quad -5$

$-1 \quad 2 \quad -7 \quad 10 \quad 6$



F DIVIDING NEGATIVE NUMBERS

In this Section we look for rules for the **division** of negative numbers.

For example, we know that $15 \div 5 = 3$, but we also need to be able to calculate:

- $15 \div -5$
- $-15 \div 5$
- $-15 \div -5$

INVESTIGATION 1 DIVIDING NEGATIVE NUMBERS

Multiplication facts can be used to help solve division problems.

For example:

- $4 \times 5 = 20$, so $20 \div 4 = 5$
- $8 \times 3 = 24$, so $24 \div 8 = 3$

What to do:

1 Copy and complete:

a $6 \times 2 = 12$, so $12 \div 6 = ...$

b $-7 \times -4 = 28$, so $28 \div -7 = ...$

c $9 \times -5 = -45$, so $-45 \div 9 = ...$

d $-3 \times 10 = -30$, so $-30 \div -3 = ...$

2 What do you suspect is the result when:

a a positive number is divided by a positive number

b a positive number is divided by a negative number

c a negative number is divided by a positive number

d a negative number is divided by a negative number?

The rules for division are identical to those for multiplication.

RULES FOR DIVISION

(positive) ÷ (positive) = (positive)
(positive) ÷ (negative) = (negative)
(negative) ÷ (positive) = (negative)
(negative) ÷ (negative) = (positive)

When the signs are the **same**, the answer is **positive**.
When the signs are **different**, the answer is **negative**.

Example 10 **Self Tutor**

Find:

a $14 \div 7$ **b** $14 \div -7$ **c** $-14 \div 7$ **d** $-14 \div -7$

a $14 \div 7 = 2$ **b** $14 \div -7 = -2$ **c** $-14 \div 7 = -2$ **d** $-14 \div -7 = 2$

EXERCISE 11F

1 Find:

a $18 \div 3$ **b** $18 \div -3$ **c** $-18 \div 3$ **d** $-18 \div -3$

e $36 \div 9$ **f** $36 \div -9$ **g** $-36 \div 9$ **h** $-36 \div -9$

i $5 \div 5$ **j** $5 \div -5$ **k** $-5 \div 5$ **l** $-5 \div -5$

m $42 \div 6$ **n** $42 \div -6$ **o** $-42 \div 6$ **p** $-42 \div -6$

2 Find:

a $-12 \div 4$ **b** $-18 \div 9$ **c** $6 \div -2$ **d** $20 \div -5$

e $-24 \div -4$ **f** $35 \div -7$ **g** $-9 \div 3$ **h** $32 \div -8$

i $-26 \div -2$ **j** $-54 \div 6$ **k** $-9 \div -9$ **l** $-49 \div -7$

m $80 \div -8$ **n** $-63 \div 7$ **o** $-96 \div -12$ **p** $121 \div -11$

3 Suppose a negative number is divided by a positive number, and then the result is divided by a negative number. Is the final answer positive or negative? Explain your answer.

INVESTIGATION 2 CALCULATOR USE

We can use our calculator to perform operations with negative numbers. The negative numbers are entered on the calculator using a key such as $\boxed{+/-}$ or $\boxed{(-)}$.

For example, to find $-3 - -2$, press $\boxed{+/-}$ 3 $\boxed{-}$ $\boxed{+/-}$ 2 $\boxed{=}$.

GRAPHICS CALCULATOR INSTRUCTIONS

What to do:

1 Use your calculator to find:

a $-10 + 8$ **b** $23 - 48$ **c** $-57 + 39$

d $-31 + 49 + -60$ **e** $19 - -83 + -43$ **f** $-124 + -71 - -94$

g 13×-17 **h** -18×-22 **i** $456 \div -24$

2 Use your calculator to solve the following problems:

a Los Angeles has an altitude of 113 m, and the Dead Sea has an altitude of -423 m. Find the difference between their altitudes.

b At the start of the month, Harry's credit card balance was $-\$347$. During the month he bought a dishwasher for \$549, then made a credit card repayment of \$250. What is his credit card balance now?

KEY WORDS USED IN THIS CHAPTER

- integer
- number line
- negative number
- opposite
- negative sign
- positive number

REVIEW SET 11A

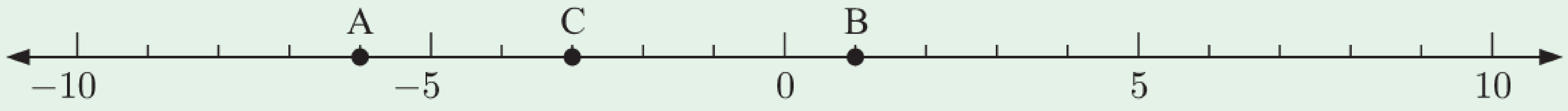
1 State whether these integers are positive, negative, or neither:

a -4 **b** 3 **c** 0 **d** -12

2 Write the opposite of:

a travelling 10 km south **b** getting 3 cm shorter.

3 Find the values at points A, B, and C.

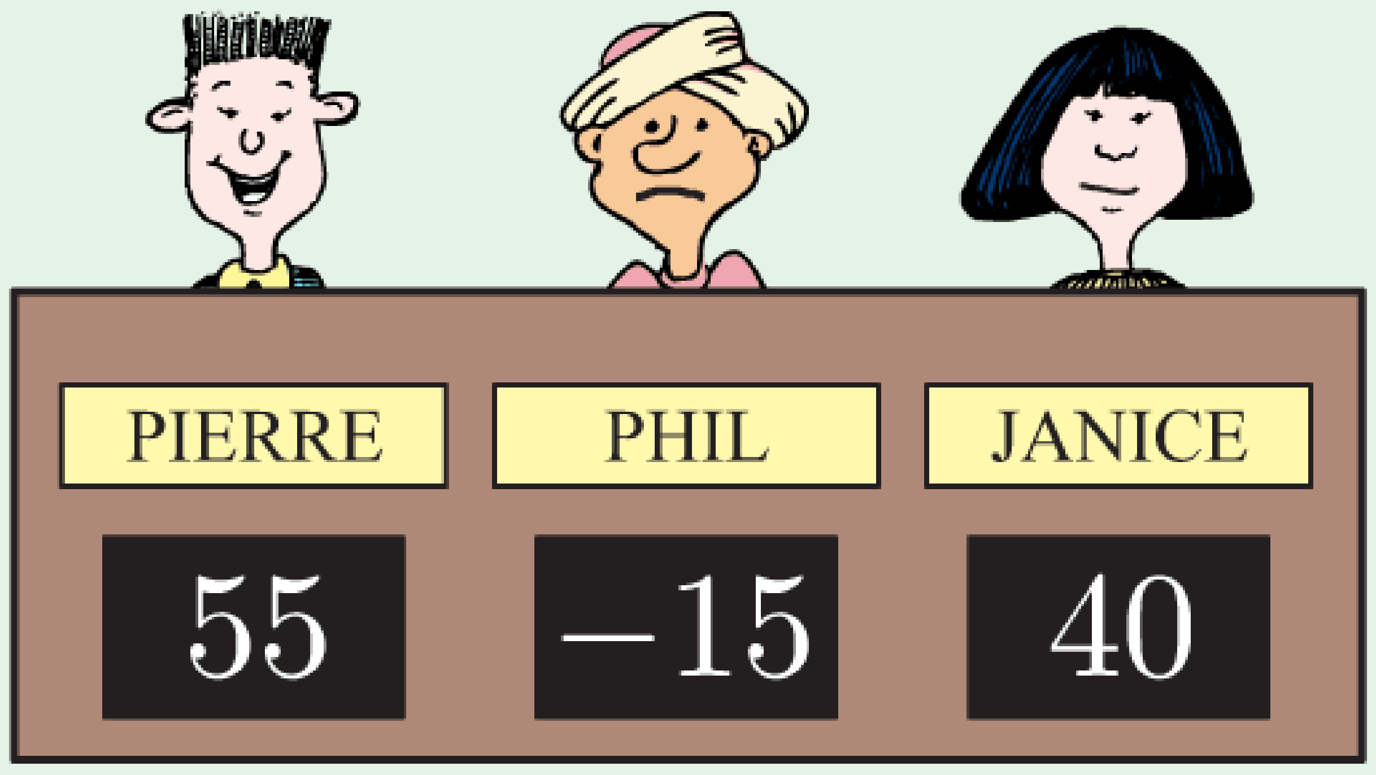


4 **a** Place the values -7 and -5 on a number line.
 b Determine which of the values in **a** is larger.

5 Use a number line to find:

a $6-10$ **b** $-4+7$ **c** $-5-6$
d $5-10+3$ **e** $-8-3+7$ **f** $1-7+9$

6 The scores of three contestants at the end of a game show are given alongside.
What is the difference between Phil's score and Janice's score?



7 Isabella ran a lemonade stall at a school festival. She made a \$10 profit in the first hour, a \$25 loss in the second hour, and a \$7 profit in the third hour. Find the total profit or loss made by Isabella over the three hours.

8 Simplify and then find:

a $3+-8$ **b** $5--5$ **c** $-6--13$
d $9+-9$ **e** $11+-7--1$ **f** $2+-10--4$

9 Find:

a 2×-7 **b** -9×-3 **c** -6×6 **d** -5×-10

10 Find:

a $-33\div 3$ **b** $16\div-4$ **c** $-72\div-9$ **d** $48\div-8$

REVIEW SET 11B

1 Find the combined effect of:

a going up 5 floors and then down 7 floors
b travelling 8 km east and then 2 km west
c getting 3°C warmer and then 9°C cooler.

2 Write the opposite of:

a $+5$ **b** -8 **c** 12

3 Place the following numbers on a number line: -9, 4, 0, -2, 8.

4 Find:

a $11+-1$ **b** $3--9$ **c** $-10--10$ **d** $-4+13+-8$

5 Find the difference between:

a 4 and -4 **b** -9 and -2 **c** -3 and 0

6 In a spelling quiz, students were awarded points for answering questions correctly, and they lost points for answering questions incorrectly. Nick scored 7 points, Teresa scored 12 points, and Brett scored -4 points.

Find the difference between:

- **a** Teresa's score and Brett's score
- **b** Brett's score and Nick's score.

7 Find:

- **a** 6×-1
- **b** -3×7
- **c** 10×-9
- **d** -11×-12

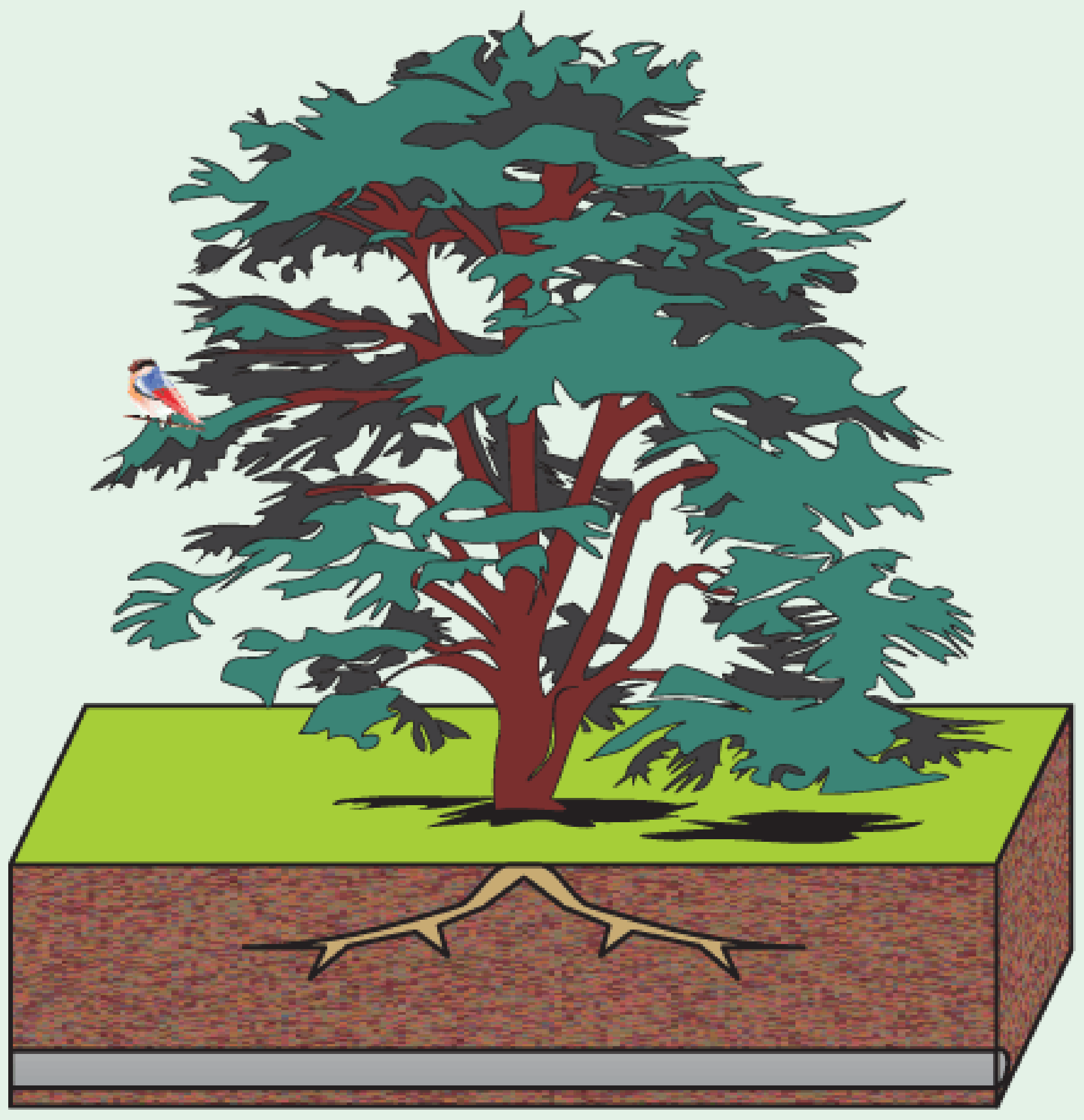
8 Answer the **Opening Problem** on page **216**.

9 Find:

- **a** $-13 + 8$
- **b** -12×5
- **c** $-2 - -8$
- **d** $-20 \div -4$
- **e** $-7 + -6 - 5$
- **f** -11×-9

10 The top of a tree is 5 metres above ground level. A bird in the tree is 3 metres above ground level, the tree's roots are 1 metre below ground level, and a pipe is 2 metres below ground level.

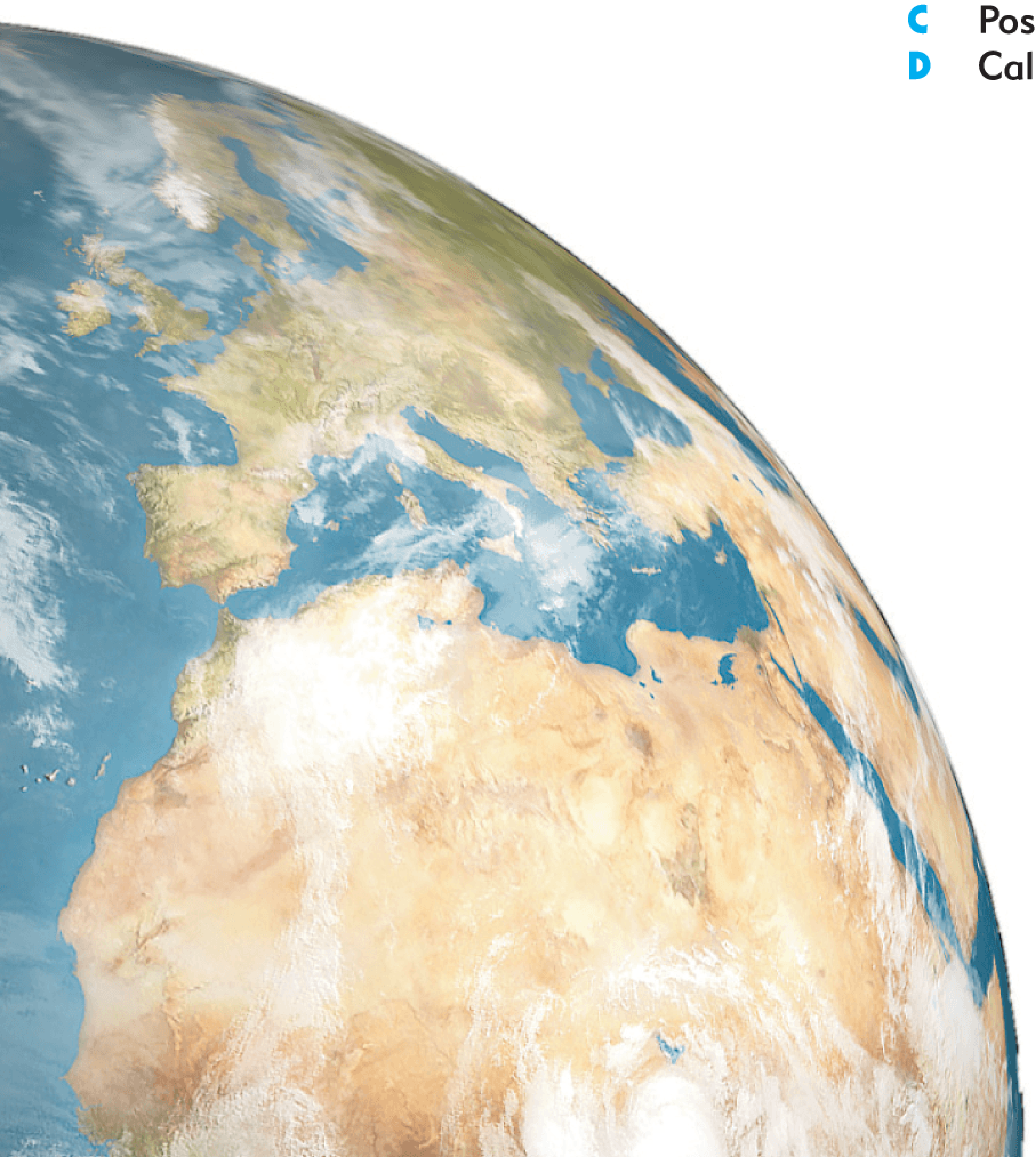
- **a** Write a positive or negative number to describe the height above ground level of each object.
- **b** Find the difference in height between the:
  - **i** tree top and the roots
  - **ii** bird and the pipe
  - **iii** roots and the pipe.



Chapter 12

Probability

Contents:



OPENING PROBLEM

There are 25 students in Mr Donnelly's class. On St Patrick's Day, he places 25 cards in a box, one of which shows a lucky 4-leaf clover. The students line up in alphabetical order ready to choose a card each.

Aaron is not happy about going first. "There are so many cards in the box," he says. "It is almost impossible for me to pick the 4-leaf clover."

Zara is not happy about going last. "The 4-leaf clover will almost certainly be gone by the time I get to pick!" she says.

Things to think about:

a What words did Aaron and Zara use to describe the *chance* of events occurring?

b Can you write a number to describe the chance of Aaron selecting the 4-leaf clover?

c Do you think Aaron and Zara are equally likely to select the 4-leaf clover?

Consider the following statements which all involve future events:

- "It is *highly likely* that it will rain today."
- "John will *probably* play computer games after school."
- "Your team has a *50-50 chance* of winning on Saturday."

For most events in the future, we cannot be certain whether they will occur or not.

All of the above statements give some indication about how *likely* the event is to occur.

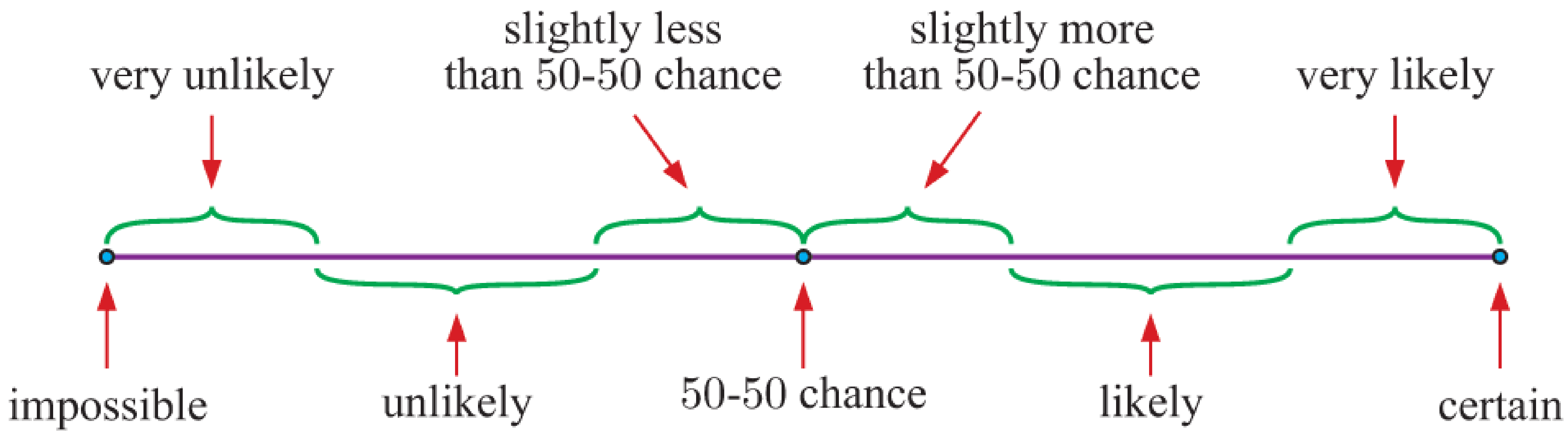
The **probability** of an event is the likelihood or chance of it occurring.

A DESCRIBING PROBABILITY

Words that are used to describe the probability of something happening in the future include:

- likely
- very likely
- slightly less than 50-50 chance
- impossible
- 50-50 chance
- unlikely
- very unlikely
- slightly more than 50-50 chance
- certain

We can place these words on a line, in order from least likely to most likely:



The probability of every event must lie somewhere between *impossible* and *certain*.

Example 1 **Self Tutor**

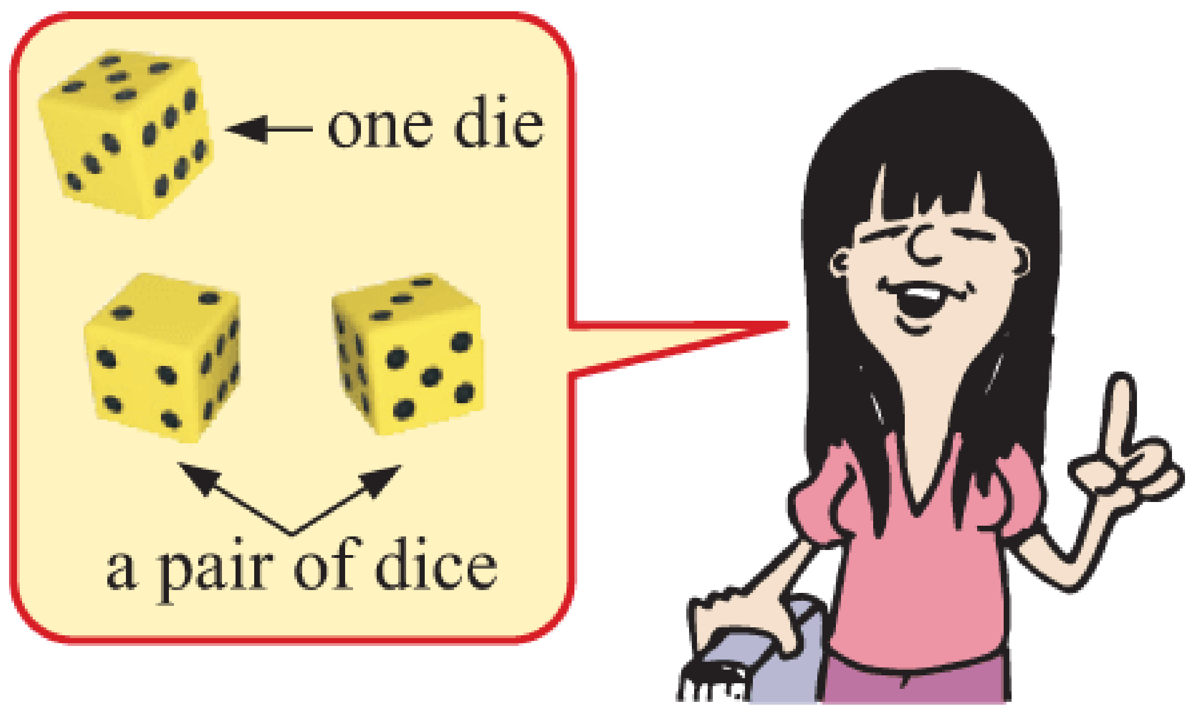
Use a word or phrase to describe the probability of these events occurring:

a A blue ball can be selected from a bag containing only red balls.
b It will rain somewhere in the world today.

a It is *impossible* for a blue ball to be selected from a bag containing only red balls.
b It is *very likely* that it will rain somewhere in the world today.

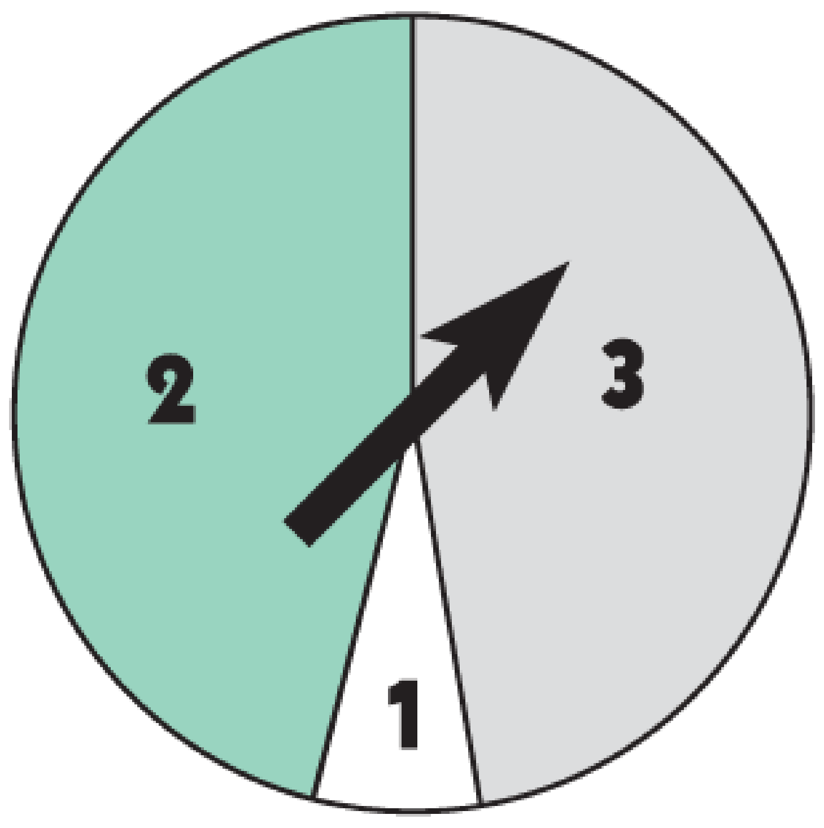
EXERCISE 12A

1 Use a word or phrase to describe the probability of these events occurring:



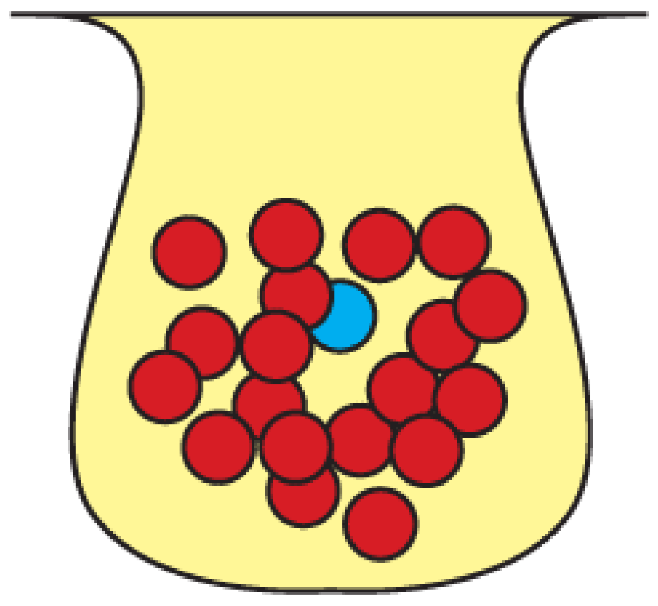
- **a** Next year there will be 30 days in February.
- **b** You will eat a meal between 5 pm and 10 pm tonight.
- **c** You will roll a '6' on your next roll with a die.
- **d** The next child born at a particular hospital will be a girl.

2 Copy and complete:



- **a** It is ...... that the spinner will stop in the white sector.
- **b** It is ...... for the spinner to stop in a sector marked 4.
- **c** It is ...... that the spinner will stop in a sector with a number.
- **d** It is ...... that the spinner will stop in a sector with a number greater than 1.

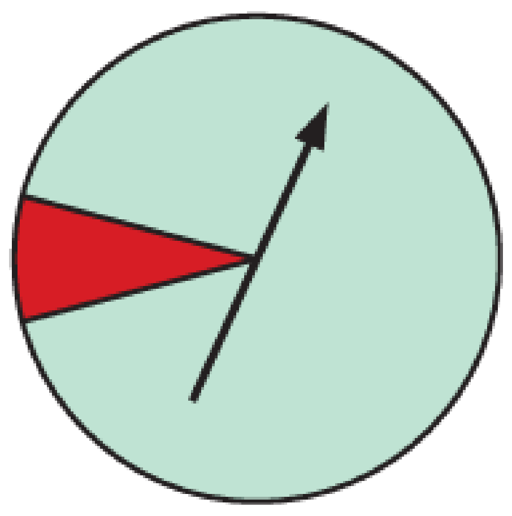
3 A bag contains 19 red marbles and one blue marble. A marble is randomly selected from the bag.
Copy and complete:



- **a** It is ...... that the marble will be blue.
- **b** It is ...... that the marble will be red.
- **c** It is ...... that the marble will be either blue or red.
- **d** It is ...... for the marble to be green.

4 For each of the following pairs of events, determine whether event A or event B is more likely to occur:

a A: The spinner alongside will land on green.
B: The spinner alongside will land on red.

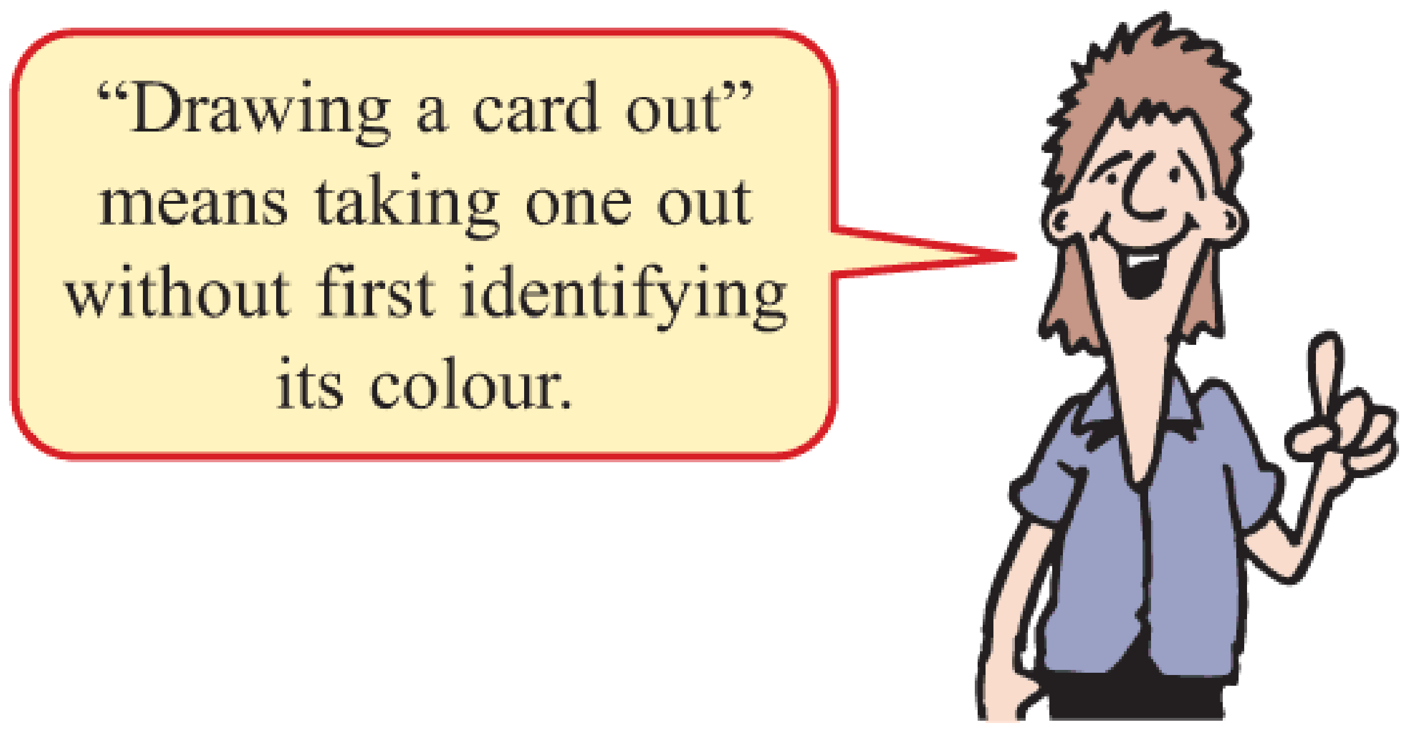


b A: A randomly selected 6 year old is taller than 120 cm.
B: A randomly selected 10 year old is taller than 120 cm.

c A: Your parents will buy vegetables next week.
B: Your parents will buy a car next week.

5 You have six blue and six white cards. You have been asked to place *four* cards in a hat. What cards would you place in the hat so that:

- **a** you will be certain of drawing out a blue card
- **b** it will be impossible to draw out a blue card
- **c** there will be a 50-50 chance of drawing out a blue card
- **d** you will be more likely to draw out a blue card than a white card?



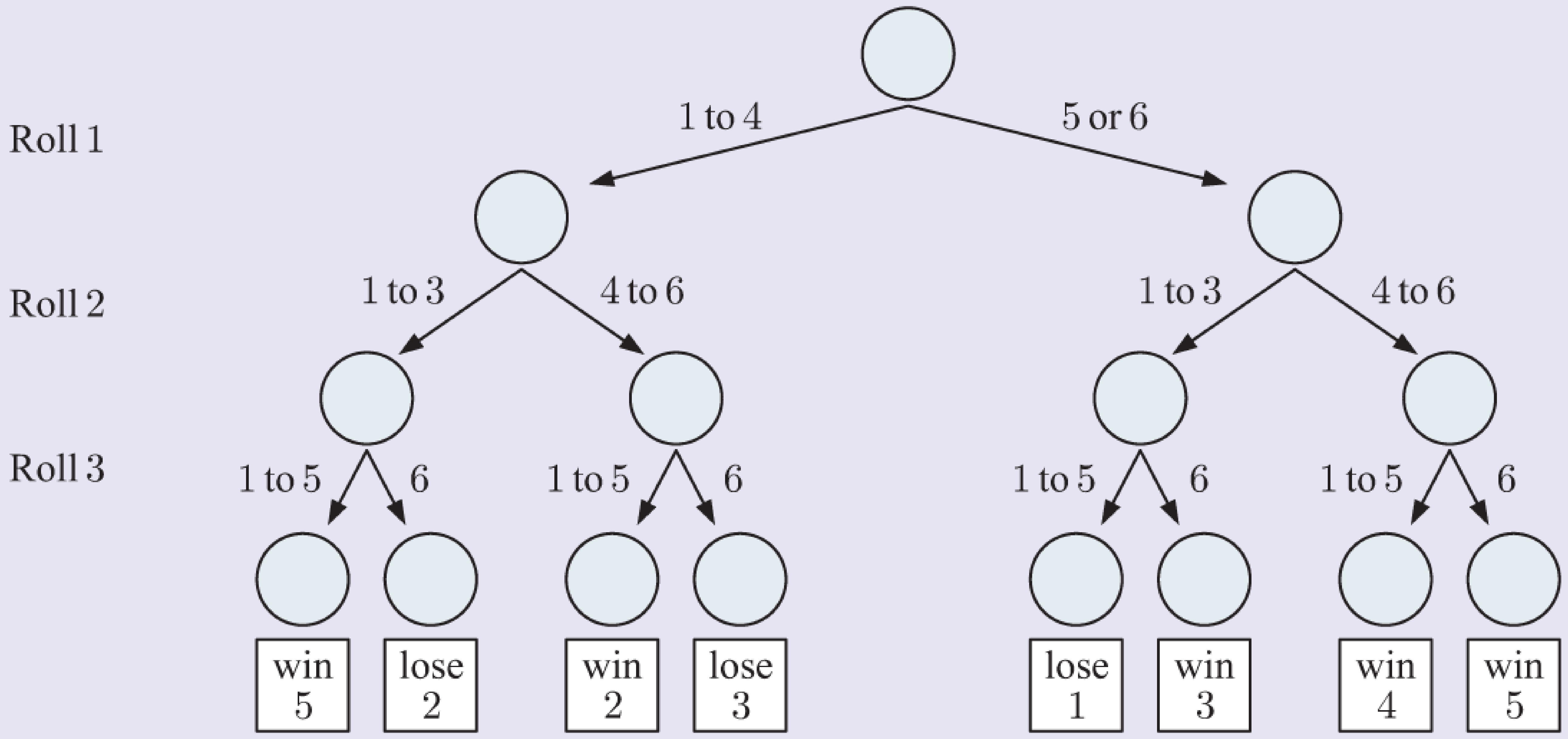
DISCUSSION

Can you think of two *impossible* events and two *certain* events in *your* life? Remember that what may be impossible for you may be possible for someone else! Discuss your ideas with your class.

GAME DICING WITH DANGER

This is a game for two to four players.

Each player takes a turn at rolling a die 3 times. The player moves his or her counter down the game tree according to the number on the die after each roll. After the third roll, the score for the player is recorded.



For example, suppose Sree rolls 5, 4, 5 and Jane rolls 2, 3, 6. Sree would win 4 points and Jane would lose 2 points.

ANIMATION

You can keep track of each player's score by filling in a score card like the one opposite.

SCORE CARD TEMPLATE

	Sree	Jane
Round 1	+4	−2
Round 2		
⋮		
Round 10		
Total		

After 10 rounds, the scores for each player are added up. The player with the highest total wins.

B ASSIGNING NUMBERS TO PROBABILITIES

Chermaine and Min are experienced gymnasts. Each time they perform a routine on the balance beam, it is *unlikely* that either of them will fall.

To *compare* the chance of each gymnast falling, we need a more accurate description than just words. We need a number that describes the probability of each gymnast falling.

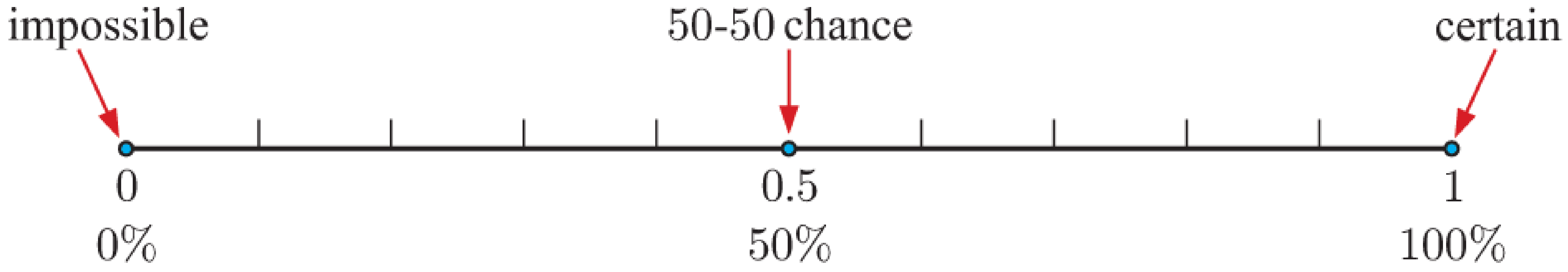
If it is **impossible** for an event to occur, we assign it the probability 0 or 0%.

If an event is **certain** to occur, we assign it the probability 1 or 100%.

The chance of any event occurring must lie between the extremes of impossible and certain.

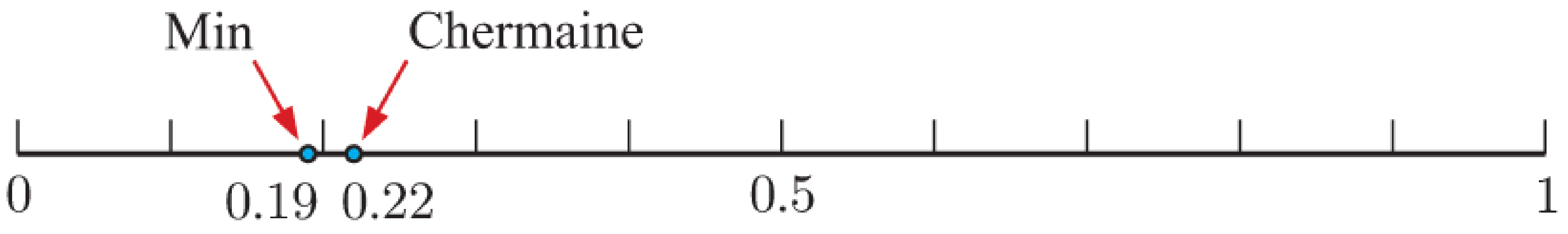
So, the probability of any event occurring lies between 0 and 1, or 0% and 100% inclusive.

We can indicate the probability of an event by placing it on a number line.



For example, in competitions in the past year, Chermaine fell in 22% of her beam routines. We can estimate that the probability she will fall in her next routine is 22% or 0.22.

In the past year, Min fell in 19% of her routines, so we estimate that the probability she will fall in her next routine is 19% or 0.19.

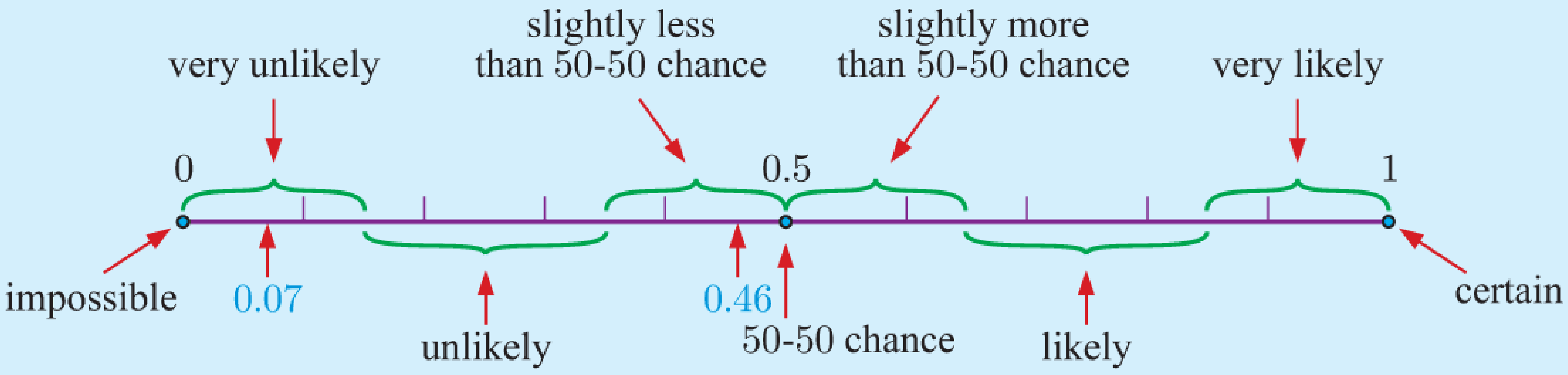


We conclude that Chermaine is more likely to fall in her next routine than Min.

Example 2 **Self Tutor**

Write a word or phrase to describe the probability value:

a 0.46 **b** 0.07



a slightly less than 50-50 chance **b** very unlikely

EXERCISE 12B

1 Match the following probability values with the most appropriate word or phrase:

a 0 **b** 0.3 **c** 0.92 **d** 0.5 **e** 0.56

A very likely **B** 50-50 chance **C** impossible

D unlikely **E** slightly more than 50-50 chance

2 Match the following events with the most appropriate probability value:

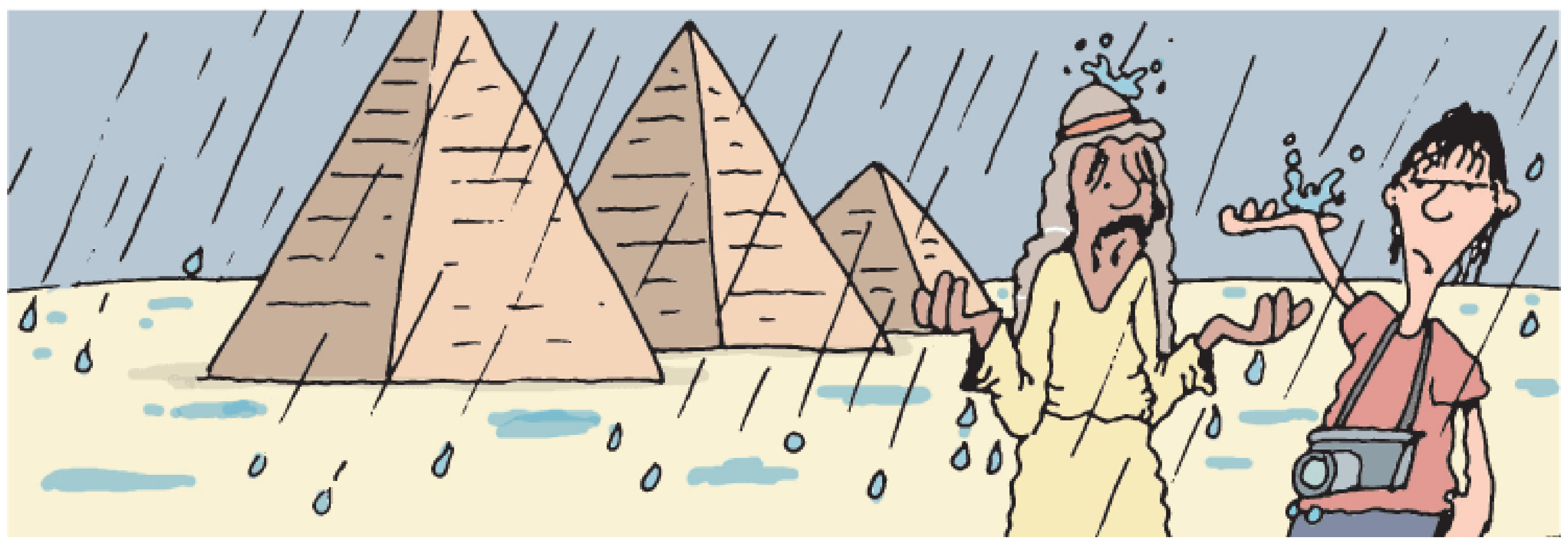
- **a** The next car that drives past your house will not be blue.
- **b** The next person you meet will be a twin.
- **c** In a school with two Year 6 classes, you are placed in the same class as your best friend.
- **d** Your birthday will be on the same date as last year.
- **e** The next point in a professional tennis match is won by the player who is receiving serve.

A 0.5 **B** 0.3 **C** 0.9 **D** 1 **E** 0.02

3 This table shows the probability of rain for some important cities on a particular day.

City	*Probability*
Berlin	18%
Johannesberg	24%
Lima	27%
Cairo	2%
Kuala Lumpur	31%
Dhaka	87%
Seattle	61%
Christchurch	67%

- **a** In which city is it:
  - **i** most likely to rain
  - **ii** least likely to rain?
- **b** Is it more likely to rain in Lima or Kuala Lumpur?
- **c** True or false?
  - **i** It is likely to rain in Christchurch.
  - **ii** It is very likely to rain in Cairo.



4 Netballers Jan, Natasha, and Ellie each shoot for goal from a particular spot. Jan has probability $\frac{4}{5}$ of shooting a goal, Natasha has probability 0.83 of shooting a goal, and Ellie has probability 58% of shooting a goal.

- **a** Write each of the probabilities as a percentage.
- **b** Write a word or phrase to describe the probability that Jan will shoot a goal.
- **c** Who is most likely to shoot a goal?

5 Each morning, Nicola has a drink with breakfast. She has milk $\frac{2}{7}$ of the time, orange juice $\frac{4}{7}$ of the time, and water $\frac{1}{7}$ of the time.

- **a** On any particular morning, which drink is Nicola most likely to have?
- **b** Find the probability that, on a particular morning, Nicola drinks *either* orange juice *or* water.

ACTIVITY WHAT WILL HAPPEN TOMORROW?

Below is a list of things which might happen tomorrow.

PRINTABLE LIST

What to do:

1 For each item on the list, write:

i a word or phrase, and **ii** a number from 0 to 1

to describe the probability of that event occurring tomorrow.

		Word or phrase	*Number*
a	You will go to school.	....	......
b	You will see a horse.	....	......
c	You will eat an apple.	....	......
d	You will break something.	....	......
e	You will receive a text message from a friend.	....	......
f	You will wear a jumper.	....	......
g	You will see one of your grandparents.	....	......
h	You will take a photograph.	....	......
i	You will use your ruler.	....	......
j	You will trip over.	....	......

2 Compare your answers with the person next to you. Explain any similarities and differences between your answers.

C POSSIBLE OUTCOMES

When we are considering the probability of a particular event occurring, it is useful to know the number of **possible outcomes**.

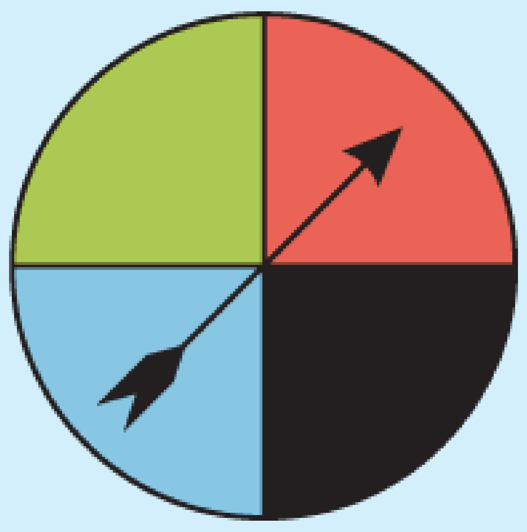
For example, when we toss a coin, there are 2 possible outcomes: *heads* and *tails*.

Heads *Tails*

Example 3 Self Tutor

a List the possible outcomes when this spinner is spun.

b How many possible outcomes are there?



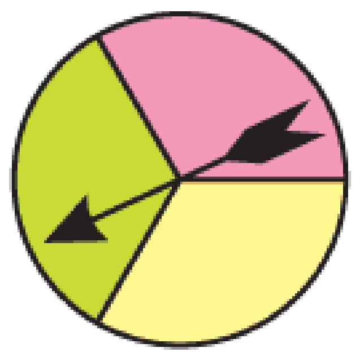
a The possible outcomes are red, black, blue, and green.

b There are 4 possible outcomes.

EXERCISE 12C

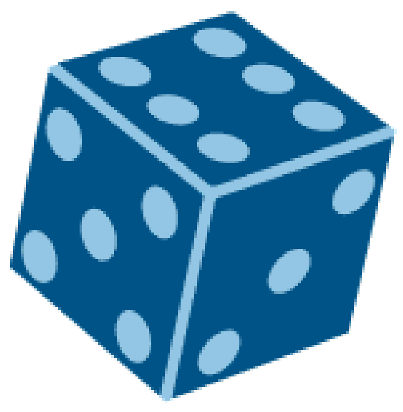
1 **a** List the possible outcomes when this spinner is spun.

b How many possible outcomes are there?

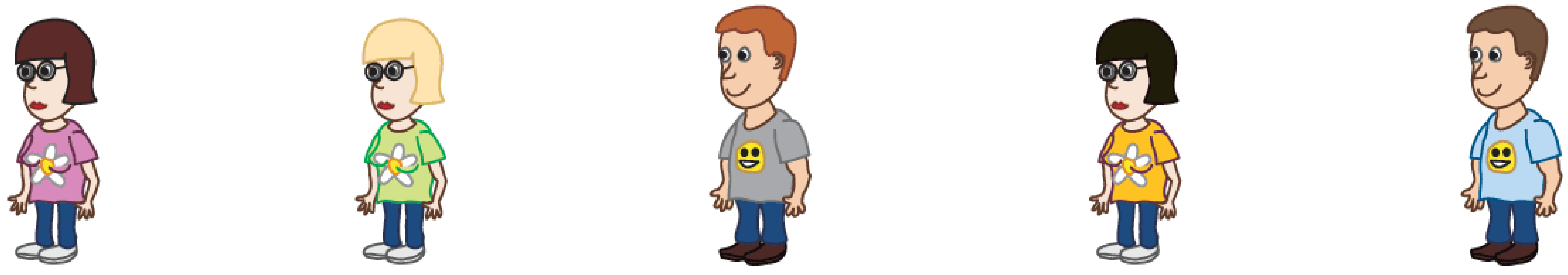


2 **a** List the possible outcomes when an ordinary 6-sided die is rolled.

b How many possible outcomes are there?



3 One of these students will be selected to read out the daily notices:

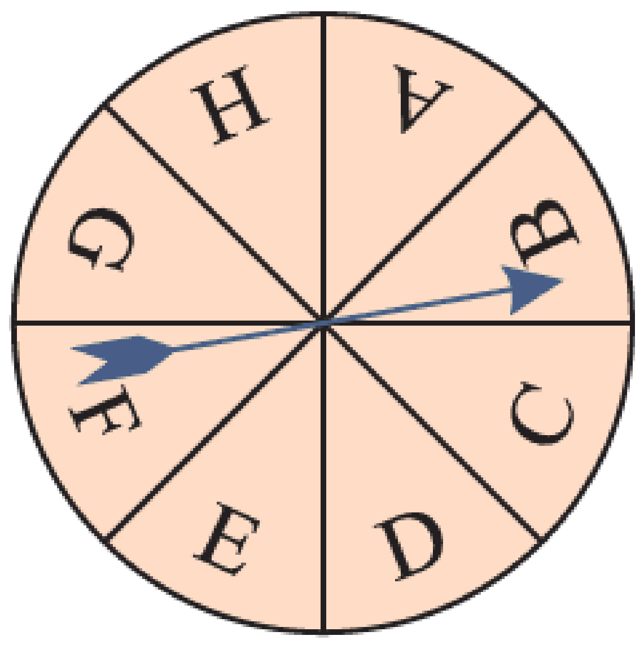


Eva Daisy Nick Ruby Jamie

a List the possible outcomes for this selection.

b How many possible outcomes are there?

4



a List the possible outcomes when this spinner is spun.

b How many possible outcomes are there?

c How many of the outcomes are vowels?

5 Look at this calendar for the month of March.

a Terence will select a Saturday during March to hold a garage sale.

i What are the possible outcomes?

ii How many possible outcomes are there?

b Janine will select a day during the week starting the 11th of March to visit her aunt.

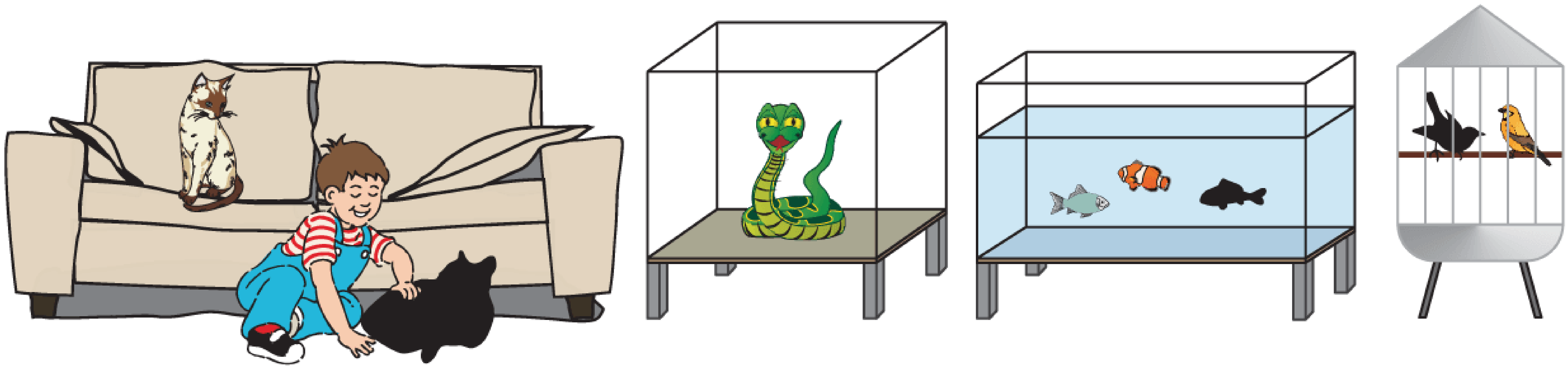
i What are the possible outcomes?

ii How many possible outcomes are there?

March

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

6 Billy has a variety of pets in his house.



Billy must select one of his pets to write about for his school project.

a How many possible outcomes are there for his selection?

b How many of the outcomes:

i are cats **ii** are black **iii** have legs?

7 Martine is choosing a South American country to visit for her next holiday.

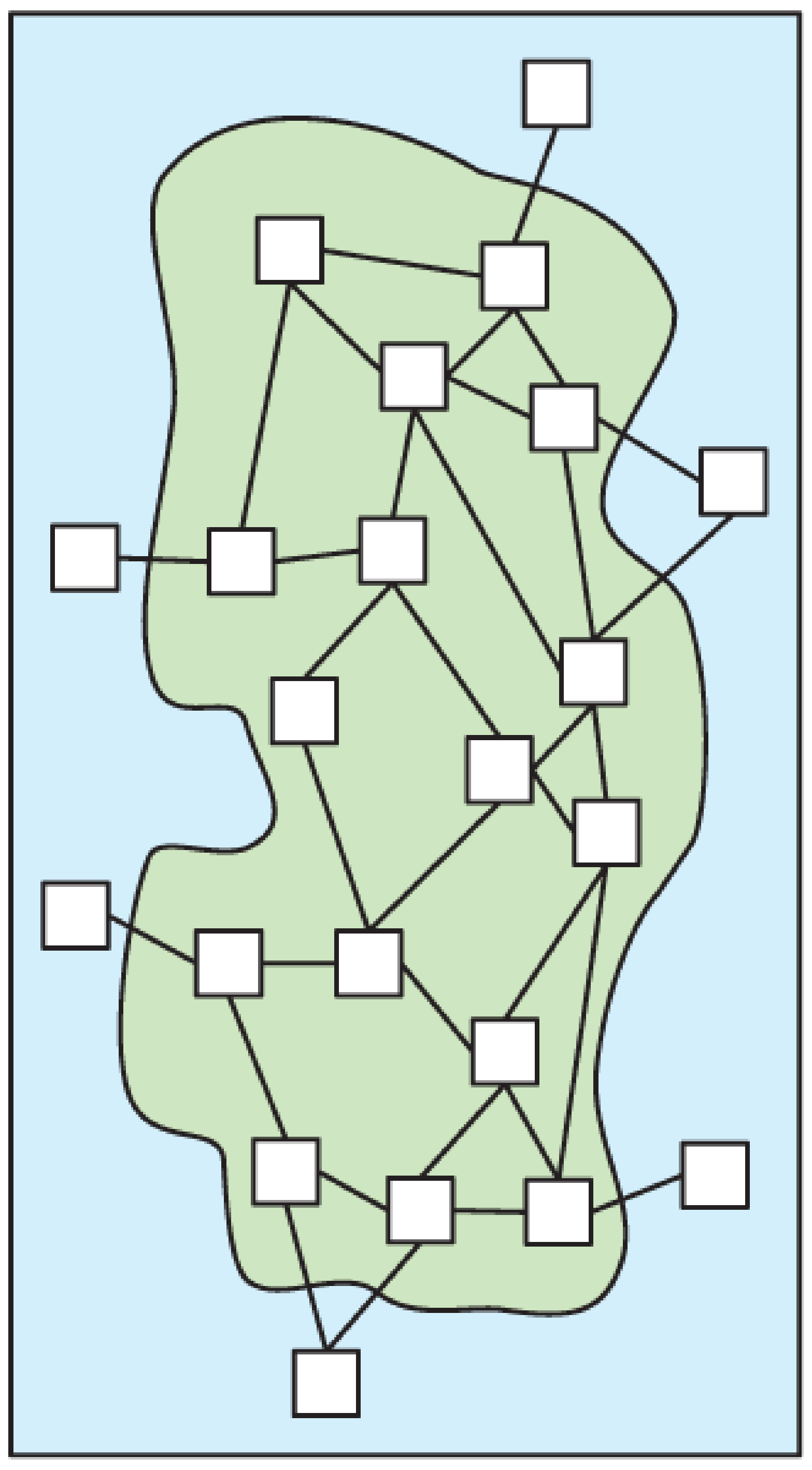
a How many possible outcomes are there?

b How many of the outcomes start with P?

8 Before starting a board game, Kiyara must choose one of the squares on the board to place her token.

a How many possible outcomes are there for this choice?

b How many of the outcomes are located in the water?



D CALCULATING PROBABILITIES

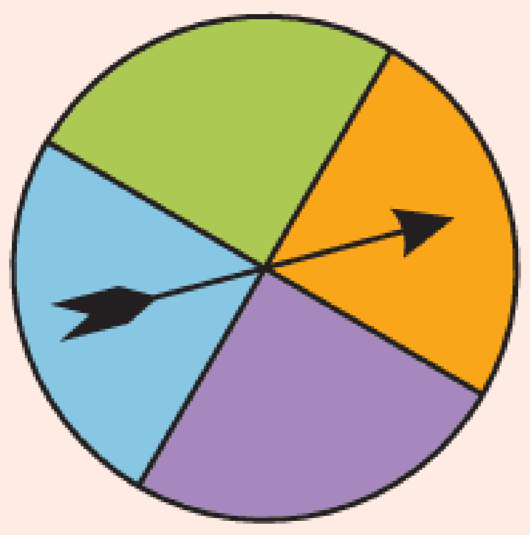
In many situations involving probability, the possible outcomes are all **equally likely**.

For example, when we roll a die, the possible outcomes are 1, 2, 3, 4, 5, and 6, and all of these outcomes are equally likely to occur.

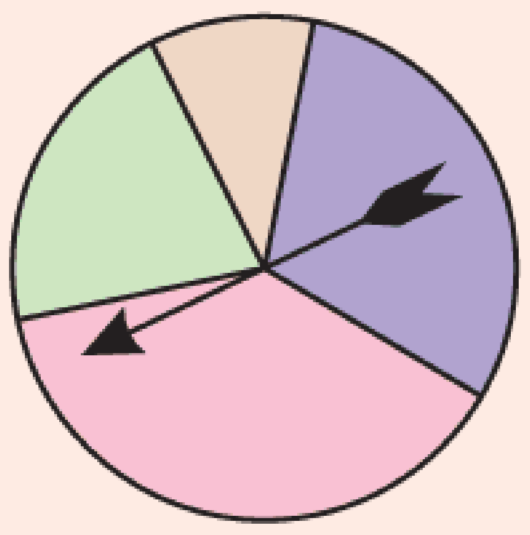
DISCUSSION EQUALLY LIKELY OUTCOMES

For each of the following situations, discuss whether the possible outcomes are equally likely:

- tossing a coin
- observing the winner of a tennis match
- recording the colour of the next car driving through the school gates
- spinning this spinner:



- spinning this spinner:



- choosing a student from a class by placing each student's name in a hat and drawing one name out
- choosing a student from a class by giving each student a tennis ball and seeing who can throw it furthest.

In probability, an **event** is an outcome or group of outcomes with a particular feature.

In situations where the possible outcomes are **equally likely**, we can calculate the probability of a particular event occurring.

$$\text{The probability of an event occurring} = \frac{\text{number of outcomes corresponding to the event}}{\text{total number of possible outcomes}}.$$

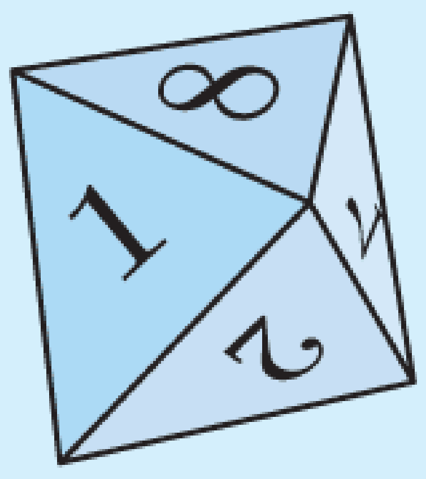
Example 4 **Self Tutor**

An eight-sided die is rolled once.

Find the probability of rolling:

a a 3 **b** an even number

c a number larger than 5.



There are 8 possible outcomes which are equally likely:

1, 2, 3, 4, 5, 6, 7, and 8.

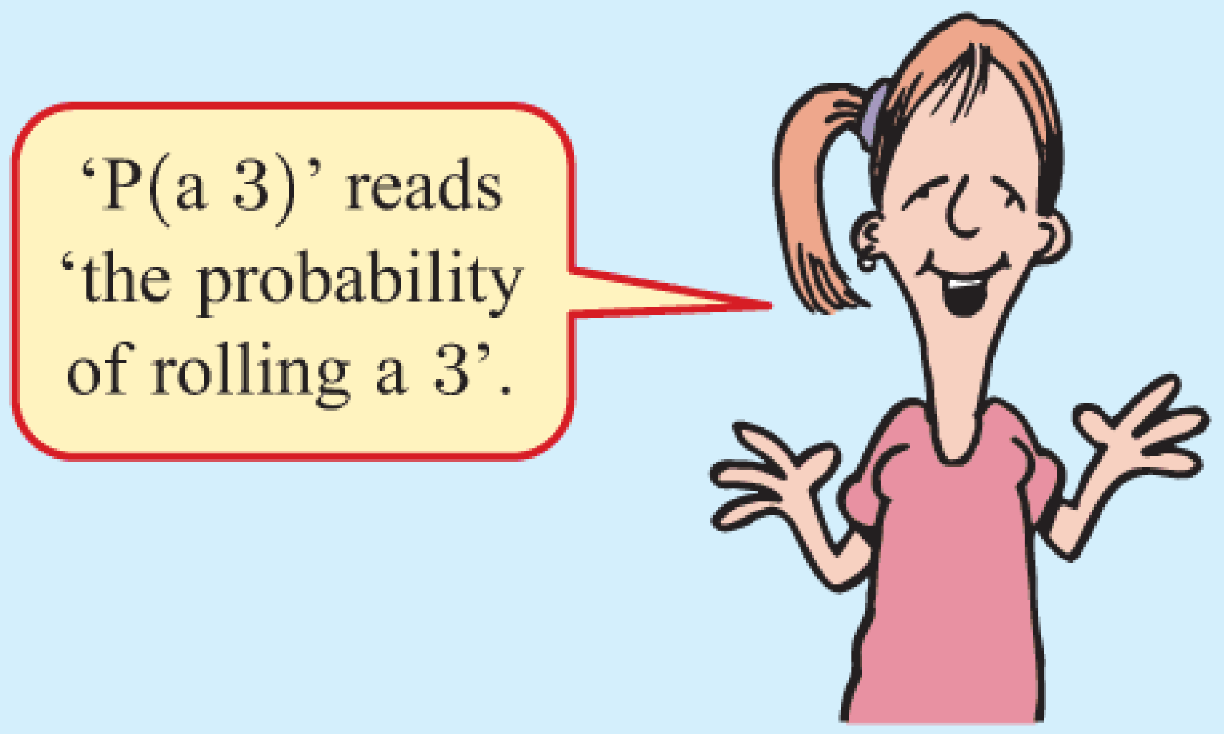
a Only 1 of the outcomes is a 3.

$\therefore$ $P(a\ 3) = \frac{1}{8}$

b The outcomes 2, 4, 6, and 8 are all even.

$\therefore$ there are 4 outcomes corresponding to the event.

$\therefore$ $P(\text{an even number}) = \frac{4}{8} = \frac{1}{2}$ {simplest form}



c The outcomes 6, 7, and 8 are greater than 5.

$\therefore$ there are 3 outcomes corresponding to the event.

$\therefore$ P(a number greater than 5) $= \frac{3}{8}$

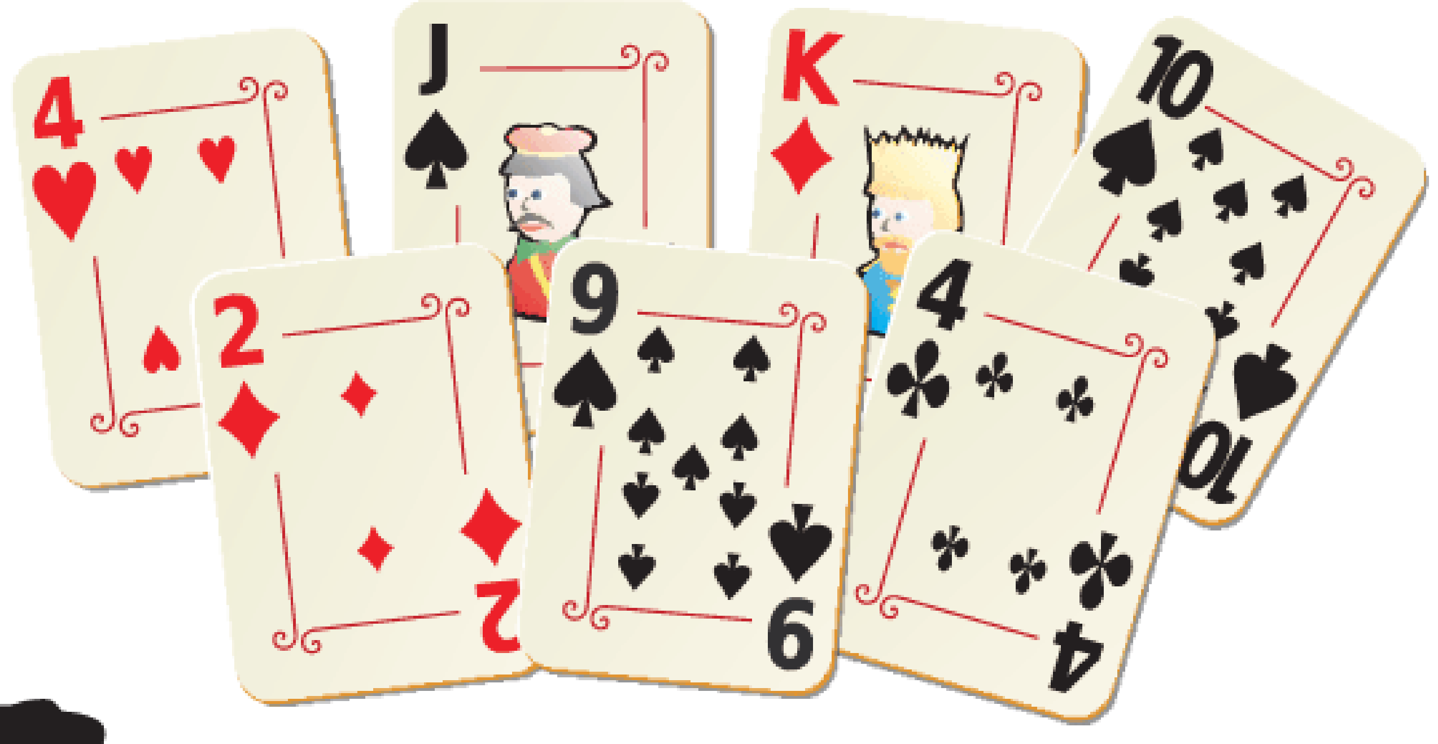
EXERCISE 12D

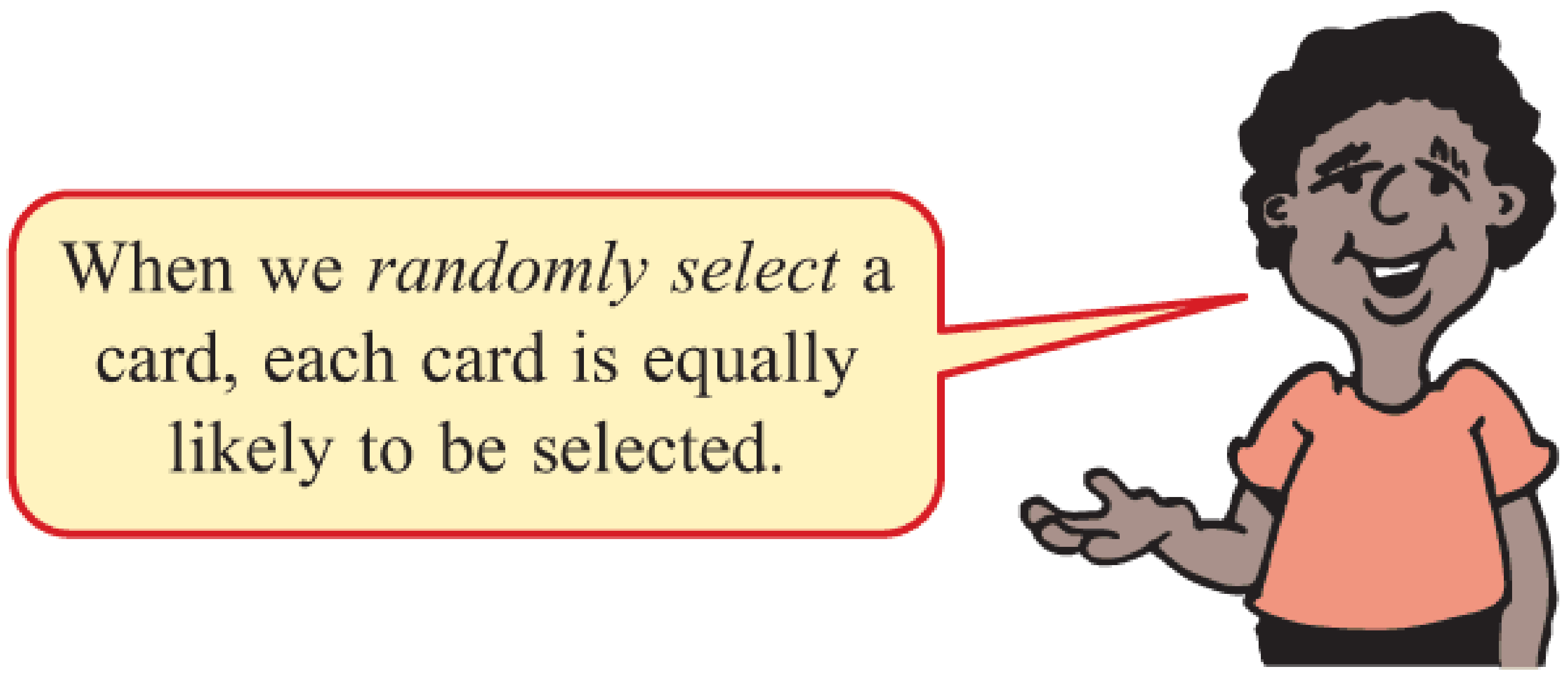
1 Suppose a coin is tossed once.

- **a** List the possible outcomes.
- **b** How many possible outcomes are there?
- **c** Find the probability of getting a head.

2 A playing card is selected at random from the group alongside.

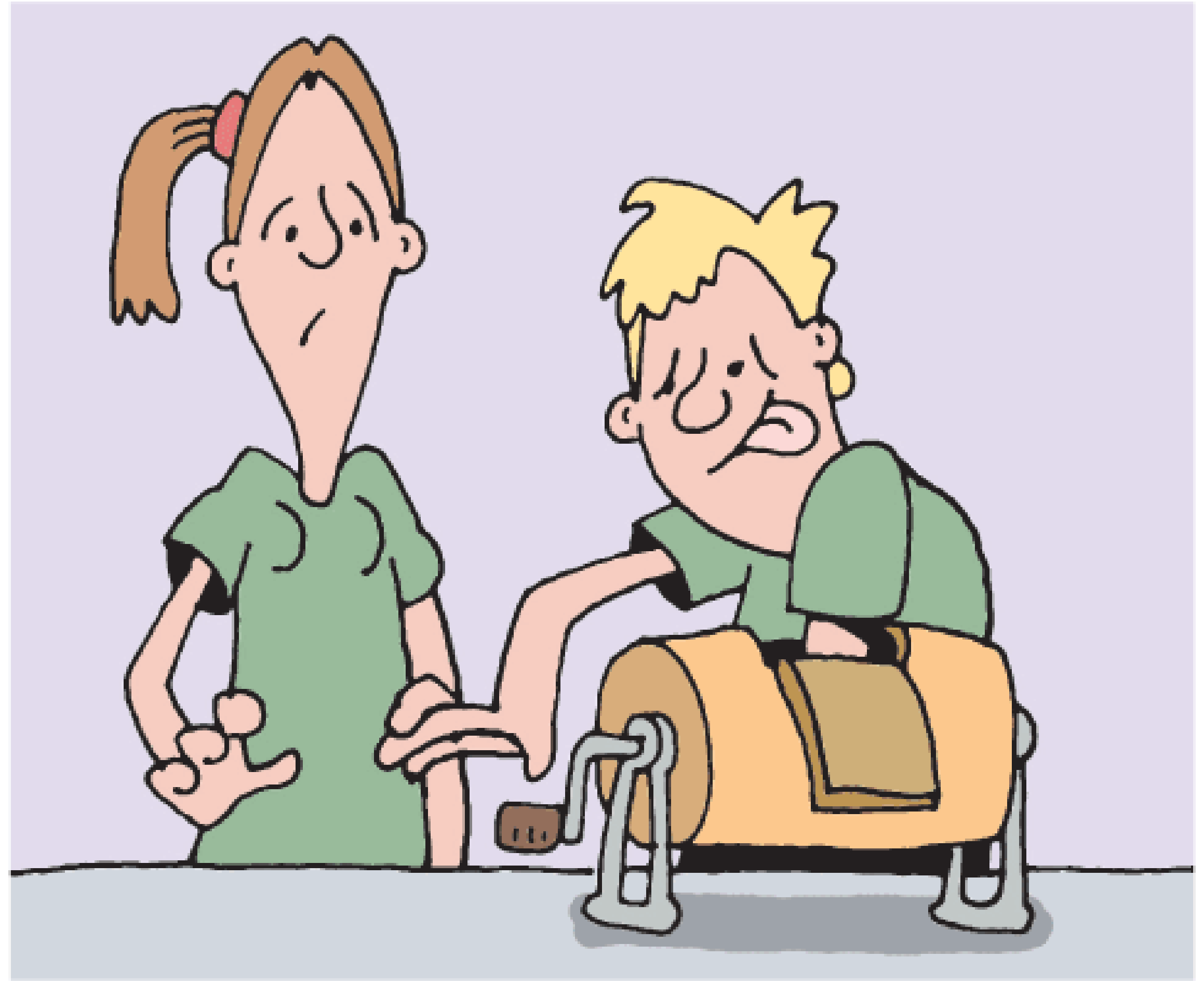
- **a** How many possible outcomes are there?
- **b** Find the probability of selecting:
  - **i** the 2 of ♦
  - **ii** a spade ♠
  - **iii** a 4
  - **iv** a black card.





3 In a school raffle, tickets numbered from 1 to 40 are placed in a barrel, and the winning ticket is drawn out at random.

- **a** How many possible outcomes are there?
- **b** Find the probability that the winning ticket number is:
  - **i** 19
  - **ii** 27 or 28
  - **iii** a single digit number
  - **iv** a prime number.



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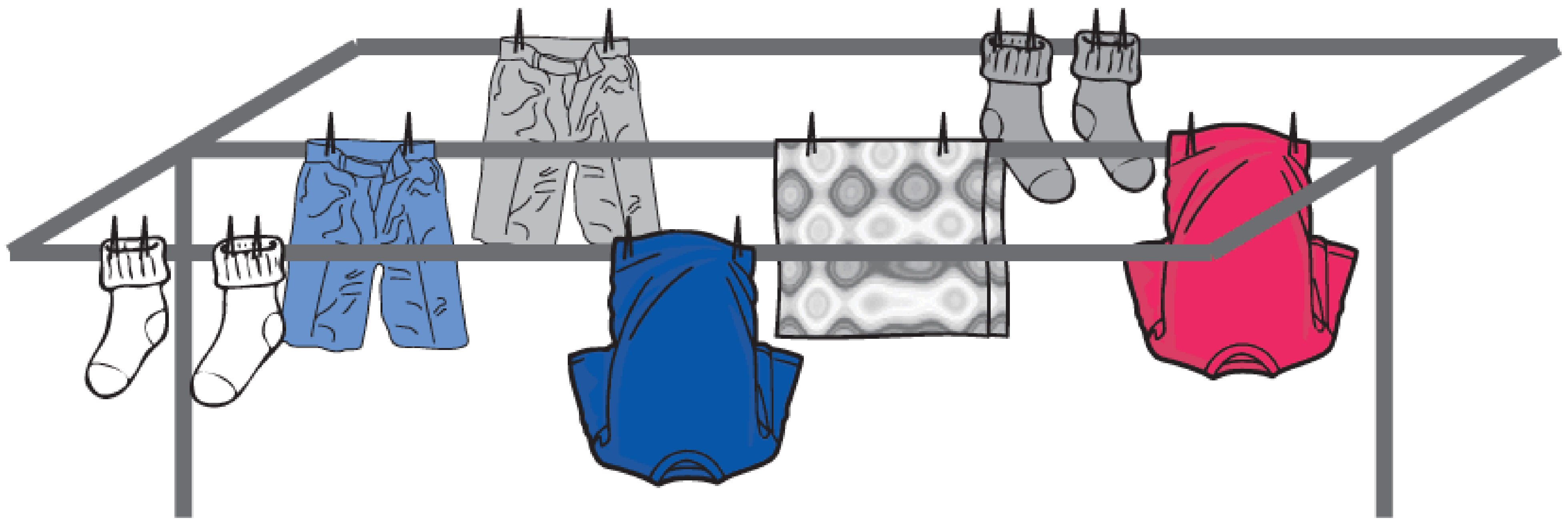


Troy hung his washing on the clothes line. Unfortunately, his dog grabbed an item at random, and pulled it off the line. Find the probability that the item was:

- **a** blue
- **b** a sock
- **c** from the middle row of the clothes line.

Example 5 **Self Tutor**

A bag contains 3 red marbles, 4 black marbles, 6 blue marbles, and 2 green marbles. A marble is selected at random from the bag. Find the probability of selecting:

a a black marble **b** a red or a blue marble.

There are $3 + 4 + 6 + 2 = 15$ possible outcomes.

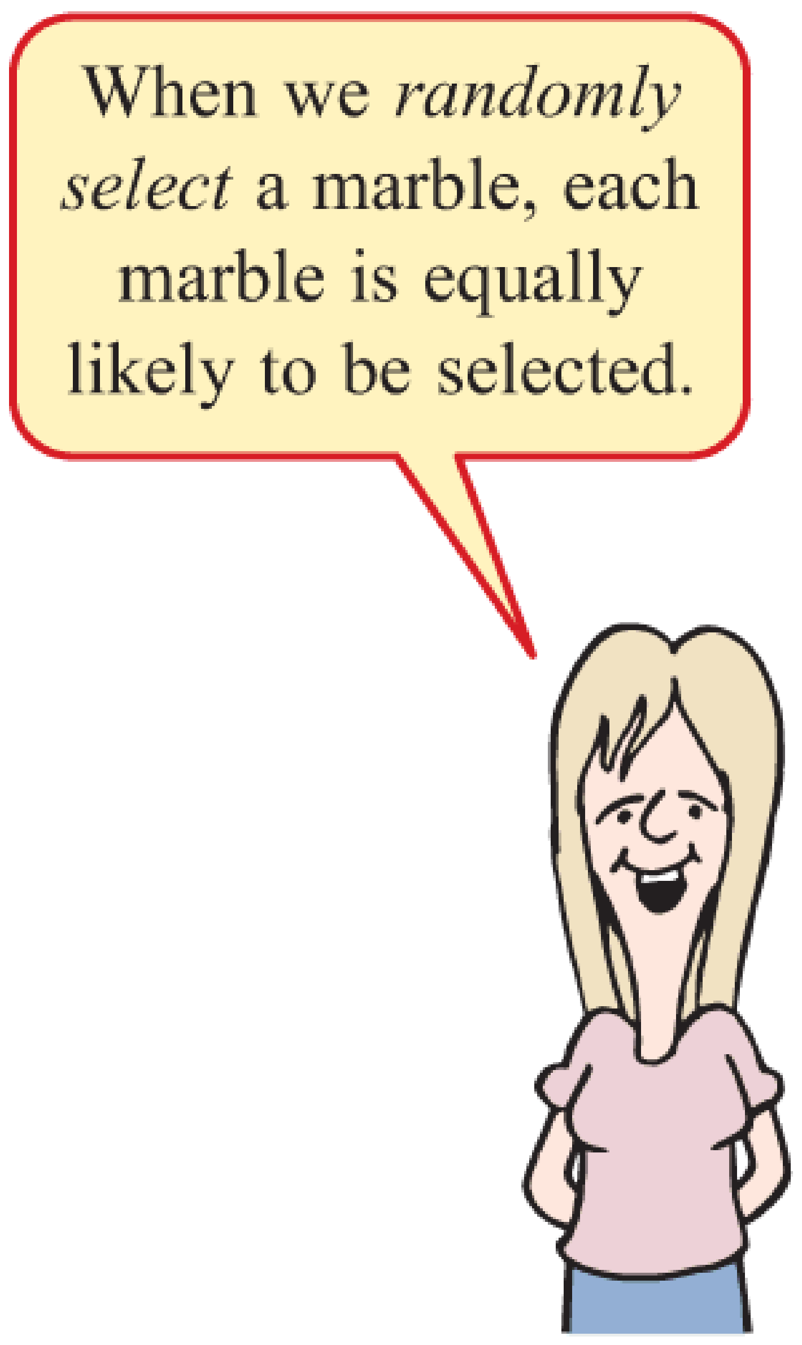
a 4 of the marbles are black.

$\therefore$ P(a black marble) $= \frac{4}{15}$

b $3 + 6 = 9$ of the marbles are red or blue.

$\therefore$ P(a red or a blue marble) $= \frac{9}{15}$

$= \frac{3}{5}$ {simplest form}



5 A box of chocolates contains 5 strawberry chocolates, 6 caramel chocolates, and 2 peppermint chocolates.

a How many chocolates are in the box?

b Prue takes a chocolate from the box at random. Find the probability that Prue selects a caramel chocolate.

6 A hockey team consists of 6 defenders, 4 attackers, and a goalkeeper.

a How many players are in the team?

b One player is chosen at random to be the team's captain. Find the probability that the chosen player is:

i the goalkeeper **ii** a defender.

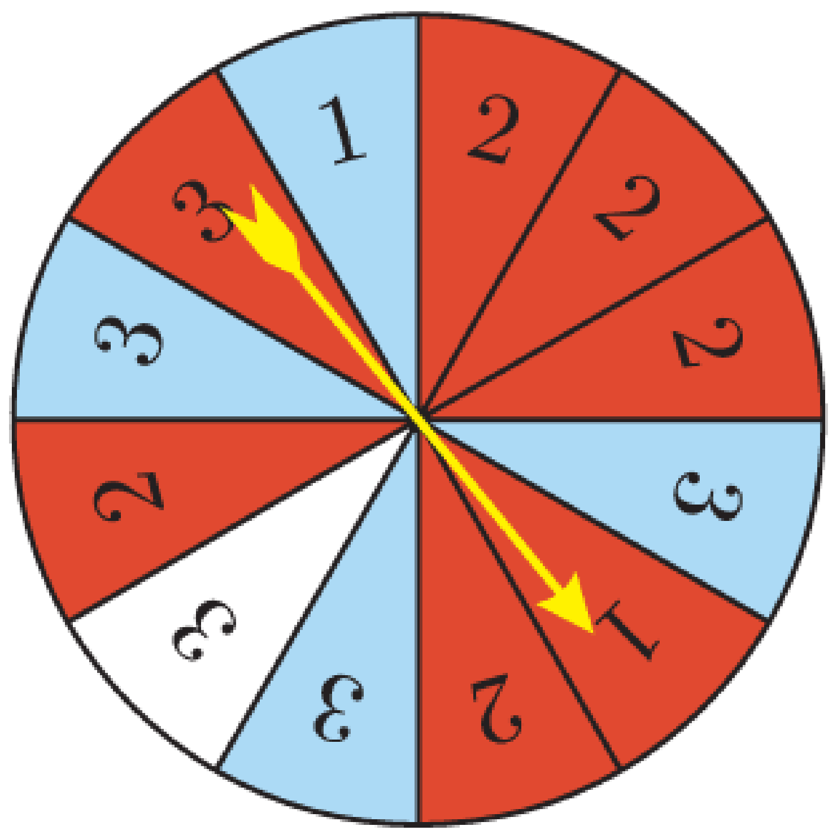
7 A kitchen drawer contains 9 forks, 5 knives, and 6 spoons. Alexander selects an item from the drawer at random.

a Find the probability that Alexander selects a knife. Give your answer as:

i a fraction **ii** a percentage **iii** a decimal.

b Use a word or phrase to describe the probability of Alexander selecting a knife.

8



The spinner alongside is spun once.

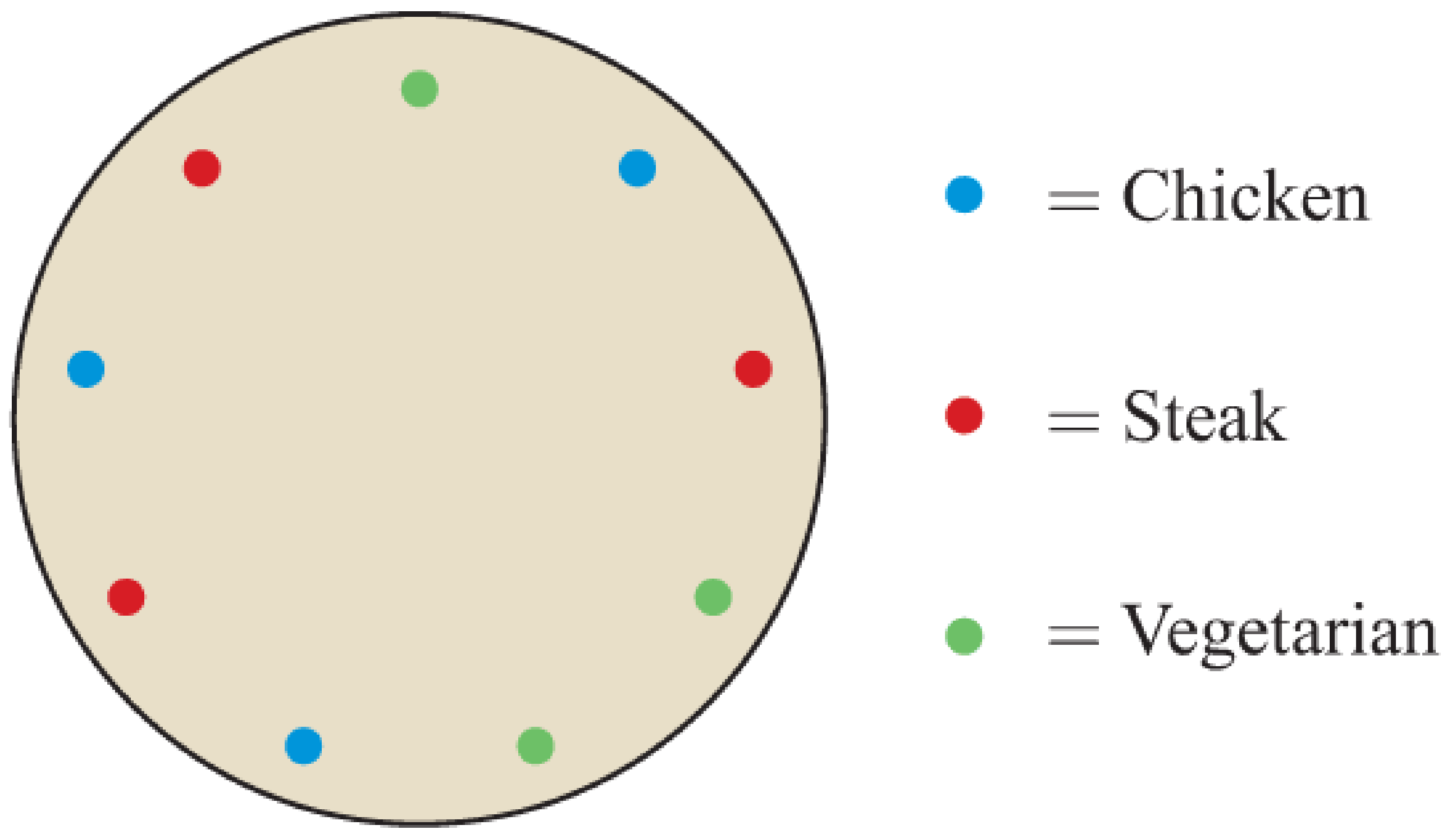
a Find the probability that the spinner lands on:

i a 3 **ii** a red section.

b Is the spinner more likely to land on a 3 or a red section?

9 Chicken, steak, and vegetarian dishes are placed around a table at a function dinner.
Scott is randomly seated at this table in front of a dish.
Find the probability that he is seated:

- **a** at a chicken dish
- **b** between two steak dishes
- **c** at a vegetarian dish, but with a steak dish next to him.



10 Ben is in Grade 6, and his sister Clarissa is in Grade 4. Each of them has been given a list of words by their teacher, and must look up the meaning of one of the words in the dictionary for homework.

Ben's words		**Clarissa's words**
exasperating	anoint	celery
pronounce	scarce	glacier
texture	junction	voyage
frugal	scent	quartet
valiant	caption	currency

Each child selects one word at random from their list.

- **a** Find the probability that:
  - **i** Ben selects a word starting with the letter s
  - **ii** Clarissa selects a word which contains the letter a.
- **b** Who is more likely to select a word containing at least 8 letters?

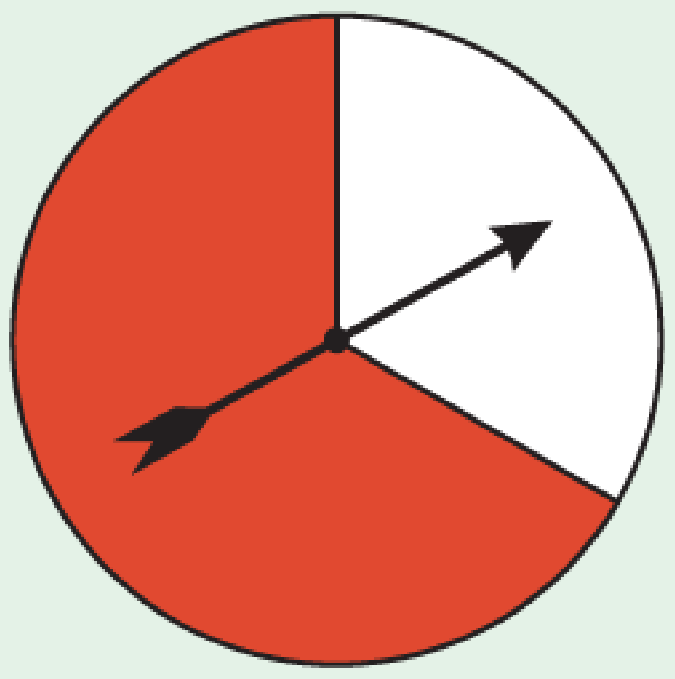
KEY WORDS USED IN THIS CHAPTER

- 50-50 chance
- event
- outcome
- randomly selected
- certain
- impossible
- possible outcomes
- unlikely
- equally likely
- likely
- probability

REVIEW SET 12A

1 Describe, using a word or phrase, the probability of these events:

- **a** The spinner alongside will land on red.
- **b** The next triangle you draw will have 4 sides.



2 You have five blue cards, five white cards, and five grey cards. You must place any three of them in a hat. What cards should you place in the hat if you want:

- **a** it to be impossible to draw out a white or grey card
- **b** to be certain of drawing out a grey card
- **c** there to be an equal chance of drawing out a blue, white, or grey card?

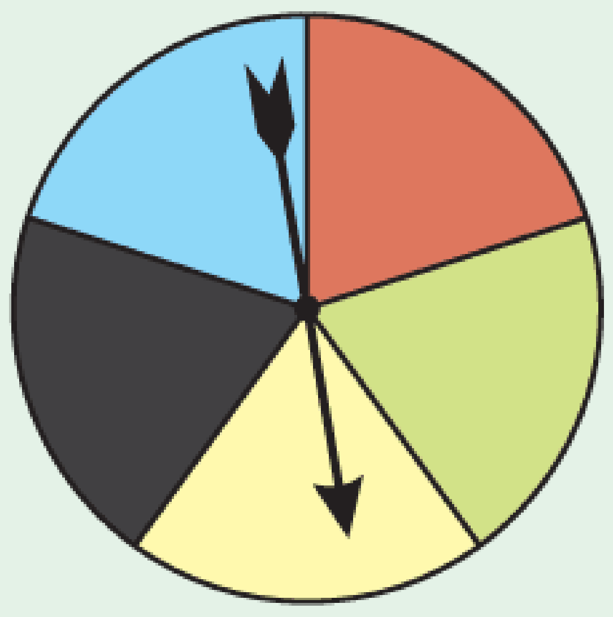
3 Write a word or phrase to describe the probability value:

a 0.14 **b** 1 **c** 0.5

4 **a** List the possible outcomes when this spinner is spun.

b How many possible outcomes are there?

c Are the outcomes equally likely?



5 Jarrod placed all the letters of the alphabet in a hat, and drew one out at random.

a How many possible outcomes are there?

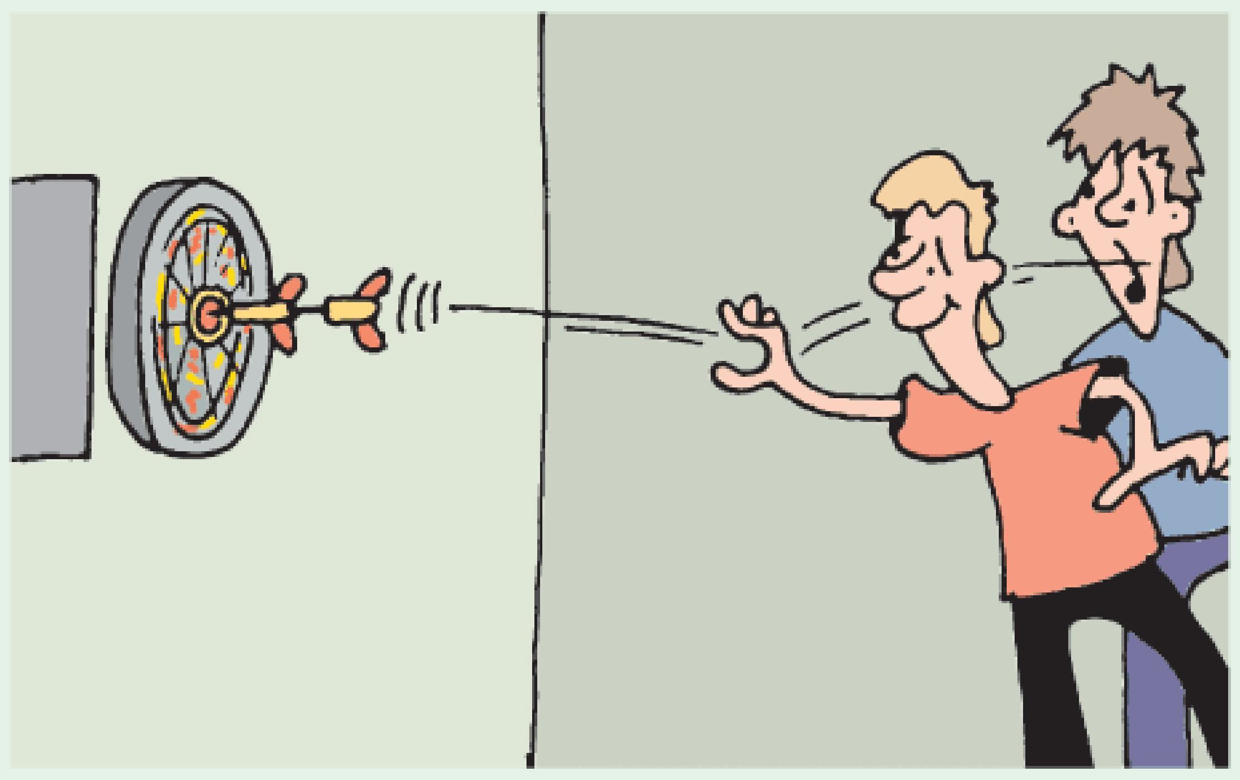
b Find the probability of drawing out:

i a vowel **ii** an X, Y, or Z.

6 Simon is playing Trent in a game of darts. Simon has probability 0.64 of winning, and Trent has probability 0.36 of winning.

a Who is more likely to win the game?

b Find the sum of the probabilities. Explain your answer.



7 A day during November is selected at random. Find the probability that the day is:

a the 17th **b** a Sunday

c a weekday.

November

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

8 Petra has 5 petunias, 7 marigolds, and 8 roses in her garden. One of the plants was damaged by a possum. Find the probability that the damaged plant was:

a a petunia **b** a marigold *or* a rose.

9 Jamela takes a mug from this cupboard at random. What is the probability that the mug is white?

10 The schedule alongside shows the doctor who is on duty at a local clinic each day during May.

If a patient goes to the clinic in May, what is the probability that the patient will be seen by Dr Thompson?

1st - 6th	Dr Jones
7th - 11th	Dr Thompson
12th - 18th	Dr Jones
19th - 23rd	Dr Ambrose
24th - 31st	Dr Thompson

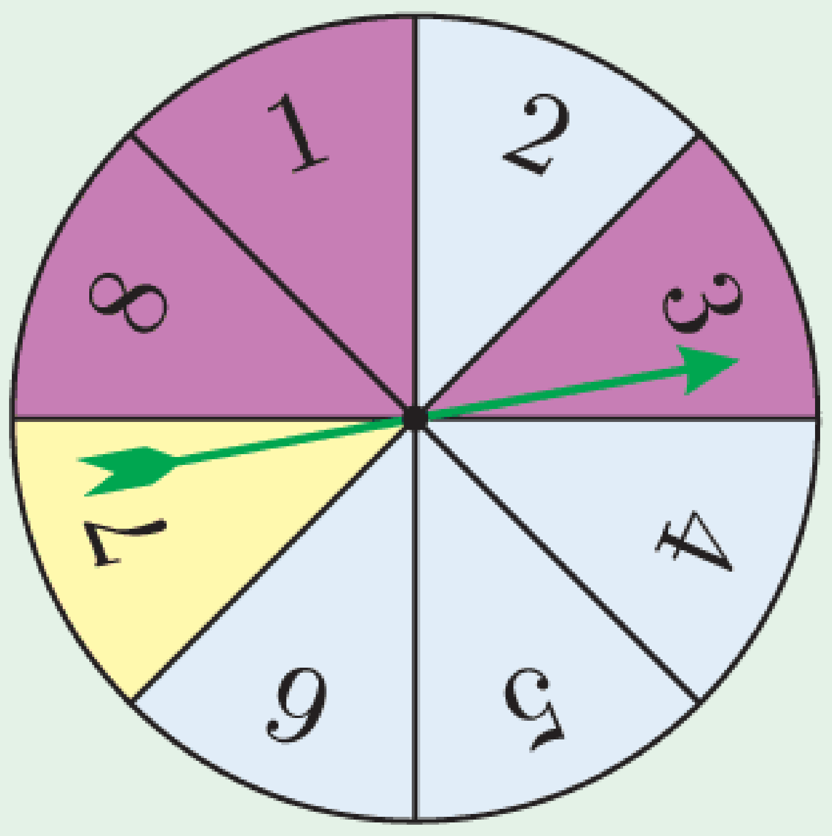
REVIEW SET 12B

1 Copy and complete these sentences about the spinner shown:

a It is ...... that the spinner will finish on a numbered sector.

b It is ...... that the spinner will finish on 'yellow'.

c It is ...... for the spinner to finish on '12'.



2 Peggy, Christina, Suzanne, and Chelsea go mountain biking together. The table alongside shows the probability that each girl will fall off her bike.

	Probability
Peggy	0.15
Christina	0.3
Suzanne	0.27
Chelsea	0.1

a Plot these probabilities on a number line.

b Which girl is most likely to fall off her bike?

c Write a word or phrase to describe the probability that Chelsea will fall off her bike.

3 A day of the week is chosen at random.

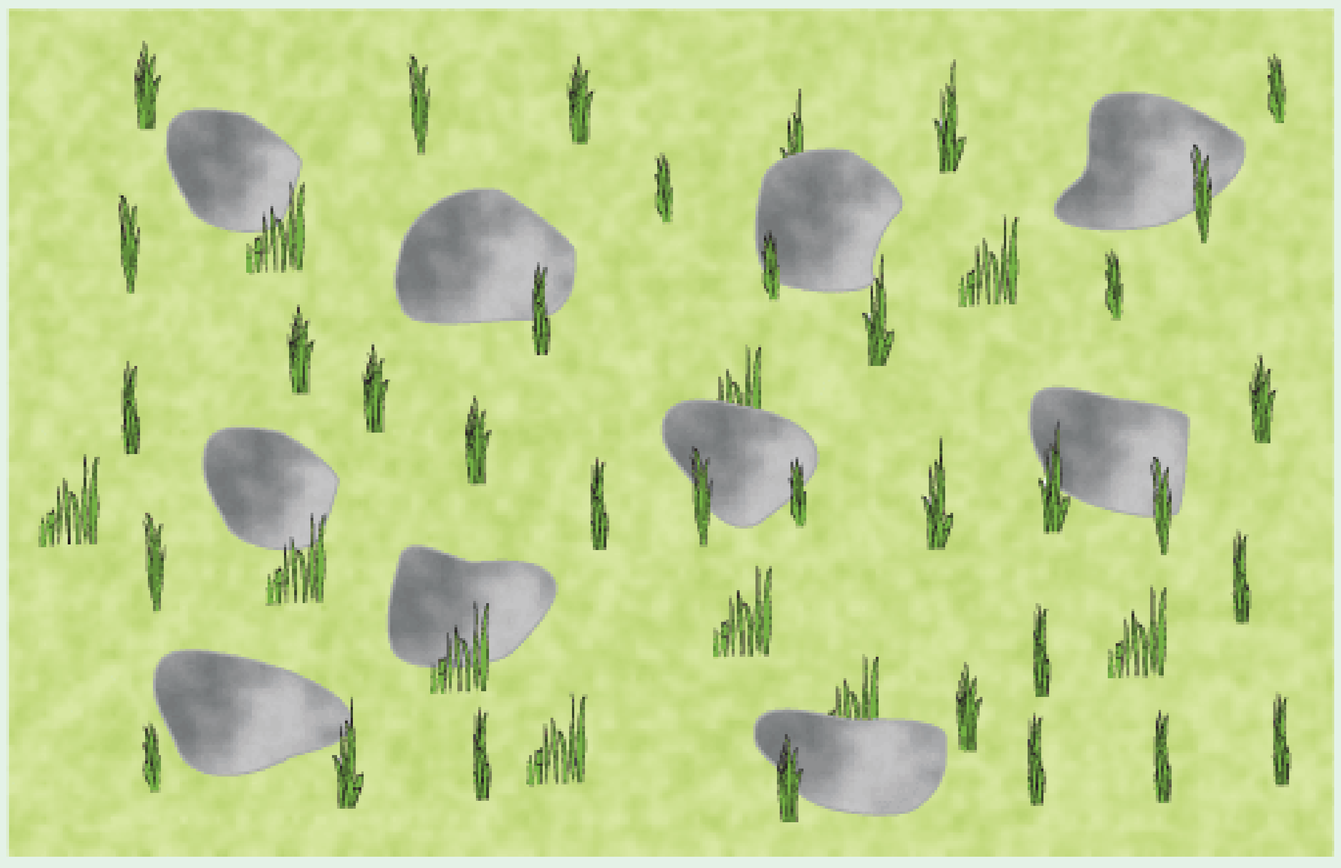
a How many possible outcomes are there?

b How many of the outcomes contain the letter U?

4 Justin is investigating a collection of 10 stones at the bottom of his garden. There are slaters under 5 of the stones, and earwigs under 3 of the stones.

Justin turns one of the stones over at random.

Find the probability that there is an earwig under the stone.



5 Bradley and Caleb have a spelling test today.

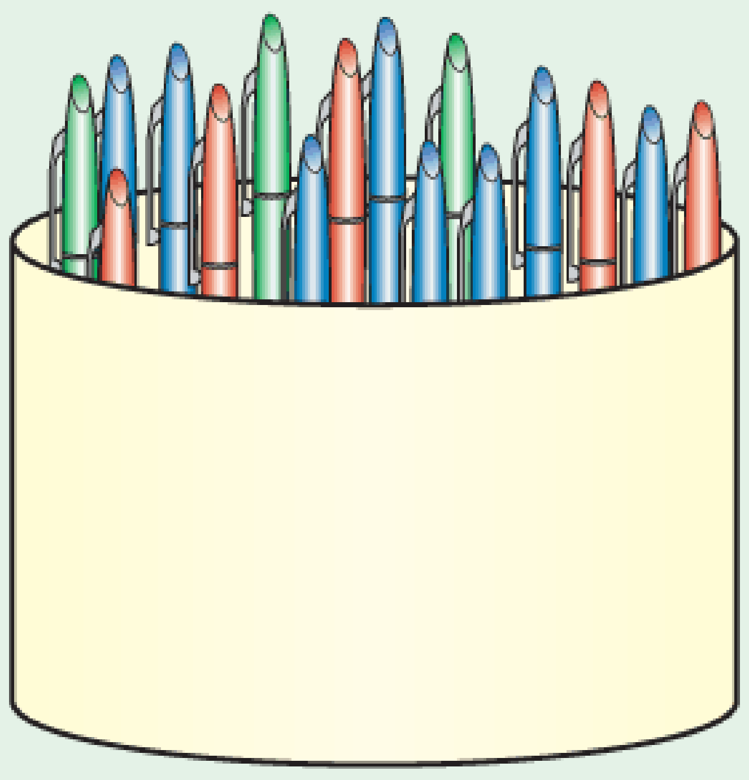
Bradley has probability $\frac{3}{5}$ of passing the test, and Caleb has probability 0.62 of passing the test.

a Write each of the probabilities as a percentage.

b Who is more likely to pass the test?

6 Liesl had to return 2 mathematics books, 3 science books, and 4 history books to the library. She accidentally left one of the books at home. Find the probability that she left a history book at home.

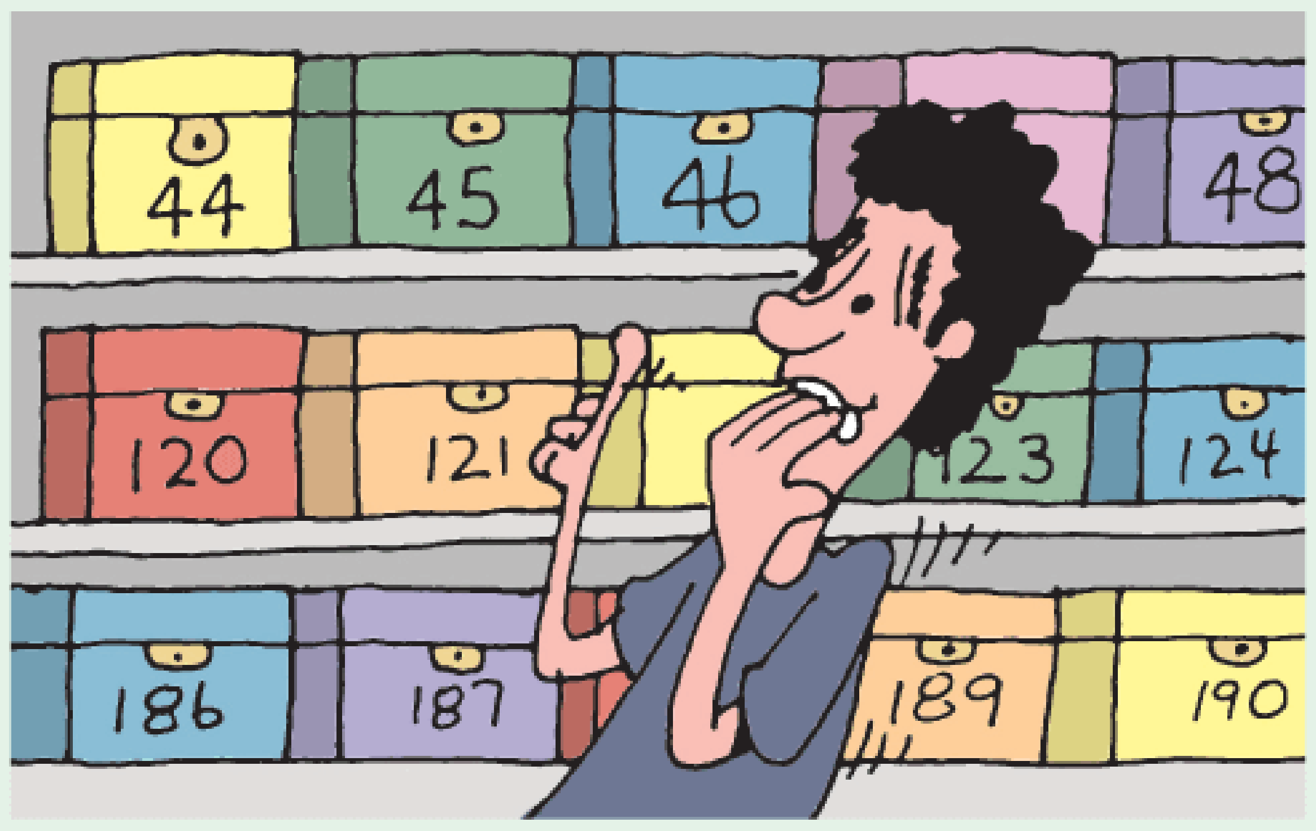
7 A jar contains 5 red pens, 8 blue pens, and 3 green pens. Satomi takes a pen from the jar at random. Find the probability that the pen is blue.



8 Georgina's mobile phone is shown alongside.
Her cat walked on the phone, and pressed one of the buttons.
Find the probability that the cat pressed:

a the '7' **b** a number.

9 Fotios has the chance to win $1 million. He is presented with 200 boxes, one of which contains the $1 million prize. The other boxes contain a $500 prize.
Fotios must select one of the boxes.



a Use a word or phrase to describe the probability that Fotios will select:
  i the $1 million prize **ii** a $500 prize **iii** a prize.

b Write a number to describe the probability of Fotios selecting:
  i the $1 million prize **ii** a $500 prize **iii** a prize.

10 Clarence the clown has a collection of balloons. He selects one at random to give to a child.

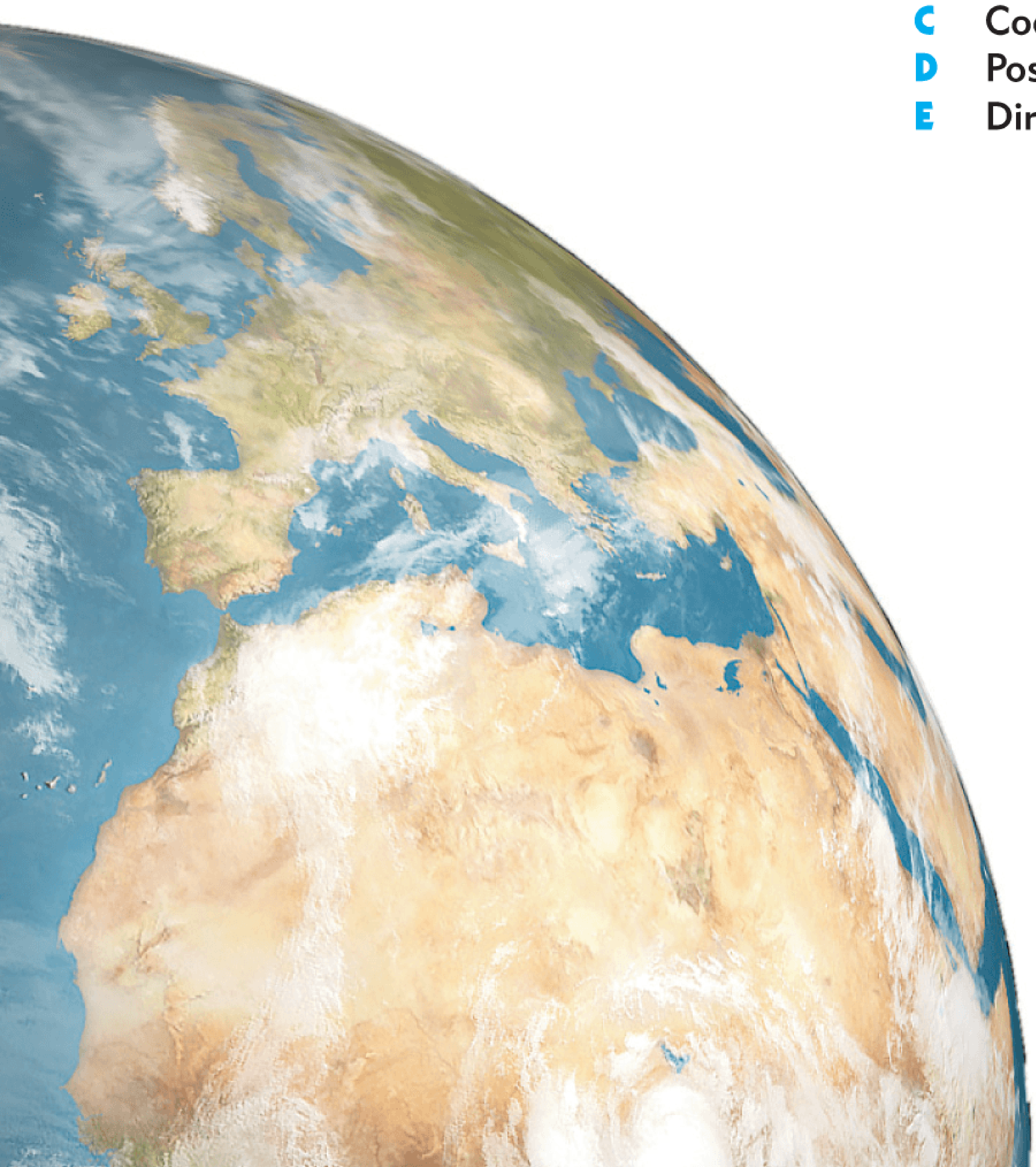
a Find the probability that the child will receive:
  i a red balloon **ii** a blue balloon
  iii a yellow balloon.

b Which colour balloon is the child most likely to receive?

c Find the sum of the probabilities in **a**. Explain your answer.

Chapter 13

Location

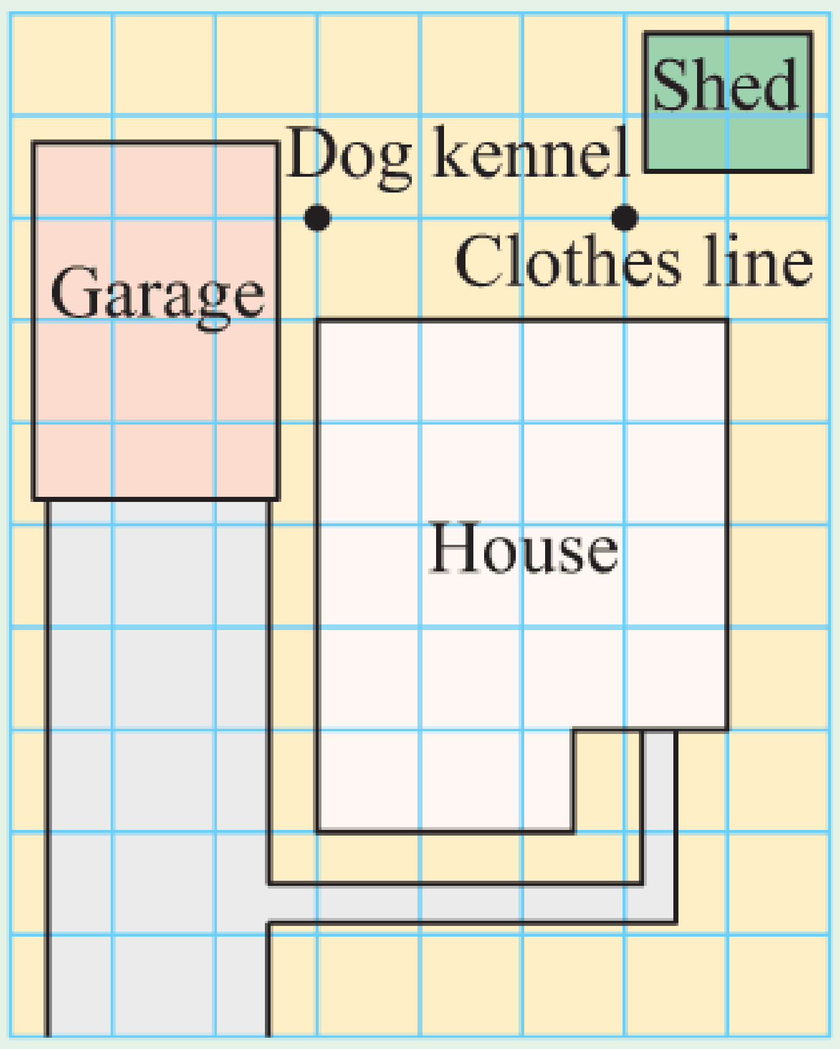


OPENING PROBLEM

Mitchell drew a map of his house on grid paper.

Things to think about:

- **a** What advantages are there in using grid paper for the map?
- **b** How can Mitchell use the grid lines to describe the exact location of the dog kennel?
- **c** Can you work out the distance between the clothes line and the dog kennel? If not, what extra information do you need?

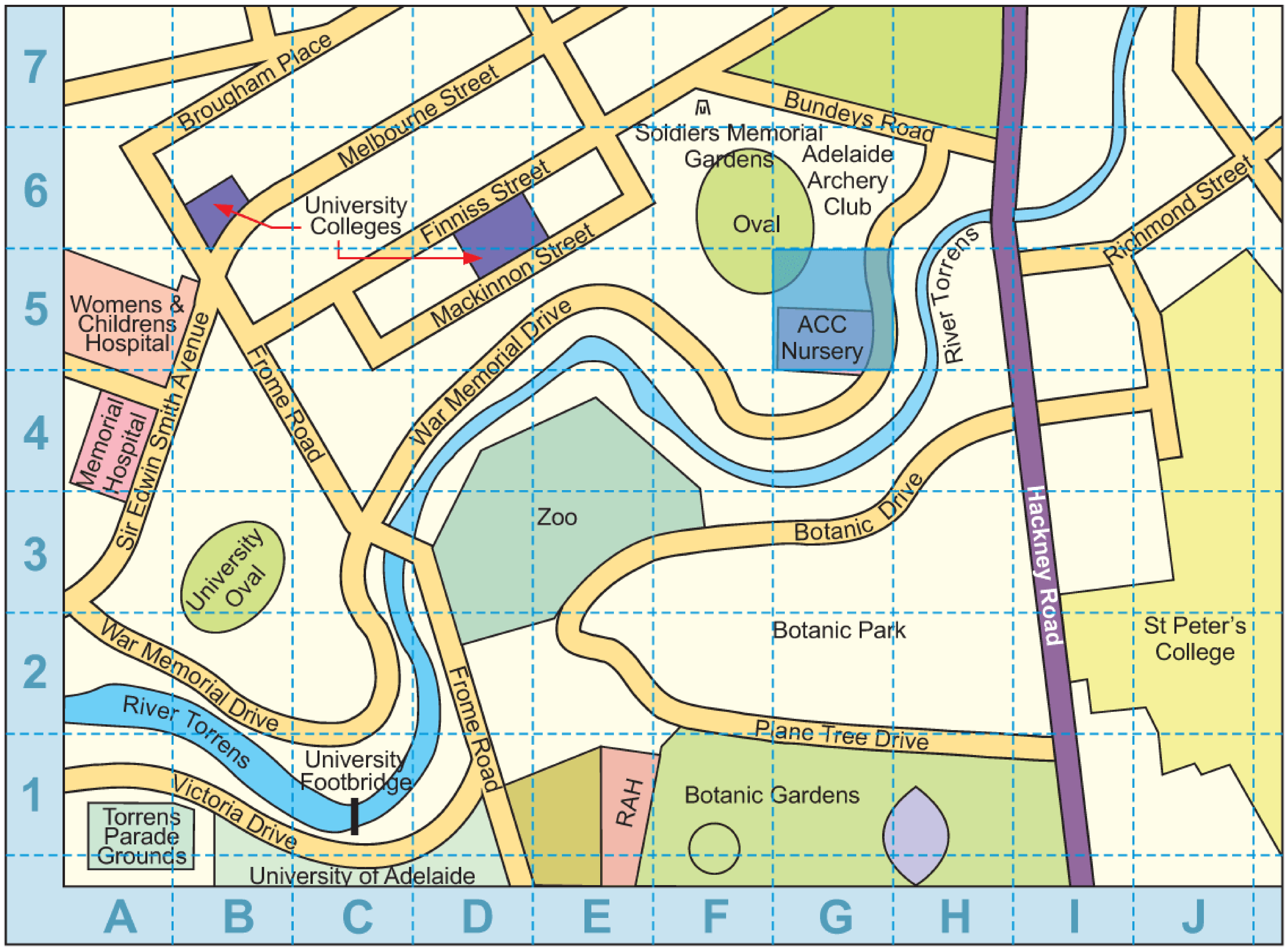


In this chapter, we will see how **map references** and **coordinates** can be used to describe the location of objects.

A MAP REFERENCES

We look at a **map** to find out where a particular town or feature is located. We can also use the map to see how far away it is, and in which direction it is.

The map below shows part of Adelaide, Australia:



Notice the blue lines drawn over the map. They are the same distance apart, and they go both horizontally and vertically across the map. These lines are called **grid lines**.

DEMO

You should also notice that there are letters along the bottom, and numbers along the side. We use these to describe particular regions of the map.

For example, suppose we want to describe the location of the ACC Nursery. The nursery is in the shaded square, which corresponds to column G and row 5. We give it the **map reference** G5.

The map reference does not tell us *exactly* where the ACC Nursery is, but rather gives the region in which it is located.

Example 1 **Self Tutor**

Look at the map of Adelaide on the previous page.

a Name the feature which is located at:
  i A5 **ii** B3.

b Determine the map reference for:
  i the Adelaide Archery Club **ii** the University Footbridge.

a **i** Womens & Childrens Hospital **ii** University Oval

b **i** G6 **ii** C1

EXERCISE 13A

1 Look at the map of Adelaide on the previous page.

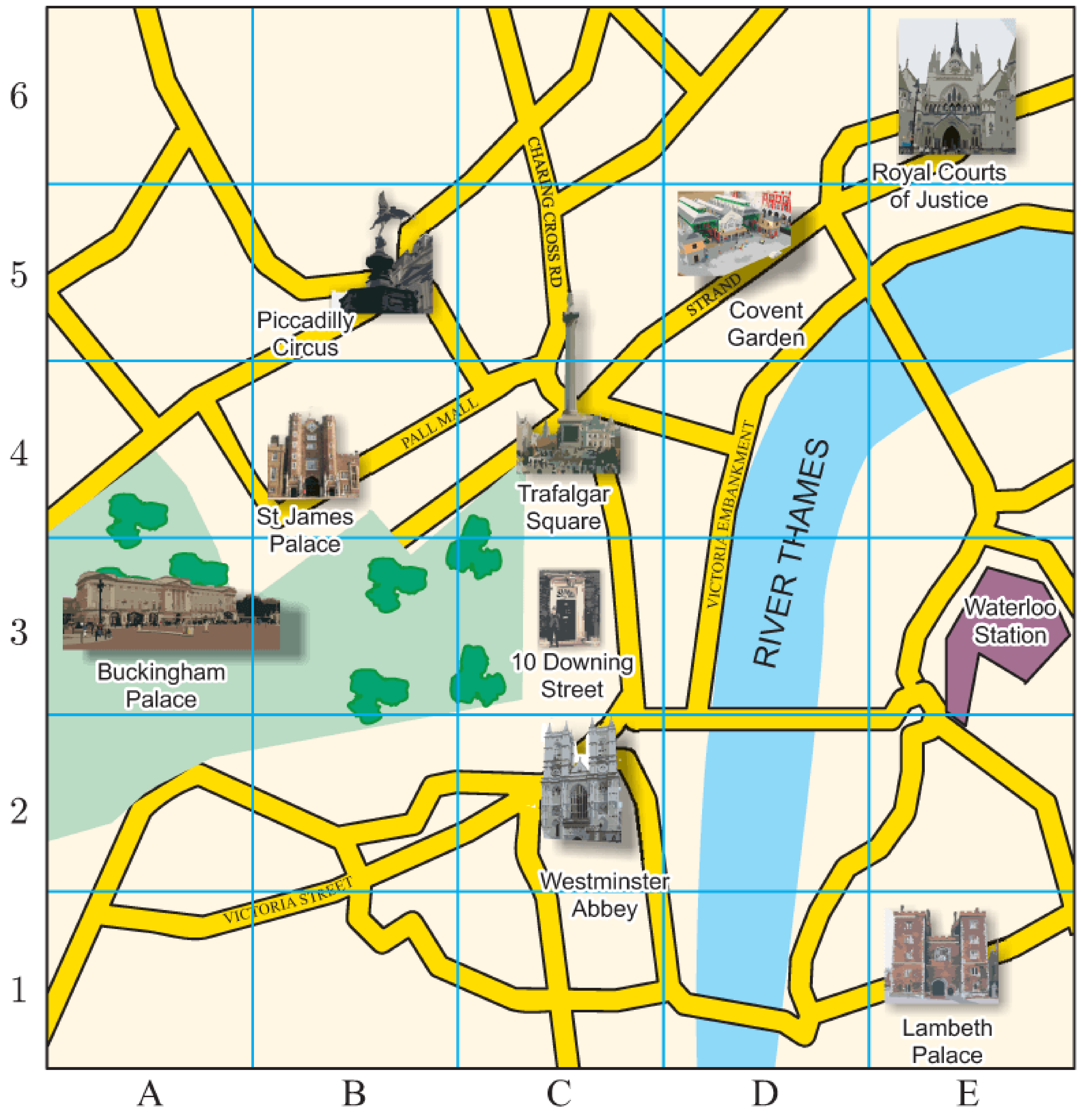
a Name the feature which is located at:
  i J2 **ii** B6 **iii** F7

b Determine the map reference for:
  i Botanic Park **ii** Royal Adelaide Hospital (RAH)

2 Look at this map of London.

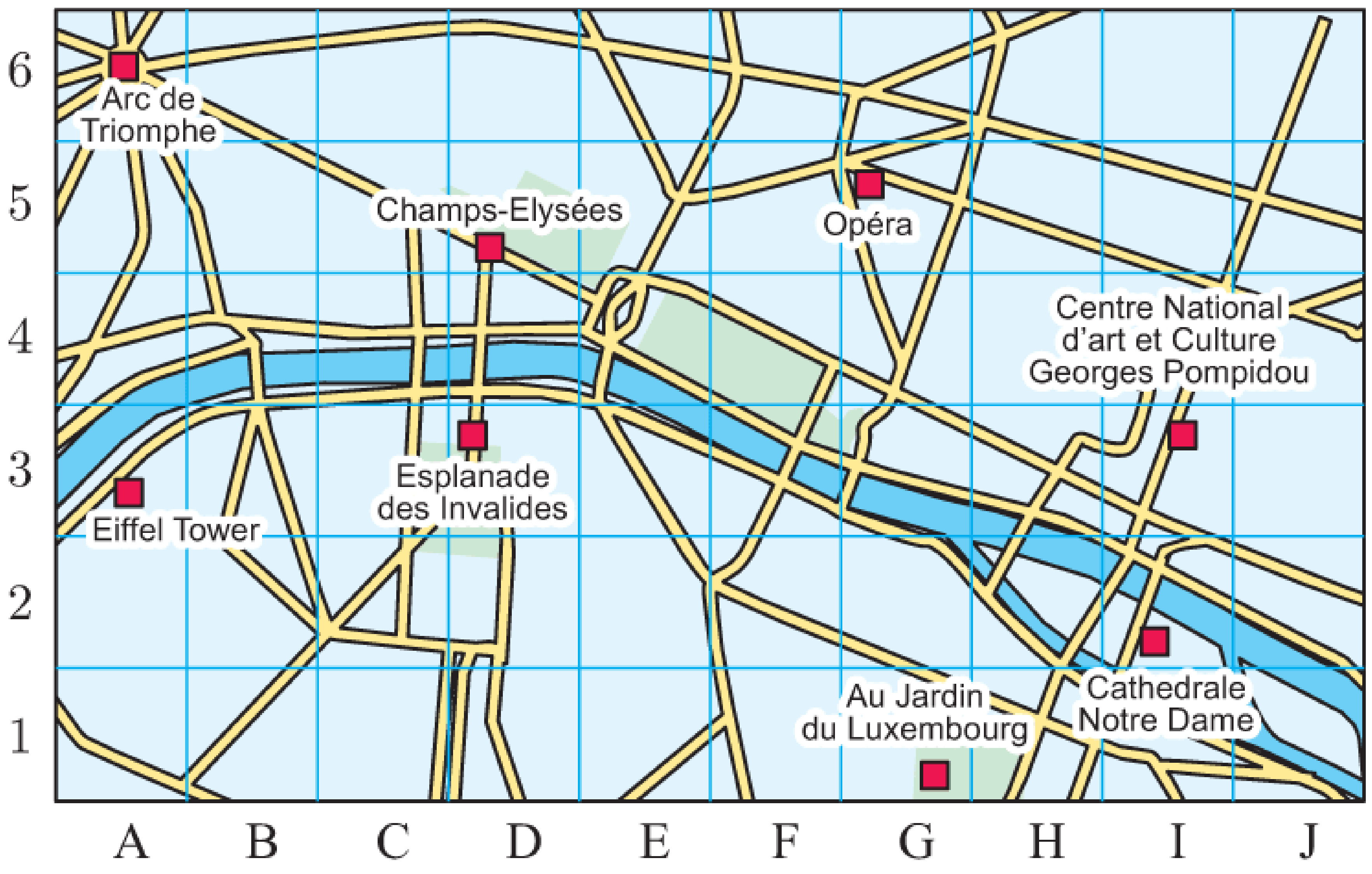
a Name the feature located at:
  i A3
  ii E6
  iii D5
  iv B5
  v E3.

b Determine the map reference for:
  i Westminster Abbey
  ii Trafalgar Square
  iii 10 Downing Street
  iv Lambeth Palace
  v St James Palace.



3 Look at this map of Paris.

- **a** Name the feature located at:
  - **i** A3
  - **ii** G1
  - **iii** I3.
- **b** Determine the map reference for:
  - **i** Opéra
  - **ii** the Arc de Triomphe
  - **iii** Cathedrale Notre Dame.



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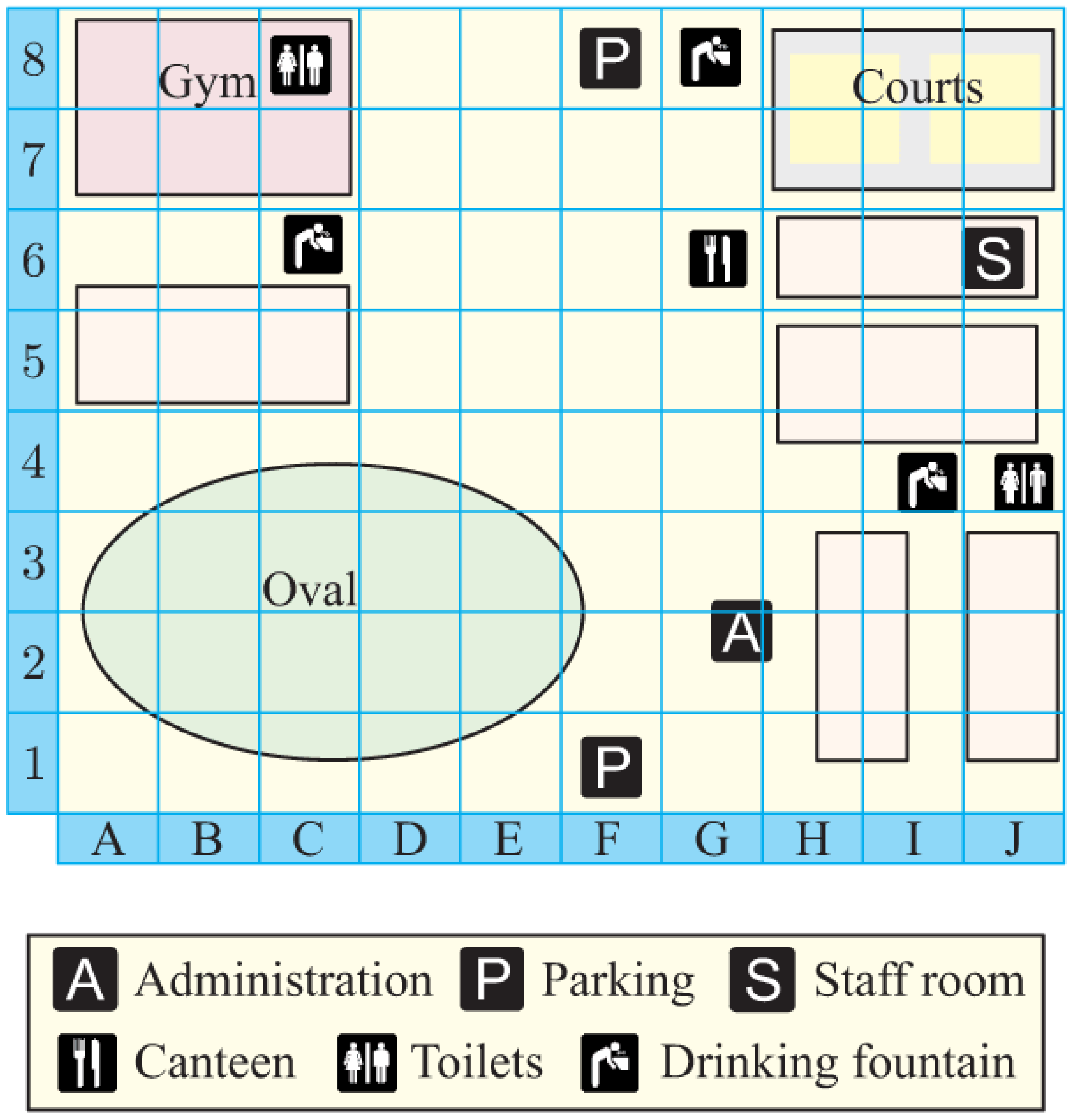


Consider the school map alongside.

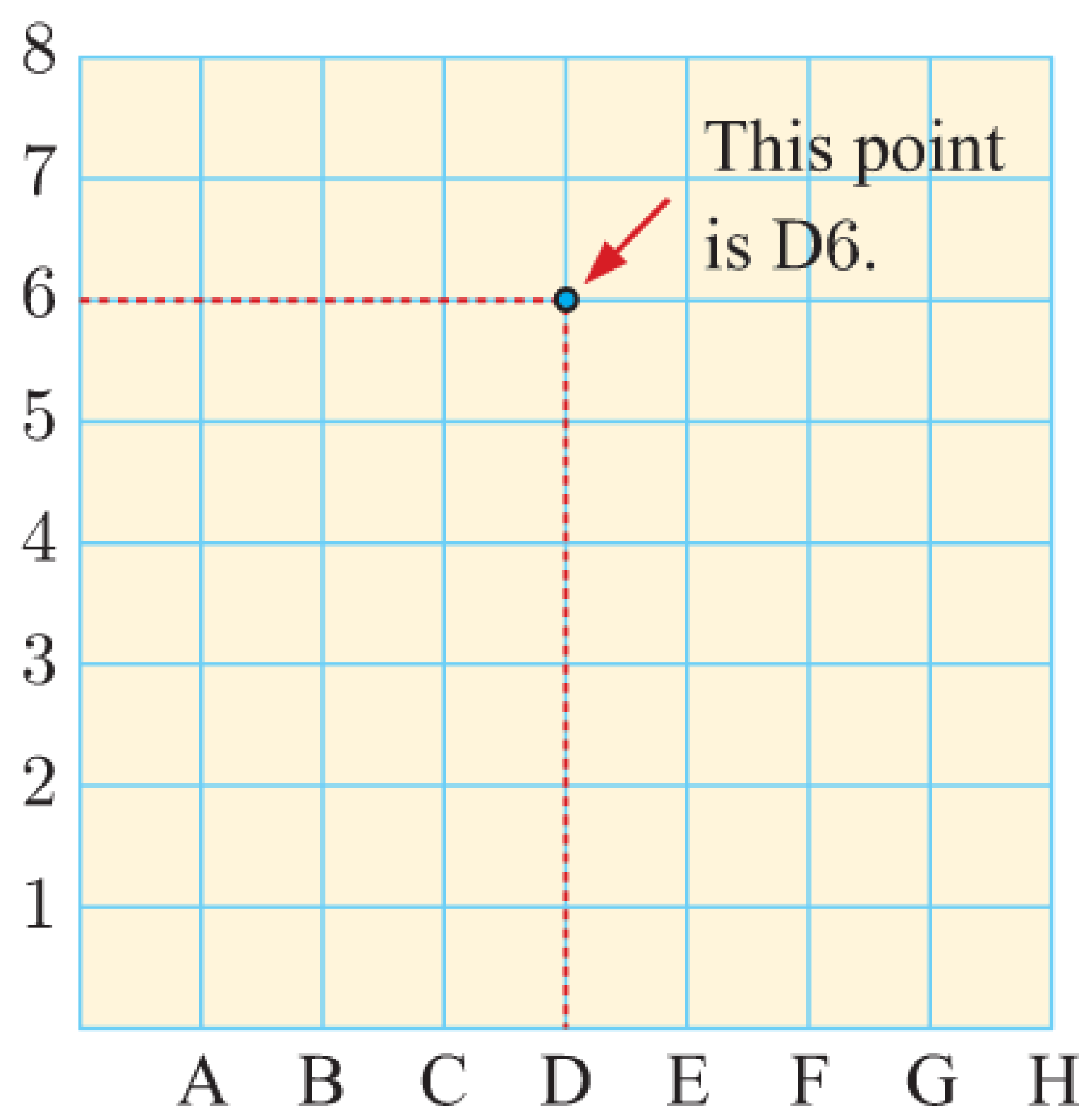
- **a** What feature is located at J6?
- **b** Where is the canteen?
- **c** Locate the parking area which is closest to the oval.
- **d**
  - **i** How many drinking fountains are there?
  - **ii** Locate the fountain which is closest to the gym.

B FINDING POINTS

In the previous Section, we saw how map references describe a square or region on a map.

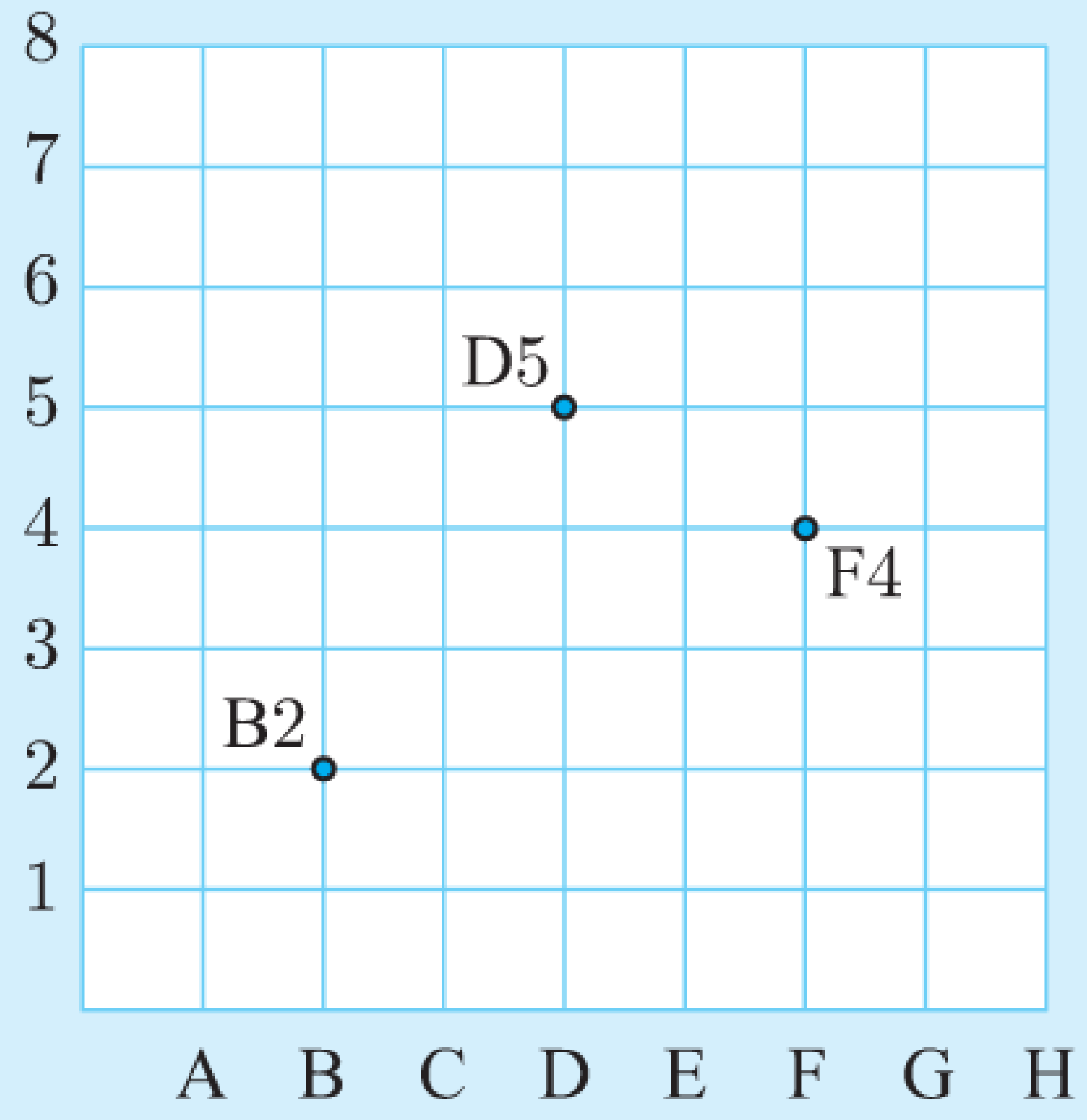
If we want to describe the *exact* position of a *point* on a map, we can place the letters and numbers *on* the grid lines, not between them.

A letter and number combination, such as D6, now refers to a specific point on the map.



Example 2 **Self Tutor**

Mark the points B2, D5, and F4 on a grid.

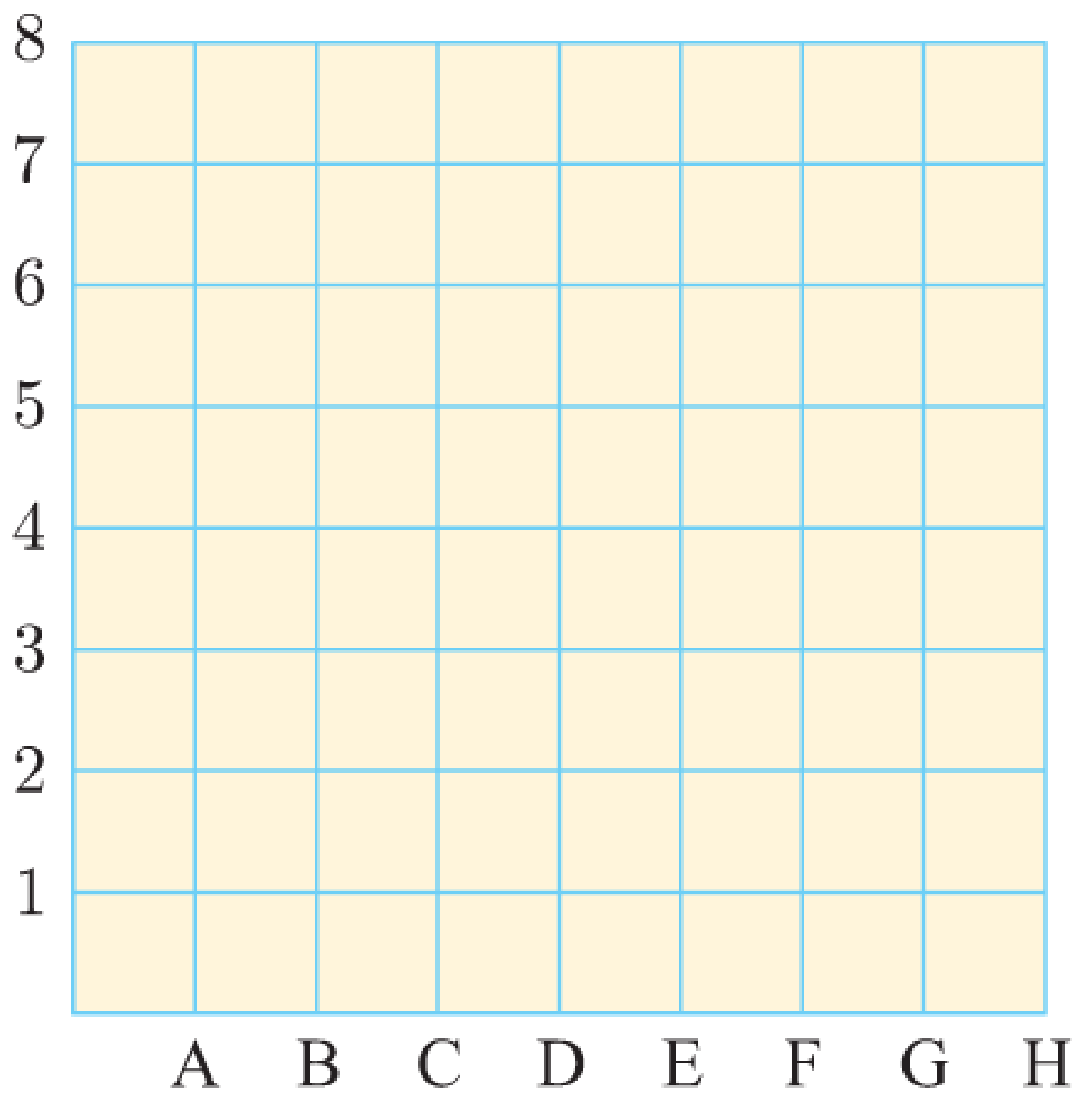


EXERCISE 13B

1 Copy the given grid, and mark these points on it:

a C3 **b** D7 **c** E8
d A1 **e** G1 **f** A2
g A8 **h** H7 **i** B4

PRINTABLE GRID



2 On a grid like the one above, plot each of the following sets of points. Join the points in the order given. Write the name of the shape which results.

a A1, C1, C4, A4, A1
b B7, C5, D8, B7
c E8, H8, G5, D5, E8
d E4, G4, H3, F1, D3, E4

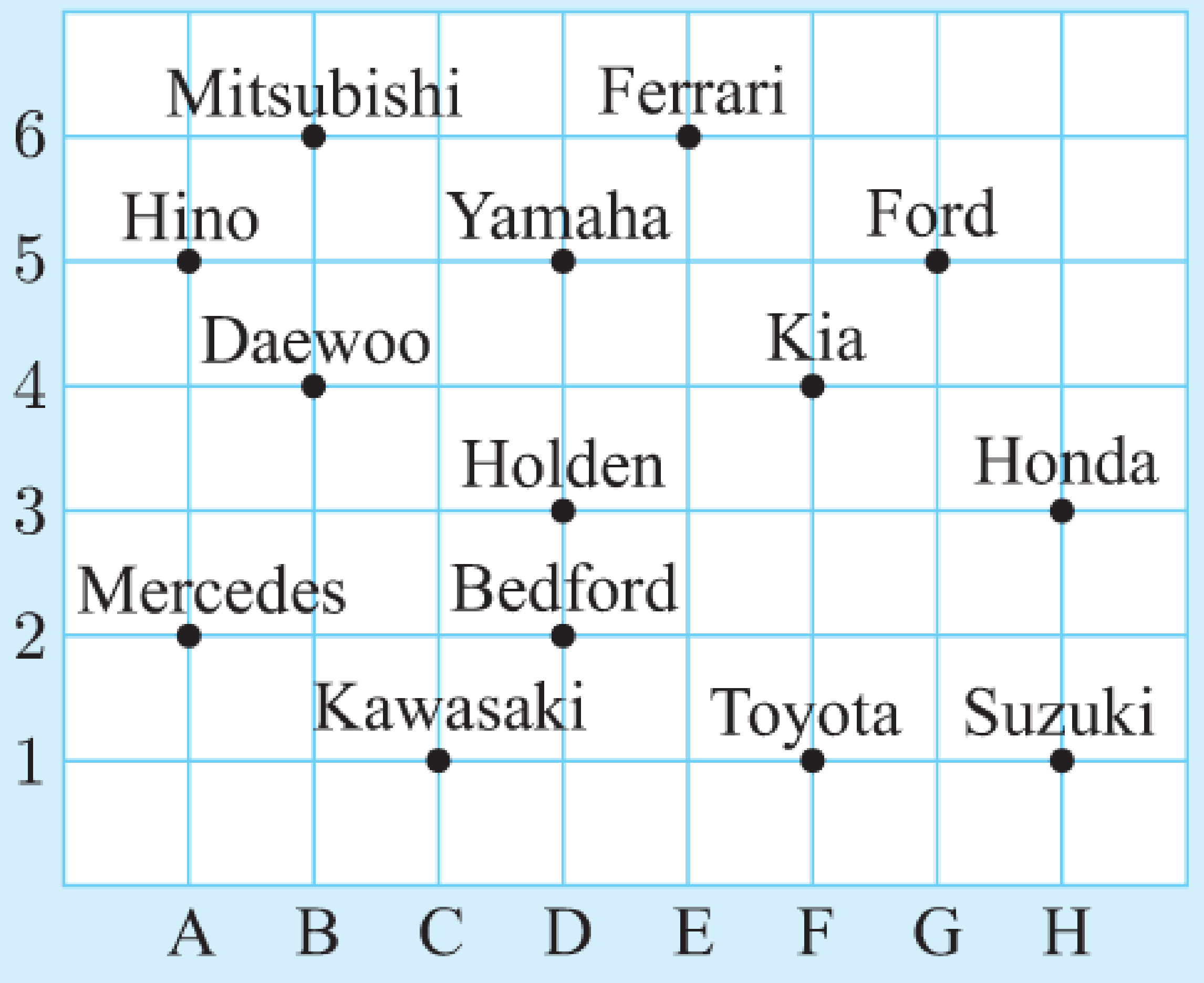
Example 3 **Self Tutor**

The grid alongside shows the layout of displays at a motor show.

a Find the position of the display for:
 i Toyota **ii** Daewoo

b Name the display which is at:
 i B6 **ii** A2.

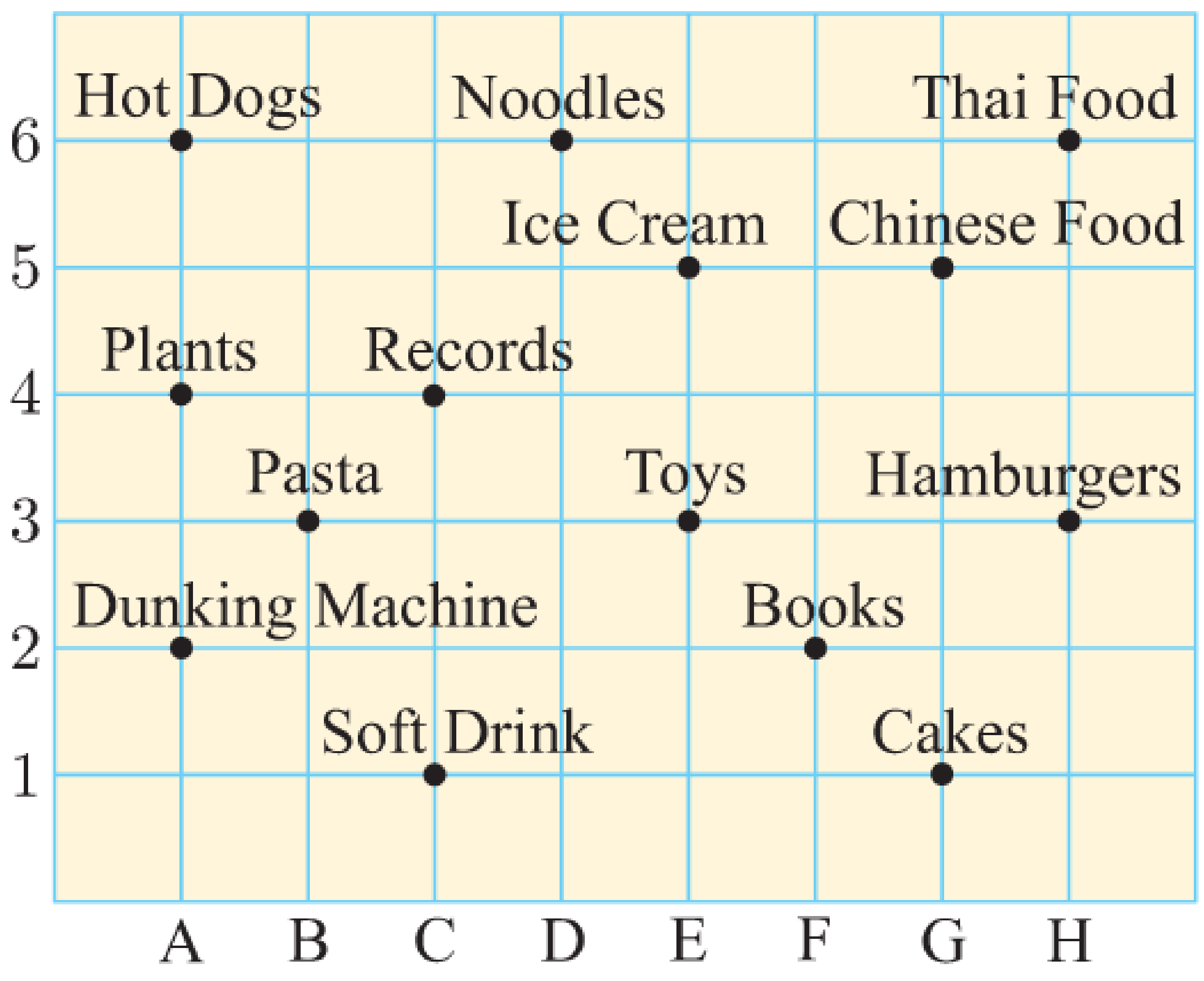
c Which display is closest to the Yamaha display? Give the grid reference for this other display.



a **i** F1 **ii** B4 **b** **i** Mitsubishi **ii** Mercedes

c Ferrari. Its grid reference is E6.

3

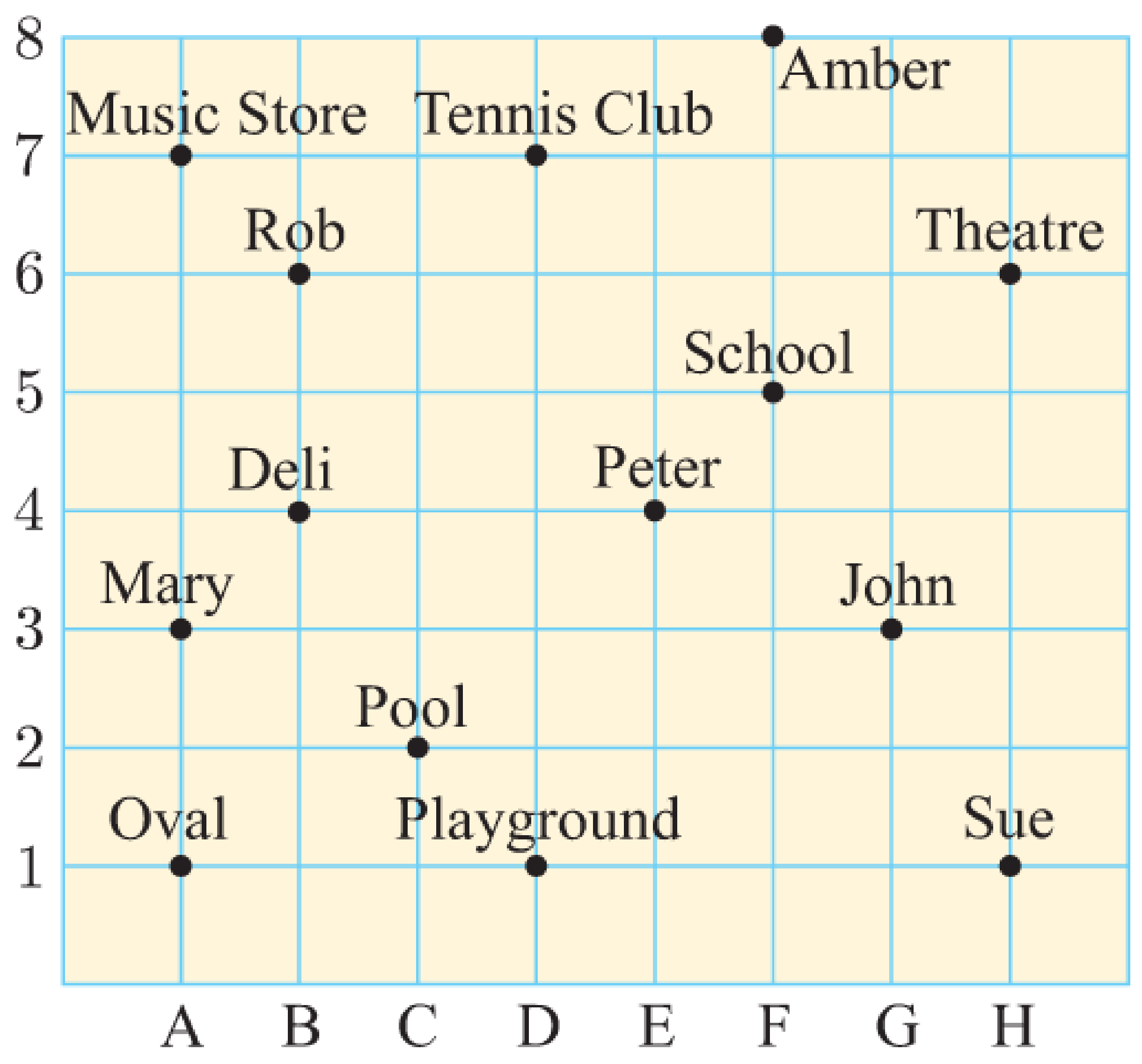


This grid shows the positions of various stalls at the local fair.

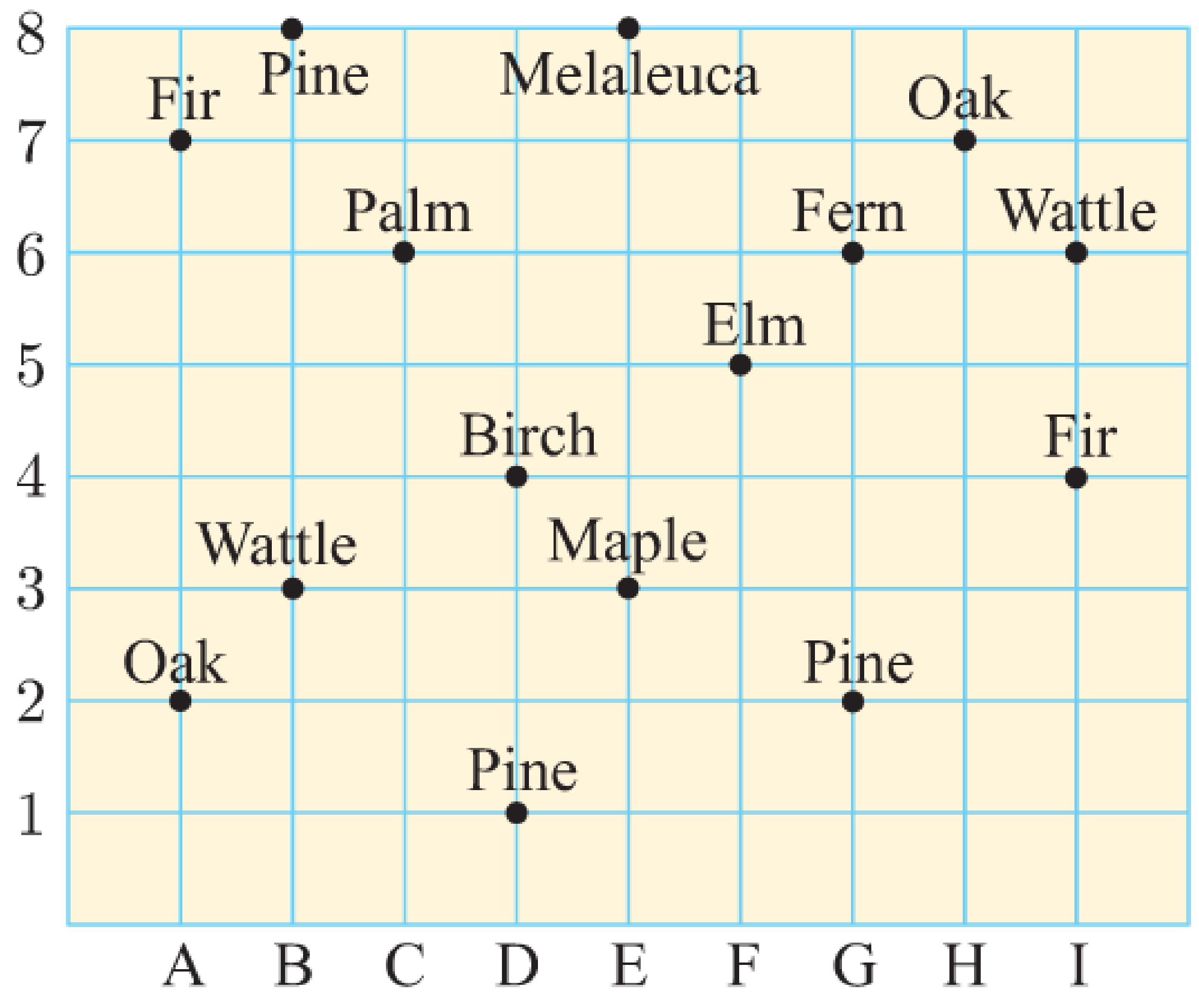
- **a** Find the position of the:
  - **i** cake stall
  - **ii** toys stall.
- **b** Name the stall at:
  - **i** B3
  - **ii** H6.

4 This grid shows the homes of a group of friends, and the positions of other important places near them.

- **a** Who lives at:
  - **i** E4
  - **ii** F8?
- **b** Find the location of the:
  - **i** tennis club
  - **ii** pool.
- **c** Name the place at F5.
- **d** Which of the friends lives closest to the music store? Give the grid reference for this person's home.



5



The grid alongside shows a field which a gardener has planted with trees.

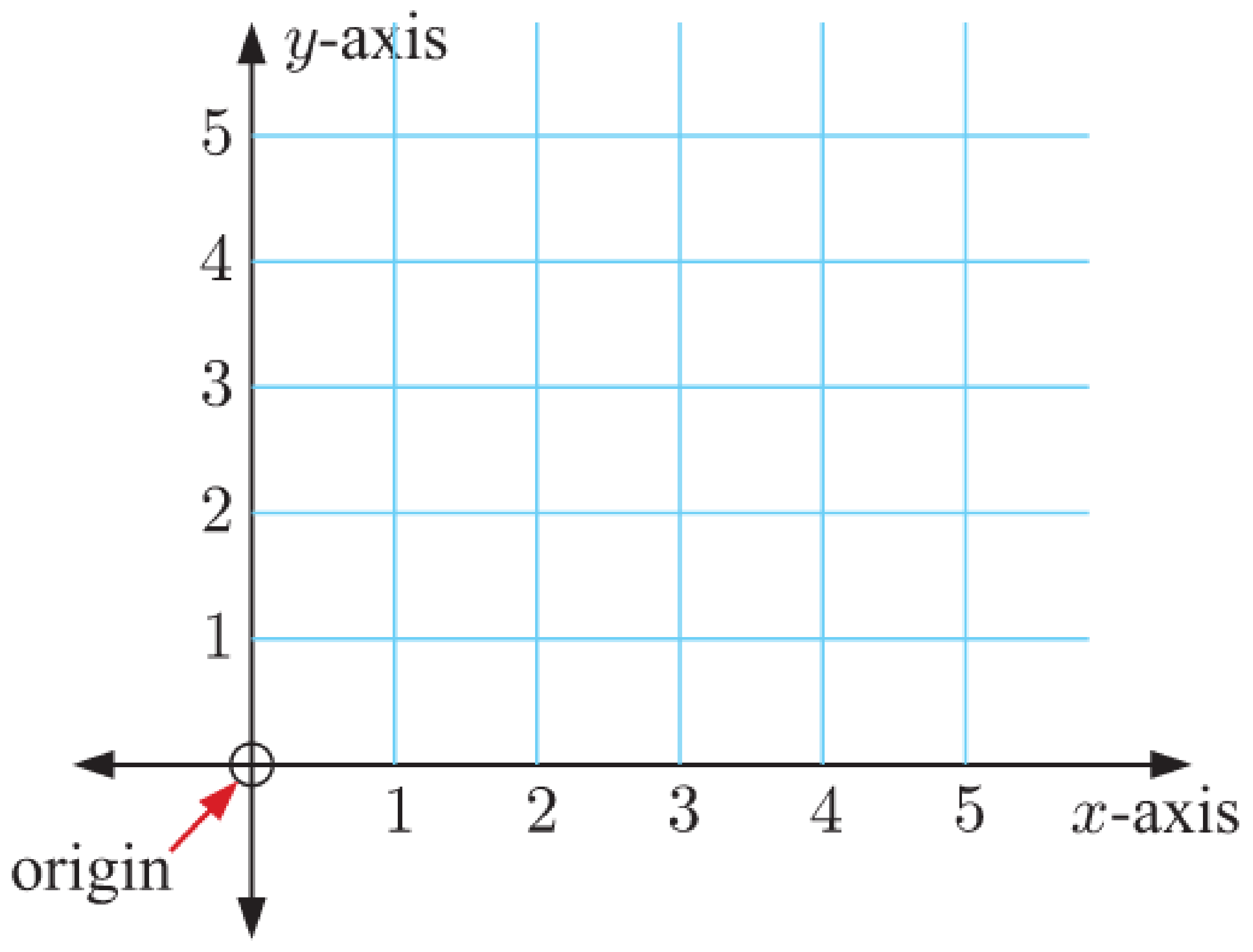
- **a** How many trees are there?
- **b** Which is the most common type of tree? Give the grid references for its locations.
- **c** Which type of tree is closest to the Birch? Give the grid reference for this tree.

C COORDINATES

Instead of using letters and numbers to locate positions, we can use numbers along both sides of the grid.

The horizontal line across the bottom is called the **x-axis**, and the vertical line along the left side is called the **y-axis**.

The horizontal axis and the vertical axis meet at the **origin**, which we mark with O or with a small circle.

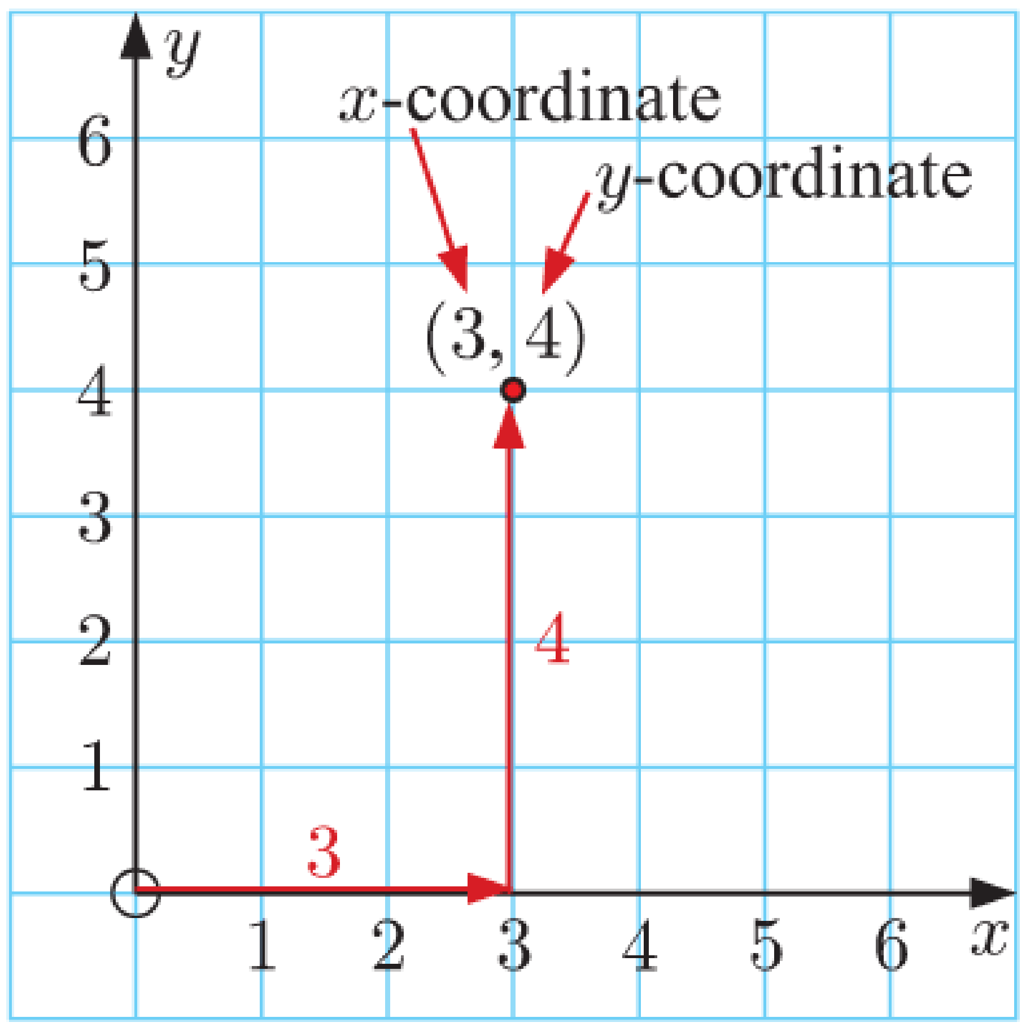


We can describe any point on the grid using a pair of numbers, known as **coordinates**. The coordinates of a point show the point's horizontal and vertical position from the origin.

Coordinates are written as an **ordered pair** of numbers. We write them in brackets, with a comma between.

For example, look at the point on this grid. To move from the origin to this point, we move 3 units horizontally, then 4 units vertically. We say that the point has coordinates (3, 4).

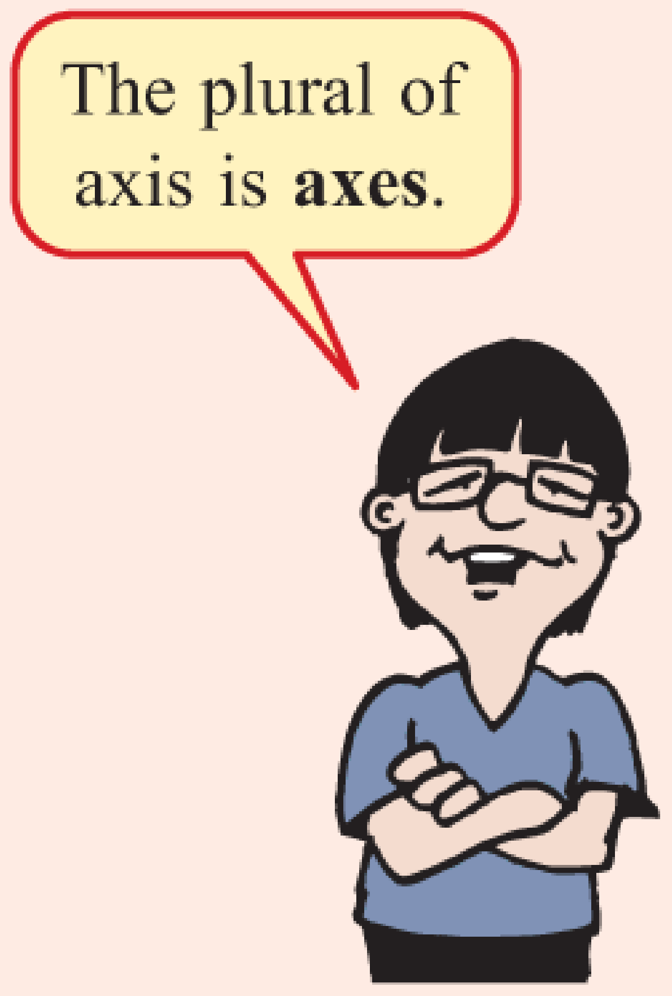
The **x-coordinate** is 3, and the **y-coordinate** is 4.



DISCUSSION

What are the advantages of using numbers on both axes, instead of a letter and a number?

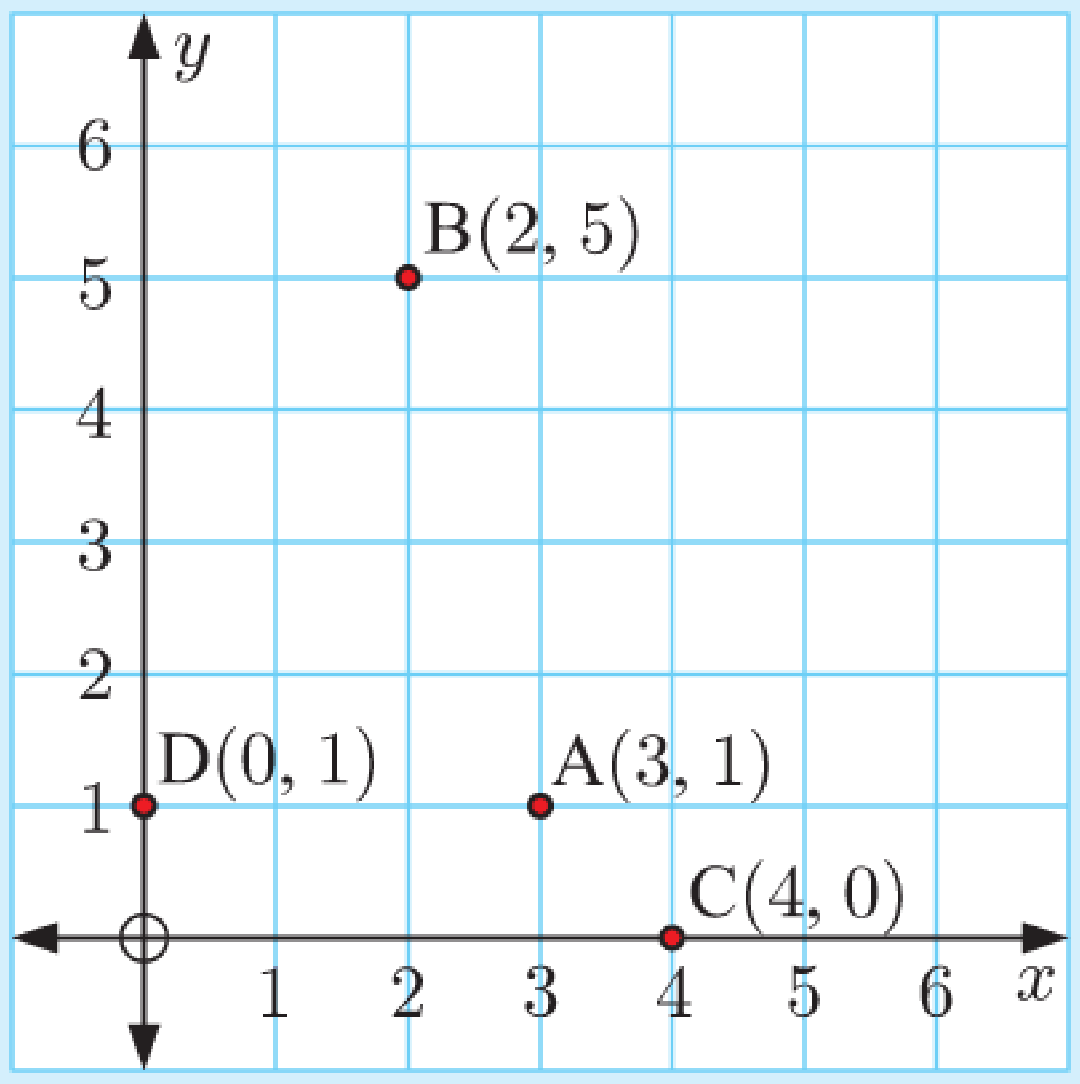
Are there any disadvantages to using two numbers?



Example 4 Self Tutor

On the same set of axes, plot and label the following points:

A(3, 1), B(2, 5), C(4, 0), D(0, 1).



EXERCISE 13C

PRINTABLE GRIDS

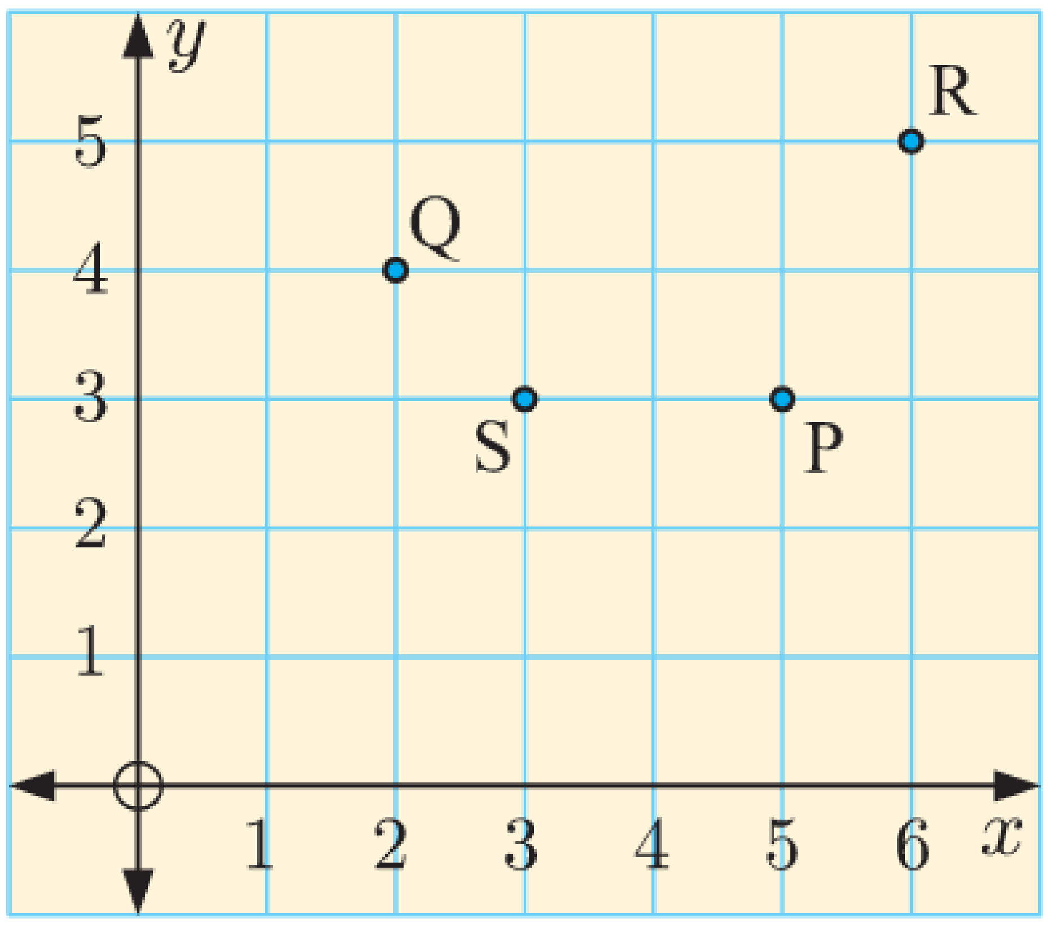
1 On the same set of axes, plot and label the following points:

a A(1, 5) **b** B(6, 3) **c** C(4, 4) **d** D(2, 0)
e E(5, 2) **f** F(0, 5) **g** G(7, 3) **h** H(8, 0)
i I(5, 8) **j** J(0, 3) **k** K(6, 0) **l** L(3, 7)

2 **a** Which of the points in **1** lie on the x-axis?
b What do you notice about the y-coordinate of each of these points?

3 **a** Which of the points in **1** lie on the y-axis?
b What do you notice about the x-coordinate of each of these points?

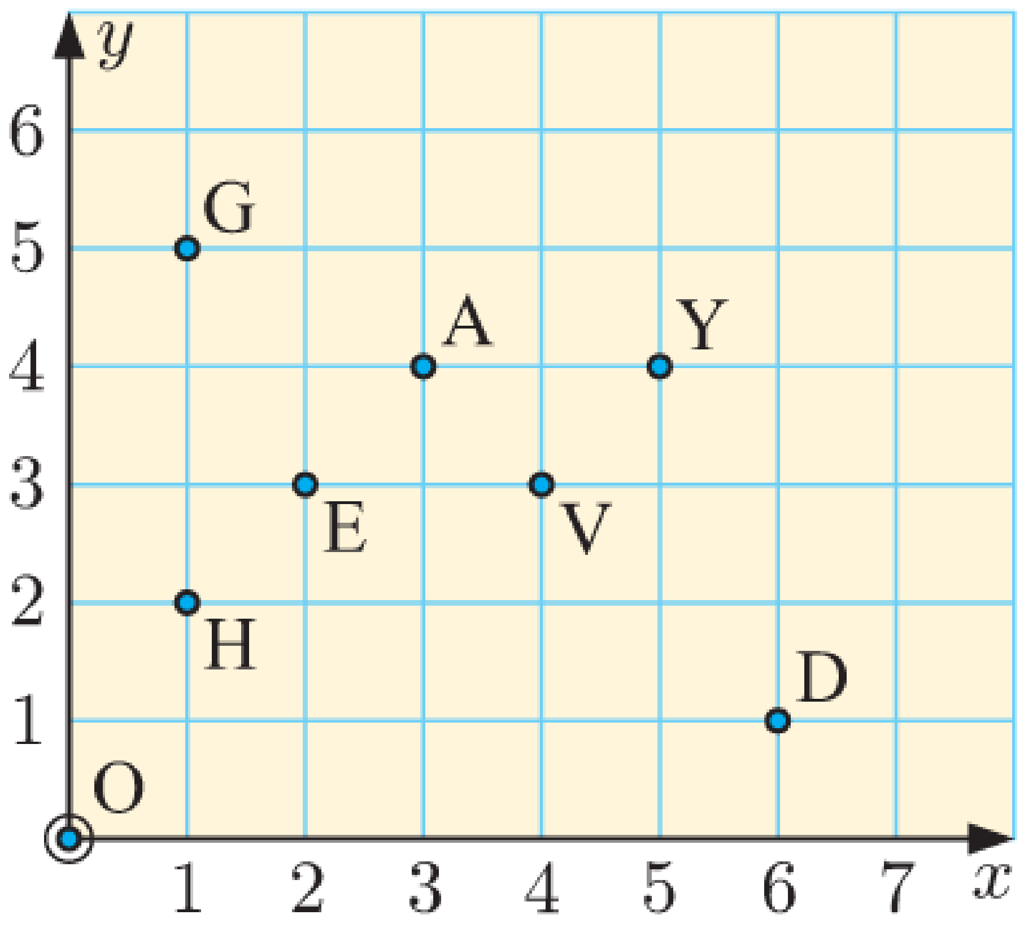
4



Write down:

a the x-coordinate of P
b the y-coordinate of Q
c the coordinates of:
i R **ii** S **iii** the origin O.

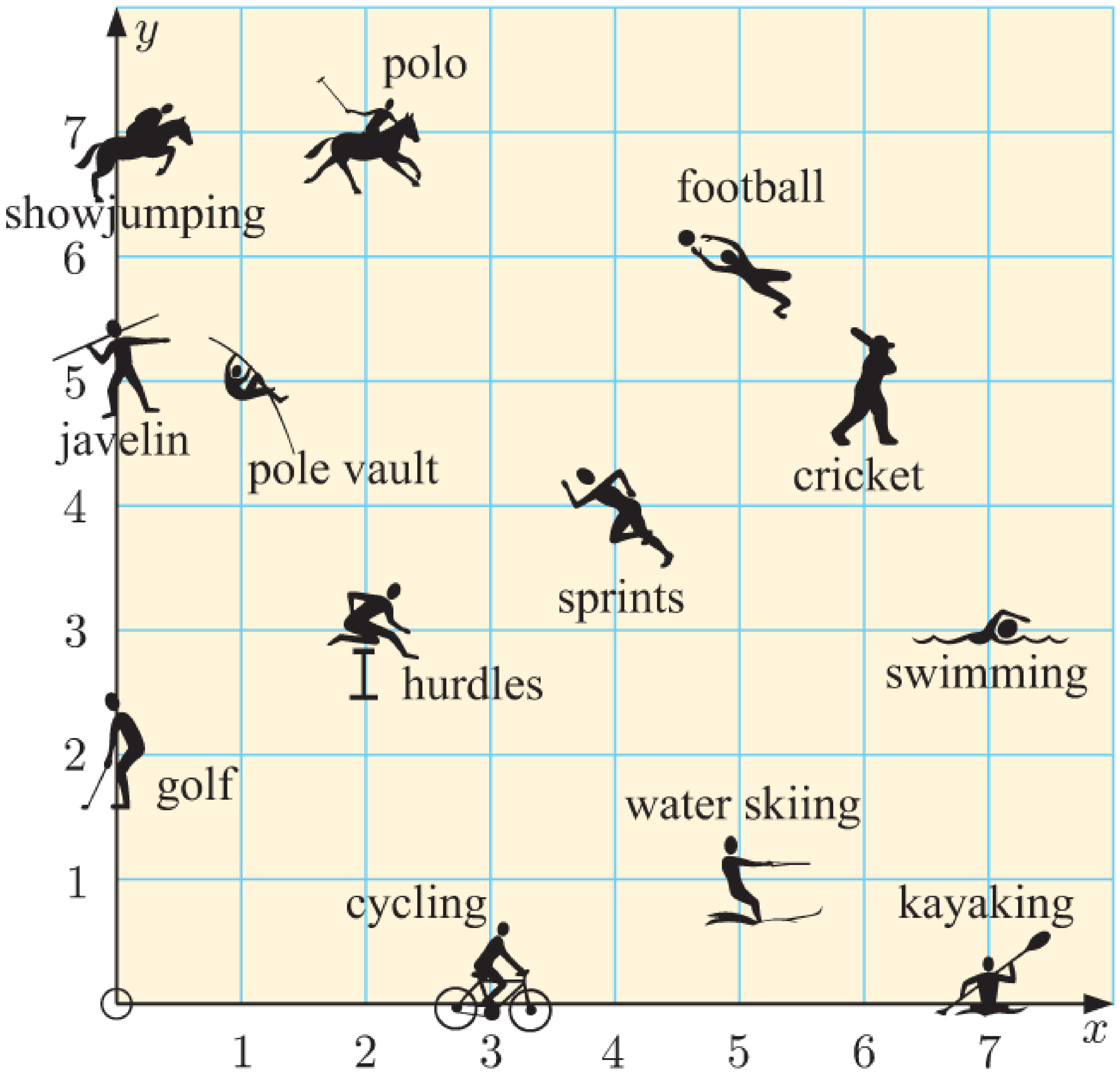
5



Write down the message given by these coordinates:

(1, 2), (3, 4), (4, 3), (2, 3)
(3, 4)
(1, 5), (0, 0), (0, 0), (6, 1)
(6, 1), (3, 4), (5, 4)

6 A map of a sporting complex is shown alongside.



a Write down the coordinates of the:

i pole vault

ii cycling

iii sprints

iv showjumping

b What sports would you find at these coordinates?

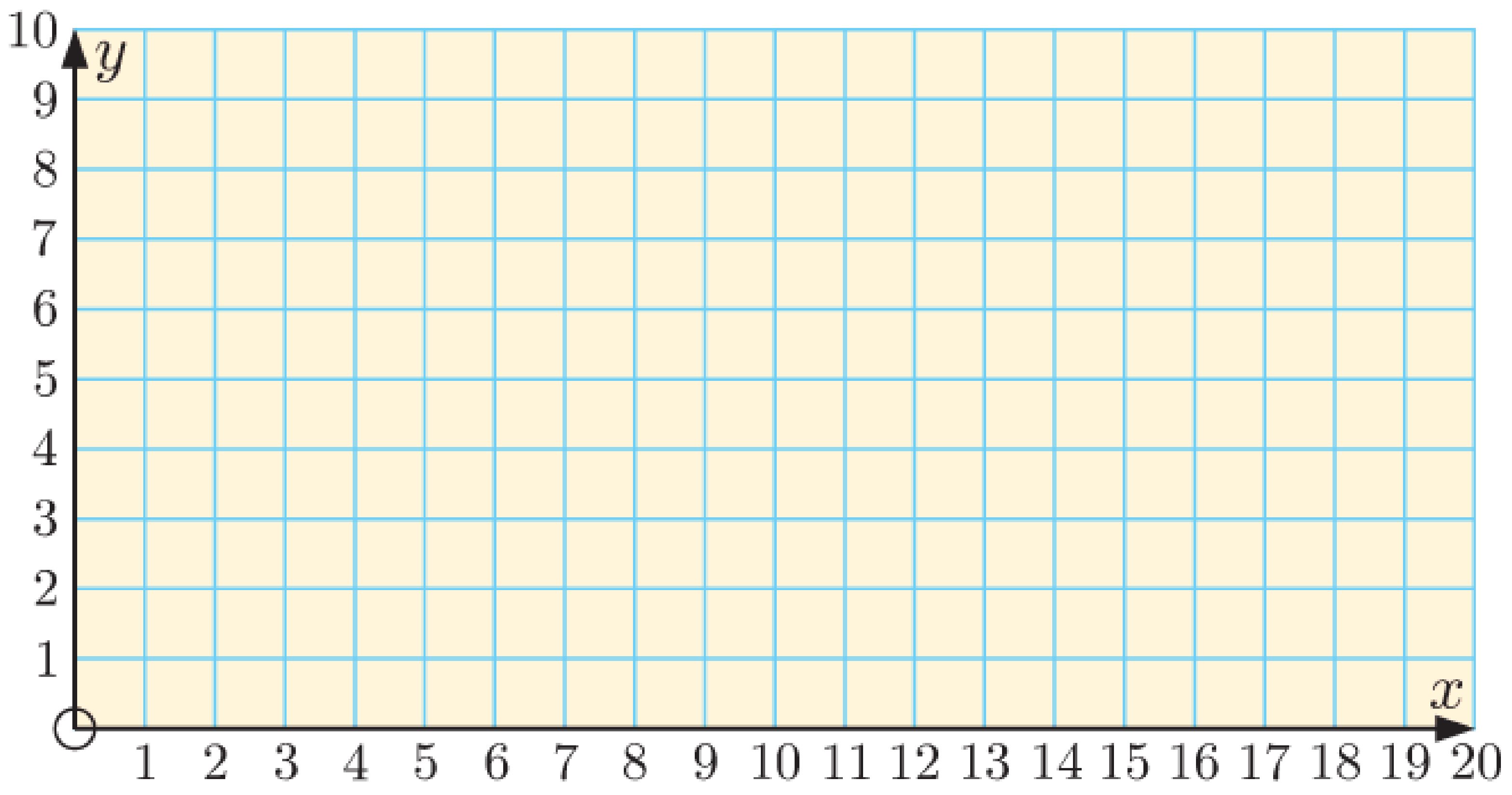
i (2, 7) **ii** (7, 0)
iii (6, 5) **iv** (0, 2)

c Which sport has the:

i same x-coordinate as the football
ii same y-coordinate as the hurdles?

7 Draw a grid with numbers up to 20 on the x-axis and numbers up to 10 on the y-axis.

PRINTABLE GRIDS



Plot the following coordinates and join them in order with straight lines:

Start with: (13, 2), (16, 2), (17, 1), (16, 0), (1, 0), (0, 2), (1, 4), (3, 4), (4, 2), (6, 2), (6, 5), (8, 7), (13, 7), (15, 5), (15, 3), (13, 2), (9, 2), (7, 4), (9, 6), (12, 6), (13, 5), (13, 4), (12, 3), (10, 3), (9, 4), (10, 5), (11, 5)

then (1, 4), (1, 6), (0, 7)

then (2, 4), (2, 6), (1, 7).

GAME FIND THE EGGS

This is a game for two people in which each player tries to find the positions of 6 "eggs" hidden by their opponent on a grid.

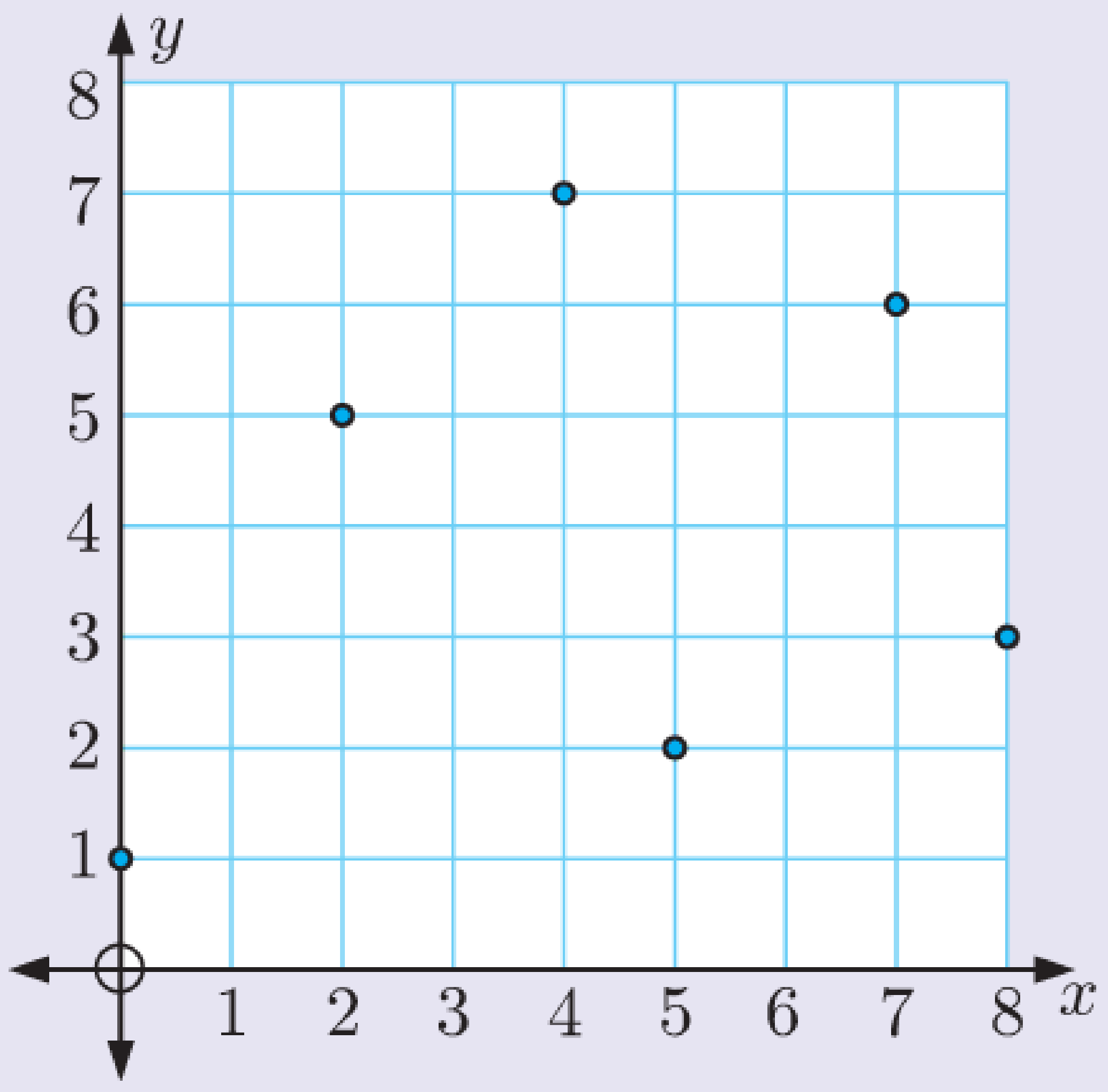
PRINTABLE GRIDS

You will need: a sheet of graph paper and a pencil for each player.

What to do:

1 Draw a grid with numbers up to 8 on each axis.

2 Mark the position of six eggs on your grid, but do not show your opponent. You may place an egg at the origin or along the axes if you wish.
An example showing how to mark the eggs on the grid is given alongside.

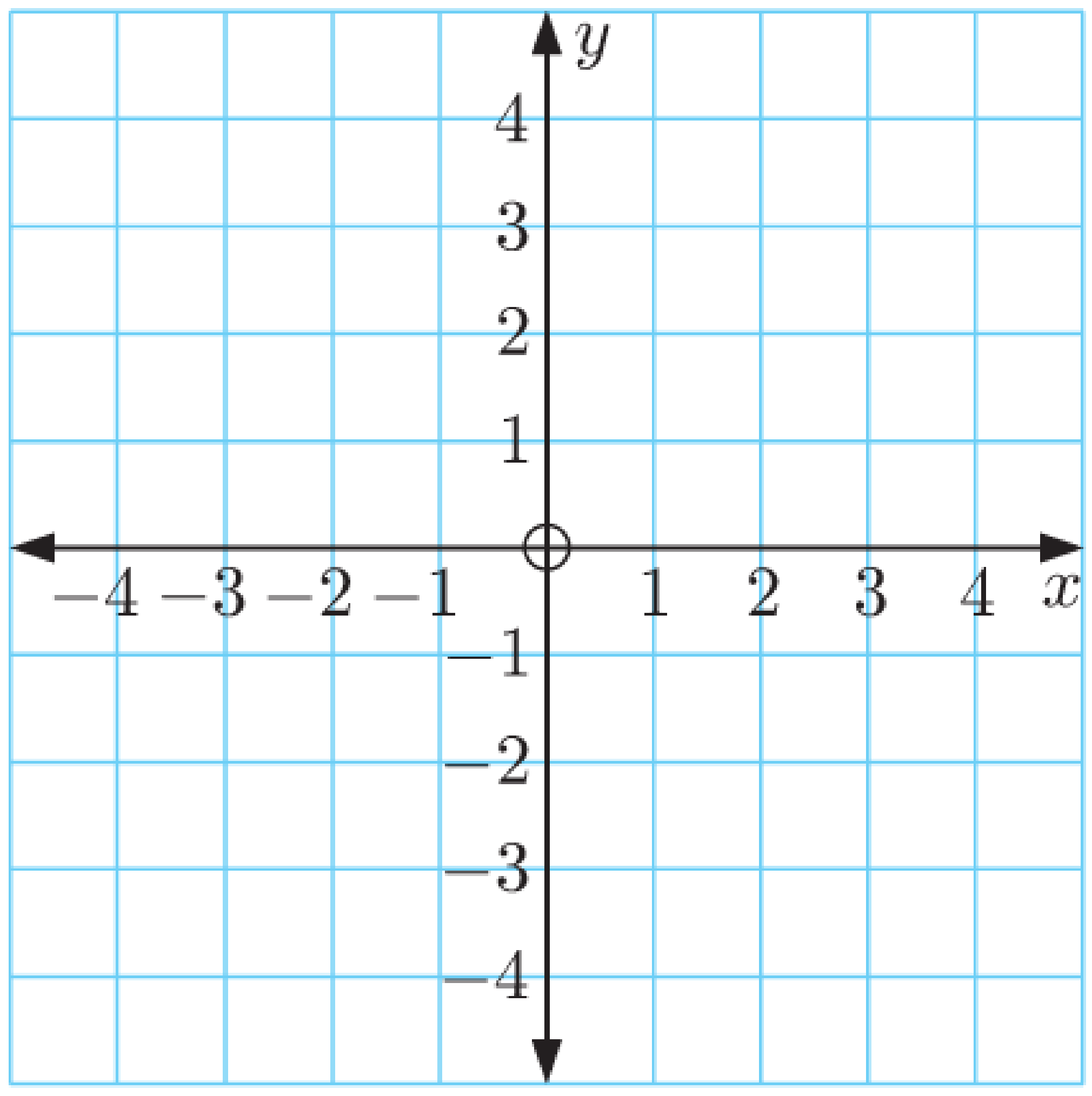


3 The first player calls out a pair of coordinates to try to locate one of their opponent's eggs. If the point has an egg on it, the opponent says "yes". If the point is *next to* an egg (either horizontally, vertically, or diagonally), the opponent says "close". Otherwise, the opponent says "no".

4 The opponent then takes his or her turn, and the players continue to take turns thereafter.

5 The winner is the person who locates all of their opponent's eggs first.

D POSITIVE AND NEGATIVE COORDINATES



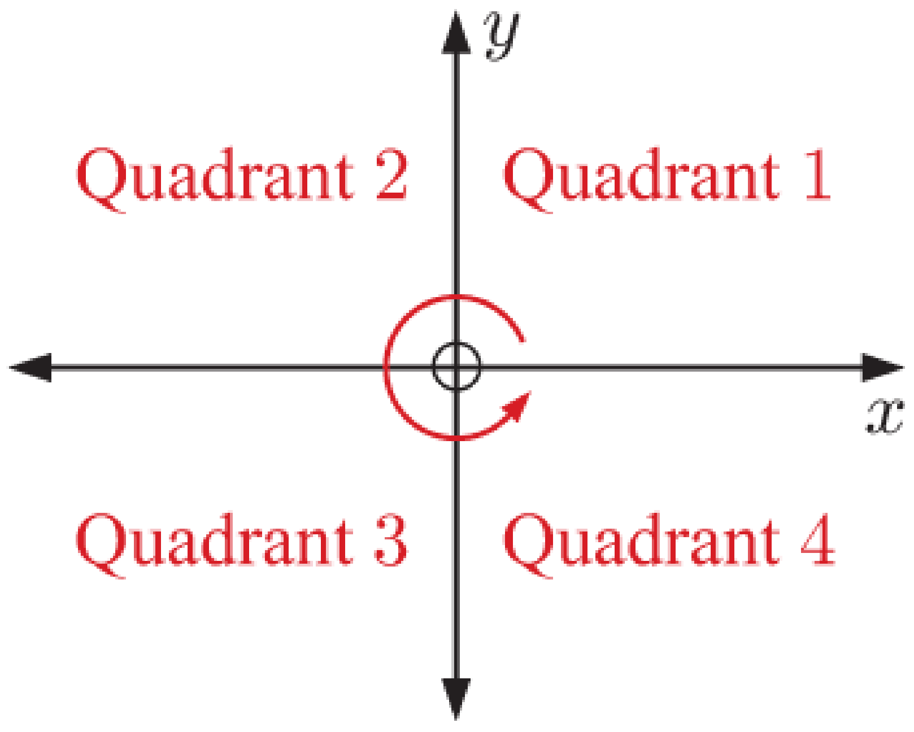
In **Chapter 11** we saw how the number line was extended in two directions to include positive and negative numbers.

In the same way, we can extend both the x-axis and the y-axis in two directions. This allows us to consider both positive and negative coordinates.

In the centre of the number plane is the origin O.

The x-axis is positive to the right of O and negative to the left of O.

The y-axis is positive above O and negative below O.



The axes divide the plane into four **quadrants**.

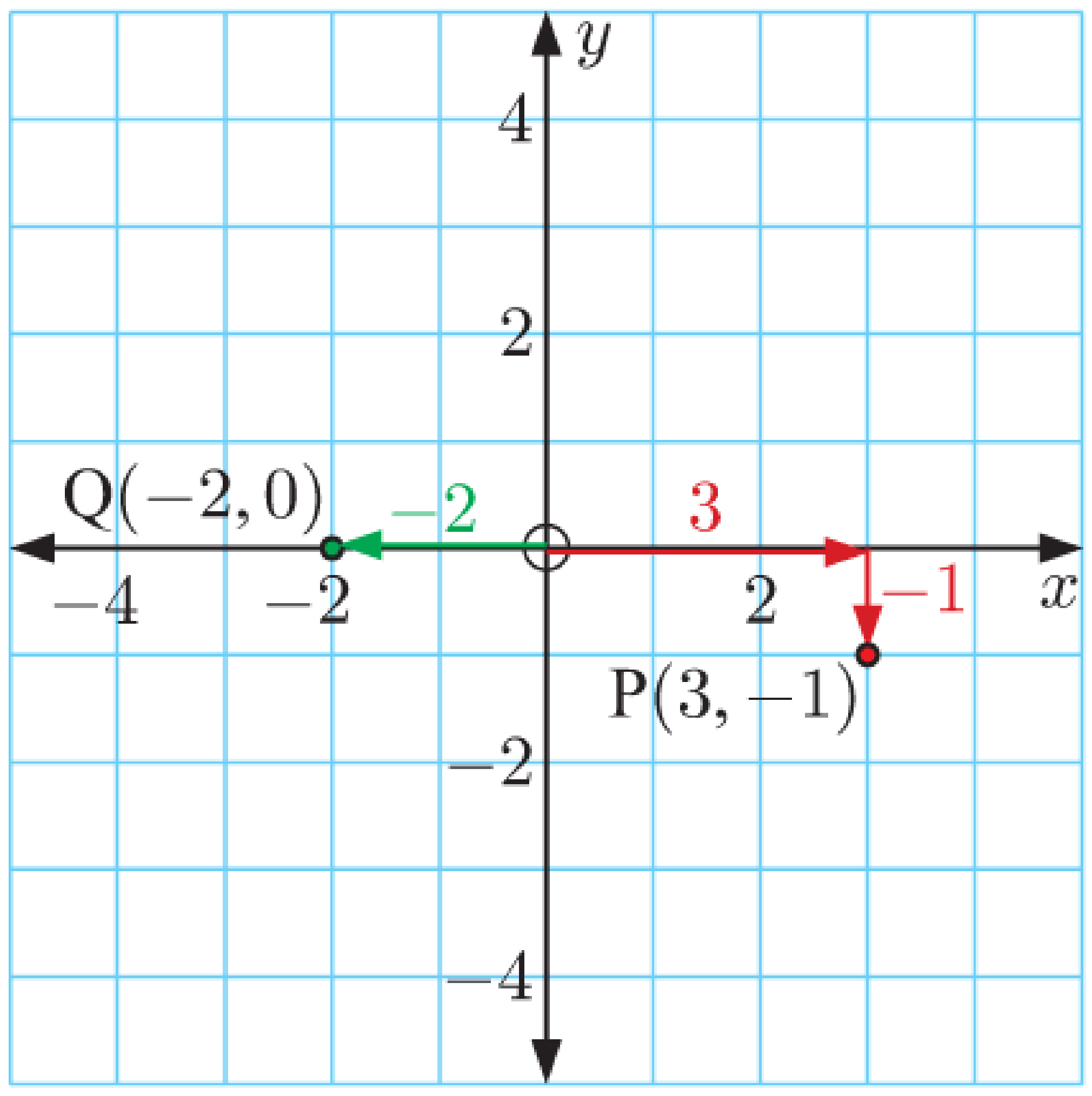
The quadrants are numbered in an anticlockwise direction, starting with the upper right hand quadrant in which x and y are both positive.

We can now describe and plot points in any of the four quadrants or on either axis.

For example:

To plot the point P(3, −1), we move 3 units to the *right* of the origin, then 1 unit *down*. P is in the fourth quadrant.

To plot the point Q(−2, 0), we move 2 units to the left, but we do not need to move up or down. Q is on the x-axis.

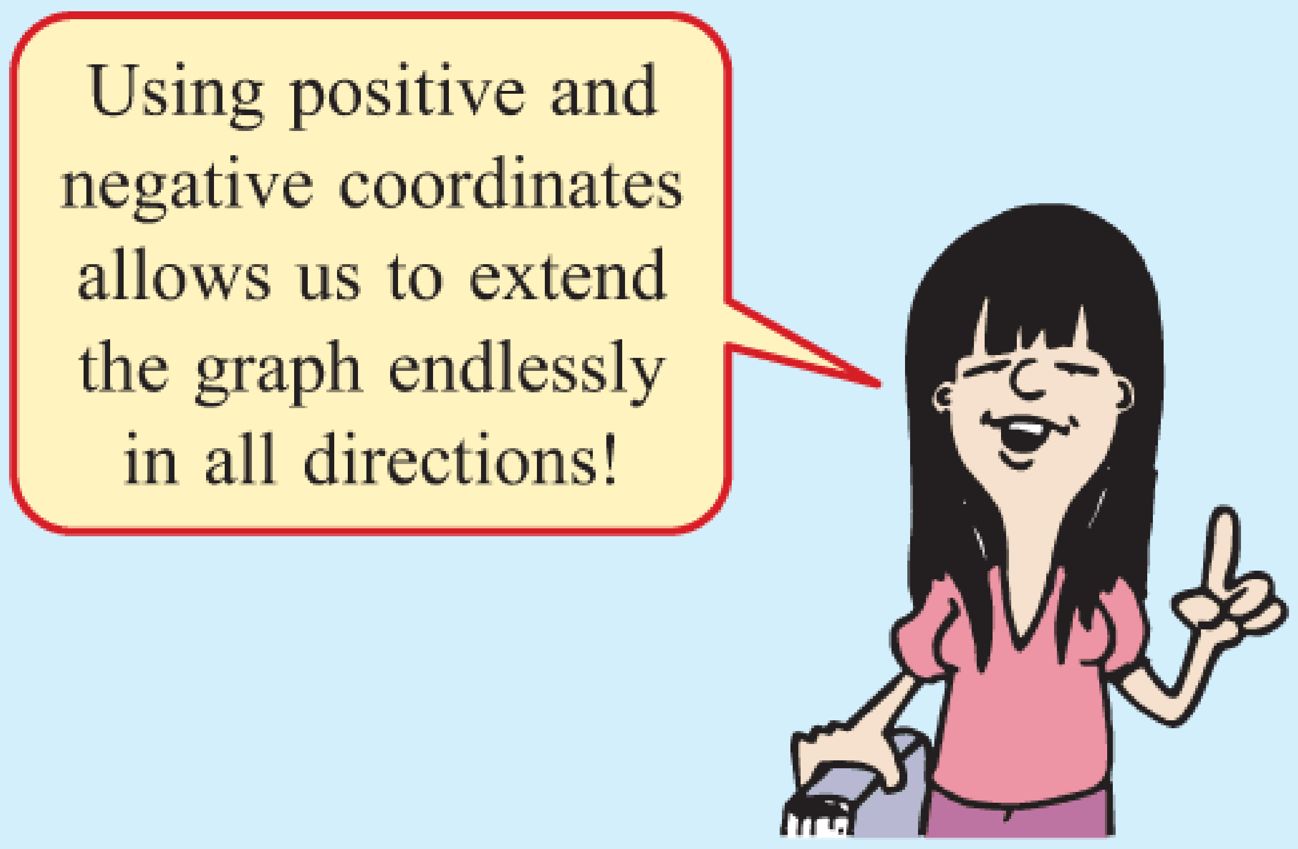


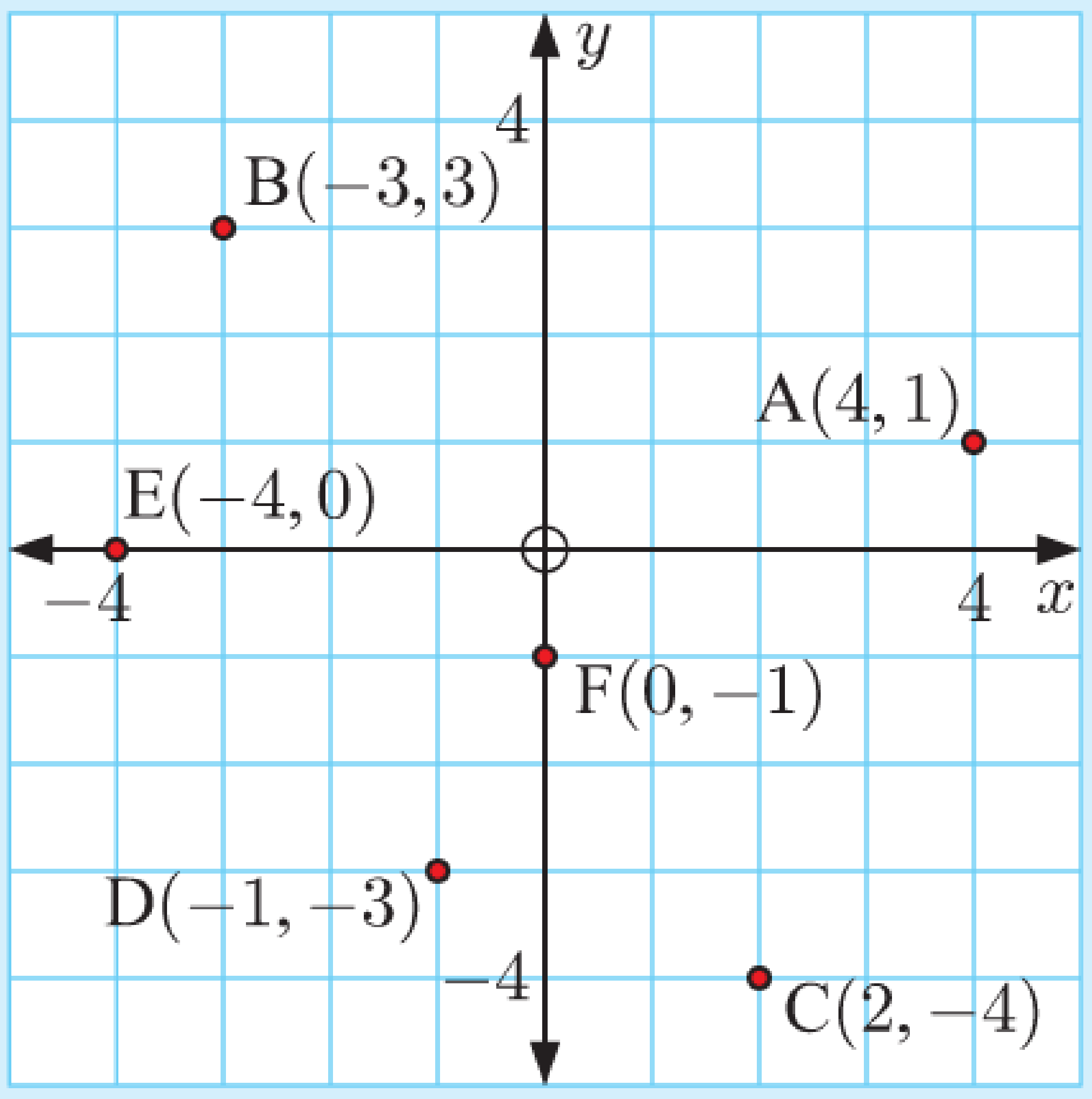
Example 5

Self Tutor

Plot the following points on a set of axes:

A(4, 1), B(−3, 3), C(2, −4), D(−1, −3), E(−4, 0), F(0, −1).





EXERCISE 13D

PRINTABLE GRIDS

1 Draw a set of axes, then plot and label the following points:

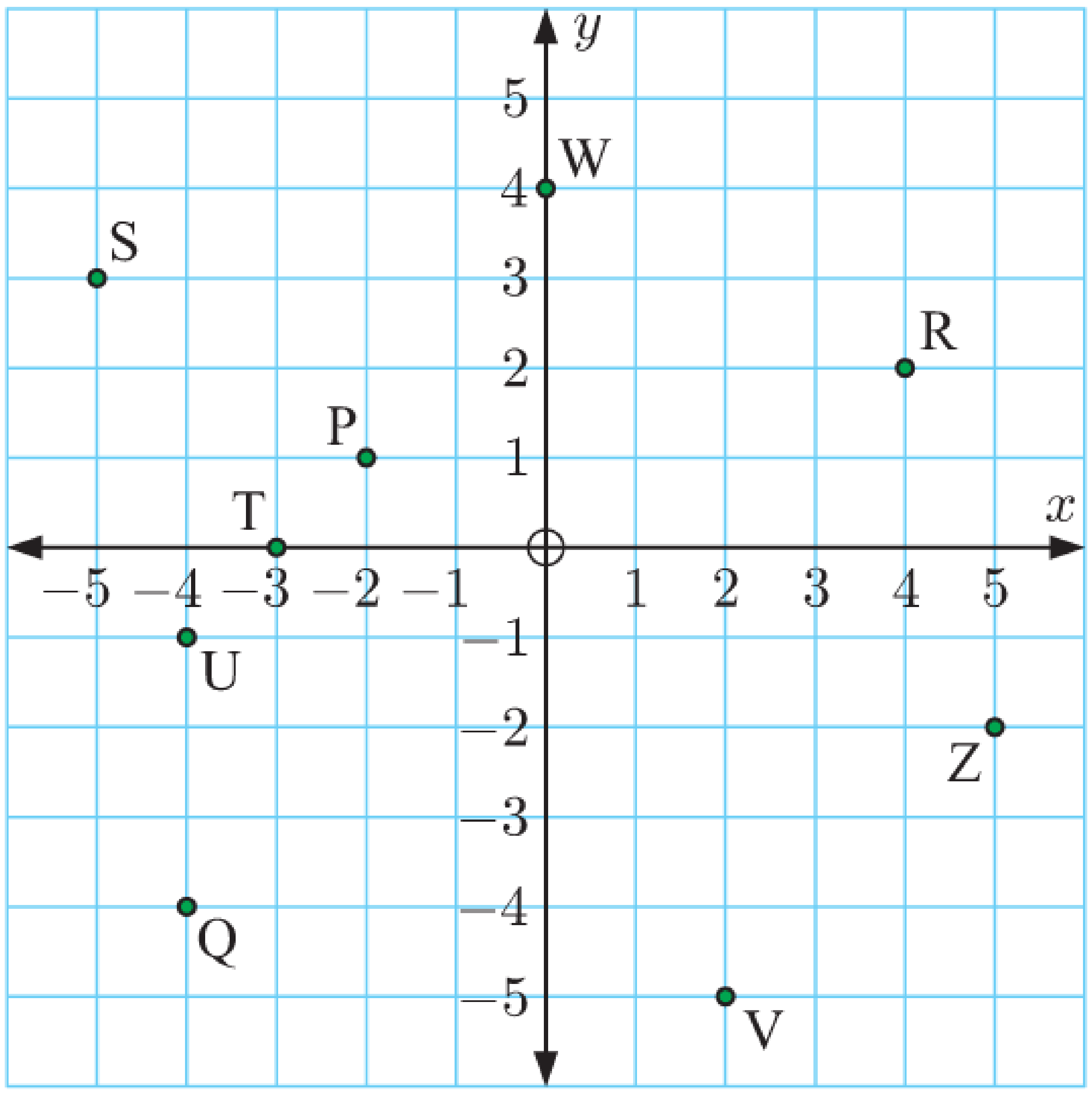
a A(5, 3) **b** B(−1, 4) **c** C(2, −2)
d D(0, −4) **e** E(3, 0) **f** F(−4, −5)
g G(−3, 1) **h** H(−5, 0) **i** I(5, −1)

2 Consider the points on the grid shown.

a Write down the:
- **i** x-coordinate of S
- **ii** y-coordinate of U
- **iii** coordinates of W.

b Name the quadrant in which you would find:
i R **ii** V **iii** U **iv** P

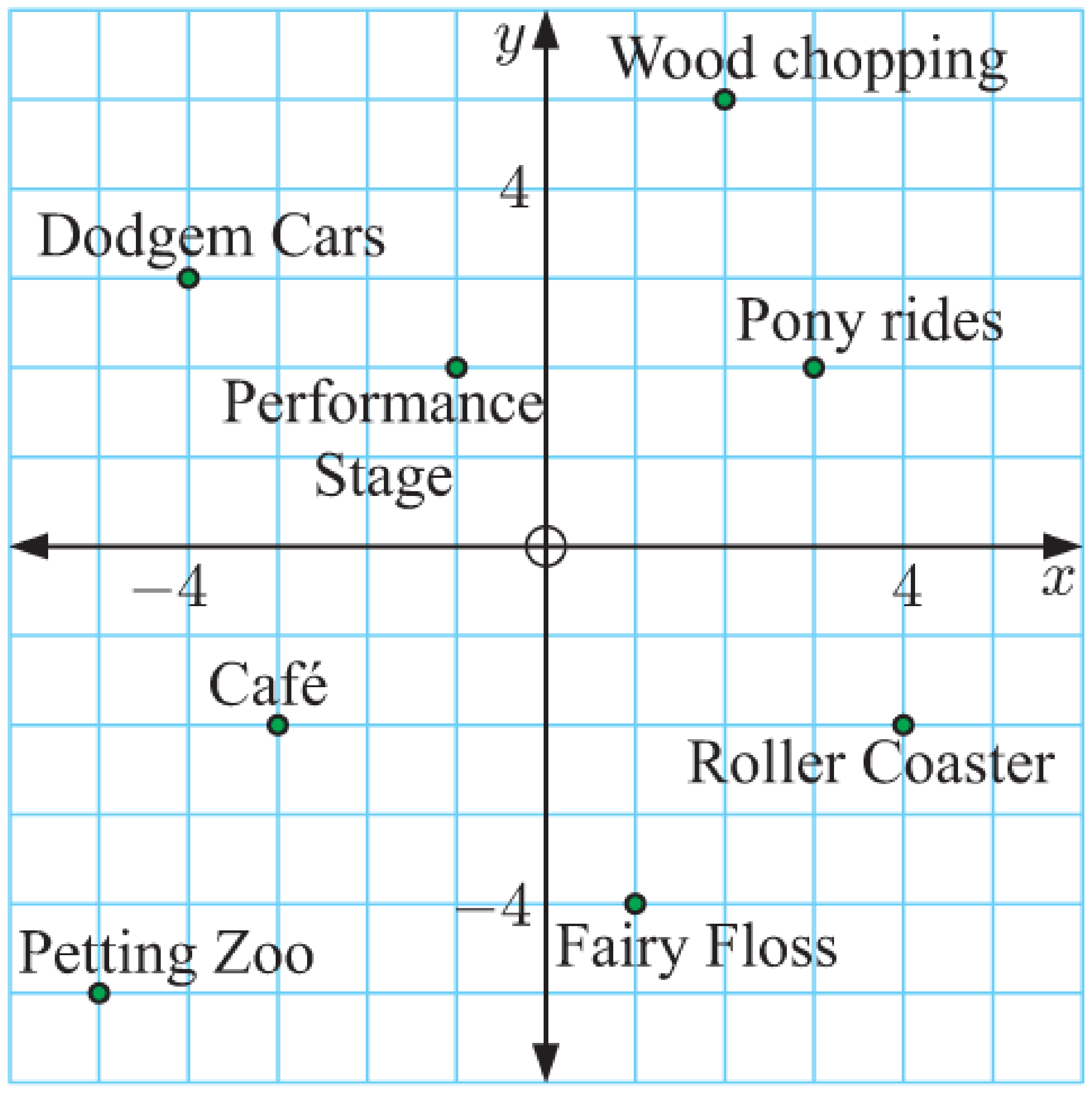
c Name the point which has:
- **i** x-coordinate −3
- **ii** y-coordinate −2
- **iii** the same x and y-coordinates.



3 This map shows the attractions at a carnival.

a Write down the coordinates of the:
i fairy floss **ii** dodgem cars
iii pony rides.

b Which feature is located at:
i (−1, 2) **ii** (4, −2)
iii (−5, −5)?

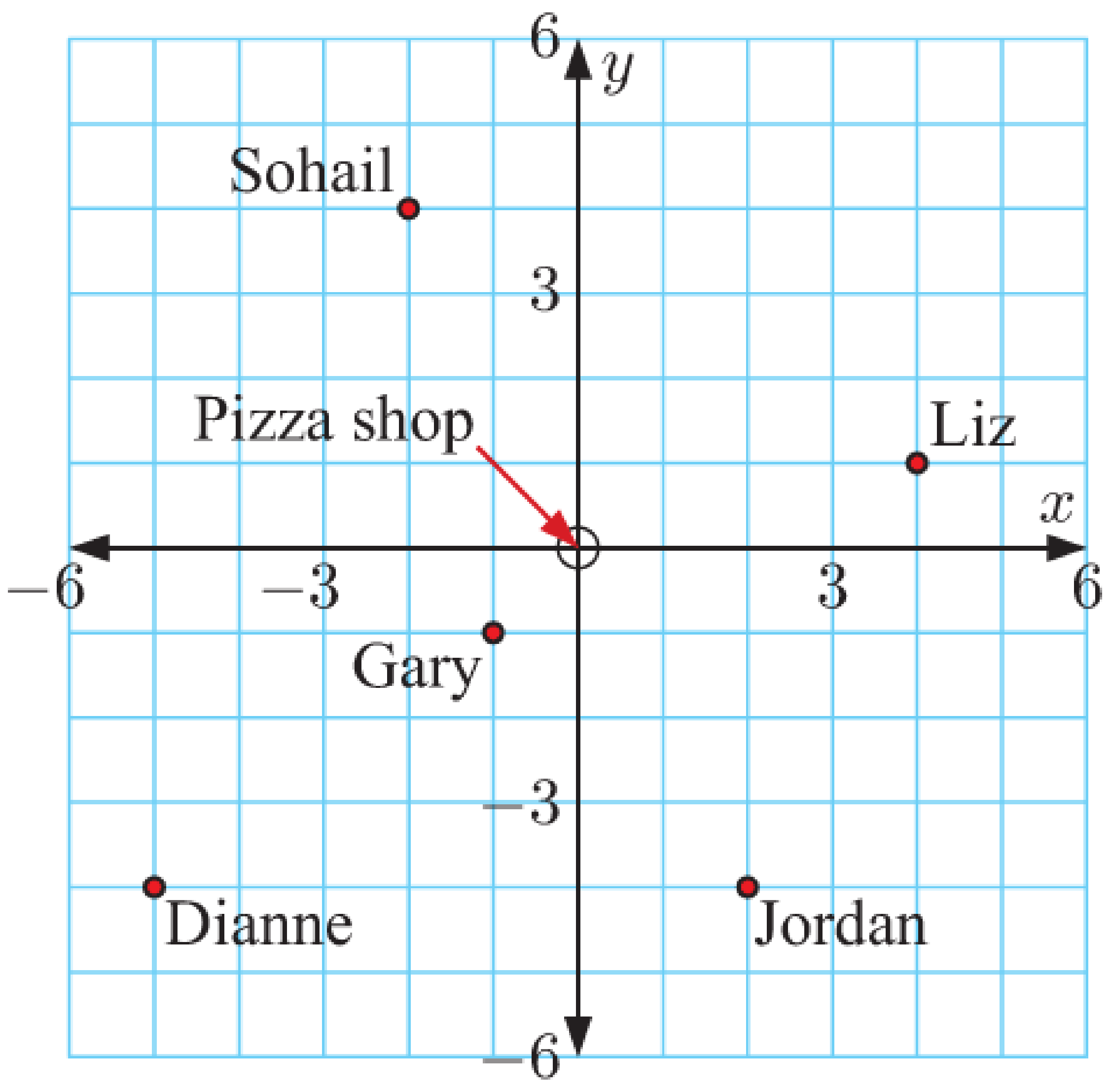


4 A group of friends are meeting at a pizza shop. This map shows the location of the friends in relation to the pizza shop.

a Find the coordinates of:
i Liz's house **ii** Sohail's house.

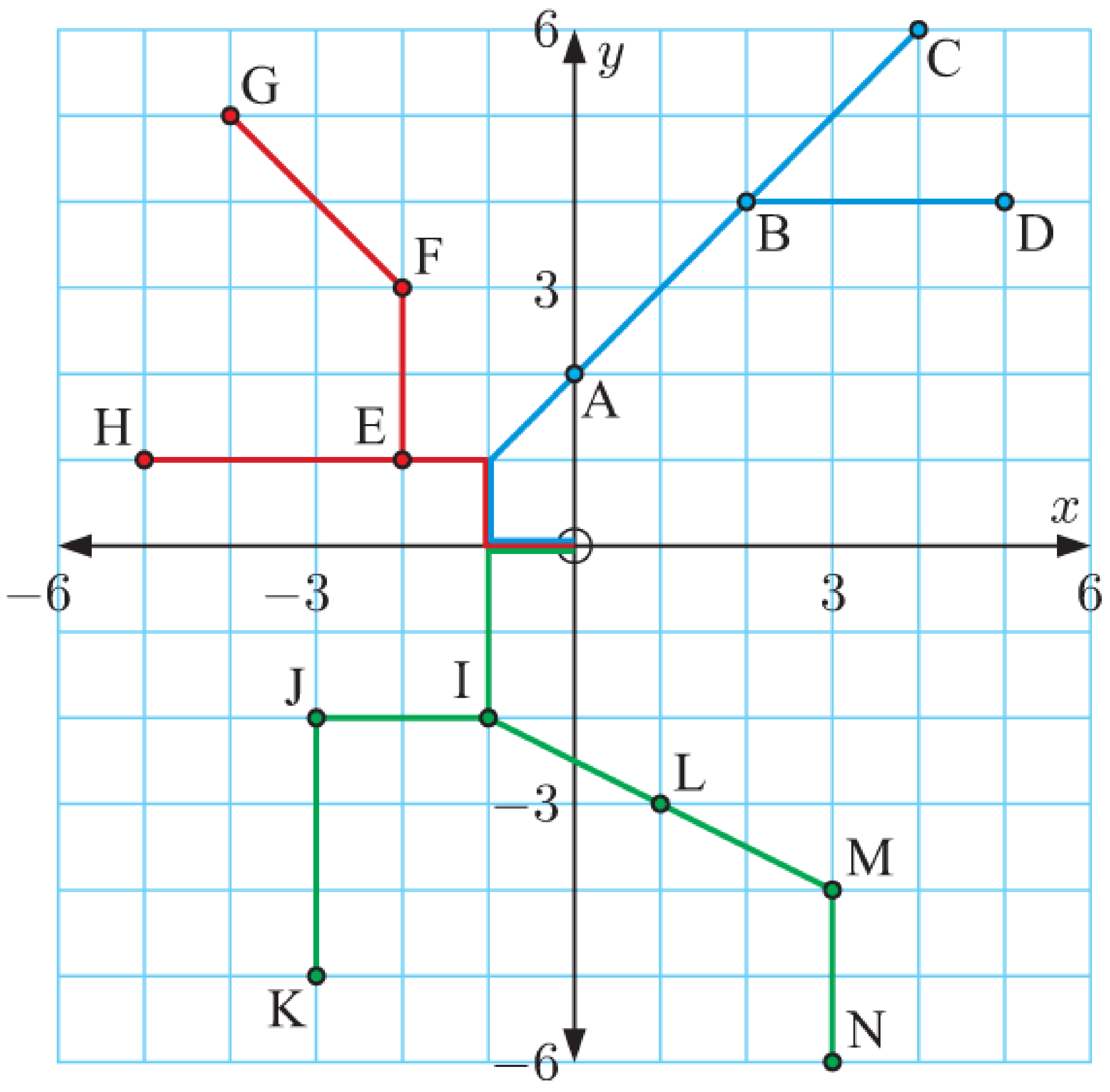
b In which quadrant is Dianne's house?

c Who lives closest to the pizza shop?



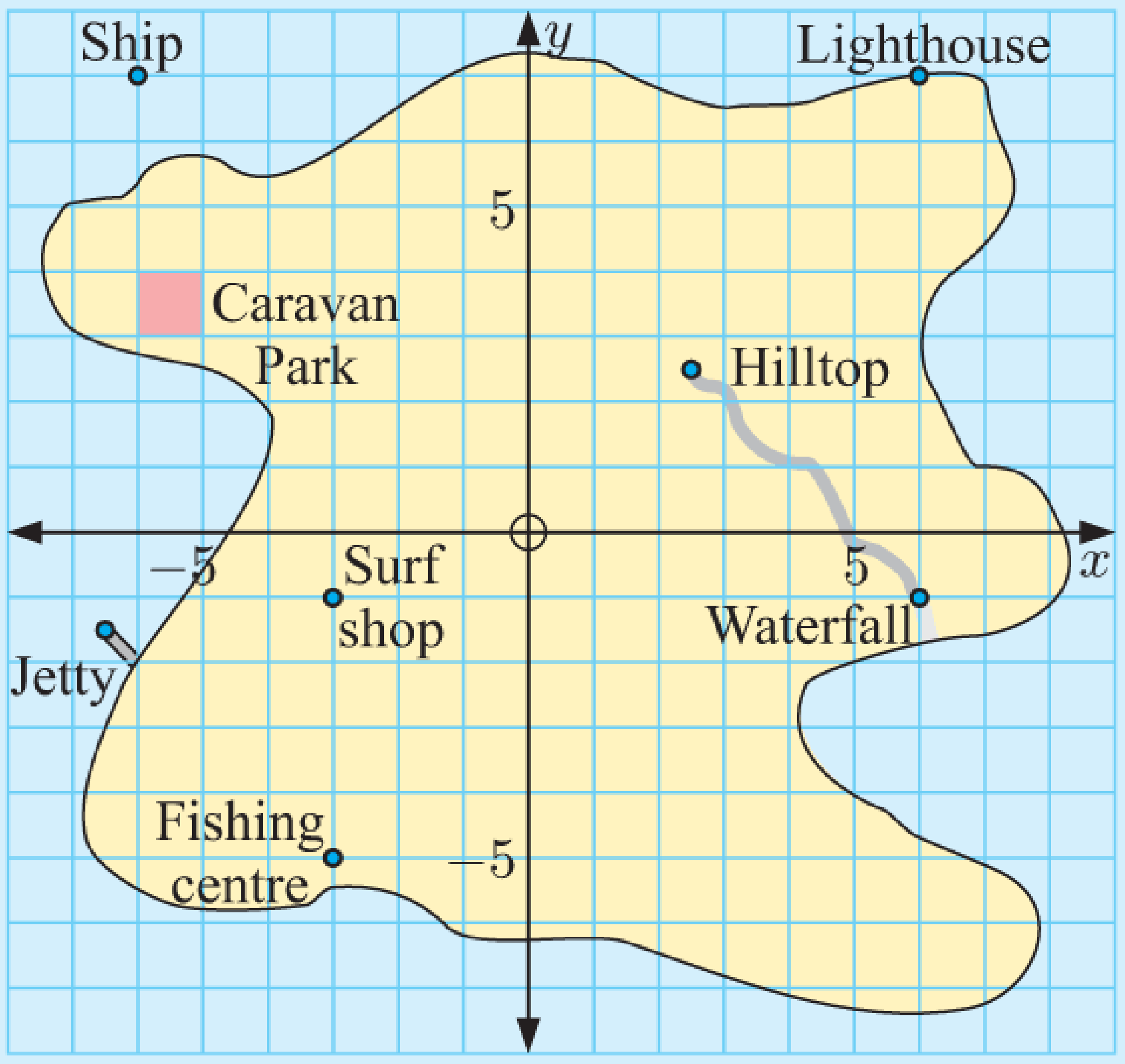
5 The grid alongside shows the train stations of a city, with the main station at the origin.

- **a** Find the coordinates of:
  - **i** station F
  - **ii** station L.
- **b** How many stations are in the fourth quadrant?
- **c** Tory lives at $(-4,\ -2)$. Which station is she closest to?



Example 6

Self Tutor



The map alongside has a scale of "1 grid unit represents 1 km".

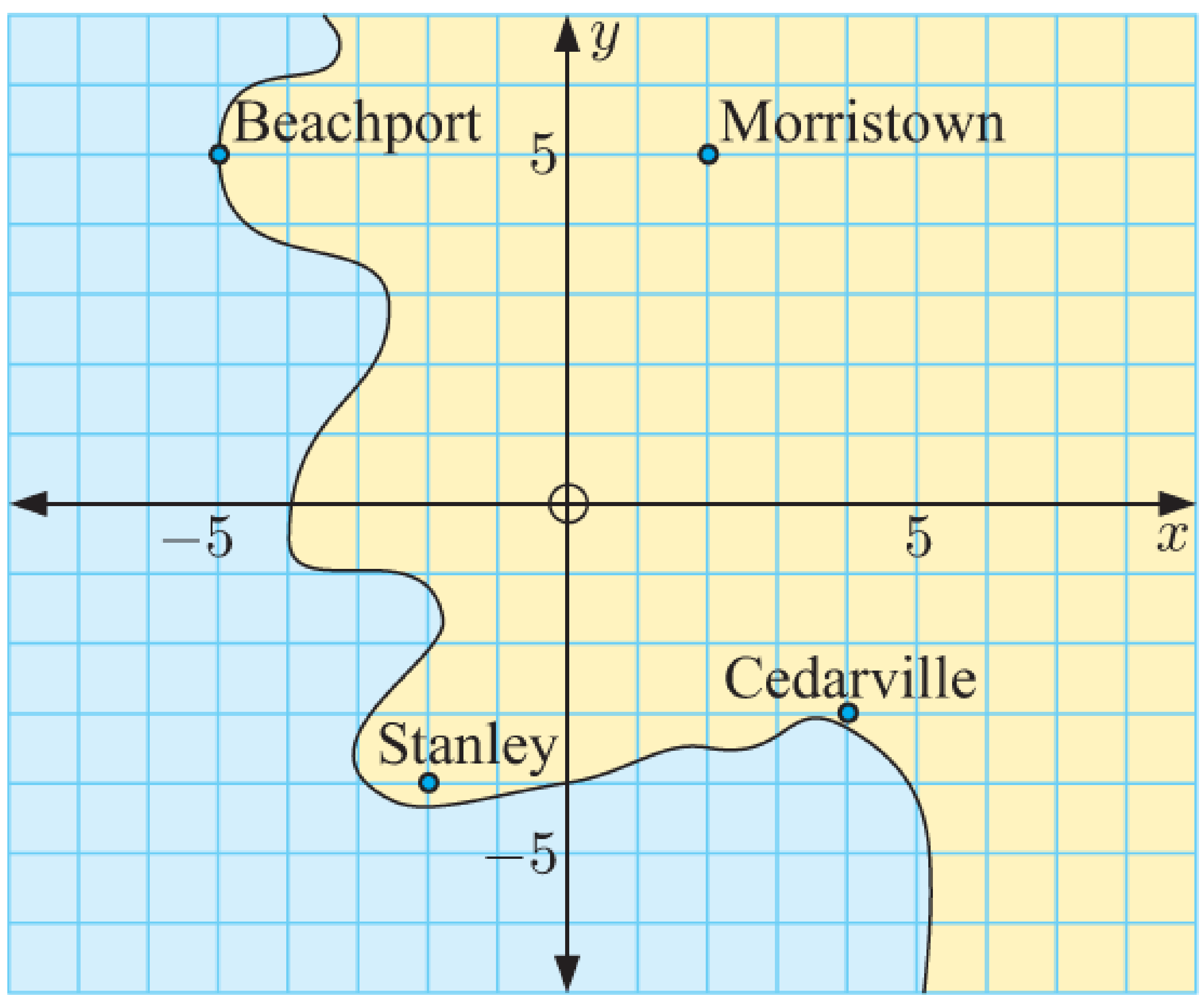
- **a** Find the actual distance between the surf shop and the fishing centre.
- **b** Monique is at $(3,\ -1)$.
  Find the actual distance between Monique and the waterfall.

The scale allows us to find actual distances represented on the map.

- **a** The surf shop and the fishing centre are 4 grid units apart.
  ∴ the actual distance between the surf shop and the fishing centre is 4 km.
- **b** $(3,\ -1)$ is 3 grid units away from the waterfall.
  ∴ the actual distance between Monique and the waterfall is 3 km.

6 The map alongside has a scale of "1 grid unit represents 10 km".

- **a** Find the actual distance between Beachport and Morristown.
- **b** Doug is currently at $(4,\ -1)$. How far is Doug from Cedarville?

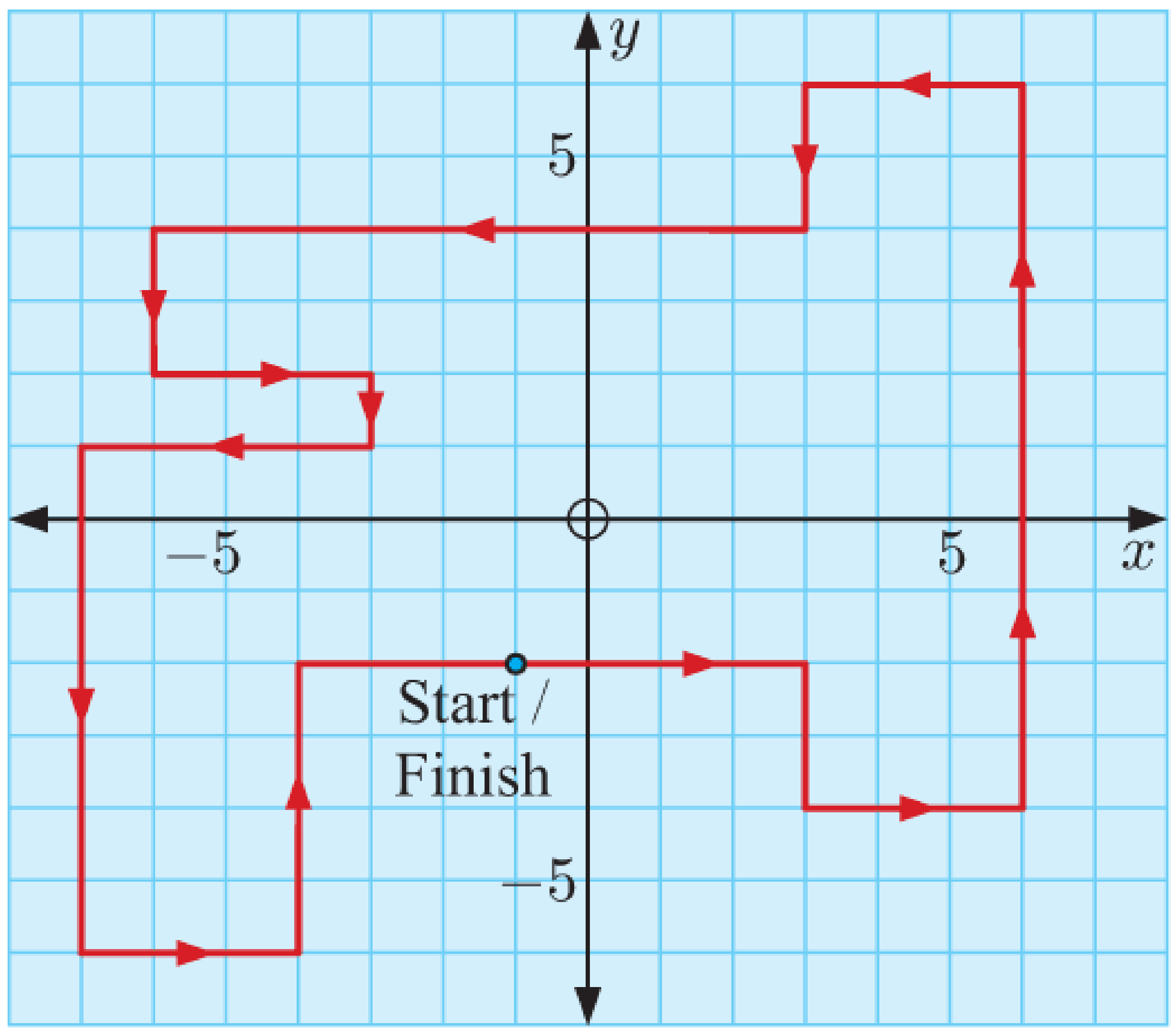


7

In this map of a recreational park, 1 grid unit represents 10 metres.

- **a** Find the coordinates of the entrance.
- **b** Find the actual distance between:
  - **i** the archery and the water skiing
  - **ii** the playground and the go-karts.
- **c** Sam is currently at $(-1, -6)$. How far is Sam from the kayaking?
- **d** Lucy is currently at $(6, 2)$.
  - **i** Is she closer to the mini-golf or the flying fox?
  - **ii** How far is she from the nearer attraction?

8



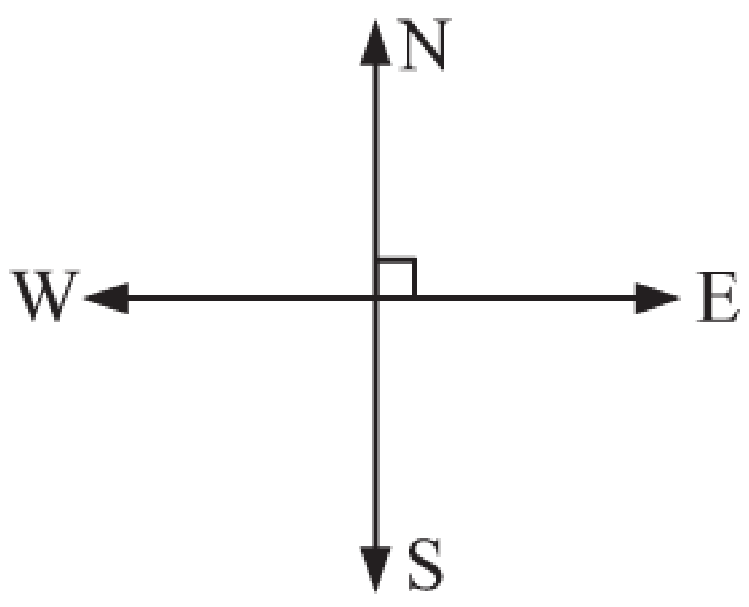
This map shows the course for a jetski competition. Each grid unit represents 100 metres.

- **a** Find the coordinates of the Start/Finish line.
- **b** Brad is on the course, and is currently at $(-7, -2)$.
  - **i** How far is he from the next corner?
  - **ii** In which direction must Brad turn at the next corner?
- **c** Find the total length of the course.

E DIRECTION

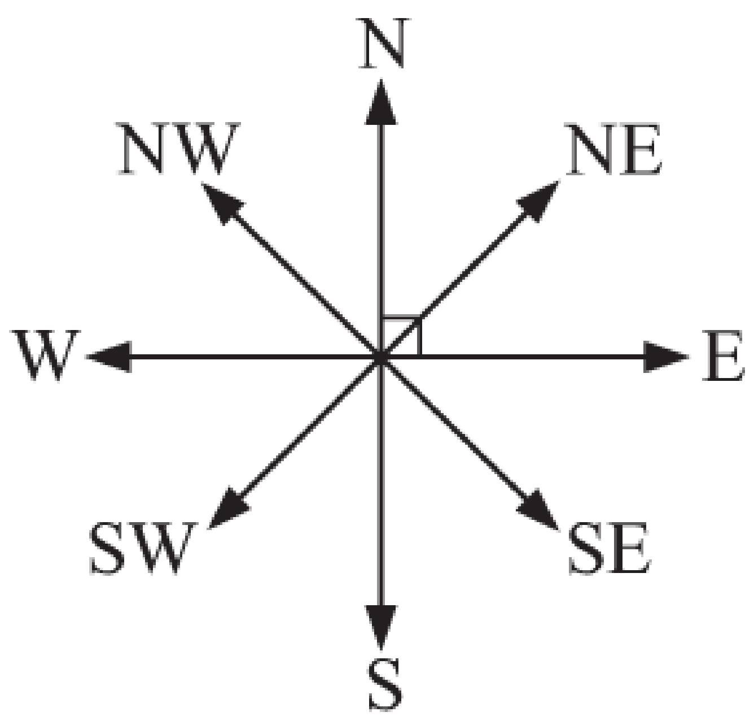
Maps and scales are useful for locating places, but they only work if we know which way up to hold the map. We therefore need some **directions**.

The most common method of giving directions is based on the four main **compass points**: **north**, **south**, **east**, and **west**.



These four main directions are often called the **cardinal points**.

Halfway between these points we have the directions **north-east**, **south-east**, **south-west**, and **north-west**.



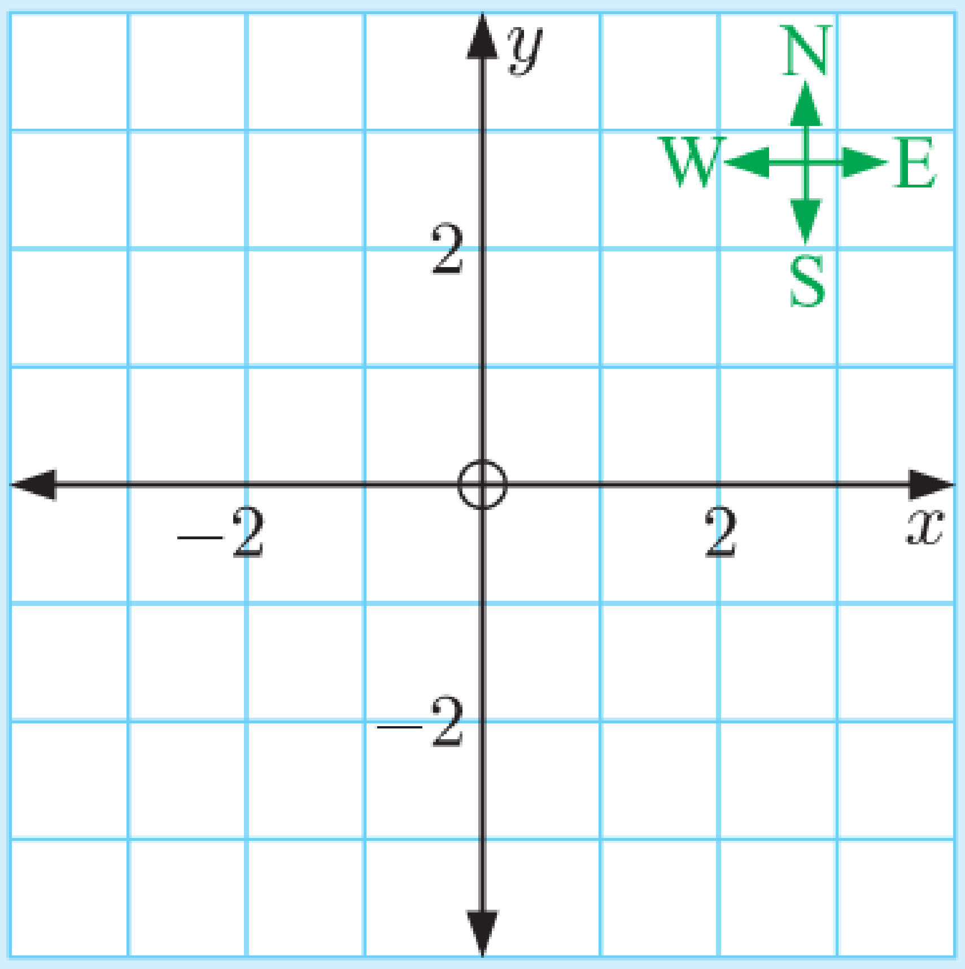
For example, south-west (SW) is halfway between south and west.

Example 7 **Self Tutor**

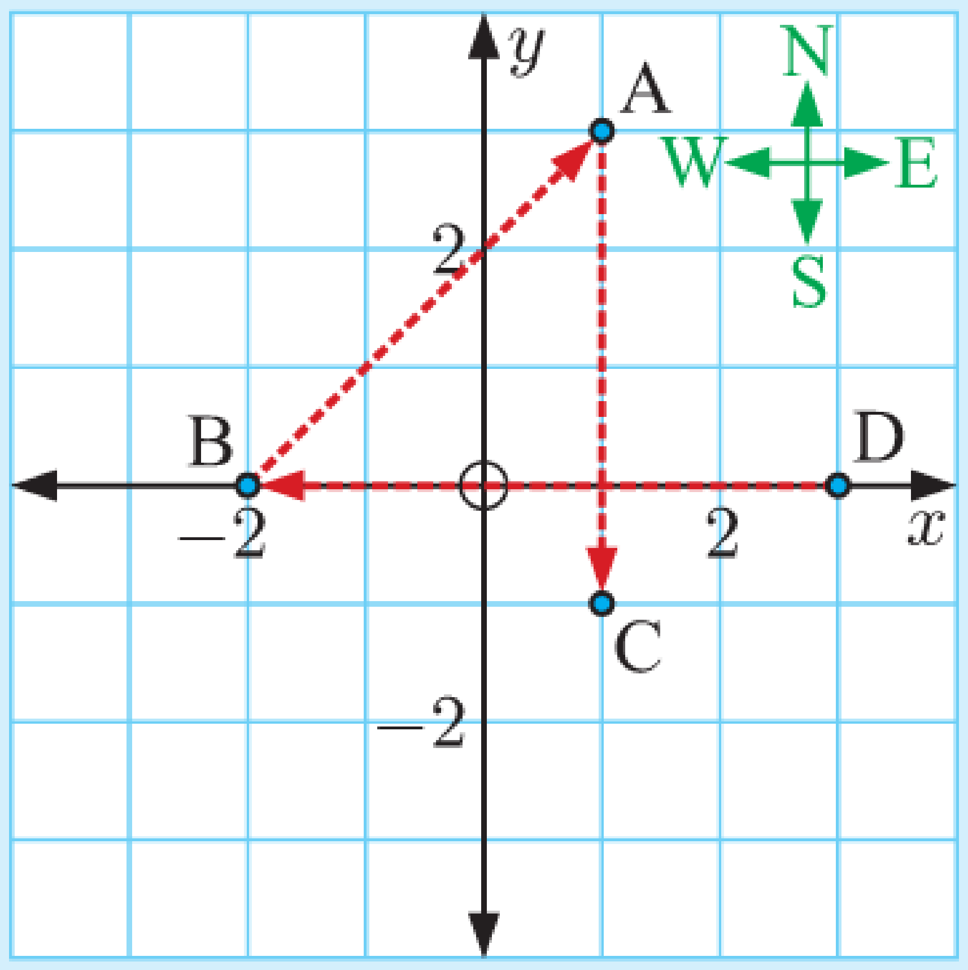
a Plot the points A(1, 3), B(−2, 0), C(1, −1), and D(3, 0) on a set of axes like the one alongside.

b In which direction must we travel to go from:

i A to C **ii** D to B **iii** B to A?



a



b **i** To go from A to C, we travel south.

ii To go from D to B, we travel west.

iii To go from B to A, we travel north-east.

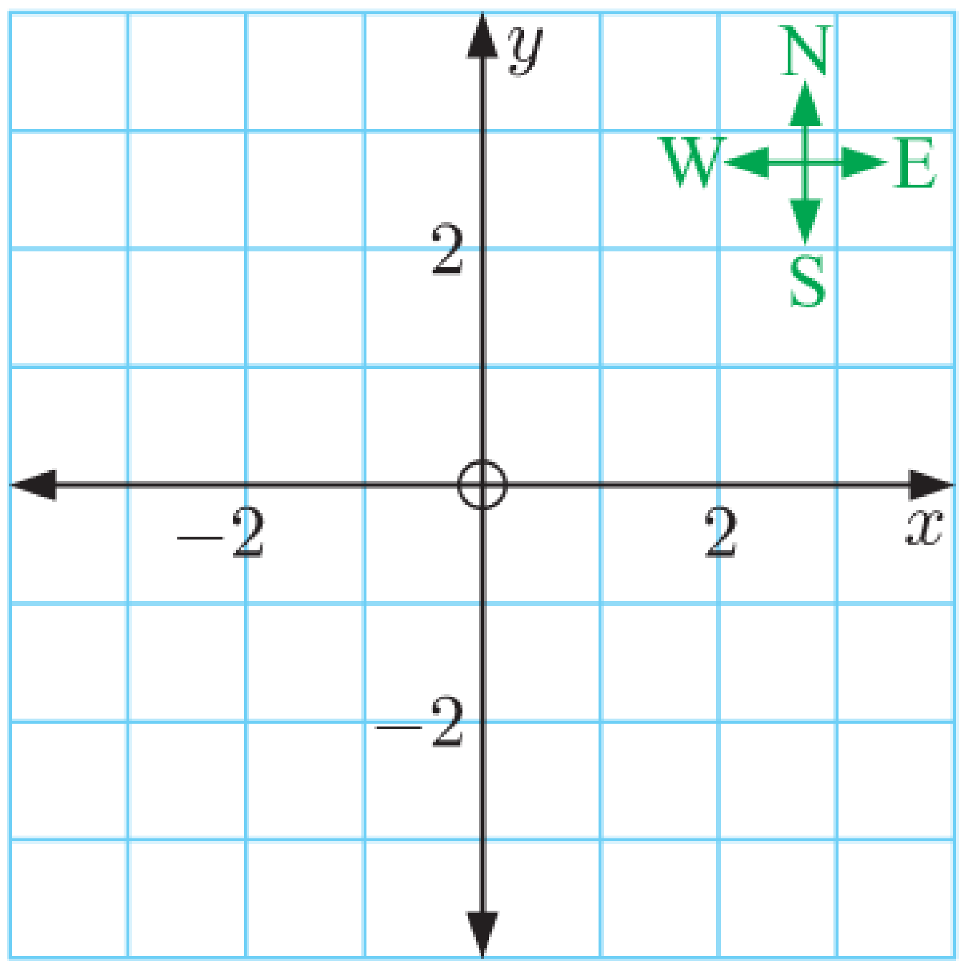
EXERCISE 13E

1 **a** Plot the points A(3, 2), B(−2, 4), C(−1, 2), and D(−2, −3) on a set of axes like the one alongside.

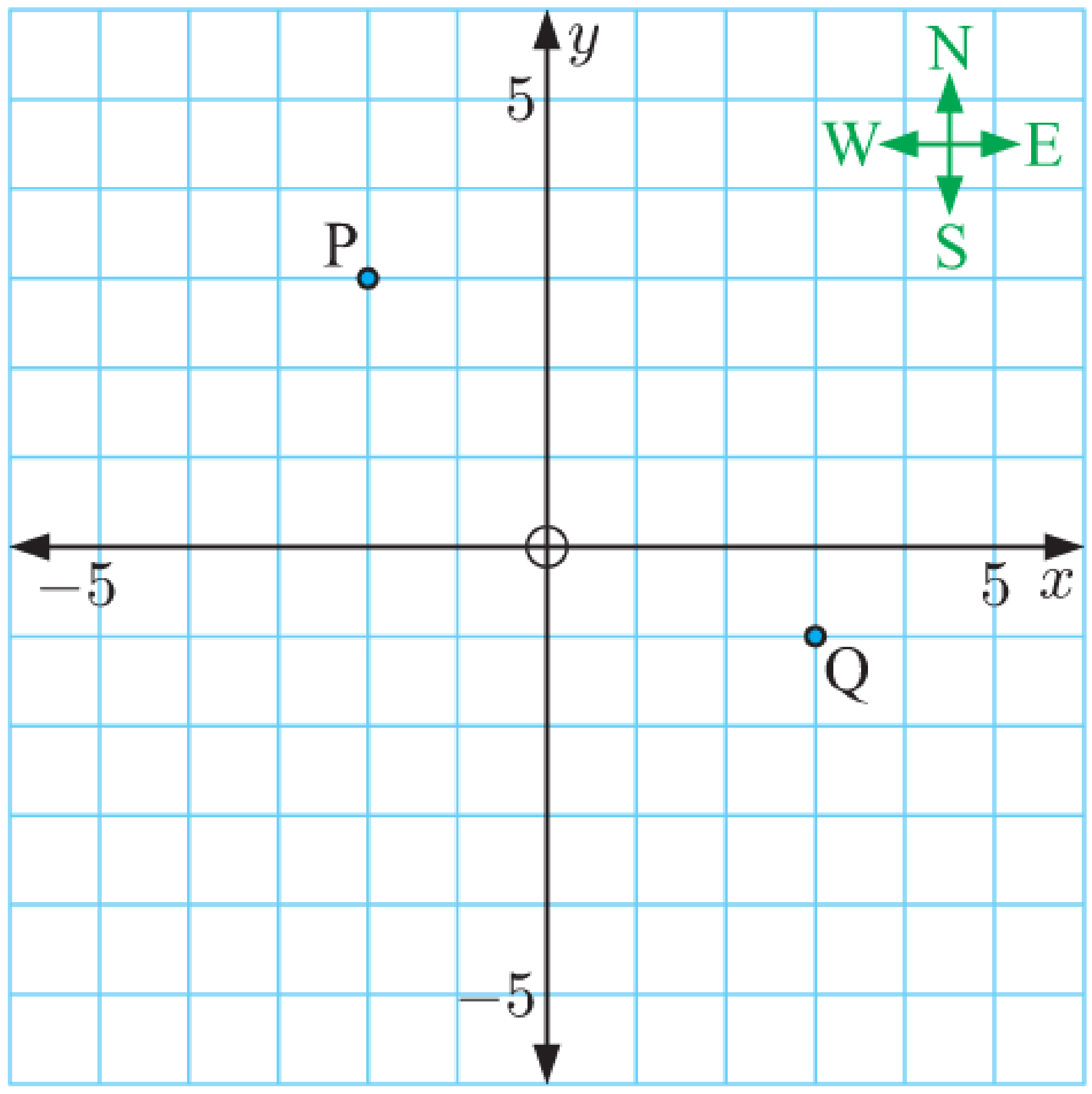
b In which direction must we travel to go from:

i D to B **ii** C to A **iii** A to D?

PRINTABLE GRIDS



2



a Write down the coordinates of P and Q.

b Which of these points is south of P?

A	(−4, 3)	**B**	(−2, 6)
C	(−2, −1)	**D**	(3, 3)

c Which of these points is west of Q?

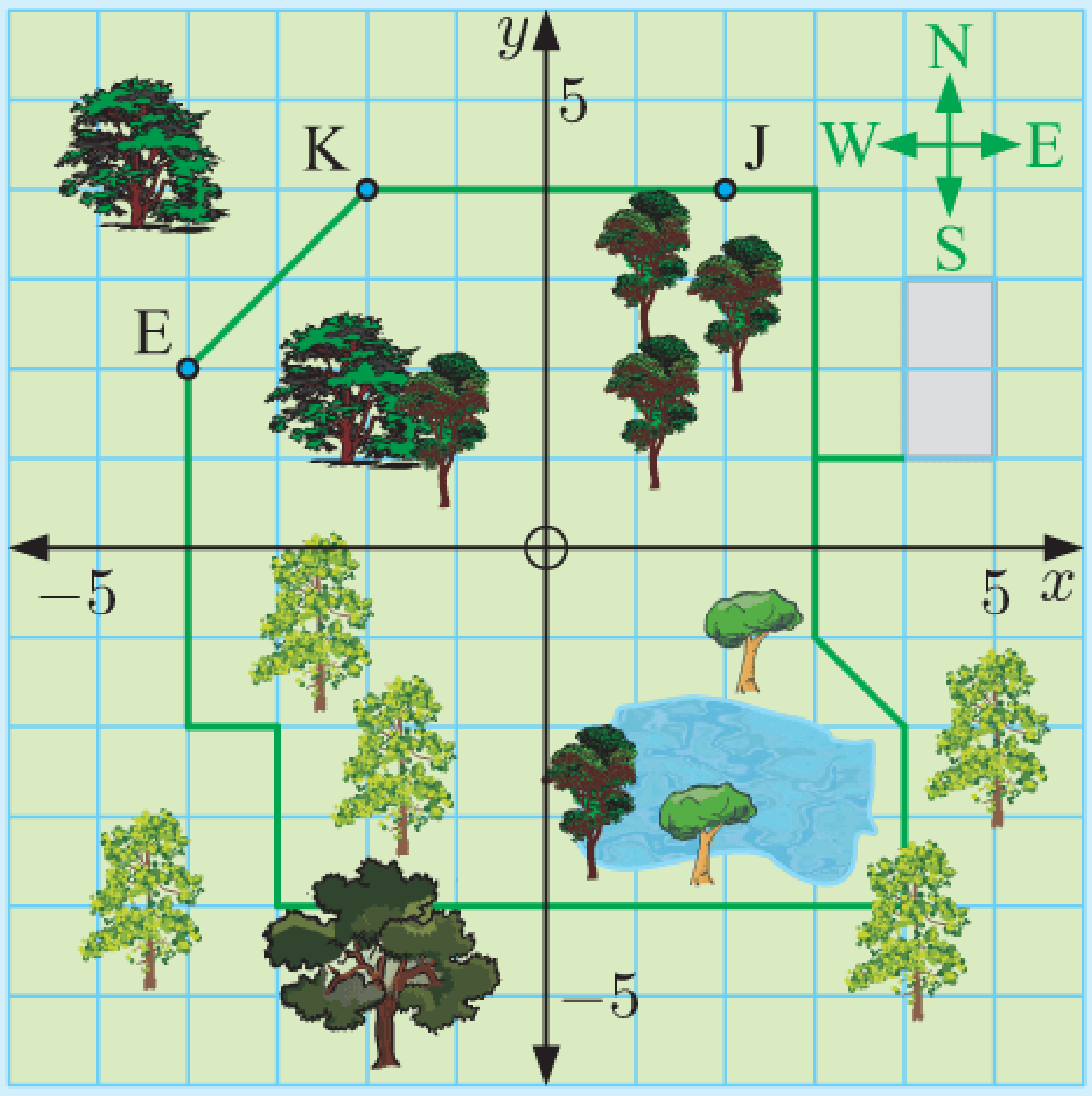
A	(5, −1)	**B**	(3, −4)
C	(3, 2)	**D**	(−4, −1)

d Which of these points is north-west of Q?

A	(3, 2)	**B**	(2, −2)
C	(0, 2)	**D**	(5, 1)

Example 8 **Self Tutor**

This map shows a walking trail. Each grid unit represents 100 m.



John is at $(2, 4)$, Kyle is at $(-2, 4)$, and Emma is at $(-4, 2)$.

a Find the actual distance between John and Kyle.

b In what direction is Emma from Kyle?

c Meg is 400 m south of Emma.

  i Find the coordinates of Meg.

  ii In what direction is John from Meg?

a On the map, John and Kyle are 4 grid units apart.

$\therefore$ the actual distance between John and Kyle is 400 m.

b Emma is south-west of Kyle.

c **i** Meg is 400 m south of Emma.

$\therefore$ on the map, Meg is 4 grid units south of Emma.

$\therefore$ Meg is at $(-4, -2)$.

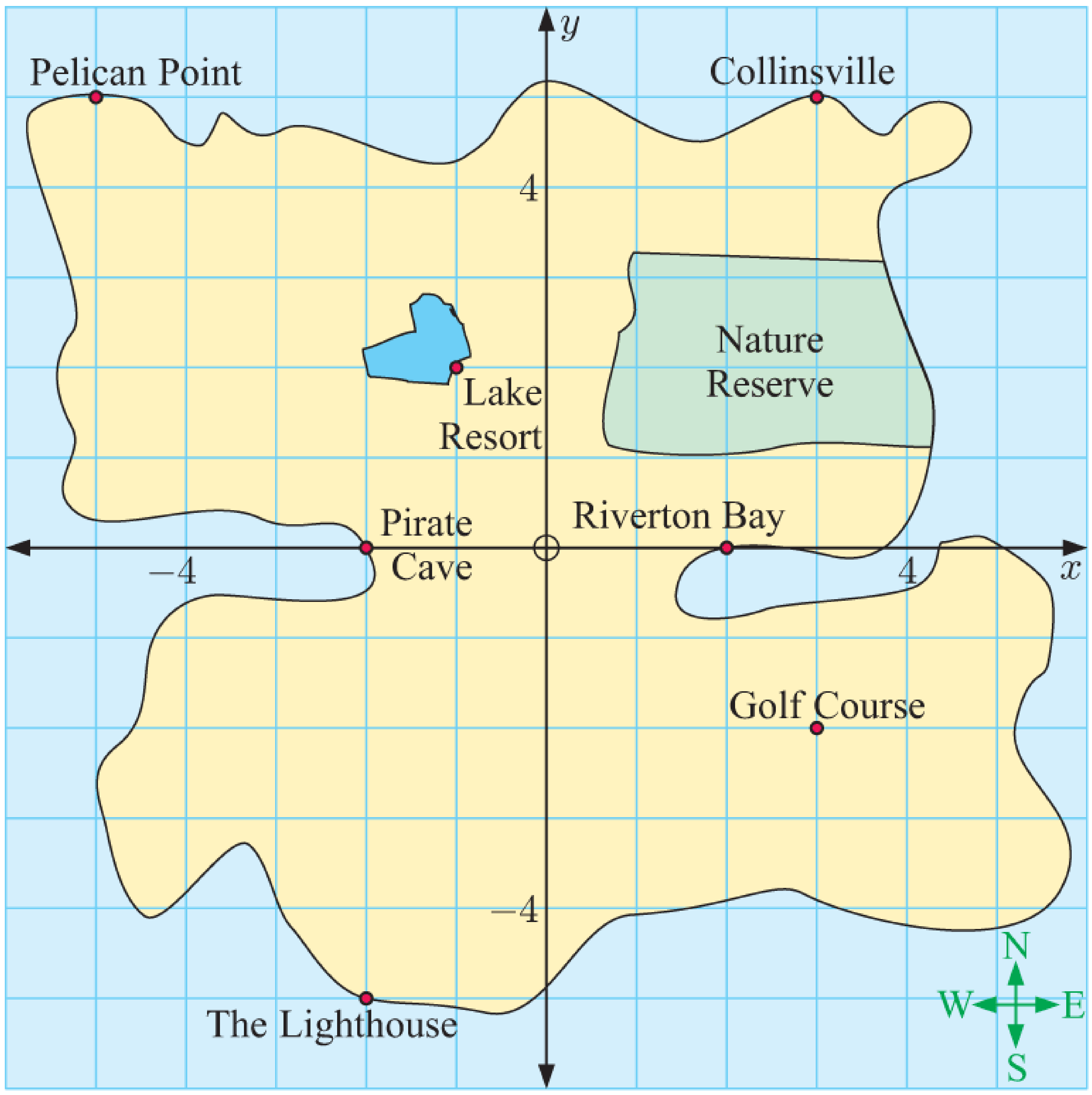
ii John is north-east from Meg.

3 On this map of an island, 1 grid unit represents 1 km.

a Find the distance between Pelican Point and Collinsville.

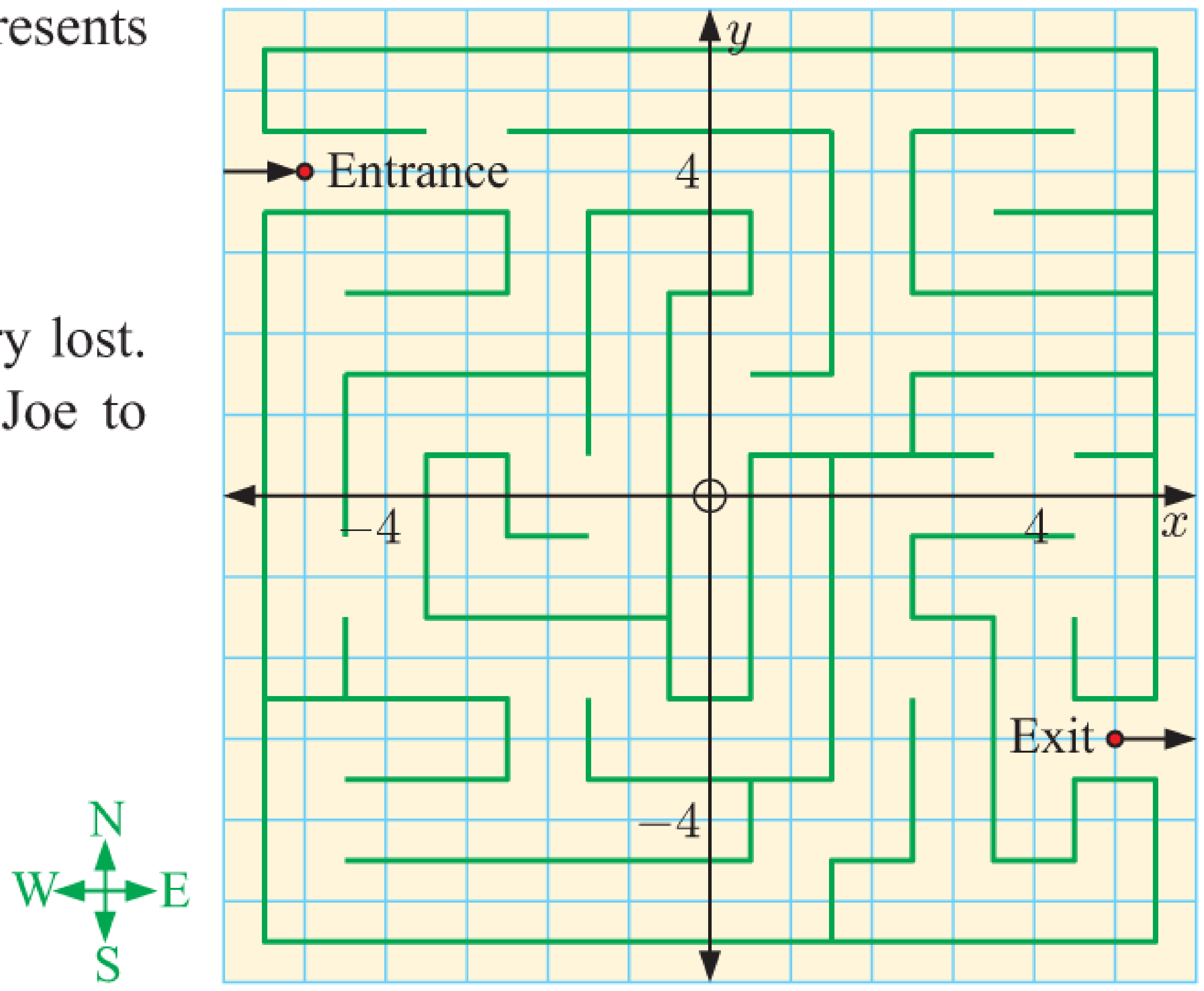
b In what direction is Pirate Cave from the lighthouse?

c There is a windmill 3 km west of the golf course. Find the coordinates of the windmill.



4 In this map of a maze, each grid unit represents 2 metres.

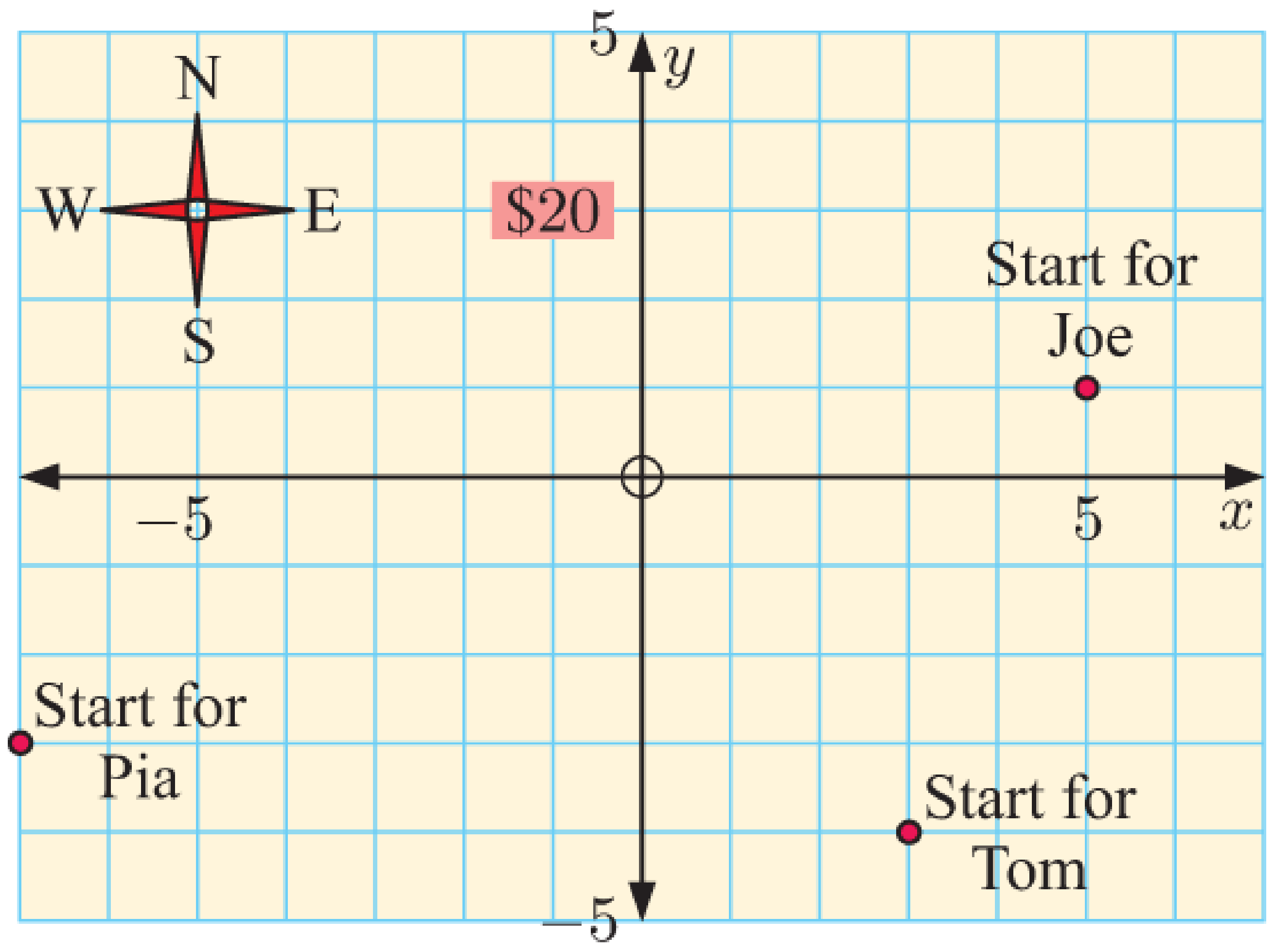
- **a** Find the coordinates of the:
  - **i** entrance
  - **ii** exit.
- **b** Joe is currently at $(-3, -3)$, and is very lost. Use directions and distances to guide Joe to the exit.



5 A \$20 note is hidden at the point shown. Tom, Pia, and Joe are each given some directions to help find the note.

For example, E30 means go east 30 metres.

Tom: N60, W50, S20, E10, N20
Pia: E30, N70, E30, S10
Joe: W70, N30, E20, S10, E10

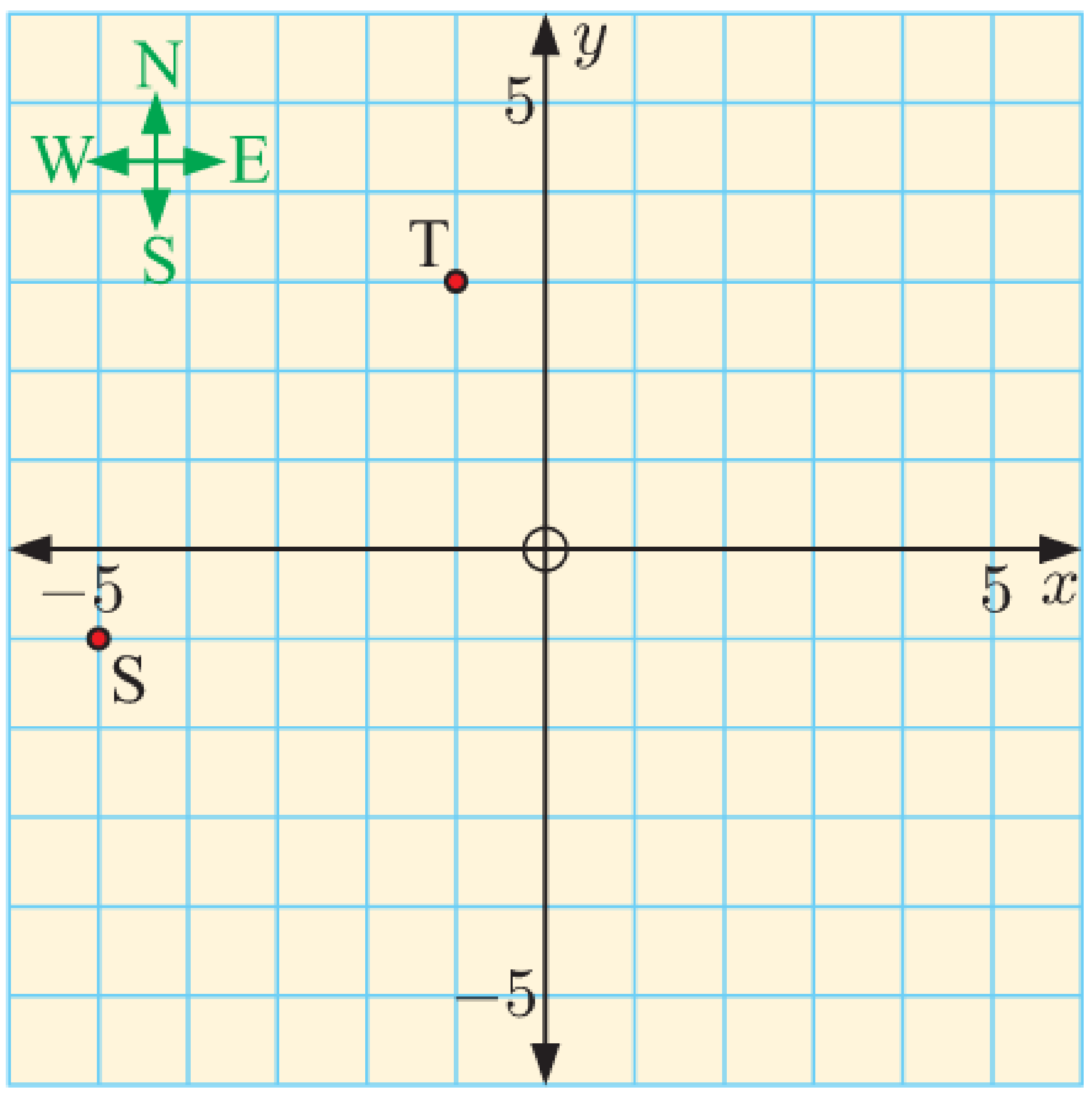


Each grid unit is 10 m

- **a** Whose directions to the \$20 note are correct?
- **b** Find the coordinates of Joe's finishing point.
- **c** Would Pia have found the note if she had walked exactly north-east from her starting point?
- **d** Would Tom have found the note if he had walked exactly north-west from his starting point?

6 Erin goes on a five day hike, starting at S. Each grid unit represents 2 kilometres.

- **a** Erin first hikes to T. In what direction does she hike?
- **b** On the second day, Erin hikes 12 kilometres east to U. Copy the grid, and mark U on it.
- **c** On the third day, Erin hikes in a south-westerly direction until she is at point V, south-east of S.
  - **i** Mark this point V on the grid.
  - **ii** What are the coordinates of V?
- **d** On the fourth day, Erin hikes 6 kilometres west to W. Mark W on the grid.
- **e** How far, and in what direction, does Erin need to hike to return to her starting position?

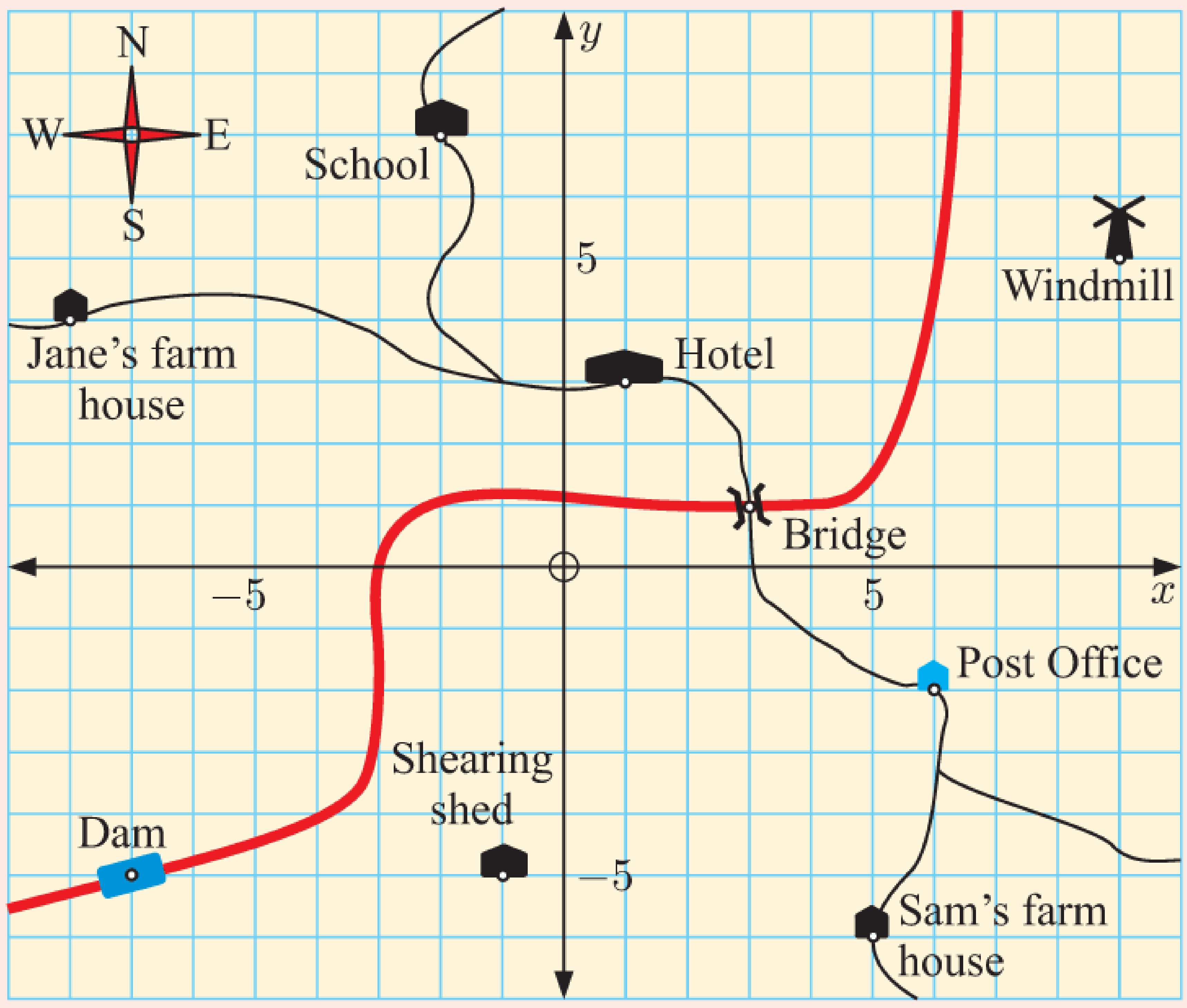


PRINTABLE GRIDS

ACTIVITY TREASURE HUNTS

What to do:

1 A wealthy farmer once owned all the land shown in the map. Fifty years later, his grandchildren Jane and Sam discovered a map and a letter he had left them. The letter told them about a fortune in gold and silver coins, hidden on the property.



Scale: 1 grid unit represents 200 m

Two sets of instructions were needed to find the exact location of the coins.

Sam's instructions: Start at your farm, travel 600 m west, then 2 km north. The treasure is now south-east of you.

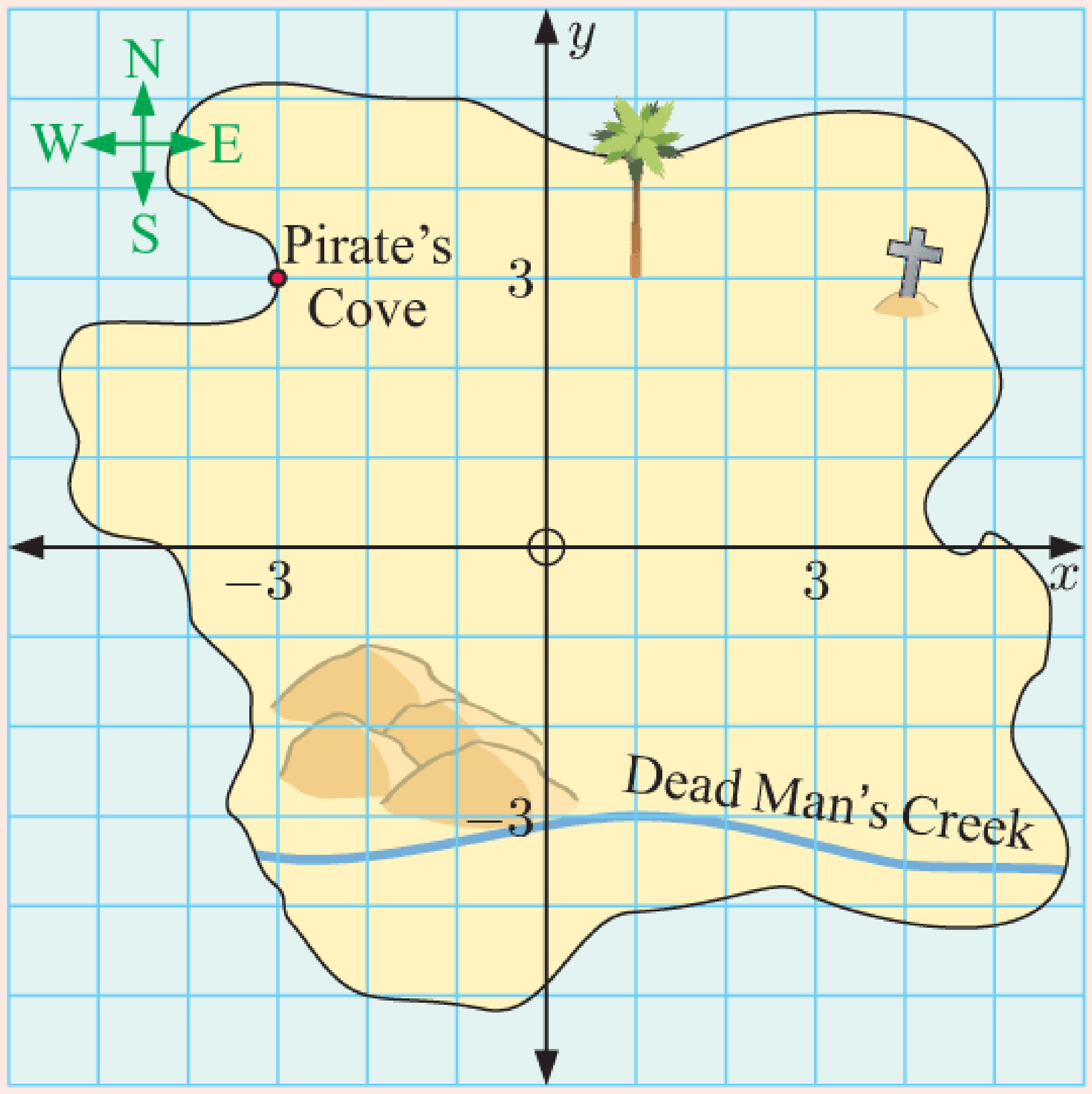
Jane's instructions: Start at your farm, travel 2 km south, then 2.2 km east. The treasure is now north-east of you.

Where did Sam and Jane find the coins?

2 You have found a treasure map and a set of clues for finding the treasure:

- From Pirate's Cove, head south for 100 paces, then south-east until you reach Dead Man's Creek.
- Face the lone palm tree, and walk 200 paces.
- Walk north-east until you are directly west of the large cross.
- The treasure is hidden 250 paces south of here.

Find the coordinates of the hidden treasure.



Scale: 1 grid unit represents 50 paces

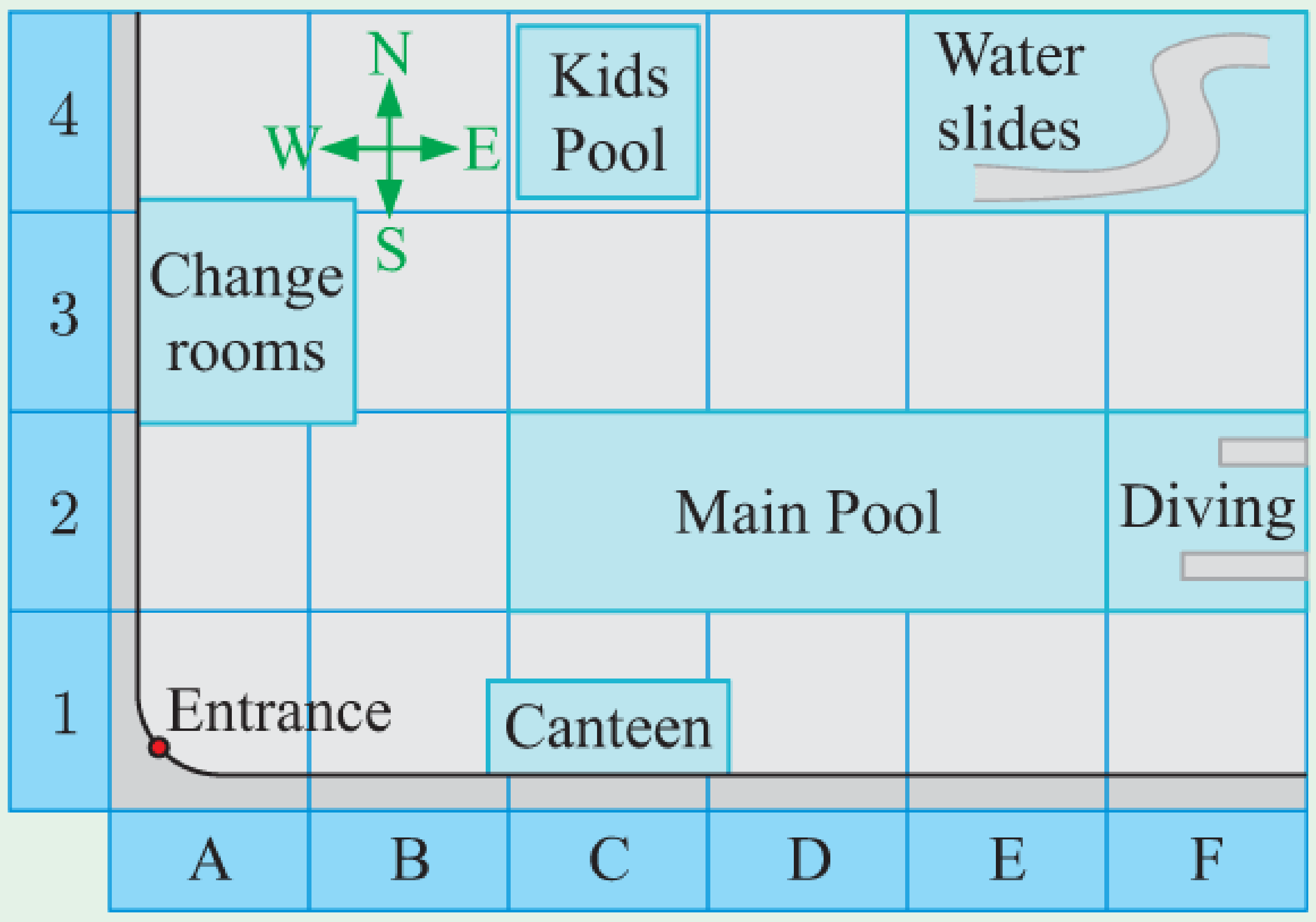
3 Draw a map of your own on grid paper. Write a set of instructions for finding a hidden treasure. Do not forget to specify a starting point. Exchange maps with a friend, and try to find each other's treasure.

KEY WORDS USED IN THIS CHAPTER

- axes
- coordinates
- east
- grid lines
- map reference
- north
- origin
- quadrant
- scale
- south
- west
- x-axis
- x-coordinate
- y-axis
- y-coordinate

REVIEW SET 13A

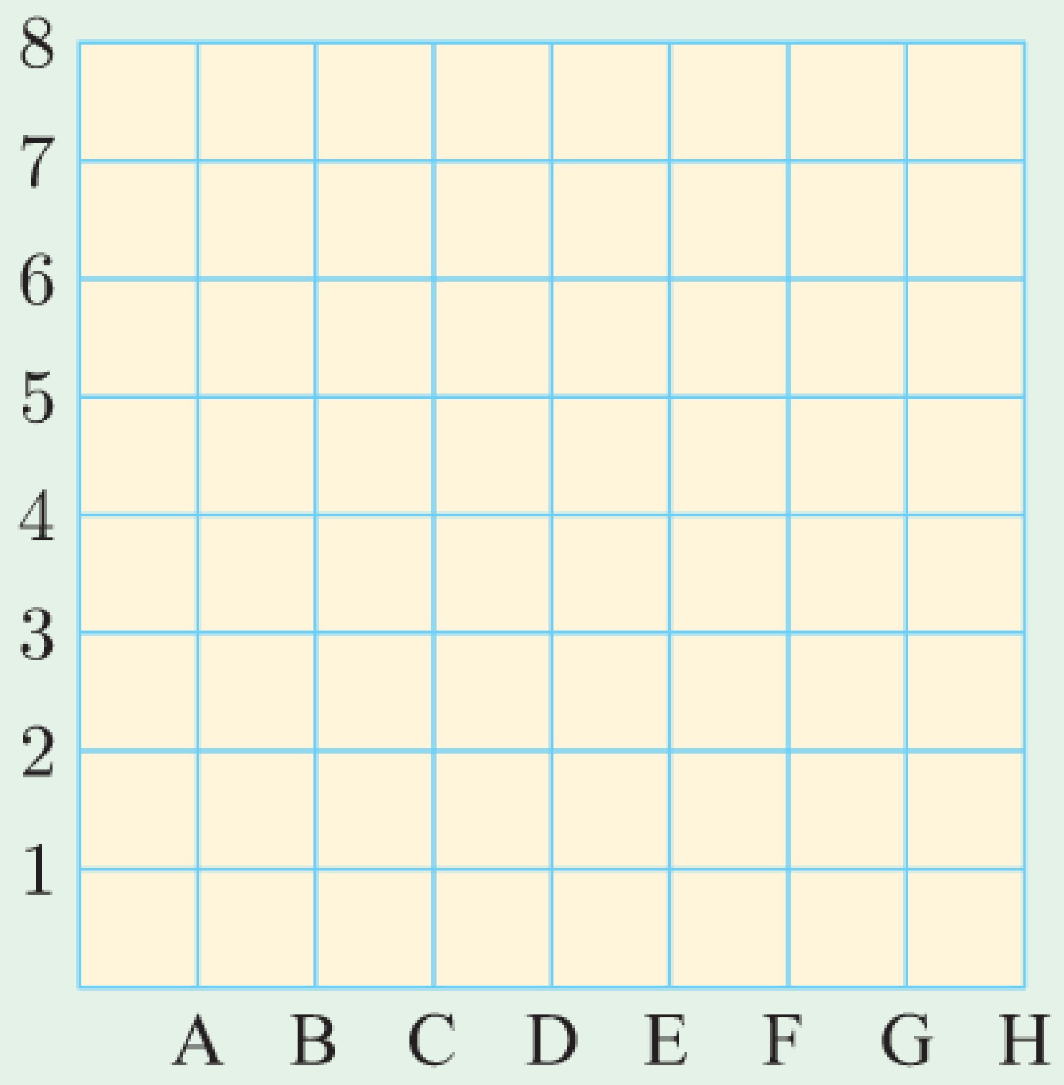
1 Look at this map of a swimming centre.



- **a** What feature is located at A3?
- **b** How many grid squares does the main pool cover?
- **c** What is the map reference for the:
  - **i** diving area
  - **ii** canteen?
- **d** In which direction are the:
  - **i** water slides from the kids pool
  - **ii** change rooms from the canteen?

2 Copy this grid, and mark these points on it:

a A4 **b** B5 **c** D1 **d** F3



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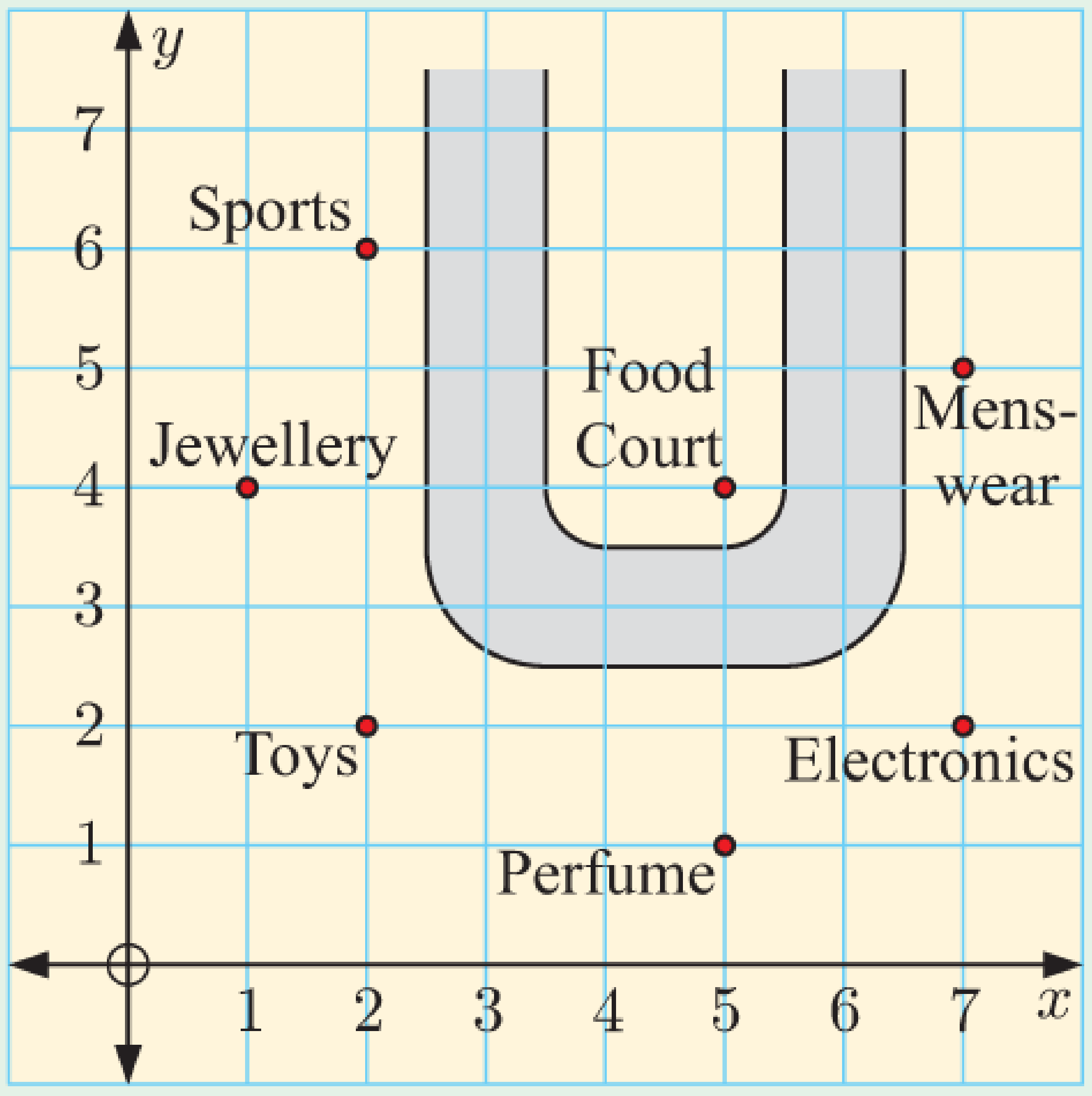
3 Draw a set of axes, then plot and label the following points:

a A(1, 5) **b** B(4, 6) **c** C(5, 2) **d** D(3, 0)

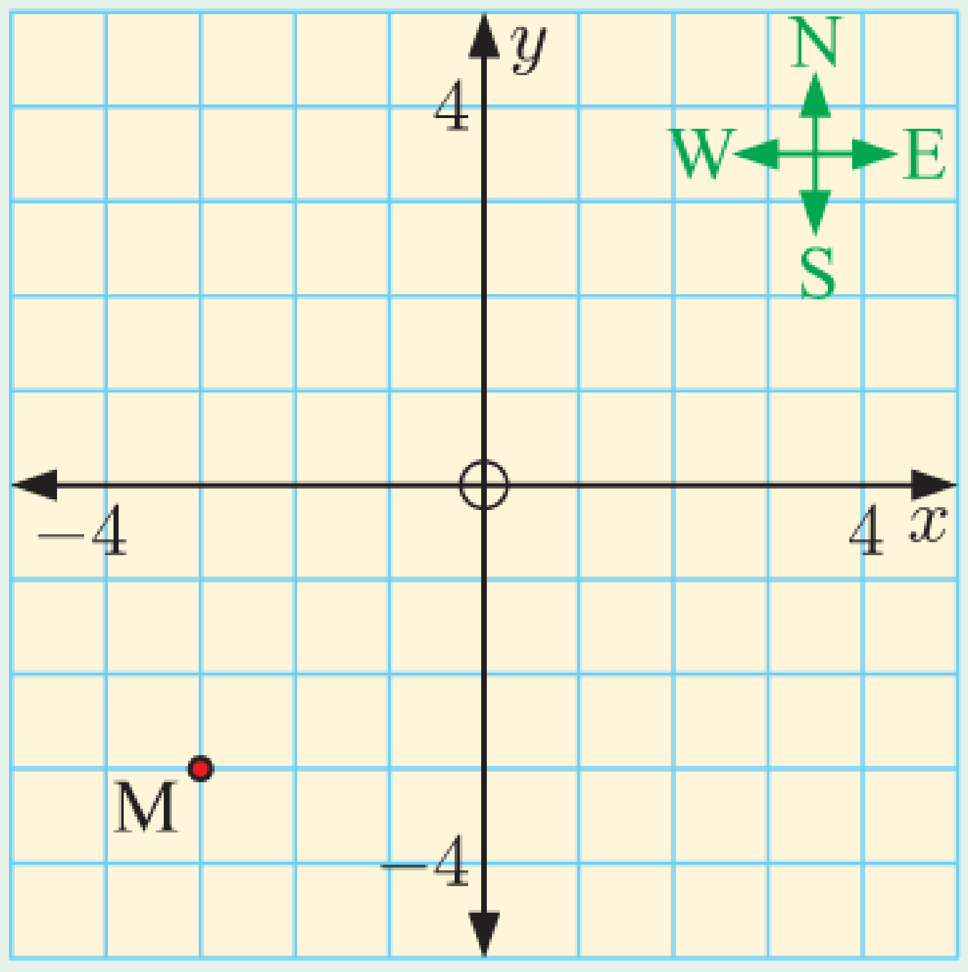
4



This map shows the various shops in a mall. Each grid unit represents 20 m.

- **a** Find the coordinates of:
  - **i** the jewellery store
  - **ii** the food court.
- **b** Which shop is located at:
  - **i** (5, 1)
  - **ii** (2, 6)?
- **c** How far is it from the Electronics store to Menswear?

5



a Write down the coordinates of M.

b Which of these points is east of M?

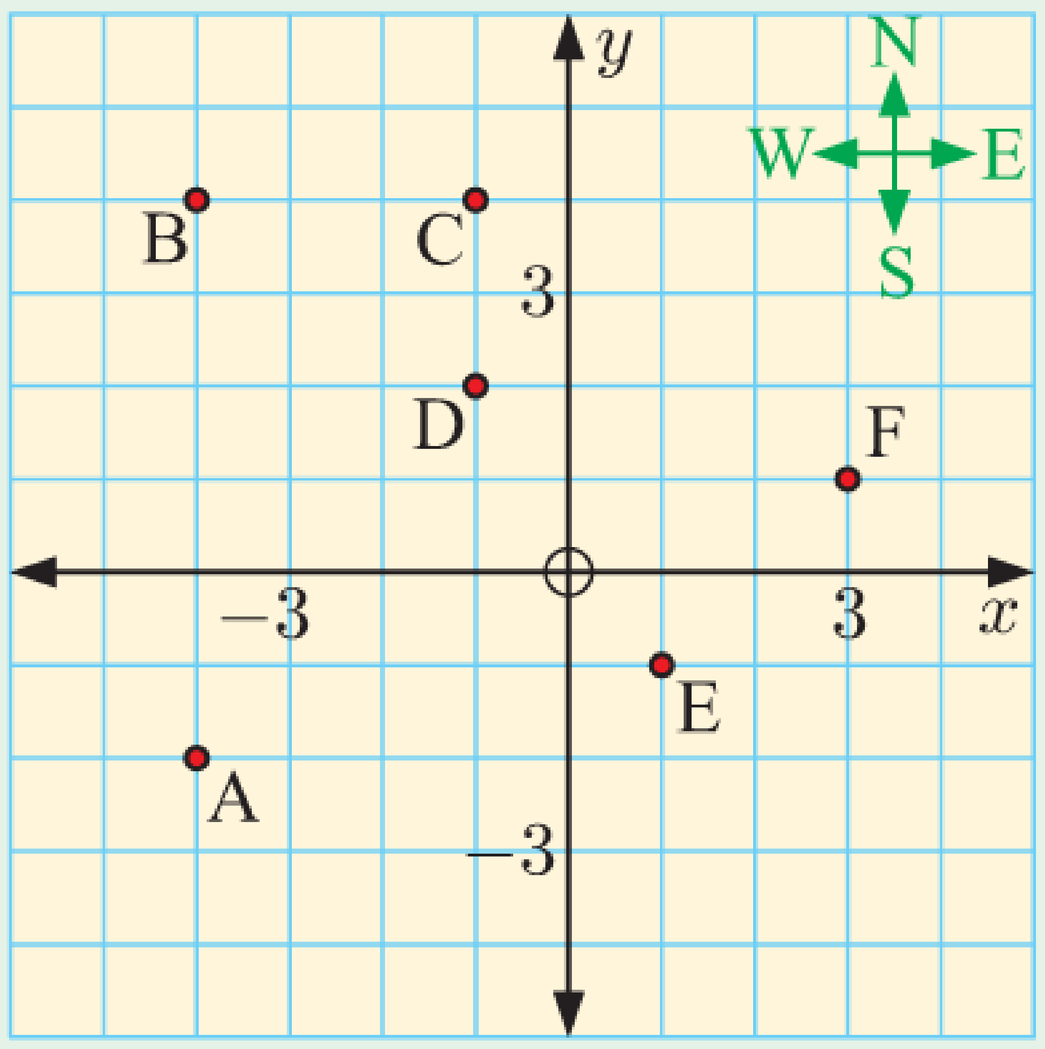
A	$(-3,\ 3)$	**B**	$(-1,\ -3)$
C	$(-5,\ -3)$	**D**	$(-3,\ -5)$

6 Consider the points alongside.

a Find the coordinates of:
  i A **ii** E.

b Name the point which has:
  i x-coordinate 3 **ii** y-coordinate 2.

c In which direction must we travel to go from:
  i C to B **ii** E to F?

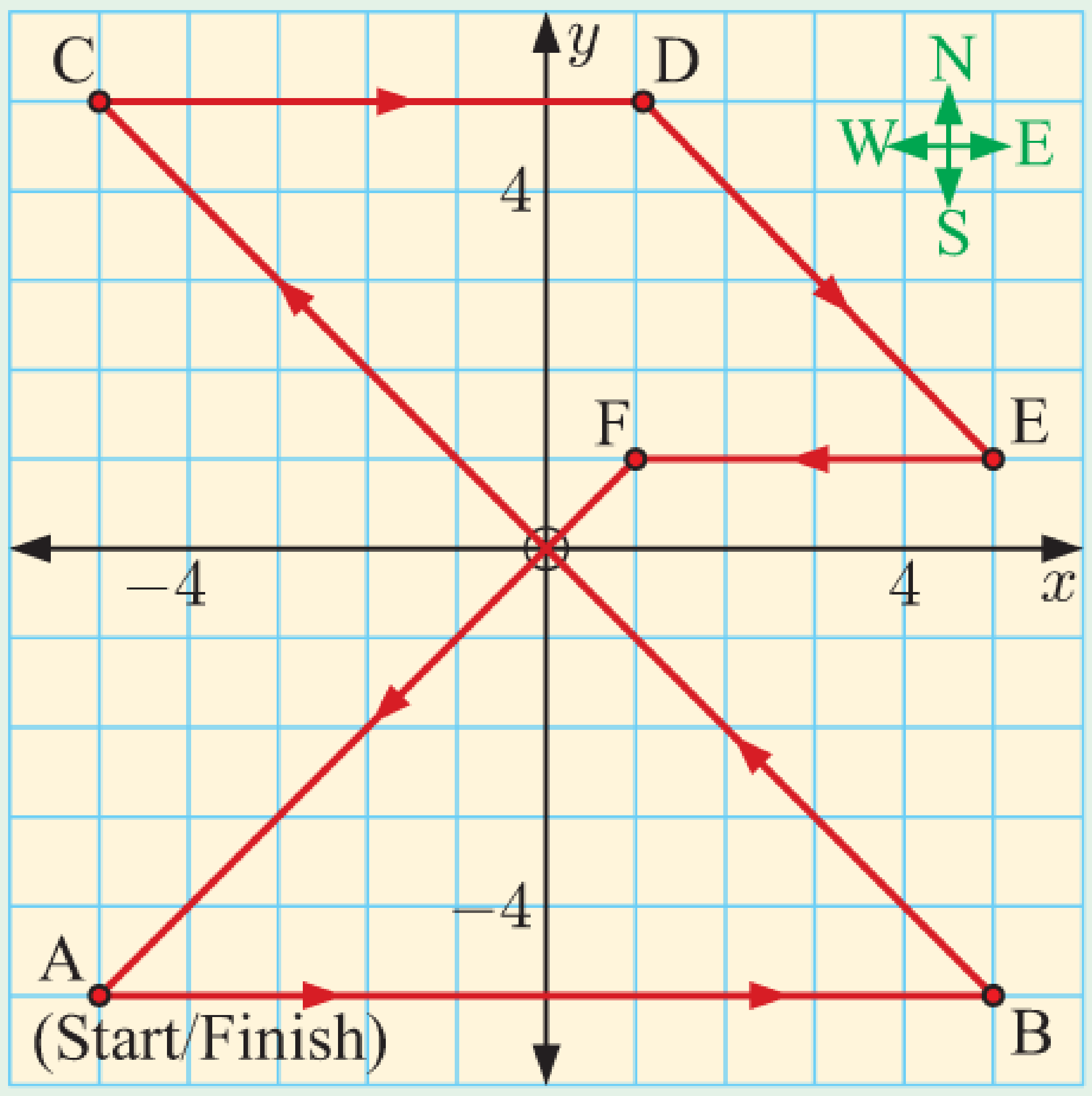


7 The map alongside shows the route for a yacht race. Each grid unit represents 100 metres.

a Find the coordinates of the Start/Finish point.

b Find the direction in which the yacht is travelling when it is sailing from:
  i B to C **ii** C to D
  iii D to E.

c The race referee is 200 metres north of the Start/Finish point. Find the coordinates of the referee.

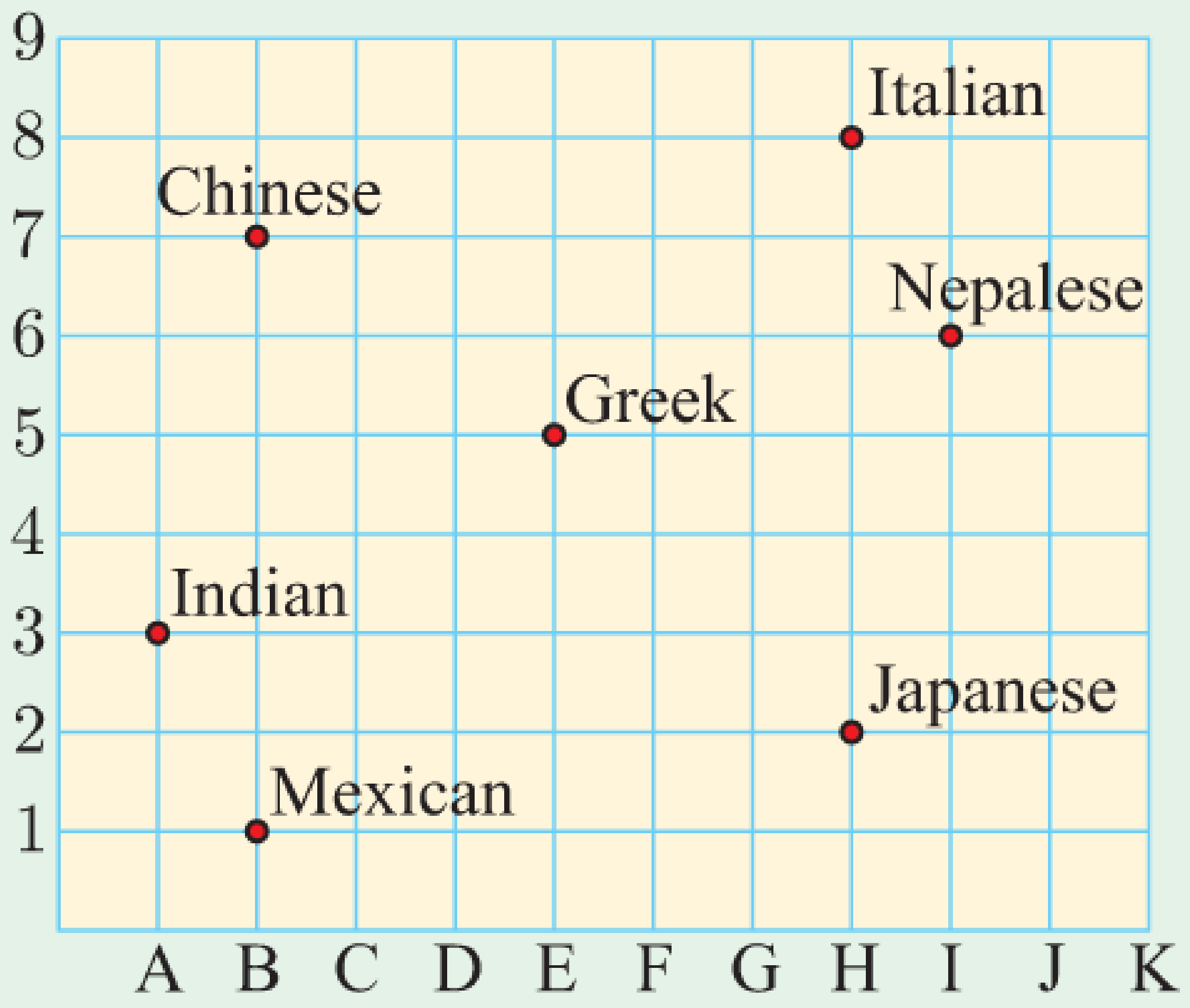


REVIEW SET 13B

1 This grid shows the positions of stalls at a Food Fair.

a Find the location of the:
  i Italian stall **ii** Chinese stall.

b Which stall is at:
  i E5 **ii** H2?



2 **a** Plot the points P(1, 4), Q(3, 5), R(4, 3), and S(2, 2) on a set of axes.
b Join [PQ], [QR], [RS], and [SP]. What shape is formed?

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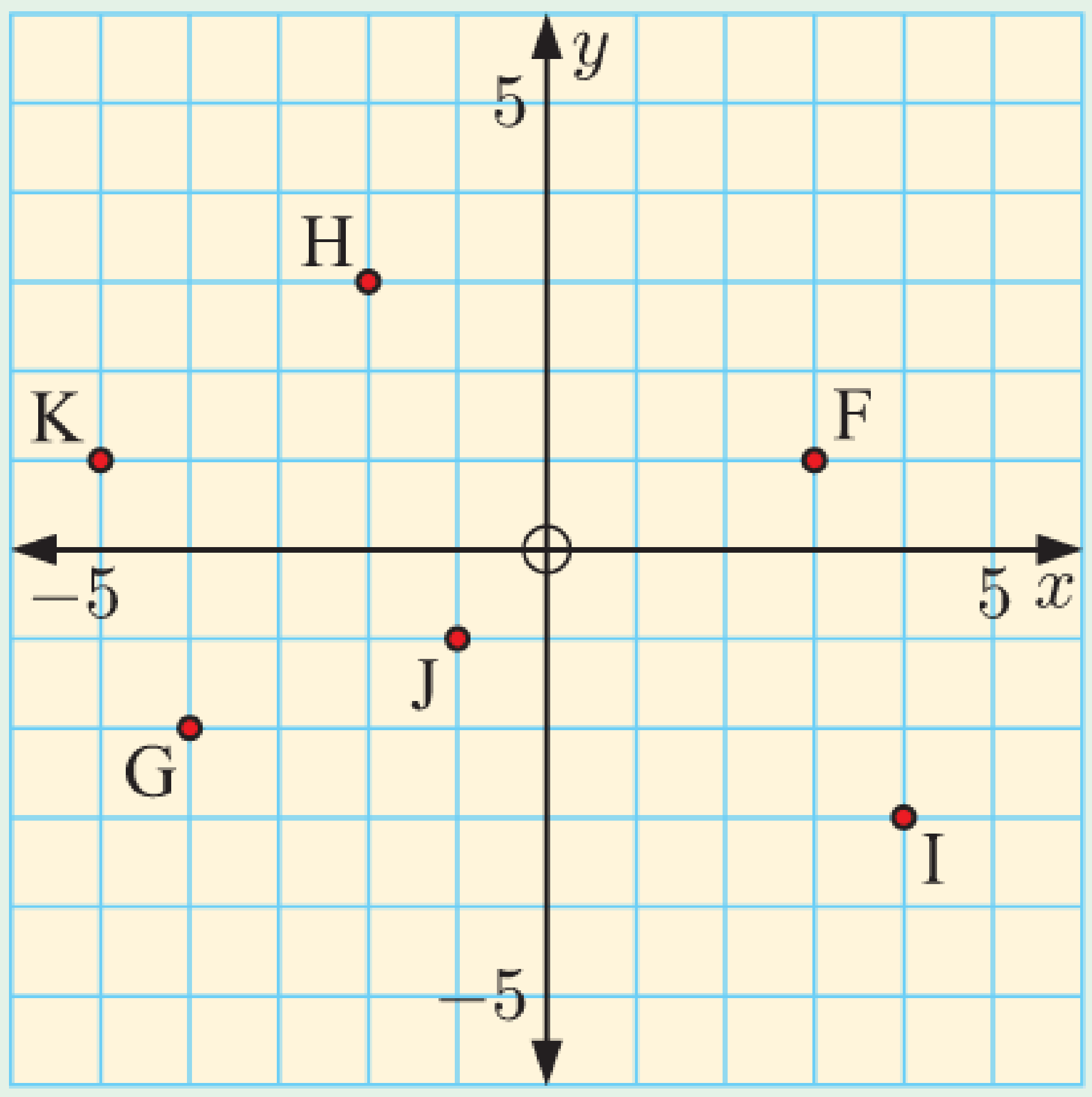
3 Draw a set of axes, then plot and label the points A(2, 3), B(−1, 5), C(3, −4), and D(−5, −3).

4 Consider the points on the axes shown.
a Write down the coordinates of each point.
b Name the quadrant in which each point lies.

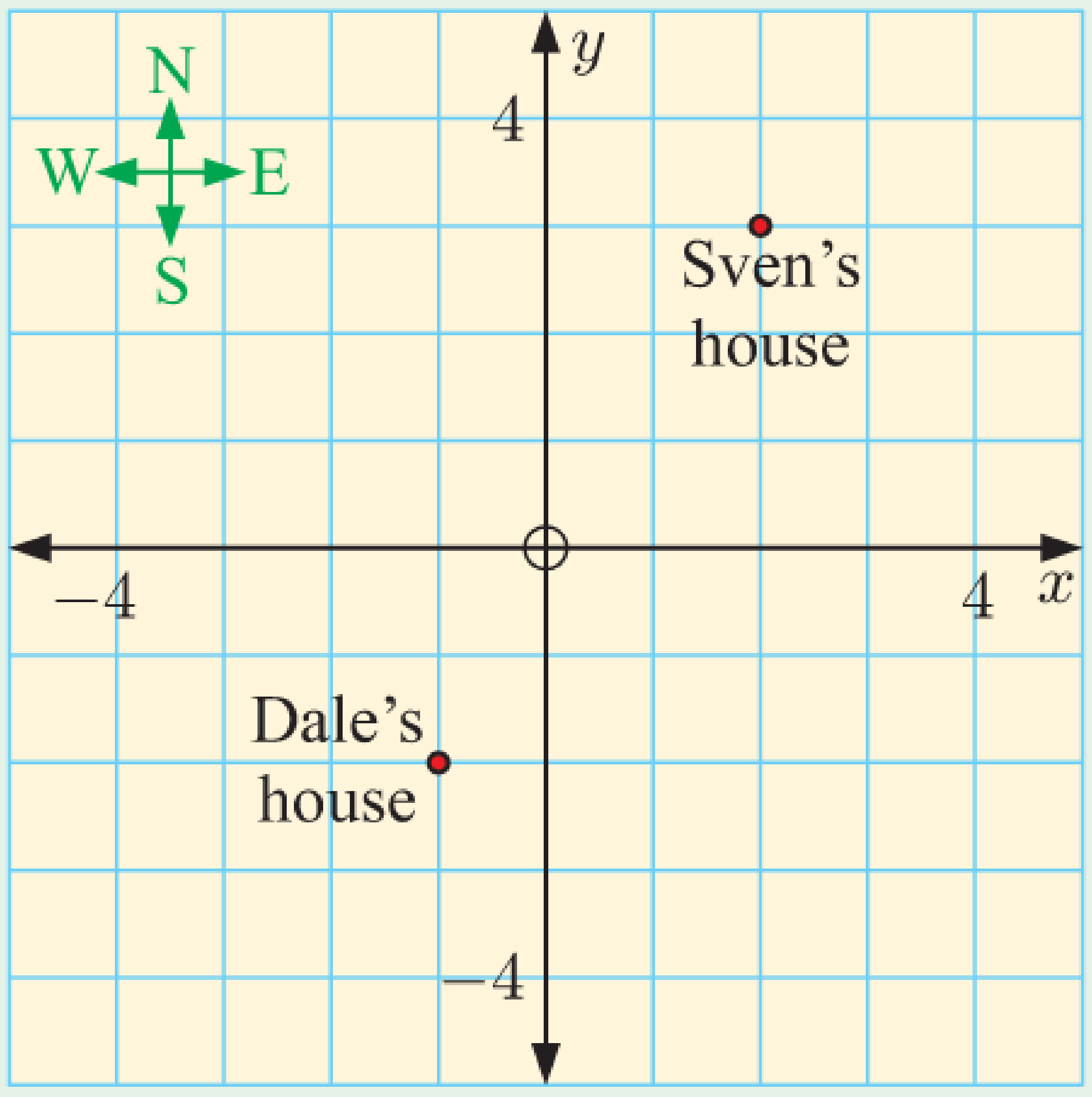


5 Construct a 10 by 10 grid, with the origin in the bottom left corner. Plot these points and join them in the given order:

(2, 3), (8, 3), (7, 2), (5, 2), (5, 1), (4, 1), (4, 2), (3, 2), (2, 3)
then (4, 3), (4, 10), (8, 4), (2, 4), (4, 10), (5, 10), (4, 9).

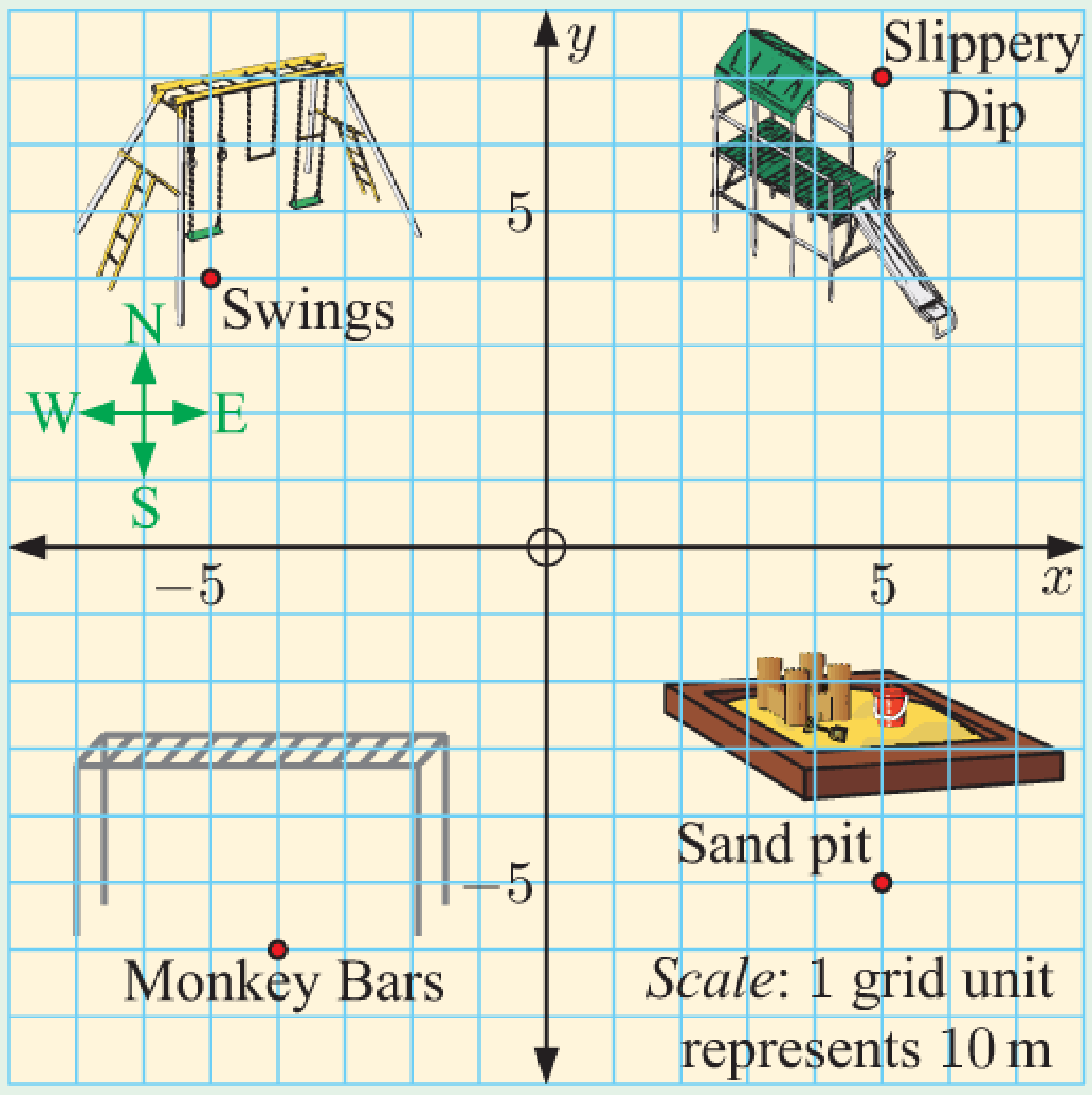
PRINTABLE GRIDS

6



On this map, 1 grid unit represents 1 km.
a Find the coordinates of:
i Sven's house **ii** Dale's house.
b Violet lives 5 km south of Sven.
i Find the coordinates of Violet's house.
ii How far is Violet's house from Dale's house?
iii In what direction is Violet's house from Dale's house?
c There is a park located directly north of Dale's house, and directly south-west of Sven's house. Find the coordinates of the park.

7 Roger buried a coin in the playground, and wrote these instructions for finding it:
Start at the monkey bars and walk 20 metres north. Then walk north-east until you are directly south of the slippery dip. Face the sand pit and walk 80 metres, then walk west until you are directly south-east of the swings.
What are the coordinates of the hidden coin?



Chapter 14

Area and volume

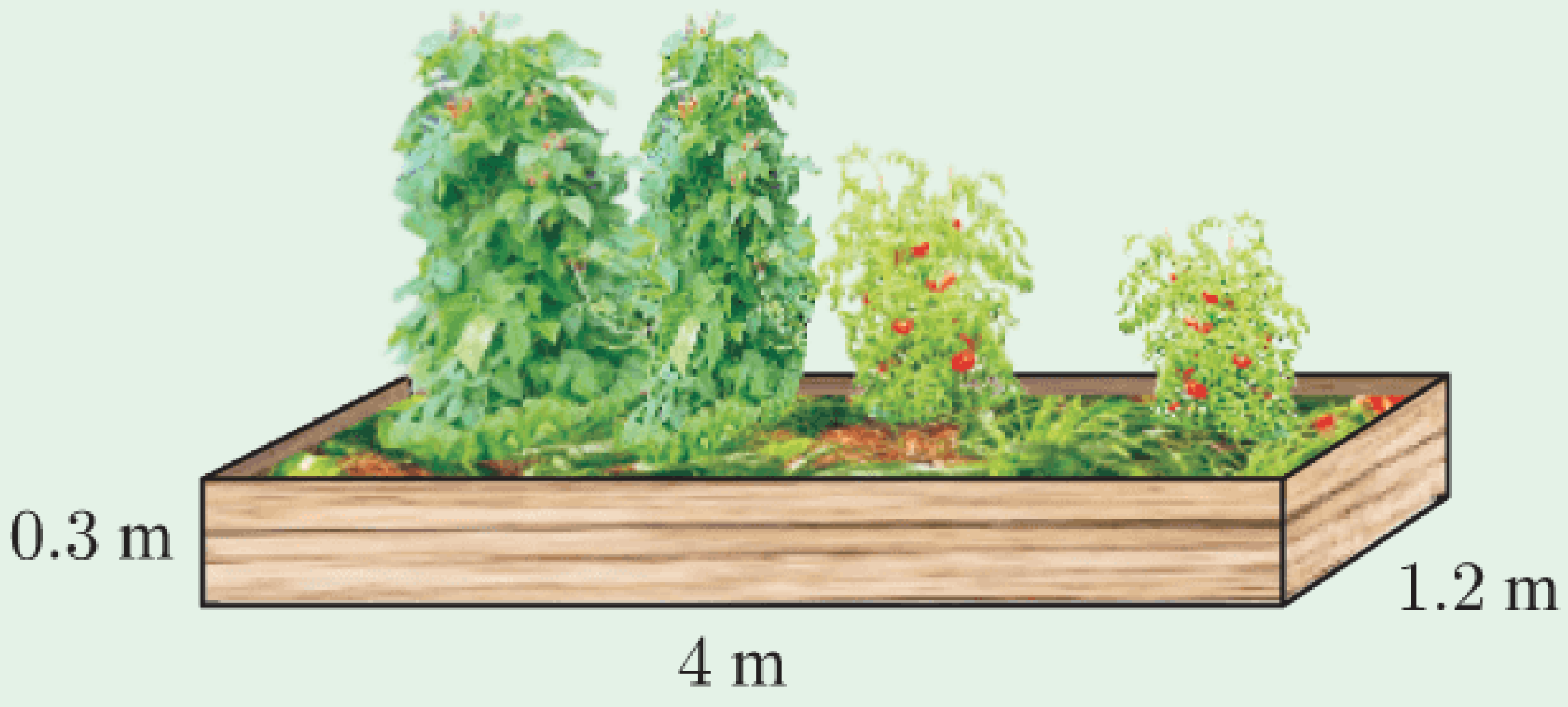
Contents:

OPENING PROBLEM

Peter is building a raised garden bed for his son's primary school. Its sides will be made of wood, and it will be 4 m long, 1.2 m wide, and 0.3 m high.

Things to think about:

- **a** What *length* of 0.3 m wide wood is needed to make the walls or *perimeter* of the garden bed?
- **b** What *area* of weed mat is needed to cover the base of the garden bed?
- **c** What *volume* of soil is needed to fill the garden bed?



In our earlier study of measurement we considered length and perimeter.

In this chapter we consider the **area** of surfaces, and the **volume** of space that objects occupy.

A AREA

All around you are objects and regions which have **surfaces**.

For example:

- in a house, the carpets, walls, ceilings, and shelves are all surfaces
- the football field, the tennis court, and the badminton court are all surfaces.

People often need to measure the amount of surface within a region. The surface may be land, a wall, or an amount of dress material.

Area is the amount of surface within a region.

We need to measure area so we know:

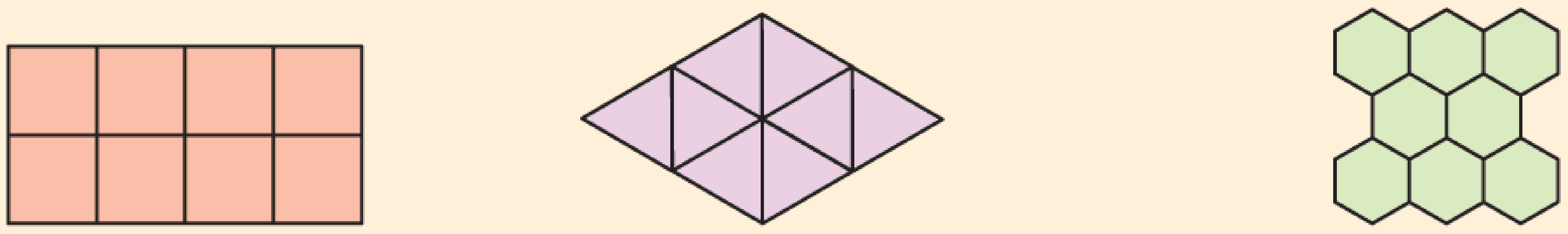
- how much paint to buy to cover the wall
- how many tiles to buy to cover the floor
- how much fertiliser to buy for the garden.

INVESTIGATION 1 CHOOSING UNITS OF AREA

When we studied length, we used mm, cm, m, and km. What should we use to measure area?

One way to measure an area is to cover it with identical shapes and then count them. It is important that the shapes can be placed together with no gaps.

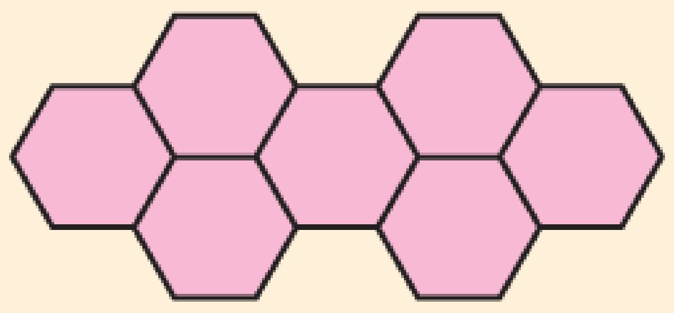
For example:



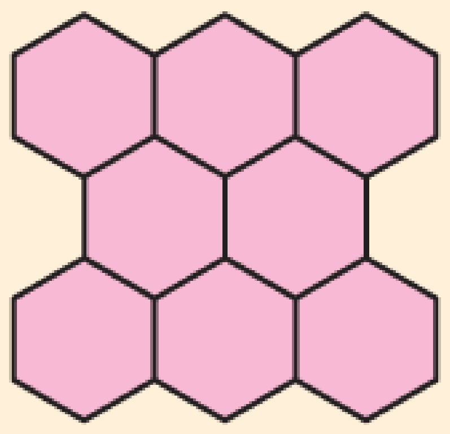
What to do:

1 For each pair of diagrams, count the number of shapes used. Which has the bigger area?

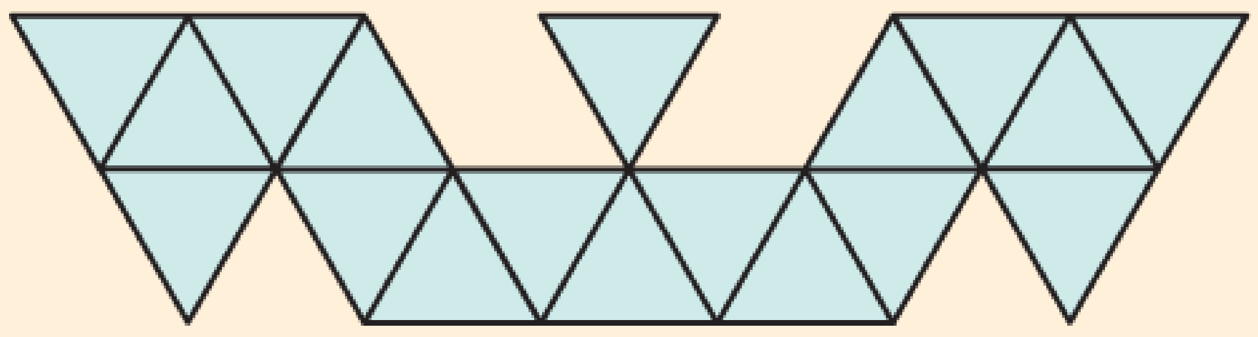
a **A**



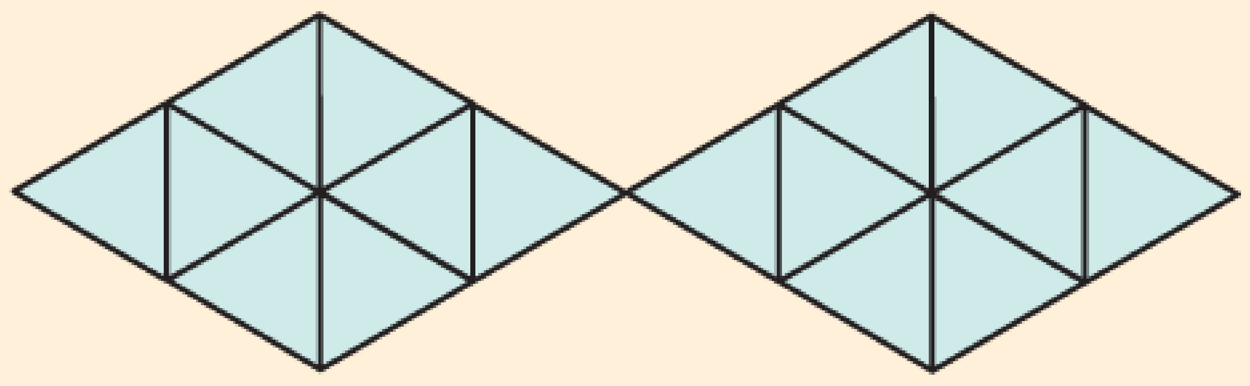
B



b **A**

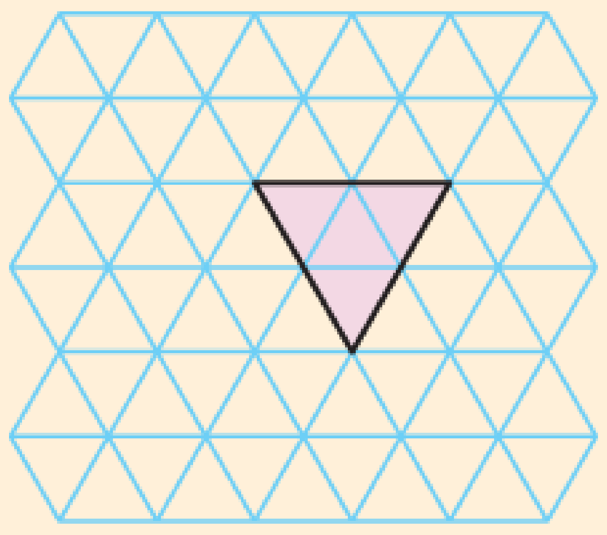


B

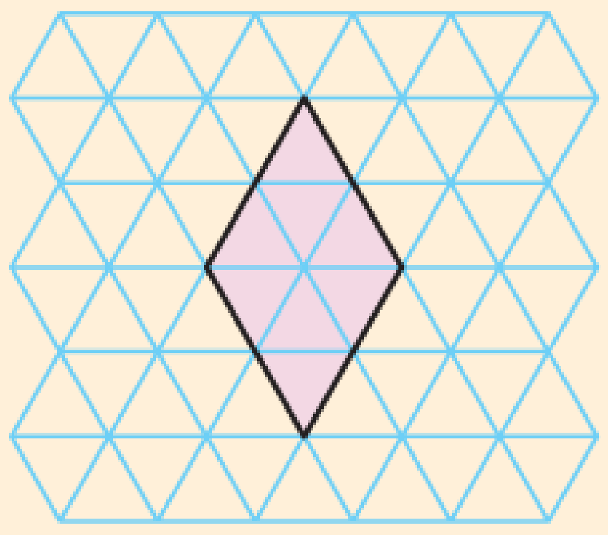


2 Use △ as a unit of area to measure the area of each shape:

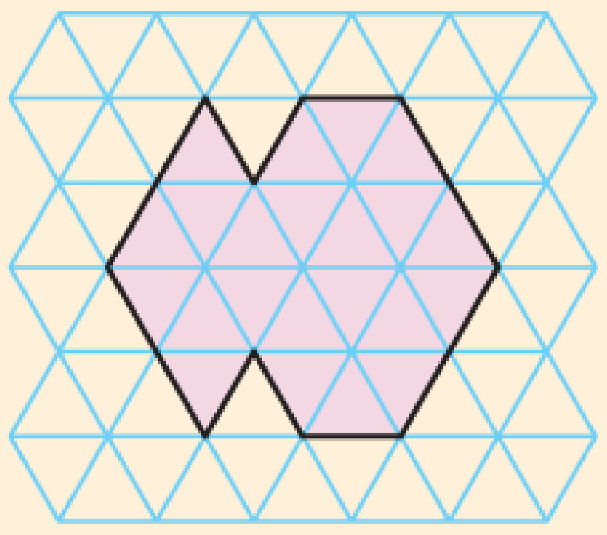
a



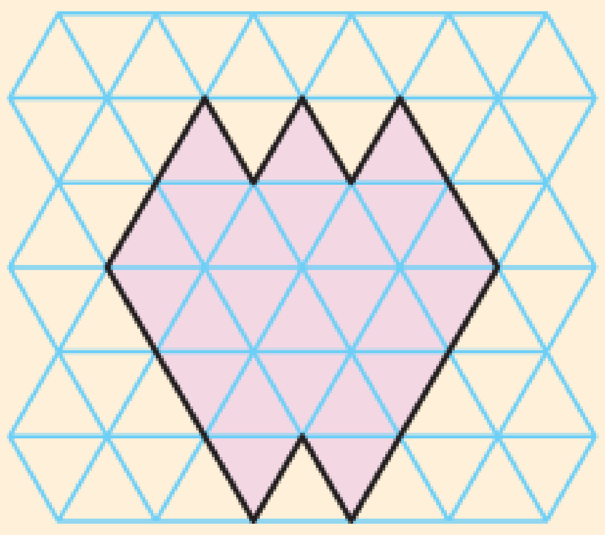
b



c



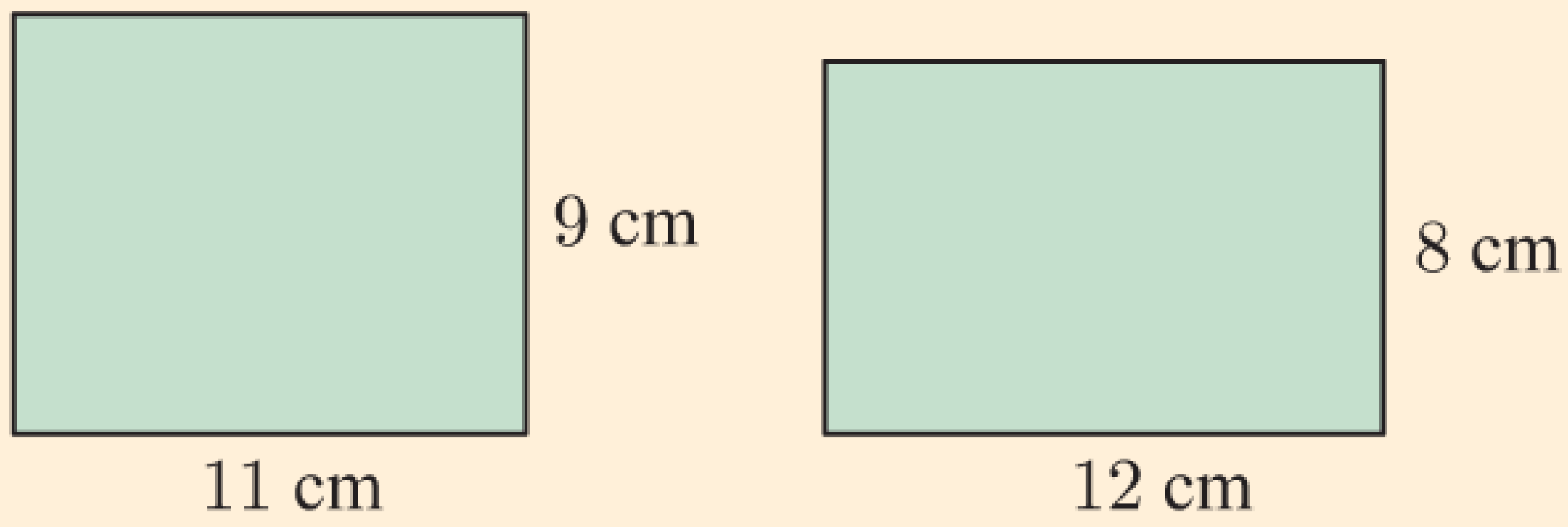
d



3 **a** Can you place circles together to cover a shape with no gaps?

b Do you think a circle would be a good choice as a unit of area? Explain your answer.

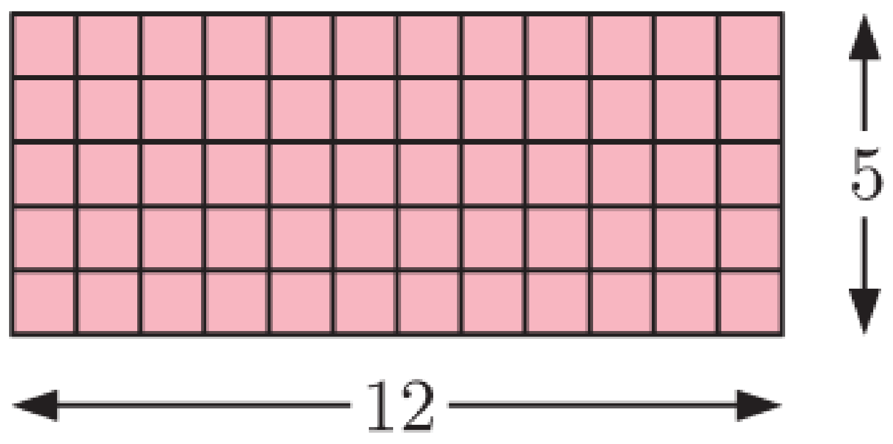
4 Suppose you wish to compare the areas of the two illustrated rectangles.
Which of the shapes above would be best to use as a measure of area? Explain your answer.



SQUARE UNITS

As you have seen, it is possible to compare area using a variety of shapes. Some shapes have advantages over others.

The **square** has been chosen as the universal unit used to measure area.



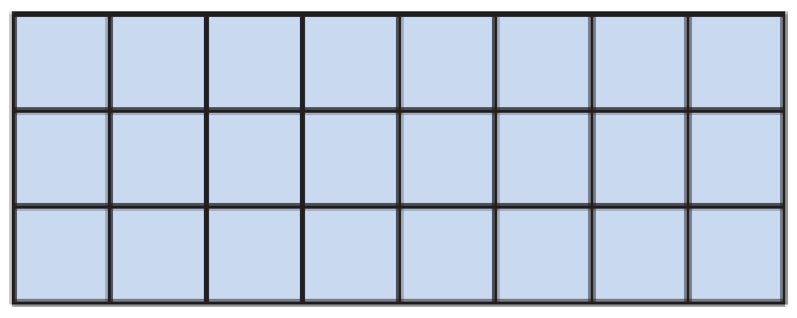
5 × 12 = 60 squares

The **area** of a closed figure, no matter what shape, is the number of square units (units^2) it encloses.

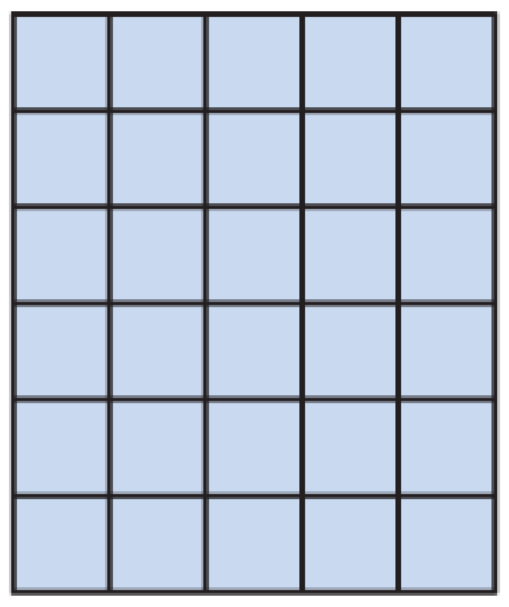
EXERCISE 14A.1

1 Find the area, in square units, of each shape:

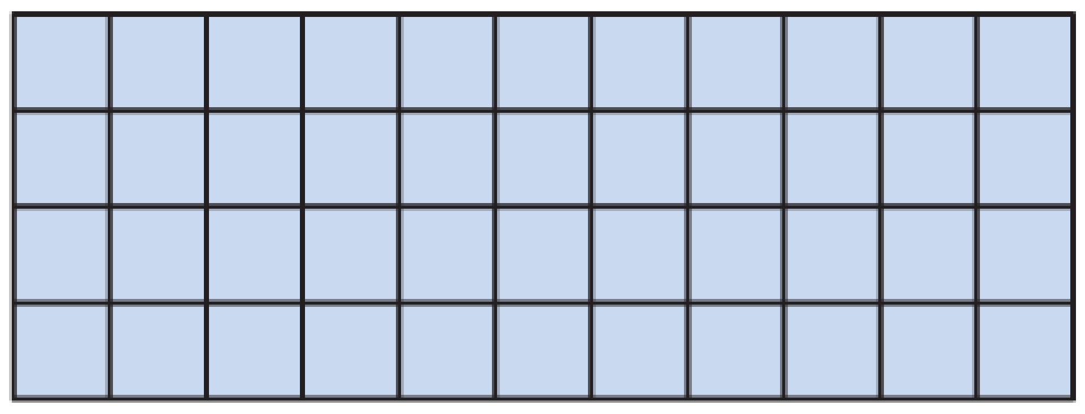
a



b



c



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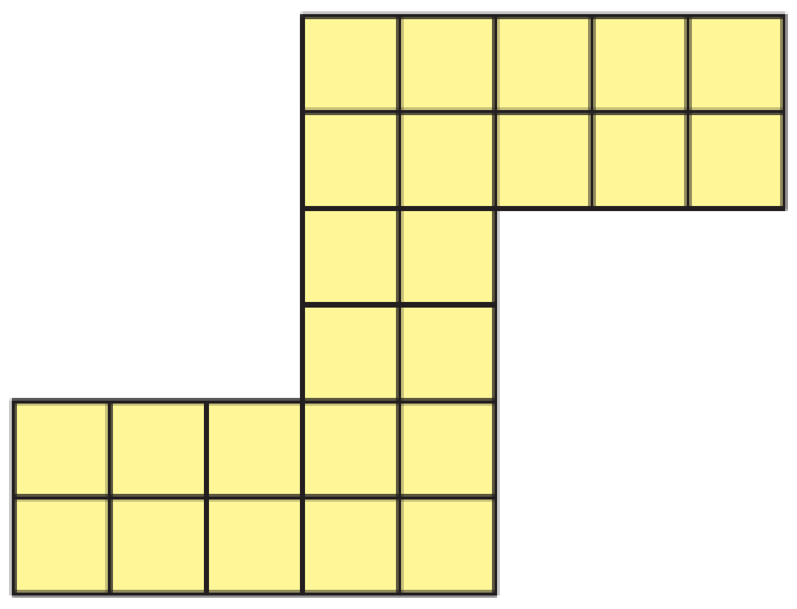
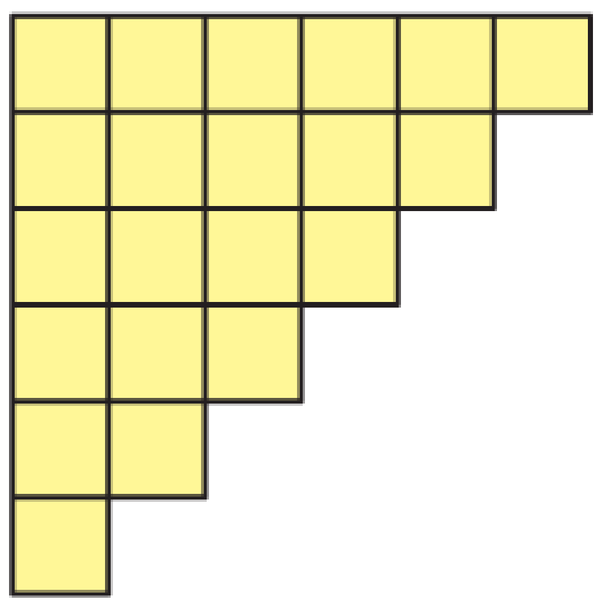
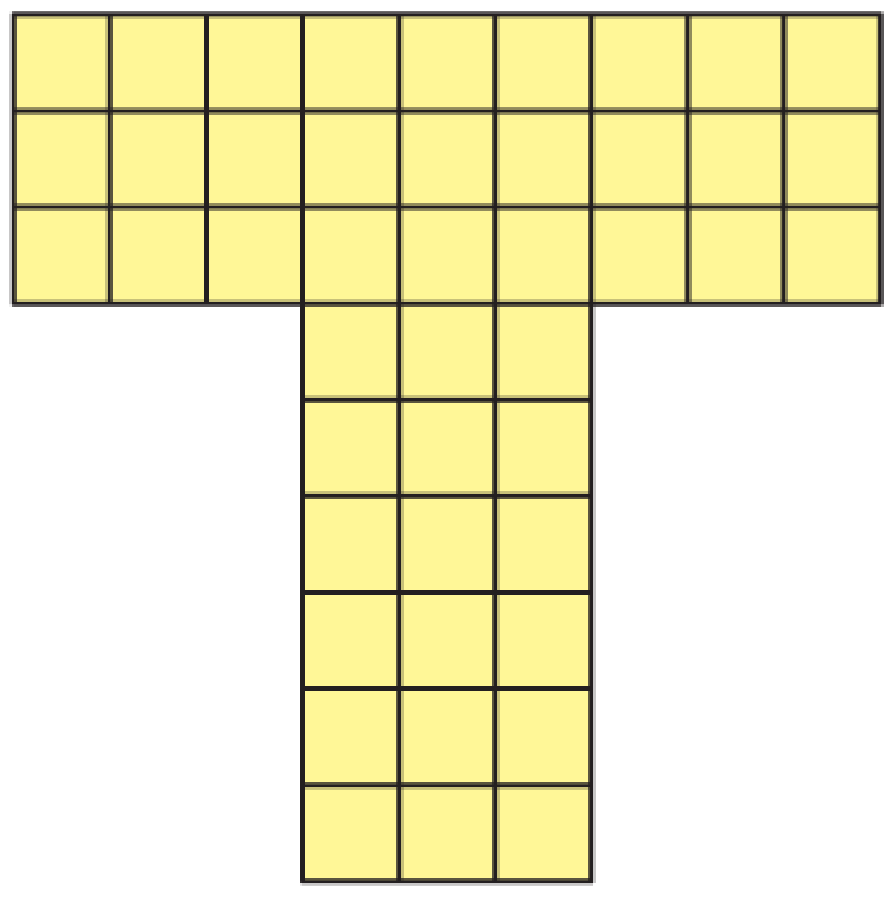


e

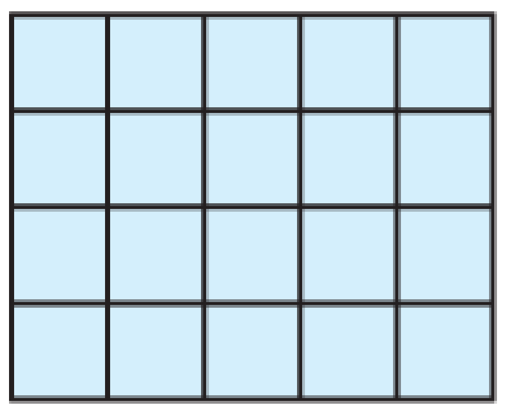


f

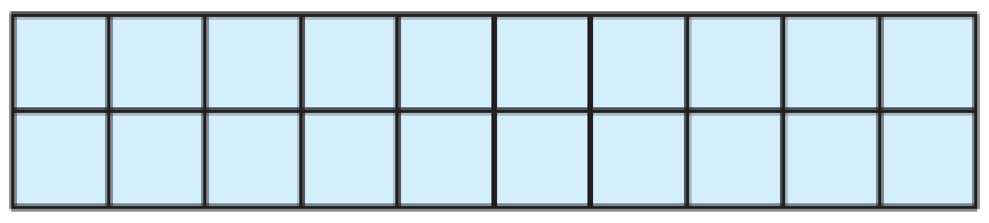


2 Consider the three shapes:

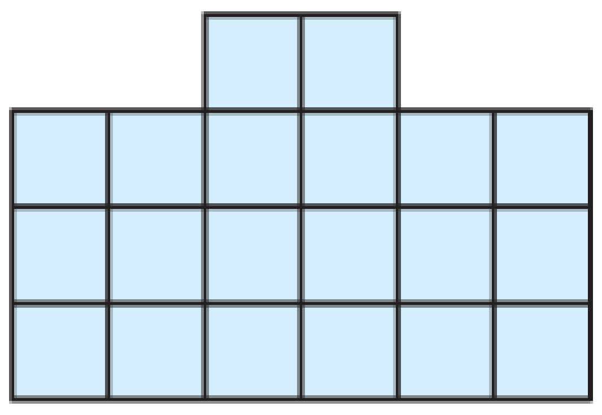
A



B



C



a Copy and complete this table:

Shape	*Perimeter* (units)	*Area* (units^2)
A		
B		
C		

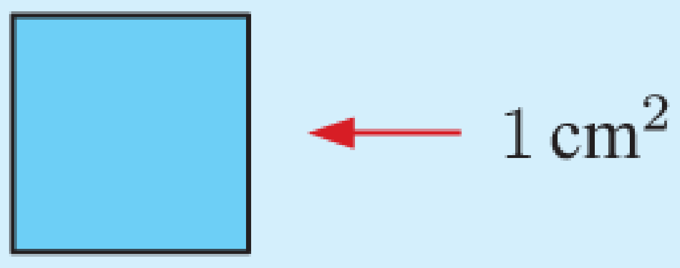
b Comment on your results.

METRIC UNITS

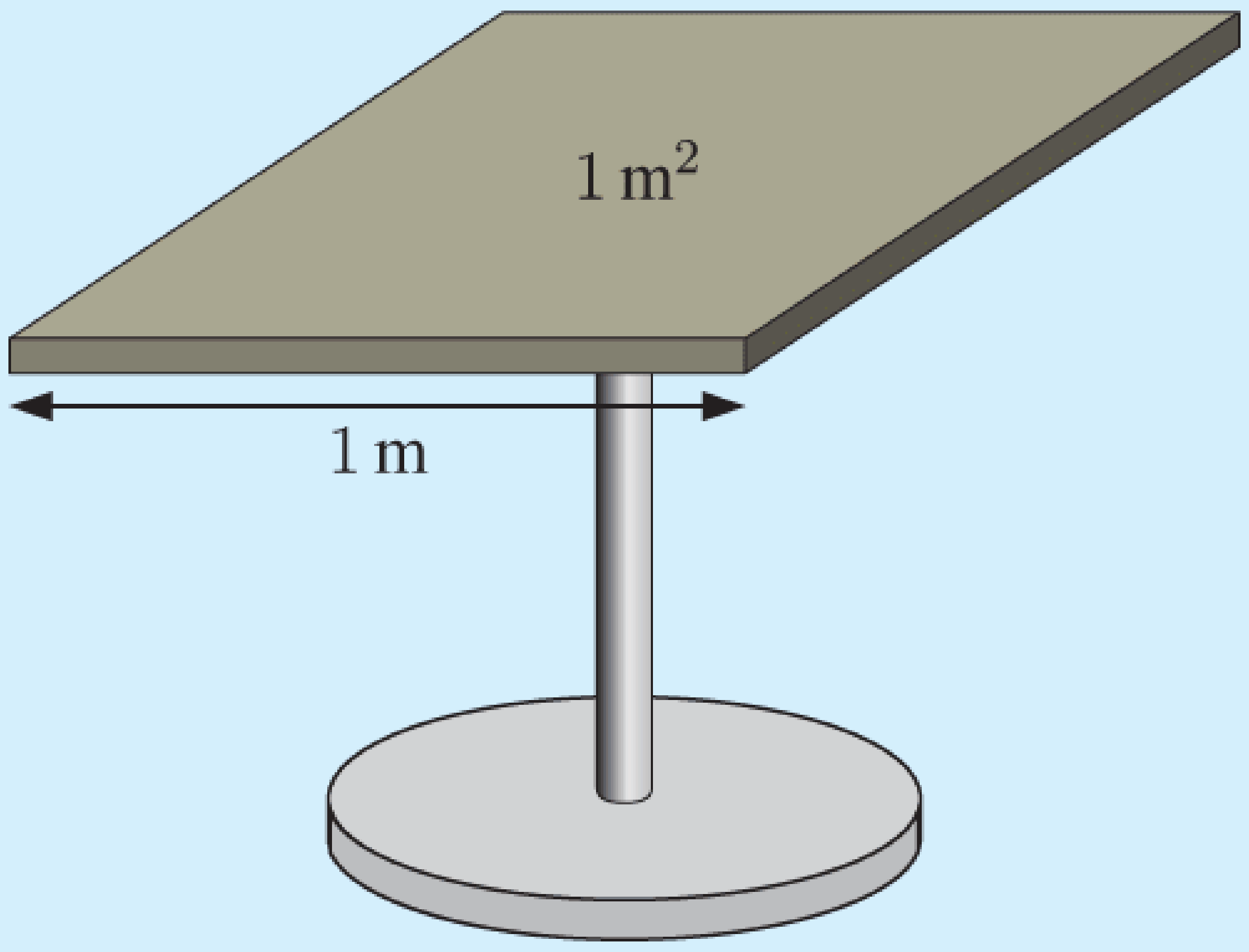
In the metric system, we use the units of length mm, cm, m, and km as a basis for the units of area.

1 square millimetre (mm^2) is the area enclosed by a square of side length 1 mm.

1 square centimetre (cm^2) is the area enclosed by a square of side length 1 cm.



1 square metre (m^2) is the area enclosed by a square of side length 1 m.



1 square kilometre (km^2) is the area enclosed by a square of side length 1 km.

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EXERCISE 14A.2

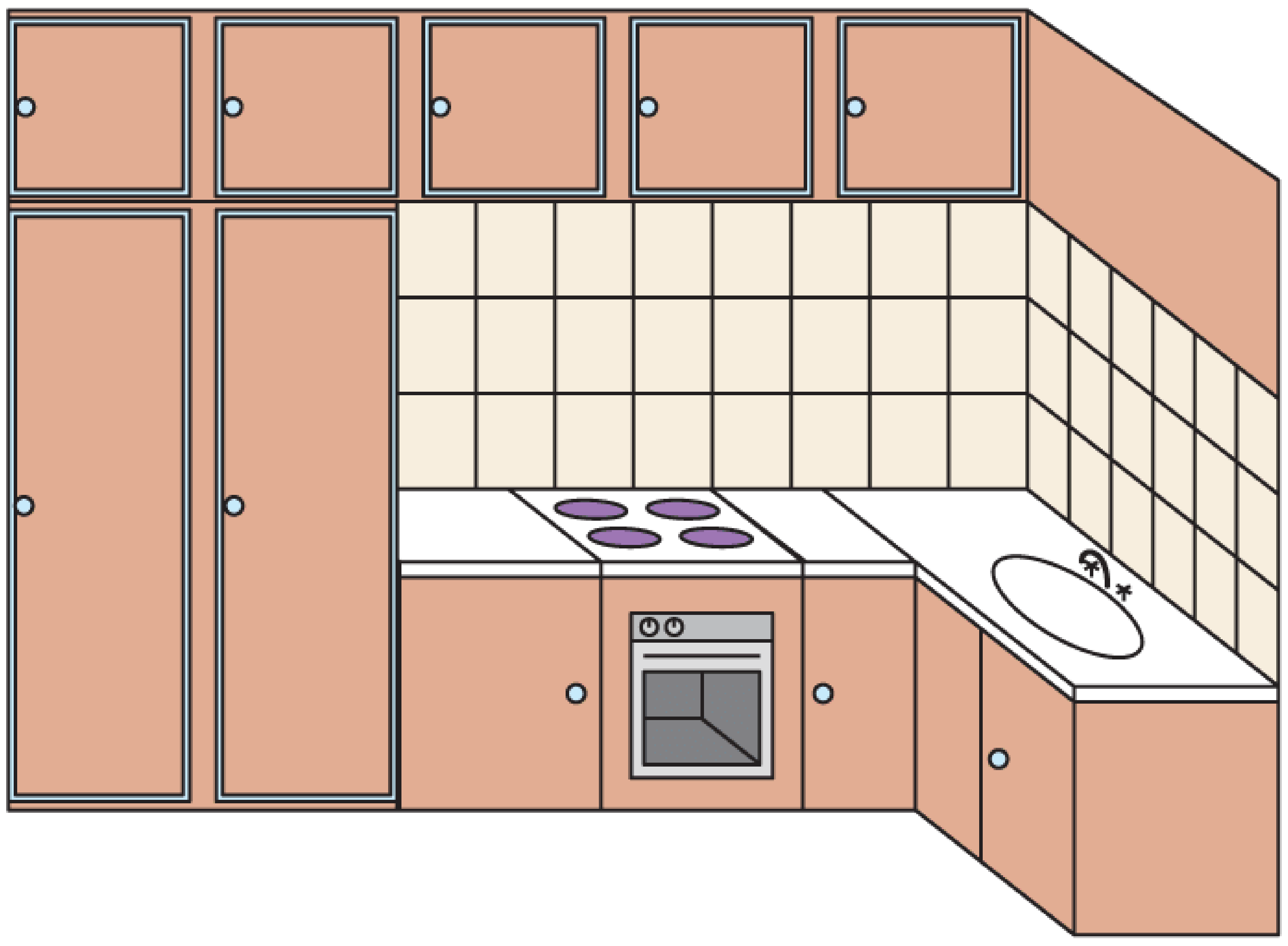
1 Match the following descriptions with the correct area:

- **a** a picnic rug
- **b** a sports stadium
- **c** New Zealand
- **d** a DVD
- **e** this dot ●
- **f** a golf course
- **g** a suburban park
- **h** a coin

A 0.8 km^2
B 110 cm^2
C 5 m^2
D 7.5 cm^2
E 600 m^2
F 7 mm^2
G 268 000 km^2
H 20 000 m^2

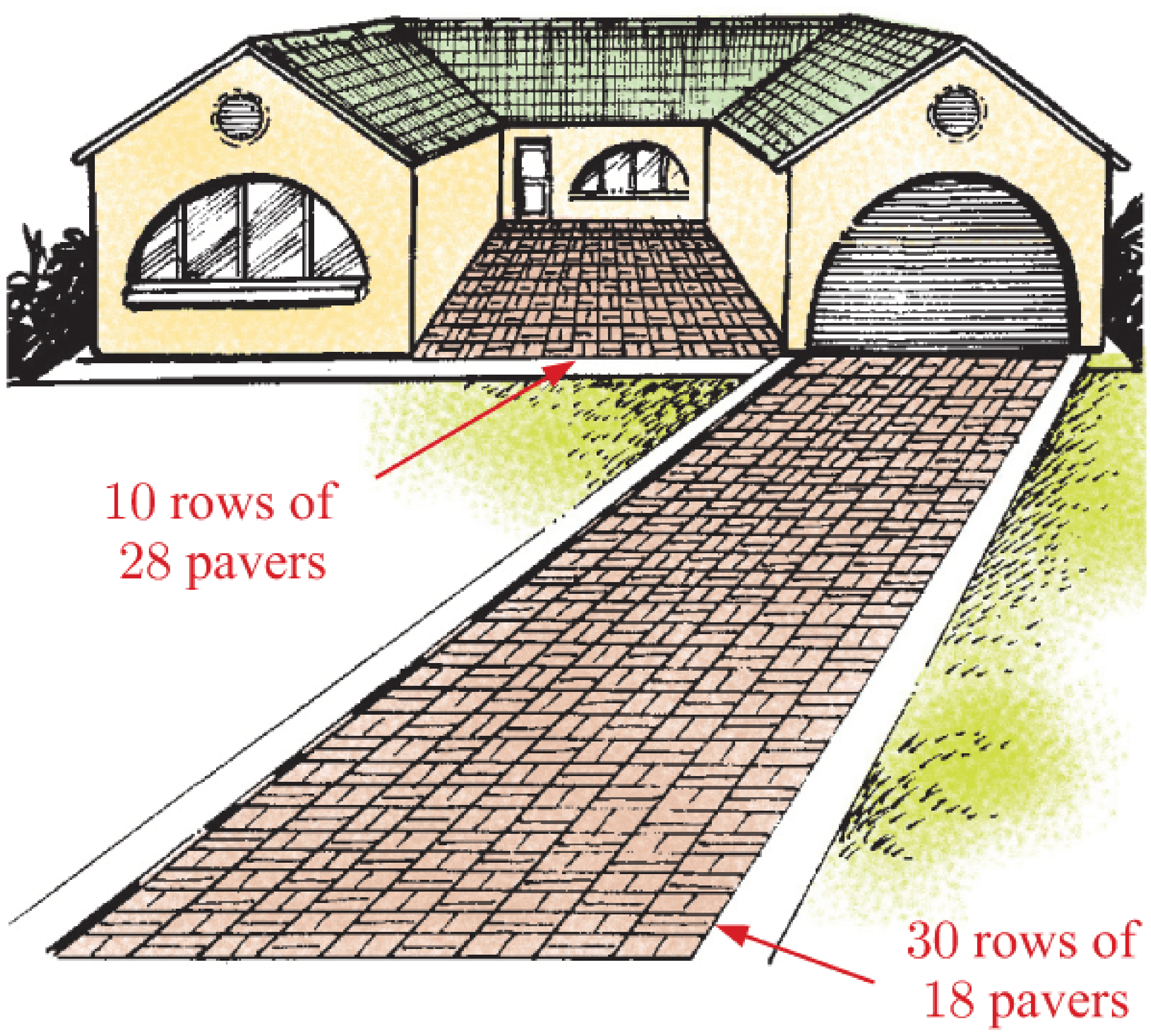
2 The diagram shows Christine's kitchen.

- **a** How many tiles have been used?
- **b** The area of 20 tiles is one square metre. How many square metres of tiles are there in Christine's kitchen?
- **c** The tiles cost £44.00 per square metre. What is their total cost?



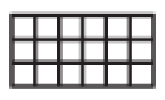
3 Look at the picture.

- **a** How many pavers have been used for:
  - **i** the driveway
  - **ii** the courtyard?
- **b** The pavers in the courtyard are the same as the pavers in the driveway. The area of 50 pavers is one square metre. How many square metres of paving are there in total?
- **c** The cost of the pavers is \$16.90 per m^2, and the cost of laying them is \$14 per m^2. Find the total cost of the paving.

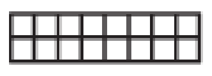


4 Each of the small squares in these shapes has area 1 square millimetre. Find the area of each shape.

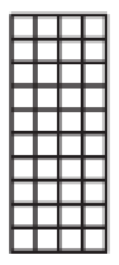
a



b



c



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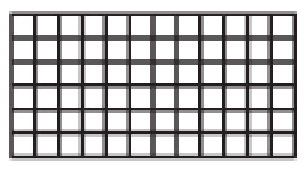


DISCUSSION

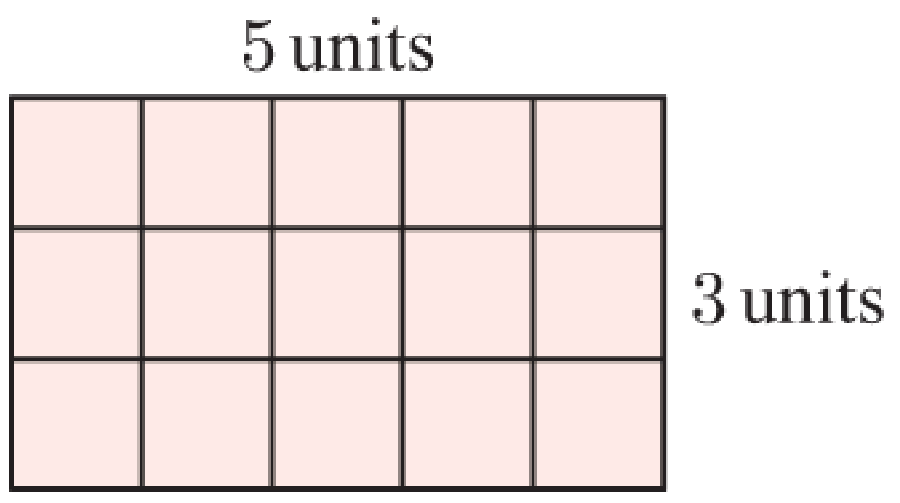
When you found the areas of the shapes in question **4**, did you count each individual square?

Is there a quicker way to find the area of each shape?

B THE AREA OF A RECTANGLE

Consider a rectangle 5 units long and 3 units wide.

The area of this rectangle is 15 units^2, and we can find this by multiplying $5 \times 3 = 15$.

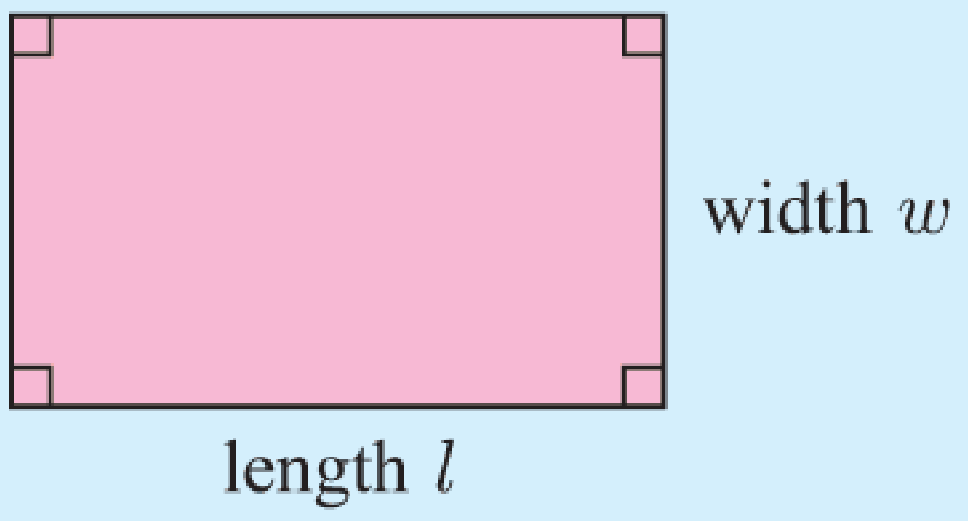


DEMO

This leads to the general rule:

Area of rectangle = length × width

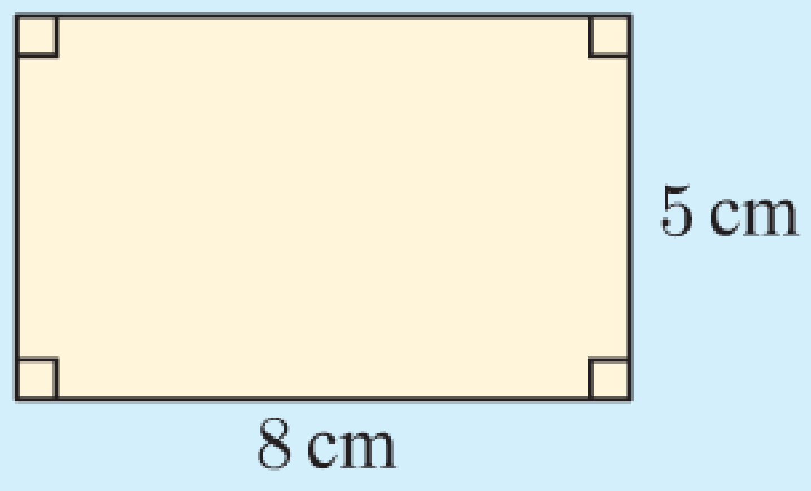
$$A = l \times w$$



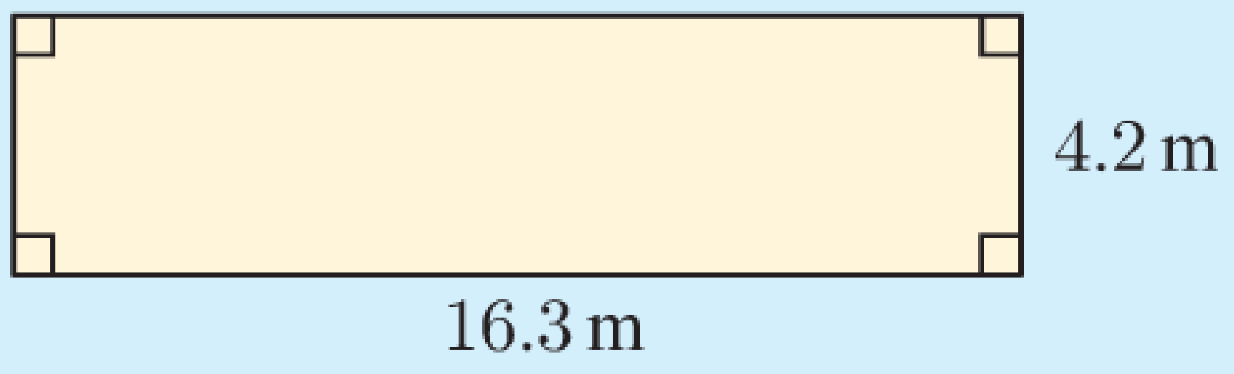
Example 1 **Self Tutor**

Find the area of each rectangle:

a



b



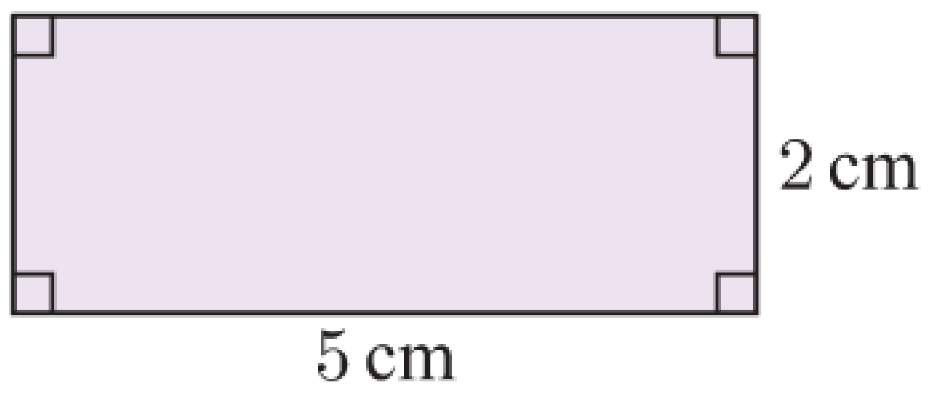
a Area $=$ length $\times$ width
$= 8 \text{ cm} \times 5 \text{ cm}$
$= 40 \text{ cm}^2$

b Area $=$ length $\times$ width
$= 16.3 \text{ m} \times 4.2 \text{ m}$
$= 68.46 \text{ m}^2$

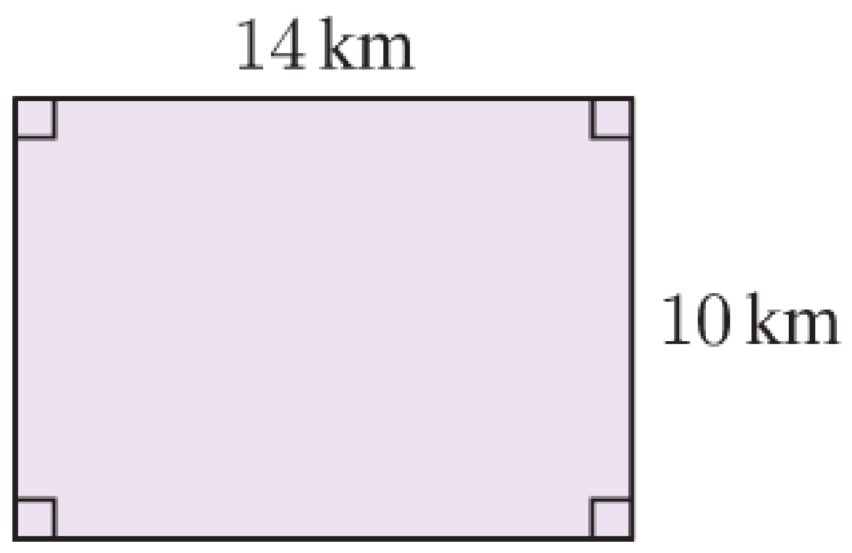
EXERCISE 14B

1 Find the area of each rectangle:

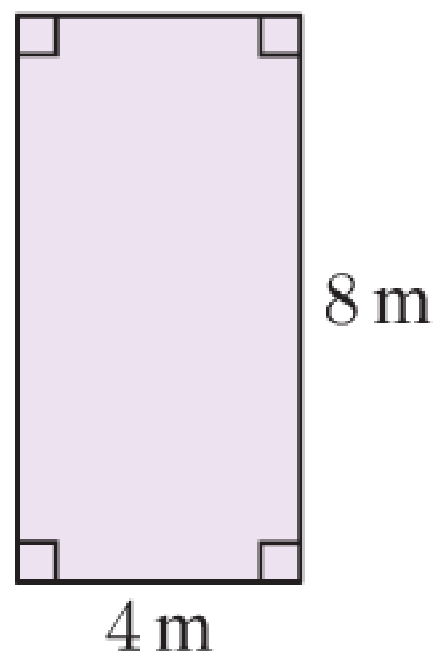
a



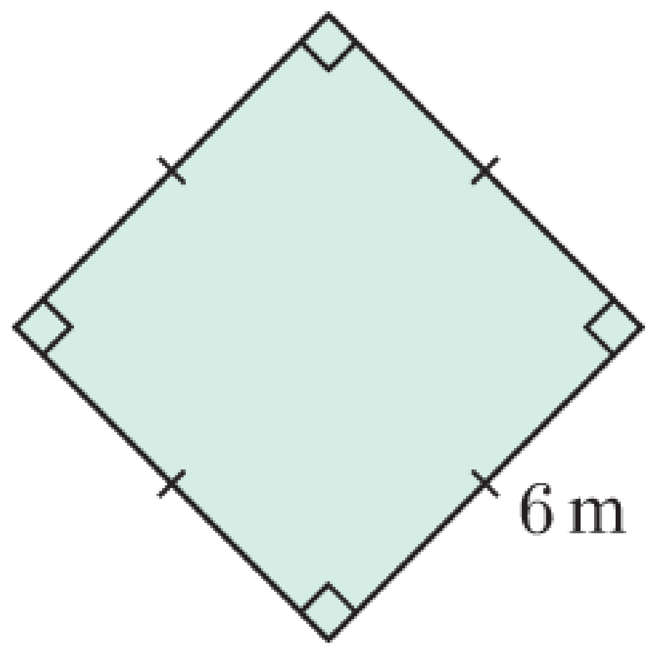
b



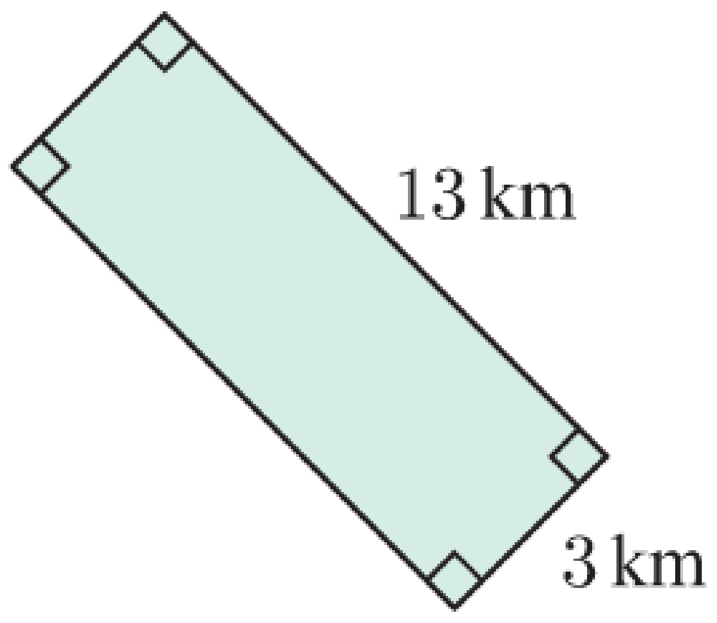
c



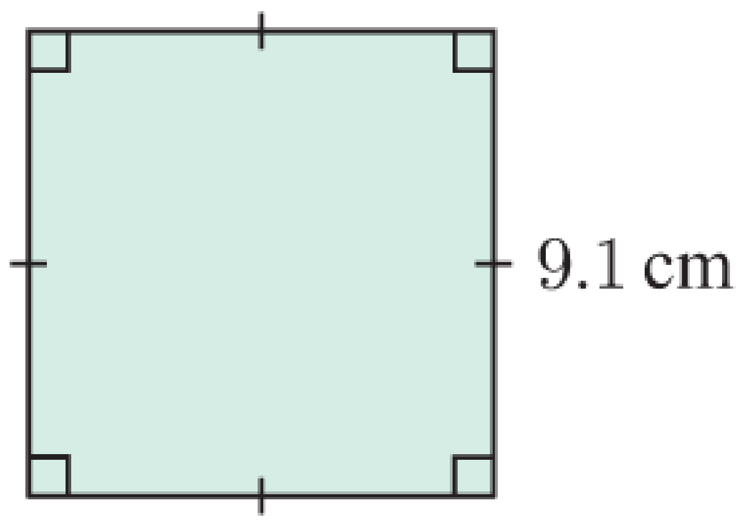
d



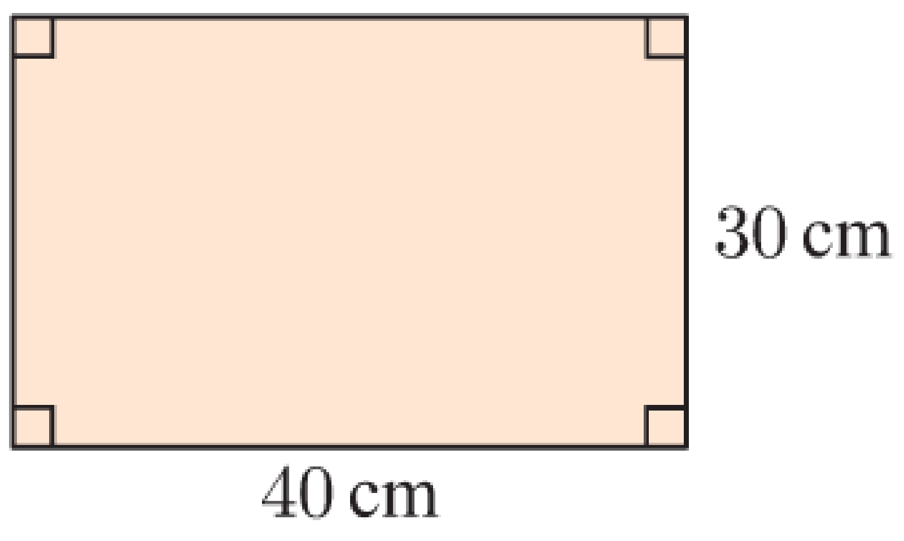
e



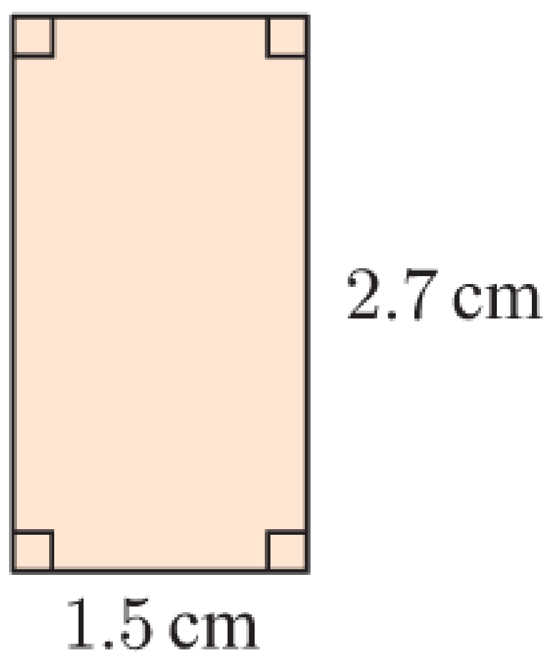
f



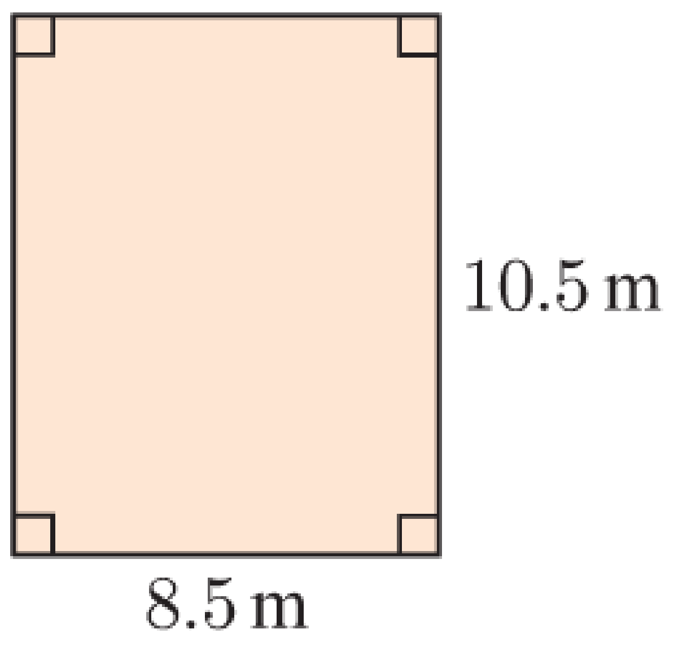
g



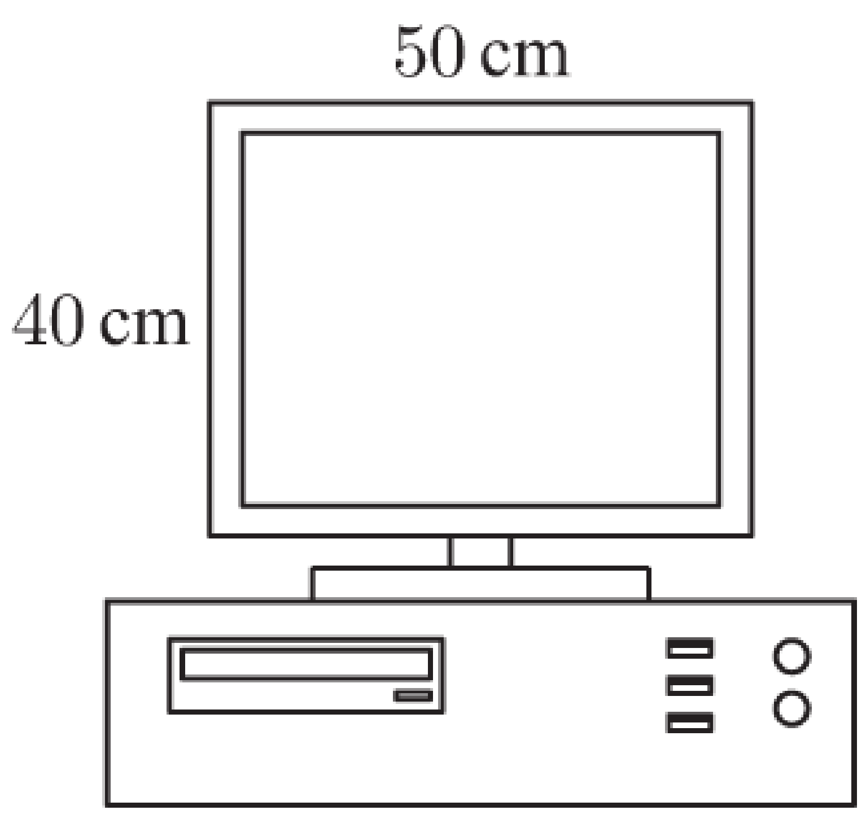
h



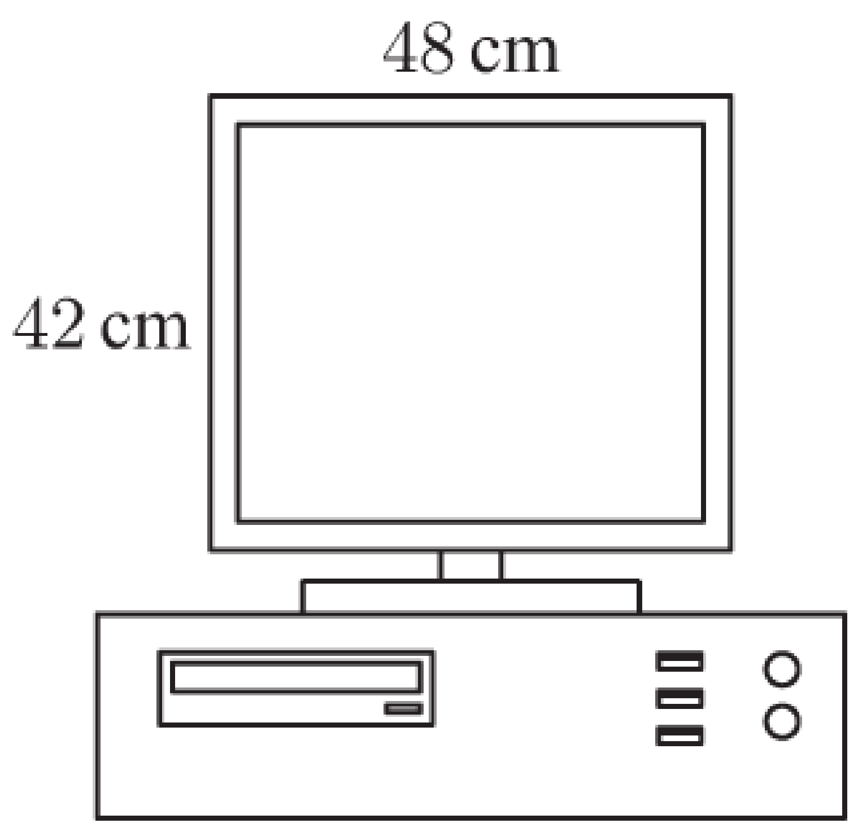
i



2 Whose computer screen is larger: Meredith's or Gemma's?



Meredith's computer



Gemma's computer

3 Tom wants to cover his 13 m by 12 m backyard with lawn. Seed for the lawn costs £7.50 per square metre. Find:

- **a** the area of the lawn
- **b** the total cost of the seed for the lawn.

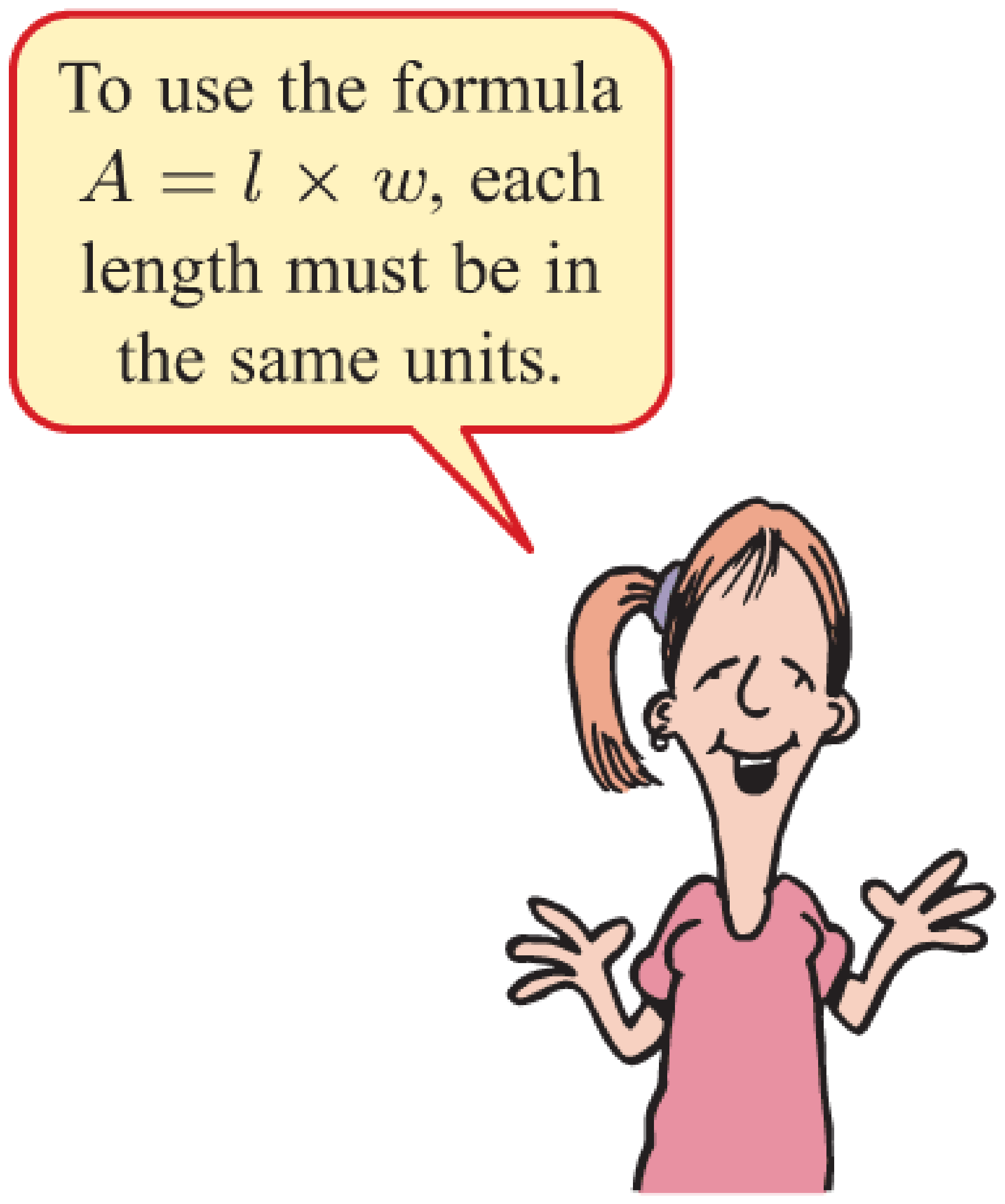
4 A 5 m by 7 m room has a 2 m by 3 m rug on the floor. Find the area of exposed floor.

5 A 6 m by 7.5 m ceiling is to be painted. One litre of paint covers 15 square metres. Find:

- **a** the area of the ceiling
- **b** the total amount of paint required.

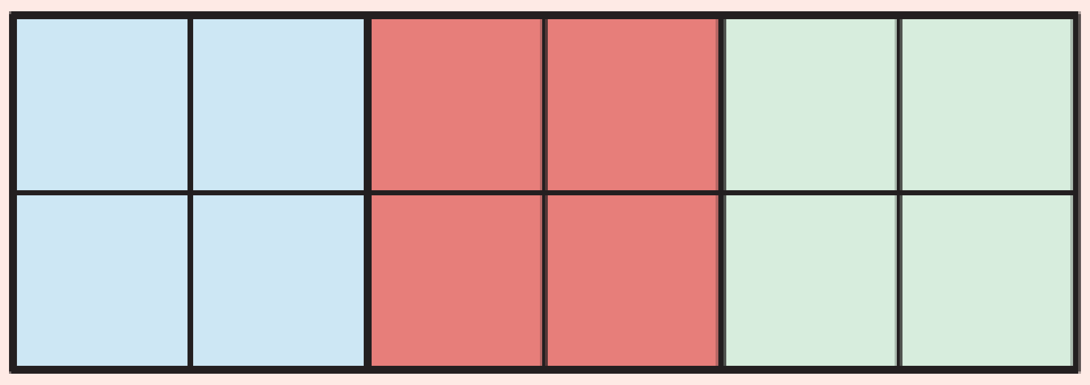
6 A 1.8 m by 9 m hallway is to be covered with floorboards which are 15 cm by 1.5 m. Each floorboard costs €21.50.

- **a** Find the area, in square metres, of each floorboard.
- **b** Find the area of the floor.
- **c** Find the number of floorboards required.
- **d** Find the total cost of the floorboards.

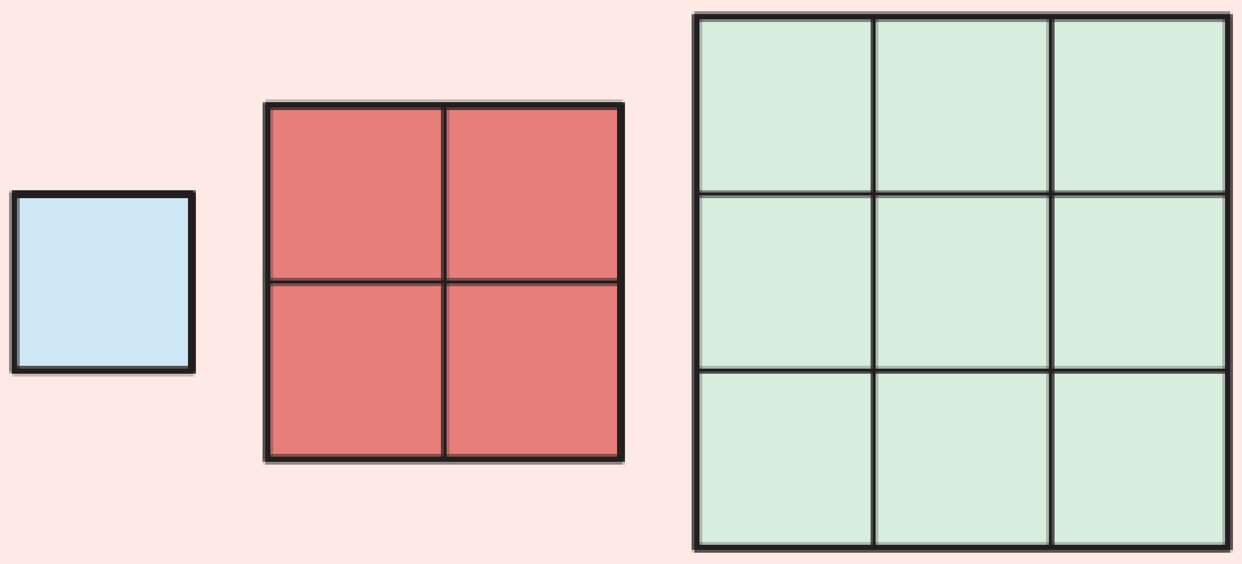


ACTIVITY 1 SQUARING A RECTANGLE

It is easy to put squares of equal size together to form a rectangle. For example, in this diagram there are three 2×2 squares put together to form a 6×2 rectangle.



It is much harder to form a rectangle if the squares have different sizes! For example, it is impossible to form a rectangle using the 1×1 square, 2×2 square, and 3×3 square opposite.



In 1925, Polish mathematician **Zbigniew Moroń** discovered that one of each of the squares 1×1, 4×4, 7×7, 8×8, 9×9, 10×10, 14×14, 15×15, and 18×18 can be used to form a rectangle.

What to do:

1. Use grid paper to draw squares with the sizes given above, or click on the icon and print the squares.
2. Cut out the squares, and arrange them to form a rectangle.

SQUARES

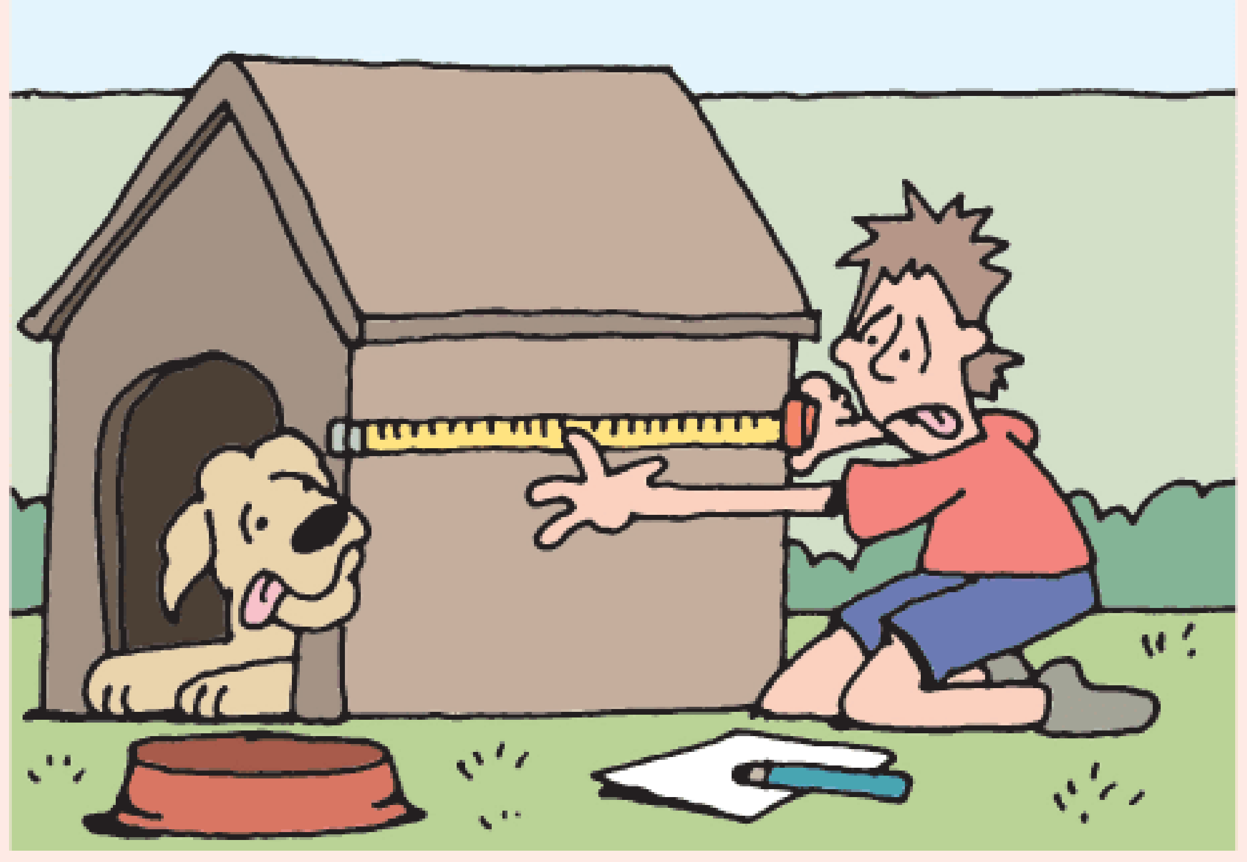
Hint: Find the total area of the squares. This can help you work out the dimensions of the rectangle.

ACTIVITY 2 WORKING WITH AREA

What to do:

1 Draw a square metre with chalk.
Estimate how many members of your class can stand on the square metre with feet entirely within it. Check your estimate.

2 Use a measuring tape to measure the dimensions of some rectangular regions. You could measure the floor area of your classroom, a tennis court, a basketball court, or the sides of a building. Calculate the area of each region.



C AREAS OF TRIANGLES AND PARALLELOGRAMS

TRIANGLES

INVESTIGATION 2 THE AREA OF A TRIANGLE

You will need:

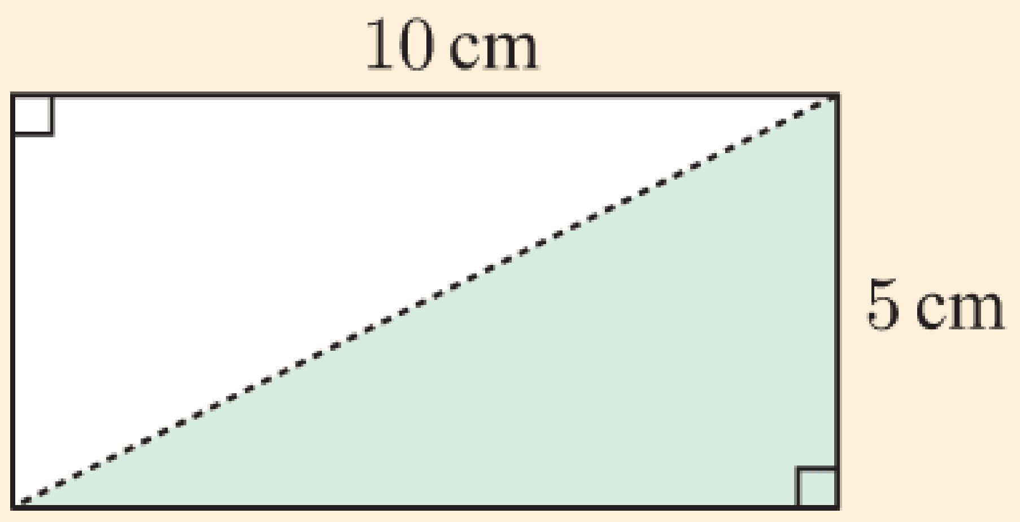
GRAPH PAPER

scissors, ruler, pencil, and square centimetre graph paper.

What to do:

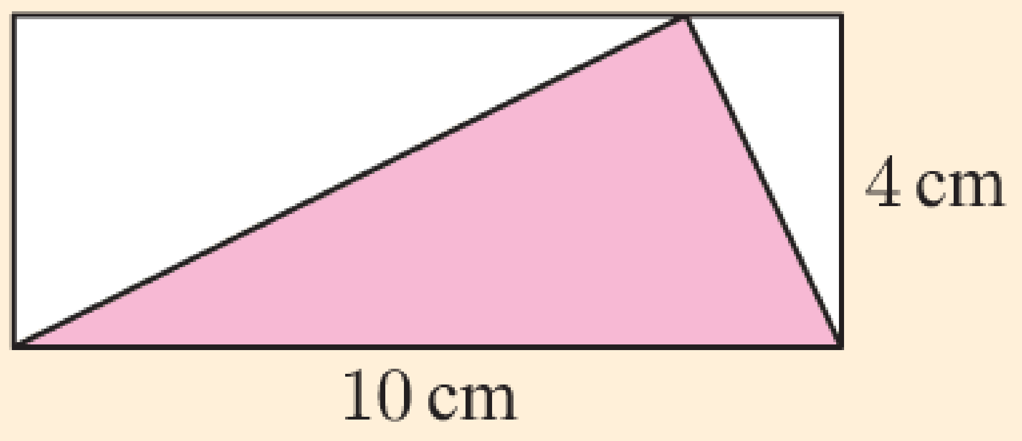
1 Draw a 10 cm by 5 cm rectangle using the graph paper. Draw in the dashed diagonal, and colour one triangle green.
Cut out the two triangles, then place one on top of the other so you can see they have identical shape.
Copy and complete:

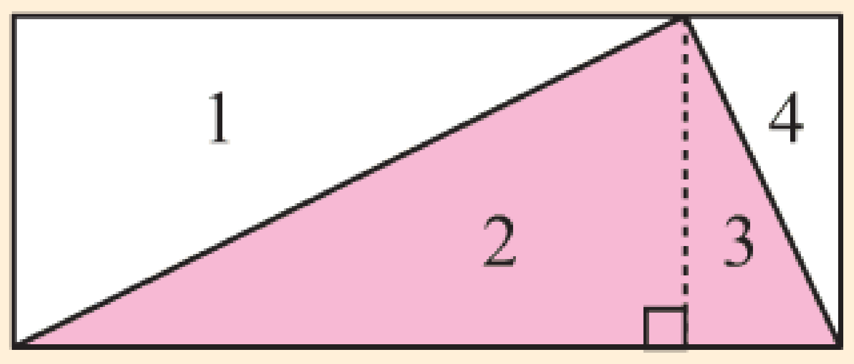
The areas of the two triangles are ....
The area of each triangle is ...... the area of the rectangle.



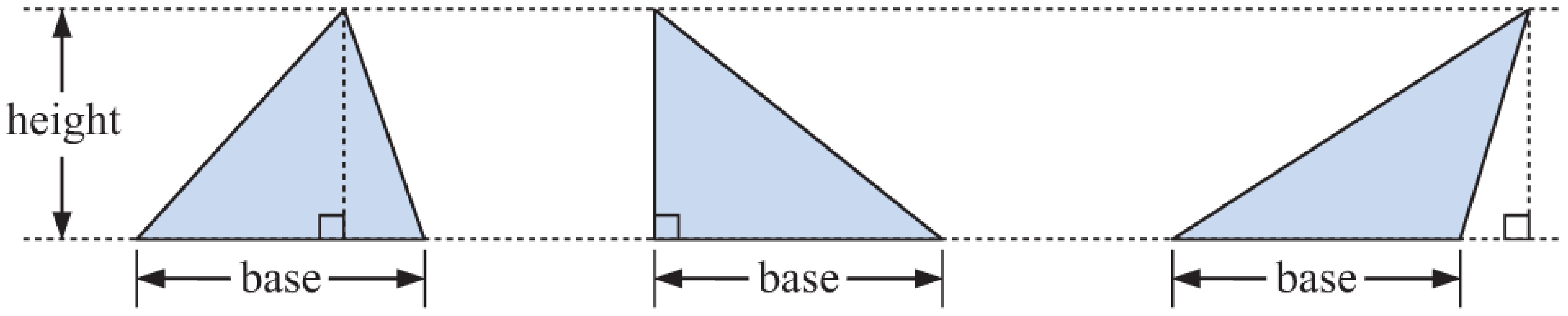
2 Draw a 10 cm by 4 cm rectangle using the graph paper.
Construct the triangles shown and colour in the pink region.
Now divide the pink triangle along the dashed line so you form four regions.
Using what you found in **1**, copy and complete:

The areas of regions 1 and 2 are ......
The areas of regions 3 and 4 are ......
So, area 2 + area 3 = area 1 + area 4.
The total area of the pink triangle is ...... the area of the rectangle.





From the **Investigation** you should have found that the area of a triangle is half the area of a rectangle which has the *same base and height* as the triangle.



Even though they have different shapes, these triangles have the same base and height. They therefore have the same area.

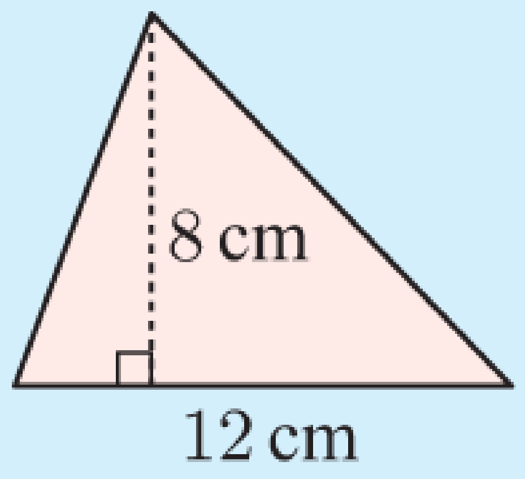
Area of triangle $= \frac{1}{2} \times$ **base** $\times$ **height** or $\frac{\textbf{base} \times \textbf{height}}{2}$.

DEMO

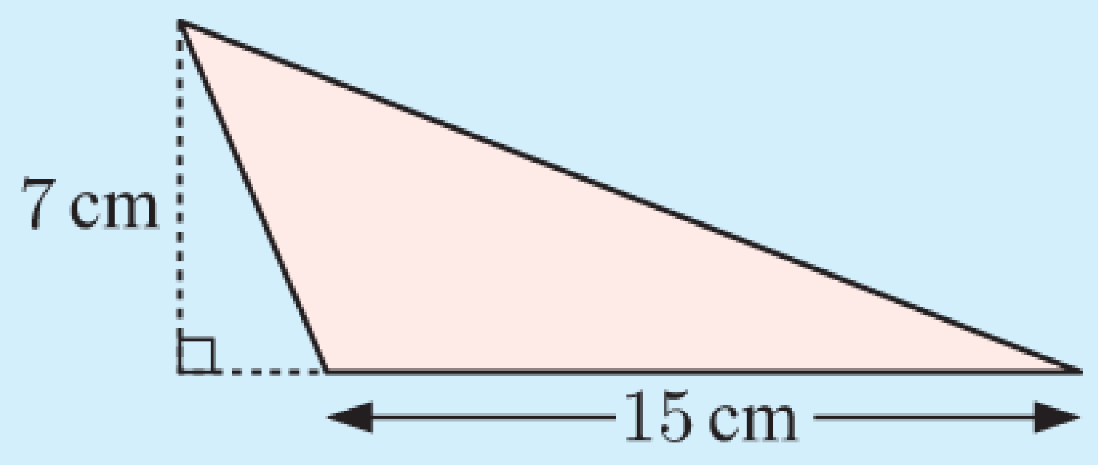
Example 2 **Self Tutor**

Find the area of each triangle:

a



b



a Area of triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \text{ cm} \times 8 \text{ cm}$

$= 48 \text{ cm}^2$

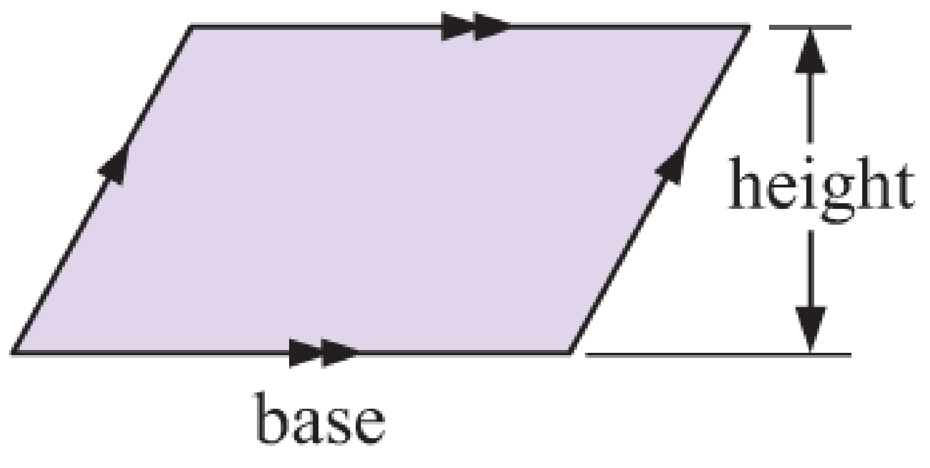
b Area of triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 15 \text{ cm} \times 7 \text{ cm}$

$= 52.5 \text{ cm}^2$

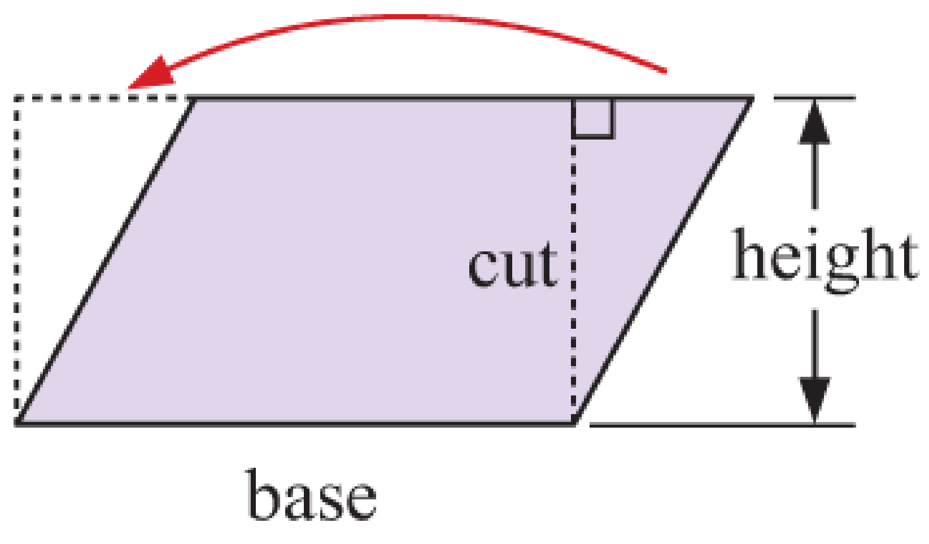
PARALLELOGRAMS

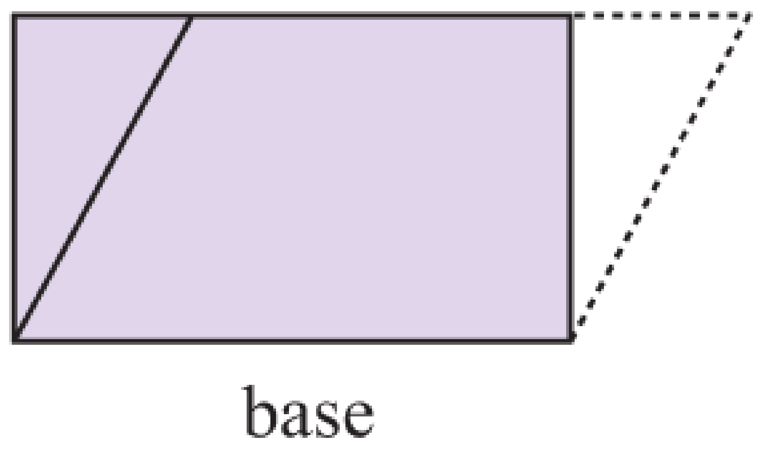


Area of parallelogram $=$ **base** $\times$ **height**

We can demonstrate this formula by cutting out a triangle from one end of the parallelogram and shifting it to the other end. The resulting shape is a rectangle with the same base and height as the parallelogram.

DEMO



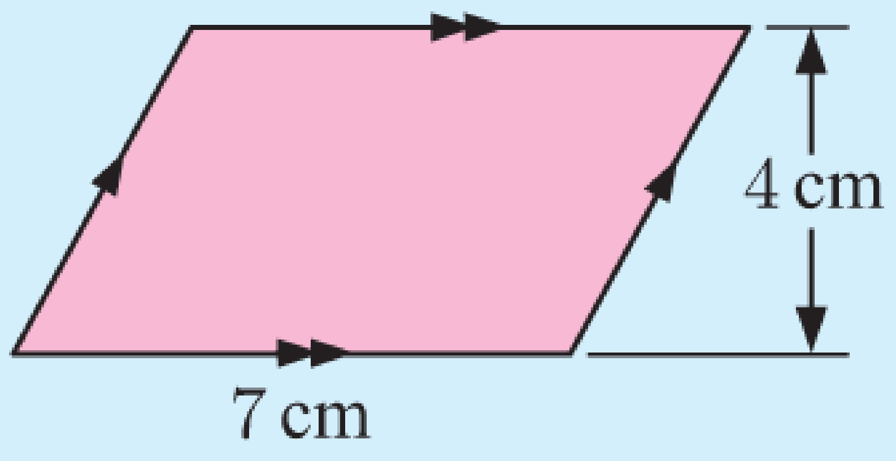


Perform this demonstration for yourself using paper and scissors.

Example 3

Self Tutor

Find the area of:

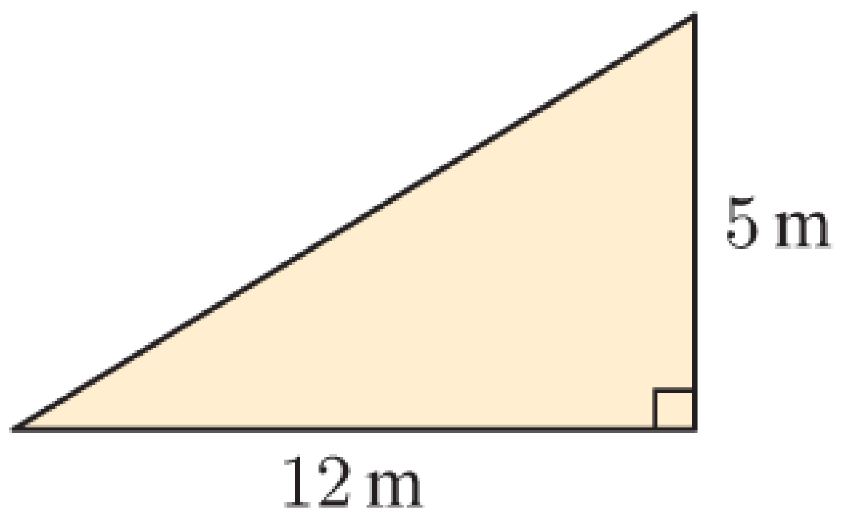


$$\begin{aligned}\text{Area} &= \text{base} \times \text{height}\\ &= 7\ \text{cm} \times 4\ \text{cm}\\ &= 28\ \text{cm}^2\end{aligned}$$

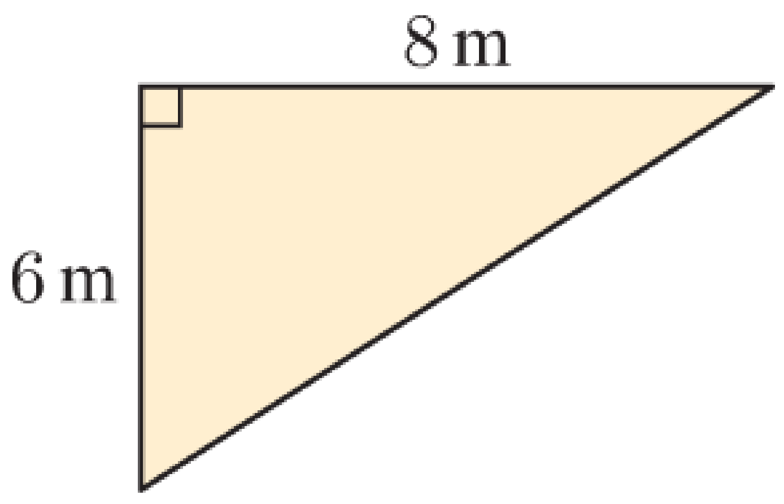
EXERCISE 14C

1 Find the area of each triangle:

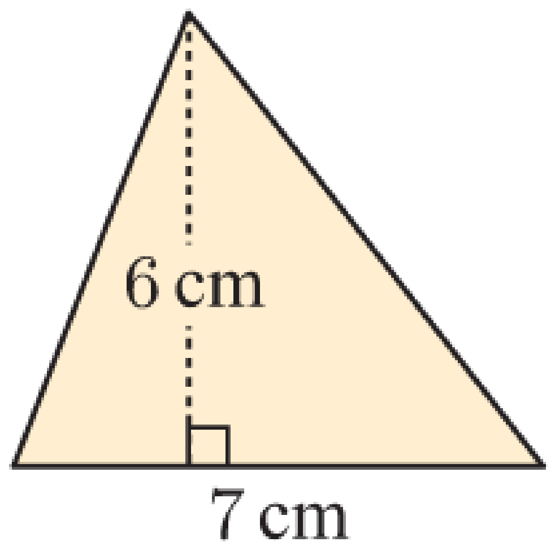
a



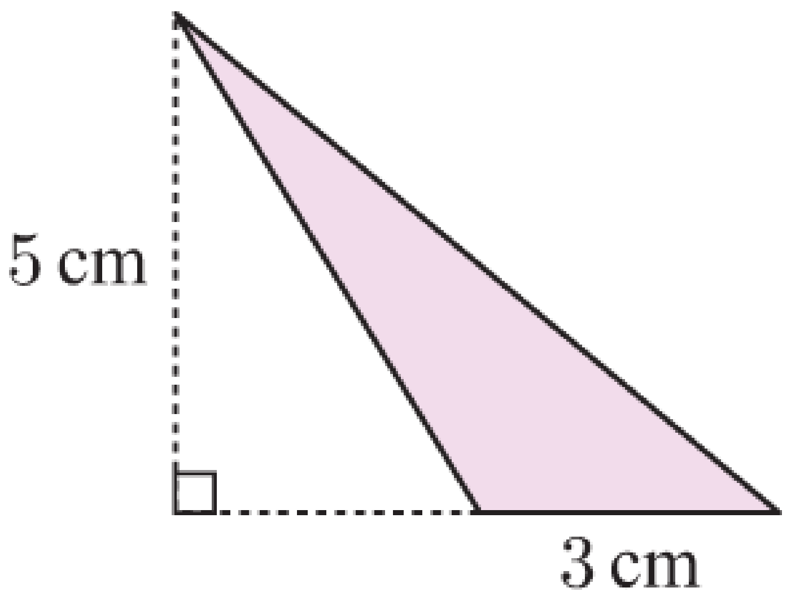
b



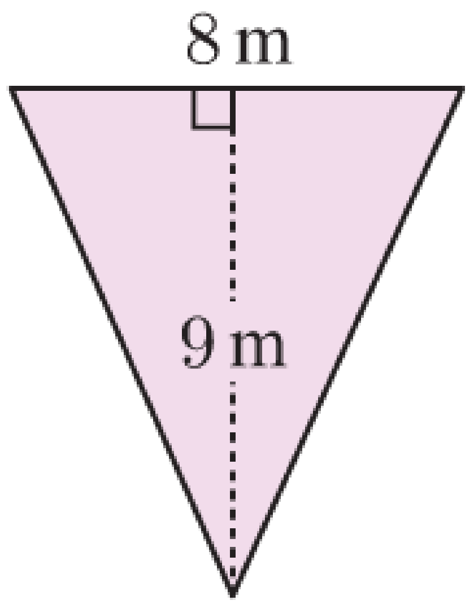
c



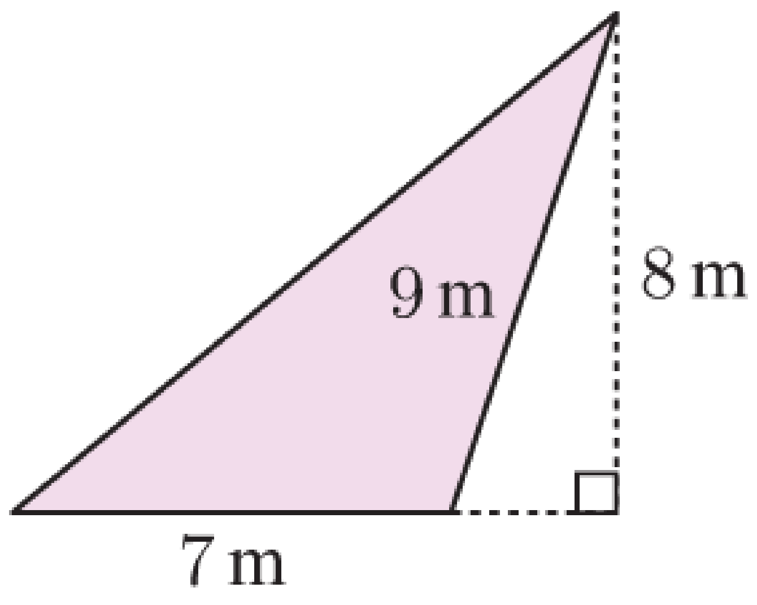
d



e

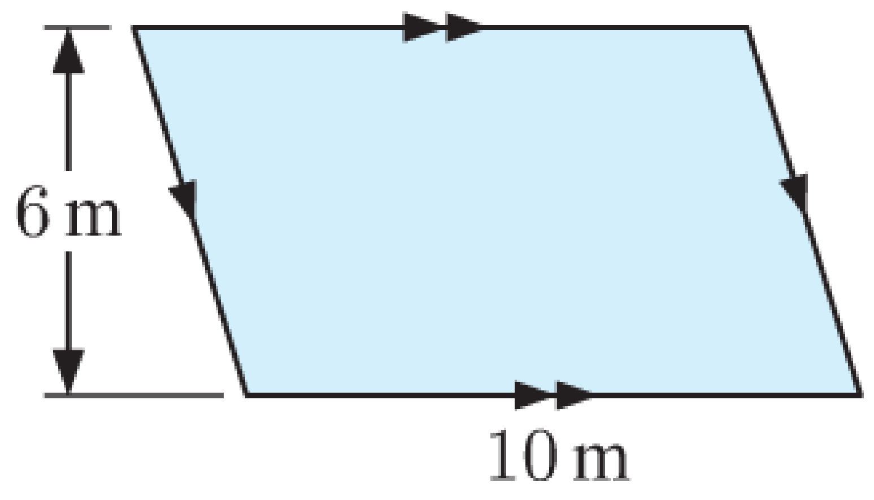


f

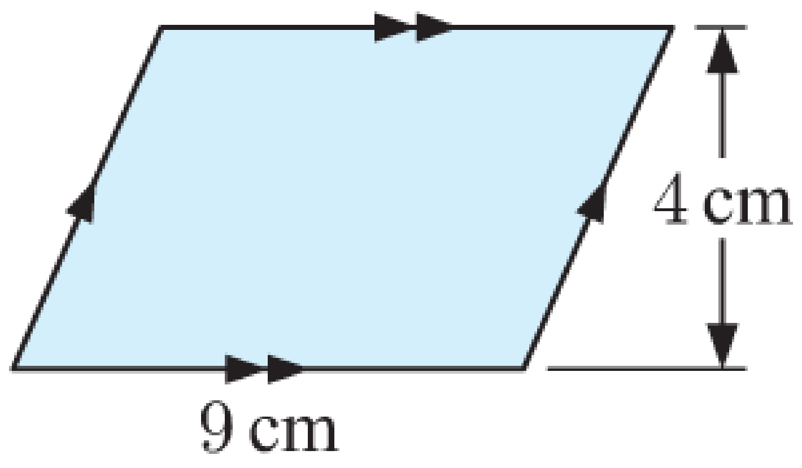


2 Find the area of each parallelogram:

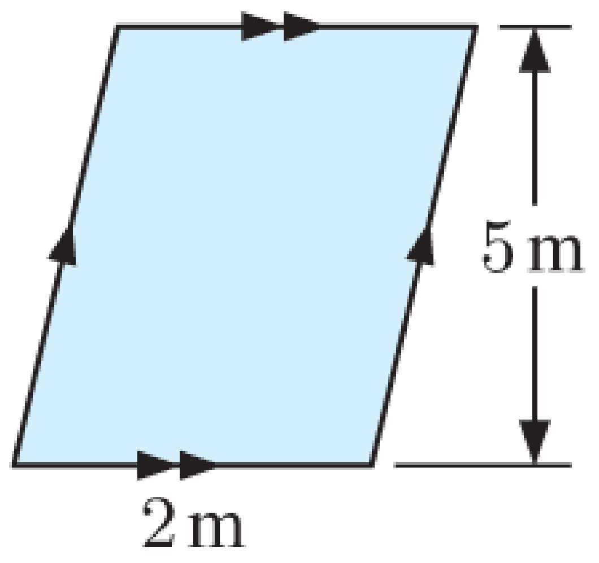
a



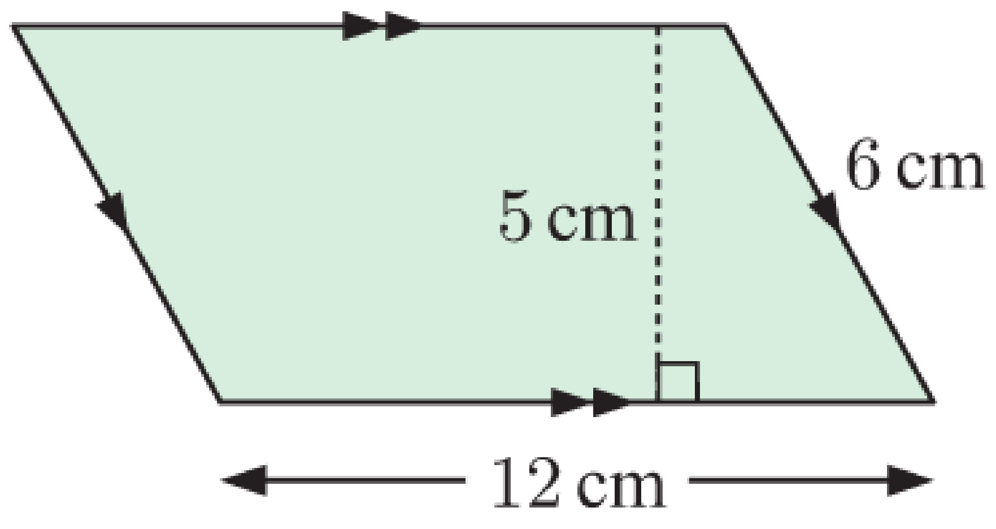
b



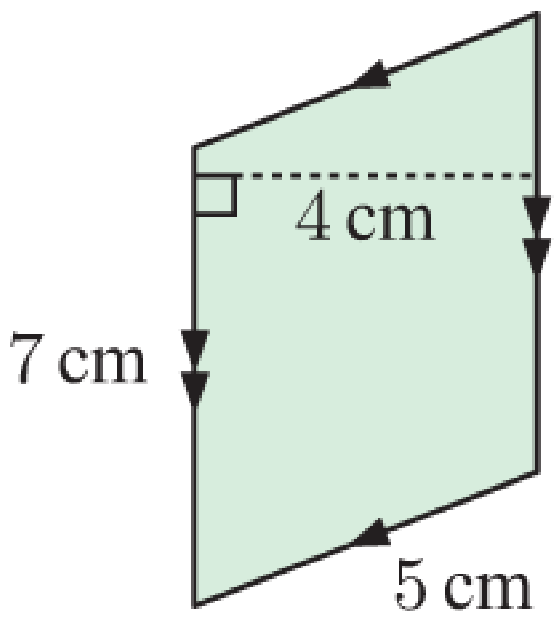
c



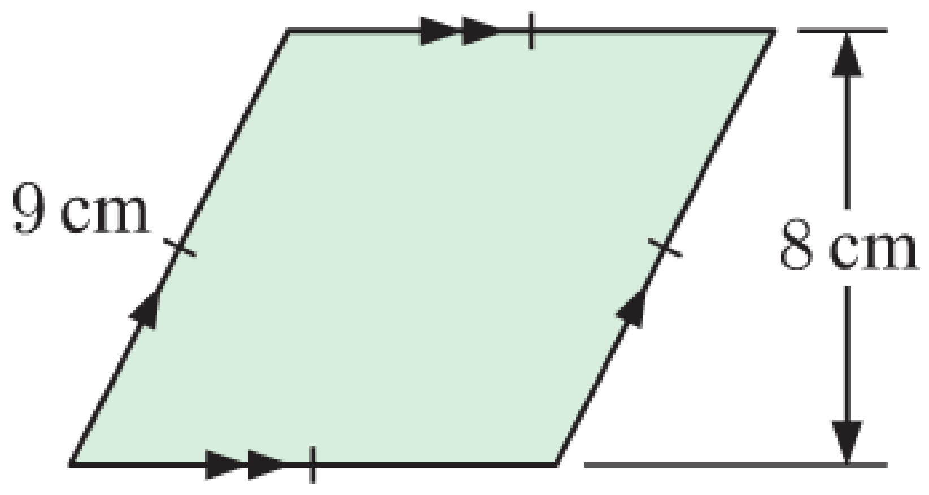
d



e



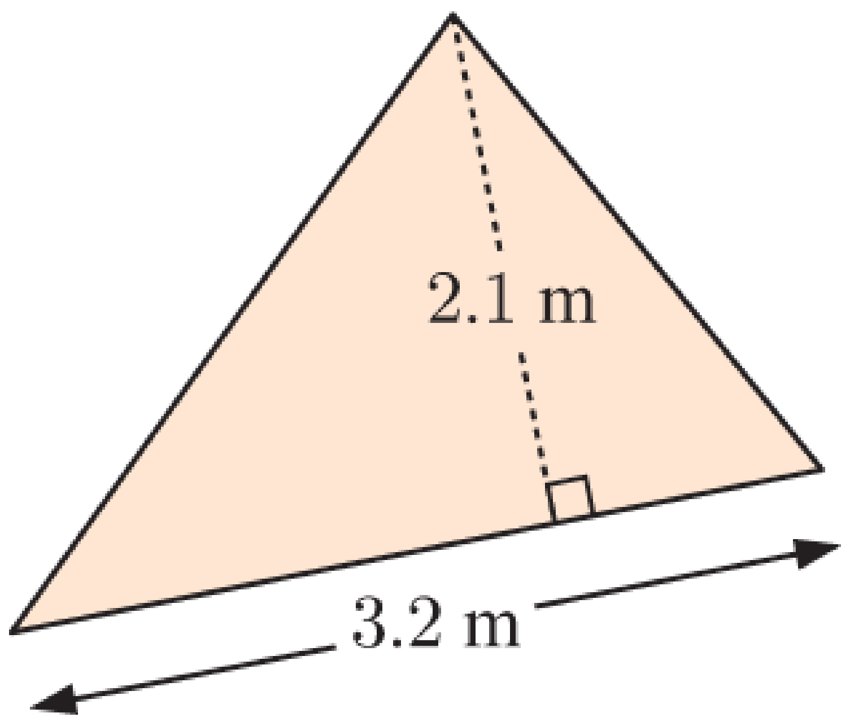
f



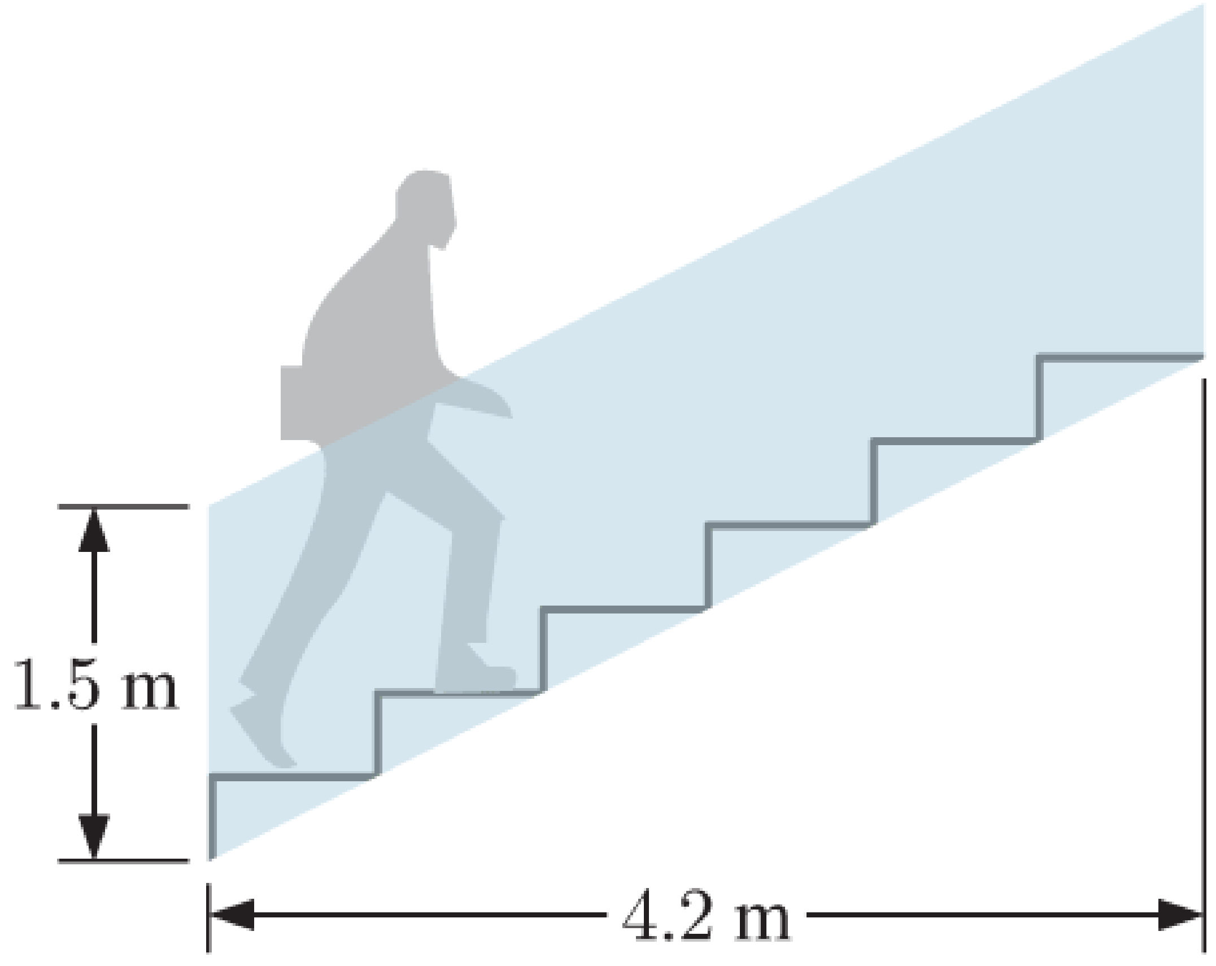
3 Alice is buying 3 identical shade sails with the dimensions shown. The shadecloth costs €17.00 per m^2.

a Find the total area of shadecloth.

b Find the total cost of the shadecloth.

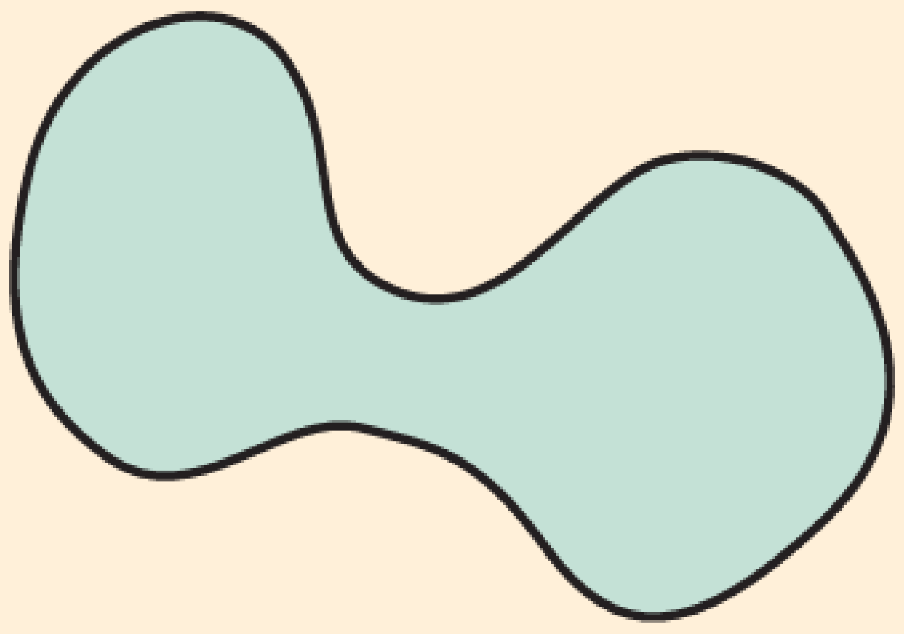


4 A perspex safety guard for one side of a staircase is made with the dimensions shown. Find the area of perspex used.



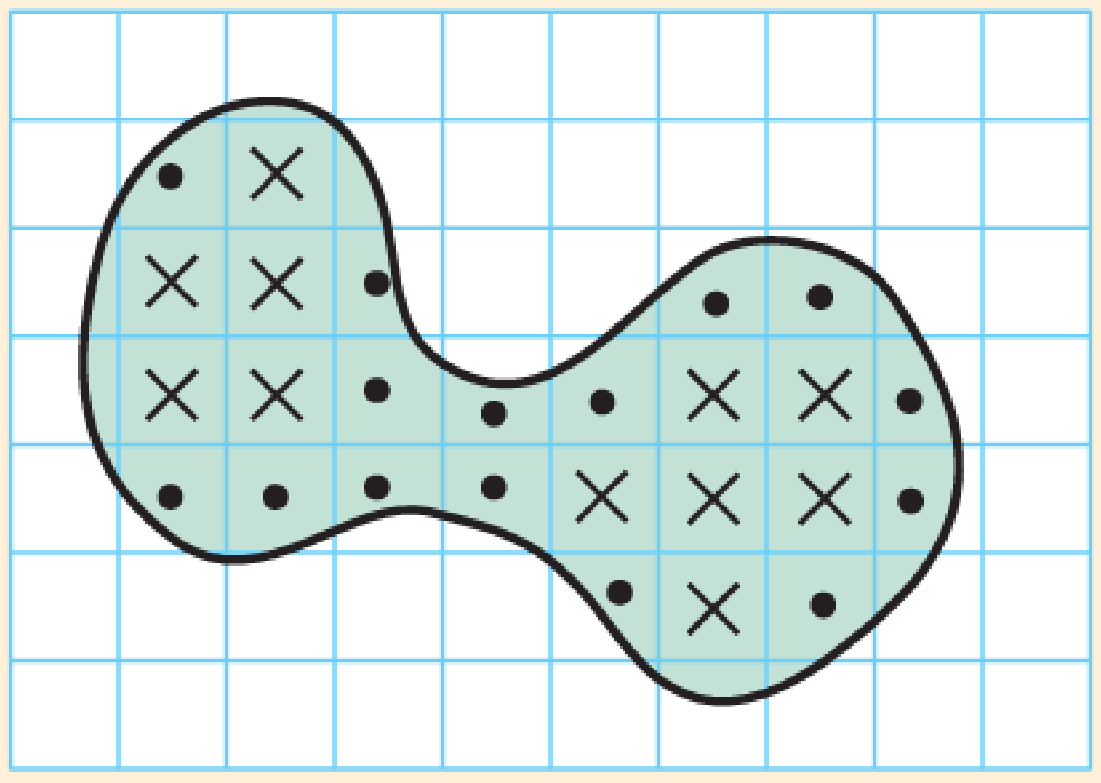
INVESTIGATION 3 AREAS OF IRREGULAR SHAPES

Have you ever thought how you could determine the area of a shape which is not regular?



For example, consider the figure alongside. We can *estimate* its area by drawing grid lines across the figure.

We count all the full squares, and as we do so we cross them out.



Now we have to make a decision about the part squares inside the shape. We can count squares which are more than half full as 1, and those less than half full as 0.

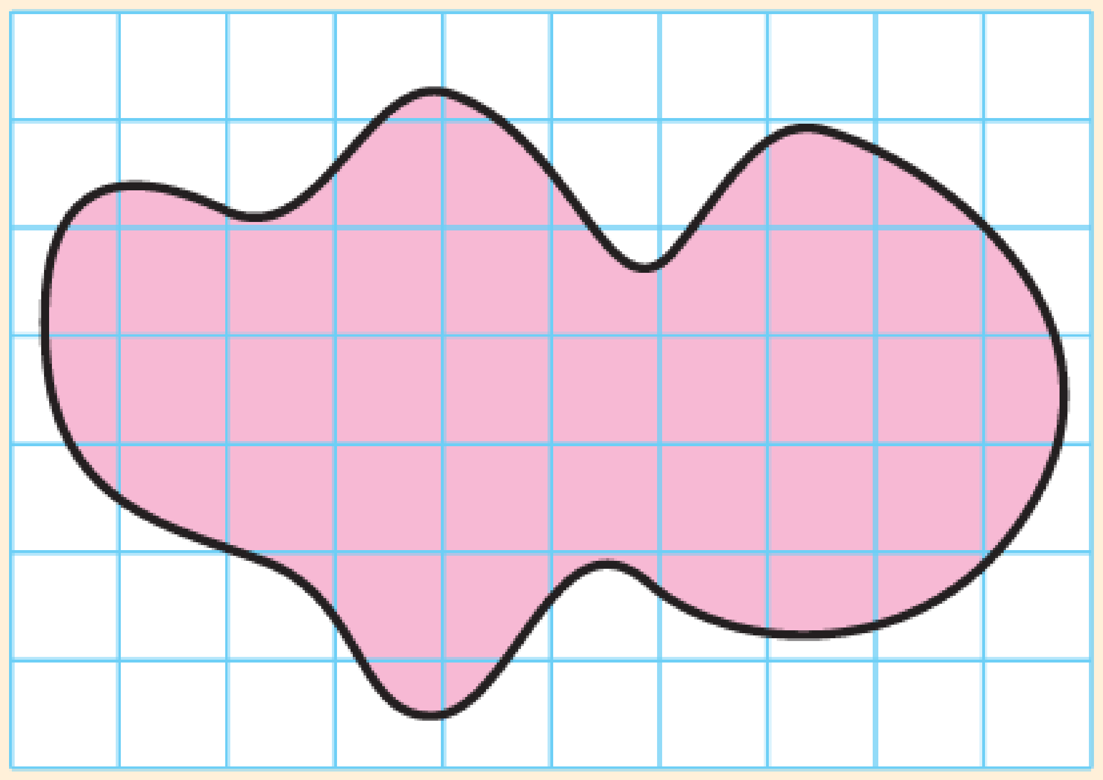
We hope that errors will cancel each other out when we add all of these together.

We thus estimate the total area to be 26 square units.

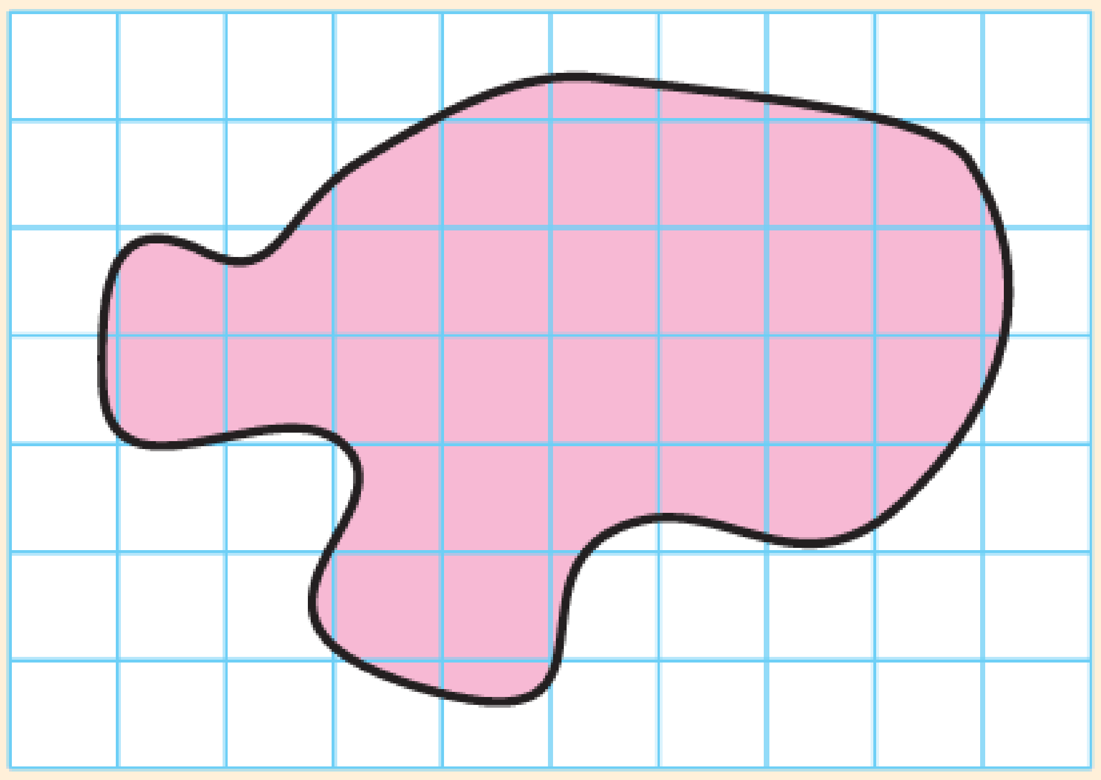
What to do:

1 Estimate the area of each shape:

a



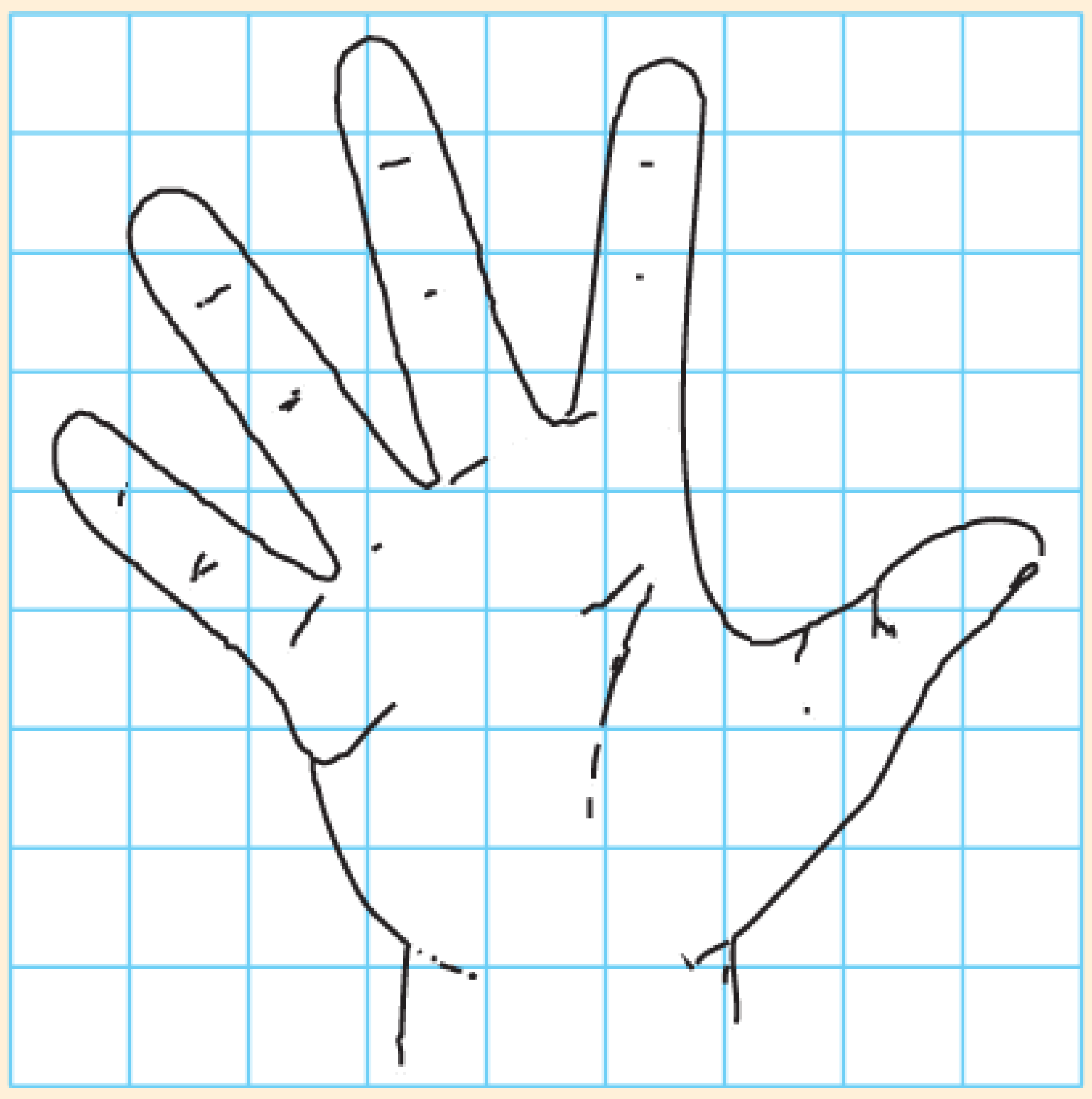
b



2 Place your hand on cm^2 grid paper and trace around the outside.

PRINTABLE GRID PAPER

a Estimate the area of your hand in cm^2.



b Do you think your estimate will be more accurate if your fingers are together or apart? Explain your answer.

D VOLUME

This stone

occupies more space than this pebble.

We say that the stone has greater *volume* than the pebble.

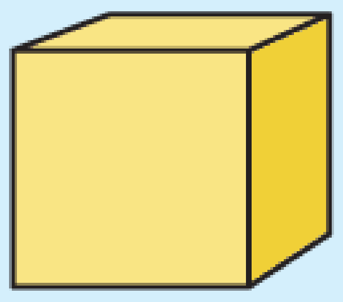
The **volume** of a solid is the amount of space it occupies.

This space is measured in **cubic units**.

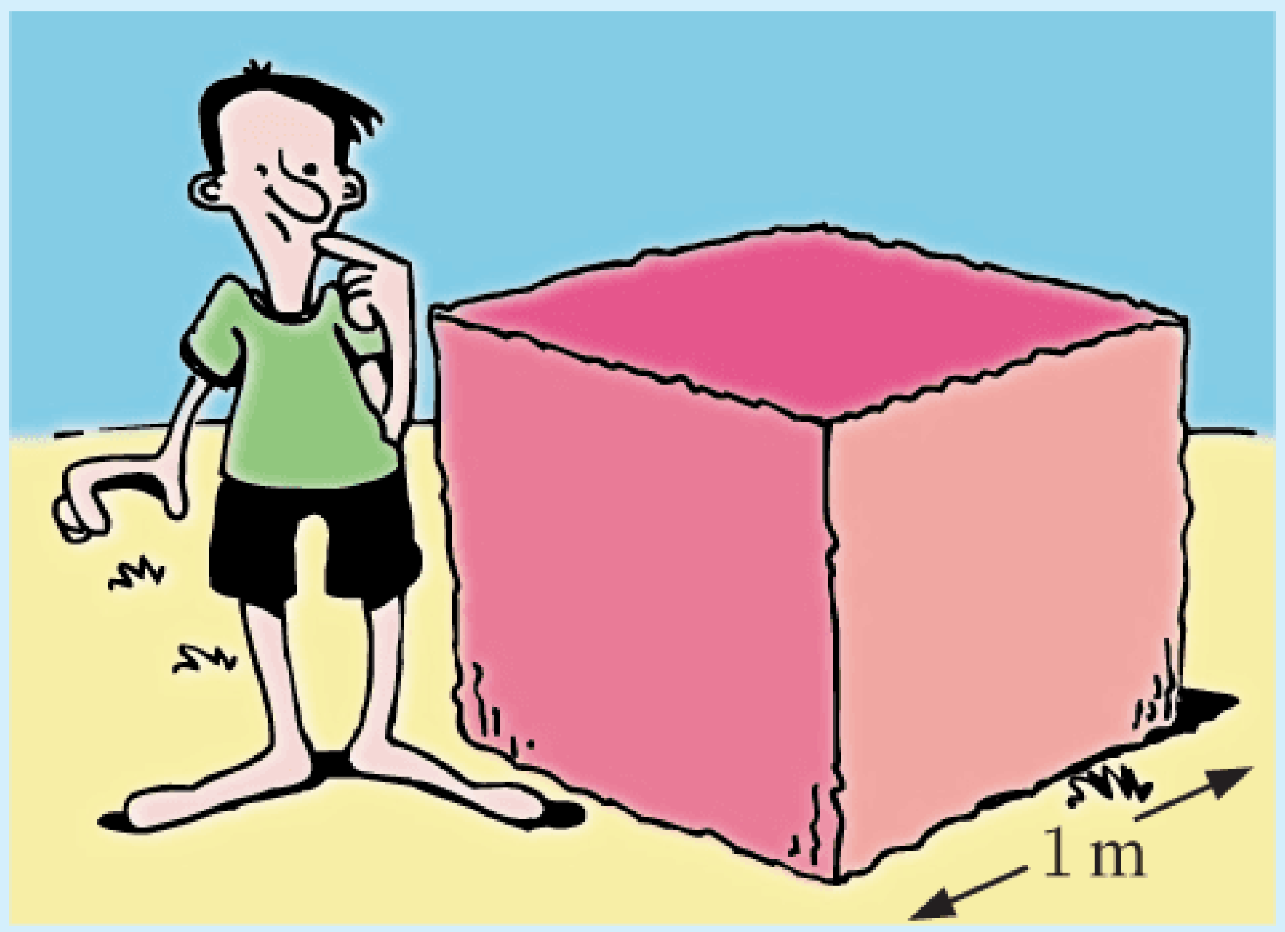
As with area, the units used for volume are related to the units used for length.

1 cubic millimetre (mm^3) is the volume of a cube with side length 1 mm.

1 cubic centimetre (cm^3) is the volume of a cube with side length 1 cm.



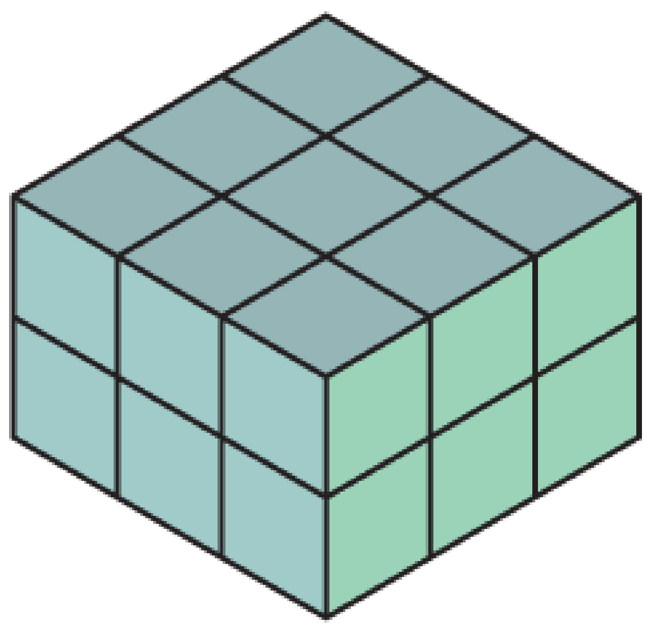
1 cubic metre (m^3) is the volume of a cube with side length 1 m.



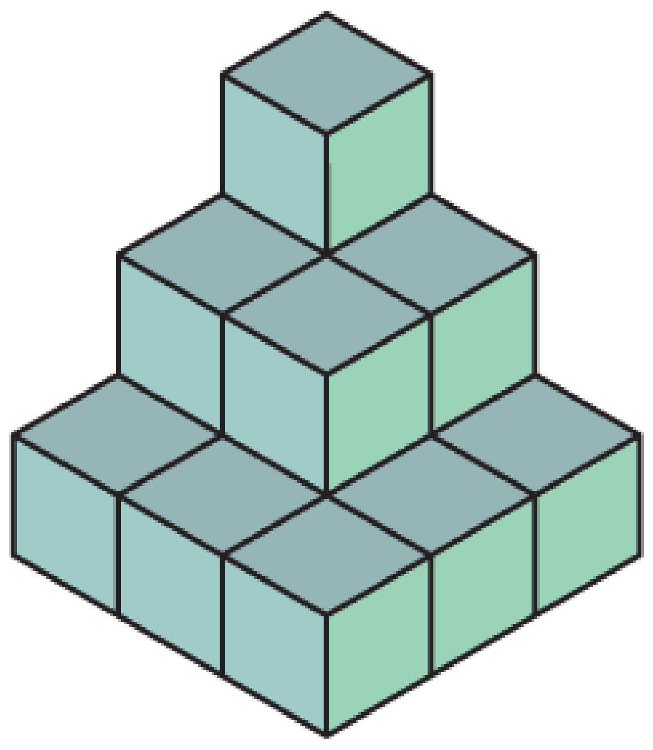
EXERCISE 14D.1

1 **a** Count the number of cubic units in each solid:

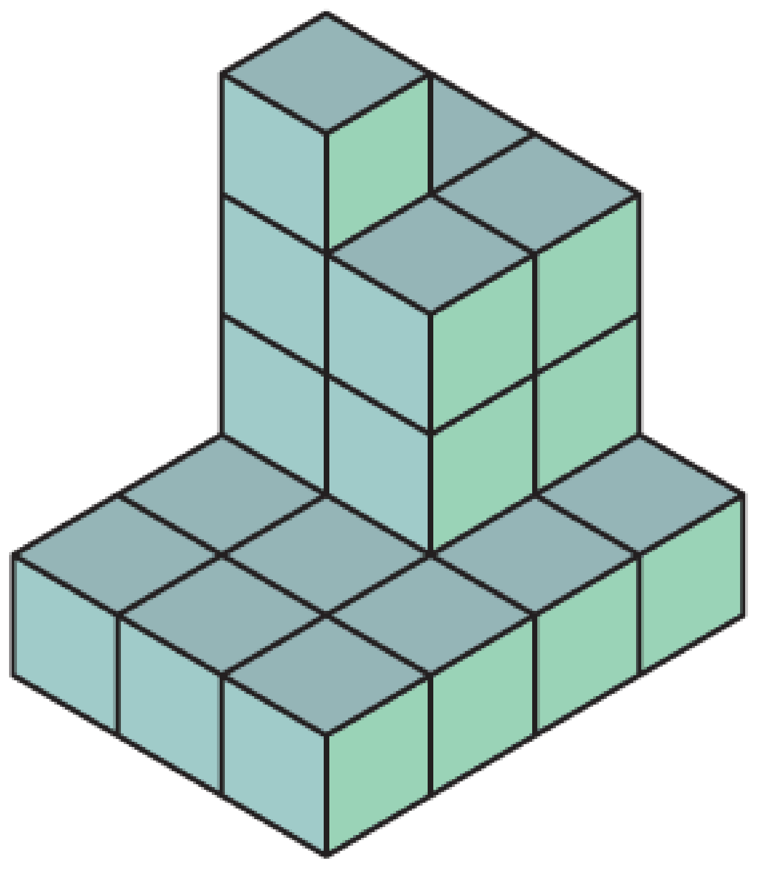
i



ii



iii



b Write down the solids in order of volume, from lowest to highest.

2 The volume of a die would be:

A 2 mm^3 **B** 2 cm^3 **C** 20 cm^3 **D** 2 m^3

3 The volume of a telephone box would be:

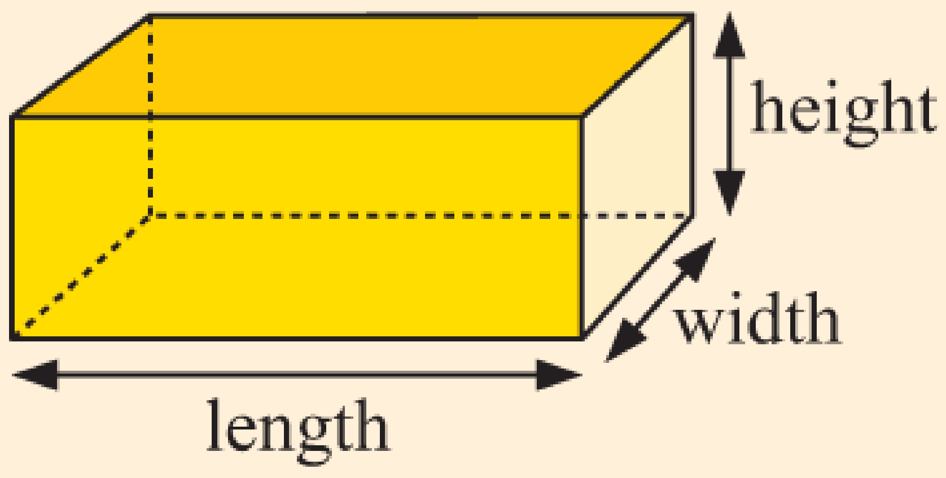
A 3 cm^3 **B** 300 cm^3 **C** 3 m^3 **D** 30 m^3

THE VOLUME OF A RECTANGULAR PRISM

INVESTIGATION 4 THE VOLUME OF A RECTANGULAR PRISM

In this Investigation we will discover a quick way to find the volume of a rectangular prism or box.

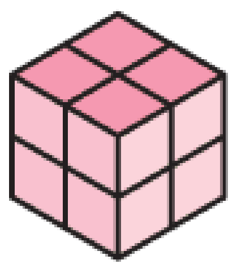
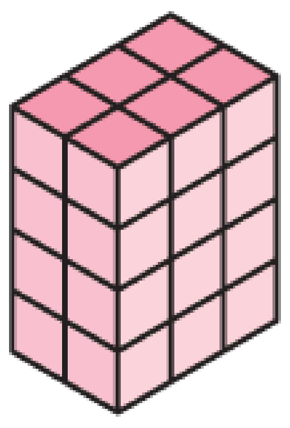
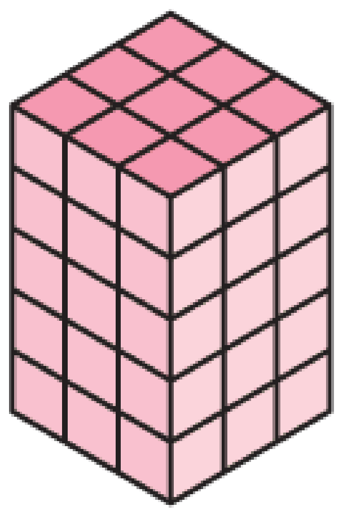
Remember that the *length* of a rectangular prism is the distance across the front face, the *height* is the height of the front face, and the *width* is the distance between the front and back faces.



What to do:

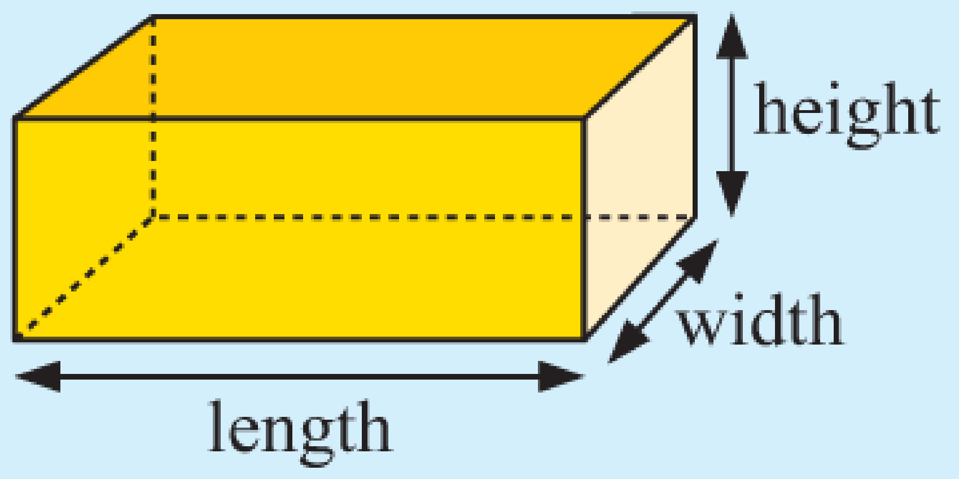
1 Copy and complete the table below.
Find the volume of each rectangular prism by counting the number of cubes.

PRINTABLE TABLE

Rectangular prism	*Volume*	*Length*	*Width*	*Height*	*Length* × *Width* × *Height*
					
					
					

2 Copy and complete: *The volume of a rectangular prism* = ...

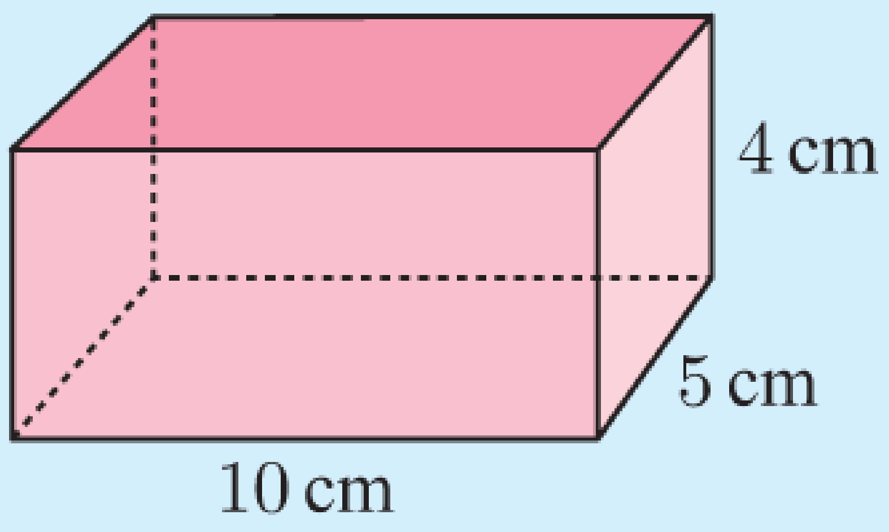
From the previous **Investigation**, you should have discovered that:



Volume of rectangular prism $=$ length $\times$ width $\times$ height

Example 4 **Self Tutor**

Find the volume of this rectangular prism:

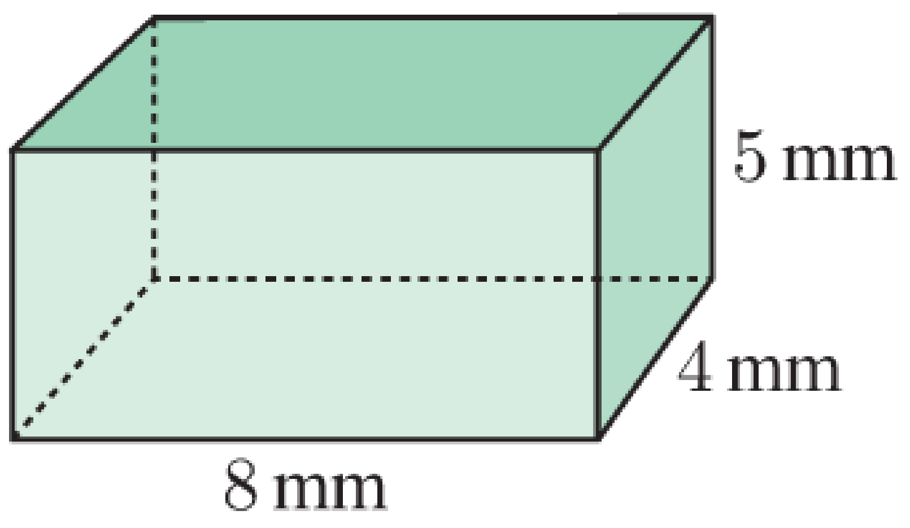


Volume
$= \text{length} \times \text{width} \times \text{height}$
$= 10 \text{ cm} \times 5 \text{ cm} \times 4 \text{ cm}$
$= 200 \text{ cm}^3$

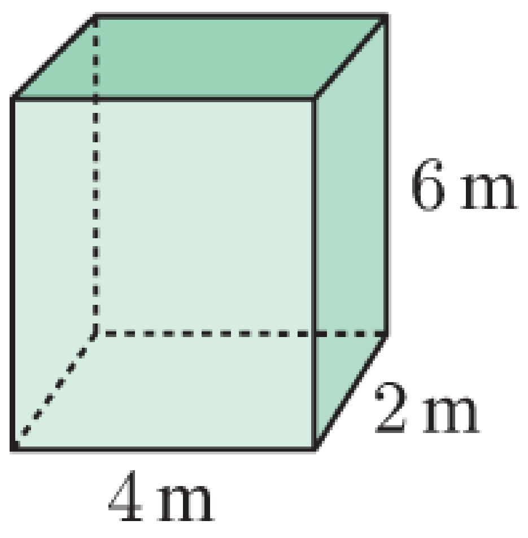
EXERCISE 14D.2

1 Find the volume of each rectangular prism:

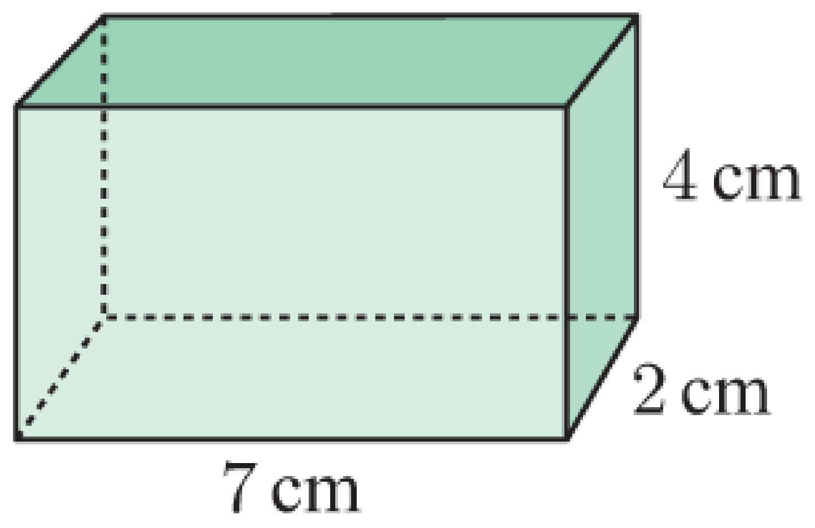
a



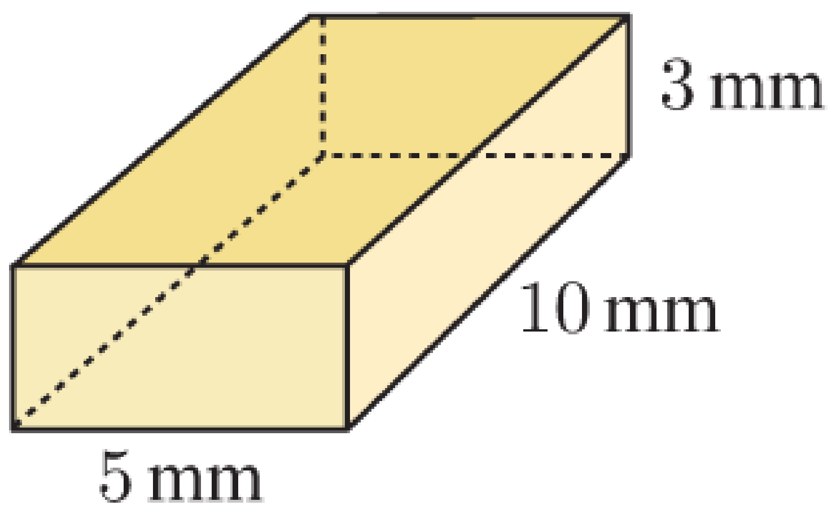
b



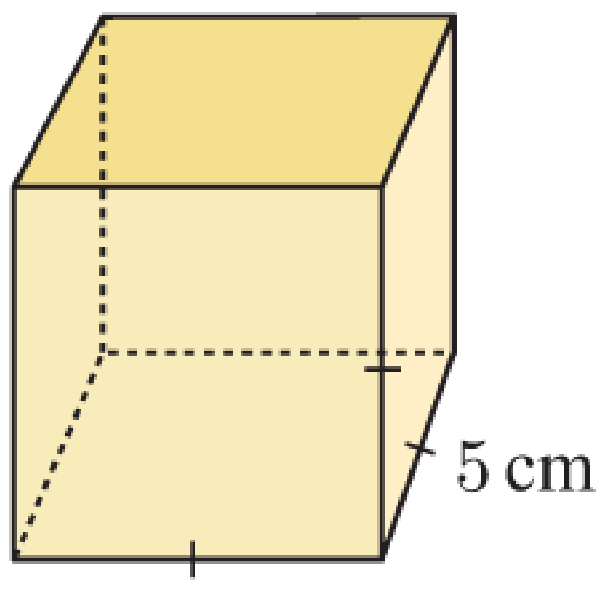
c



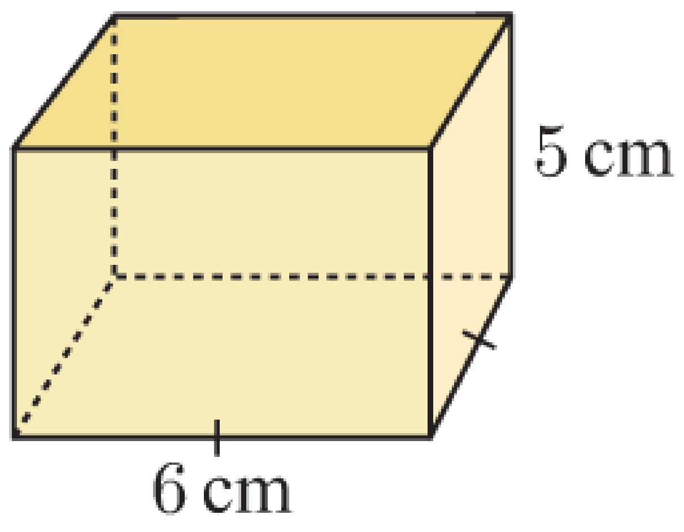
d



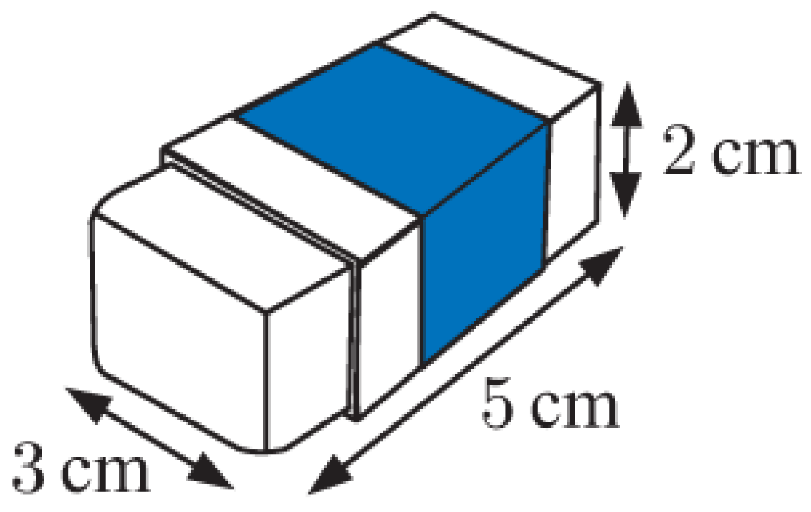
e



f

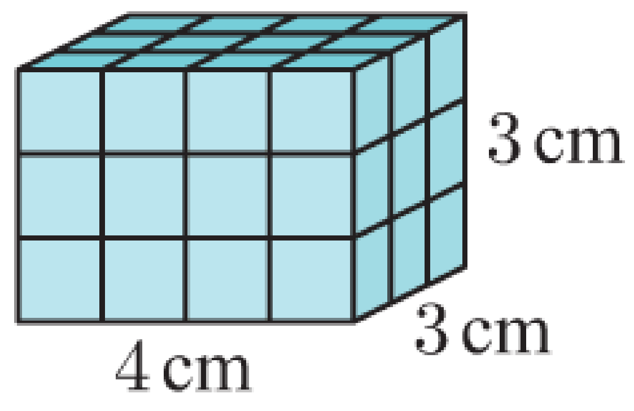


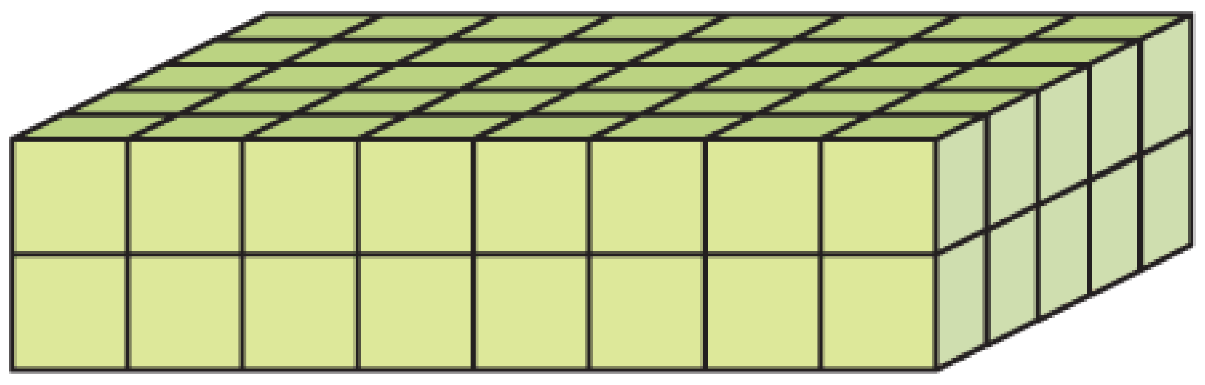
2 Find the volume of this eraser.



3 A shoe box is 30 cm long, 16 cm wide, and 10 cm high. Find the volume of air inside the shoe box.

4 The rectangular prism alongside has a volume of 36 cm^3.
Show that there are exactly 8 different rectangular prisms with whole number sides that have a volume of 36 cm^3.
You do not need to draw them.



5 

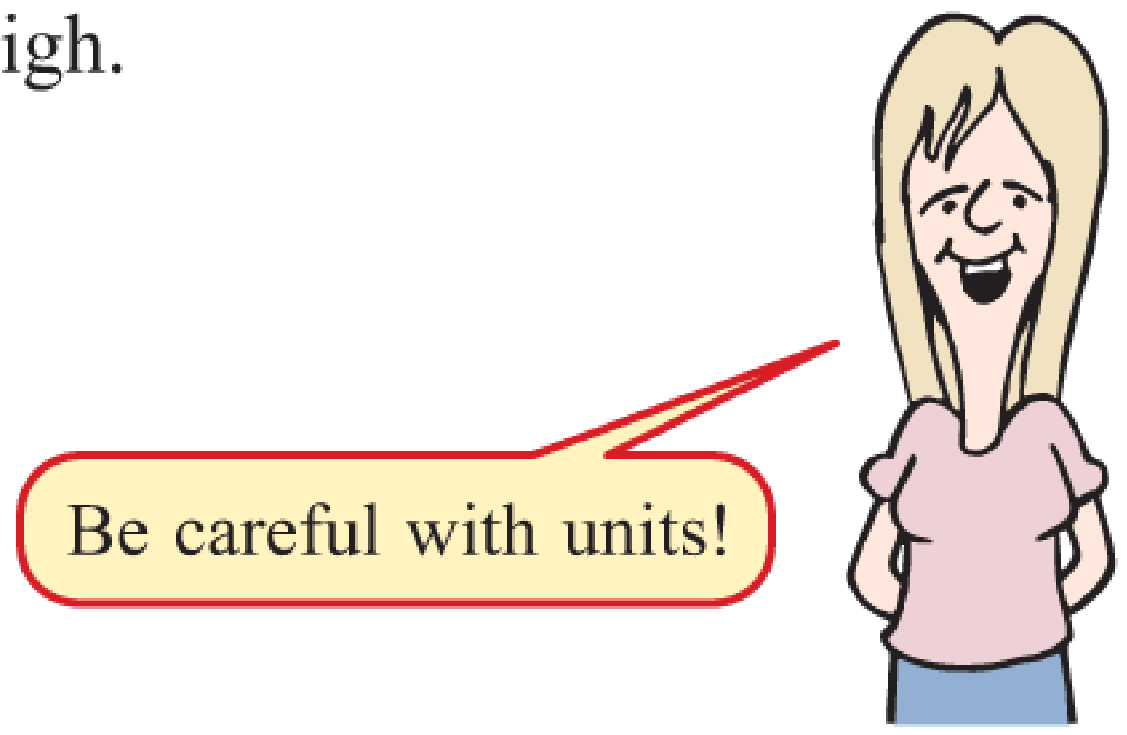
Find:

a the volume of this prism

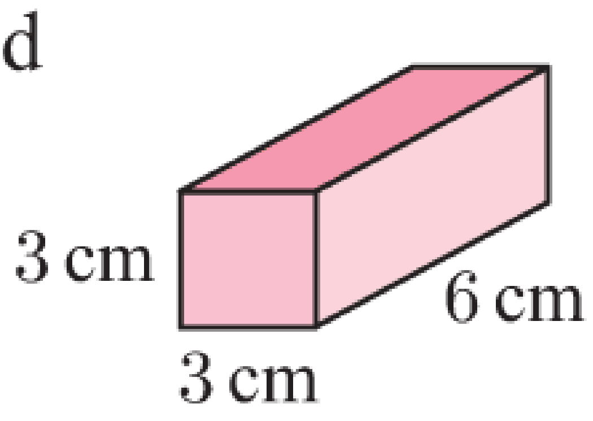
b the sum of the areas of its six faces.

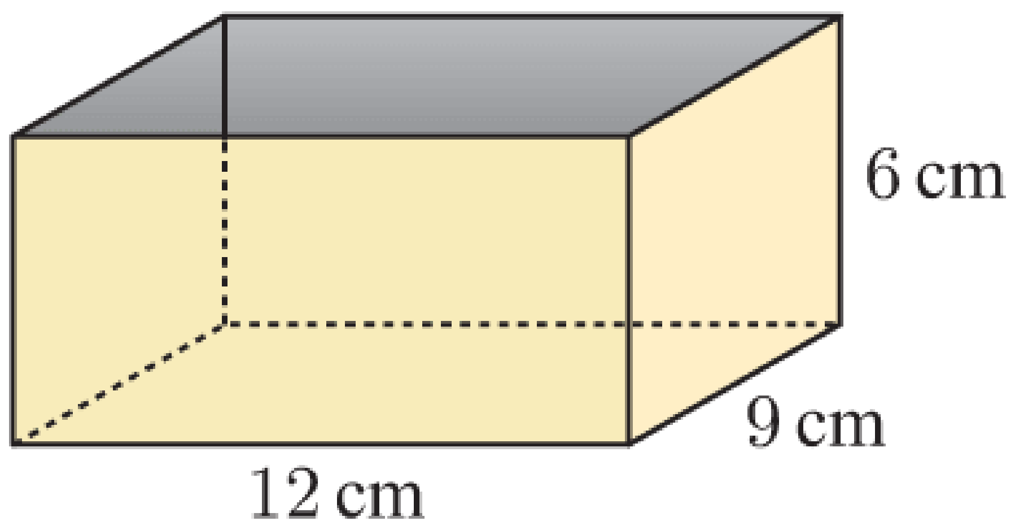
6 Answer the **Opening Problem** on page **270**.

7 An industrial vat measures 2.2 m by 3.1 m by 1.1 m high.
It is filled with dye to a level 15 cm from the top.
What volume of dye is in the vat?



8 How many small pink boxes can be packed into the large container?
Illustrate your answer.



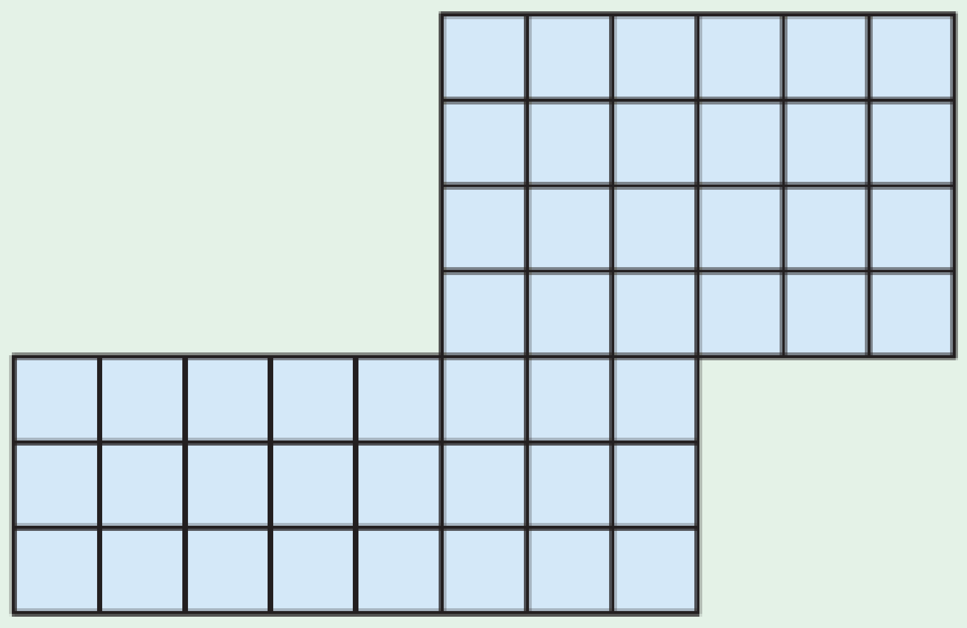


KEY WORDS USED IN THIS CHAPTER

- area
- square unit
- cubic unit
- volume
- rectangular prism

REVIEW SET 14A

1 Find the area, in square units, of this shape:



2 A rectangular garden is 6 m long and 4.5 m wide. Find the area of the garden.

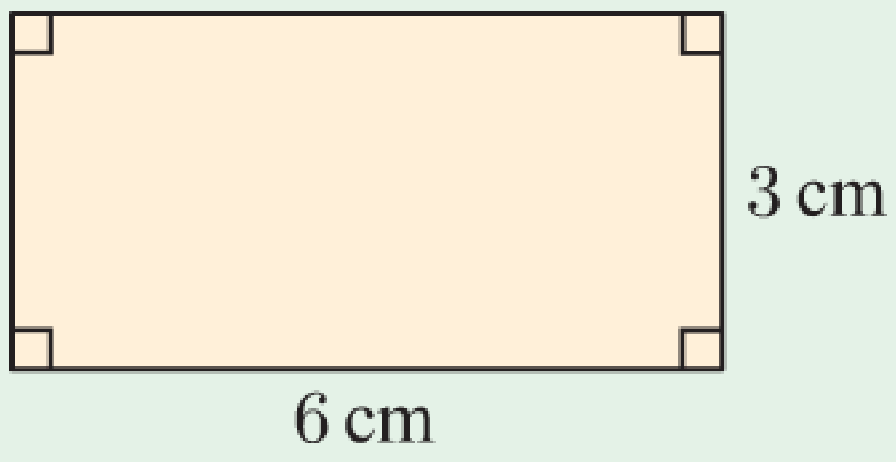
3 The area of a credit card would be:

A 4 cm^2 **B** 40 mm^2 **C** 4 m^3

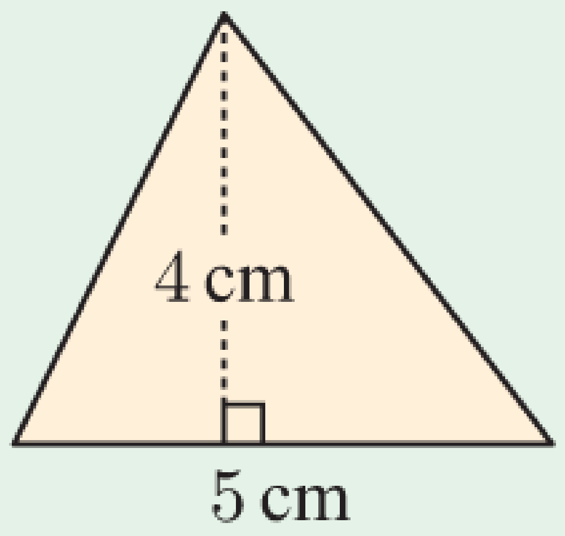
D 4 mm^2 **E** 40 cm^2

4 Find the area of each polygon:

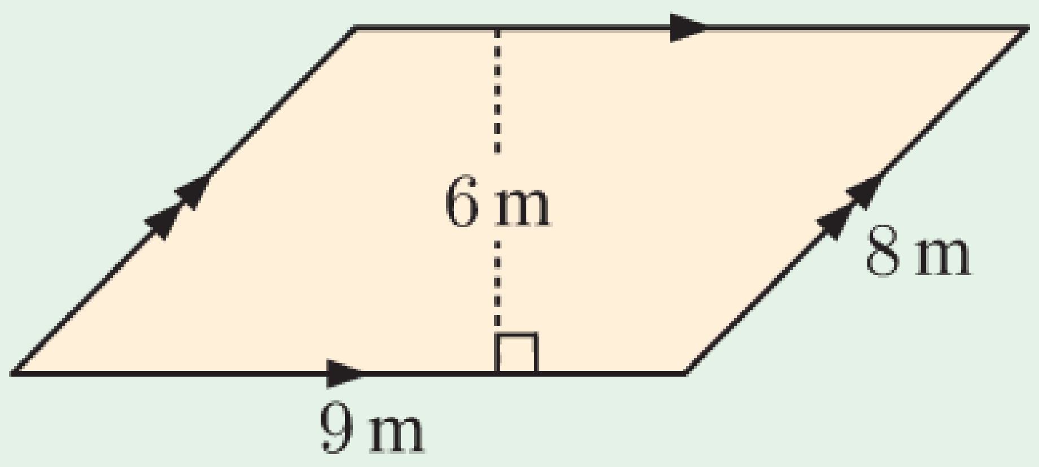
a



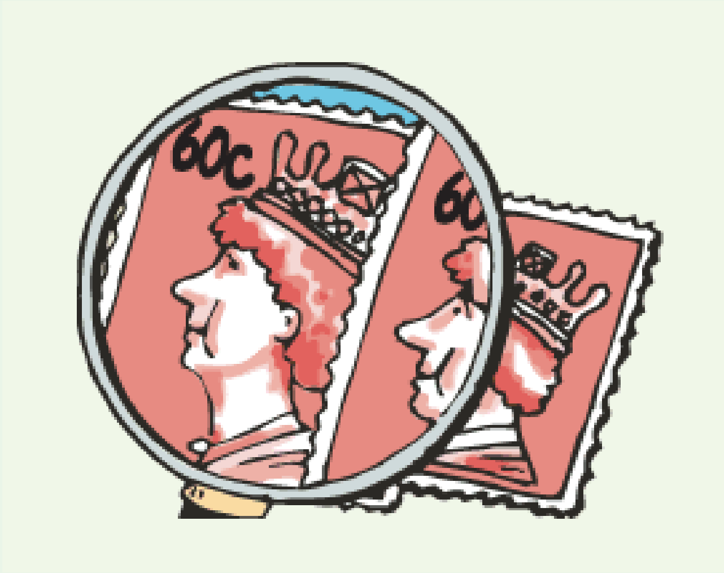
b



c

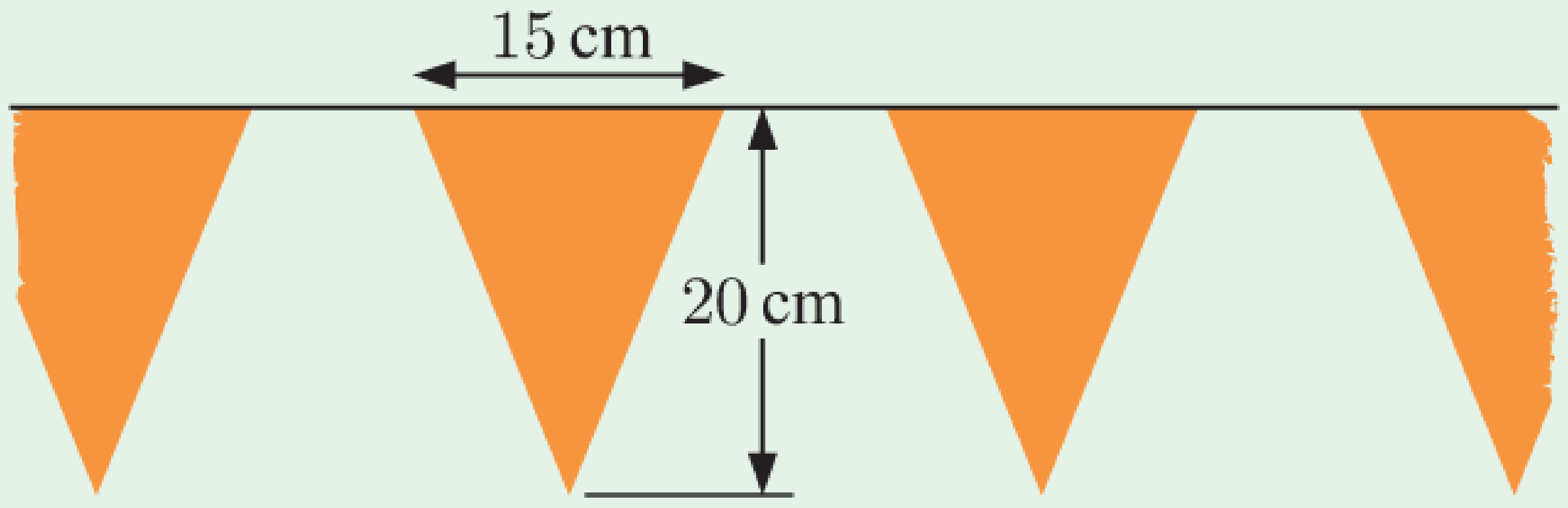


5 A particular 60-cent postage stamp is 3 cm long and 2 cm wide.



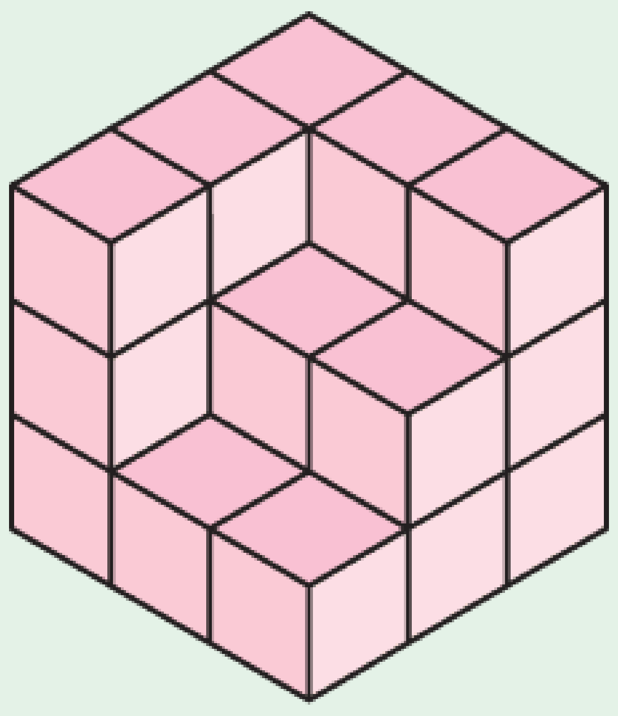
- **a** Find the area of one of the postage stamps.
- **b** How many of these stamps can fit on a 20 cm by 30 cm sheet?

6 A yachting line contains 30 flags with the dimensions shown. Find the area of material required to make the flags on the line.

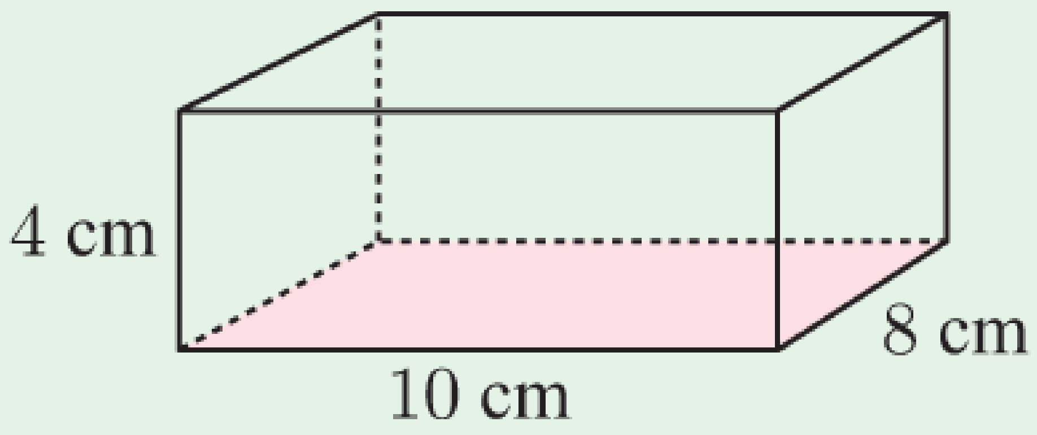


7 Find the volume of each solid:

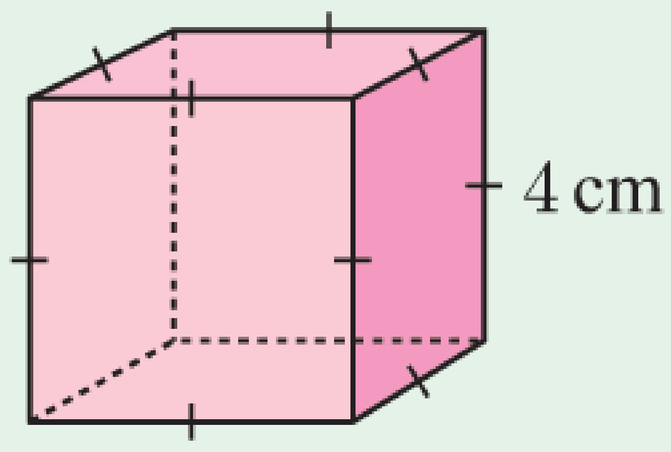
a



b



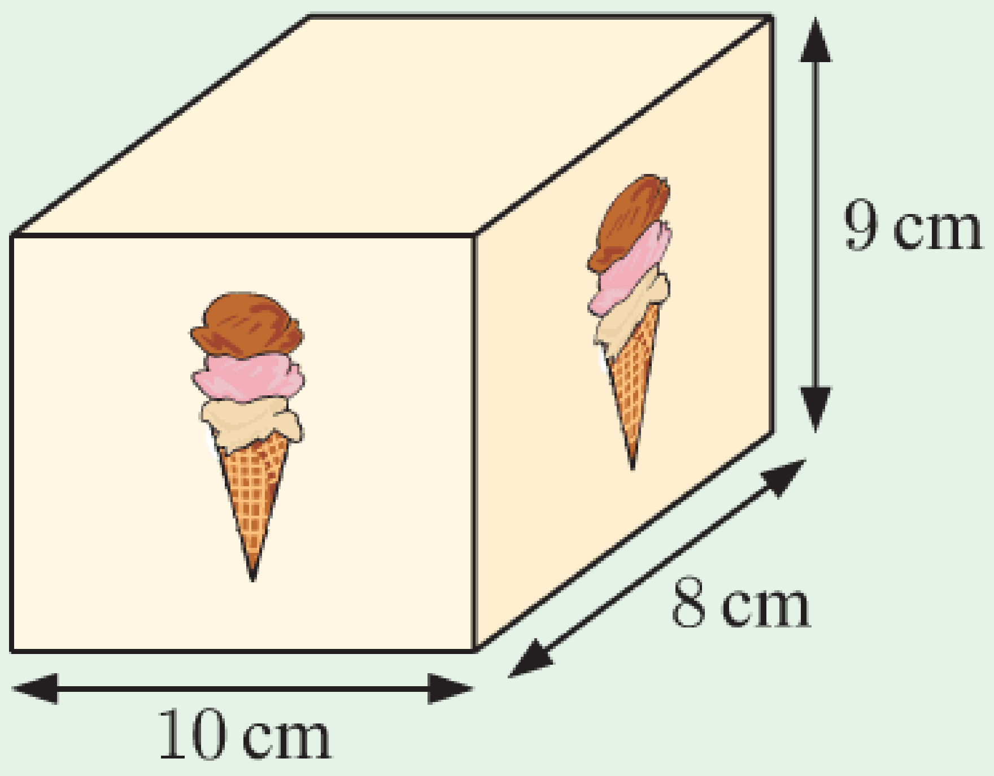
c



8 A book is 15 cm long, 10 cm wide, and 2 cm high. Find the volume of the book.

9 An ice cream container is 10 cm long, 8 cm wide, and 9 cm high.

- **a** Find the area of the base of the container.
- **b** Find the volume of the container.



10 How many 10 cm × 6 cm × 10 cm boxes can fit into a container with dimensions 60 cm × 60 cm × 60 cm?

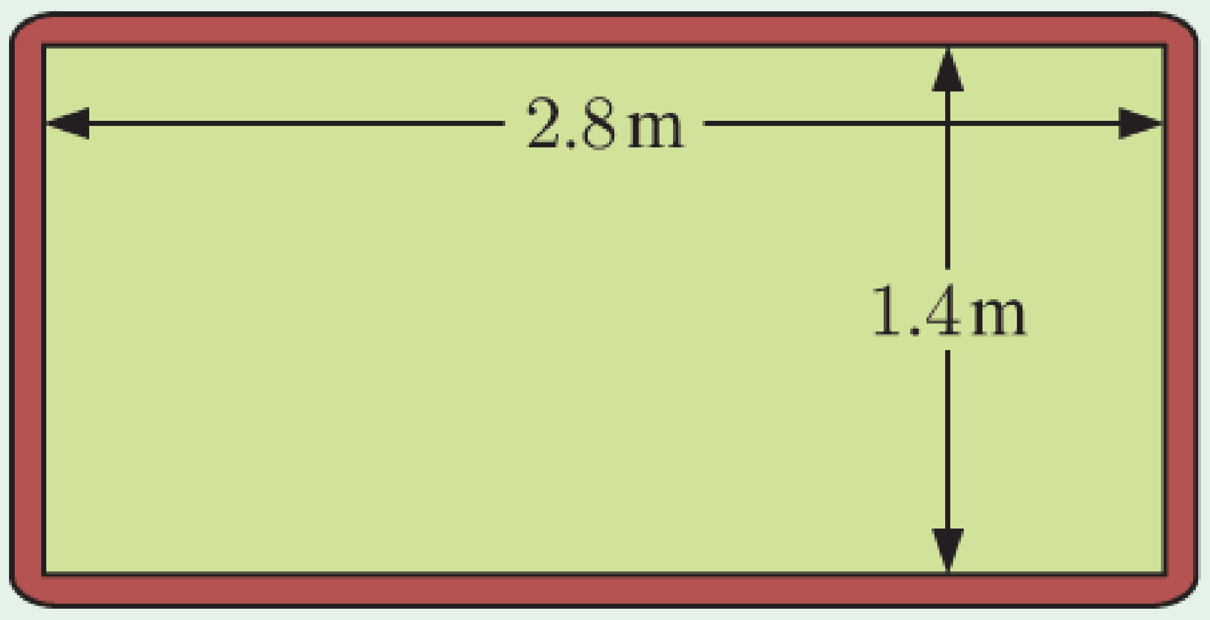
REVIEW SET 14B

1 A square has sides of length 8 cm. Find the area of the square.

2 The volume of a mattress would be:

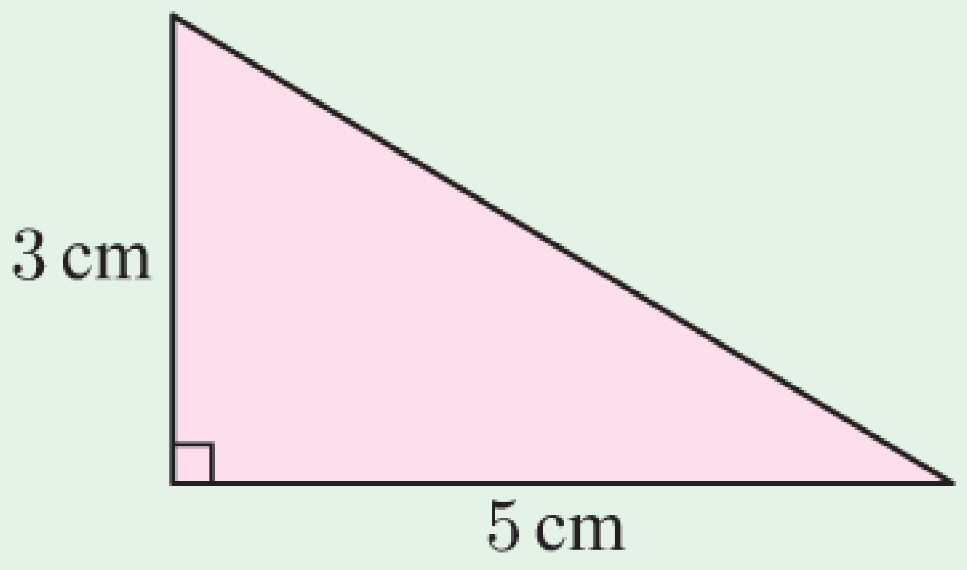
A 6 cm^3 **B** 600 cm^3 **C** 0.6 m^3 **D** 60 m^3

3 A billiard table has the dimensions shown.
The cloth which covers the table costs £89 per square metre.
Find the cost of covering the table.

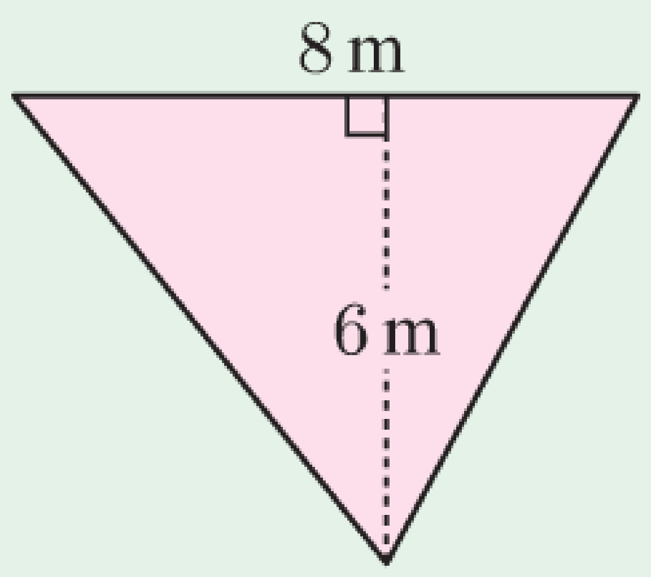


4 Find the area of each polygon:

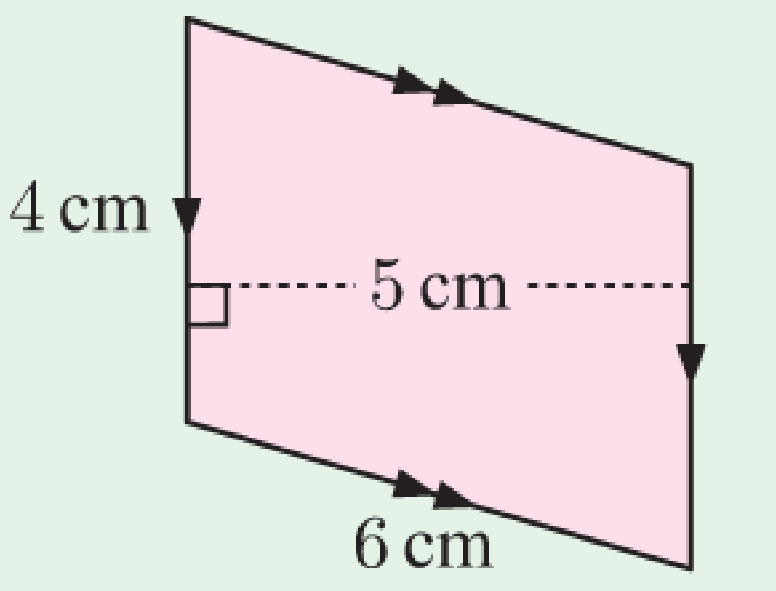
a



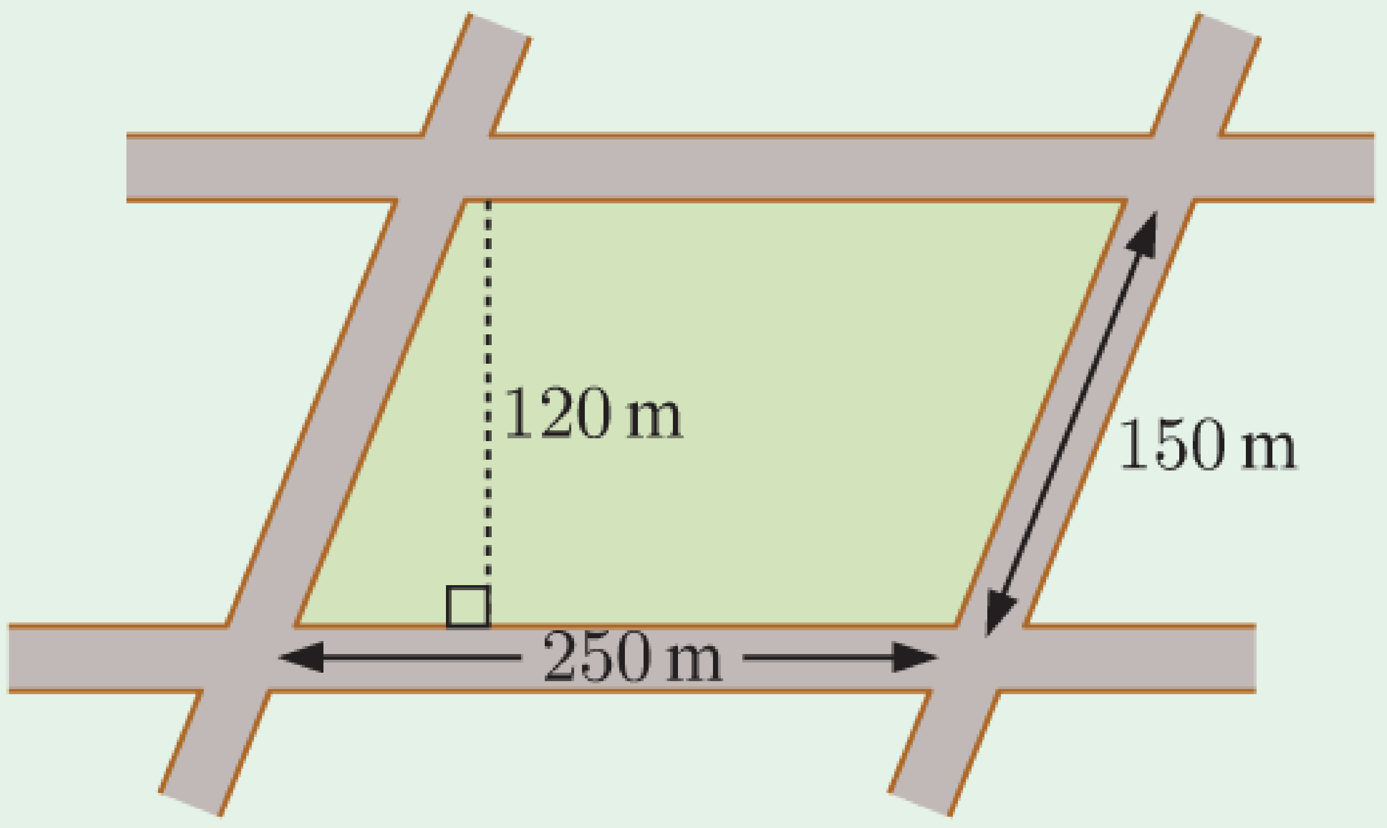
b



c

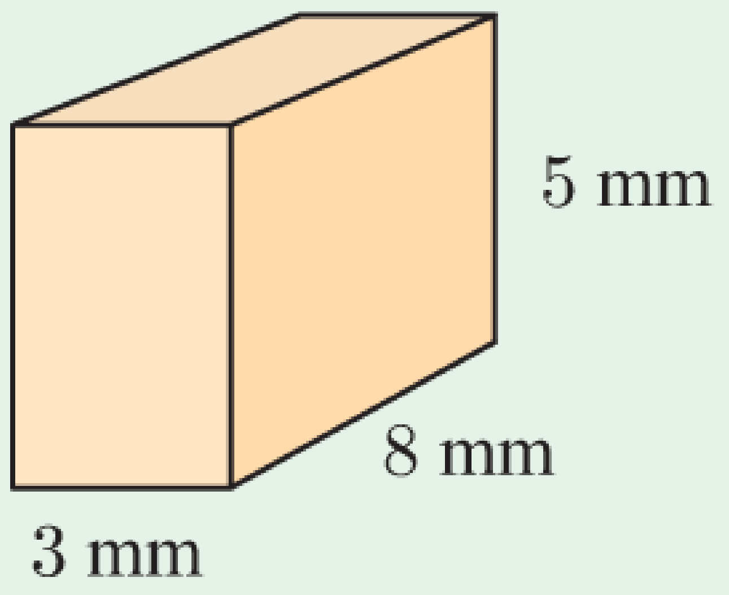


5 A park is surrounded by two sets of parallel roads as illustrated. Find the area of the park.

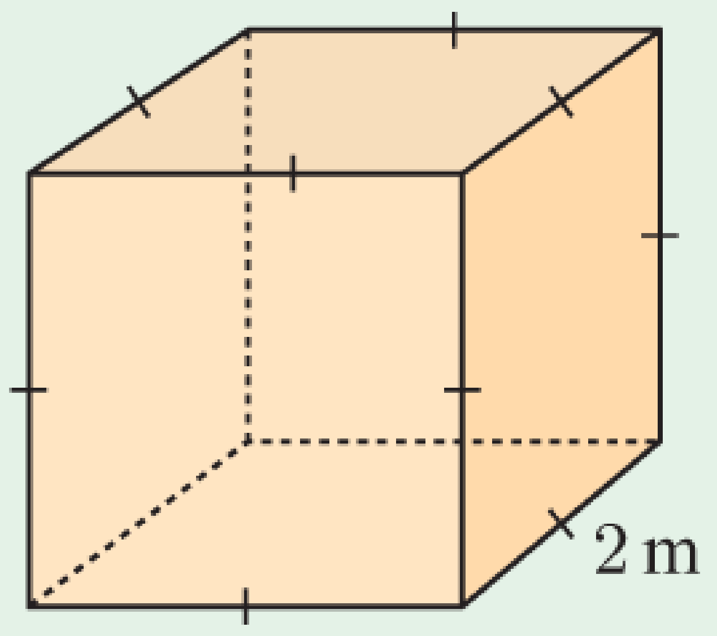


6 Find the volume of each rectangular prism:

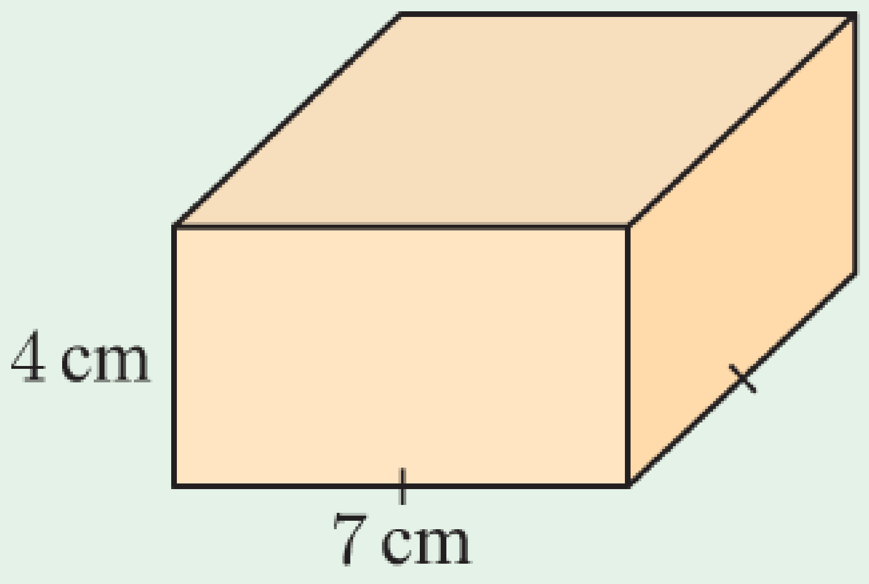
a



b

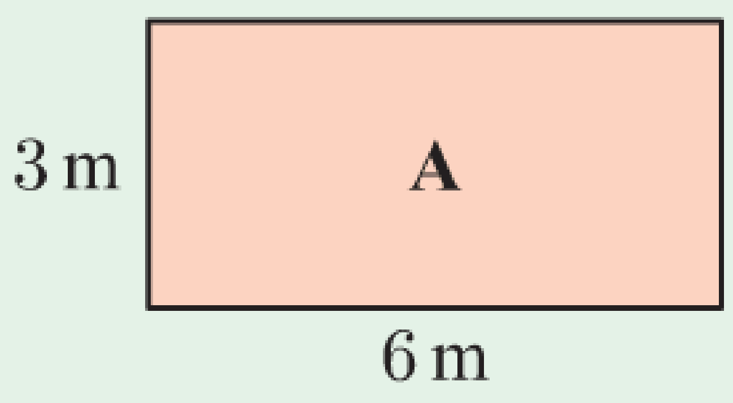


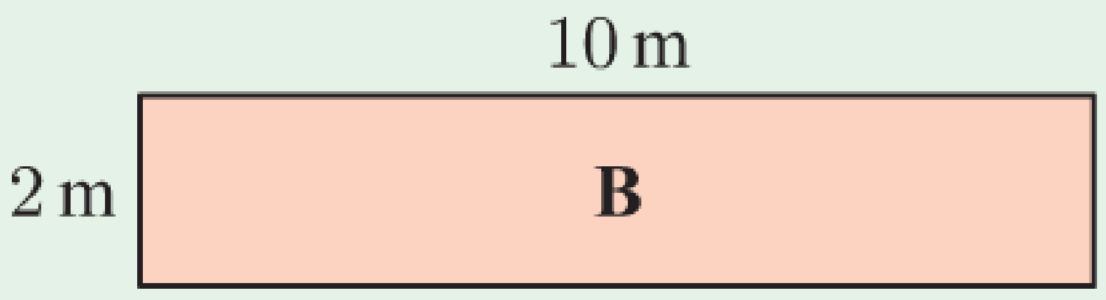
c

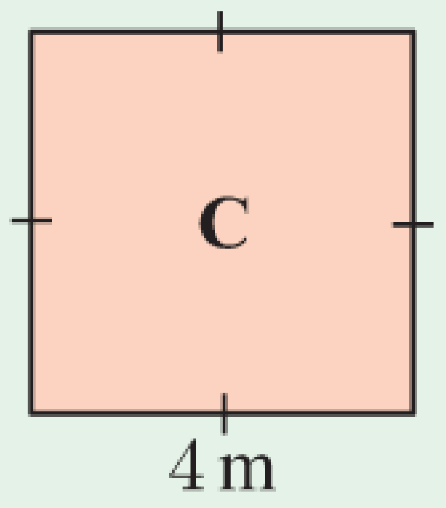


7 A rectangular garden shed is 5 m long, 4 m wide, and 4.5 m high. Find the volume of air inside the shed.

8 Which of these rectangles has the largest area?







9 This large rectangle has been divided into three smaller rectangles.

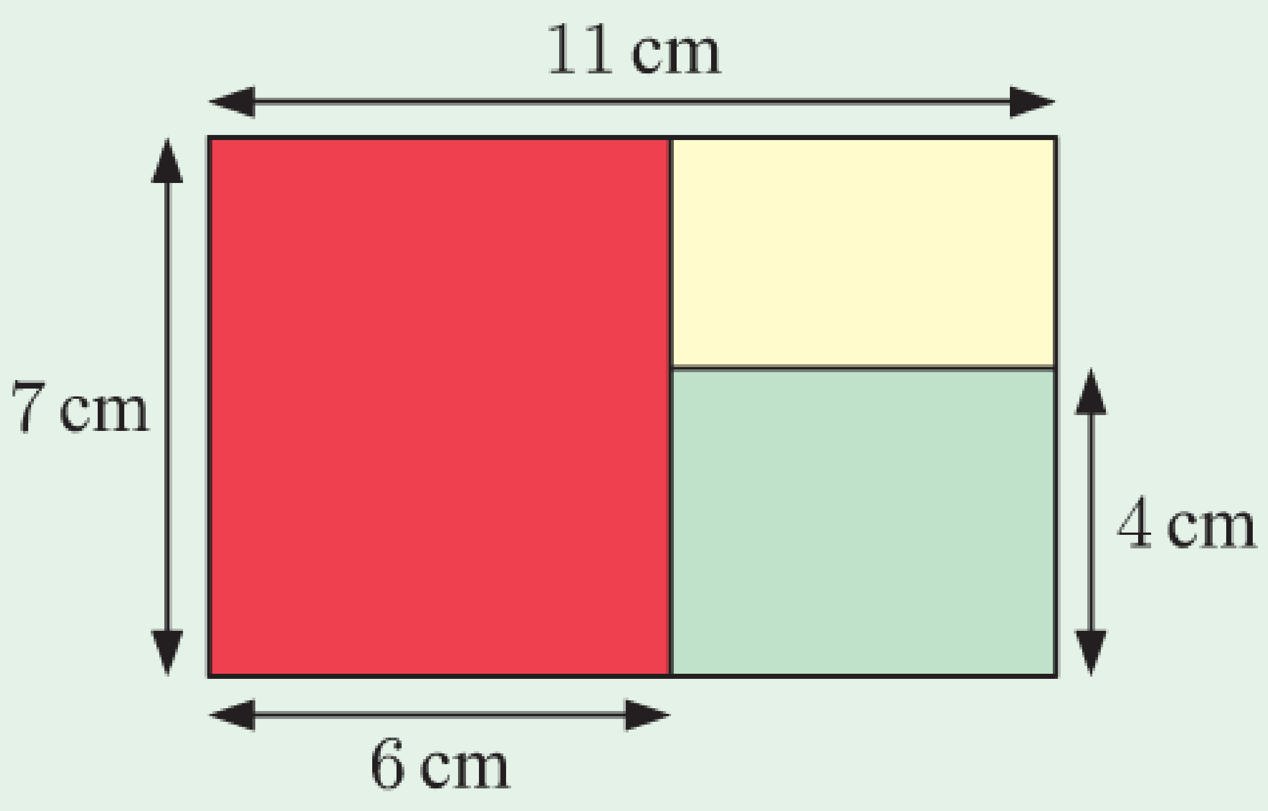
a Find the area of the large rectangle.

b Find the area of the:

i red rectangle **ii** green rectangle

iii yellow rectangle.

c Find the sum of the areas in **b**. Compare this with your answer to **a**.

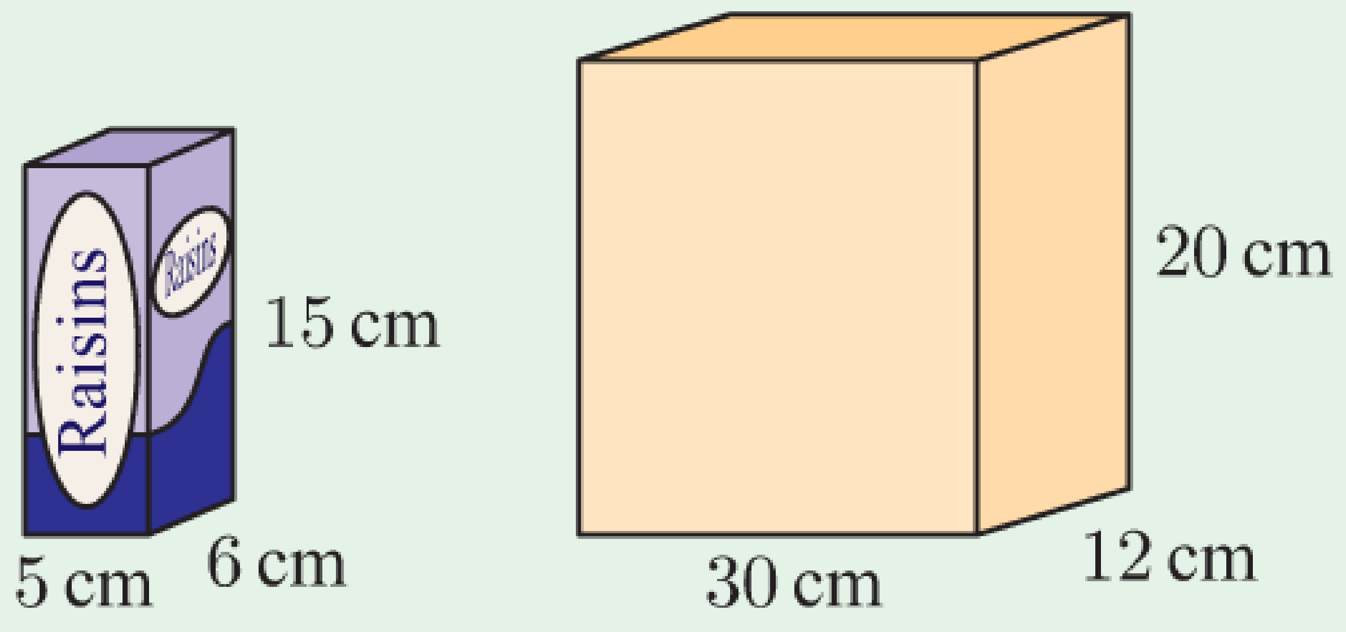


10 Small boxes of raisins with dimensions 5 cm by 6 cm by 15 cm, need to be packed into the larger box shown.

a Find the volume of the box of raisins.

b Find the volume of the large box.

c Find the maximum number of raisin boxes that will fit into the large box.

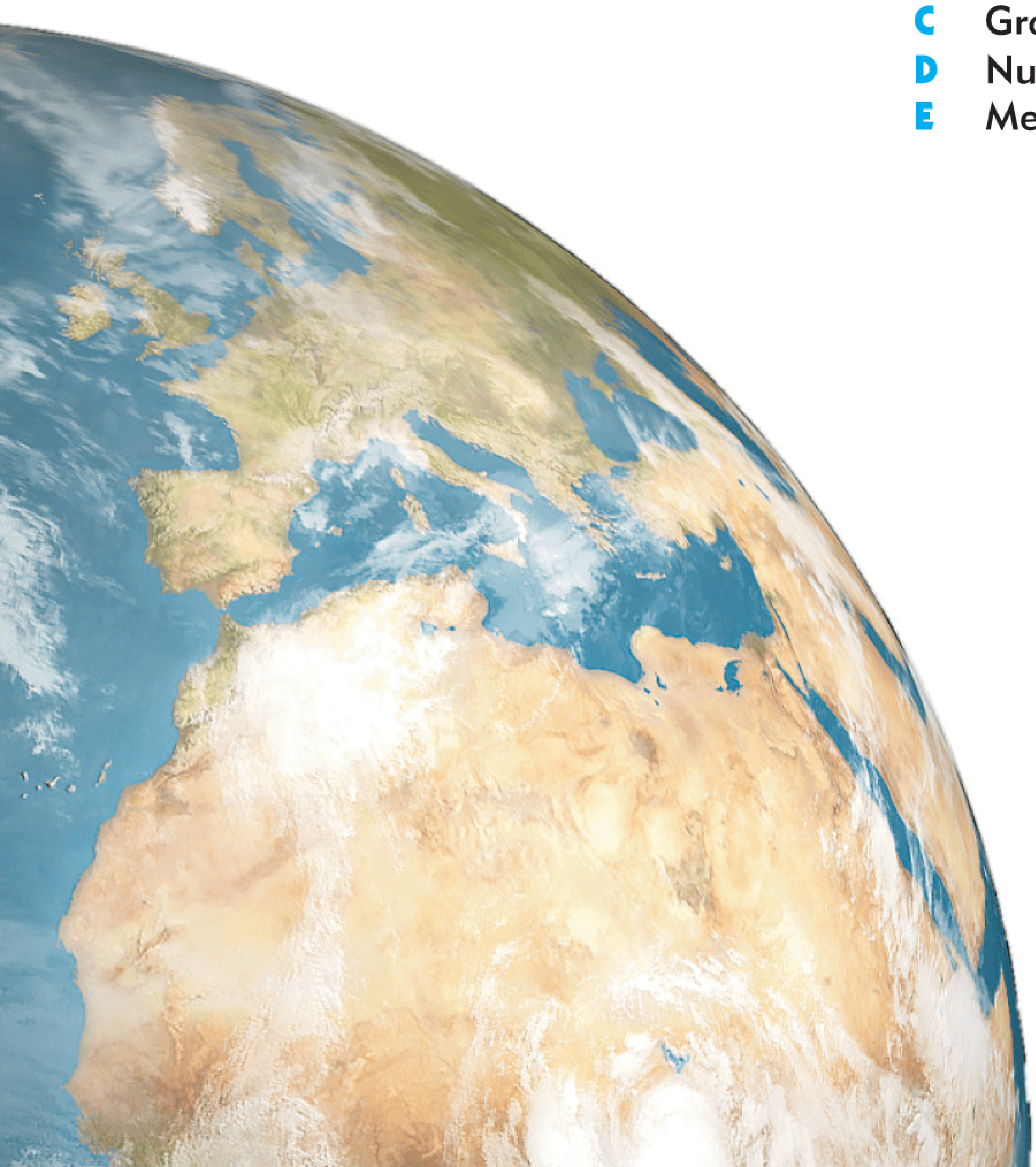


d Illustrate a method of packing the maximum number of raisin boxes into the large box.

Chapter 15

Statistics

Contents:

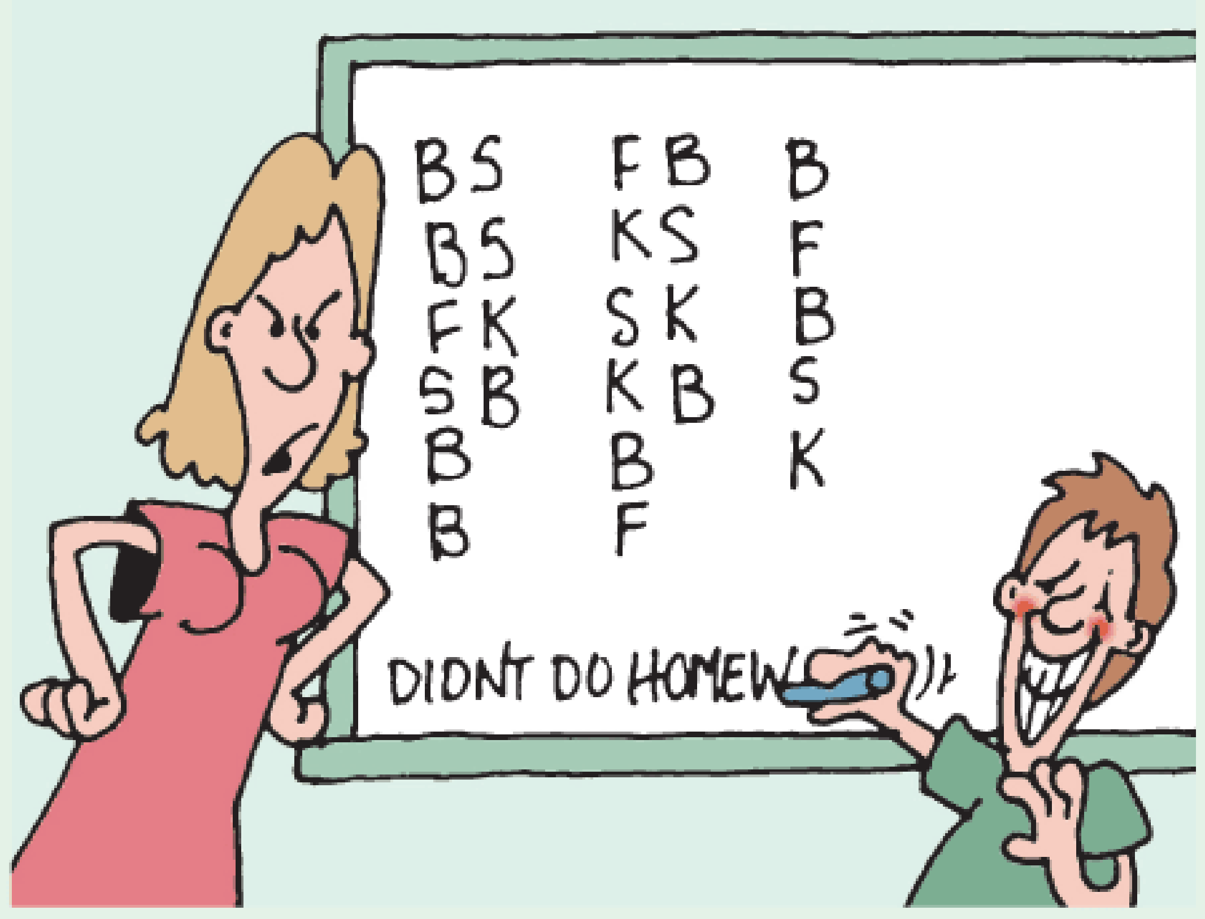


OPENING PROBLEM

A teacher asked her students "Which room at home do you usually do your homework in?"

She recorded the answers on the board, using the code B = Bedroom, K = Kitchen, F = Family room, S = Study:

BBFSB FKSKB BFBSK
BSSKB FBSKB



Things to think about:

- **a** Which room was most popular for doing homework?
- **b** How can we organise and display this information so it is easier to analyse?

When we collect facts or information about something, we call this information **data**.

Statistics is the study of collecting, organising, and analysing data.

Many groups such as schools, businesses, and government departments, collect information. The information is used to decide whether changes are needed, or whether changes that have been made have been successful.

A SAMPLES AND POPULATIONS

These are important words used in statistics:

Population	The whole group of objects or people from whom we are collecting data.
Sample	A group chosen to take part in a survey, or to be measured or tested.
Random sample	A sample selected so that every person or object in the population has equal chance of being selected.
Inference	A conclusion you make based on your survey or investigation.

For example, suppose we conduct a survey on how much chocolate students at your school eat.

- The *population* is the students at your school.
- A *sample* could be chosen by selecting 10 students at random from the school roll.
- An *inference* might be that most students eat chocolate at least once a week.

EXERCISE 15A

1 How could you select a random sample of:

- **a** 400 adults
- **b** bottles of soft drink at a factory
- **c** 30 students at a school
- **d** words from the English language?

2 How could you randomly select:

- **a** one ticket out of 5 tickets
- **b** one of the letters A or B
- **c** one of the numbers 1, 2, 3, 4, 5, or 6
- **d** one card from a pack of 52 playing cards?

Example 1 **Self Tutor**

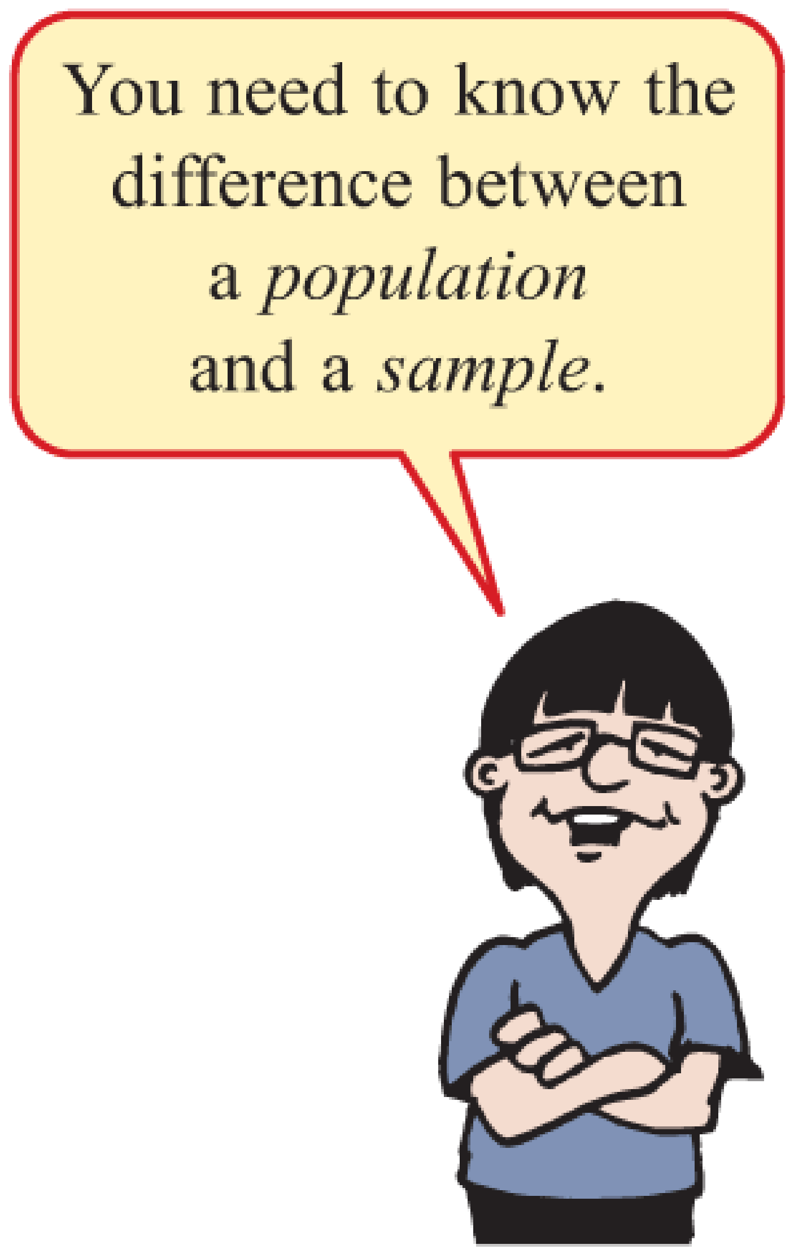
From a school of 400 students, a random sample of 60 students was selected. 13 students in the sample had blue eyes.

a How many students were in the population?
b How many students were in the sample?
c What fraction of the sample had blue eyes?
d Estimate how many students in the school have blue eyes.

a There were 400 students in the population.
b There were 60 students in the sample.
c 13 out of the 60 students in the sample had blue eyes, so the fraction of the sample with blue eyes was $\frac{13}{60}$.
d We assume that the fraction of the population with blue eyes is also $\frac{13}{60}$.

$$\frac{13}{60} \text{ of } 400$$
$$= \frac{13}{60} \times 400$$
$$\approx 87 \qquad \{\textit{Calculator}: \quad 13 \boxed{\div} 60 \boxed{\times} 400 \boxed{=}\}$$

So, we estimate that 87 students in the school have blue eyes.



3 From a colony of 10 000 ants, 300 were collected and examined for red eye colour. 36 ants in the sample had red eyes.

a How many ants were in the population?
b How large was the sample?
c What fraction of the sample had red eyes?
d Estimate the total number of red-eyed ants in the population.

4 Of the 750 people who attended the opening night of a new play, 50 people were randomly selected for a survey. 33 of them said that they liked the play.

a How many people were in the population?
b How large was the sample?
c What percentage of the people sampled did *not* like the play?
d Estimate the total number of people attending the opening night who did *not* like the play.

DISCUSSION

1 Discuss why:

- **a** a hat manufacturer would like to know the head measurements of people in different age groups
- **b** the manager of your school canteen would be interested in the types and quantities of food you eat
- **c** your school keeps records of what is bought by the school population throughout the year
- **d** meteorologists are interested in temperature, rainfall, and atmospheric pressure measurements throughout the world.

2 For each of the situations listed in **1**, discuss how the information could be collected.

3 Discuss how you would gather data in each of these situations:

- **a** You wish to convince your local council that traffic lights should be installed near the school.
- **b** You own a lawn mowing business, and want to expand your business to a new area.

4 Collect samples from newspapers, magazines, packaging, and letterbox deliveries which invite you to provide information. In what ways are you tempted by companies to provide them with information?

5 How do people tempt you to supply information online?

B CATEGORICAL DATA

Categorical data is data which can be placed in categories.

Rosemary stood at a street intersection one morning, and recorded the colour of each car going past. She recorded the data in five categories: red (R), blue (B), grey (G), white (W), and other (O).

She recorded these results for a sample of 40 cars:

BGWWR BGWRW OOBBG
BBGGW WWWOG WOBWW
BGRWR RWWRB

TALLY AND FREQUENCY TABLE

Having collected her categorical data, Rosemary needs to **organise** it using a **tally and frequency table**.

The **tally** is used to count the data in each category. For each data value, we place a stroke | in the appropriate category. After we have put four strokes in a category, the fifth is shown by a diagonal stroke 𝍸 which completes a 'bundle' of five.

The **frequency** gives the total number for each category.

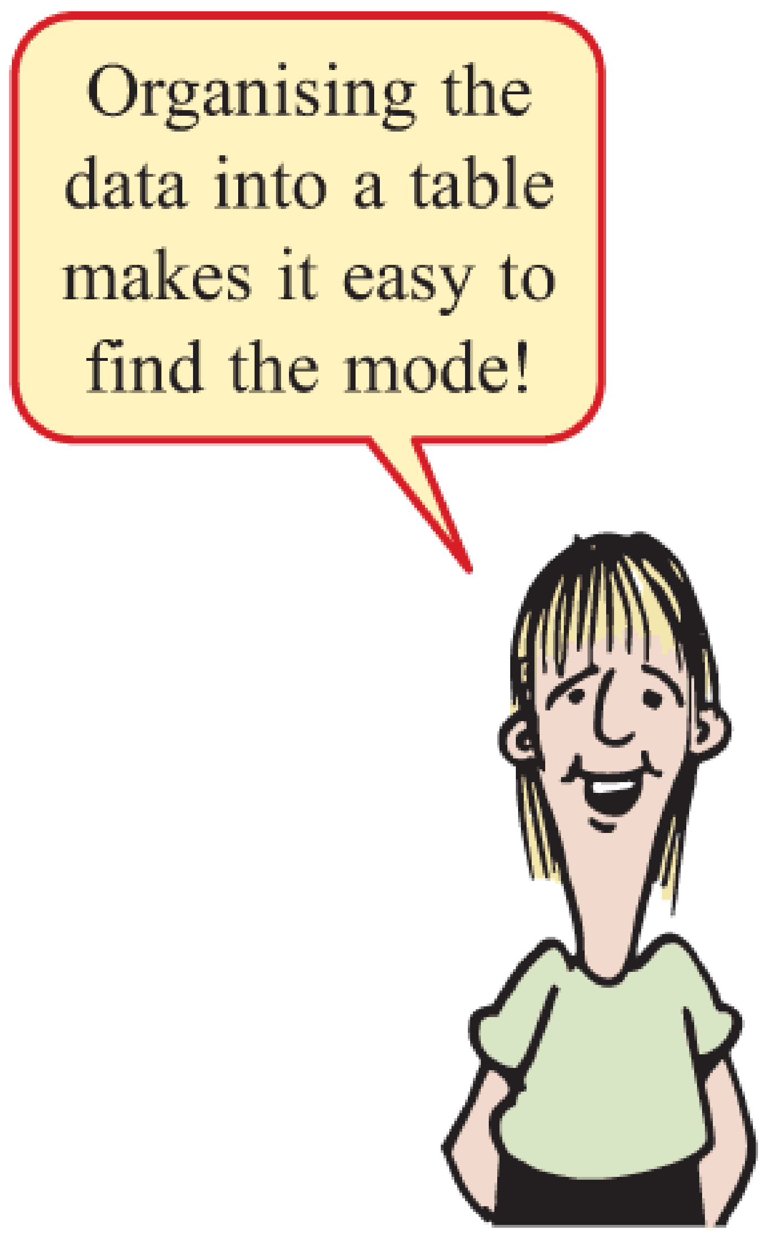
Rosemary's car colour data

Colour	*Tally*	*Frequency*
Red (R)	𝍸 |	6
Blue (B)	𝍸 ||||	9
Grey (G)	𝍸 ||	7
White (W)	𝍸 𝍸 ||||	14
Other (O)	||||	4
	Total	40

THE MODE

The **mode** is the most frequently occurring category.

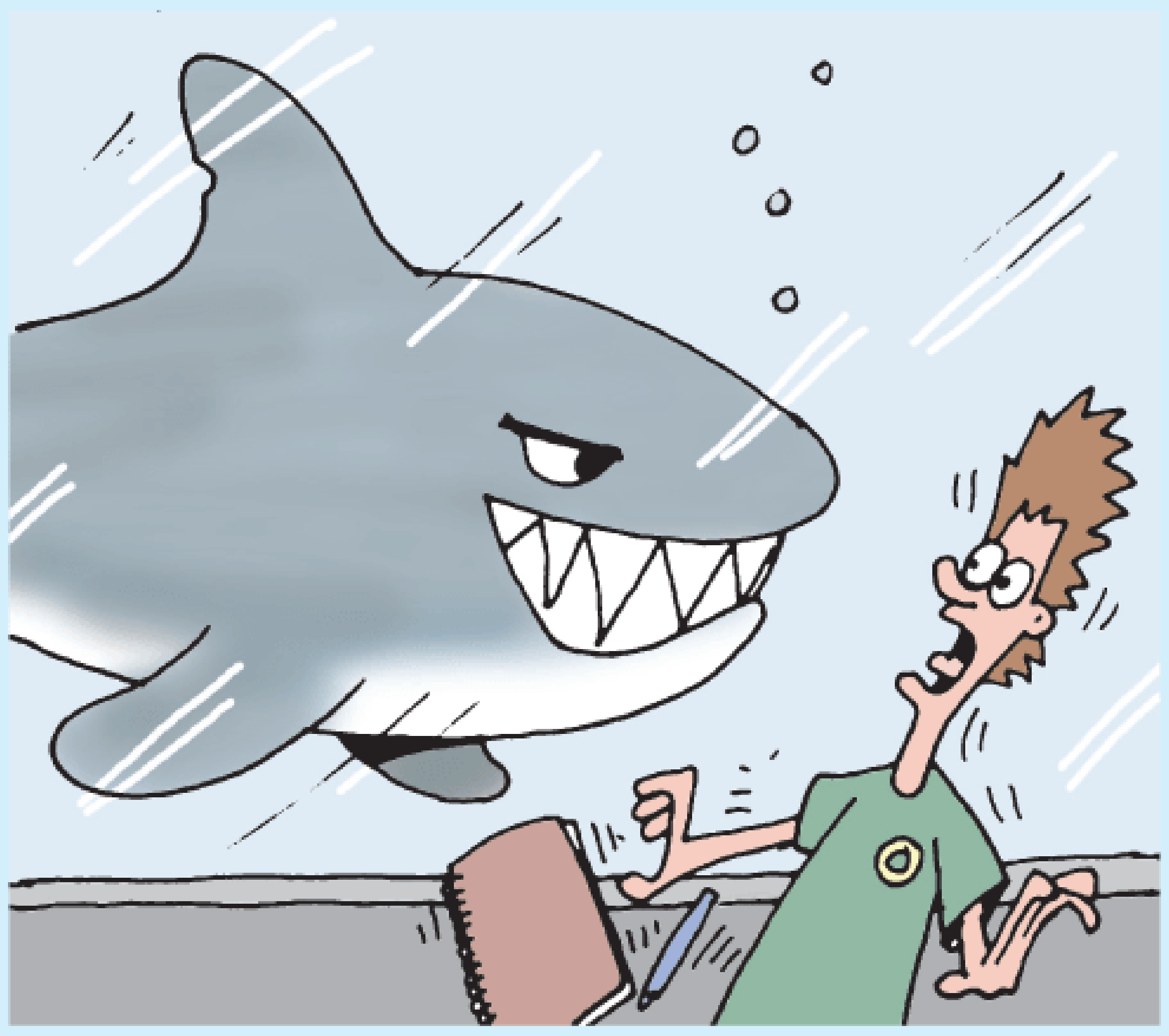
For Rosemary's car colour data, the mode is 'white'.



Example 2 Self Tutor

A class of students went on an excursion to the aquarium. Each student was asked to name their favourite animal. Data was recorded in the 4 categories: shark (Sh), seahorse (Se), dolphin (D), and otter (O). The results were:

D Sh Se D O Sh D O D Sh Se D Sh O D
Se O D Sh D O Sh D Se O D Sh Sh O D



a Draw a tally and frequency table to organise the data.

b Find the mode of the data.

a

Animal	*Tally*	*Frequency*
Shark (Sh)	𝍸 |||	8
Seahorse (Se)	||||	4
Dolphin (D)	𝍸 𝍸 |	11
Otter (O)	𝍸 ||	7
	Total	30

b The mode is 'dolphin', as this category occurs most frequently.

EXERCISE 15B

1 The eye colour of each student in a class was recorded using the categories brown (Br), blue (Bl), green (Gn), and grey (Gr). The results were:

Br Bl Gn Bl Gn Br Br Bl Gn Gr Br Gr Br Br Bl Br Bl Br Gr Gn
Br Bl Br Gn Gr Br Bl Gn

a Draw a tally and frequency table for the data.

b Find the mode of the data.

2 A cinema owner records the last 50 products sold at the candy bar. The results are shown below, where P = popcorn, S = soft drink, I = ice cream, and C = chips:

CPSPI PSCIP IPCSC PCSPC SPIPC
IPCCP PCSPI CPCPS CIIPP CSCPI

a Draw a tally and frequency table for the data.

b Find the mode of the data.

3 Tourists staying in a city hotel were surveyed to find out what they thought of the hotel's service. They were asked to choose E = excellent, G = good, S = satisfactory, or U = unsatisfactory.

The results were:

EGGSE USSGG SGUGG ESGUG SSEGG

a Draw a tally and frequency table for the data.

b What is the mode of the data?

c Suggest a reason why this survey would be carried out.

4 Brody, Cooper, Hailey, and Maria played board games on the weekend.

They recorded which person won each game.

a Draw a tally and frequency table for the data.

b How many games did Cooper win?

c Who won the most games?

Game	*Winner*
1	Hailey
2	Cooper
3	Maria
4	Maria
5	Hailey
6	Brody
7	Maria
8	Hailey
9	Cooper
10	Hailey
11	Maria
12	Hailey
13	Brody
14	Cooper
15	Hailey

C GRAPHS OF CATEGORICAL DATA

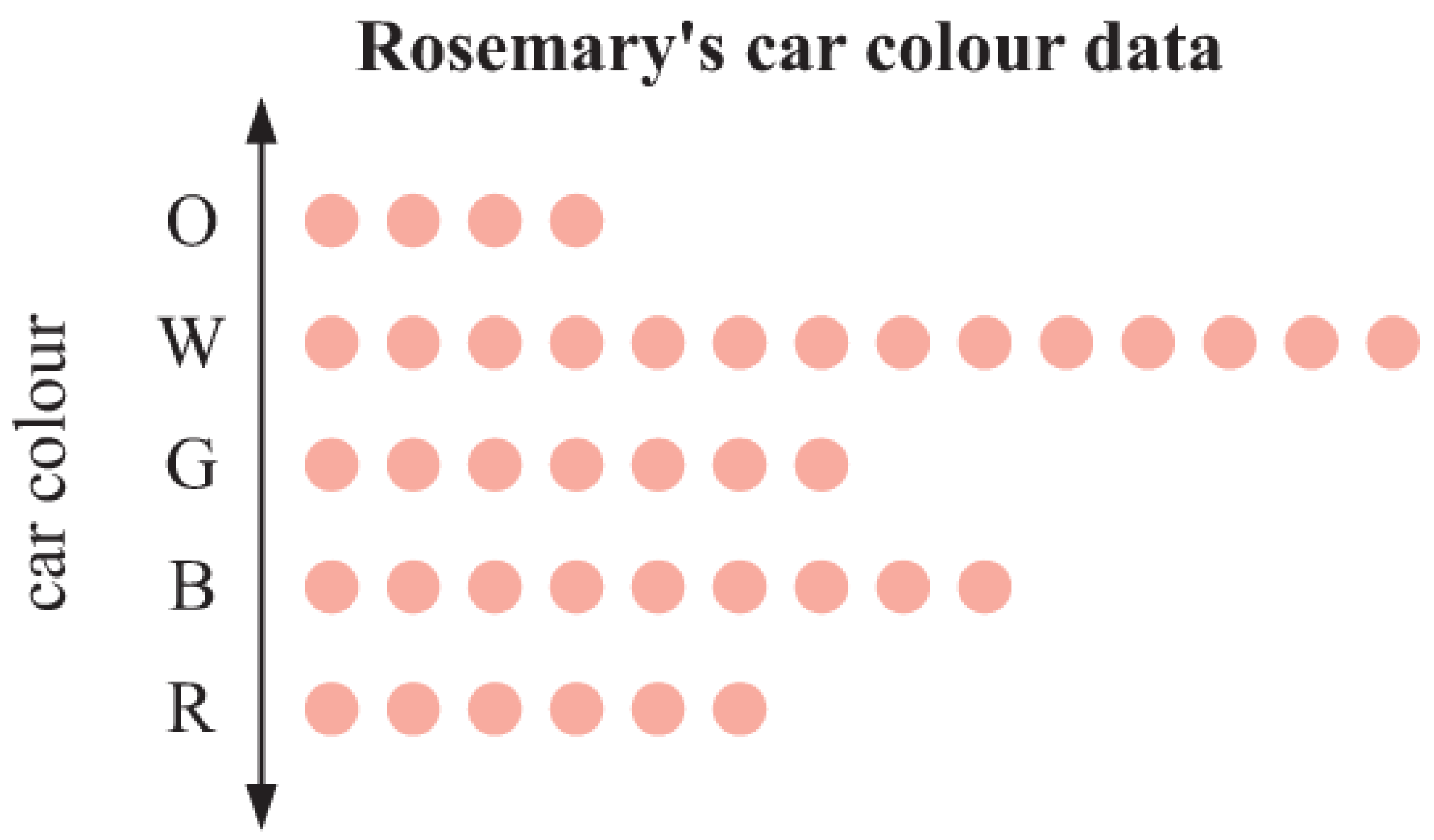
Categorical data is often displayed using **dot plots**, **column graphs**, and **pie charts**.

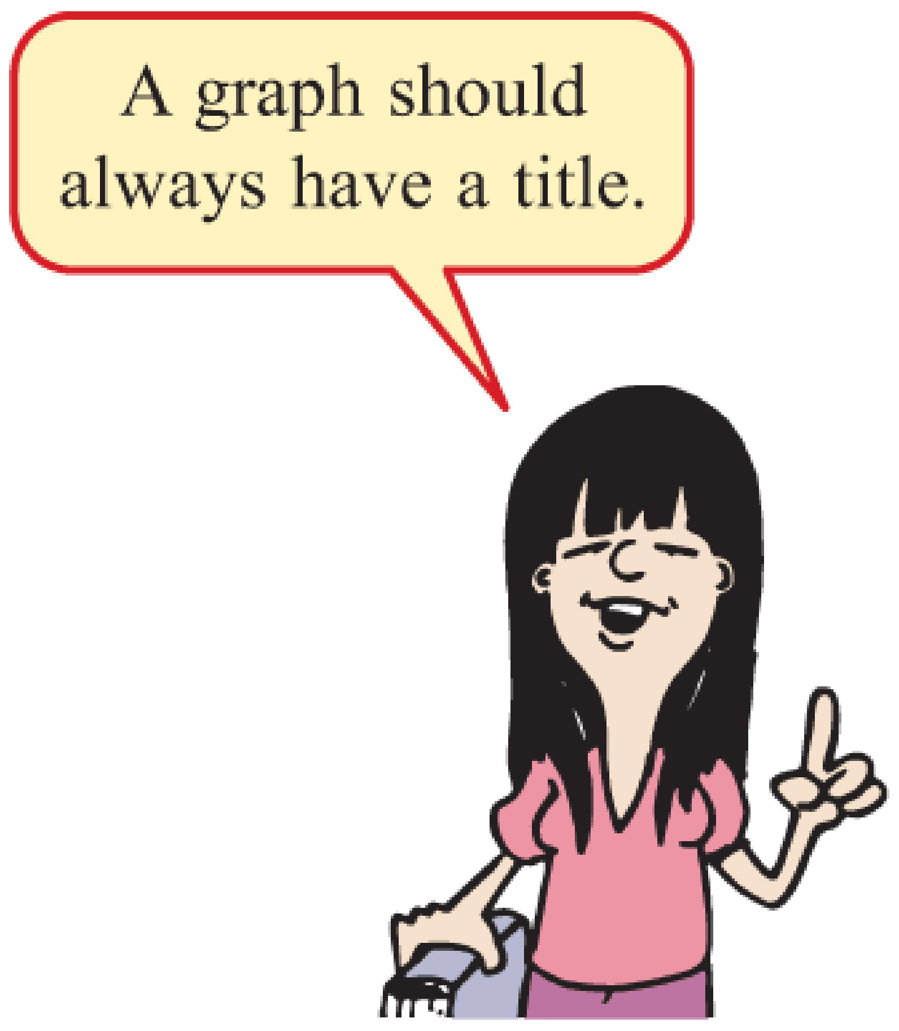
DOT PLOTS

A **dot plot** is a graph in which each dot represents one data value.

Dot plots may be **horizontal** or **vertical**.

A horizontal dot plot for Rosemary's car colour data is shown below.





Check that there is one dot for each car recorded in the data.

The mode is the category with the most dots. In this case, the mode is 'white'.

Example 3 **Self Tutor**

At recess time, the sales of drinks at the canteen were recorded over a ten minute period. The categories recorded were O = orange juice, S = soy milk, C = cola, and I = iced tea.

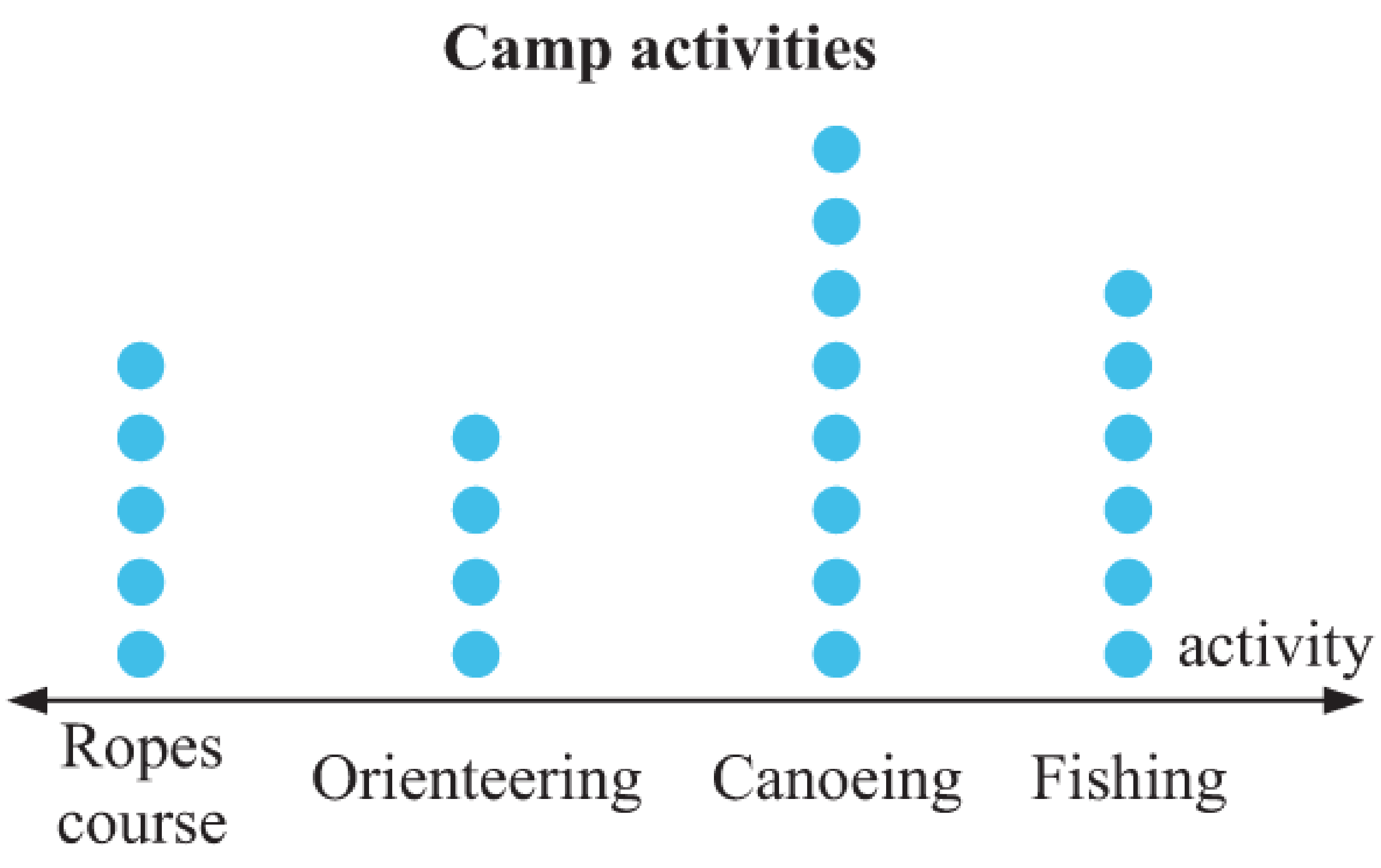
OSSCI OCISO IOCSO OOOSC SOCOS SOOCO OIOIS

a Draw a horizontal dot plot of the data. **b** What is the mode?

a

b The mode is 'orange juice'.

EXERCISE 15C.1

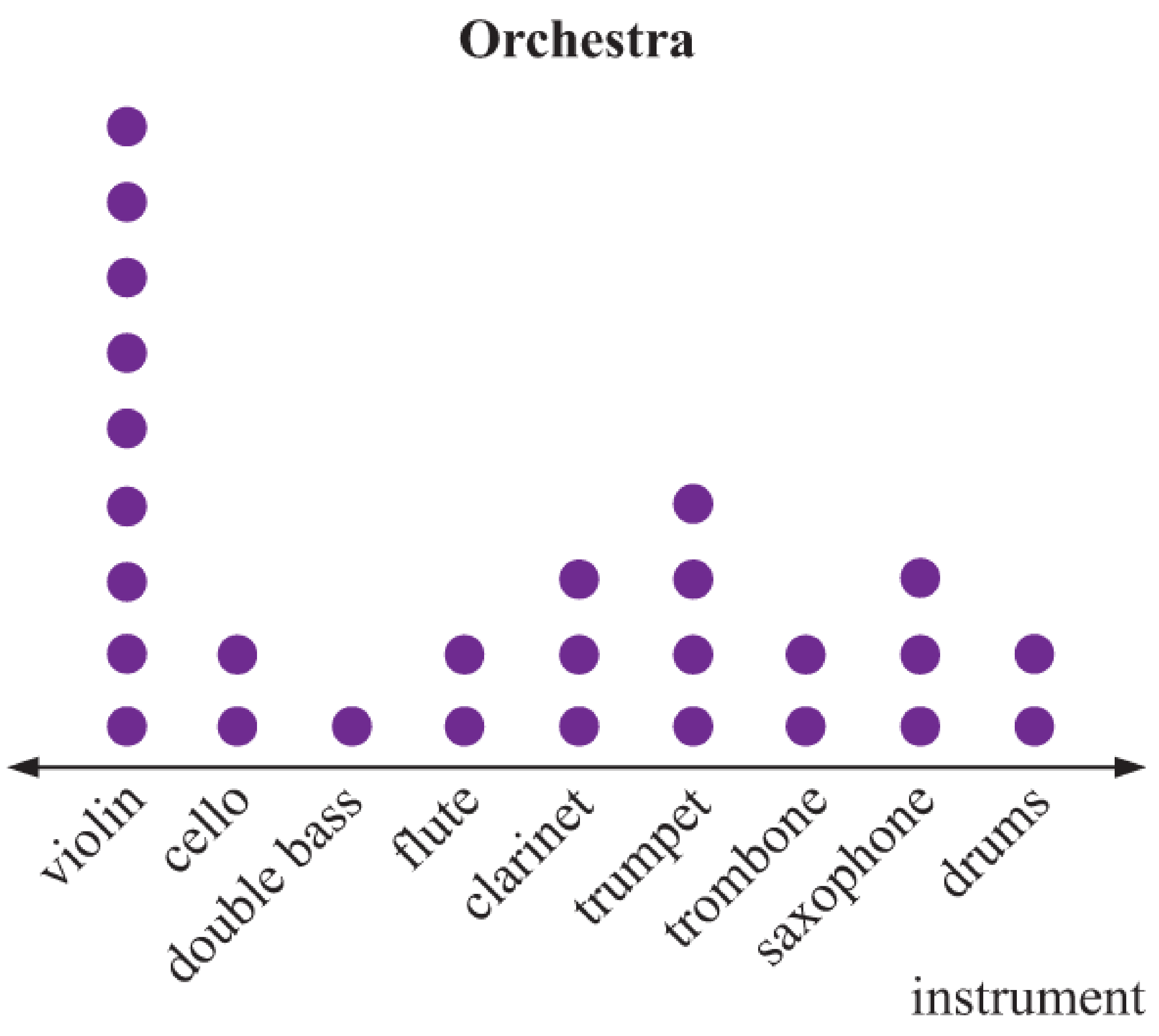


1 Students at a school camp were given the choice of several afternoon activities to participate in. This vertical dot plot shows how many students chose each activity.

- **a** How many students chose orienteering?
- **b** How many students were at the camp?
- **c** Find the mode of the data.

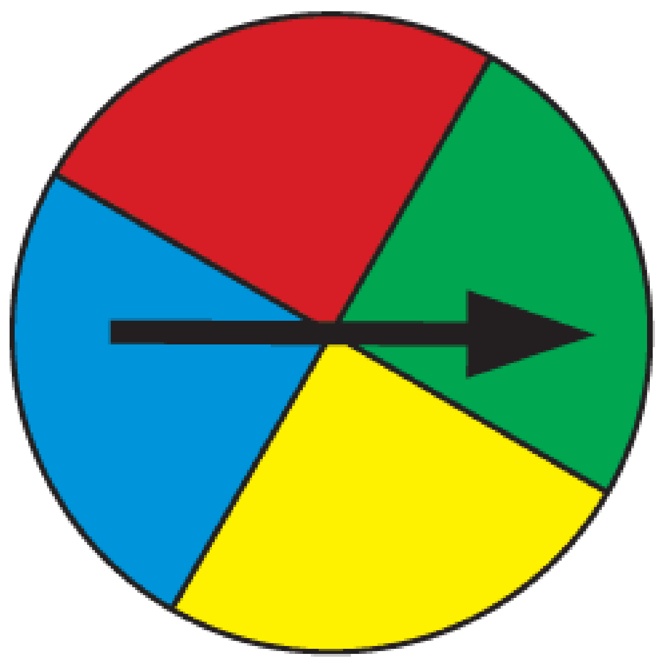
2 This dot plot shows the musicians playing various instruments in an orchestra.

- **a** Find the mode of the data.
- **b** How many musicians are in the orchestra?
- **c** How many musicians play stringed instruments?



3 A spinner with red, blue, yellow, and green sectors is spun 30 times. The results are given below:

RBRYG YGBBR RGBYY
GRGBR YRRGB GRGBR



- **a** Draw a vertical dot plot of the data.
- **b** Find the mode of the data.

4 Lucas had a Halloween party. Each of the children at the party dressed up as a ghost, a pumpkin, a witch, or a mummy.

- **a** How many children were at the party?
- **b** Draw a horizontal dot plot to show how many children wore each costume.
- **c** Find the mode of the data.

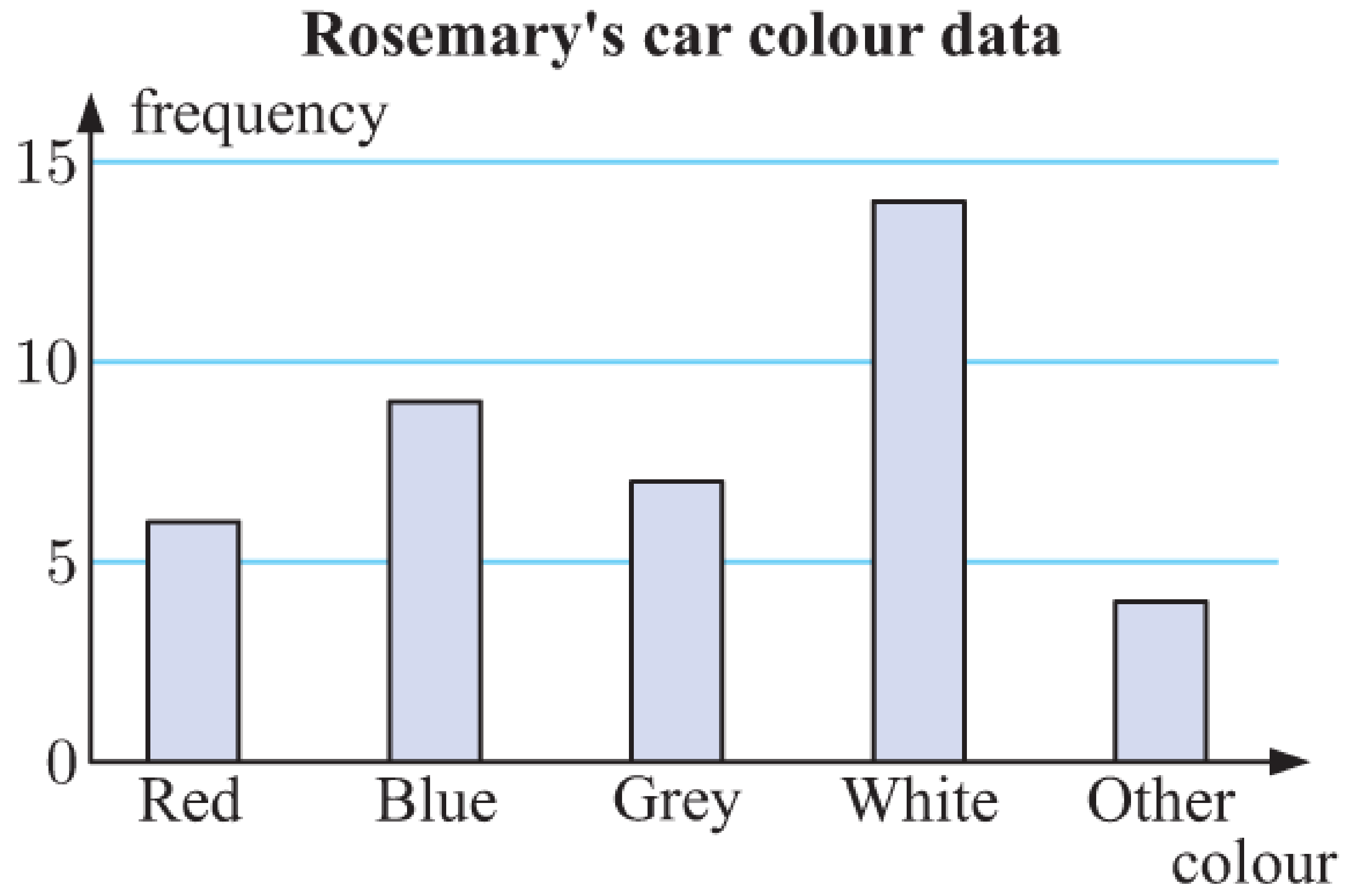
COLUMN GRAPHS

Column graphs consist of rectangular columns of equal width. The height of each column represents the frequency of the category.

The column graph for Rosemary's car colour data is shown alongside.

The mode is the category with the highest column.

STATISTICS PACKAGE



Example 4

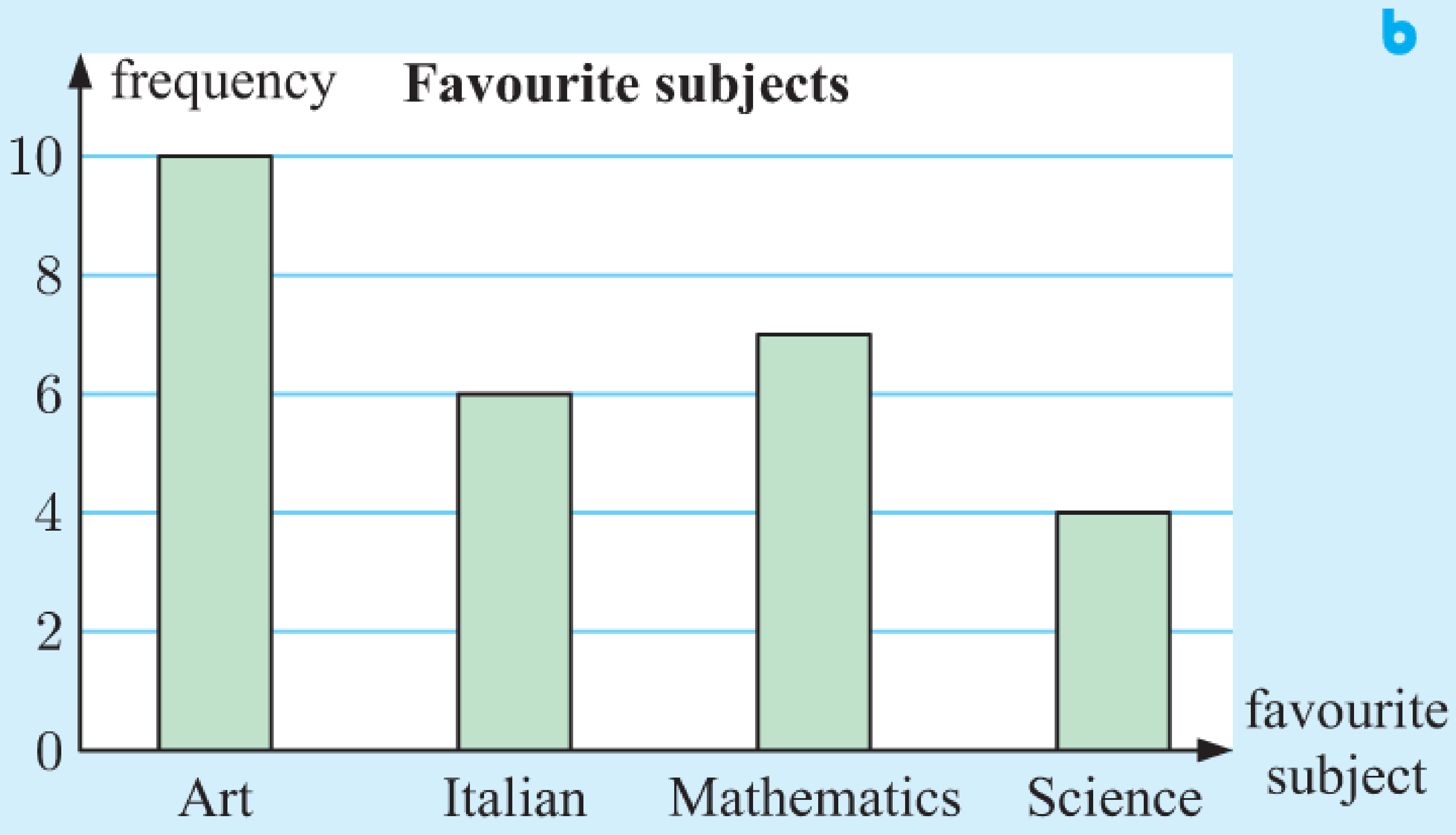
Self Tutor

The table alongside shows the favourite subjects of students in a class.

- **a** Draw a column graph to display the data.
- **b** Find the mode of the data.

Favourite subject	*Frequency*
Art	10
Italian	6
Mathematics	7
Science	4

a

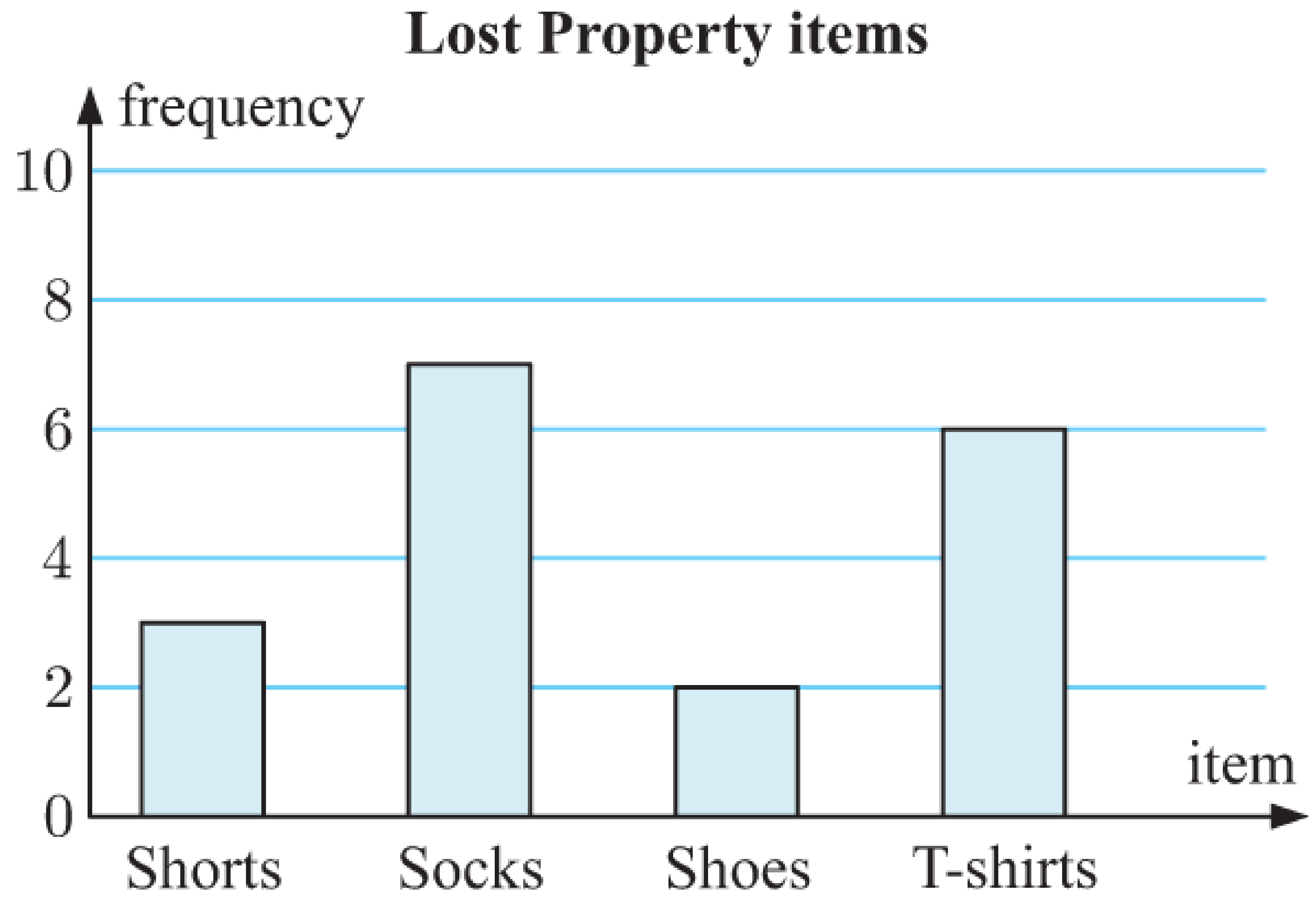


b The mode of the data is 'Art'.

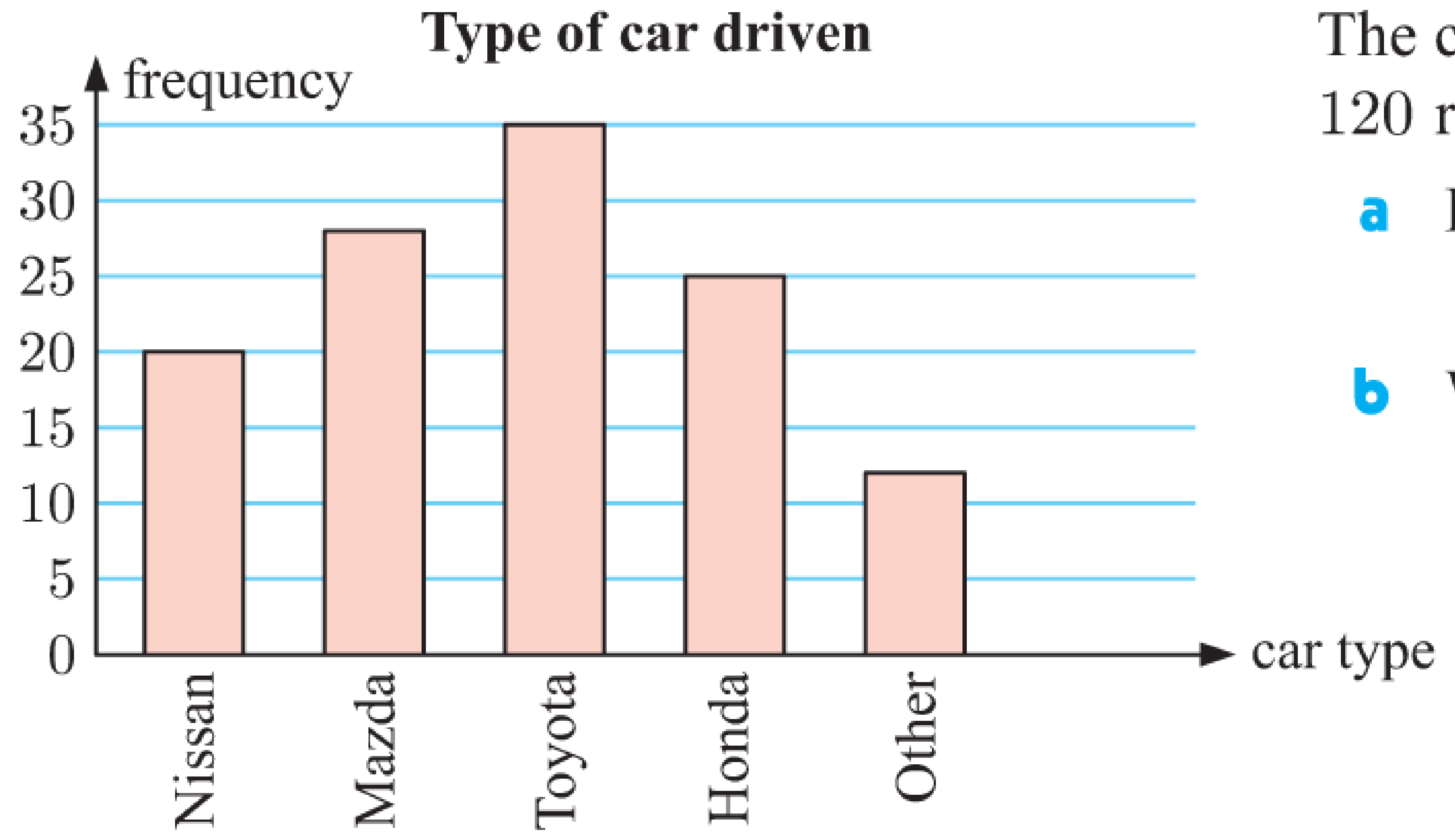
EXERCISE 15C.2

1 This column graph shows the items handed in to Lost Property at a gym.

- **a** How many T-shirts were handed in?
- **b** Find the mode of the data.



2



The column graph shows the types of car driven by 120 randomly selected people.

a How many people in the sample drive a:
  i Nissan **ii** Honda?

b Which make of car is most popular?

3 This table describes the stalls at a market.

a How many stalls are at the market?

b Draw a column graph to display the data.

c Find the mode of the data.

d How many more Clothing stalls are there than Art stalls?

Stall type	*Number of stalls*
Food	5
Art	4
Craft	8
Clothing	6

4 Adults completing a survey were asked whether they worked full-time (F), part-time (P), are unemployed (U), or retired (R). The results were:

F R P P F U P F R F P F U P F R F R F R U F R P

a How many adults took part in the survey?

b Draw a tally and frequency table for the data.

c Draw a column graph to display the data.

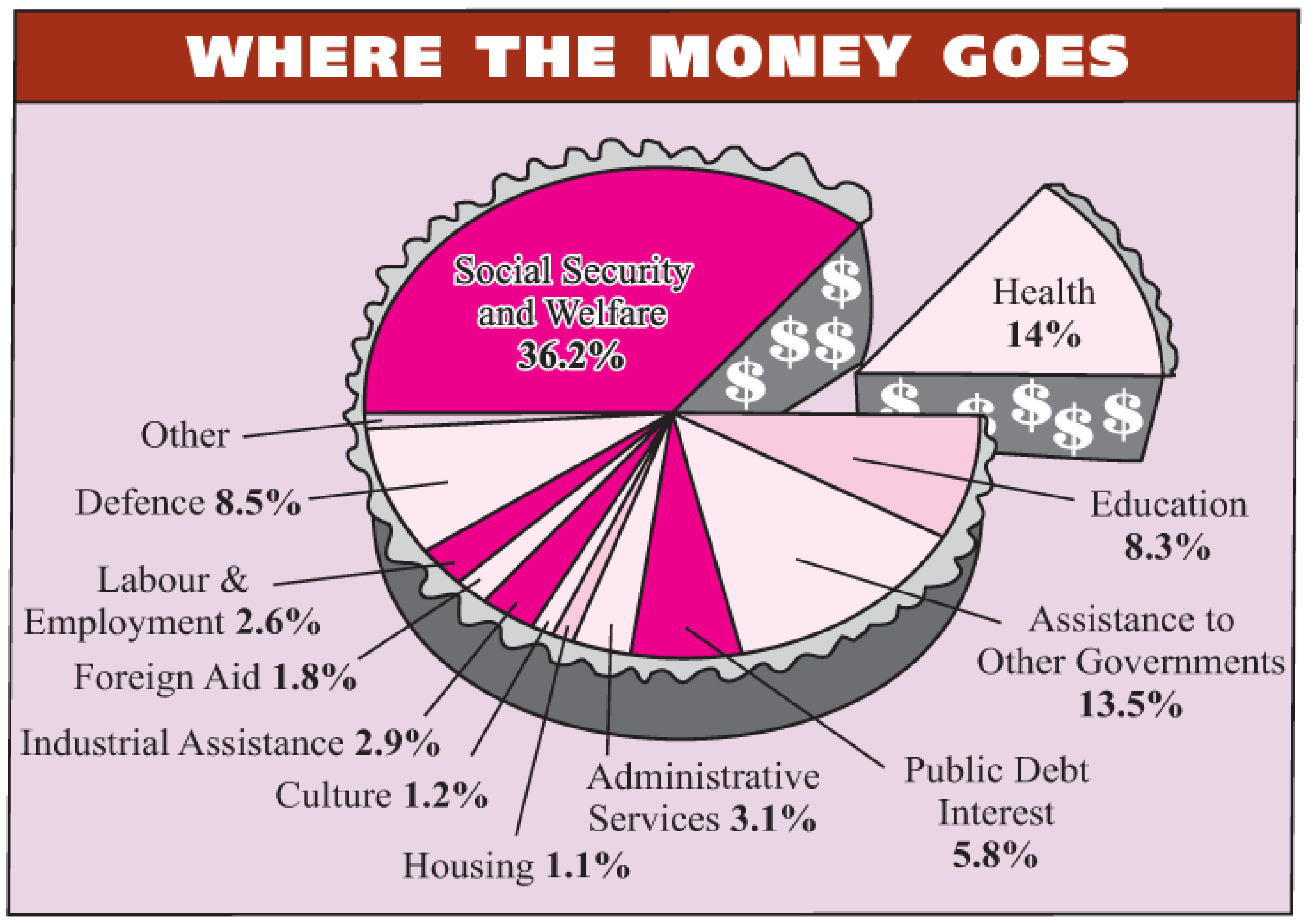
d Find the mode of the data.

e How many of the adults worked either full-time or part-time?

PIE CHARTS

A **pie chart** displays how a quantity is divided up. A full circle represents the whole quantity. We divide the circle into **sectors** or wedges to show each category.

This pie chart shows how the budget of a country is distributed.



We see that:

- 14% of the country's budget is used for Health
- the total given in foreign aid and assistance to other governments $= 1.8\% + 13.5\% = 15.3\%$.

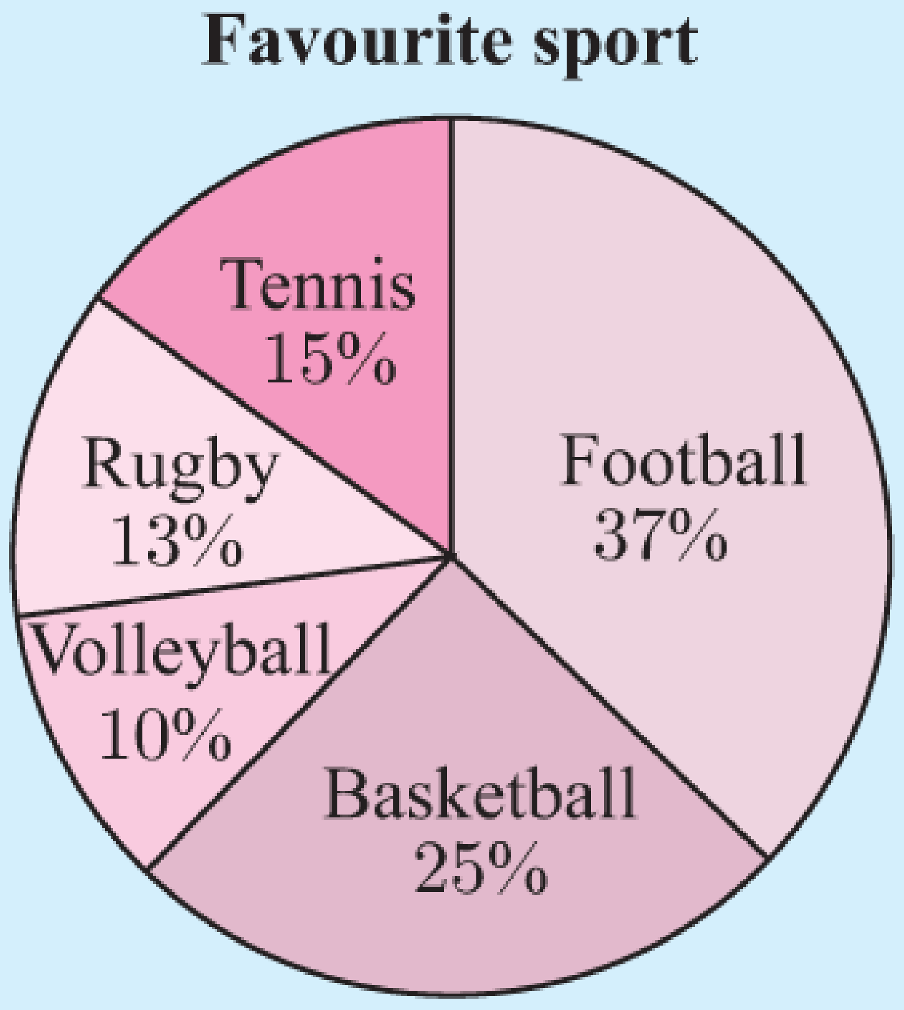
Example 5 **Self Tutor**

120 Year 6 students were asked the question: "*What is your favourite sport*?" This pie chart shows the results.

a Which is the most popular sport?

b True or false? More than half of the students chose football or basketball.

c How many students chose volleyball?



a The largest sector indicates the most popular sport, and this is football.

b 37% + 25% = 62% of the students chose football or basketball. This is more than half, so the statement is true.

c 10% of the students chose volleyball.

$\therefore$ the number of students who chose volleyball

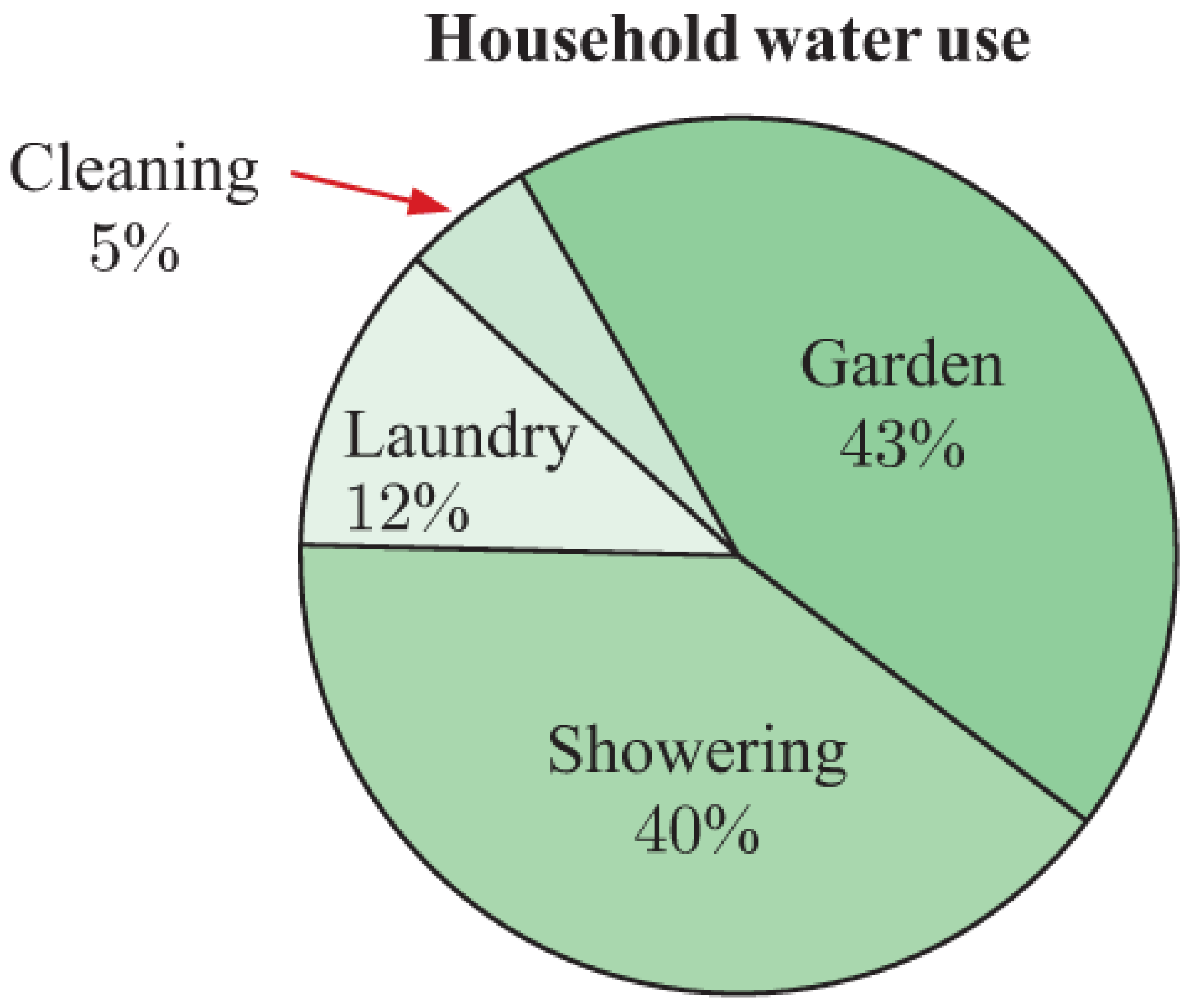
$= 10\%$ of 120

$= 0.1 \times 120$

$= 12$

EXERCISE 15C.3

1



The pie chart alongside illustrates how water is used by a household.

a For what purpose is the most water used?

b True or false? Less than one fifth of the water is used for laundry and cleaning.

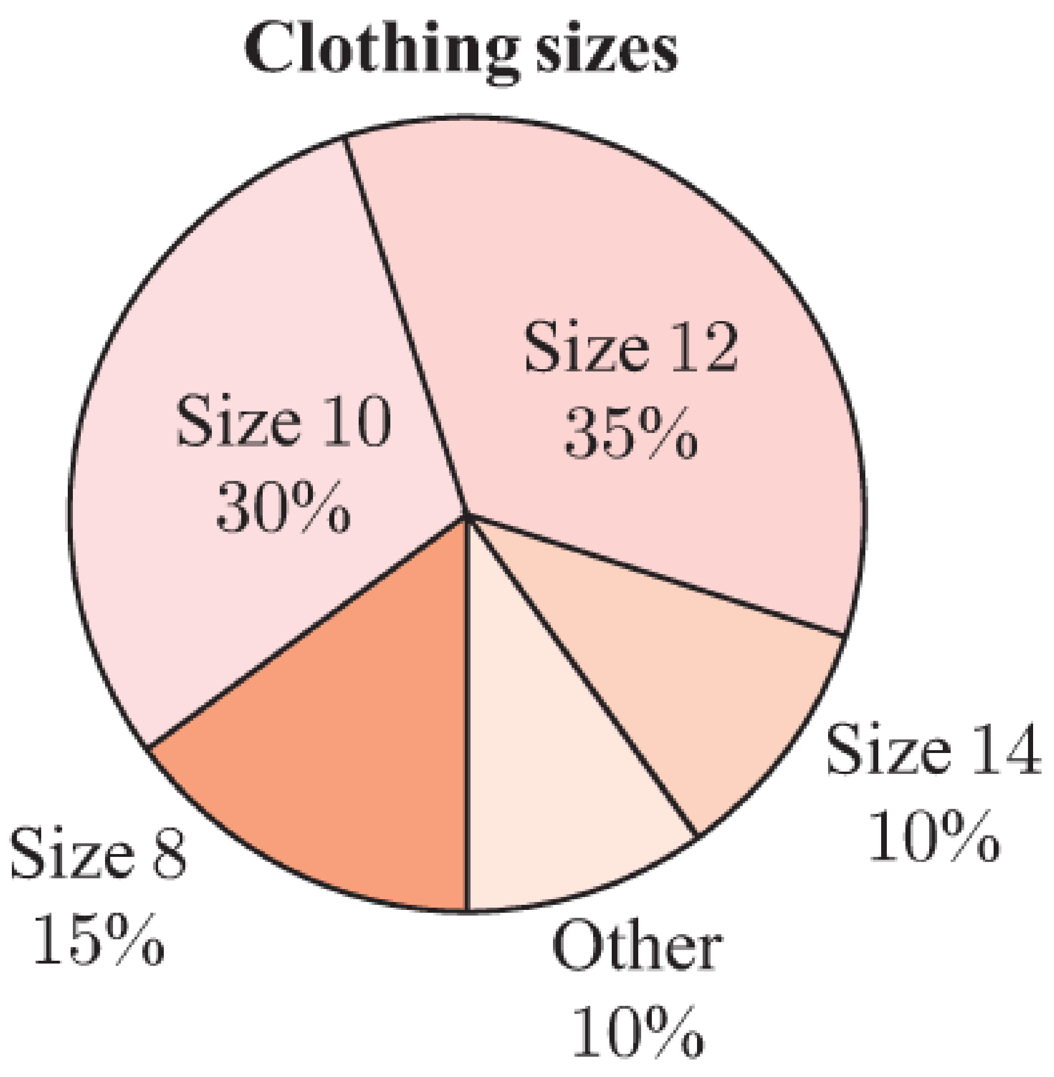
c The household used 400 kilolitres of water during a particular period. How much was used for showering?

2 The pie chart alongside shows the clothing size of Grade 6 boys.

a What size is most commonly worn?

b True or false? More than half of Grade 6 boys wear size 12 or 14.

c There are 200 Grade 6 boys in a school. Estimate how many would wear size 10 clothing.



CONSTRUCTING PIE CHARTS

To draw a pie chart, we need to divide the circle into sectors of the correct size. There are 360° in a circle, so if we divide 360° by our sample size, we know what angle each individual in the sample represents.

We use a protractor to draw the angles in the pie chart.

Example 6

Self Tutor

Draw a pie chart for Rosemary's car colour data.

Car colour	*Frequency*
Red	6
Blue	9
Grey	7
White	14
Other	4
Total	40

There are 40 cars in total, so each car represents $\frac{1}{40}$th of the circle. There are 360° in a circle, so each car represents $\frac{1}{40}$th of 360°, which is 9°.

There are 6 red cars, so the 'red' sector has size $6 \times 9° = 54°$.

Likewise:

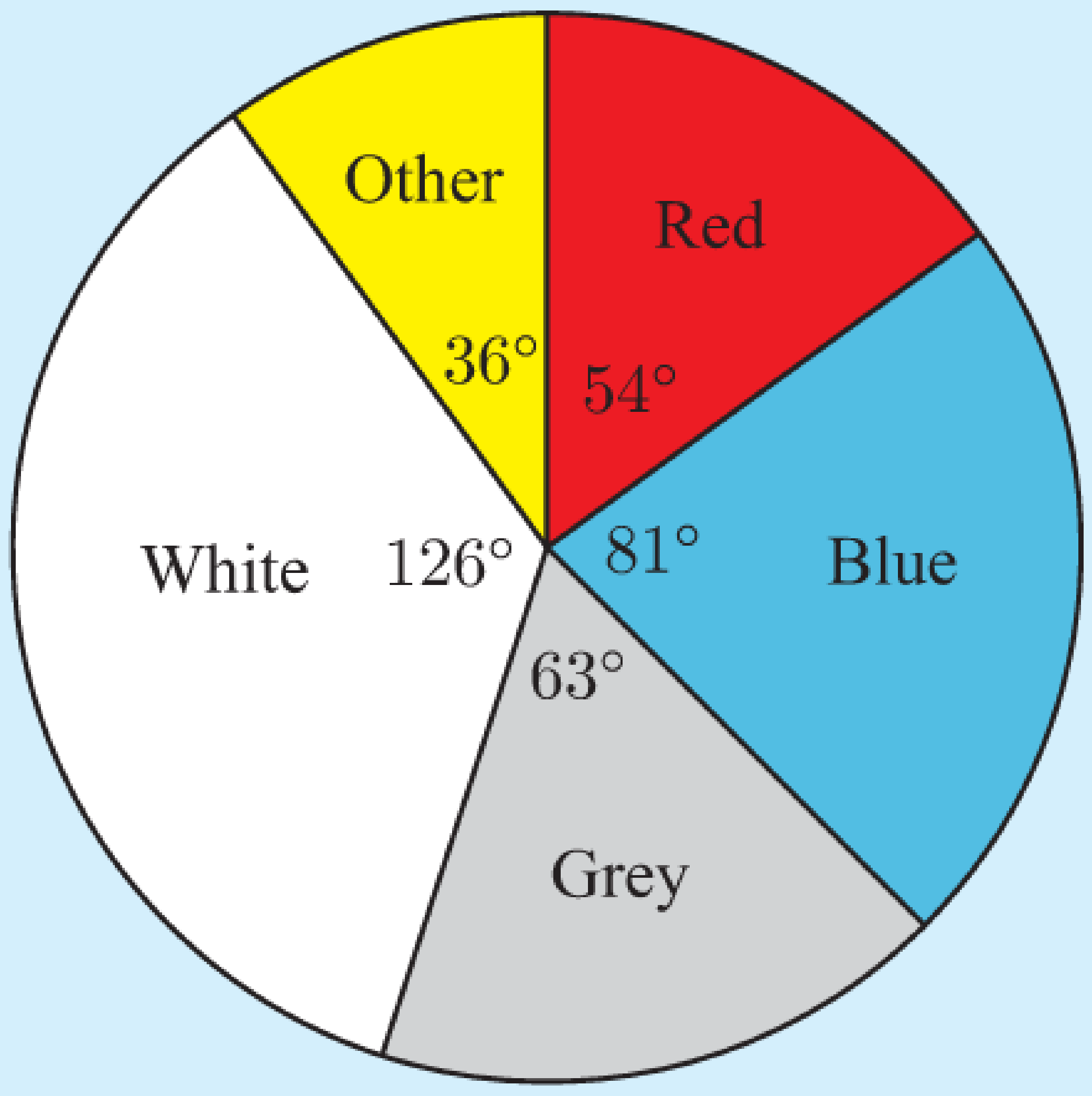
the 'blue' sector has size $9 \times 9° = 81°$

the 'grey' sector has size $7 \times 9° = 63°$

the 'white' sector has size $14 \times 9° = 126°$

the 'other' sector has size $4 \times 9° = 36°$.

Car colour data



EXERCISE 15C.4

1 A table tennis club has 36 members. The table alongside shows their playing levels.

Playing level	*Frequency*
A Grade	9
B Grade	5
C Grade	10
Junior	12
Total	36

- **a** In a pie chart, what angle does each member represent?
- **b** Find the sector size for each category.
- **c** Draw a pie chart for the data.

2 30 children in the school library were asked "What is your favourite type of book?" The responses are shown in this table.

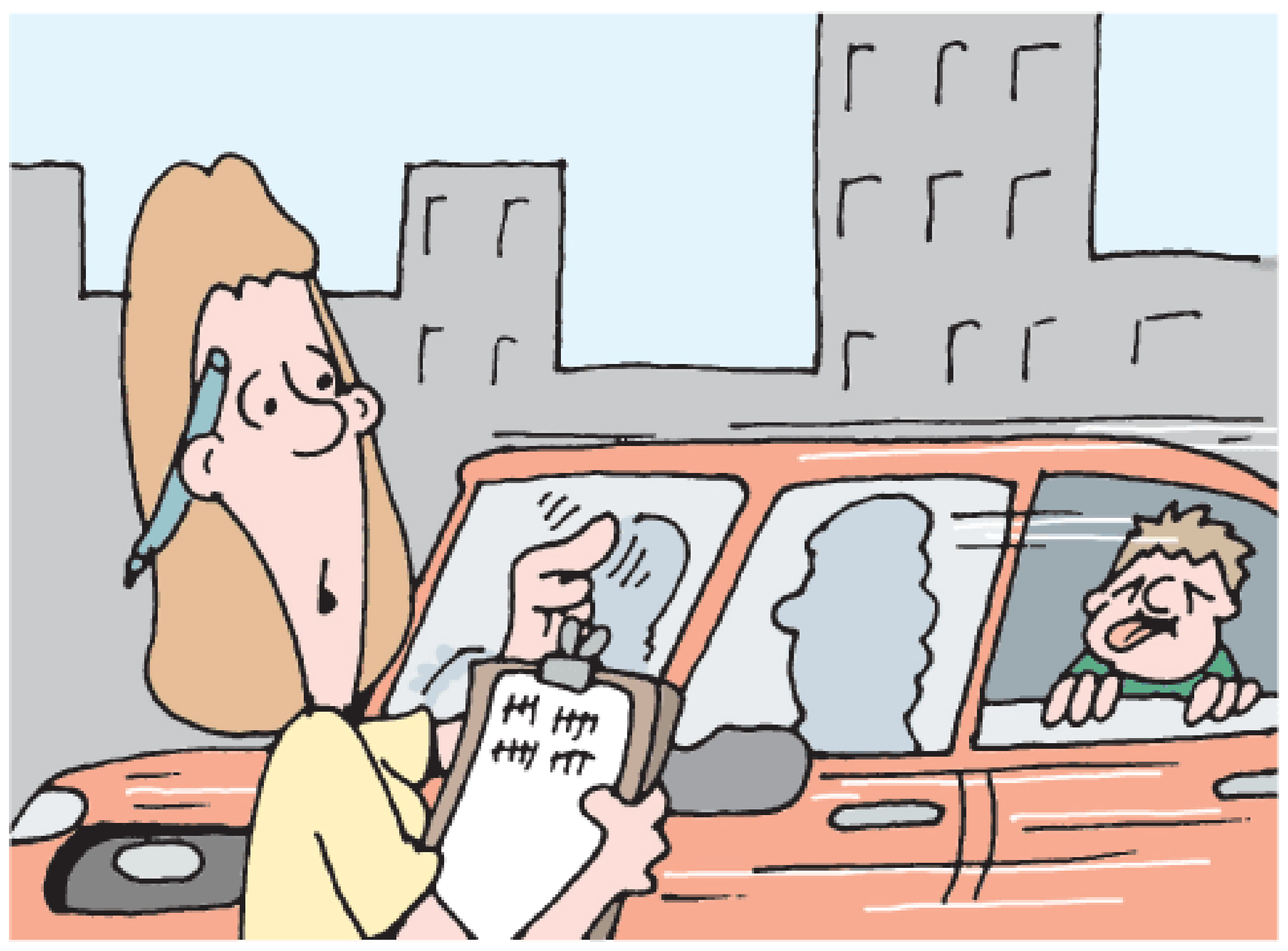
Type	*Frequency*
Non-fiction	6
Crime	5
Adventure	5
Fantasy	10
Horror	4

- **a** Draw a pie chart to display the data.
- **b** Find the mode of the data.

D NUMERICAL DATA

Numerical data is data which is in number form.

While Rosemary was standing at the intersection, she also recorded the *number of people* in the cars going past. The data she collected is numerical data.



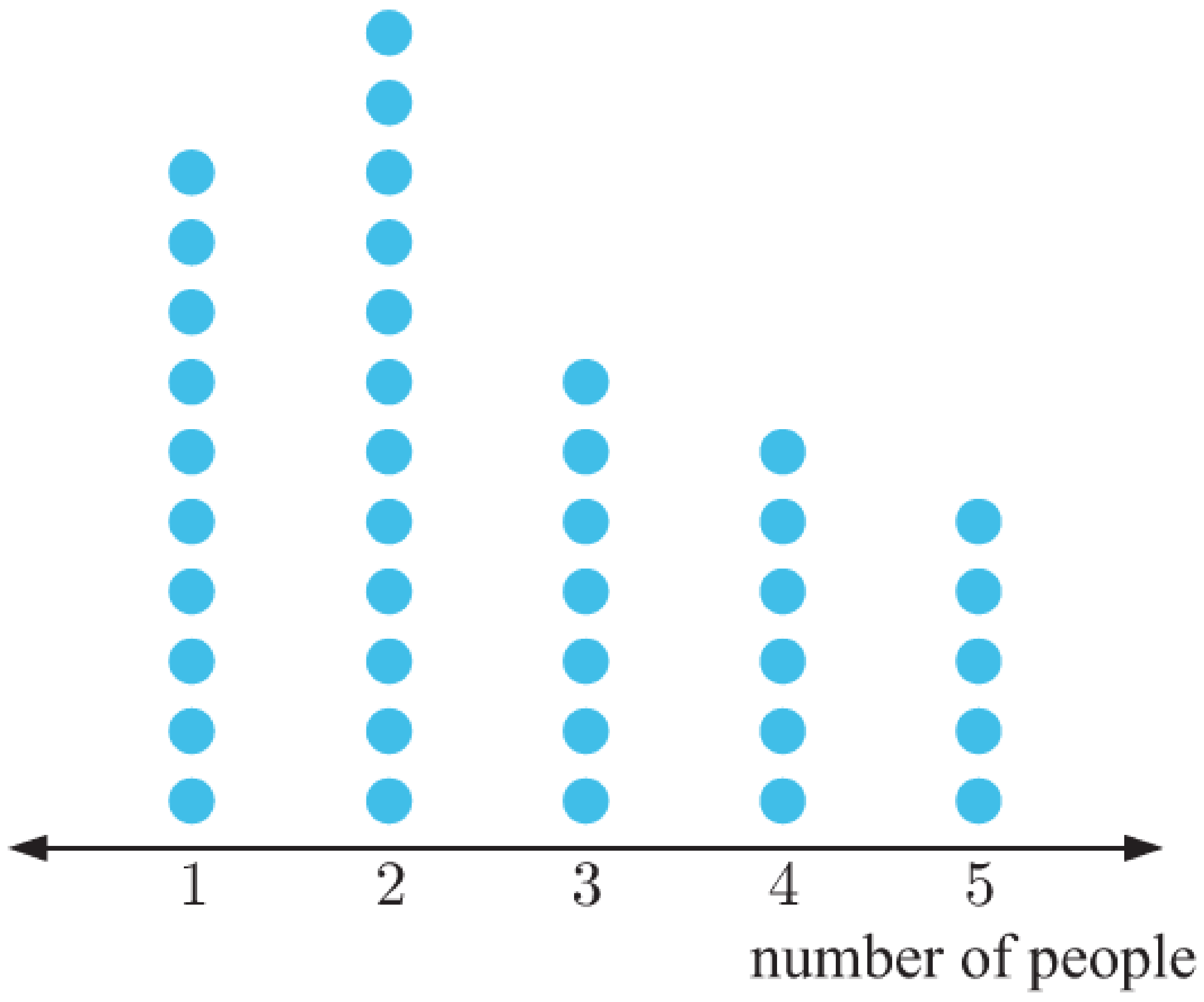
Number of people in cars

2	1	2	3	2	4	2	5	3	1
1	3	2	4	1	3	2	2	1	5
3	4	1	2	5	2	4	1	2	5
5	1	3	2	4	1	3	2	1	4

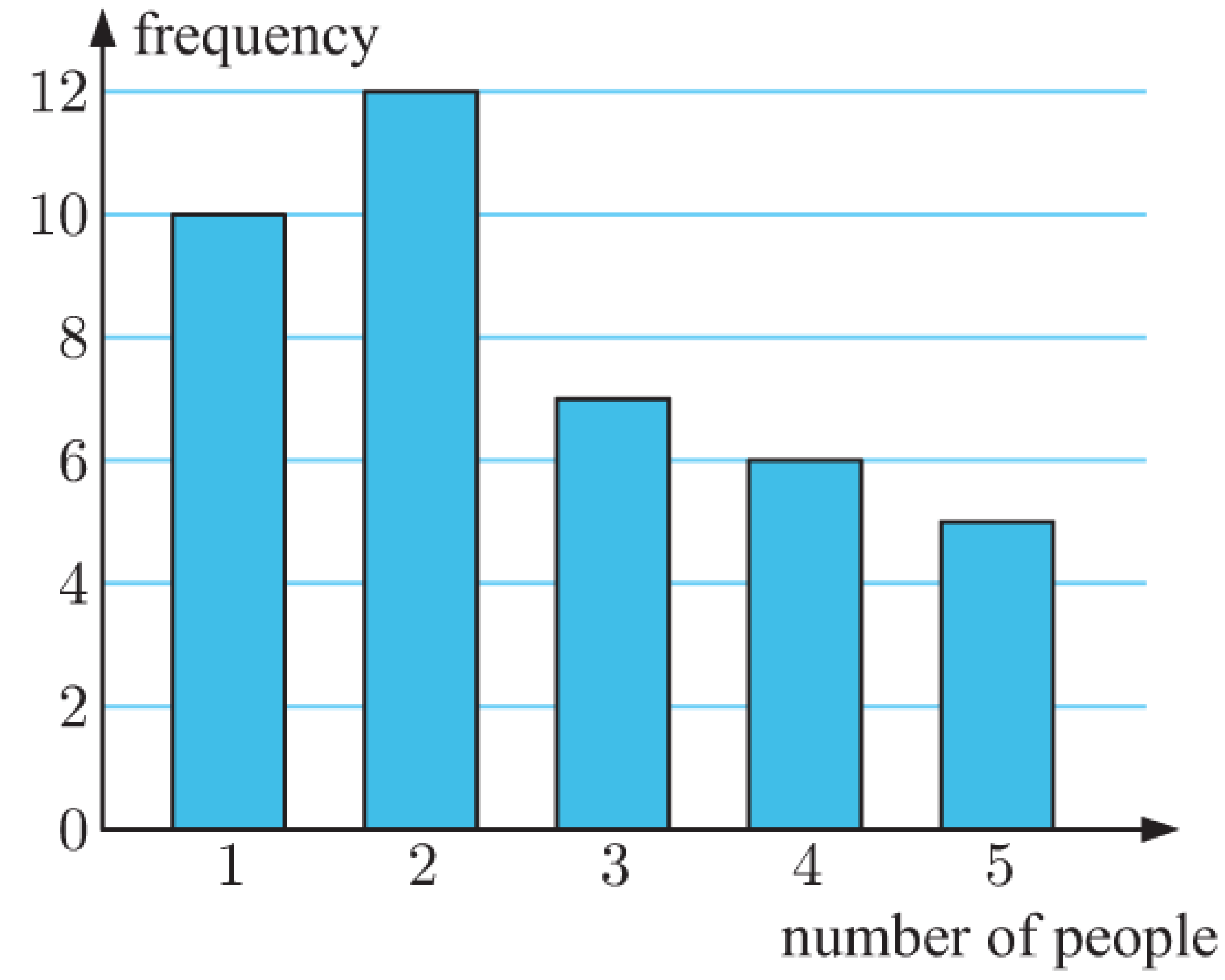
We can organise this numerical data using a tally and frequency table, and display the data using a dot plot or column graph.

Number of people	*Tally*	*Frequency*
1	𝍸 𝍸	10
2	𝍸 𝍸 ||	12
3	𝍸 ||	7
4	𝍸 |	6
5	𝍸	5
	Total	40

Number of people in each car



Number of people in each car



As with categorical data, the **mode** is the most frequently occurring value. In this case, the mode is 2 people.

If there are two values which occur most frequently, we say they are both modes and that the data is **bimodal**.

Example 7 **Self Tutor**

An exceptional hockey player scored the following number of goals over a 25 match period:

4 3 6 1 5 8 4 2 2 4 6 0 5 1 9 3 7 2 6 6 8 3 6 2 10

a Organise the data in a tally and frequency table.

b Draw a column graph to display the data.

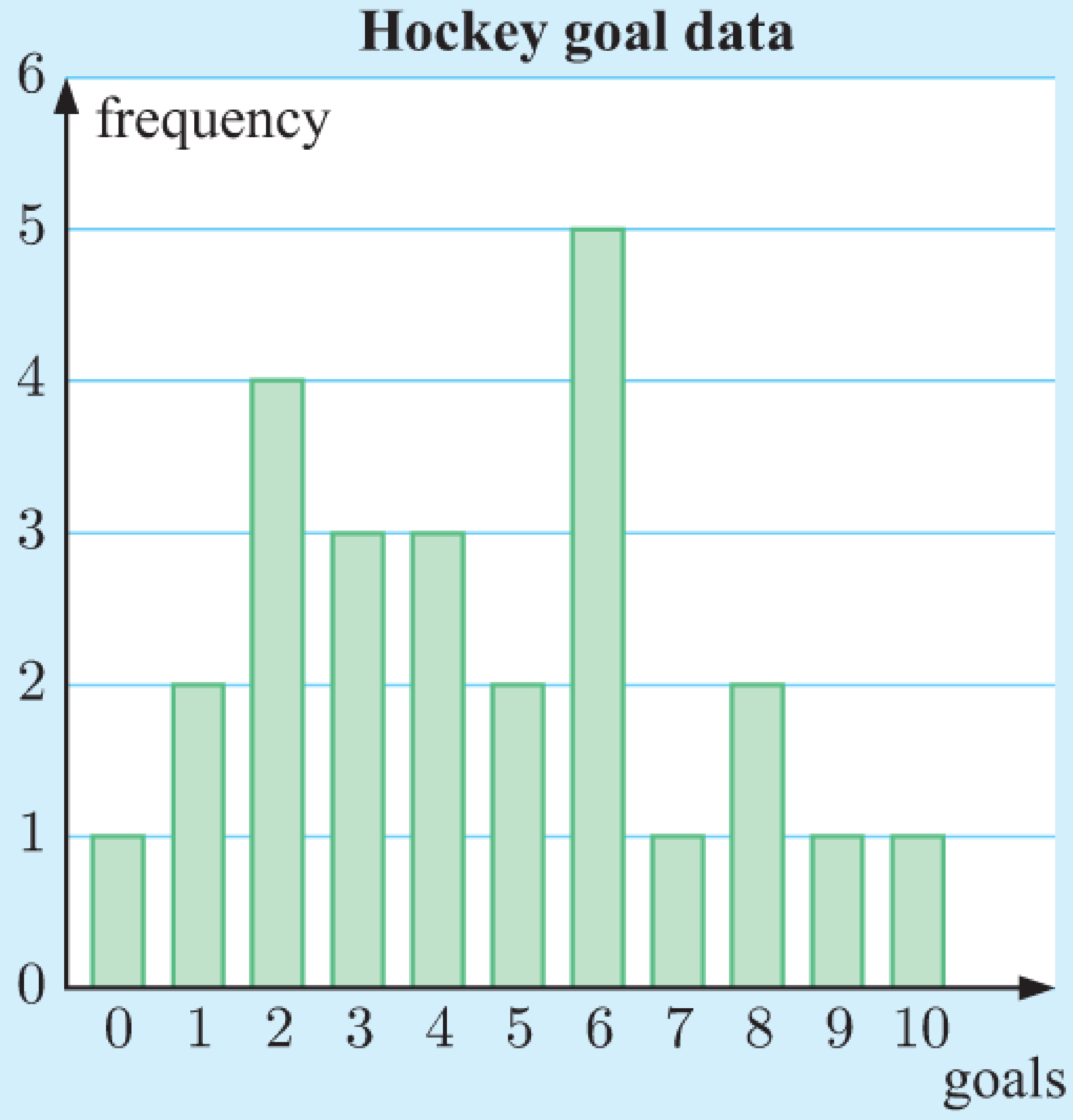
c On how many occasions did the player score 5 or more goals in a match?

d Find the mode of the data.

a

Goals	*Tally*	*Frequency*
0	|	1
1	||	2
2	||||	4
3	|||	3
4	|||	3
5	||	2
6	卌	5
7	|	1
8	||	2
9	|	1
10	|	1

b



c The player scored 5 or more goals on $2+5+1+2+1+1=12$ occasions.

d The mode of the data is 6 goals.

EXERCISE 15D

1 The number of children in 30 families is shown below:

0, 4, 6, 2, 1, 3, 2, 4, 0, 2, 1, 2, 5, 0, 2, 3, 1, 4, 2, 1, 2, 4, 3, 3, 0, 4, 5, 2, 2, 4

a Draw a tally and frequency table for the data.

b Use your table to find the number of families with:

- **i** exactly two children
- **ii** at least three children.

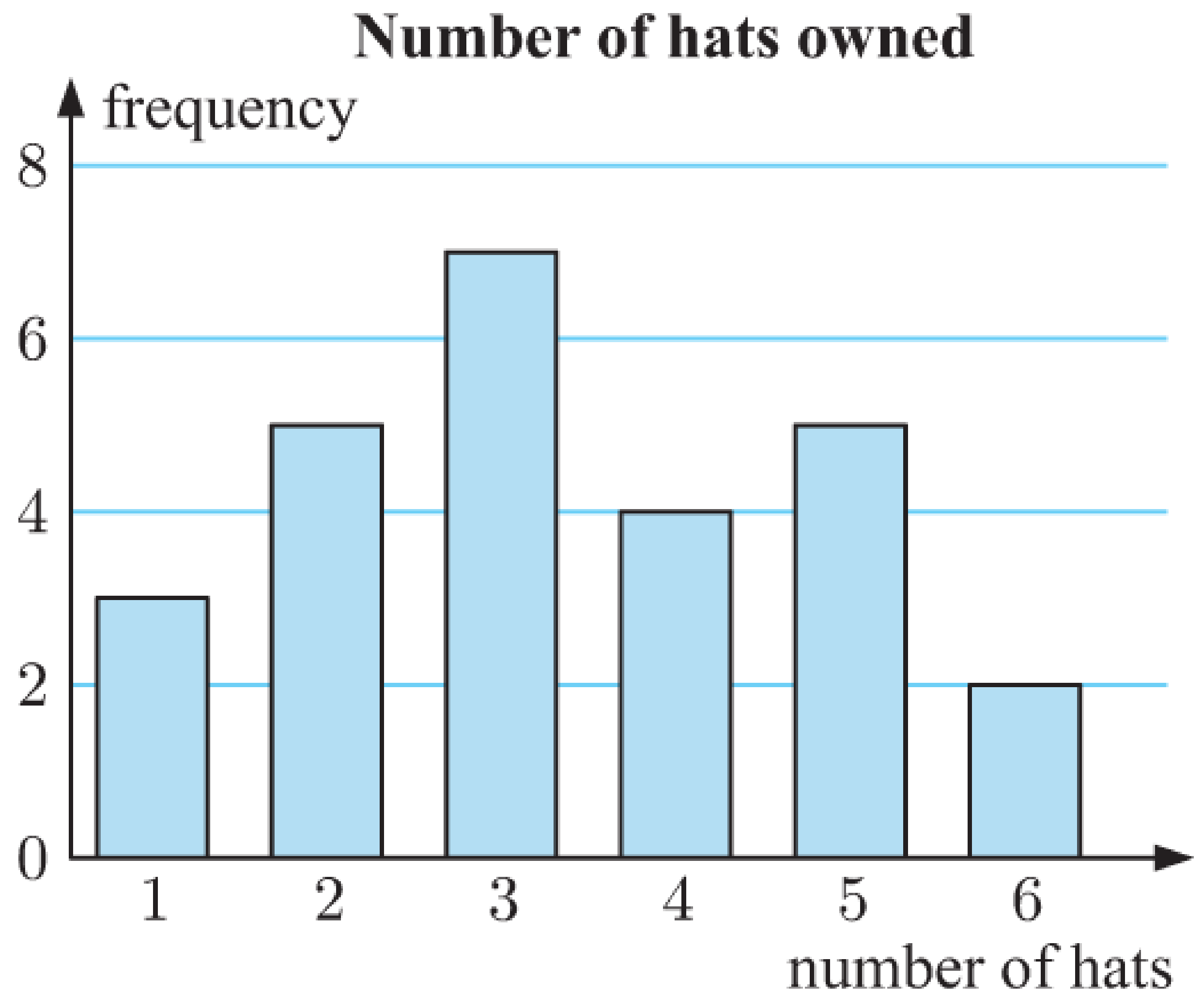
c What is the mode of the data? Explain what this means.

2 A class of students was asked how many hats they owned. Their answers are shown in the column graph.

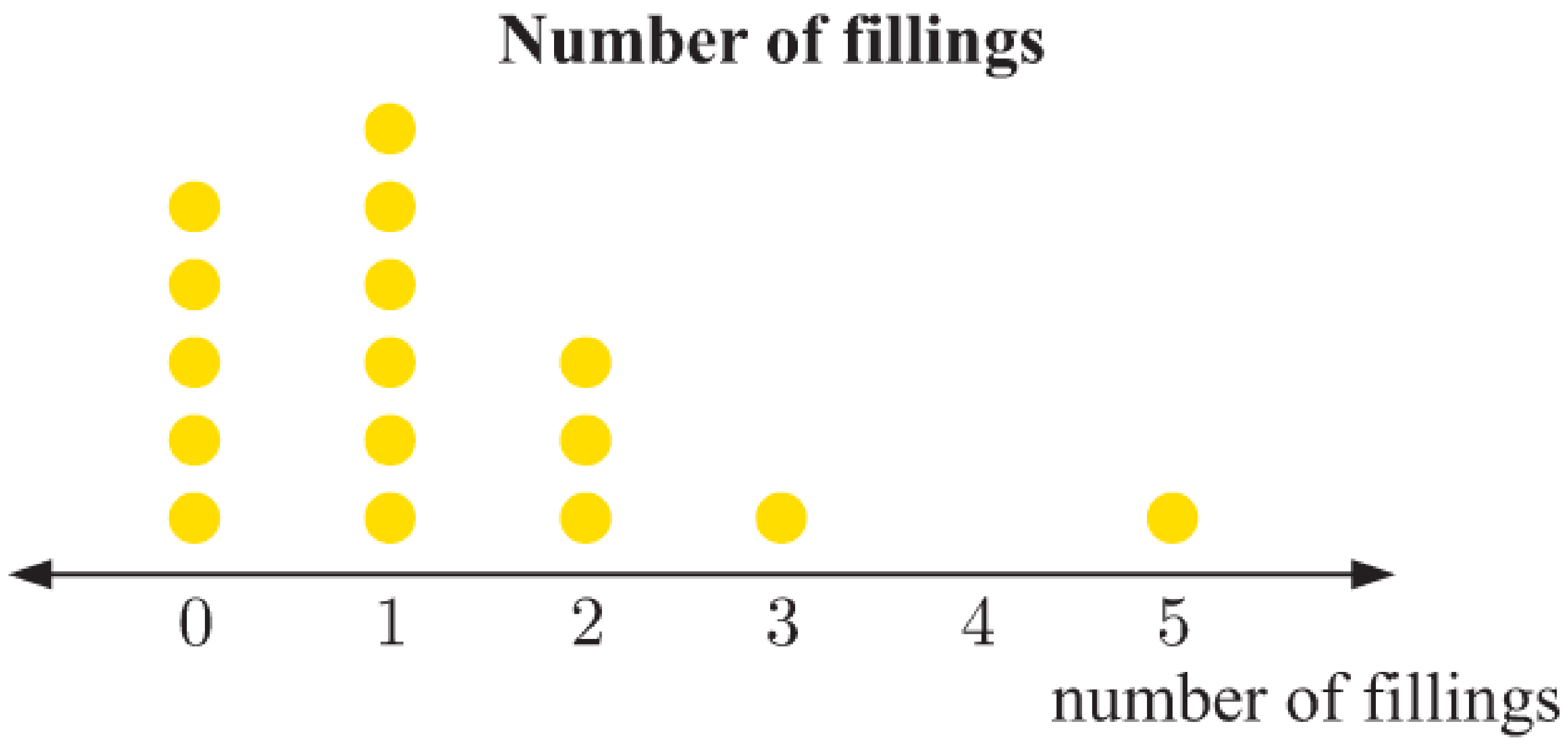
a How many students own 2 hats?

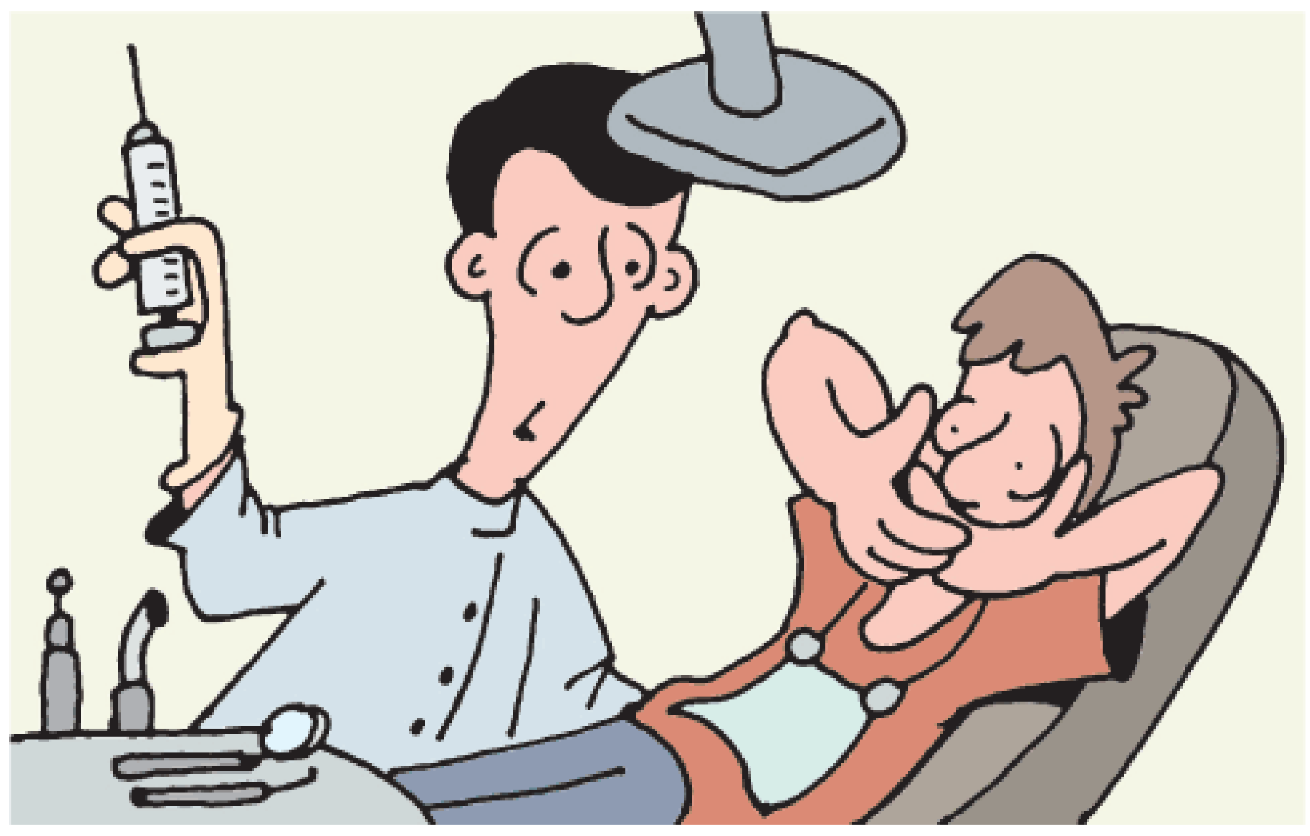
b How many students are in the class?

c Find the mode of the data.



3 This dot plot shows the number of fillings that a group of children received at their last dental appointment.





- **a** How many children received at least 2 fillings?
- **b** Find the mode of the data.

4 A group of athletes ran laps of the athletics track at the end of their training session. The table alongside shows the number of laps they ran.

Number of laps	*Frequency*
2	1
3	2
4	5
5	7
6	3

- **a** Draw a vertical dot plot to display the data.
- **b** Find the mode of the data.
- **c** How many athletes ran less than 4 laps?
- **d** How many athletes were in the group?

5 The numbers of home runs scored by a baseball team in each match of a season were:

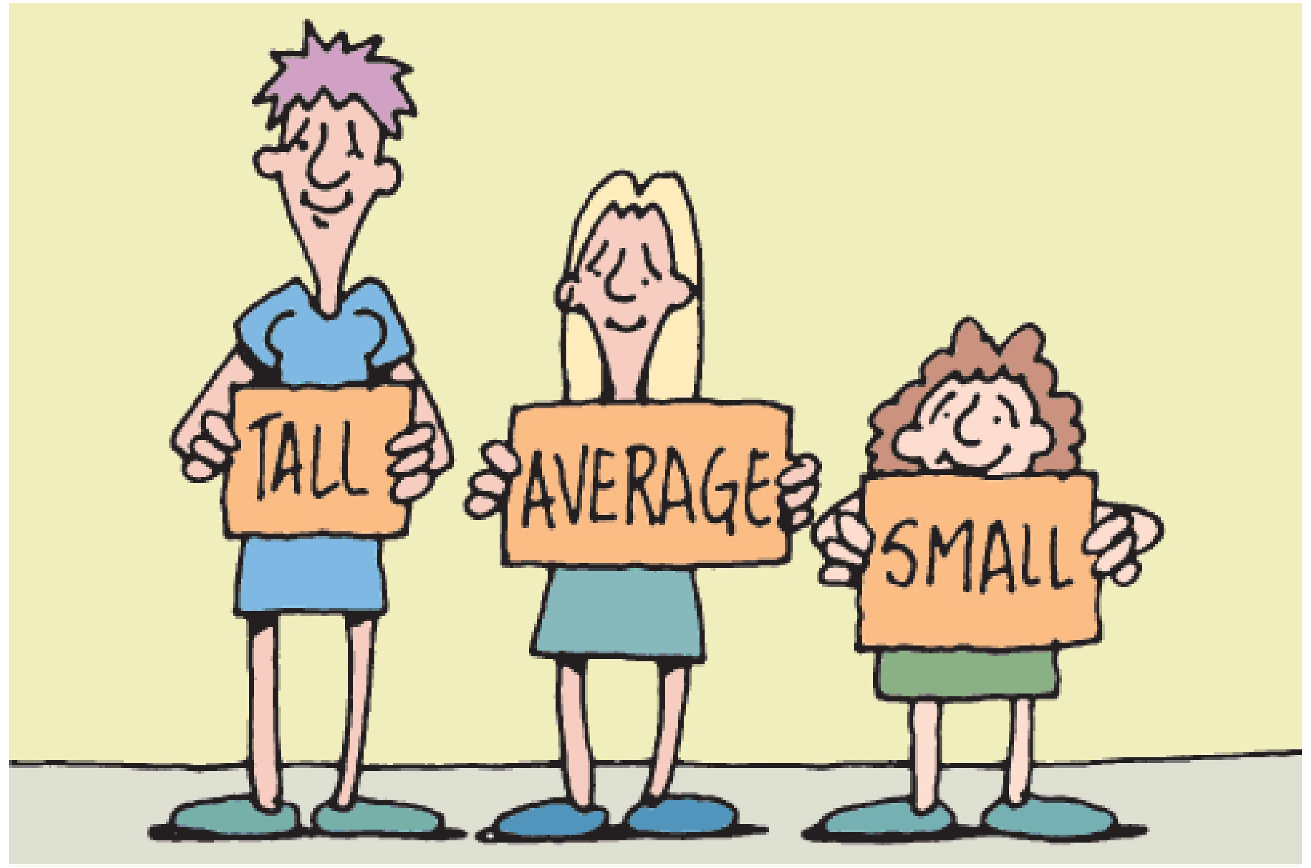
3 0 4 2 0 3 3 1 2 1 1 2 3 3 2 2 5 0 2 1

- **a** Complete a tally and frequency table for the given data.
- **b** Draw a column graph to display the data.
- **c** Find the number of games in which the team scored exactly 3 home runs.
- **d** In what fraction of games did the team score no home runs?
- **e** In what percentage of games did the team score at least 4 home runs?

E MEAN OR AVERAGE

We have all heard the word “average”. We talk about:

- the average speed of a car
- average height and weight
- the average score for a test
- the average wage or income
- the average temperature in summer.



DISCUSSION

List some other situations in which we talk about averages.

What do we actually mean by the word “average”?

In statistics, the **mean** or **average** of a set of numerical data is an important measure of its **centre**.

The **mean** or **average** of a set of numerical data is given by:

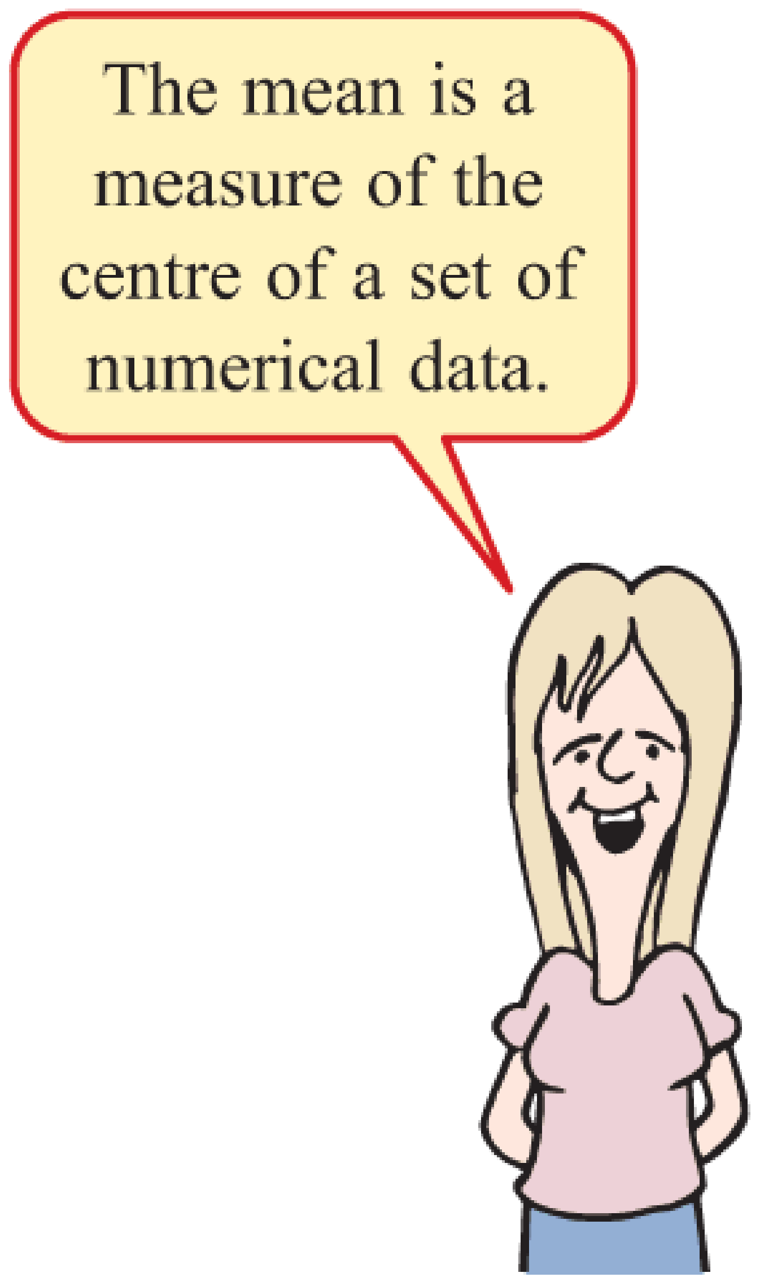
$$\textbf{mean} = \frac{\textbf{sum of the data values}}{\textbf{the number of data values}}$$

Example 8 **Self Tutor**

Find the mean of 7, 11, 15, 6, 11, 19, 23, 0, and 7.

There are 9 data values.

$$\begin{aligned}\text{mean} &= \frac{\text{sum of data values}}{\text{number of data values}}\\ &= \frac{7+11+15+6+11+19+23+0+7}{9}\\ &= \frac{99}{9}\\ &= 11\end{aligned}$$



EXERCISE 15E

1 Find the mean of each data set:

a 4, 5, 8, 11

b 2, 2, 6, 7, 8

c 6, 9, 5, 12, 9, 7

d 13, 7, 6, 14, 11, 16, 10

e 5, 9, 1, 2, 0, 3, 9, 3

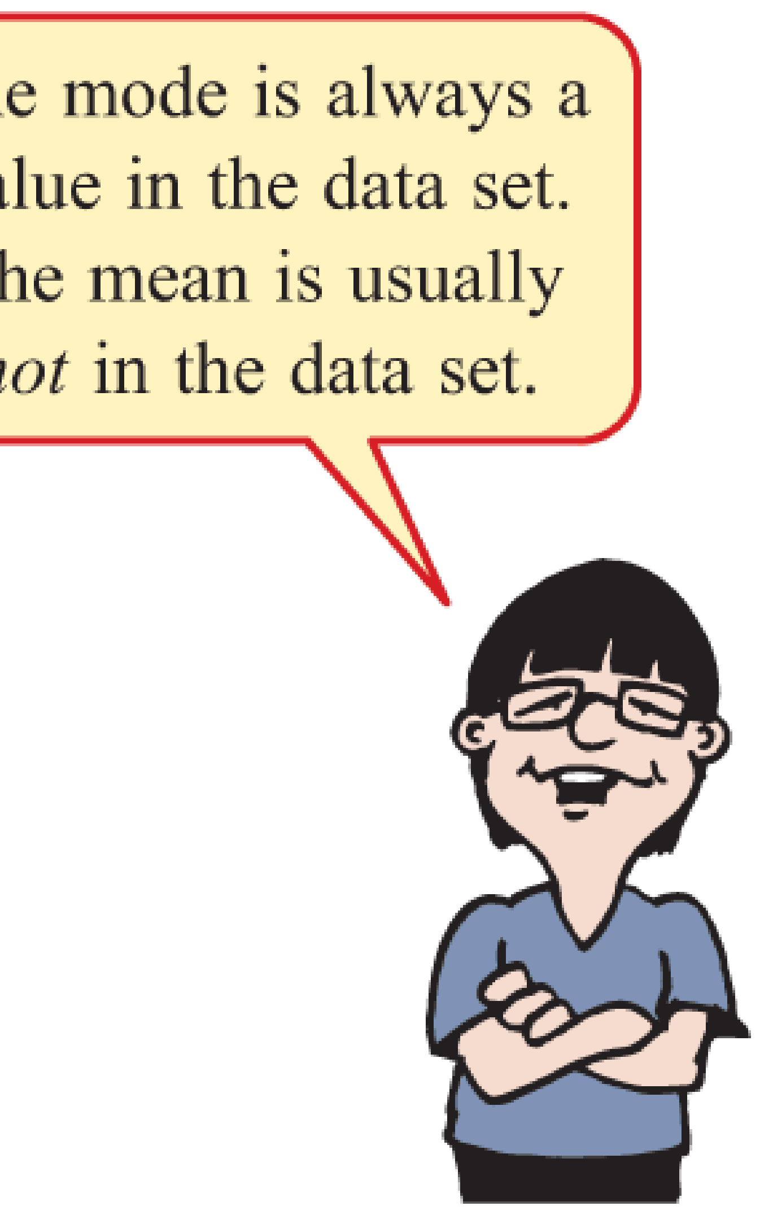
f 3, 4, 4, 6, 7, 8, 8, 10, 11, 12

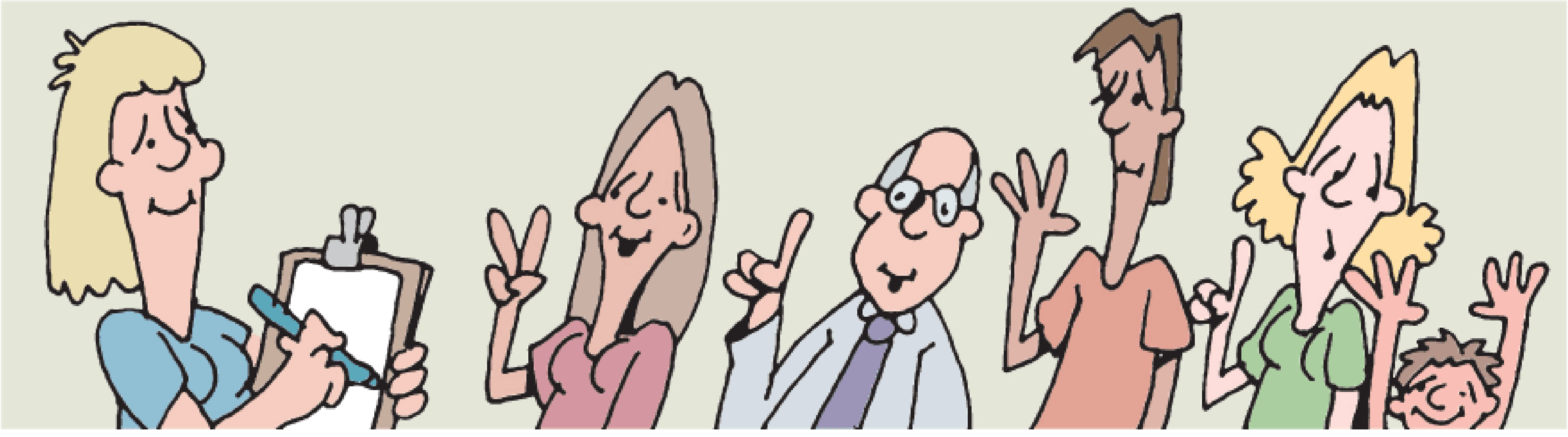
2 People at a sweets store were asked how many chocolates they ate last week. The responses were:

2 5 0 6 1 8 0 2 5 4 1 2

a Find the average number of chocolates eaten.

b Find the mode of the data.

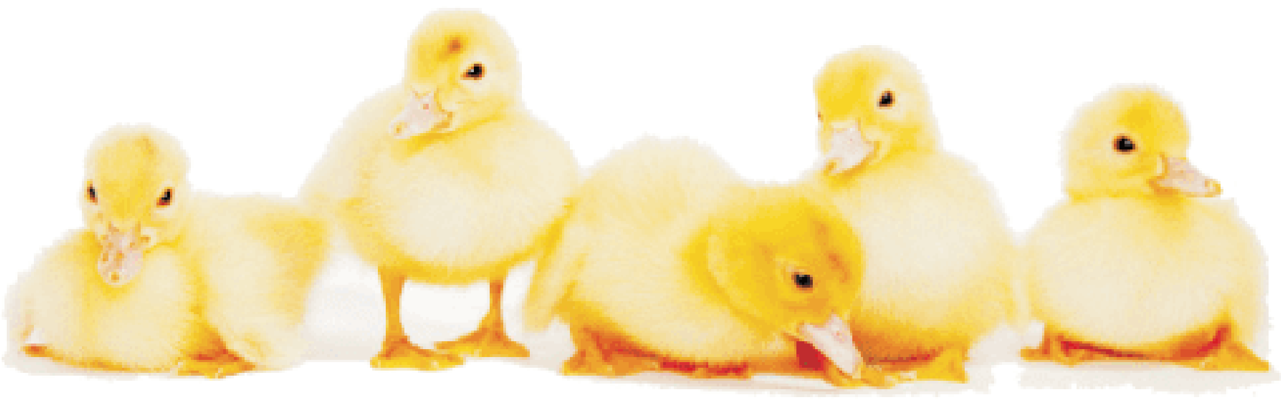




3 The weights of a group of newborn ducklings are:

50 g, 55 g, 52 g, 61 g, 59 g, 59 g.

Find the average birthweight of the ducklings.



4 In a skijumping competition, Lars jumps the following distances:
110 m, 112 m, 118 m, 103 m, 122 m.
Calculate the average length of Lars' ski jumps.

5 Olivia records how long it takes her to cycle to school each day for 15 days. The results, in minutes, are:

17 14 22 23 19 24 25 20 17 18 23 16 22 22 18

Find the average length of time it takes Olivia to cycle to school.

6 The data alongside shows the number of letters which were delivered to each house on a street one day.

2	1	0	1	2
0	1	3	2	1
1	4	0	0	2
4	1	1	3	1

- **a** Draw a vertical dot plot to display the data.
- **b** How many houses received more than 2 letters?
- **c** Find the mode of the data.
- **d** Find the mean of the data.

7 Cameron played 16 cricket matches for his local club this season. The number of wickets he took in each match is given below:

0 3 4 2 3 5 4 4 3 0 4 2 1 5 6 2

- **a** Draw a column graph of the data.
- **b** Find the mode of the data.
- **c** **i** Find Cameron's average number of wickets per match.
  - **ii** Did Cameron take this average number of wickets in any particular game? Discuss your answer.

INVESTIGATION THE MODE AND THE MEAN

In this Investigation we think about when it is useful to talk about the mode of a data set, and when it is useful to talk about the mean.

What to do:

1 The ages of tourists in two tour buses are given below:

Bus A: 40 55 37 48 59 53 56 52 37
Bus B: 53 42 50 41 47 36 55 41 40 45 50

- **a** Find the number of tourists in: **i** bus A **ii** bus B.
- **b** Find the mode of the ages of tourists in: **i** bus A **ii** bus B.
- **c** Find the average age of the tourists in: **i** bus A **ii** bus B.
- **d** On average, which bus has the younger tourists?
- **e** Is it more meaningful to talk about the mode or the mean in this case? Explain your answer.

2 The next day, the tour buses are driving with different groups of tourists.

Bus A contains six 18 year old university students.
Bus B contains a 51 year old teacher and five 12 year old students.

- **a** Find the number of tourists in: **i** bus A **ii** bus B.
- **b** Find the mode of the ages of tourists in: **i** bus A **ii** bus B.
- **c** Find the average age of the tourists in: **i** bus A **ii** bus B.
- **d** On average, which bus has the younger tourists?
- **e** Is it more meaningful to talk about the mode or the mean in this case? Explain your answer.

3 Discuss your results with your class.

ACTIVITY A STATISTICAL EXPERIMENT

In this Activity, you will grow wheat over a 21 day period in a controlled experiment. You will use 6 grains of wheat in each of 4 plots.

You will need: 4 saucers or coffee jar lids, cotton wool, 24 grains of wheat, an eye dropper, water, diluted liquid fertiliser.

What to do:

1. Layer the cotton wool three quarters of the way up each lid. Place 6 grains of wheat at equal distances apart in each lid.
2. Label the lids as plots 1, 2, 3, and 4. Saturate each plot with 15 mL of water.
3. Over a 3 week period, perform these steps:
    - In plot 1 squeeze 2 drops of water onto each grain of wheat every Monday, Wednesday, and Friday.
    - In plot 2 squeeze 2 drops of water onto each grain of wheat every weekday.
    - In plot 3 squeeze 2 drops of water and 1 drop of diluted fertiliser onto each grain every Monday, Wednesday, and Friday.
    - In plot 4 squeeze 2 drops of water and 1 drop of diluted fertiliser onto each grain every weekday.
4. Place all the plots in the same safe, sheltered place, with plenty of light.
5. Every Monday, Wednesday, and Friday, record the mean height of any germinating seeds for each plot. Avoid handling any shoots. Make a table to summarise your results.
6. Use graphs and the language of statistics to comment on your results.

Global context

click here

Clothing sizes

Statement of inquiry:	Collecting data can help businesses to make informed decisions.
Global context:	Fairness and development
Key concept:	Logic
Related concepts:	Representation, Justification
Objectives:	Knowing and understanding, Applying mathematics in real-life contexts
Approaches to learning:	Thinking, Communication

KEY WORDS USED IN THIS CHAPTER

- average
- categorical data
- column graph
- dot plot
- inference
- mean
- mode
- numerical data
- pie chart
- population
- random sample
- sample
- tally and frequency table

REVIEW SET 15A

1 Suggest how you could select:

a a random sample of houses in a suburb **b** a random student from your class.

2 Yuka surveyed the orders of the first 20 customers at her sushi shop on Friday. Her results are shown in the table. Draw a column graph of her data.

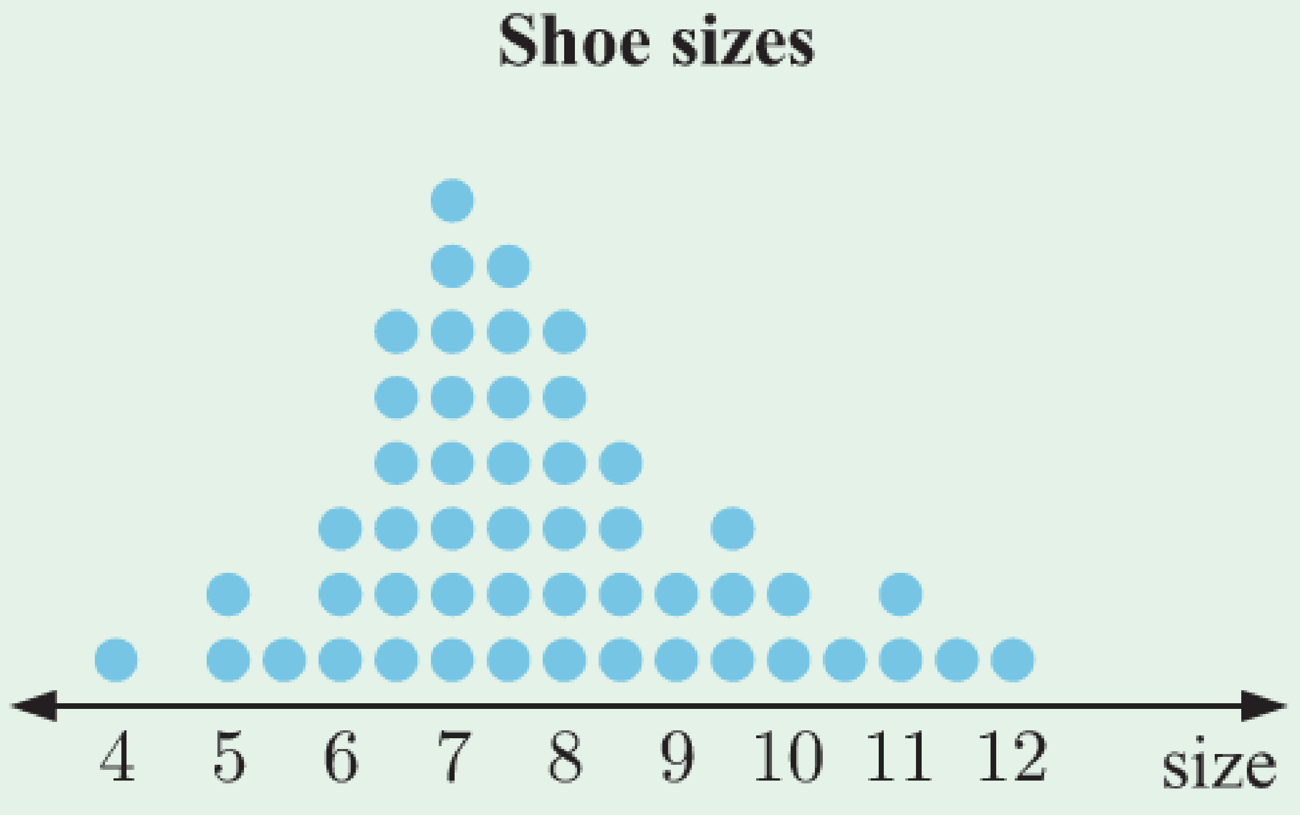
Food	*Frequency*
Tekkamaki	7
Hamachi	1
Kappa Maki	4
Ebi Nigiri	5
Maguro Nigiri	3

3 This dot plot shows the shoe sizes for the students in Grade 6 at a school.

a How many students are in Grade 6 at this school?

b How many of the students have shoe size 9 or more?

c Find the mode of the data.



4 The times, in minutes, for the customers at a restaurant to receive their meals are:

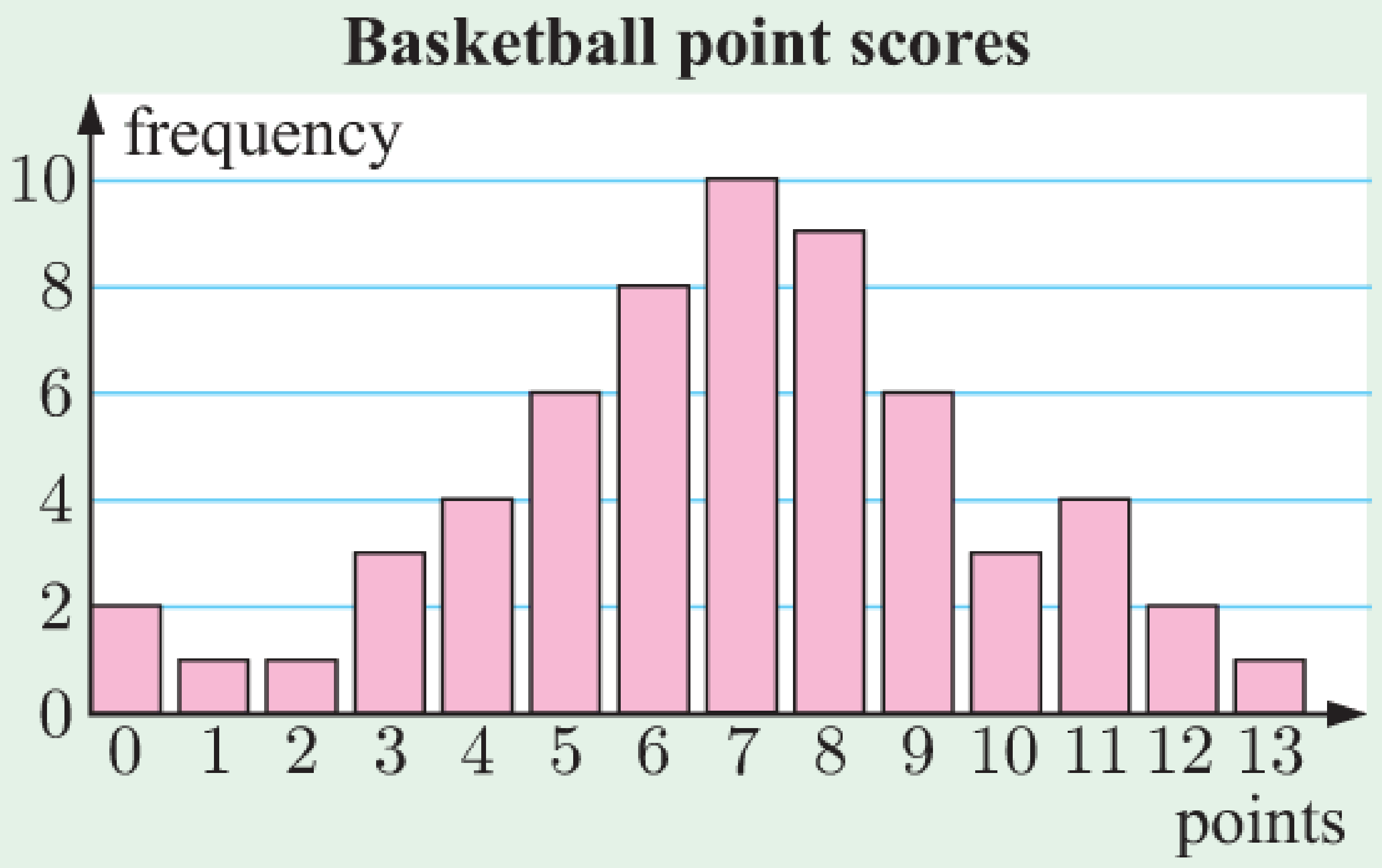
19 28 31 8 22 18 35 24 15 9 28 17 28 20 13

Find the average time for the customers to receive their meals.

5 The column graph shows the number of points scored by a basketballer over 60 matches.

a What point score occurred most frequently?

b On how many occasions were 10 or more points scored?



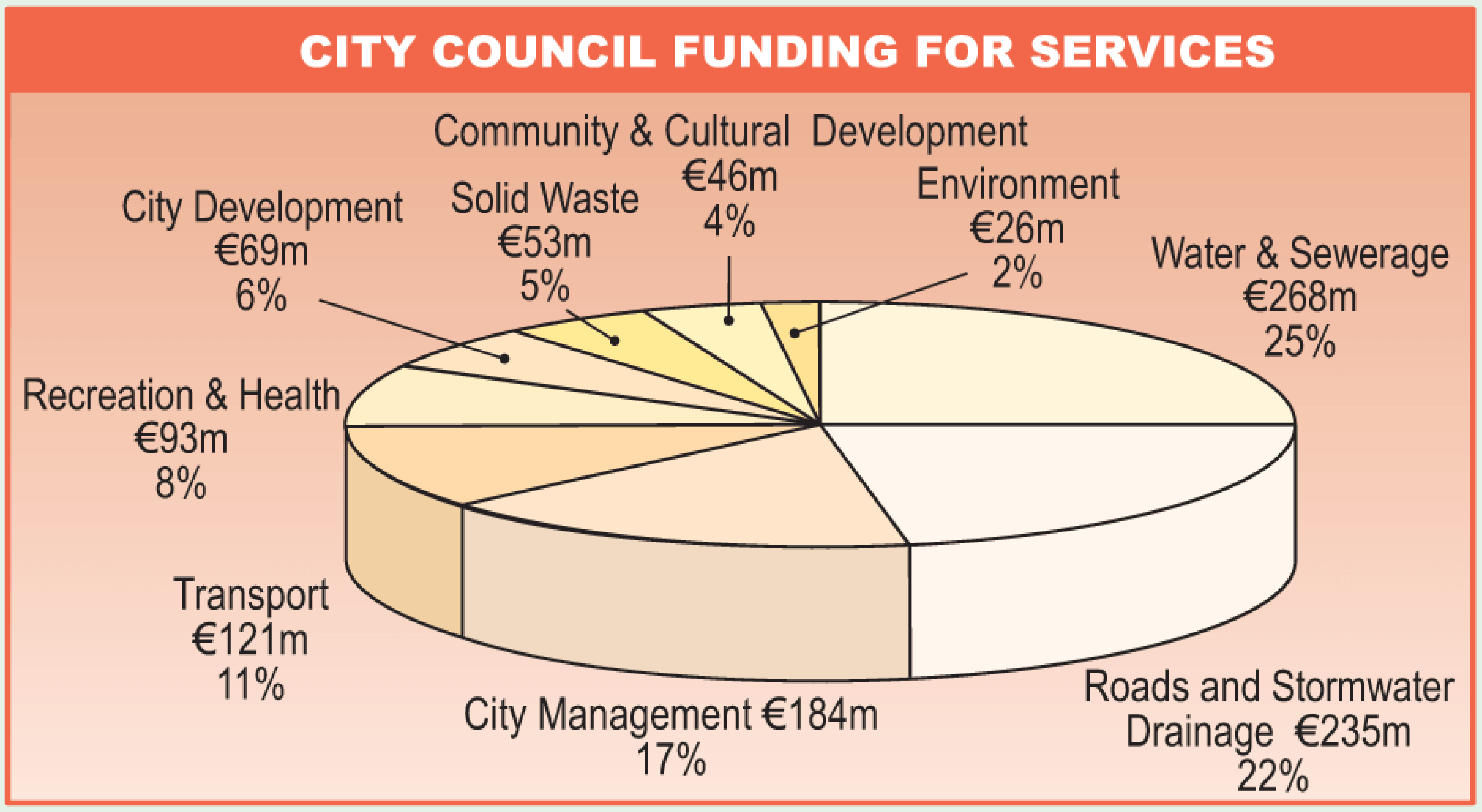
6 Don wants to know how many flowers of each colour there are in his garden.

a Draw a tally and frequency table for the data.

b Find the mode of the data.

c Draw a vertical dot plot to display the data.

7 This pie chart shows both the percentages and the actual amounts a council spent in various departments.



- **a** Comment on the usefulness of having both percentages and amounts shown.
- **b** What percentage of total funding is spent on:
  - **i** Recreation and Health
  - **ii** either Transport or City Management?
- **c** On what service is the smallest amount spent?
- **d** On what service is one quarter of the funding spent?
- **e** Using the percentage given, find the sector angle for Solid Waste.

8 The time taken by a farmer to plough, fertilise, and seed each of his paddocks is given below, in hours:

7 24 9 12 13
28 18 27 24 25

Find the mean of this data.

REVIEW SET 15B

1 From a plantation of 5000 trees, 200 were randomly selected and inspected. It was found that 24 of the trees in the sample were unhealthy.

- **a** How many trees form the population?
- **b** How large was the sample?
- **c** What percentage of the trees in the sample were unhealthy?
- **d** Estimate the number of unhealthy trees in the plantation.

2 Kirsty surveyed the bird life in her area using the categories magpie (M), sparrow (S), kookaburra (K), wren (W), and galah (G).

During a 30 minute period she recorded these birds:

M M S K S G G G G W M G G S S S K
S S S S K M S S S M W S S S W S S
S S S M M S S S K M S S W G G G

a Draw a tally and frequency table for the data.

b Find the mode of the data.

3 The children in a class were asked their favourite rainy day pastime.

The results are given in this table.

Draw a horizontal dot plot of this data.

Rainy day pastime	*Frequency*
Watching TV	5
Reading	6
Computer games	9
Playing music	2
Other	4

4 Sixty people whose houses had been burgled were asked where they were at the time of the burglary. The responses are shown alongside.

Draw a pie chart to display the data.

Response	*Frequency*
At home	12
At work	20
Shopping	5
On holidays	10
Visiting friends	13
Total	60

5 This dot plot shows the number of phone calls Susan received at work each day for 4 weeks.

a Find the mode of the data.

b On how many days did Susan receive less than 4 calls?

Telephone calls

6 Tina rolled a die 35 times, and recorded these results.

a Draw a vertical dot plot to display the data.

b How many times did Tina roll a number greater than 4?

7 A Grade 6 class was given a mathematics test out of 10 marks. Their results were:

8 7 6 9 10 6 7 9 8 5 9 8 7 7 7 9 4 8 7 9

Find the mean score for the class.

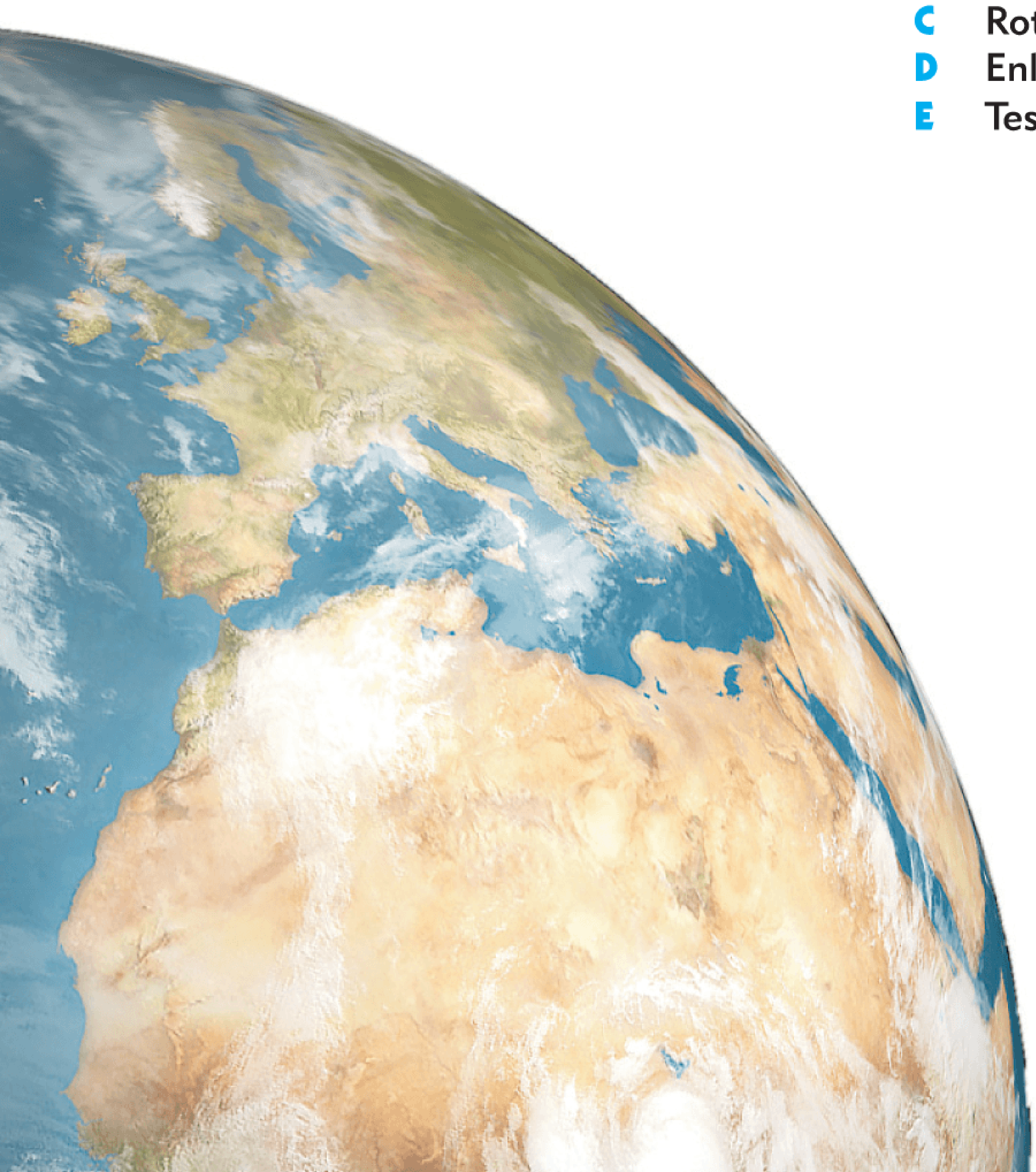
8 The ages of children at a party were: 8 7 8 4 7 3 7 5 6 6 7 4

- **a** Organise the data in a tally and frequency table.
- **b** How many children attended the party?
- **c** How many of the children were aged 7 or 8?
- **d** Display the data on a column graph.
- **e** Find the mode of the data.
- **f** Find the mean of the data.

Chapter 16

Transformations

Contents:



OPENING PROBLEM

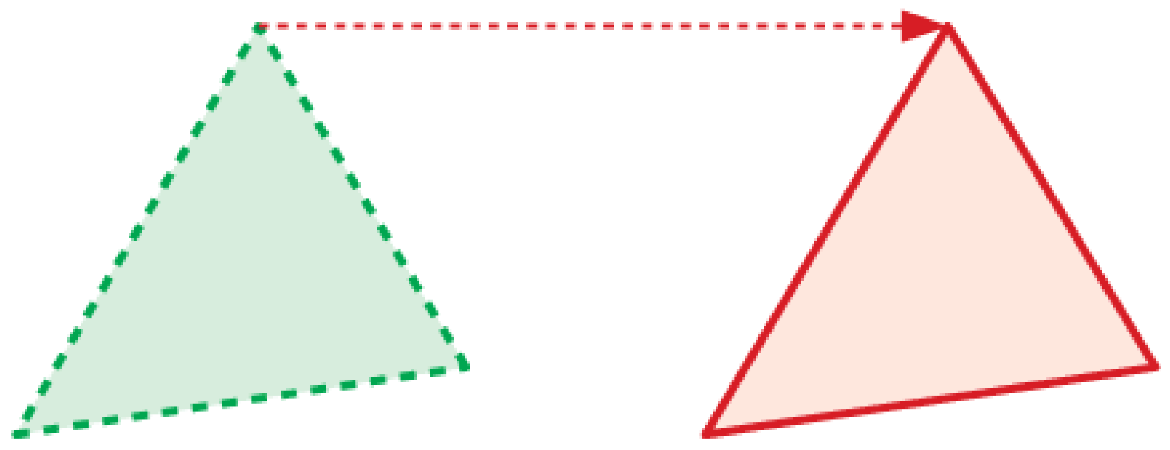
Melissa designed this logo for the Port Thomas Sailing Club.

Things to think about:

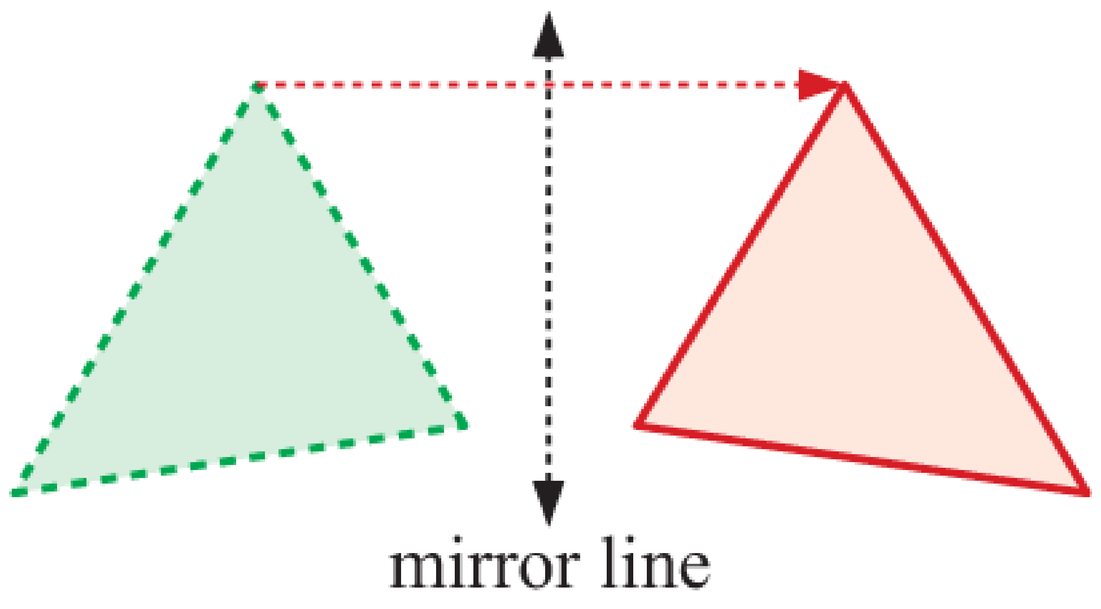
- **a** Are all the boats on the logo the same size and shape?
- **b** How did Melissa *transform* the red boat to create the:
  - **i** yellow boat
  - **ii** orange boat
  - **iii** green boats?

In this chapter we will study four ways in which figures are **transformed**. They are:

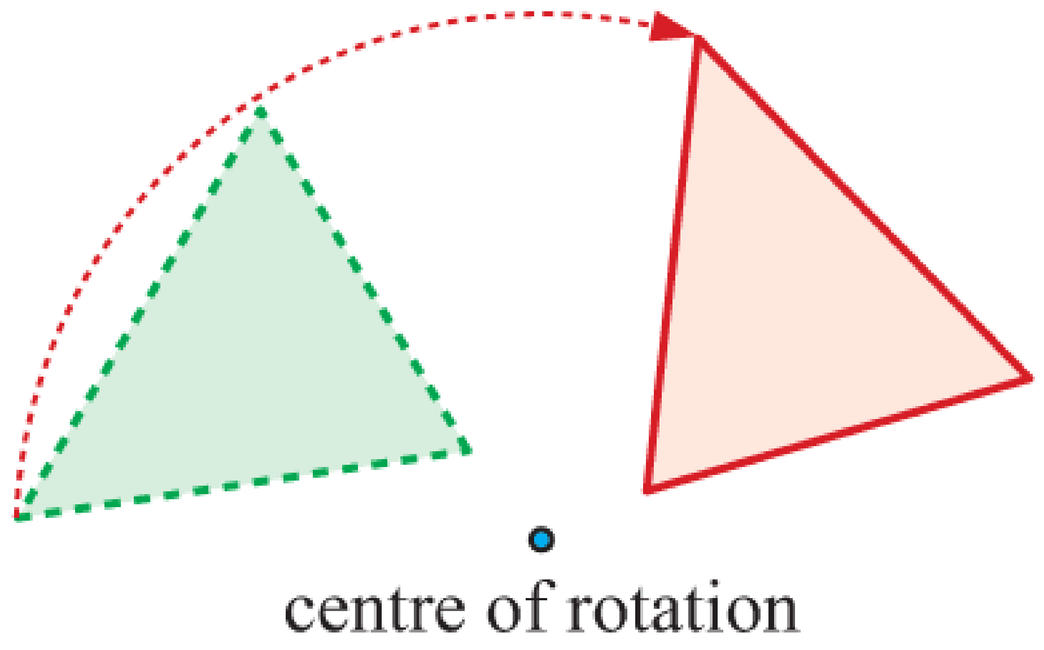
- **translation**



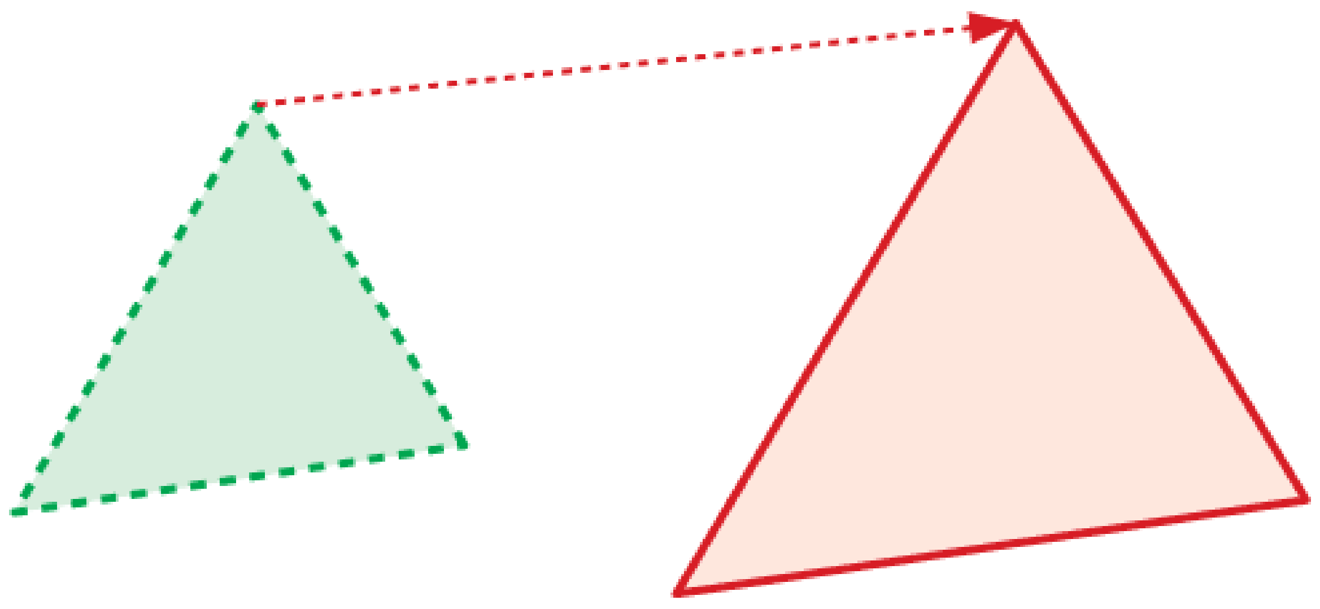
- **reflection**



- **rotation**



- **enlargement and reduction**



When we perform a transformation, the original shape is called the **object**. The shape which results from the transformation is called the **image**.

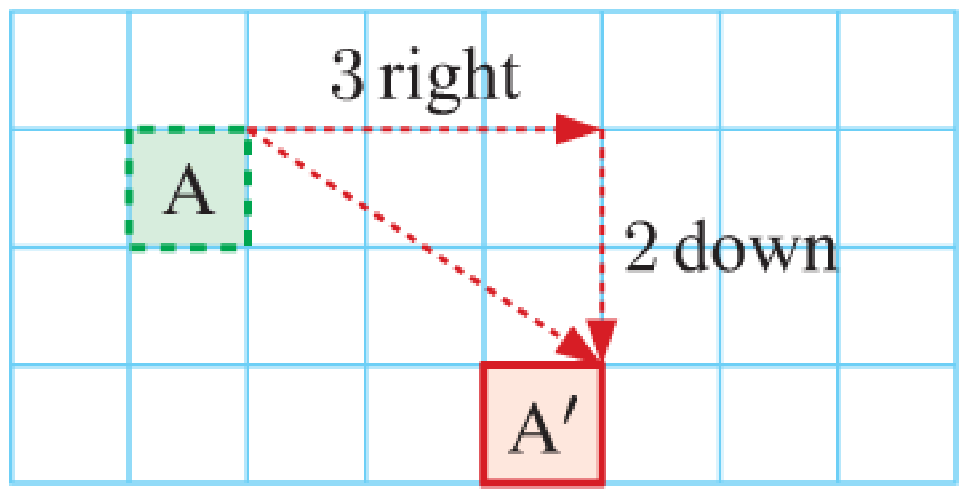
If the object is A then we label the image A′.

A TRANSLATIONS

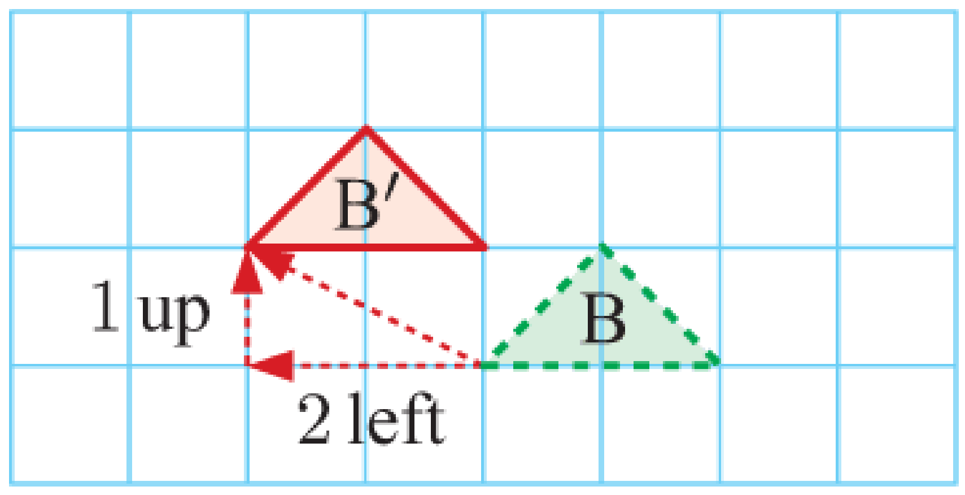
A **translation** moves every point on a figure the same distance in the same direction.

We can describe a translation using a horizontal step (right or left) followed by a vertical step (up or down).

Here are two examples of translation:



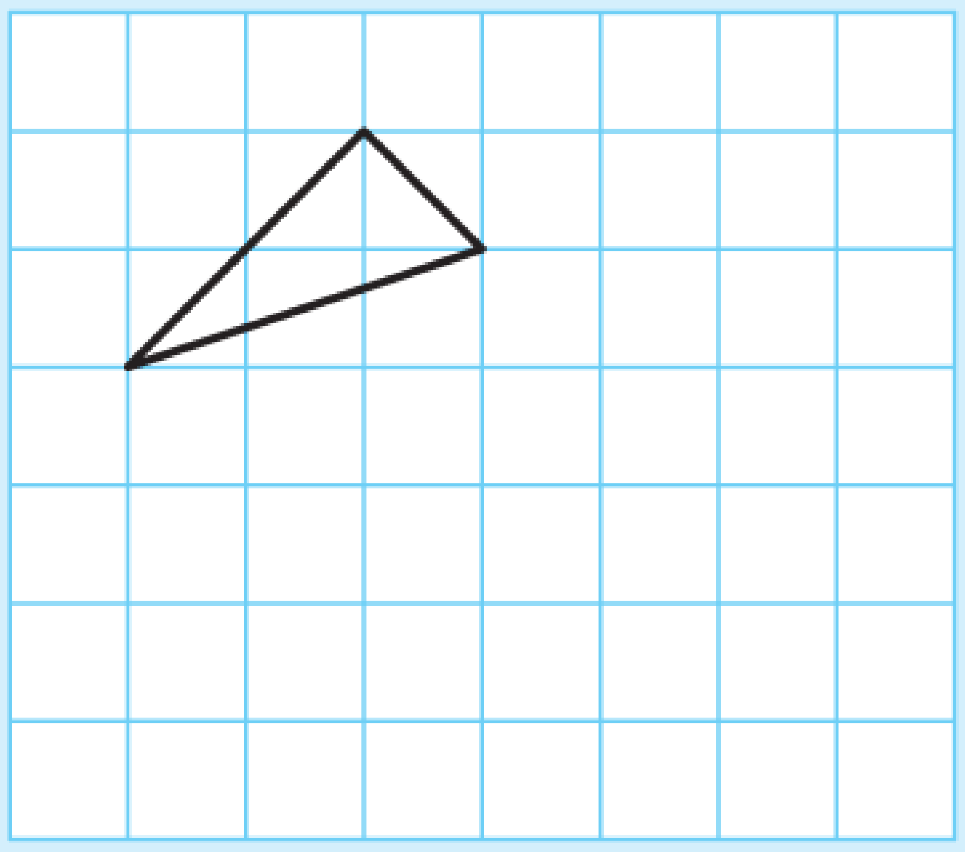
To translate A to A′, we move 3 units right and 2 units down.

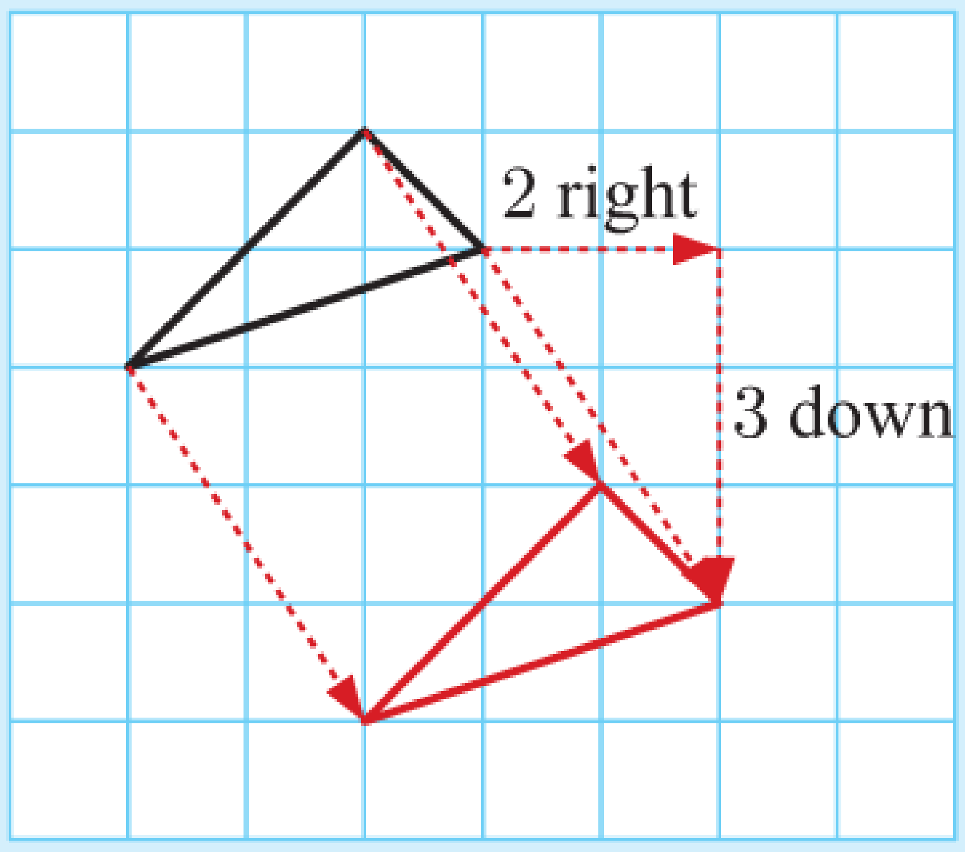


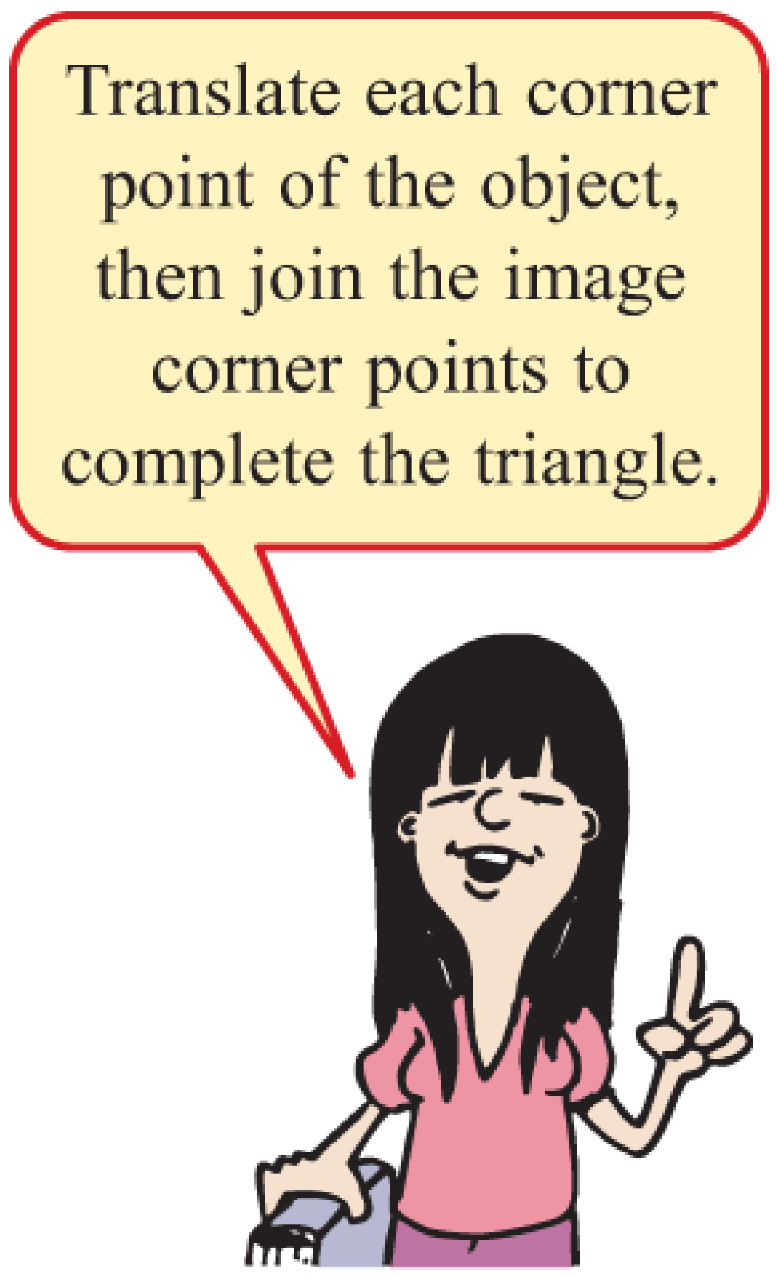
To translate B to B′, we move 2 units left and 1 unit up.

Example 1 **Self Tutor**

Translate this figure 2 units right and 3 units down.



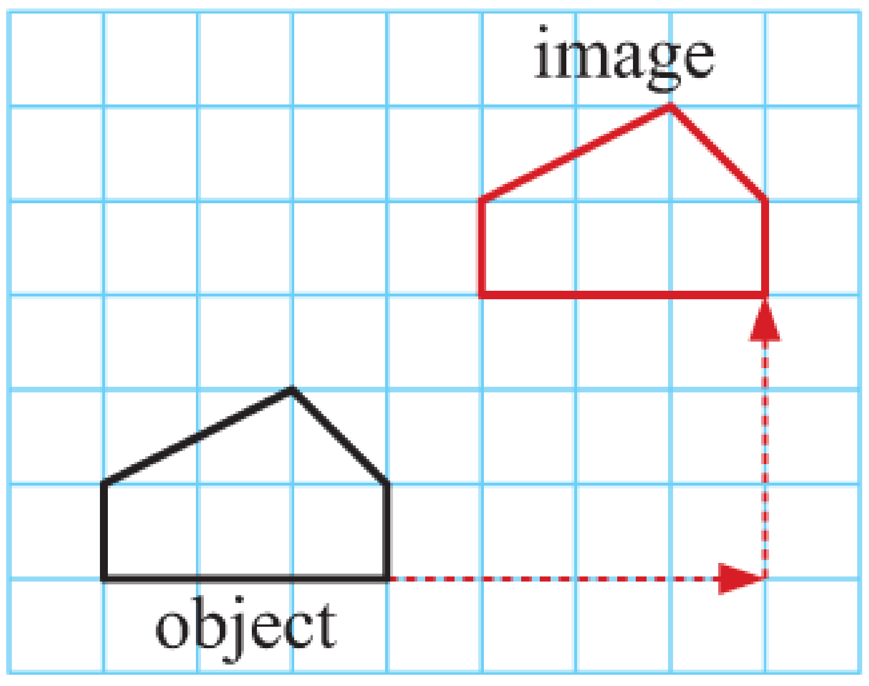




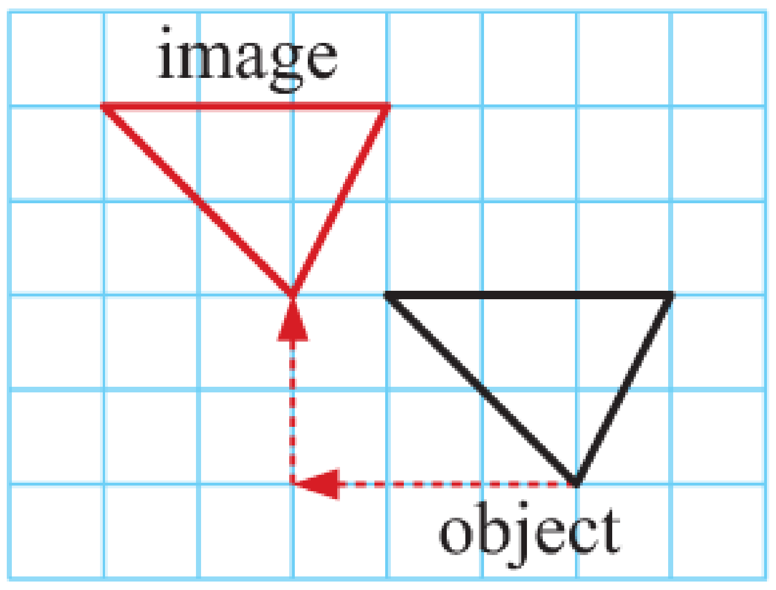
EXERCISE 16A

1 Describe each of these translations using a horizontal step and a vertical step:

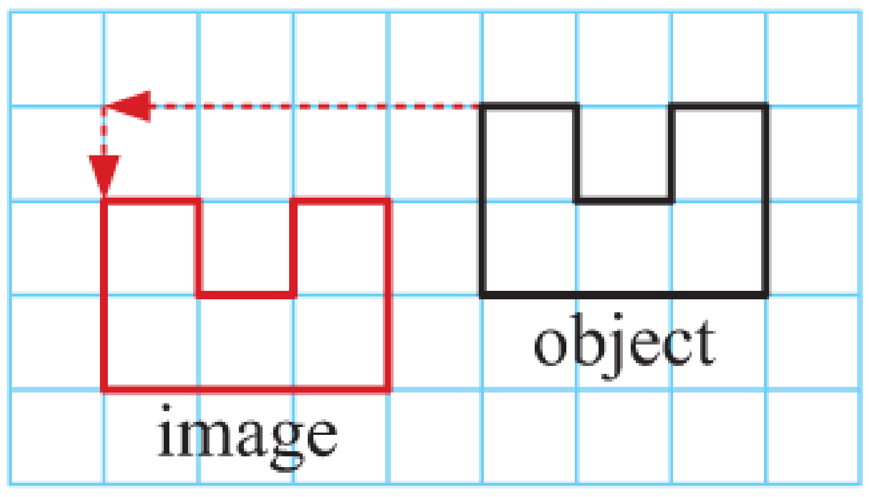
a



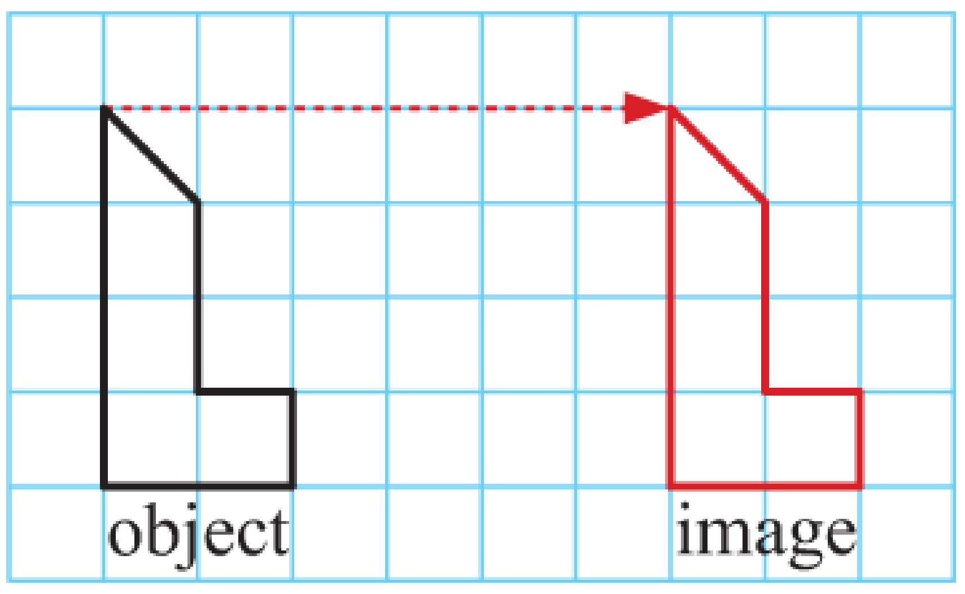
b



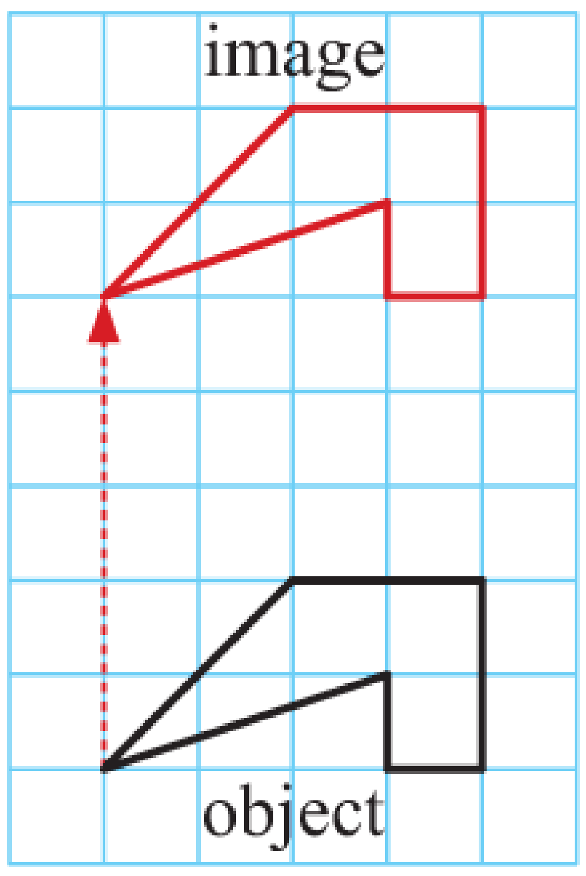
c



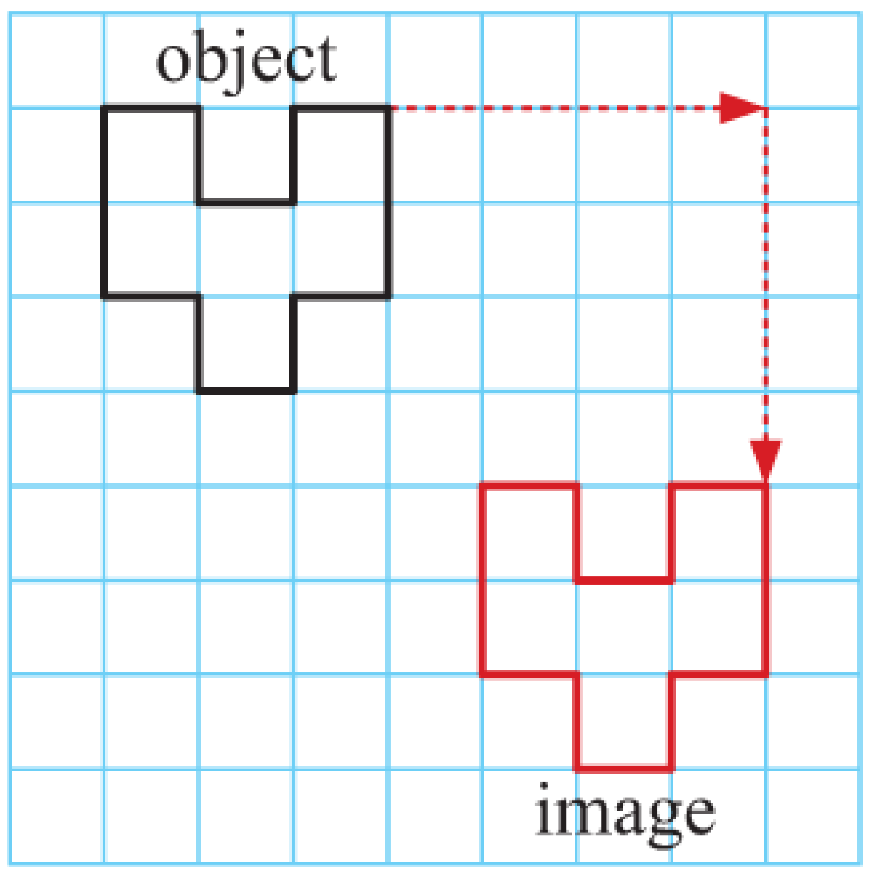
d



e

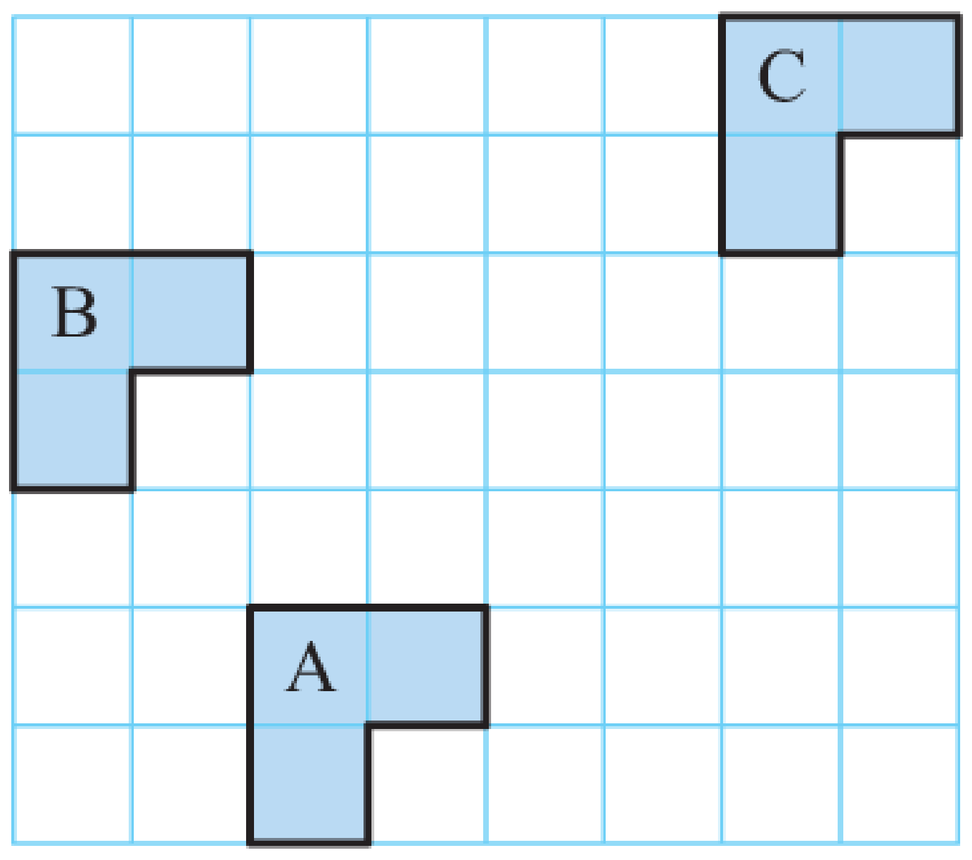


f

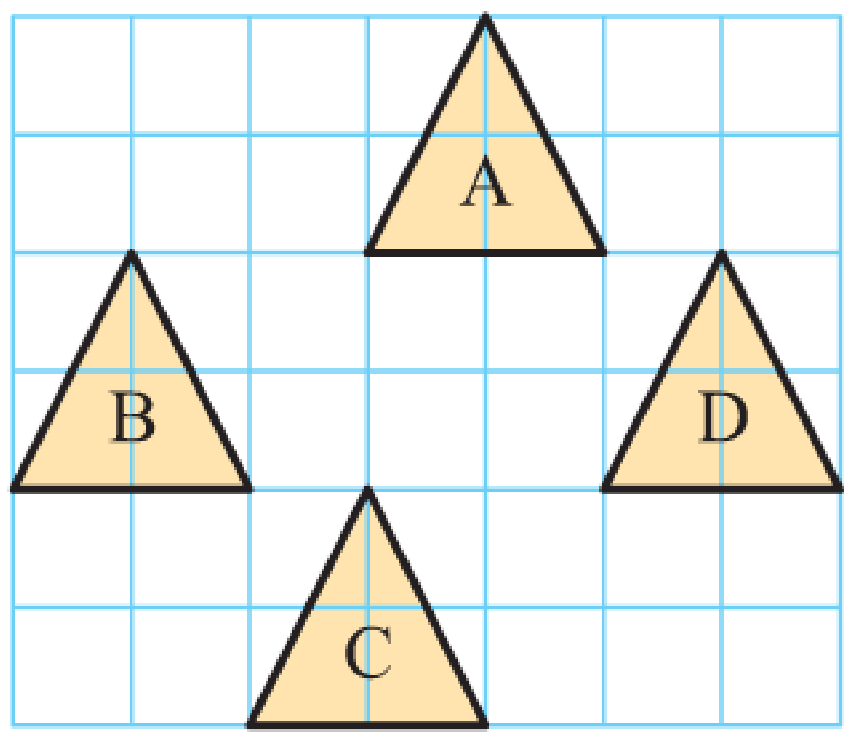


2 Describe the translation from:

a A to B **b** B to C.



3



Describe the translation from:

a A to B **b** A to C

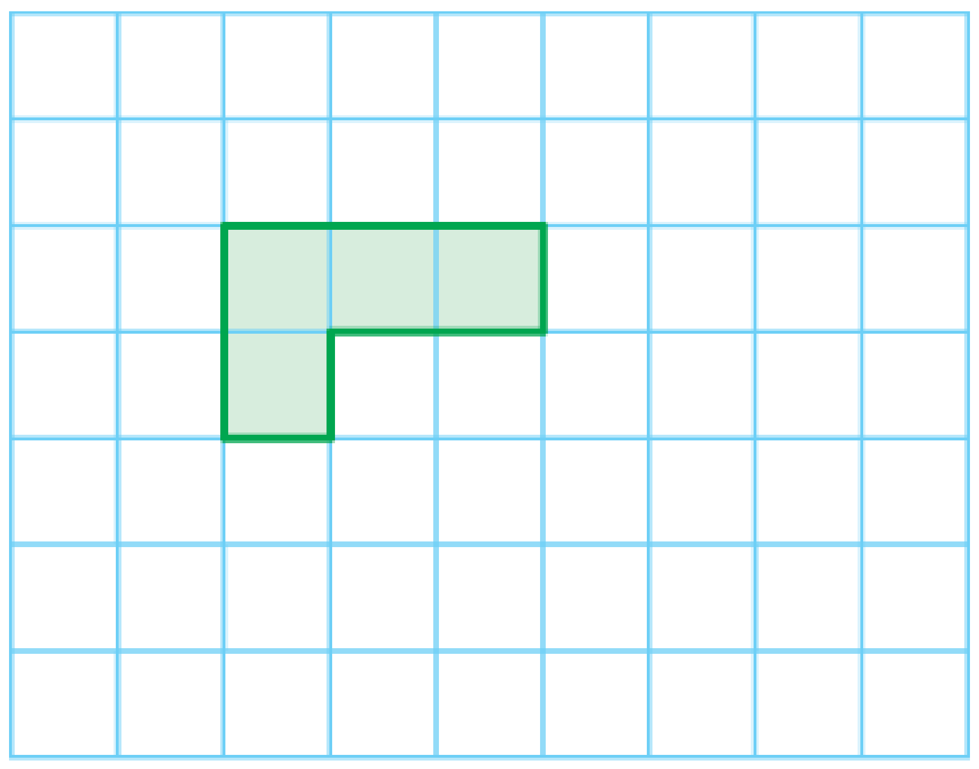
c D to B **d** B to C

e C to D **f** A to D.

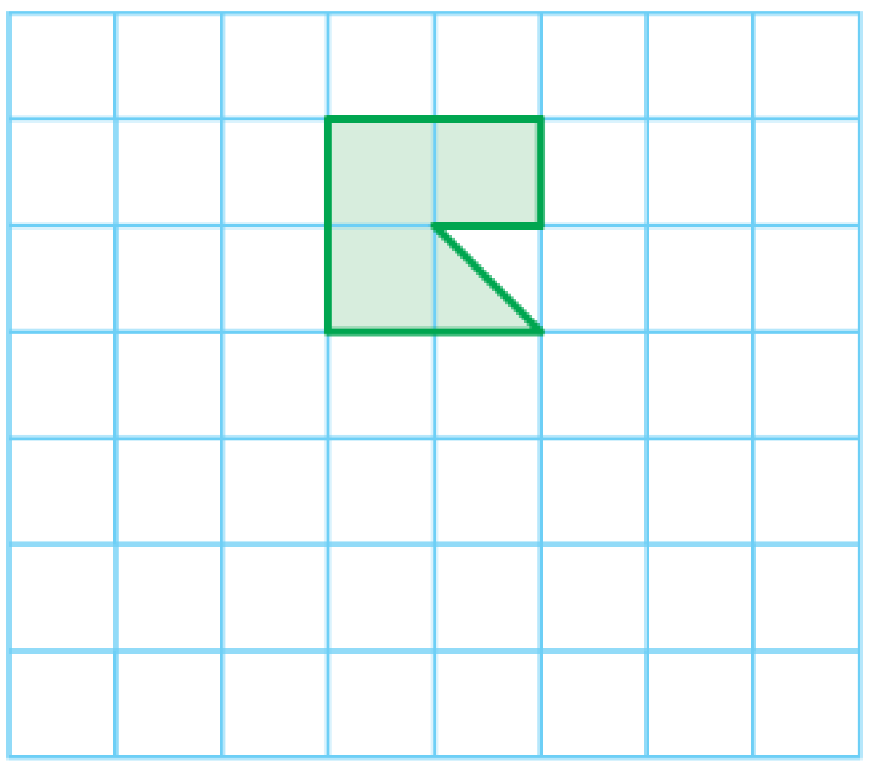
4 For each of these figures, perform the translation given:

PRINTABLE DIAGRAMS

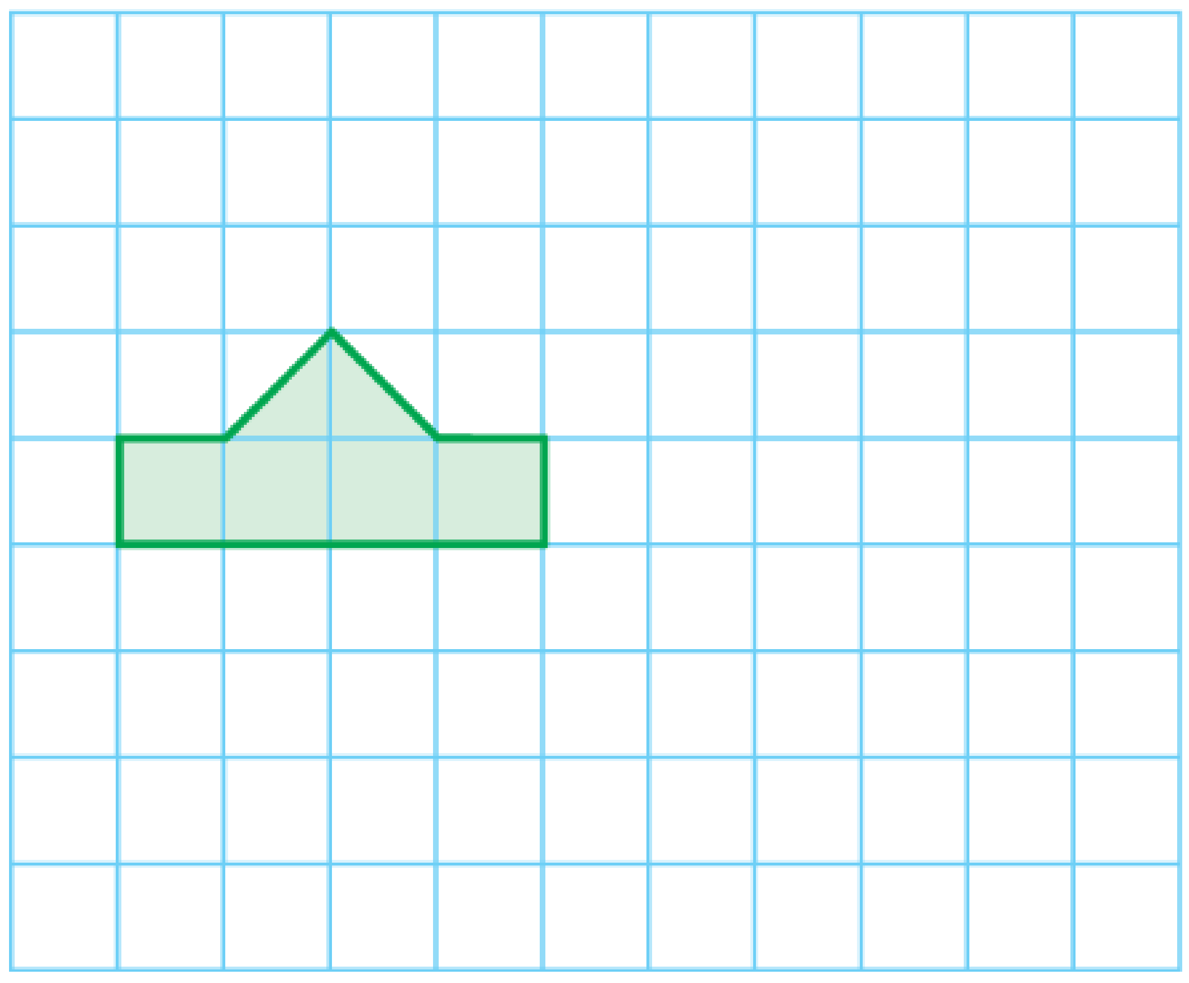
a 3 units right, 2 units down



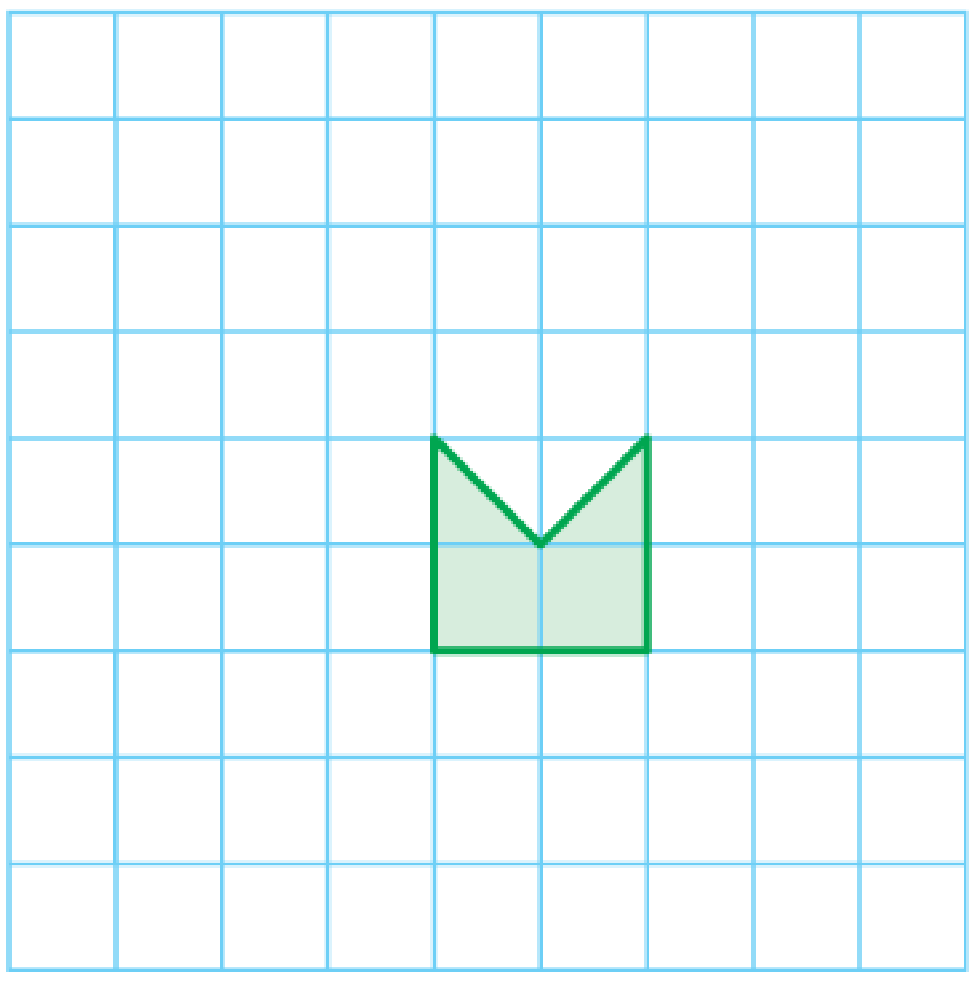
b 1 unit left, 4 units down



c 5 units right, 3 units down

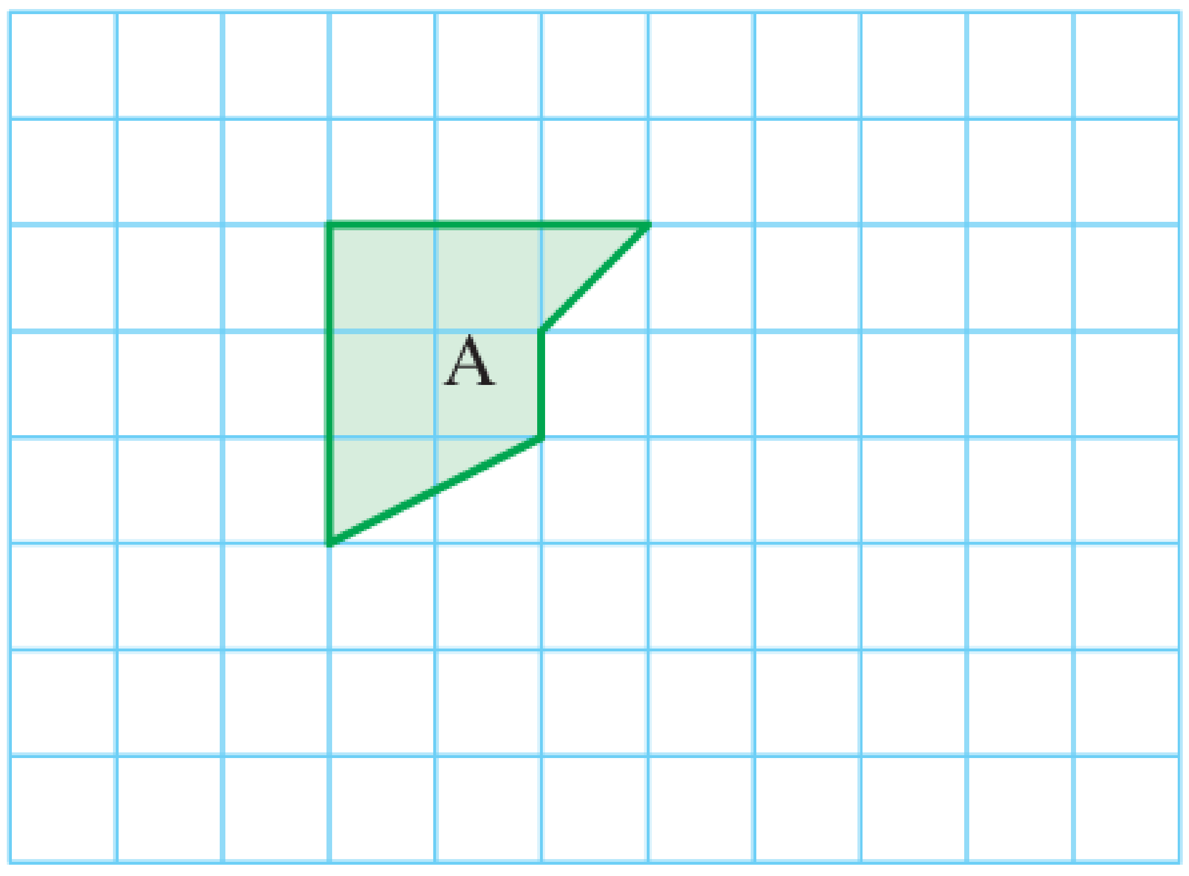


d 2 units left, 3 units up



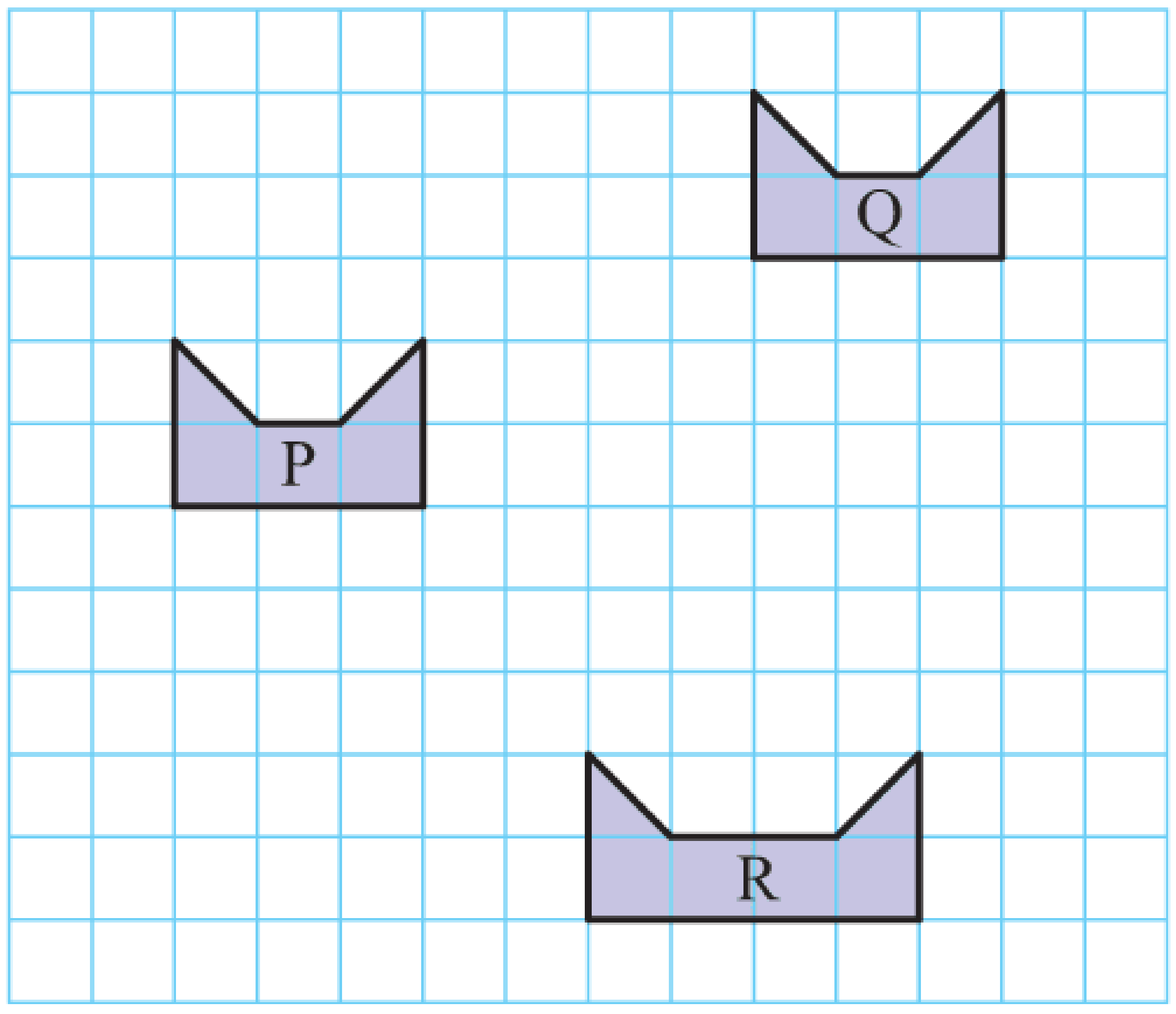
5 **a** Translate figure A 3 units right and 1 unit down to give A′.

b What translation is needed to shift A′ back to A?

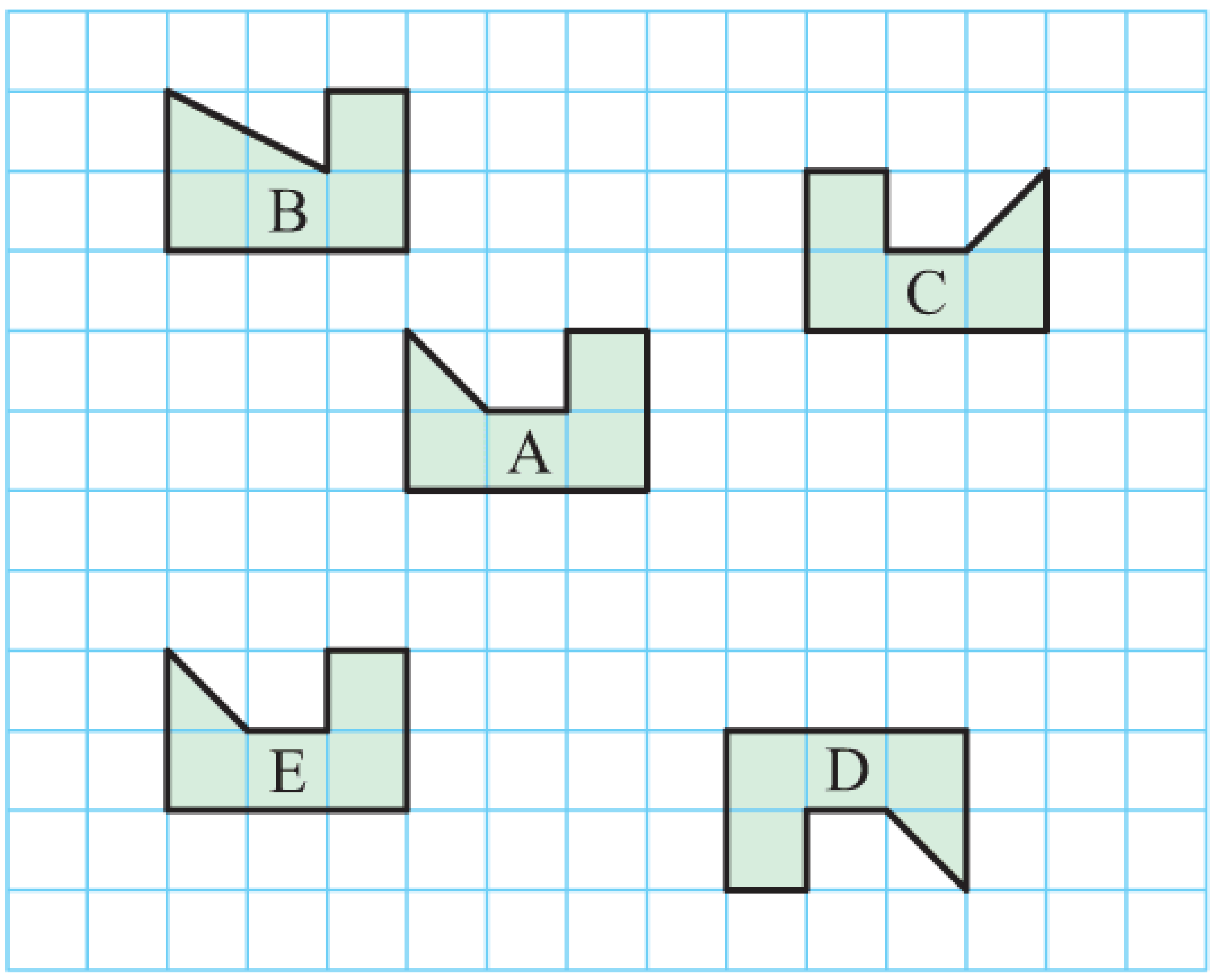


6 Look at the figures P, Q, and R.

- **a** Describe the translation from P to Q.
- **b** Explain why R is not a translation of P.



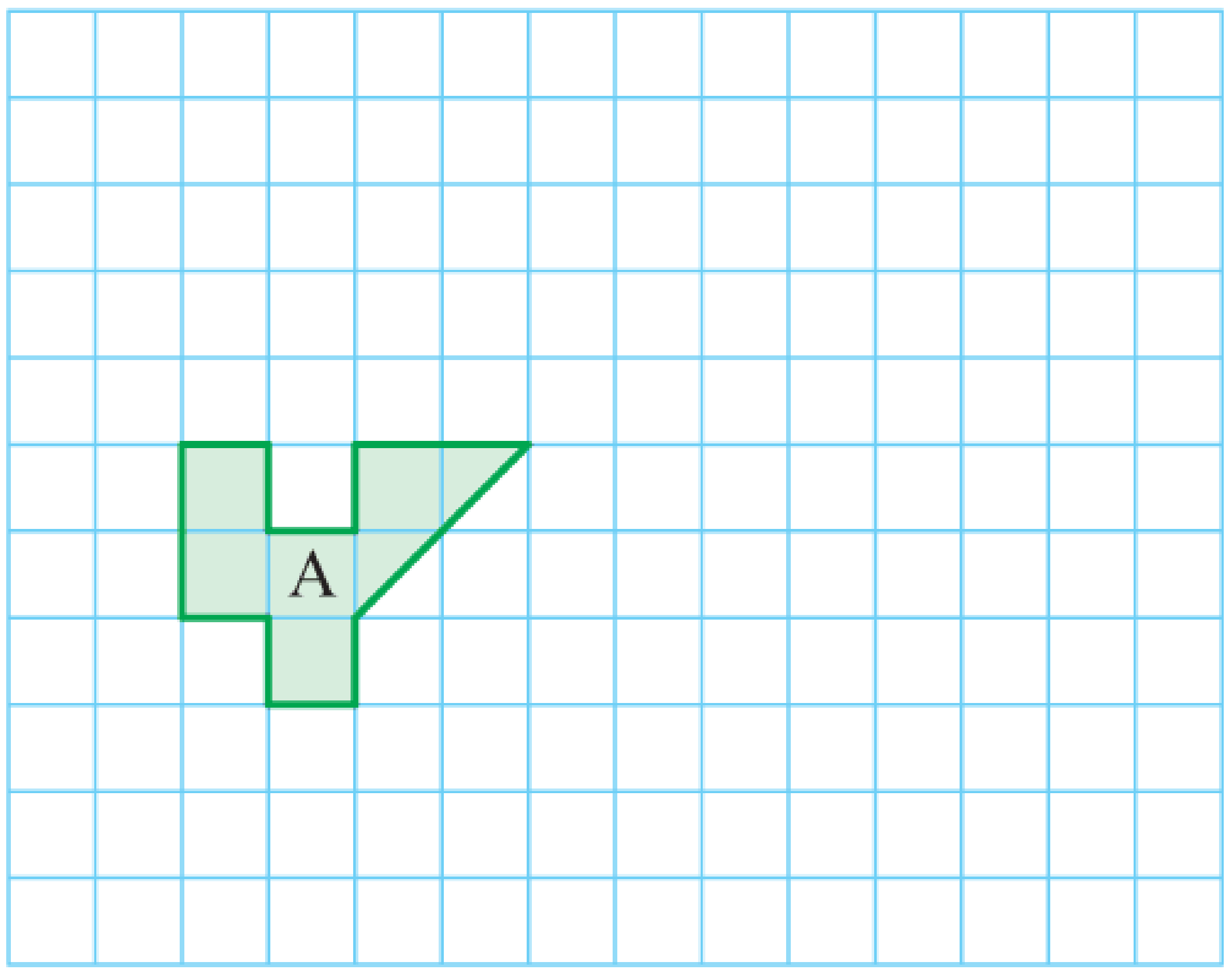
7



- **a** Which of the figures B, C, D, or E is a translation of A?
- **b** Describe the translation from A to this figure.

8

- **a** Translate figure A 4 units right and 3 units down to produce figure B.
- **b** Translate A 6 units right and 5 units up to produce figure C.
- **c** What translation is needed to shift figure B to figure C?



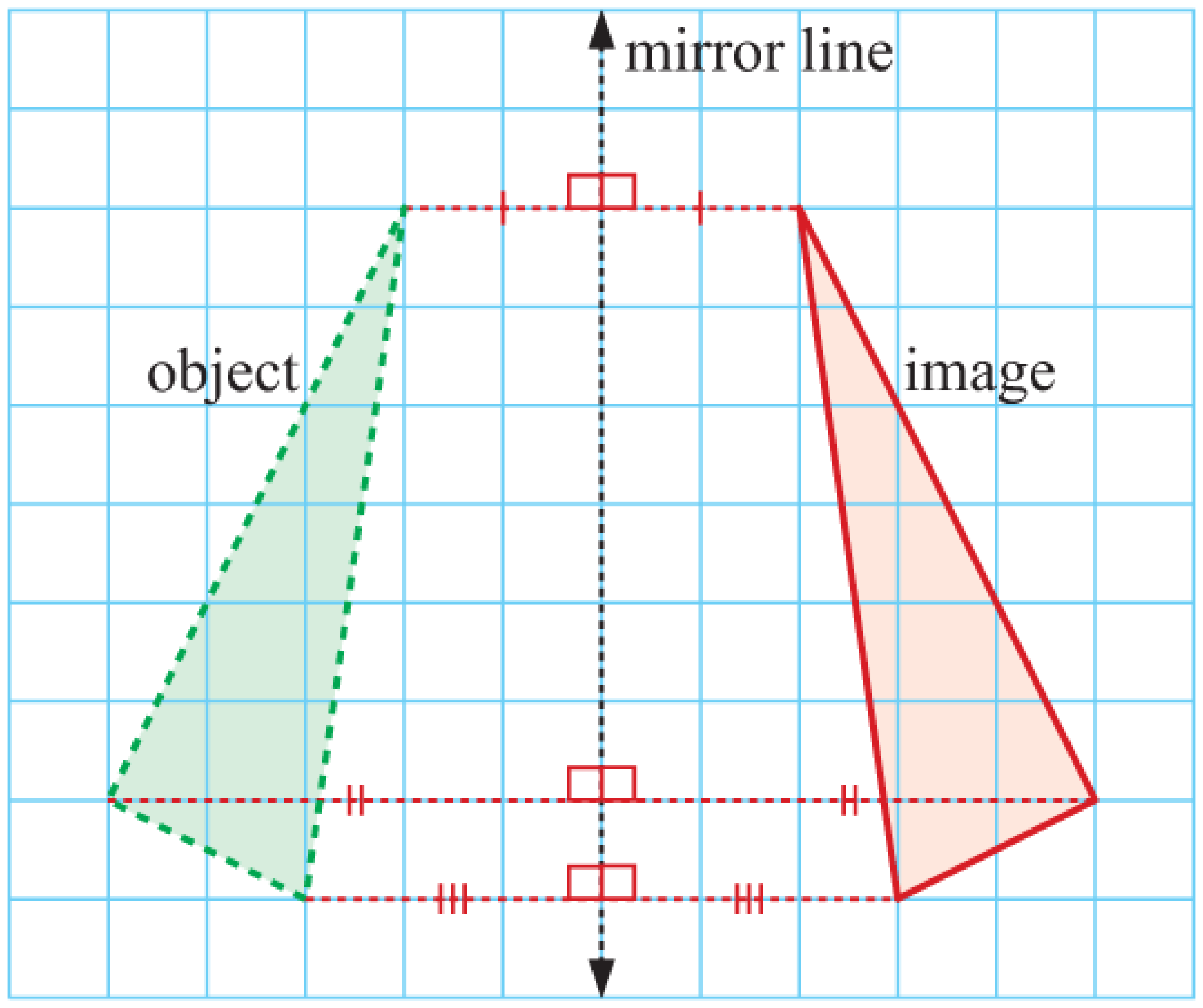
B REFLECTIONS

A **reflection** of a figure gives its mirror image.

To reflect a figure, we need a **mirror line**.

We draw lines at right angles to the mirror line which pass through key points on the object.

The image of each point is the same distance from the mirror line as the object point, but on the opposite side of the mirror line.



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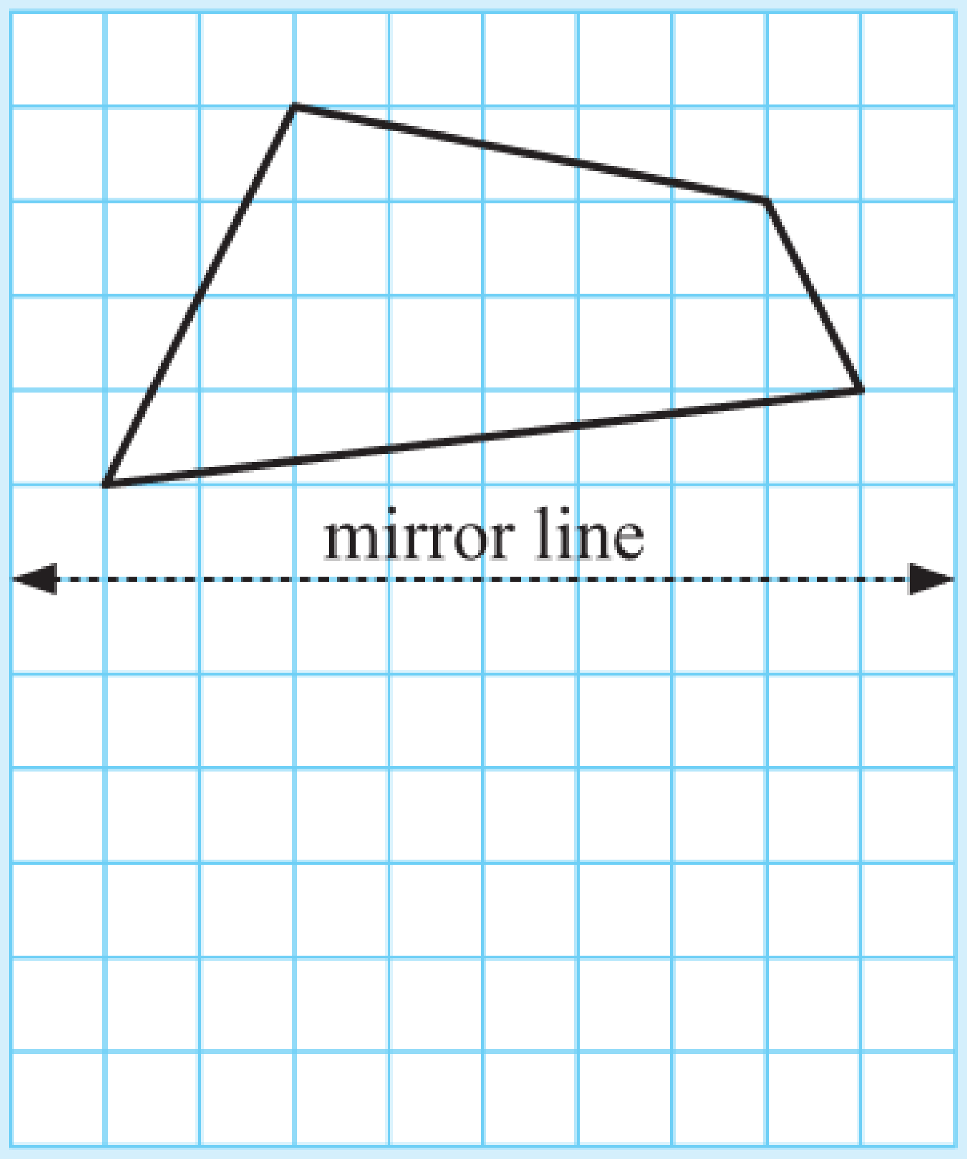
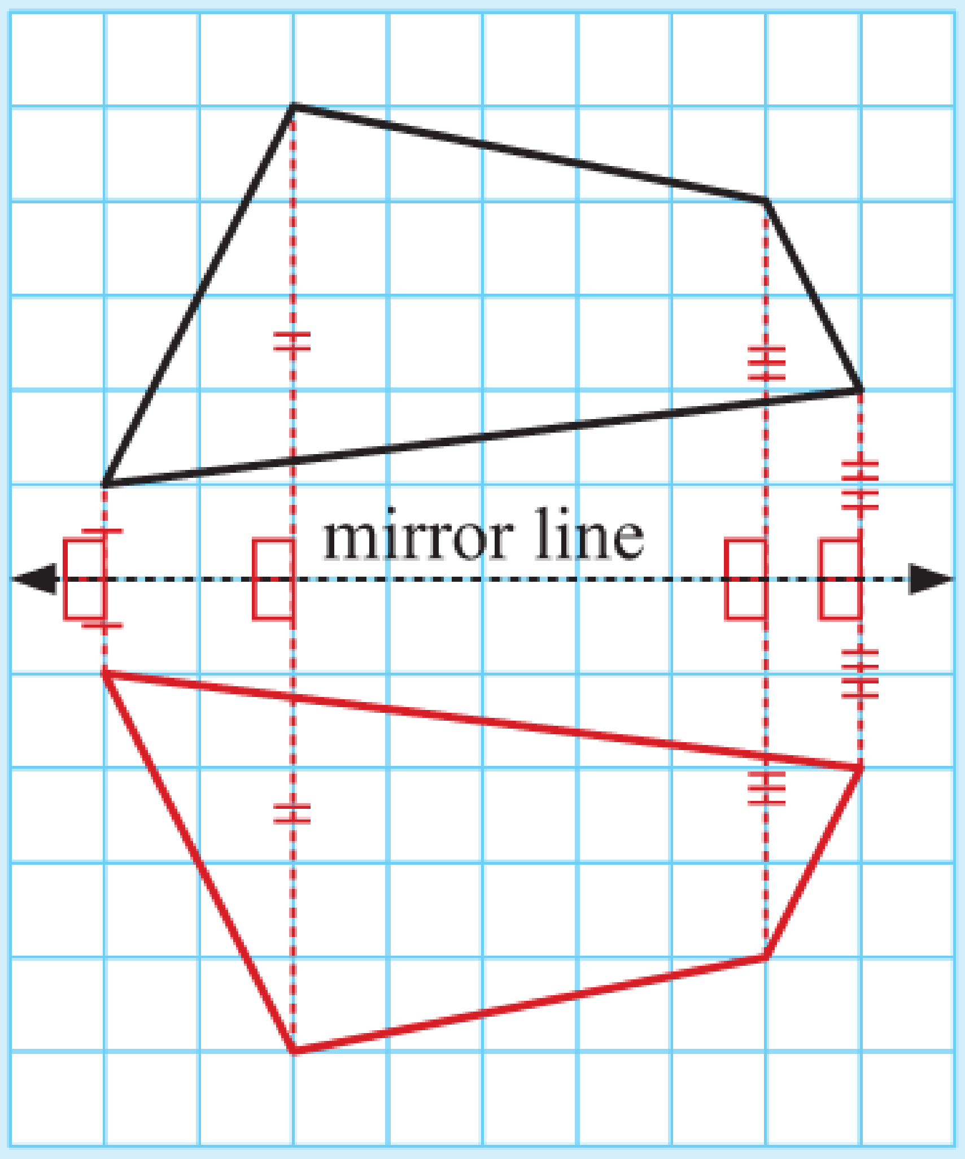
Example 2

Self Tutor

Reflect this figure in the mirror line.



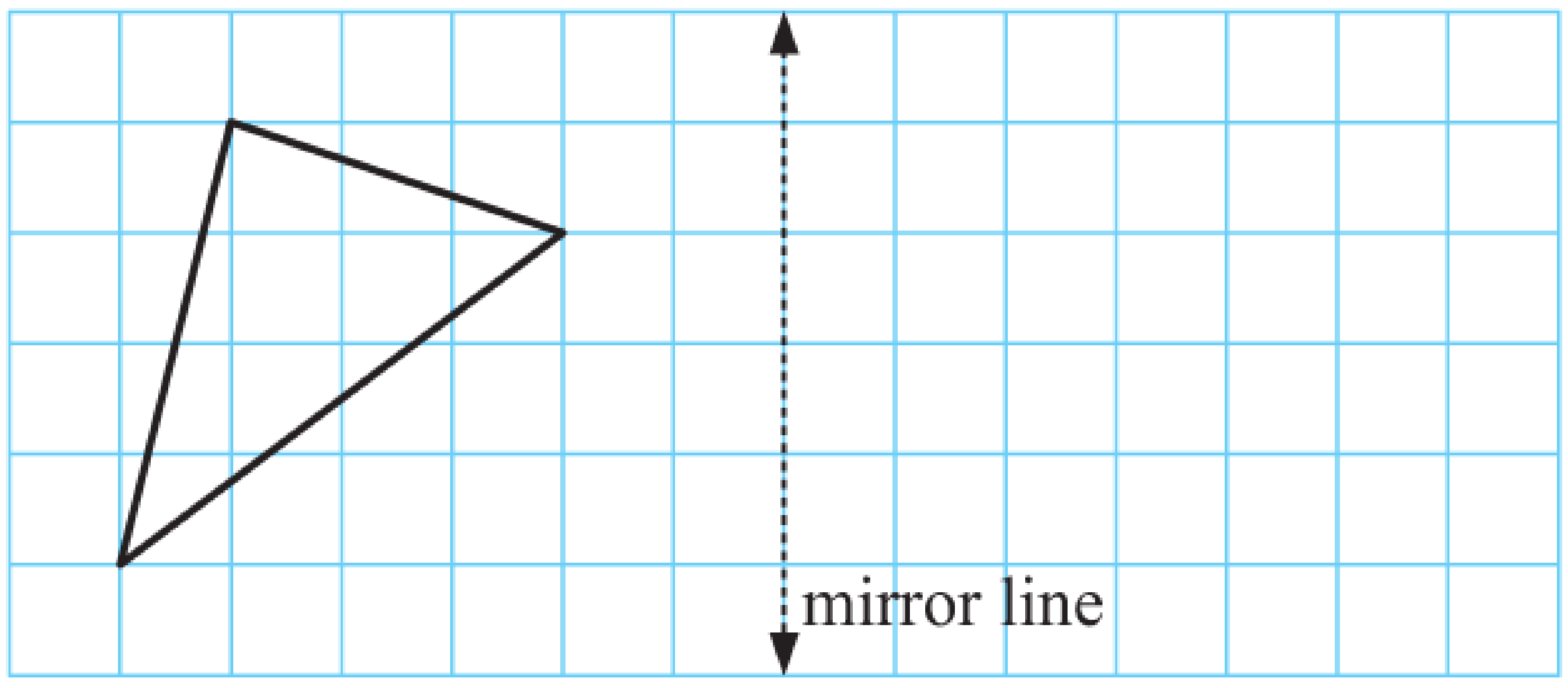


EXERCISE 16B

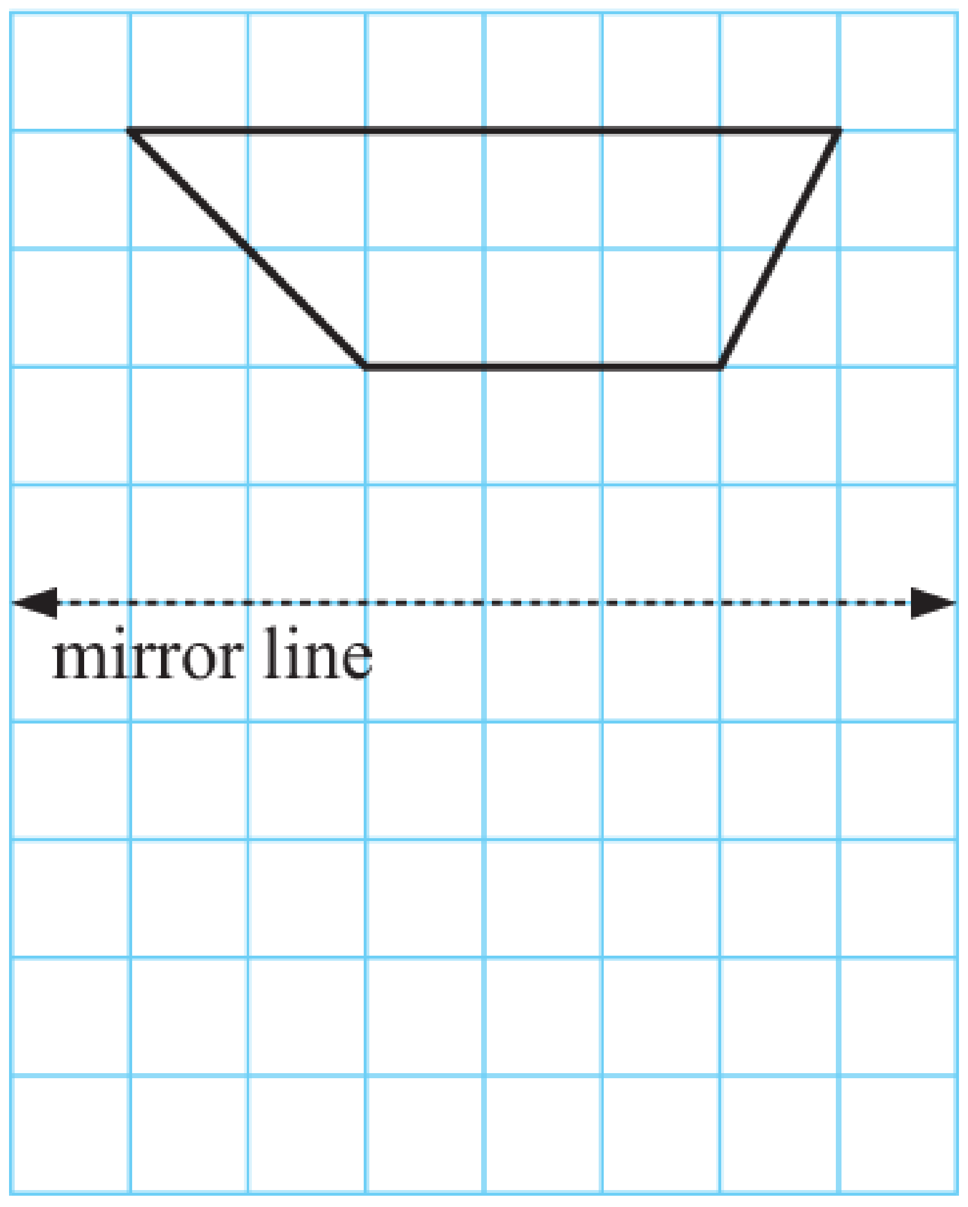
1 Reflect each figure in the mirror line:

PRINTABLE DIAGRAMS

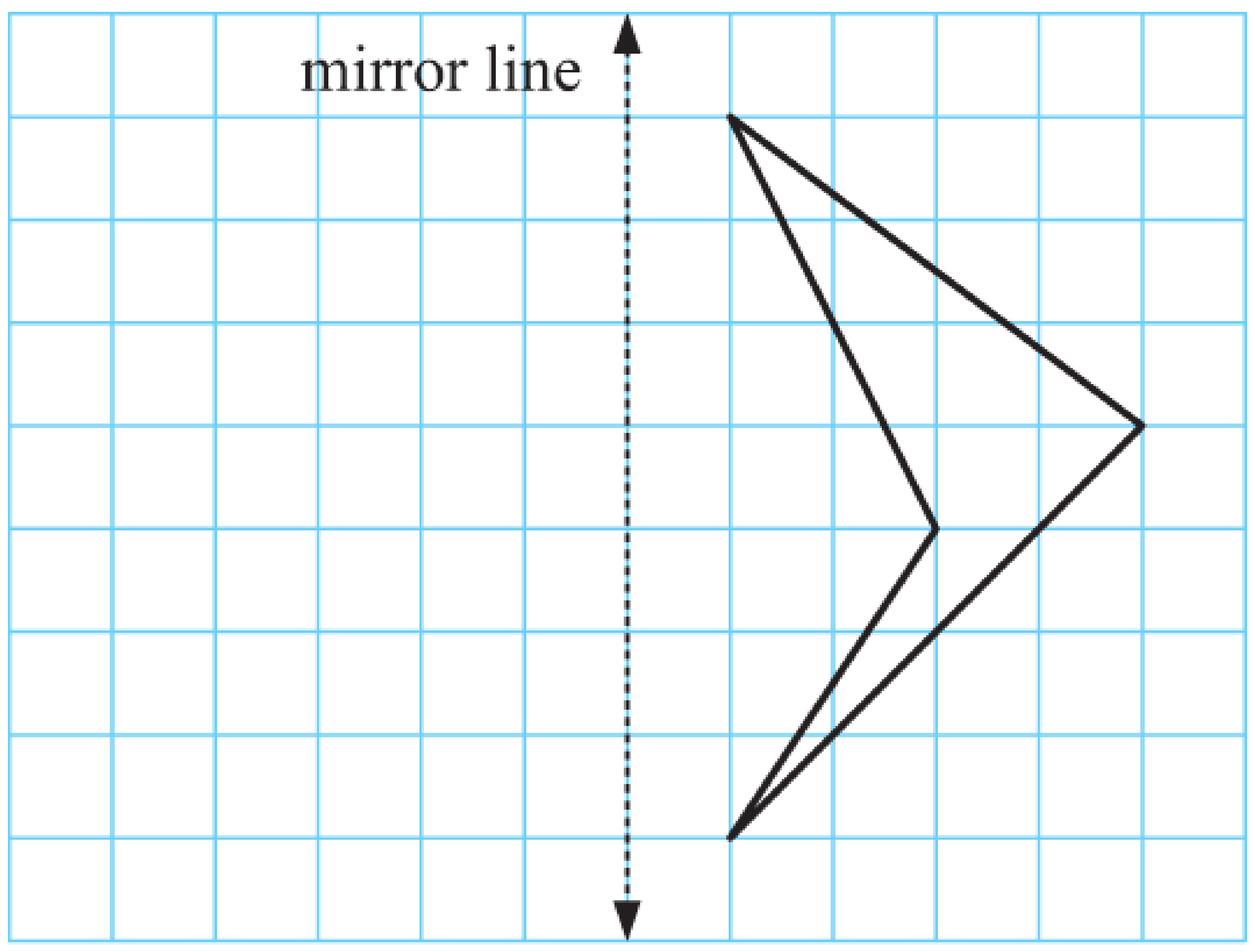
a



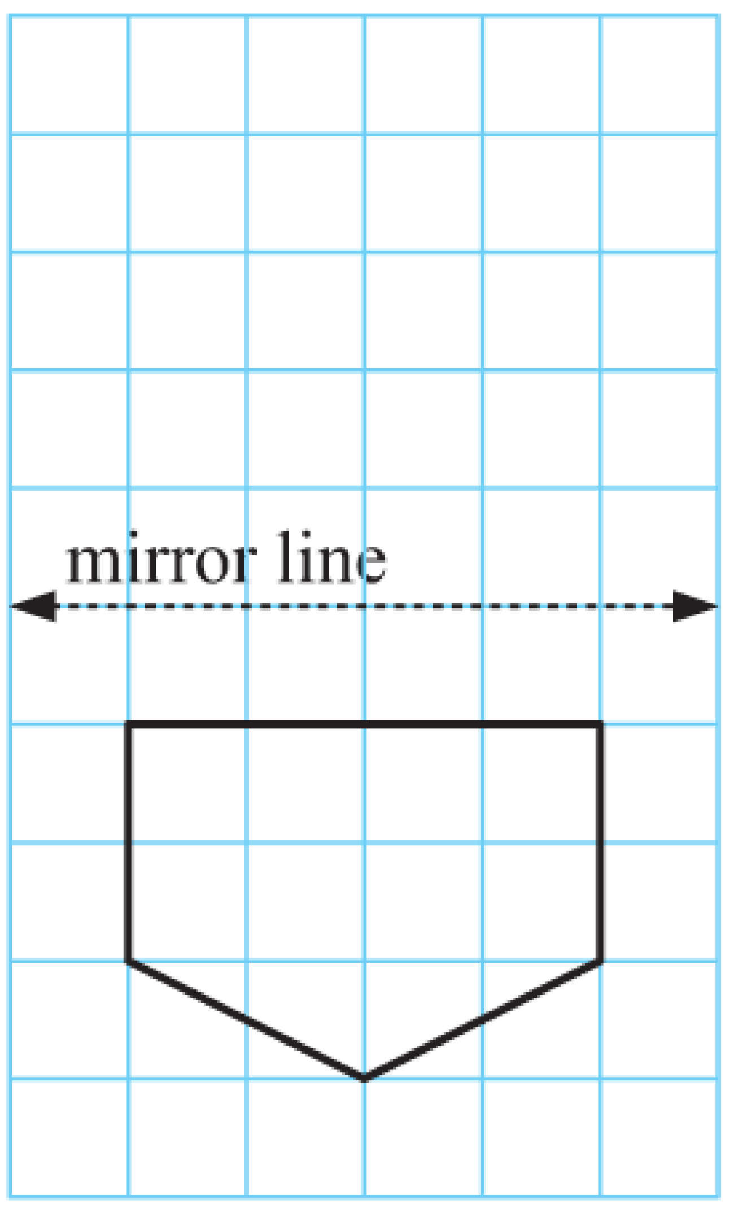
b



c

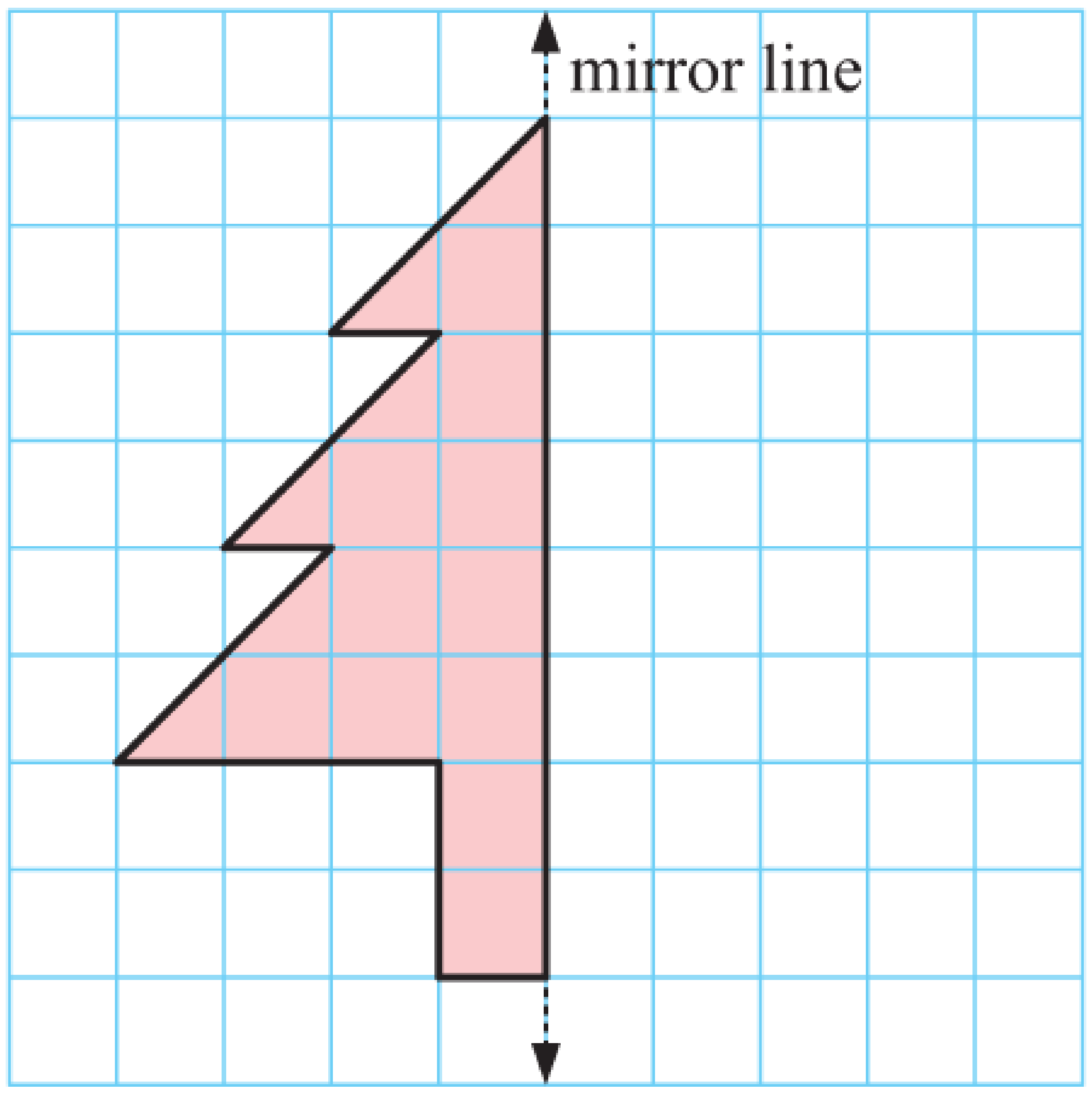


d

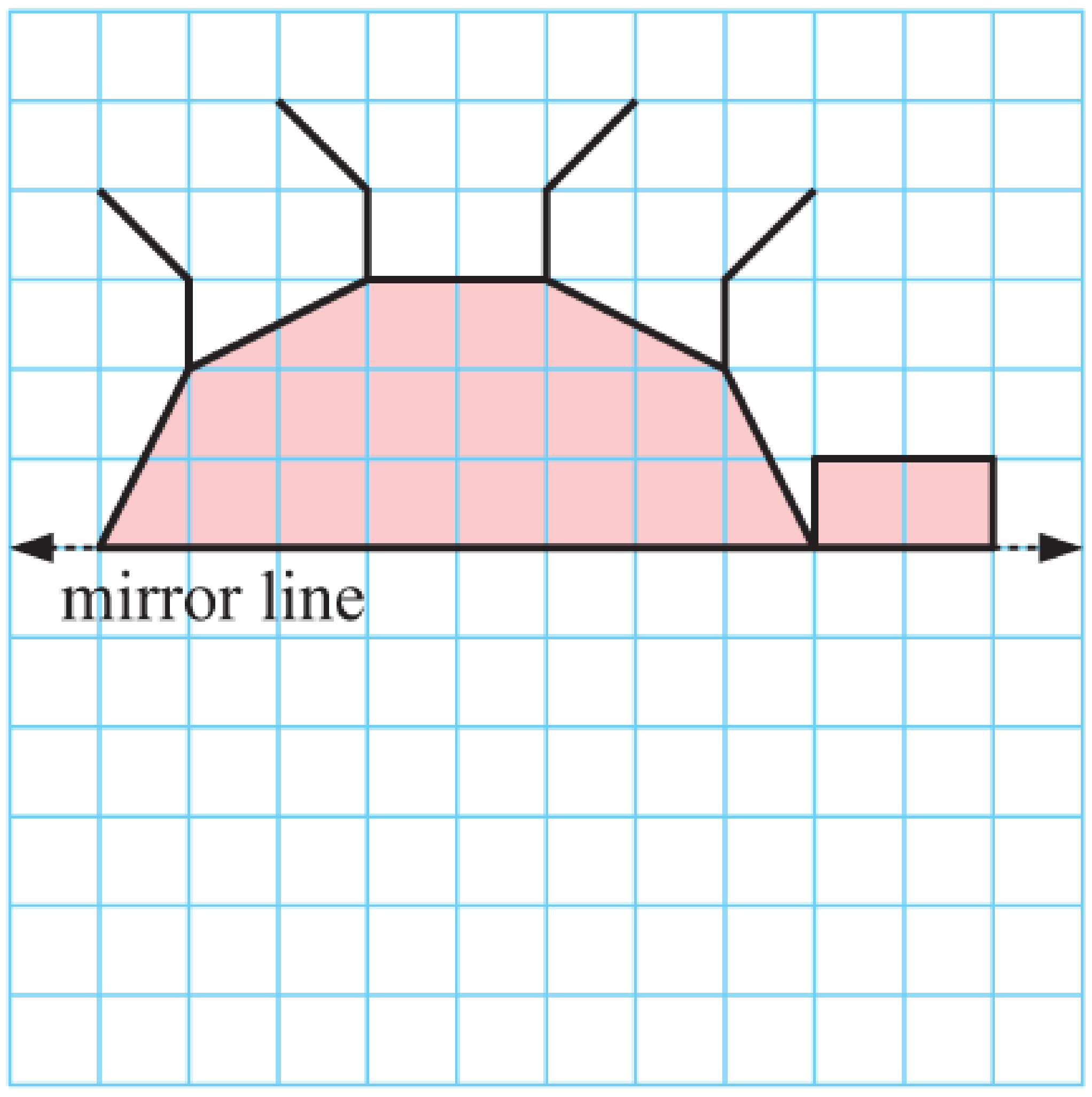


2 Reflect in the mirror line to complete the picture:

a

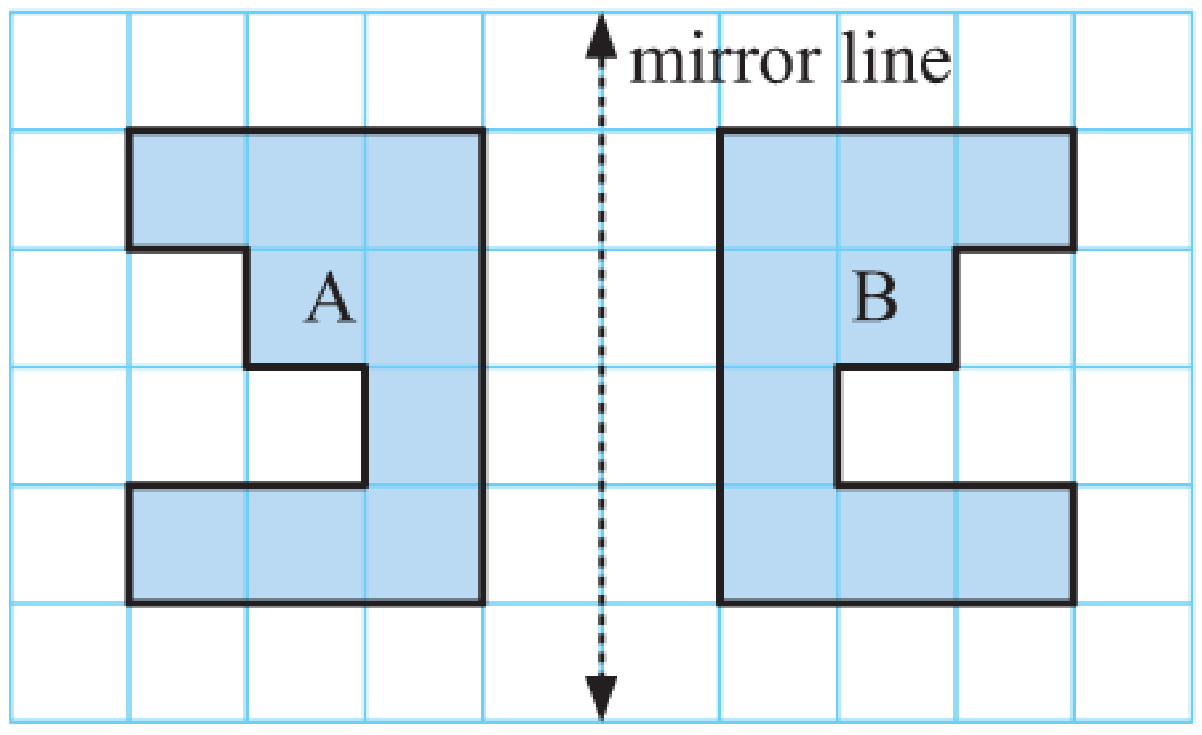


b

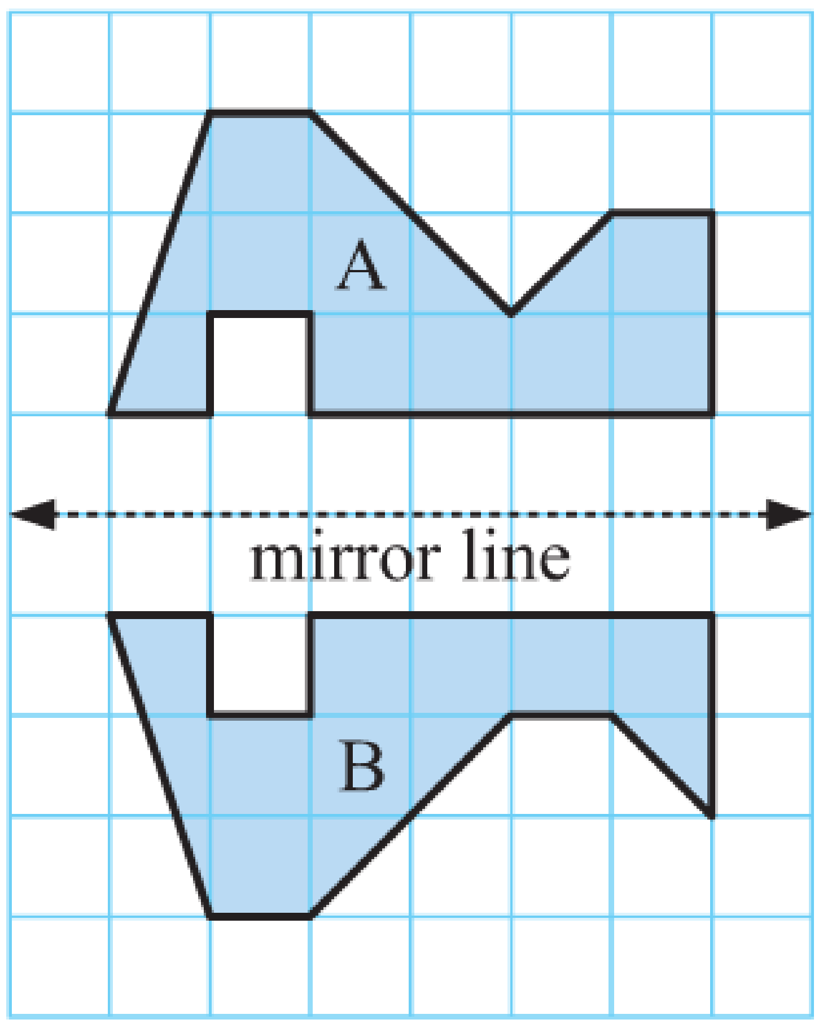


3 Is B a reflection of A in each diagram? If B is not a reflection, give a reason for your answer.

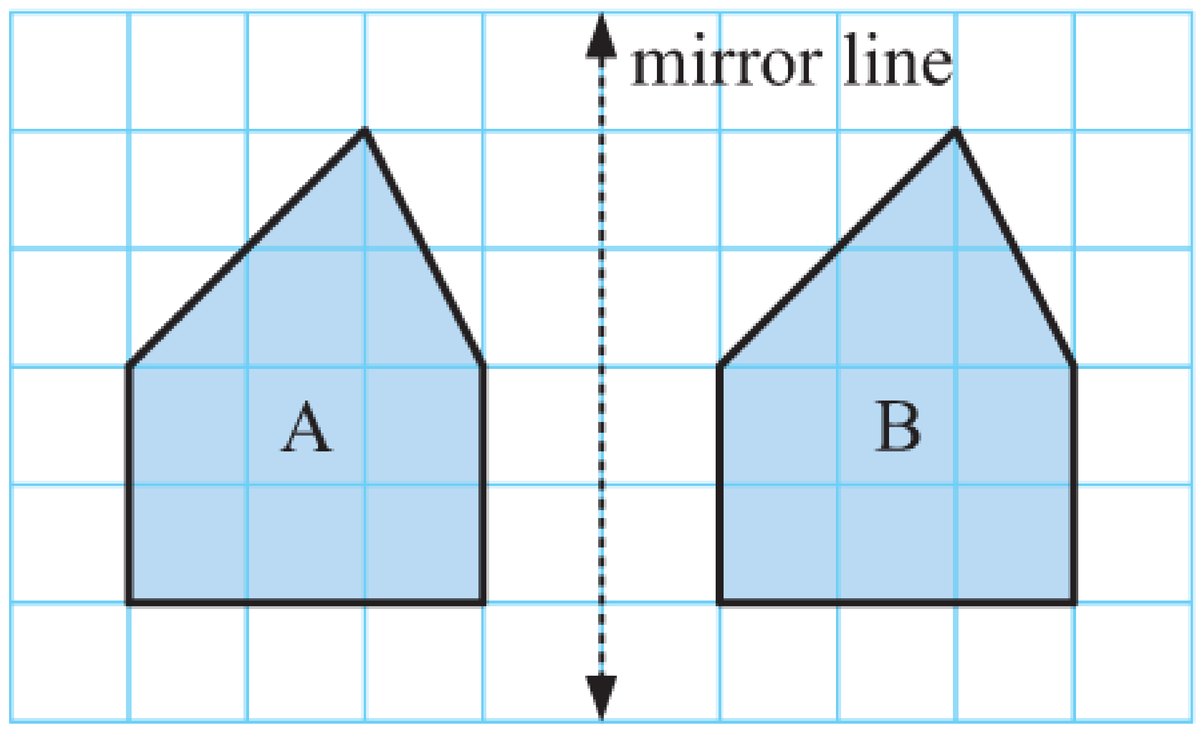
a



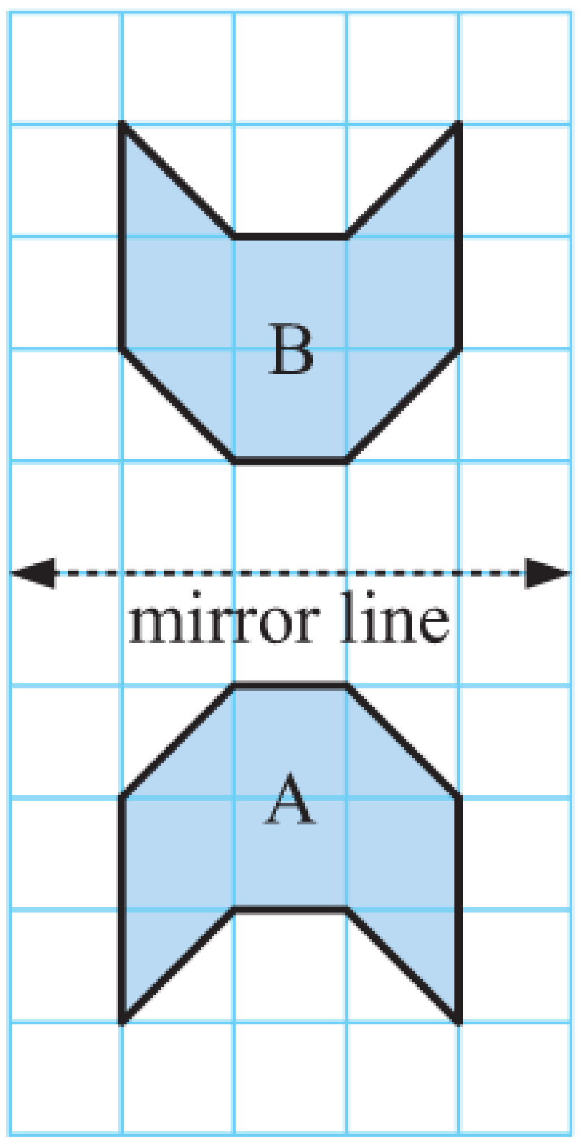
b



c

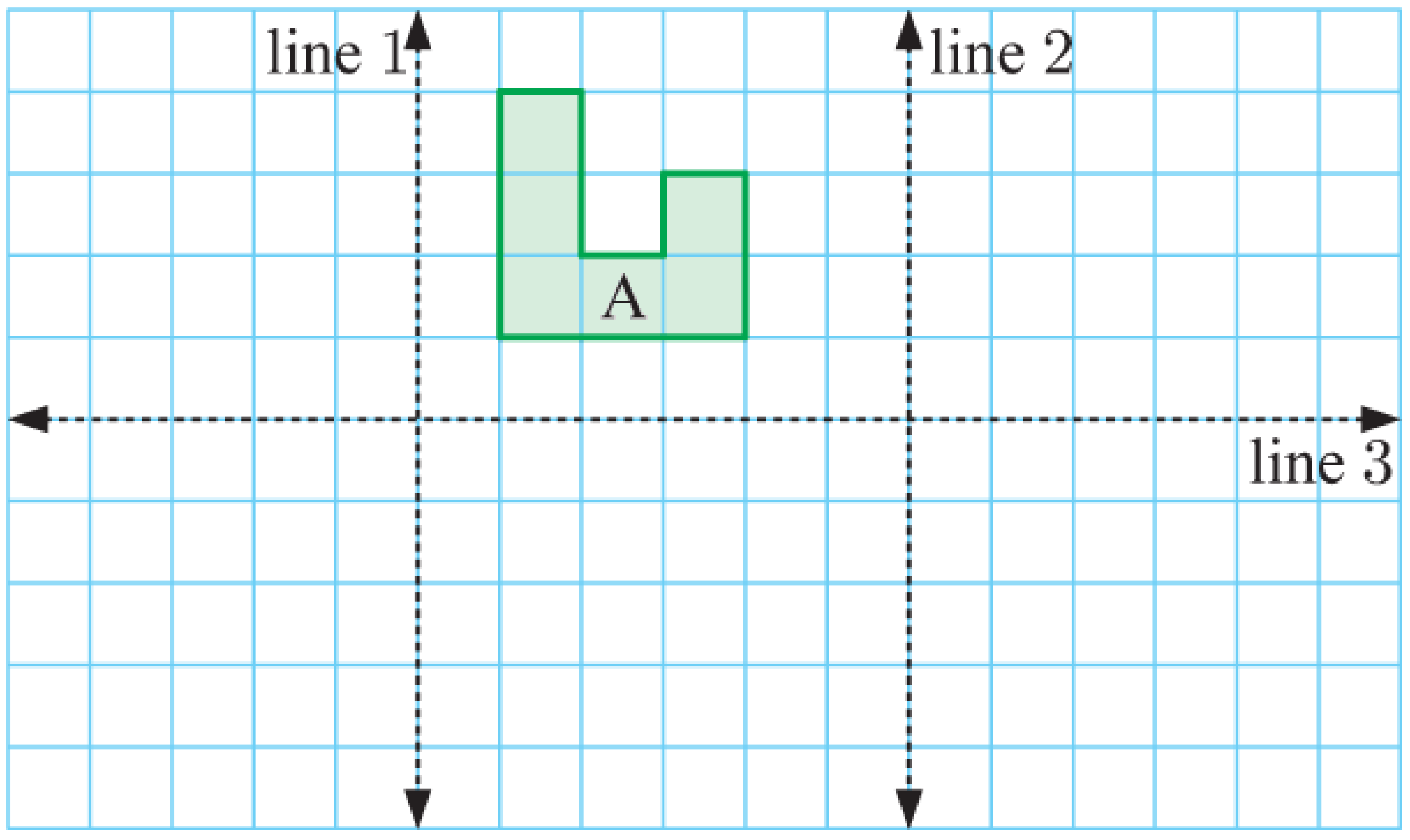


d



4 On a single diagram, show the image when figure A is reflected in:

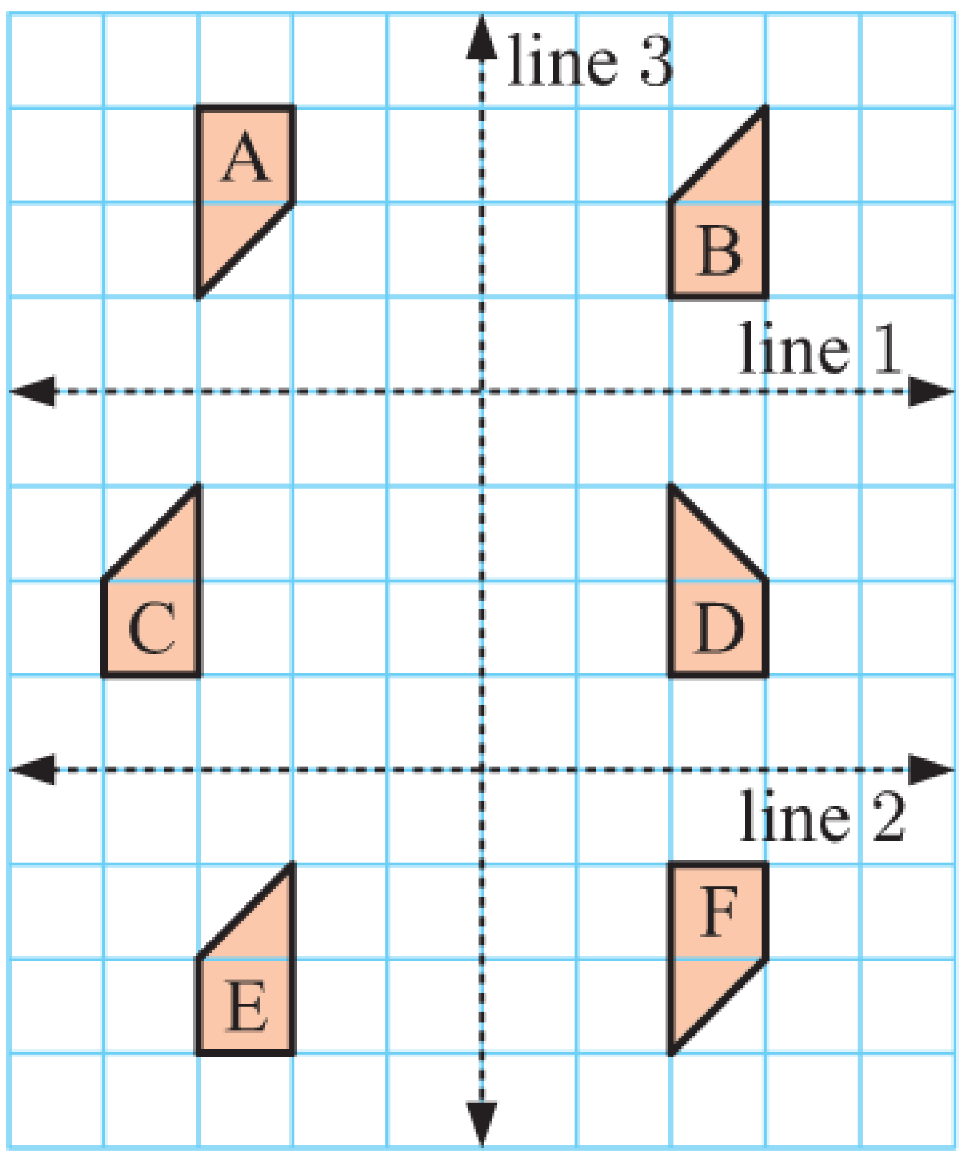
a line 1 **b** line 2 **c** line 3.



5 Julian is playing Sebastian in a game of chess. Sebastian is playing white, and Julian is playing black. Julian moves second, and his strategy on each turn is to perform the mirror image of Sebastian's previous turn.

It is now Julian's turn. Which piece will he move, and which square will he move it to?

6



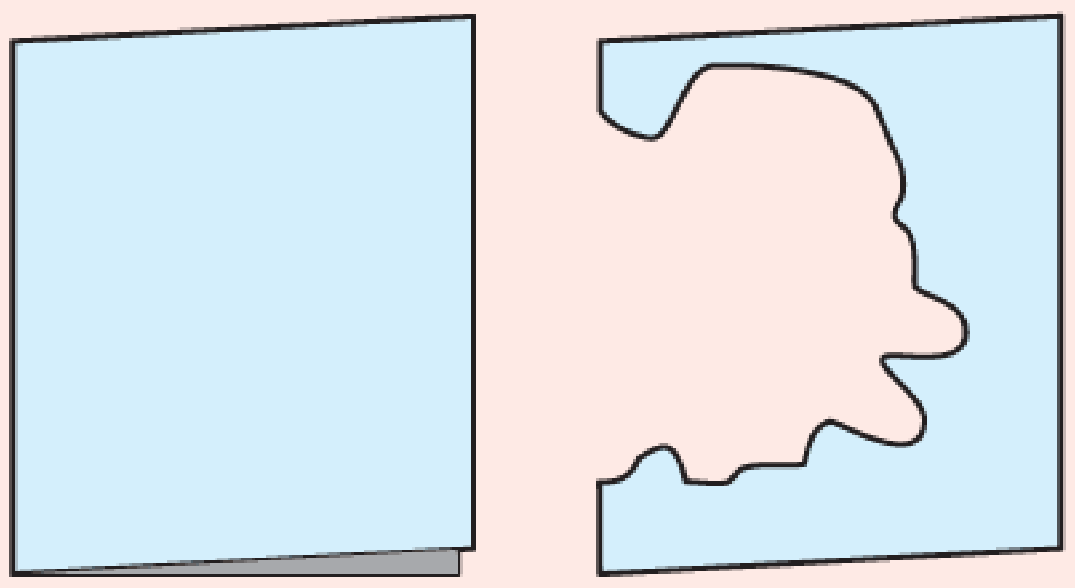
Find the two figures which are reflections of each other in one of the mirror lines. Name the mirror line in this case.

ACTIVITY 1 USING REFLECTIONS TO MAKE PICTURES

You will need: paper, scissors, pencil.

What to do:

1. Take a piece of paper and fold it in half.
2. Cut out a shape along the fold line.
3. Open out the sheet of paper and observe the shapes revealed.



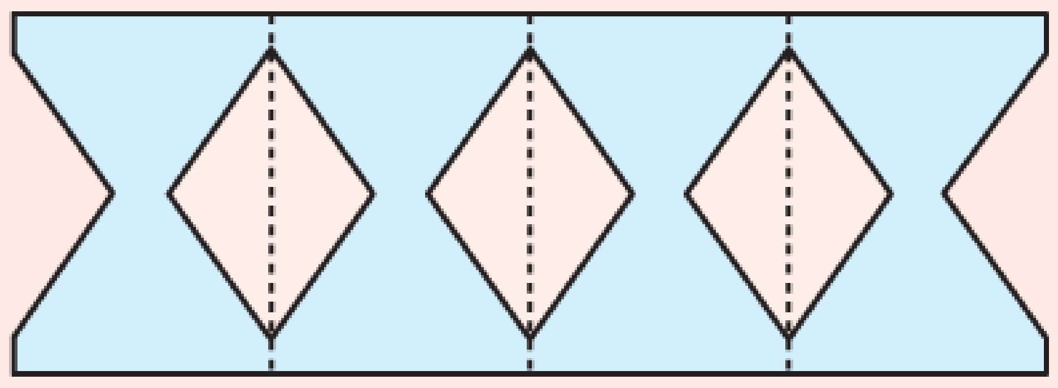
4 Record any observations about reflections that you notice.

5 Try the following:

- **a** Fold the paper twice before cutting out your shape.
- **b** Fold the paper three times before cutting out your shape.

In each case record your observations about reflections.

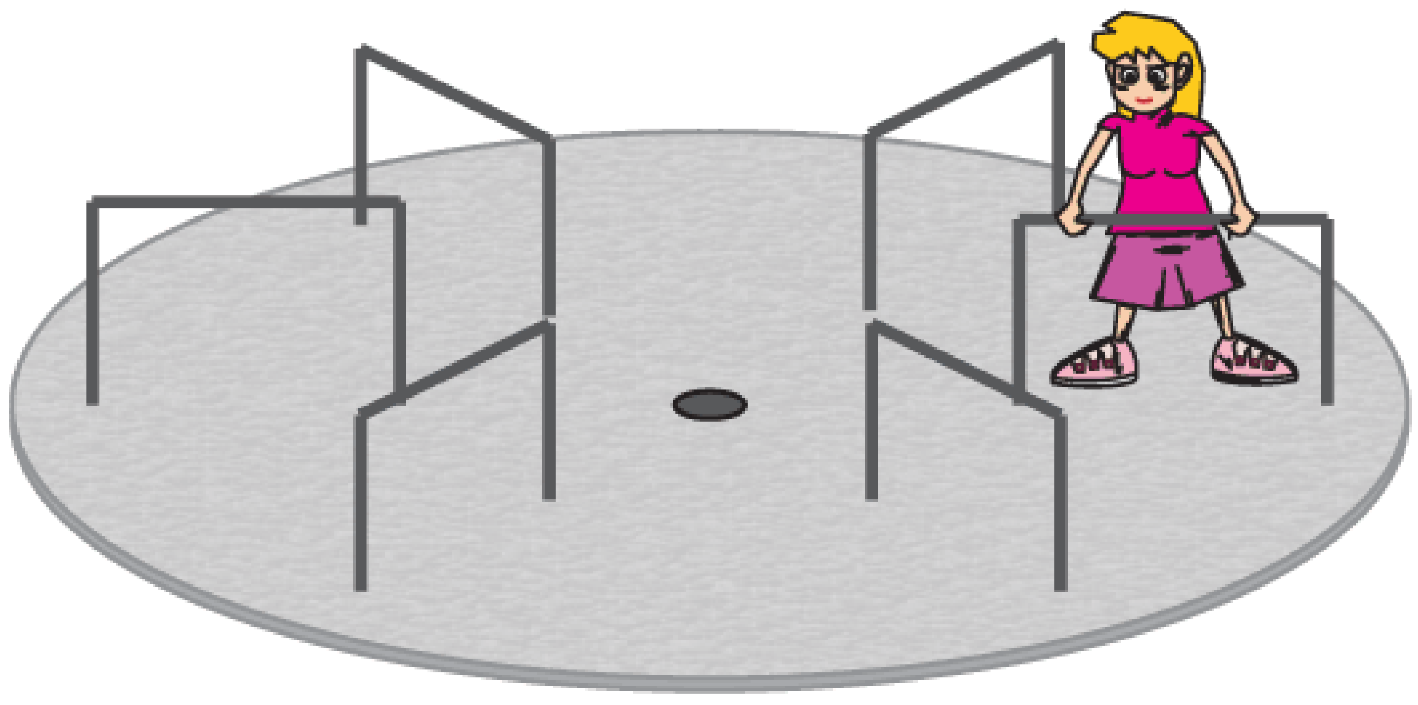
6 Make the pattern alongside by folding a piece of paper a number of times and cutting out a shape. How many folds do you need, and what shape do you need to cut out?



C ROTATIONS

When a child is riding a playground roundabout, she is **rotating** about the centre of the ride. She moves around in a circle, and is always the same distance from the centre.

In mathematics, we rotate figures in the same way.

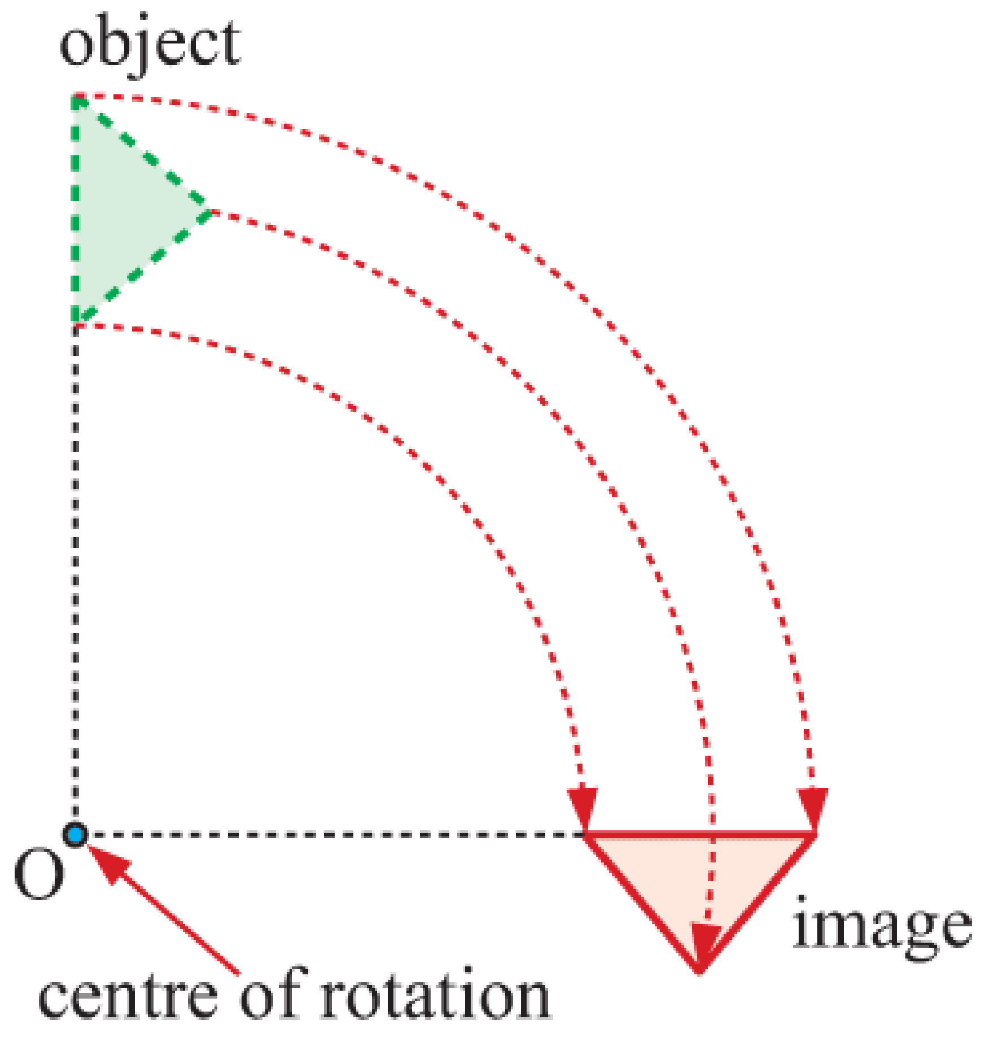


A **rotation** turns a figure through a given angle about a point.

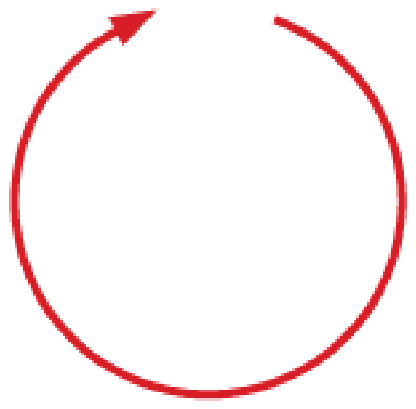
The point about which a figure rotates is called the **centre of rotation**.

The centre of rotation is often labelled O.

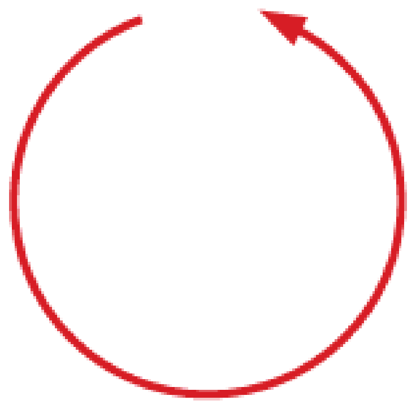
Each point on the image will be the same distance from the centre of rotation as the corresponding point on the object.



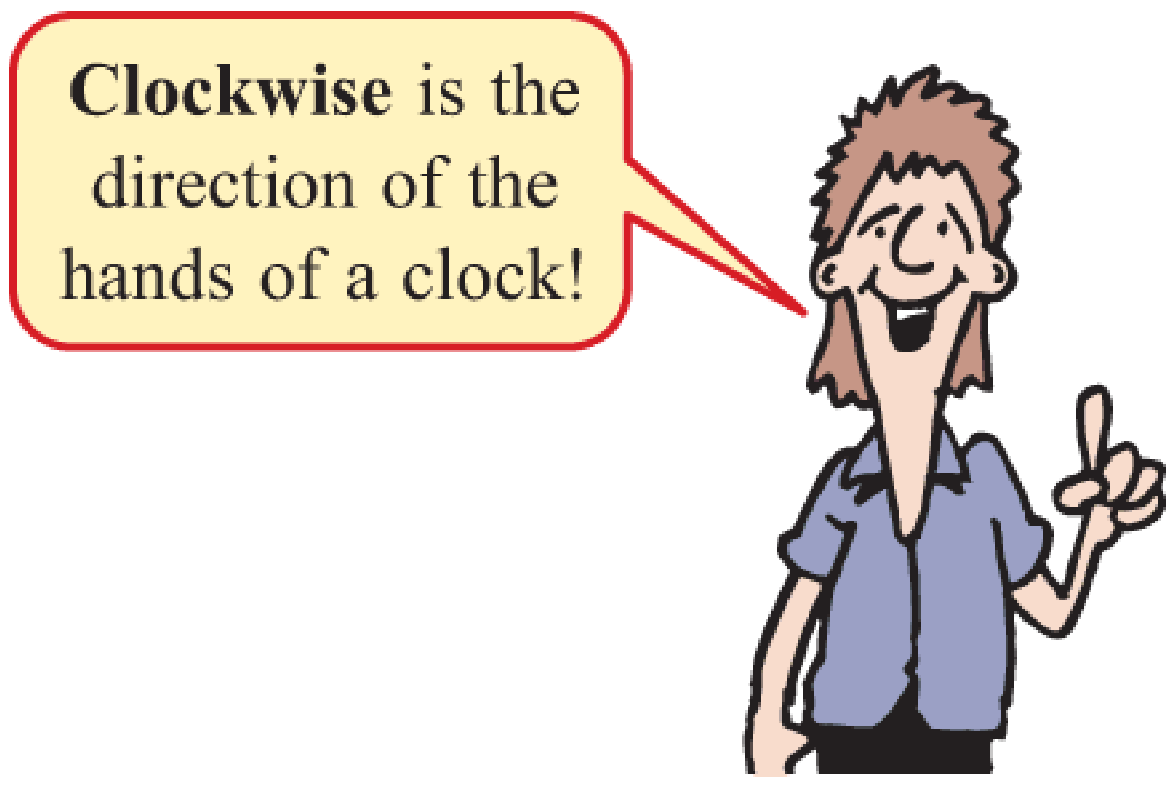
Rotations can be performed in a **clockwise** or **anticlockwise** direction.



clockwise



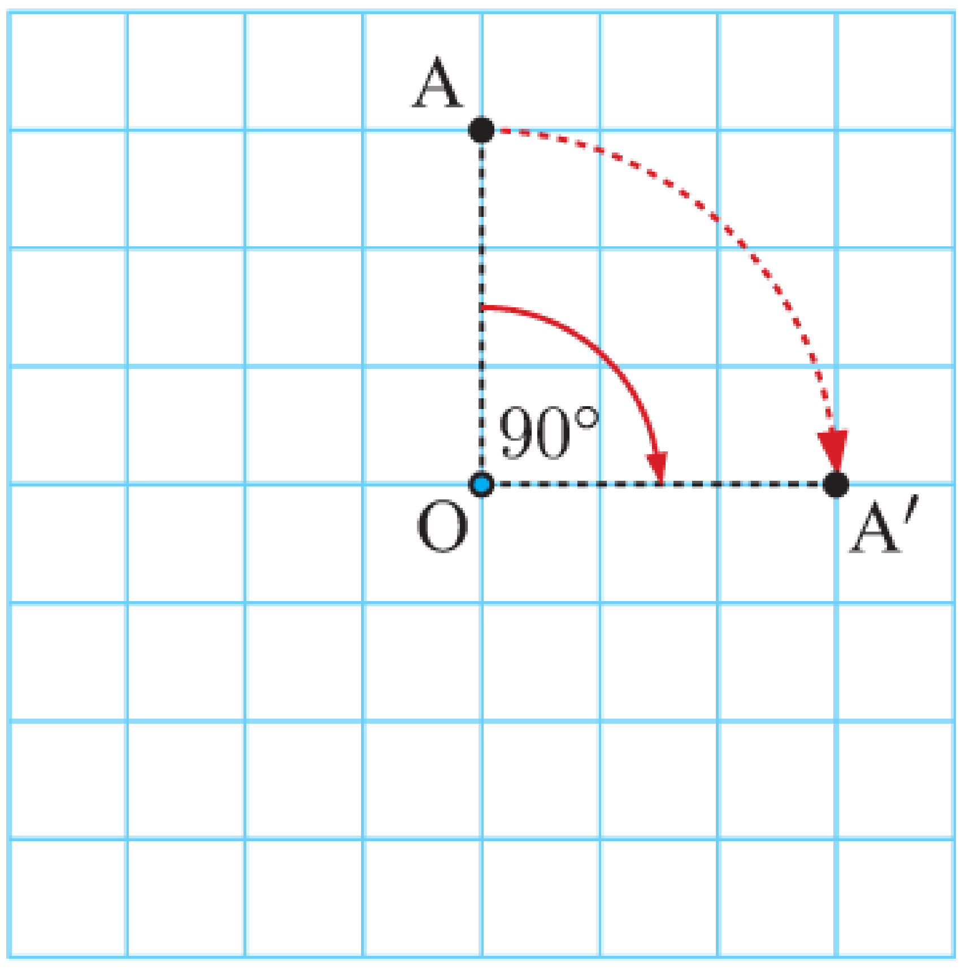
anticlockwise

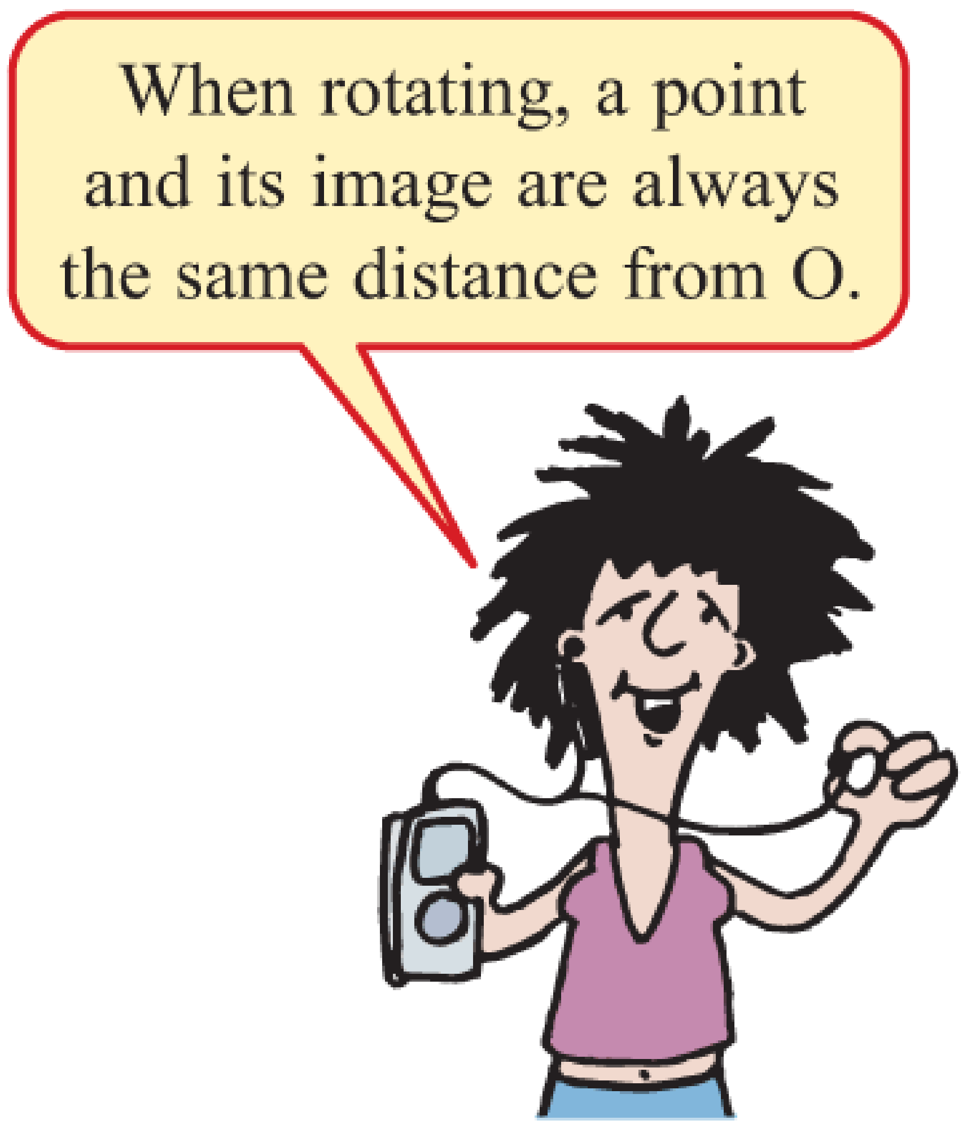


For example:

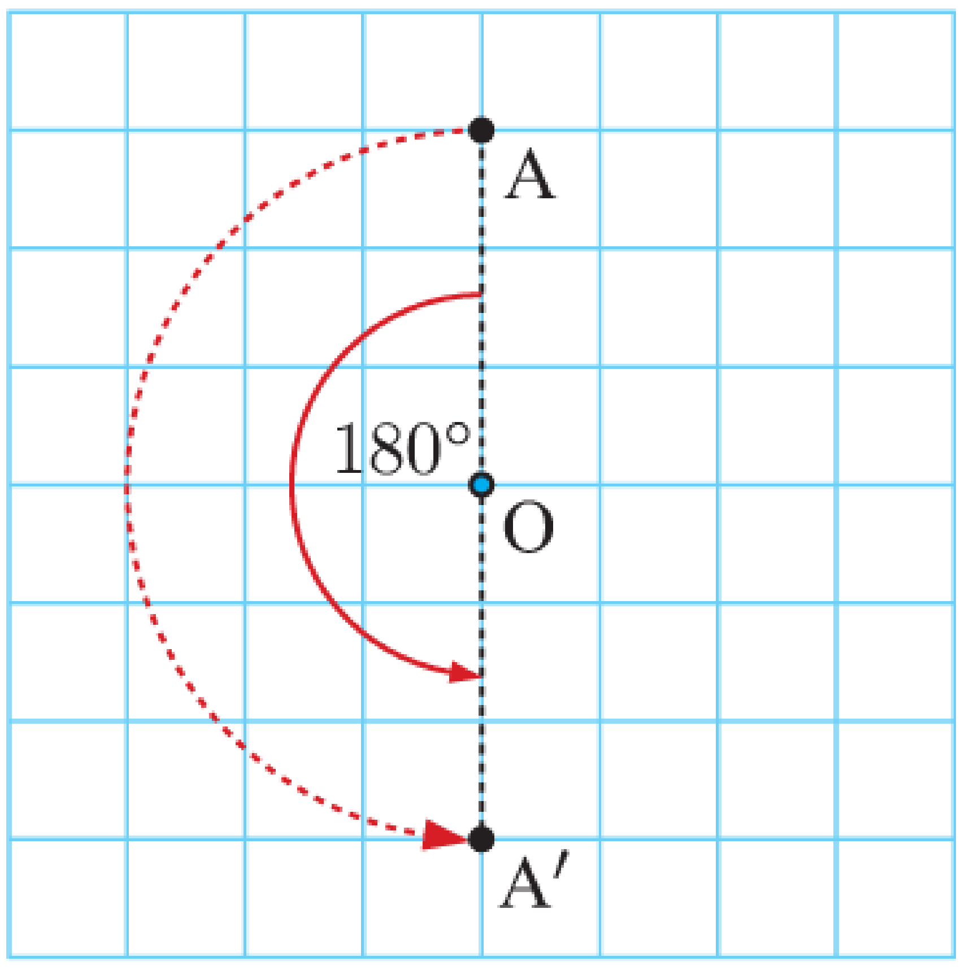
- To rotate point A 90° clockwise about O, we start at A, then move clockwise in a circle about O, through an angle of 90°.

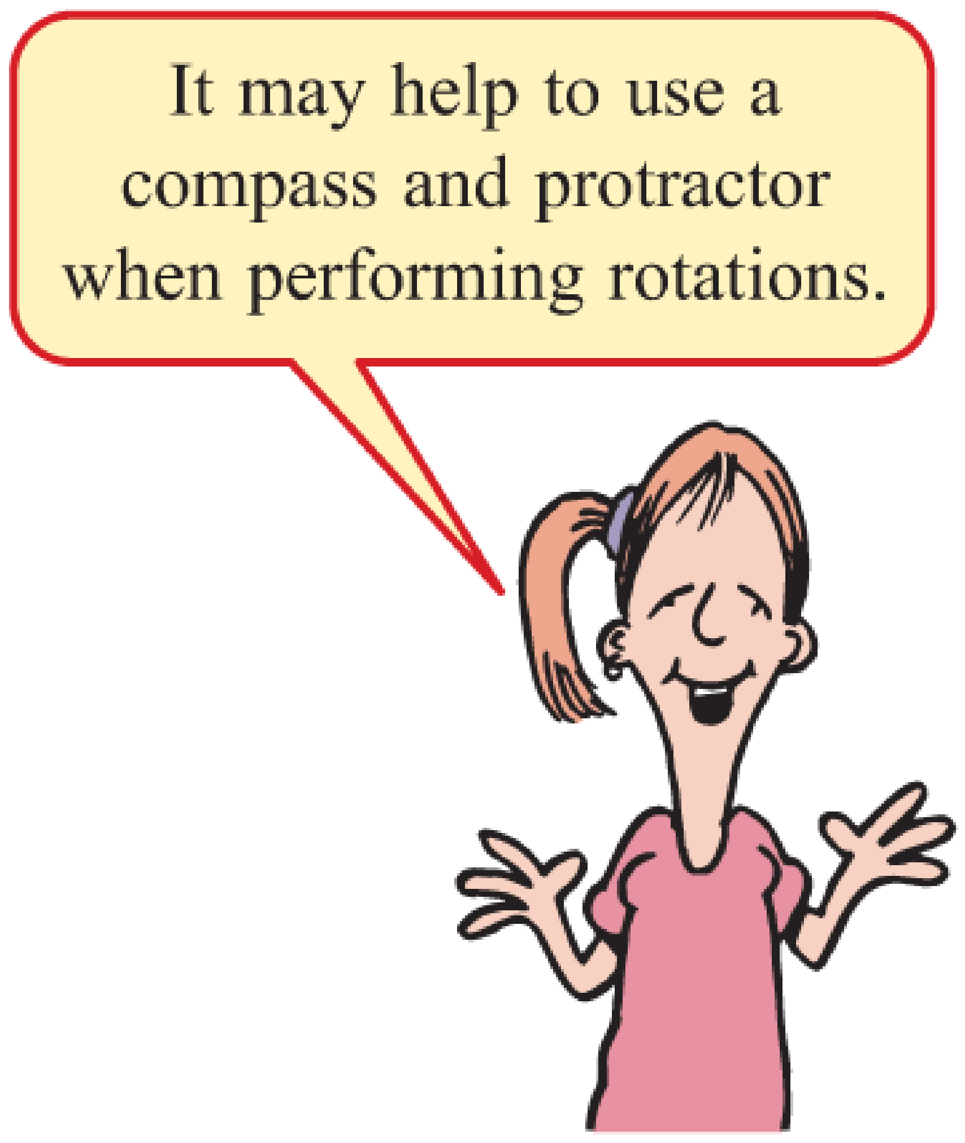
DEMO





- To rotate point A 180° anticlockwise about O, we start at A, then move anticlockwise in a circle about O, through an angle of 180°.

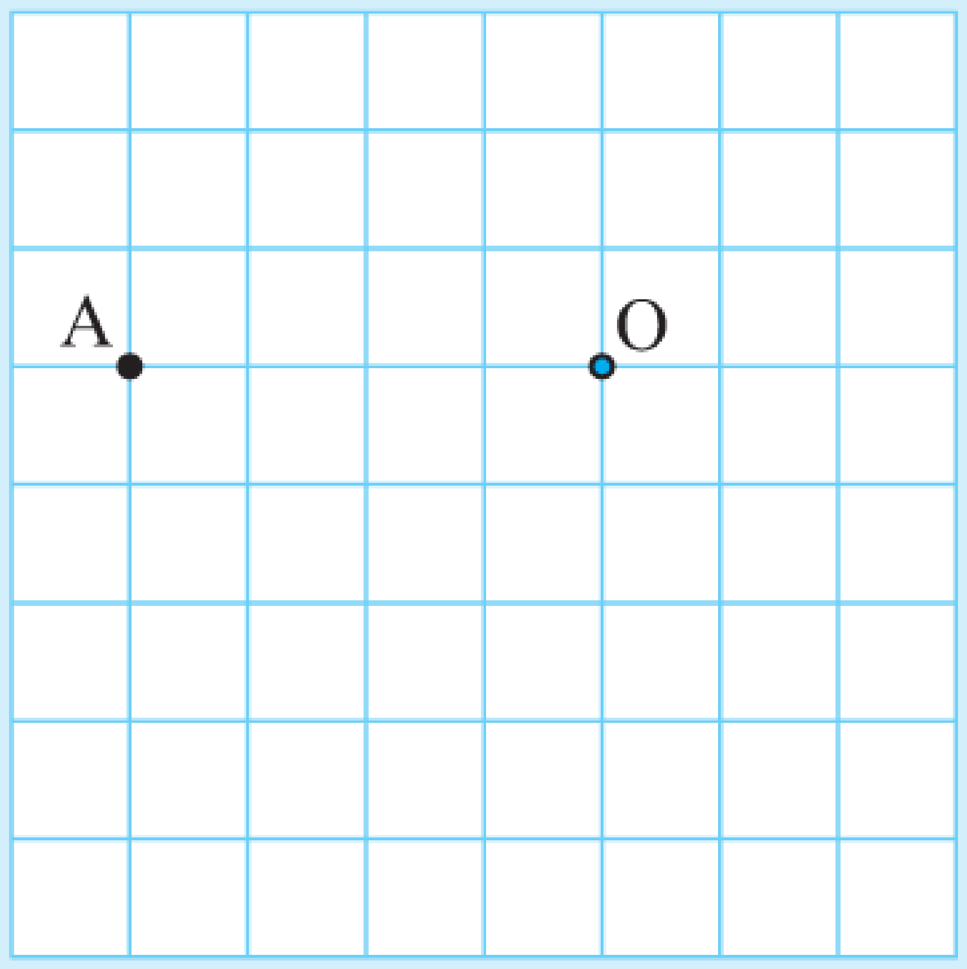




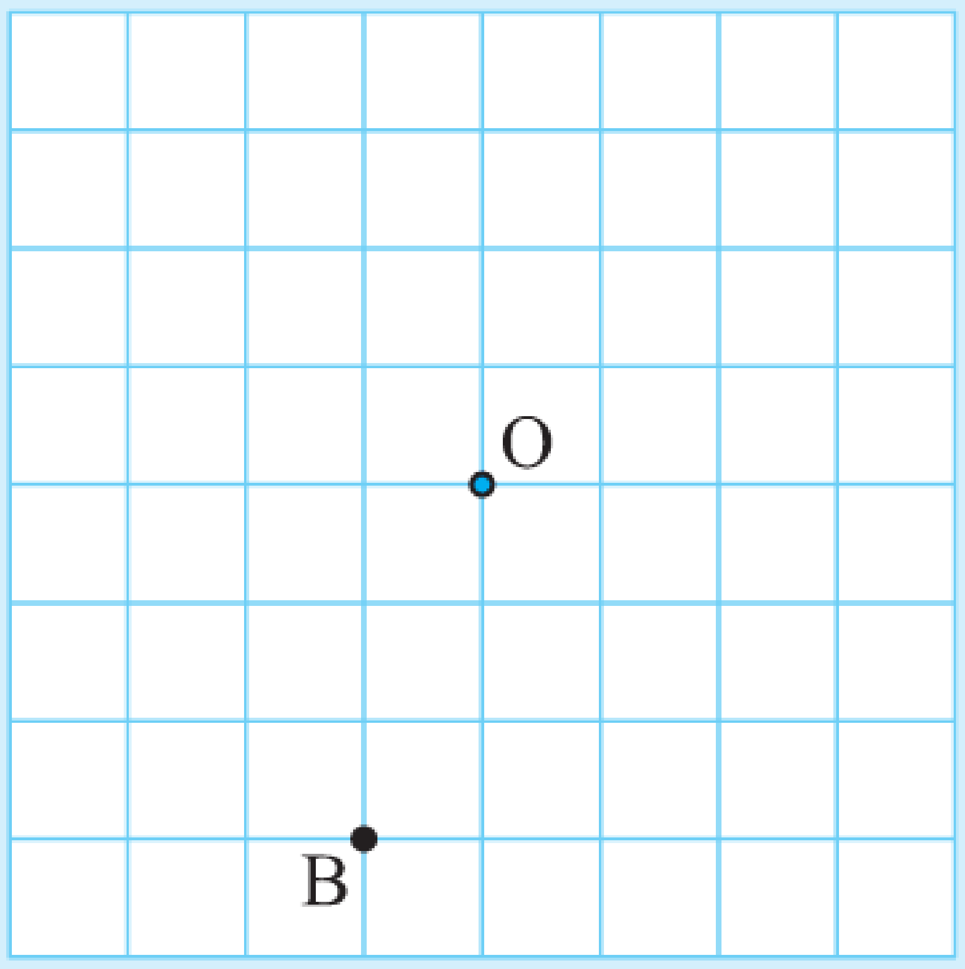
Example 3

Self Tutor

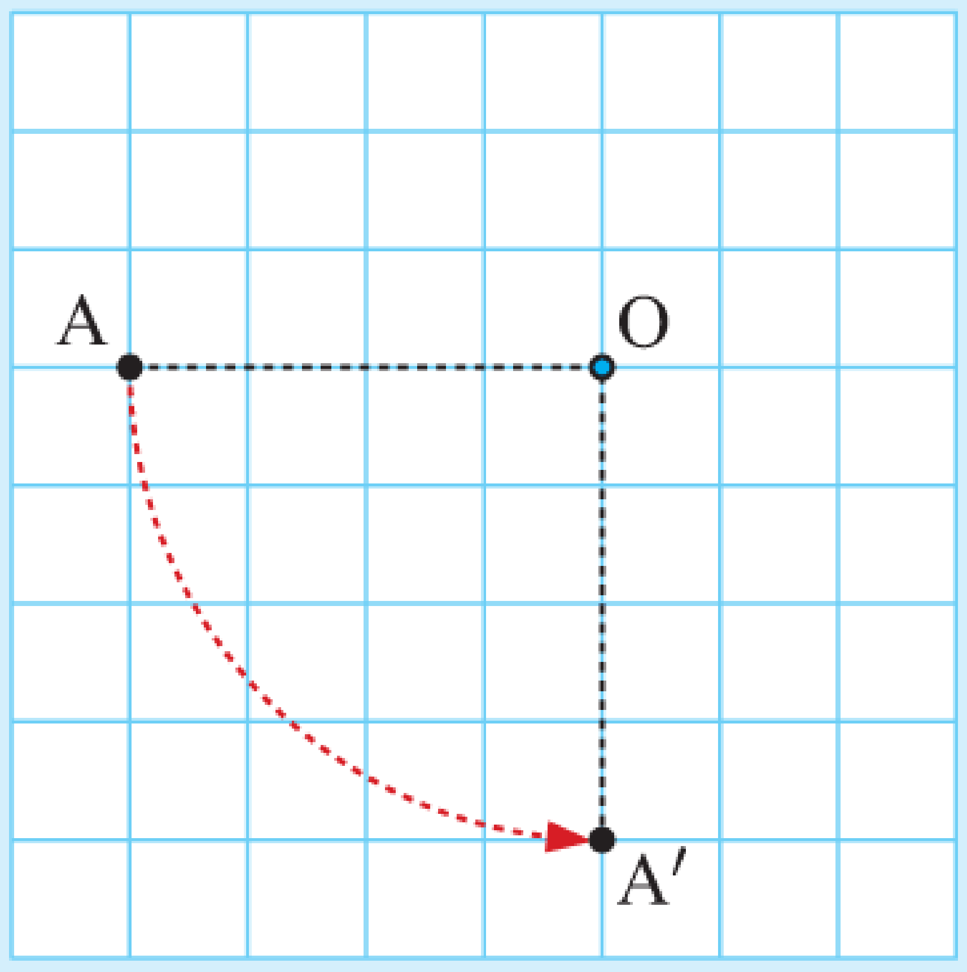
a Rotate A 90° anticlockwise about O.



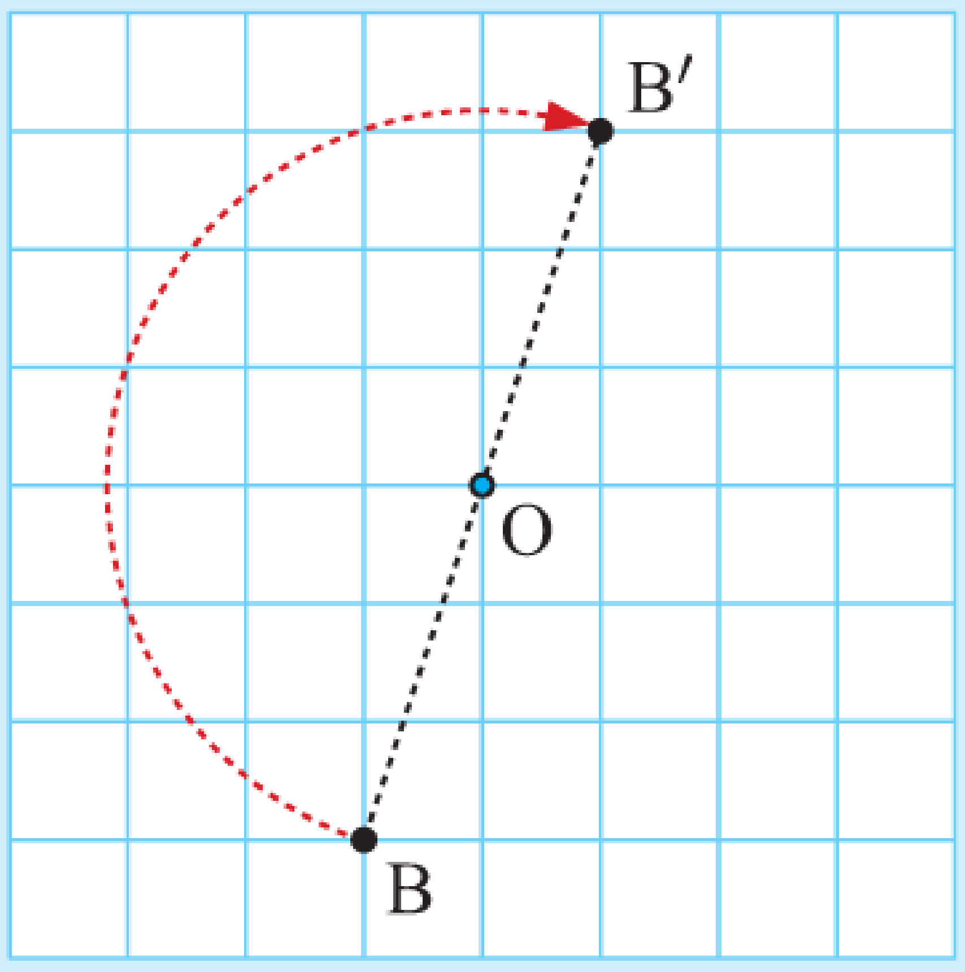
b Rotate B 180° clockwise about O.



a



b



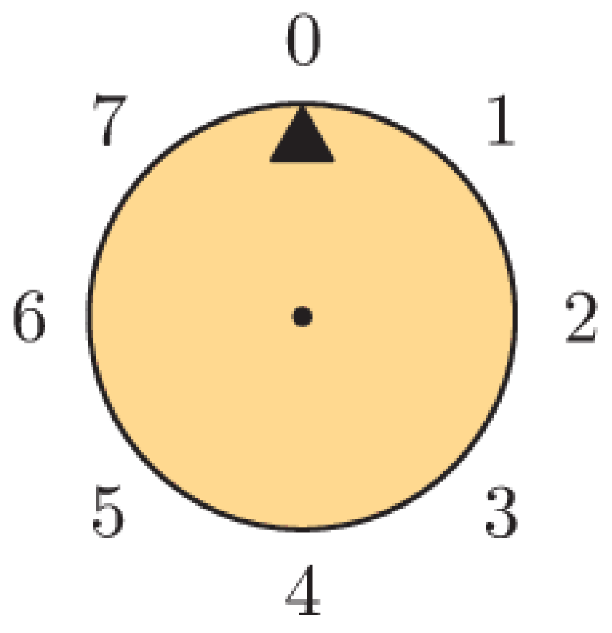
DISCUSSION

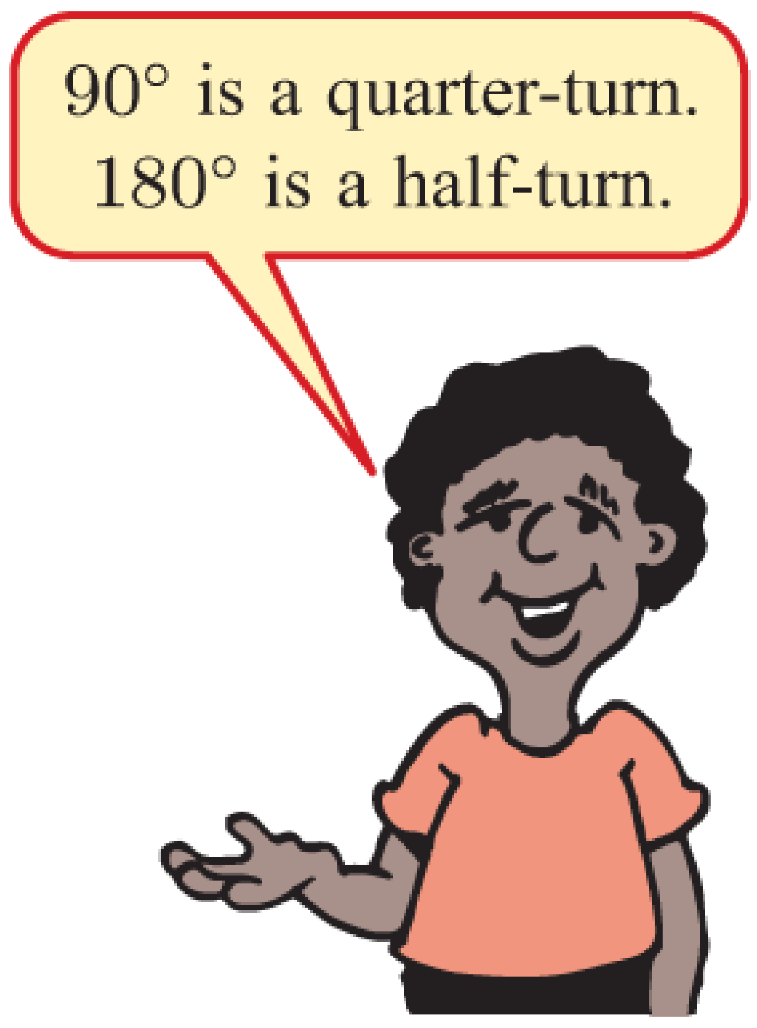
Will a rotation 180° clockwise always give the same result as a rotation 180° anticlockwise?

EXERCISE 16C

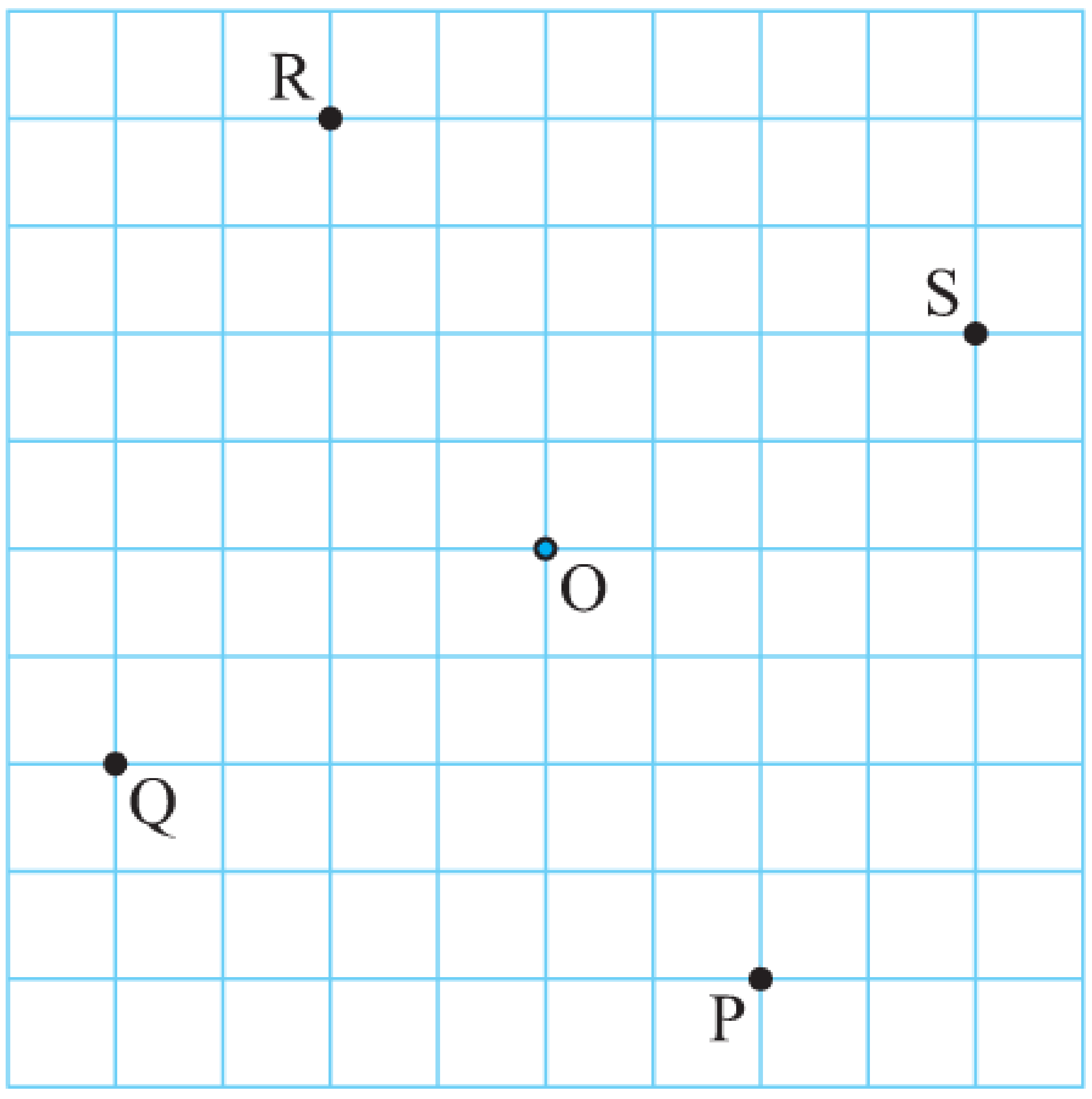
1 Find the position of the arrow on this dial if the arrow is turned through:

a 90° clockwise **b** 90° anticlockwise
c 180° clockwise **d** 180° anticlockwise.





2



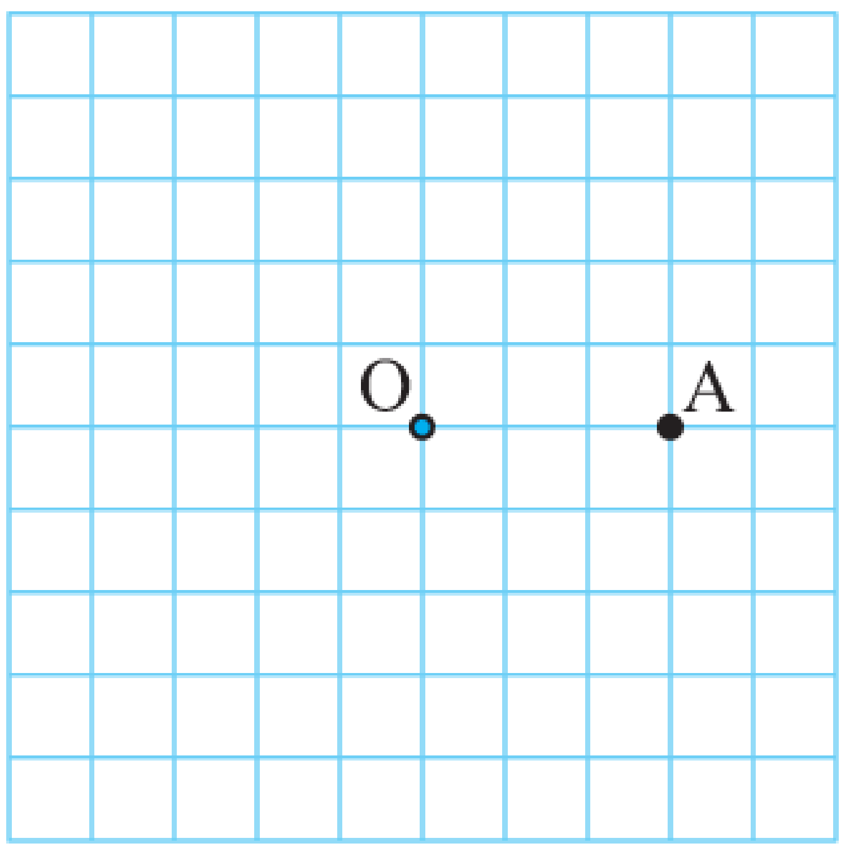
Which of these points is the image when P is rotated about O:

a 90° clockwise **b** 90° anticlockwise
c 180° anticlockwise?

3 Rotate each point about O in the direction given:

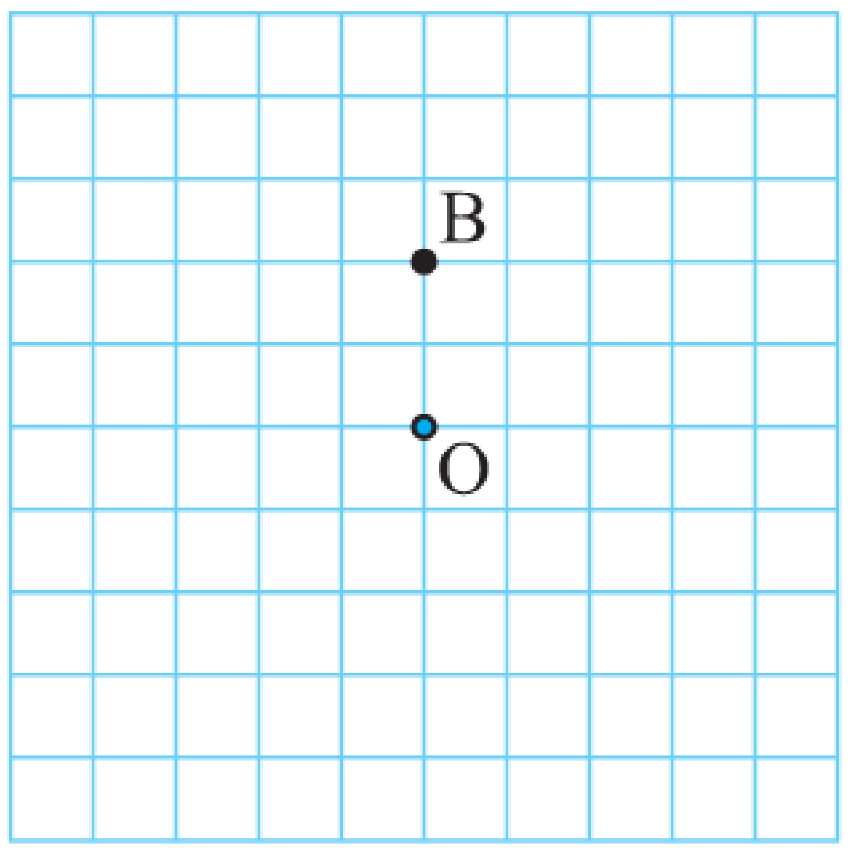
PRINTABLE DIAGRAMS

a



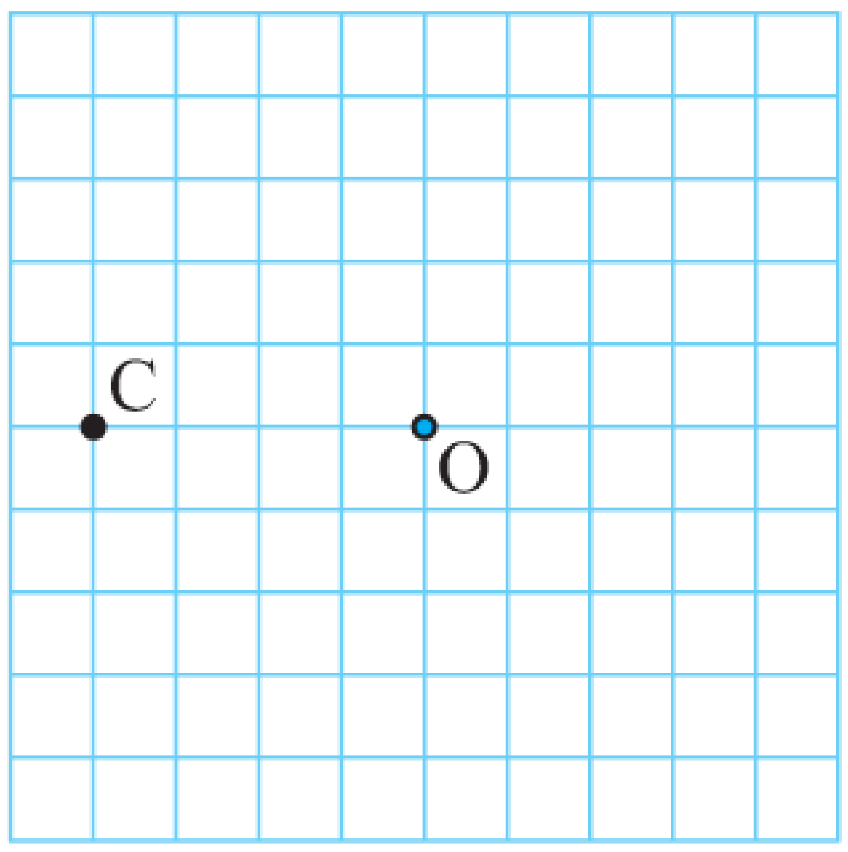
90° clockwise

b



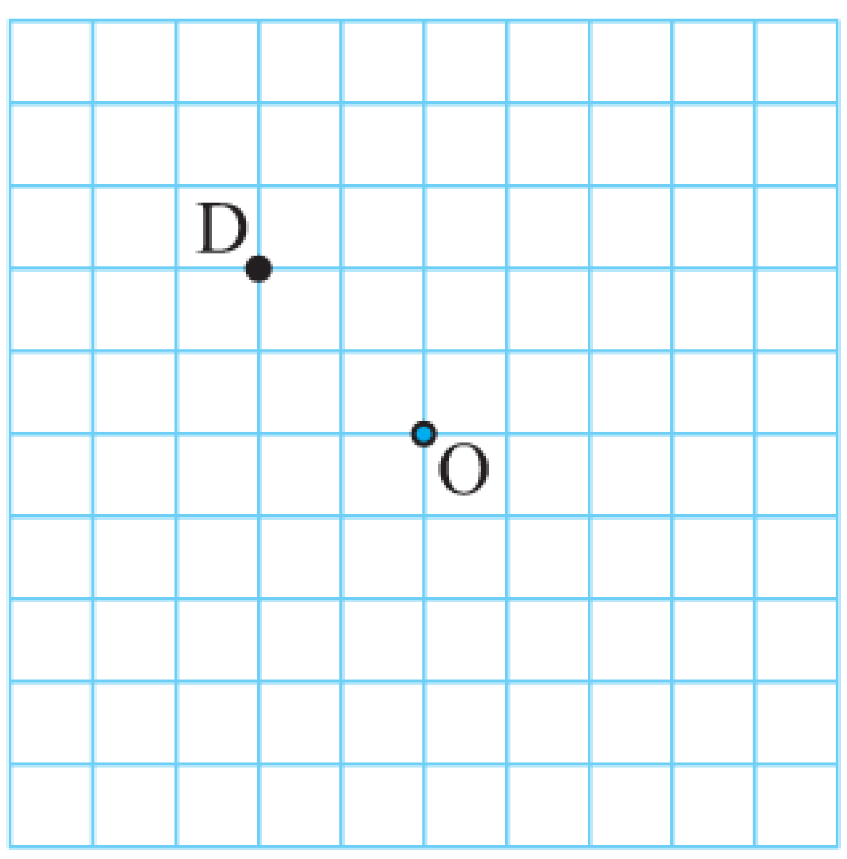
90° anticlockwise

c



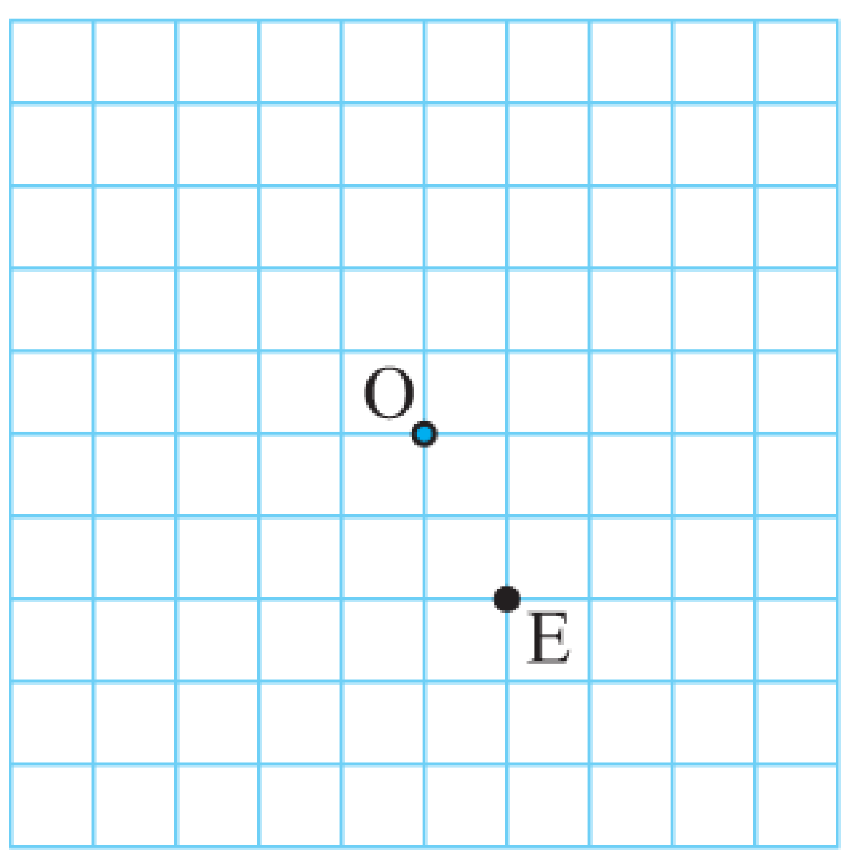
180° clockwise

d



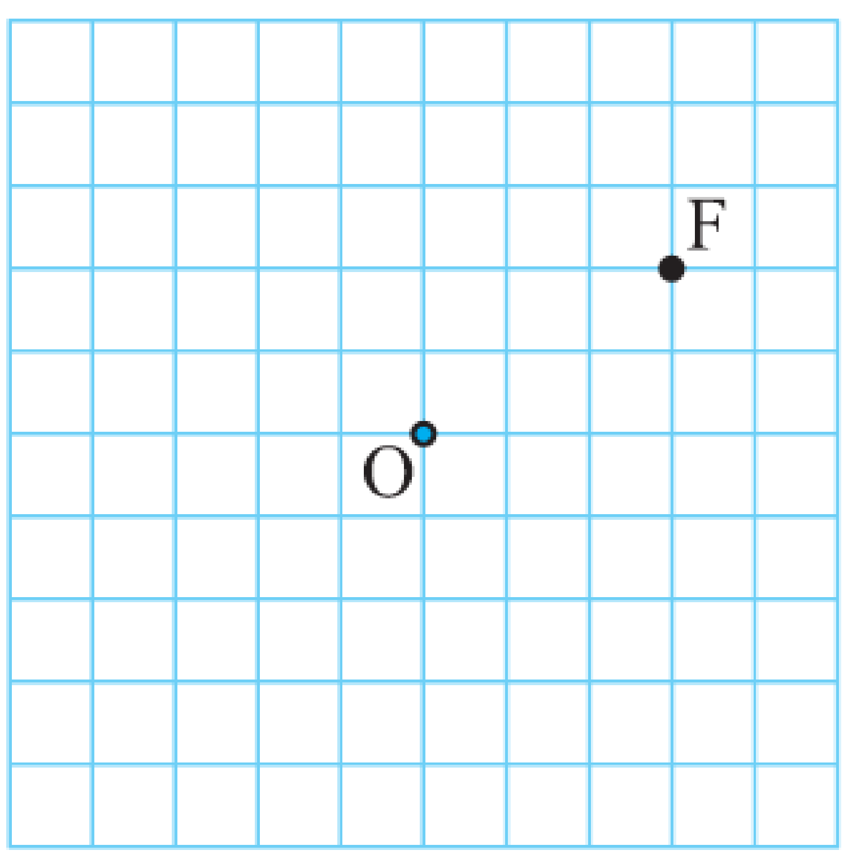
180° anticlockwise

e



90° clockwise

f

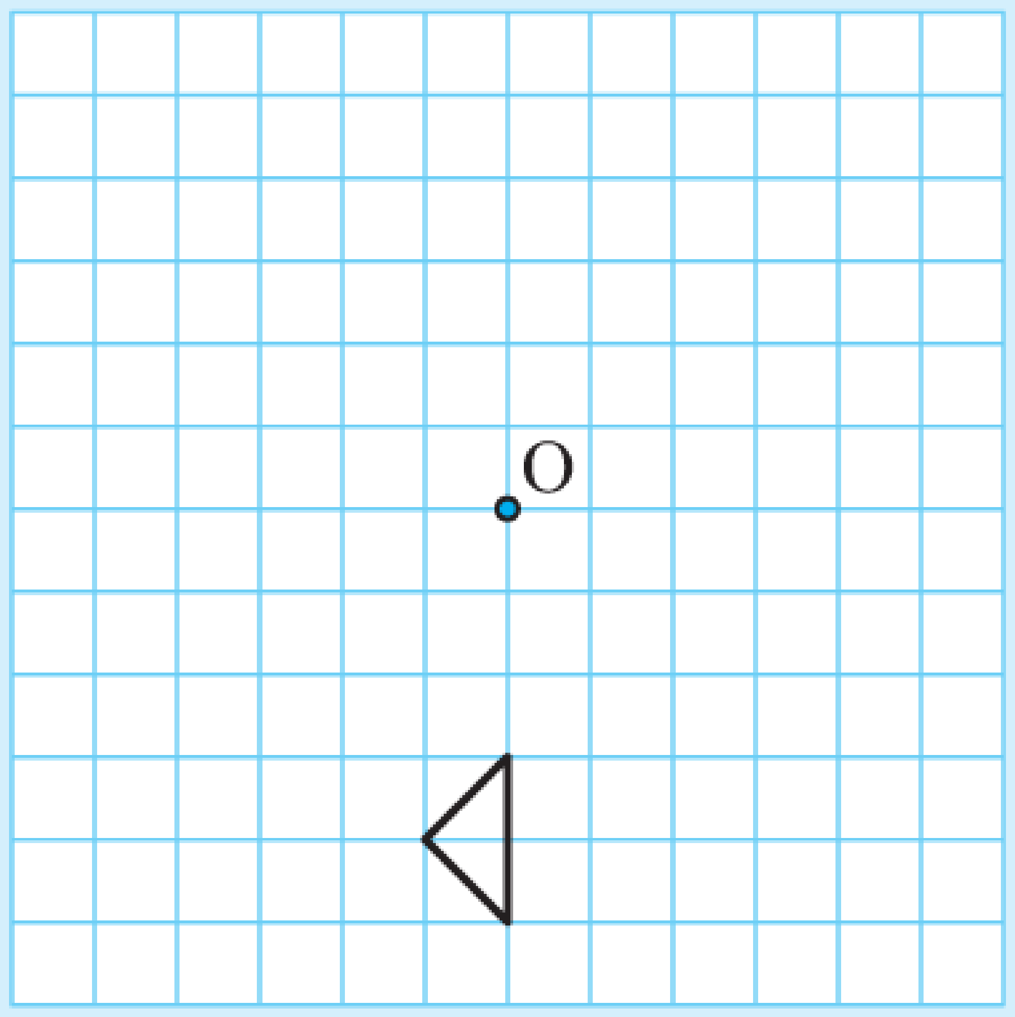


90° anticlockwise

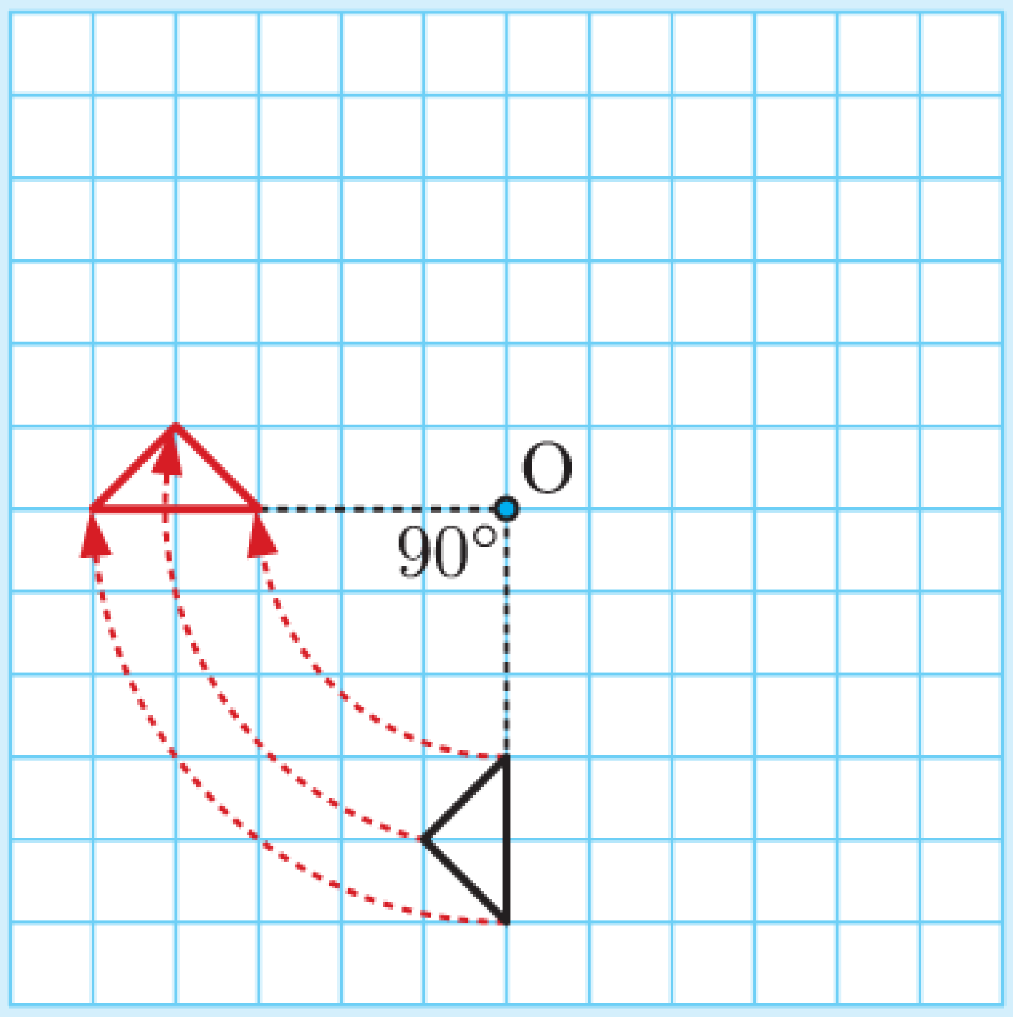
Example 4

Self Tutor

Rotate this figure 90° clockwise about O.

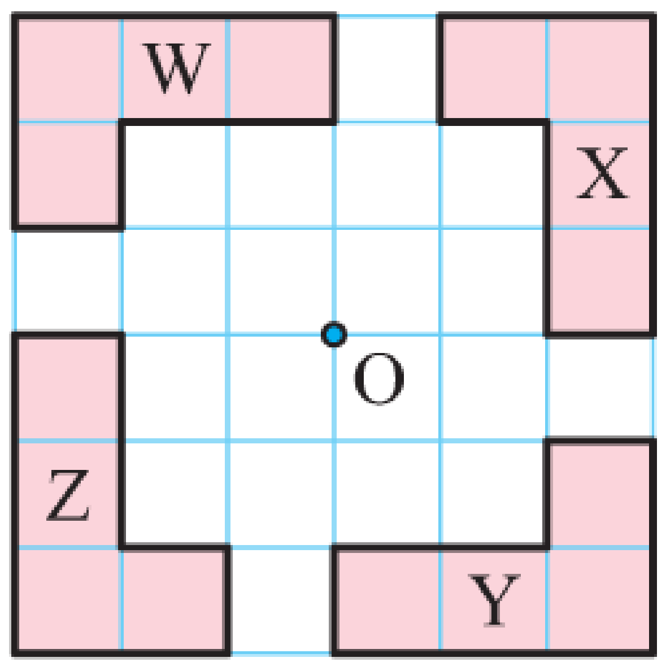


We rotate each corner of the figure 90° clockwise about O, then complete the sides of the triangle.



4 The shape W is rotated about the point O. What turn is needed to rotate W onto:

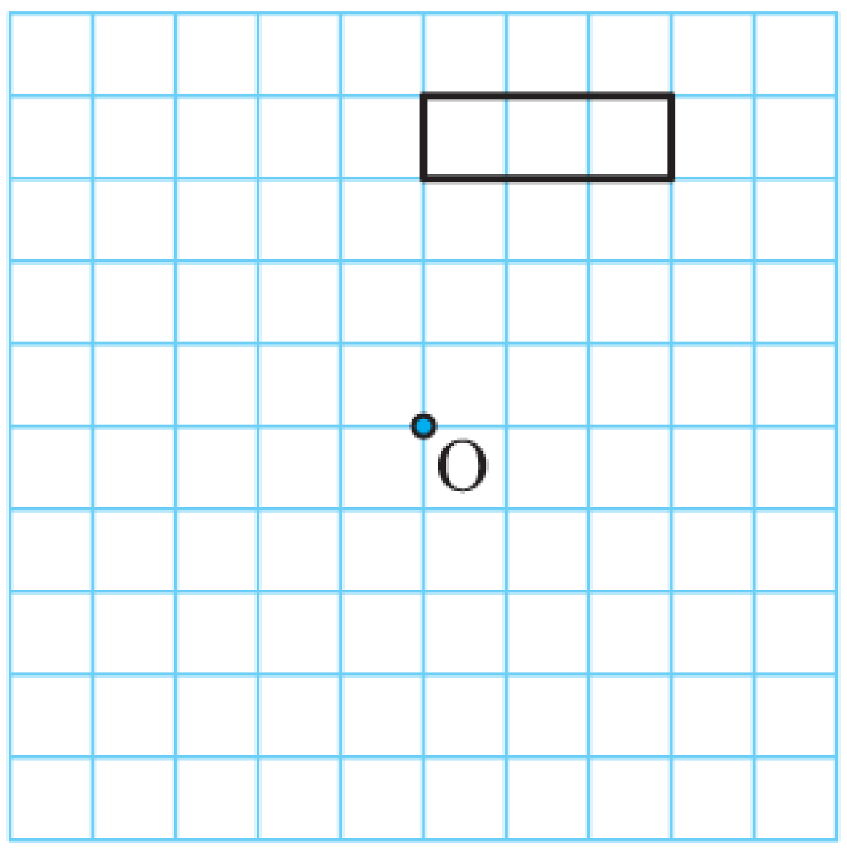
a X **b** Y **c** Z?



5 Rotate each shape about O in the direction given:

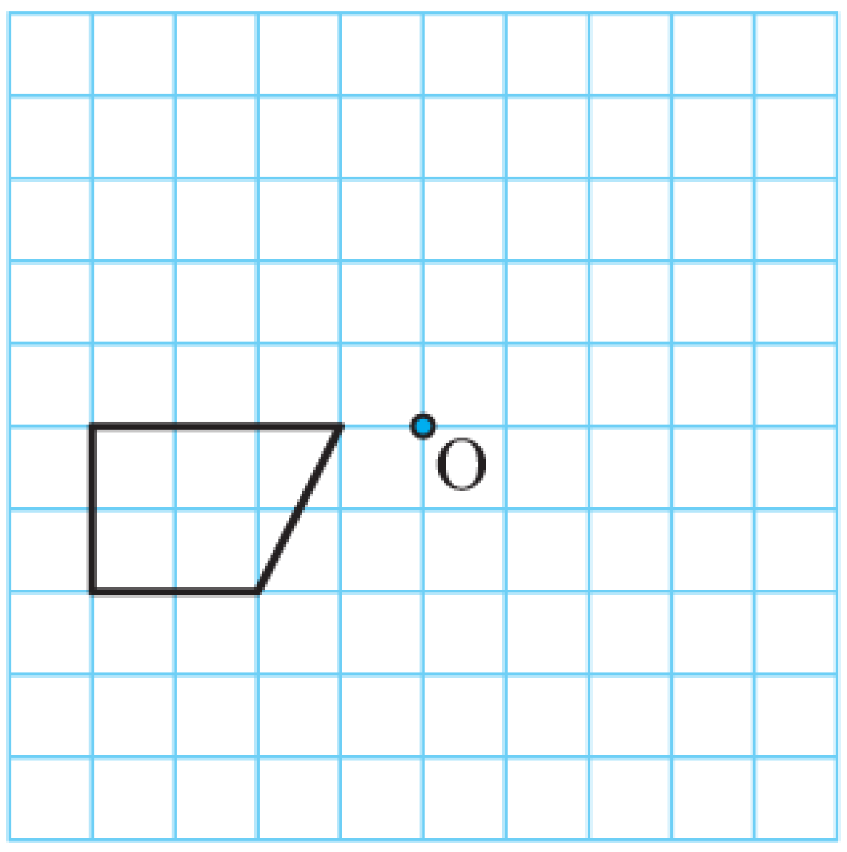
PRINTABLE DIAGRAMS

a



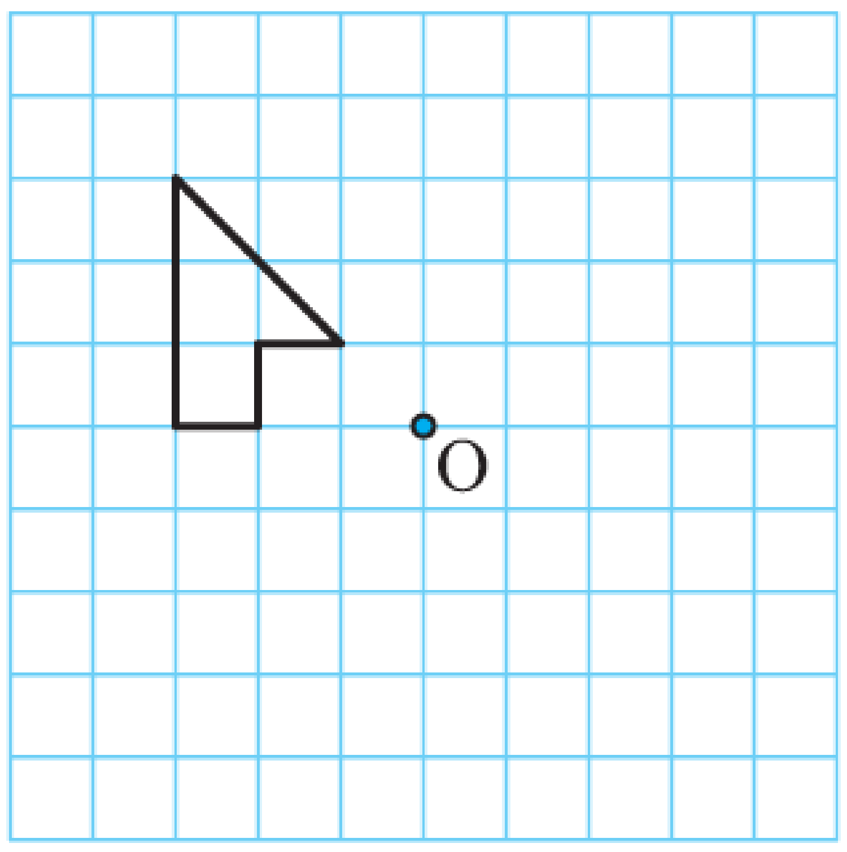
90° clockwise

b



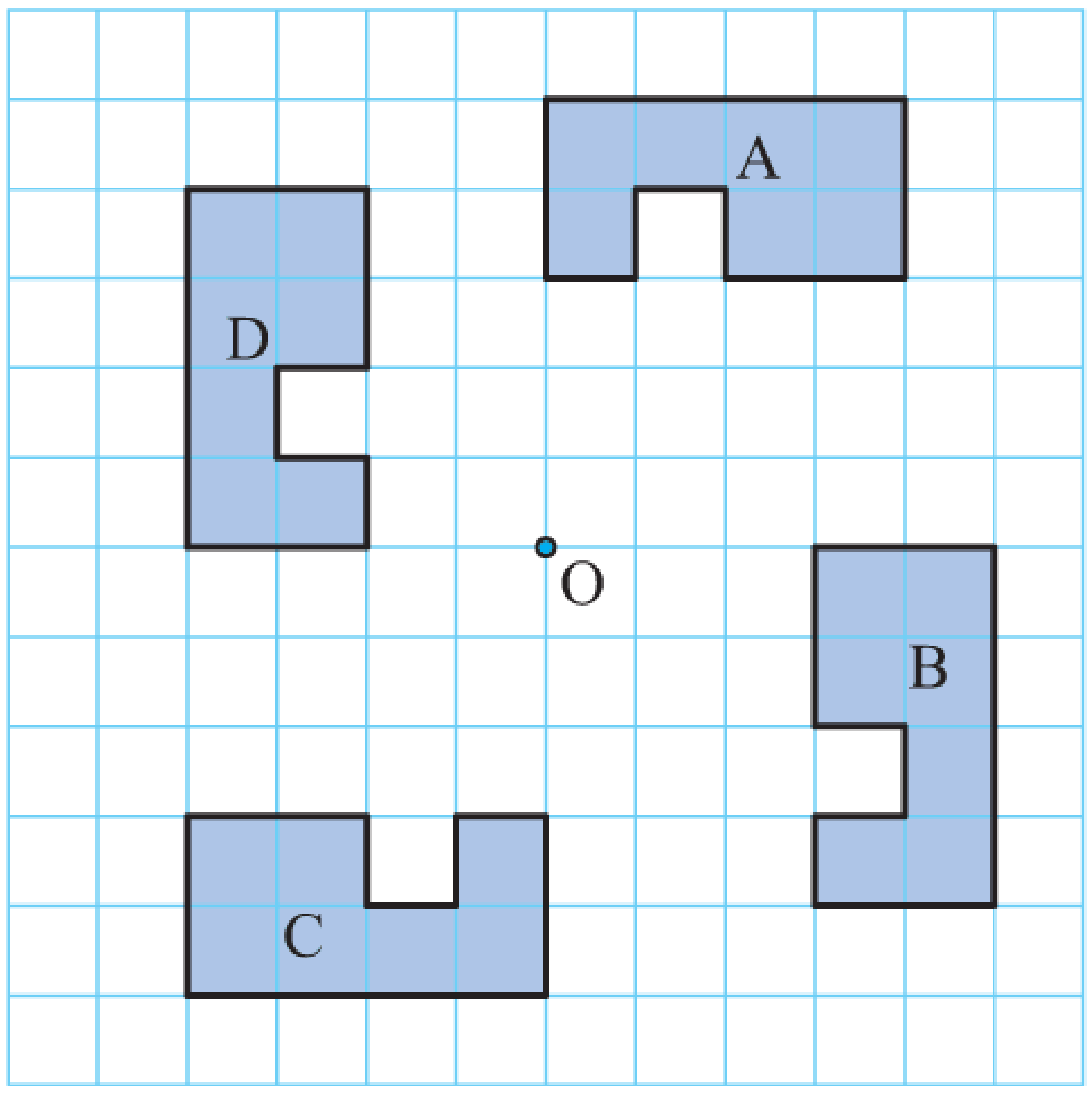
180° clockwise

c



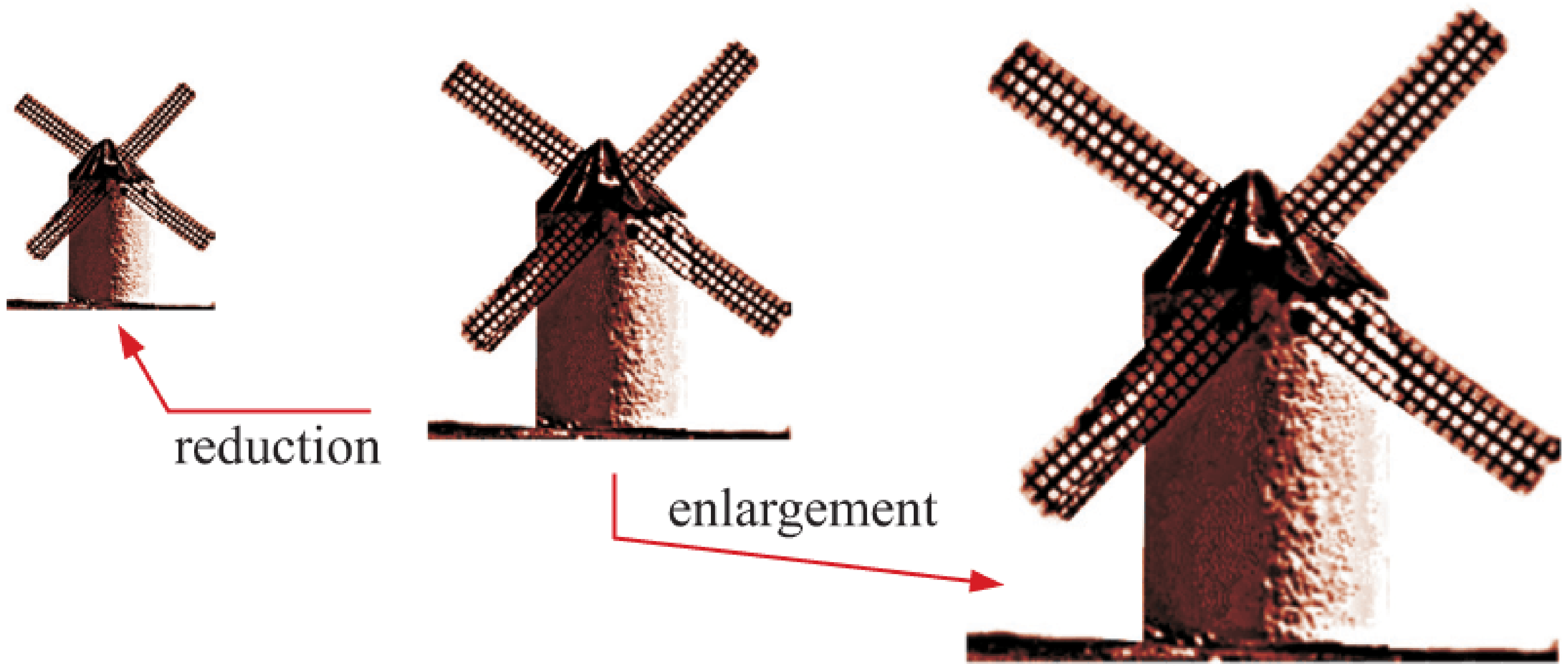
90° anticlockwise

6 Which of these figures is a rotation of A about O?

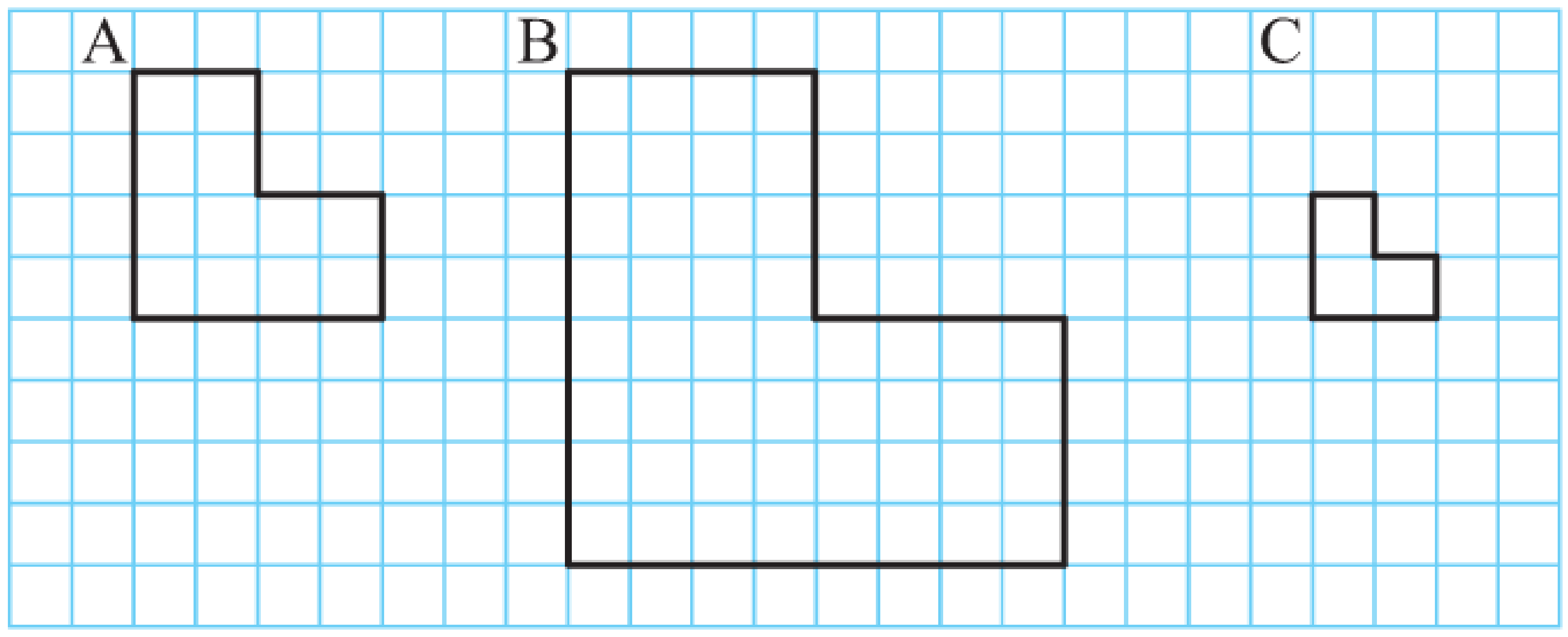


D ENLARGEMENTS AND REDUCTIONS

If you want to make a picture bigger or smaller, you can **enlarge** or **reduce** it using a photocopier.



To describe an enlargement or reduction, we use a **scale factor**. The scale factor tells us the amount by which an object is enlarged or reduced.



For example, look at the figures in the grid alongside.

The side lengths in figure B are all twice the corresponding side lengths in figure A. We say that B is an **enlargement** of A with scale factor 2.

The sides lengths in figure C are all half the side lengths in figure A. We say that C is a **reduction** of A with scale factor $\frac{1}{2}$.

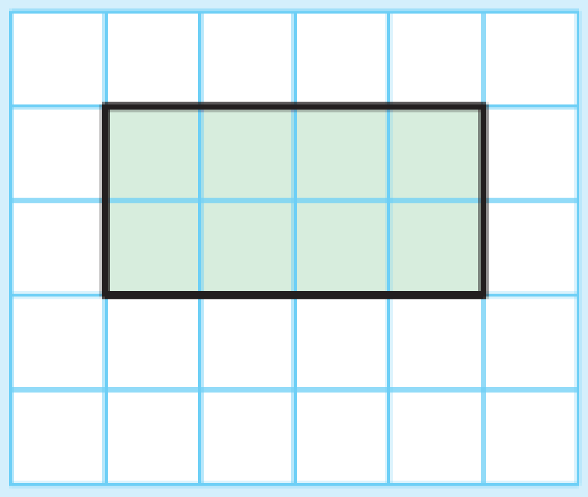
> If the scale factor is greater than 1, an **enlargement** occurs.
>
> If the scale factor is less than 1, a **reduction** occurs.

Example 5

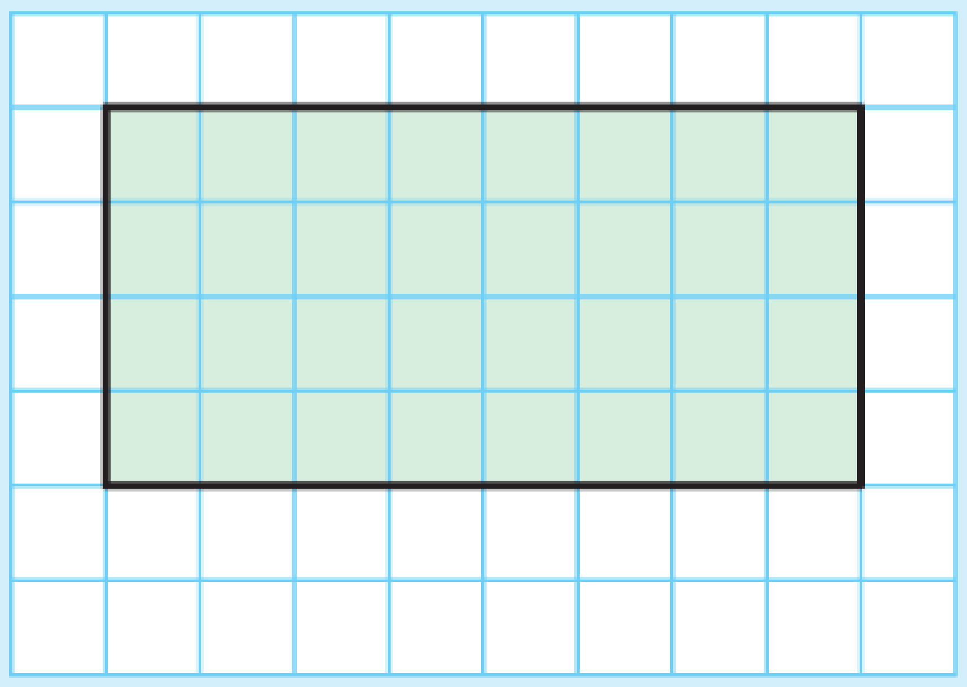
Self Tutor

Enlarge or reduce the figure using a scale factor of:

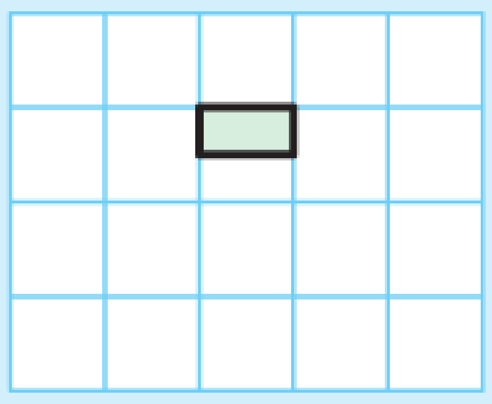
a 2 **b** $\frac{1}{4}$.



a For a scale factor of 2, the side lengths are *doubled*.



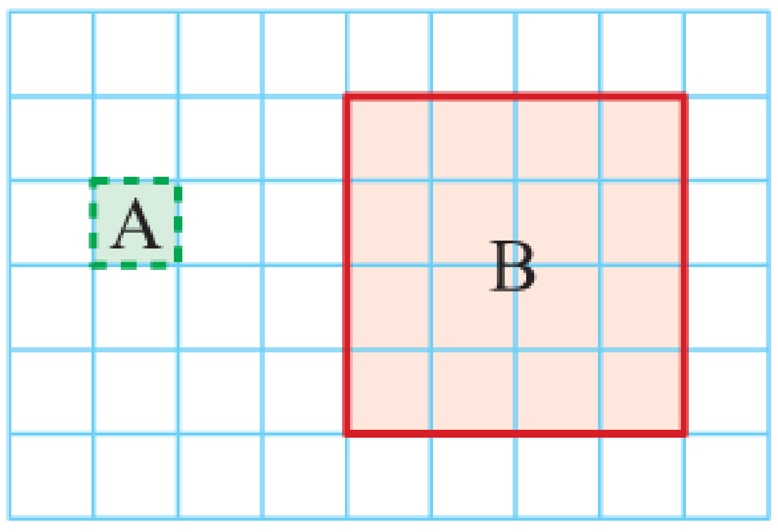
b For a scale factor of $\frac{1}{4}$, the side lengths are *quartered*.



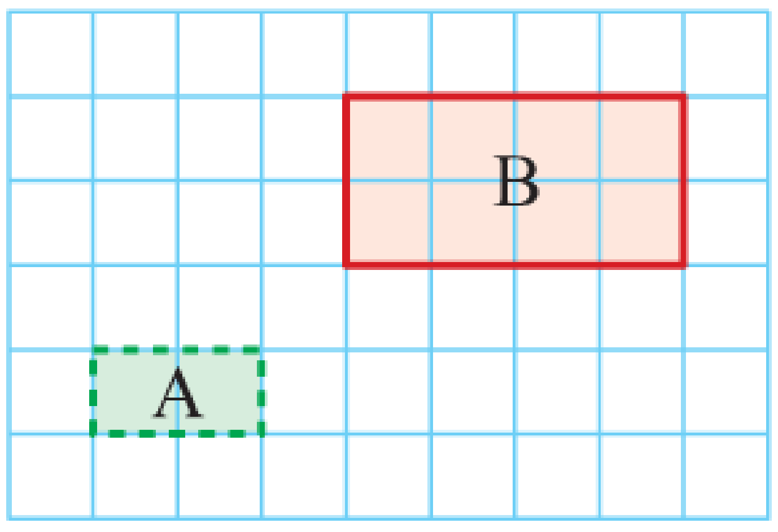
EXERCISE 16D

1 In these diagrams, A has been enlarged to B. Find the scale factor.

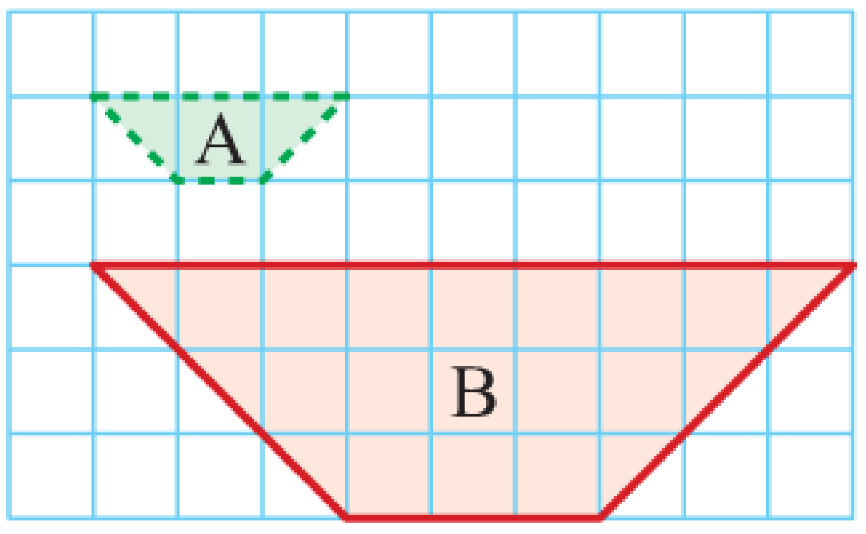
a



b

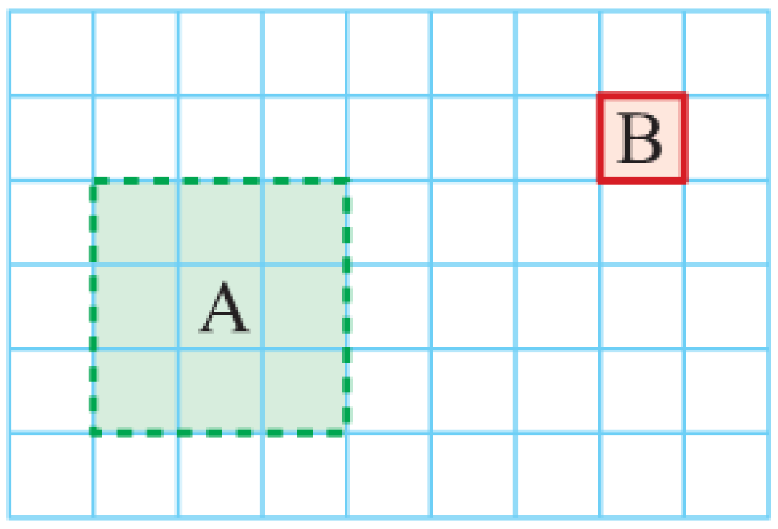


c

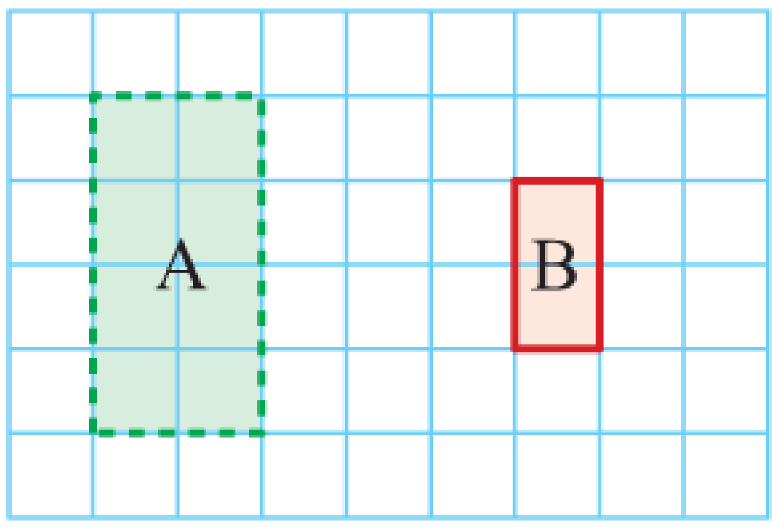


2 In these diagrams, A has been reduced to B. Find the scale factor.

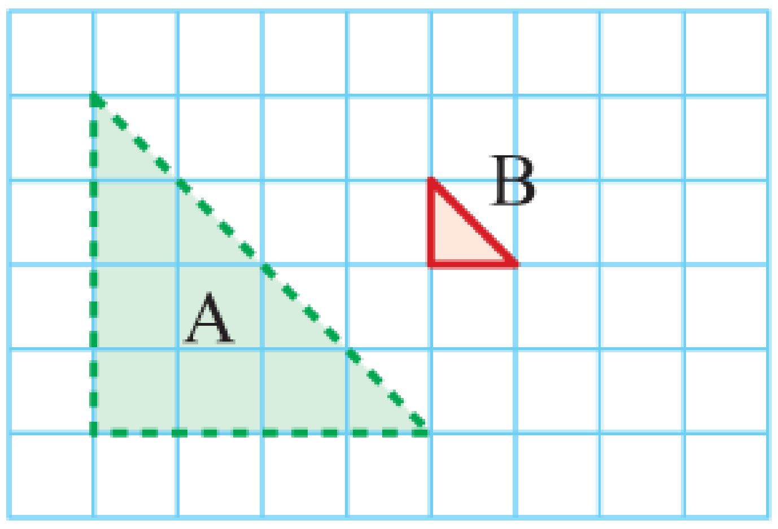
a



b



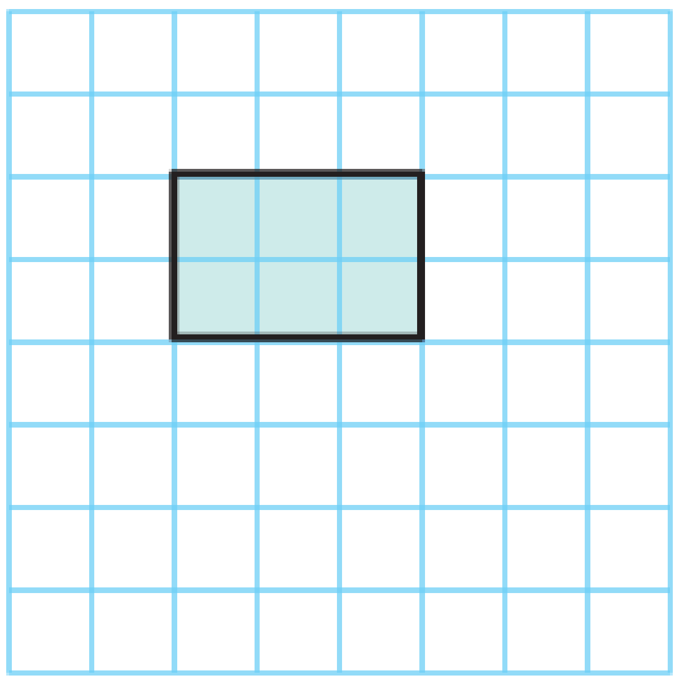
c



3 Enlarge or reduce each object by the scale factor given:

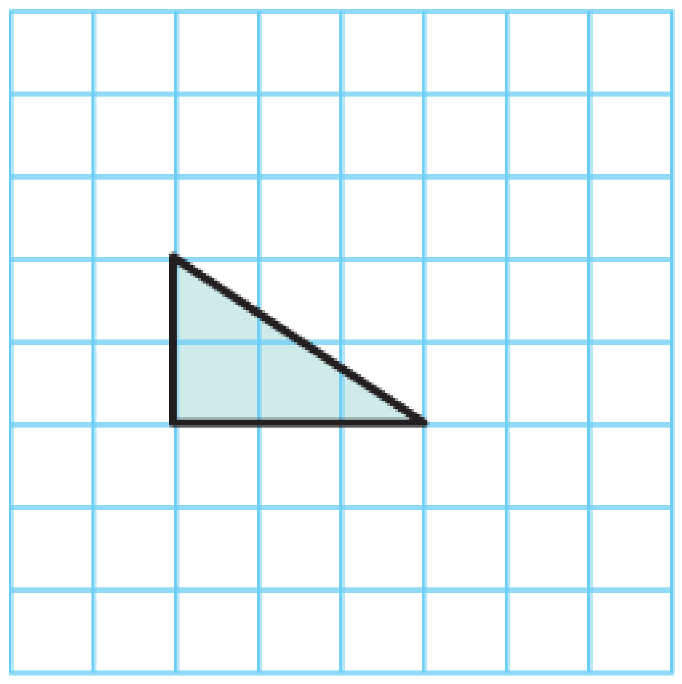
PRINTABLE WORKSHEET

a



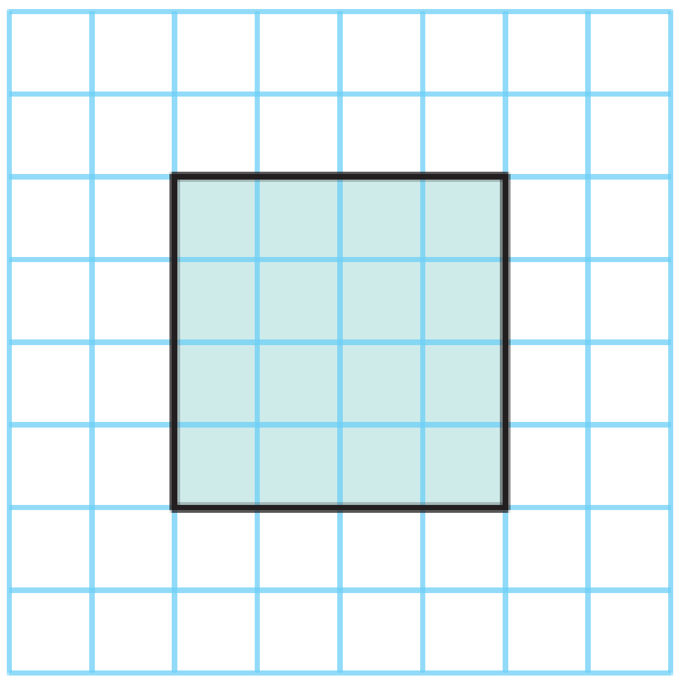
scale factor 2

b



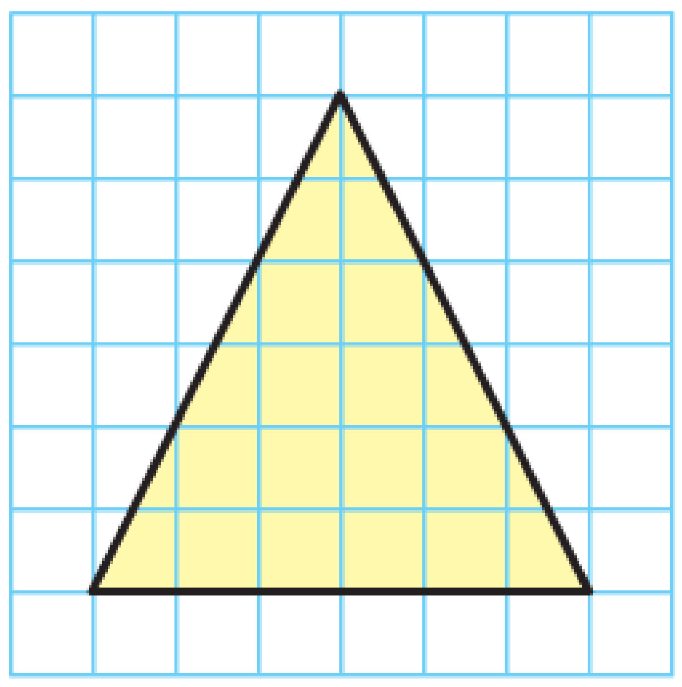
scale factor 3

c



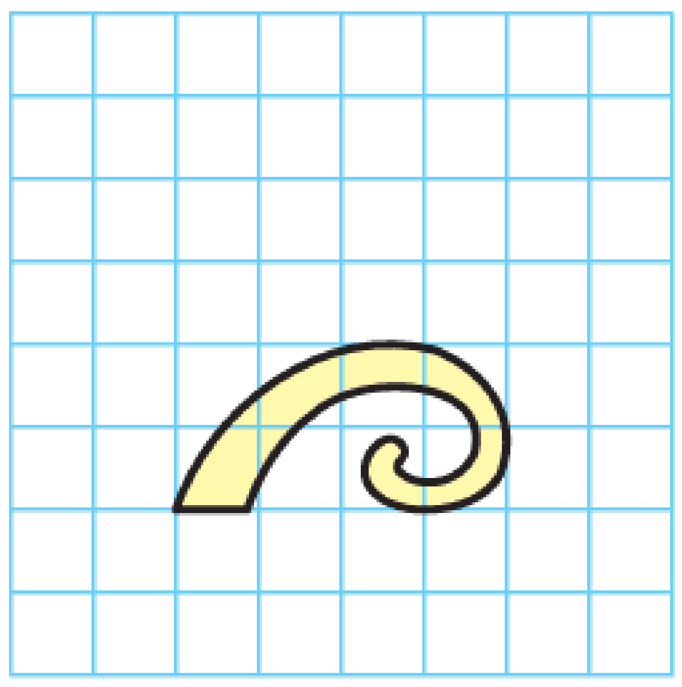
scale factor $\frac{1}{2}$

d



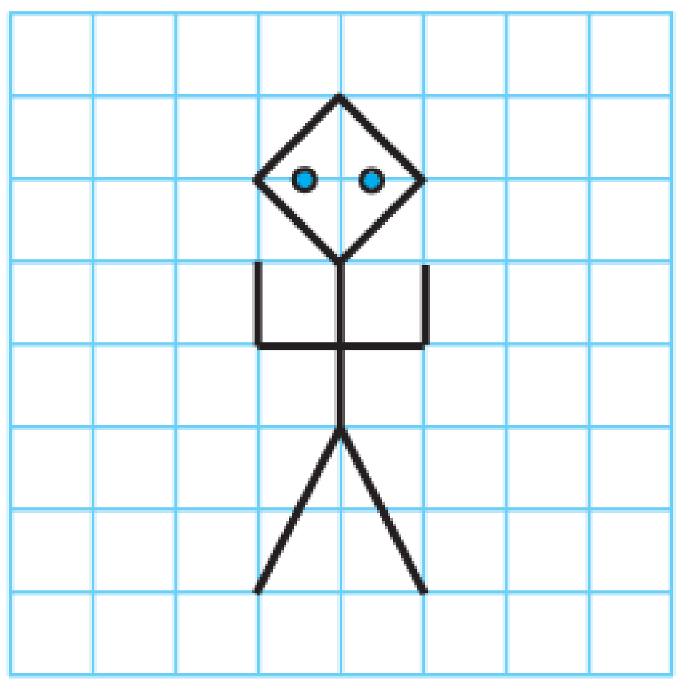
scale factor $\frac{1}{3}$

e



scale factor 4

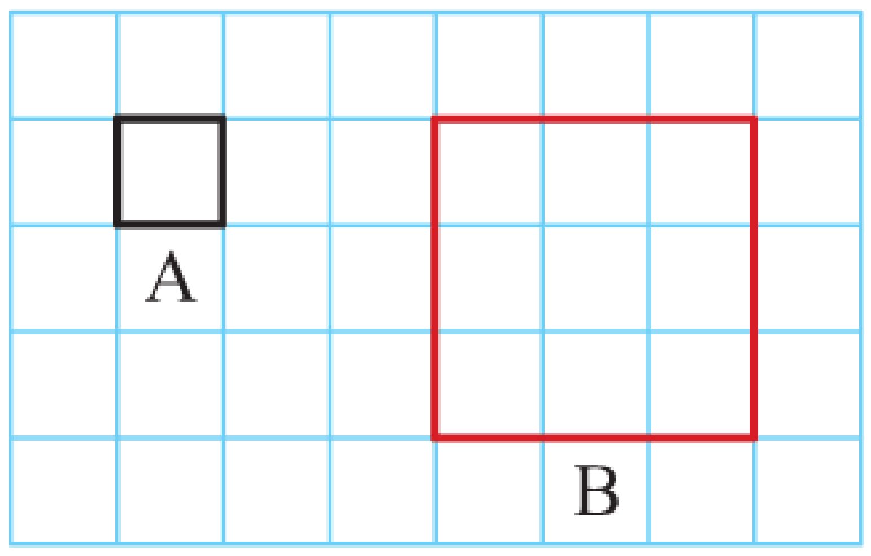
f



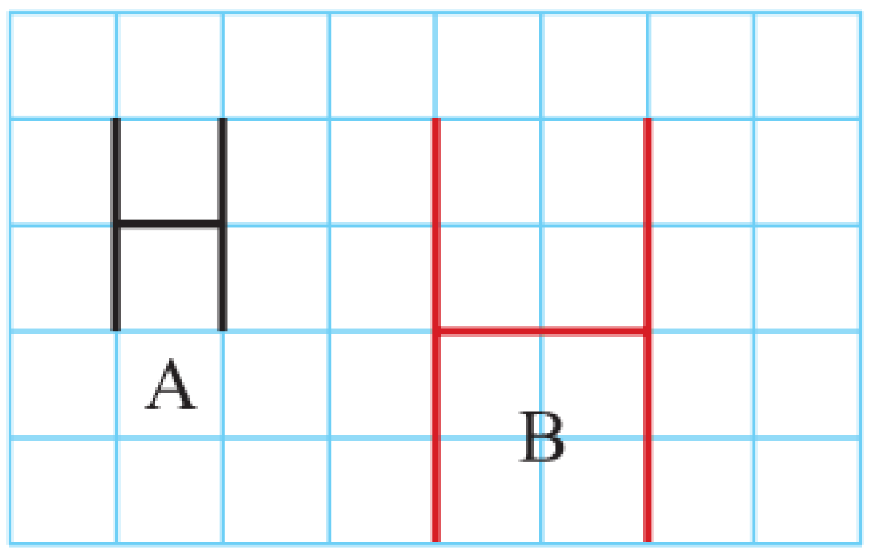
scale factor 2

4 **a** Find the scale factor when A is transformed to B:

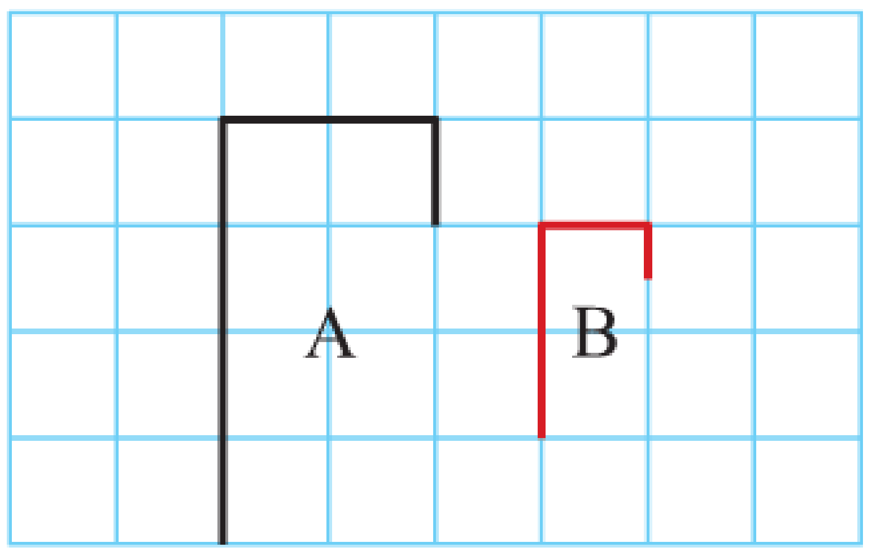
i



ii



iii



b For each diagram in **a**, write down the scale factor which transforms B to A.

E TESSELLATIONS

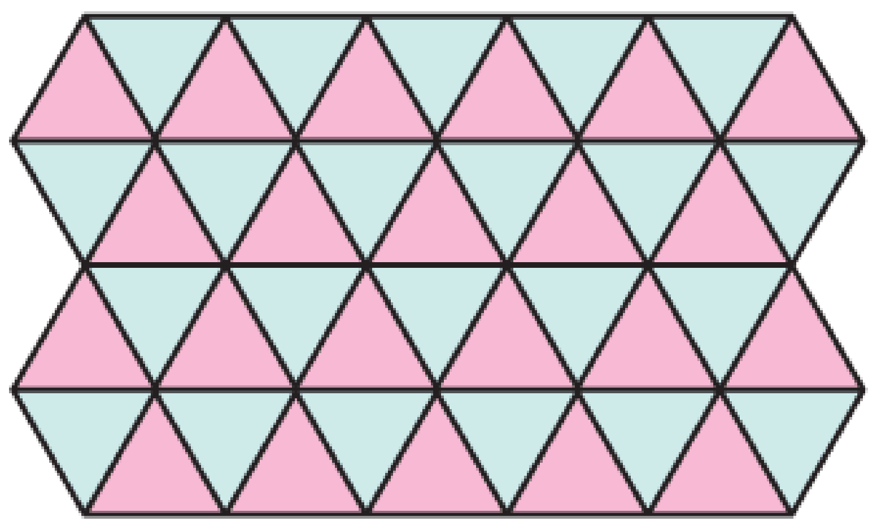
A **tessellation** is a pattern made using figures which are identical in shape and size. They must cover an area without leaving any gaps.

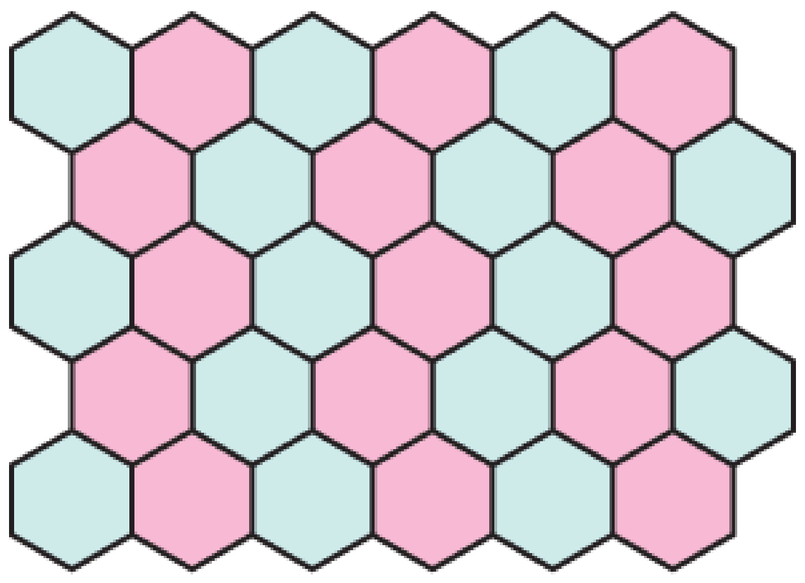
The photograph alongside shows a tessellation of bricks used to pave a footpath.

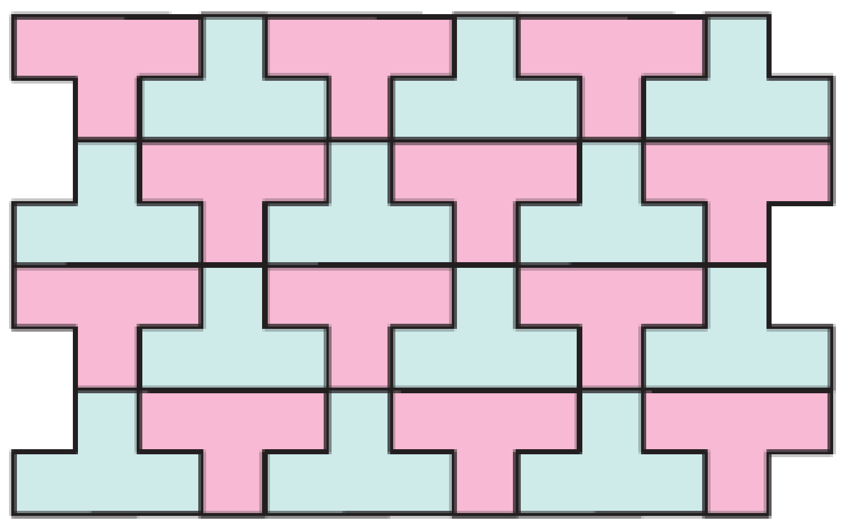
Tessellations are also found in carpets, tiles, weaving, and wallpaper.

This brick design is *not* a tessellation, as it is constructed from two different brick sizes.

The following tile patterns are all tessellations:



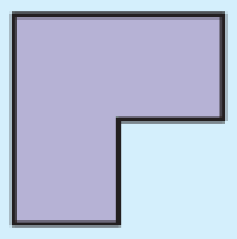




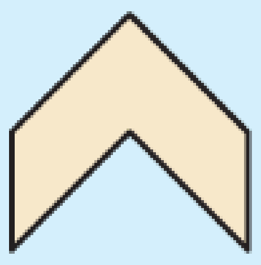
Example 6 **Self Tutor**

Draw a tessellation of each shape:

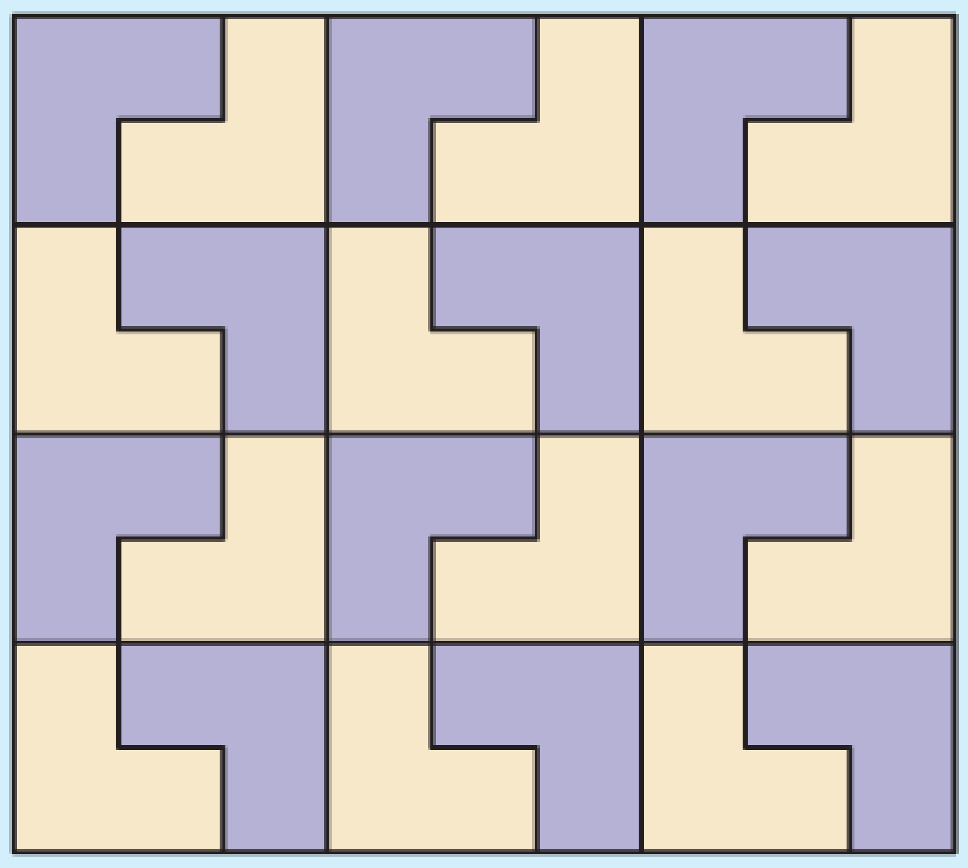
a



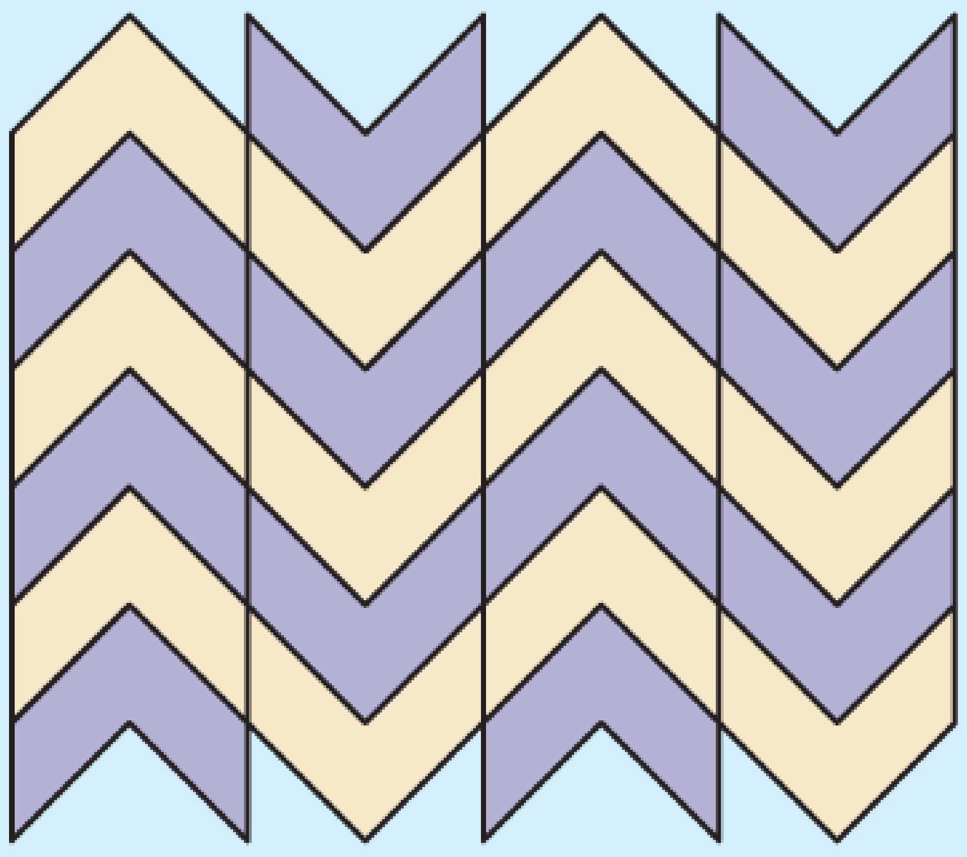
b



a



b

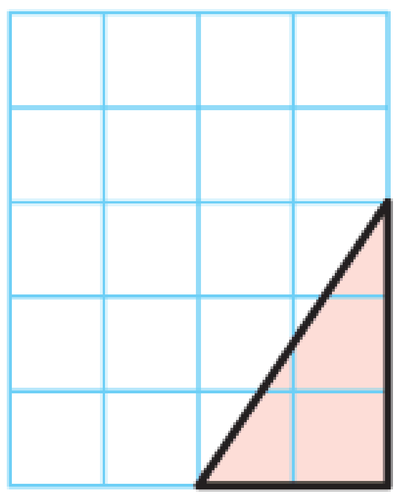


For a tessellation, the shapes must:
- be identical in size and shape
- fit together with no gaps

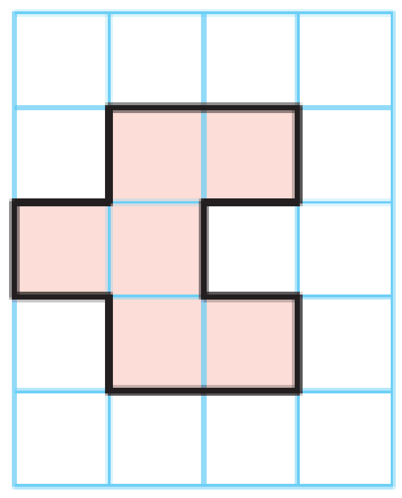
EXERCISE 16E

1 Draw a tessellation of each shape:

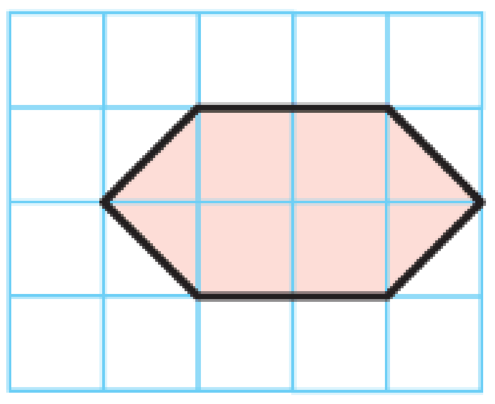
a



b

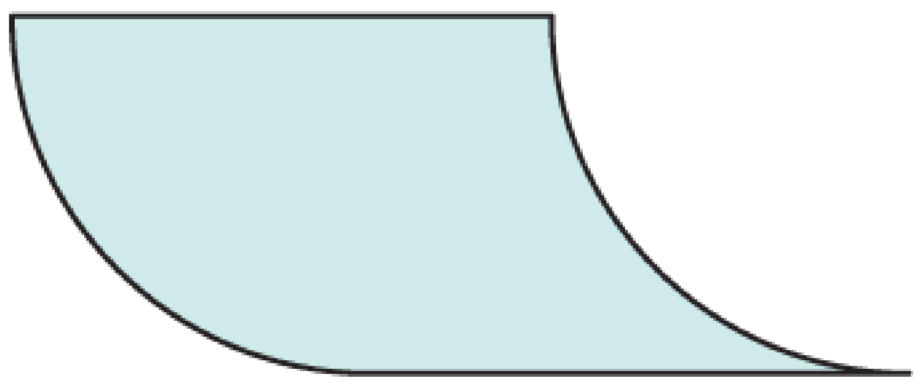


c

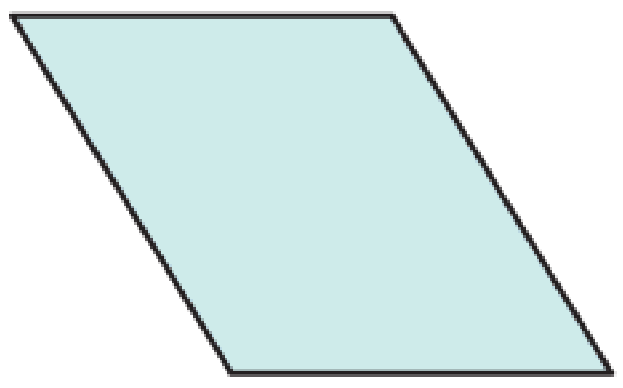


2 Draw a tessellation of each shape:

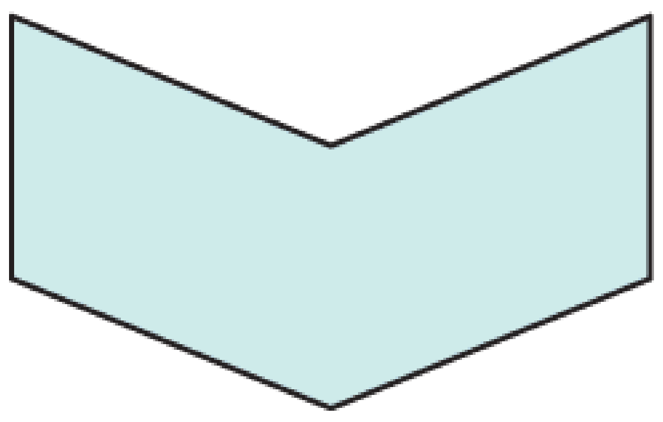
a



b



c

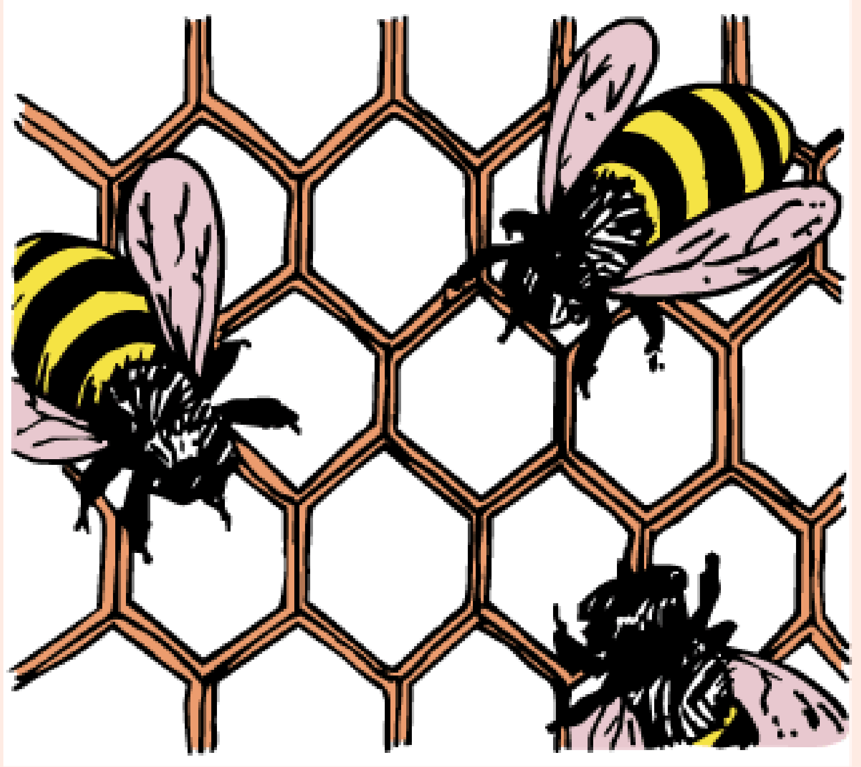


DISCUSSION IN GOOD SHAPE

1 Why are the cells in a beehive shaped the way they are?

2 Look at the shapes of paving blocks. Explain what advantage some shapes have over others.

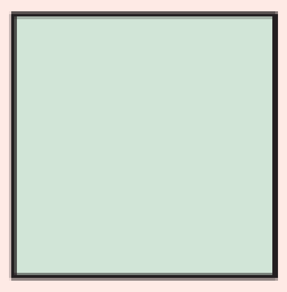
3 When building walls, what are the advantages of rectangular bricks over square bricks?



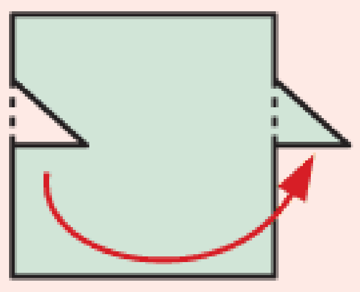
ACTIVITY 2 CREATING TESSELLATIONS

What to do:

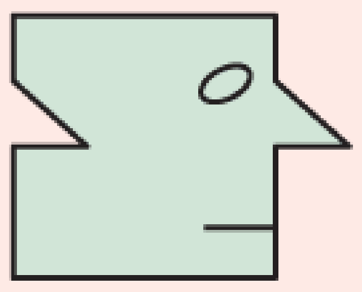
1 Draw a square.



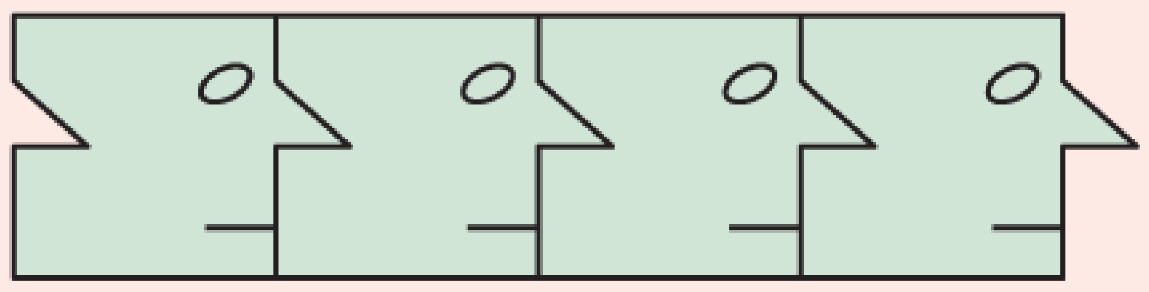
2 Cut a piece from one side and “glue” it onto the opposite side.



3 Erase any unwanted lines and add features.



4 Photocopy this several times and cut out each face. Combine them to form a tessellation.



5 Make your own tessellation pattern, and produce a full page pattern with 3 cm by 3 cm tiles. Be creative and colourful. You could use a computer drawing package.

Global context

click here

The Ishtar Gate

Statement of inquiry:	Understanding transformations can help us see patterns in the world around us.
Global context:	Personal and cultural expression
Key concept:	Form
Related concepts:	Representation, Space
Objectives:	Knowing and understanding, Applying mathematics in real-life contexts
Approaches to learning:	Research, Thinking

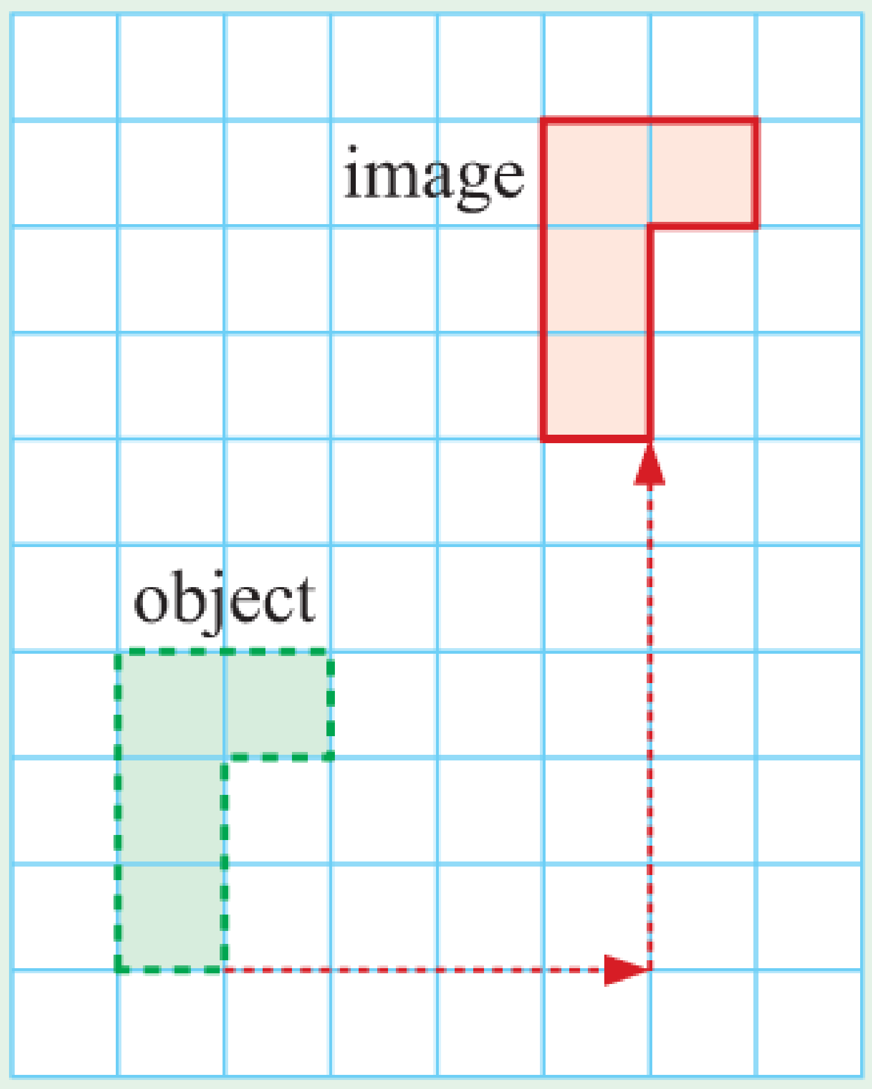
KEY WORDS USED IN THIS CHAPTER

- anticlockwise
- enlargement
- object
- rotation
- transformation
- centre of rotation
- image
- reduction
- scale factor
- translation
- clockwise
- mirror line
- reflection
- tessellation

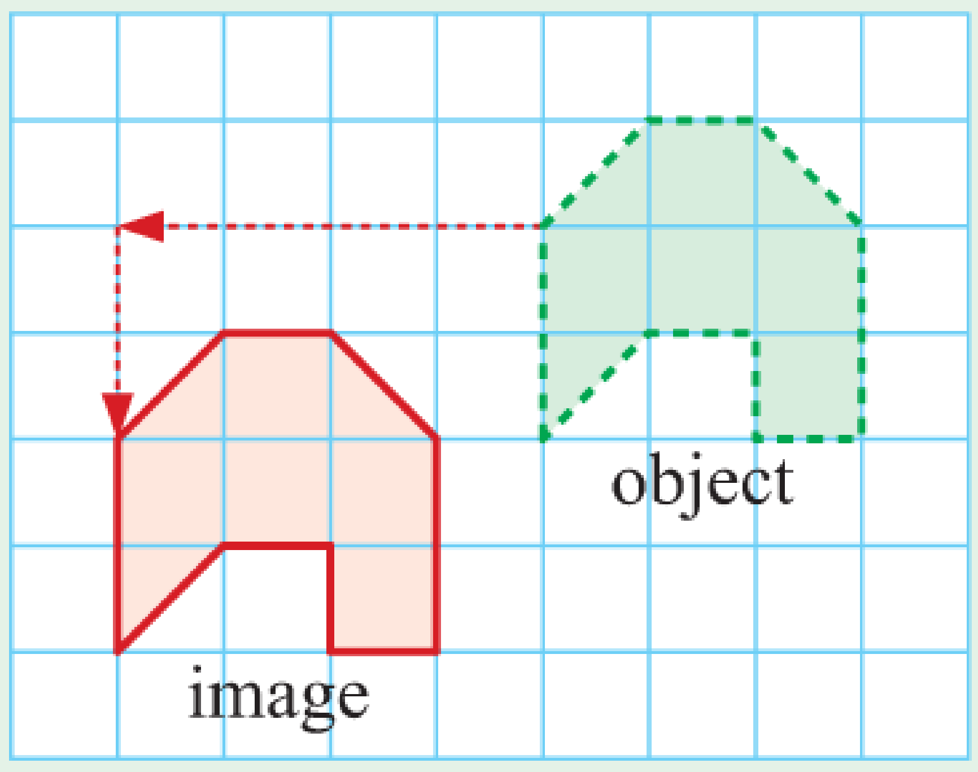
REVIEW SET 16A

1 Describe each of these translations using a horizontal step and a vertical step:

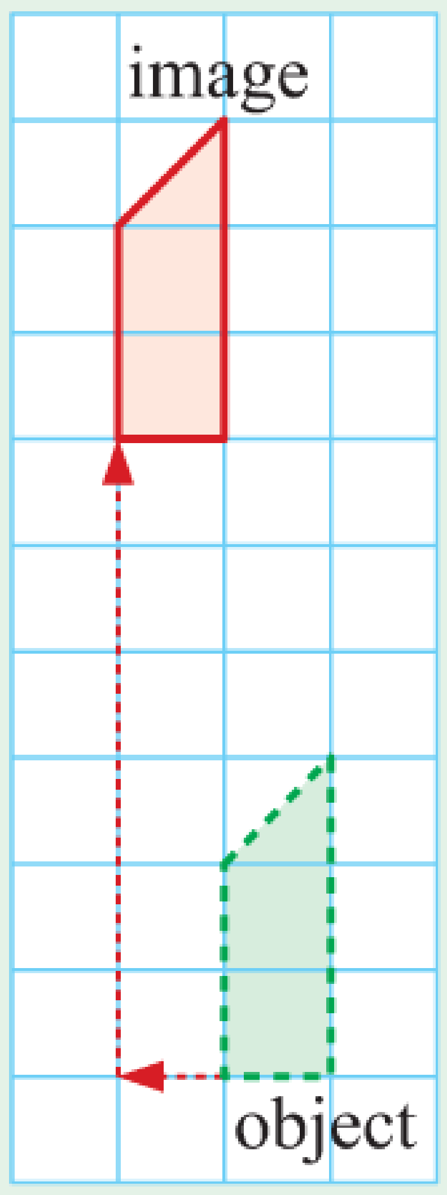
a



b

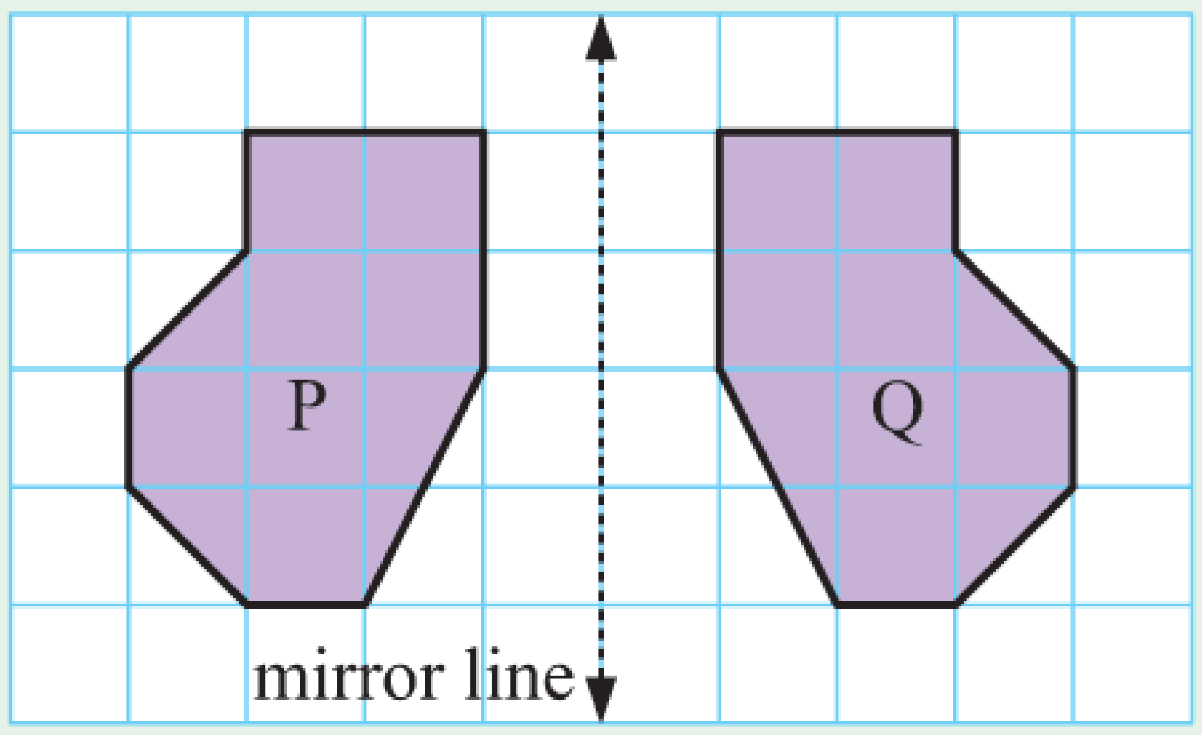


c

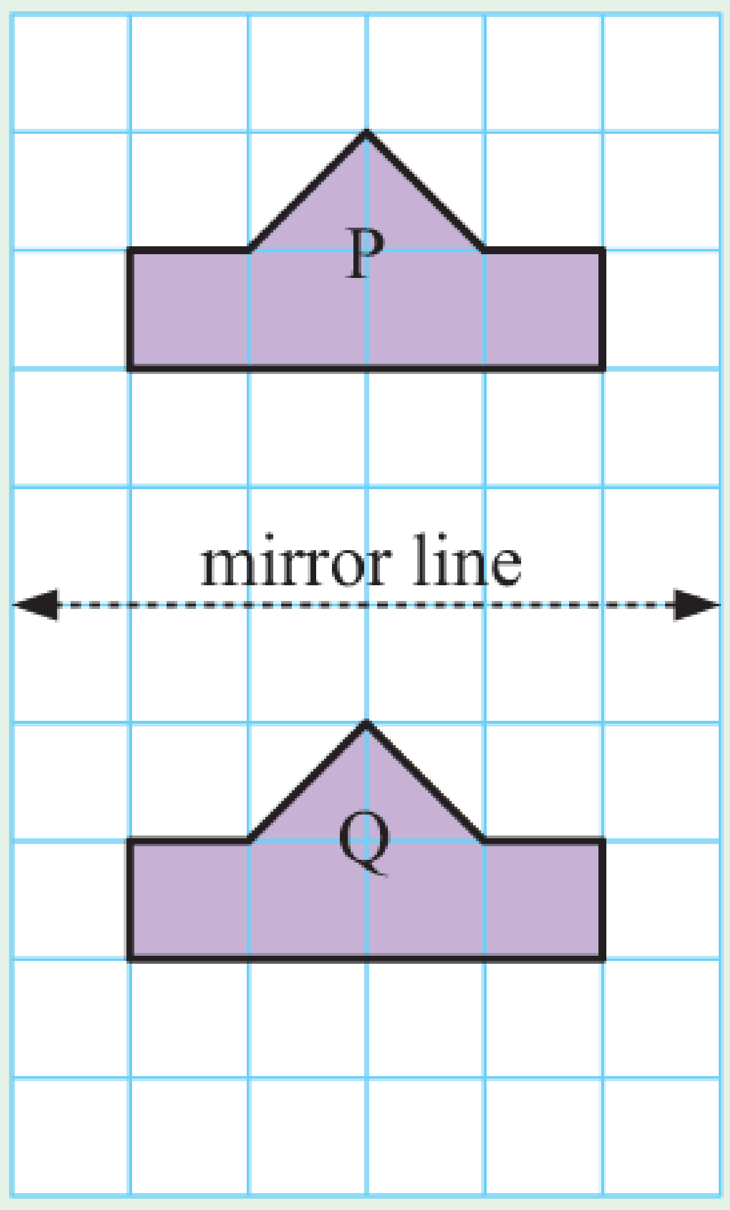


2 Is Q a reflection of P in each diagram? If Q is not a reflection, give a reason for your answer.

a



b

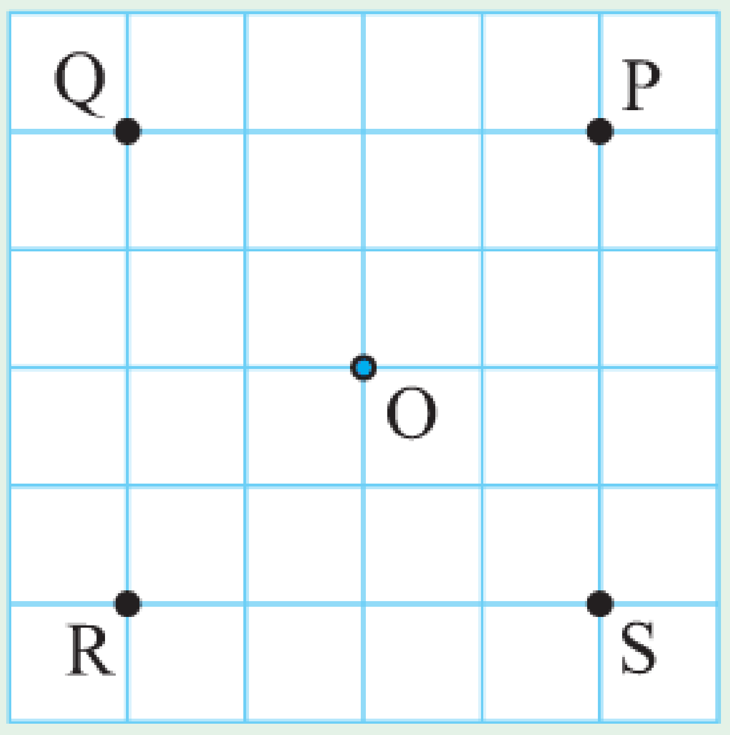


3 Which of these points is the image when P is rotated about O:

a 90° clockwise

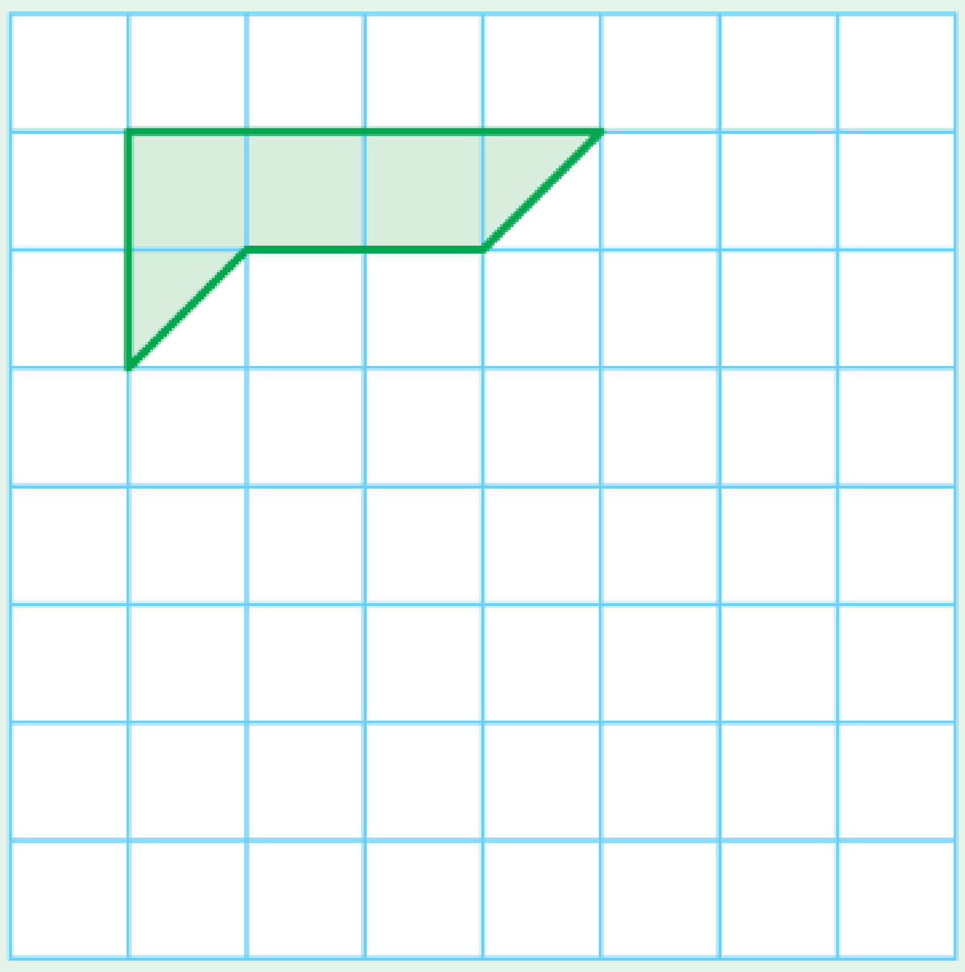
b 90° anticlockwise

c 180° anticlockwise?

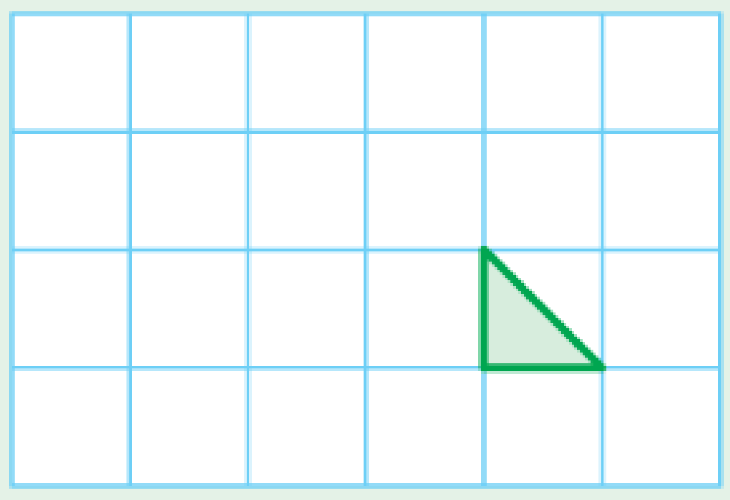


4 For each of these figures, perform the translation given:

a 2 units right, 4 units down



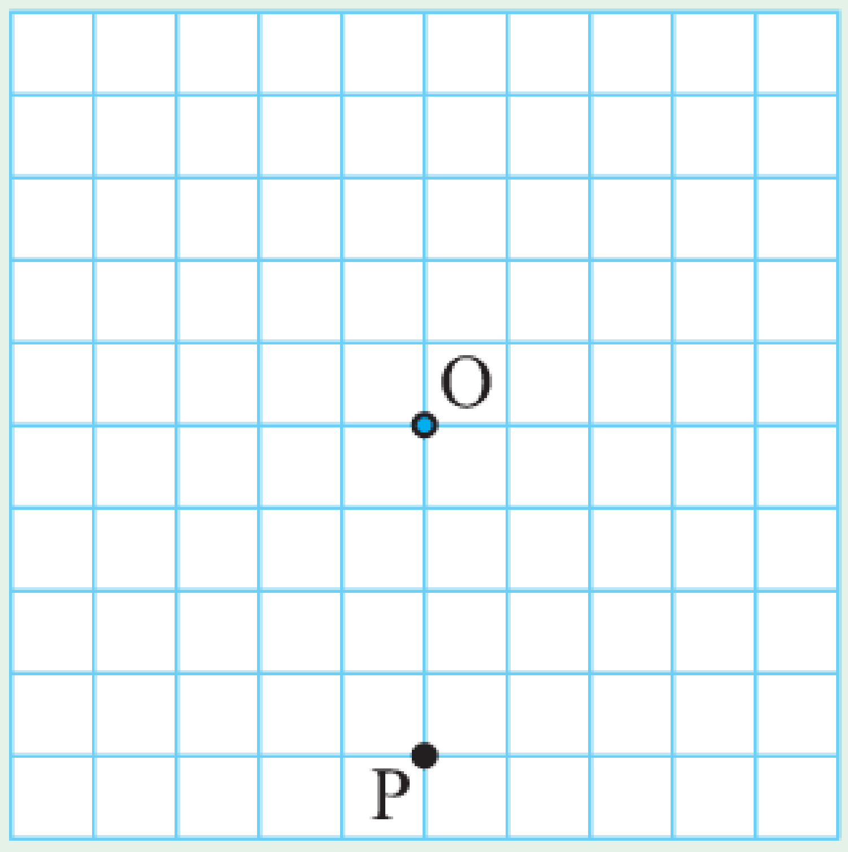
b 3 units left, 1 unit up



PRINTABLE DIAGRAMS

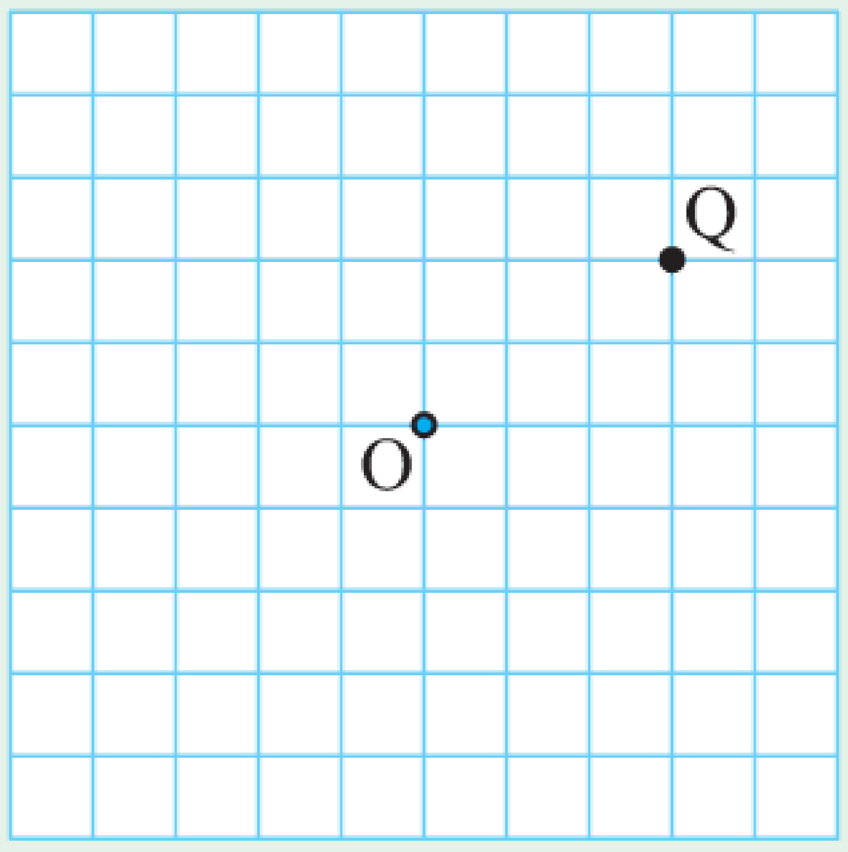
5 Rotate each point about O in the direction given:

a



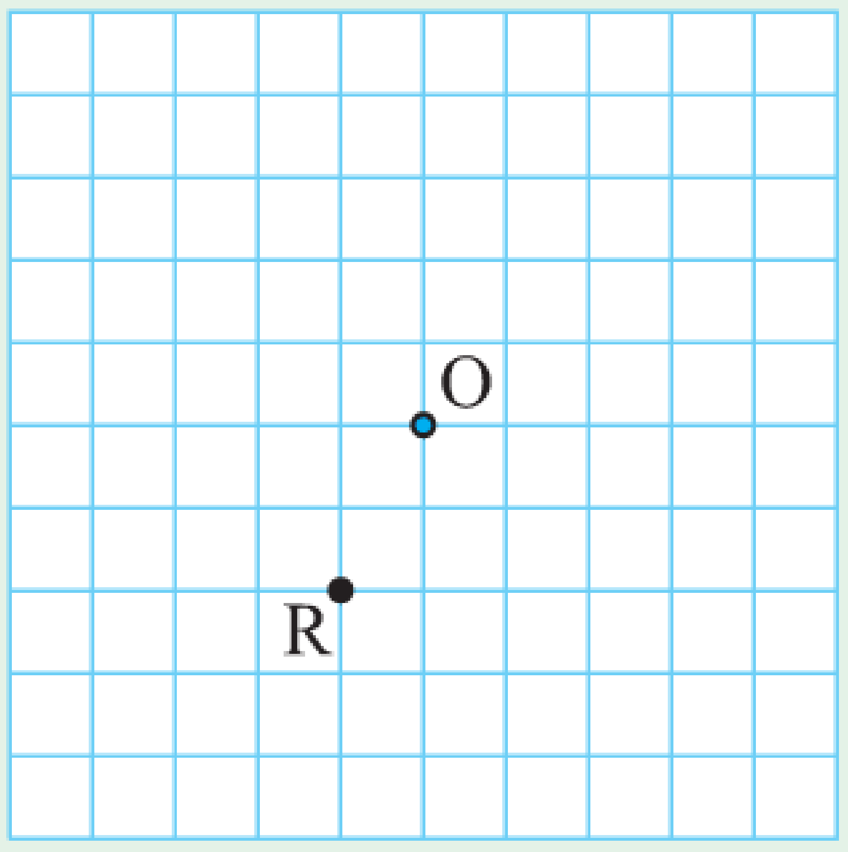
90° clockwise

b



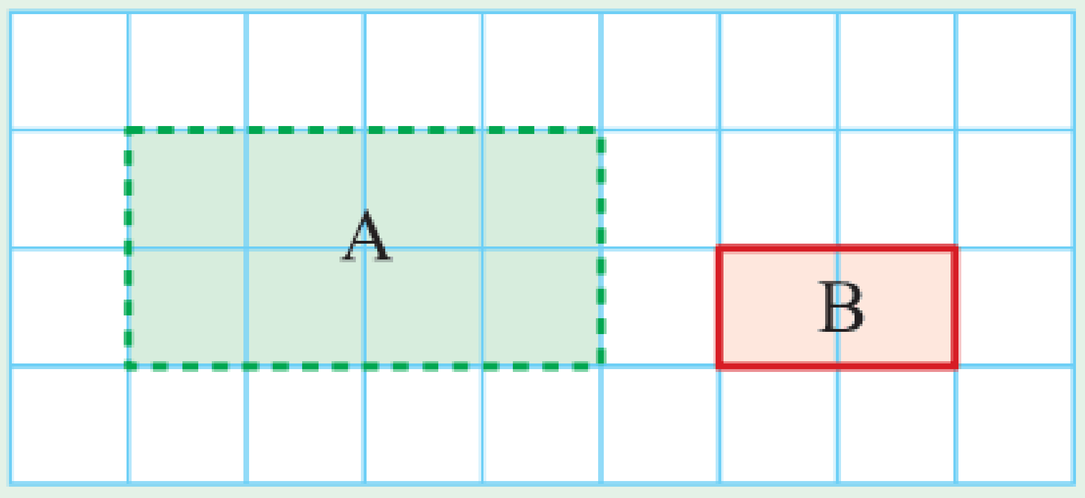
90° anticlockwise

c



180° clockwise

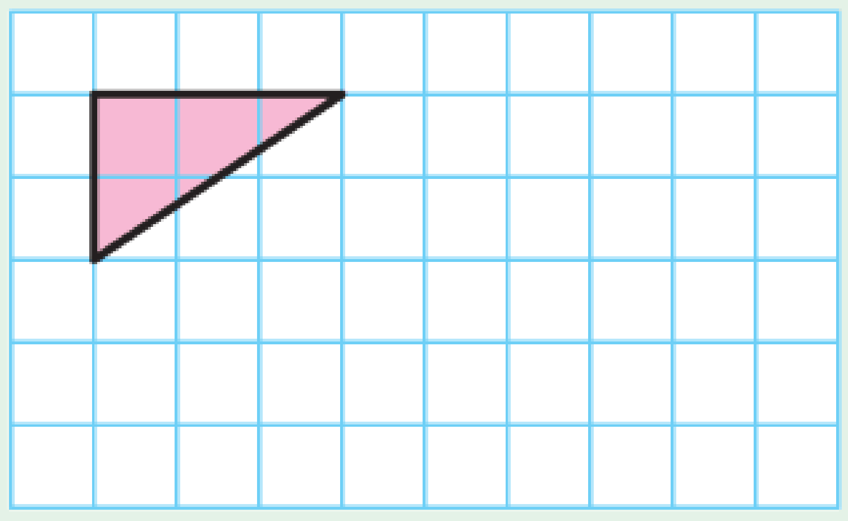
6



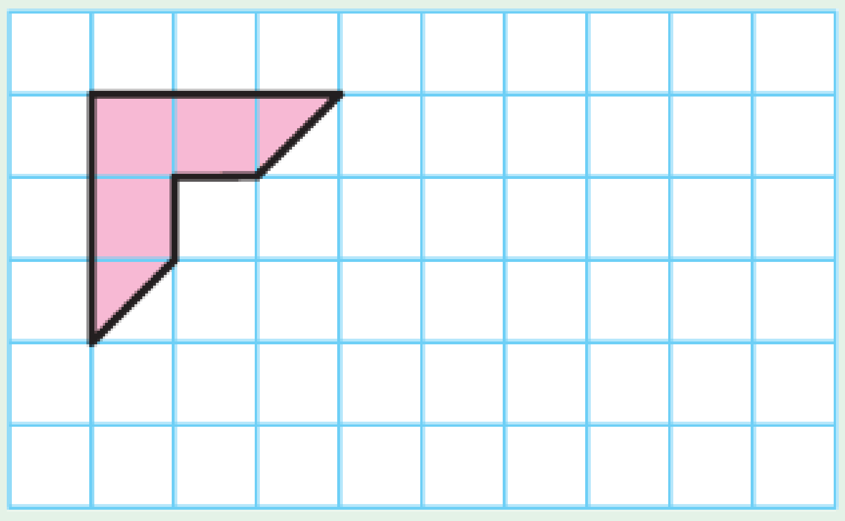
In the diagram alongside, figure A has been reduced to figure B.
Find the scale factor.

7 Draw a tessellation of each shape:

a



b

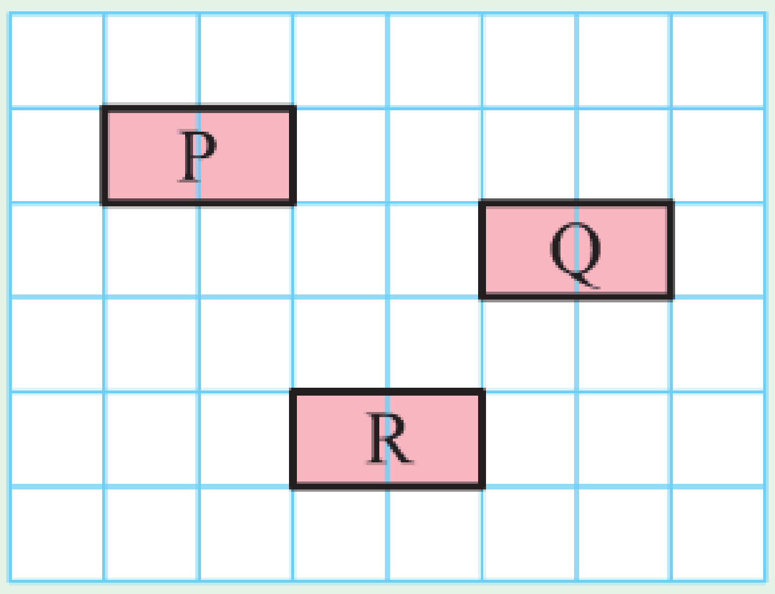


REVIEW SET 16B

1 Describe the translation from:

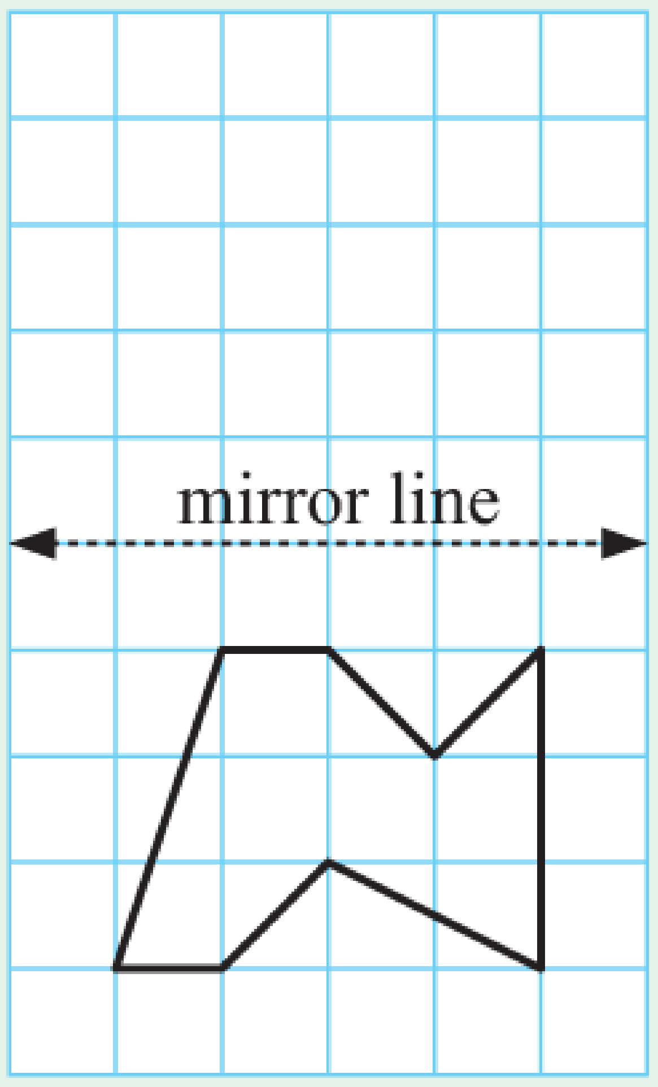
a P to Q

b Q to R.

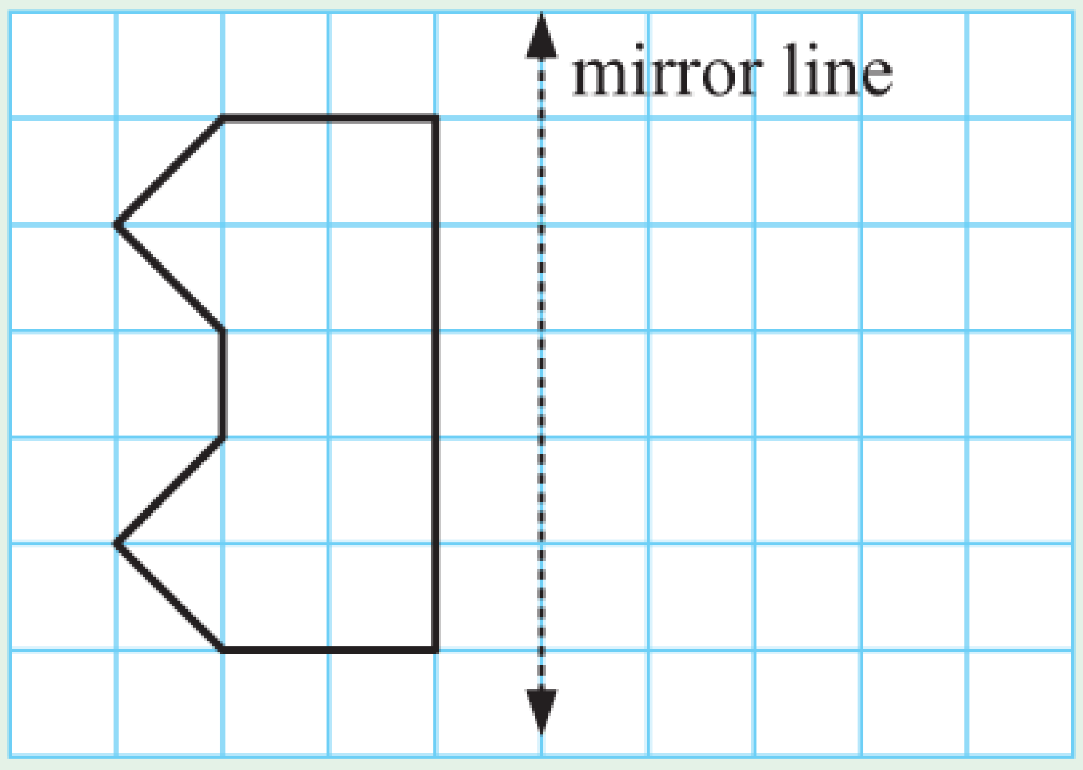


2 Reflect each figure in the mirror line:

a



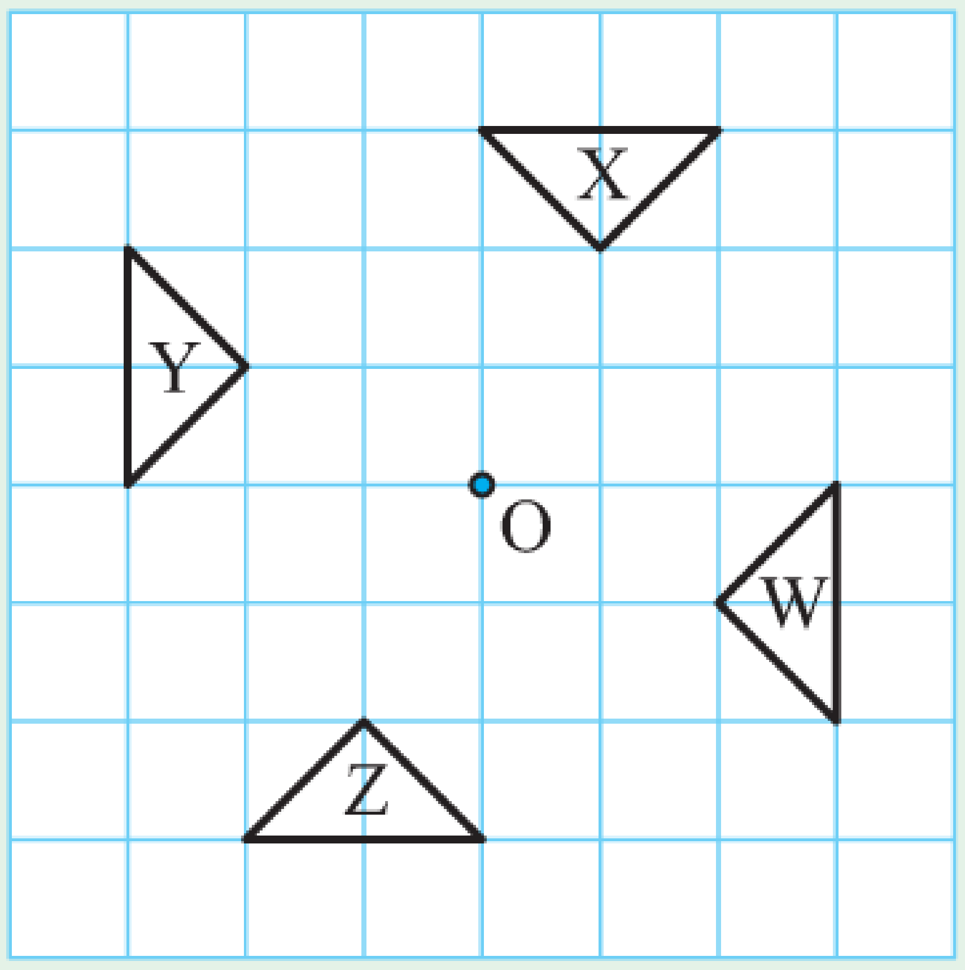
b



PRINTABLE DIAGRAMS

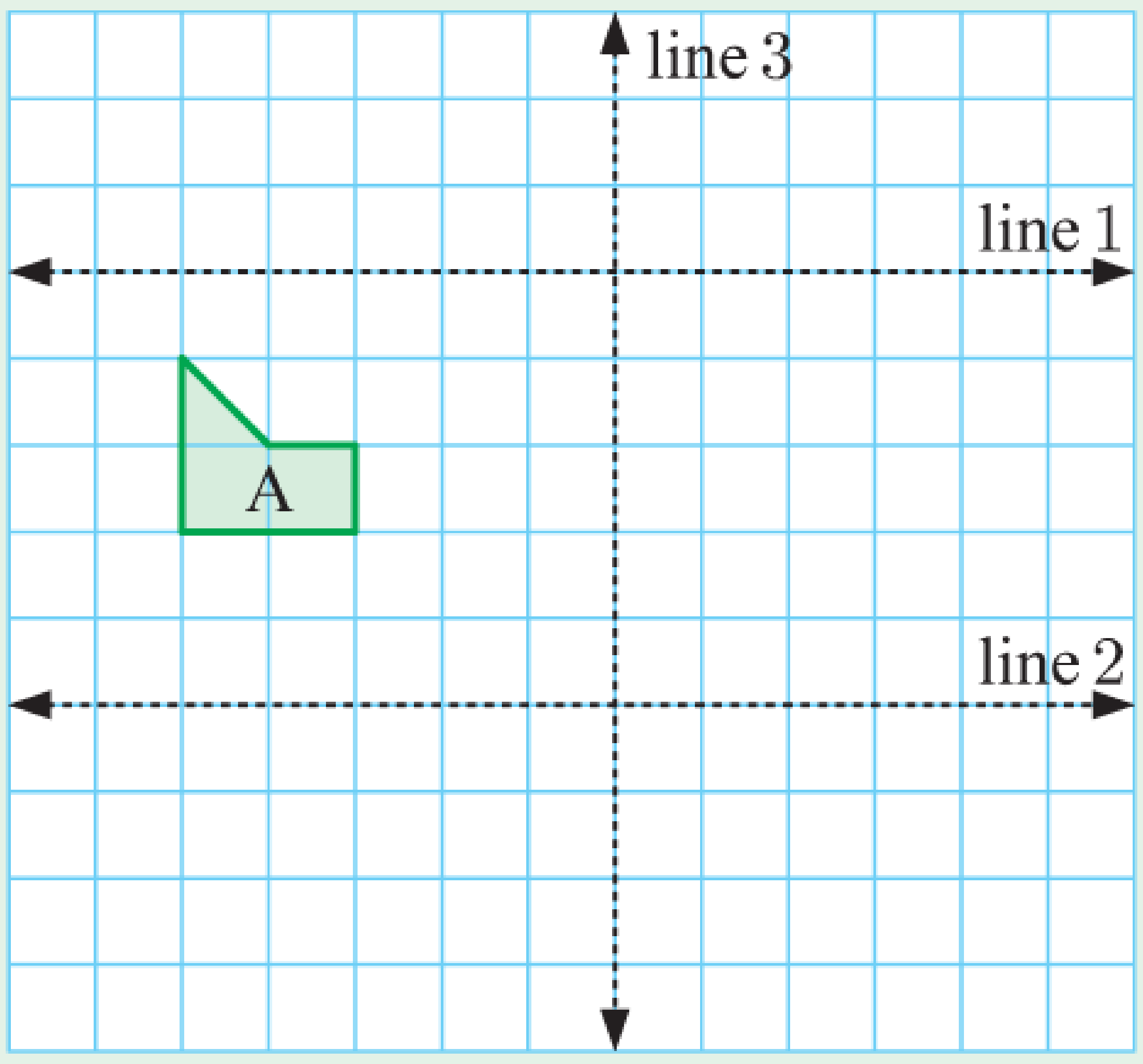
3 If the figure W is rotated about the point O, what turn is needed to rotate W onto:

a X **b** Y **c** Z?



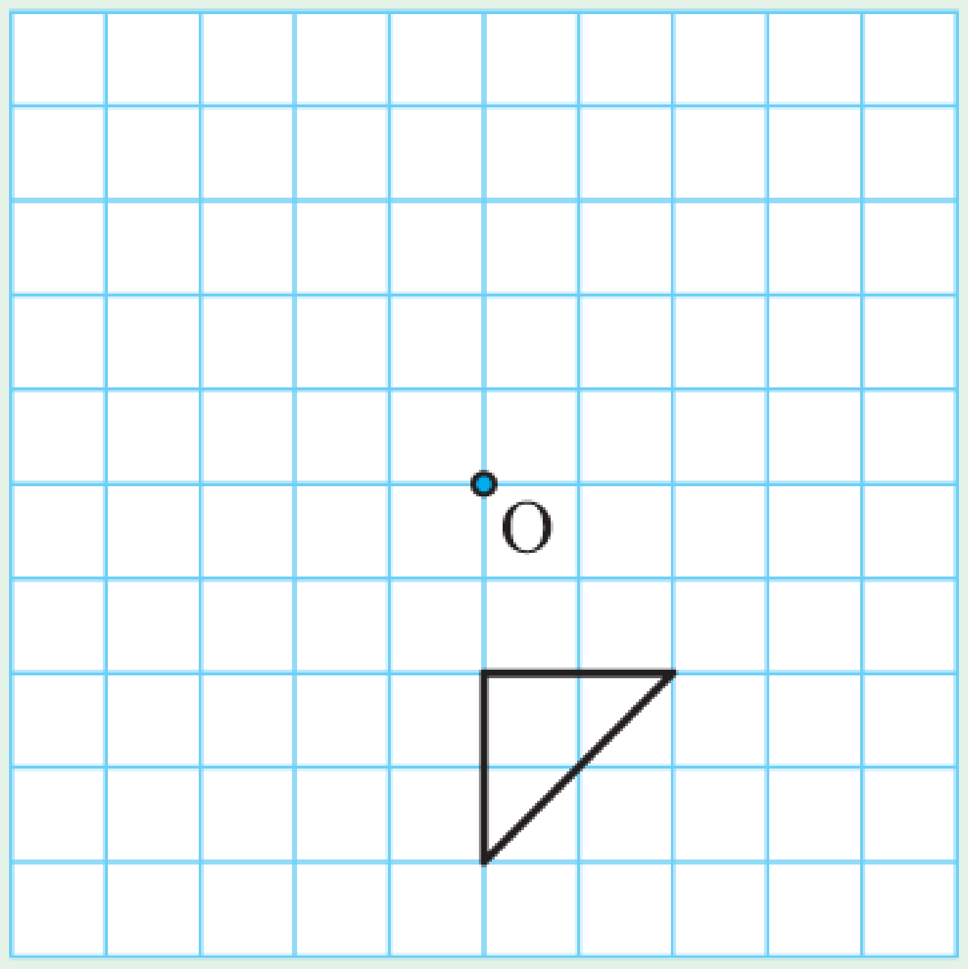
4 On a single diagram, show the image when figure A is reflected in:

a line 1 **b** line 2 **c** line 3.



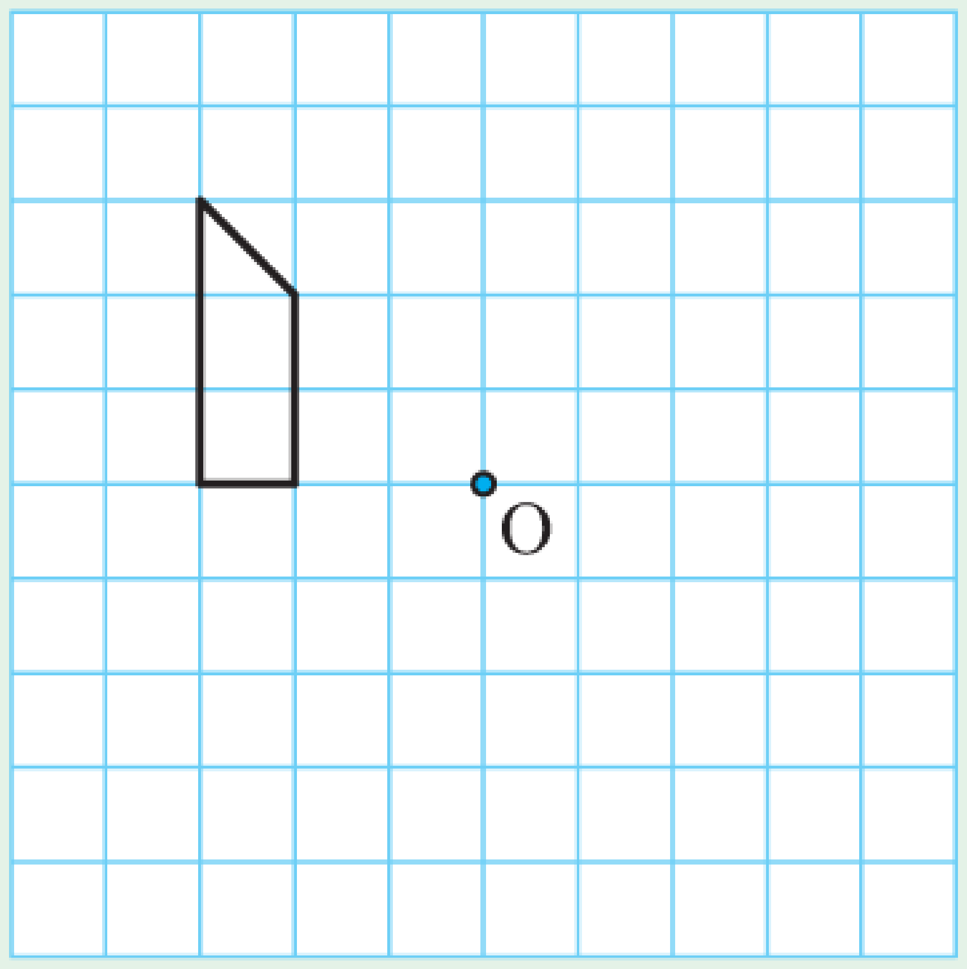
5 Rotate each shape about O in the direction given:

a



90° anticlockwise

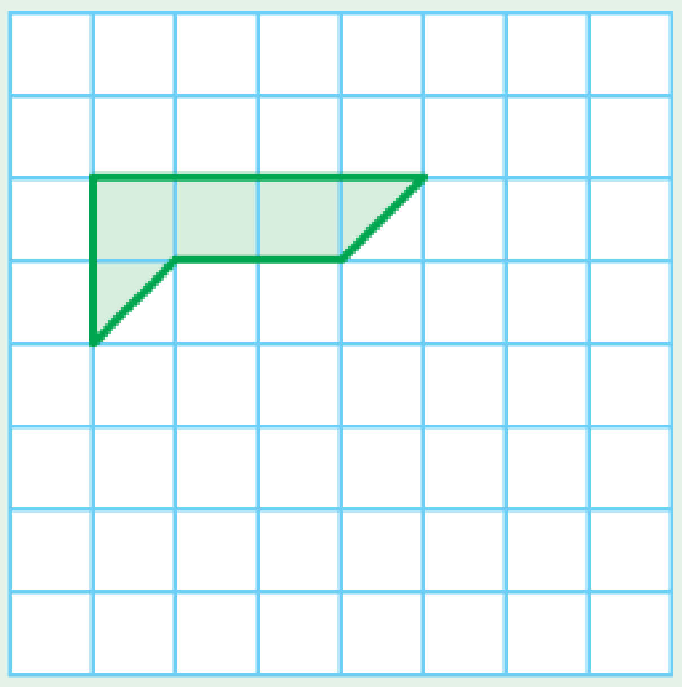
b



180° clockwise

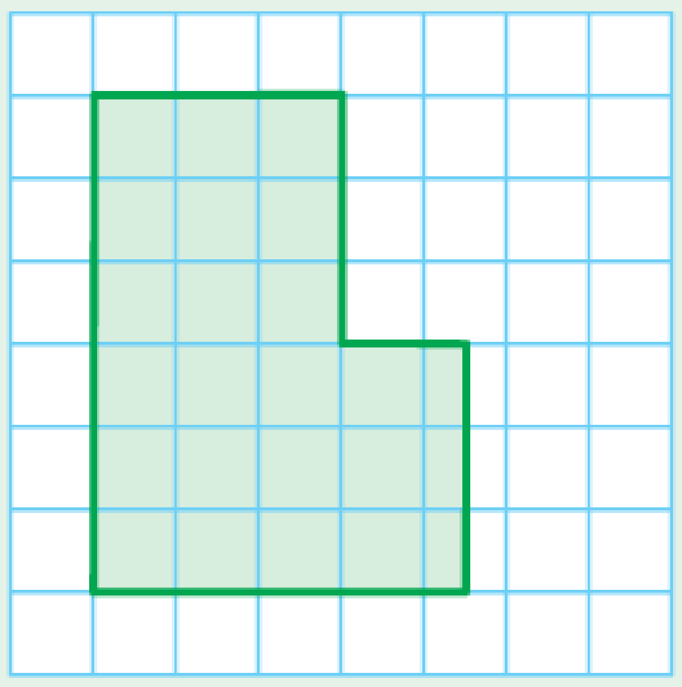
6 Enlarge or reduce each figure by the scale factor given:

a



scale factor 2

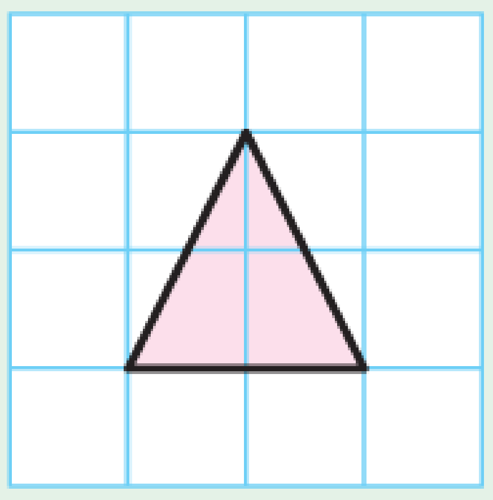
b



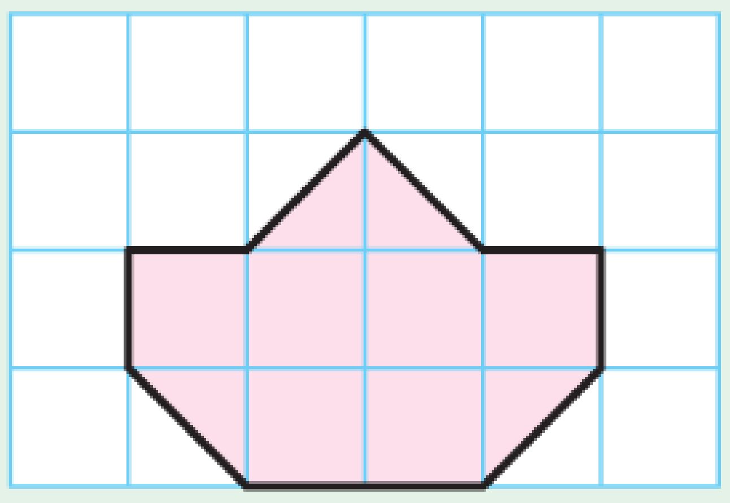
scale factor $\frac{1}{3}$

7 Draw a tessellation of each shape:

a



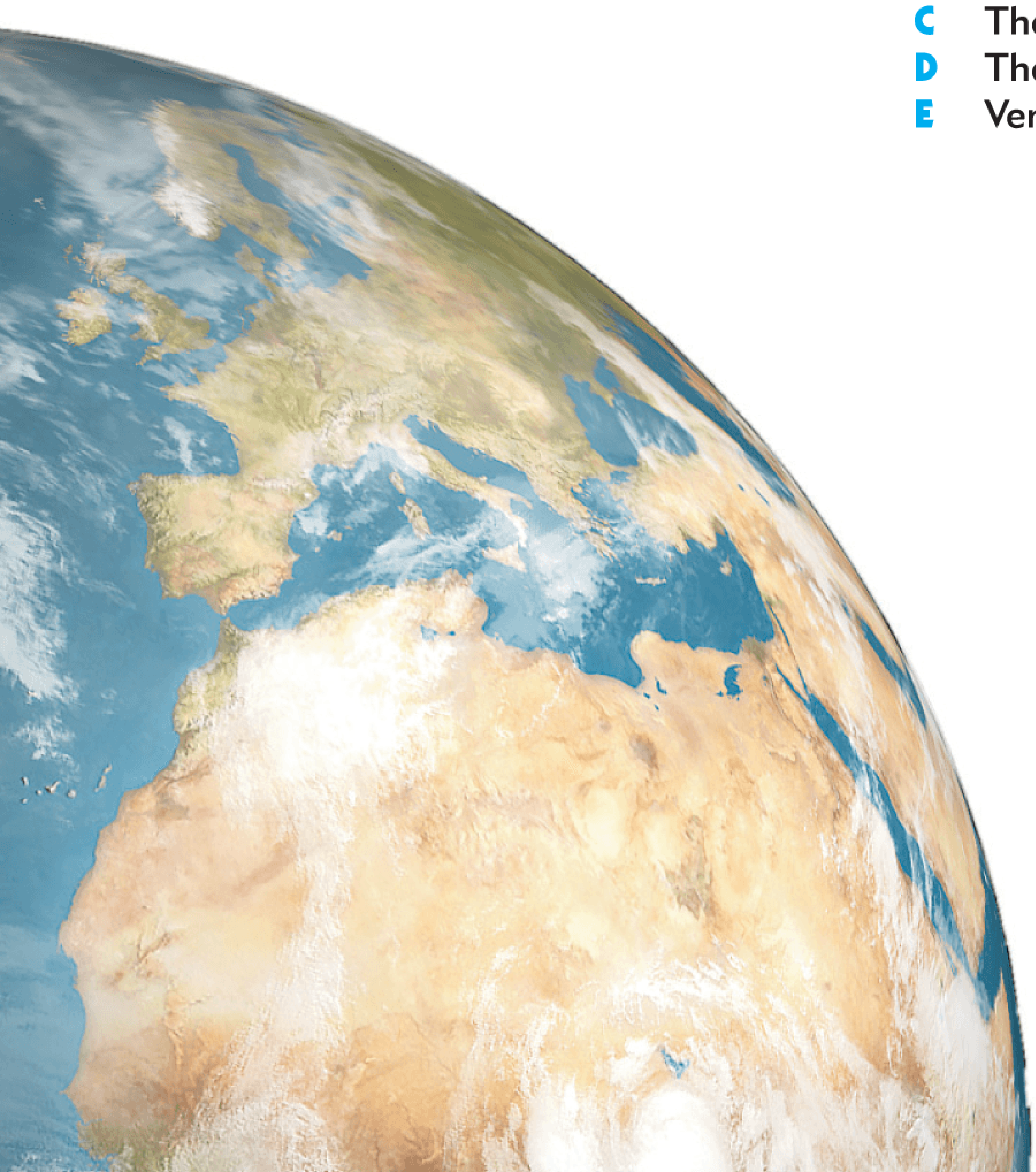
b



Chapter 17

Sets

Contents:



OPENING PROBLEM

The diagram shows two circles A and B which overlap. How many dots are there in:

a circle A

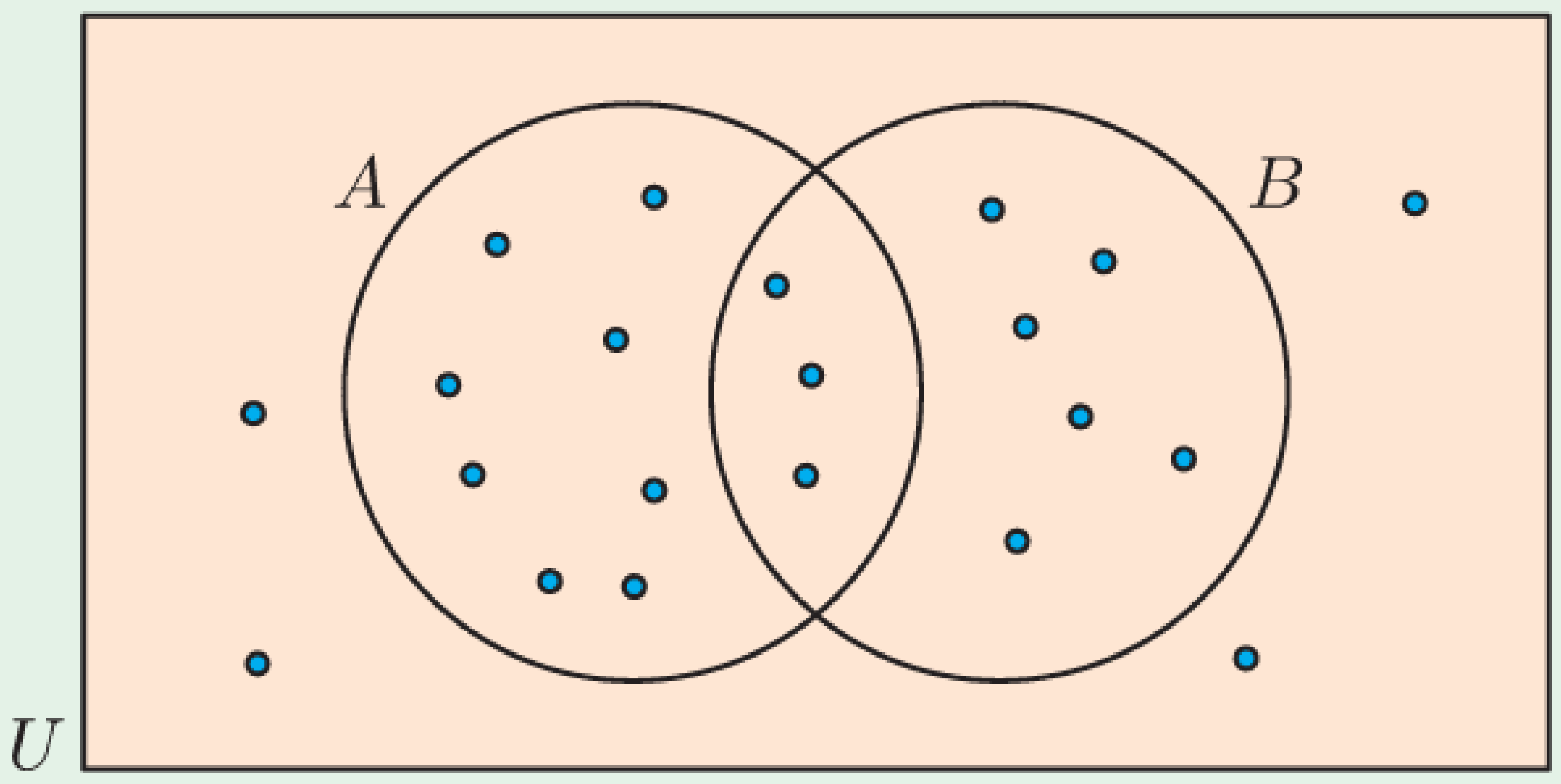
b circle B

c circle A *and* circle B

d circle A *or* circle B

e *neither* circle A *nor* circle B

f exactly one of the two circles?



In mathematics, it is often useful to talk about a group of numbers, or a group of objects. We call these groups **sets**.

A SETS AND THEIR ELEMENTS

A **set** is a group of objects or symbols.

The objects or symbols in the set are called its **elements** or **members**.

We use a capital letter like A to identify a set.

When we list its elements, we write them in curly brackets.

For example:

- The prime numbers less than 13 are 2, 3, 5, 7, and 11.
  We can write these numbers as the set $A = \{2, 3, 5, 7, 11\}$.
- $\{a, e, i, o, u\}$ is the set of all vowels in the English alphabet.
- {blue, grey, hazel, brown, green} is the set of eye colours of students in a class.

We can also use words to help define a set.

For example, the set of all multiples of three which are less than 13 could be written as

$$M = \{\text{multiples of 3 which are} < 13\} \quad \text{or} \quad M = \{3, 6, 9, 12\}.$$

SET NOTATION

$\in$ means	"*is an element of*" or "*is in*"
$\notin$ means	"*is not an element of*" or "*is not in*"
$n(A)$ means	"*the number of elements in set A*"

For example, if $A = \{2, 3, 5, 7, 11\}$ then $5 \in A$, $8 \notin A$, and $n(A) = 5$.

EMPTY SETS

An **empty set** or **null set** is a set which contains no elements.

The set of all prime numbers between 8 and 10 is an example of an empty set.

The symbols $\{ \; \}$ or $\varnothing$ are used to represent an empty set.

Example 1 **Self Tutor**

Let P be the set of prime numbers between 10 and 30, and F be the set of factors of 21.

a List the elements of: **i** P **ii** F.

b **i** Is $23 \in P$? **ii** Is $4 \in F$?

c Find: **i** $n(P)$ **ii** $n(F)$.

a **i** $P = \{11, 13, 17, 19, 23, 29\}$ **ii** $F = \{1, 3, 7, 21\}$

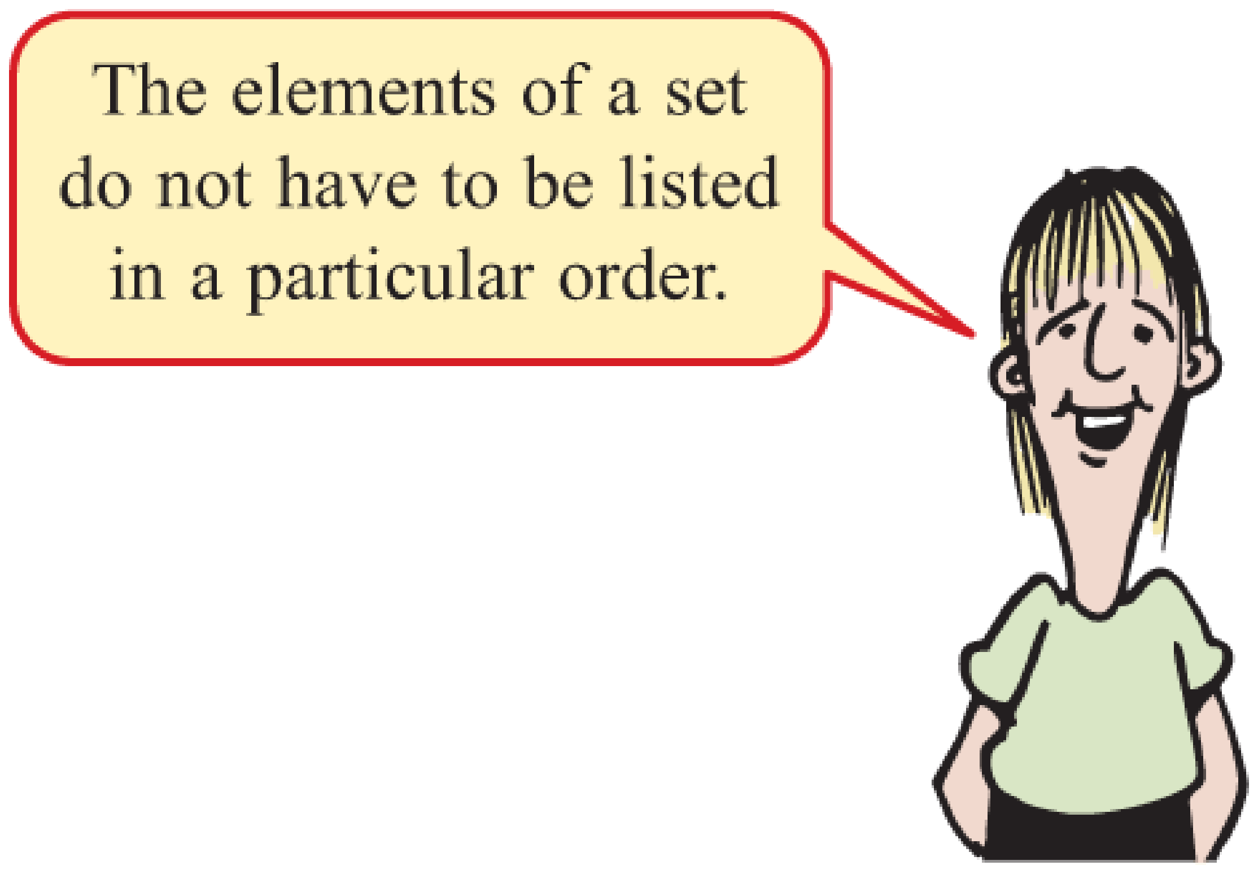
b **i** Yes, $23 \in P$. **ii** No, $4 \notin F$.

c **i** $n(P) = 6$ {there are 6 elements in P} **ii** $n(F) = 4$ {there are 4 elements in F}

EQUAL SETS

Two sets are **equal** if they have exactly the same elements.

For example, if $A = \{2, 3, 8\}$ and $B = \{3, 8, 2\}$, then $A = B$.



EXERCISE 17A

1 Suppose $S = \{2, 5, 6, 9, 11, 12, 15, 17, 18, 20\}$.

a Use $\in$ or $\notin$ to complete these statements:

i 5 .... S **ii** 7 .... S **iii** 13 .... S **iv** 20 .... S

b Find $n(S)$.

2 List the elements of the set of all:

a positive even numbers less than 14

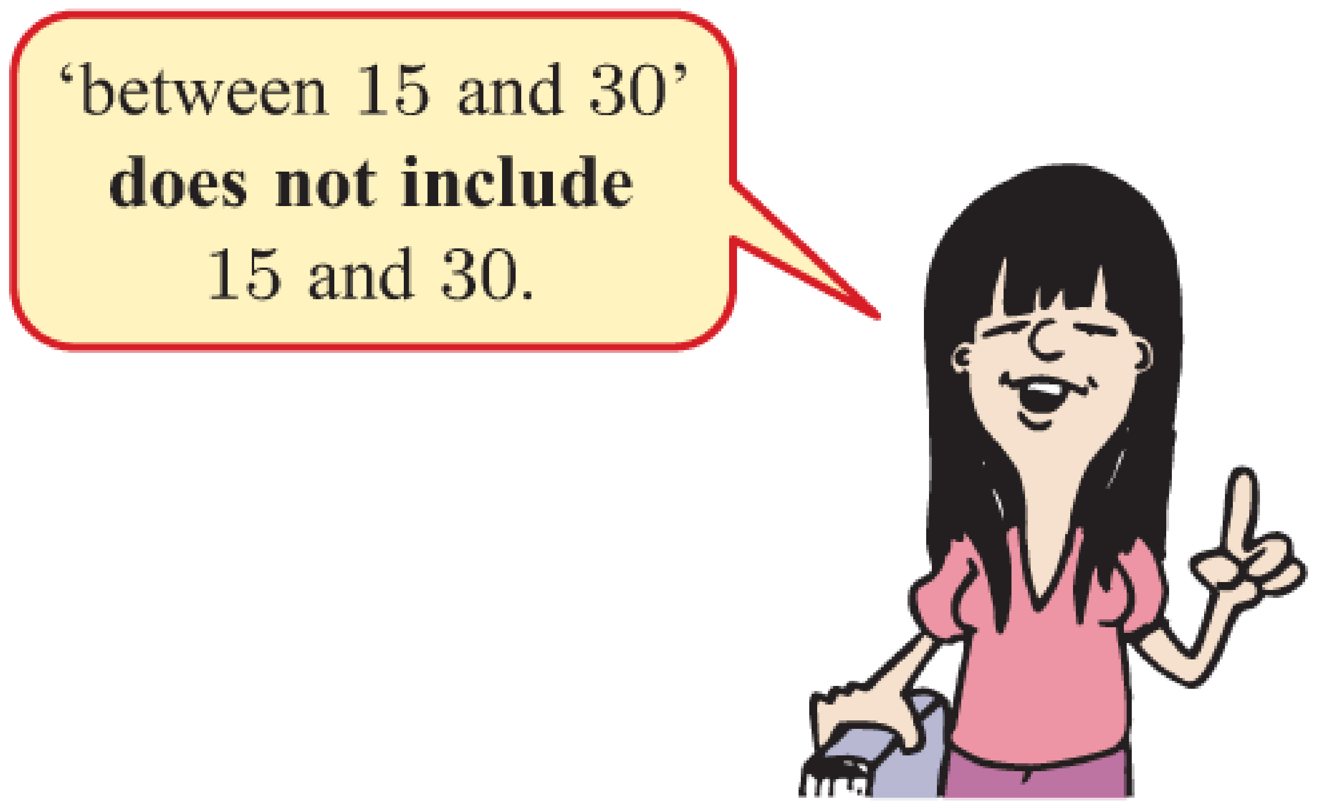
b months of the year

c positive odd numbers between 15 and 30

d prime numbers less than 23

e medal colours at the Olympic games

f factors of 12.



3 For each of the following sets A:

i list the elements of A **ii** find $n(A)$.

a A = {composite numbers less than 18}

b A = {letters between G and T in the English alphabet}

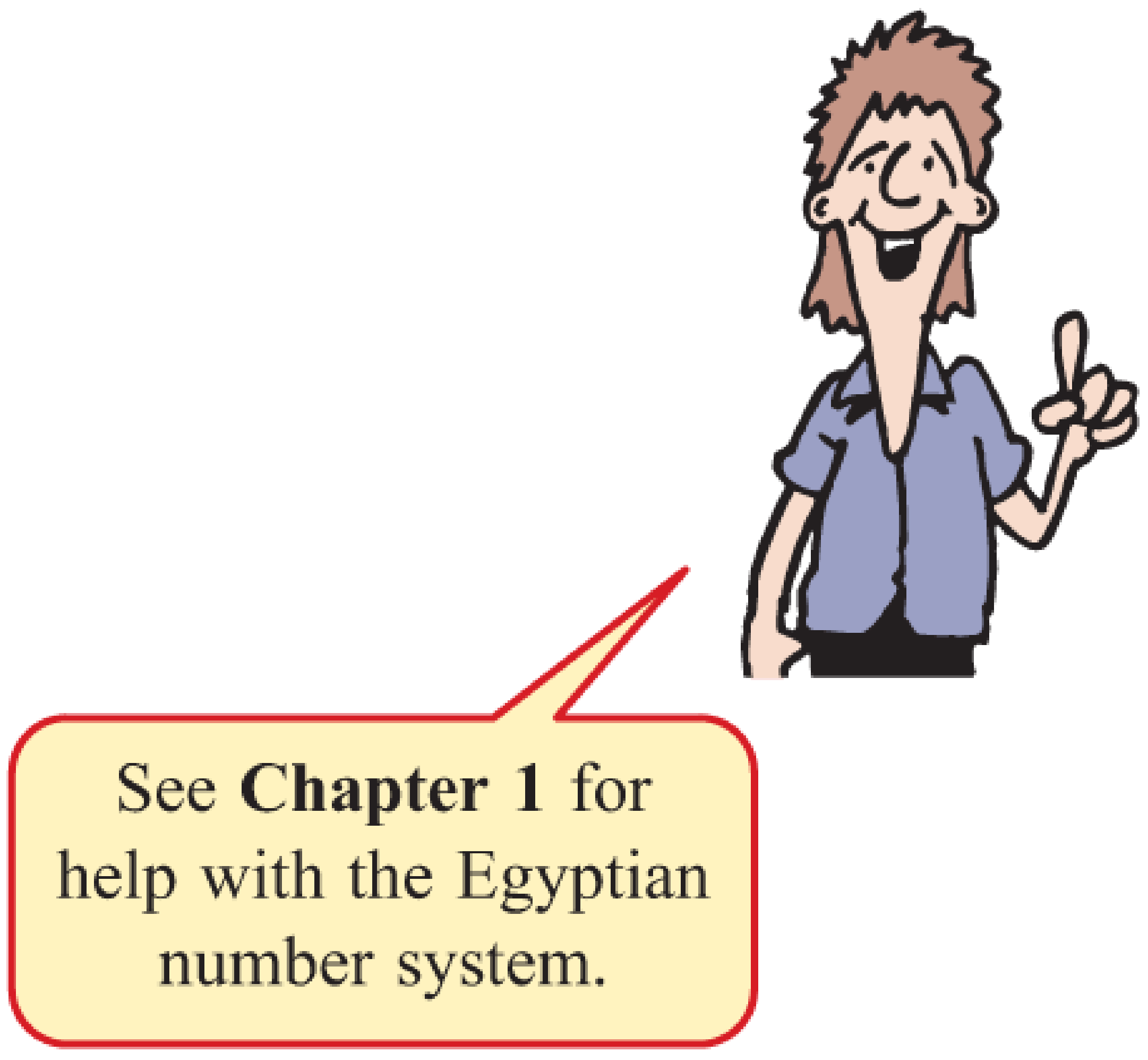
c A = {multiples of 7 between 20 and 60}

d A = {factors of 32}

e A = {symbols used in the Egyptian number system}

f A = {square numbers between 5 and 50}

g A = {prime numbers which are divisible by 6}



4 Let F be the set of factors of 28, and M be the set of multiples of 6 which are less than 50.

a List the elements of: **i** F **ii** M.

b **i** Is $8 \in F$? **ii** Is $24 \in M$?

c Find: **i** $n(F)$ **ii** $n(M)$.

5 Suppose P is the set of all prime numbers between 0 and 15, and Q is the set of all factors of 27.

a List the elements of: **i** P **ii** Q.

b Find $n(P)$.

c Is $12 \in Q$?

d Find the number which is an element of both sets.

6 Which two of these sets are equal?

A {red, white, yellow, green}

B {white, green, blue, red}

C {green, red, white}

D {blue, white, red, black, green}

E {blue, white, red, green}

F {yellow, red, white, blue}

7 Suppose $A = \{3, 5, 1, 4\}$ and $B = \{1, 3, 4, \square\}$ are equal sets. Find $\square$.

8 Let $P =$ {even numbers between 10 and 20} and $Q =$ {multiples of 2 between 11 and 19}. Are P and Q equal sets?

9 True or false?

a If $A = B$, then $n(A) = n(B)$.

b If $n(A) = n(B)$, then $A = B$.

B SUBSETS

Consider the sets $M = \{2, 7, 8\}$ and $N = \{1, 2, 3, 5, 7, 8, 11\}$. Notice that every element of M is also an element of N. We say that M is a *subset* of N.

Set A is a **subset** of set B if every element of A is also an element of B.

If A is a subset of B, we write $A \subseteq B$.

DISCUSSION SUBSETS

- Explain why these statements are true:
  - The empty set $\varnothing$ is a subset of any given set. For example, $\varnothing \subseteq \{1, 2, 3\}$.
  - Any set is a subset of itself. For example, $\{1, 2, 3\} \subseteq \{1, 2, 3\}$.
- Consider two sets A and B.
  - Suppose $A = B$. Is $A \subseteq B$ or $B \subseteq A$?
  - Suppose $A \subseteq B$ and $B \subseteq A$. Is $A = B$?

Example 2 Self Tutor

List all the subsets of $\{1, 2, 3\}$.

The subsets of $\{1, 2, 3\}$ are:

$\varnothing$ {the empty set}
$\{1\}, \{2\}, \{3\}$ {the subsets containing one element}
$\{1, 2\}, \{2, 3\}, \{1, 3\}$ {the subsets containing two elements}
$\{1, 2, 3\}$ {any set is a subset of itself}

There are 8 subsets in all.

EXERCISE 17B

To answer "Is $A \subseteq B$?" we ask ourselves "Are the elements of A also elements of B?"

1 True or false?

- **a** $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}$
- **b** $\{1, 2, 3, 4\} \subseteq \{4, 3, 2, 1\}$
- **c** $\{1, 2, 3, 4\} \subseteq \{2, 3, 4\}$
- **d** $\{2, 4, 6\} \subseteq$ {even numbers}

2 Which of these sets is a subset of {Tim, Melanie, Jack, Lucy, George}?

- **A** {George, Melanie, Dianne}
- **B** {Melanie, Jack, Sophie, Tim}
- **C** {Jack, George, Lucy, Frank, Melanie, Tim}
- **D** {Lucy, Tim, George, Melanie}

3 Let $M = \{3, 4, 5\}$. List the subsets of M which contain exactly:

- **a** one element
- **b** two elements
- **c** three elements.

4 Let $N = \{1, 2, 3, 4\}$. List the subsets of N which contain exactly:

- **a** two elements
- **b** three elements.

5 List all the subsets of {♥, ♠, ♣, ♦}.

6 Let $P =$ {square numbers between 0 and 10}, and $Q =$ {factors of 36}.
Show that P is a subset of Q.

7 Suppose A is the set of all students in your class, and B is the set of all students in your school. Explain why A is a subset of B.

ACTIVITY 1 SETS

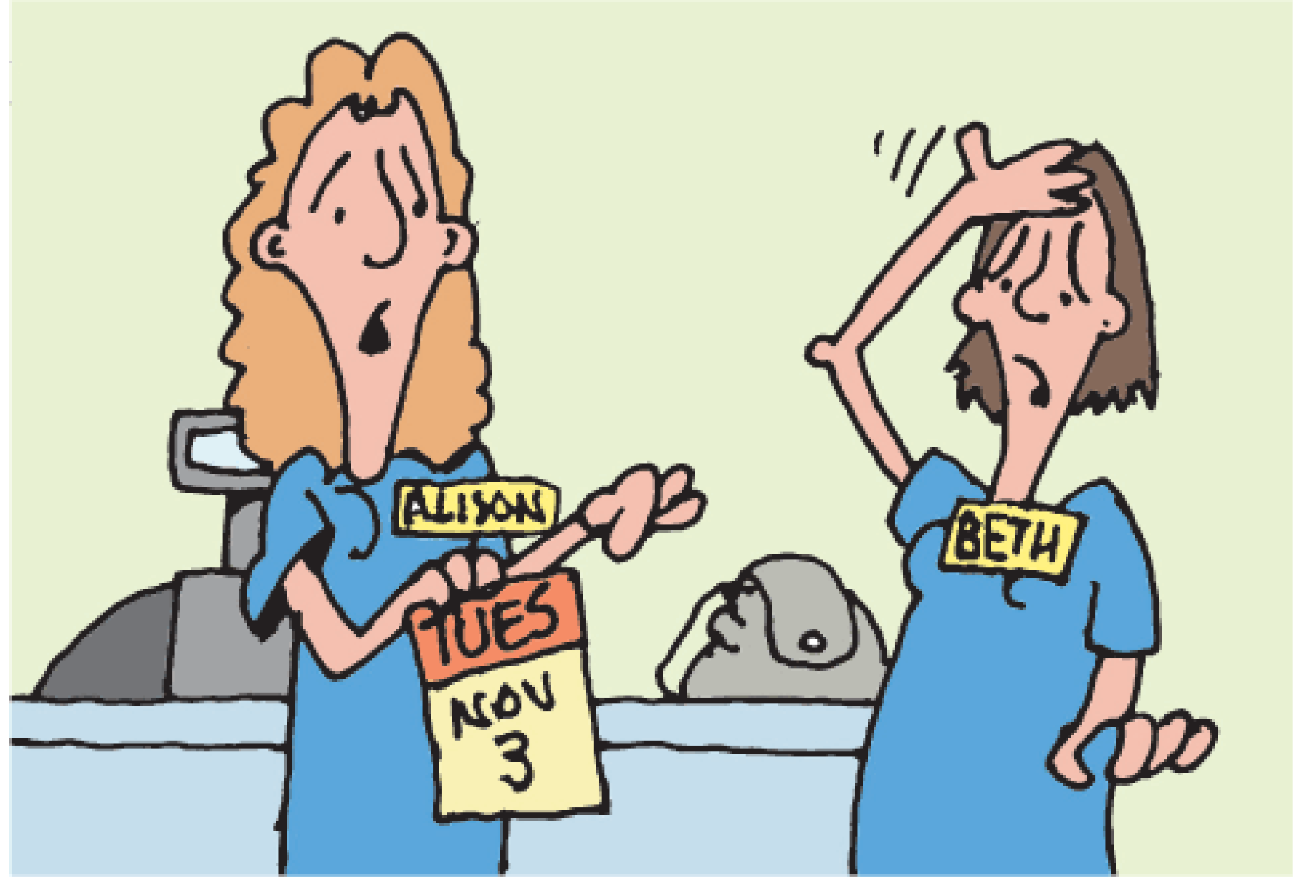
What to do:

1 Write down the set of all:

- **a** sports you play
- **b** colours you like
- **c** pets you own
- **d** countries you have visited
- **e** letters in your name
- **f** languages you can speak.

2 Compare your sets with those of your classmates. Look for any equal sets or subsets.

C THE INTERSECTION OF SETS



Alison and Beth are friends who work in the same shop.

The set of days that Alison will work this week is $A = \{\text{Tuesday, Thursday, Saturday}\}$. The set of days that Beth will work this week is
$B = \{\text{Monday, Thursday, Saturday, Sunday}\}$.

Alison and Beth will be working together on Thursday and Saturday, because these days appear in **both** sets. The set $\{\text{Thursday, Saturday}\}$ is called the **intersection** of A and B.

The **intersection** of two sets A and B is the set of all elements which are common to both set A **and** set B.

The intersection of sets A and B is written $A \cap B$.

Example 3 **Self Tutor**

Suppose $P = \{+, *, \blacktriangleright, \#, @\}$ and $Q = \{-, \times, *, \blacktriangleleft, @, \bullet\}$. Find $P \cap Q$.

$*$ and @ are in both sets P and Q, so $P \cap Q = \{*, @\}$.

Example 4 **Self Tutor**

Let $A = \{\text{factors of } 12\}$ and $B = \{\text{factors of } 18\}$.

a Find $A \cap B$. **b** What does $A \cap B$ represent?

a $A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{1, 2, 3, 6, 9, 18\}$
So, $A \cap B = \{1, 2, 3, 6\}$.

b This set represents the common factors of 12 and 18.

EXERCISE 17C

1 Find $A \cap B$ for:

- **a** $A = \{a, e, i, o, u\}$ and $B = \{a, r, e, s, t\}$
- **b** $A = \{5, 6, 8, 10, 12\}$ and $B = \{3, 6, 9, 10\}$
- **c** $A = \{\div, \times, \blacktriangleright, \#, +, \%\}$ and $B = \{-, @, +, \bullet\}$
- **d** $A = \{7, 13, 23, 28, 42, 56, 64, 75\}$ and $B = \{12, 23, 29, 37, 42, 51, 67, 75, 83\}$

2 Let M be the set of all letters used to write the word APARTMENT, and N be the set of all letters used to write the word PROSPECTOR.

- **a** List the elements of: **i** M **ii** N.
- **b** Find $M \cap N$.
- **c** What does $M \cap N$ represent?

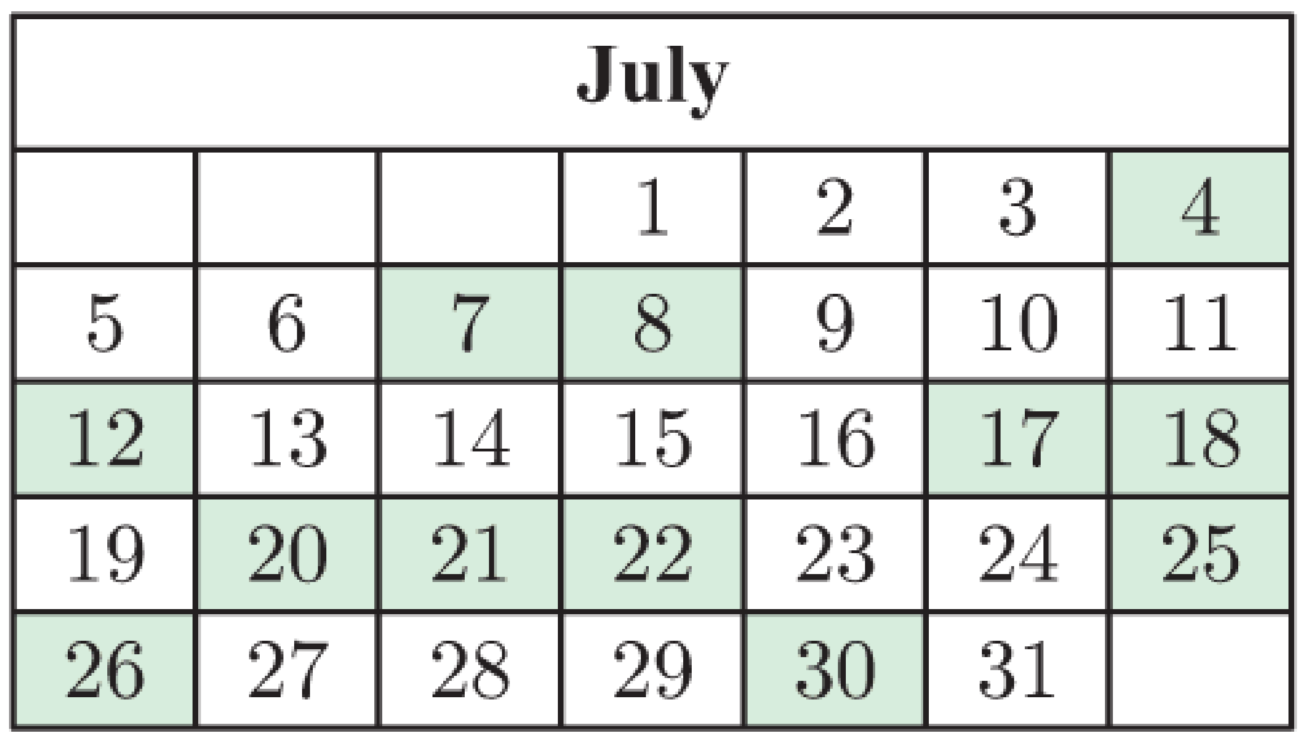
3 Let $P = \{\text{prime numbers less than } 18\}$ and $Q = \{\text{odd numbers between } 0 \text{ and } 18\}$.

- **a** List the elements of: **i** P **ii** Q.
- **b** Find $P \cap Q$.
- **c** Find $n(P \cap Q)$.

4 Let $F = \{\text{factors of } 60\}$ and $G = \{\text{factors of } 80\}$.

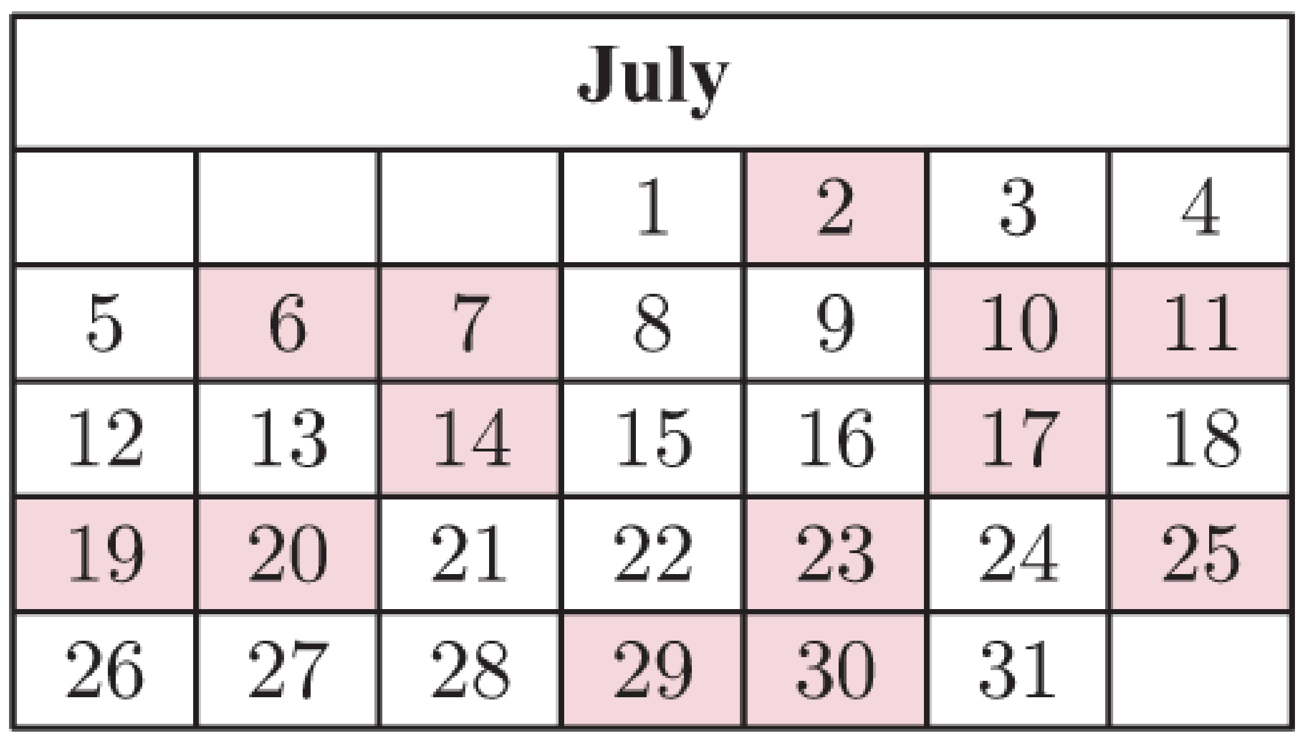
- **a** Find $F \cap G$.
- **b** What does $F \cap G$ represent?

5 Friends Jade and Kurt are arranging a date to meet in July. The shaded dates on the calendars indicate when each friend is available to meet.



July						
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Jade's calendar



July						
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Kurt's calendar

Let J be the set of dates when Jade is available, and K be the set of dates when Kurt is available.

- **a** List the elements of: **i** J **ii** K.
- **b** Find $J \cap K$.
- **c** On how many days during July is it possible for Jade and Kurt to meet?

D THE UNION OF SETS

In the previous Section, we used the sets:

- $A = \{\text{Tuesday, Thursday, Saturday}\}$ as the set of days Alison is working this week
- $B = \{\text{Monday, Thursday, Saturday, Sunday}\}$ as the set of days Beth is working this week.

Their friend Fiona will visit the shop on any day that Alison or Beth is working. We can see that Alison or Beth will be working on Monday, Tuesday, Thursday, Saturday, and Sunday.

The set {Monday, Tuesday, Thursday, Saturday, Sunday} is called the **union** of A and B.

The **union of sets** A and B is the set of all elements which are in A **or** B.

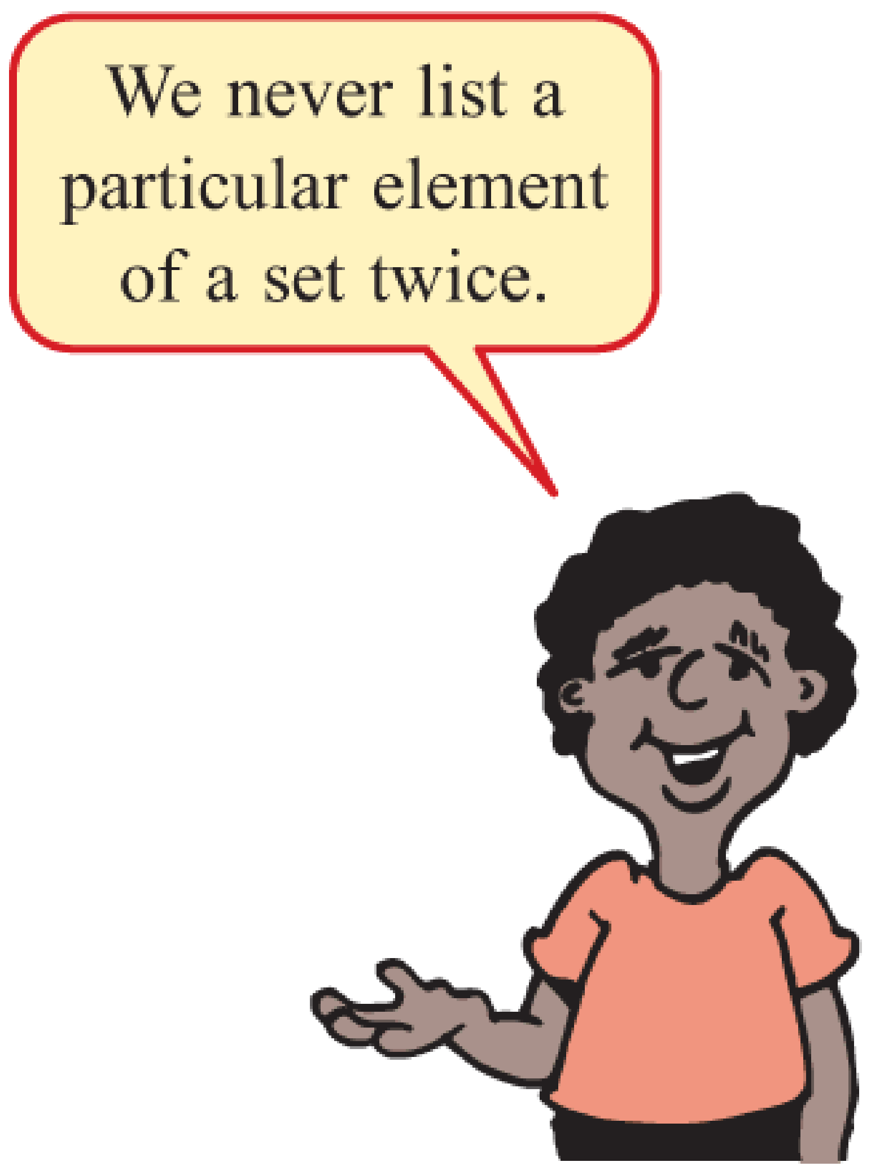
The union of sets A and B is written $A \cup B$.

Elements in both A and B **are included** in the union of A and B.

Example 5 **Self Tutor**

Suppose $P = \{1, 3, 6, 10, 11\}$ and $Q = \{2, 6, 7, 10, 13\}$.
Find $P \cup Q$.

$P \cup Q = \{1, 2, 3, 6, 7, 10, 11, 13\}$



EXERCISE 17D

1 Find $A \cup B$ for:

- **a** $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7, 8\}$
- **b** $A = \{a, c, d, f, m\}$ and $B = \{b, c, e, f, g\}$
- **c** $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7, 9\}$
- **d** $A = \{*, \#, !, \times\}$ and $B = \{\#, :, 5, \times, +\}$

2 On Saturday, the Knights are playing against the Lions. Let K be the set of colours in the Knights uniform, and L be the set of colours in the Lions uniform.

- **a** List the elements of: **i** K **ii** L.
- **b** Find $K \cup L$.

3 Let $A = \{\text{even numbers between 1 and 20}\}$ and $B = \{\text{multiples of 3 between 1 and 20}\}$.

- **a** List the elements of: **i** A **ii** B.
- **b** Find $A \cup B$.
- **c** Find $n(A \cup B)$.

4 Twins Mark and Stephen are organising a joint birthday party. They each make a list of people they want to invite to the party.
Let M be the set of people Mark wants to invite, and S be the set of people Stephen wants to invite.

- **a** List the elements of: **i** M **ii** S.
- **b** Find $M \cap S$. What does this set represent?
- **c** Find $M \cup S$. What does this set represent?
- **d** How many guests will be invited to Mark and Stephen's party?

MARK
Michael
Bradley
Craig
Sally
Alistair
Kylie
Emma
Nigel

STEPHEN
William
Nigel
Kylie
David
Sam
Craig
Luke

DISCUSSION

Suppose set A has 6 elements, and set B has 10 elements.

What can you say about the number of elements in:

- $A \cap B$
- $A \cup B$?

E VENN DIAGRAMS

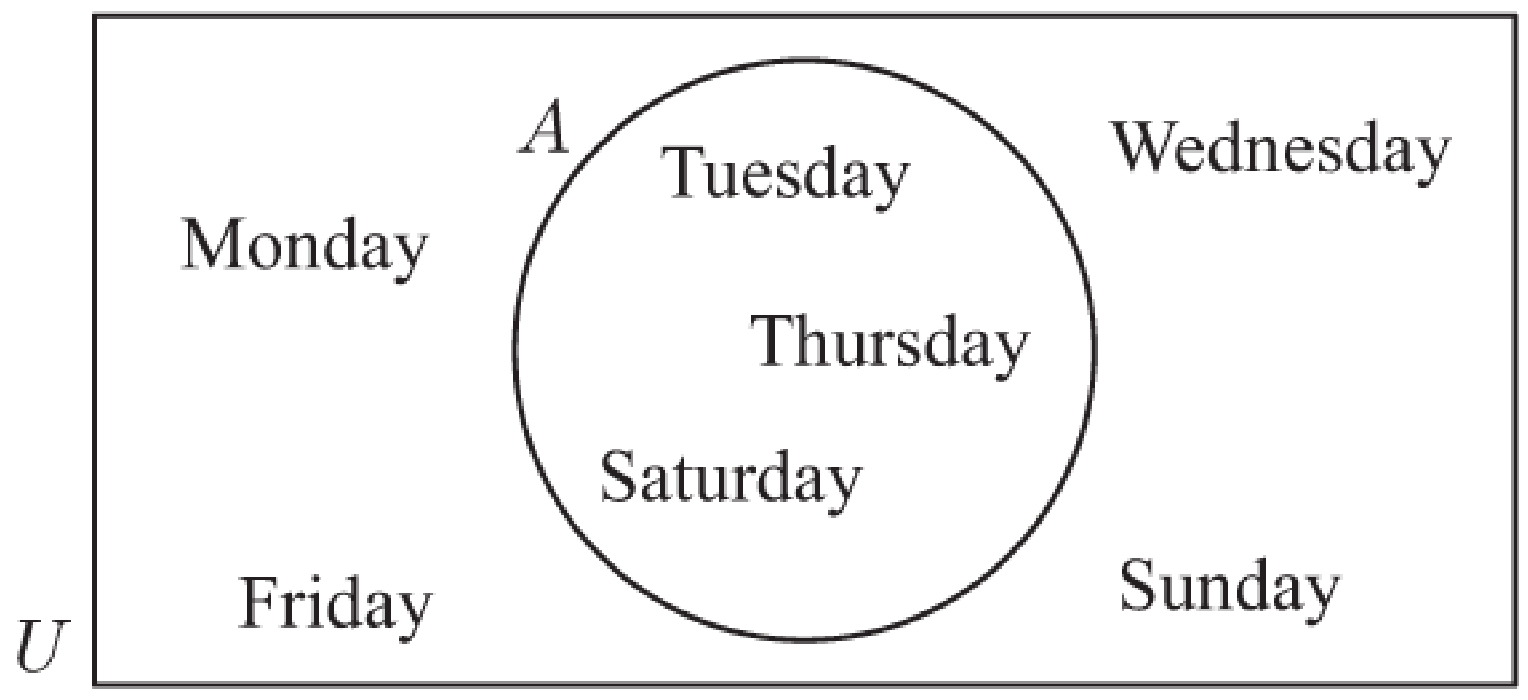
In any problem involving sets:

> The **universal set** U is the set which contains all of the elements we are considering.

For example, when we are considering the days of the week, the universal set is
$U = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}$.
The set $A = \{\text{Tuesday, Thursday, Saturday}\}$ is a subset of the universal set.

> A **Venn diagram** shows the relationship between sets. The universal set is represented by a rectangle, and the other sets are represented by circles within it.

For the sets U and A described above, the Venn diagram is shown alongside.



ACTIVITY 2 VENN DIAGRAMS

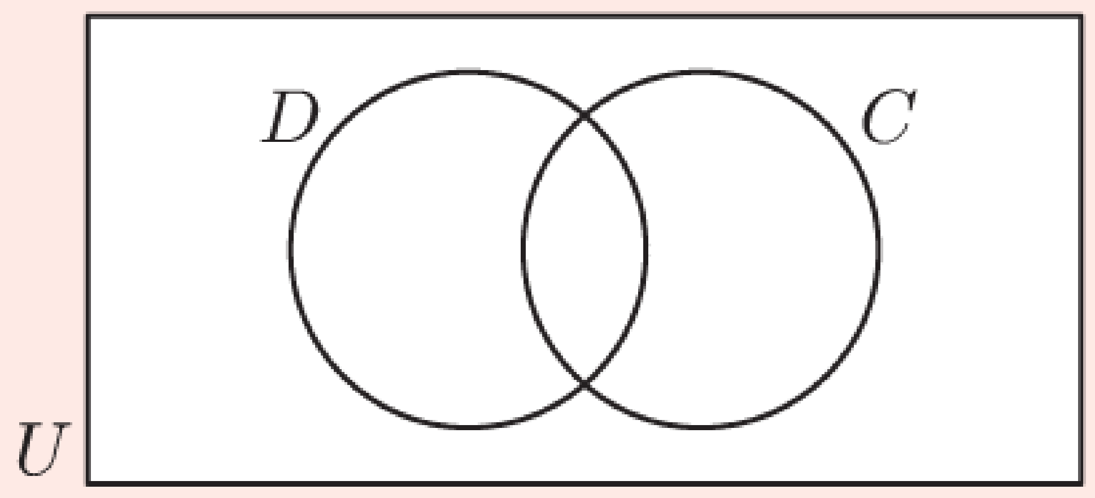
Click on the icon to load a demonstration of a Venn diagram.

DEMO

20 people are asked whether they own a cat or a dog, or both. The information is sorted onto a Venn diagram which consists of two overlapping circles within a rectangle.

Circle D represents the people who own a dog. Circle C represents the people who own a cat.

The circles overlap because some people own both a cat *and* a dog.



What to do:

1. Start the demonstration. Press continue to see how each person's response is added to the diagram.
2. Identify the region of the Venn diagram which represents people who own:
   - a dog
   - *neither* a dog *nor* a cat
   - a dog *or* a cat.
   - a cat
   - a dog *but not* a cat
   - a dog *and* a cat
   - a cat *but not* a dog

Example 6 **Self Tutor**

Consider the single digit numbers 0, 1, 2, 3, 4, ..., 9.

Let $P = \{\text{prime numbers less than } 9\}$ and $F = \{\text{factors of } 6\}$.

a State the universal set.

b List the elements of P and F.

c Find: **i** $P \cap F$ **ii** $P \cup F$

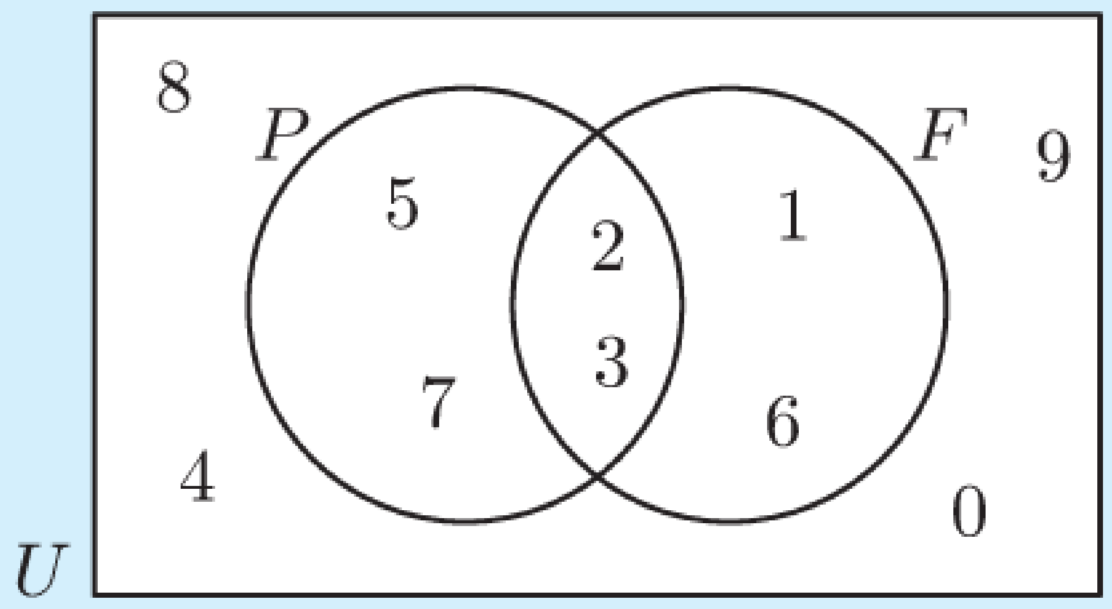
d Illustrate the sets on a Venn diagram.

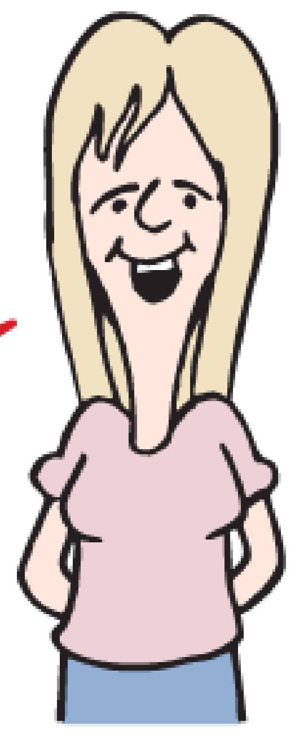
a $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b $P = \{2, 3, 5, 7\}$ and $F = \{1, 2, 3, 6\}$

c **i** $P \cap F = \{2, 3\}$ **ii** $P \cup F = \{1, 2, 3, 5, 6, 7\}$

d

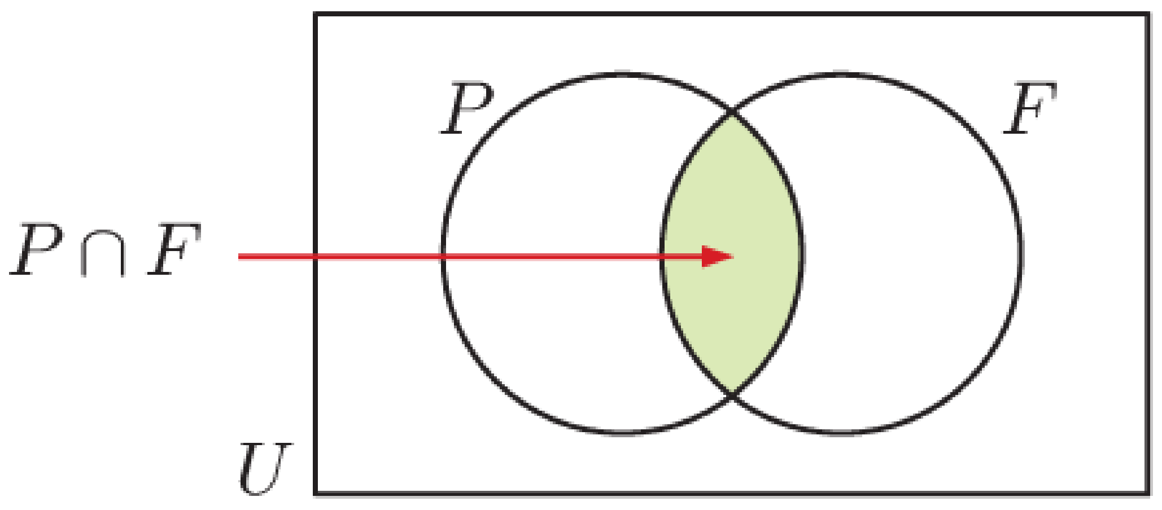




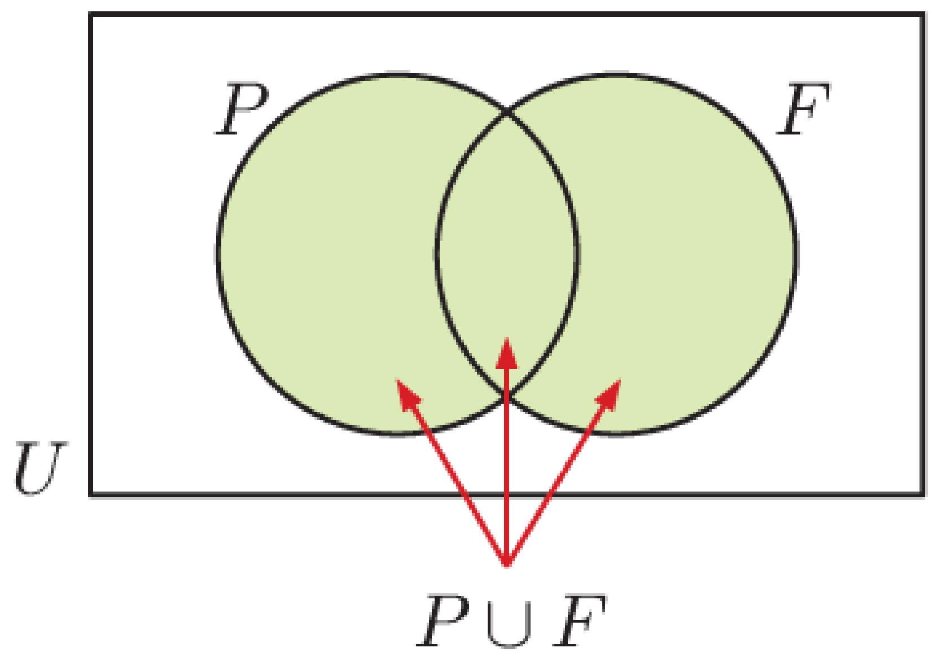
It is a good idea to put elements in the intersection on the Venn diagram first.

From **Example 6**, we can see that:

- The **intersection** of two sets is represented by the region where the two circles overlap.



- The **union** of the two sets is shown in the Venn diagram alongside.
  An element in the union $P \cup F$ could be in P or F or in both sets. It belongs to *at least one* of the sets.



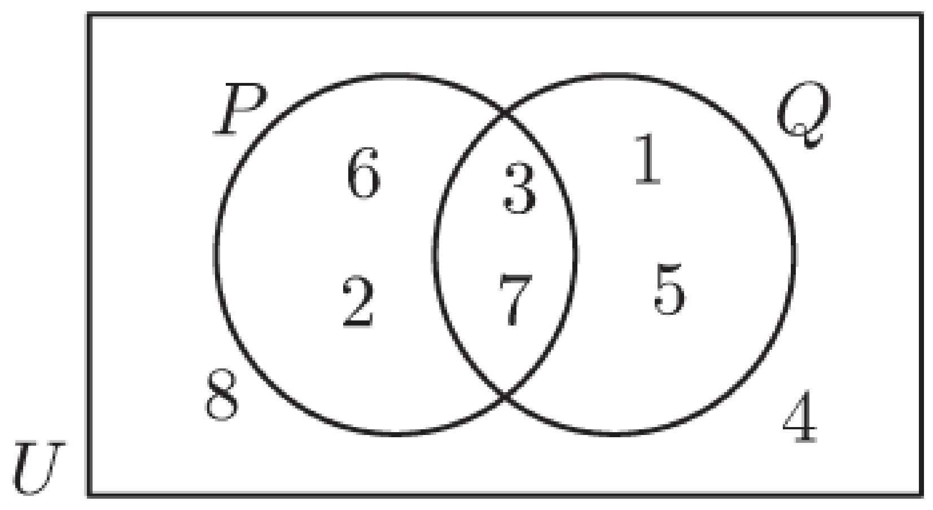
EXERCISE 17E.1

1 Consider the sets $U = \{0, 1, 2, 3, ..., 9\}$, $A = \{3, 5, 6, 7, 9\}$, and $B = \{1, 4, 6, 8, 9\}$.

  a Find: **i** $A \cap B$ **ii** $A \cup B$

  b Illustrate the sets on a Venn diagram.

2



Using this Venn diagram, list the elements of:

a P **b** Q **c** U

d $P \cap Q$ **e** $P \cup Q$.

3 Suppose $U = \{a, b, c, d, e, f, g, h, i, j, k\}$, $A = \{b, c, e, g, i, k\}$, and $B = \{a, b, d, e, f, g, k\}$.

- **a** List the elements of: **i** $A \cap B$ **ii** $A \cup B$.
- **b** Illustrate the sets on a Venn diagram.

4 $U = \{1, 2, 3, 4, 5, 6, ..., 17\}$, $M = \{\text{multiples of 3 which are less than 16}\}$, and $F = \{\text{factors of 15}\}$.

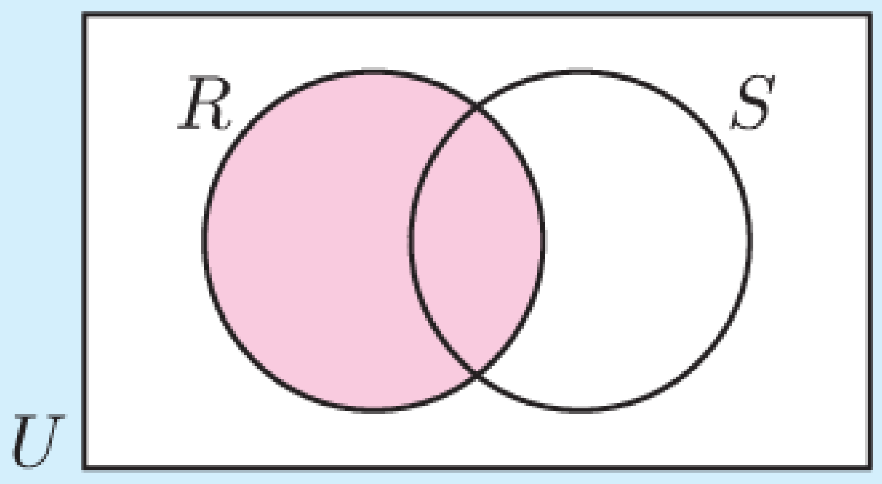
- **a** List the elements of: **i** M **ii** F
- **b** Find: **i** $M \cap F$ **ii** $M \cup F$ **iii** $n(M \cap F)$ **iv** $n(M \cup F)$
- **c** Illustrate the sets on a Venn diagram.

Example 7

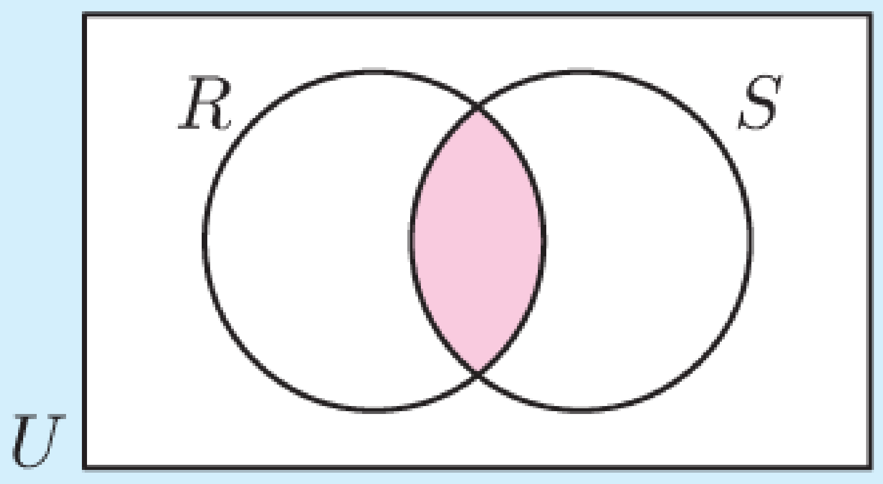
Self Tutor

Describe, in words, the shaded region:

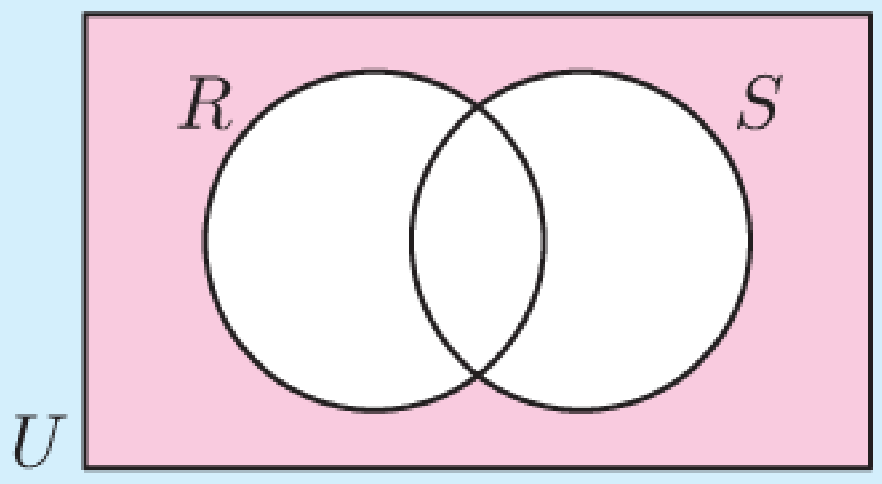
a



b



c



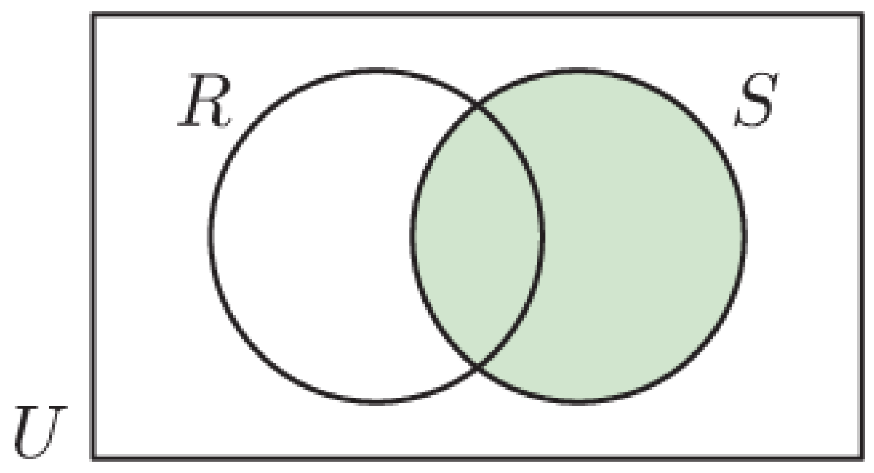
a The elements in set R.

b The elements in both set R and set S.

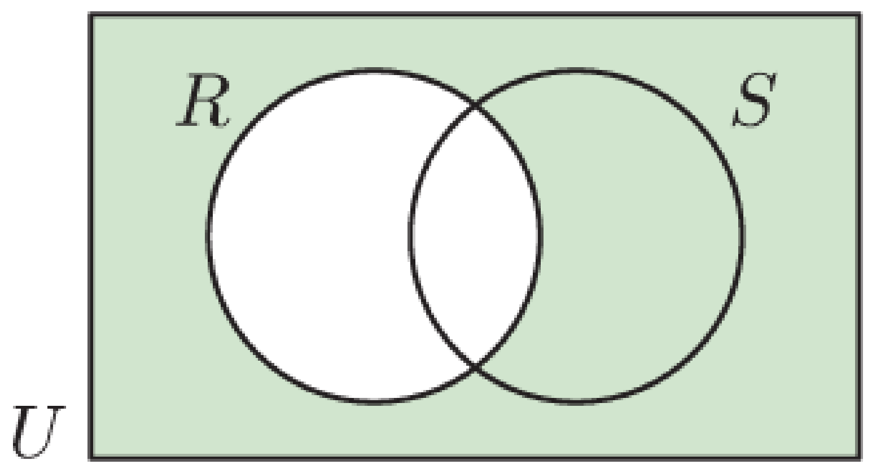
c The elements in neither set R nor set S.

5 Describe, in words, the shaded region:

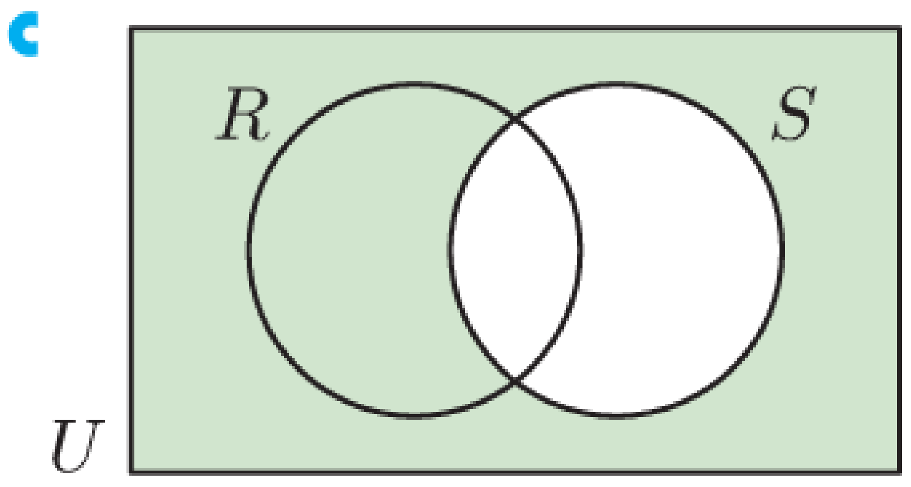
a



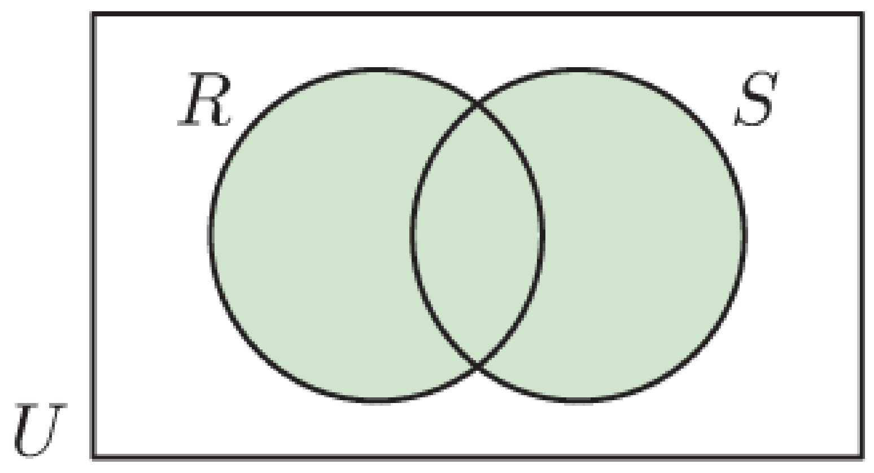
b



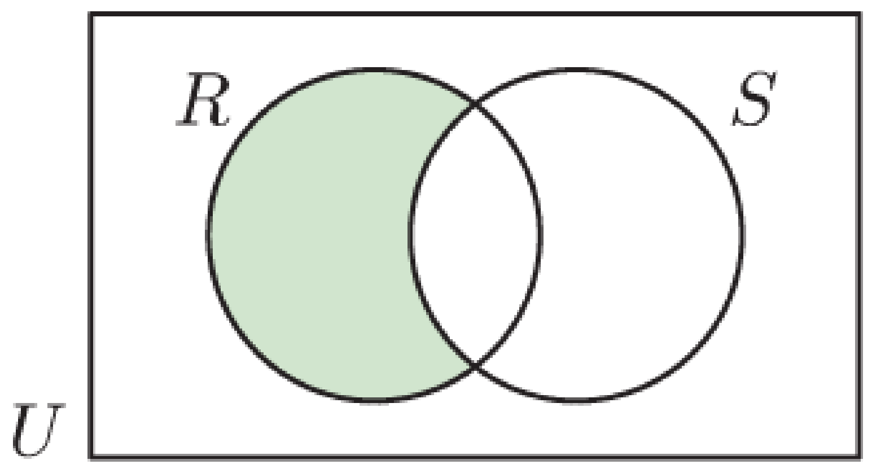
c



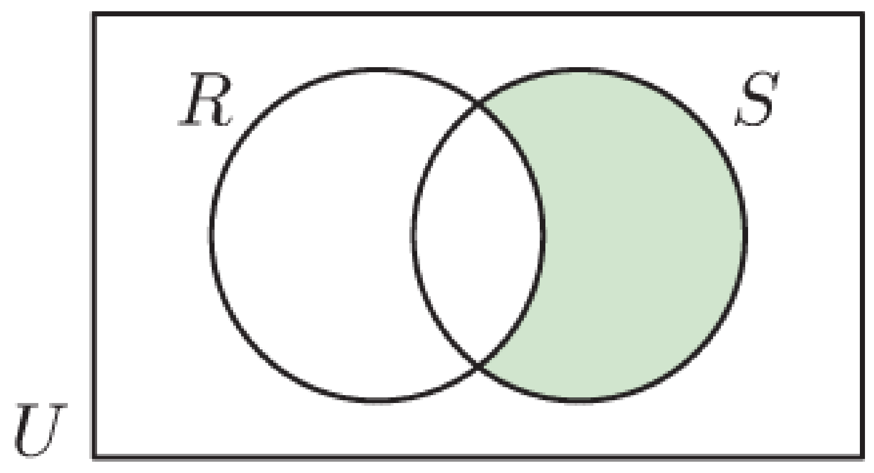
d



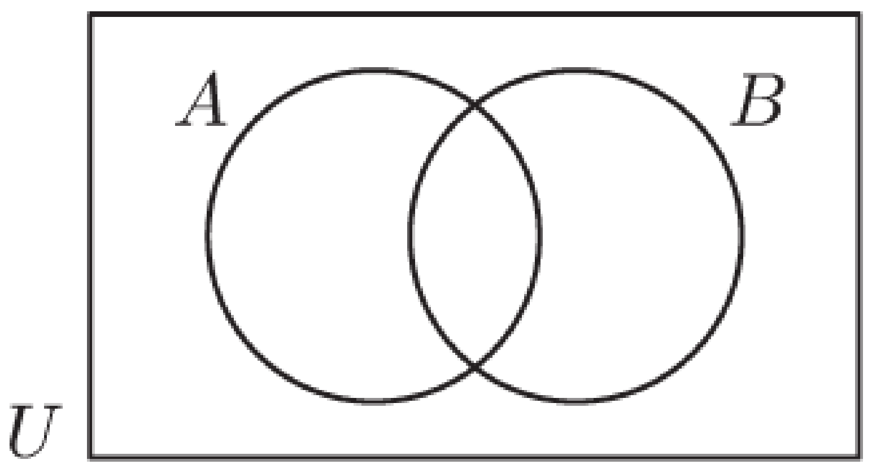
e



f



6 On separate Venn diagrams like the one illustrated, shade the region which represents the elements:



PRINTABLE VENN DIAGRAMS

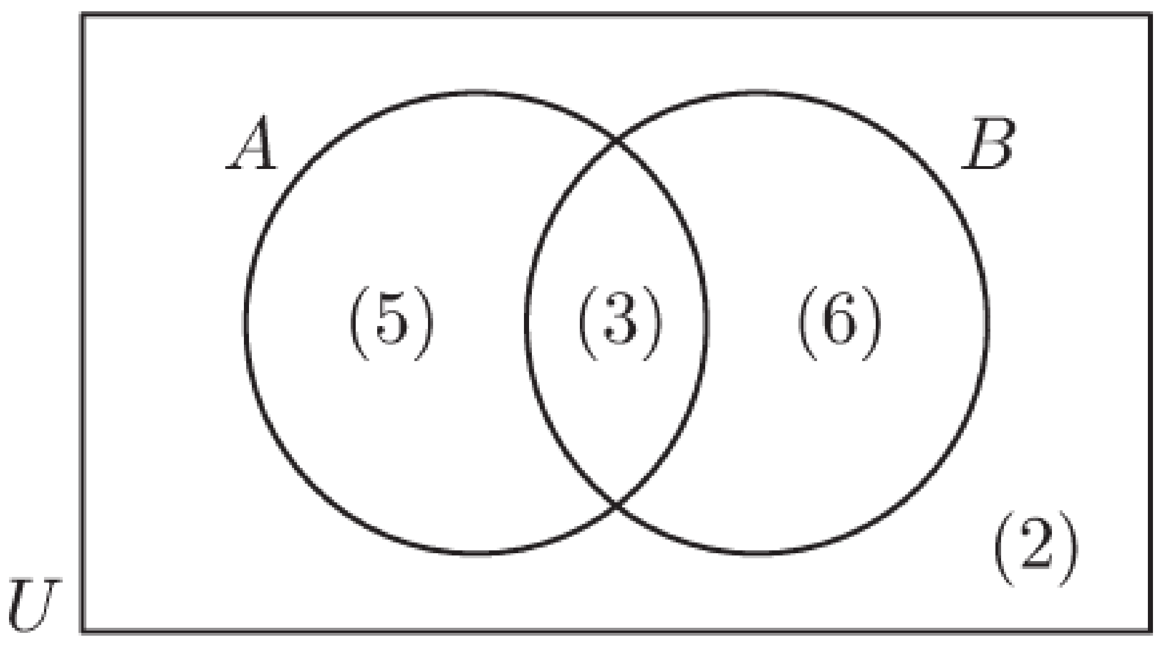
- **a** in A
- **b** in B
- **c** not in A
- **d** not in B
- **e** in both A and B
- **f** in A but not in B
- **g** in either A or B
- **h** in U.

NUMBERS IN REGIONS

We are often more interested in the *number* of elements in the regions of a Venn diagram, rather than the elements themselves.

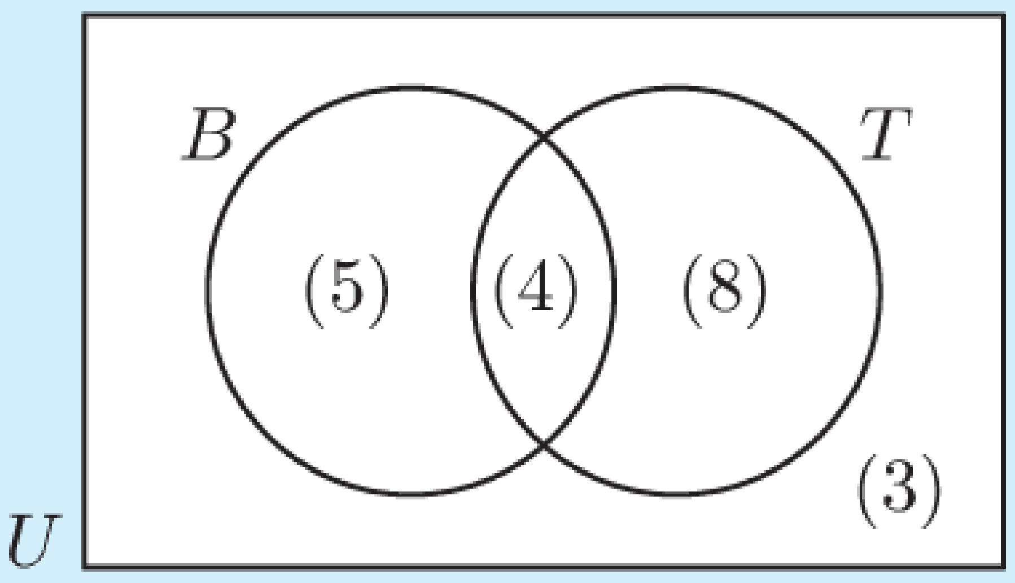
We indicate the number of elements in a region by placing brackets around the number.

For example, in the Venn diagram alongside there are 3 elements in $A \cap B$, and $3 + 6 = 9$ elements in B.



Example 8 **Self Tutor**

This Venn diagram shows the sports played by the students in a class. B represents the students who play basketball, and T represents the students who play tennis.



a How many students are there in the class?

b How many students play:

- **i** basketball
- **ii** tennis
- **iii** basketball and tennis
- **iv** neither basketball nor tennis
- **v** basketball but not tennis
- **vi** basketball or tennis?

a There are $5 + 4 + 8 + 3 = 20$ students in the class.

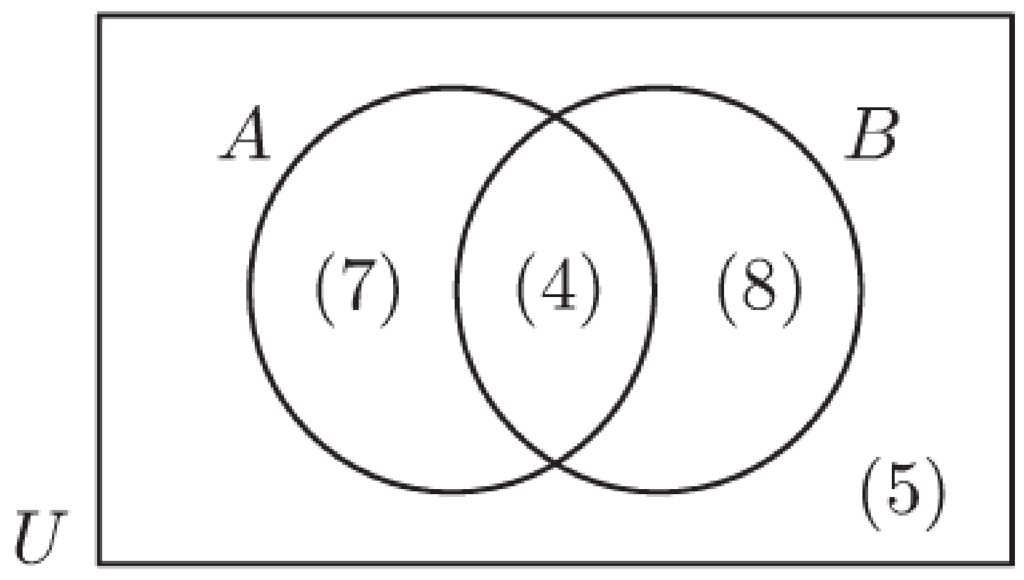
b
- **i** $5 + 4 = 9$ students play basketball.
- **ii** $4 + 8 = 12$ students play tennis.
- **iii** 4 students play both basketball and tennis.
- **iv** 3 students play neither basketball nor tennis.
- **v** 5 students play basketball but not tennis.
- **vi** $5 + 4 + 8 = 17$ students play basketball or tennis.

The (3) means that 3 students were not in either set B or T.

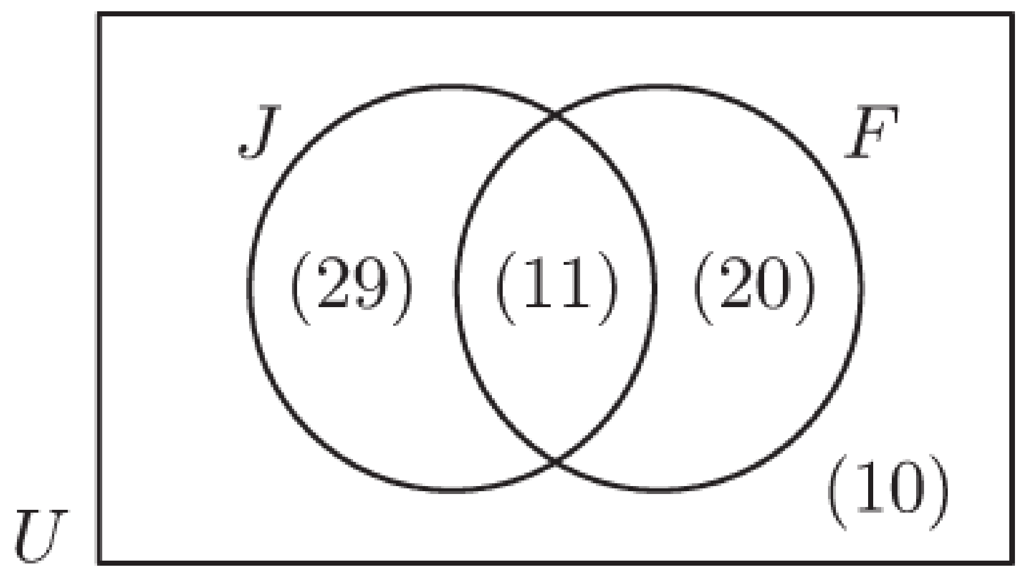
EXERCISE 17E.2

1 For the Venn diagram alongside, find the number of elements:

- **a** in A
- **b** in B
- **c** in A, but not in B
- **d** in either A or B
- **e** in neither A nor B
- **f** not in B.



2 This Venn diagram represents the people at a conference.
J represents the people who understand Japanese.
F represents the people who understand French.

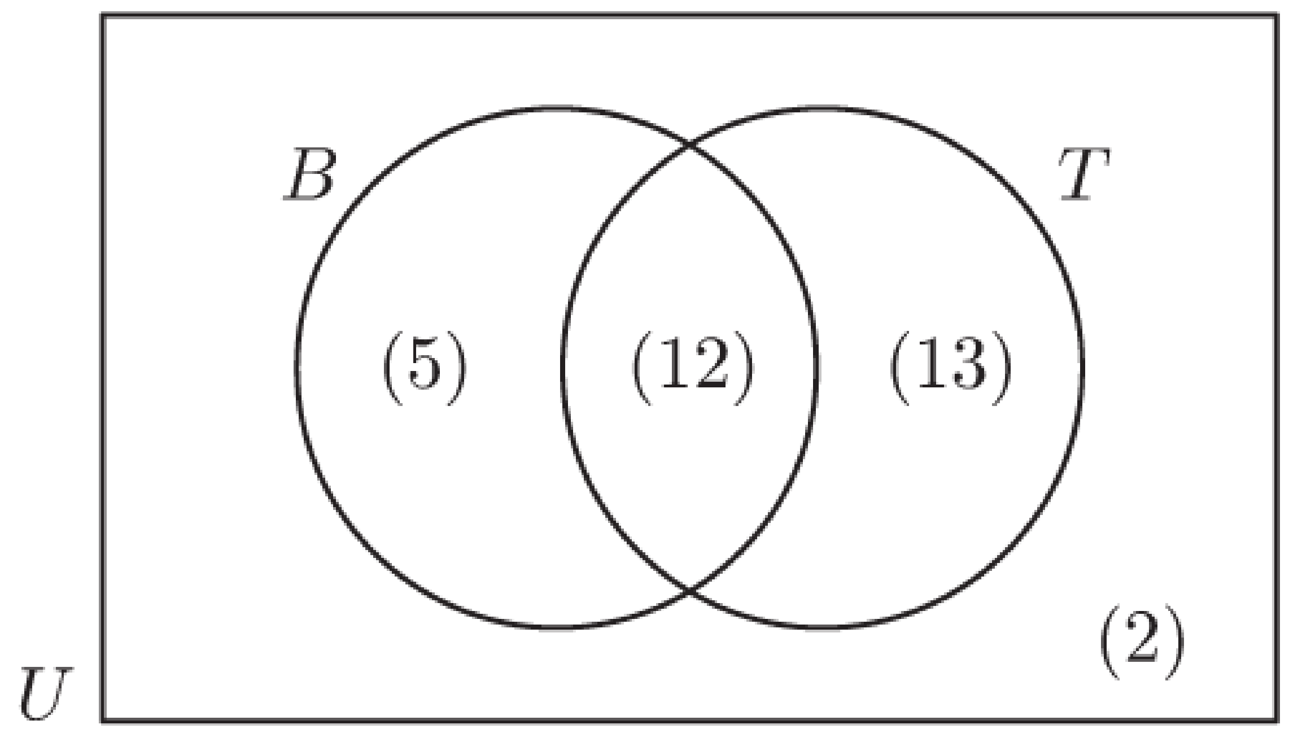


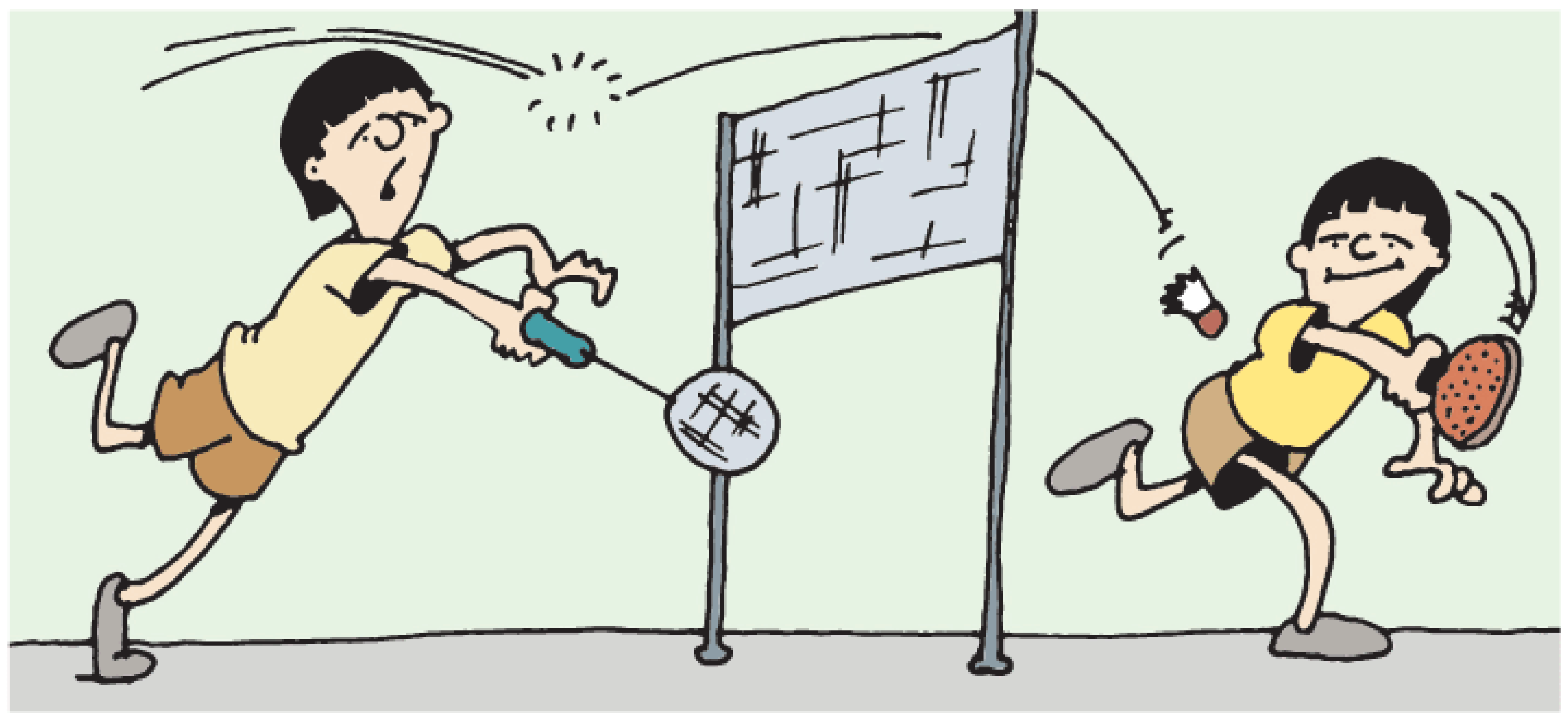
a How many people are at the conference?

b How many people at the conference understand:

- **i** Japanese
- **ii** French
- **iii** both Japanese and French
- **iv** Japanese but not French
- **v** neither Japanese nor French
- **vi** Japanese or French?

3 This Venn diagram shows the members of a youth club in Xi'an. B represents the members who play badminton, and T represents the members who play table tennis.



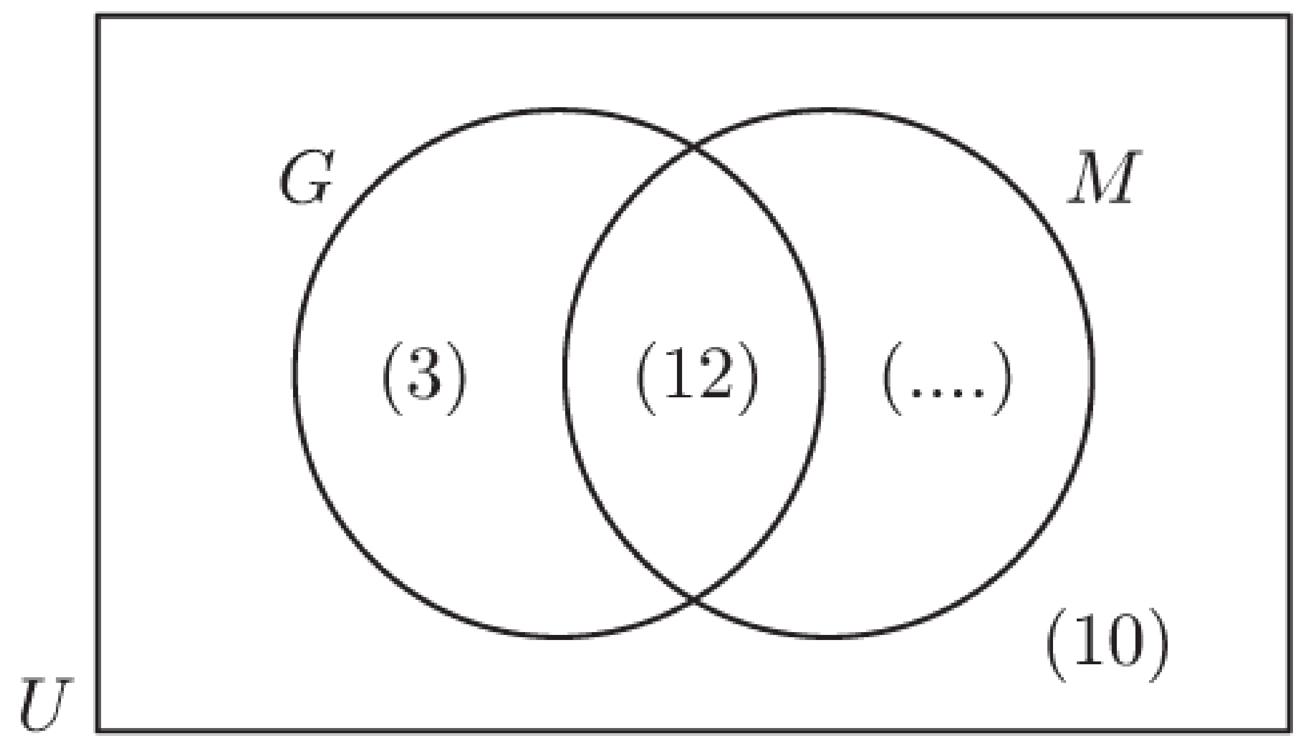


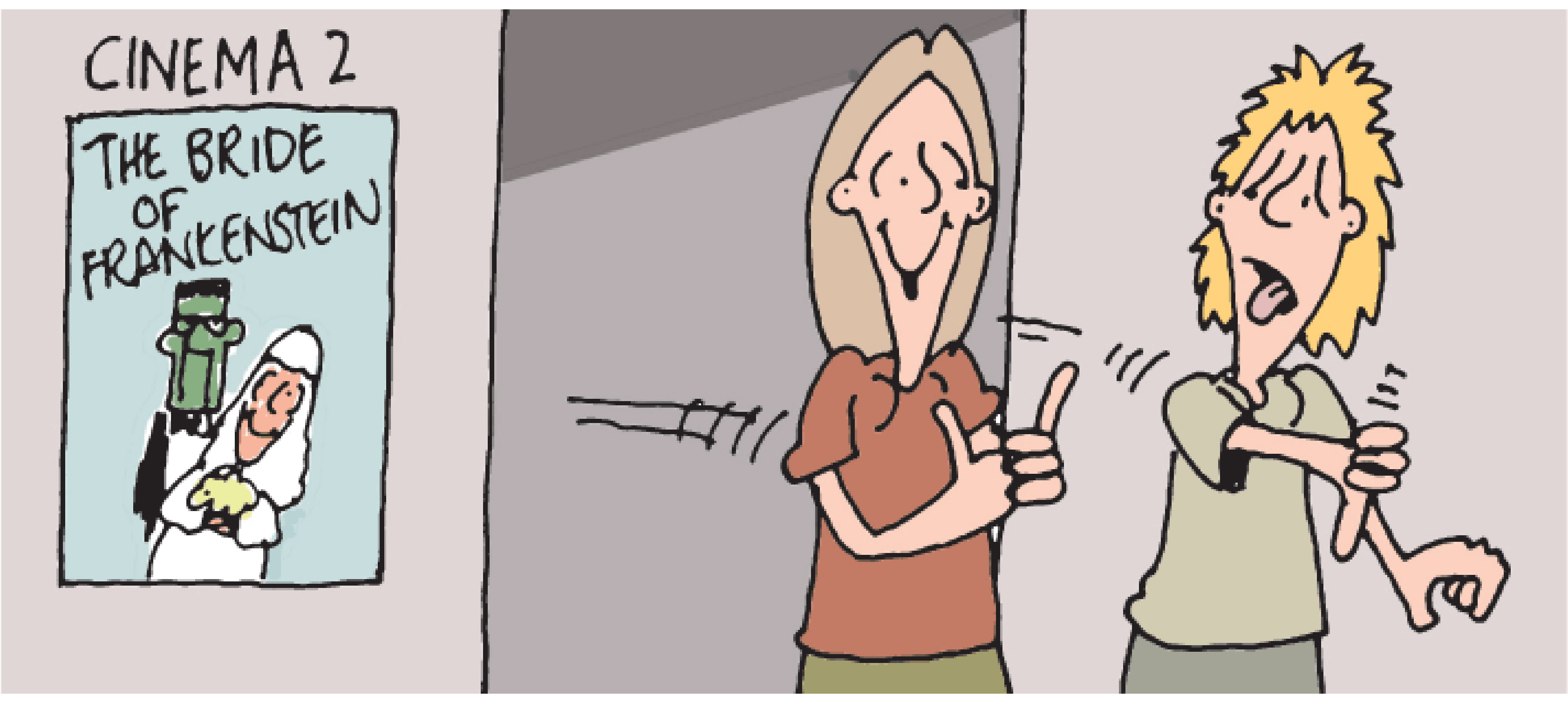
a How many members are in the club?

b How many of the club's members play:

- **i** badminton
- **ii** table tennis
- **iii** badminton, but not table tennis
- **iv** both of the sports
- **v** neither of the sports?

4 During one year, Gemma and Megan watched 30 movies together. In the Venn diagram, G represents the movies Gemma liked, and M represents the movies Megan liked.





a Copy and complete the Venn diagram.

b How many of the movies were liked by:

- **i** Gemma
- **ii** Megan
- **iii** both Gemma and Megan
- **iv** Gemma, but not Megan?

c Do you think that Gemma and Megan have a similar taste in movies? Explain your answer.

KEY WORDS USED IN THIS CHAPTER

- element
- intersection
- set
- universal set
- empty set
- member
- subset
- Venn diagram
- equal sets
- null set
- union

REVIEW SET 17A

1 Let $P = \{\text{blue, purple, orange, green, black, red}\}$.

a Use $\in$ or $\notin$ to complete these statements:

- **i** pink .... P
- **ii** green .... P

b Find $n(P)$.

2 Using set notation, list the elements of the set of all:

- **a** multiples of 8 which are less than 50
- **b** prime numbers between 25 and 40.

3 Let $A = \{\text{even numbers between 25 and 35}\}$.

- **a** List the elements of A.
- **b** Find $n(A)$.

4 Suppose $A = \{c,\ f,\ m,\ s,\ v\}$, $B = \{d,\ f,\ j,\ p,\ s,\ w,\ z\}$, and $C = \{f,\ p,\ w\}$.

- **a** List the elements of:
  - **i** $A \cup B$
  - **ii** $A \cap C$.
- **b** Is C a subset of B? Explain your answer.

5 Suppose $C = \{1,\ 6,\ 7,\ \square,\ 8\}$ and $D = \{7,\ 6,\ 8,\ 1,\ 4\}$. If $C = D$, find $\square$.

6 Find $A \cup B$ for:

- **a** $A = \{\text{dog, bird, sheep, giraffe, tiger}\}$ and $B = \{\text{sheep, otter, bird, bear}\}$
- **b** $A = \{!,\ @,\ \div,\ \blacktriangleright,\ \bullet,\ \#\}$ and $B = \{+,\ \blacktriangleright,\ *,\ \%,\ !\}$.

7 Suppose $U = \{1,\ 2,\ 3,\ ....,\ 20\}$, $E = \{\text{positive even numbers less than 20}\}$, and $F = \{\text{factors of 20}\}$.

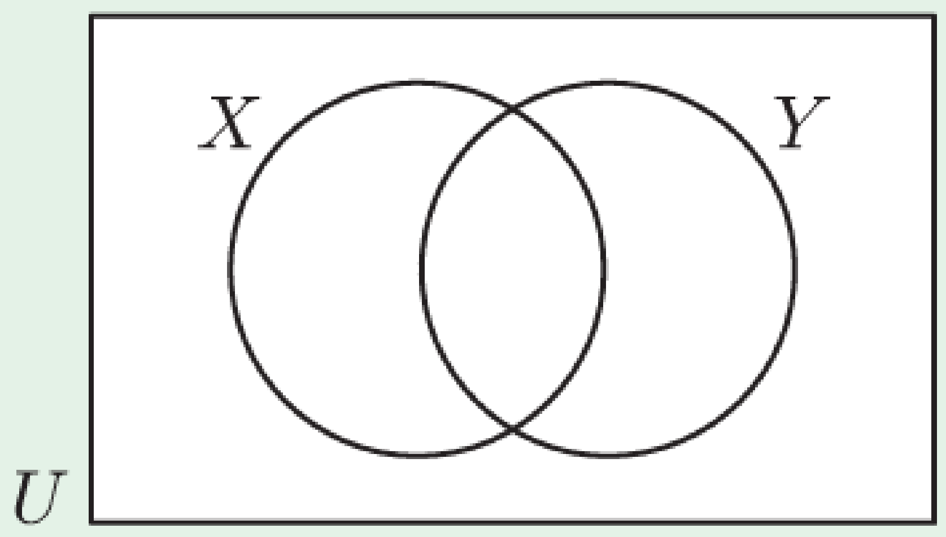
- **a** List the elements of:
  - **i** E
  - **ii** F
  - **iii** $E \cap F$
  - **iv** $E \cup F$.
- **b** Illustrate the sets on a Venn diagram.

8 Suppose $U = \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\}$, $A = \{2,\ 3,\ 6,\ 8,\ 9\}$, and $B = \{1,\ 3,\ 5,\ 7,\ 8\}$.

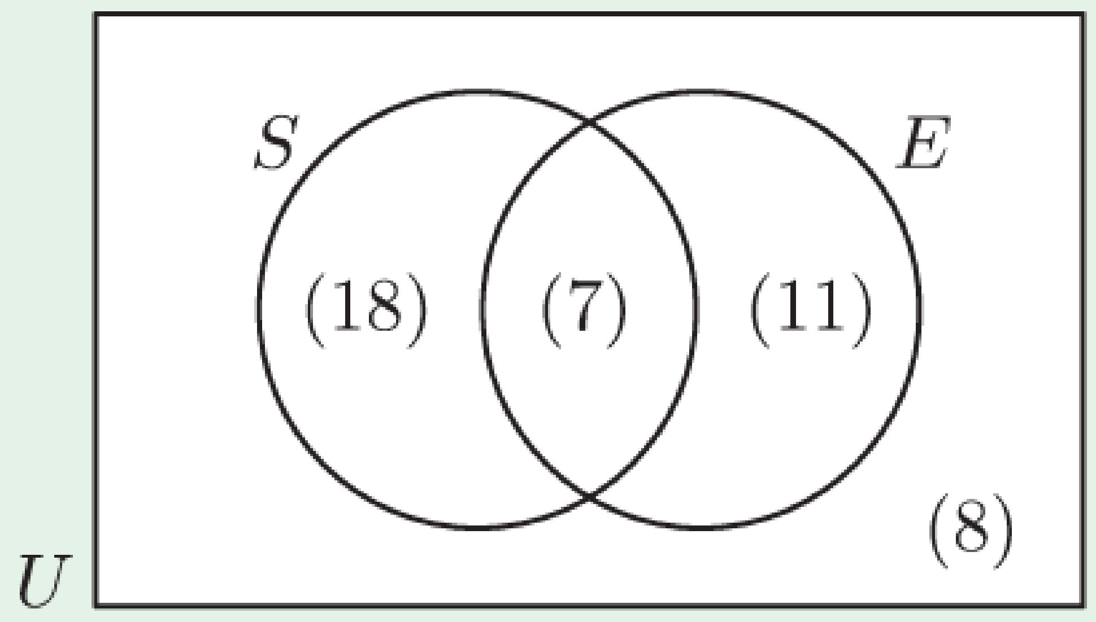
- **a** Illustrate A, B, and U on a Venn diagram.
- **b** Find: **i** $n(A \cap B)$ **ii** $n(A \cup B)$

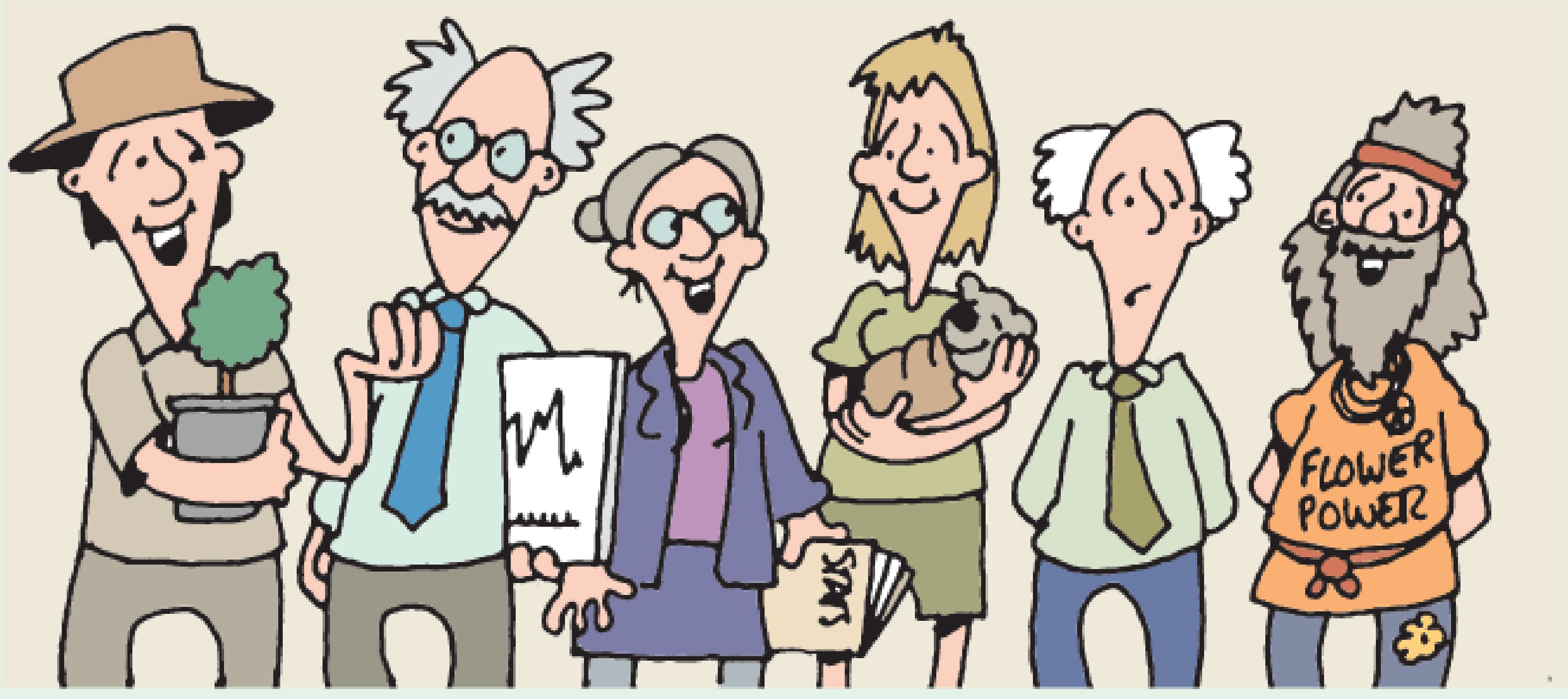
9 On separate Venn diagrams like the one illustrated, shade the regions representing:

- **a** in X
- **b** not in Y
- **c** in both X and Y
- **d** in neither X nor Y.



10 This Venn diagram represents the people attending a convention on climate change. S represents all of the scientists, and E represents all of the environmentalists.





- **a** How many people are at the convention?
- **b** How many people at the convention are:
  - **i** scientists
  - **ii** environmentalists
  - **iii** both scientists and environmentalists
  - **iv** neither scientists nor environmentalists
  - **v** environmentalists but not scientists?

REVIEW SET 17B

1 Using set notation, list the elements of the set of all:

a square numbers between 10 and 60 **b** factors of 18.

2 Which two of these are equal sets?

A {pear, apple, orange, tomato} **B** {tomato, pear, banana, apple}

C {apple, orange, grape, banana} **D** {pear, tomato, apple, banana}

3 Let $M = \{\text{multiples of 3 less than 40}\}$ and $N = \{\text{multiples of 4 less than 40}\}$.

a List the elements of:

i M **ii** N

b Find $M \cap N$.

4 Let $K = \{1, 5, 7, 9\}$. List all the subsets of K which contain:

a two elements **b** three elements.

5 Let X be the set of all letters used to write the word ISOSCELES, and Y be the set of all letters used to write the word SCALENE.

a List the elements of:

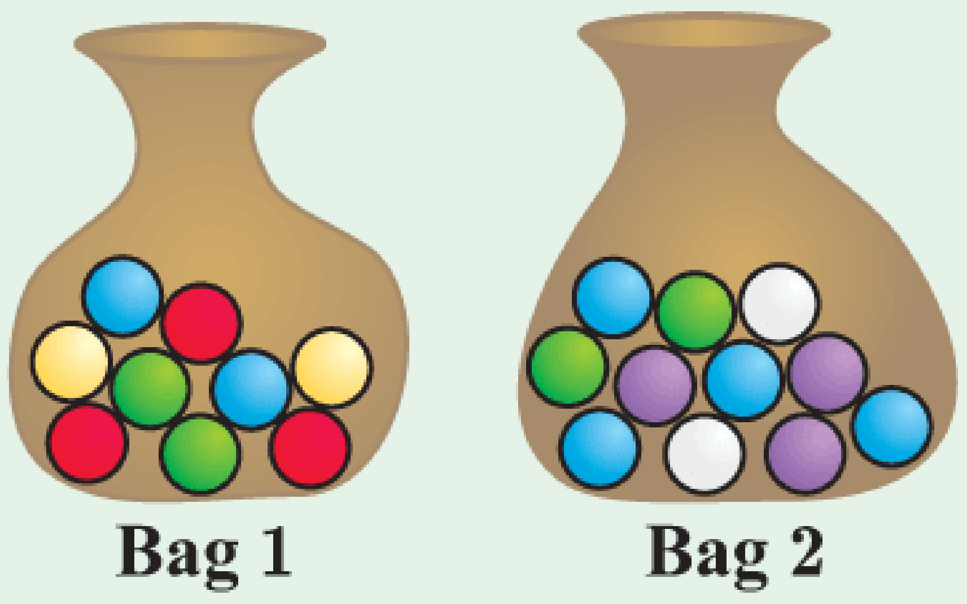
i X **ii** Y

b Find $X \cup Y$. **c** Find $n(X \cup Y)$.

6 Find $S \cap T$ for:

a $S = \{5, 7, 8, 12, 15, 21, 24, 29\}$ and $T = \{9, 12, 16, 22, 24\}$

b $S = \{\text{colours of balls in bag 1}\}$ and
$T = \{\text{colours of balls in bag 2}\}$

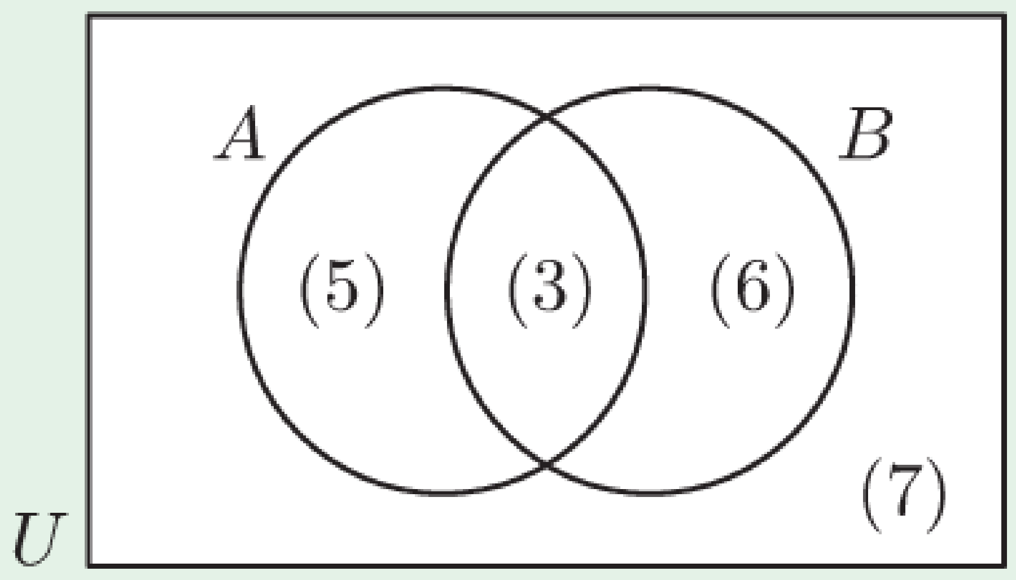


7 For the Venn diagram shown, find the number of elements in:

a A **b** B

c both A and B **d** either A or B

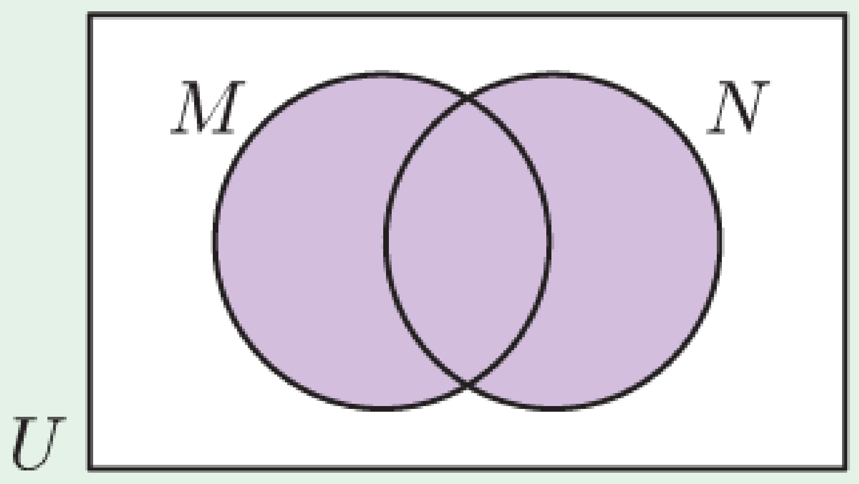
e U.



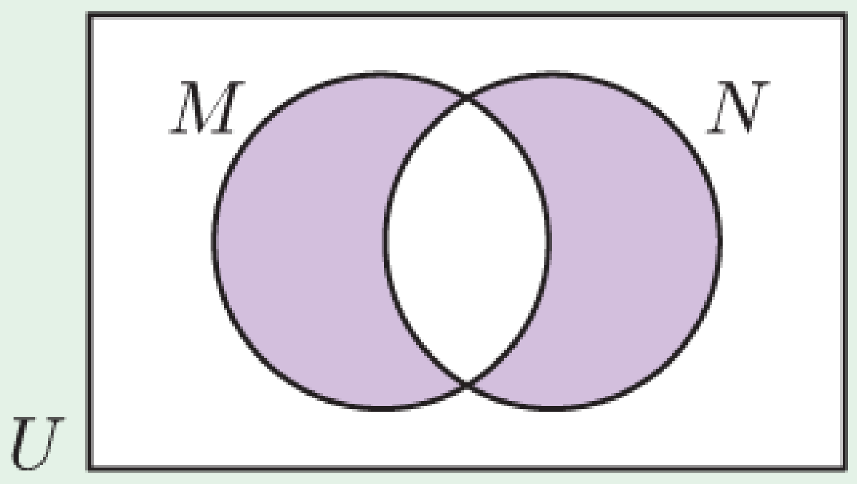
8 Consider the positive integers 1, 2, 3, ..., 12. Let $F = \{\text{factors of 12}\}$ and $P = \{\text{prime numbers less than 12}\}$. Represent these two sets on a Venn diagram.

9 Describe, in words, the shaded region:

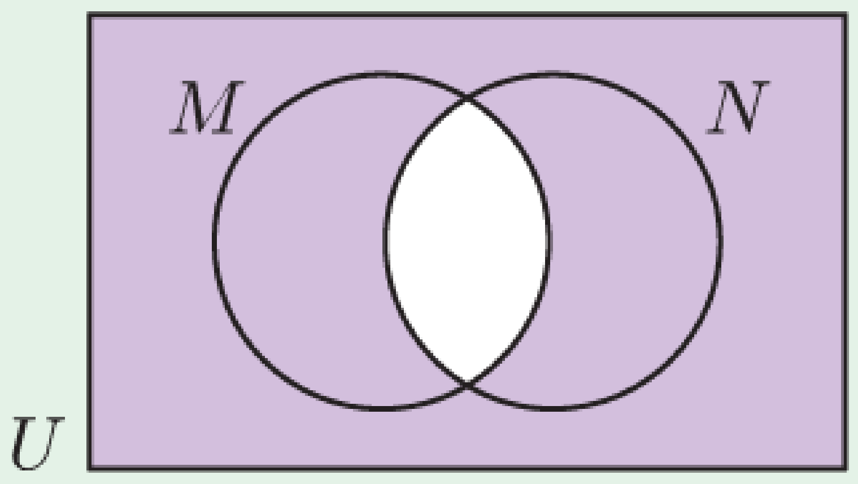
a



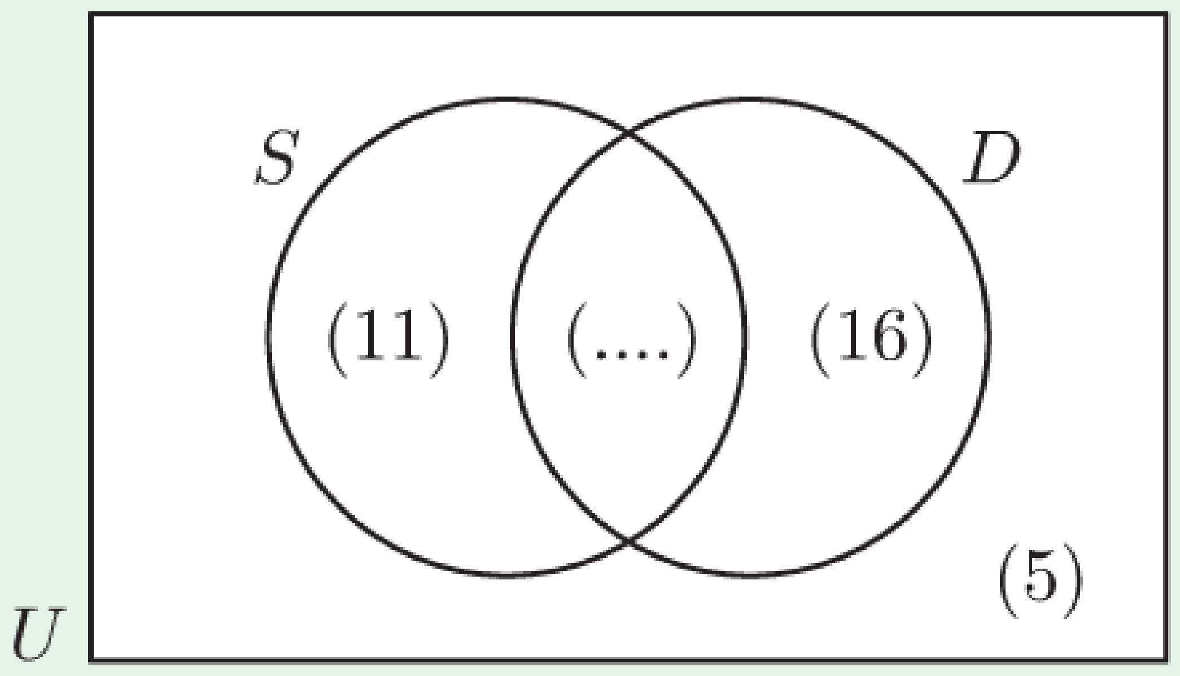
b

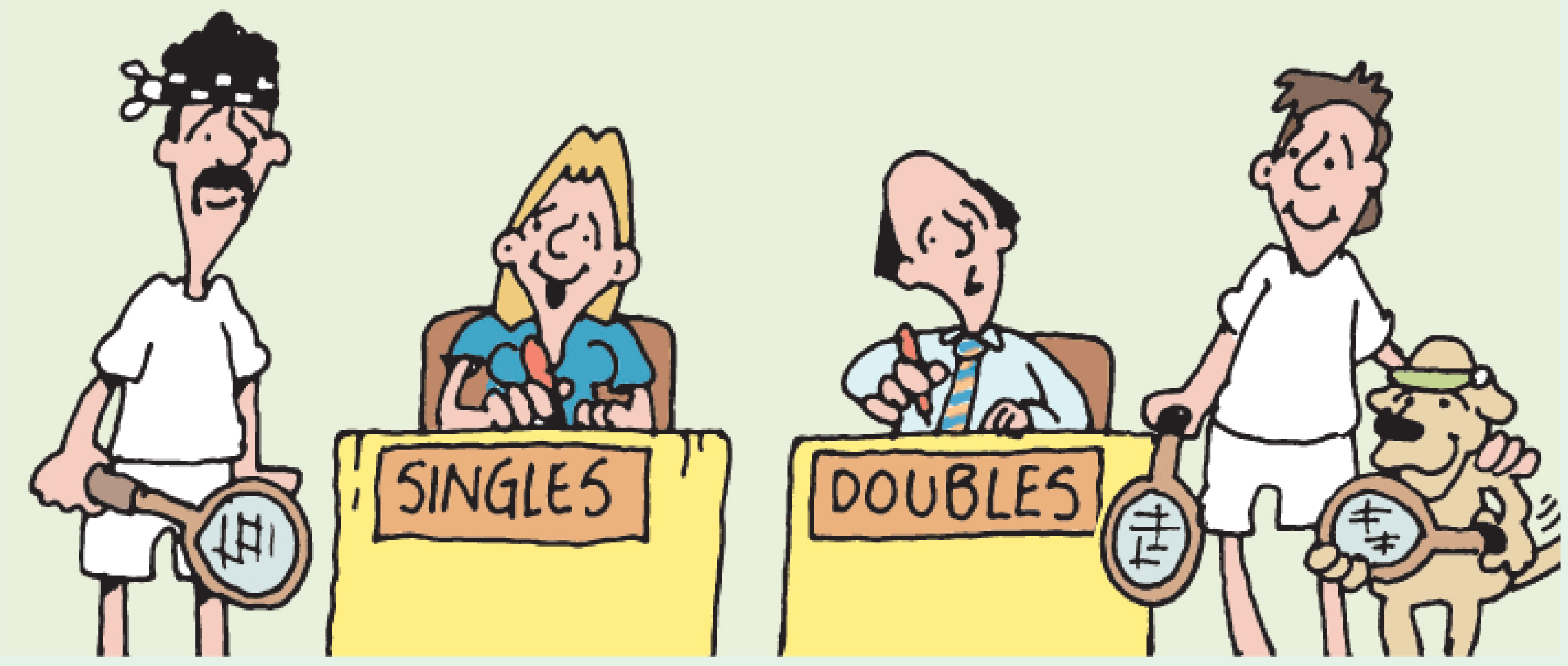


c



10 A tennis club with 40 members holds a tournament. Some members are entered to play singles, and some to play doubles.





- **a** Copy and complete the Venn diagram.
- **b** How many members are entered to play:
  - **i** singles
  - **ii** doubles
  - **iii** both singles and doubles
  - **iv** singles but not doubles
  - **v** at least one of the events
  - **vi** exactly one of the events?

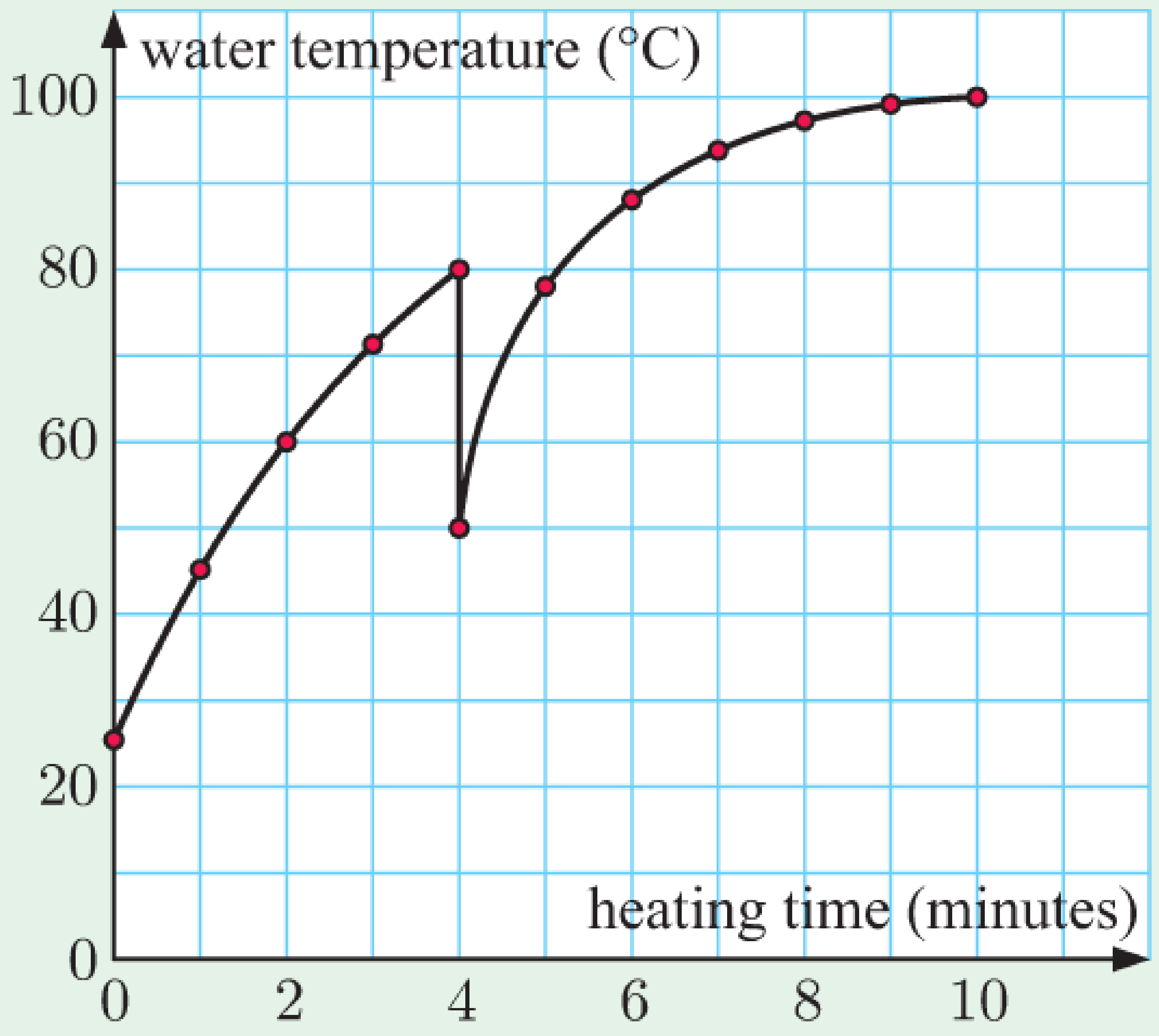
Chapter 18

Line graphs

Contents:

OPENING PROBLEM

Jun is heating some water on the stove. She measures its temperature at one minute intervals from the time she turns the stove on. After a while, she realises she does not have enough water, so she adds some cold water. She plots her temperature readings on a graph.



Things to think about:

- **a** What is the room temperature?
- **b** What is the water temperature after 2 minutes?
- **c** When did Jun add the cold water?
- **d** What is the maximum temperature the water reaches?

A LINE GRAPHS

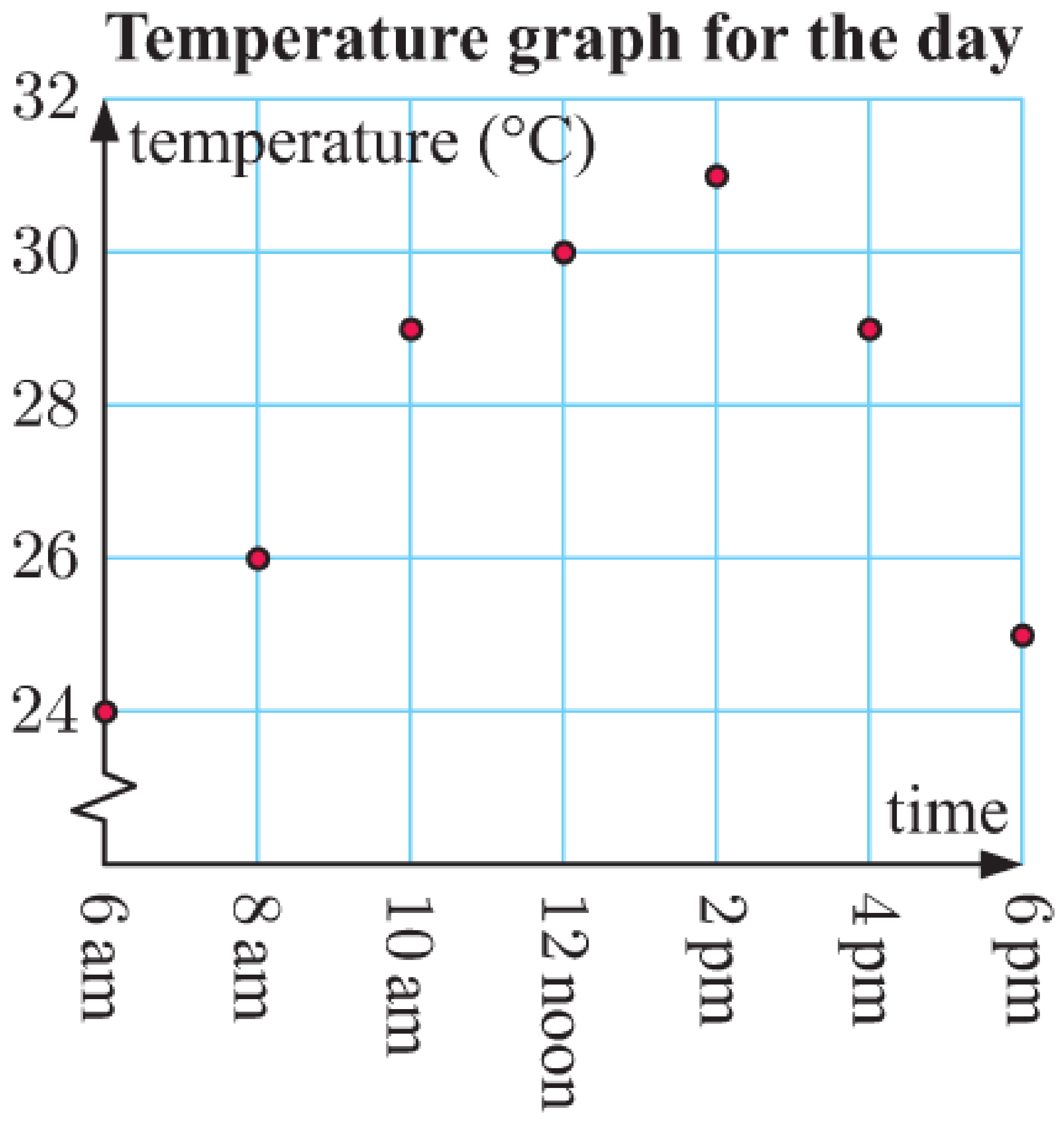
Line graphs consist of straight line segments or curves.

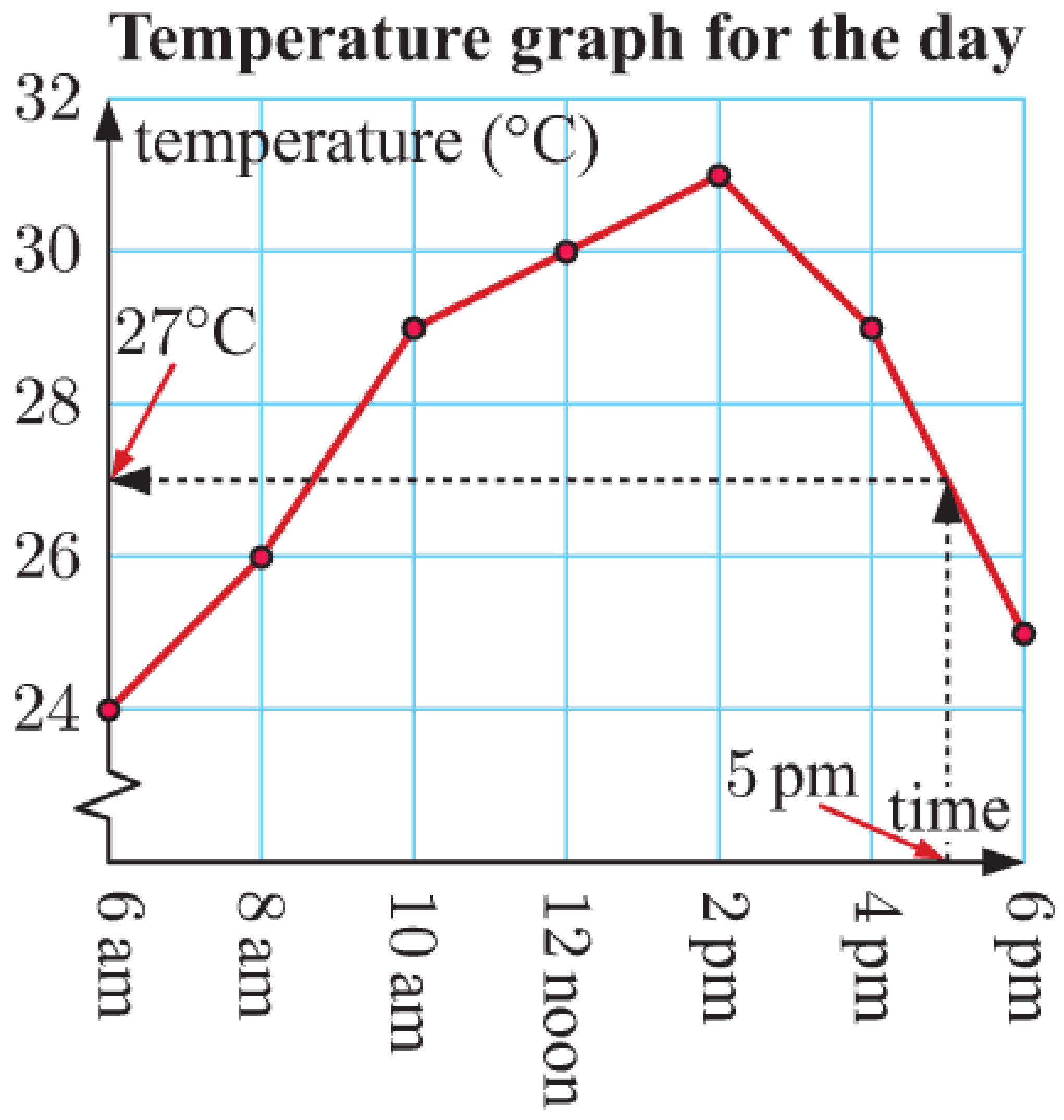
Line graphs are used to show how one quantity varies in relation to another.

For example, suppose we record the temperature outside at 2-hour intervals, and obtain these results:

Time	6 am	8 am	10 am	noon	2 pm	4 pm	6 pm
Temperature (°C)	24	26	29	30	31	29	25

We can graph this data by plotting the values on a set of axes. We plot the variable *time* on the horizontal axis, and the variable *temperature* on the vertical axis.





The graph on the left is a **point graph** of the temperature data. However, we know that the temperature has a value at all times. We therefore join the dots together to create the **line graph** on the right.

From both graphs we can determine information such as:

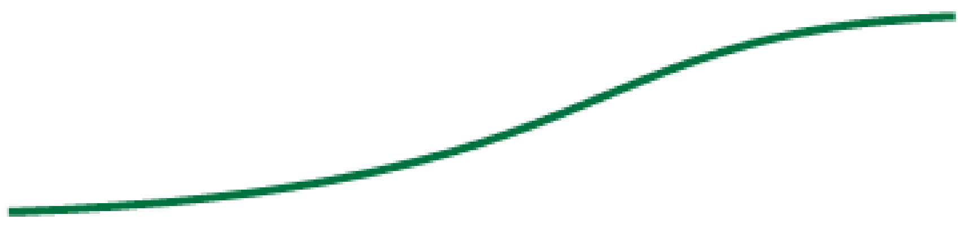
- the highest temperature recorded for the day was at 2 pm
- the lowest temperature recorded was at 6 am
- the temperature at 8 am was 26°C.

The line graph is more useful than the point graph because values *in between* data points can be **estimated** from the graph.

For example, using the dotted line on the line graph, we estimate the temperature at 5 pm to be 27°C.

INCREASING AND DECREASING GRAPHS

If the line or curve slopes *upwards* from left to right, we say the graph is **increasing**.



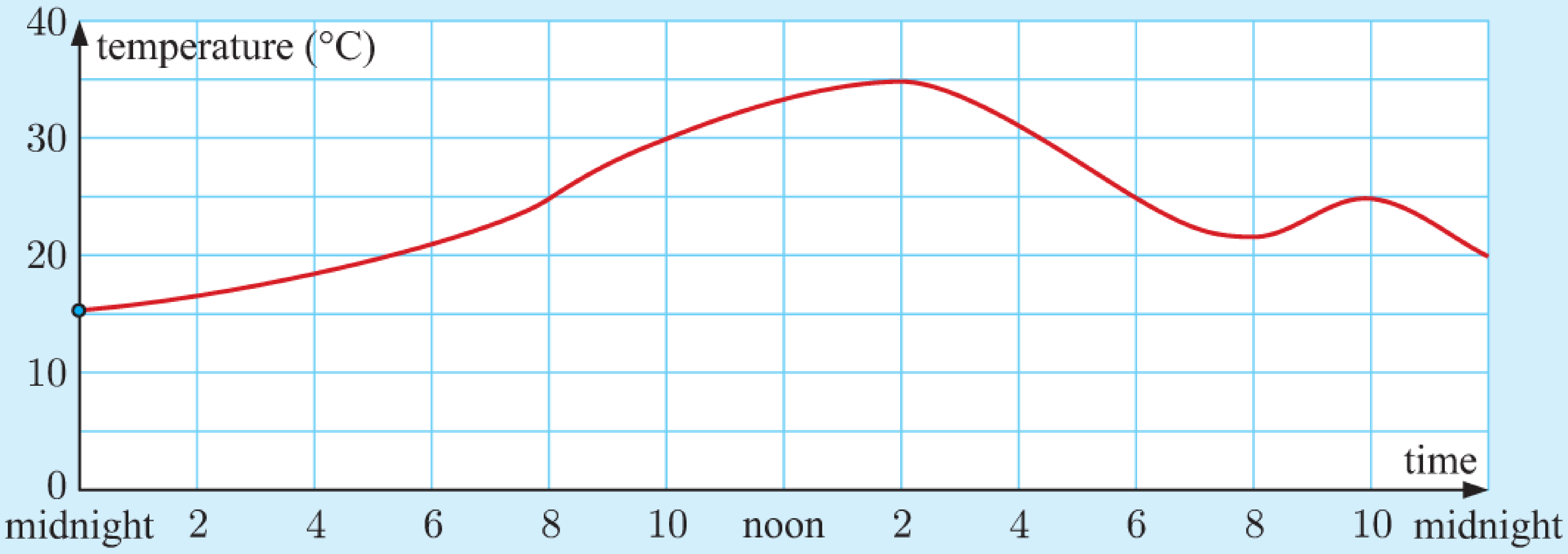
If the line or curve slopes *downwards* from left to right, we say the graph is **decreasing**.

Graphs may be increasing in some sections, and decreasing in other sections.

For example, the temperature graph on the previous page is increasing from 6 am to 2 pm, and then decreasing from 2 pm to 6 pm.

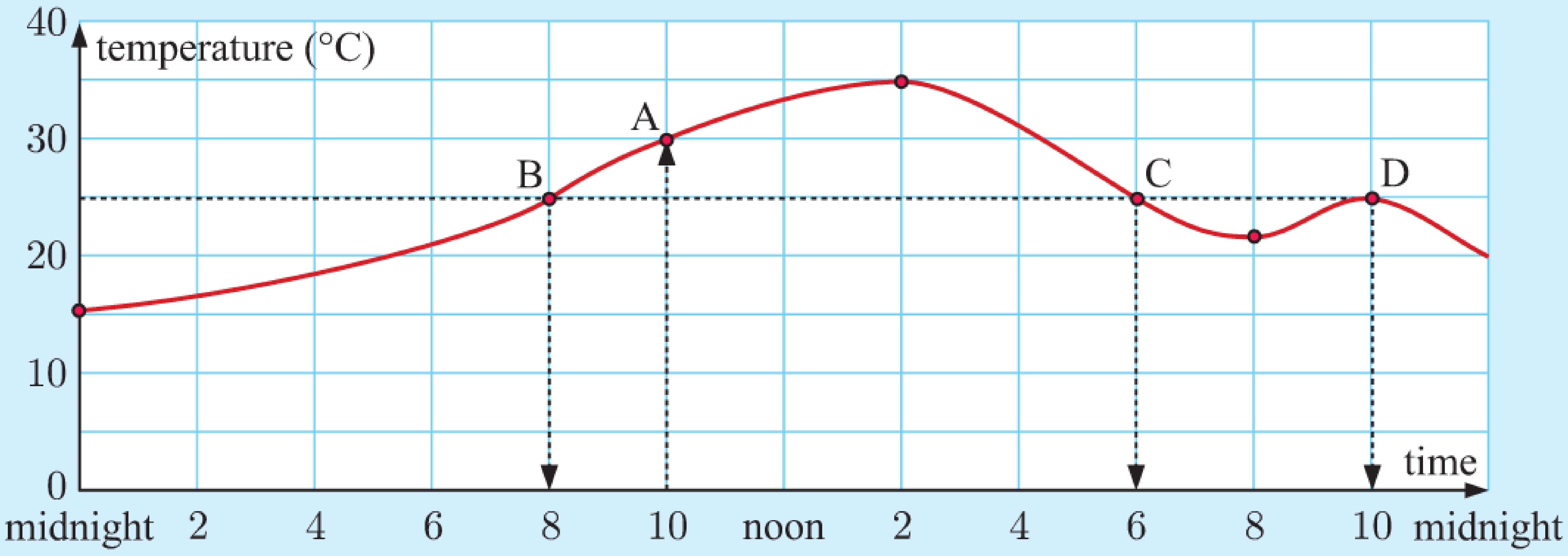
Example 1

Self Tutor



Use this line graph to find:

- **a** the temperature at 10 am
- **b** the times when the temperature was 25°C
- **c** the periods when the temperature was: **i** increasing **ii** decreasing
- **d** the maximum temperature and when it occurred.



a The temperature at 10 am was 30°C. {point A}

b The temperature was 25°C at 8 am, 6 pm, and 10 pm. {points B, C, and D}

c The temperature was:
  - **i** increasing from midnight to 2 pm, and from 8 pm to 10 pm
  - **ii** decreasing from 2 pm to 8 pm, and from 10 pm to midnight.

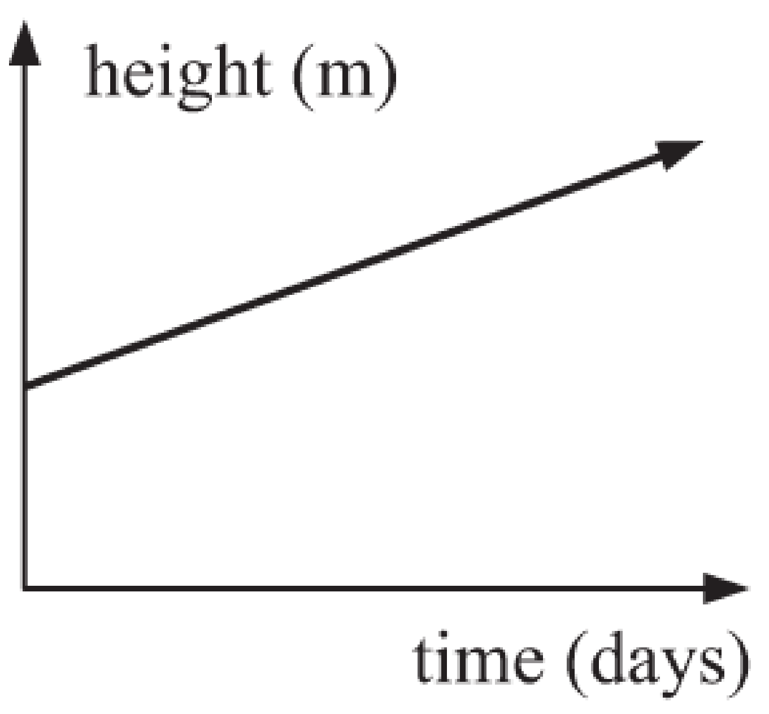
d The maximum temperature was 35°C at 2 pm.

EXERCISE 18A

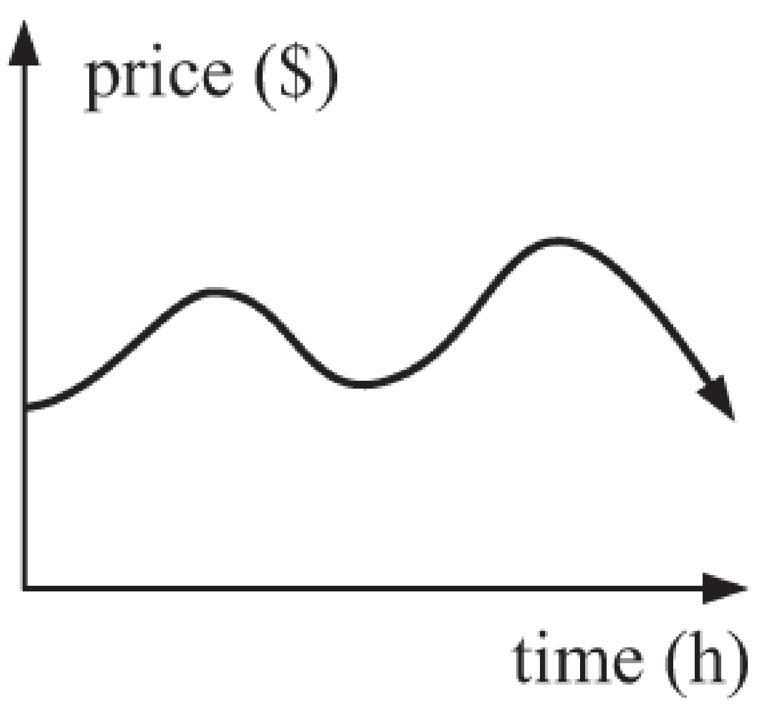
1 State whether each line graph is:

A increasing **B** decreasing **C** increasing in some sections and decreasing in others.

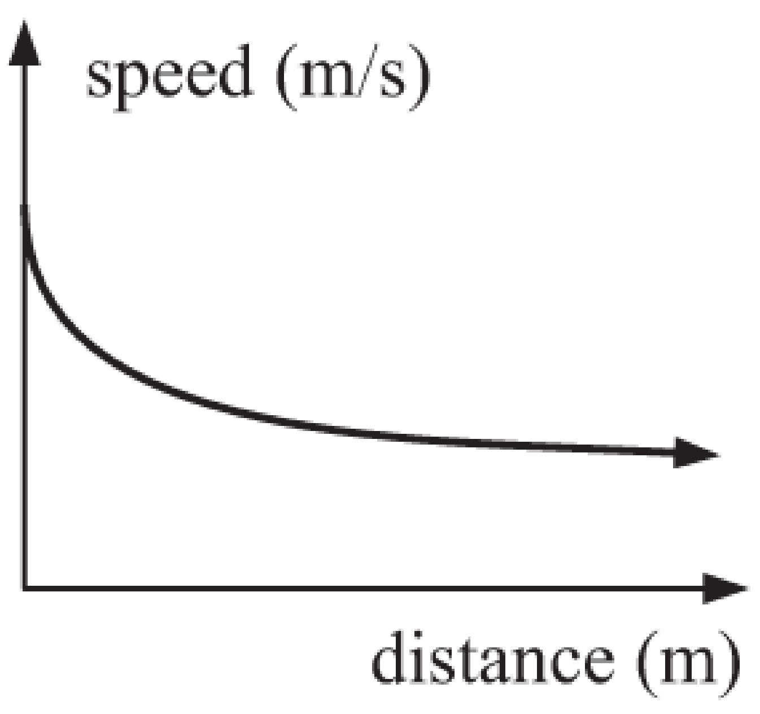
a



b



c



2 Managers of a café conduct a customer count to help them decide how to roster their staff. The results are shown in the line graph.

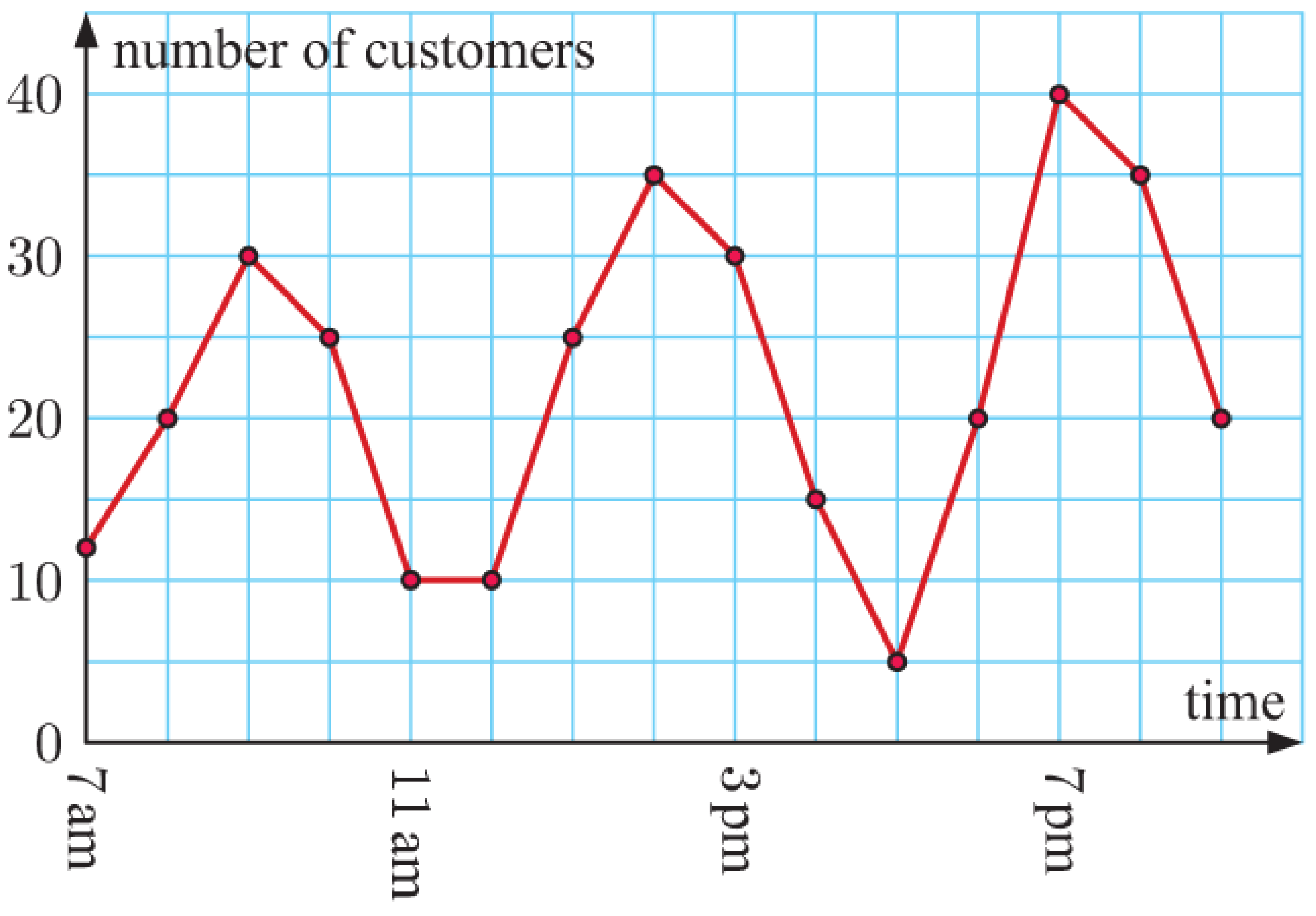
a At what time was the number of people in the café greatest?

b At what time was the number of people in the café least?

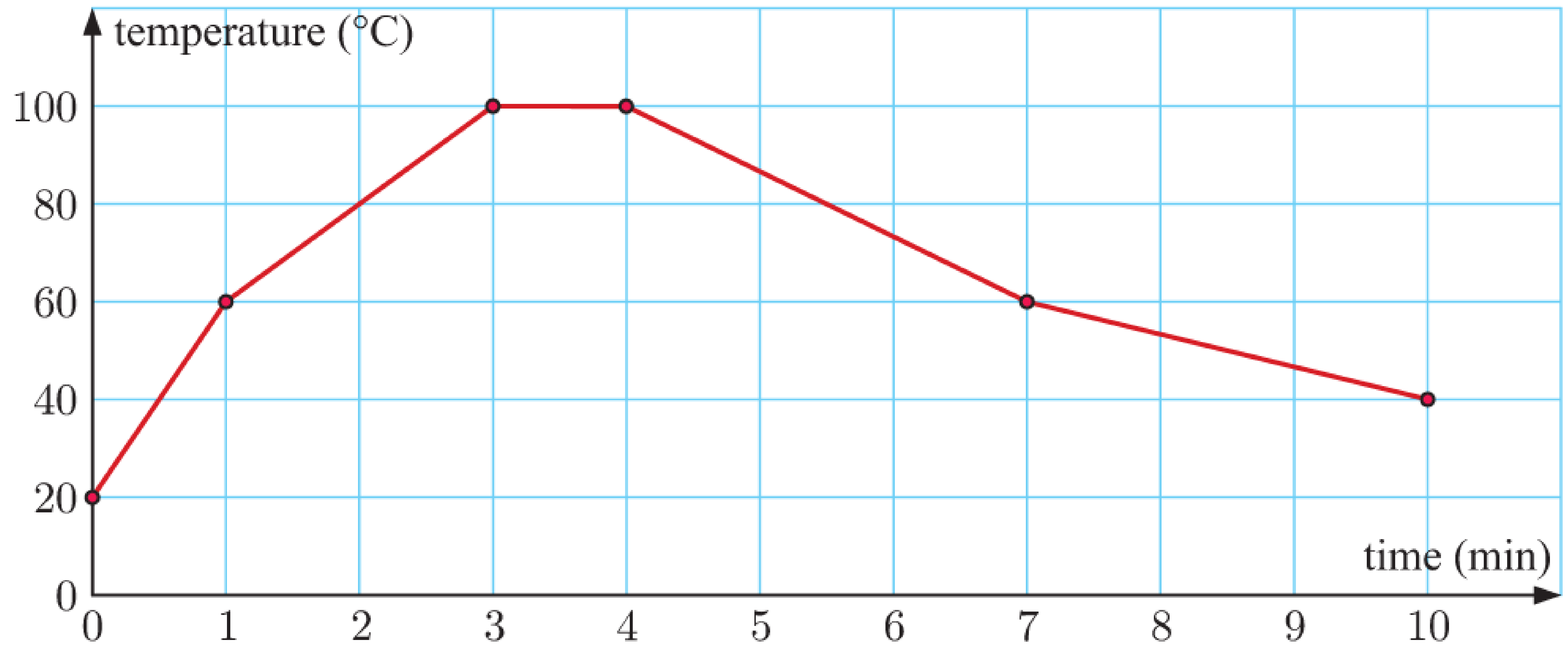
c How many people were in the café at 10 am?

d Describe what happened between 6 pm and 7 pm.

e Estimate the number of people in the café at 4:30 pm.



3 The temperature of water in a kettle is graphed over a 10 minute period.



a What was the room temperature when the kettle was switched on?

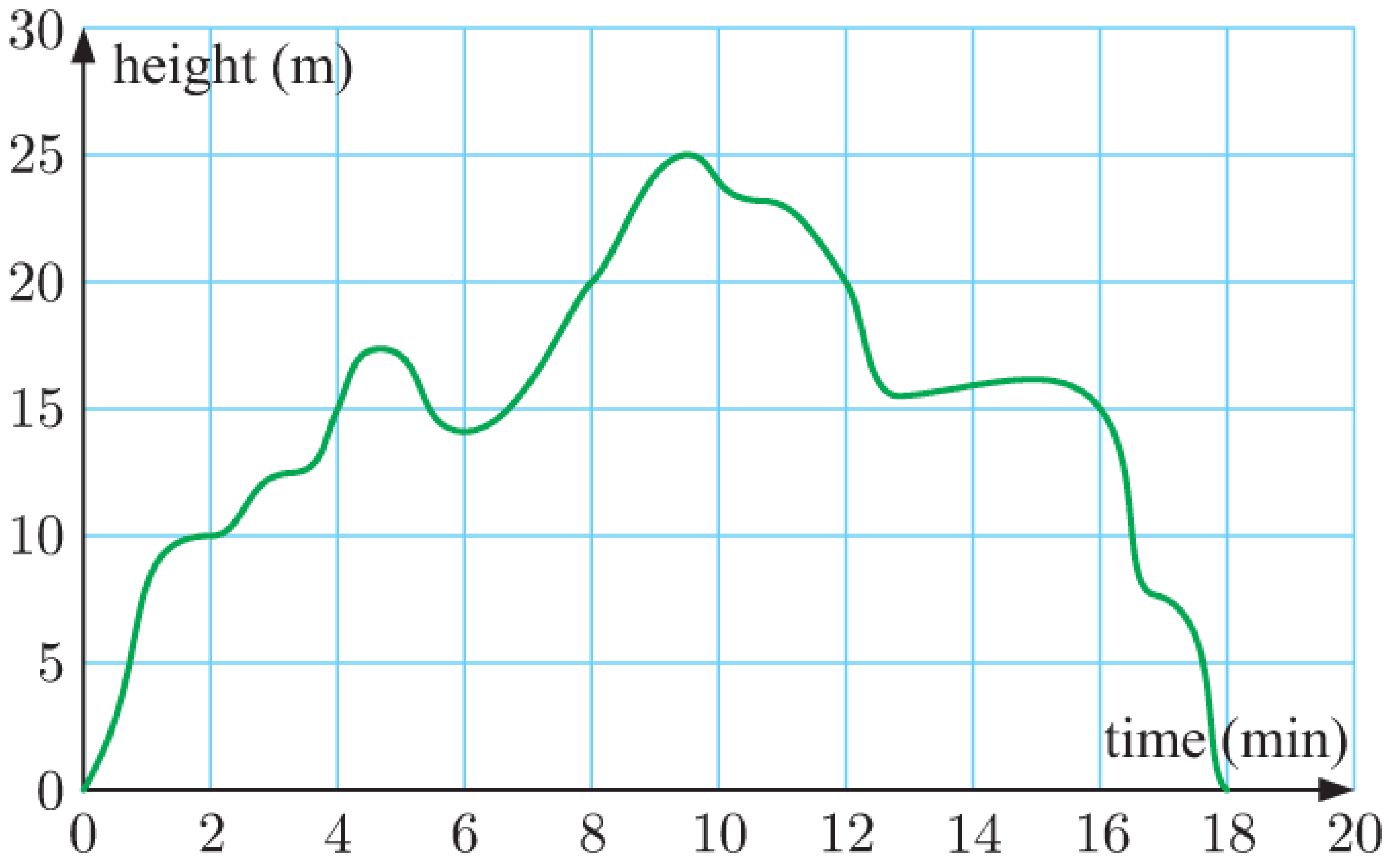
b How long did it take for the water to boil?

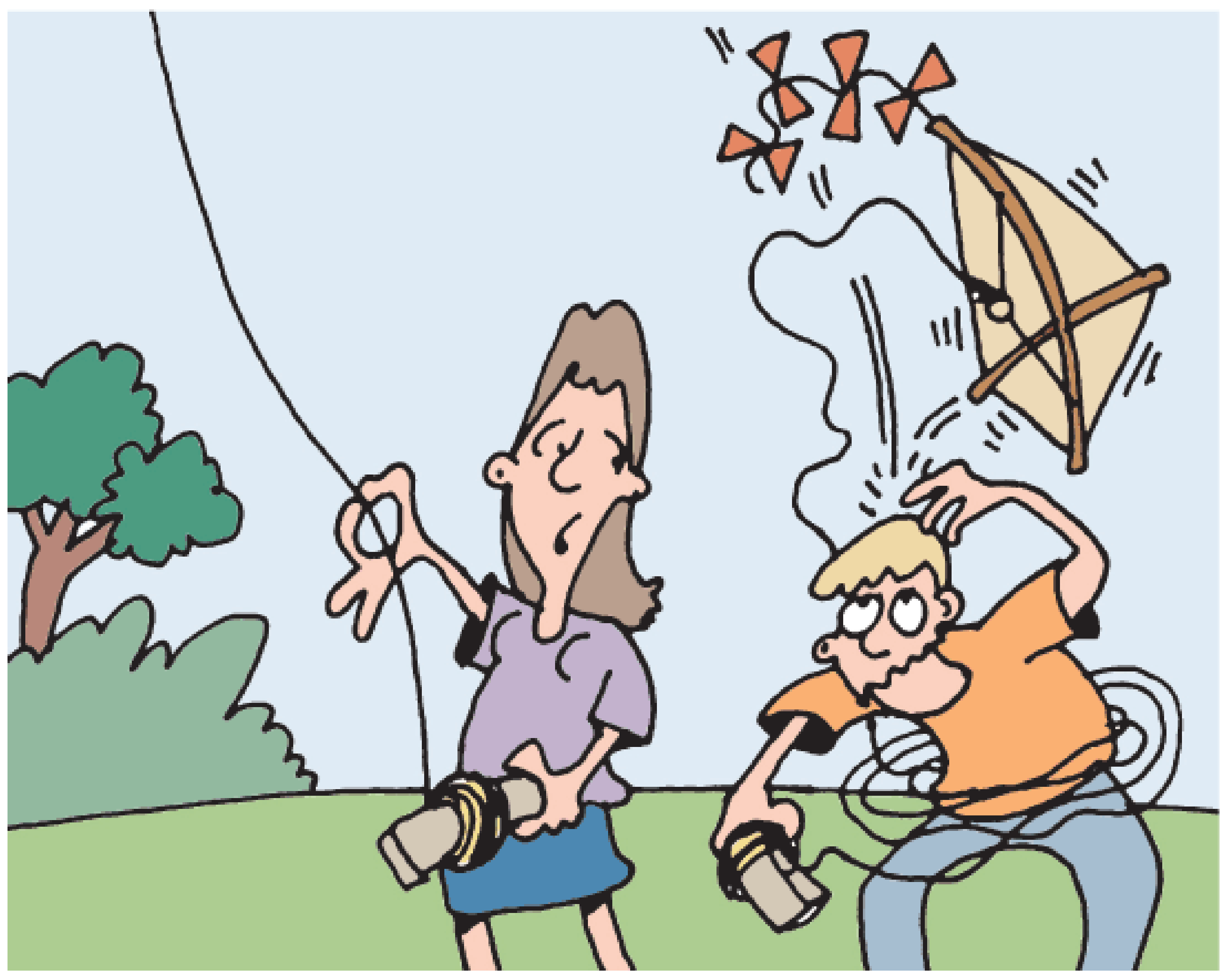
c For how long did the water boil?

d At what times was the water temperature 60°C?

e During what period was the temperature decreasing?

4 Caitlin flies a kite one afternoon. This line graph shows the height of the kite over time.





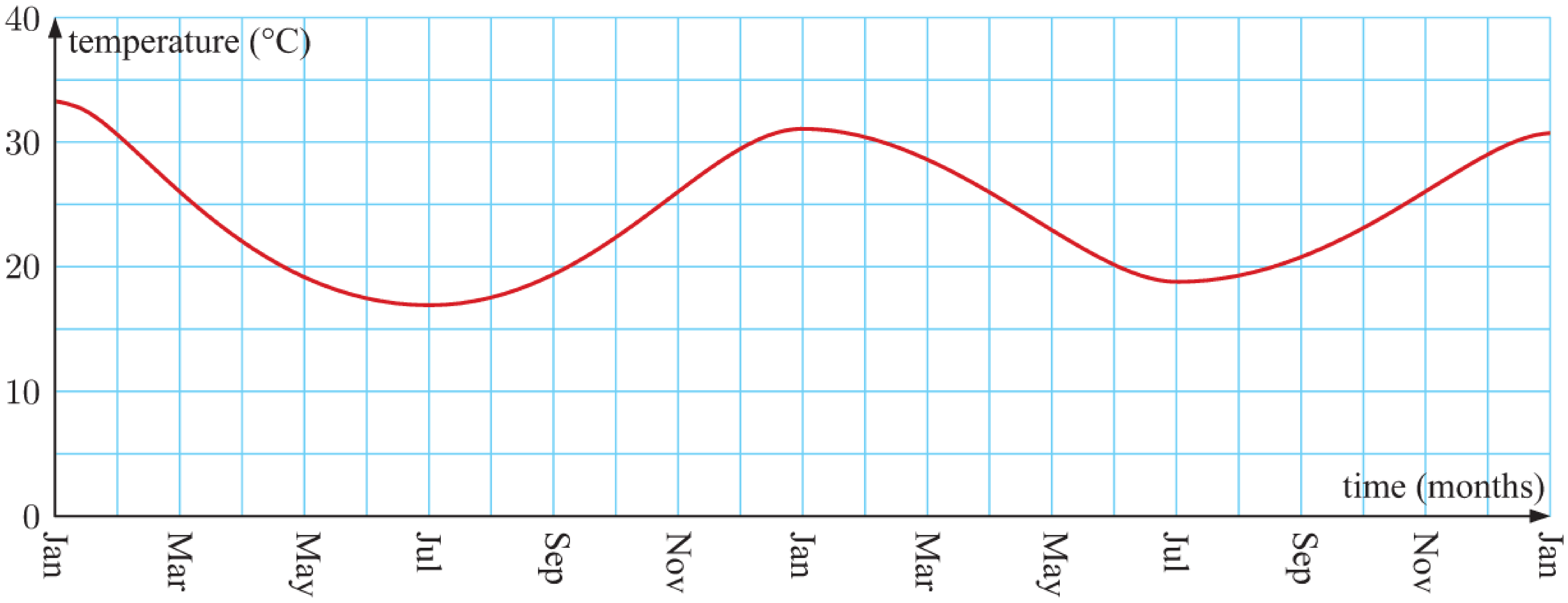
a For how long was the kite in the air?

b How high was the kite after 4 minutes?

c What was the greatest height reached by the kite?

d At what times was the kite 20 metres high?

5 The graph below shows the average daily temperature for Johannesburg over a 2 year period.



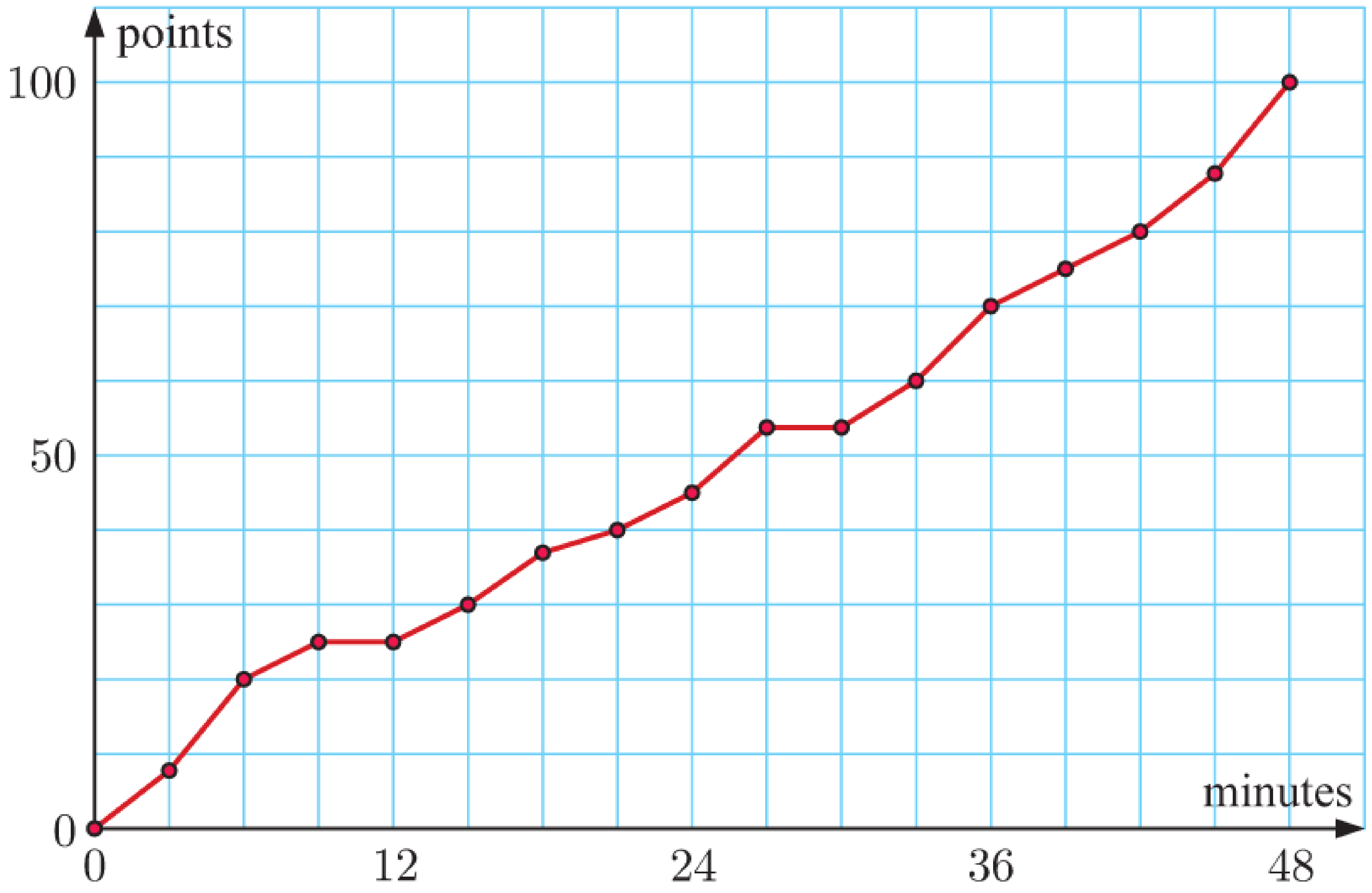
a In which month was the average temperature:

i highest **ii** lowest?

b What happens to the temperature in Johannesburg from July to January?

c In which month (or months) was the average temperature about 26°C?

6 The graph below shows the progress of a basketball team during a match. Their points were recorded every three minutes.



Use the graph to determine:

a the total number of points at the end of the:

i first quarter **ii** second quarter

iii third quarter **iv** match.

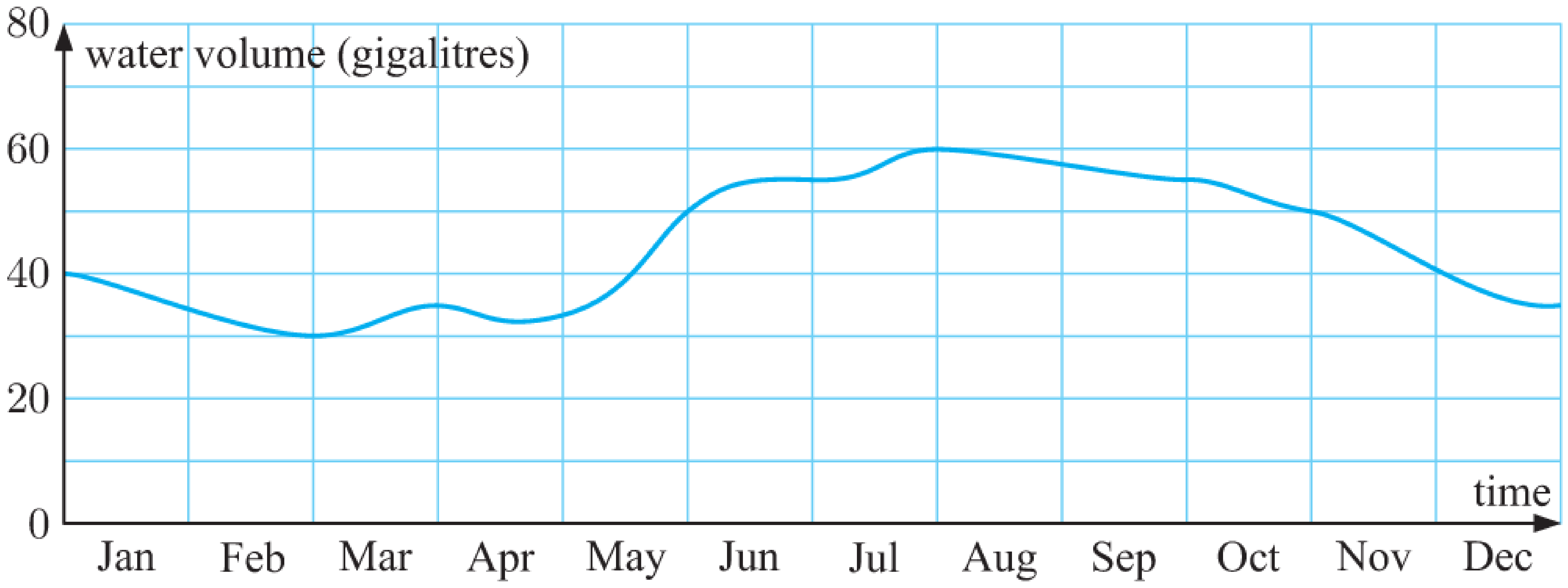
b In which quarter did the team score the most points?

c During which time intervals was the team unable to score?

d During which time interval did the team's score reach 80?

7 Answer the **Opening Problem** on page **346**.

8 The volume of water in a New Zealand lake over the course of a year is shown below.



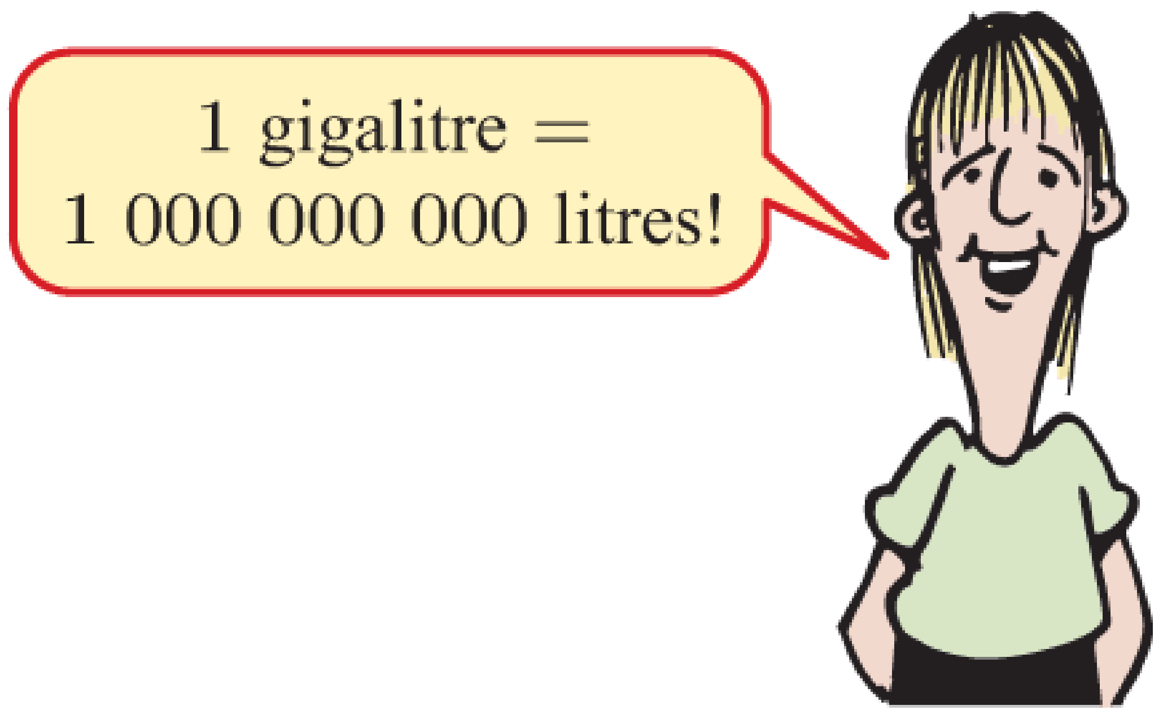
a Estimate the volume of water in the lake at the start of July.

b When was the water volume least?

c What was the maximum water volume?

d How much water was taken from the lake between the start of January and the start of March?

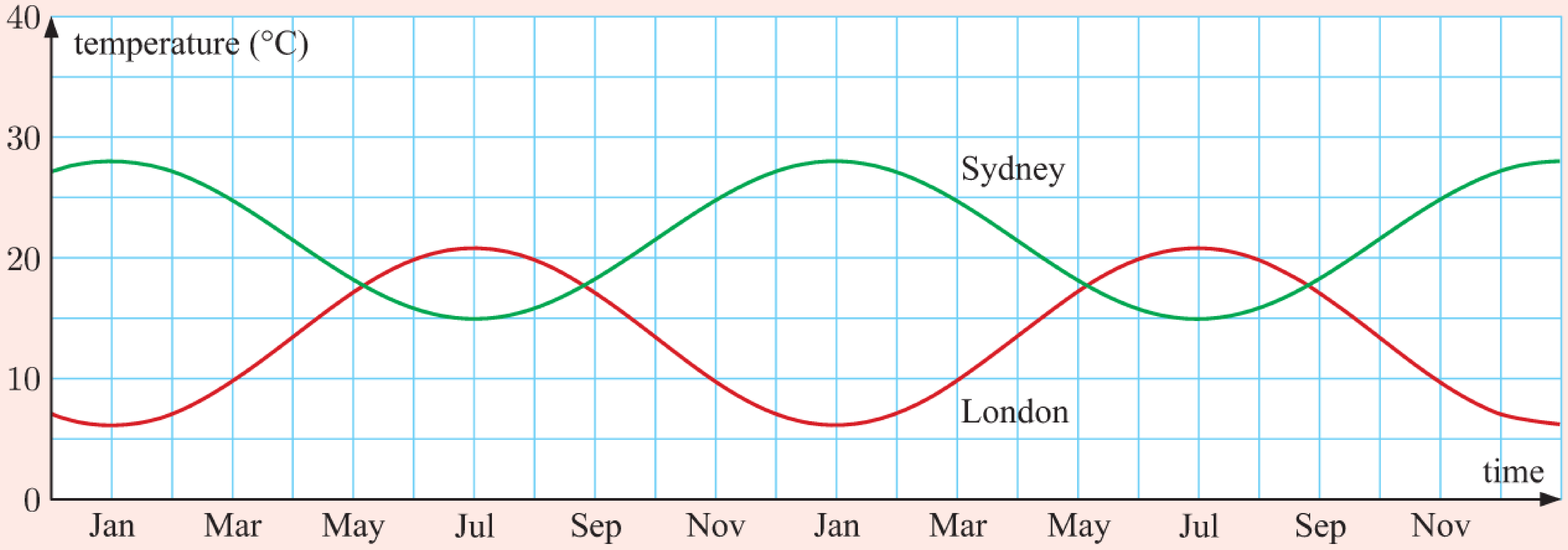
e During which months was there at least 50 gigalitres of water in the lake?



ACTIVITY 1 TEMPERATURE GRAPHS

When we want to compare two sets of data, we can place the information on the same graph.

The graph below shows the average temperature for London and Sydney over a two year period.



What to do:

1 Explain why the two curves are increasing and decreasing at different times.

2 In which month was the average temperature a minimum in:
  a London **b** Sydney?

3 In which month was the average temperature a maximum in:
  a London **b** Sydney?

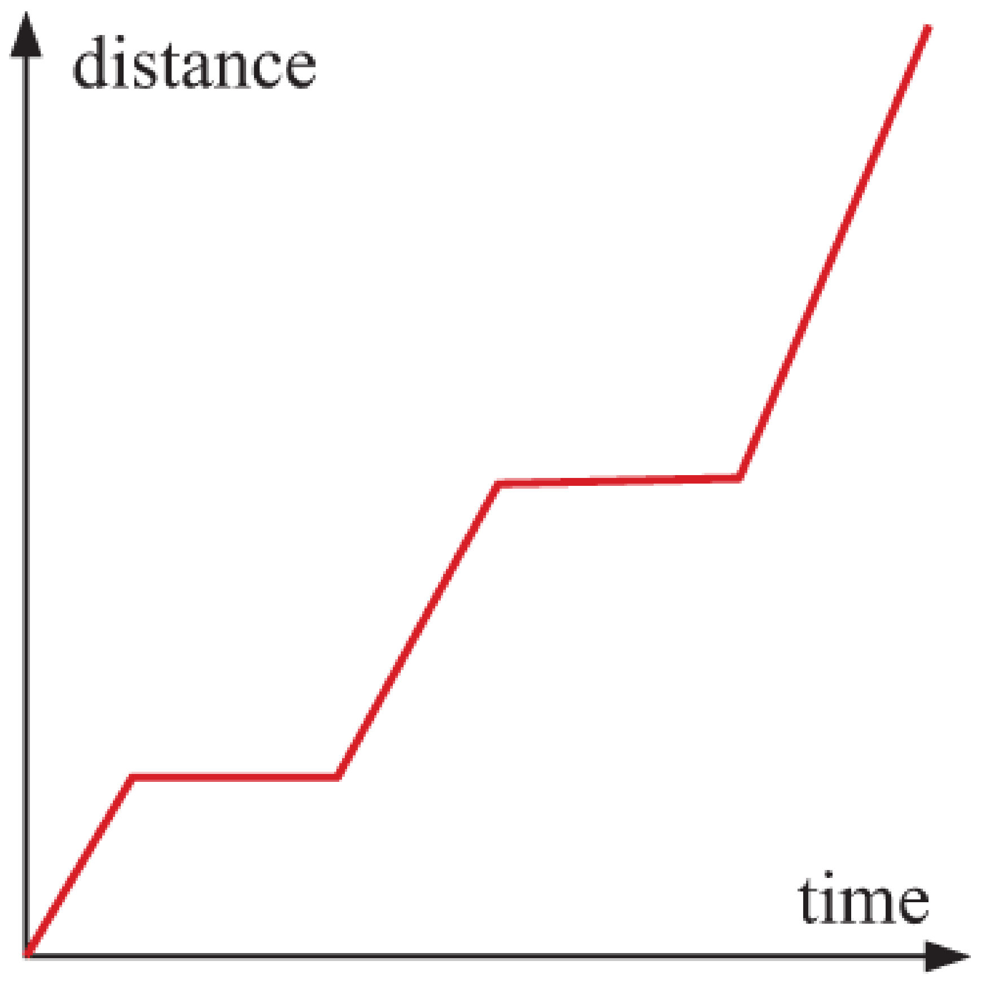
4 In which months are London and Sydney the same temperature?

5 Overall, which city is hotter? Explain your answer.

B TRAVEL GRAPHS

The **travel graph** for a journey shows the relationship between the distance travelled and the time taken to travel that distance.

For these graphs, time is on the x-axis, and distance is on the y-axis.



Example 2 **Self Tutor**

The graph shows the progress of Juen as he cycles to school.

a How long did Juen take to cycle to school?

b How far is it from his home to the school?

c On his way to school, Juen was stopped at a set of traffic lights.

   i How far are these lights from his home?

   ii For what length of time was he stopped at these lights?

d Juen's friend lives on the way to school and 3 km from Juen's house. How long did it take Juen to reach his friend's house?

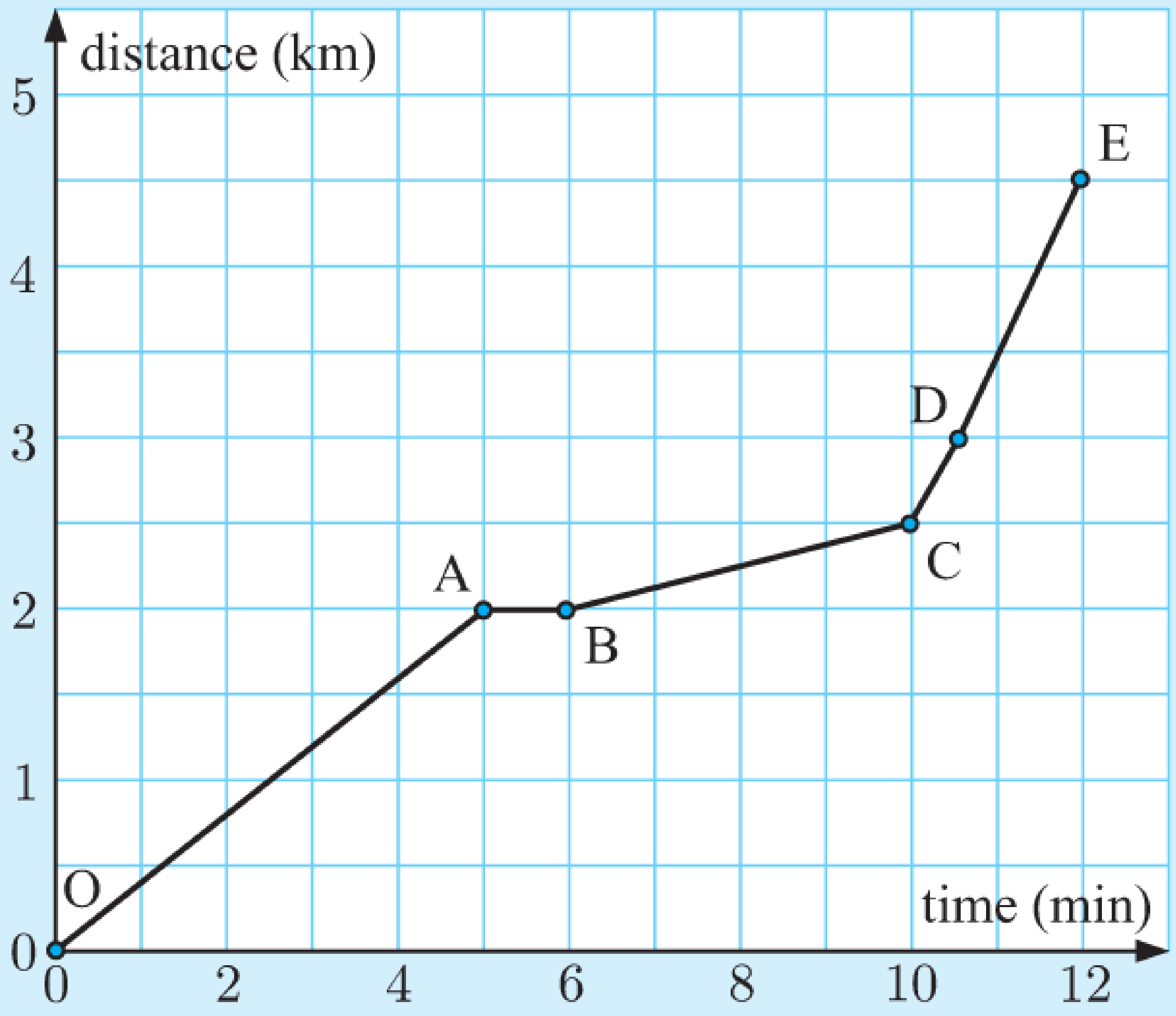
e How far did Juen cycle:

   i in the first 5 minutes

   ii between the 6th and 10th minutes

   iii between the 10th and 12th minutes?

f There is a steep hill on Juen's way to school. When did he reach the top?



a 12 minutes {point E}

b $4\frac{1}{2}$ km {point E}

c **i** The lights are 2 km from Juen's home.

   ii He was stopped for 1 minute. {between points A and B}

d $10\frac{1}{2}$ minutes {point D}

e **i** 2 km {O to A}

   ii $\frac{1}{2}$ km {B to C}

   iii 2 km {C to E}

f Juen rode quite slowly from B to C. We assume this is the hill. He reached the top after 10 minutes. {point C}

EXERCISE 18B

1 The graph alongside shows the distance travelled by a family car on a holiday journey.

   a How many hours did the trip take?

   b What was the total distance of the trip?

   c How far had they travelled after 3 hours?

   d How long did it take them to travel the first 100 km?

   e What distance was travelled in the:

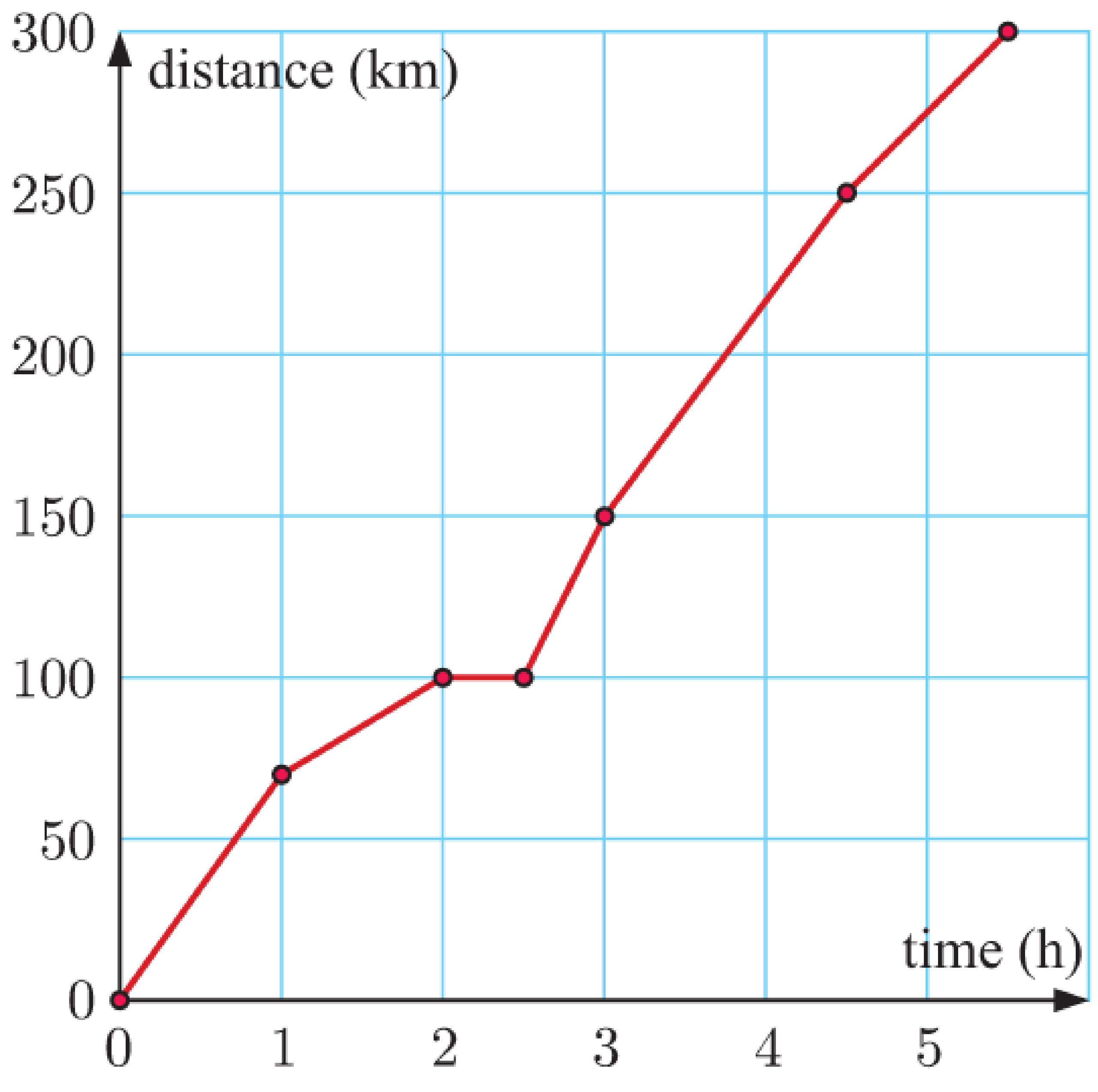
      i first hour

      ii second hour

      iii third hour

      iv last 2 hours?

   f Describe what happened during the third hour.

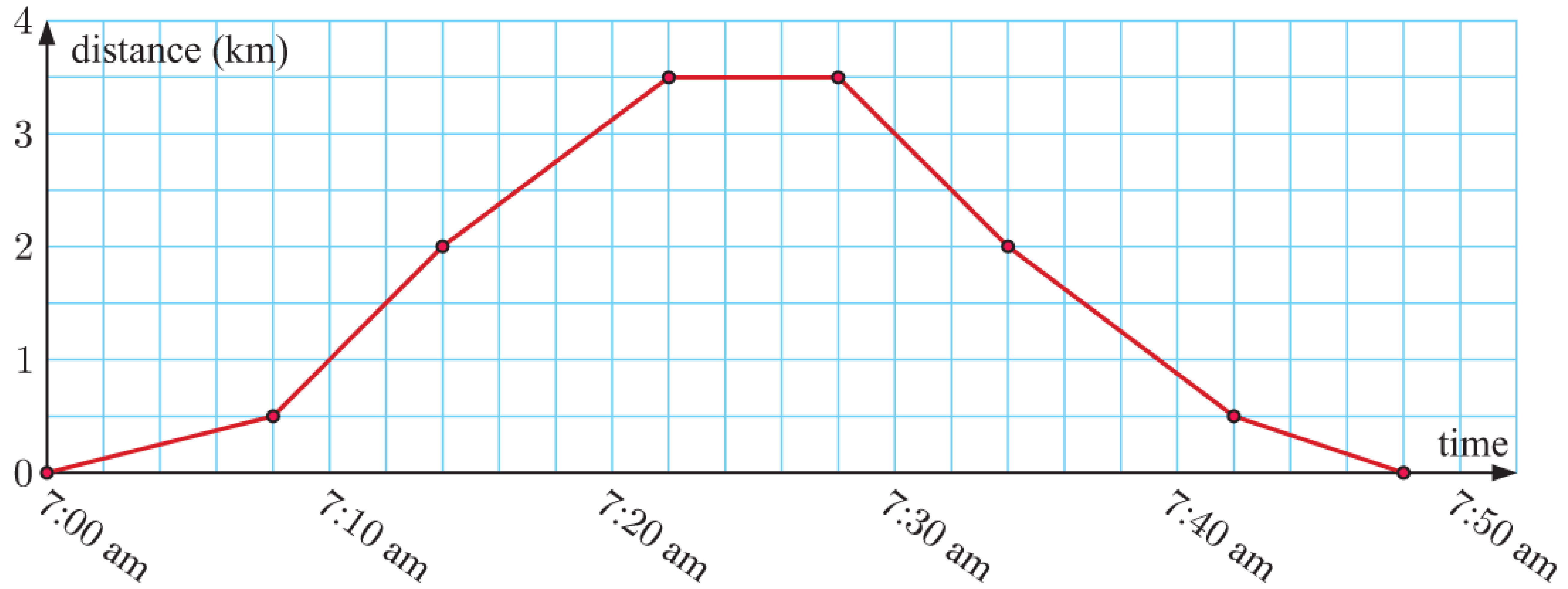


2 Two families travel 900 km by car from Boston to Cleveland. Their journeys are shown on the graph alongside.

Use the graph to determine:

- **a** which family arrived in Cleveland first
- **b** how long the Williams family were stopped for lunch
- **c** the distance travelled by the Murphy family in the first 5 hours.

3 The graph below indicates the distance of a jogger from her home at various times:

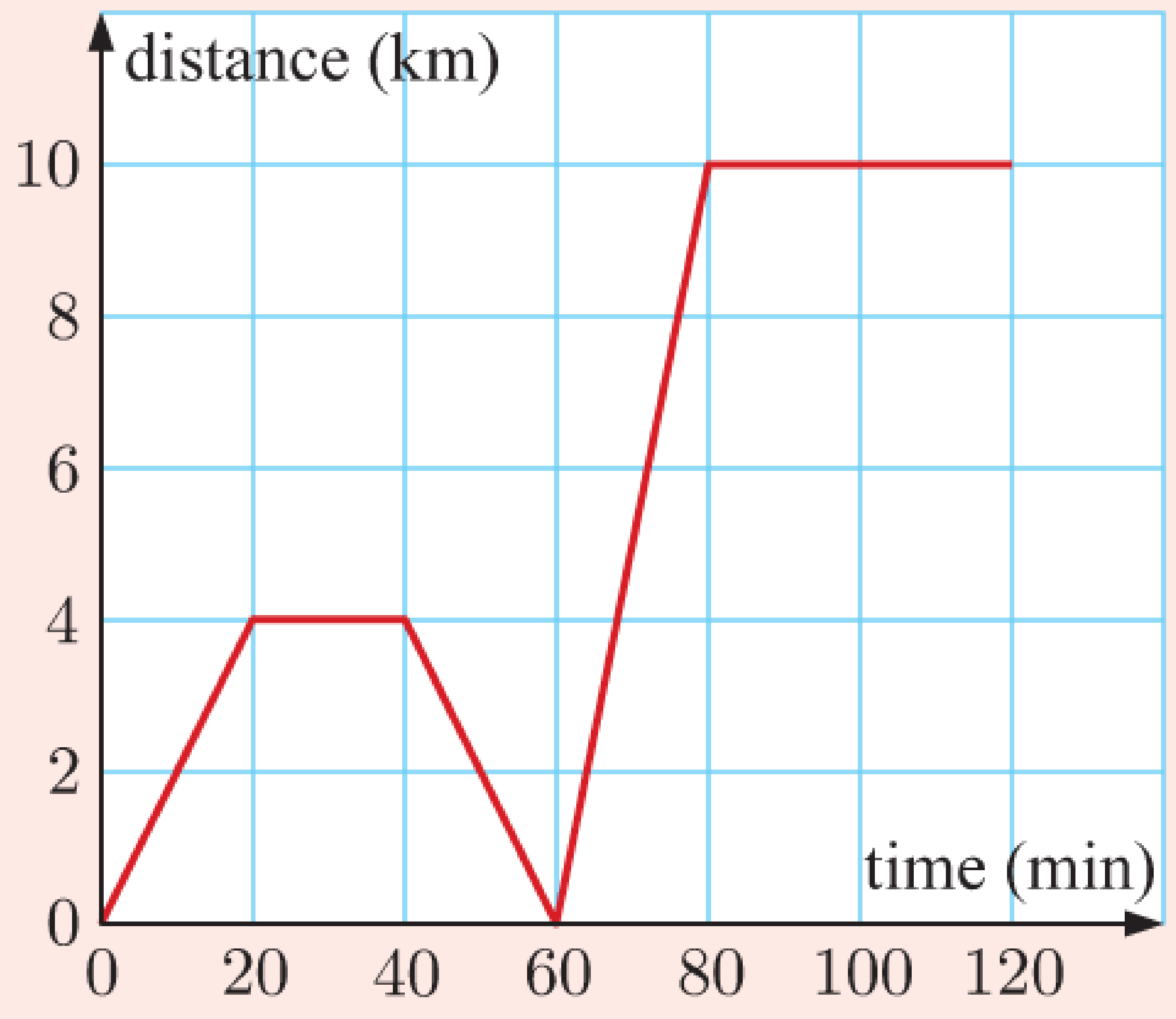


Use the graph to determine:

- **a** at what time the jogger arrived back home
- **b** the total distance travelled by the jogger
- **c** the total time the jogger was away from home
- **d** for how long the jogger rested at the halfway point
- **e** how far the jogger was from home at: **i** 7:10 am **ii** 7:30 am
- **f** how far the jogger travelled between: **i** 7 am and 7:14 am **ii** 7:32 am and 7:42 am.

ACTIVITY 2 WRITING A STORY TO FIT THE GRAPH

The graph alongside shows the distance of Taylor from home over two hours.



There could be many reasons for the graph taking this form.

One story describing the graph is:

"Taylor caught the train to her friend's house. Her plan was to go directly from her friend's house to their netball match, but Taylor realised that she had left her netball shoes at home. She went back home on the train to get her shoes, and since she was running late, her mum drove her to the netball game, arriving just in time."

What to do:

1. Draw your own travel graph, and write a short story to describe it.
2. Swap graphs with a friend. Write a short story using your friend's graph.

C CONVERSION GRAPHS

Conversion graphs are special line graphs which enable us to convert from one quantity to another. We can use conversion graphs to convert between currencies, and between units of measurement.

Conversion graphs are usually straight lines, but they do not always pass through the origin.

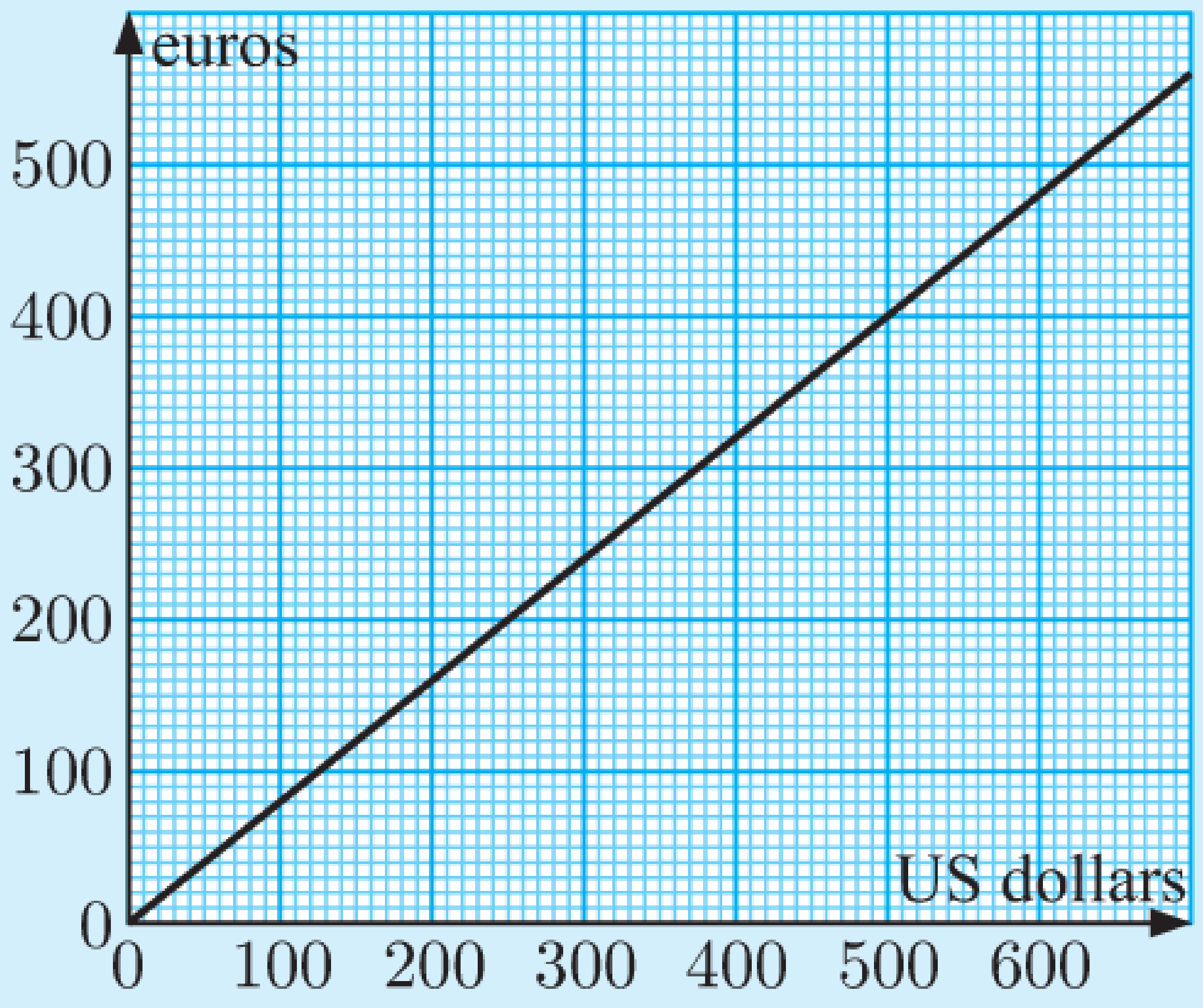
Example 3 **Self Tutor**

The graph shows the relationship between United States dollars and euros on a particular day.

Convert:

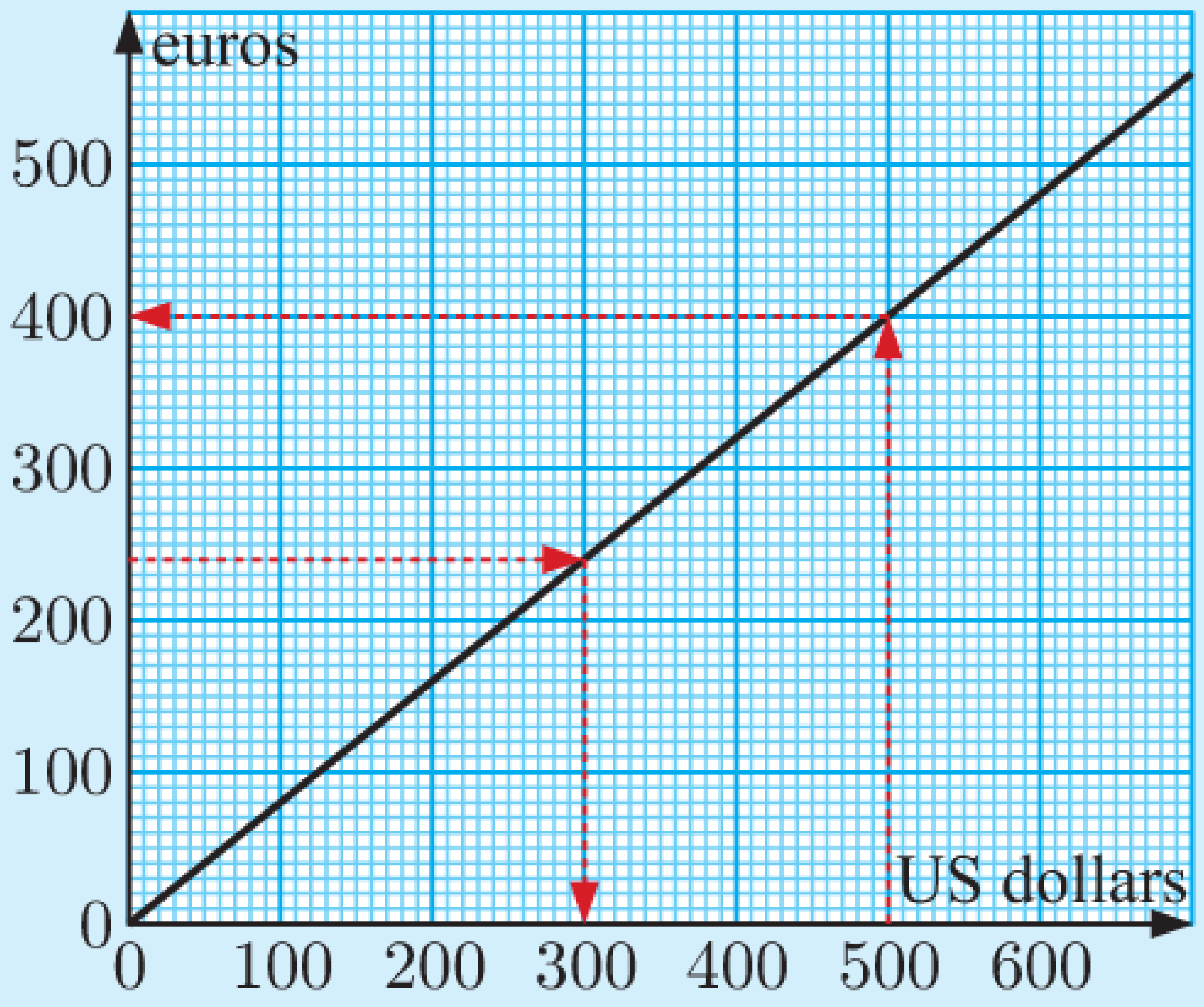
a 500 US dollars to euros

b 240 euros to US dollars.



a 500 US dollars is equivalent to 400 euros.

b 240 euros is equivalent to 300 US dollars.



EXERCISE 18C

1 Use the currency conversion graph in **Example 3** to convert:

a 200 US dollars to euros **b** 280 euros to US dollars.

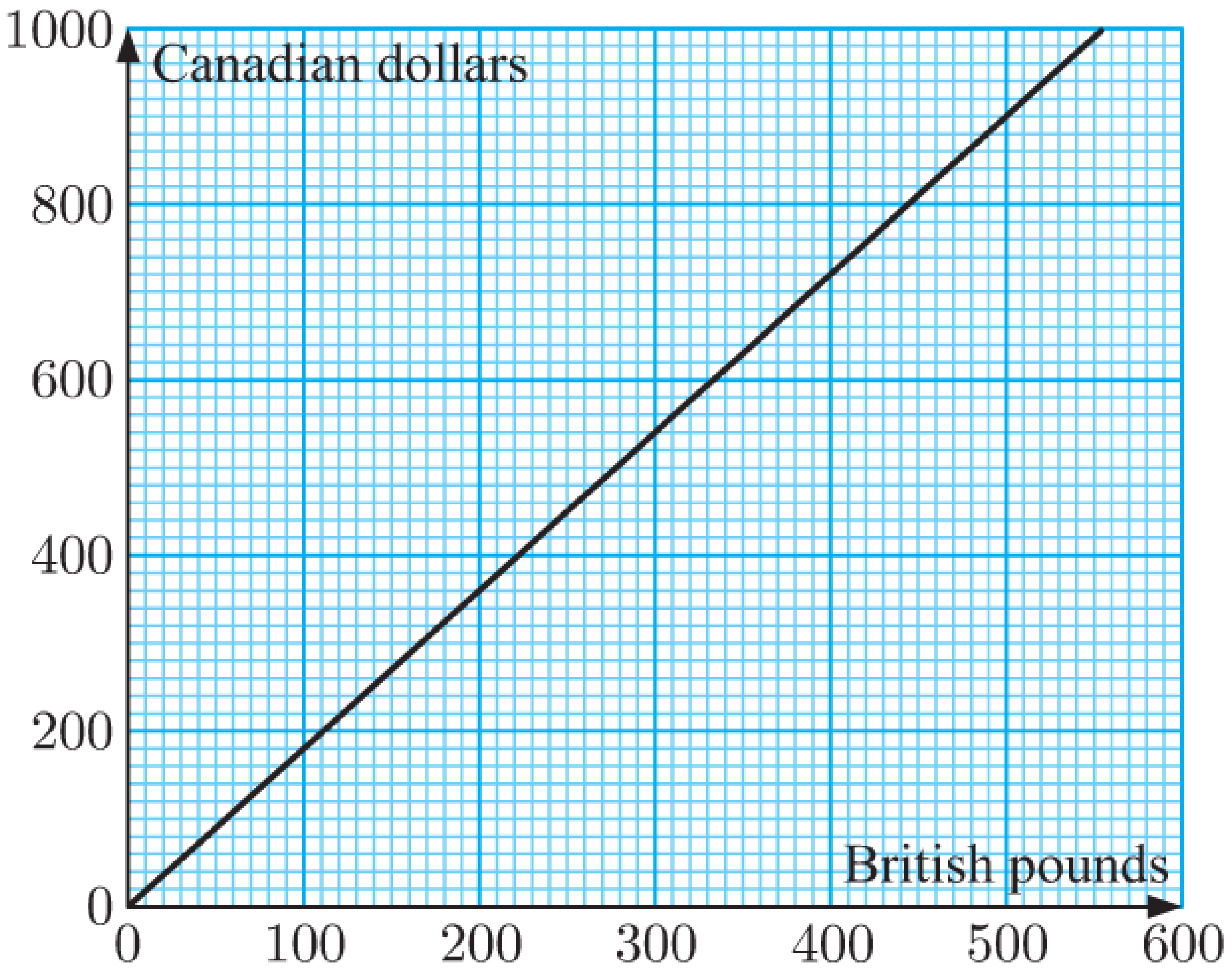
2 This graph shows the relationship between British pounds and Canadian dollars. Convert:

a 500 British pounds to Canadian dollars

b 300 British pounds to Canadian dollars

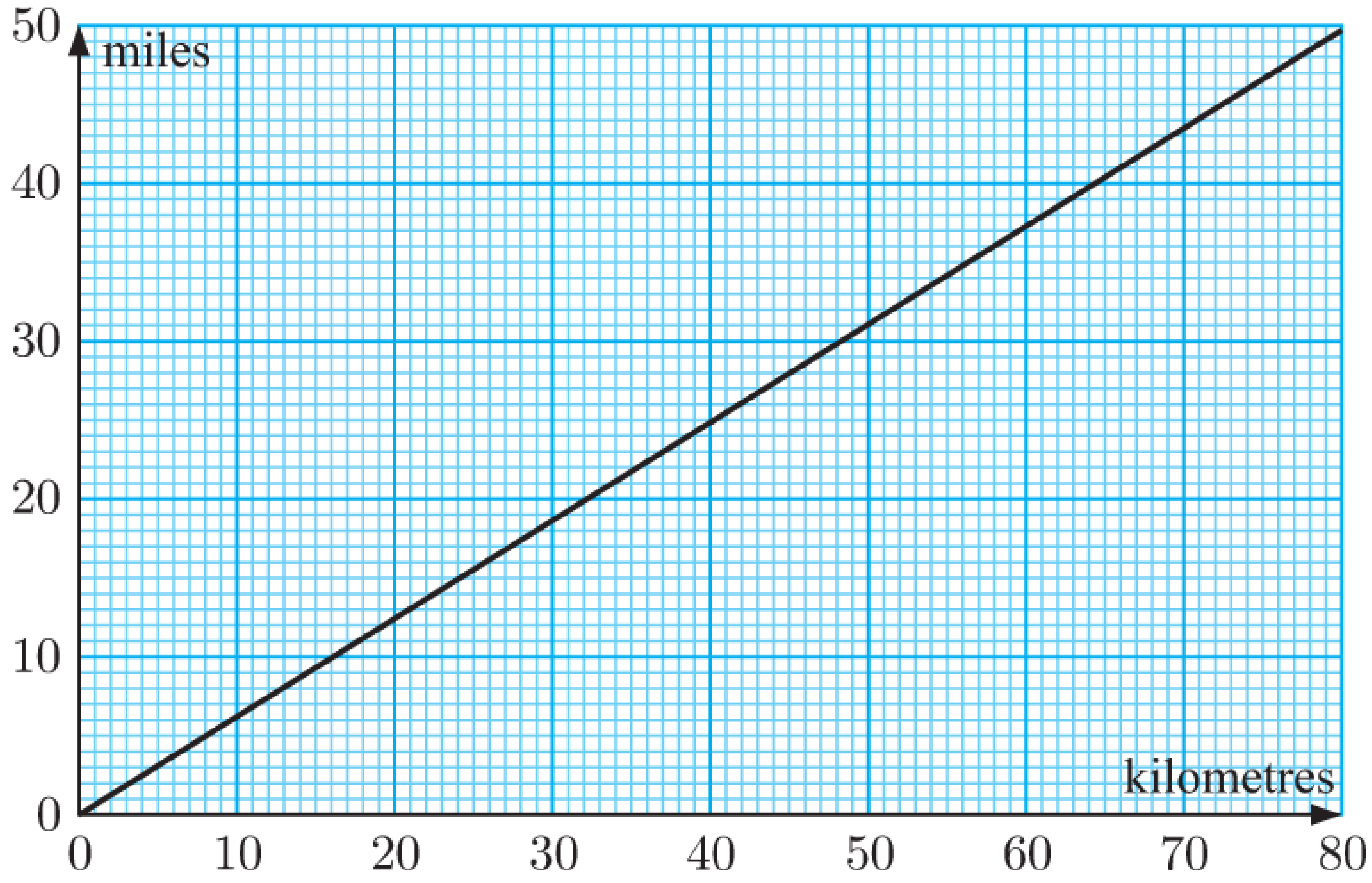
c 450 Canadian dollars to British pounds

d 720 Canadian dollars to British pounds.

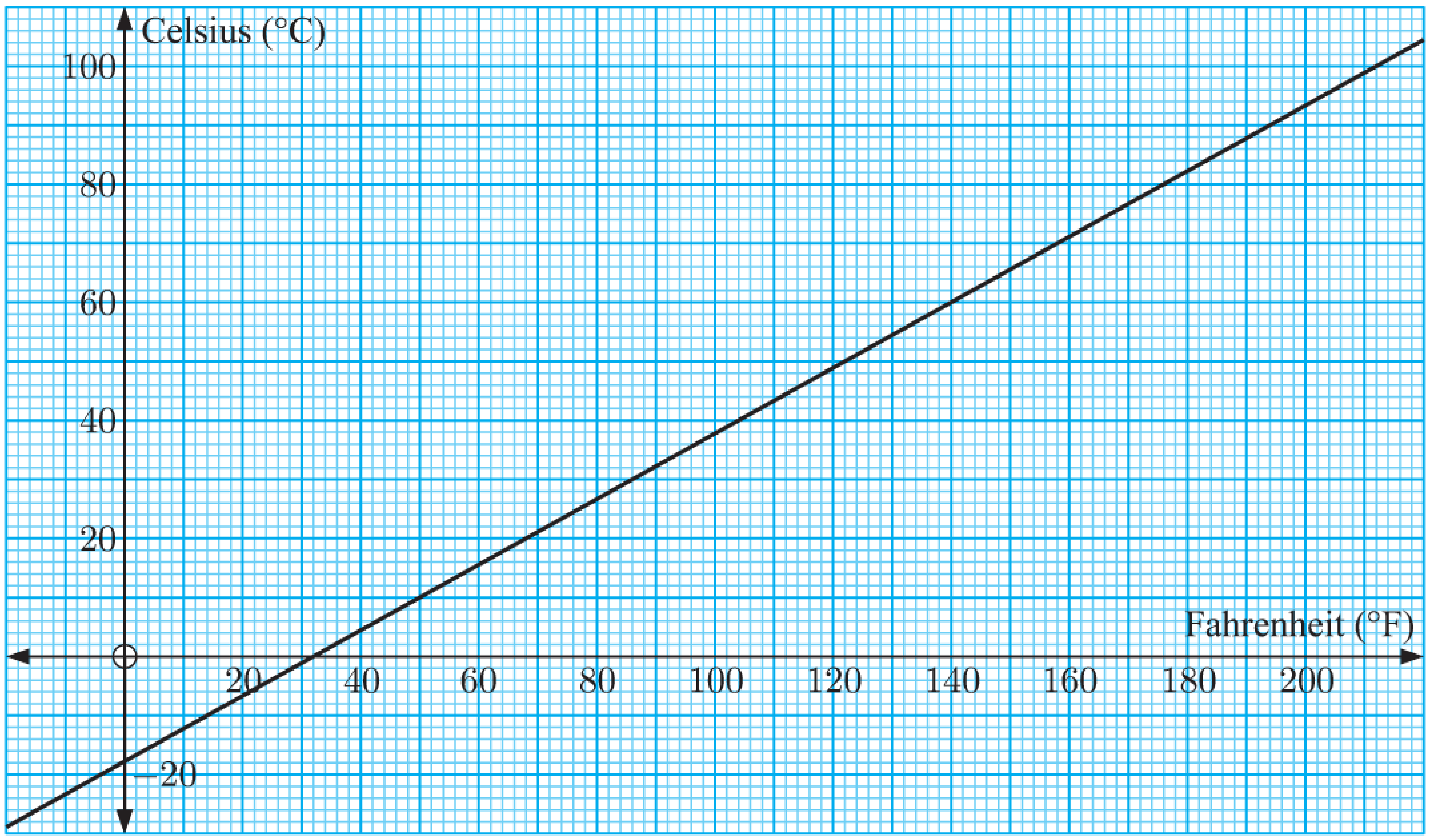


3 This graph shows the relationship between lengths in miles and kilometres. Convert:

- **a** 45 km to miles
- **b** 28 km to miles
- **c** 48 miles to km
- **d** 30 miles to km.



4 Fahrenheit and Celsius are two scales for measuring temperature. The graph below shows how to convert from one unit to the other.



- **a** Convert:
  - **i** 40°F to °C
  - **ii** 60°C to °F.
- **b** Water boils at 100°C. Find the equivalent temperature in °F.
- **c** Suppose the temperature today is 85°F. Write this temperature in °C.

ACTIVITY 3 CURRENCY TRENDS

Over a period of a month, collect the **currency exchange rates** which compare your local currency with either the US dollar or the euro. These are easily available on the internet.

1 Graph the currency exchange rate against time, updating the graph each day.

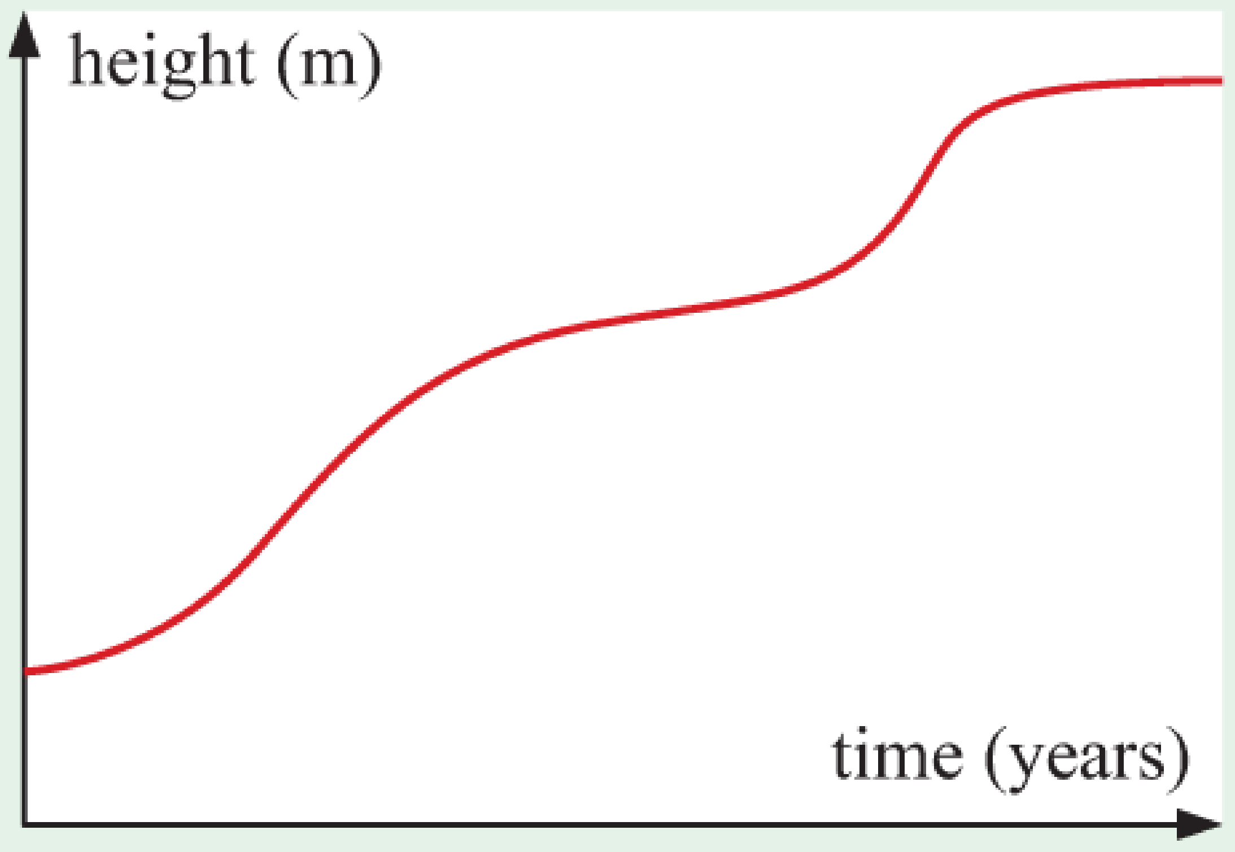
2 Discuss how the currency exchange rate affects the conversion graph between the currencies.

KEY WORDS USED IN THIS CHAPTER

- conversion graph
- line graph
- decreasing
- point graph
- increasing
- travel graph

REVIEW SET 18A

1 State whether this line graph is increasing, decreasing, or increasing in some sections and decreasing in others.



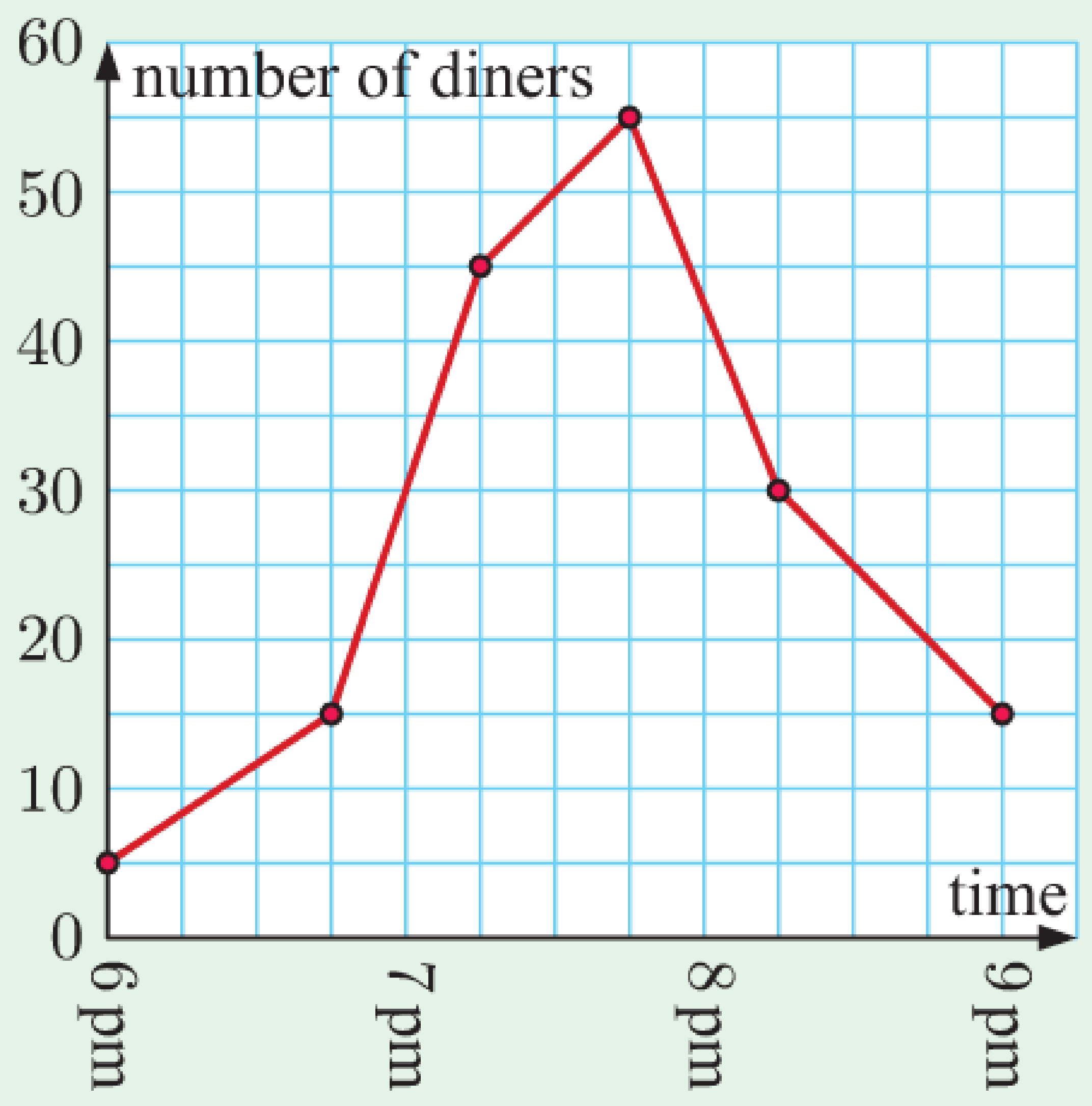
2 The number of diners in a restaurant was recorded throughout an evening. The results are shown in the line graph.

a At what time was the number of diners in the restaurant:

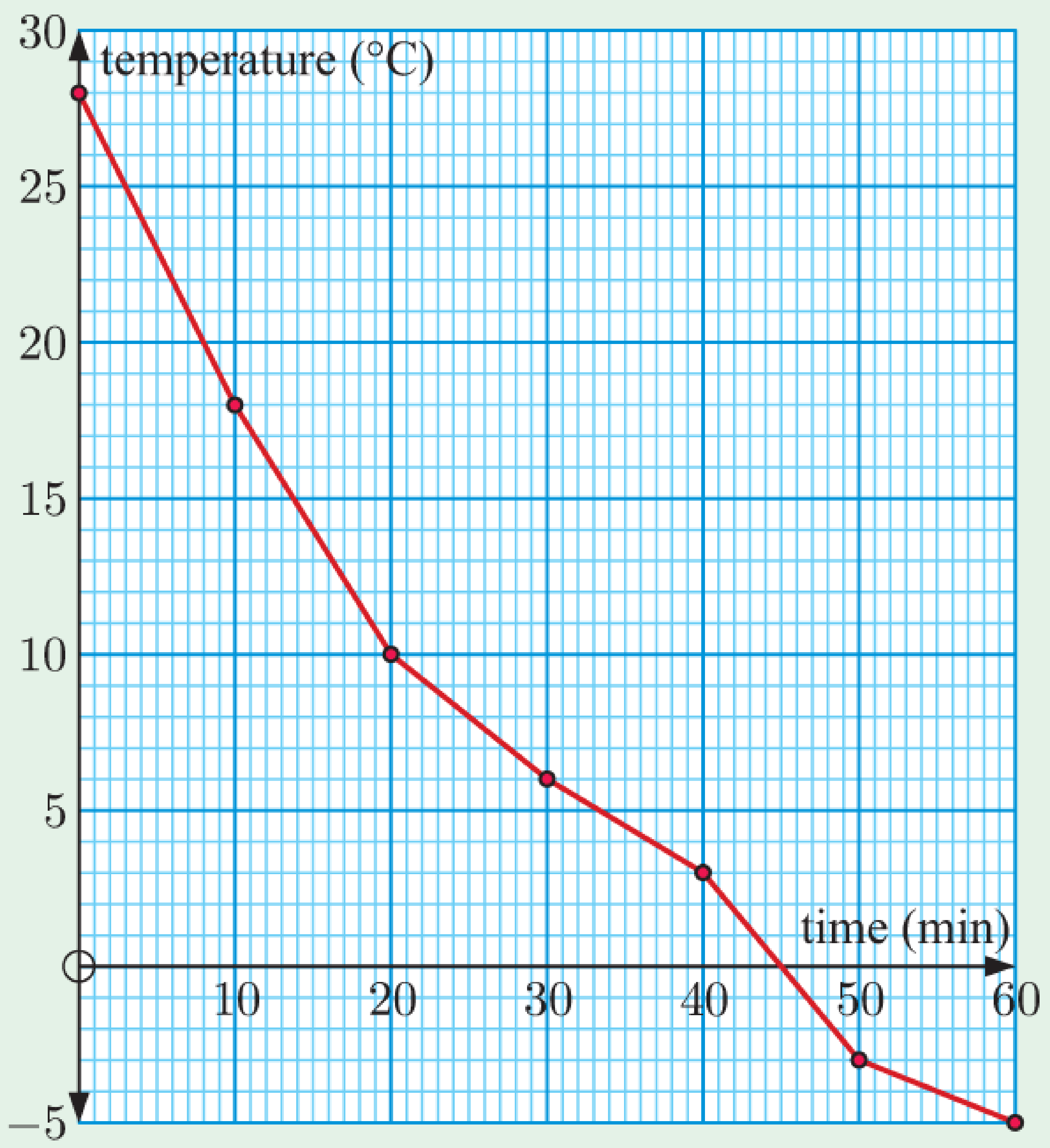
i greatest **ii** least?

b Estimate when there were 25 diners in the restaurant.

c Estimate the number of diners in the restaurant at 8 pm.



3



Some chocolate custard was placed into an ice cream churn. Its temperature was recorded at 10-minute intervals and the results graphed alongside. Find:

a the temperature of the liquid when it was first placed in the ice cream churn

b the time taken for the temperature to drop to 0°C

c the temperature of the custard after 10 minutes in the ice cream churn

d the fall in temperature during the first 15 minutes.

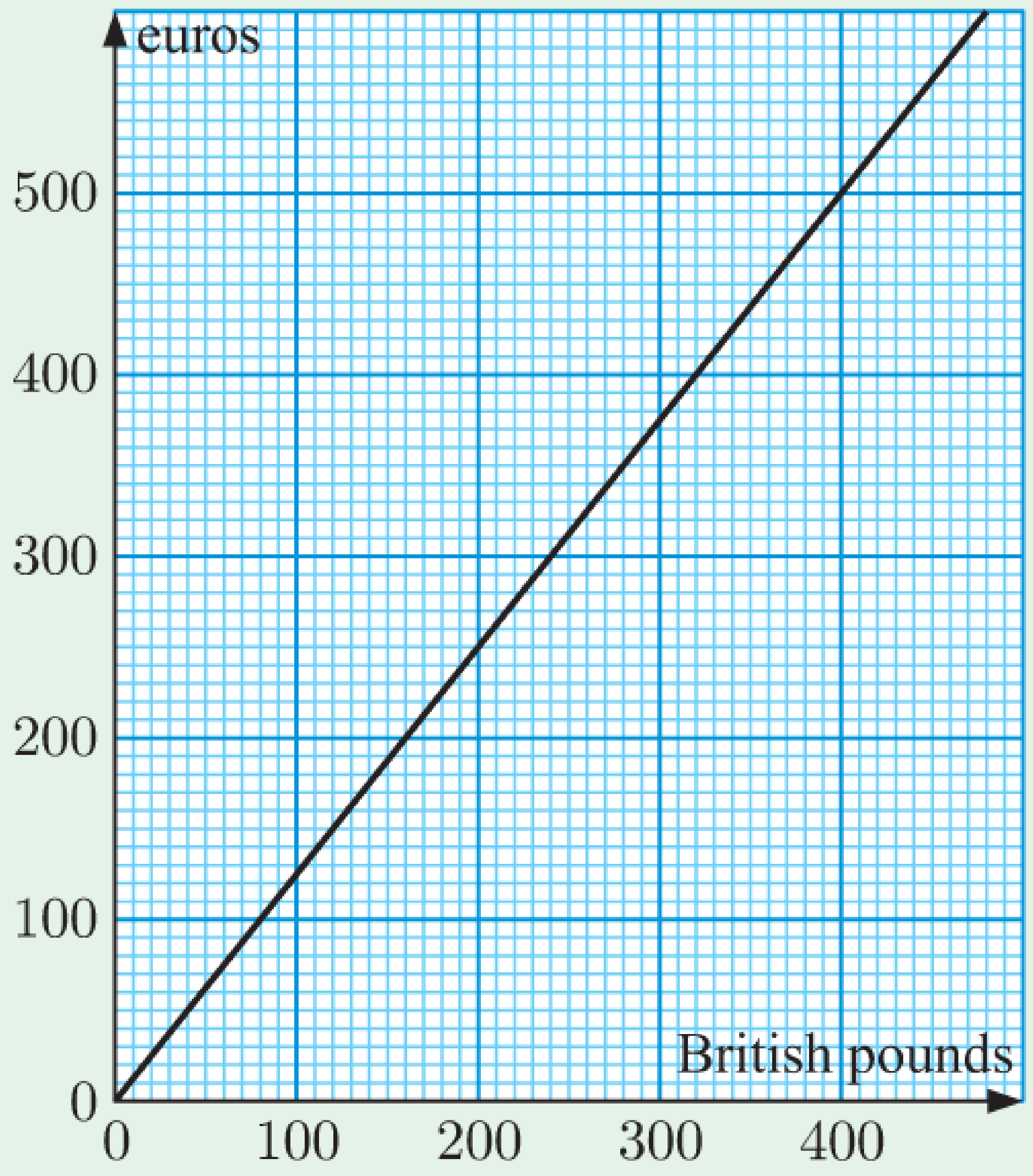
4 This graph shows the relationship between British pounds and euros.
Convert:

a 200 British pounds to euros

b 360 British pounds to euros

c 500 euros to British pounds

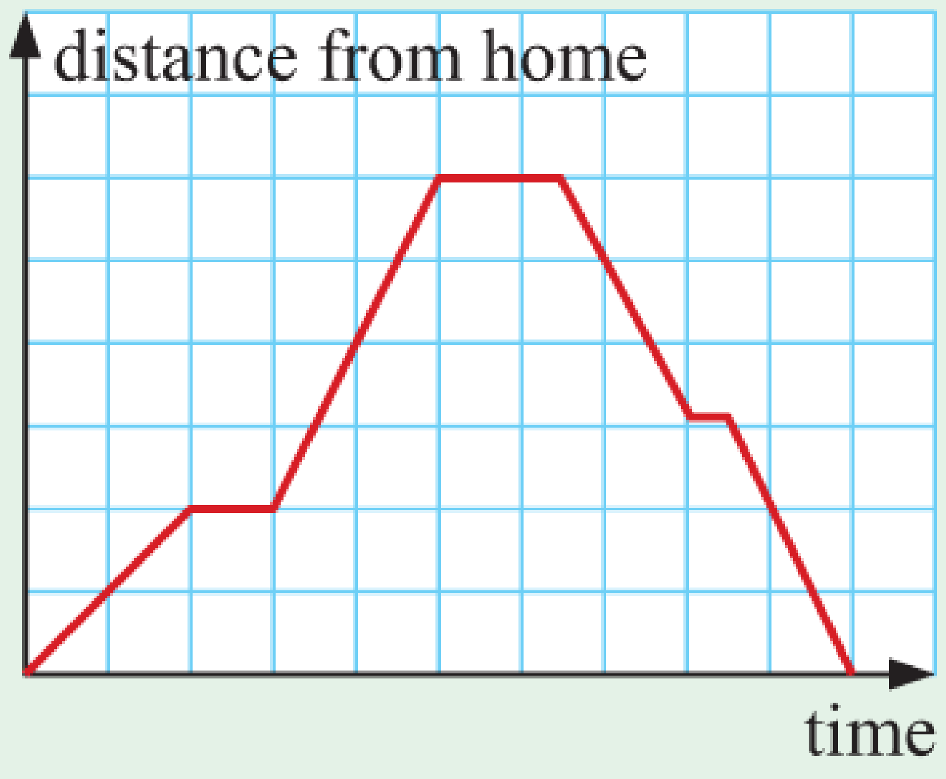
d 150 euros to British pounds.



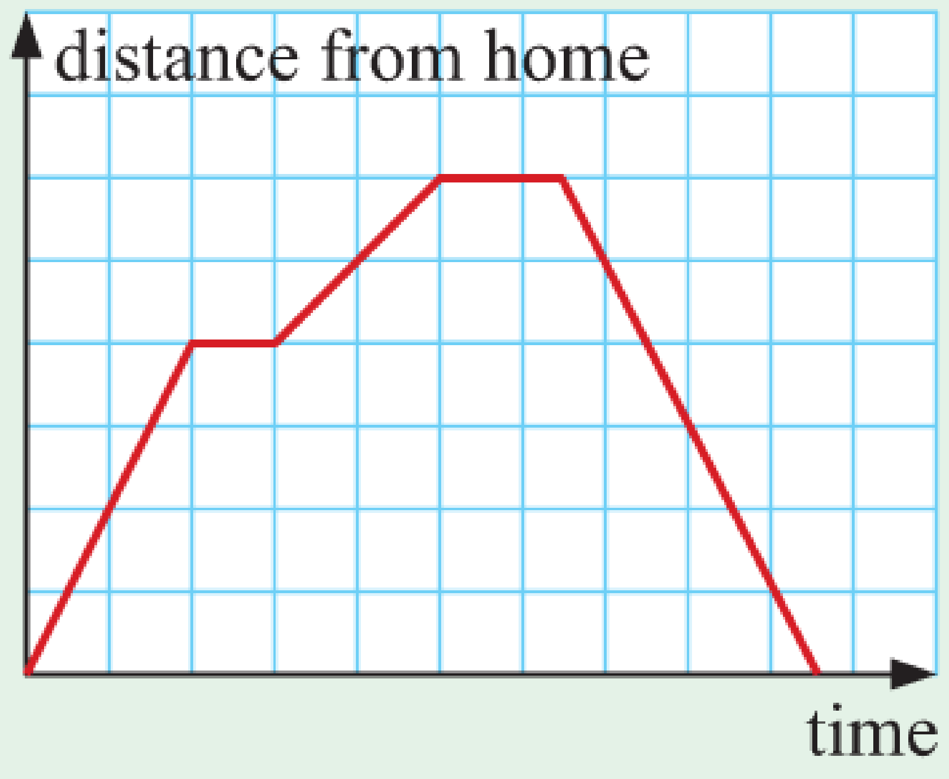
5 Select the travel graph which best matches this story:

Jane left home, walking quite quickly. She stopped at a set of traffic lights. She then climbed a steep hill, walking more slowly. At the top she stopped for a drink. She then returned home, pausing briefly about halfway home.

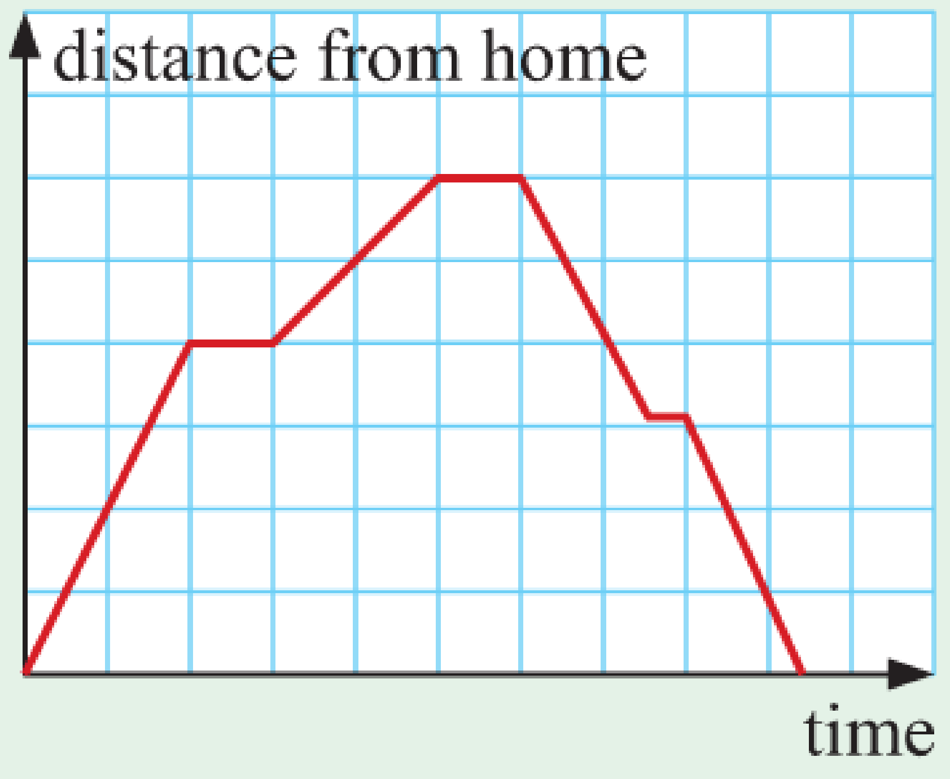
A



B



C



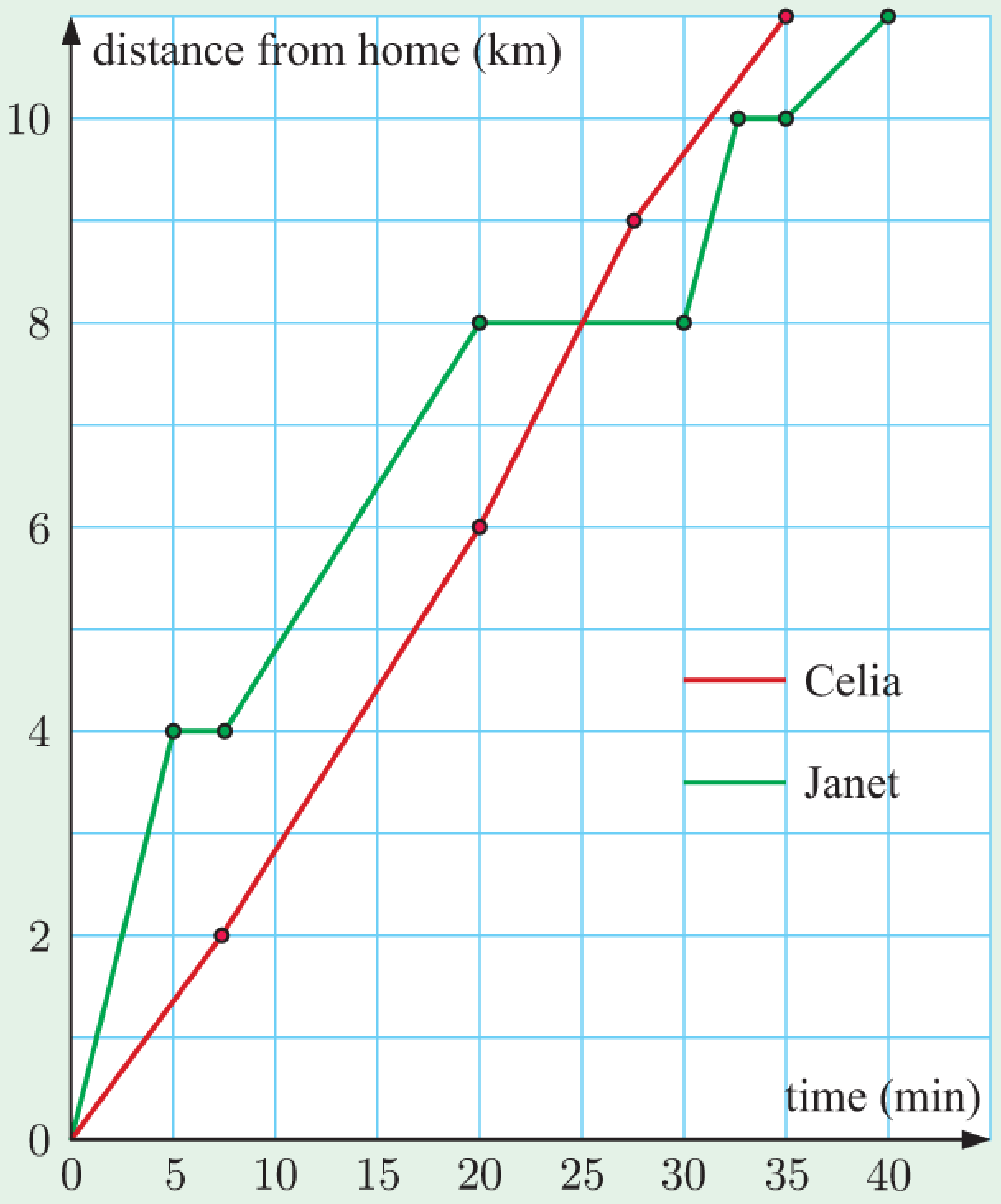
6 Celia and Janet are flatmates who also work in the same office. One day, Celia decides to ride her bicycle to work, while Janet drives her car. The travel graph shows their journeys.

a How far is it from the girls' flat to their office?

b Who arrives at the office first?

c How many times was Janet stopped on the way to work?

d How far was Janet from work when she stopped for 10 minutes while a freight train passed?

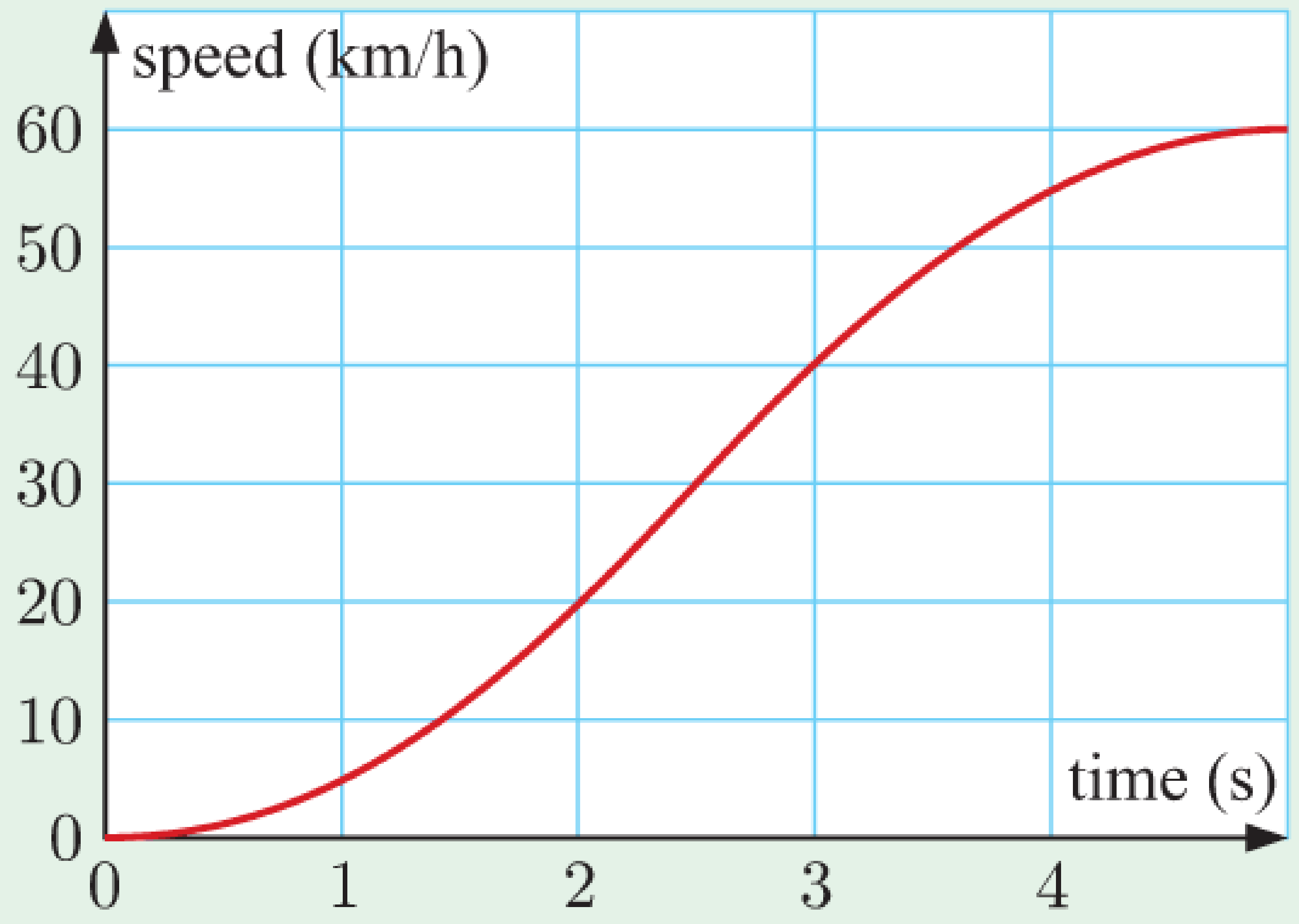


REVIEW SET 18B

1 The graph opposite shows the speed of a car as it accelerates from traffic lights.

a Estimate the speed after 3 seconds.

b How long does it take for the car to reach 20 km/h?



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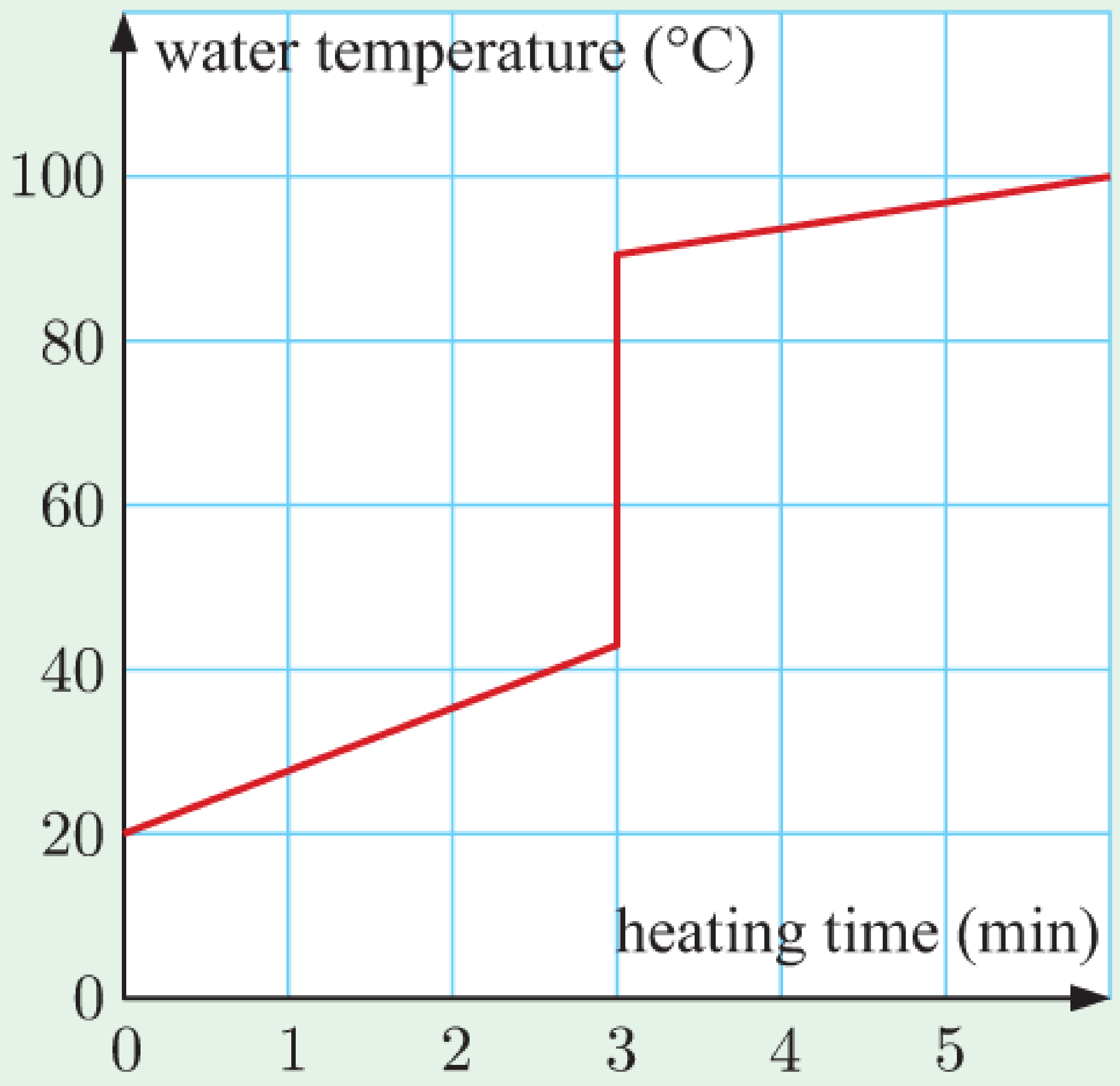


Joy is heating water on her stove. She is in a hurry, and decides to speed up the process by adding some boiling water straight from the kettle.

a What was the initial temperature of the water?

b At what time did Joy add water from her kettle?

3 The population of salmon in a lake over a 10 year period is displayed in the graph alongside.

a At what time was the population greatest?

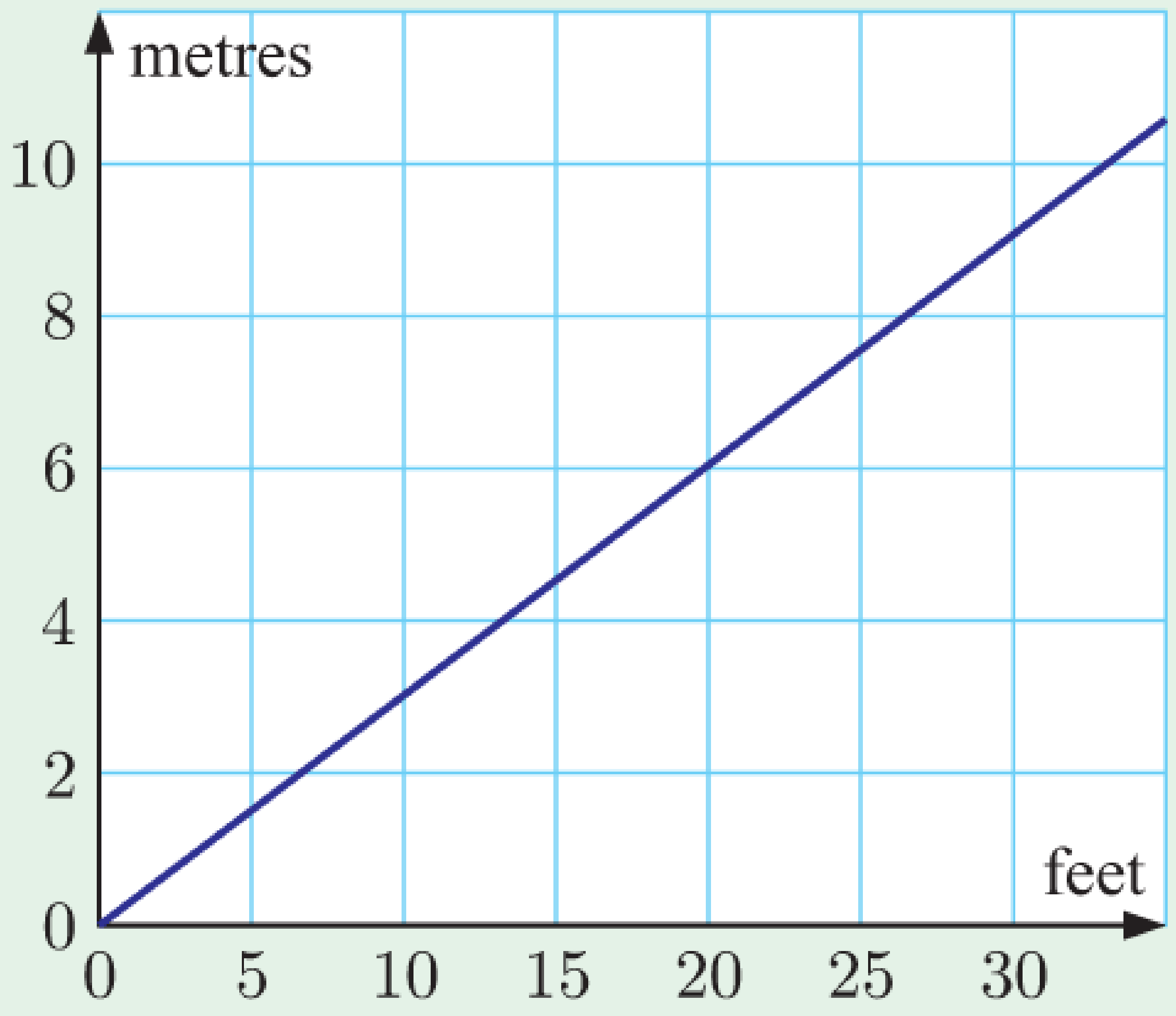
b What was the minimum population, and when did it occur?

c At what times was the population:

i increasing **ii** decreasing?

d Estimate the times when the population was 350.

4



This graph shows the relationship between distances measured in metres and feet.

Convert:

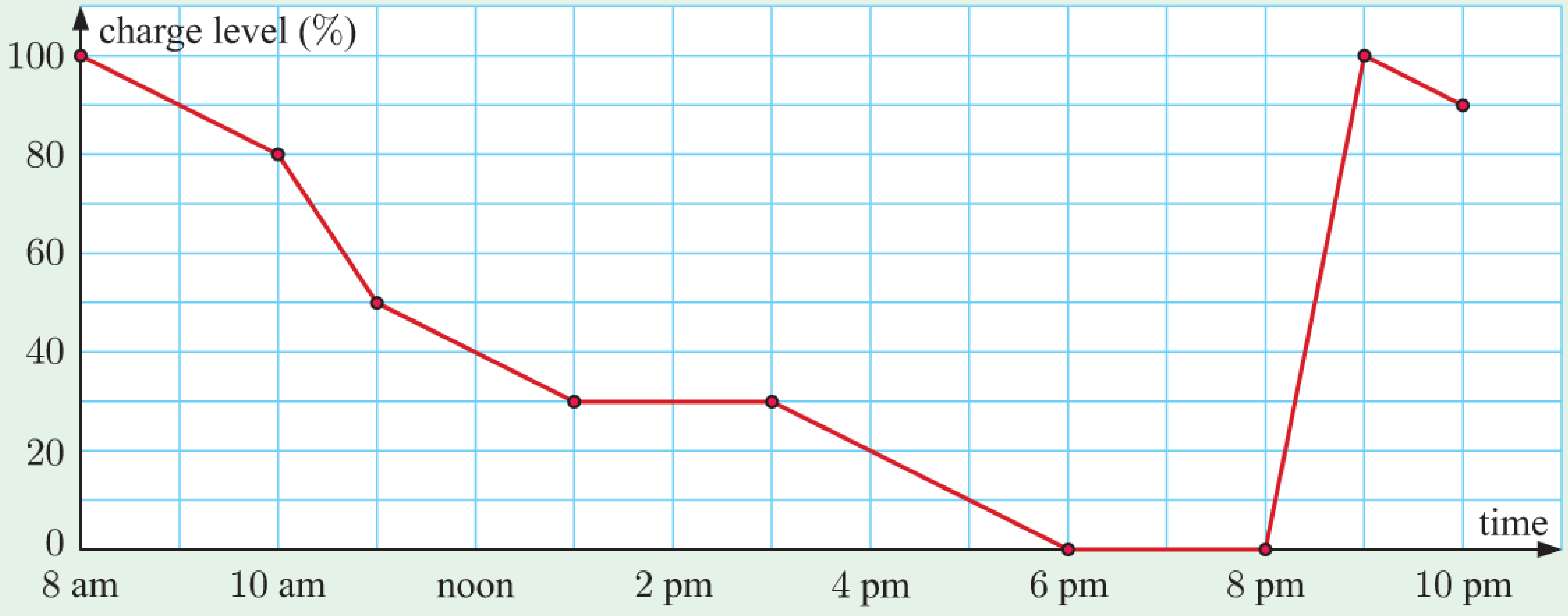
a 20 feet to metres

b 5 feet to metres

c 8 metres to feet

d 2 metres to feet.

5 The graph below shows the battery charge level of Jessica’s laptop during a day.



a What was the charge level at noon?

b During which hour was Jessica using her laptop the most?

c At 1 pm, Jessica turned off her laptop to conserve the battery. For how long was the laptop turned off?

d At what time did the laptop run out of battery charge?

e At what times was the laptop fully charged?

6 Two long distance runners Hissam and Mohammed decided to have a race one afternoon. The distance of each runner from the starting point is shown on the graph.

Use the graph to determine:

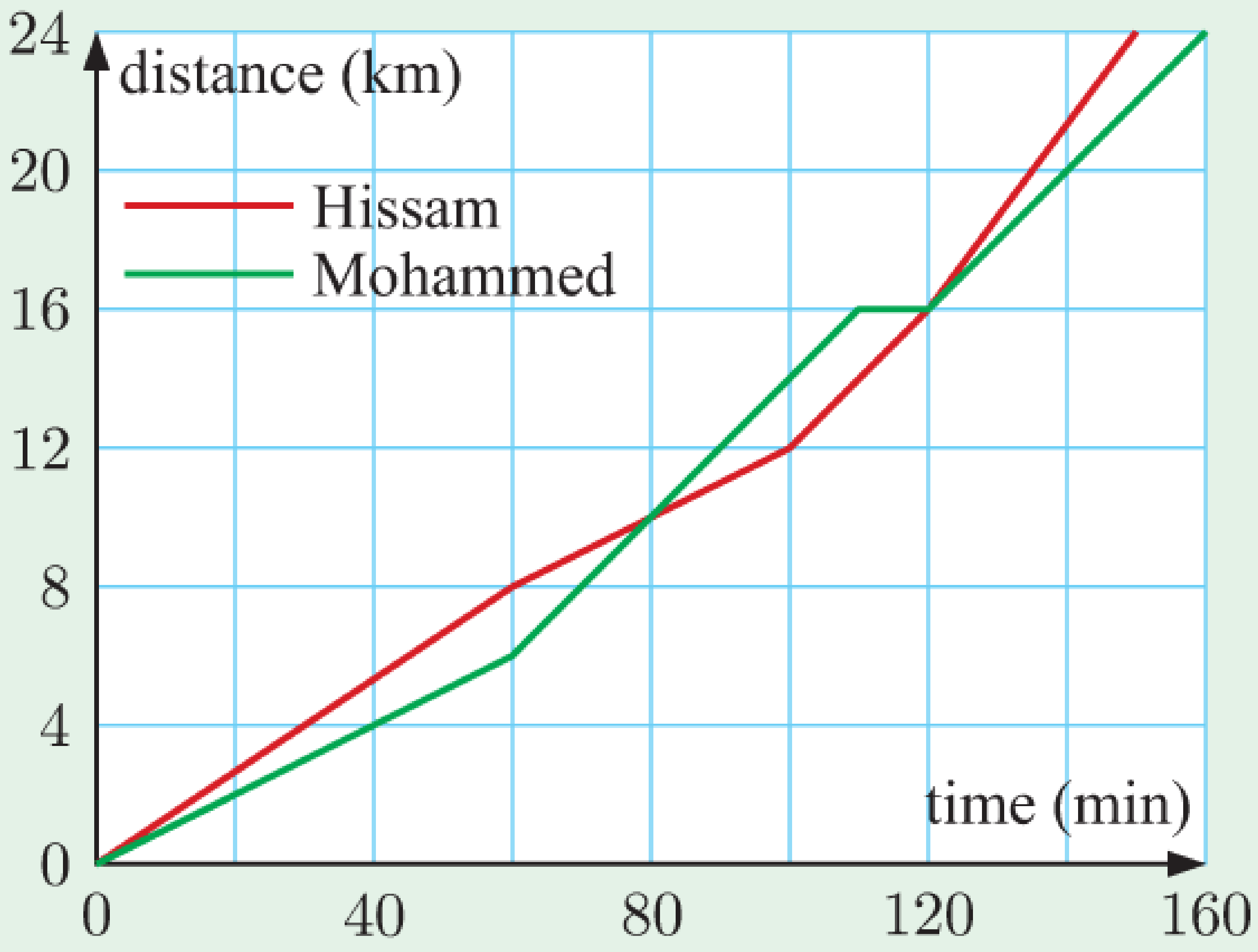
a the distance of the race

b who was leading the race after 1 hour

c the time at which Mohammed overtook Hissam

d how far Mohammed had run before he developed cramp and had to stop and rest

e who won the race.



ANSWERS

EXERCISE 1A.1

1 **a** 75 **b** 340 **c** 1608 **d** 4162
e 31 427 **f** 21 123 423

2

3 **B** = **E** = 134 **4** 27 Egyptian symbols

EXERCISE 1A.2

1 **a** 13 **b** 26 **c** 204 **d** 3240 **e** 723 **f** 5259

2 **a** ΔIIII **b** ΔΔΔI **c** ΔΔΓII
d 𐅄III **e** 𐅄ΔΓIII **f** 𐅄ΔΔΔΔΓIIII
g 𐅅𐅄Γ **h** XXXX𐅄ΔΔΔII **i** 𐅆𐅅HI
j 𐅆XXHH𐅄ΔIIII

3 15 symbols

EXERCISE 1A.3

1 **a** 8 **b** 14 **c** 16 **d** 31 **e** 110
f 81 **g** 125 **h** 216 **i** 62 **j** 1156
k 550 605 **l** 720 **m** 419 **n** 555 501 **o** 2 300 000

2 **a** XVIII **b** XXXIV **c** LXV **d** CXLI
e CCLXXIX **f** CMII **g** MXLVI **h** MMDLI
i $\overline{\text{V}}$MXXXII **j** $\overline{\text{X}}\overline{\text{X}}\overline{\text{X}}$MCMLXVII

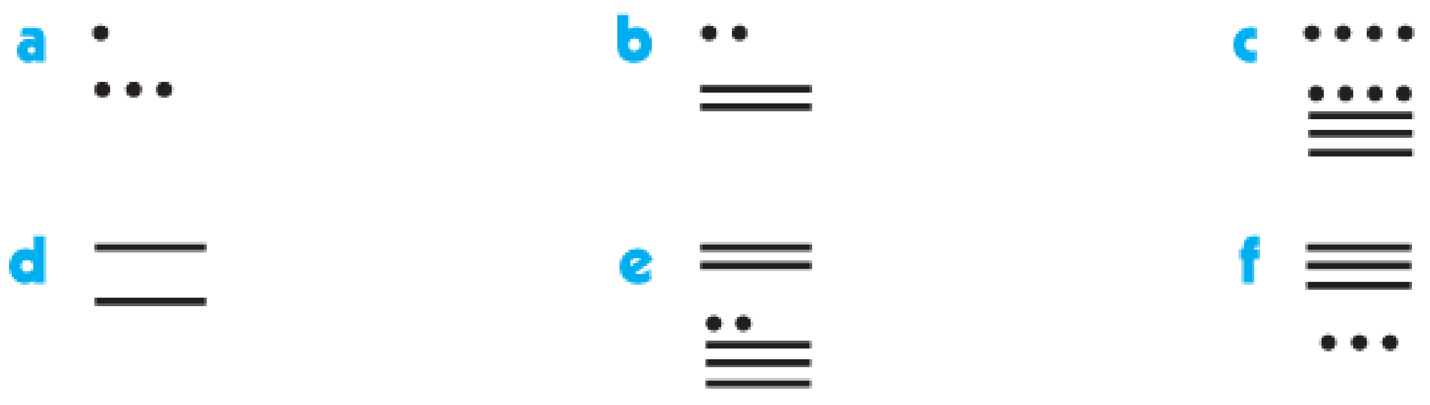
3 88 = LXXXVIII **4** 1500 = MD

5 **a** DCCVIII swords **b** MCC denarii

EXERCISE 1A.4

1 **a** 14 **b** 120 **c** 218 **d** 168 **e** 313 **f** 380

2

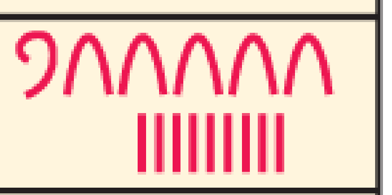
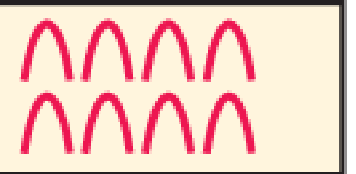


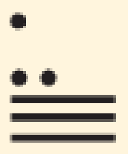
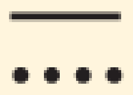
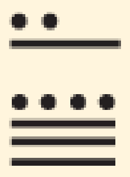
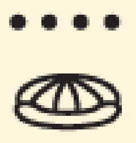
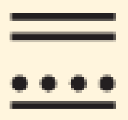
EXERCISE 1A.5

1 **a** 765 **b** 3248 **c** 9999

2 **a** 四百九十七 **b** 八千四百 **c** 千百十一

3

	Words	Numeral	Roman	Egyptian
a	thirty seven	37	XXXVII	
b	one hundred and four	104	CIV	
c	one hundred and fifty nine	159	CLIX	
d	eighty	80	LXXX	
e	two hundred and nine	209	CCIX	

	Words	Mayan	Chinese-Japanese
a	thirty seven		三十七
b	one hundred and four		百四
c	one hundred and fifty nine		百五十九
d	eighty		八十
e	two hundred and nine		二百九

EXERCISE 1B.1

1 **a** 5 **b** 4 **c** 0 **d** 1

2 **a** 9 **b** 0 **c** 7 **d** 8

3 **a** 8 **b** 80 **c** 8 **d** 800 **e** 80
f 8000 **g** 800 **h** 8000 **i** 8 **j** 80 000
k 8000 **l** 80 000

4 **a** 3 thousands, 5 ten thousands, 8 tens
b 3 thousands, 5 hundreds, 8 units
c 3 units, 5 ten thousands, 8 tens
d 3 hundreds, 5 thousands, 8 hundred thousands

5 **a** 1 ten, 4 units, 7 thousands
b 1 thousand, 4 hundreds, 7 units
c 1 unit, 4 ten thousands, 7 hundreds
d 1 hundred thousand, 4 thousands, 7 tens

EXERCISE 1B.2

1 **a** 86 **b** 674 **c** 9638 **d** 50 240
e 27 003 **f** 500 375 **g** 73 298 **h** 809 302

2 **a** $9 \times 100 + 7 \times 10 + 5 \times 1$ **b** $6 \times 100 + 8 \times 10$
c $3 \times 1000 + 8 \times 100 + 7 \times 10 + 4 \times 1$
d $9 \times 1000 + 8 \times 10 + 3 \times 1$
e $5 \times 10\,000 + 6 \times 1000 + 7 \times 100 + 4 \times 10 + 2 \times 1$

f 7 × 10 000 + 5 × 1000 + 7 × 1
g 6 × 100 000 + 8 × 100 + 2 × 10 + 9 × 1
h 3 × 100 000 + 5 × 10 000 + 4 × 1000 + 7 × 100 + 1 × 10 + 8 × 1

3 **a** 27 **b** 80 **c** 608 **d** 1016
e 8200 **f** 19 538 **g** 75 403 **h** 602 818

4 **a** 7 **b** 13 **c** 21 **d** 299 **e** 4007 **f** 9997

EXERCISE 1C.1

1 **a** 80 **b** 50 000 000 (50 million) **c** 600
d 400 000 **e** 70 000 **f** 2

2 **a** 3 000 000, 600 000, 40 000, 8000, 500, 90, 7
b 30 000 000, 4 000 000, 800 000, 60 000, 5000, 200, 70, 1
c 200 000 000, 90 000 000, 3 000 000, 100 000, 40 000, 8000, 700, 50, 6

3 **a** five million, seven hundred and eighty four thousand, two hundred and fourteen
b forty three million, twenty nine thousand, three hundred and six
c one hundred and ninety eight million, three thousand, six hundred and twenty

4 **a** 37 000 000 times
b 200 000 000 bread buns and 17 000 000 kilograms of beef
c 150 000 000 years **d** 21 240 657 Volkswagen 'Beetles'

EXERCISE 1C.2

1 **a**

Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
				3	1	8	2	7	4	0	6	5	9	3

b

Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
		4	9	0	8	2	7	5	6	2	3	1	1	5

c

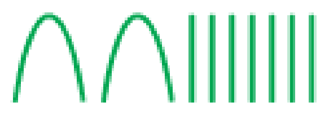
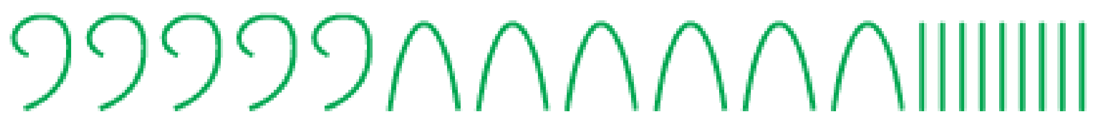
Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
	3	2	4	0	3	9	7	6	8	1	3	2	1	4

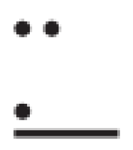
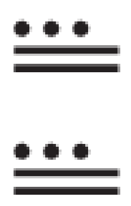
2 **a** two billion, five million, seventeen thousand, three hundred and sixty nine
b thirty billion, five hundred and eight million, four hundred and fifty seven thousand, one hundred and twelve
c seven trillion, twenty six billion, four hundred and eleven million, eighty thousand, nine hundred and forty three

3 **a** 1 427 000 000 km **b** 3 843 000 000 email accounts
c 7 062 186 320 people **d** 1 099 511 627 776 bytes

REVIEW SET 1A

1 **a** 165 **b** 2634

2 **a**  **b** 

3 **a** 18 **b** 79 **4** **a**  **b** 

5 **a** 476 **b** 359 **6** **a** 6 **b** 6 **c** 2

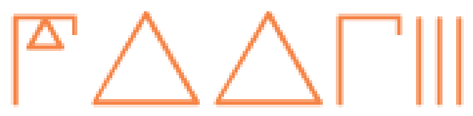
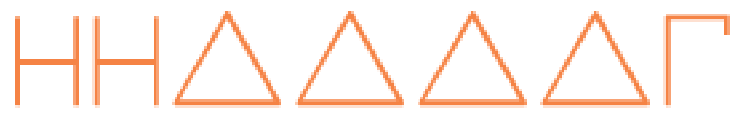
7 1 × 10 000 + 7 × 1000 + 3 × 100 + 4 × 1

8 **a** 23 **b** 991

9 **a** six million, three hundred and seventeen thousand, six hundred and ninety four
b seven billion, eight hundred and five million, thirty six thousand, five hundred and twenty seven

10 5 890 000 metres

REVIEW SET 1B

1 **a**  **b** 

2 **a** 253 **b** 1 202 305 **3** 188 = CLXXXVIII

4 **a** 三百八十六 **b** 二千百十三 **5** **a** 200 **b** 154

6 **a** 400 **b** 40 000 **c** 400 000

7 **a** 2 hundreds, 5 thousands, 9 units
b 2 tens, 5 hundreds, 9 ten thousands
c 2 ten thousands, 5 hundred thousands, 9 tens

8 2497

9 **a**

Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
					6	2	8	4	5	8	0	7	1	2

b

Trillions			*Billions*			*Millions*			*Thousands*			*Units*		
H	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*	*H*	*T*	*U*
		8	6	0	3	5	9	1	2	7	7	9	4	6

10 129 085 403 formal votes

EXERCISE 2A.1

1 **a** 19 **b** 32 **c** 41 **d** 98
2 **a** 4 **b** 9 **c** 19 **d** 157
3 **a** 23 **b** 19 **c** 17 **d** 20
4 **a** 16 **b** 29 **c** 20 **d** 91 **e** 138 **f** 85
g 257 **h** 144 **i** 90

EXERCISE 2A.2

1 **a** 757 **b** 933 **c** 3995 **d** 4697 **e** 1644
f 1597 **g** 13 059 **h** 5240

2 **a** 79 **b** 107 **c** 748 **d** 696 **e** 2155
f 6565 **g** 814 **h** 4955 **i** 4619

3 **a** 82 **b** 44 **c** 315 **d** 109 **e** 665
f 453 **g** 5183 **h** 3656

4 **a** 34 **b** 48 **c** 66 **d** 22 **e** 182
f 476 **g** 376 **h** 3767 **i** 2417

5 **a**
$$\begin{array}{r} 2\,3\,9 \\ +\quad 4\,7\,8 \\ {\scriptstyle 1\,1\;\;} \\ \hline 7\,1\,7 \end{array}$$

b
$$\begin{array}{r} \overset{4}{\cancel{5}}\;\overset{16}{\cancel{6}}\;3 \\ -\quad 2\;8\;1 \\ \hline 2\;8\;2 \end{array}$$

c correct

d
$$\begin{array}{r} 5\;\overset{8}{\cancel{9}}\;\overset{9}{\cancel{0}}\;\overset{10}{\cancel{0}} \\ -\quad 3\;8\;1\;4 \\ \hline 2\;0\;8\;6 \end{array}$$

e
$$\begin{array}{r} 3\,1\,1 \\ 1\,9\,7 \\ +\quad 6\,4\,8 \\ {\scriptstyle 1\,1\,1\;\;} \\ \hline 1\,1\,5\,6 \end{array}$$

f correct

g correct

h
$$\begin{array}{r} 3\;\overset{1}{\cancel{2}}\;\overset{10}{\cancel{1}}\;\overset{15}{\cancel{5}} \\ -\quad 3\;1\;8\;6 \\ \hline 2\;9 \end{array}$$

EXERCISE 2A.3

1 495 cm **2** $432 **3** 39 minutes
4 £60 000 **5** €446 **6** 361 ice creams
7 £16 **8** 3923 km **9** 173 days
10 **a** **i** 8 fish **ii** 23 fish **b** 117 fish

EXERCISE 2B.1

1 **a** 54 **b** 143 **c** 70 **d** 240
2 **a** 4 **b** 4 **c** 11 **d** 15
3 **a** 500 **b** 5000 **c** 50 000 **d** 6900
e 69 000 **f** 690 000 **g** 12 300 **h** 246 000
i 96 000 **j** 490 000 **k** 49 000 **l** 490 000
4 **a** 200 **b** 20 **c** 2 **d** 5700
e 570 **f** 57 **g** 24 300 **h** 2430
i 243 **j** 4500 **k** 450 **l** 45
m 72 000 **n** 7200 **o** 720 **p** 600 000
q 60 000 **r** 6000
5 **a** 130 **b** 1900 **c** 2100 **d** 19 000 **e** 2100
f 97 000 **g** 27 000 **h** 12 000
6 **a** 6 **b** 60 **c** 600 **d** 6000 **e** 35
f 350 **g** 3500 **h** 35 000 **i** 33 **j** 330
k 3300 **l** 330 000
7 **a** 3 **b** 30 **c** 300 **d** 3000 **e** 5
f 50 **g** 500 **h** 5000 **i** 4 **j** 40
k 400 **l** 4000

EXERCISE 2B.2

1 **a** 216 **b** 236 **c** 875 **d** 336
e 682 **f** 546 **g** 602 **h** 3496
2 **a** 120 **b** 148 **c** 496 **d** 1272
e 405 **f** 2744 **g** 14 580 **h** 23 112

EXERCISE 2B.3

1 **a** 14 **b** 54 **c** 21 **d** 84
e 75 **f** 291 **g** 901 **h** 619
2 **a** 12 **b** 25 **c** 52 **d** 48
e 29 **f** 208 **g** 41 **h** 817
3 **a** 28 with remainder 2 **b** 38 with remainder 3
c 162 with remainder 2 **d** 438 with remainder 6

EXERCISE 2B.4

1 4800 pine trees **2** 90 kg **3** $80 each **4** $1700
5 82 minutes **6** 216 people **7** 81 minutes
8 **a** 300 rooms **b** £45 000 **c** £900 000
9 **a** 6000 m **b** 7000 m **c** 14 000 m

EXERCISE 2C

1 €26 **2** 54 goats **3** $168 **4** $2.24
5 **a** €1240 **b** €1860 **6** **a** $1990 **b** failed by $10
7 426 km **8** 600 grams

EXERCISE 2D

1 **a** 2^3 **b** 7^2 **c** 9^4 **d** 13^2 **e** 3^5 **f** 4^7
2 **a** $2^3 \times 3^2$ **b** $4^2 \times 7^4$ **c** $5^3 - 6^3$ **d** $9^3 + 8^4$
e $3^3 \times 5^1 \times 7^2$ **f** $13^2 - 2^4 + 5^2$
3 **a** 10^2 **b** 10^4 **c** 10^5 **d** 10^6 **e** 10^9 **f** 10^{12}
4 **a** 9 **b** 8 **c** 16 **d** 27 **e** 64 **f** 16
g 125 **h** 81 **i** 288 **j** 108 **k** 675 **l** 64 000
5 **a** 3^2 **b** are equal **c** 2^5

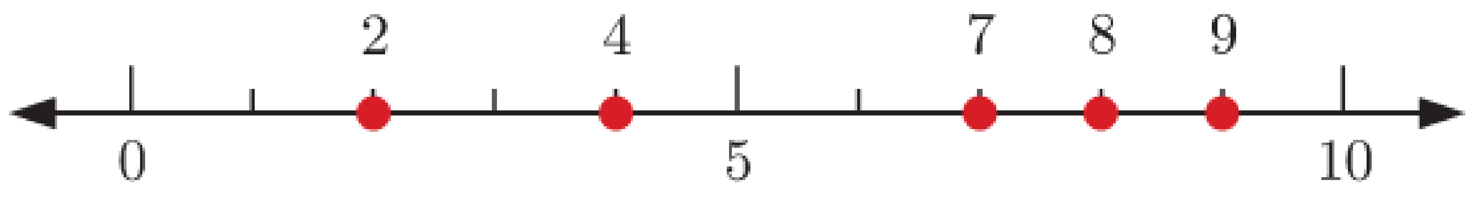
EXERCISE 2E

1 **a** 8 **b** 10 **c** 10 **d** 4 **e** 6 **f** 5
g 18 **h** 14 **i** 3 **j** 25 **k** 16 **l** 3
2 **a** 10 **b** 26 **c** 36 **d** 4 **e** 16 **f** 0
g 31 **h** 44 **i** 12
3 **a** 2 **b** 6 **c** 35 **d** 2 **e** 6 **f** 5
g 31 **h** 16 **i** 14 **j** 11 **k** 3 **l** 8
4 **a** 1 **b** 7 **c** 16 **d** 11 **e** 27 **f** 49
g 18 **h** 36 **i** 1 **j** 125 **k** 86 **l** 45
5 **a** $4 + 18 \div 3 = 10$ **b** $6 \times 7 - 12 = 30$
c $(17 + 3) \div 5 = 4$ **d** $(18 - 2) \div 8 = 2$
e $3^3 - 2^2 = 23$ **f** $4 + (21 \div 7) = 7$
6 **a** Derrick performed the addition first, but working should be done from left to right as the expression contains only + and − operations.
b 11

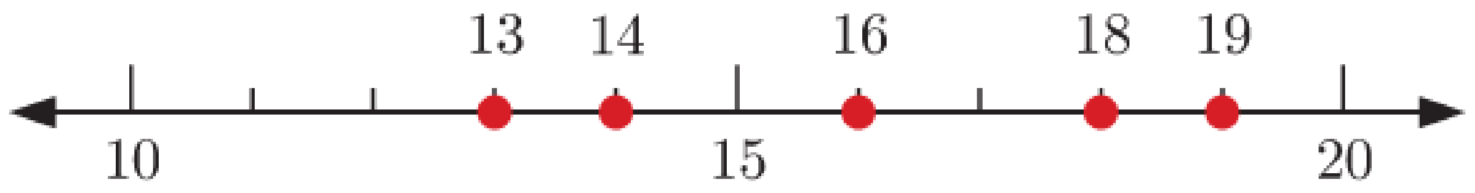
EXERCISE 2F

1 **a** □ = 8 **b** □ = 13 **c** □ = 40 **d** □ = 100
e □ = 50, △ = 80 **f** □ = 50, △ = 125

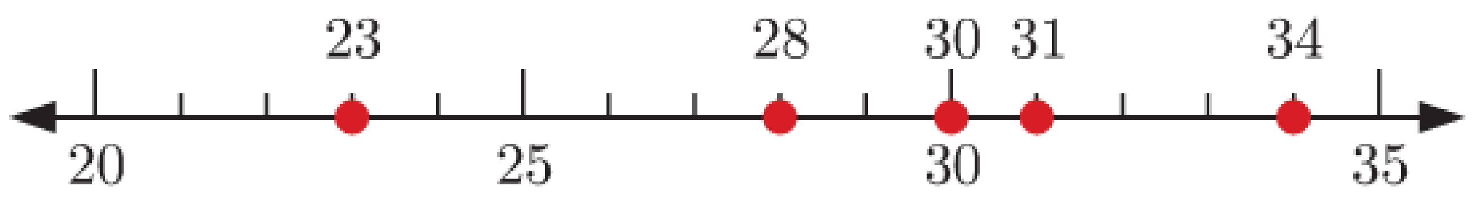
2 **a**



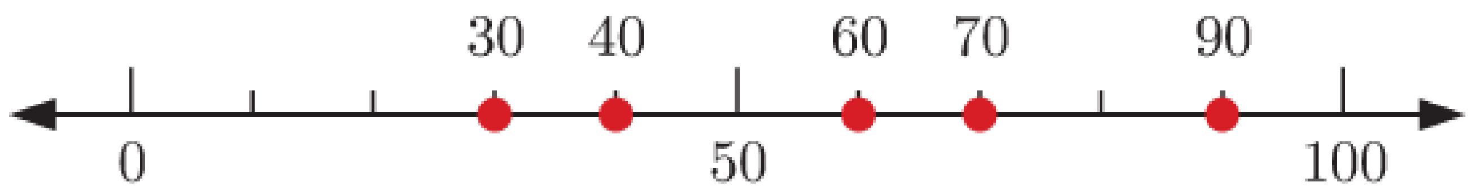
b



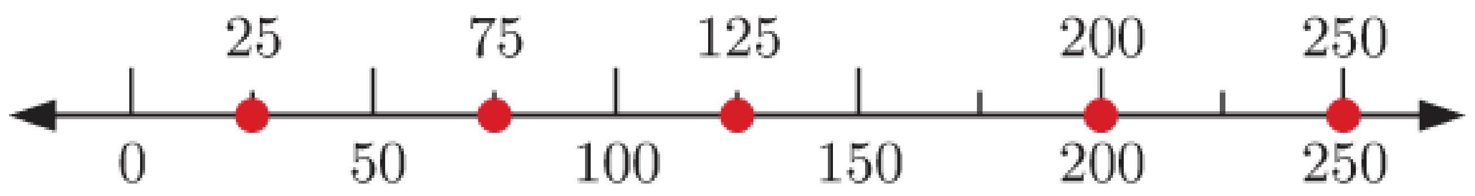
c



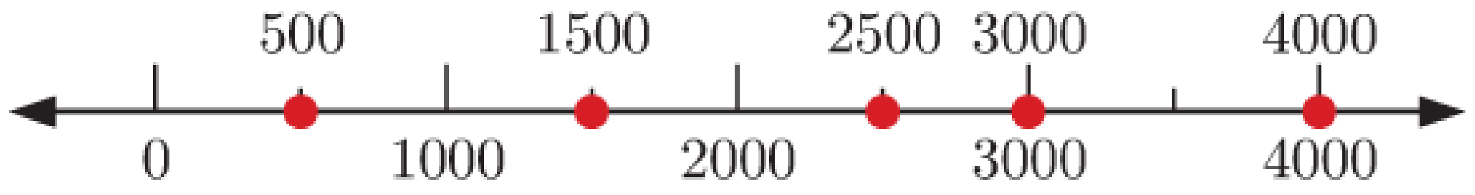
d



e

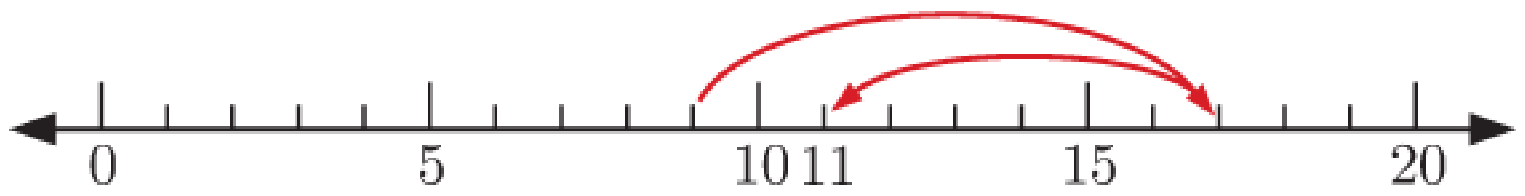


f



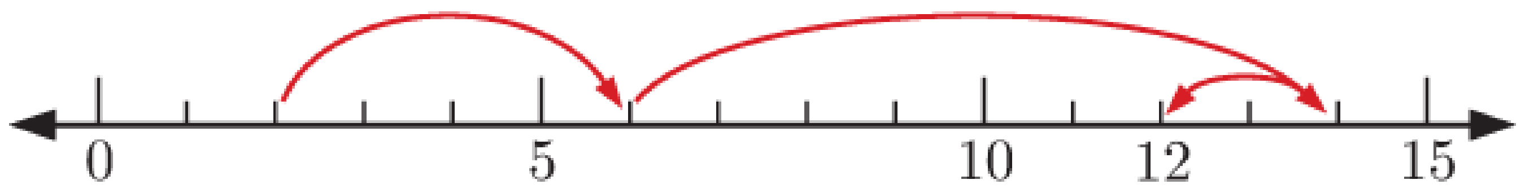
3 **a** $3 + 6 + 9 = 18$ **b** $11 + 9 - 13 = 7$
c $2 + 2 \times 4 = 10$ **d** $22 - 3 \times 3 = 13$
e $3 \times 200 - 100 = 500$ **f** $10 + 60 - 40 + 20 = 50$

4 **a**



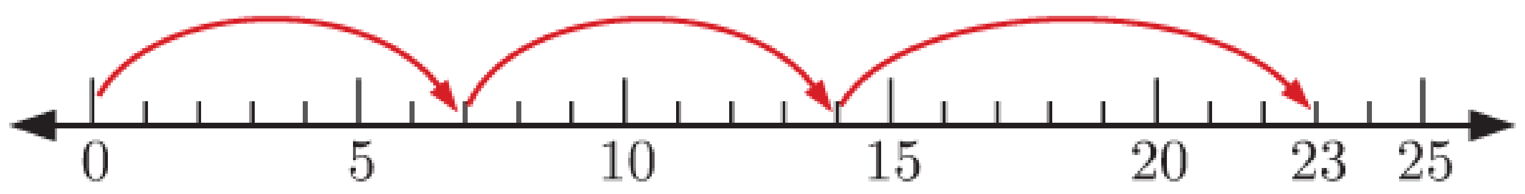
$9 + 8 - 6 = 11$

b



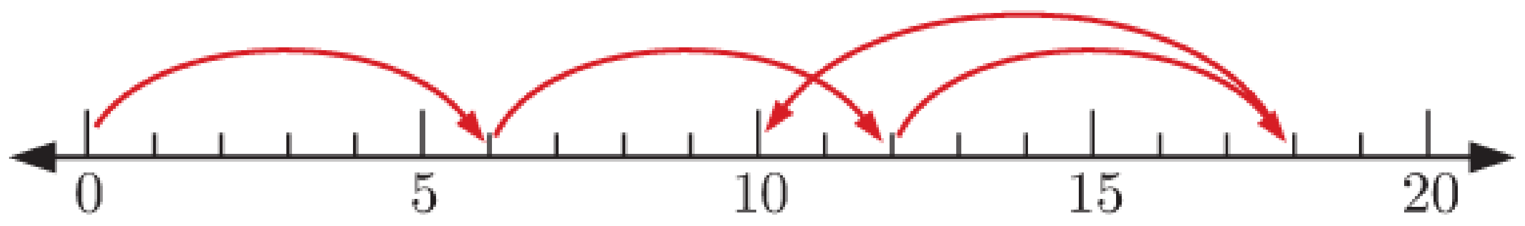
$2 + 4 + 8 - 2 = 12$

c



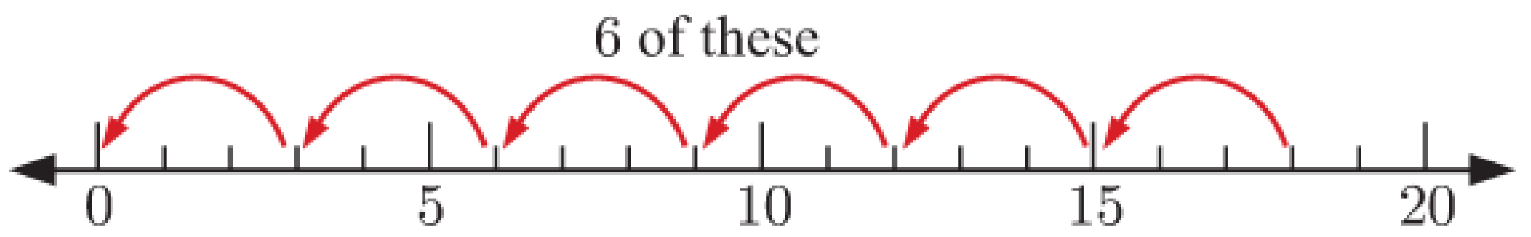
$2 \times 7 + 9 = 23$

d



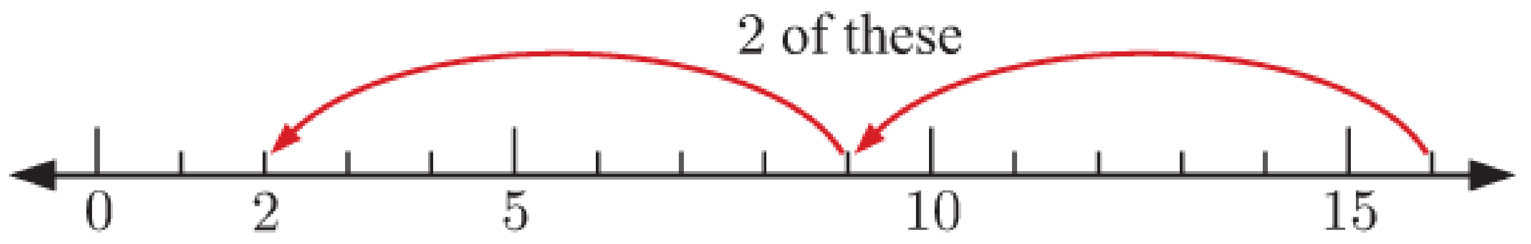
$3 \times 6 - 8 = 10$

e



$18 \div 3 = 6$

f



$16 \div 7 = 2$ with remainder 2

EXERCISE 2G

1 **a** 40 **b** 70 **c** same **d** 130 **e** 460
f same **g** 820 **h** same **i** 6740

2 **a** 20 **b** 50 **c** 40 **d** 70 **e** 100
f 210 **g** 310 **h** 500 **i** 890 **j** 3660
k 7440 **l** 8710 **m** 9610 **n** 14 080 **o** 30 120
p 47 780 **q** 69 570 **r** 70 100

3 **a** 500 **b** 7600 **c** 3000

4 **a** 100 **b** 200 **c** 600 **d** 800 **e** 1100
f 2700 **g** 7000 **h** 13 200 **i** 27 700 **j** 38 500
k 55 400 **l** 85 100

5 **a** 1000 **b** 0 **c** 1000 **d** 5000
e 8000 **f** 7000 **g** 10 000 **h** 9000
i 13 000 **j** 8000 **k** 246 000 **l** 500 000

6 **a** 20 000 **b** 50 000 **c** 50 000 **d** 80 000
e 90 000 **f** 50 000 **g** 90 000 **h** 100 000

7 **a** 200 000 **b** 300 000 **c** 700 000 **d** 700 000
e 100 000 **f** 500 000 **g** 300 000 **h** 100 000

8 **a** 40 musicians **b** 60 singers **c** £580 **d** €4100
e 700 kg **f** $25 000 **g** 35 600 km
h 40 000 km **i** £460 000 **j** 3 500 000 people

REVIEW SET 2A

1 **a** 18 **b** 78 **c** 104 **d** 23

2 **a** 758 **b** 7961 **c** 3250

3 **a** 67 **b** 79 **c** 208 **d** 270 **e** 1700 **f** 4600

4 **a** 224 **b** 272 **c** 522 **d** 128

5 84 points **6** **a** 3400 **b** 59

7 **a** 336 **b** 508 **c** 611

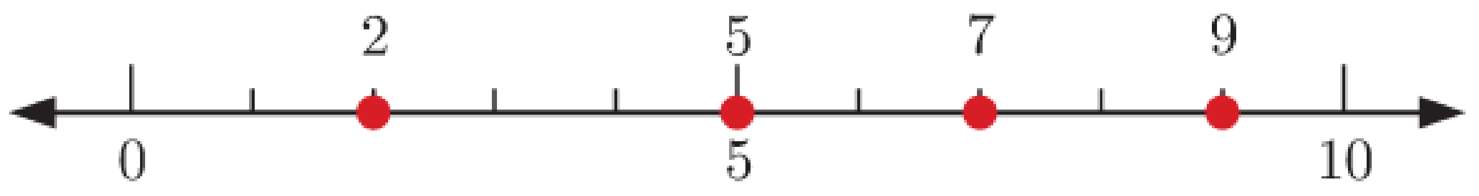
8 **a** 6^4 **b** $2^3 \times 7^5$ **9** £29 **10** €2688

11 **a** 500 **b** 800 **c** 2300 **d** 49 500

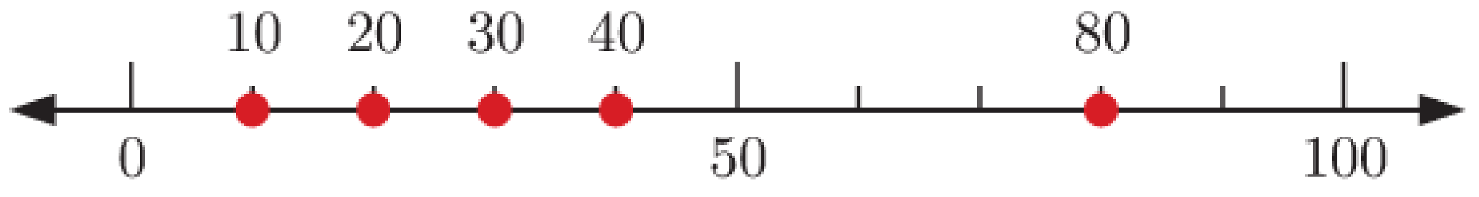
12 **a** 3 **b** 13 **c** 12 **d** 24 **e** 8 **f** 9

13 ☐ = 15

14 **a**



b



15 **a** 40 **b** 4000 **c** 460 000 **d** 900 000

REVIEW SET 2B

1 **a** 53 **b** 463 **c** 1794 **2** **a** 448 **b** 2843

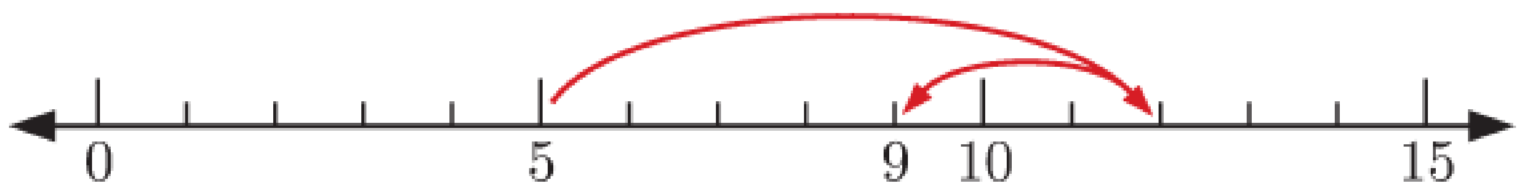
3 **a** 34 **b** 77 **c** 56 **d** 61 with remainder 4

4 $1 000 000

5 **a** 897 **b** 51 **c** 518 with remainder 6

6 No, he is $25 short. **7** 15 sections **8** €728

9



$5 + 7 - 3 = 9$

10 **a** $2^2 + 7^3$ **b** $11^3 - 3^4$

11 **a** 3000 **b** 10 000 **c** 24 000 **d** 313 000

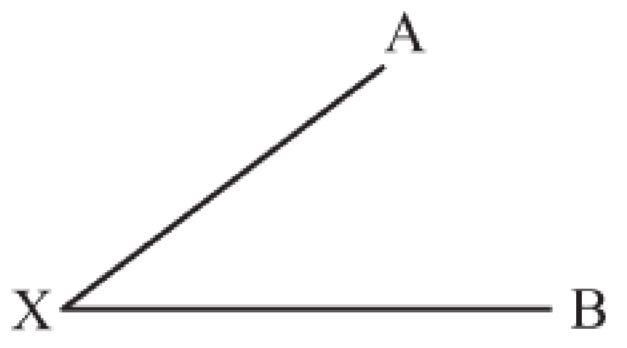
12 **a** 16 **b** 3 **c** 32 **13** **a** $17 **b** $81

14 $2 \times 8 \div 4 + 2 = 6$ **15** **a** £80 **b** 17 000 people

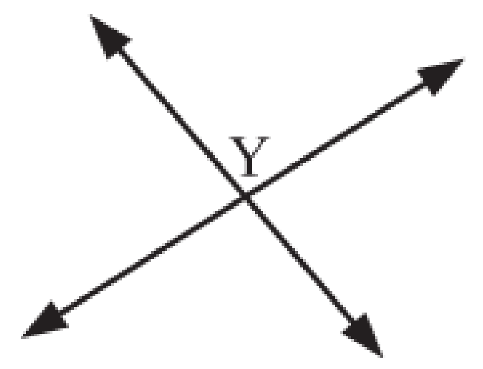
EXERCISE 3A

1 **Note:** Answers are examples only; many answers are possible.
a a corner of a door **b** the top edge of a door

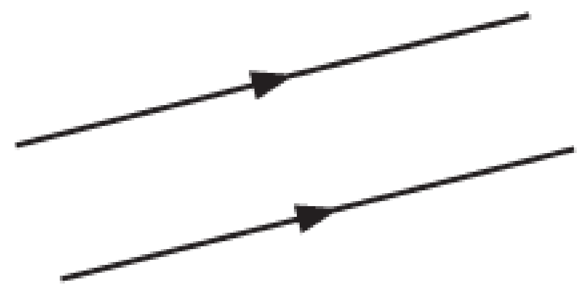
2 **a** A point where two straight line segments meet.
Example, point X in:



b The point at which two straight lines or line segments cross over each other.
Example, point Y in:



c Two lines which are always a fixed distance apart.
Example:



3 **a** (LM) or (ML) **b** (CD), (DC), (CE), (EC), (DE), or (ED)

4 **a** B **b** C

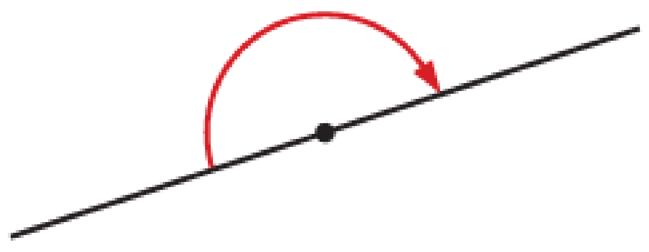
5 **a** C **b** B and C **c** (BE) and (CD)

6 **a** B **b** [BC] **c** [AB]

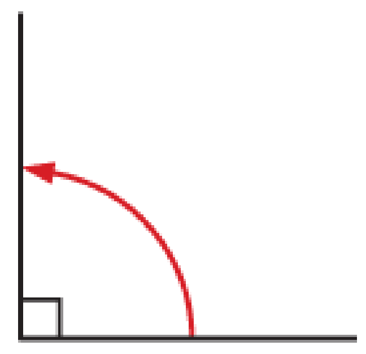
EXERCISE 3B

1 **a** 75° **b** 60° **c** 128° **d** 103° **e** 27°
f 23° **g** 135° **h** 155° **i** 87° **j** 96°

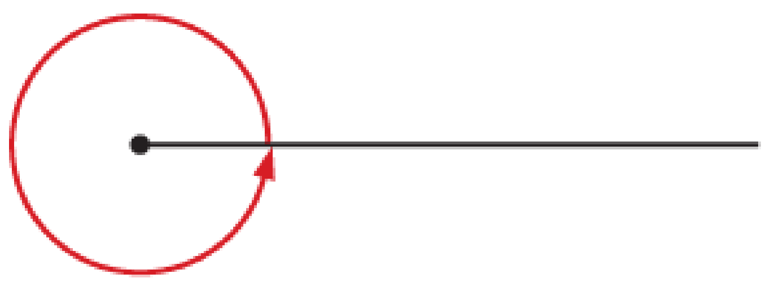
2 **a**



b



c



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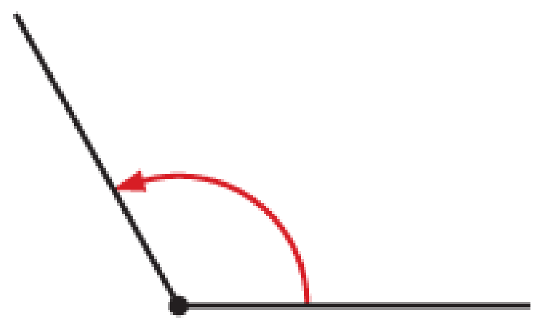

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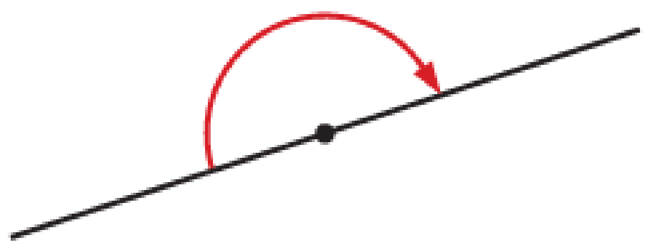

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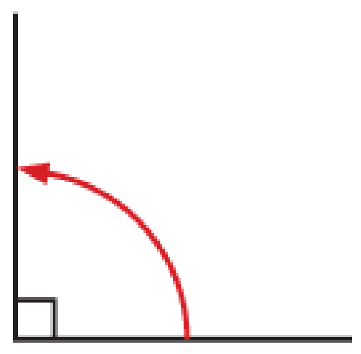
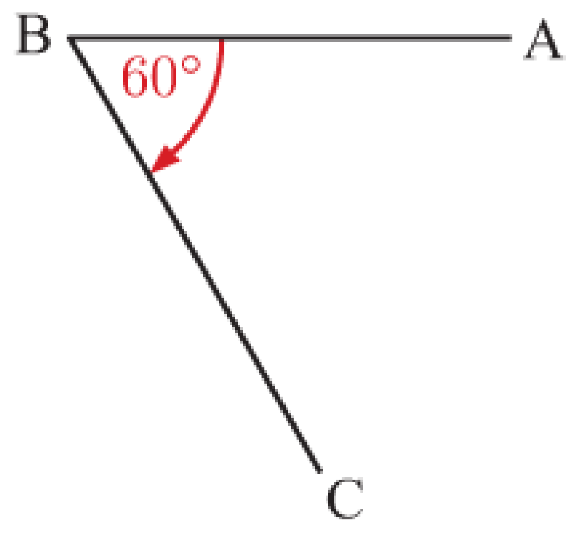


3 **a** $\widehat{ABC}$ (or $\widehat{CBA}$), acute **b** $\widehat{PQR}$ (or $\widehat{RQP}$), obtuse
c $\widehat{KLM}$ (or $\widehat{MLK}$), acute **d** $\widehat{CBD}$ (or $\widehat{DBC}$), acute
e $\widehat{XZW}$ (or $\widehat{WZX}$), obtuse **f** $\widehat{LJN}$ (or $\widehat{NJL}$), acute

4 33°

5 **a** $\widehat{ABC} = 67^\circ$, $\widehat{ACB} = 30^\circ$, $\widehat{BAC} = 83^\circ$
b $\widehat{DEF} = 33^\circ$, $\widehat{EFD} = 42^\circ$, $\widehat{FDE} = 105^\circ$

6 **a**



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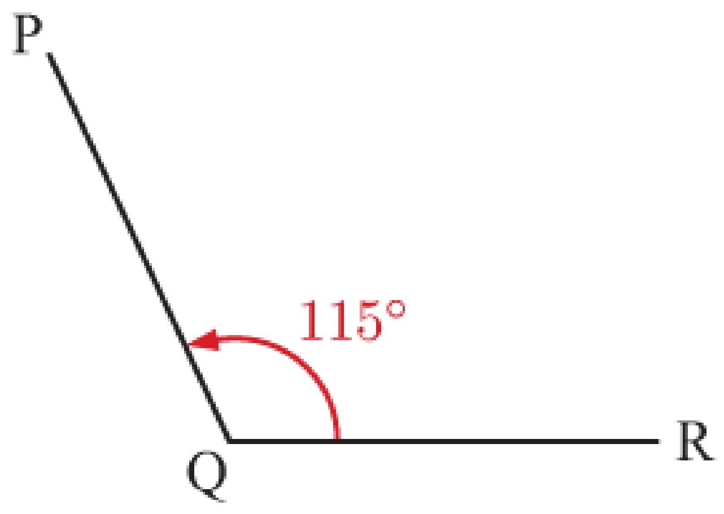

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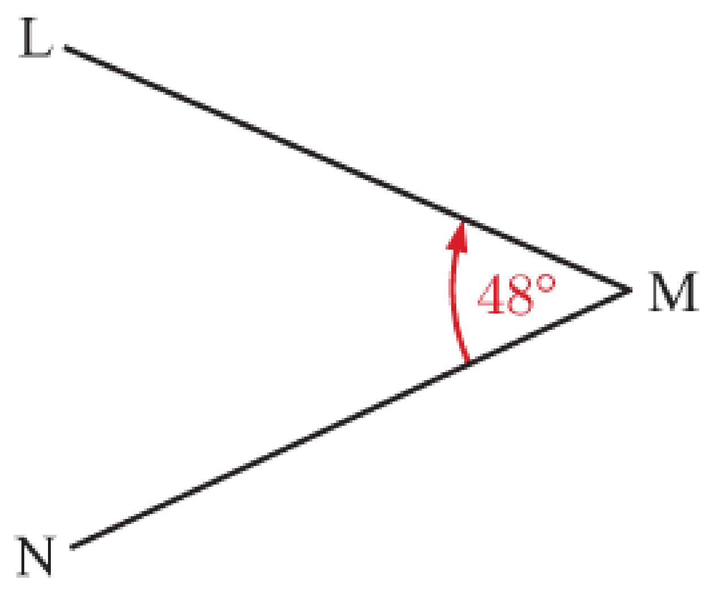

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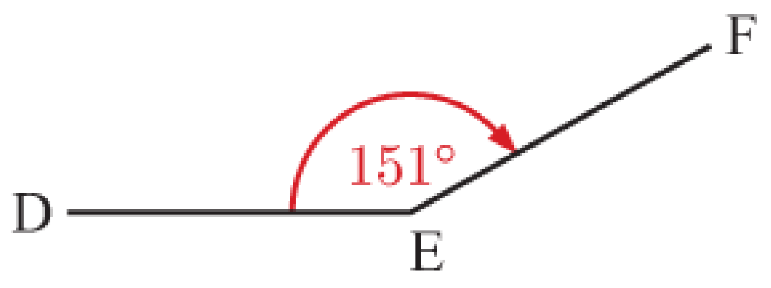

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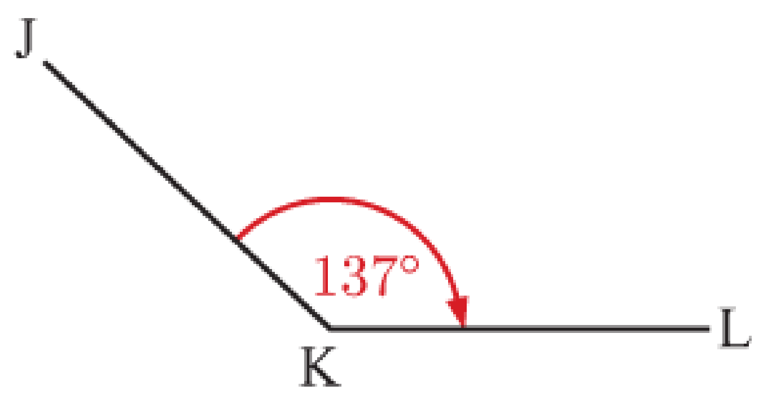

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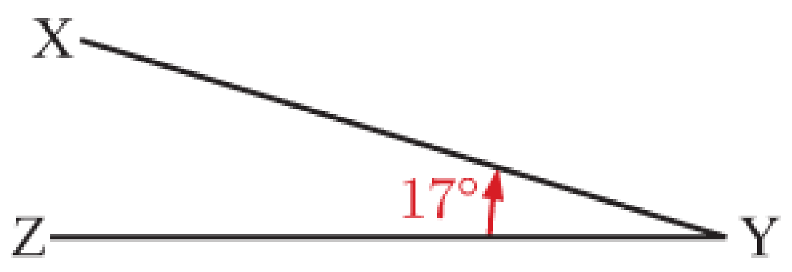


7 **a** acute **b** reflex **c** right **d** obtuse
e acute **f** straight **g** reflex **h** obtuse

8 **a** **i** $A\widehat{B}C = 40°$, $X\widehat{Y}Z = 28°$ **ii** $A\widehat{B}C$
b **i** $A\widehat{B}C = 105°$, $X\widehat{Y}Z = 118°$ **ii** $X\widehat{Y}Z$
c **i** $A\widehat{B}C = 90°$, $X\widehat{Y}Z = 106°$ **ii** $X\widehat{Y}Z$

9 **a** **i** 51° **ii** acute **b** **i** 100° **ii** obtuse
c **i** 90° **ii** right **d** **i** 20° **ii** acute
e **i** 180° **ii** straight **f** **i** 114° **ii** obtuse

EXERCISE 3C

1 **a** $x = 25$ **b** $x = 56$ **c** $y = 45$ **d** $m = 50$
e $n = 107$ **f** $p = 50$ **g** $x = 60$ **h** $x = 90$
i $x = 32$

2 **a** $a = 270$ **b** $b = 120$ **c** $c = 318$ **d** $d = 89$
e $e = 120$ **f** $f = 81$ **g** $x = 90$ **h** $x = 148$
i $g = 112$

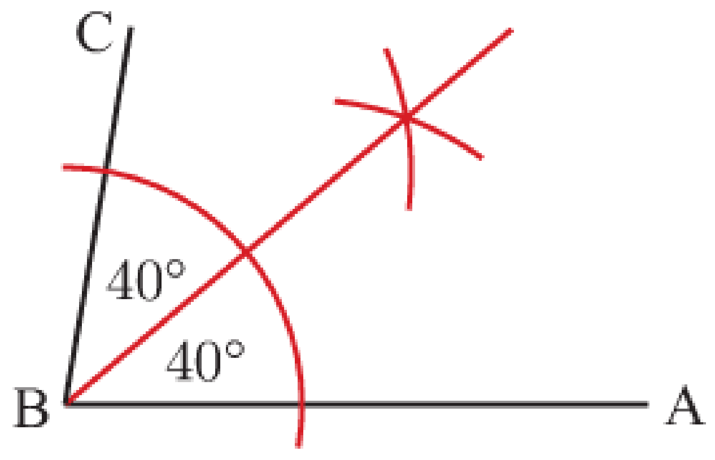
EXERCISE 3D

1 **a** $D\widehat{B}E$ **b** $F\widehat{B}D$ **c** $G\widehat{B}E$

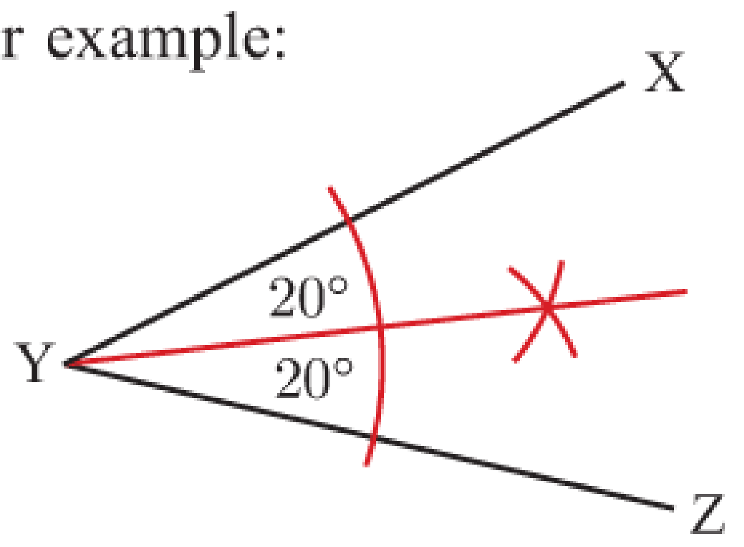
2 **a** $a = 114$ **b** $b = 97$ **c** $c = 27$ **d** $d = 90$
e $e = 100$ **f** $f = 124$ **g** $g = 41$ **h** $h = 60$
i $i = 68$ **j** $j = 61$, $k = 67$ **k** $l = 72$, $m = 41$
l $x = 93$, $y = 47$, $z = 40$

EXERCISE 3E

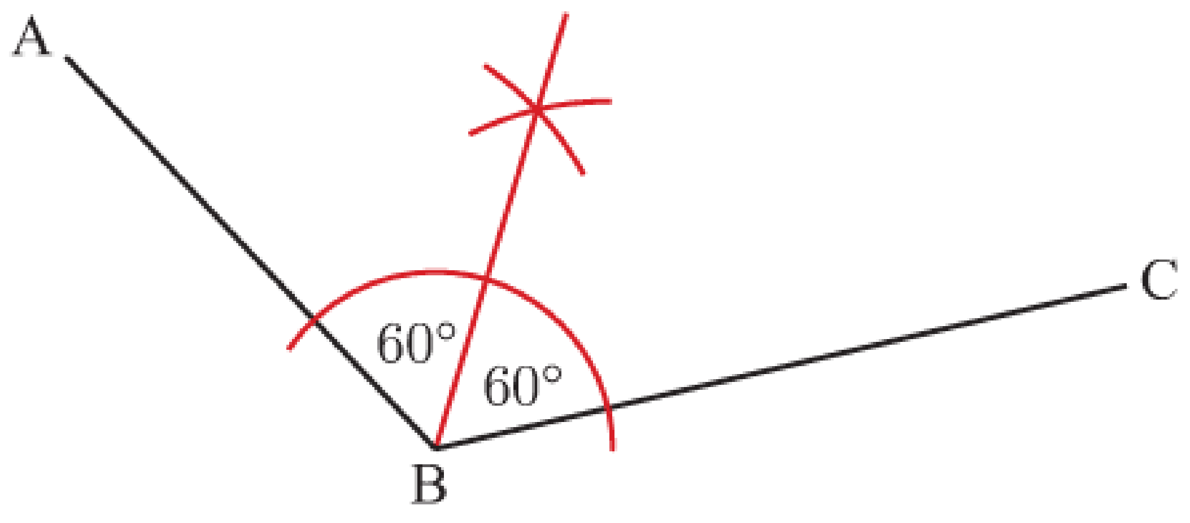
1 **a**, **b**



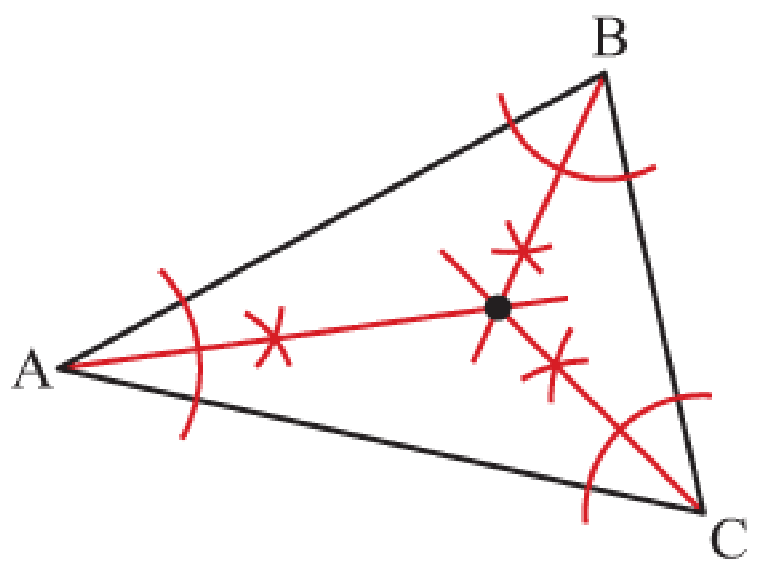
2 For example:



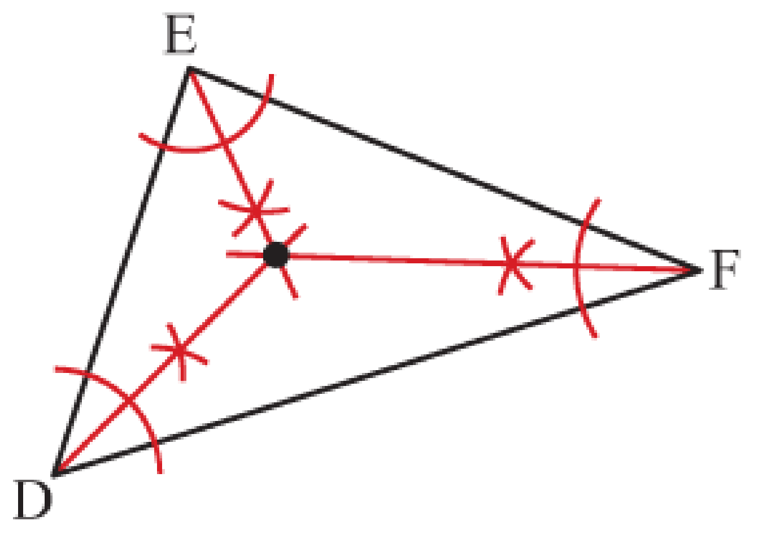
3 For example:



4 **a**

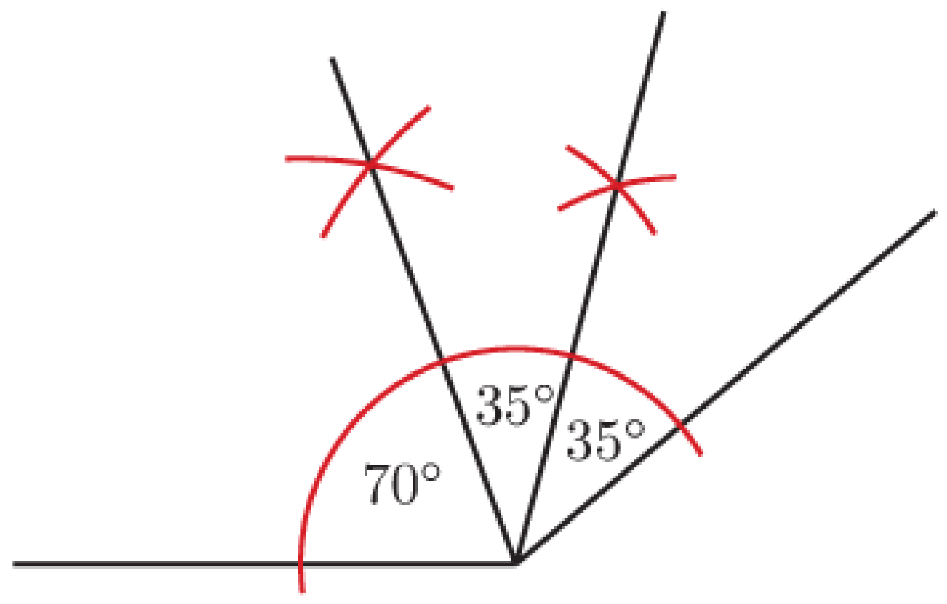


b



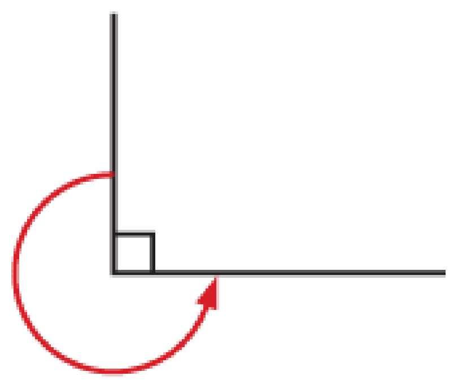
d "The three angle bisectors of a triangle all meet at the same point."

5 **a**, **b**

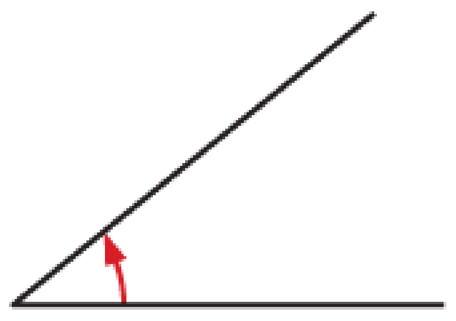


REVIEW SET 3A

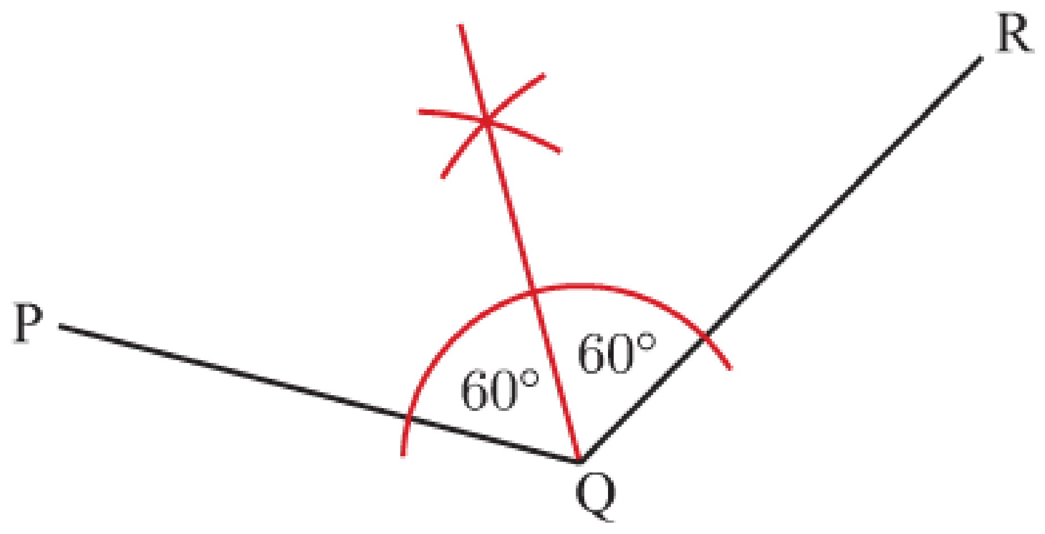
1 **a**



b

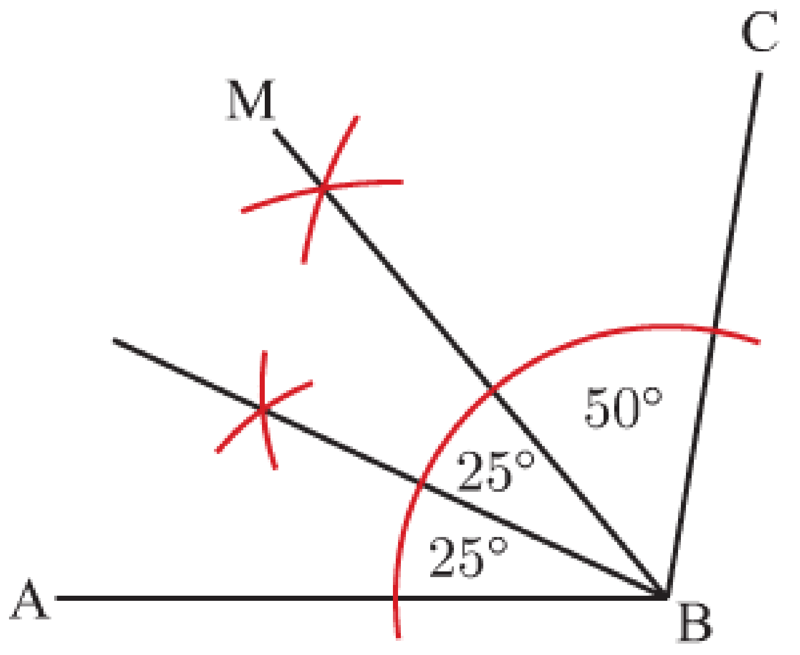


2 **a** $A\widehat{B}C$ (or $C\widehat{B}A$), acute **b** $J\widehat{K}L$ (or $L\widehat{K}J$), obtuse
3 **a** obtuse **b** right **c** reflex **d** acute
4 **a** $x = 26$ **b** $x = 111$ **5** 18°
6 **a** 32°, acute **b** 91°, obtuse
7 **a** $a = 145$ **b** $x = 112$ **c** $x = 98$ **d** $k = 47$
8 **a** **i** $U\widehat{O}T$ **ii** $R\widehat{O}S$ **b** **i** 82° **ii** 123°
9



REVIEW SET 3B

1 (PQ) or (QP) **2** A **3** $X\widehat{O}Y$ (or $Y\widehat{O}X$)
4 **a** $E\widehat{O}D$ (or $D\widehat{O}E$)
b $A\widehat{O}B$ and $E\widehat{O}D$, $B\widehat{O}C$ and $F\widehat{O}E$, $C\widehat{O}D$ and $A\widehat{O}F$
5 **a** $a = 90$ **b** $a = 35$
6 $J\widehat{K}L = 82°$, $K\widehat{L}J = 55°$, $L\widehat{J}K = 43°$
7 $x = 48$, $y = 42$
8 **a** (AE), (EA), (EC), (CE), (AC), or (CA)
b (AB) and (DC) **c** A and C
9 **a** $a = 37$, $b = 122$ **b** 238° **c** reflex
10 **a**, **b**, **c**



EXERCISE 4A

1 **a** 7 **b** 7 **c** 0 **d** undefined **e** 18 **f** 7
g undefined **h** 0 **i** 15 **j** 23 **k** 6 **l** 30

2 **a** 73 **b** 0 **c** undefined **d** 0 **e** undefined
f 0 **g** 3 **h** 125 **i** 0 **j** 45 **k** 0 **l** 0
m 0 **n** 0 **o** 235 **p** undefined

EXERCISE 4B

1 $5^2 = 25$, $6^2 = 36$

2 $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$

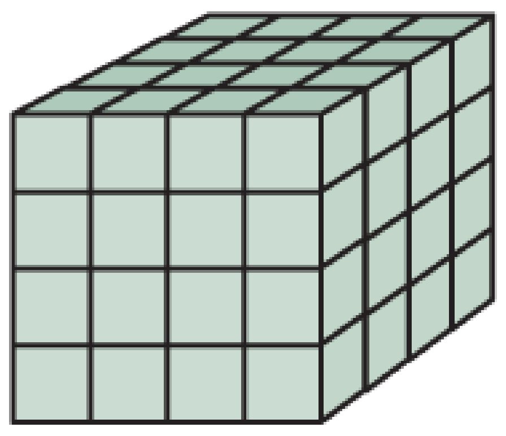
3 **a** $12^2 = 144$ **b** $15^2 = 225$ **c** $22^2 = 484$

4 **Note:** Other answers are possible.
9 and 16, as $3^2 + 4^2 = 9 + 16 = 25$

5 **a** 0 **b** 0 and 1

EXERCISE 4C

1 **a**



$4 \times 4 \times 4 = 4^3 = 64$ blocks

b 64

2 $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$,
$6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$, $10^3 = 1000$

3 180 **4** 1 and 8, as $1^3 + 2^3 = 1 + 8 = 9 = 3^2$

5 25 and 100, as $5^2 + 10^2 = 25 + 100 = 125 = 5^3$

6 **a** 0 **b** 0 and 1

EXERCISE 4D.1

1 **a** yes **b** no **c** yes **d** no **e** no **f** yes
g no **h** yes

2 **a** even **b** odd **c** odd **d** even **e** even **f** odd
g odd **h** even

3 56 **4** Zero is even, as it is divisible by 2. **5** 24

6 **a** 8, 10, 12 or 8, 6, 4
b 17, 19, 21, 23, 25 or 17, 15, 13, 11, 9

7 **a** 2 and 8 **b** 1 and 19, 3 and 17, 5 and 15, 7 and 13
c 2, 4, 14; 2, 6, 12; 2, 8, 10; and 4, 6, 10

8 **a** even **b** even **c** odd **d** odd **e** even
f odd **g** even

EXERCISE 4D.2

1 **a** yes **b** yes **c** no **d** yes **e** yes

2 **a** no **b** yes **c** no **d** yes **e** no

3 **a** yes **b** yes **c** no **d** yes **e** yes

4 **a** yes **b** no **c** yes **d** no **e** yes

5 **a** no **b** yes **c** no **d** yes **e** no

6 **a** yes **b** no **c** yes **d** no **e** yes

7 **a** ☐ = 0, 2, 4, 6, or 8 **b** ☐ = 2, 5, or 8
c ☐ = 2 or 6 **d** ☐ = 0 or 5

8 **a** ☐ = 1, 4, or 7 **b** ☐ = 2, 5, or 8
c ☐ = 1, 4, or 7 **d** ☐ = 2, 5, or 8

EXERCISE 4E

1 **a** yes **b** no **c** no **d** yes

2 **a** 1, 5 **b** 1, 2, 3, 6 **c** 1, 7 **d** 1, 2, 4, 8
e 1, 3, 9 **f** 1, 2, 5, 10 **g** 1, 11 **h** 1, 2, 3, 4, 6, 12

3 **a** yes **b** no **c** yes **d** no **e** yes
f no **g** no **h** yes

4 **a** $22 = 2 \times 11$ **b** $45 = 9 \times 5$ **c** $30 = 3 \times 10$
d $49 = 7 \times 7$ **e** $72 = 12 \times 6$ **f** $85 = 5 \times 17$

5 **a** 1, 3, 5, 15 **b** 1, 2, 3, 6, 9, 18
c 1, 23 **d** 1, 2, 3, 4, 6, 8, 12, 24
e 1, 3, 5, 9, 15, 45 **f** 1, 2, 4, 8, 16, 32, 64
g 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
h 1, 2, 4, 5, 10, 20, 25, 50, 100

6 **a** 4 factors **b** 2 factors **c** 6 factors **d** 9 factors
e 2 factors **f** 8 factors **g** 12 factors **h** 8 factors

7 Every natural number greater than 1 has at least a factor of 1 and a factor of itself.

8 **a** The factors of 32 are 1, 2, 4, 8, 16, 32.
The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.
b 1, 2, 4, 8, and 16 **c** 16

EXERCISE 4F

1

1	neither
2	prime
3	prime
4	composite
5	prime
6	composite
7	prime
8	composite
9	composite
10	composite

11	prime
12	composite
13	prime
14	composite
15	composite
16	composite
17	prime
18	composite
19	prime
20	composite

21	composite
22	composite
23	prime
24	composite
25	composite
26	composite
27	composite
28	composite
29	prime
30	composite

31	prime
32	composite
33	composite
34	composite
35	composite
36	composite
37	prime
38	composite
39	composite
40	composite

2 one (It is 2.)

3 **a** 53, 59 **b** 61, 67 **c** 97, 101, 103, 107, 109

4 In addition to being divisible by 1 and itself, it is also divisible by 5. $\therefore$ it is composite.

5 25 and 27

6 **a** 4 and 9 are composites and their sum is $4 + 9 = 13$ which is a prime. Other answers are possible.
b No. Each composite number has at least 3 different factors and so the product of two of them has at least 3 different factors. So, the product cannot be a prime.

7 **a** 1, 2, 4, 5, 10, 20 **b** 2 and 5 **c** $20 = 2 \times 2 \times 5$

8 **a** 1, 3, 9, 27 **b** 3 **c** $27 = 3 \times 3 \times 3$

EXERCISE 4G

1 **a** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
b 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
c 11, 22, 33, 44, 55, 66, 77, 88, 99, 110

2 **a** 12, 15, 18 **b** 48, 60

3 **a** 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84
b **i** no **ii** yes, $21 = 7 \times 3$ **iii** no **iv** no
v yes, $63 = 7 \times 9$

4 **a** 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
b 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
c 40, 80 **d** 40

5 198

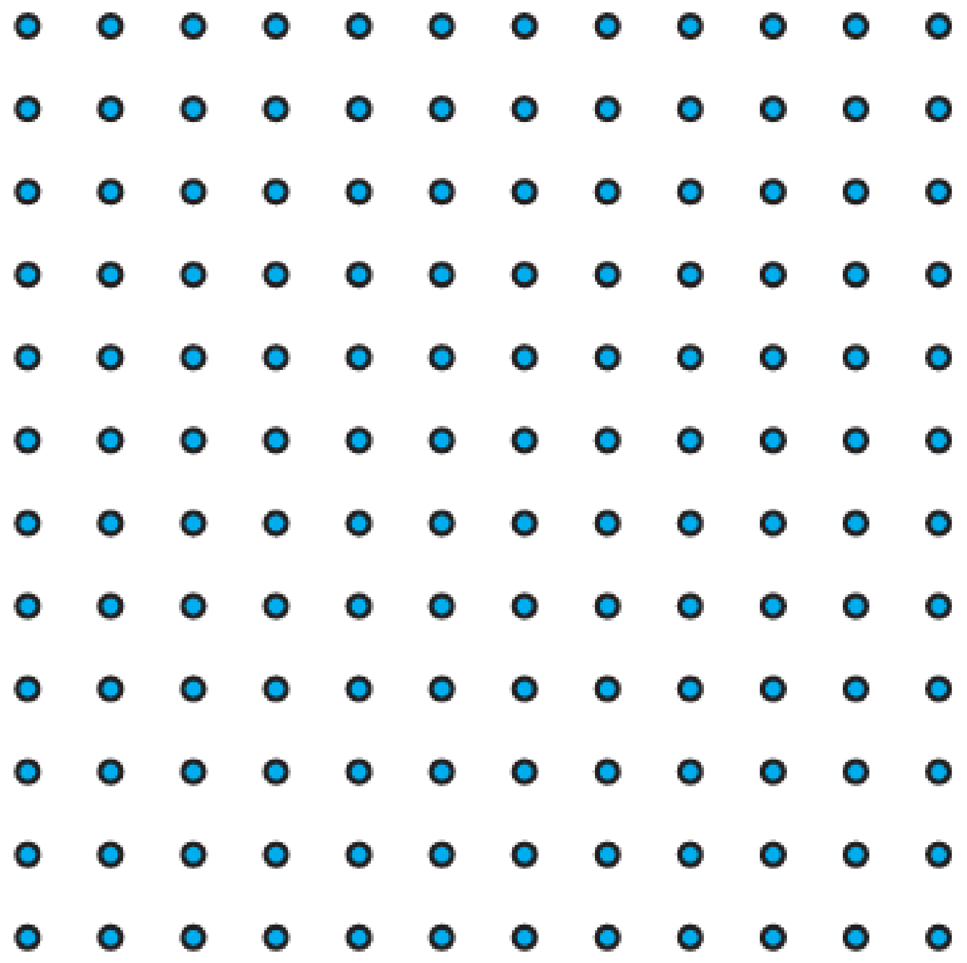
REVIEW SET 4A

1 **a** 32 **b** 0 **c** 14 **d** 0

2 **a** no **b** yes

3 **a** $\square = 2, 5,$ or 8 **b** $\square = 0, 2, 4, 6,$ or 8

4 **a**



b 144

5 **a** composite **b** composite **c** prime **d** composite

6 **a** 65 **b** 16 and 49 (as $4^2 + 7^2 = 16 + 49 = 65$)

7 **a** 28 **b** composite **c** 1, 2, 4, 7, 14, 28

8 **a** The factors of 16 are 1, 2, 4, 8, 16.
The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.
b 1, 2, 4, and 8 **c** 8

9 7 cubic numbers **10** 54, 60, 66

REVIEW SET 4B

1 **a** 243 **b** 0 **c** 243 **d** undefined **2** 36

3 2, 4, 24 (or 2, 6, 22 or 2, 8, 20 or 2, 10, 18
or 2, 12, 16 or 4, 6, 20 or 4, 8, 18
or 4, 10, 16 or 4, 12, 14 or 6, 8, 16
or 6, 10, 14 or 8, 10, 12)

4 1, 2, 3, 6, 9, 18, 27, 54 **5** 18 natural numbers

6 **a** no **b** yes **c** no **d** yes **7** 144

8 **a** 1, 2, 3, 5, 6, 10, 15, 30 **b** 2, 3, 5 **c** $30 = 2 \times 3 \times 5$

9 **a** **i** 16 **ii** 44 **iii** 60 **iv** 76
b ...... is always divisible by 4.

10 **a** 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78
b 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 **c** 24, 48, 72
d 24

EXERCISE 5A

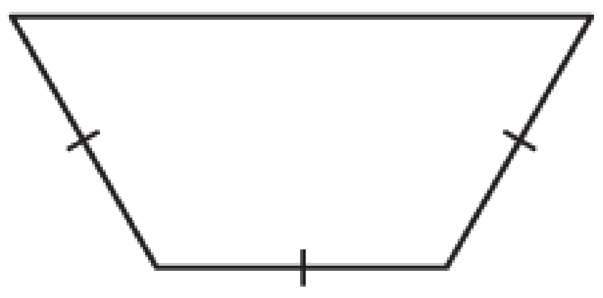
1 **a** triangle **b** quadrilateral **c** hexagon **d** pentagon
e heptagon **f** octagon **g** nonagon **h** decagon

2 **a** not closed **b** sides are not all straight
c it crosses itself **d** sides are not straight

3 **a** Not regular, as angles are not all equal.
b Regular, as sides are equal *and* angles are equal.
c Not regular, as angles are not all equal.
d Not regular, as sides are not all equal.
e Not regular, as sides are not all equal.
f Regular, as sides are equal *and* angles are equal.

4 **Note:** Other answers are possible.

a



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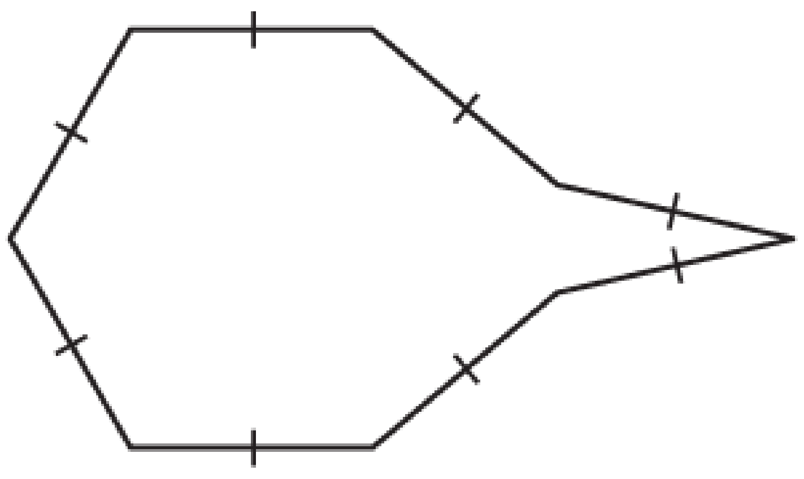
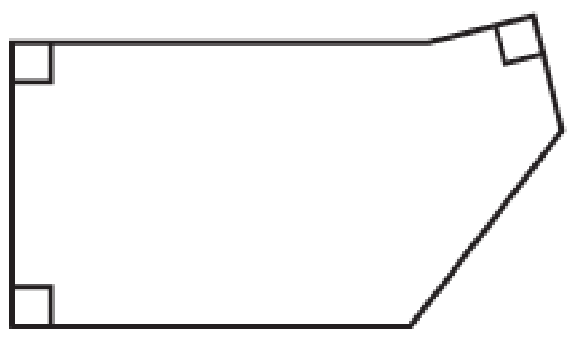


c

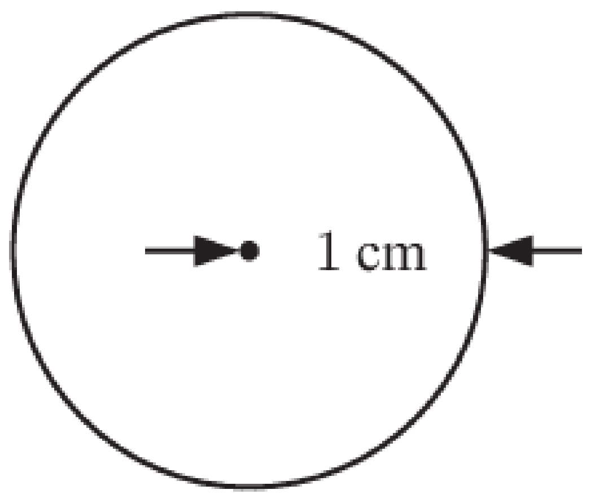


5 **a** Regular; all sides are 4.5 cm long, and all angles are 60°.
b Not regular; all sides are 4 cm long, but all angles are not equal in size.
c Regular; all sides are 2.5 cm long, and all angles are 108°.
d Not regular; sides are not equal in length. Some are 4.5 cm, others are 4 cm.

EXERCISE 5B

1 A circle does not have straight line sides.

2 **a**



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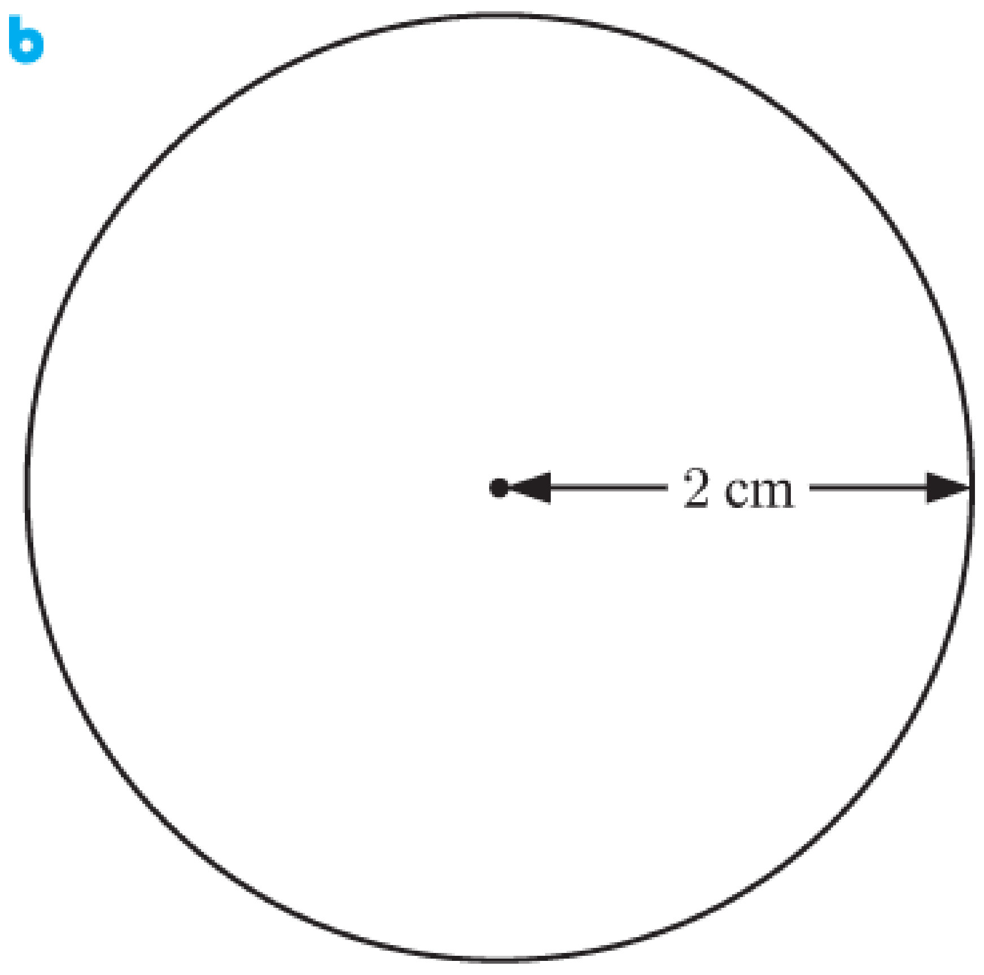
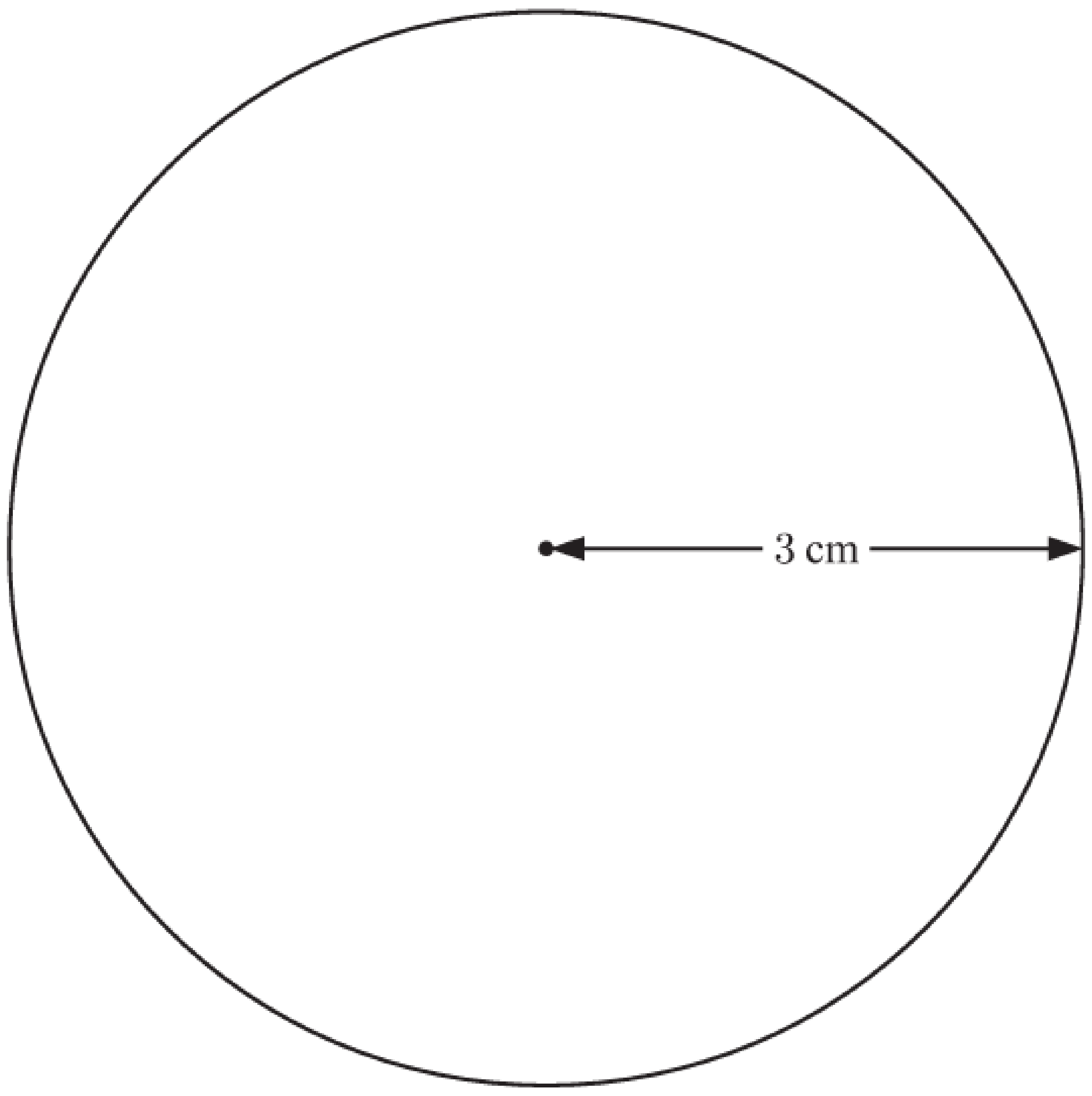


c



3 **a** distance is 2 cm
b distance is more than 0 cm but less than 2 cm
c distance is more than 2 cm

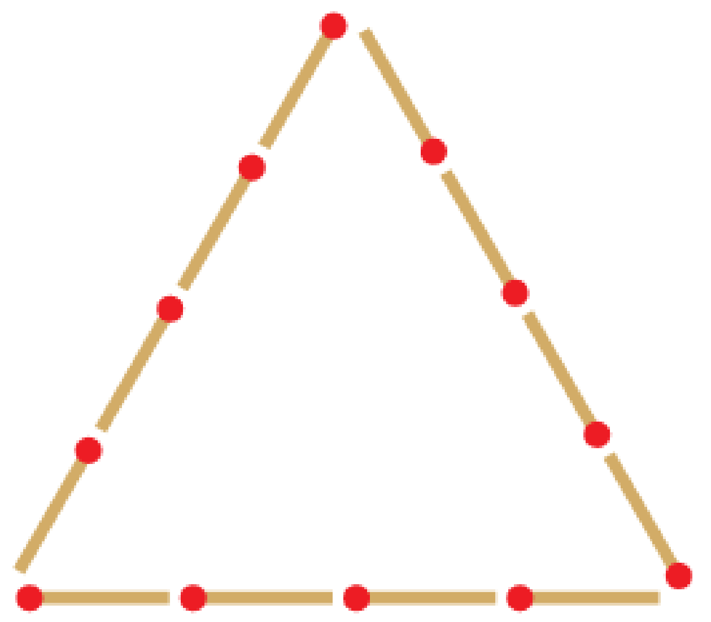
EXERCISE 5C

1 **a** 16 triangles **b** 20 triangles

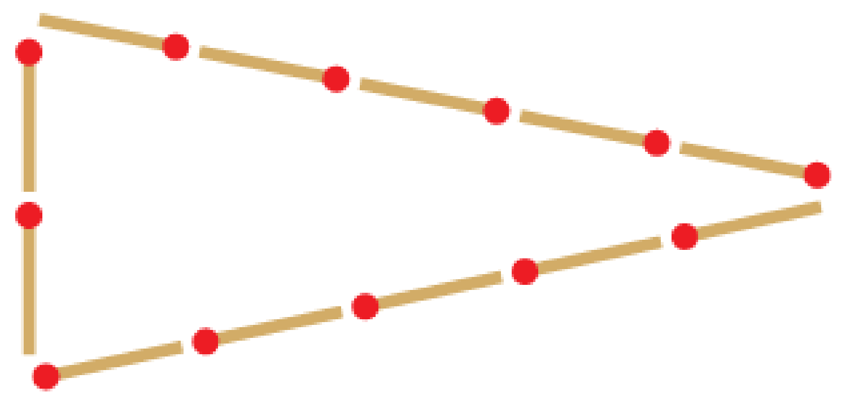
2 **a** scalene **b** equilateral **c** isosceles **d** equilateral
e scalene **f** isosceles

3 **a** isosceles; with two sides 4 cm, other side 3 cm
b equilateral; with all sides 5 cm
c scalene (3 cm, 4 cm, 6 cm)
d isosceles; with two sides 2 cm, other side 3 cm

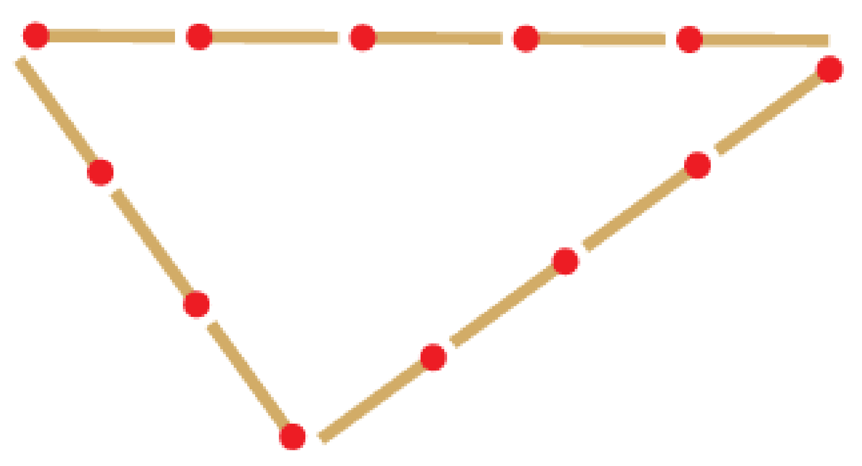
4 **a**



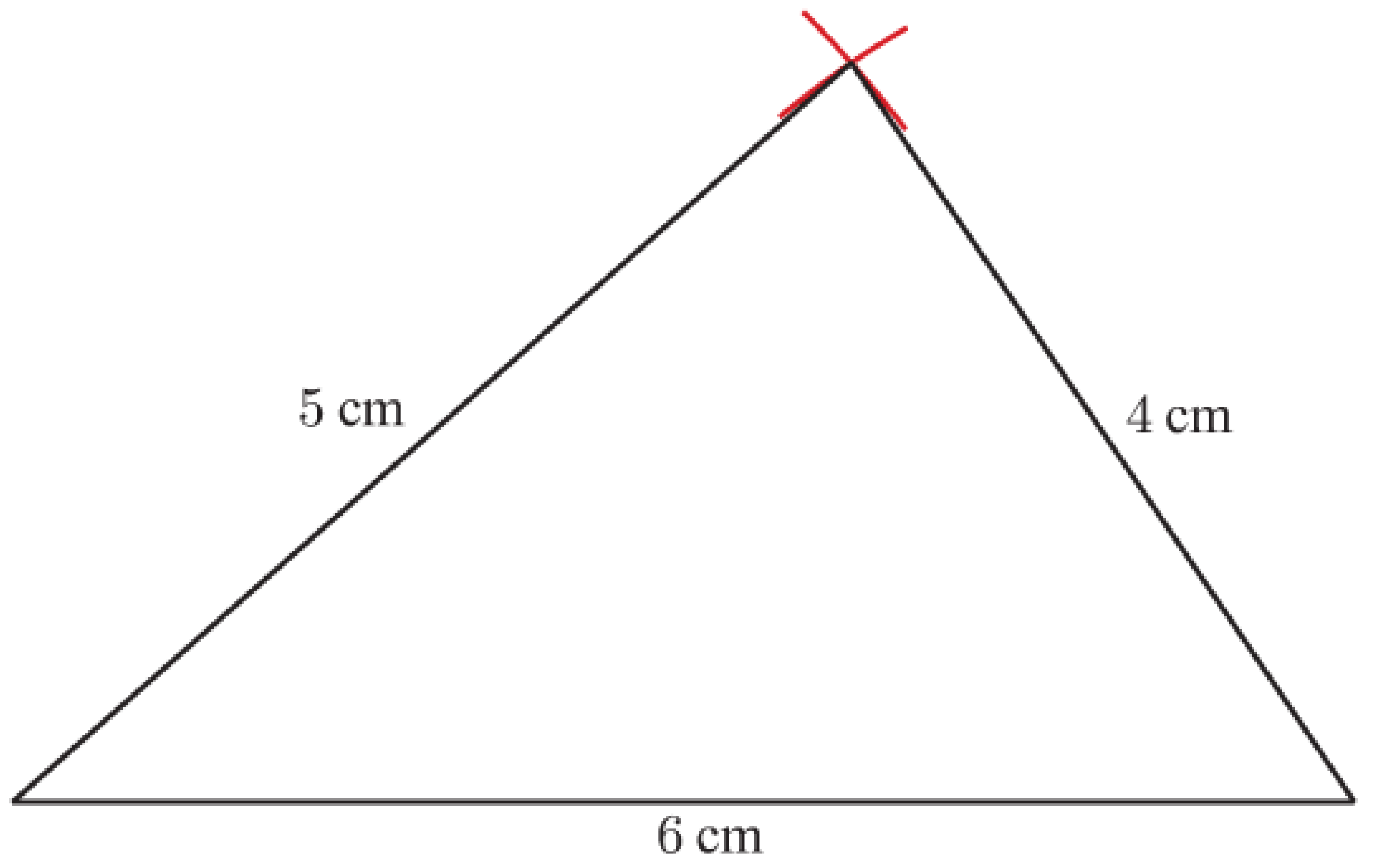
b



c



5 **a**



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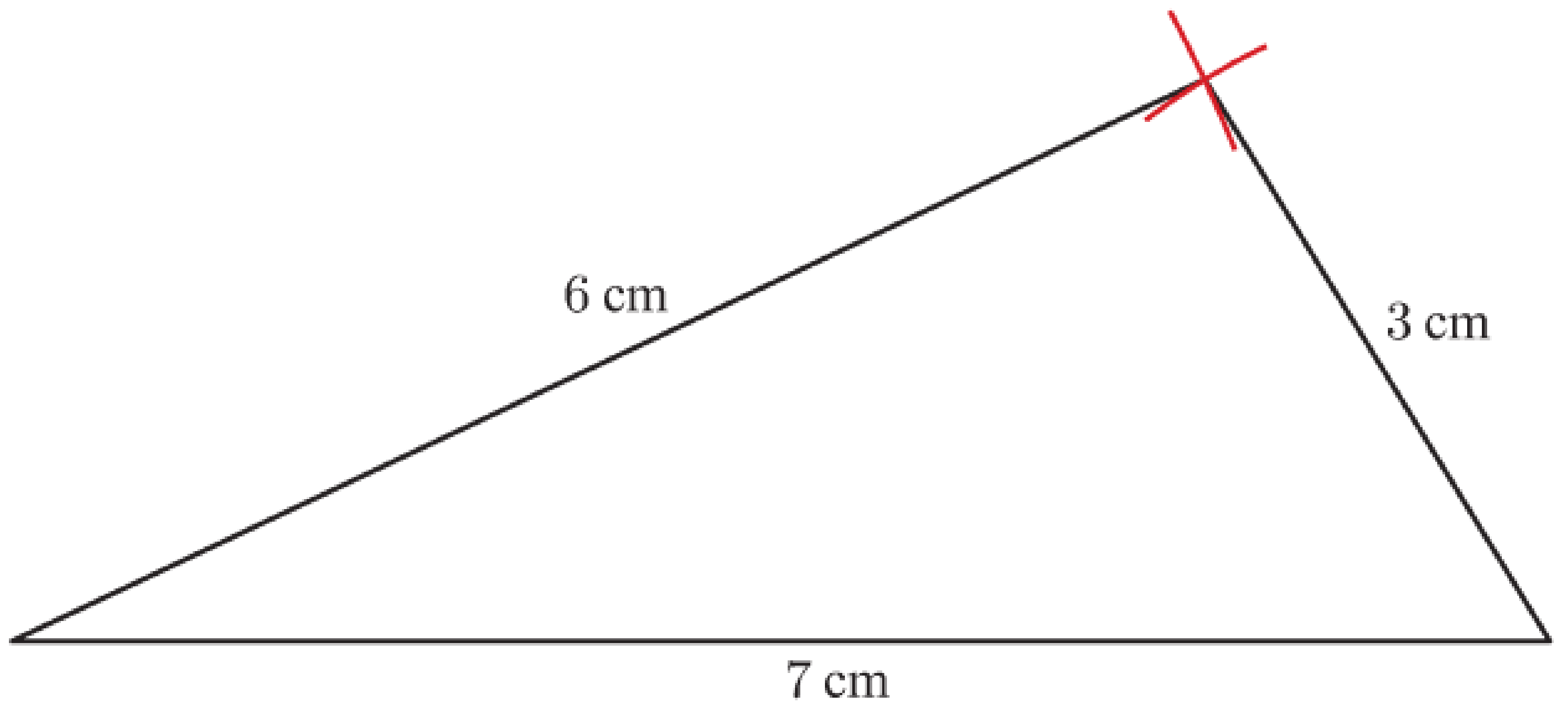
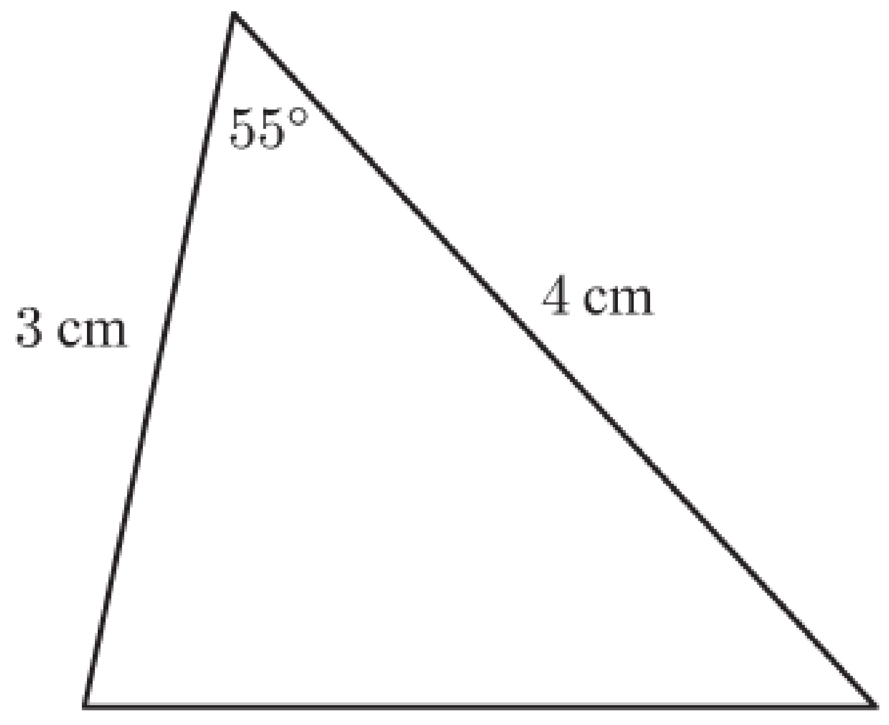
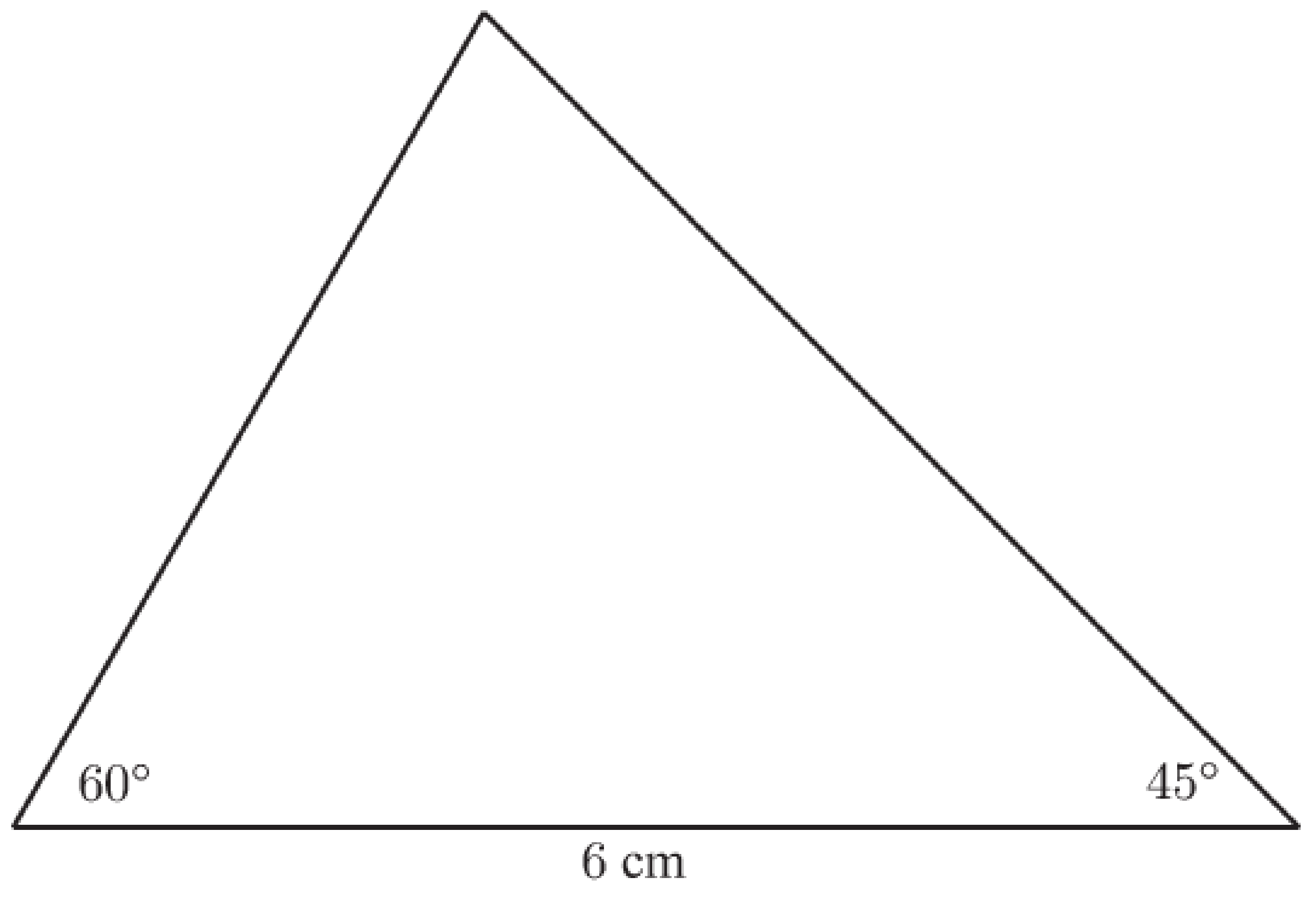


6 It is not possible. The sum of the shorter sides must be greater than the third side.

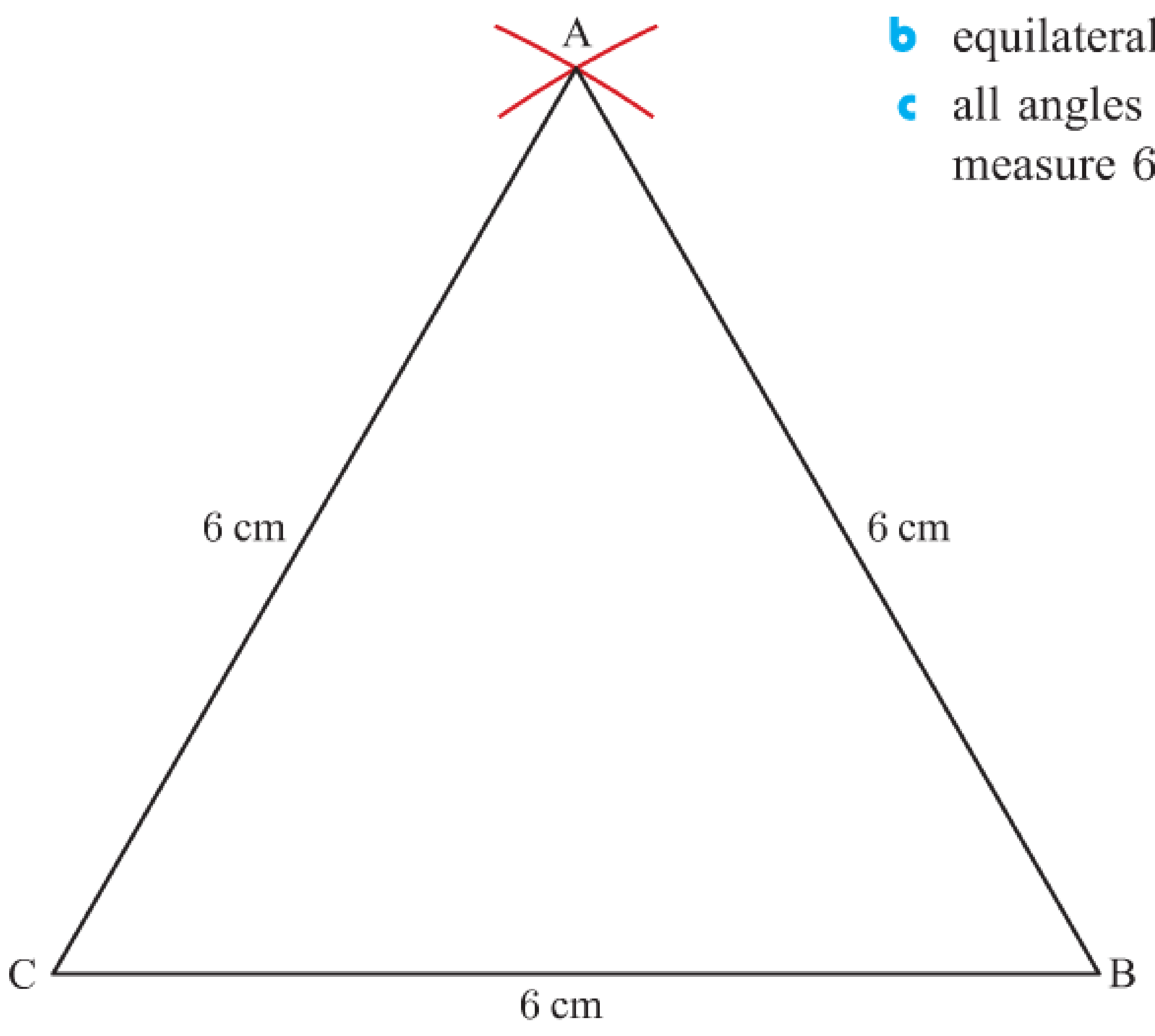
7 **a**



b



8 **a**



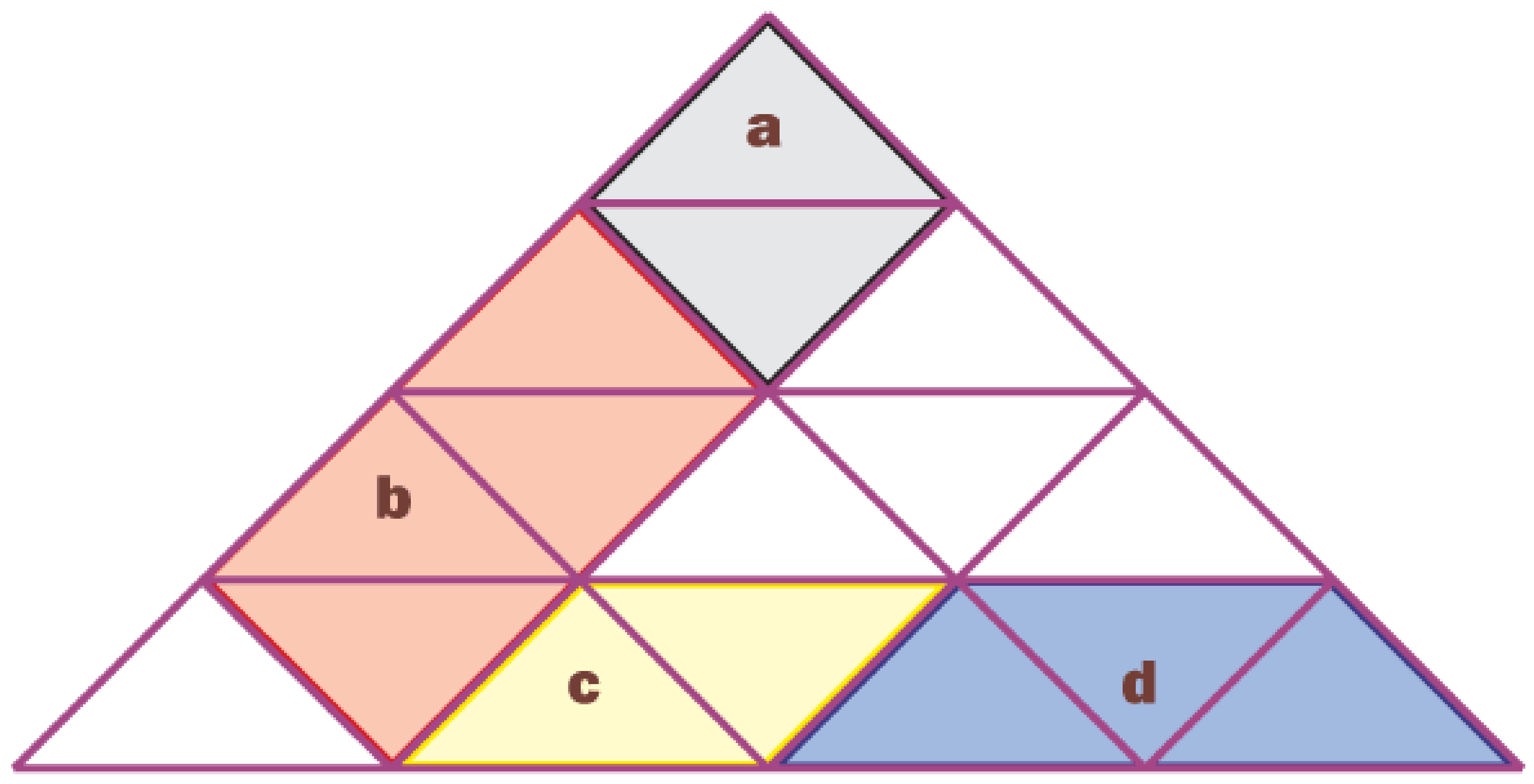
b equilateral

c all angles measure 60°

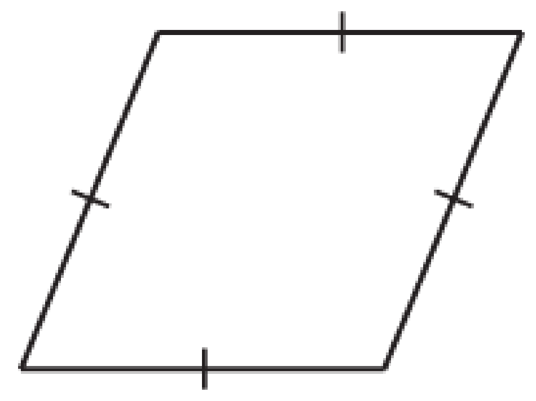
d All angles of an equilateral triangle measure 60°.

EXERCISE 5D

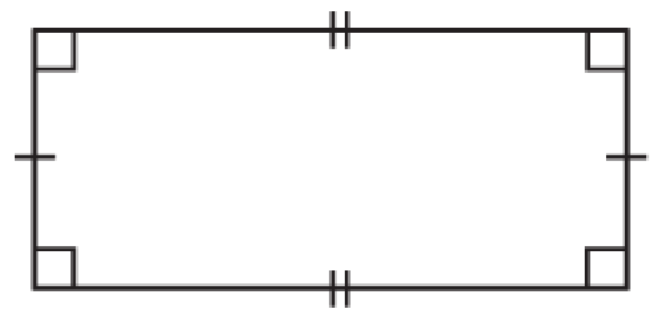
1



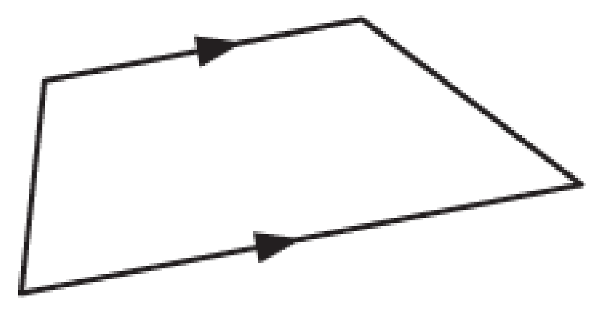
2 **a**



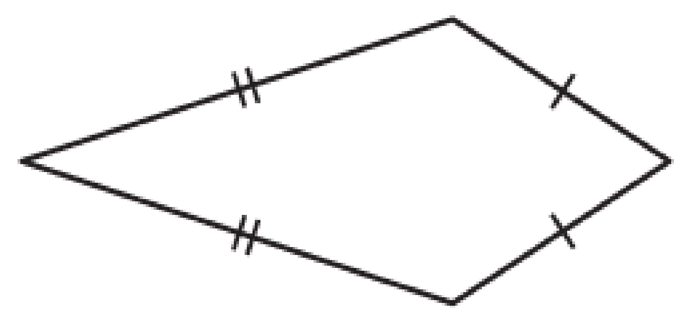
b



c

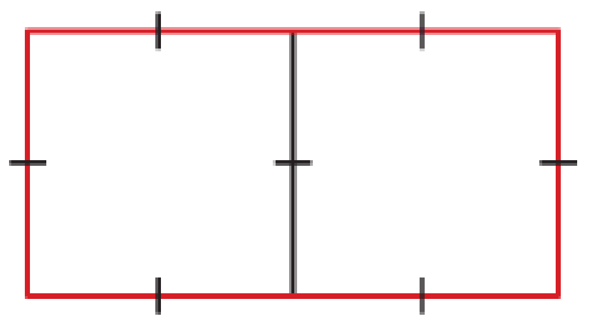


d

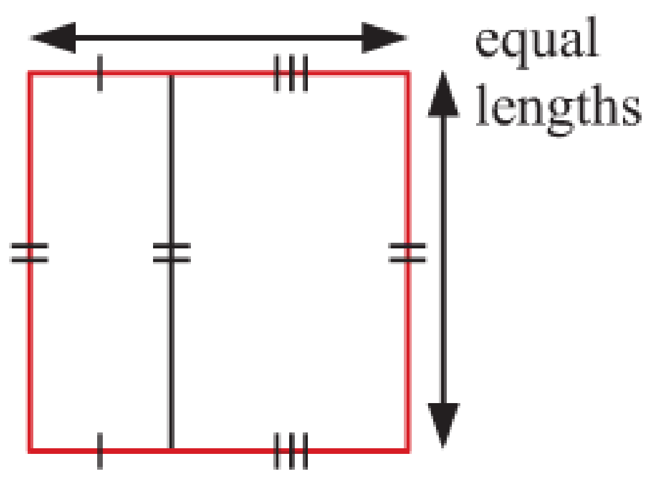


3 **a** square **b** trapezium **c** rhombus **d** rectangle
e kite **f** trapezium **g** parallelogram **h** square

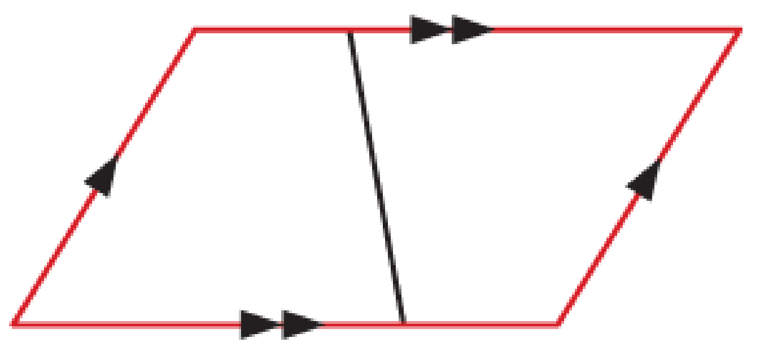
4 **a**



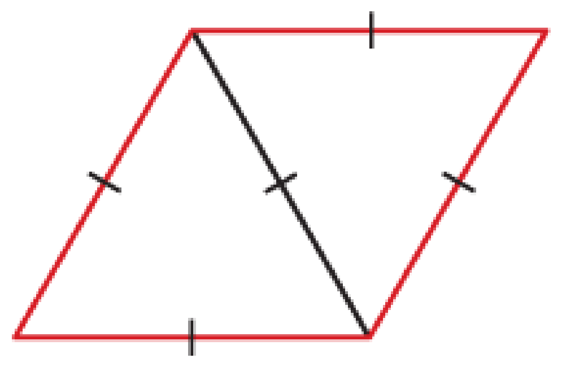
b



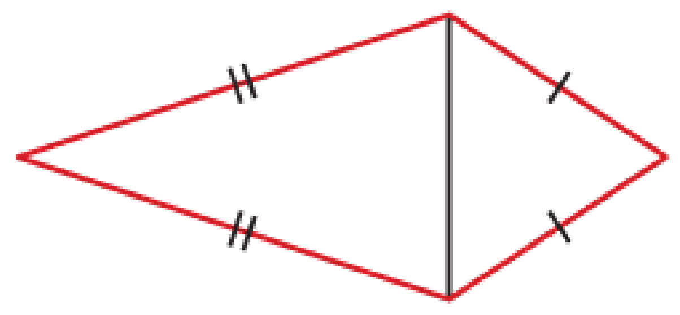
c



d



e

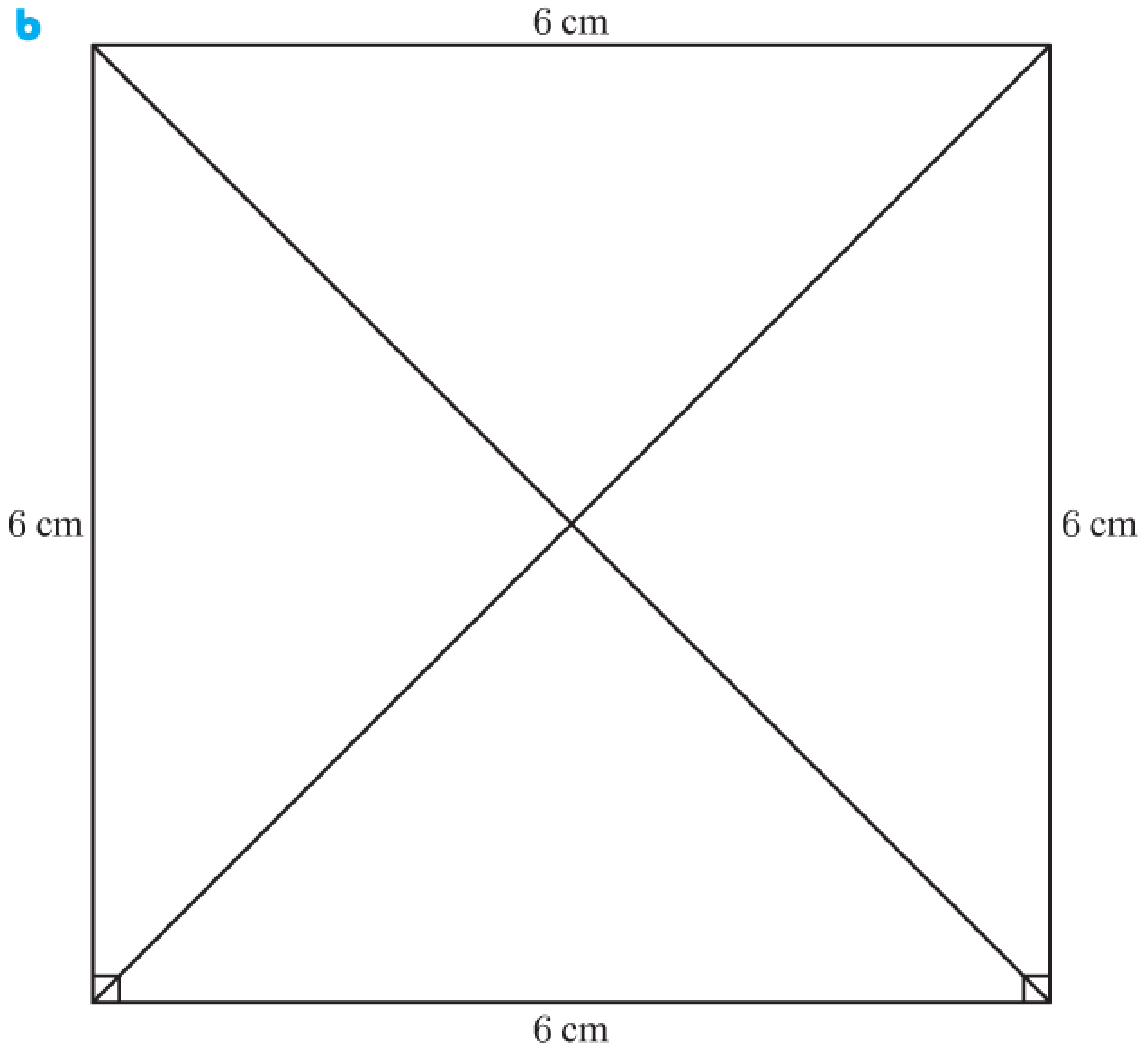


5 **a** True; it is a rhombus with all angles 90°. **b** false

c True; it is a parallelogram with all sides equal in length and all angles 90°.

d True; it is a parallelogram with all angles 90°.

6 **a**, **b**



c They are equal in length; about 8.5 cm.

EXERCISE 5E

1 **a** cylinder **b** cube **c** cone **d** sphere
e rectangular prism **f** square-based pyramid
g triangular prism **h** triangular-based pyramid

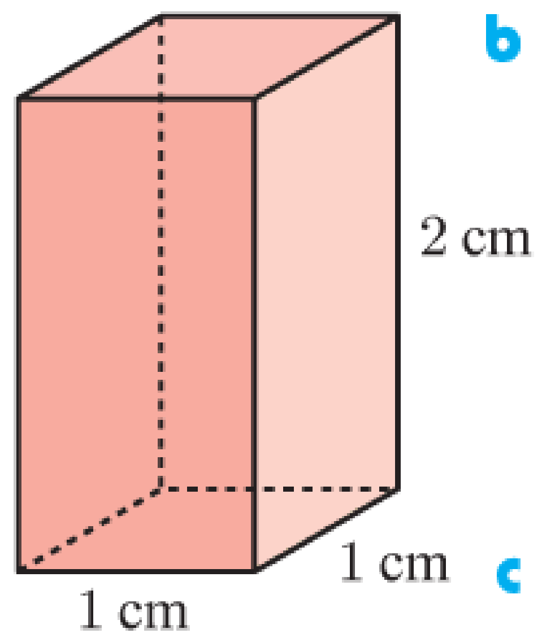
2 **a** rectangular prism **b** cylinder **c** triangular prism

3 **a** **A** **b** **B** **c** **A** **d** **C**

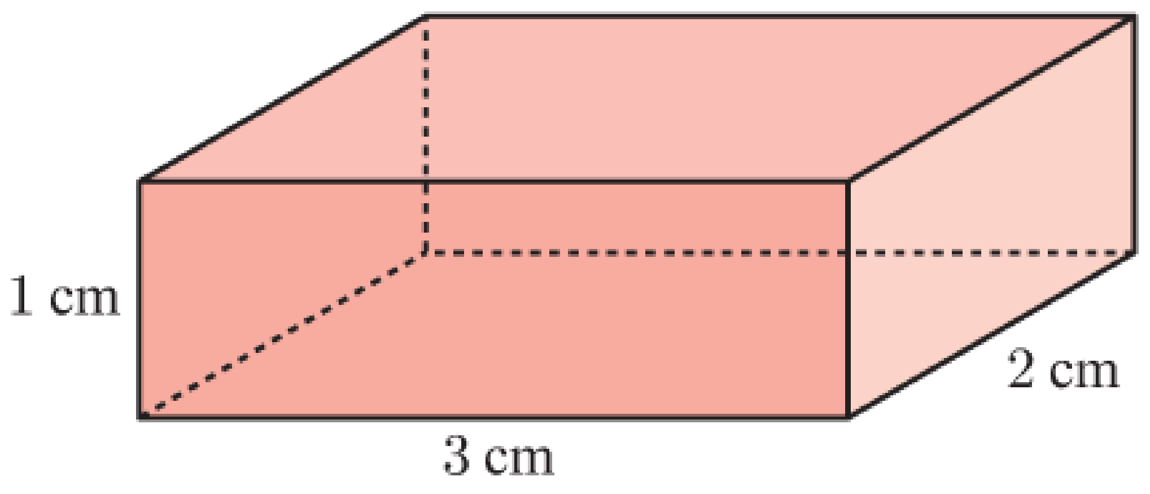
4 **a** a rectangular prism **b** a triangular-based pyramid

EXERCISE 5F

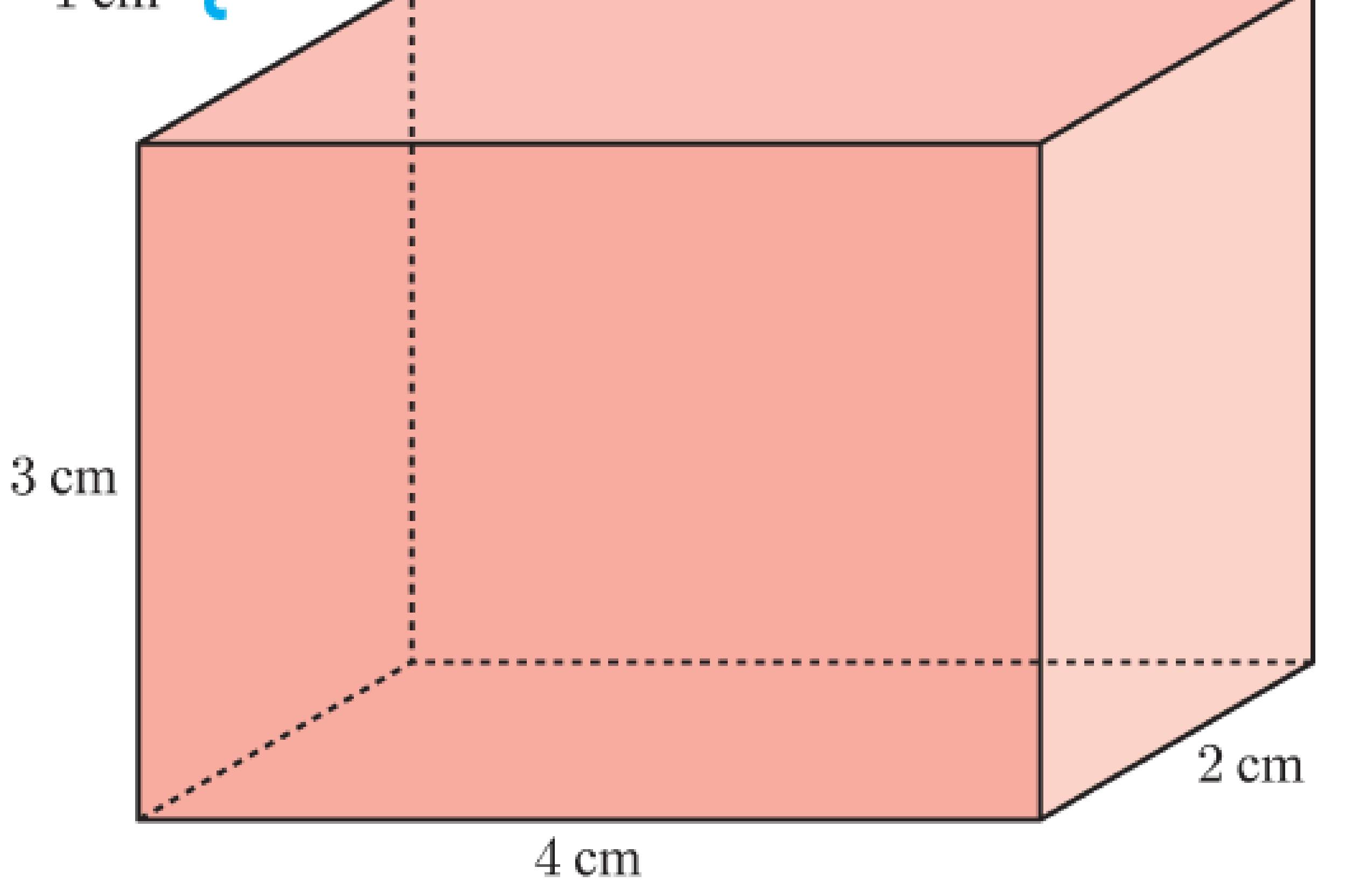
1 **a**



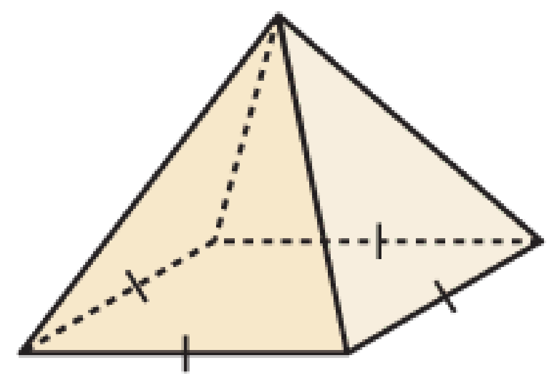
b



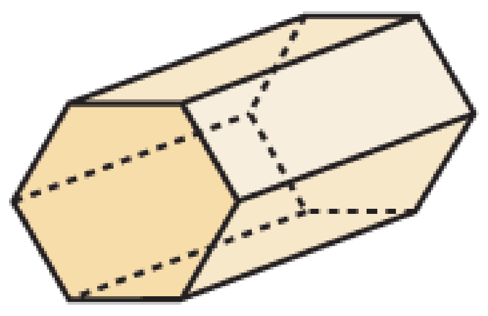
c



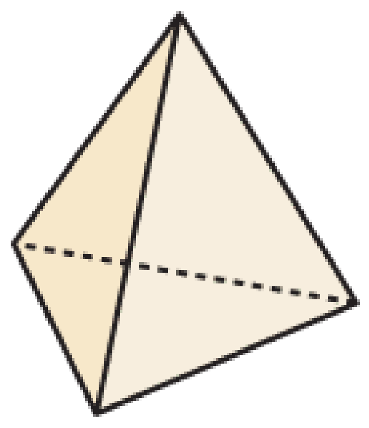
2 **a**



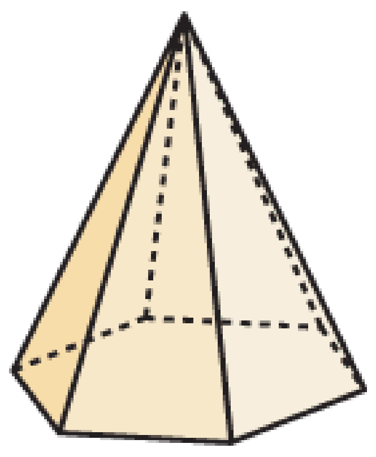
b



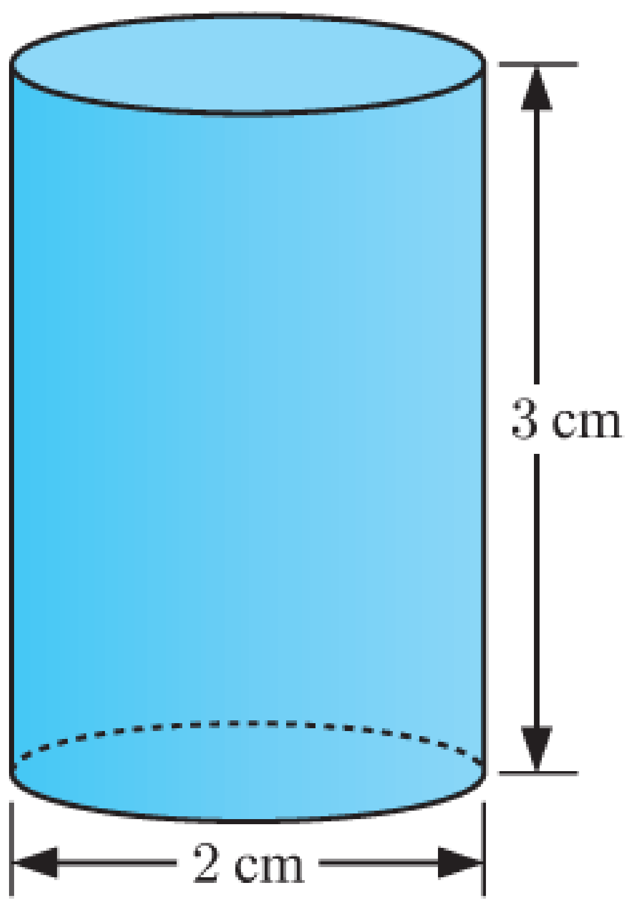
c



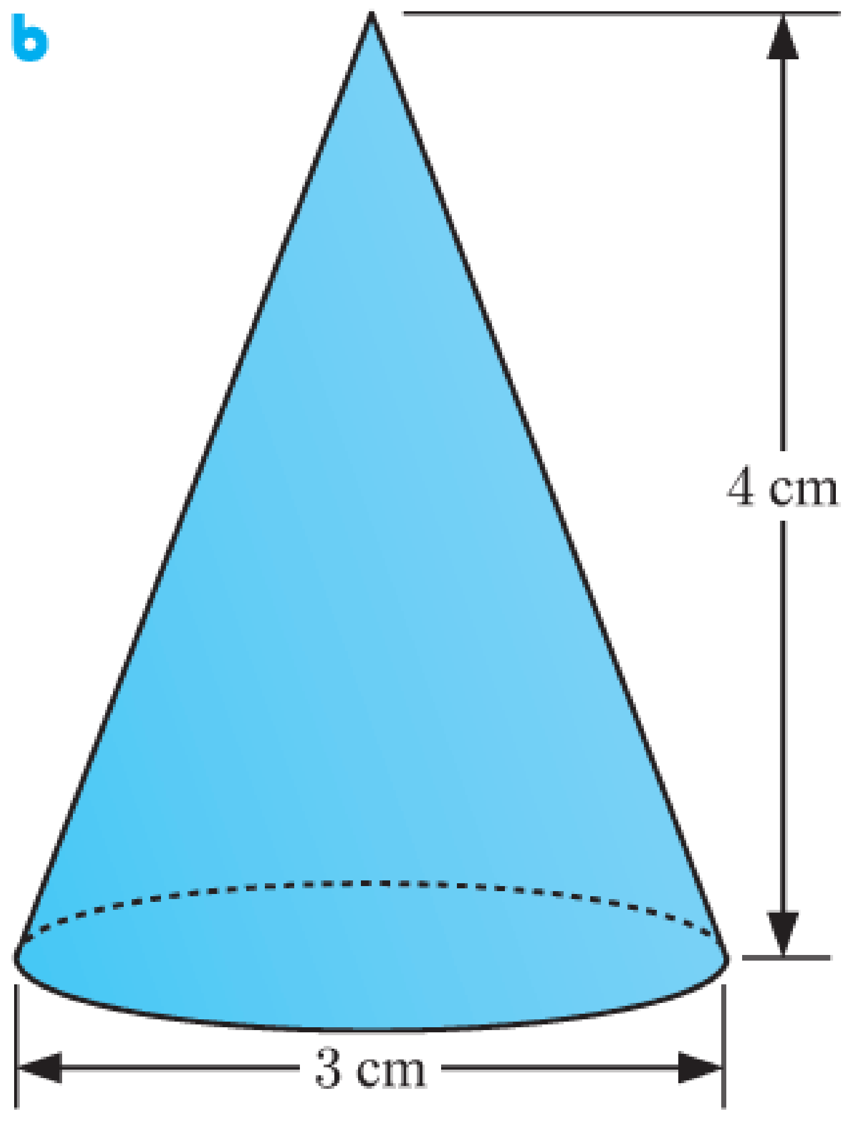
d



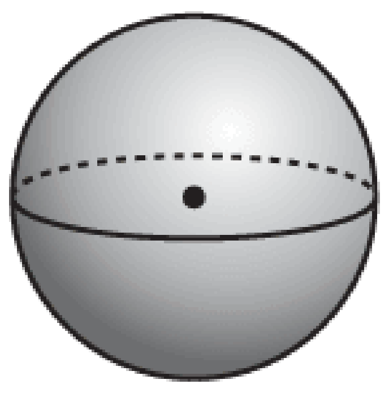
3 **a**



b

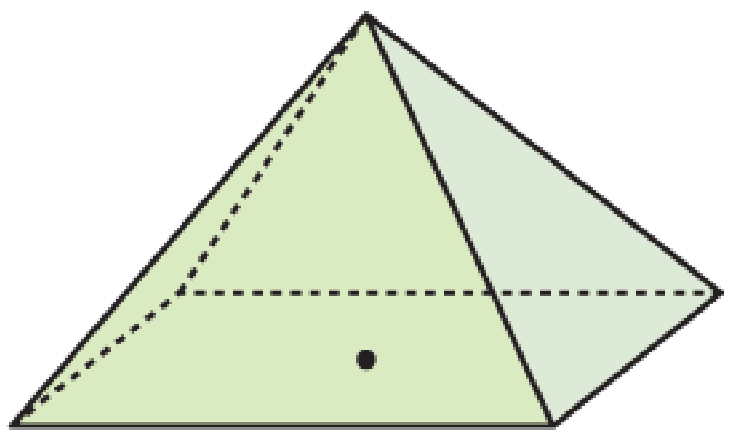


4



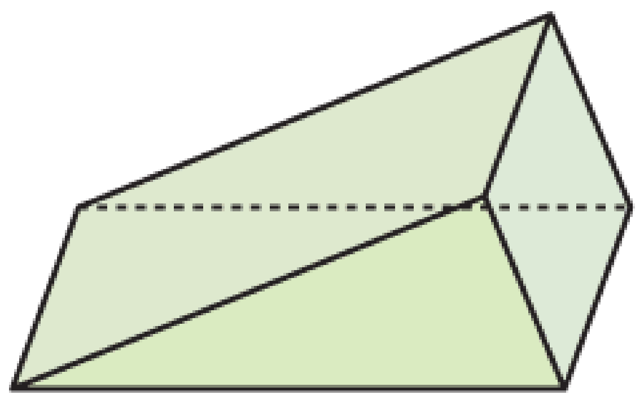
EXERCISE 5G

1 **a**



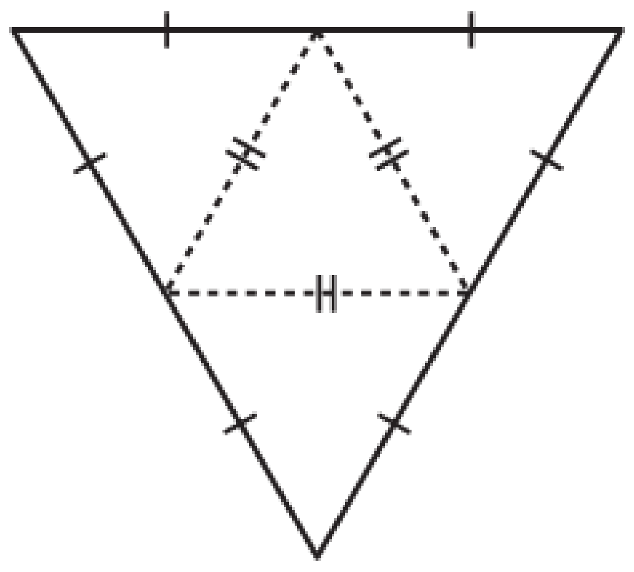
rectangular-based pyramid

b

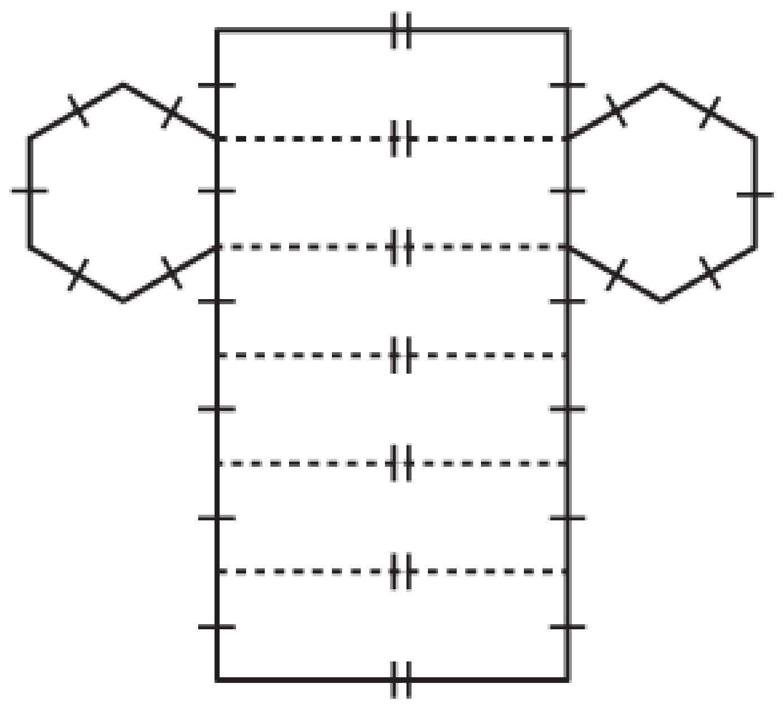


triangular prism

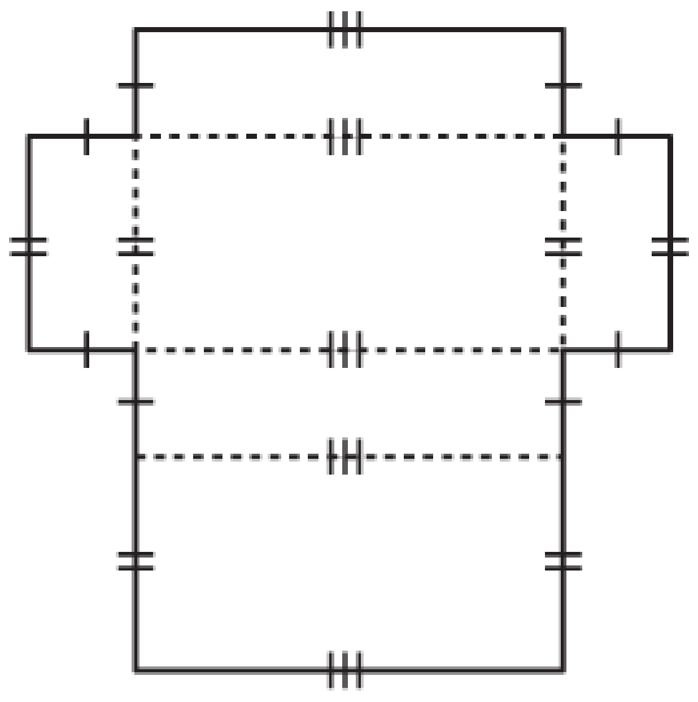
2 **a**



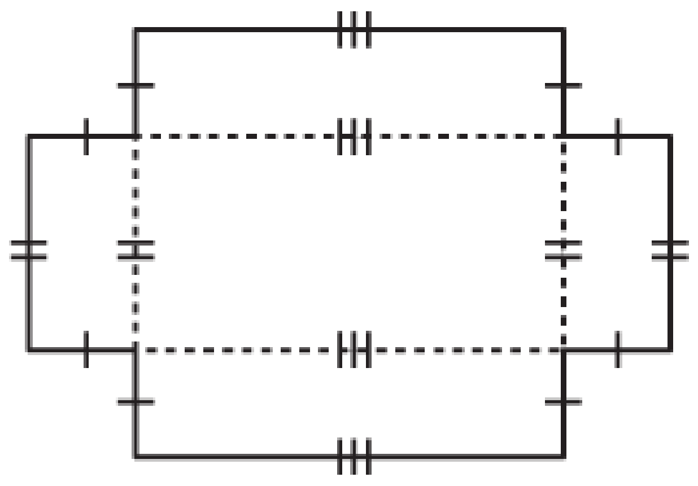
b



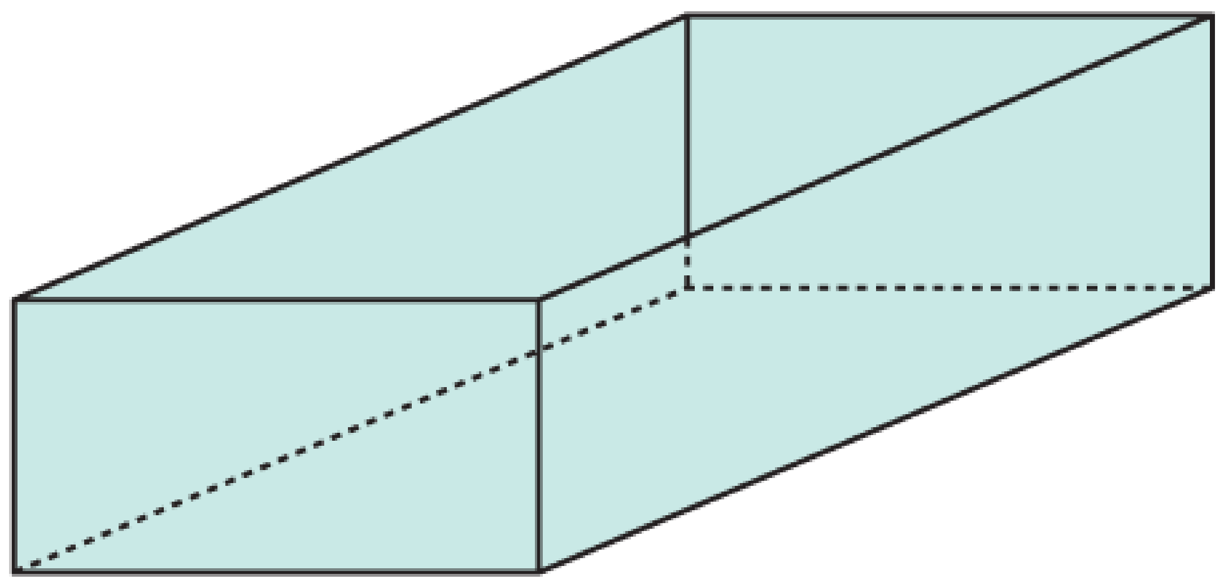
3 **a**



b



c

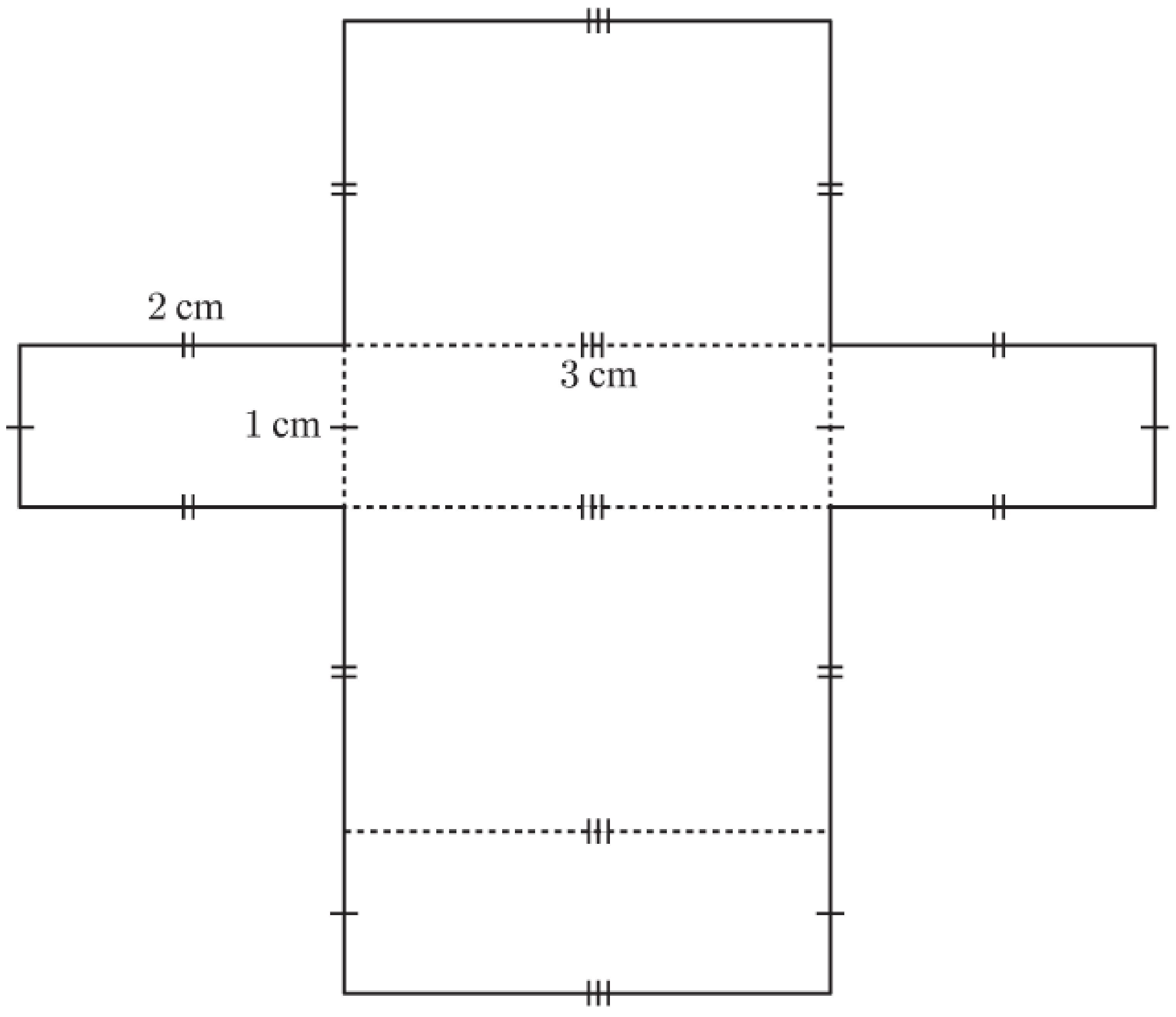


4 **a** **B** and **3** **b** **A** and **4** **c** **D** and **1** **d** **C** and **2**

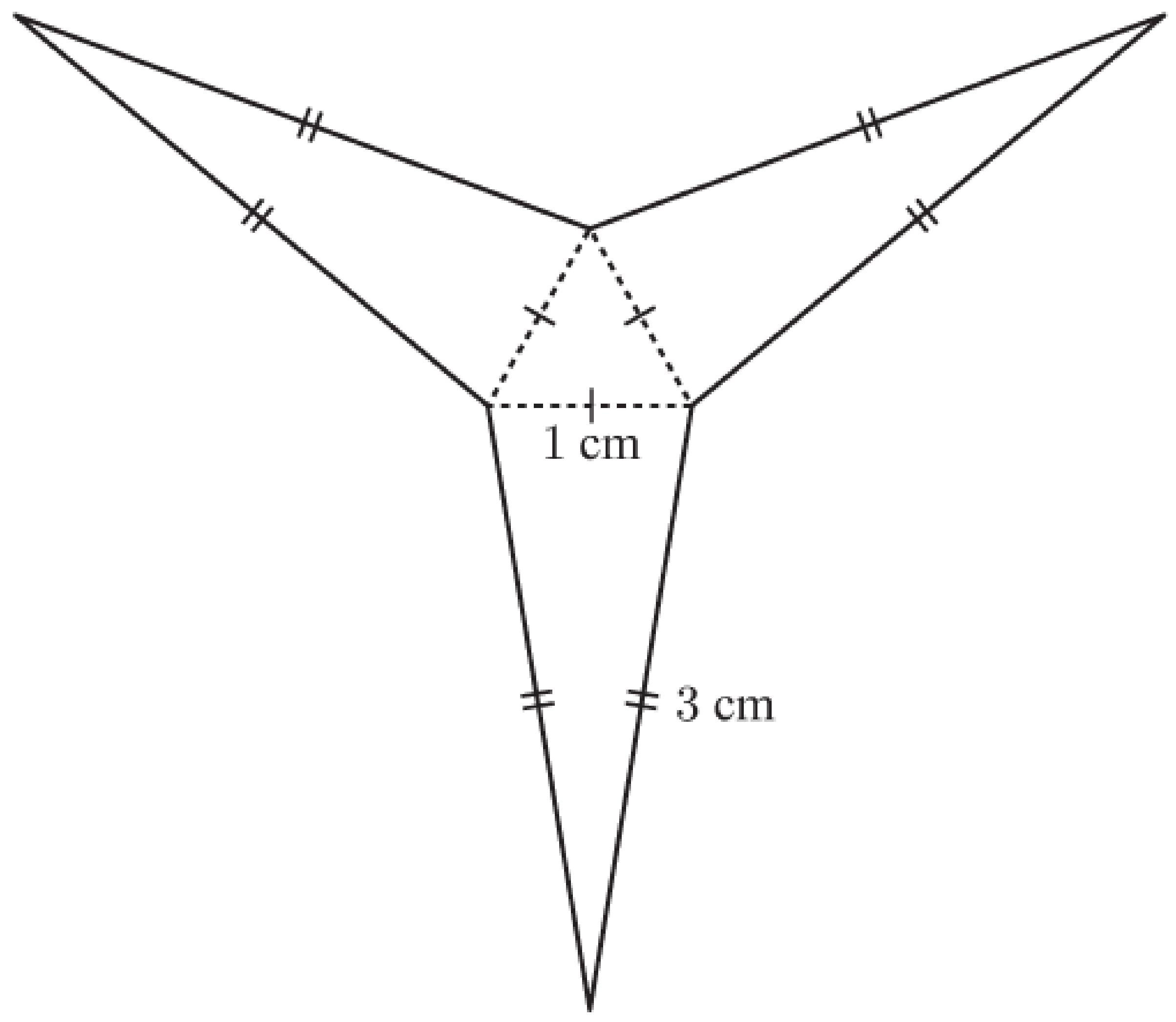
5 **a** The sides of the triangles are not long enough to meet at the apex.
b Eric's, as the sides which meet at the apex are longer than Derek's and the square base is smaller.

6 **a** yes **b** no **c** yes **d** yes **e** no **f** no

7 **a**



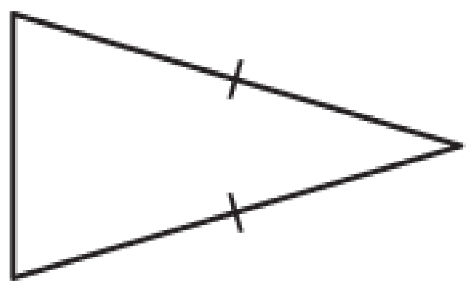
b



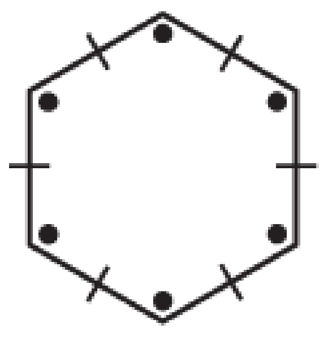
REVIEW SET 5A

1 **a** hexagon **b** quadrilateral **c** dodecagon

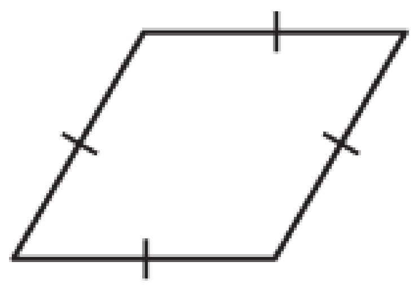
2 **a**



b

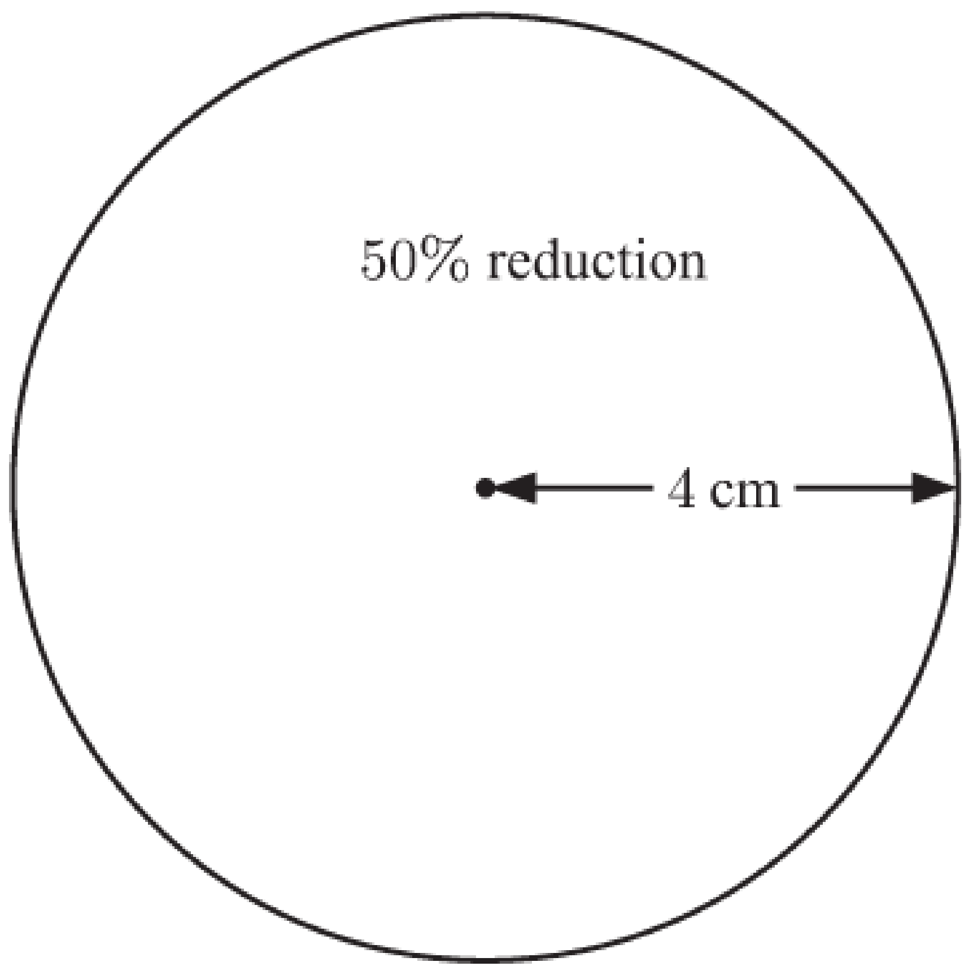


c

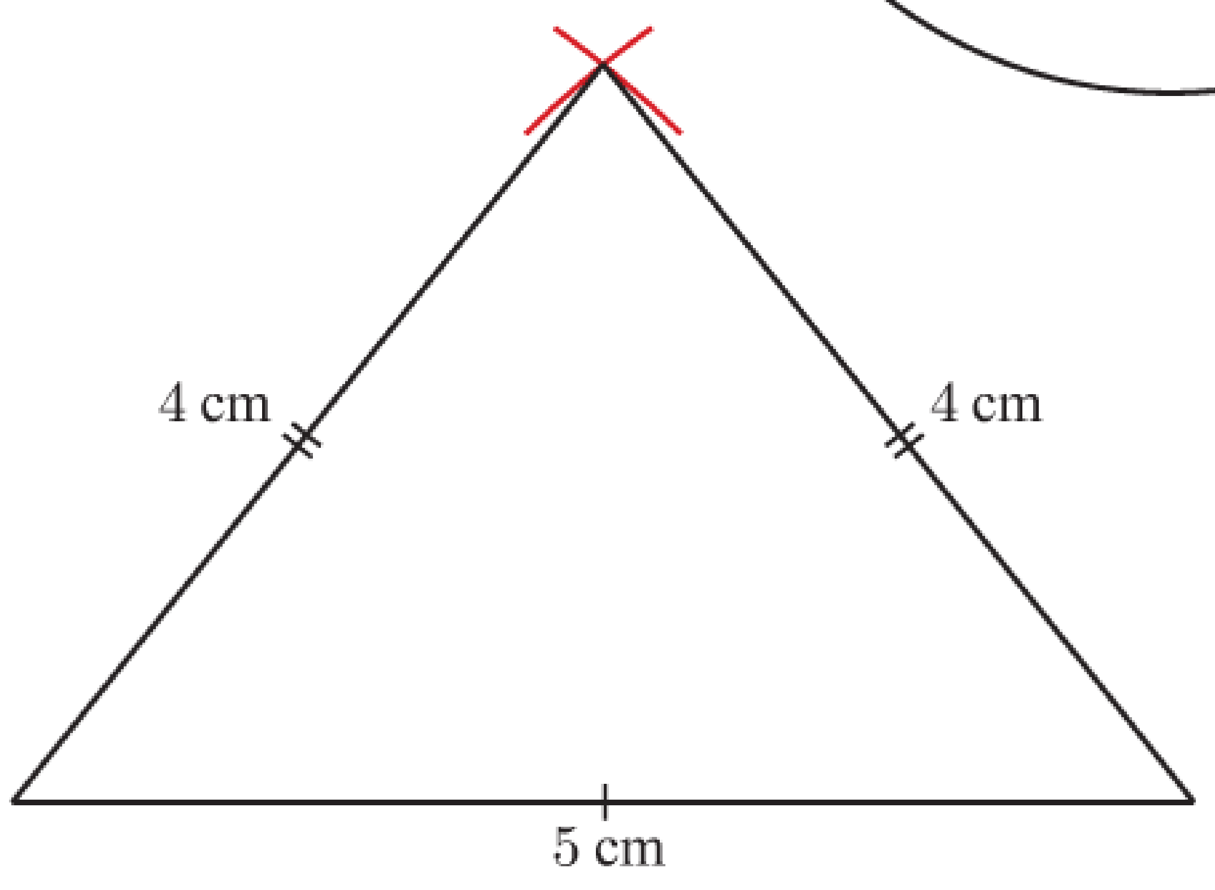


3 **a** scalene
b isosceles
c equilateral

4



5



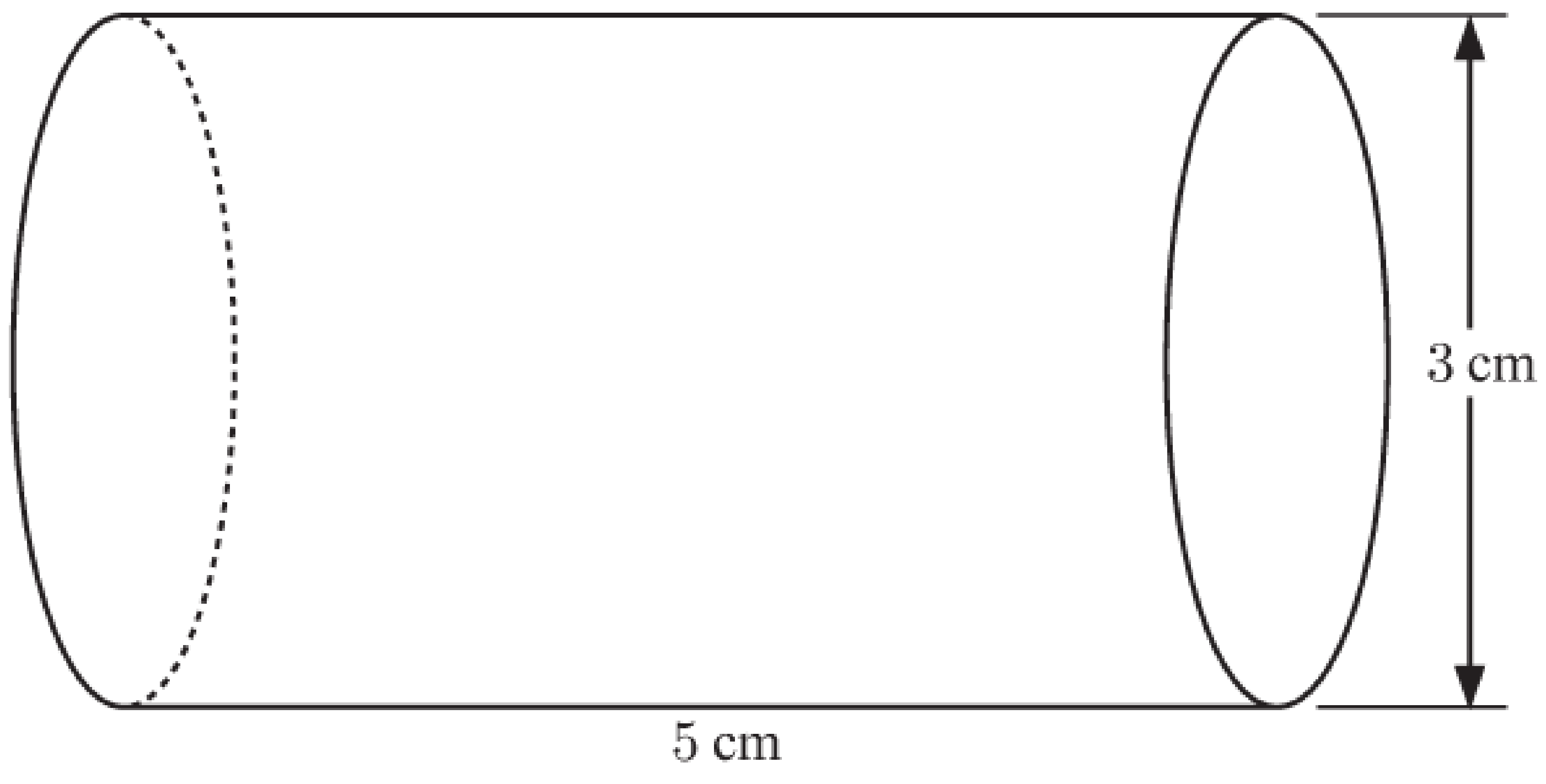
6 **a** square **b** kite **c** rhombus **d** trapezium

7 **a** sphere **b** cone **c** pentagonal-based pyramid

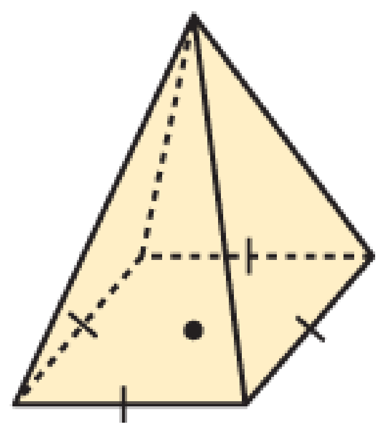
8 **a** An equilateral triangle has all three sides equal in length.
∴ it satisfies the requirements of an isosceles triangle of having two equal sides.

b A square has all four sides equal in length.
∴ it satisfies the requirements of a kite of having two pairs of adjacent sides which are equal in length.

9

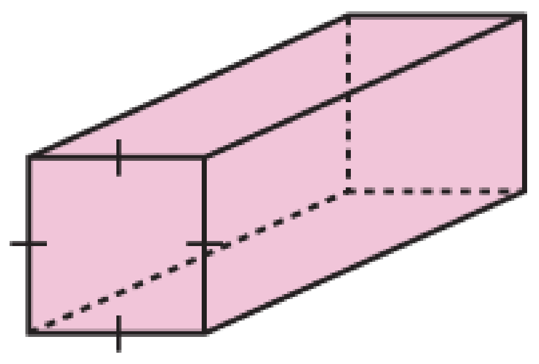


10 **a**



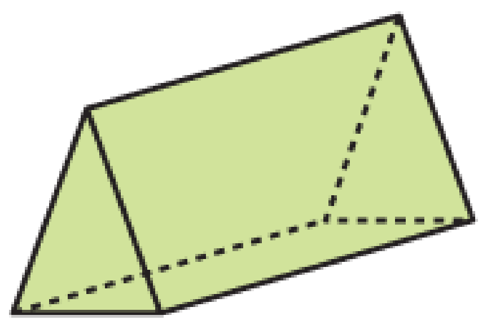
square-based pyramid

b



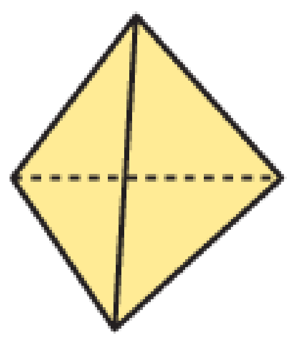
square prism

c



triangular prism

d

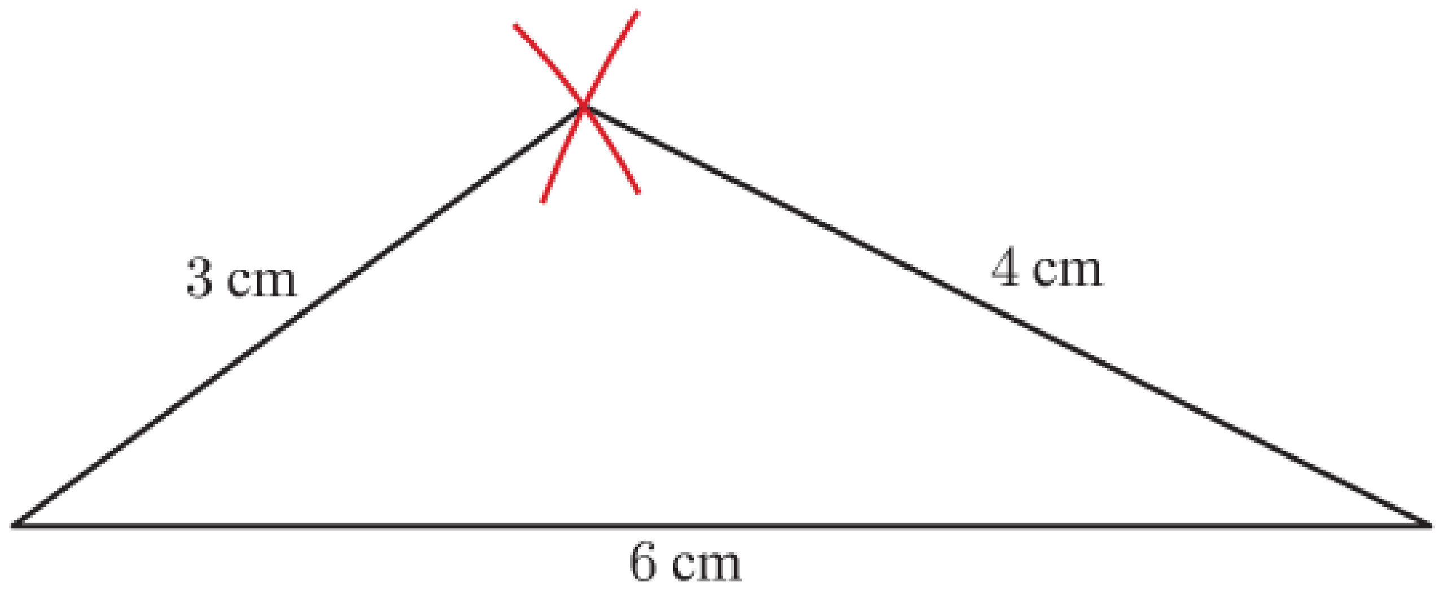


triangular-based pyramid

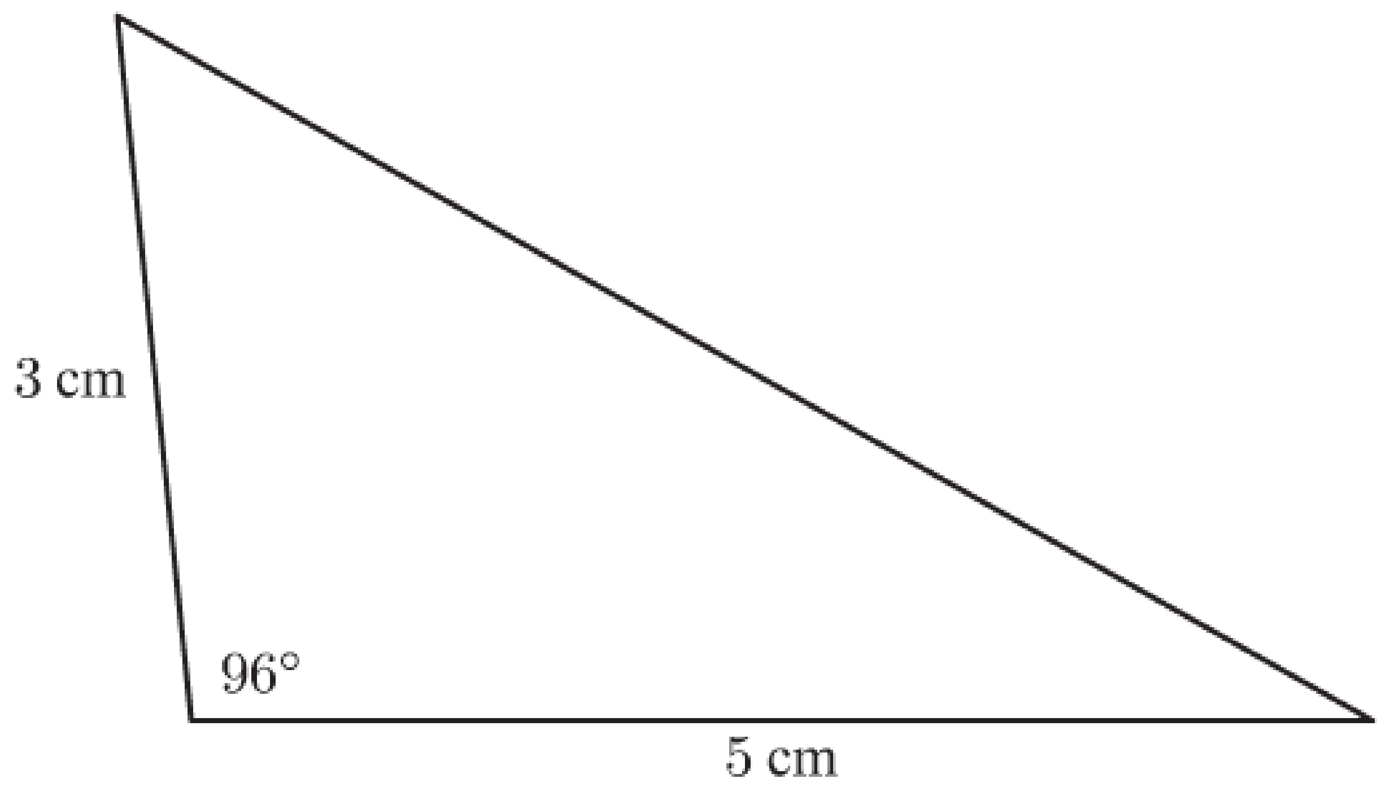
REVIEW SET 5B

1 **a** Not regular; angles are not all equal (to 90°).
b Not regular; sides are not all equal.

2

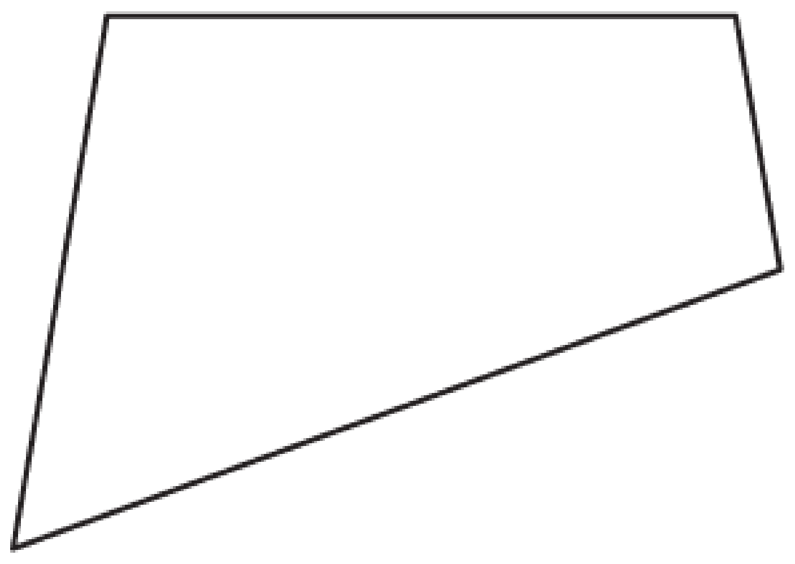


3



4 **a** a square, four identically shaped isosceles triangles, an octagon
b isosceles **c** a square-based pyramid

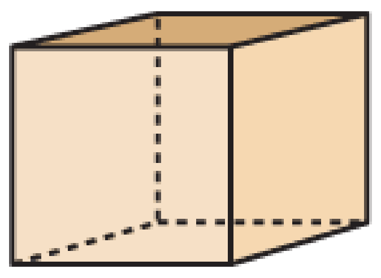
5 scalene **6**



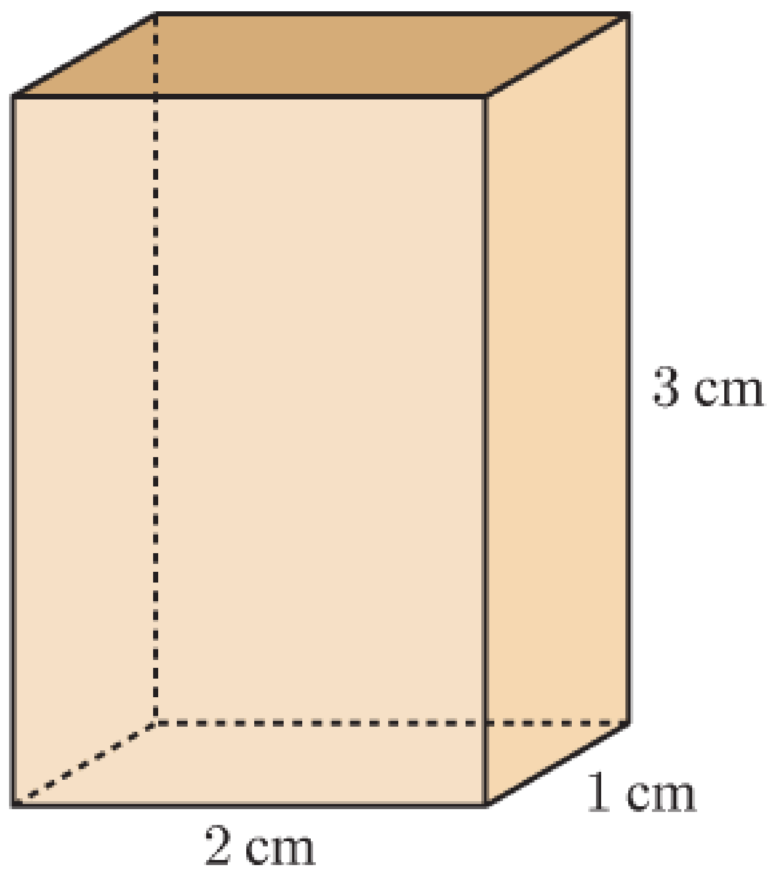
7 **a** sphere **b** rectangular prism

8 **a** Q **b** T **c** S **d** R **e** P

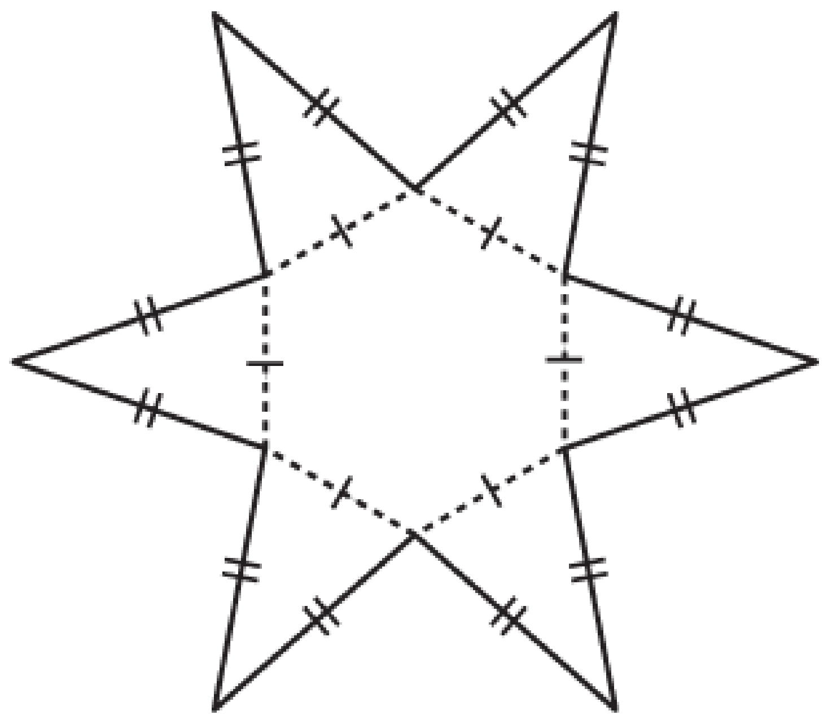
9 **a**



b



10



EXERCISE 6A

1 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$ **e** $\frac{3}{8}$ **f** $\frac{5}{8}$
g $\frac{2}{7}$ **h** $\frac{3}{10}$ **i** $\frac{1}{100}$

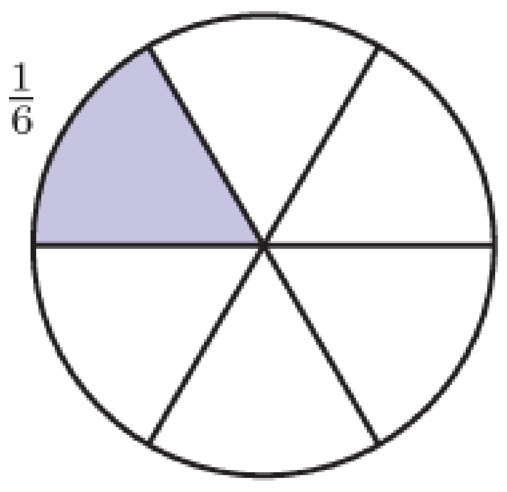
2 **a** one third **b** two ninths **c** three fifths
d seven eighths **e** four ninths **f** five sevenths
g five twelfths **h** seventeen twentieths
i eleven thirtieths **j** four twenty fifths
k three hundredths **l** ninety seven hundredths

3 **a** 2 **b** 4 **c** 3 **d** 1

4 **a** 3 **b** 5 **c** 7 **d** 8

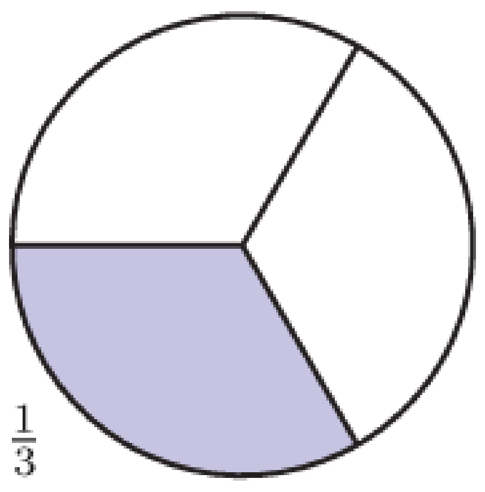
5 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{2}{3}$ **e** $\frac{3}{4}$ **f** $\frac{1}{4}$
g $\frac{1}{8}$ **h** $\frac{3}{8}$ **i** $\frac{1}{16}$ **j** $\frac{3}{16}$ **k** $\frac{5}{16}$ **l** $\frac{11}{16}$

6 **a**



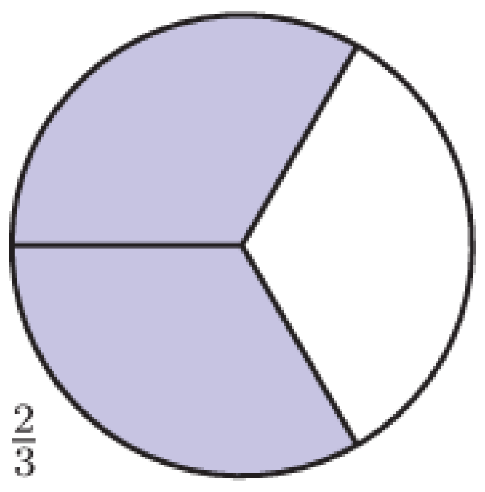
$\frac{1}{6}$

b



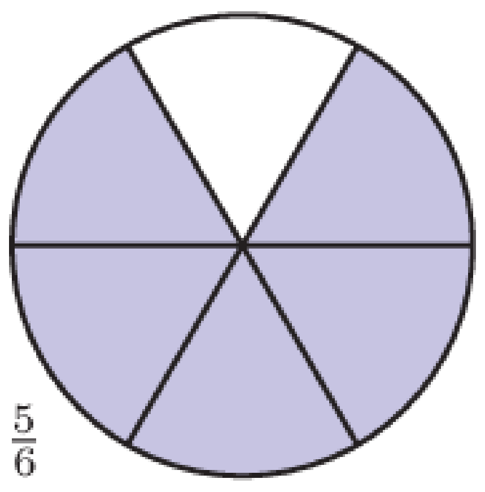
$\frac{1}{3}$

c



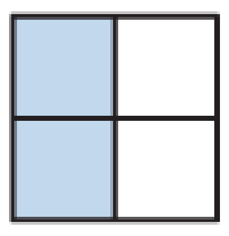
$\frac{2}{3}$

d



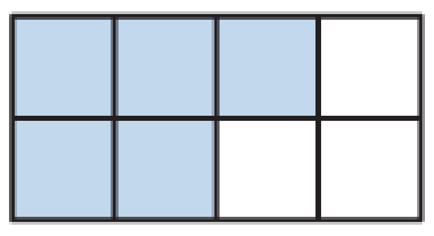
$\frac{5}{6}$

7 **a**



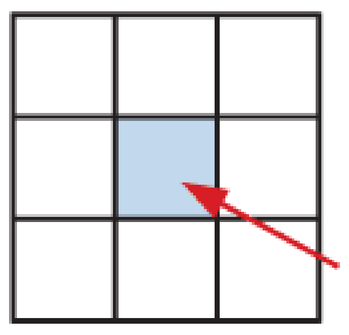
$\frac{2}{4}$

b



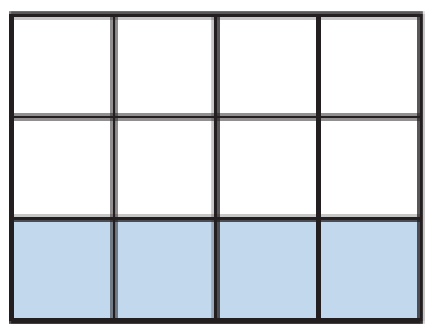
$\frac{5}{8}$

c



$\frac{1}{9}$

d



$\frac{4}{12}$

8 No, as the eight smaller divisions are not equal in size (area).

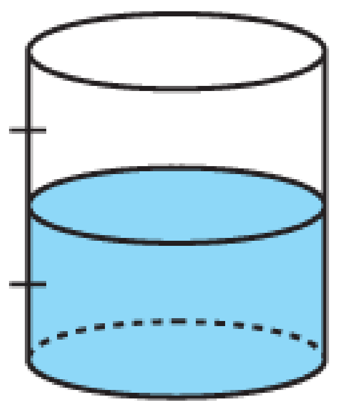
9 **a** $\frac{6}{10}$ **b** $\frac{6}{11}$ **c** $\frac{8}{15}$ **10** **a** $\frac{5}{8}$ **b** $\frac{3}{8}$

11 **a** **i** $\frac{6}{11}$ **ii** $\frac{5}{11}$ **b** **i** $\frac{4}{11}$ **ii** $\frac{7}{11}$

12 **a** **i** $\frac{2}{7}$ **ii** $\frac{5}{7}$ **b** **i** $\frac{4}{7}$ **ii** $\frac{3}{7}$

13 **a** $\frac{1}{4}$ **b** $\frac{3}{5}$ **c** $\frac{1}{6}$

14 **a**

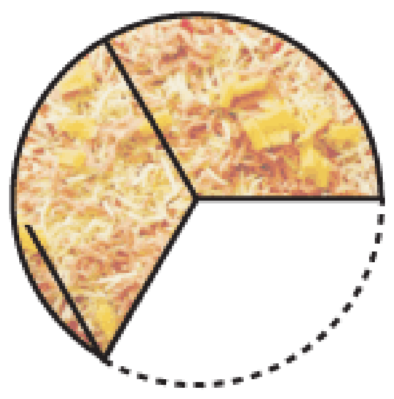


$\frac{1}{2}$ full

b

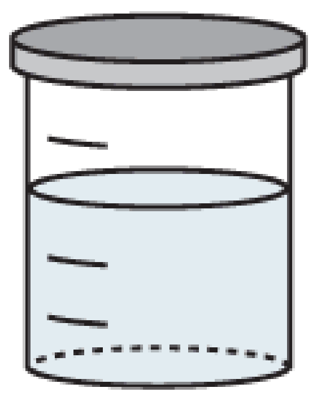
$\frac{3}{4}$ full

c



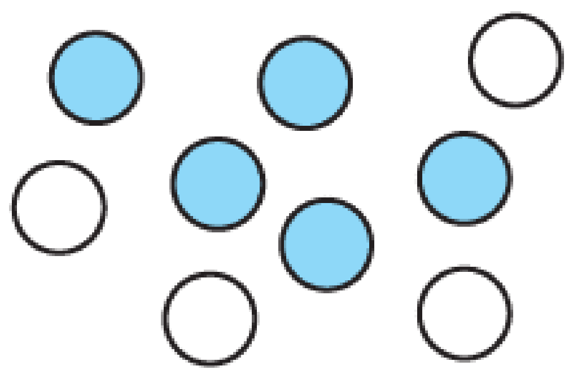
$\frac{1}{3}$ missing

d



$\frac{3}{5}$ full

e



$\frac{5}{9}$ are blue

EXERCISE 6B

1 **a** $\frac{4}{5}$ **b** $\frac{1}{7}$ **c** $\frac{3}{10}$ **d** $\frac{8}{9}$ **e** $\frac{2}{11}$ **f** $\frac{12}{13}$

2 **a** $\frac{2}{3}$ **b** Each person gets $\frac{2}{3}$ of a pizza.
c 2 pizzas $\div$ 3 people $= \frac{2}{3}$ of a pizza each

3 **a** $1 \div 3$ **b** $2 \div 5$ **c** $7 \div 8$ **d** $3 \div 4$
e $8 \div 13$ **f** $11 \div 20$

4 **a** $20 \div 5 = 4$ **b** $27 \div 3 = 9$ **c** $55 \div 11 = 5$
d $7 \div 7 = 1$ **e** $24 \div 12 = 2$ **f** $19 \div 19 = 1$
g $0 \div 8 = 0$ **h** $108 \div 9 = 12$

EXERCISE 6C

1 **a** proper fraction **b** improper fraction
c proper fraction **d** mixed number
e improper fraction **f** proper fraction
g mixed number **h** improper fraction

2 **a** $2\frac{1}{2}$ m **b** $1\frac{3}{4}$ L **c** $4\frac{1}{4}$ kg

3 **a** 7 halves **b** $3\frac{1}{2} = \frac{7}{2}$

4 **a** $3\frac{1}{4}$ **b** 13 quarters **c** $3\frac{1}{4} = \frac{13}{4}$

5 **a** $\frac{5}{4}$ **b** $\frac{5}{2}$ **c** $\frac{11}{3}$ **d** $\frac{17}{6}$ **e** $\frac{8}{5}$ **f** $\frac{16}{3}$
g $\frac{13}{2}$ **h** $\frac{19}{8}$ **i** $\frac{25}{6}$ **j** $\frac{37}{10}$

6 **a** 4 sandwiches **b** 1 quarter **c** $\frac{17}{4} = 4\frac{1}{4}$

7 **a** $1\frac{1}{3}$ **b** $2\frac{1}{4}$ **c** $1\frac{5}{6}$ **d** $3\frac{1}{5}$ **e** $4\frac{3}{4}$ **f** $7\frac{1}{2}$
g $4\frac{2}{3}$ **h** $2\frac{3}{7}$ **i** $3\frac{3}{10}$ **j** $4\frac{3}{8}$

8 $3\frac{4}{5}$ carrots

EXERCISE 6D

1 **a** 5 **b** 18 **c** 4 **d** 5 **e** 11 **f** 6
g 5 **h** 6 **i** 7 **j** 15 **k** 24 **l** 50

2 **a** 10 people **b** 5 lollies **c** 7 drinks **d** 65 g
e €19 **f** 15 minutes

3 5 games **4** 50 students **5** 80 cars **6** $180

7 **a** 12 plants **b** 84 plants

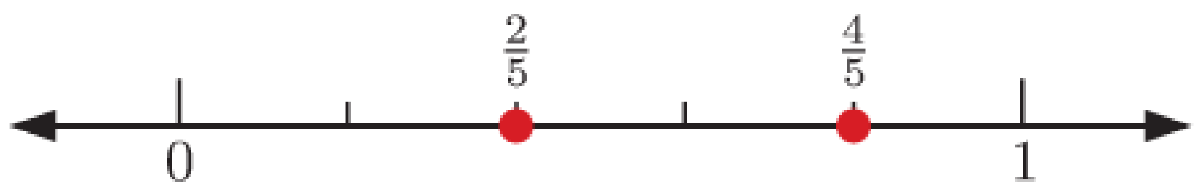
8 **a** 6 **b** 18 **c** 18 **d** 21 **e** 12 **f** 45
g 84 **h** 64 **i** 260

9 22 school children **10** 6 hours **11** 14 goals

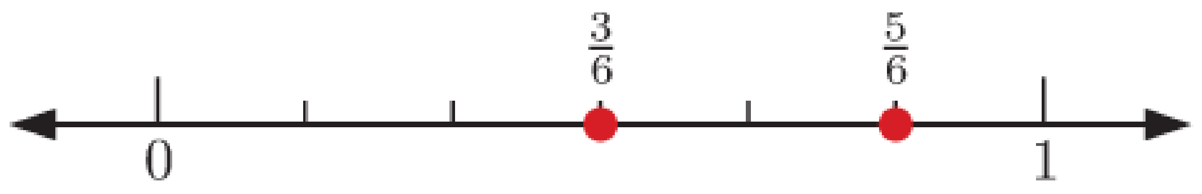
12 **a** 1875 kg **b** 30 boxes

EXERCISE 6E

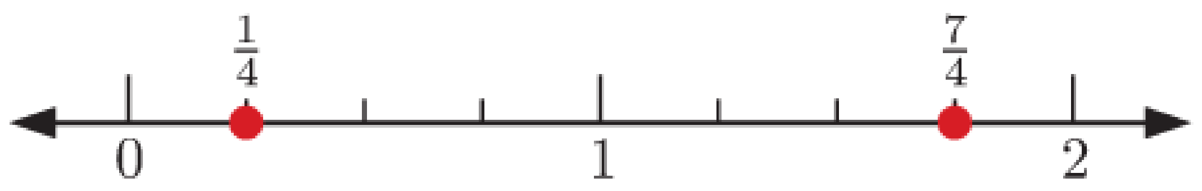
1 **a**



b



c



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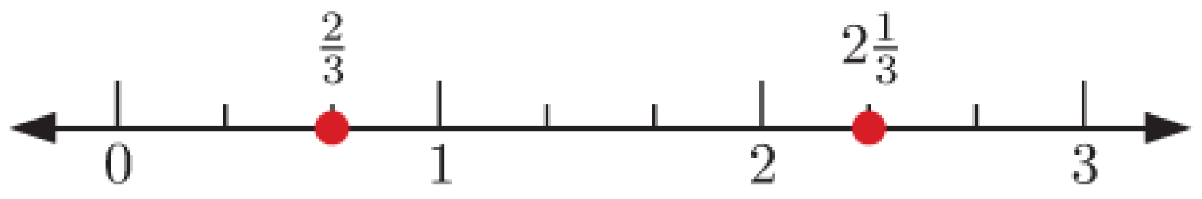

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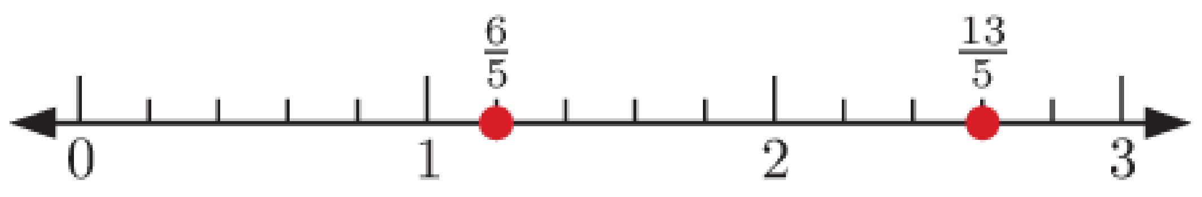

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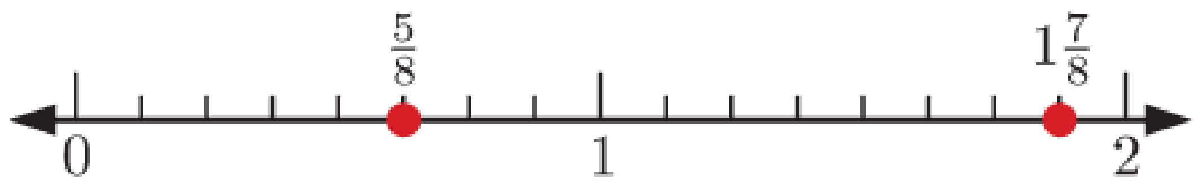

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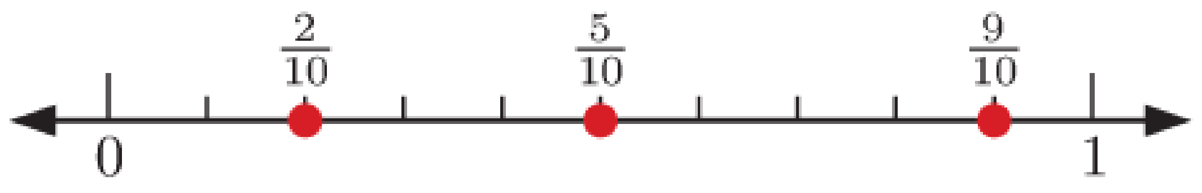

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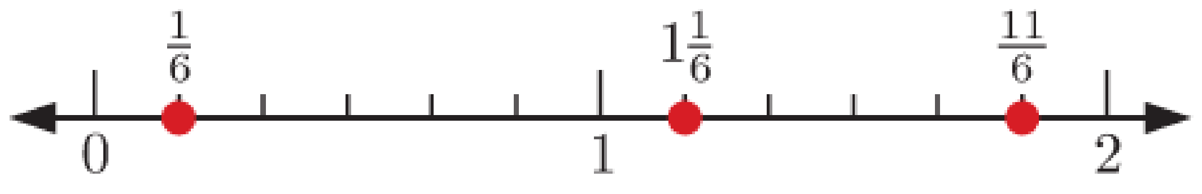
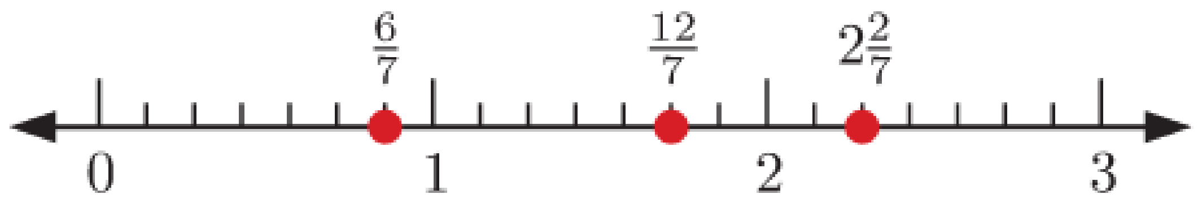
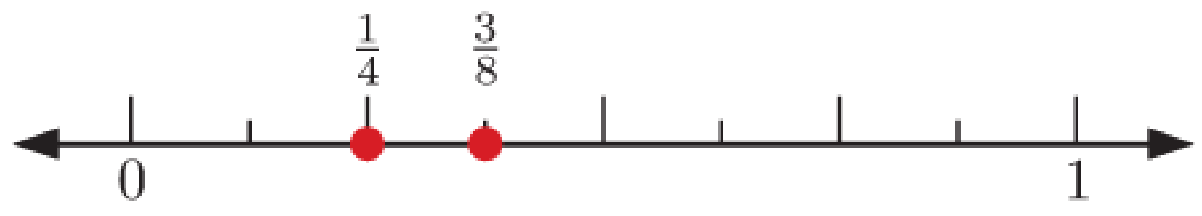


i



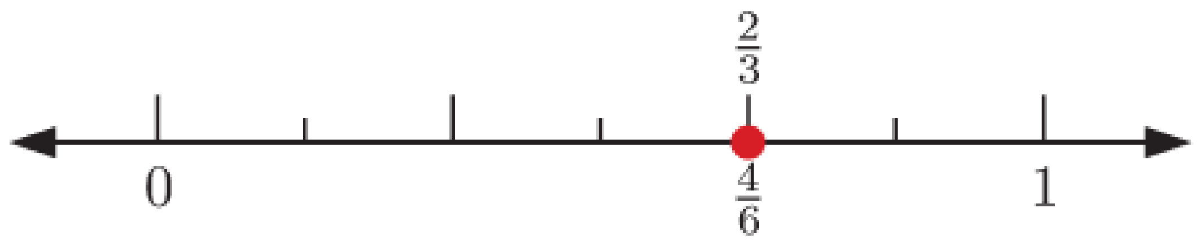
2 **a** $\frac{4}{5}$ **b** $\frac{4}{7}$ **c** $\frac{3}{8}, \frac{7}{8}$ **d** $\frac{3}{4}, 1\frac{1}{4}$
e $\frac{5}{9}, 1\frac{7}{9}$ **f** $\frac{2}{3}, 1\frac{1}{3}, 2\frac{2}{3}$ **g** $2\frac{2}{3}, 3\frac{1}{3}$ **h** $3\frac{1}{4}, 4\frac{1}{4}, 4\frac{3}{4}$

3 **a**, **b**



c $\frac{3}{8}$, as it is to the right of $\frac{1}{4}$ on the number line. Also, $\frac{1}{4}$ is at $\frac{2}{8}$ and so $\frac{3}{8}$ is larger.

4 **a**



b They are at the same place on the number line and so must be equal.

EXERCISE 6F.1

1 **a** $\frac{18}{30}$ **b** $\frac{3}{5}$ **2** **a** $\frac{20}{60}$ **b** $\frac{1}{3}$

3 **a** $\frac{1}{4} = \frac{2}{8}$ **b** $\frac{5}{8} = \frac{15}{24}$ **c** $\frac{16}{22} = \frac{8}{11}$ **d** $\frac{24}{30} = \frac{8}{10}$
e $\frac{2}{7} = \frac{10}{35}$ **f** $\frac{6}{7} = \frac{36}{42}$

4 **a** $\frac{6}{8}$ **b** $\frac{9}{12}$ **c** $\frac{12}{16}$ **d** $\frac{15}{20}$

5 **a** $\frac{8}{20}$ **b** $\frac{12}{30}$ **c** $\frac{20}{50}$ **d** $\frac{2}{5}$

6 **a** $\frac{2}{8}$ **b** $\frac{4}{8}$ **c** $\frac{6}{8}$ **d** $\frac{8}{8}$ **e** $\frac{5}{8}$

7 **a** $\frac{15}{30}$ **b** $\frac{24}{30}$ **c** $\frac{25}{30}$ **d** $\frac{9}{30}$ **e** $\frac{6}{30}$ **f** $\frac{20}{30}$
g $\frac{30}{30}$ **h** $\frac{18}{30}$ **i** $\frac{7}{30}$ **j** $\frac{39}{30}$

8 **a** $\frac{50}{100}$ **b** $\frac{25}{100}$ **c** $\frac{80}{100}$ **d** $\frac{90}{100}$ **e** $\frac{28}{100}$ **f** $\frac{26}{100}$
g $\frac{100}{100}$ **h** $\frac{85}{100}$ **i** $\frac{17}{100}$ **j** $\frac{122}{100}$

EXERCISE 6F.2

1 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{1}{5}$ **e** $\frac{1}{3}$ **f** $\frac{1}{6}$
g $\frac{3}{5}$ **h** $\frac{2}{3}$ **i** $\frac{6}{7}$ **j** $\frac{3}{4}$

2 **C**

EXERCISE 6G

1 **a** $\frac{7}{12}$ **b** $\frac{4}{5}$ **c** $\frac{13}{9}$ **d** $\frac{11}{7}$ **e** $5\frac{1}{4}$ **f** $4\frac{5}{6}$

2 Keith ($\frac{2}{3}$ more)

3 **a** $\frac{3}{4}$ **b** $\frac{3}{6}$ **c** $\frac{7}{8}$ **d** $\frac{5}{8}$ **e** $\frac{2}{3}$ **f** equal
g $\frac{4}{3}$ **h** $\frac{6}{5}$ **i** $\frac{15}{4}$

4 rent ($\frac{1}{9}$ more) **5** **a** $\frac{2}{5}$ **b** $\frac{3}{10}$ **c** in Trent's cage

EXERCISE 6H.1

1 **a** $\frac{3}{4}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$ **d** $\frac{7}{8}$ **e** $\frac{4}{5}$ **f** $\frac{2}{7}$
g $\frac{4}{11}$ **h** $\frac{17}{20}$ **i** $\frac{9}{25}$

2 **a** $1\frac{1}{5}$ **b** $1\frac{3}{10}$ **c** $1\frac{4}{7}$ **d** $1\frac{4}{15}$ **e** $2\frac{3}{13}$ **f** $1\frac{9}{14}$

3 **a** $3\frac{5}{9}$ **b** $2\frac{7}{10}$ **c** $5\frac{2}{7}$ **d** $2\frac{1}{6}$ **e** $5\frac{2}{15}$ **f** $8\frac{5}{17}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** $\frac{3}{4}$ **e** $\frac{1}{2}$ **f** $\frac{4}{5}$

5 **a** $4\frac{1}{3}$ **b** $2\frac{2}{5}$ **c** $8\frac{5}{7}$ **d** $2\frac{5}{8}$ **e** $9\frac{5}{9}$ **f** $8\frac{7}{10}$

6 $\frac{7}{9}$ **7** $3\frac{1}{2}$ pages **8** **a** $\frac{3}{10}$ **b** 6 kg

9 **a** $4\frac{2}{5}$ bags **b** $1\frac{1}{5}$ bags

EXERCISE 6H.2

1 **a** $\frac{1}{4}$ **b** $\frac{5}{6}$ **c** $\frac{3}{8}$ **d** $\frac{5}{12}$ **e** $\frac{7}{30}$ **f** $\frac{44}{49}$

2 **a** $1\frac{1}{10}$ **b** $1\frac{7}{12}$ **c** $1\frac{11}{45}$ **d** $1\frac{43}{100}$

3 **a** $2\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $1\frac{3}{8}$

4 **a** $3\frac{7}{8}$ **b** $1\frac{1}{6}$ **c** $3\frac{3}{10}$ **d** $1\frac{3}{4}$ **e** $3\frac{16}{21}$ **f** $3\frac{8}{15}$

5 **a** $\frac{5}{9}$ **b** $1\frac{3}{8}$ **c** $2\frac{5}{7}$ **d** $2\frac{7}{12}$

6 $\frac{5}{9}$ **7** $\frac{5}{8}$ **8** $\frac{7}{10}$ of a tub **9** $5\frac{3}{4}$ hours

10 $1\frac{5}{8}$ tonnes

REVIEW SET 6A

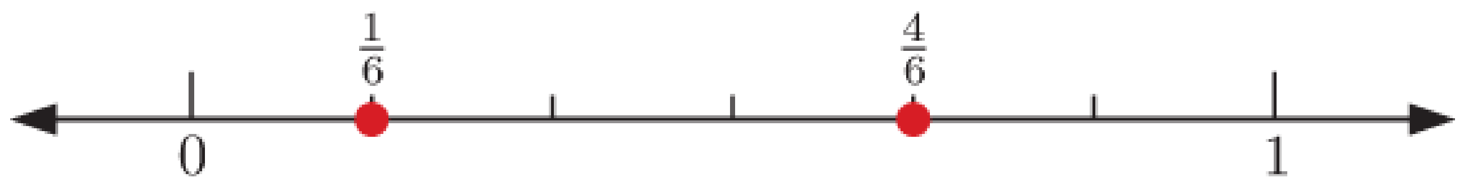
1 **a** $\frac{3}{5}$ **b** $\frac{5}{12}$ **c** $1\frac{8}{9}$ **2** $\frac{2}{7}$

3 **a** $\frac{6}{11}$ **b** $\frac{15}{19}$ **4** **a** $1\frac{4}{5}$ **b** $4\frac{1}{3}$ **c** $5\frac{5}{6}$

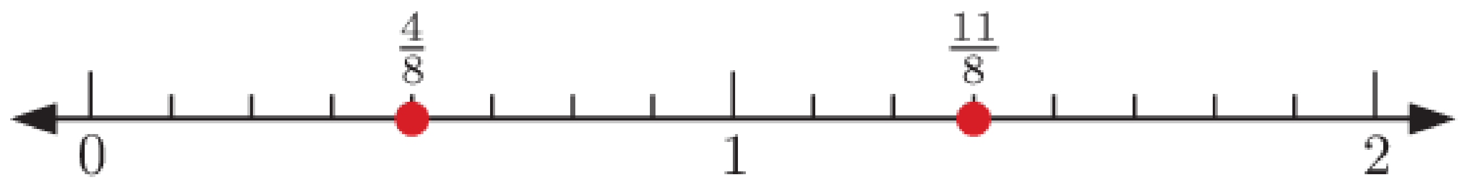
5 **a** $\frac{3}{8} = \frac{12}{32}$ **b** $\frac{18}{21} = \frac{6}{7}$ **c** $\frac{16}{72} = \frac{2}{9}$

6 **a** £50 **b** 40 g **c** 21 cm

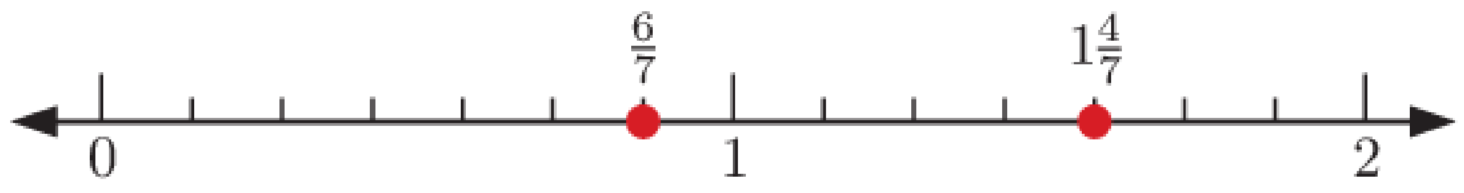
7 **a**



b



c

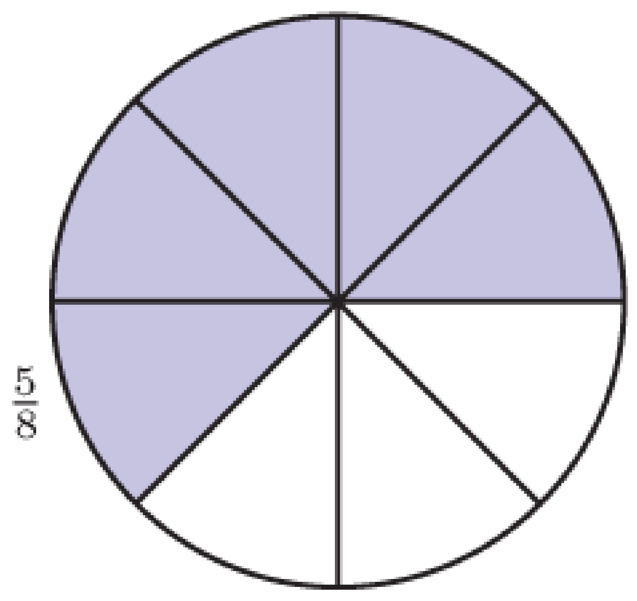


8 $\frac{7}{10}$ **9** **a** $\frac{6}{10}$ **b** $\frac{19}{7}$ **c** $\frac{22}{25}$ **d** $\frac{17}{3}$

10 **a** $\frac{4}{7}$ **b** $1\frac{6}{11}$ **c** $\frac{5}{8}$ **d** $4\frac{2}{9}$

REVIEW SET 6B

1



2 **a** $2\frac{1}{4}$ **b** $\frac{9}{4}$

3 **a** $40 \div 8 = 5$
b $72 \div 9 = 8$
c $99 \div 11 = 9$

4 **a** $\frac{23}{6}$ **b** $\frac{31}{7}$ **c** $\frac{27}{5}$

5 **a** $\frac{1}{8}$ **b** $\frac{5}{9}$ **c** $\frac{4}{5}$

6 **a** $\frac{10}{12}$ **b** $\frac{25}{30}$ **c** $\frac{40}{48}$

7 5 days

8 $\frac{10}{21}$ (as $\frac{3}{7} = \frac{9}{21}$)

9 **a** $4\frac{2}{5}$ **b** $1\frac{7}{10}$ **c** $1\frac{11}{18}$

10 **a** $7\frac{5}{6}$ sausages **b** $2\frac{5}{6}$ sausages

EXERCISE 7A

1 **a** zero point six **b** zero point four five
c zero point nine zero eight **d** eight point three
e six point zero eight **f** ninety six point zero two
g five point eight six four
h thirty four point zero zero three
i seven point five eight one **j** sixty point two six four

2 **a** 8.37 **b** 21.05 **c** 9.004 **d** 38.206

3 **a** 5 and 6 **b** 13 and 14 **c** 9 and 10 **d** 6 and 7
e 19 and 20 **f** 32 and 33 **g** 0 and 1 **h** 8 and 9

4 **a** 1 **b** 2 **c** 1 **d** 2 **e** 3 **f** 3
g 1 **h** 2 **i** 3

5

	Number	*thousands*	*hundreds*	*tens*	*units*		*tenths*	*hundredths*	*thousandths*	*Decimal number*
a	$\frac{8}{10} + \frac{3}{100}$					.	8	3		0.83
b	$4 + \frac{1}{10} + \frac{2}{100} + \frac{8}{1000}$				4	.	1	2	8	4.128
c	$9 + \frac{4}{1000}$				9	.	0	0	4	9.004
d	$28 + \frac{6}{10} + \frac{9}{100} + \frac{9}{1000}$			2	8	.	6	9	9	28.699
e	$\frac{5}{100} + \frac{6}{1000}$					.	0	5	6	0.056
f	$139 + \frac{7}{100} + \frac{7}{1000}$		1	3	9	.	0	7	7	139.077

6

	thousands	*hundreds*	*tens*	*units*		*tenths*	*hundredths*	*thousandths*	*Decimal number*
a					.	8			0.8
b					.	0	4		0.04
c					.	0	0	3	0.003
d			7	0	.	8			70.8
e				5	.	0	6		5.06
f	9	0	0	0	.	0	0	2	9000.002
g		2	0	9	.	0	4		209.04
h	8	0	0	0	.	4	0	2	8000.402
i			6	0	.	8	9		60.89

7 **a** 3 hundreds or 300 **b** 3 tenths or $\frac{3}{10}$
c 3 tens or 30 **d** 3 thousandths or $\frac{3}{1000}$
e 3 units or 3 **f** 3 hundredths or $\frac{3}{100}$
g 3 thousands or 3000 **h** 3 ten thousandths or $\frac{3}{10\,000}$

8 **a** 5 thousandths or $\frac{5}{1000}$ **b** 5 hundreds or 500
c 5 tenths or $\frac{5}{10}$ **d** 5 hundredths or $\frac{5}{100}$
e 5 thousands or 5000 **f** 5 units or 5
g 5 ten thousands or 50 000 **h** 5 ten thousandths or $\frac{5}{10\,000}$

9 **a** $5 + \frac{4}{10}$ **b** $1 \times 10 + 4 + \frac{9}{10}$ **c** $2 + \frac{3}{100}$
d $3 \times 10 + 2 + \frac{8}{10} + \frac{6}{100}$ **e** $2 + \frac{2}{10} + \frac{6}{100} + \frac{4}{1000}$
f $1 + \frac{3}{10} + \frac{8}{1000}$ **g** $3 + \frac{2}{1000}$ **h** $\frac{9}{10} + \frac{5}{100} + \frac{2}{1000}$
i $4 + \frac{2}{100} + \frac{4}{1000}$ **j** $2 + \frac{9}{10} + \frac{7}{100} + \frac{3}{1000}$
k $2 \times 10 + \frac{8}{10} + \frac{1}{100} + \frac{6}{1000}$
l $7 + \frac{7}{10} + \frac{7}{100} + \frac{7}{1000}$ **m** $9 + \frac{8}{1000}$
n $1 \times 100 + 5 \times 10 + 4 + \frac{4}{10} + \frac{5}{100} + \frac{1}{1000}$
o $8 \times 100 + 8 + \frac{8}{10} + \frac{8}{1000}$ **p** $\frac{6}{100} + \frac{4}{1000}$

10 **a** 0.6 **b** 0.2 **c** 0.43 **d** 0.71 **e** 0.809
f 0.09 **g** 0.007 **h** 0.052 **i** 0.568 **j** 0.0023
k 4.387 **l** 0.0308 **m** 0.3033 **n** 5.555 **o** 0.200 05

11 **a** 0.23 **b** 0.79 **c** 0.307 **d** 0.117 **e** 0.6
f 0.703 **g** 4.69 **h** 0.54 **i** 0.4672 **j** 0.36

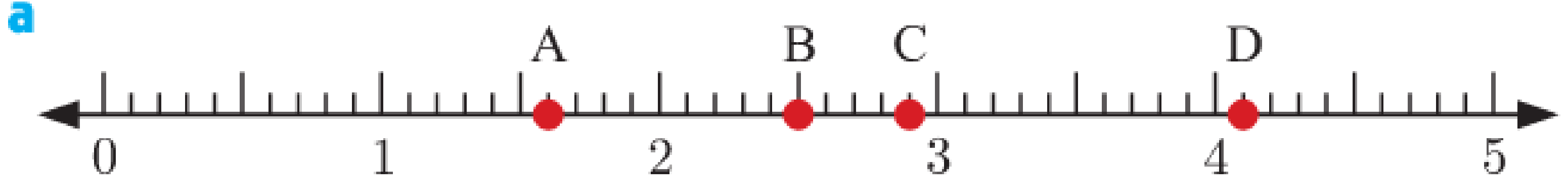
12 **a** \$1.30 **b** \$10.95 **c** \$0.45 **d** \$37.08

EXERCISE 7B

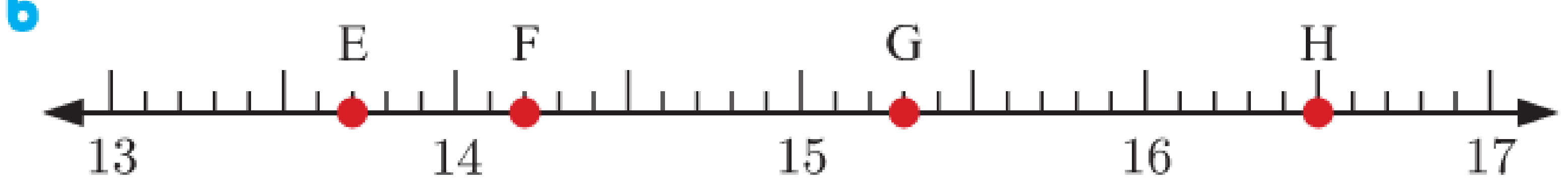
1 **a** M is 0.2, N is 0.7 **b** M is 2.6, N is 2.3
c M is 6.1, N is 6.8 **d** M is 21.5, N is 21.4

2 **a** M is 0.16, N is 0.25 **b** M is 0.09, N is 0.24
c M is 1.63, N is 1.77 **d** M is 3.47, N is 3.59

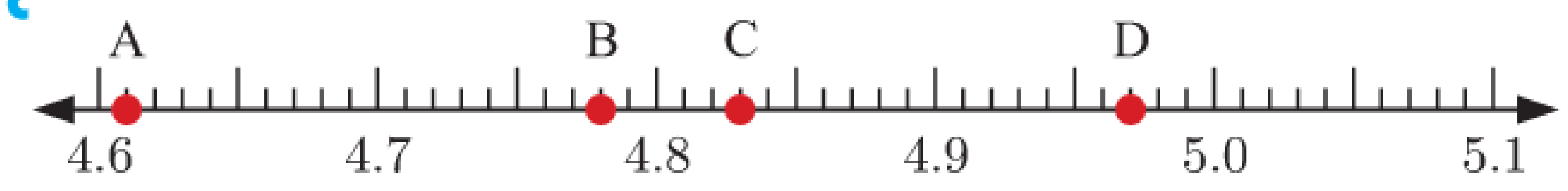
3 **a**



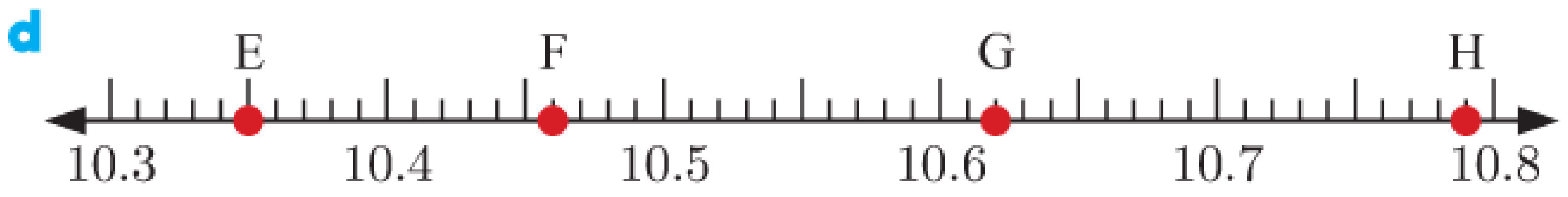
b



c



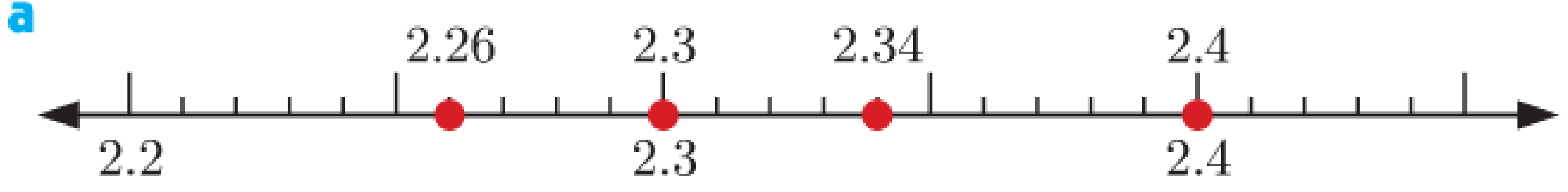
d



EXERCISE 7C

1 **a** A is 3.7, B is 3.2, and A > B
b A is 22.8, B is 22.3, and A > B
c A is 5.47, B is 5.48, and A < B
d A is 14.03, B is 14.06, and A < B
e A is 2.362, B is 2.367, and A < B
f A is 9.174, B is 9.172, and A > B

2 **a**



b 2.26, 2.3, 2.34, 2.4

3 **a** 0.7 < 0.8 **b** 0.06 > 0.05 **c** 0.2 > 0.19
d 4.01 < 4.1 **e** 0.81 > 0.803 **f** 2.5 = 2.50
g 0.304 < 0.34 **h** 0.03 < 0.2 **i** 6.05 < 60.50
j 0.29 = 0.290 **k** 5.01 < 5.016 **l** 1.15 > 1.035

m $21.021 < 21.210$ **n** $8.09 = 8.090$ **o** $0.904 < 0.94$

4 **a** 0.4, 0.6, 0.8 **b** 0.1, 0.4, 0.9 **c** 0.06, 0.09, 0.14
d 0.46, 0.5, 0.51 **e** 1.06, 1.59, 1.61 **f** 0.206, 2.06, 2.6
g 0.0905, 0.095, 0.905 **h** 15.05, 15.5, 15.55

5 **a** 0.9, 0.8, 0.4, 0.3 **b** 0.51, 0.5, 0.49, 0.47
c 0.61, 0.609, 0.6, 0.596 **d** 0.42, 0.24, 0.04, 0.02
e 6.277, 6.271, 6.27, 6.027 **f** 0.311, 0.31, 0.301, 0.031
g 8.880, 8.088, 8.080, 8.008 **h** 7.61, 7.061, 7.06, 7.01

EXERCISE 7D

1 **a** 2.4 **b** 3.6 **c** 4.9 **d** 7.8 **e** 0.6 **f** 4.3
2 **a** 4.24 **b** 2.73 **c** 5.63 **d** 4.38 **e** 6.52 **f** 1.09
3 **a** 4 **b** 6 **c** 7 **d** 13 **e** 21
4 **a** 0.5 **b** 0.49 **5** **a** 3.8 **b** 3.79
6 **a** 5 **b** 5.2 **c** 5.18 **d** 5.184
7 **a** 3.9 **b** 4 **c** 6.1 **d** 0.462 **e** 2.95 **f** 0.176
8 **a** 4.3 kg **b** 32.7°C **c** 2.94 m **d** 87 seconds

EXERCISE 7E.1

1 **a** $\frac{1}{10}$ **b** $\frac{9}{10}$ **c** $\frac{19}{100}$ **d** $\frac{67}{100}$ **e** $\frac{7}{100}$ **f** $\frac{191}{1000}$
g $\frac{523}{1000}$ **h** $\frac{49}{1000}$ **i** $4\frac{3}{10}$ **j** $\frac{87}{100}$ **k** $\frac{1}{100}$ **l** $5\frac{271}{1000}$
2 **a** $\frac{4}{5}$ **b** $\frac{1}{2}$ **c** $\frac{13}{50}$ **d** $\frac{7}{20}$ **e** $\frac{1}{4}$ **f** $\frac{53}{500}$
g $\frac{3}{200}$ **h** $\frac{7}{40}$ **i** $\frac{5}{8}$ **j** $7\frac{3}{5}$ **k** $4\frac{14}{25}$ **l** $3\frac{19}{20}$

EXERCISE 7E.2

1 **a** 0.3 **b** 0.21 **c** 0.77 **d** 0.319 **e** 4.91 **f** 2.137
2 **a** 5 **b** 2 **c** 25 **d** 125 **e** 5
f 4 **g** 2 **h** 8 **i** 25 **j** 4
3 **a** 0.2 **b** 0.15 **c** 0.85 **d** 0.36 **e** 0.84
f 0.26 **g** 0.62 **h** 0.276 **i** 0.024 **j** 0.364
k 0.022 **l** 0.072 **m** 0.544 **n** 0.225 **o** 0.375
p 5.75 **q** 7.55 **r** 3.168
4 **a** $\frac{1}{2} = 0.5$ **b** $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$
c $\frac{1}{4} = 0.25$, $\frac{2}{4} = 0.5$, $\frac{3}{4} = 0.75$
d $\frac{1}{8} = 0.125$, $\frac{2}{8} = 0.25$, $\frac{3}{8} = 0.375$, $\frac{4}{8} = 0.5$,
$\frac{5}{8} = 0.625$, $\frac{6}{8} = 0.75$, $\frac{7}{8} = 0.875$
5 0.63, $\frac{13}{20}$, $\frac{7}{10}$, $\frac{18}{25}$, 0.74
6 **a** 0.5 g **b** 0.2 kg **c** 0.35 m **d** 0.46 cm
e 4.6 s **f** $1.95 **g** 4.92 t **h** 7.875 L

EXERCISE 7F

1 **a** 0.9 **b** 3.3 **c** 1.13 **d** 1.03 **e** 1.53
f 27.82 **g** 18.43 **h** 4.7 **i** 0.444 **j** 2.955
k 0.7006 **l** 1.8 **m** 1.13 **n** 13.3 **o** 10.92
2 **a** 0.52 **b** 1.34 **c** 3.12 **d** 0.8 **e** 1.5 **f** 0.4
g 1.4 **h** 2.3 **i** 2.26 **j** 2.67 **k** 0.01 **l** 9.02
3 **a** 44.2 **b** 14.38 **c** 11.211 **d** 8.452
4 **a** 6.1 **b** 22.18 **c** 1.02 **d** 167.5 **e** 58.63
f 2.014
5 $17.10 **6** 0.37 m **7** 237.4 m **8** 27.95 kg
9 13.079 m **10** €8.10

EXERCISE 7G

1 **a** 20 **b** 63 **c** 2 **d** 0.1 **e** 2.38 **f** 606
g 600 **h** 920 **i** 70 **j** 54 **k** 7040 **l** 5.798
m 7000 **n** 6200 **o** 700 **p** 380 **q** 6750 **r** 82.4

2

	Number	× 10	× 100	× 1000
a	0.009	0.09	0.9	9
b	0.12	1.2	12	120
c	0.5	5	50	500
d	4.6	46	460	4600
e	19.07	190.7	1907	19 070

3 **a** $9 \times 100 = 900$ **b** $33 \times 10 = 330$
c $3.4 \times 10 = 34$ **d** $0.02 \times 100 = 2$
e $0.003 \times 10 = 0.03$ **f** $5.64 \times 1000 = 5640$
4 **a** $18 **b** $180 **c** $1800
5 **a** €456 **b** €4560 **c** €45 600

EXERCISE 7H

1 **a** 0.2 **b** 0.63 **c** 0.02 **d** 0.001
e 5.402 **f** 60.6 **g** 0.06 **h** 0.092
i 0.007 **j** 0.5 **k** 1.66 **l** 3.007
2 **a** 0.007 **b** 0.0062 **c** 0.0561 **d** 0.499
e 0.701 **f** 6.8549

3

	Number	÷ 10	÷ 100	÷ 1000
a	8	0.8	0.08	0.008
b	4.6	0.46	0.046	0.0046
c	50	5	0.5	0.05
d	19.07	1.907	0.1907	0.019 07
e	231.4	23.14	2.314	0.2314

4 **a** $6 \div 10 = 0.6$ **b** $33 \div 100 = 0.33$
c $3.4 \div 10 = 0.34$ **d** $0.2 \div 100 = 0.002$
e $49 \div 100 = 0.49$ **f** $634.1 \div 1000 = 0.6341$
5 $275.65 **6** **a** £1.82 **b** £18.20

EXERCISE 7I

1 **a** 2.1 **b** 3.2 **c** 2.5 **d** 10.8 **e** 0.42
f 0.24 **g** 0.06 **h** 0.52 **i** 0.96 **j** 1.25
k 0.056 **l** 0.0036
2 **a** 4 **b** 3 **c** 0.6 **d** 0.2 **e** 0.1
f 0.6 **g** 0.4 **h** 0.15 **i** 0.06
3 **a** 21.7 **b** 39.9 **c** 172.2 **d** 32.66 **e** 29.2 **f** 182
4 16.1 kg **5** 93.6 min **6** €58.75 **7** 36 kg

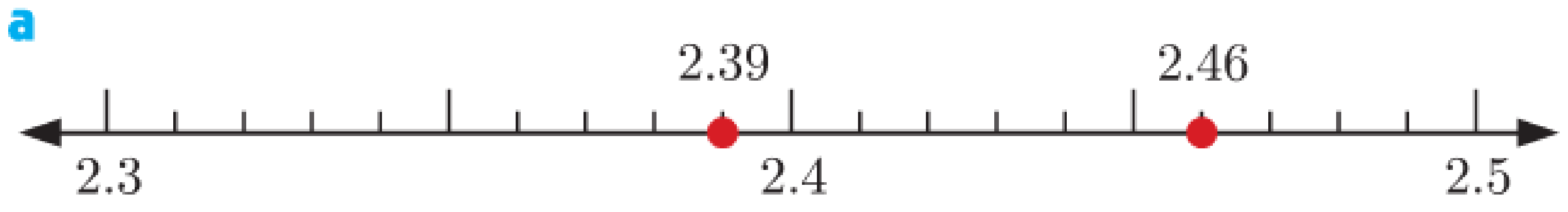
EXERCISE 7J

1 **a** 0.8 **b** 1.5 **c** 0.42 **d** 0.51 **e** 3.02
f 0.41 **g** 0.08 **h** 20.4
2 2.15 kg **3** $2.29
4 **a** 2.65 **b** 1.22 **c** 0.85 **d** 0.425 **e** 3.25
f 1.475 **g** 1.205 **h** 1.264
5 3.35 km

REVIEW SET 7A

1 **a** 7 and 8 **b** 15 and 16 **c** 41 and 42 **d** 67 and 68
2 **a** 0.73 **b** 0.107 **c** 5.069
3 **a** A is 2.47, B is 2.49 **b** A is 0.064, B is 0.065
4 **a** 3.9 **b** 3.86
5 **a** $0.57 > 0.41$ **b** $0.09 < 0.1$ **c** $3.07 < 3.7$
6 **a** $\frac{23}{100}$ **b** $\frac{1}{5}$ **c** $\frac{59}{1000}$ **d** $\frac{17}{25}$
7 **a** 0.71 **b** 0.8 **c** 0.95 **d** 0.162
8 **a** 0.68 **b** 2.23 **c** 20.24 **d** 5.51
9 1937.88 tonnes

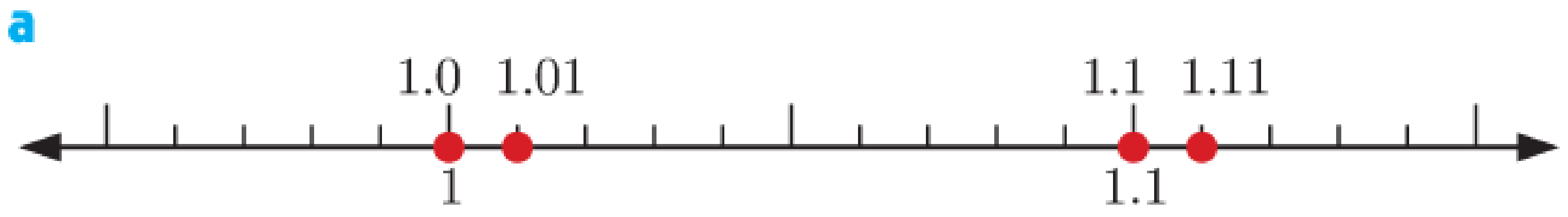
10 a 62 b 215.8 c 0.56 d 0.042
11 a 2.7 b 0.48 c 0.003 d 0.0375
12 $109.90
13 a



b 4.85 c 4.9
14 a 26.11 kg b 1.09 kg c £26.10

REVIEW SET 7B

1 a 1 b 2 c 2 d 3
2 16.574 3 0.031 4 3 thousandths or $\frac{3}{1000}$
5 a A is 0.81, B is 0.87 b A is 2.372, B is 2.374
6 $2+\frac{4}{100}+\frac{9}{1000}$
7 a $203 \div 100 = 2.03$ b $2.03 \times 1000 = 2030$
c $0.203 \div 100 = 0.002\,03$
8 a 0.92 b 2.56 c 9.54 d 7.17
9 a



b 1.11, 1.1, 1.01, 1.0
10 a 0.52 b 1.09 c 12.37
11 a 0.85 km b 0.12 L c 4.42 kg
12 a 0.42 b 36.8 c 2.9 d 1.58
13 a i 57.05 s ii 57.21 s b 0.16 s
14 a $21.75 b $28.25 c $108.75

EXERCISE 8A

1 a 24 cm b 13 cm c 10.2 cm d 16.8 cm
e 25.6 cm f 18.5 cm
2 a 35°C b 37.4°C c 38.3°C d 35.7°C
3 a $\frac{3}{4}$ full b $\frac{1}{4}$ full c $\frac{9}{16}$ full
4 a 120 km/h b 95 km/h c 65 km/h
5 a 45.2 kg b 71.6 kg c 63.65 kg
6 a 700 mL b 350 mL c 650 mL

EXERCISE 8B

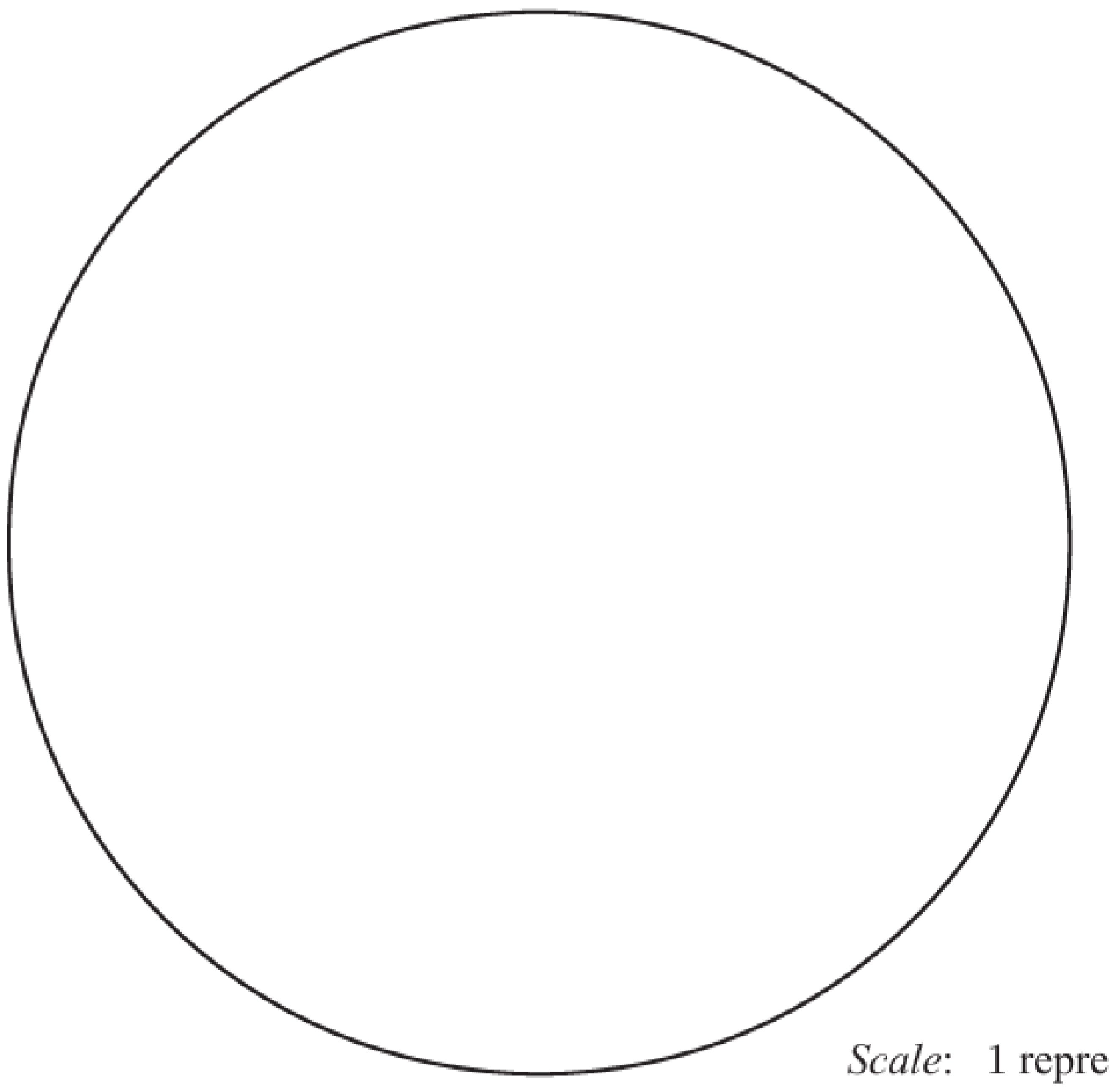
1 a cm b mm c mm d cm e cm f m
2 a **B** b **D** c **C**
3 a 7.6 m b 0.4 m c 2000 m d 0.25 m
e 2.763 m f 250 m g 4700 m h 90 m
4 a 55 000 cm b 4.7 cm c 130 cm
d 43.5 cm e 137.7 cm f 870 000 cm
g 29 000 cm h 196 000 cm
5 a 25 mm b 1830 mm c 490 mm d 920 mm
6 a 3.371 km b 21.901 km c 2.67 km d 0.388 km
7 a i 4 cm ii 40 mm b i 5.5 cm ii 55 mm
c i 4.5 cm ii 45 mm d i 4.7 cm ii 47 mm
8 226.5 cm 9 23.5 mm
10 a 3580 m b 1038 m c 5674.22 m d 48.916 m
11 a 11 700 m b 11.7 km
12 7.8 cm, 95 mm, 0.13 m, 29.2 cm
13 a i 26 slats ii 15.6 m b 13.2 m

EXERCISE 8C

1 a 14 m b 100 km c 28 cm d 104 mm
e 25 km f 123 cm g 92 mm h 48 m
2 a 100 m b 20 km c 90 mm d 138 cm
3 a 70 cm b 50 m c 80 cm d 144 m
4 a **A**: 11.3 cm, **B**: 9.9 cm, **C**: 11.9 cm b **C**
5 a ≈ 103 mm b ≈ 90 mm c ≈ 115 mm
6 760 m 7 a 9.6 m b €44.64
8 a 3.9 km b 19.5 km 9 9 cm 10 56 cm

EXERCISE 8D

1 a i 6 m ii 14 m iii 16.4 m iv 1.6 m
b i 1 m ii 9 cm iii 2.8 cm iv 6.1 cm
2 a i 200 m ii 290 m iii 120 m iv 630 m
b i 10 cm ii 35 mm iii 4 mm iv 2.16 cm
3 a **B** b **D** c **C** d **A**
4 a 4.5 m b 1.5 m c 4.7 m
5 a 12 m b 5 m c 2 m by 4 m
d 2.4 m by 1.2 m and 3.6 m by 1.2 m
6 a 5.3 m b 2.3 m
7 a 5 km b i 21 km ii 9.5 km iii 10.5 km
8

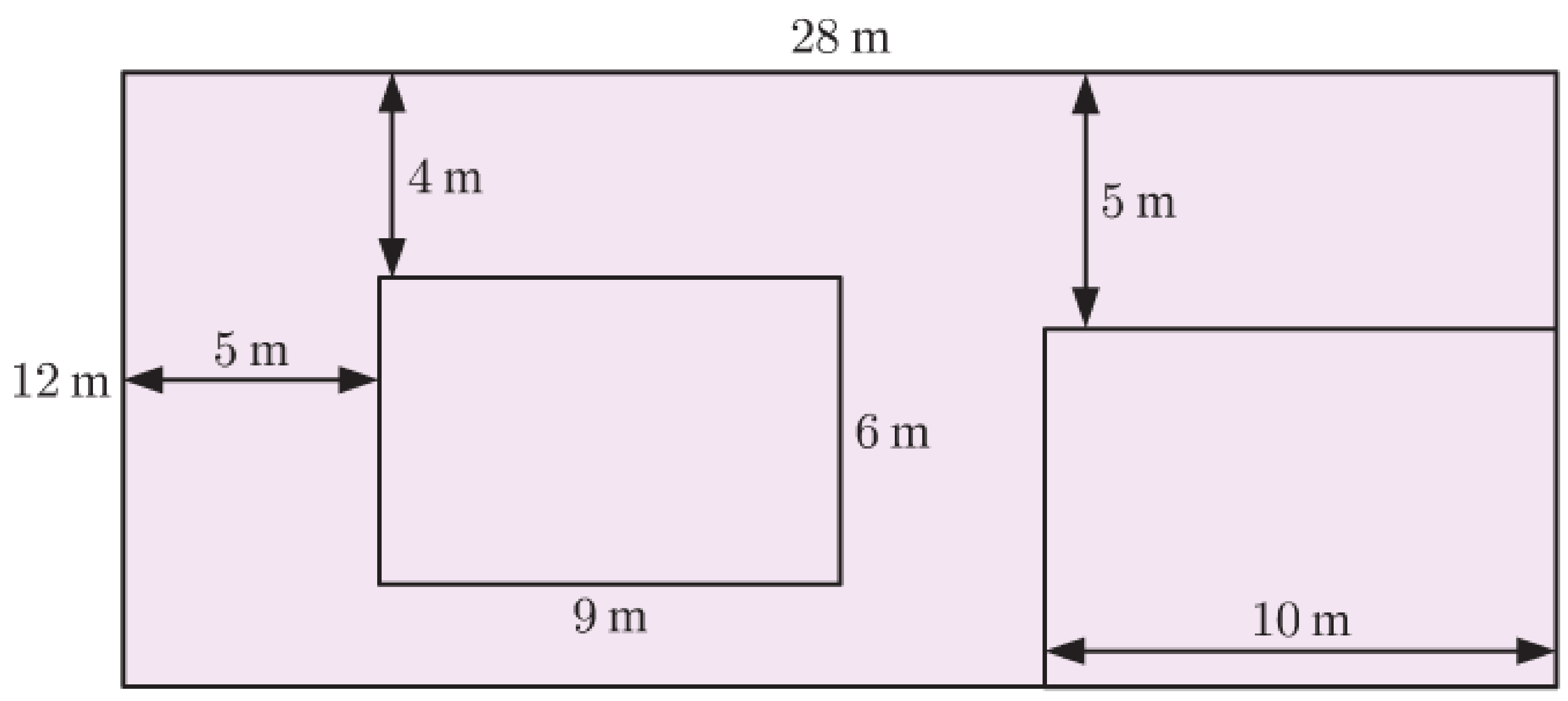


Scale: 1 represents 50

9

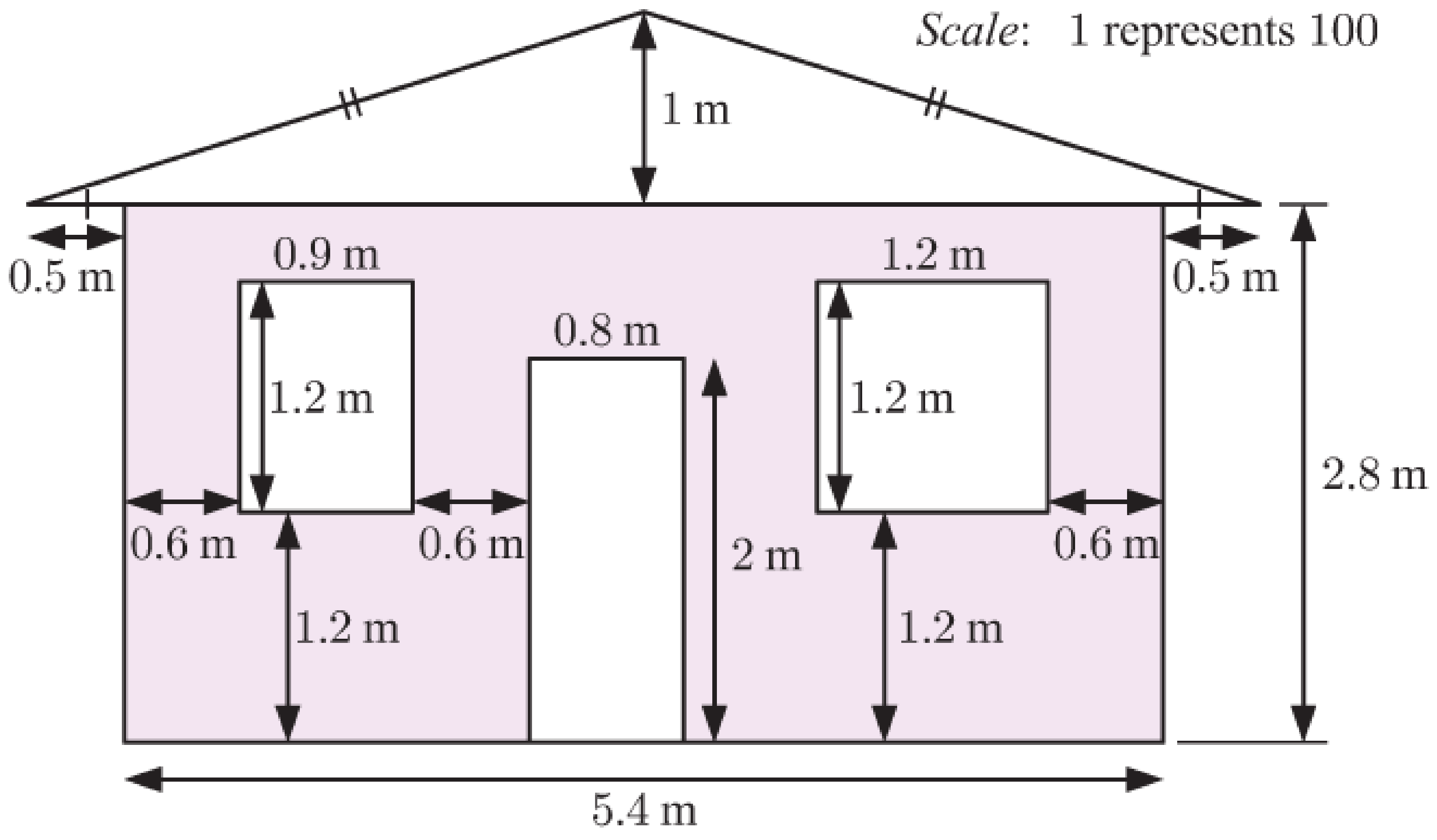
Scale: 1 represents 2000

10



Scale: 1 represents 400

11



12 **a** 1 represents 1000 **b** 1 represents 125

13 **a** 16 mm **b** 1 represents 50 000 000

c Alice Springs **d** ≈ 1475 km

e The distance travelled by car will be greater than 1475 km as the route by car is not a direct straight line the whole way.

EXERCISE 8E

1 **a** kg **b** tonnes **c** g **d** g **e** kg **f** mg

g g **h** kg **i** kg **j** g **k** g **l** tonnes

2 **a** **D** **b** **C** **c** **A** **d** **E** **e** **B** **f** **F**

3 **a** 8000 g **b** 3200 g **c** 14 200 g **d** 0.38 g

e 4.25 g **f** 75.42 g **g** 6 800 000 g **h** 560 000 g

4 **a** 13.87 kg **b** 3400 kg **c** 0.786 kg **d** 0.003 496 kg

5 24 kg **6** 7 tonnes

7 Yes, as only 2.86 g of the powder is needed.

8 **a** 924 kg **b** less than 1 tonne

9 40 kg **10** **a** 63 tonnes **b** 90 tonnes

REVIEW SET 8A

1 **a** 110 km/h **b** $\frac{7}{8}$ full **c** 600 mL **d** 650 g

2 **a** 3.56 m **b** 0.45 km **c** 7630 mm

3 **a** 49 cm **b** 33.4 km **4** **a** 35.2 m **b** $580.80

5 **a** 6207 m **b** 9.384 m

6 **a** 21 200 g **b** 0.0212 tonnes **7** **a** 9 cm **b** 15 cm

8 **a** **i** 19 km **ii** 32 km **iii** 61 km

b **i** 10 cm **ii** 4.4 cm **iii** 26 cm

9 270 kg

10 **a** 40 cm by 22 cm

b **C** - The room would be more like 4 m by 2.2 m and the stamp 40 mm by 22 mm.

REVIEW SET 8B

1 **a** $\frac{1}{8}$ full **b** 75 km/h **c** 3.3 kg **d** 3.5 L

2 **a** 54 cm **b** 15 m **3** 2.2 km

4 47.1 mm, 21 cm, 0.35 m, 423 mm **5** 1 represents 20 000

6 **a** **i** 40 mm **ii** 4 cm **b** **i** 28 mm **ii** 2.8 cm

7 **a** 3.2 kg **b** 4600 mg **c** 700 000 g

8 20 tonnes **9** **a** 1300 m **b** 3900 m **c** £9360

10 **a** **i** 3.4 m by 3.0 m **ii** 8.6 m by 1.0 m

b **i** 10 mm by 25 mm **ii** No, it would be too wide.

EXERCISE 9A

1 **a** 1970 **b** **i** 1983 **ii** 1989 **iii** 2002

c 18 years **d** **i** 7 years old **ii** 41 years old

2 **a** 2012 **b** Valéry Giscard d'Estaing **c** 5 years

d François Mitterrand, by 2 years

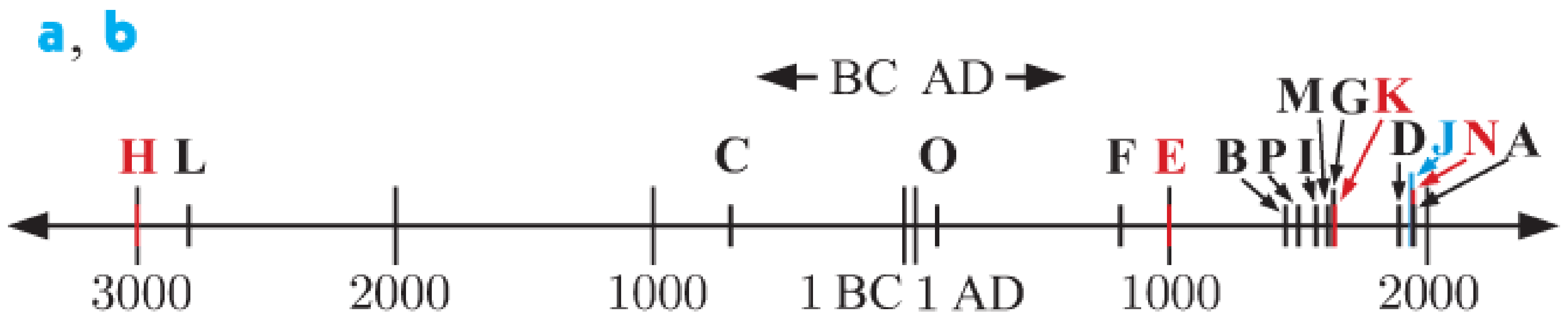
3 **a** 1910 **b** Elizabeth II **c** 9 years

4 **a** pop up toaster **b** lawn mower **c** 1855

d 1901 **e** 89 years

5 **a** 200 AD **b** 600 BC **c** **i** 1800 years **ii** 800 years

6 **a**, **b**



c A **d** 3 events **e** 4641 years **f** 501 years

EXERCISE 9B

1 **a** 600 seconds **b** 144 hours **c** 2 years

d 35 days **e** 4 minutes **f** 84 months

g 20 weeks **h** 14 400 seconds **i** 504 hours

2 8 minutes

3 No, he practised for only 5 hours and 10 minutes.

4 **a** 444 min **b** 4663 min **c** 18 216 min **d** 24 977 min

5 **a** 2438 s **b** 12 927 s **c** 51 163 s **d** 82 331 s

6 **a** 720 h **b** 744 h

7 **a** 365 days in 2010 **b** 8784 h in 2012

c 525 600 min in 2013

8 **a** 1461 days **b** 35 064 h **c** 2 103 840 min

9 **a** 3 min 20 s **b** 2 days 2 h **c** 2 h 30 min

d 4 weeks 2 days **e** 3 years 4 months

10 5 h 50 min

EXERCISE 9C

1 **a** 27 days **b** 43 days **c** 117 days **d** 111 days

e 68 days **f** 179 days

2 **a** 45 days **b** $6.20 **3** yes **4** 27th May 2012

5 **a** 7:00 am **b** 3:00 pm **c** 6:49 am **d** 6:36 pm

e 10:32 pm **f** 4:09 pm **g** 11:05 am **h** 6:42 am

i 3:17 pm **j** 10:44 pm Sun

6 7:45 am **7** 10:50 am

8 **a** 2 h 20 min **b** 5 h 35 min **c** 6 h 16 min

d 11 h 27 min

9 3 h 44 min **10** 2 h 42 min **11** 7 h 39 min

12 2 min 51 s

EXERCISE 9D

1 **a** 0313 hours **b** 1117 hours **c** 0000 hours

d 1247 hours **e** 1741 hours **f** 1200 hours

g 2019 hours **h** 2359 hours

2 **a** 3:00 am **b** 6:30 am **c** 6:00 pm

d 12:00 pm (midday) **e** 6:15 am **f** 3:45 pm

g 8:17 pm **h** 11:48 pm

3 **a** 0930 hours **b** 1240 hours **c** 1915 hours

4 **a** 60 minutes or more is not possible.

b 0713 hours is correct.

c 2400 hours or more is not possible.

5 **a** flight BA10 **b** 1820 hours

EXERCISE 9E

1 **a** 11:50 am **b** 9:45 am **c** Sport

d **i** 45 min **ii** 50 min **e** 6 h 10 min

2 **a** 9:00 am **b** volleyball **c** 1 h 30 min **d** 2 h 15 min

3 **a** 3:00 am **b** 10:27 pm **c** 0.5 m at 8:58 am

d 1.0 m at 4:37 pm

4 **a** 8:30 am **b** North Park **c** 60 min
d 3 h 55 min **e** 8 h 45 min
5 **a** 6 bus services **b** 10:15 am **c** 5:30 pm
d **i** 3 h 10 min **ii** 5 h 40 min **e** 8 h 30 min
f bus E **g** bus A
6 **a** **i** arrival time **ii** departure time **b** 4:50 pm
c 5:27 pm **d** 6:20 pm **e** **i** 4:11 pm **ii** 4:36 pm

REVIEW SET 9A

1 **a** **i** 1968 **ii** 1970 **iii** 1978 **iv** 1984
b **i** 25 years **ii** 8 years
2 **a** 720 min **b** 168 h **c** 3 days **3** 2 h 48 min
4 **a** **i** 8 h 35 min **ii** 4 h 45 min **iii** 14 h
b **i** 40 min **ii** 40 min **iii** 30 min **c** 6:40 pm
5 12:10 pm **6** **a** 176 days **b** £2640 **c** £360
7 **a** 198 min **b** 552 s
8 **a** 0032 hours **b** 1015 hours **c** 1749 hours
9 **a** 8:00 am **b** 10:30 am **c** 1:00 pm **d** 6 h 15 min
10 **a** 7:57 pm **b** 23 min **c** **i** 2 h 15 min **ii** 10:35 pm

REVIEW SET 9B

1 **a** 3100 BC **b** 400 years **2** **a** 4 pm **b** 4:39 pm
3 3 h 39 min **4** **a** 720 h **b** 366 days
5 5:12 pm **6** **a** 0715 hours **b** 2125 hours
7 **a** 195 days **b** \$3510 **c** yes, \$1939
8 **a** 2006 **b** Toyota **c** Mazda **d** 4 years
9 **a** 4:15 am **b** 1:00 pm **c** 11:35 pm
10 **a** 30 min **b** **i** 10 min **ii** 15 min **iii** 20 min
c **i** 10:10 am **ii** 10:22 am **d** 2 buses

EXERCISE 10A

1 **a** **i** $\frac{14}{100}$ **ii** 14% **b** **i** $\frac{67}{100}$ **ii** 67%
c **i** $\frac{95}{100}$ **ii** 95% **d** **i** $\frac{60}{100}$ **ii** 60%
e **i** $\frac{46}{100}$ **ii** 46% **f** **i** $\frac{40}{100}$ **ii** 40%
2 **a** 50% **b** 85% **c** 25%
3 **a** Player 2 **b** Player 4 **c** Player 1 **d** Player 3
4 **a** **i** $\frac{11}{100}$ **ii** $\frac{23}{100}$ **iii** $\frac{39}{100}$ **iv** $\frac{27}{100}$
b **i** 11% **ii** 23% **iii** 39% **iv** 27%
c 100%; this represents all of the symbols in the circle.
5 **a** $36\% + 21\% + 16\% + 10\% + 7\% + 5\% + 1\% + 4\% = 100\%$
b paper **c** **i** 36% **ii** 5% **iii** 11%

EXERCISE 10B.1

1 **a** $\frac{59}{100}$ **b** $\frac{13}{100}$ **c** $\frac{3}{100}$ **d** $\frac{97}{100}$
2 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{9}{10}$ **d** $\frac{1}{20}$ **e** $\frac{11}{50}$ **f** $\frac{37}{50}$
g $\frac{3}{20}$ **h** $\frac{13}{20}$ **i** $\frac{1}{4}$ **j** $\frac{4}{5}$ **k** $\frac{7}{20}$ **l** $\frac{3}{4}$
m $\frac{1}{25}$ **n** $\frac{12}{25}$ **o** $\frac{14}{25}$ **p** $\frac{16}{25}$

EXERCISE 10B.2

1 **a** 21% **b** 53% **c** 91% **d** 8% **e** 30% **f** 70%
g 0% **h** 100%
2 **a** 50% **b** 26% **c** 20% **d** 82% **e** 15% **f** 60%
g 28% **h** 95% **i** 48% **j** 76%
3 **a** 14.5% **b** 23.1% **c** 75.9% **d** 20.6%

4 **a** $\frac{2}{1} = 2$ **b** 200%

5

	Number	*Fraction*	*Fraction with denominator 100*	*Percentage*
a	4	$\frac{4}{20}$	$\frac{20}{100}$	20%
b	9	$\frac{9}{20}$	$\frac{45}{100}$	45%
c	15	$\frac{15}{20}$	$\frac{75}{100}$	75%
d	3	$\frac{3}{20}$	$\frac{15}{100}$	15%
e	1	$\frac{1}{20}$	$\frac{5}{100}$	5%
f	11	$\frac{11}{20}$	$\frac{55}{100}$	55%
g	20	$\frac{20}{20}$	$\frac{100}{100}$	100%

6 **a** $\frac{1}{5} = 20\%$
$\frac{2}{5} = 40\%$
$\frac{3}{5} = 60\%$
$\frac{4}{5} = 80\%$
$\frac{5}{5} = 100\%$

b $\frac{1}{4}$ is 25%
$\frac{2}{4} = \frac{1}{2}$ is 50%
$\frac{3}{4}$ is 75%
$\frac{4}{4} = 1$ is 100%

c $\frac{1}{3}$ is $33\frac{1}{3}\%$
$\frac{2}{3}$ is $66\frac{2}{3}\%$
$\frac{3}{3}$ is 100%

d 1 is 100%
$\frac{1}{2}$ is 50%
$\frac{1}{4}$ is 25%
$\frac{1}{8}$ is $12\frac{1}{2}\%$
$\frac{1}{16}$ is $6\frac{1}{4}\%$

EXERCISE 10C.1

1 **a** 0.1 **b** 0.5 **c** 0.25 **d** 0.05
e 0.33 **f** 0.57 **g** 0.94 **h** 0.06
i 0.4 **j** 0.11 **k** 0.01 **l** 0.9
2 **a** 0.175 **b** 0.816 **c** 0.607 **d** 0.094
e 0.039 **f** 0.043 **g** 0.017 **h** 0.008
3 **a** **i** 0.71 **ii** $\frac{71}{100}$ **b** **i** 0.65 **ii** $\frac{13}{20}$
c **i** 0.3 **ii** $\frac{3}{10}$ **d** **i** 0.08 **ii** $\frac{2}{25}$

EXERCISE 10C.2

1 **a** 37% **b** 89% **c** 15% **d** 49% **e** 73%
f 11% **g** 5% **h** 2%
2 **a** 20% **b** 70% **c** 90% **d** 40% **e** 7.4%
f 73.9% **g** 8.6% **h** 0.1%

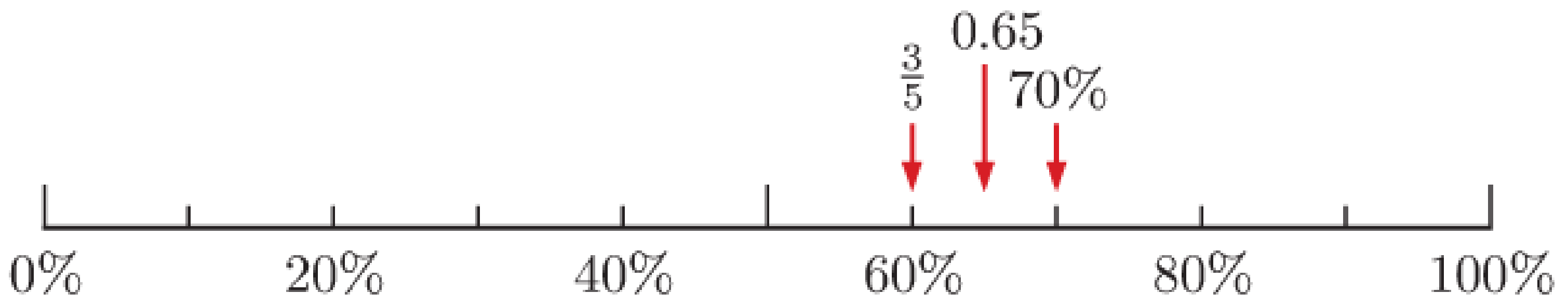
3

	Percent	*Fraction*	*Decimal*
a	20%	$\frac{1}{5}$	0.2
b	40%	$\frac{2}{5}$	0.4
c	50%	$\frac{1}{2}$	0.5
d	75%	$\frac{3}{4}$	0.75
e	85%	$\frac{17}{20}$	0.85
f	8%	$\frac{2}{25}$	0.08
g	35%	$\frac{7}{20}$	0.35
h	84%	$\frac{21}{25}$	0.84
i	100%	$\frac{1}{1} = 1$	1.00
j	15%	$\frac{3}{20}$	0.15

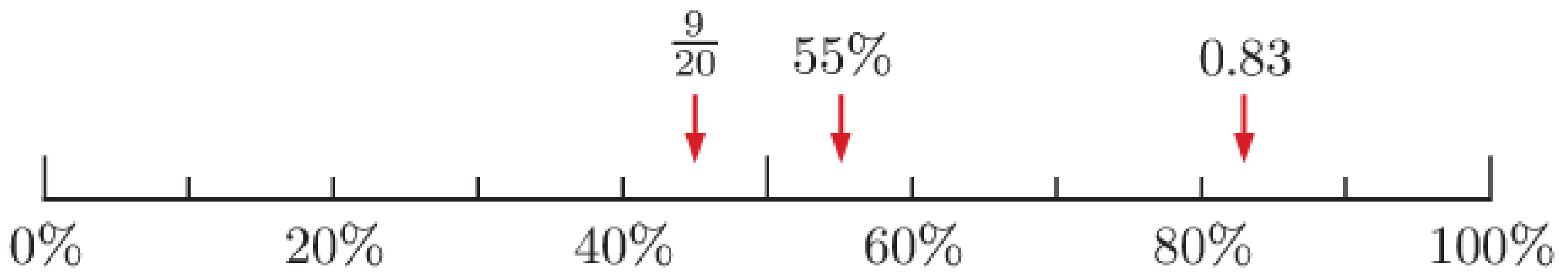
4 **a** $\frac{7}{25}$, 0.28 **b** 80%, 0.8 **c** 45%, $\frac{9}{20}$ **d** 25%, $\frac{1}{4}$

EXERCISE 10D

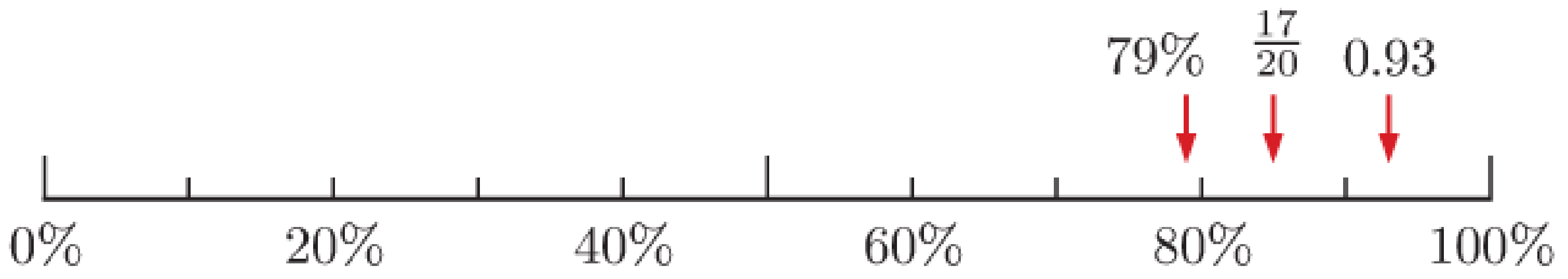
1 **a**



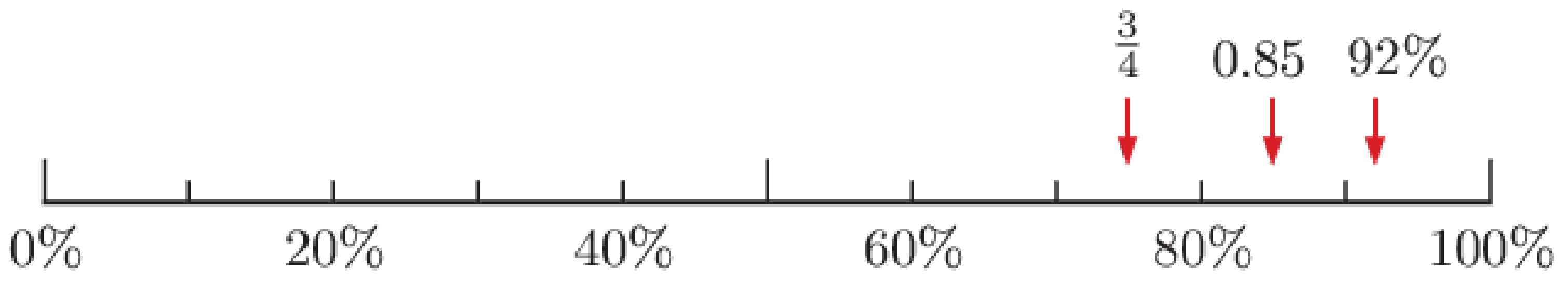
b



c



d



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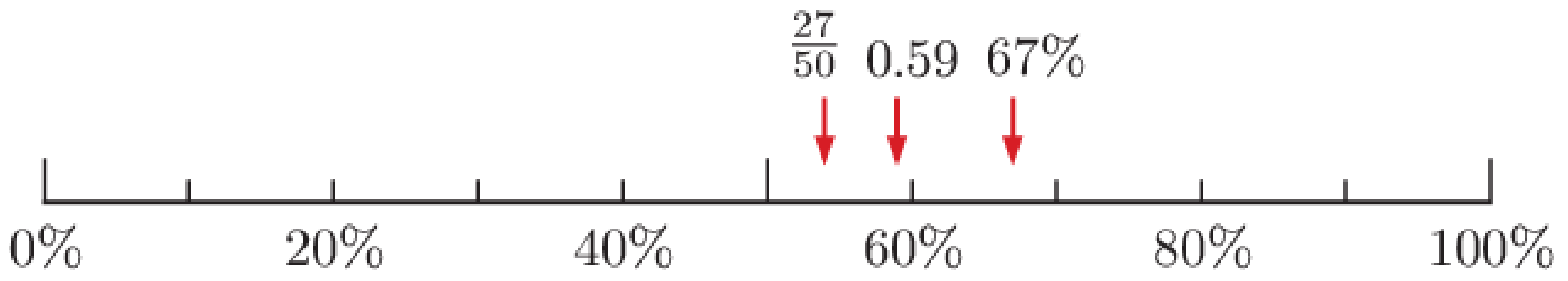

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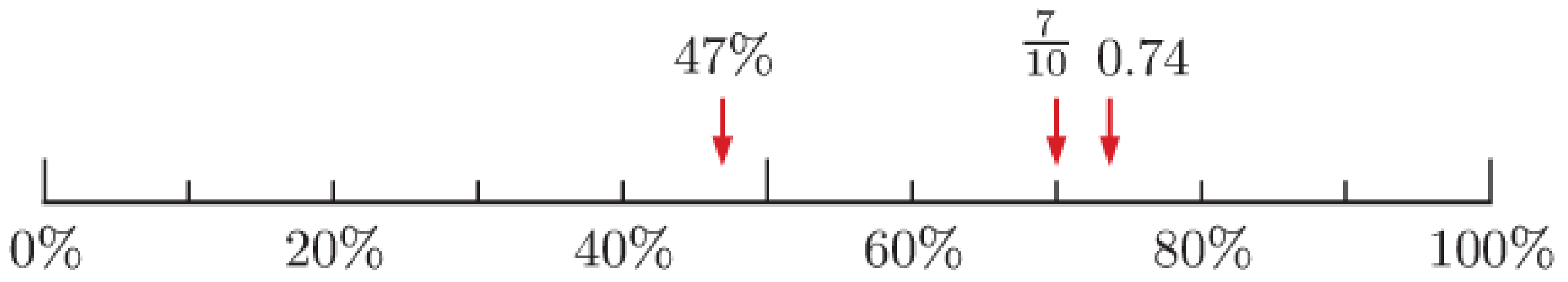

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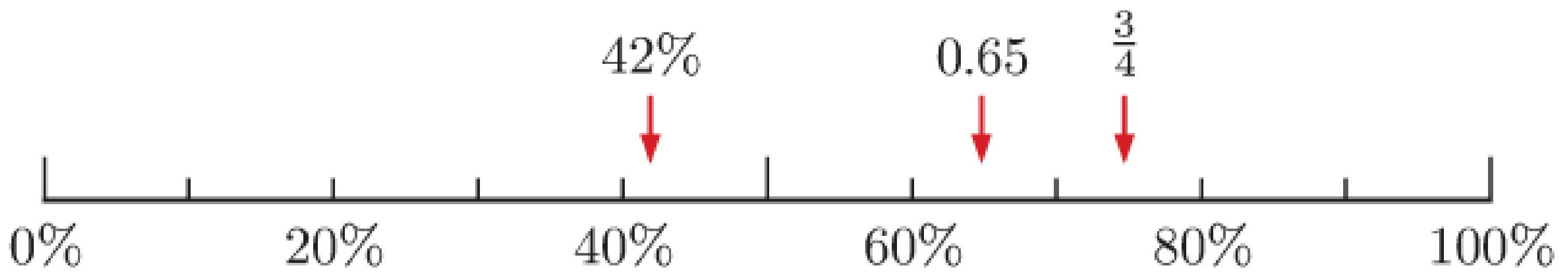

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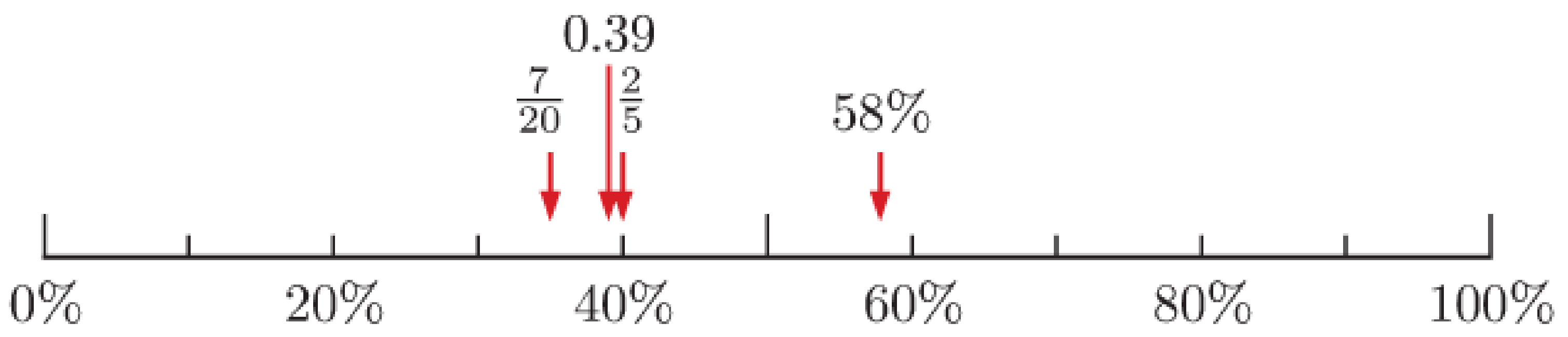
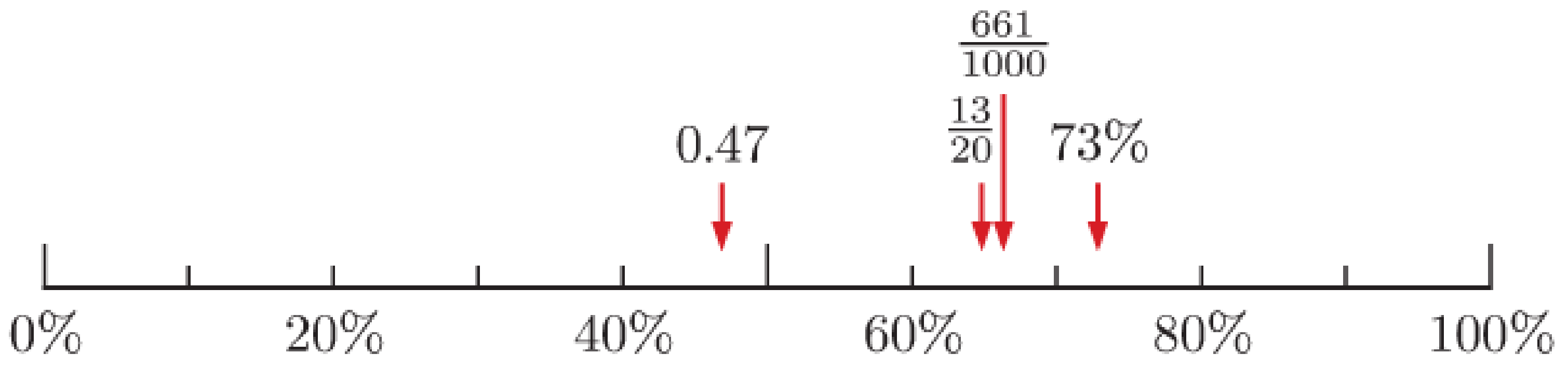


i



2 **a** **i** $\frac{28}{100} = 0.28 = 28\%$ **ii** $\frac{45}{100} = 0.45 = 45\%$

iii $\frac{68}{100} = 0.68 = 68\%$

b **i** $\frac{58}{100} = 0.58 = 58\%$ **ii** $\frac{74}{100} = 0.74 = 74\%$

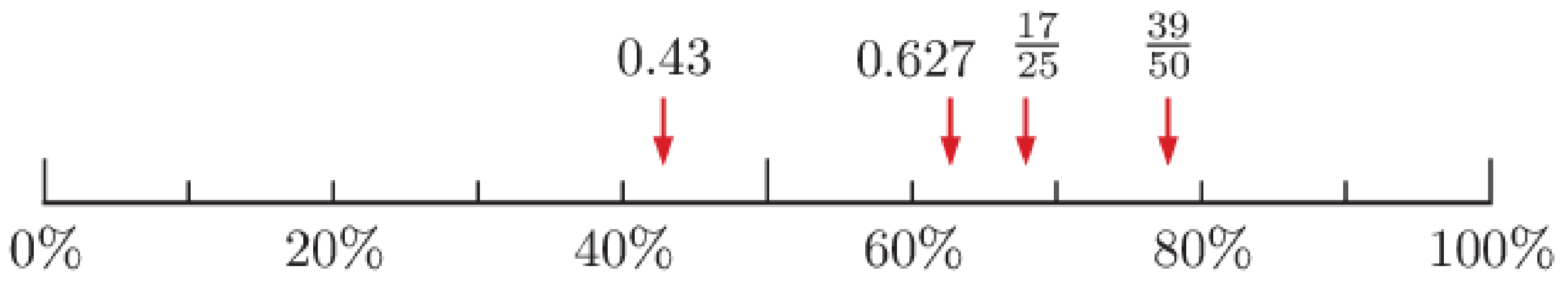
iii $\frac{89}{100} = 0.89 = 89\%$

c **i** $\frac{22}{100} = 0.22 = 22\%$ **ii** $\frac{37}{100} = 0.37 = 37\%$

iii $\frac{55}{100} = 0.55 = 55\%$

3 **a** **i** 68% **ii** 43% **iii** 78% **iv** 62.7%

b



c 0.43, 0.627, $\frac{17}{25}$, $\frac{39}{50}$

EXERCISE 10E

1 **a** 85% **b** 44% **c** 74% **d** 69%

2 **a** 20% **b** 20% **c** 75% **d** 35% **e** 5% **f** 21%
g 16% **h** 36% **i** 7% **j** 4% **k** 2% **l** 0.1%

3 **a** 36% **b** 81% **c** 33% **d** 70% **e** 62% **f** 15%

4 52% **5** 24%

6 **a** **i** 24% **ii** 34% **iii** 42%
b 24% + 34% + 42% = 100%

7 **a** 85% **b** yes

8 **a** Smith's: 56%, Jones': 55% **b** Smith's
c **i** 48% **ii** 52%

EXERCISE 10F

1 **a** \$30 **b** 200 people **c** 8.1 kg
d 240 L **e** 4.9 cm **f** 15.75 s

2 **a** 27 cents **b** 45 cm **c** 700 g **d** 18 min
e 4800 L **f** 16.8 mm **g** 26.4 hours **h** 98 cents

3 97 students **4** 1215 tonnes **5** 67.5 g

6 **a** 70% **b** **i** 720 acres **ii** 1680 acres

7 360 kg **8** **a** 1 worker **b** 15 workers

9 **a** \$450 **b** \$3250

10 **a** **i** 33% **ii** 67% **b** **i** \$148.50 **ii** \$301.50

REVIEW SET 10A

1 **a** $\frac{40}{100}$ **b** 40%

2 **a** 10% + 60% + 1% + 15% + 6% + 8% = 100%
b **i** 60% **ii** 14% **c** **i** $\frac{1}{10}$ **ii** $\frac{3}{20}$

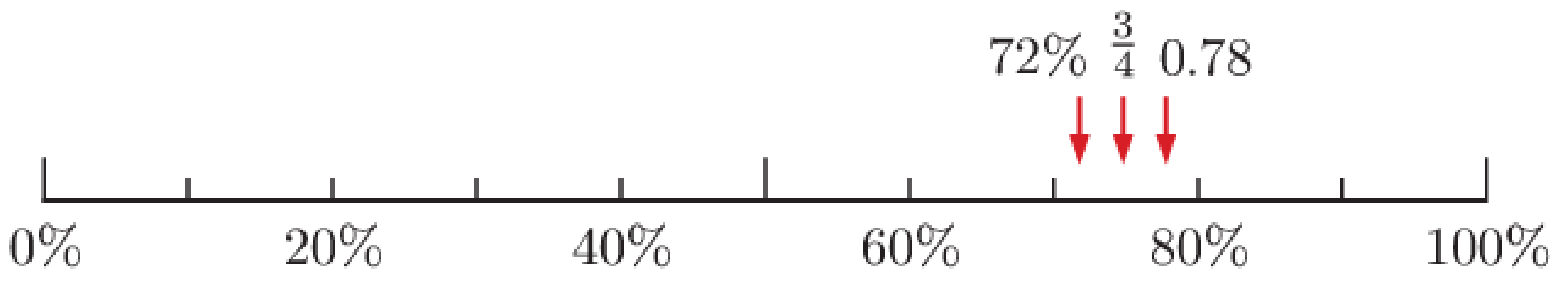
3 **a** 47% **b** 30.6%

4 **a** $\frac{31}{100}$ **b** $\frac{4}{25}$ **c** $\frac{47}{50}$

5 **a** £27 **b** 48 cm **6** 54% **7** 22.1%

8 **a** 0.81 **b** 0.02 **c** 0.108 **9** 31%

10



11 **a** 84 households **b** 168 households

12 **a** oil **b** 5% mustard
c **i** 60 mL lemon juice **ii** 280 mL oil

REVIEW SET 10B

1 **a** X: 39, V: 61 **b** X: $\frac{39}{100}$, V: $\frac{61}{100}$
c X: 39%, V: 61% **d** 39% + 61% = 100%

2 **a** 70% **b** 15%

3 **a** 9% **b** 13.6% **c** 70.2% **4** 17%

5 $\frac{37}{50}$, 0.74 **6** 30 students

7 **a** 27% **b** 72% **c** 65%

8 **a** 52% **b** 29% **9** 30%

10 **a** 30 mL cordial **b** 680 mL water

11 **a** **i** $\frac{10}{100}$ **ii** 0.1 **iii** 10%
b **i** $\frac{35}{100}$ **ii** 0.35 **iii** 35%
c **i** $\frac{62}{100}$ **ii** 0.62 **iii** 62%

12 4%

EXERCISE 11A

1 **a** losing €20 **b** 10 minutes late
c moving 2 steps to the left **d** 6 m below sea level

2

	Statement	*Number*	*Opposite of statement*	*Opposite number*
a	losing by 3 points	-3	winning by 3 points	$+3$
b	a clock is 5 minutes fast	$+5$	a clock is 5 minutes slow	-5
c	4 m above the water	$+4$	4 m below the water	-4
d	2°C below zero	-2	2°C above zero	$+2$
e	an increase of 6 kg	$+6$	a decrease of 6 kg	-6

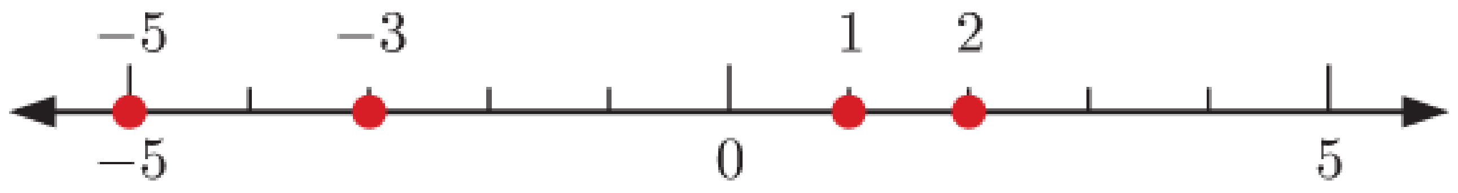
3 **a** -7 **b** $+12$ **c** -15
4 **a** -7 **b** $+32$ **c** -40

EXERCISE 11B

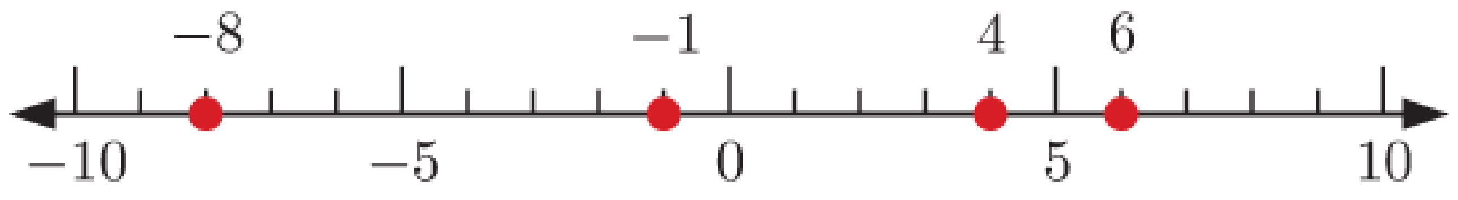
1 **a** up 2 floors **b** down 1 floor **c** up 4 floors
d down 7 floors **e** down 3 floors
2 **a** 4 km east **b** 2 km west **c** 1 km east **d** 8 km west
3 **a** 3°C warmer **b** 5°C cooler **c** 7°C warmer
d 1°C cooler
4 **a** earning $30 **b** spending £10 **c** earning €20
d no change

EXERCISE 11C

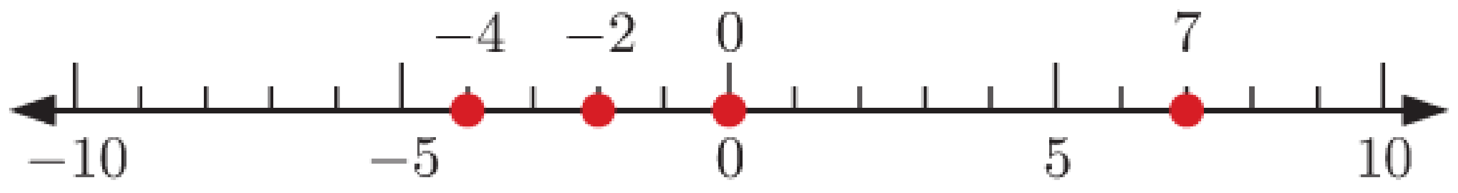
1 **a** positive **b** negative **c** negative **d** positive
e negative **f** neither **g** positive **h** negative
2 **a** -7 **b** $+1$ **c** -2 **d** $+4$ **e** -11 **f** $+13$
3 A: 3, B: -4, C: 9, D: -7
4 **a**



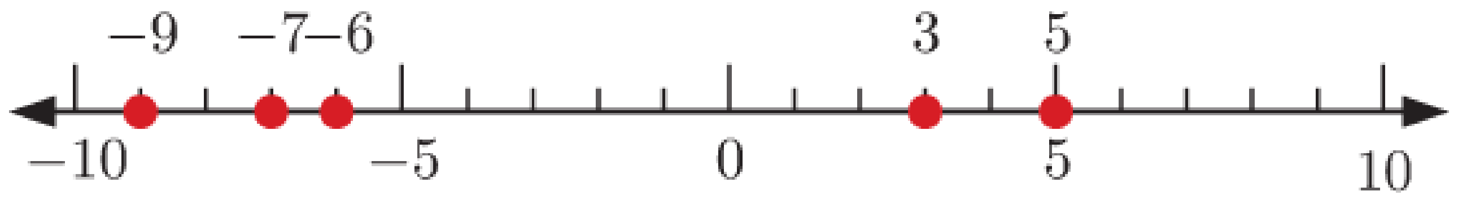
b



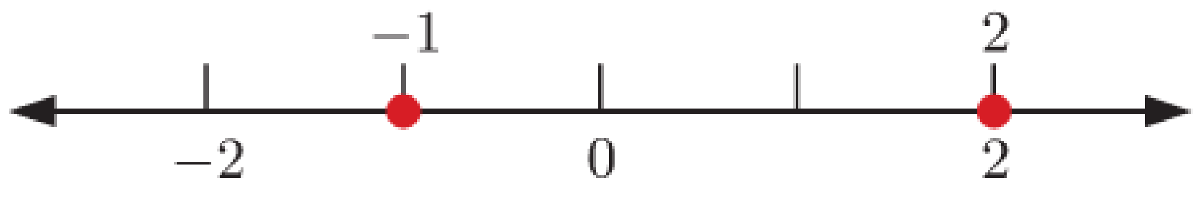
c



d

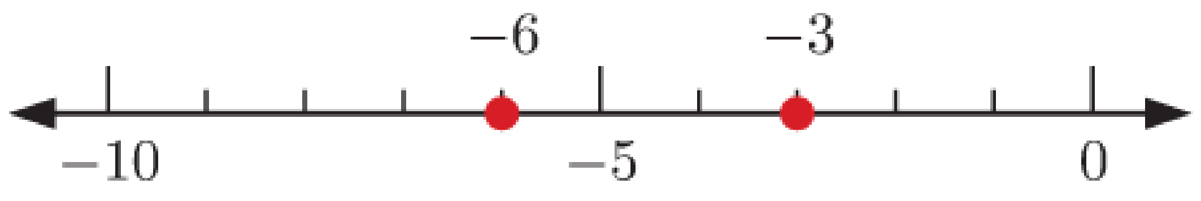


5 **a** **i**



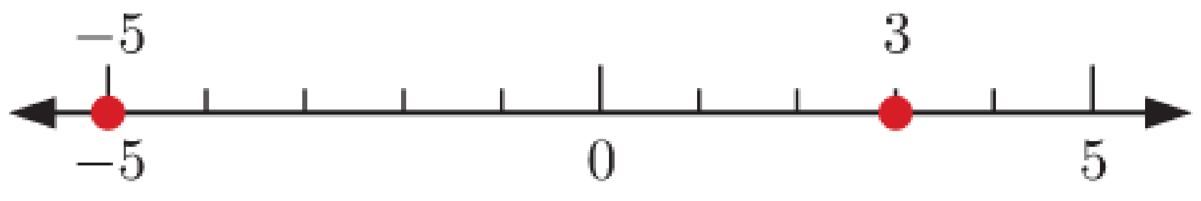
ii 2

b **i**



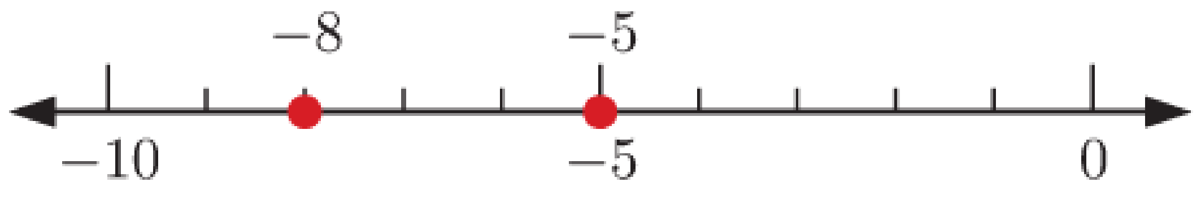
ii -3

c **i**



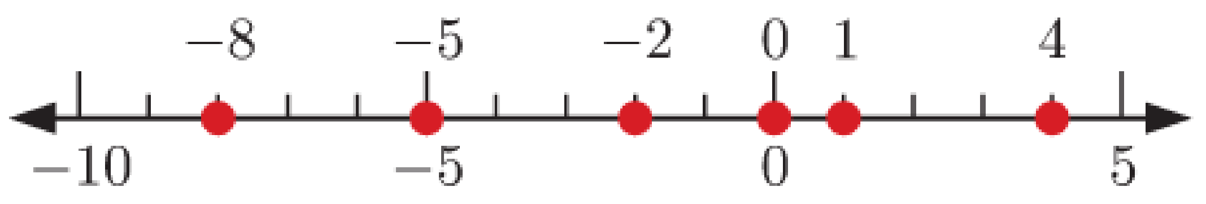
ii 3

d **i**



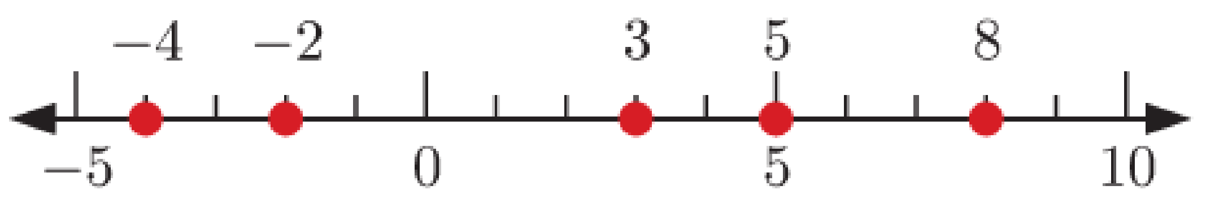
ii -5

6 **a**



b $-8, -5, -2, 0, 1, 4$

7 **a**



b Hachirogata, Japan and New Orleans, USA
c **i** Dublin, Ireland **ii** Hachirogata, Japan

EXERCISE 11D.1

1 **a** -2 **b** 3 **c** -6 **d** -7 **e** -8 **f** -5
g -3 **h** -13 **i** -8 **j** 0 **k** -4 **l** -13
2 **a** -1 **b** -5 **c** 6 **d** -6 **e** -9 **f** -16
3 3 floors below ground level **4** -1°C
5 **a** **i** losing by 8 points **ii** winning by 5 points
b Lost the game by 4 points.

EXERCISE 11D.2

1 **a** 4 **b** 8 **c** -8 **d** -4 **e** -4 **f** 14
g -14 **h** 4
2 **a** 6 **b** 8 **c** -8 **d** 6 **e** -3 **f** -13
g 5 **h** 7 **i** -9 **j** 13 **k** -7 **l** -8
3 **a** 6 **b** -1 **c** 8 **d** -6 **e** -5 **f** 0
4 **a** 7 **b** 9 **c** 4 **d** 6 **e** 19 **f** 4
g 14 **h** 2 **i** 12
5 **a** seagull: $+5$, kayaker: 0, diver: -2, dolphin: -6
b **i** 2 m **ii** 11 m **iii** 6 m **iv** 4 m
6 **a** $-\$50 + -\$40 = -\$90$
This is the same as spending $90.
b $-20°C + 13°C = -7°C$
This is the same as a fall in temperature of 7°C.
c $£310 + -£97 = £213$
This is the same as a profit of £213.
d $-5°C + -8°C = -13°C$
This is the same as a fall in temperature of 13°C.
e 21 km + -29 km = -8 km
This is the same as a journey of 8 km south.
f 13 points + -17 points = -4 points
This is the same as losing by 4 points.
g -3 kg + -2 kg = -5 kg
This is the same as a loss of 5 kg.
7 **a** **i** 860 points **ii** 260 points **iii** 1200 points
b Team A: 1250 points, Team B: 650 points
8 **a** **i** Tuesday **ii** Saturday **iii** Wednesday **iv** Sunday
b **i** 5°C **ii** 3°C **iii** 9°C
9 **a** $-3, -1, +2, +5$ **b** **i** 5 shots **ii** 2 shots **c** -1
10 **a** 5 children **b** 4 children **c** Claire's guess
d **i** 5 lollies **ii** 22 lollies
e **i** 74 lollies **ii** 95 lollies

EXERCISE 11E

1 **a** 10 **b** -10 **c** -10 **d** 10 **e** -10 **f** -10
g 10 **h** 10
2 **a** -8 **b** -15 **c** -28 **d** -9 **e** 18 **f** -40
g 7 **h** -44 **i** -25 **j** 64 **k** -63 **l** -120
m -56 **n** 66 **o** -45 **p** 84

3 **a** 60 marks **b** -36 marks

4 -1×30, 1×-30, -2×15, 2×-15, -3×10, 3×-10, -5×6, 5×-6

EXERCISE 11F

1 **a** 6 **b** -6 **c** -6 **d** 6 **e** 4 **f** -4
g -4 **h** 4 **i** 1 **j** -1 **k** -1 **l** 1
m 7 **n** -7 **o** -7 **p** 7

2 **a** -3 **b** -2 **c** -3 **d** -4 **e** 6 **f** -5
g -3 **h** -4 **i** 13 **j** -9 **k** 1 **l** 7
m -10 **n** -9 **o** 8 **p** -11

3 The final answer is positive.

1st step: negative $\div$ positive $=$ $\boxed{\text{negative}}$

2nd step: $\boxed{\text{negative}}$ $\div$ negative $=$ positive

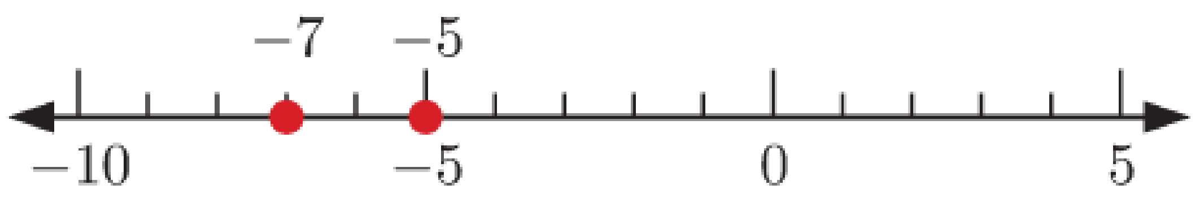
REVIEW SET 11A

1 **a** negative **b** positive **c** neither **d** negative

2 **a** travelling 10 km north **b** getting 3 cm taller

3 A: -6, B: 1, C: -3

4 **a**



b -5

5 **a** -4 **b** 3 **c** -11 **d** -2 **e** -4 **f** 3

6 55 points **7** \$8 loss

8 **a** -5 **b** 10 **c** 7 **d** 0 **e** 5 **f** -4

9 **a** -14 **b** 27 **c** -36 **d** 50

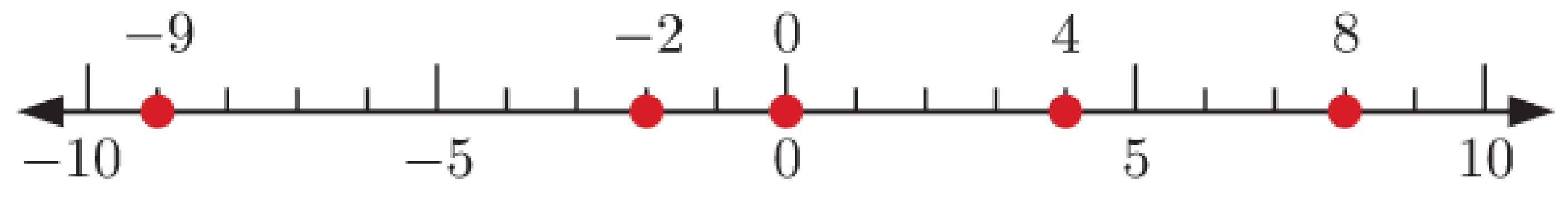
10 **a** -11 **b** -4 **c** 8 **d** -6

REVIEW SET 11B

1 **a** down 2 floors **b** 6 km east **c** 6°C cooler

2 **a** -5 **b** $+8$ **c** -12

3



4 **a** 10 **b** 12 **c** 0 **d** 1

5 **a** 8 **b** 7 **c** 3

6 **a** 16 points **b** 11 points

7 **a** -6 **b** -21 **c** -90 **d** 132

8 **a** subtraction **b** yes **c** -2 **d** $+4$

9 **a** -5 **b** -60 **c** 6 **d** 5 **e** -18 **f** 99

10 **a** tree top: $+5$, bird: $+3$, roots: -1, pipe: -2
b **i** 6 m **ii** 5 m **iii** 1 m

EXERCISE 12A

1 **a** impossible **b** very likely **c** unlikely
d 50-50 chance

2 **a** very unlikely **b** impossible **c** certain
d very likely

3 **a** very unlikely **b** very likely **c** certain
d impossible

4 **a** A **b** B **c** A

5 **a** 4 blue cards **b** 4 white cards
c 2 blue and 2 white cards
d 3 blue and 1 white card, or 4 blue cards

EXERCISE 12B

1 **a** **C** **b** **D** **c** **A** **d** **B** **e** **E**

2 **a** **C** **b** **E** **c** **A** **d** **D** **e** **B**

3 **a** **i** Dhaka **ii** Cairo **b** Kuala Lumpur
c **i** true **ii** false

4 **a** Jan 80%, Natasha 83%, Ellie 58% **b** likely
c Natasha

5 **a** orange juice **b** $\frac{5}{7}$

EXERCISE 12C

1 **a** yellow, green, and pink **b** 3 outcomes

2 **a** 1, 2, 3, 4, 5, and 6 **b** 6 outcomes

3 **a** Eva, Daisy, Nick, Ruby, and Jamie **b** 5 outcomes

4 **a** A, B, C, D, E, F, G, and H **b** 8 outcomes
c 2 outcomes

5 **a** **i** March 2nd, 9th, 16th, 23rd, and 30th
ii 5 outcomes
b **i** March 11th, 12th, 13th, 14th, 15th, 16th, and 17th
ii 7 outcomes

6 **a** 8 outcomes
b **i** 2 outcomes **ii** 3 outcomes **iii** 4 outcomes

7 **a** 13 outcomes **b** 2 outcomes

8 **a** 22 outcomes **b** 6 outcomes

EXERCISE 12D

1 **a** heads and tails **b** 2 outcomes **c** $\frac{1}{2}$

2 **a** 7 outcomes **b** **i** $\frac{1}{7}$ **ii** $\frac{3}{7}$ **iii** $\frac{2}{7}$ **iv** $\frac{4}{7}$

3 **a** 40 outcomes
b **i** $\frac{1}{40}$ **ii** $\frac{2}{40} = \frac{1}{20}$ **iii** $\frac{9}{40}$ **iv** $\frac{12}{40} = \frac{3}{10}$

4 **a** $\frac{2}{9}$ **b** $\frac{4}{9}$ **c** $\frac{3}{9} = \frac{1}{3}$

5 **a** 13 chocolates **b** $\frac{6}{13}$

6 **a** 11 players **b** **i** $\frac{1}{11}$ **ii** $\frac{6}{11}$

7 **a** **i** $\frac{5}{20} = \frac{1}{4}$ **ii** 25% **iii** 0.25 **b** unlikely

8 **a** **i** $\frac{5}{12}$ **ii** $\frac{7}{12}$ **b** a red section

9 **a** $\frac{3}{9} = \frac{1}{3}$ **b** $\frac{1}{9}$ **c** $\frac{2}{9}$

10 **a** **i** $\frac{2}{10} = \frac{1}{5}$ **ii** $\frac{3}{5}$ **b** Ben $\left(\frac{3}{10} > \frac{1}{5}\right)$

REVIEW SET 12A

1 **a** likely **b** impossible

2 **a** 3 blue cards **b** 3 grey cards **c** one of each colour

3 **a** very unlikely **b** certain **c** 50-50 chance

4 **a** blue, red, green, yellow, black **b** 5 outcomes **c** yes

5 **a** 26 outcomes **b** **i** $\frac{5}{26}$ **ii** $\frac{3}{26}$

6 **a** Simon
b Sum is 1. It is certain that either Trent or Simon wins the game.

7 **a** $\frac{1}{30}$ **b** $\frac{4}{30} = \frac{2}{15}$ **c** $\frac{22}{30} = \frac{11}{15}$

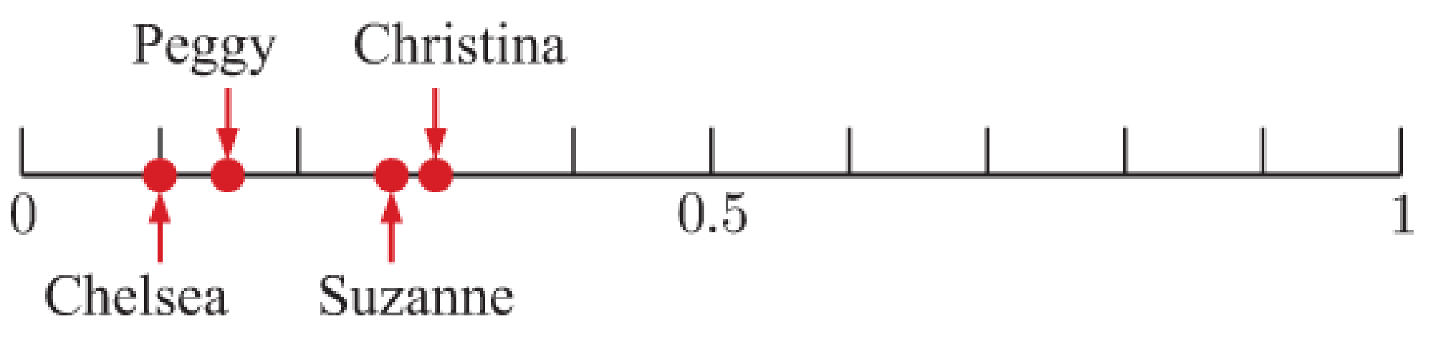
8 **a** $\frac{5}{20} = \frac{1}{4}$ **b** $\frac{15}{20} = \frac{3}{4}$

9 $\frac{5}{11}$ **10** $\frac{13}{31}$

REVIEW SET 12B

1 **a** certain **b** very unlikely **c** impossible

2 **a**



b Christina **c** very unlikely

3 **a** 7 outcomes **b** 4 outcomes **4** $\frac{3}{10}$

5 **a** Bradley: $\frac{3}{5} = 60\%$, Caleb: $0.62 = 62\%$ **b** Caleb

6 $\frac{4}{9}$ **7** $\frac{8}{16} = \frac{1}{2}$ **8** **a** $\frac{1}{18}$ **b** $\frac{10}{18} = \frac{5}{9}$

9 **a** **i** very unlikely **ii** very likely **iii** certain

b **i** $\frac{1}{200}$ **ii** $\frac{199}{200}$ **iii** $\frac{200}{200} = 1$

10 **a** **i** $\frac{3}{14}$ **ii** $\frac{5}{14}$ **iii** $\frac{6}{14} = \frac{3}{7}$ **b** yellow

c Sum is 1. The child is certain to receive either a red, blue, or yellow balloon.

EXERCISE 13A

1 **a** **i** St Peter's College **ii** University College
iii Soldiers Memorial Gardens

b **i** G2 **ii** E1

2 **a** **i** Buckingham Palace **ii** Royal Courts of Justice
iii Covent Garden **iv** Piccadilly Circus
v Waterloo Station

b **i** C2 **ii** C4 **iii** C3 **iv** E1 **v** B4

3 **a** **i** Eiffel Tower **ii** Au Jardin du Luxembourg
iii Centre National d'art et Culture Georges Pompidou

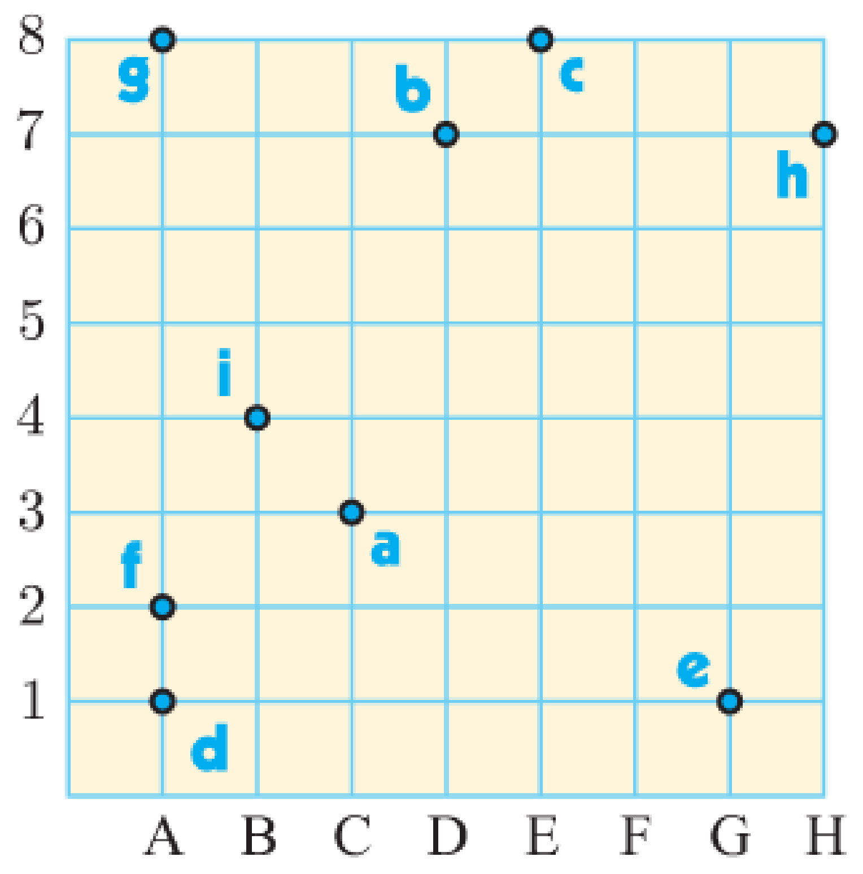
b **i** G5 **ii** A6 **iii** I2

4 **a** staff room **b** G6 **c** F1

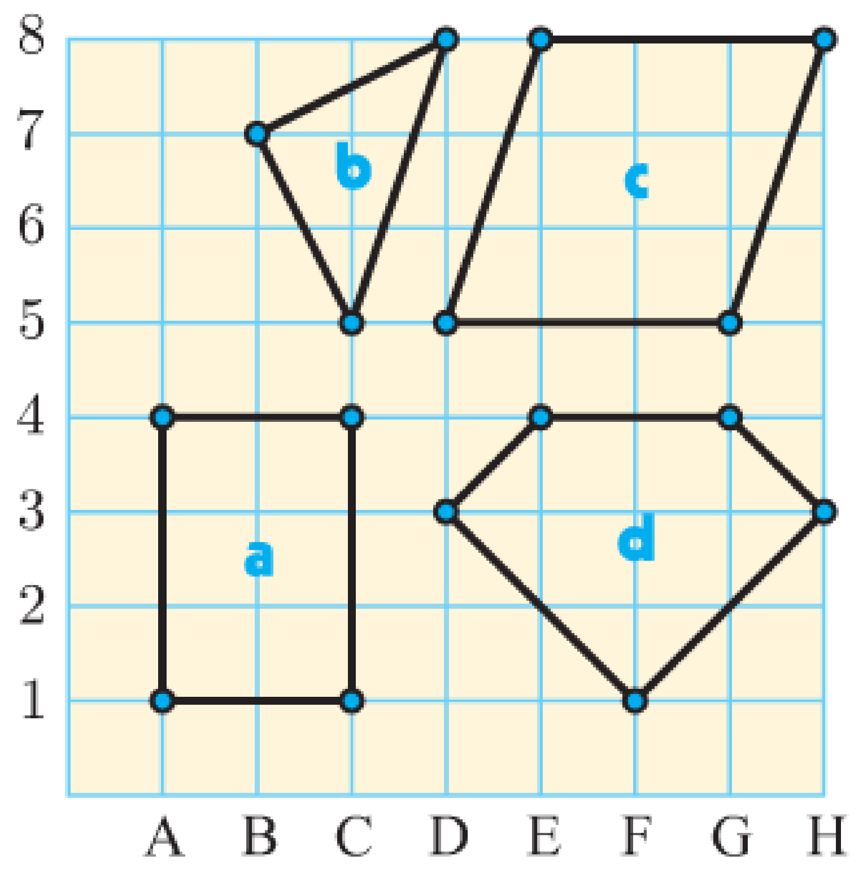
d **i** 3 drinking fountains **ii** C6

EXERCISE 13B

1



2



a rectangle
b triangle
c parallelogram
d pentagon

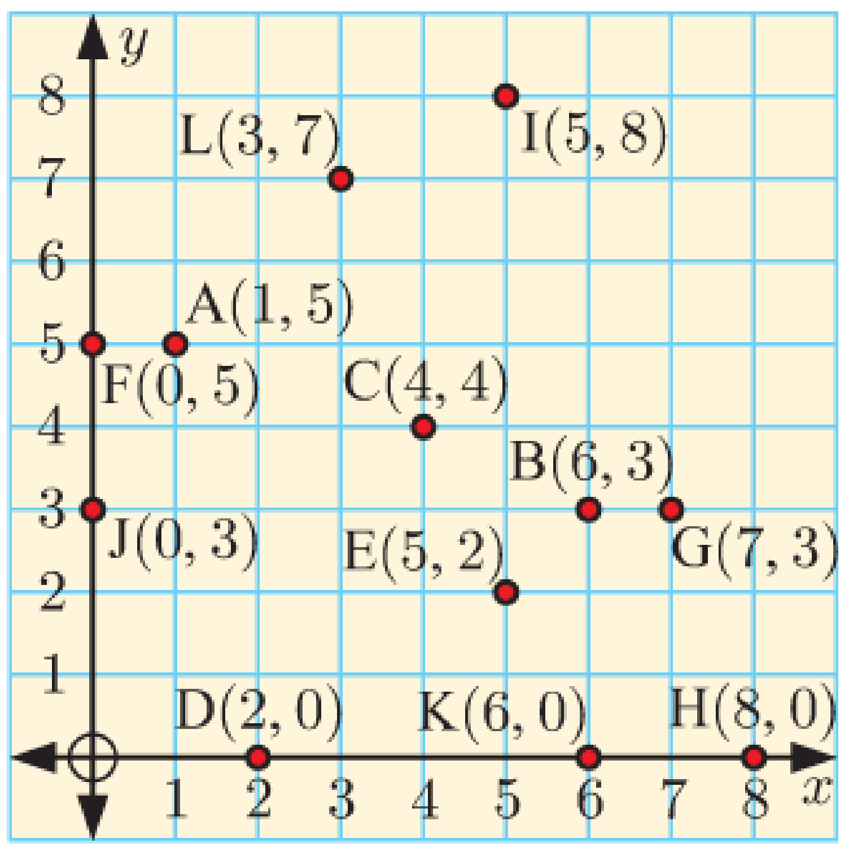
3 **a** **i** G1 **ii** E3 **b** **i** pasta stall **ii** Thai food stall

4 **a** **i** Peter **ii** Amber
b **i** D7 **ii** C2 **c** school **d** Rob; B6

5 **a** 15 trees **b** Pine tree; B8, D1, and G2
c Maple tree; E3

EXERCISE 13C

1



2 **a** D, H, and K **b** The y-coordinate is zero.

3 **a** F and J **b** The x-coordinate is zero.

4 **a** 5 **b** 4 **c** **i** R(6, 5) **ii** S(3, 3) **iii** O(0, 0)

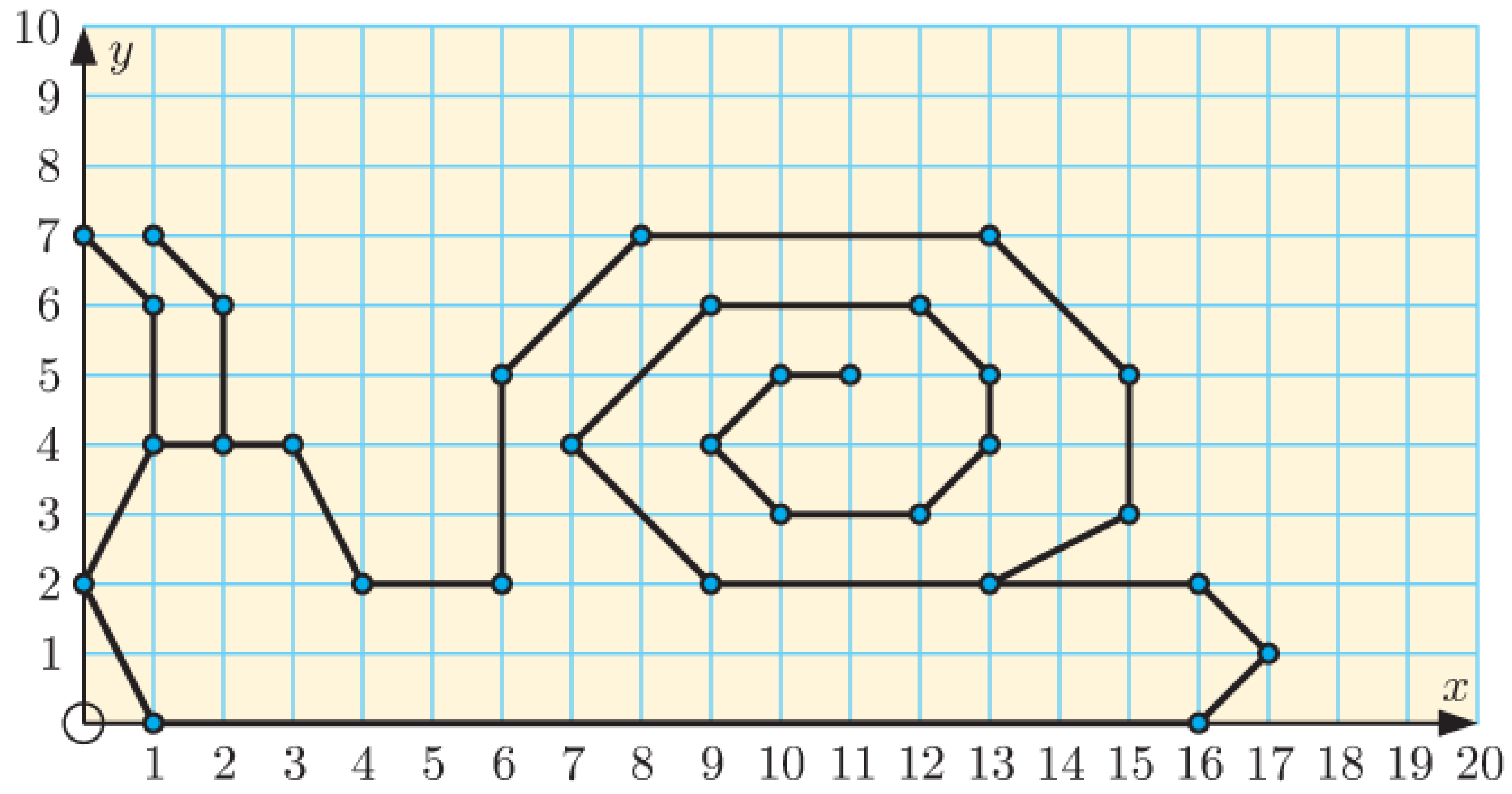
5 HAVE A GOOD DAY

6 **a** **i** (1, 5) **ii** (3, 0) **iii** (4, 4) **iv** (0, 7)

b **i** polo **ii** kayaking **iii** cricket **iv** golf

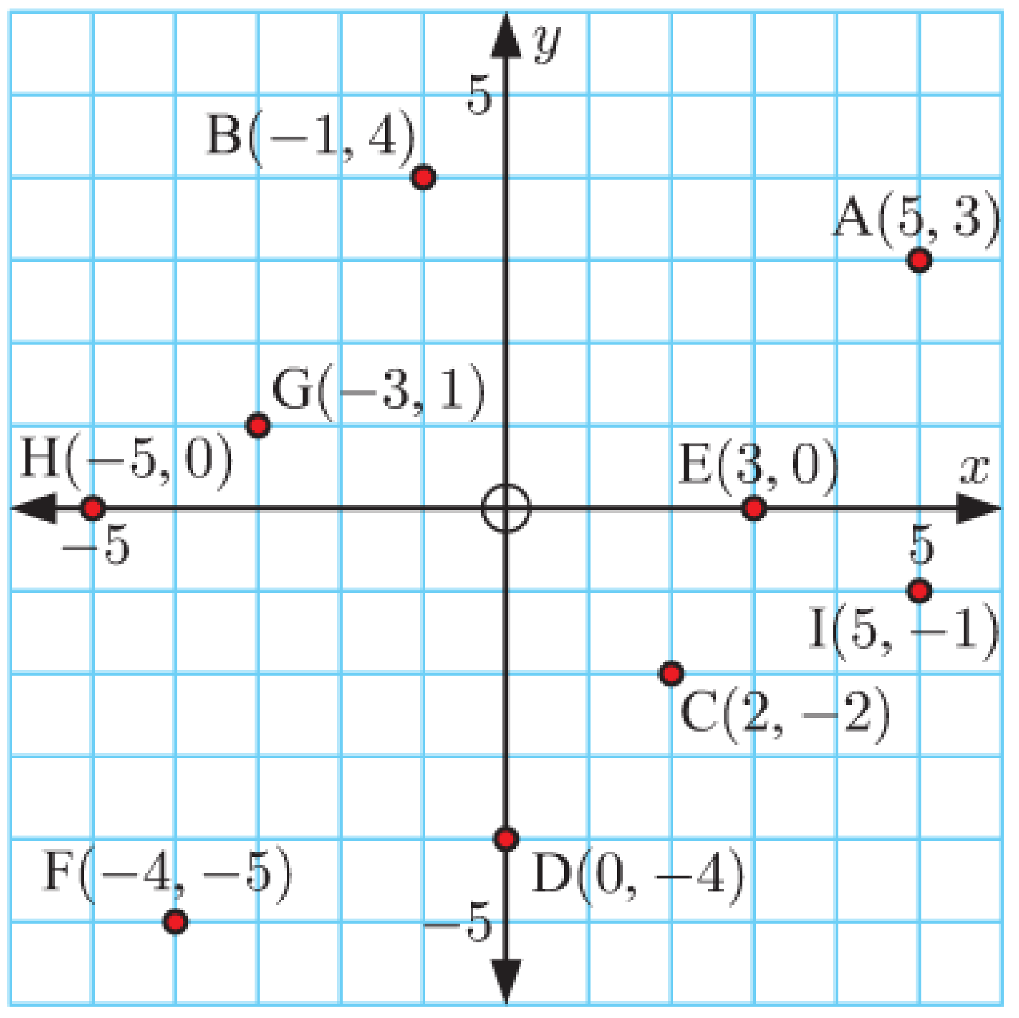
c **i** water skiing **ii** swimming

7



EXERCISE 13D

1



2 **a** **i** −5 **ii** −1 **iii** W(0, 4)

b **i** 1st quadrant **ii** 4th quadrant **iii** 3rd quadrant
iv 2nd quadrant

c **i** T **ii** Z **iii** Q

3 **a** **i** (1, −4) **ii** (−4, 3) **iii** (3, 2)

b **i** performance stage **ii** roller coaster **iii** petting zoo

4 **a** **i** (4, 1) **ii** (−2, 4) **b** 3rd quadrant **c** Gary

5 **a** **i** (−2, 3) **ii** (1, −3) **b** 3 stations **c** station J

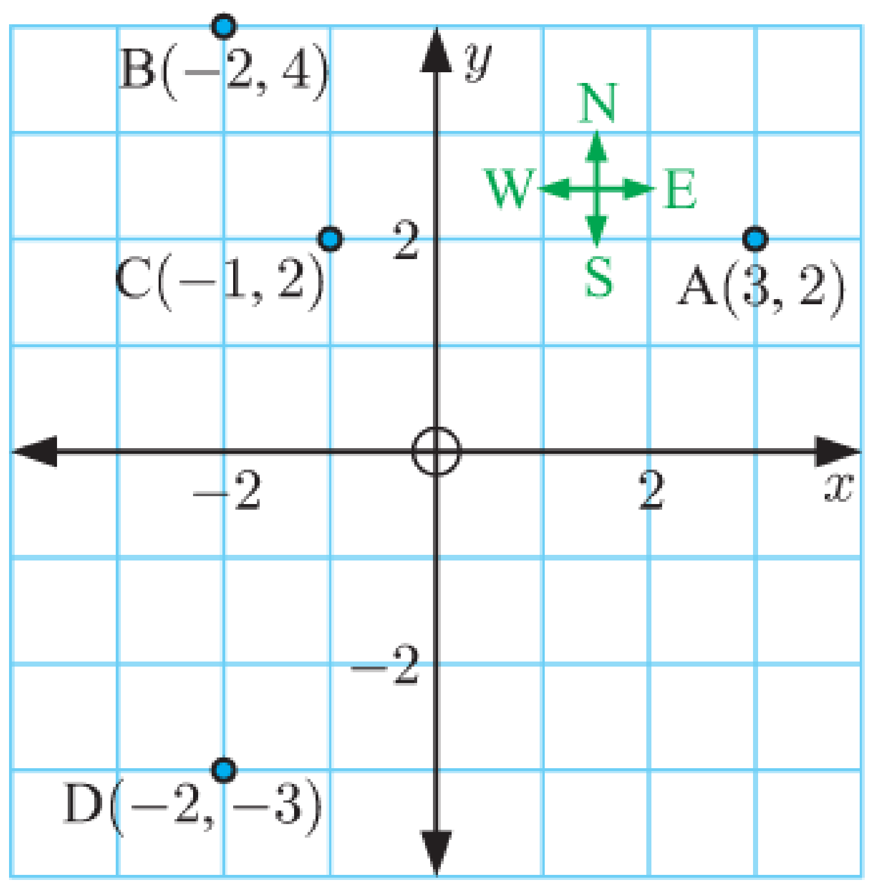
6 **a** 70 km **b** 20 km

7 **a** (−6, −7) **b** **i** 70 m **ii** 70 m
c 60 m **d** **i** flying fox **ii** 30 m

8 **a** (−1, −2) **b** **i** 400 m **ii** left
c 6000 m (or 6 km)

EXERCISE 13E

1 **a**



b **i** north **ii** east **iii** south-west

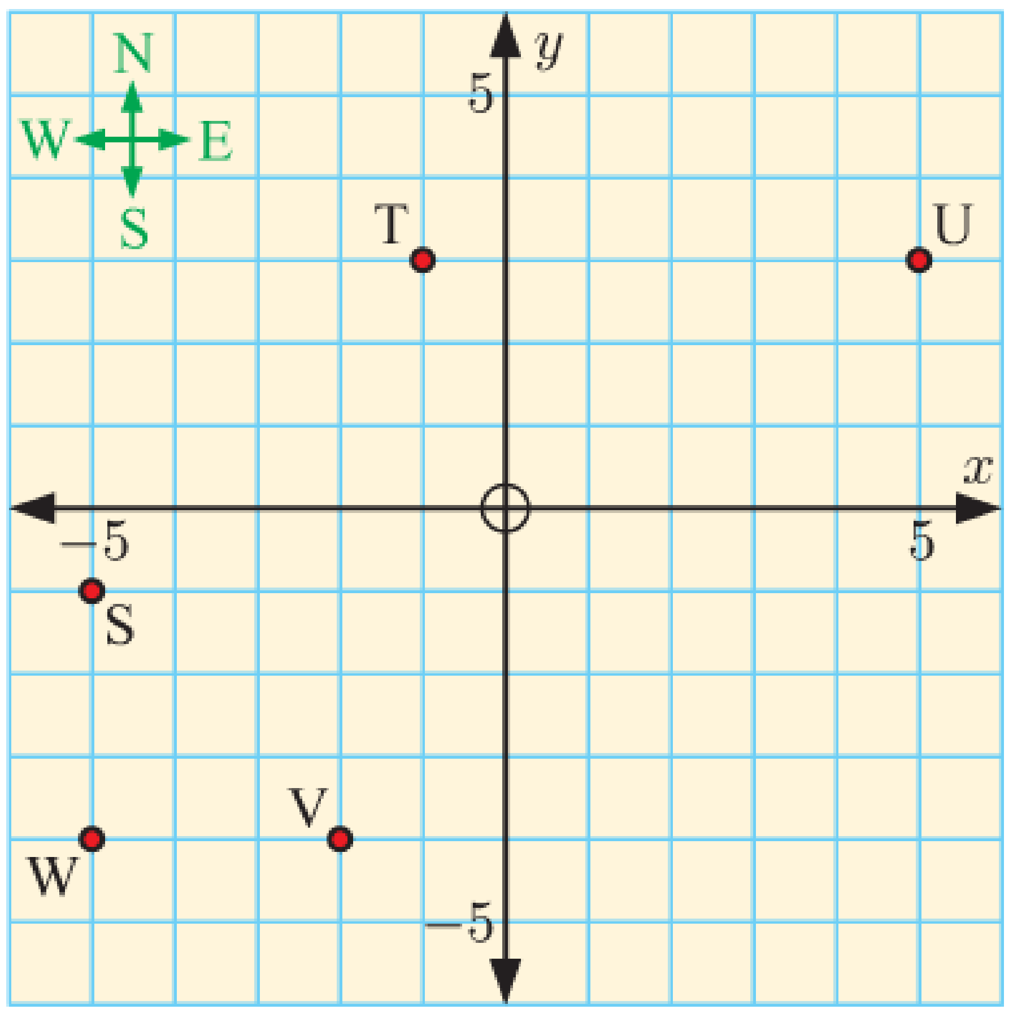
2 **a** P(−2, 3), Q(3, −1) **b** C **c** D **d** C

3 **a** 8 km **b** north **c** (0, −2)

4 **a** **i** (−5, 4) **ii** (5, −3)
b Joe needs to travel 4 m west, then 4 m south, then 12 m east, then 2 m north, then 2 m east, then 8 m north, then 6 m east, then 2 m south, then 2 m west, then 4 m south, then 2 m east to get to the exit.

5 **a** Pia's directions **b** (1, 3) **c** yes **d** no

6 **a** north-east
b, **c** **i**, **d**

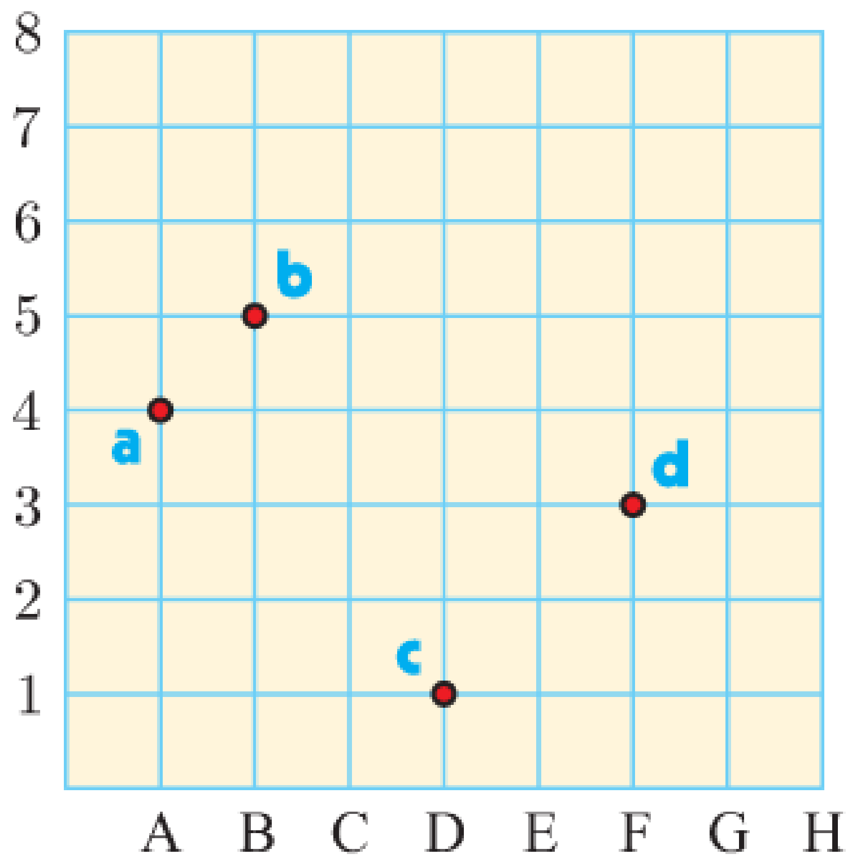


c **ii** V(−2, −4) **e** 6 km north

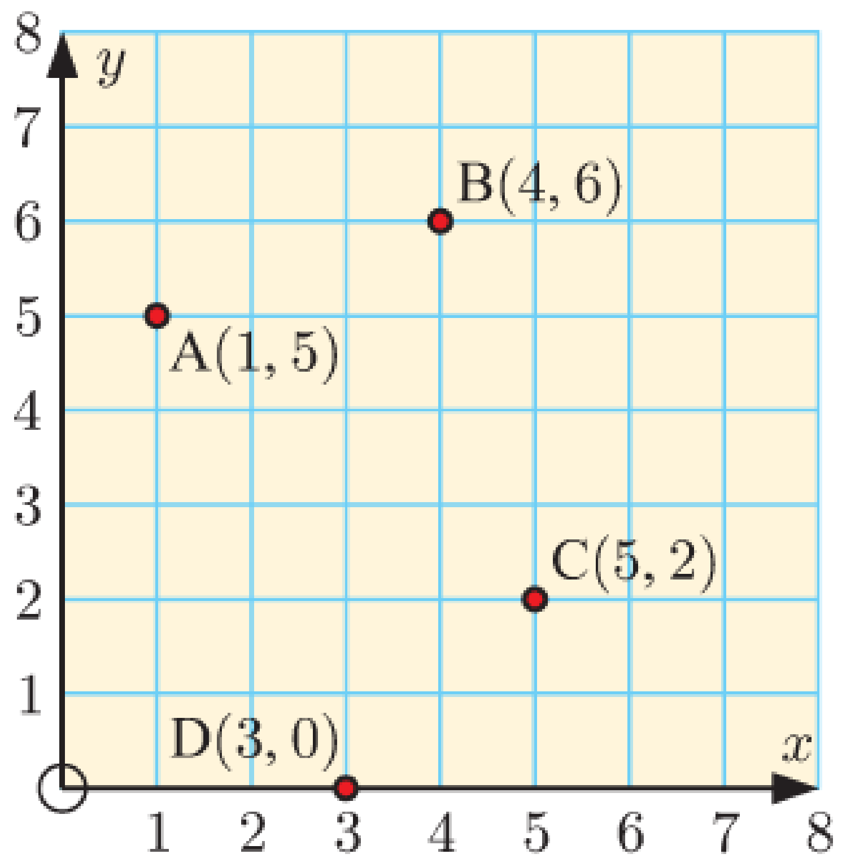
REVIEW SET 13A

1 **a** change rooms **b** 4 grid squares
c **i** F2 **ii** C1 **d** **i** east **ii** north-west

2



3



4 **a** **i** (1, 4) **ii** (5, 4)
b **i** the perfume shop **ii** the sports shop **c** 60 m

5 **a** M(−3, −3) **b** B

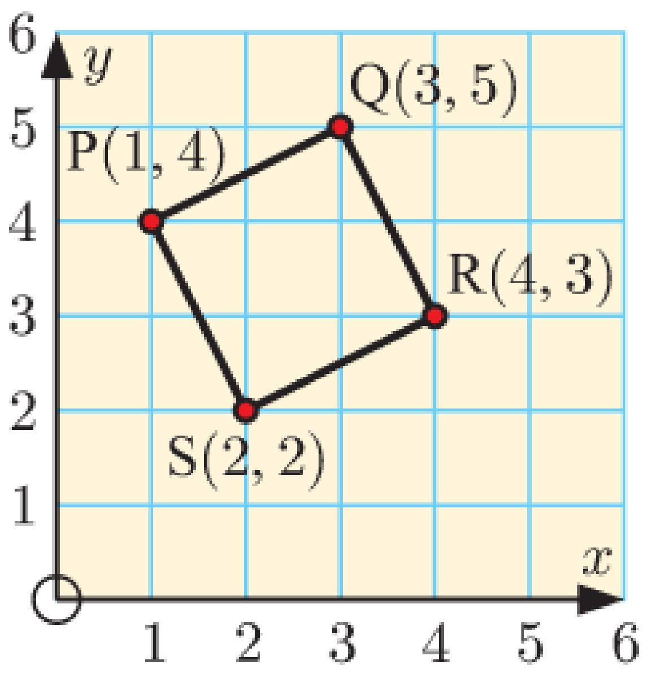
6 **a** **i** A(−4, −2) **ii** E(1, −1) **b** **i** F **ii** D
c **i** west **ii** north-east

7 **a** (−5, −5) **b** **i** north-west **ii** east **iii** south-east
c (−5, −3)

REVIEW SET 13B

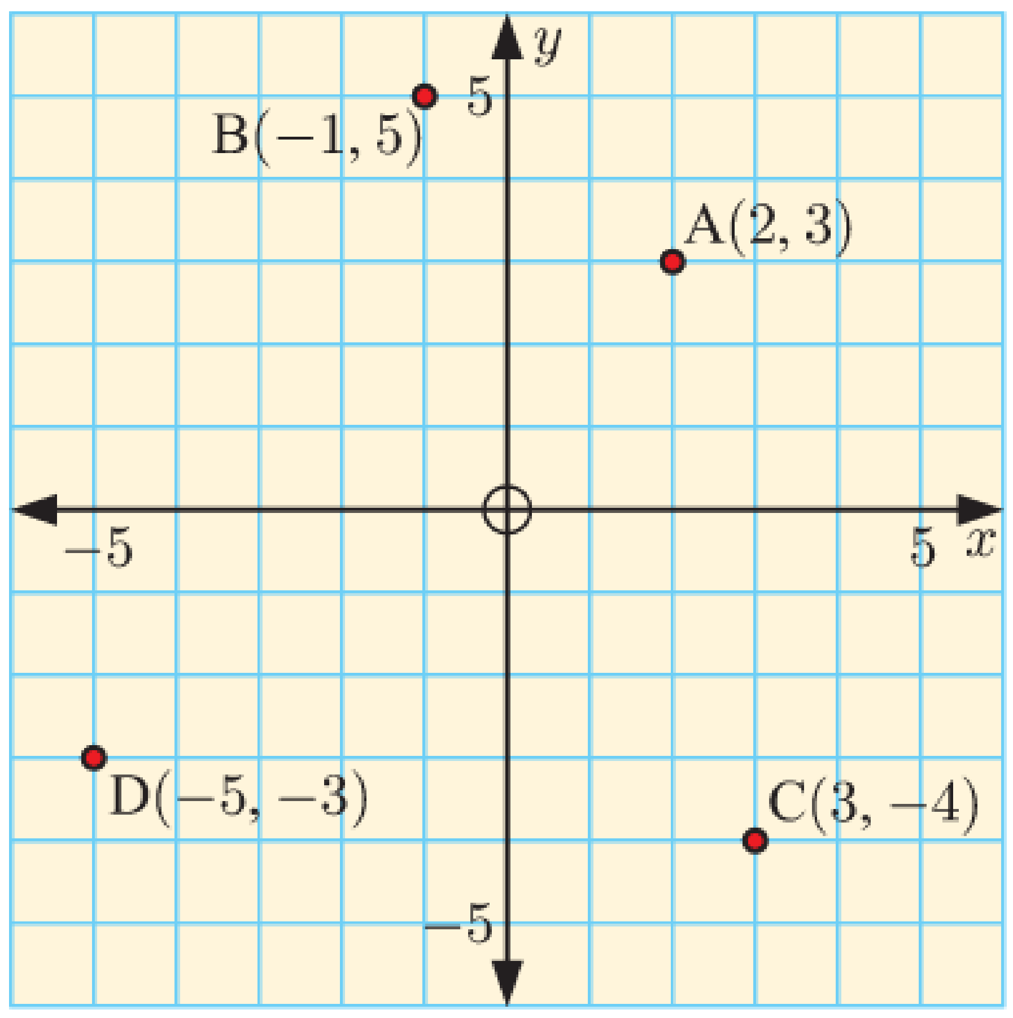
1 **a** **i** H8 **ii** B7 **b** **i** Greek stall **ii** Japanese stall

2 **a**



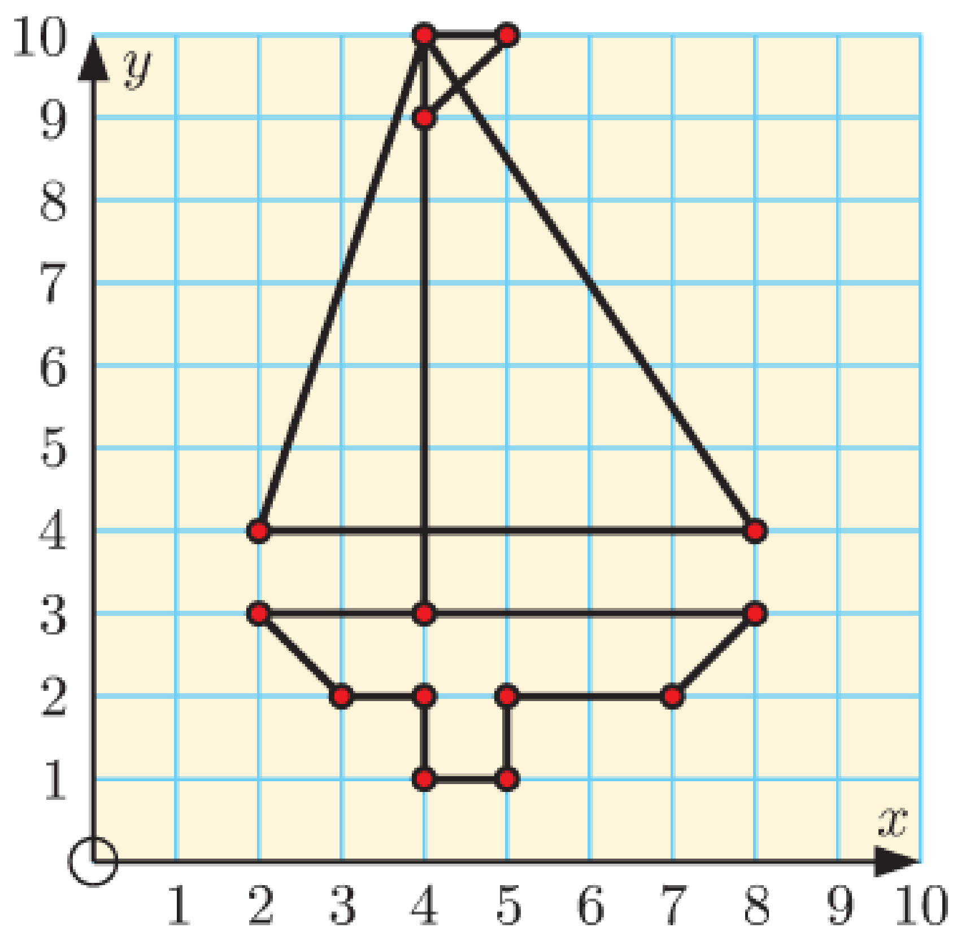
b a square

3



4 **a** F(3, 1), G(−4, −2), H(−2, 3), I(4, −3), J(−1, −1), K(−5, 1)
b F lies in the 1st quadrant.
H and K lie in the 2nd quadrant.
G and J lie in the 3rd quadrant.
I lies in the 4th quadrant.

5



6 **a** **i** (2, 3) **ii** (−1, −2)
b **i** (2, −2) **ii** 3 km **iii** east **c** (−1, 0)

7 (2, −3)

EXERCISE 14A.1

1 **a** 24 units^2 **b** 30 units^2 **c** 44 units^2
d 24 units^2 **e** 21 units^2 **f** 45 units^2

2 **a**

Shape	*Perimeter* (units)	*Area* (units^2)
A	18	20
B	24	20
C	20	20

b The perimeter can vary for different shapes with the same area.

EXERCISE 14A.2

1 **a** **C** **b** **H** **c** **G** **d** **B** **e** **F** **f** **A**
g **E** **h** **D**

2 **a** 42 tiles **b** 2.1 m^2 **c** £92.40

3 **a** **i** 540 pavers **ii** 280 pavers
b 16.4 m^2 **c** \$506.76

4 **a** 18 mm^2 **b** 16 mm^2 **c** 40 mm^2 **d** 72 mm^2

EXERCISE 14B

1 **a** 10 cm^2 **b** 140 km^2 **c** 32 m^2
d 36 m^2 **e** 39 km^2 **f** 82.81 cm^2
g 1200 cm^2 **h** 4.05 cm^2 **i** 89.25 m^2

2 Gemma's computer screen is larger.

3 **a** 156 m^2 **b** £1170 **4** 29 m^2

5 **a** 45 m^2 **b** 3 litres

6 **a** 0.225 m^2 **b** 16.2 m^2 **c** 72 floorboards **d** €1548

EXERCISE 14C

1 **a** 30 m^2 **b** 24 m^2 **c** 21 cm^2 **d** 7.5 cm^2
e 36 m^2 **f** 28 m^2

2 **a** 60 m^2 **b** 36 cm^2 **c** 10 m^2 **d** 60 cm^2
e 28 cm^2 **f** 72 cm^2

3 **a** 10.08 m^2 **b** €171.36 **4** 6.3 m^2

EXERCISE 14D.1

1 **a** **i** 18 $units^3$ **ii** 14 $units^3$ **iii** 21 $units^3$
b solid **ii**, solid **i**, solid **iii**

2 **B** **3** **C**

EXERCISE 14D.2

1 **a** 160 mm^3 **b** 48 m^3 **c** 56 cm^3 **d** 150 mm^3
e 125 cm^3 **f** 180 cm^3

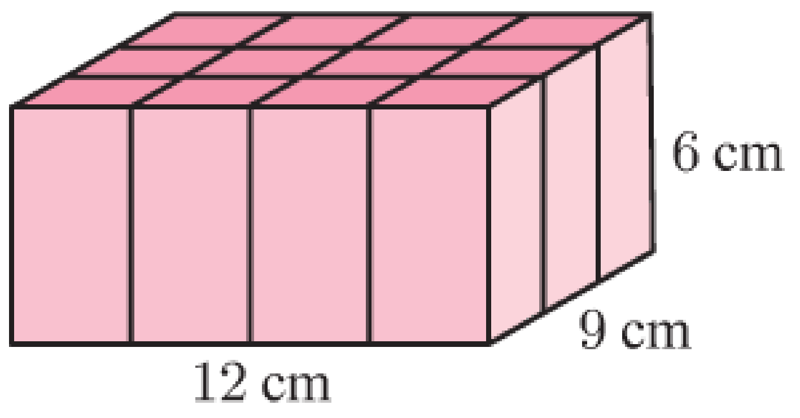
2 30 cm^3 **3** 4800 cm^3

4 **Hint:** $1 \times 1 \times 36$, $1 \times 2 \times 18$, $1 \times 3 \times 12$, $1 \times 4 \times 9$, $1 \times 6 \times 6$, $2 \times 2 \times 9$, $2 \times 3 \times 6$, $3 \times 3 \times 4$

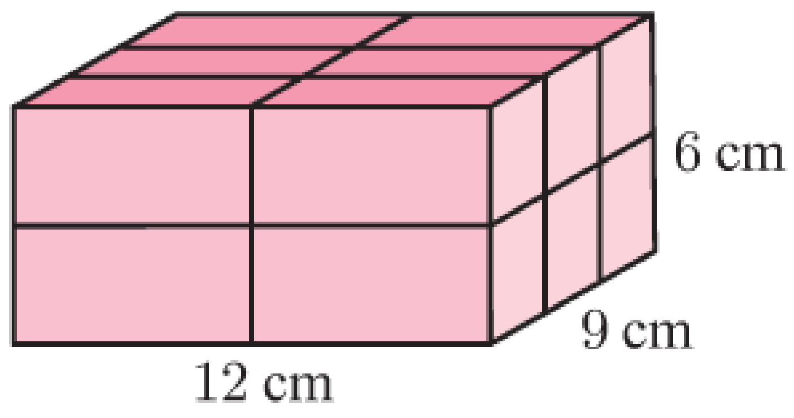
5 **a** 80 $units^3$ **b** 132 $units^2$

6 **a** 10.4 m **b** 4.8 m^2 **c** 1.44 m^3 **7** 6.479 m^3

8 12 boxes



or



REVIEW SET 14A

1 48 $units^2$ **2** 27 m^2 **3** **E**

4 **a** 18 cm^2 **b** 10 cm^2 **c** 54 m^2

5 **a** 6 cm^2 **b** 100 stamps **6** 4500 cm^2

7 **a** 21 $units^3$ **b** 320 cm^3 **c** 64 cm^3 **8** 300 cm^3

9 **a** 80 cm^2 **b** 720 cm^3 **10** 360 boxes

REVIEW SET 14B

1 64 cm^2 **2** **C** **3** £348.88

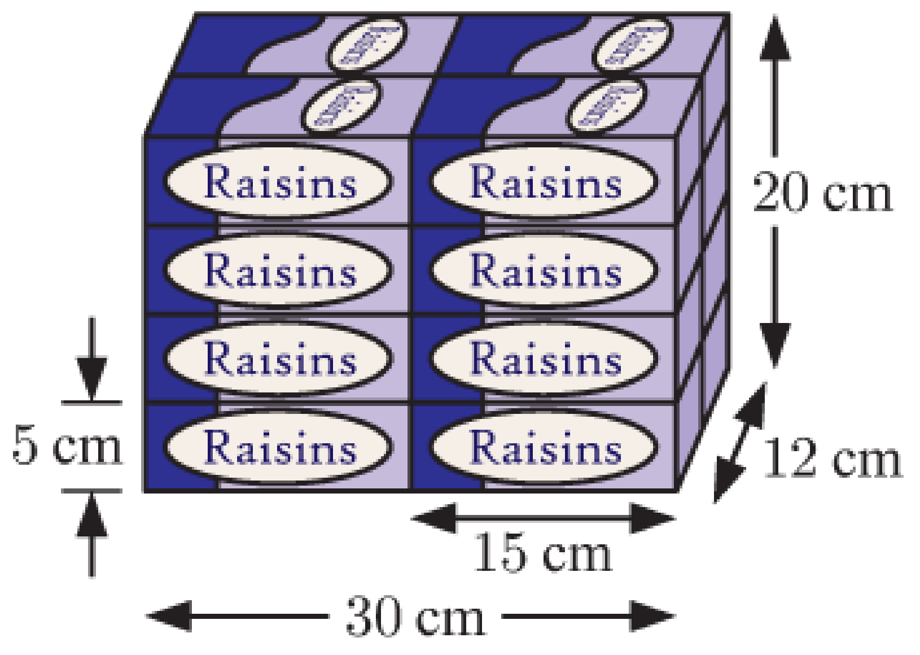
4 **a** 7.5 cm^2 **b** 24 m^2 **c** 20 cm^2 **5** 30 000 m^2

6 **a** 120 mm^3 **b** 8 m^3 **c** 196 cm^3

7 90 m^3 **8** rectangle **B**

9 **a** 77 cm^2 **b** **i** 42 cm^2 **ii** 20 cm^2 **iii** 15 cm^2
c $42 + 20 + 15 = 77$ cm^2

10 **a** 450 cm^3
b 7200 cm^3
c 16 boxes
d



EXERCISE 15A

1 **a** Choose 400 names from the electoral roll.
b Select every 100th bottle, for example.
c Choose 30 names from a list of students at the school.
d Select a random page of a dictionary, and randomly select a word on that page.

2 **a** Place the tickets in a hat, select one at random.
b Toss a coin, if it lands heads select A, if it lands tails select B.
c Roll a die, and select the number that appears.
d Shuffle the pack, and select the card on top.

3 **a** 10 000 ants **b** 300 ants **c** $\frac{36}{300} = \frac{3}{25}$ **d** 1200 ants

4 **a** 750 people **b** 50 people **c** 34% **d** 255 people

EXERCISE 15B

1 **a**

Colour	*Tally*	*Frequency*
Brown (Br)	卌 卌 |	11
Blue (Bl)	卌 ||	7
Green (Gn)	卌 |	6
Grey (Gr)	||||	4
	Total	28

b brown

2 **a**

Product	*Tally*	*Frequency*
Popcorn (P)	卌 卌 卌 |||	18
Soft drink (S)	卌 |||	8
Ice cream (I)	卌 ||||	9
Chips (C)	卌 卌 卌	15
	Total	50

b popcorn

3 **a**

Result	*Tally*	*Frequency*
Excellent (E)	||||	4
Good (G)	卌 卌 |	11
Satisfactory (S)	卌 ||	7
Unsatisfactory (U)	|||	3
	Total	25

b good
c The owner of the hotel may want to know if guests are satisfied with the service so improvements can be made if service is seen as unsatisfactory.

4 **a**

Winner	*Tally*	*Frequency*
Brody	||	2
Cooper	|||	3
Hailey	卌 |	6
Maria	||||	4
	Total	15

b 3 games
c Hailey

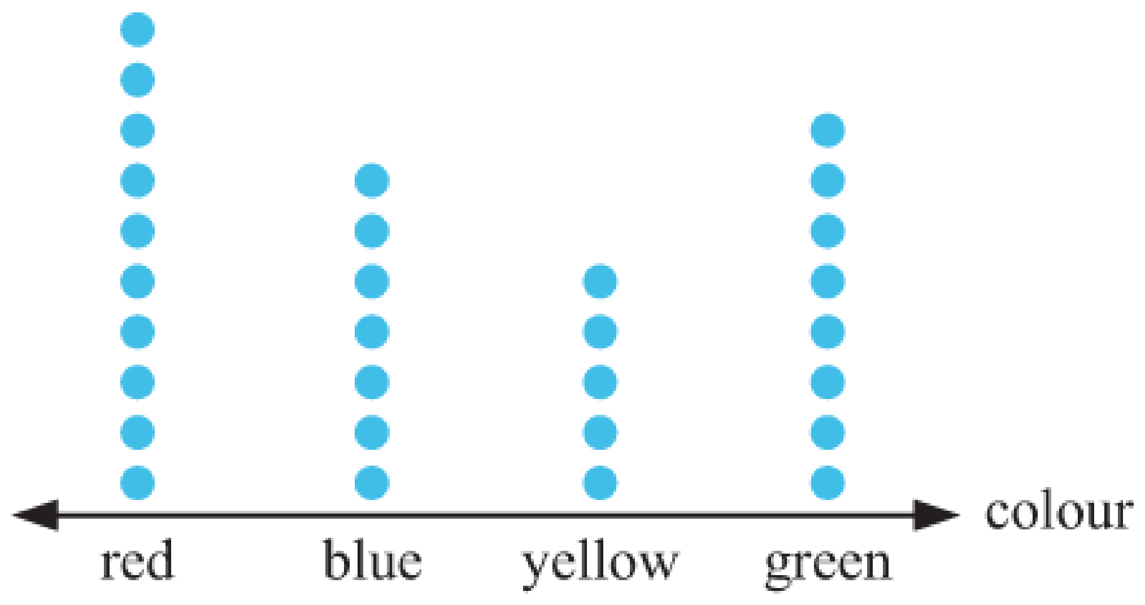
EXERCISE 15C.1

1 **a** 4 students **b** 23 students **c** canoeing

2 **a** violin **b** 28 musicians **c** 12 musicians

3 **a**

Spinner results

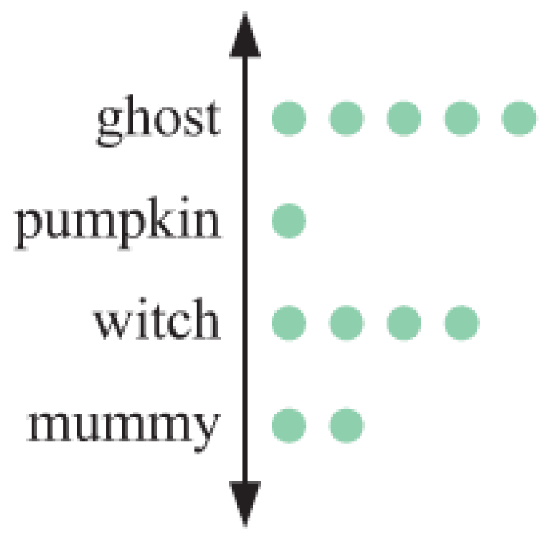


b red

4 **a** 12 children

b

Halloween party costumes



c ghost

EXERCISE 15C.2

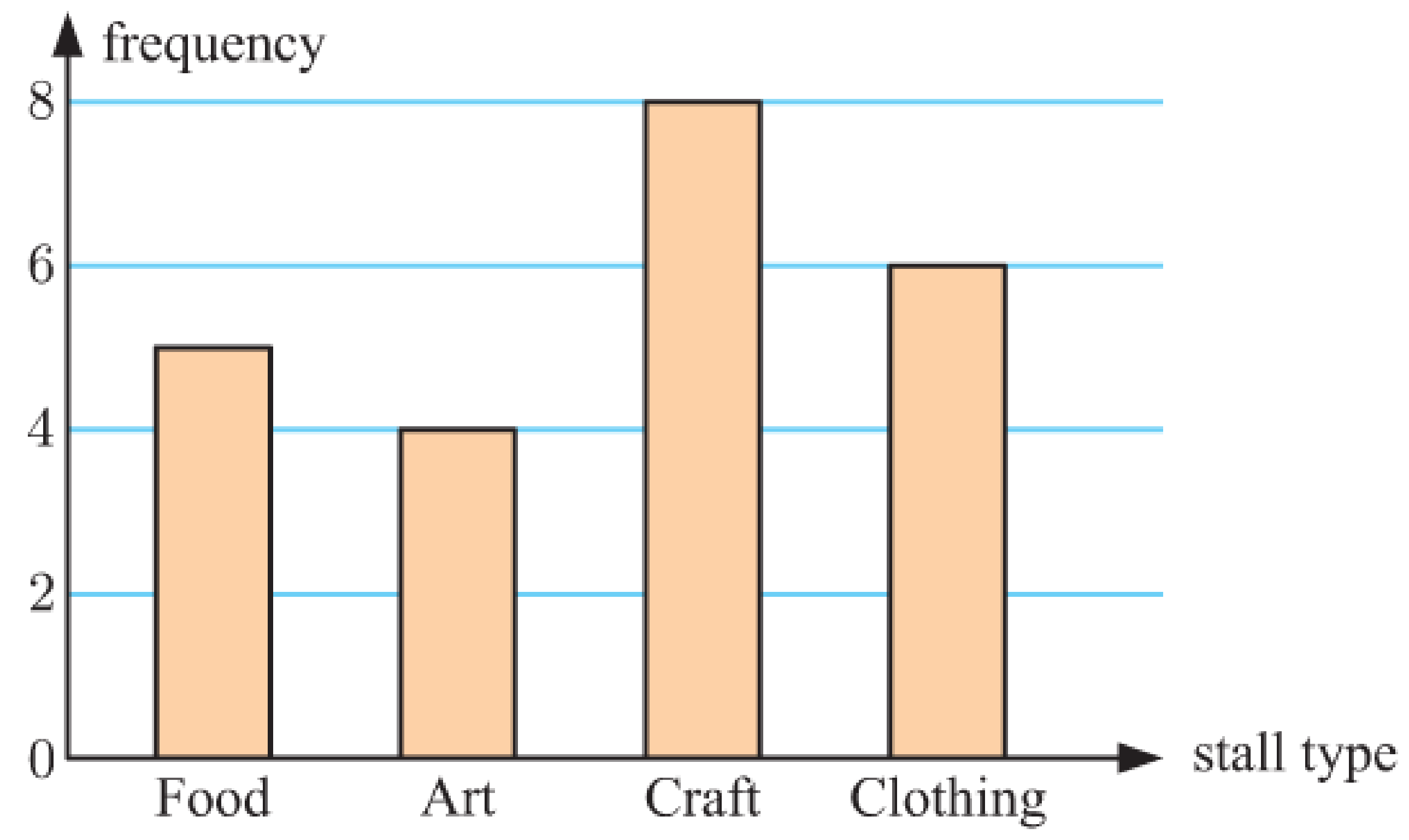
1 **a** 6 T-shirts **b** socks

2 **a** **i** 20 people **ii** 25 people **b** Toyota

3 **a** 23 stalls

b

Stalls at market



c Craft **d** 2 stalls

4 **a** 24 adults

b

Employment type	*Tally*	*Frequency*
Full-time (F)	𝍸 ||||	9
Part-time (P)	𝍸 |	6
Unemployed (U)	|||	3
Retired (R)	𝍸 |	6
	Total	24

c

Employment type of adults

d full-time **e** 15 adults

EXERCISE 15C.3

1 **a** garden

b True, as $12\% + 5\% = 17\%$ which is less than 20% (or $\frac{1}{5}$).

c 160 kilolitres

2 **a** size 12

b False, as $35\% + 10\% = 45\%$ which is less than 50% (or $\frac{1}{2}$).

c 60 boys

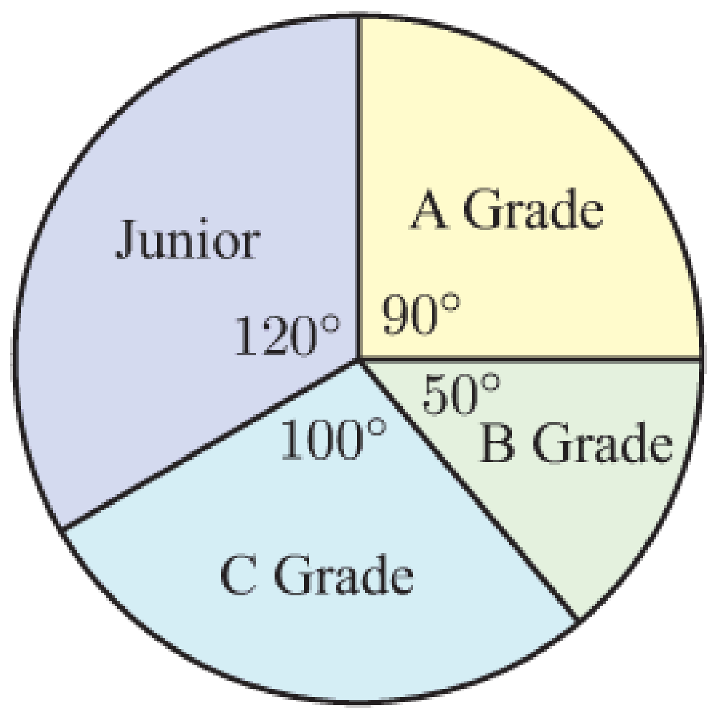
EXERCISE 15C.4

1 **a** 10°

b A Grade: 90°,
B Grade: 50°,
C Grade: 100°,
Junior: 120°

c

Members' playing levels



2 **a**

Favourite type of book

b Fantasy

EXERCISE 15D

1 **a**

Children	*Tally*	*Frequency*
0	||||	4
1	||||	4
2	𝍸 ||||	9
3	||||	4
4	𝍸 |	6
5	||	2
6	|	1
	Total	30

b **i** 9 families **ii** 13 families

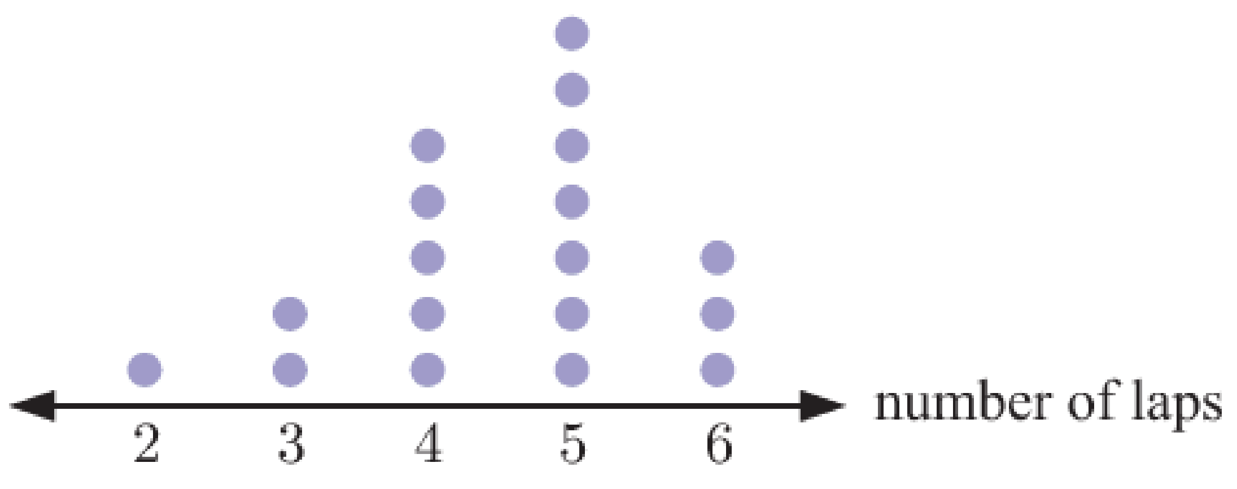
c The mode is 2 children. This is the most common number of children in the families surveyed.

2 **a** 5 students **b** 26 students **c** 3 hats

3 **a** 5 children **b** 1 filling

4 **a**

Number of laps

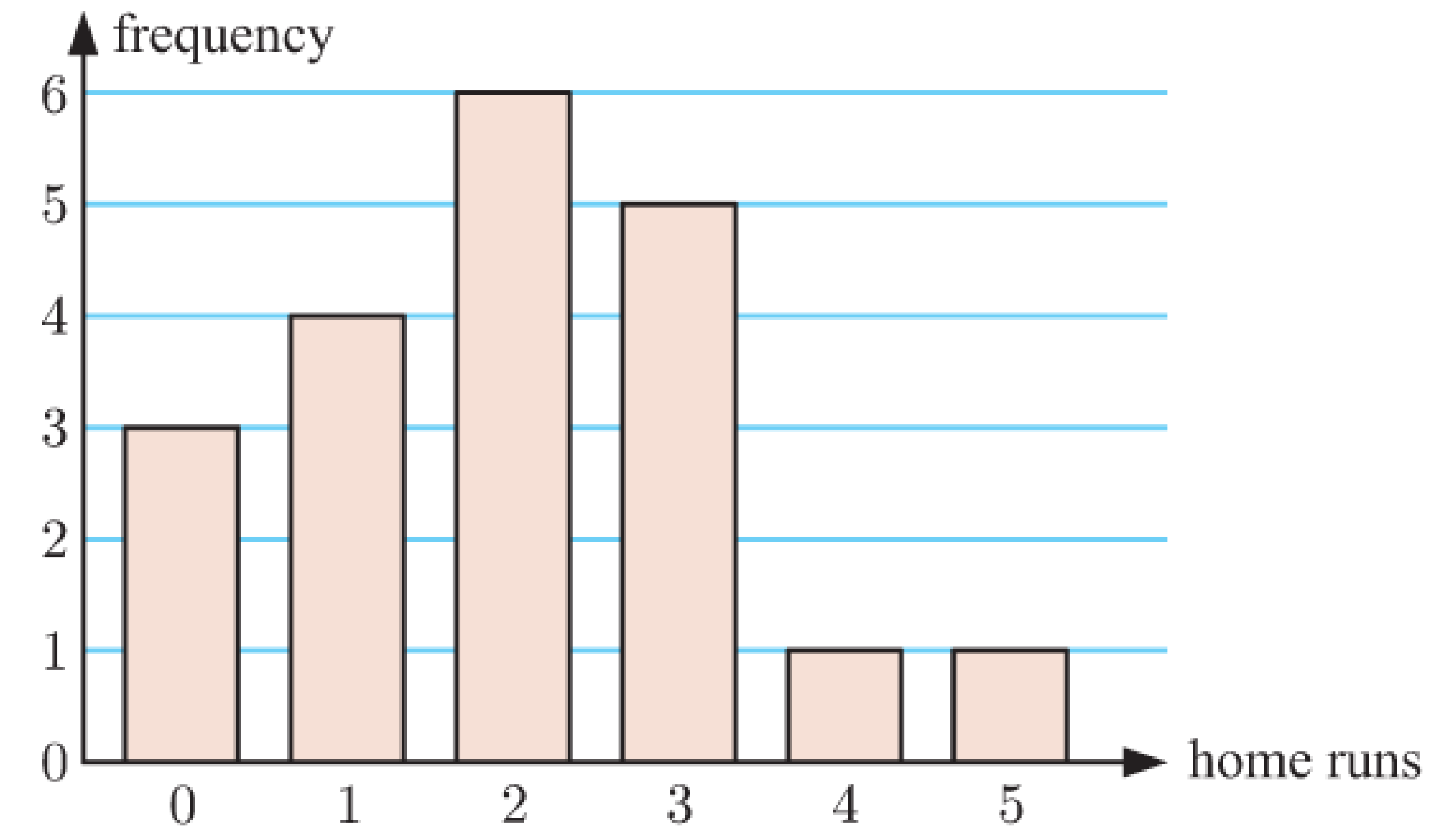


b 5 laps **c** 3 athletes **d** 18 athletes

5 **a**

Home runs	*Tally*	*Frequency*
0	|||	3
1	||||	4
2	𝍸 |	6
3	𝍸	5
4	|	1
5	|	1
	Total	20

b **Home runs scored by baseball team**



c 5 games **d** $\frac{3}{20}$ **e** 10%

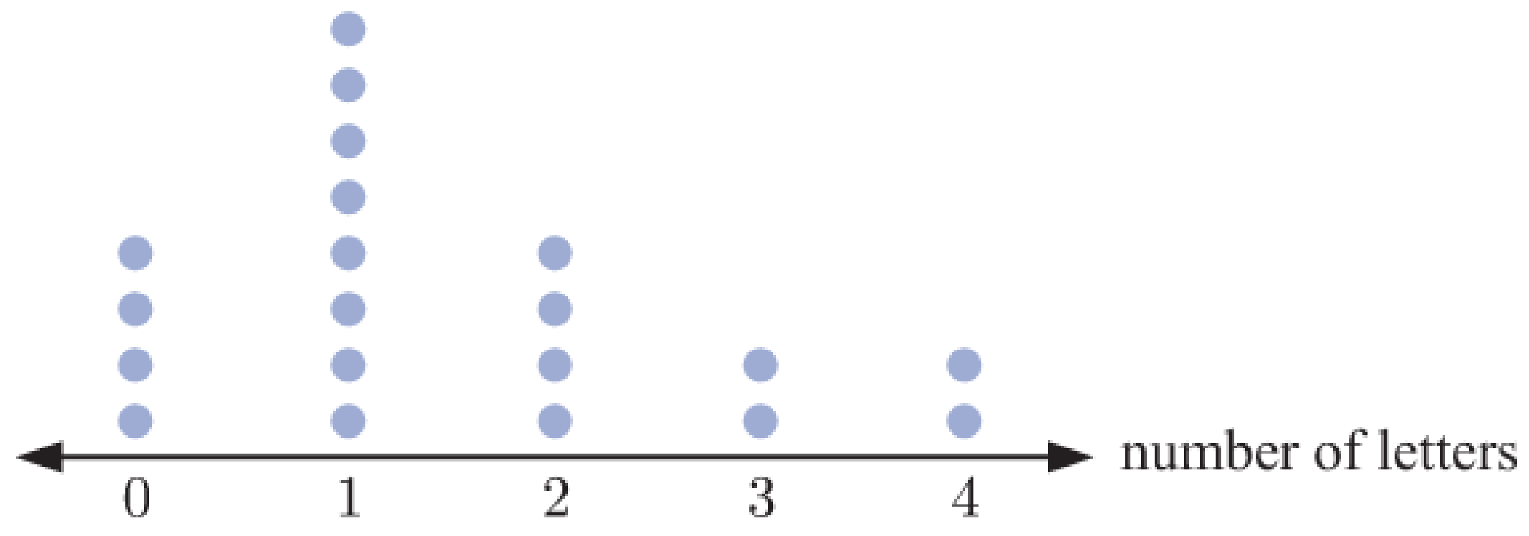
EXERCISE 15E

1 **a** 7 **b** 5 **c** 8 **d** 11 **e** 4 **f** 7.3

2 **a** 3 chocolates **b** 2 chocolates

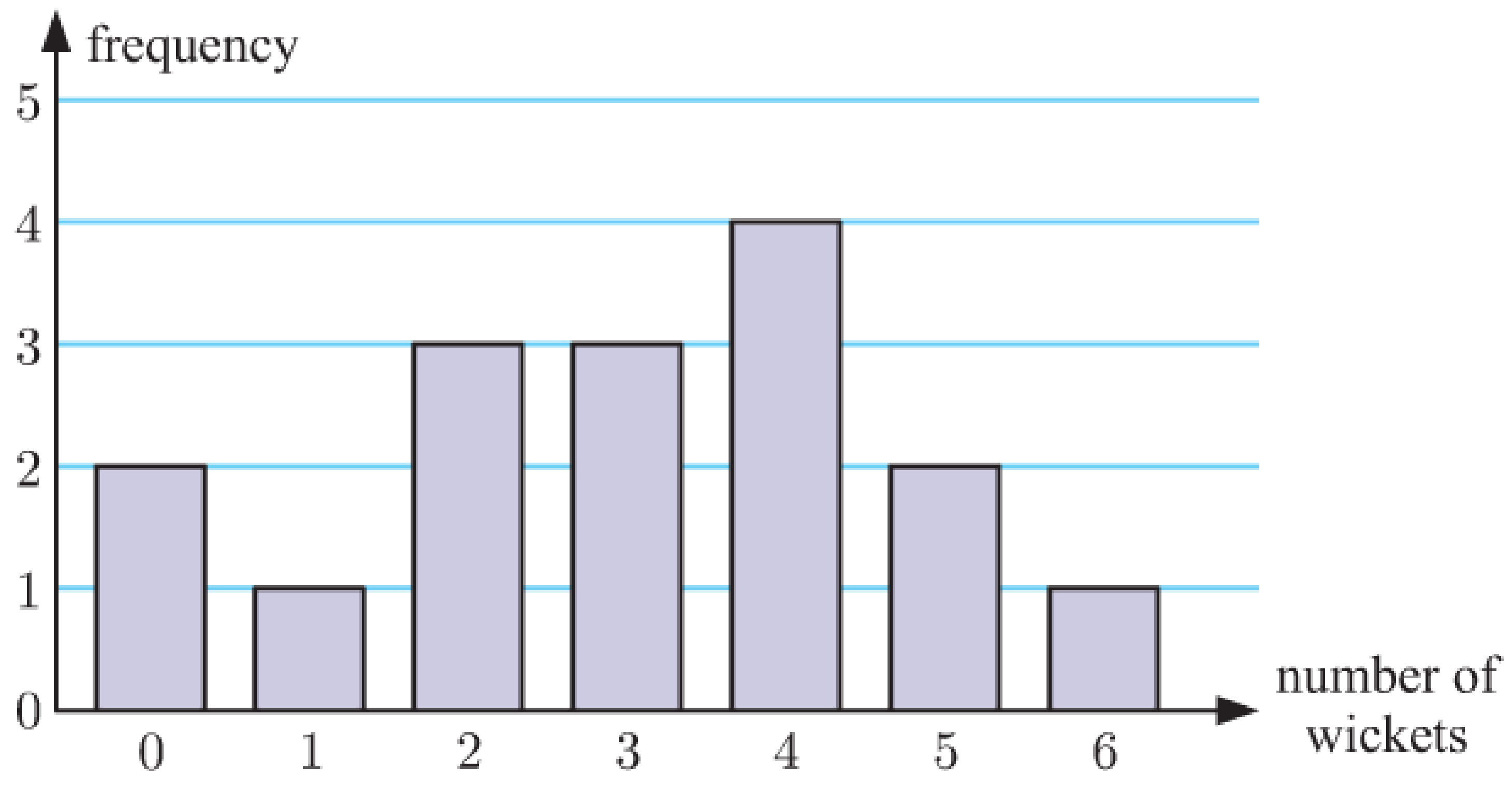
3 56 g **4** 113 m **5** 20 minutes

6 **a** **Number of letters delivered**



b 4 houses **c** 1 letter **d** 1.5 letters

7 **a** **Number of wickets taken**



b 4 wickets

c **i** 3 wickets

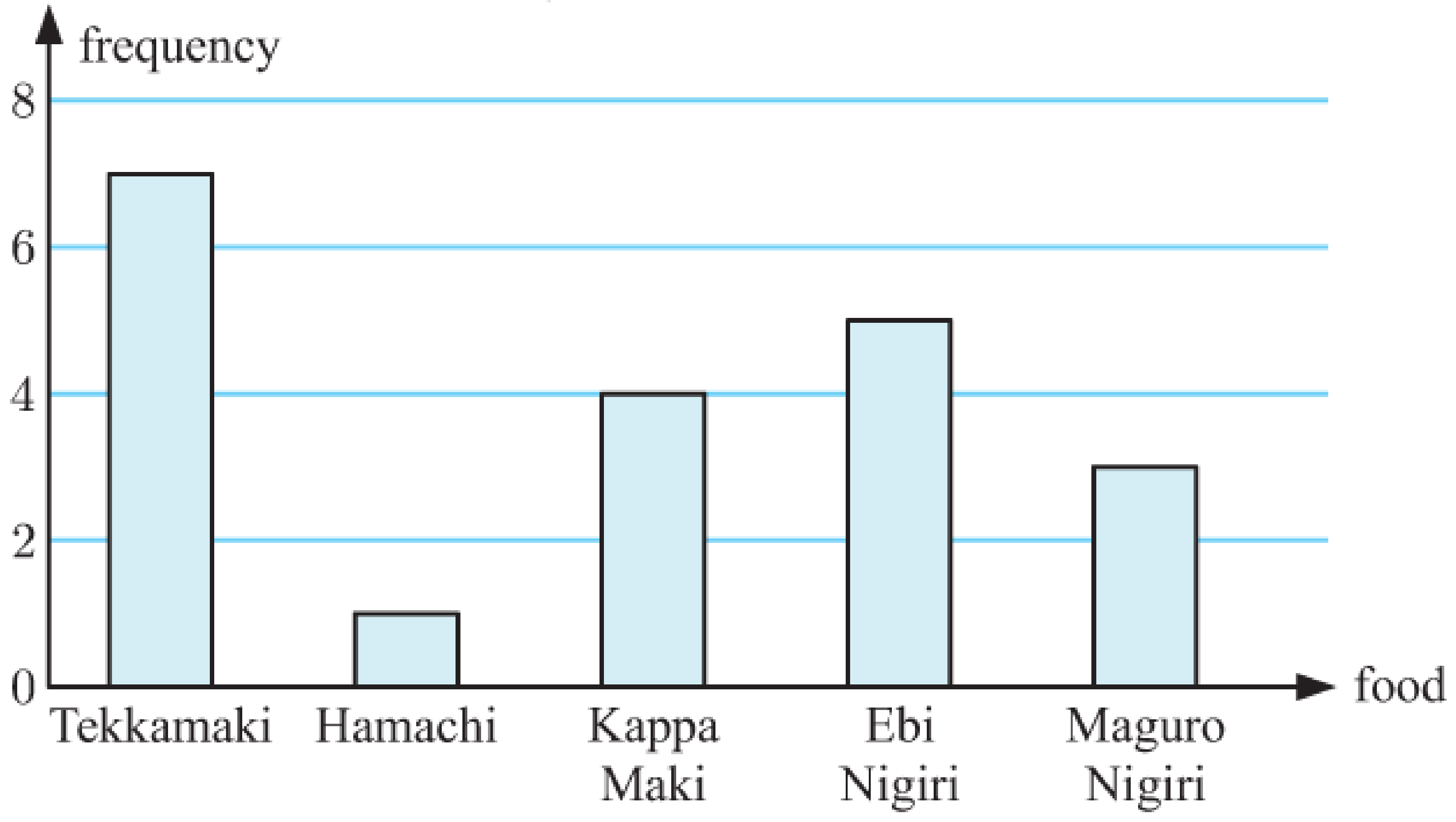
ii Yes, Cameron took 3 wickets in 3 games. The mean value is not always one of the data values but in this case it is one of the values.

REVIEW SET 15A

1 **a** Pick a house at random from every street in the suburb.

b Draw a name at random from a bag containing each name on a separate piece of paper.

2 **Sushi orders**



3 **a** 50 students **b** 12 students **c** size 7

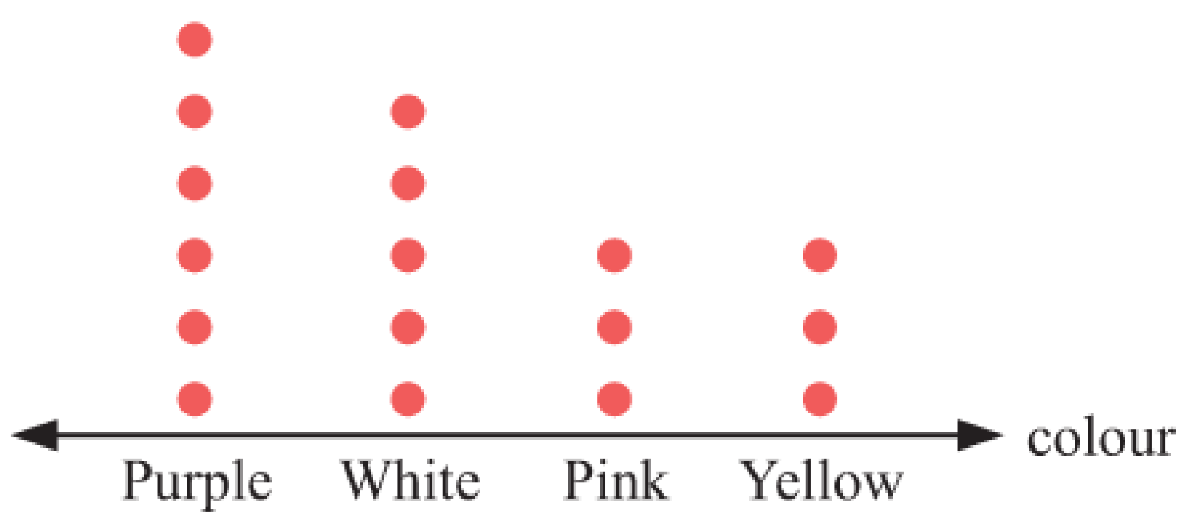
4 21 minutes **5** **a** 7 points **b** 10 occasions

6 **a**

Flower colour	*Tally*	*Frequency*
Purple	𝍸 |	6
White	𝍸	5
Pink	|||	3
Yellow	|||	3
	Total	17

b purple

c **Flower colours**



7 **a** It is easier to compare the actual amounts spent on each area.

b **i** 8% **ii** 28% **c** Environment

d Water & Sewerage **e** 18°

8 18.7 hours

REVIEW SET 15B

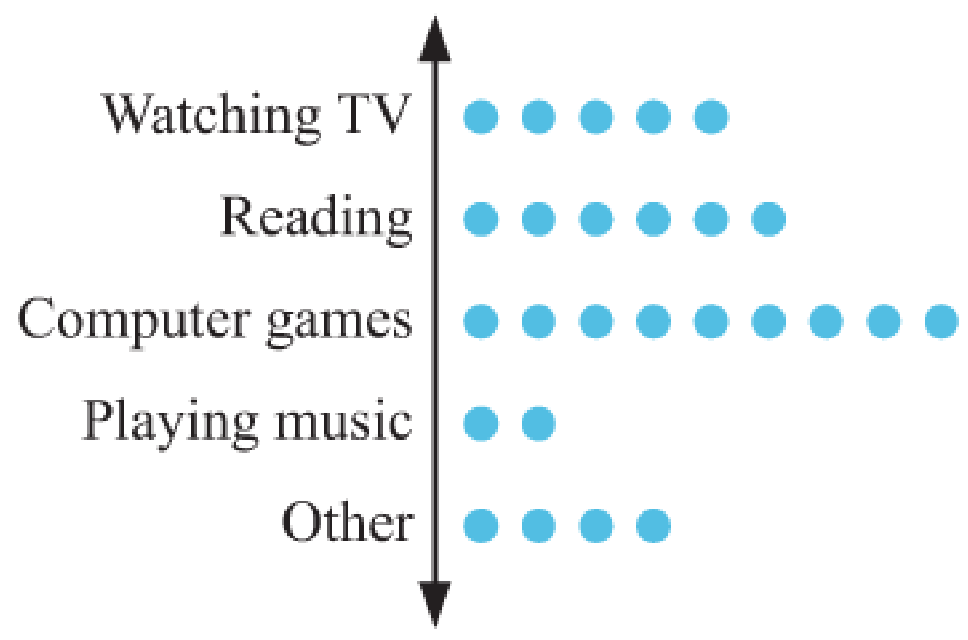
1 **a** 5000 trees **b** 200 trees **c** 12% **d** 600 trees

2 **a**

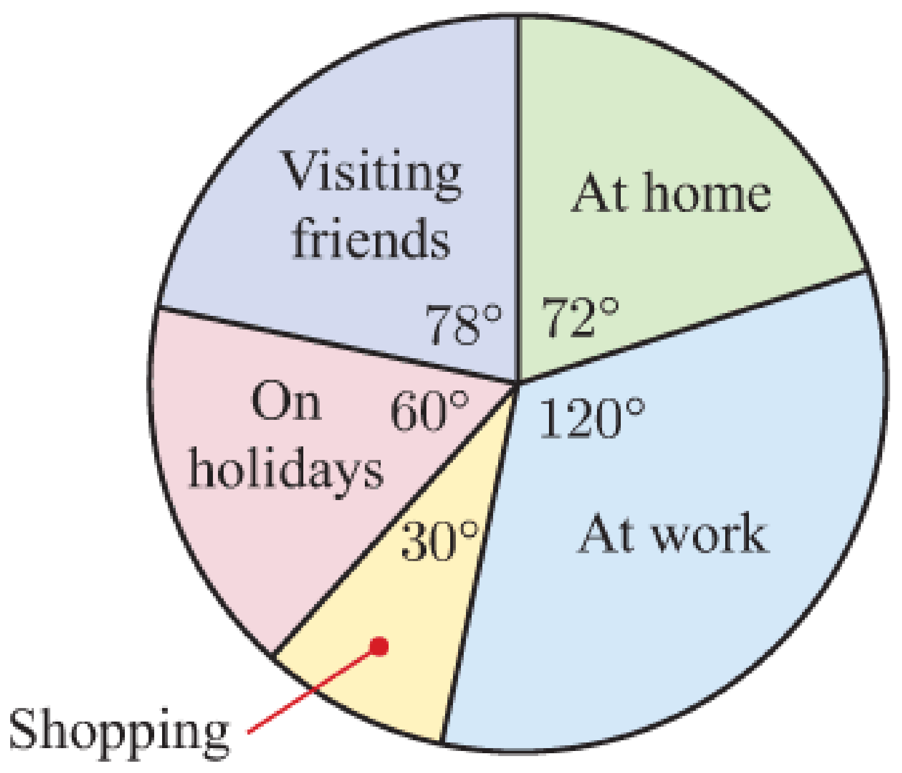
Species	*Tally*	*Frequency*
Magpie (M)	𝍸 |||	8
Sparrow (S)	𝍸 𝍸 𝍸 𝍸 𝍸	25
Kookaburra (K)	||||	4
Wren (W)	||||	4
Galah (G)	𝍸 ||||	9
	Total	50

b sparrow

3 **Rainy day pastime**

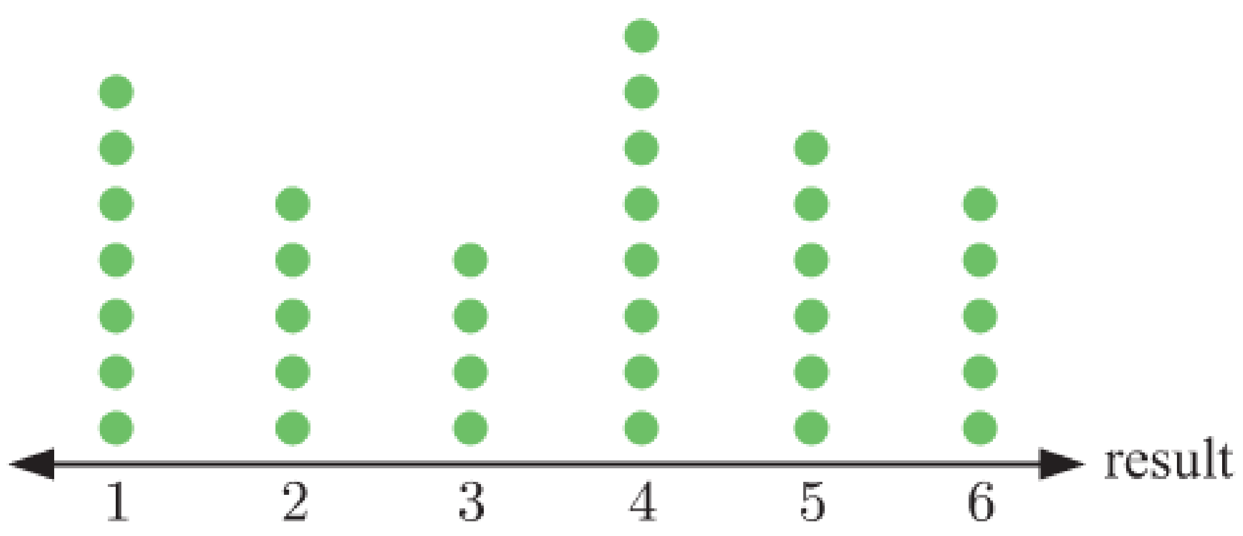


4 **Location at time of burglary**



5 **a** 5 phone calls **b** 6 days

6 **a** **Die roll results**



b 11 times

7 7.5 marks

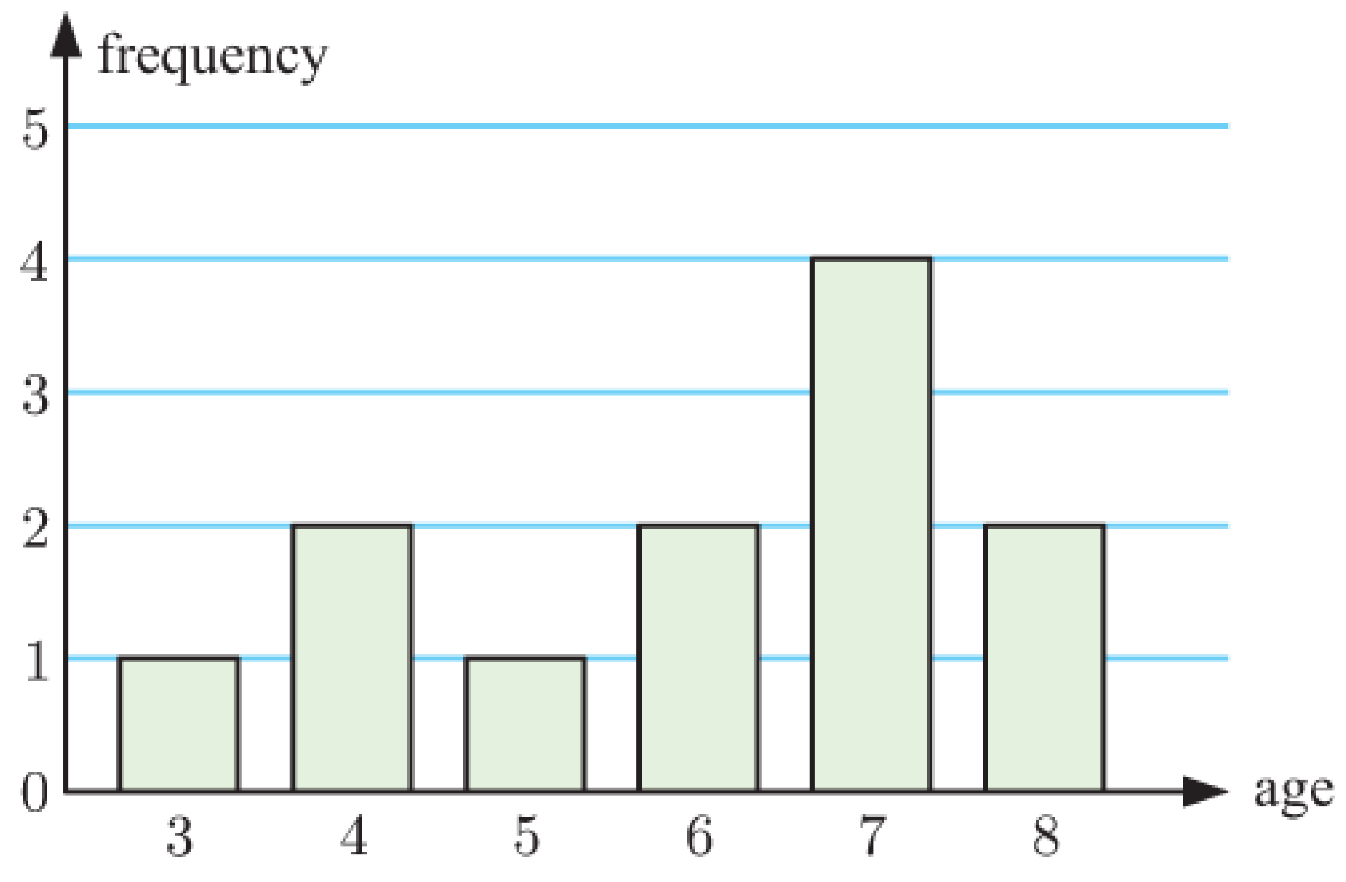
8 **a**

Age	*Tally*	*Frequency*
3	|	1
4	||	2
5	|	1
6	||	2
7	||||	4
8	||	2
	Total	12

b 12 children

c 6 children

d **Ages of children at party**



e 7 years **f** 6 years

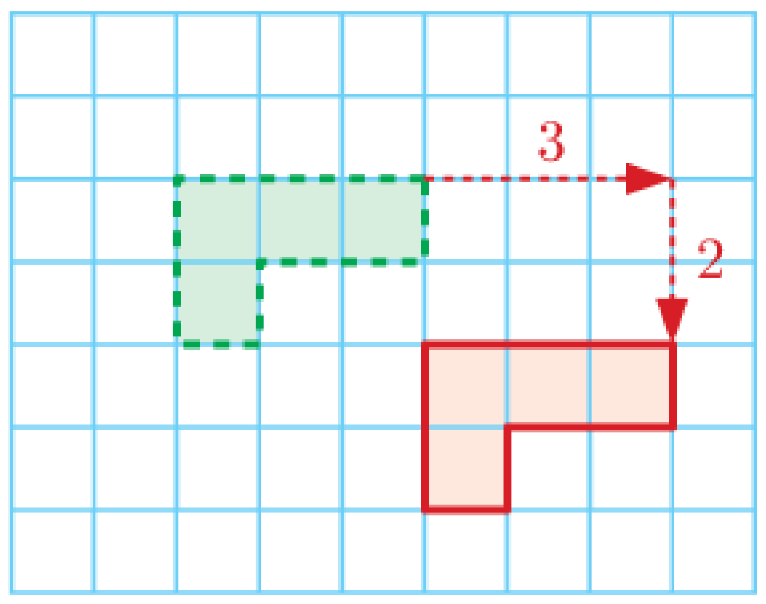
EXERCISE 16A

1 **a** 4 units right and 3 units up **b** 3 units left and 2 units up
c 4 units left and 1 unit down **d** 6 units right
e 5 units up **f** 4 units right and 4 units down

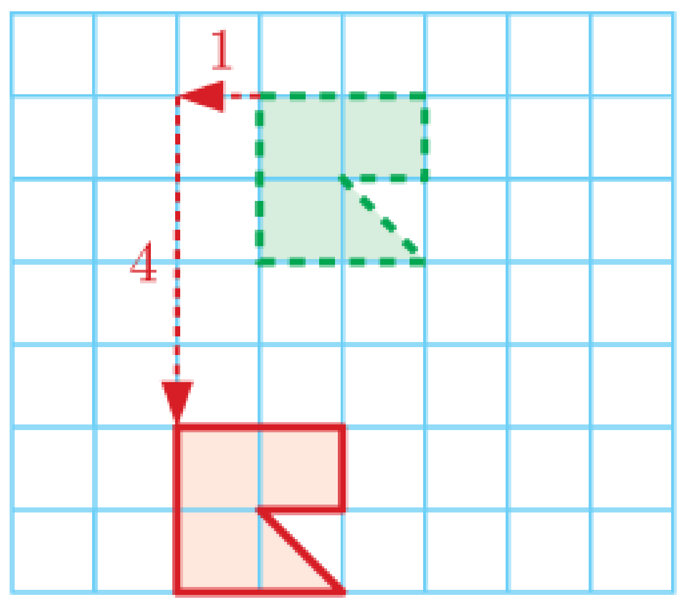
2 **a** 2 units left and 3 units up **b** 6 units right and 2 units up

3 **a** 3 units left and 2 units down
b 1 unit left and 4 units down
c 5 units left
d 2 units right and 2 units down
e 3 units right and 2 units up
f 2 units right and 2 units down

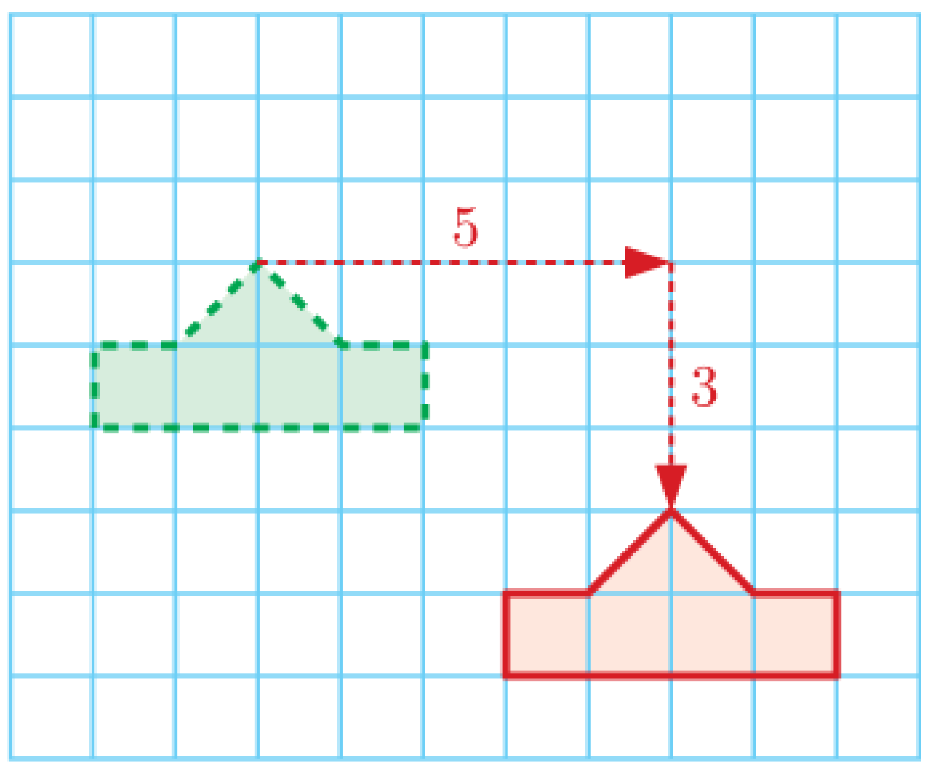
4 **a**



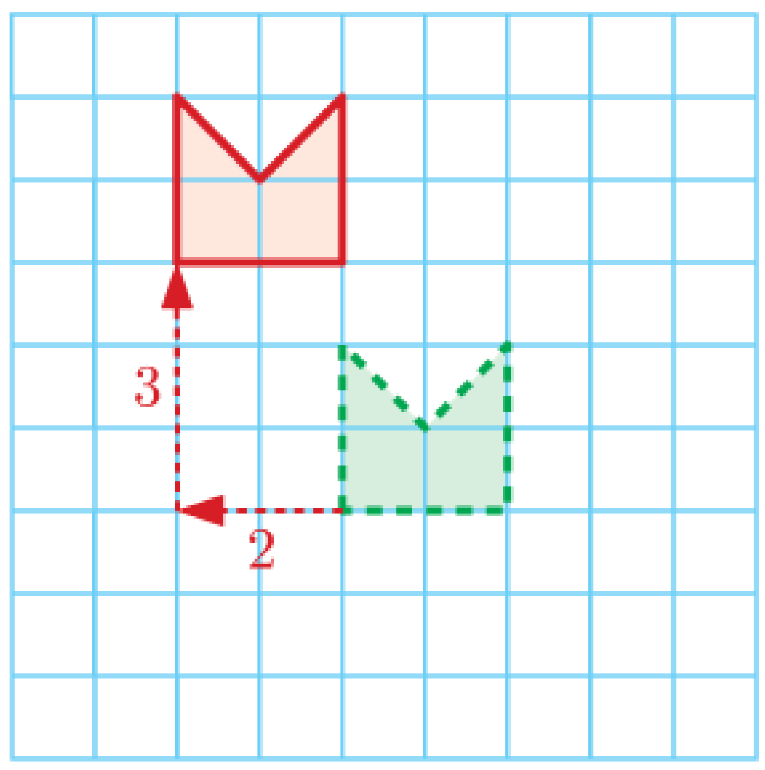
b



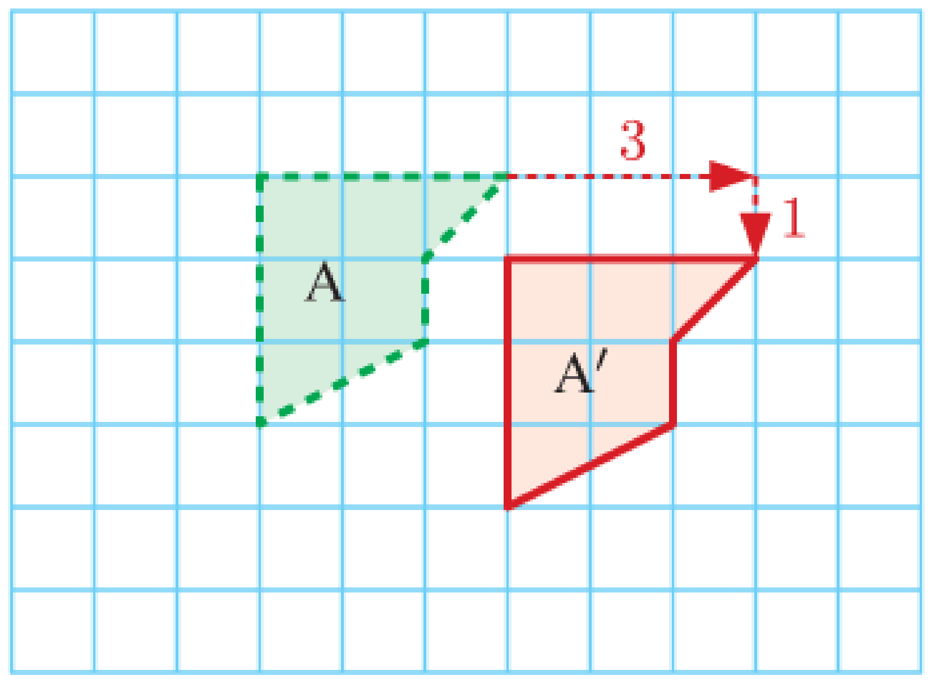
c



d



5 **a**

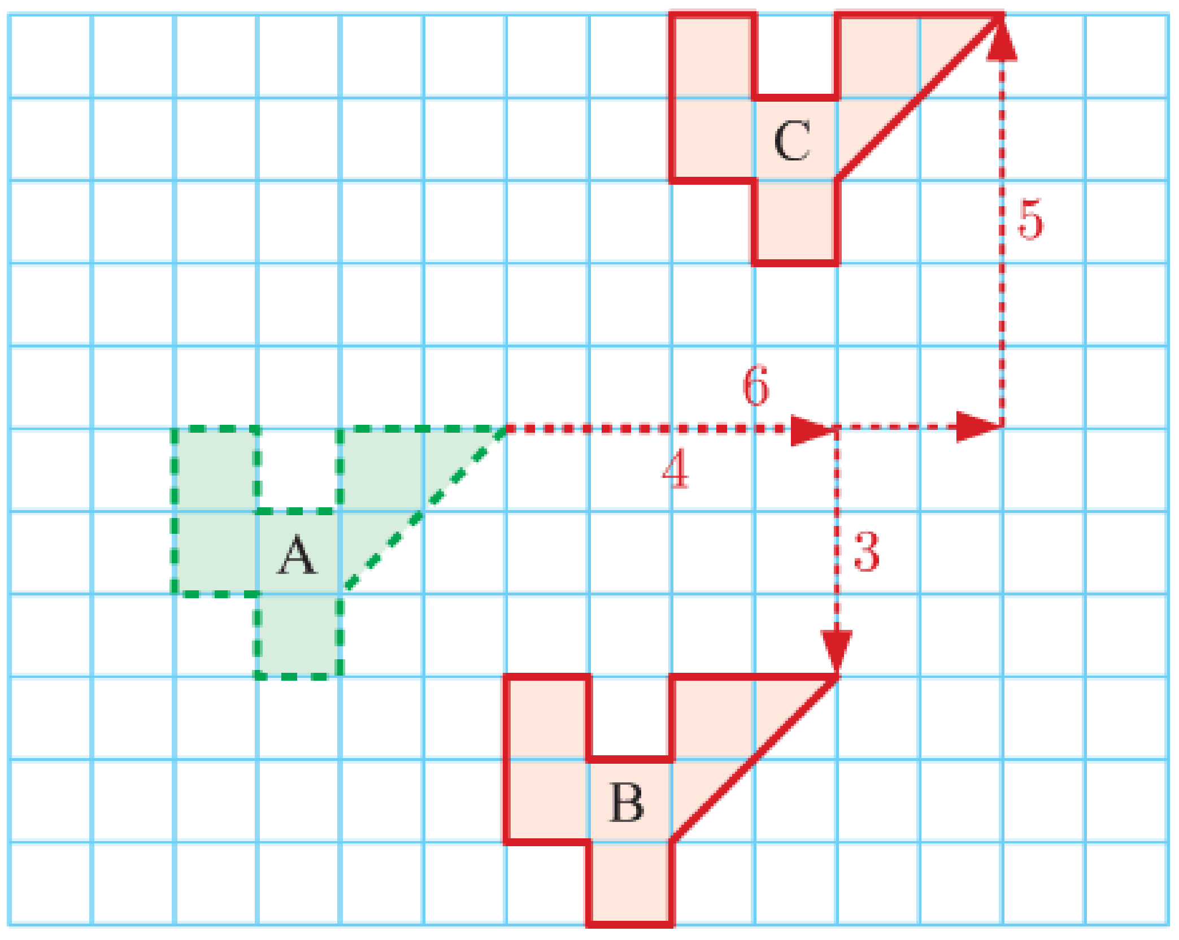


b 3 units left and 1 unit up

6 **a** 7 units right and 3 units up
b R is not an identical shape to P.
∴ R is not a translation of P.

7 **a** E **b** 3 units left and 4 units down

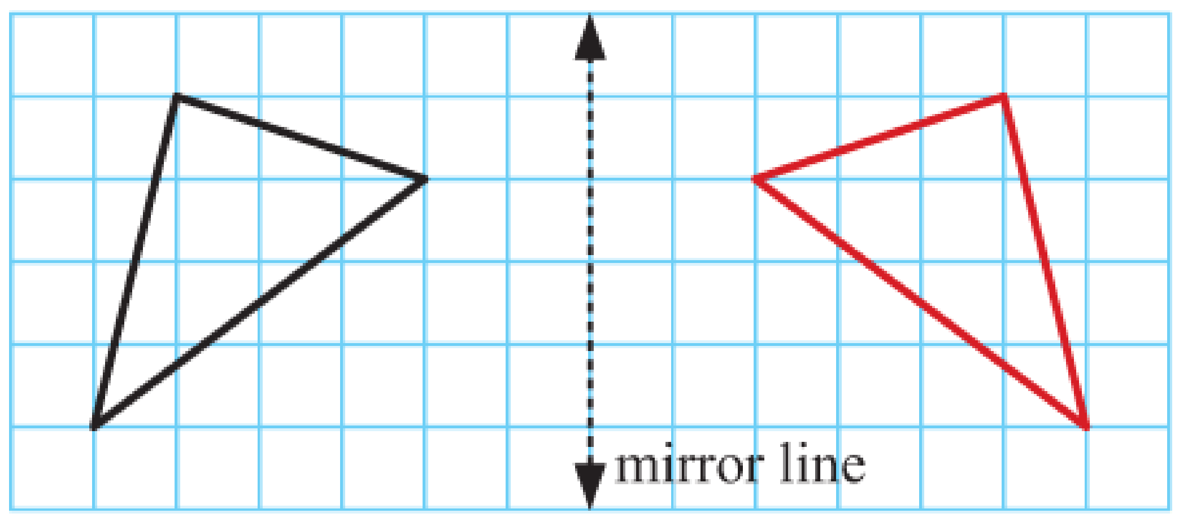
8 **a**, **b**



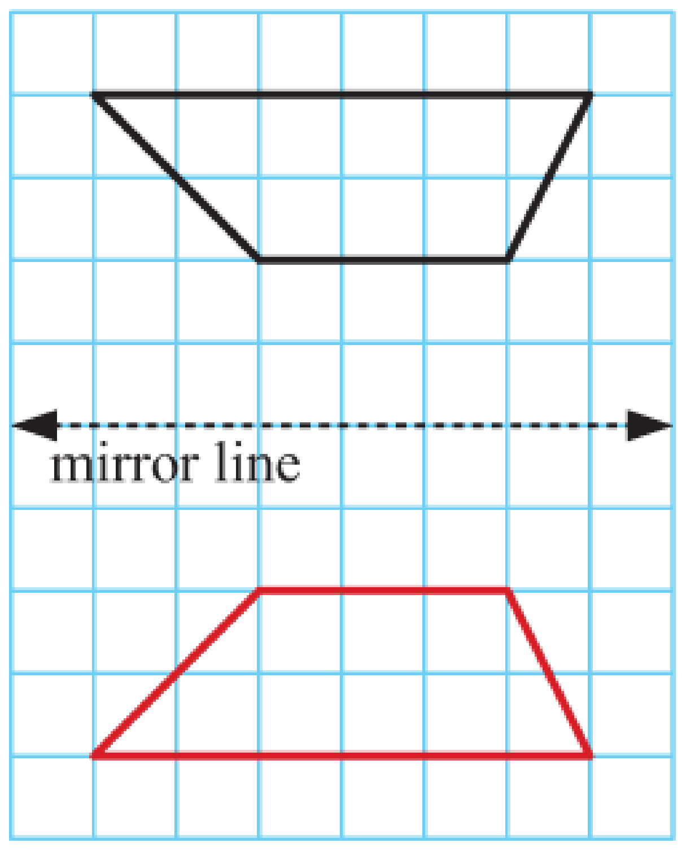
c 2 units right and 8 units up

EXERCISE 16B

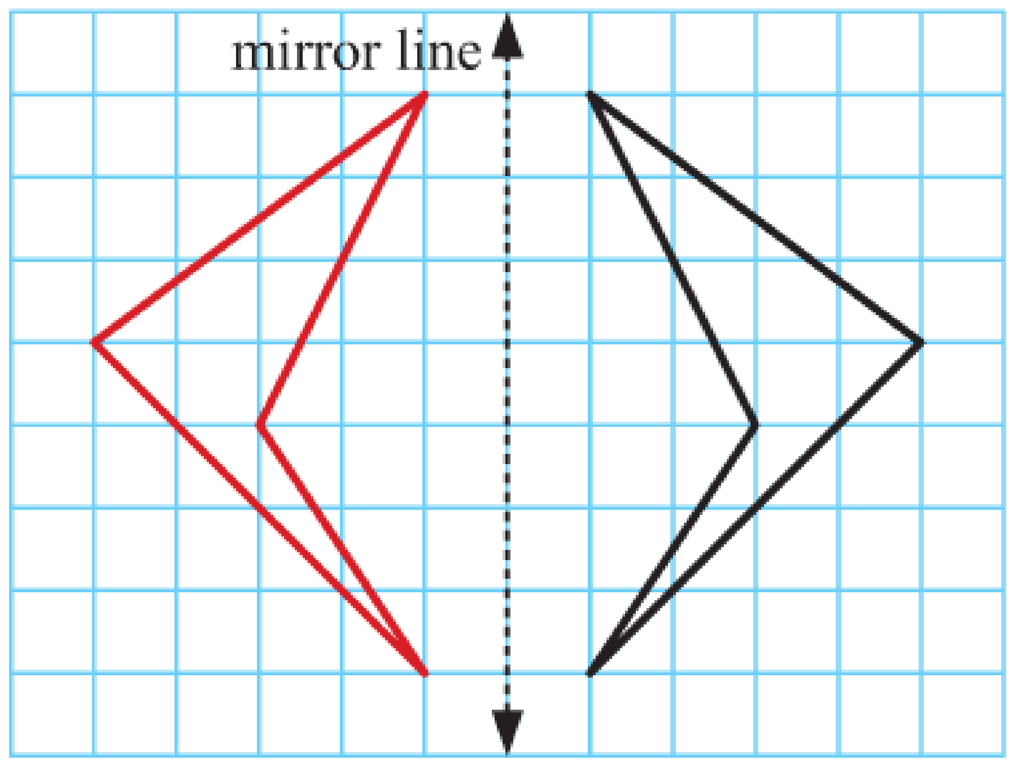
1 **a**



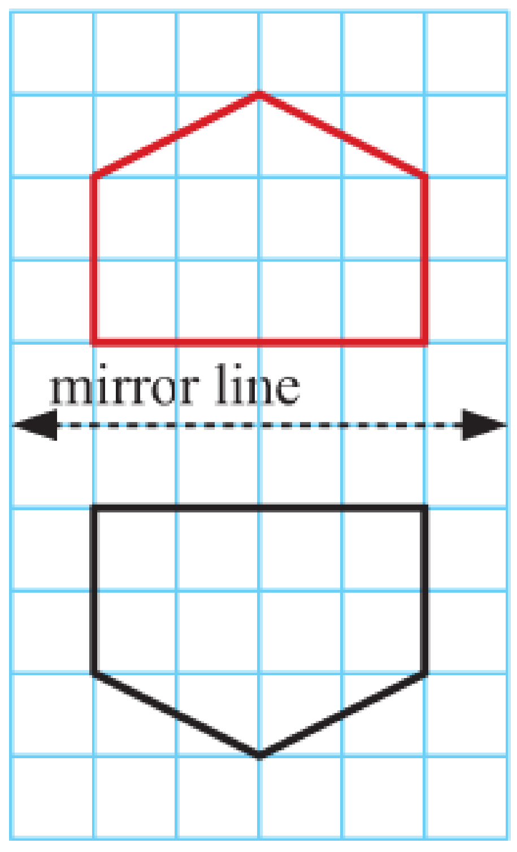
b



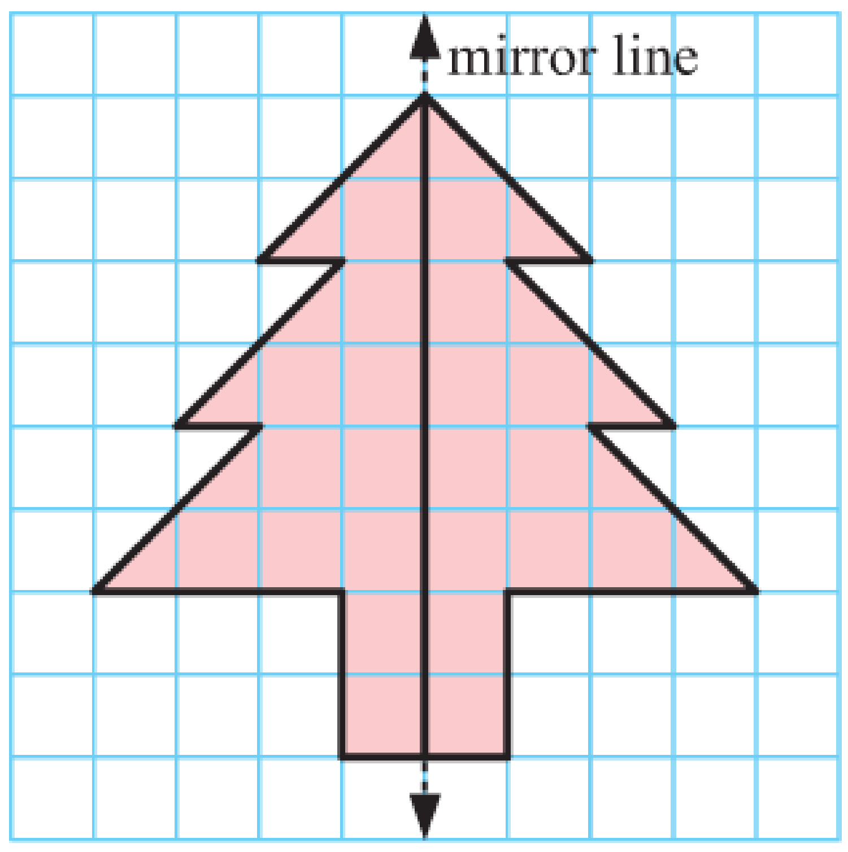
c



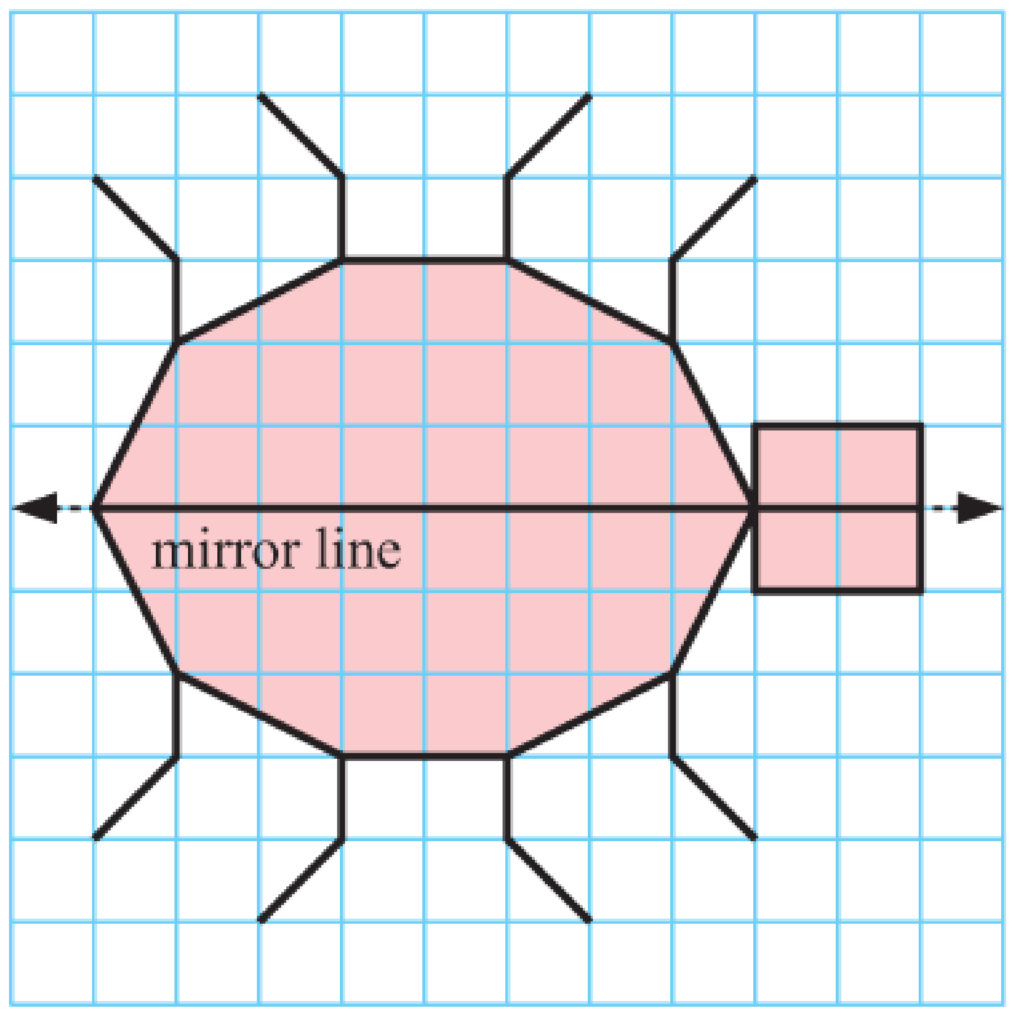
d



2 **a**



b



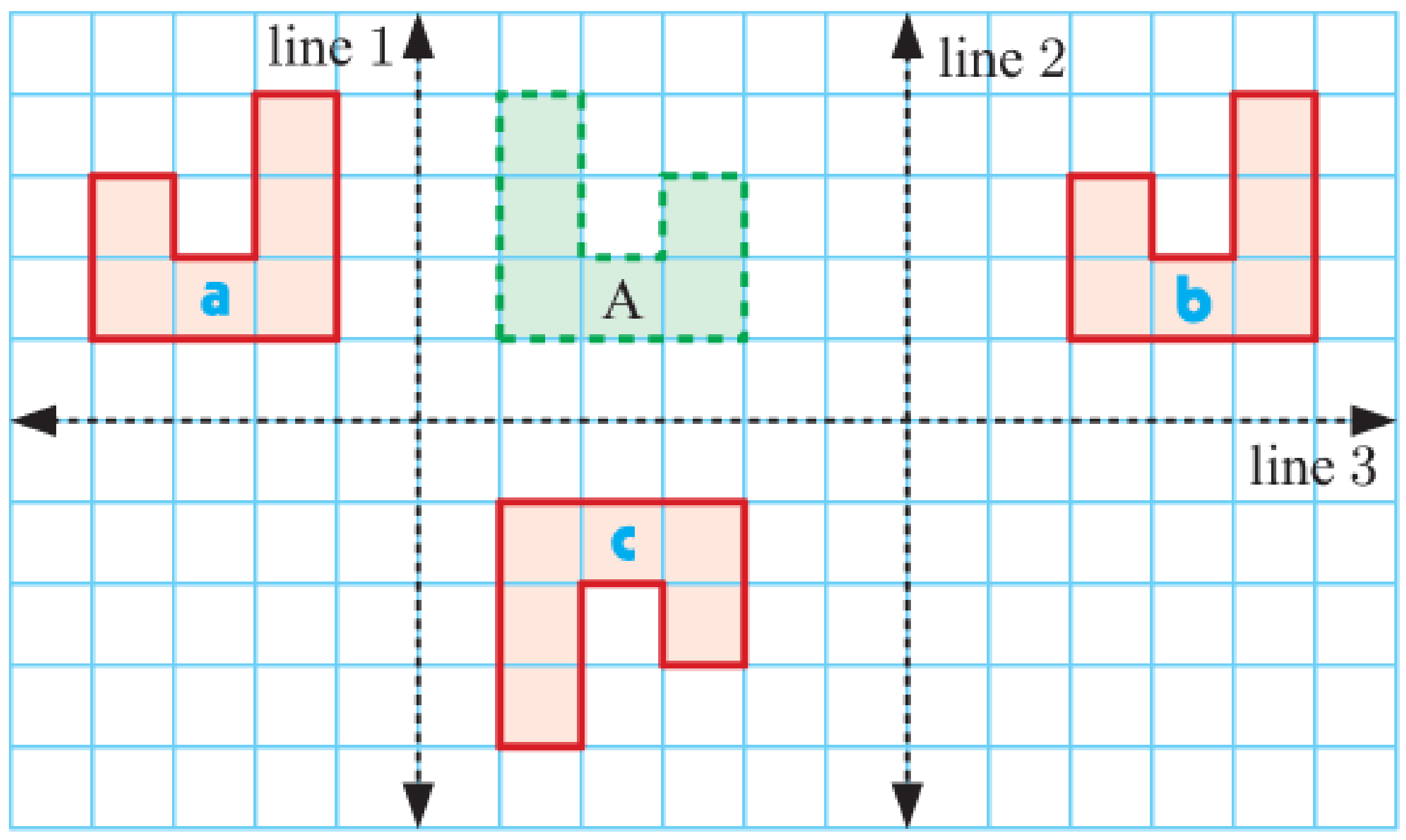
3 **a** yes

b No, as A and B do not have the same shape.

c No, B is a translation of A, but not a reflection.

d yes

4



5 F1 moves to H3

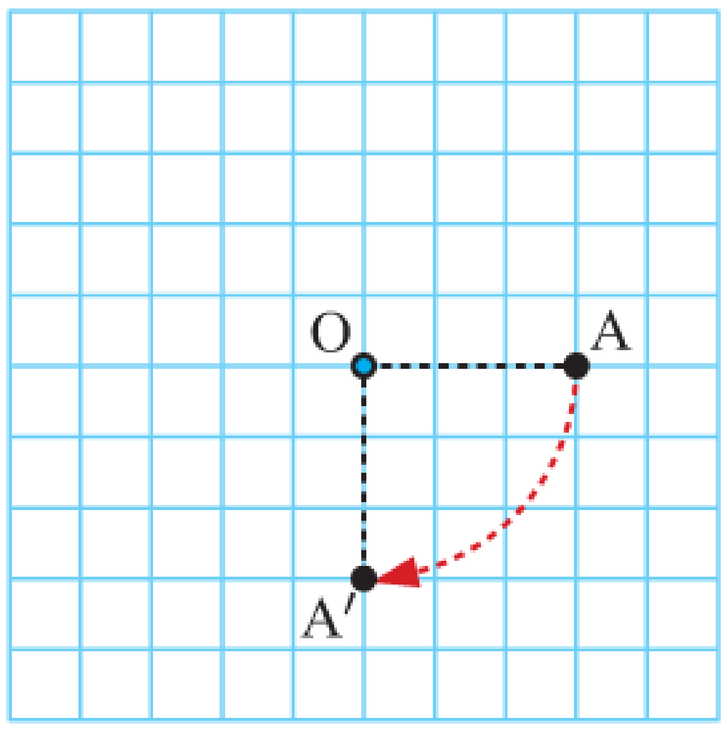
6 D and F, line 2

EXERCISE 16C

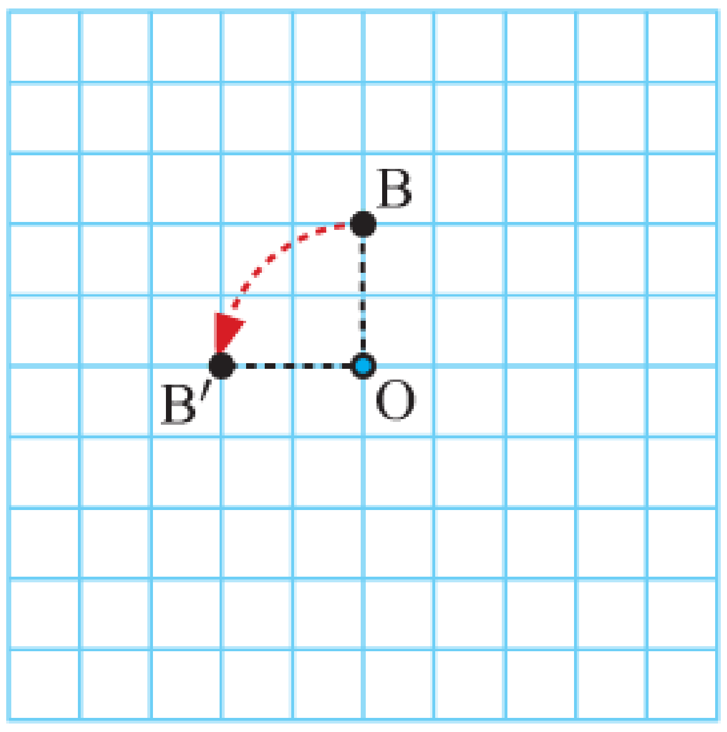
1 **a** 2 **b** 6 **c** 4 **d** 4

2 **a** Q **b** S **c** R

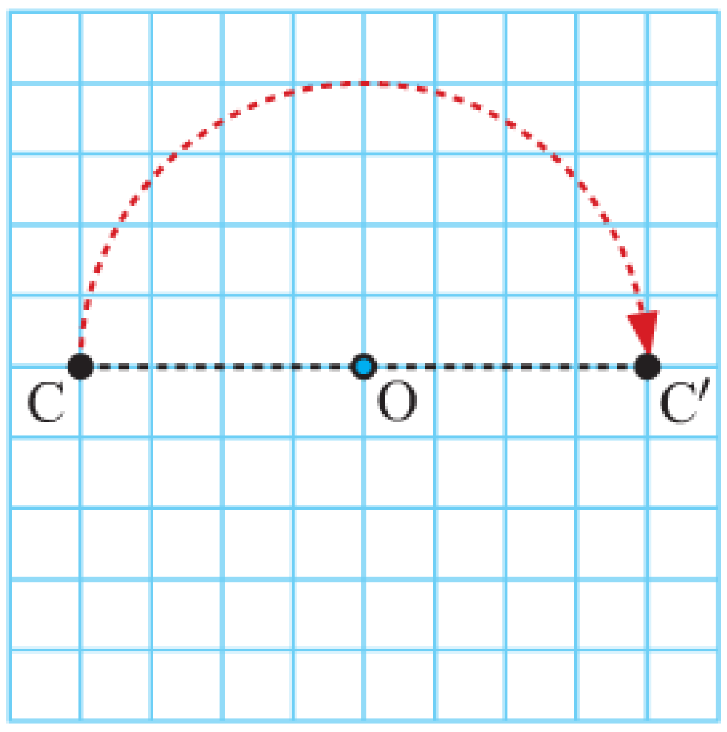
a



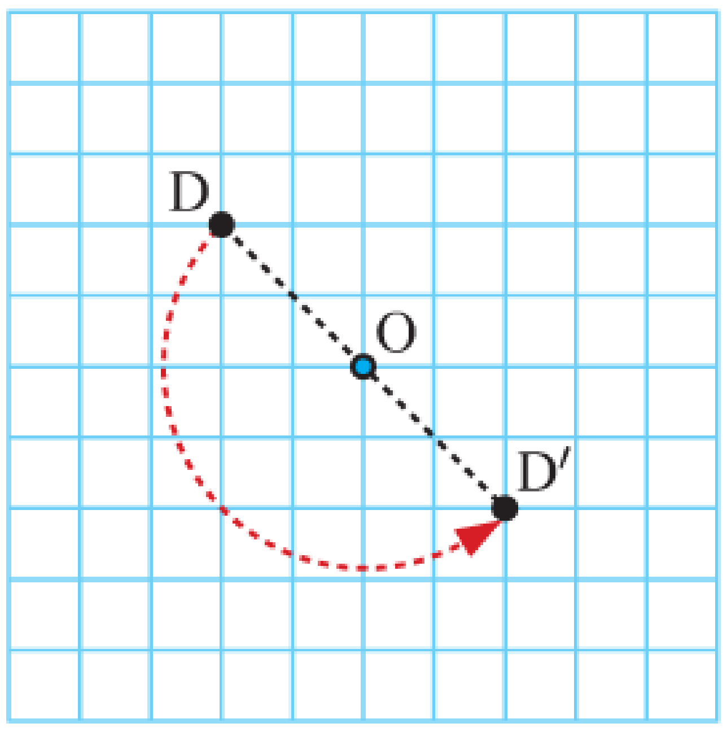
b



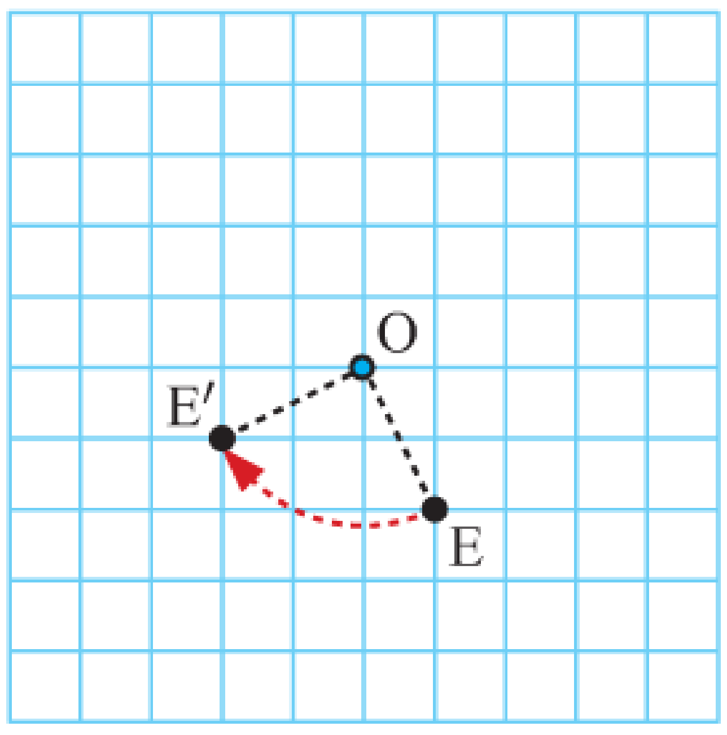
c



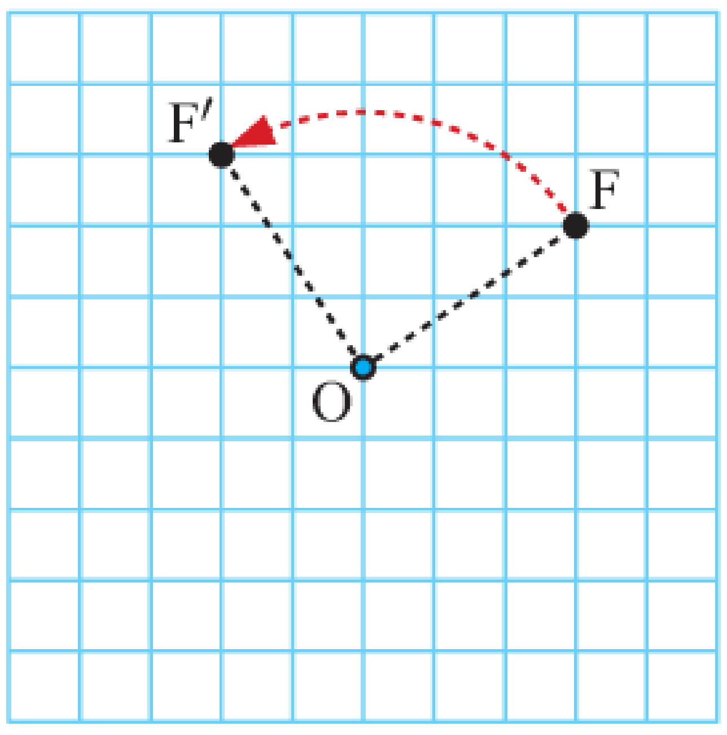
d



e



f

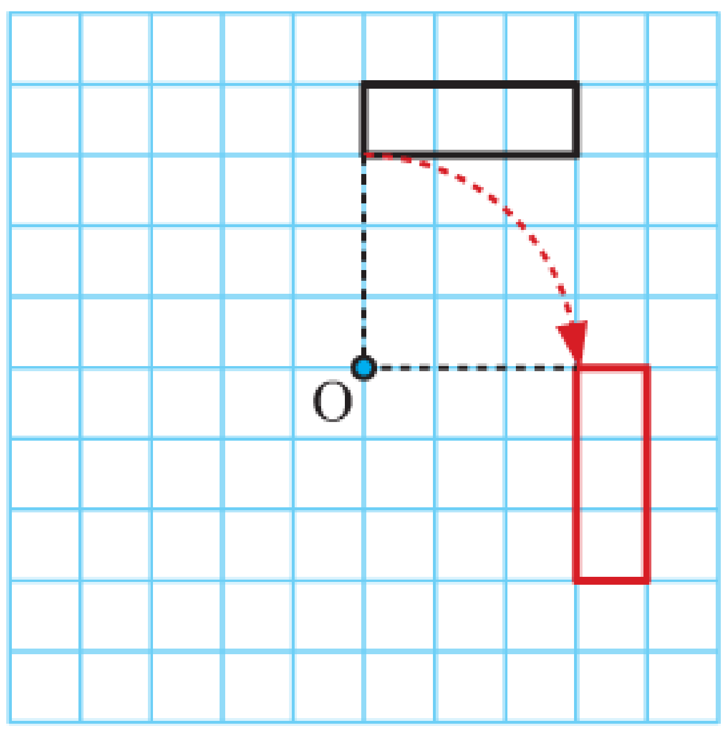


4 **a** 90° clockwise

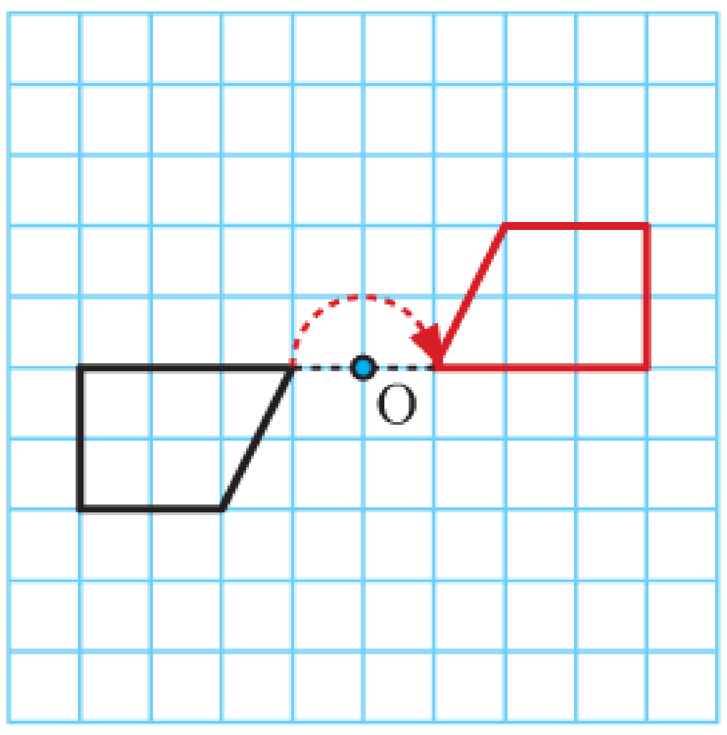
b 180° clockwise or 180° anticlockwise

c 90° anticlockwise

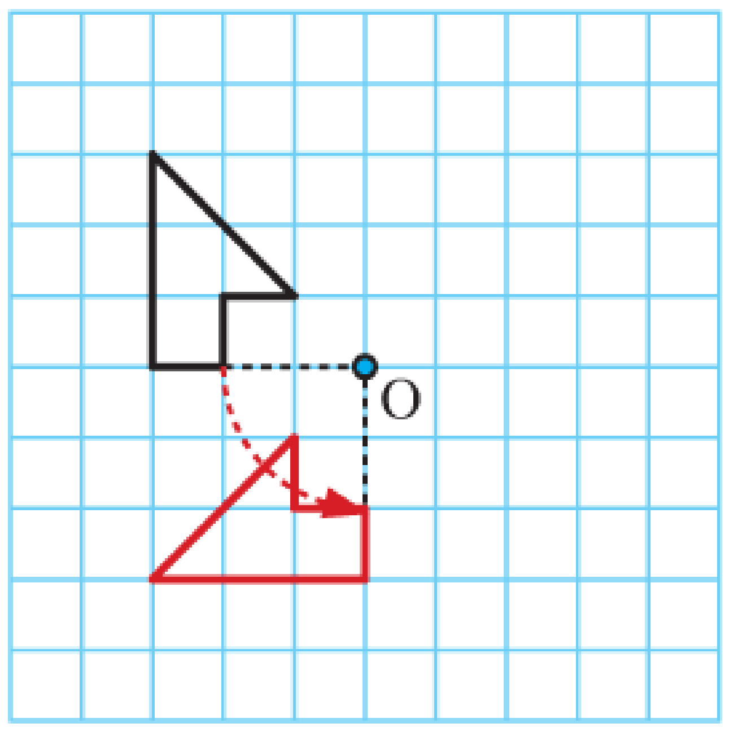
5 **a**



b



c

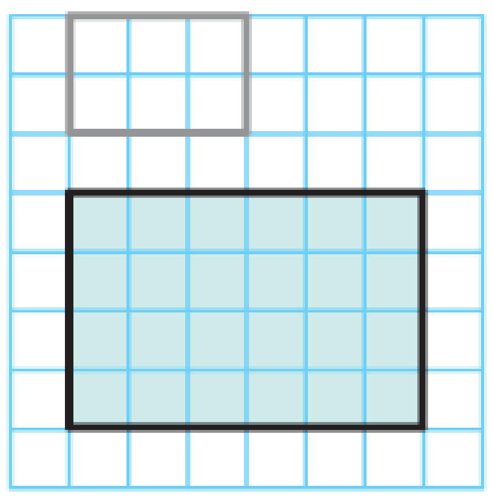


6 C

EXERCISE 16D

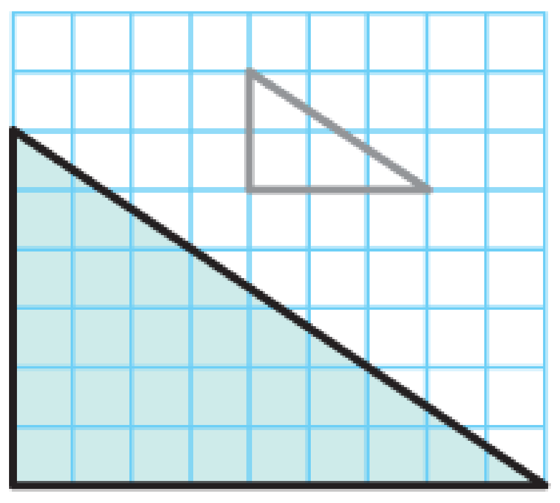
1 **a** 4 **b** 2 **c** 3 **2** **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

3 **a**



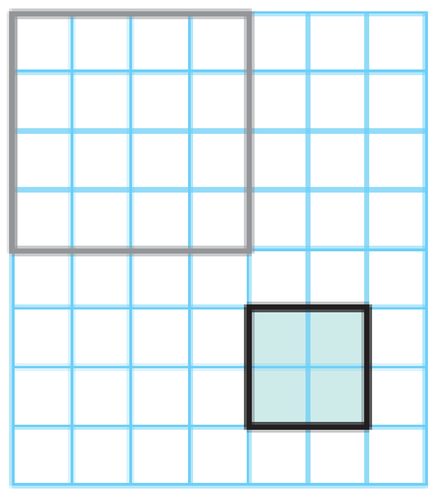
scale factor 2

b



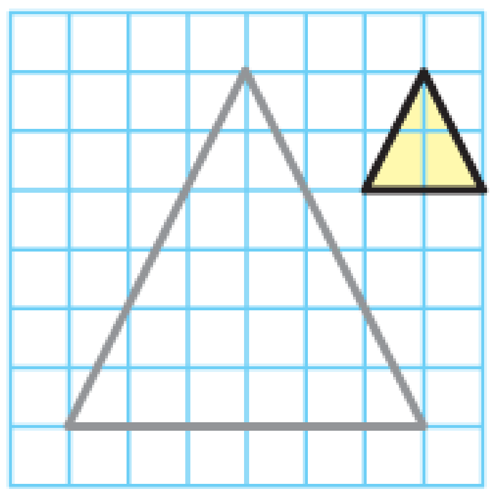
scale factor 3

c



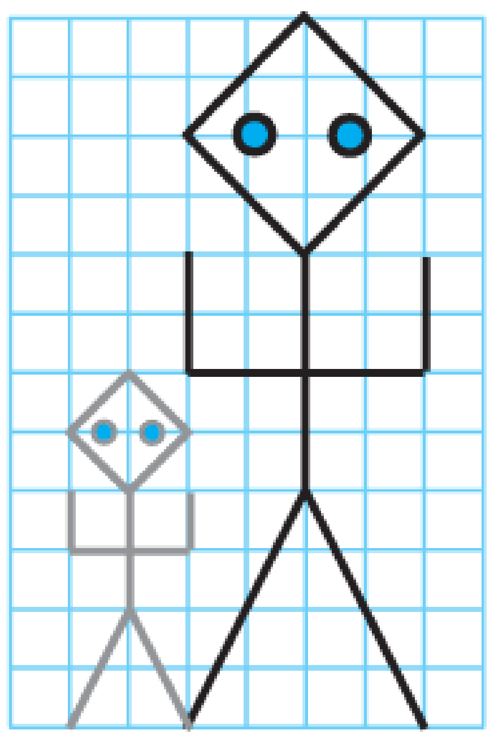
scale factor $\frac{1}{2}$

d



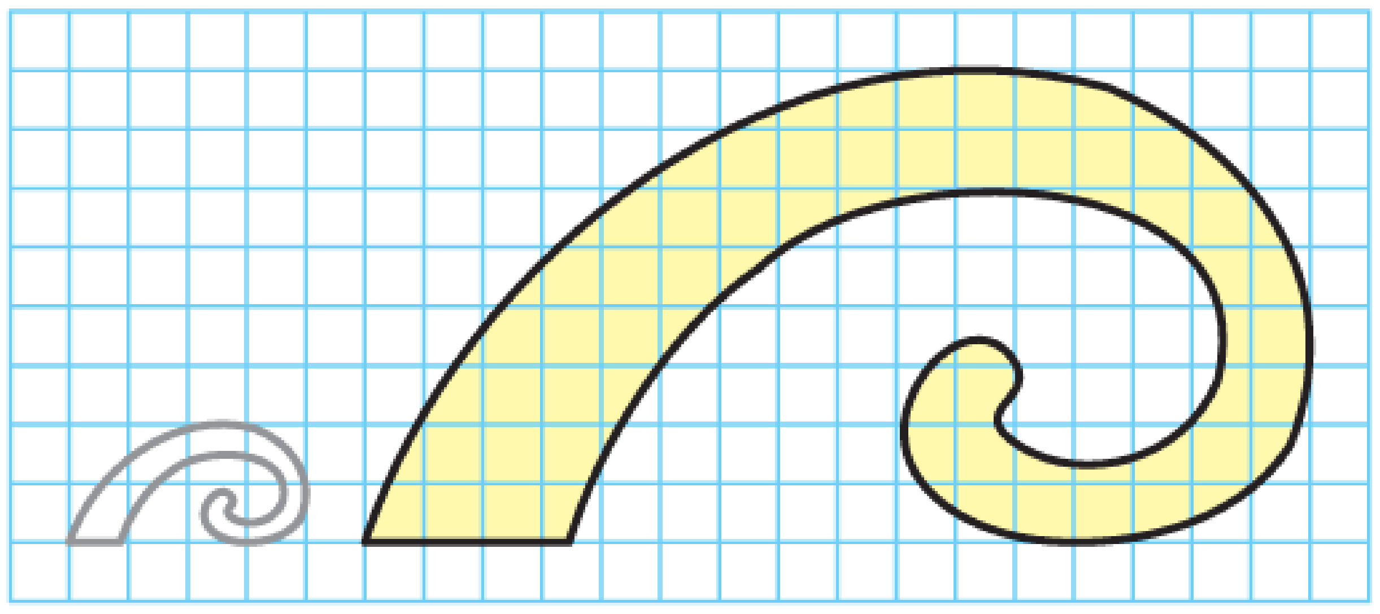
scale factor $\frac{1}{3}$

f



scale factor 2

e

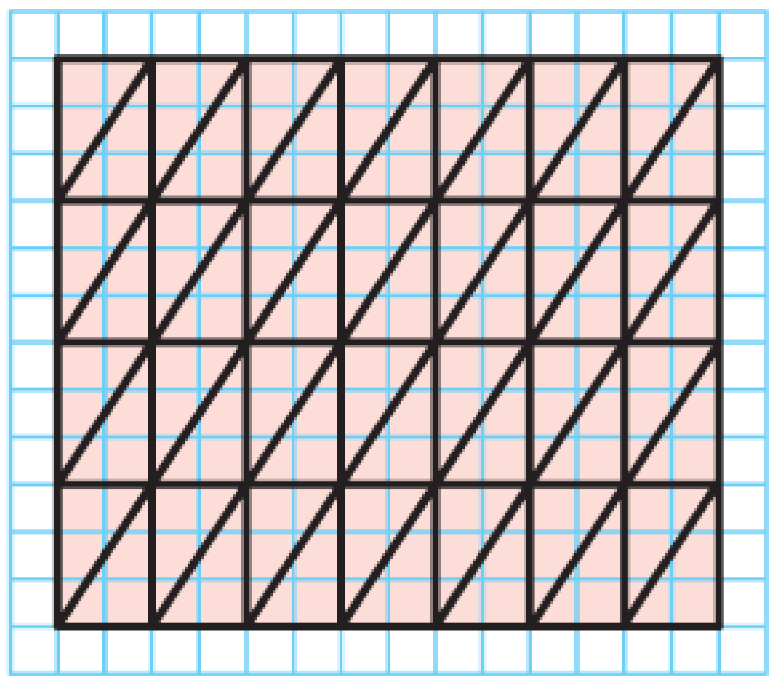


scale factor 4

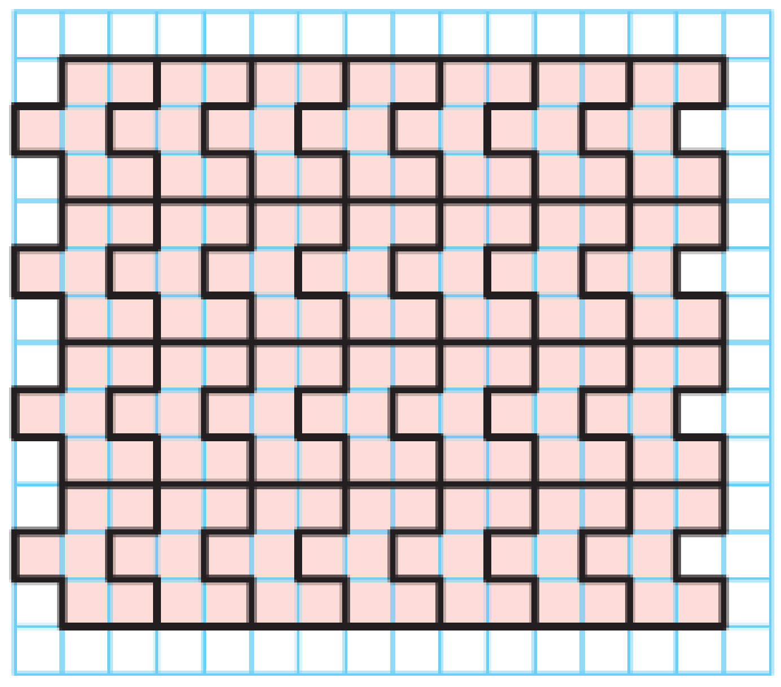
4 **a** **i** 3 **ii** 2 **iii** $\frac{1}{2}$ **b** **i** $\frac{1}{3}$ **ii** $\frac{1}{2}$ **iii** 2

EXERCISE 16E

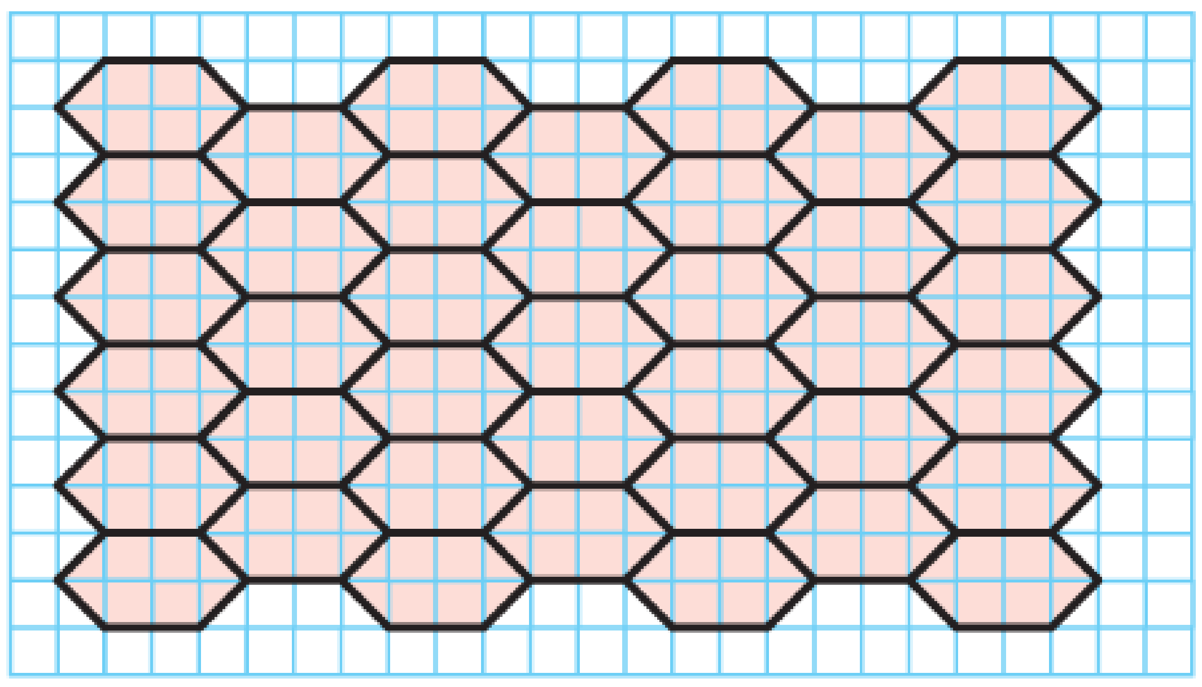
1 **a**



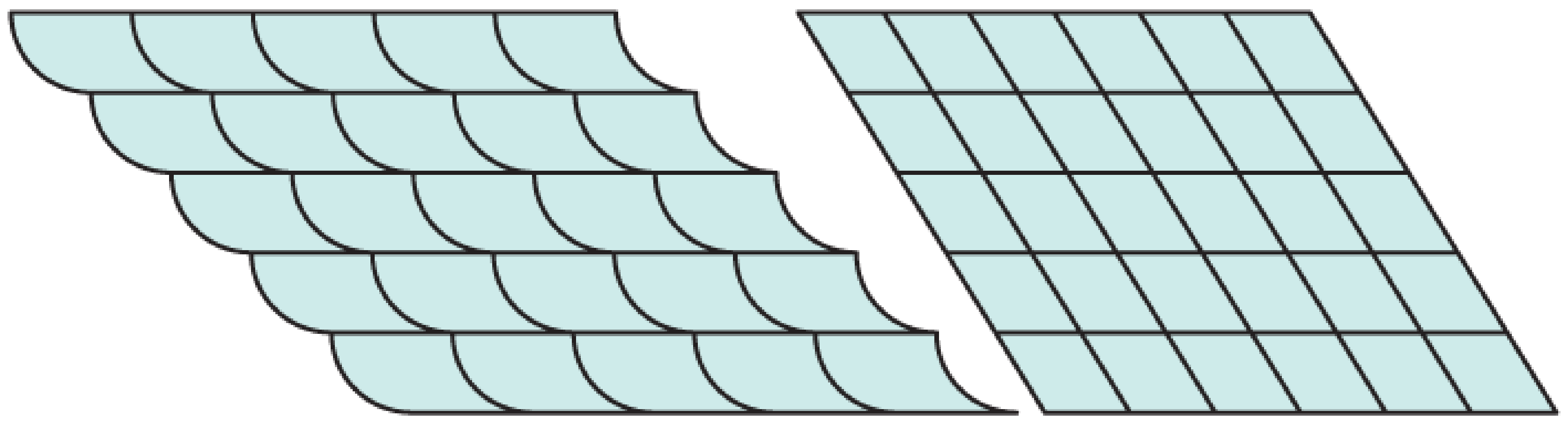
b



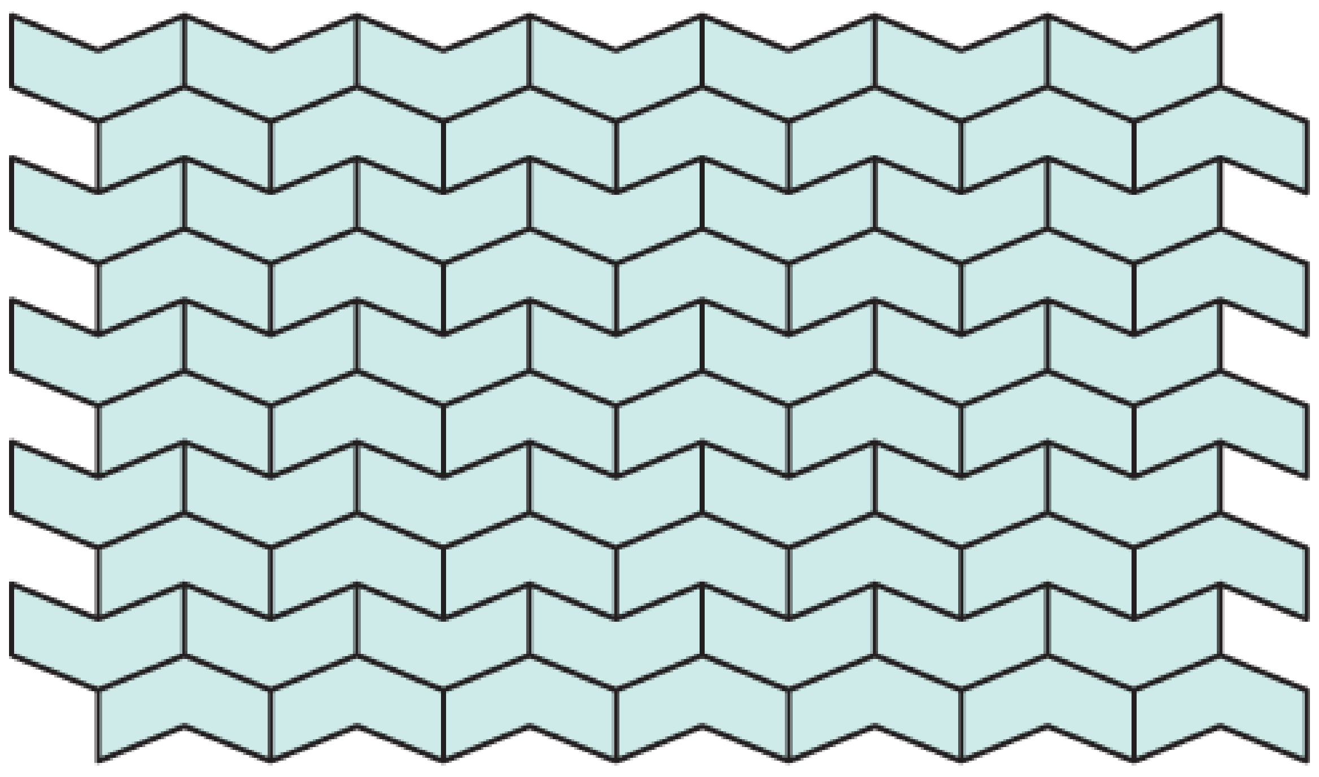
c



2 **a** **b**



c



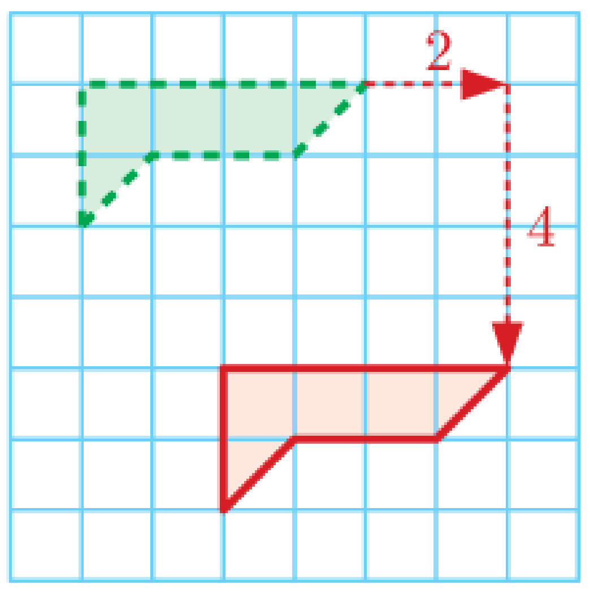
REVIEW SET 16A

1 **a** 4 units right and 5 units up
b 4 units left and 2 units down **c** 1 unit left and 6 units up

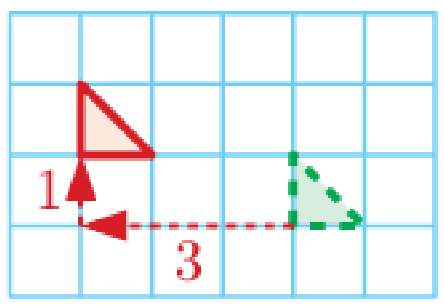
2 **a** yes **b** No, Q is a translation of P, but not a reflection.

3 **a** S **b** Q **c** R

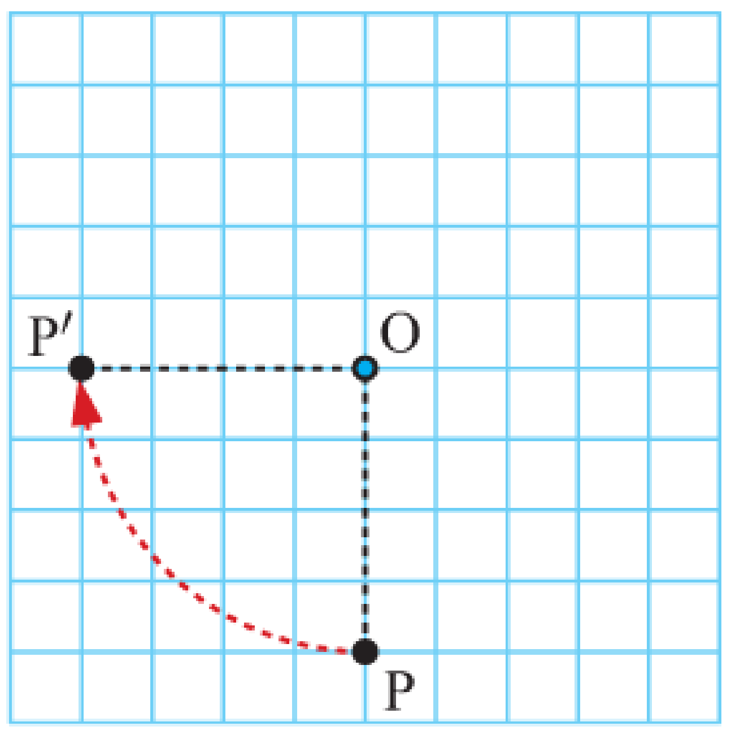
4 **a**



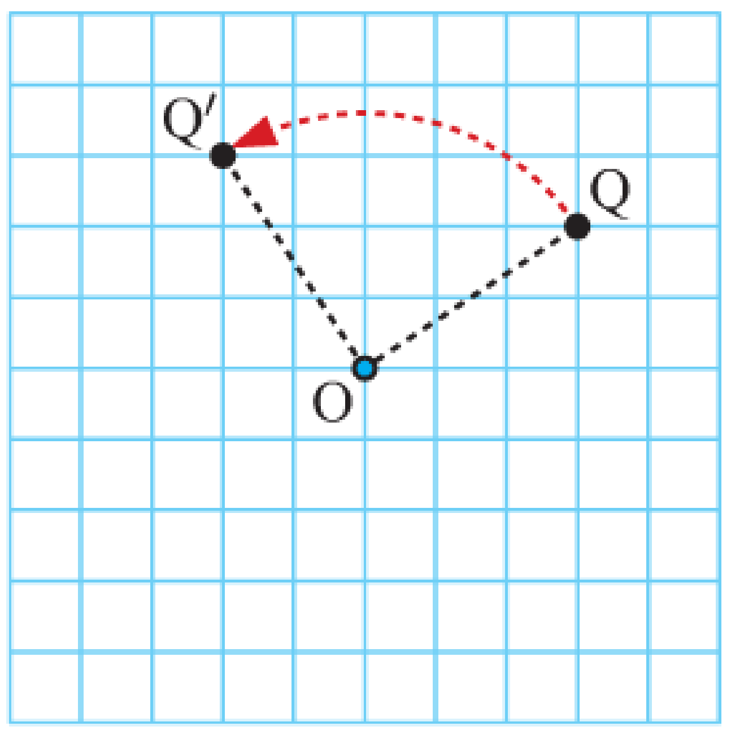
b



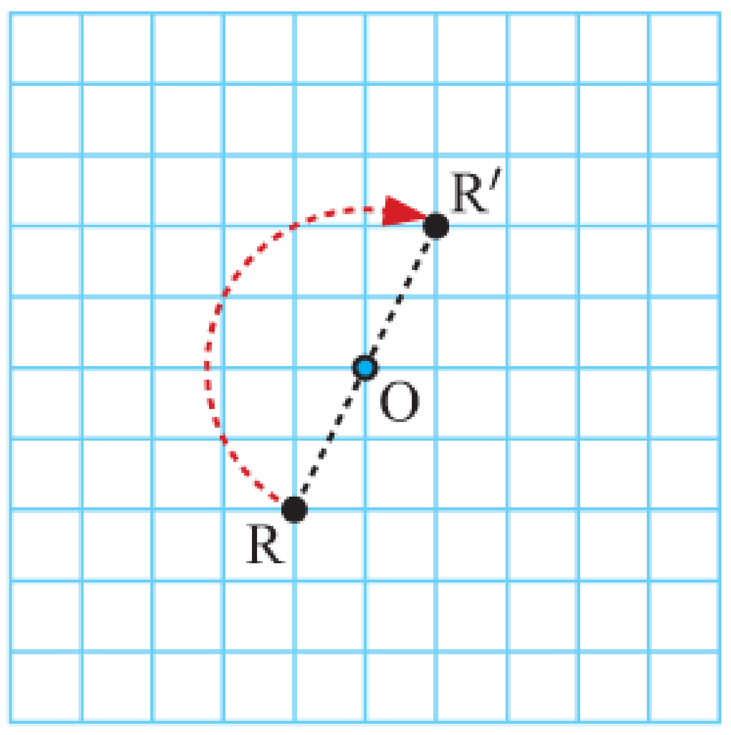
5 **a**



b

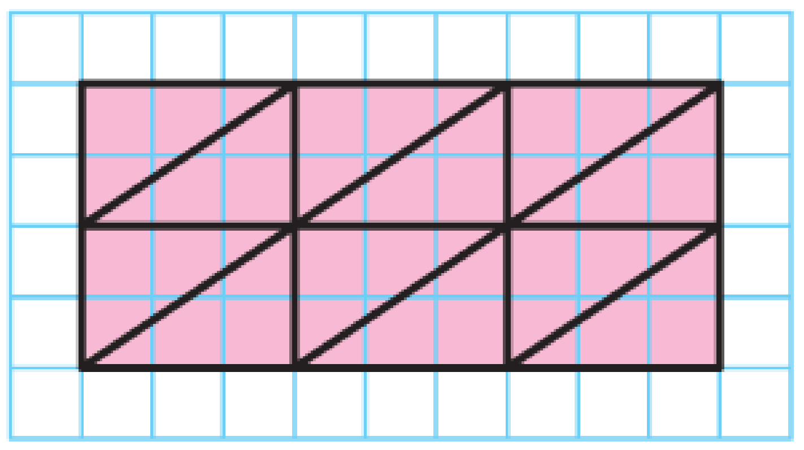


c

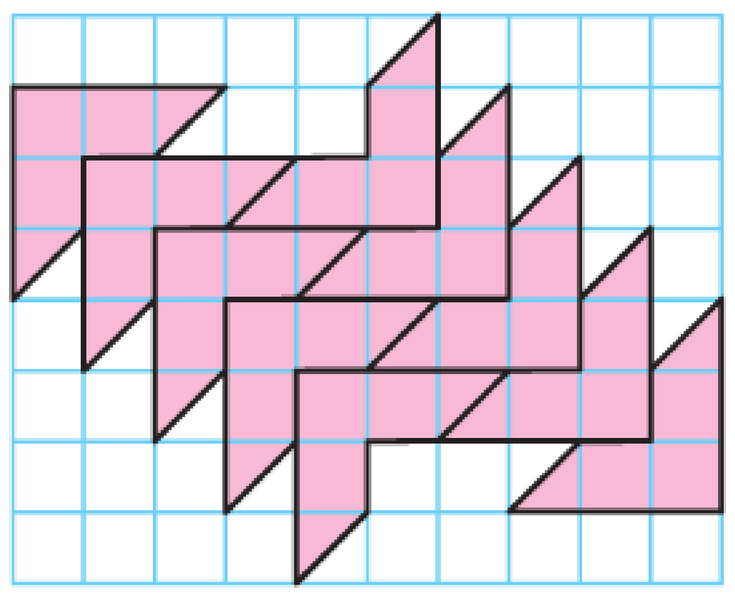


6 $\frac{1}{2}$

7 **a**



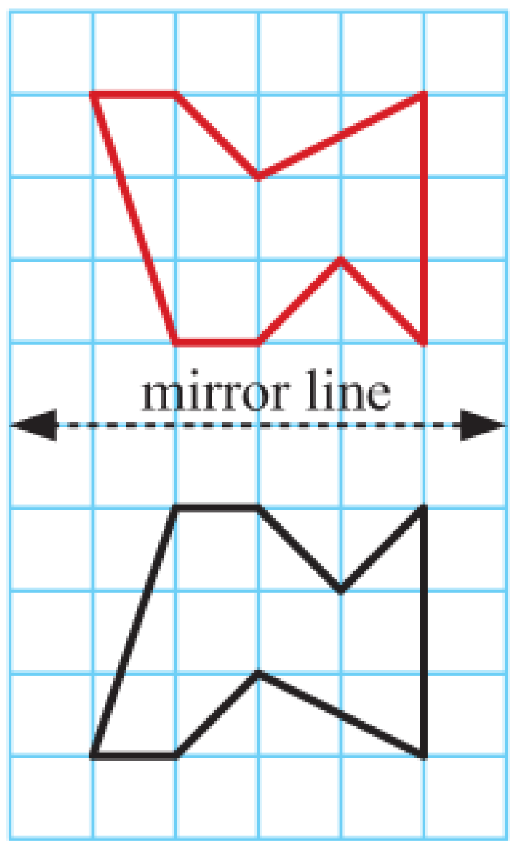
b



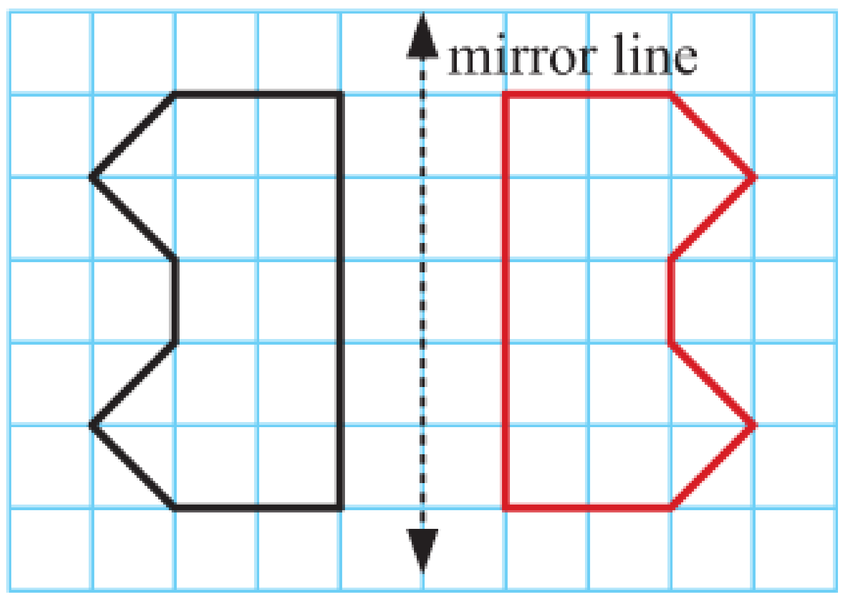
REVIEW SET 16B

1 **a** 4 units right, 1 unit down
b 2 units left, 2 units down

2 **a**

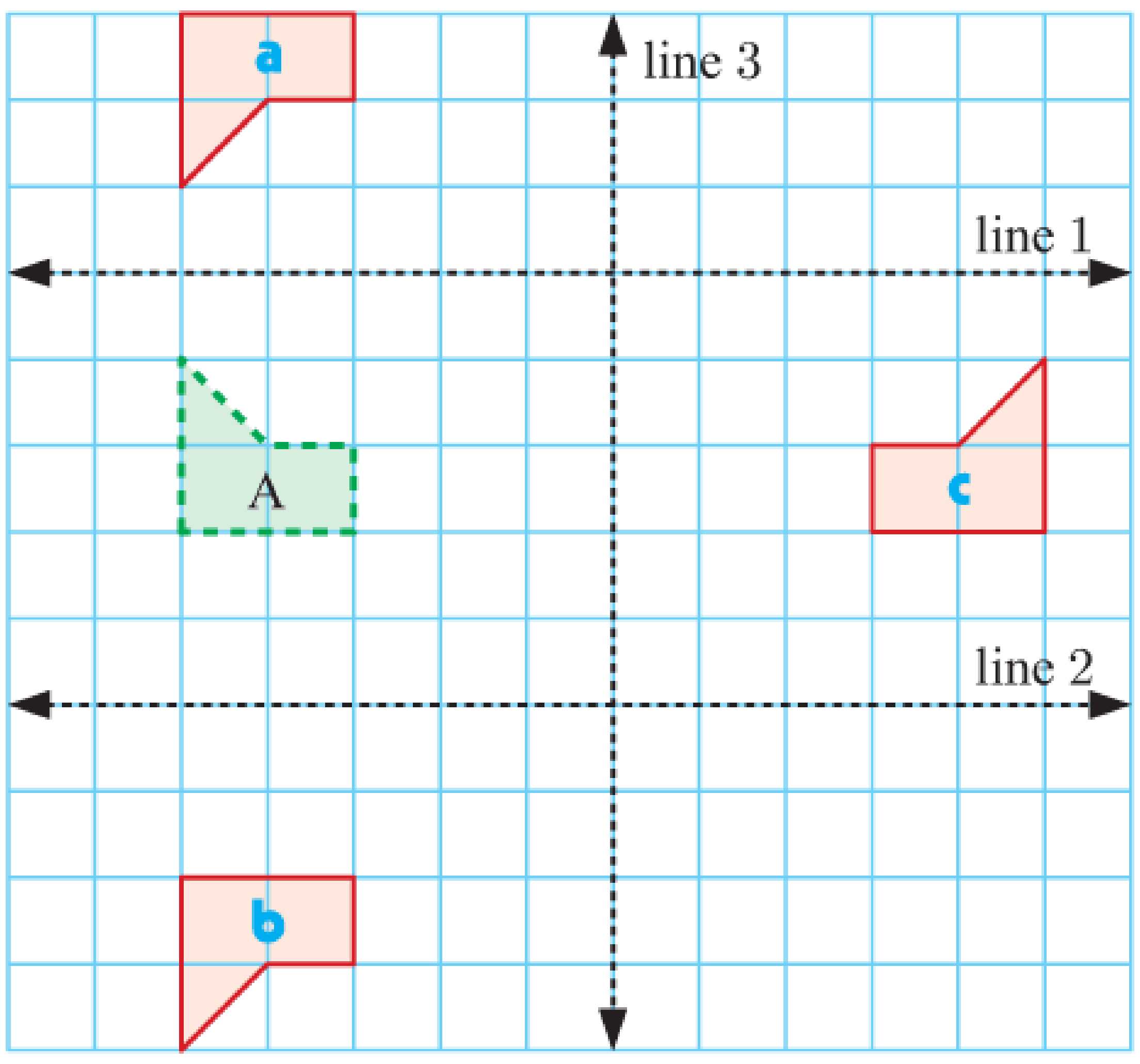


b

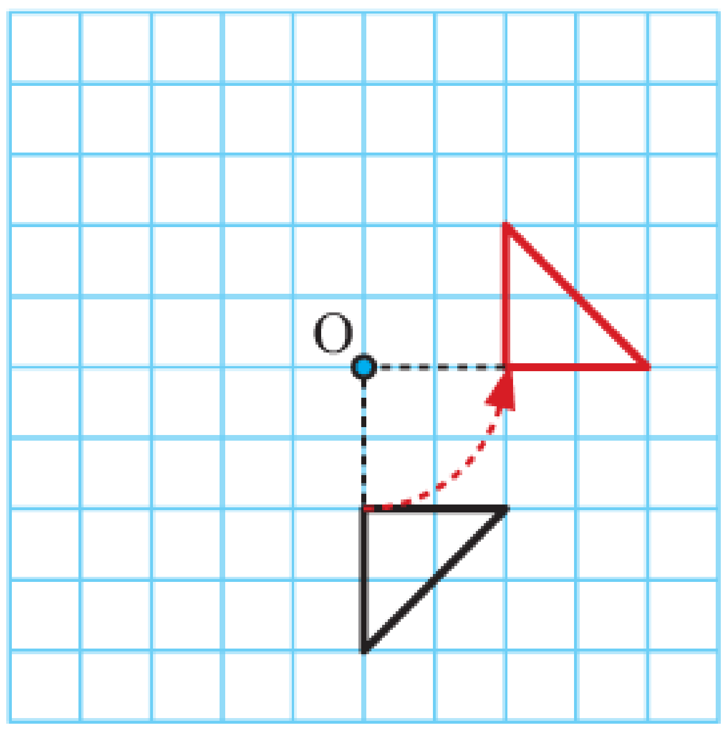


3 **a** 90° anticlockwise
b 180° clockwise or anticlockwise
c 90° clockwise

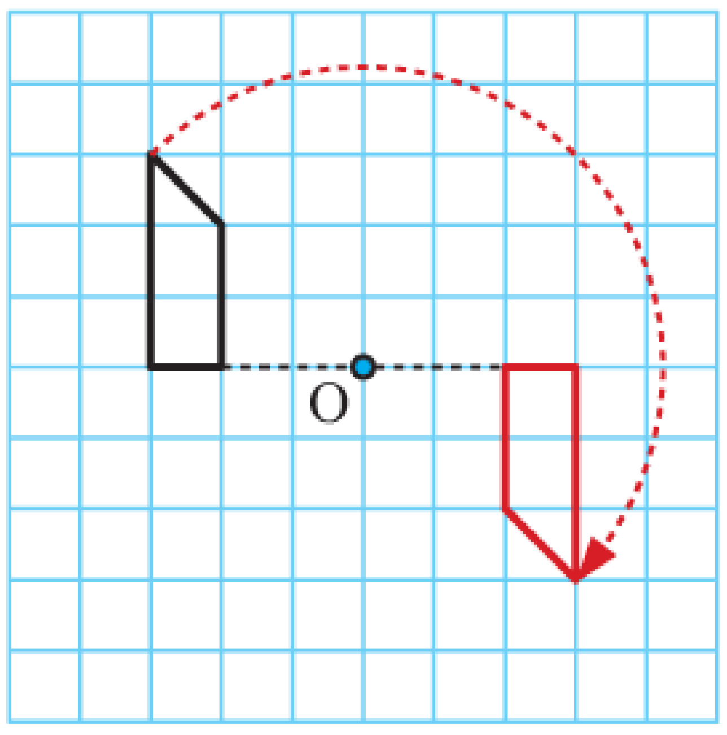
4



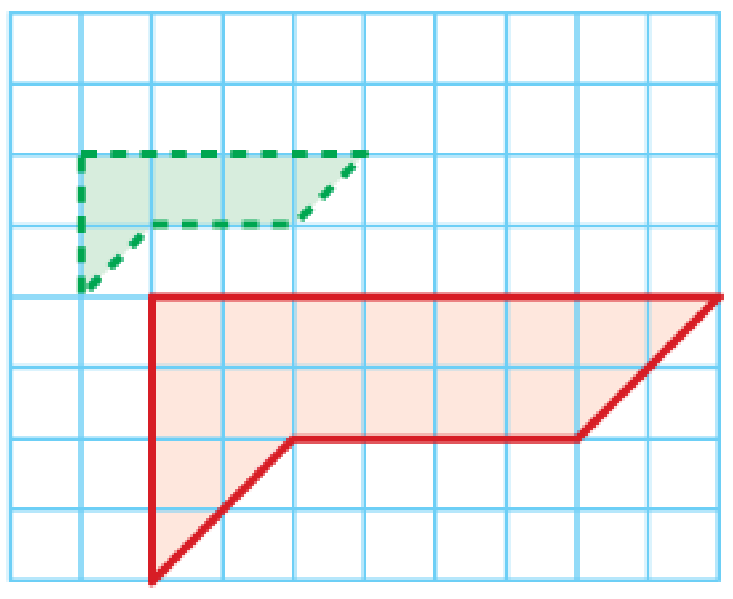
5 **a**



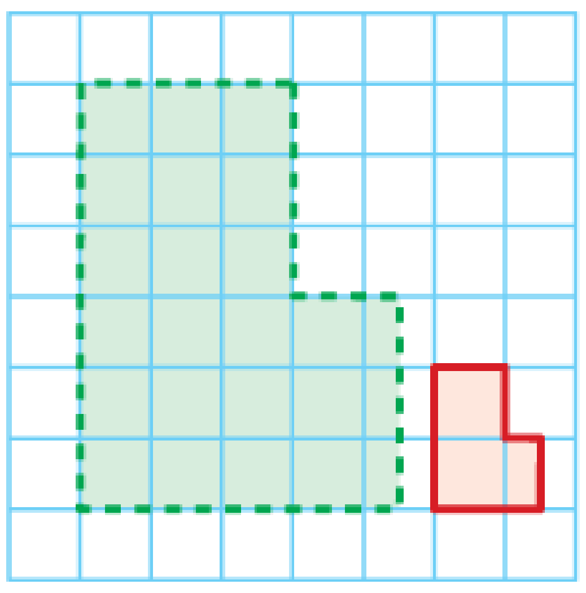
b



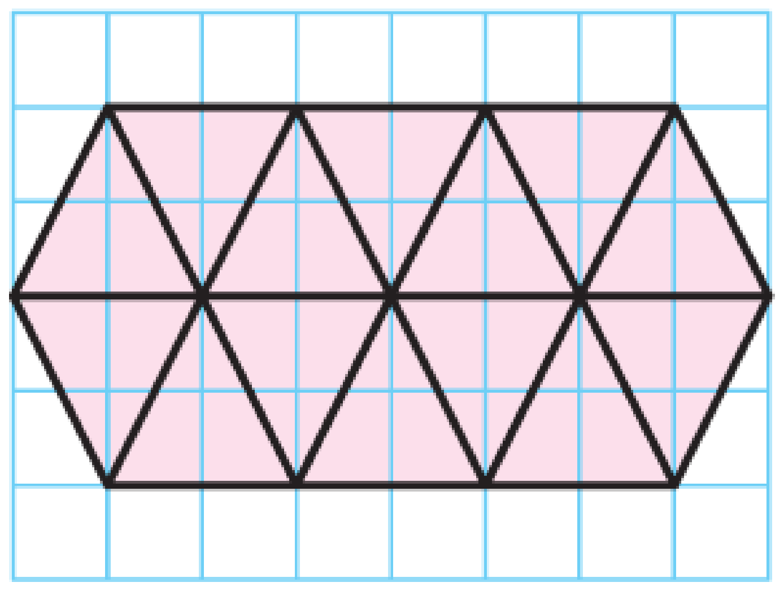
6 **a**



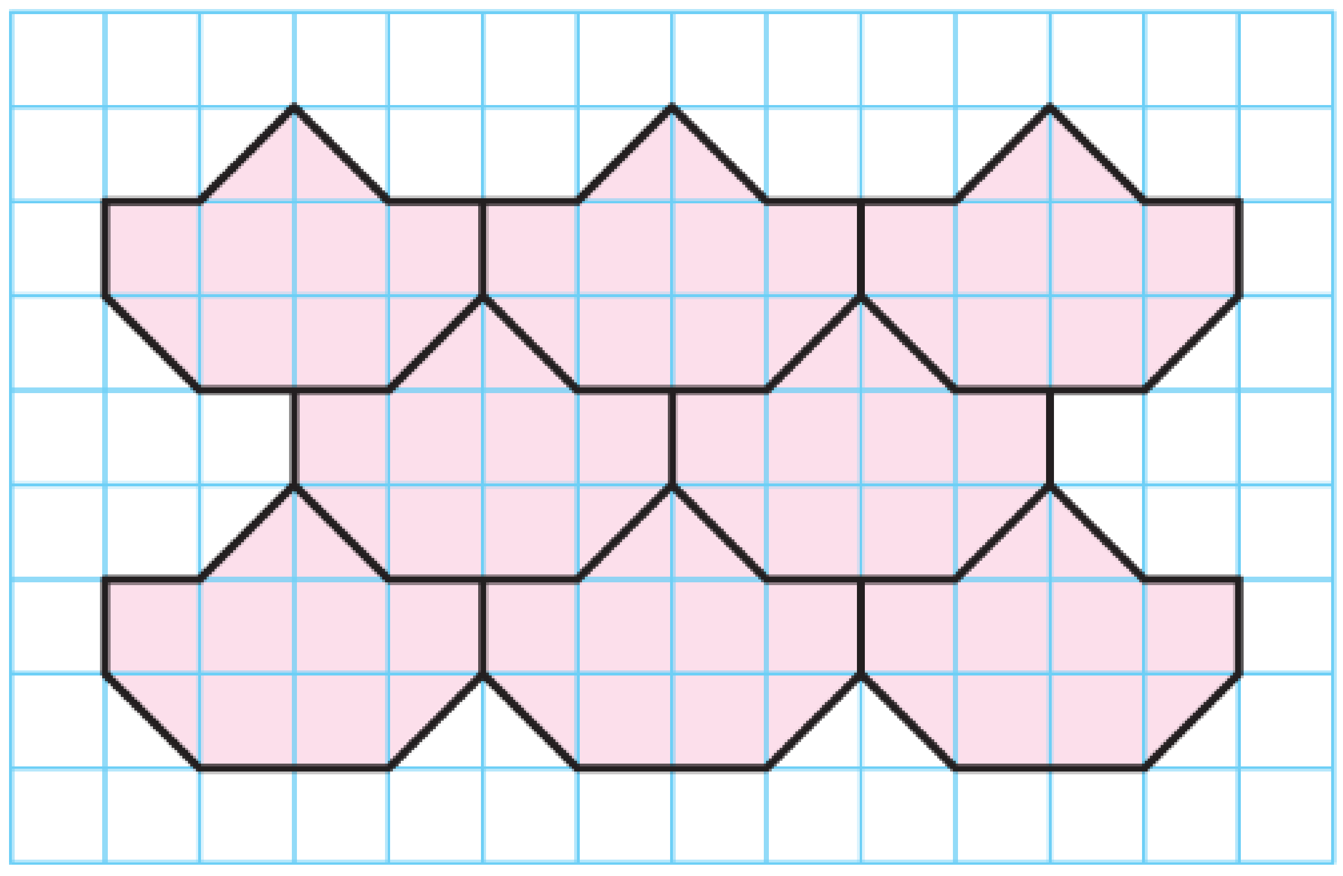
b



7 **a**



b



EXERCISE 17A

1 **a** **i** $5 \in S$ **ii** $7 \notin S$ **iii** $13 \notin S$ **iv** $20 \in S$
b $n(S) = 10$

2 **a** {2, 4, 6, 8, 10, 12}
b {January, February, March, April, May, June, July, August, September, October, November, December}
c {17, 19, 21, 23, 25, 27, 29}
d {2, 3, 5, 7, 11, 13, 17, 19} **e** {gold, silver, bronze}
f {1, 2, 3, 4, 6, 12}

3 **a** **i** $A = \{4, 6, 8, 9, 10, 12, 14, 15, 16\}$ **ii** $n(A) = 9$
b **i** $A = \{\text{H, I, J, K, L, M, N, O, P, Q, R, S}\}$ **ii** $n(A) = 12$
c **i** $A = \{21, 28, 35, 42, 49, 56\}$ **ii** $n(A) = 6$
d **i** $A = \{1, 2, 4, 8, 16, 32\}$ **ii** $n(A) = 6$
e **i** $A = \{$  $\}$
ii $n(A) = 8$
f **i** $A = \{9, 16, 25, 36, 49\}$ **ii** $n(A) = 5$
g **i** $A = \{\ \}$ or $\varnothing$ **ii** $n(A) = 0$

4 **a** **i** $F = \{1, 2, 4, 7, 14, 28\}$
ii $M = \{6, 12, 18, 24, 30, 36, 42, 48\}$
b **i** no **ii** yes **c** **i** $n(F) = 6$ **ii** $n(M) = 8$

5 **a** **i** $P = \{2, 3, 5, 7, 11, 13\}$ **ii** $Q = \{1, 3, 9, 27\}$
b $n(P) = 6$ **c** no **d** 3

6 **B** and **E** **7** $\square = 5$ **8** yes **9** **a** true **b** false

EXERCISE 17B

1 **a** true **b** true **c** false **d** true **2** **D**

3 **a** {3}, {4}, {5} **b** {3, 4}, {3, 5}, {4, 5} **c** {3, 4, 5}

4 **a** {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}
b {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}

5 ∅, {♥}, {♠}, {♣}, {♦}, {♥, ♠}, {♥, ♣}, {♥, ♦}, {♠, ♣}, {♠, ♦}, {♣, ♦}, {♥, ♠, ♣}, {♥, ♠, ♦}, {♥, ♣, ♦}, {♠, ♣, ♦}, {♥, ♠, ♣, ♦}

6 $P = \{1, 4, 9\}$, $Q = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
Every element of P is also an element of Q, so $P \subseteq Q$.

7 Every student is your class is also a student in your school. Therefore, every element in A is also an element in B, so $A \subseteq B$.

EXERCISE 17C

1 **a** $A \cap B = \{a, e\}$ **b** $A \cap B = \{6, 10\}$
c $A \cap B = \{+\}$ **d** $A \cap B = \{23, 42, 75\}$

2 **a** **i** $M = \{\text{A, P, R, T, M, E, N}\}$
ii $N = \{\text{P, R, O, S, E, C, T}\}$
b $M \cap N = \{\text{P, R, T, E}\}$
c This set represents the letters which are in both of the words APARTMENT and PROSPECTOR.

3 **a** **i** $P = \{2, 3, 5, 7, 11, 13, 17\}$
ii $Q = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$
b $P \cap Q = \{3, 5, 7, 11, 13, 17\}$ **c** $n(P \cap Q) = 6$

4 $F = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$
$G = \{1, 2, 4, 5, 8, 10, 16, 20, 40, 80\}$
a $F \cap G = \{1, 2, 4, 5, 10, 20\}$
b This set represents the common factors of 60 and 80.

5 **a** **i** $J = \{4, 7, 8, 12, 17, 18, 20, 21, 22, 25, 26, 30\}$
ii $K = \{2, 6, 7, 10, 11, 14, 17, 19, 20, 23, 25, 29, 30\}$
b $J \cap K = \{7, 17, 20, 25, 30\}$ **c** 5 days

EXERCISE 17D

1 **a** $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
b $A \cup B = \{a, b, c, d, e, f, g, m\}$
c $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
d $A \cup B = \{*, \#, !, \times, :, 5, +\}$

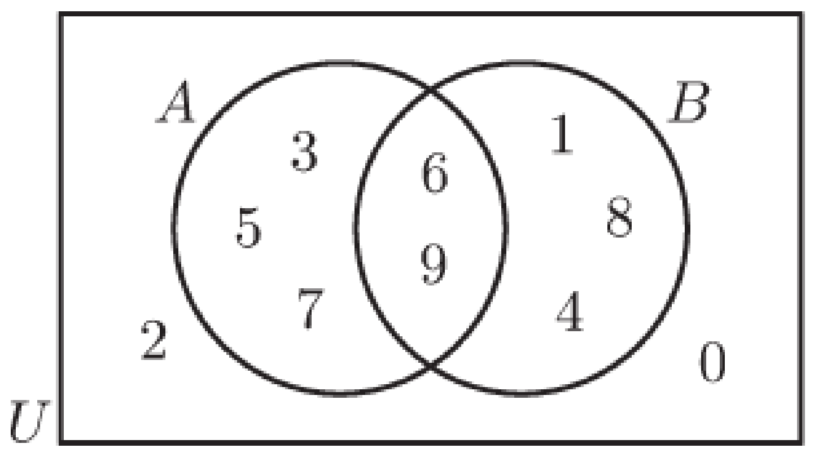
2 **a** **i** $K = \{$white, red, black$\}$ **ii** $L = \{$red, yellow, blue$\}$
b $K \cup L = \{$white, red, black, yellow, blue$\}$

3 **a** **i** $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$
ii $B = \{3, 6, 9, 12, 15, 18\}$
b $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$
c $n(A \cup B) = 12$

4 **a** **i** $M = \{$Michael, Bradley, Craig, Sally, Alistair, Kylie, Emma, Nigel$\}$
ii $S = \{$William, Nigel, Kylie, David, Sam, Craig, Luke$\}$
b $M \cap S = \{$Craig, Kylie, Nigel$\}$
This set represents the people who both Mark and Stephen want to invite.
c $M \cup S = \{$Michael, Bradley, Craig, Sally, Alistair, Kylie, Emma, Nigel, William, David, Sam, Luke$\}$
This set represents the people who either Mark or Stephen want to invite.
d 12 guests

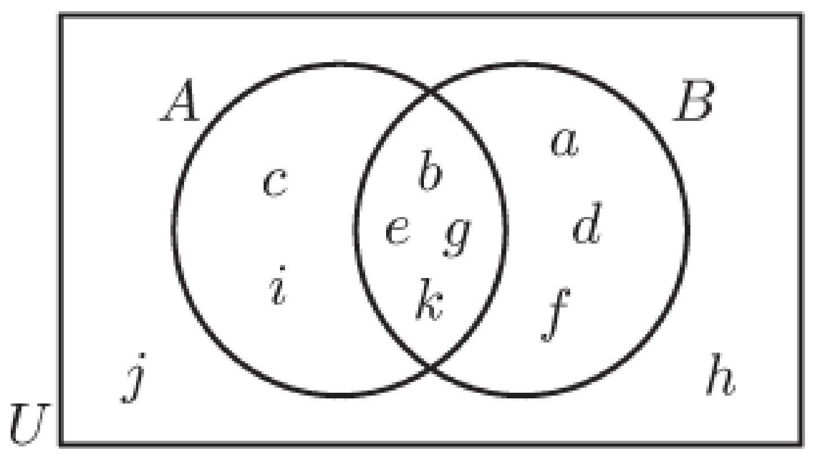
EXERCISE 17E.1

1 **a** **i** $A \cap B = \{6, 9\}$ **ii** $A \cup B = \{1, 3, 4, 5, 6, 7, 8, 9\}$
b



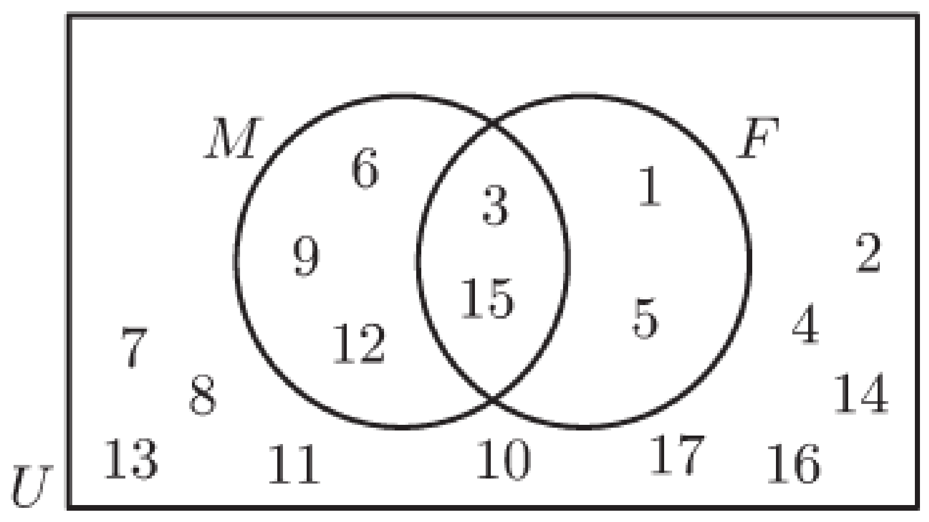
2 **a** $P = \{2, 3, 6, 7\}$ **b** $Q = \{1, 3, 5, 7\}$
c $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ **d** $P \cap Q = \{3, 7\}$
e $P \cup Q = \{1, 2, 3, 5, 6, 7\}$

3 **a** **i** $A \cap B = \{b, e, g, k\}$
ii $A \cup B = \{a, b, c, d, e, f, g, i, k\}$
b



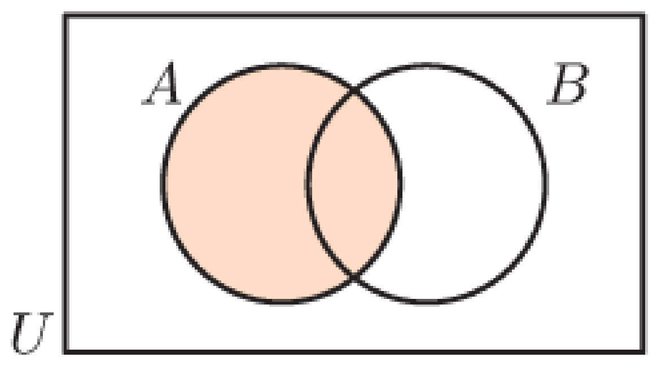
4 **a** **i** $M = \{3, 6, 9, 12, 15\}$ **ii** $F = \{1, 3, 5, 15\}$
b **i** $M \cap F = \{3, 15\}$
ii $M \cup F = \{1, 3, 5, 6, 9, 12, 15\}$
iii $n(M \cap F) = 2$ **iv** $n(M \cup F) = 7$

c

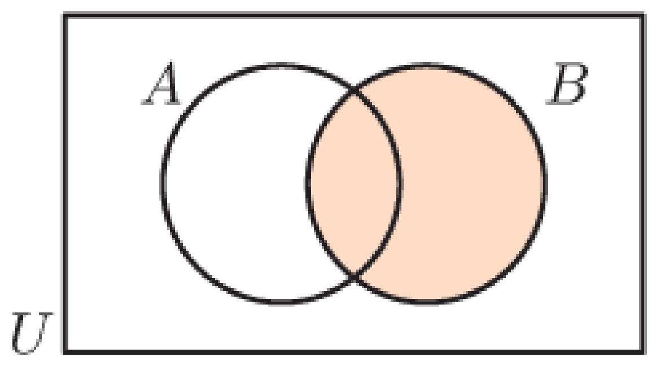


5 **a** The elements in set S. **b** The elements not in set R.
c The elements not in set S.
d The elements in either set R or set S.
e The elements in set R but not set S.
f The elements in set S but not set R.

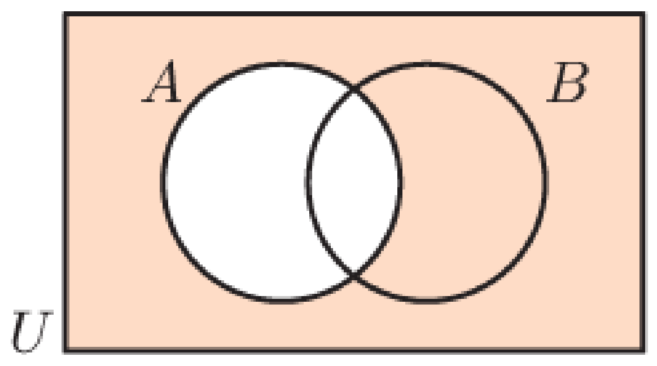
6 **a**



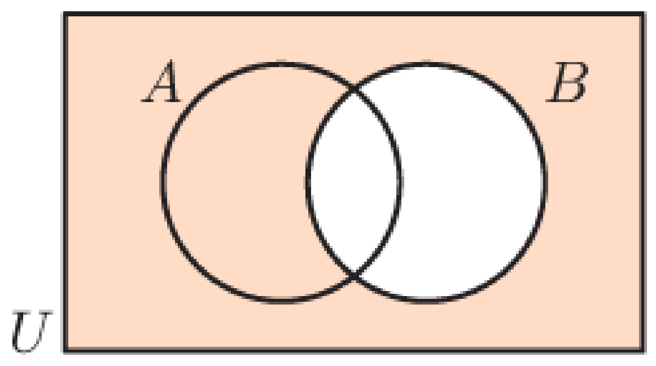
b



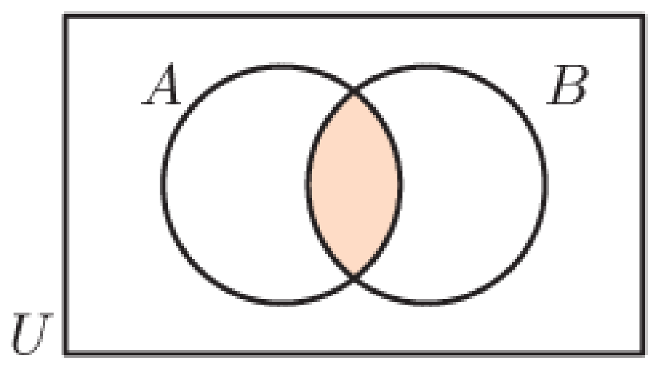
c



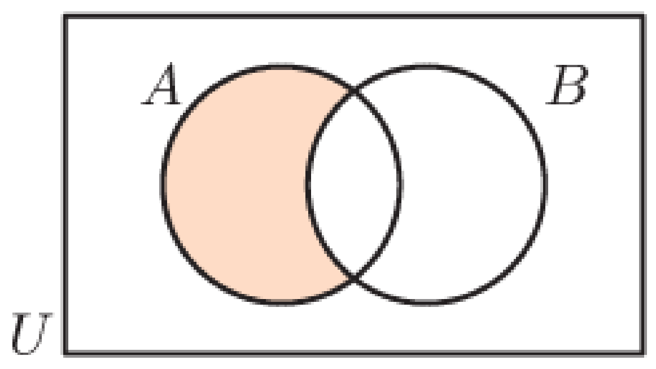
d



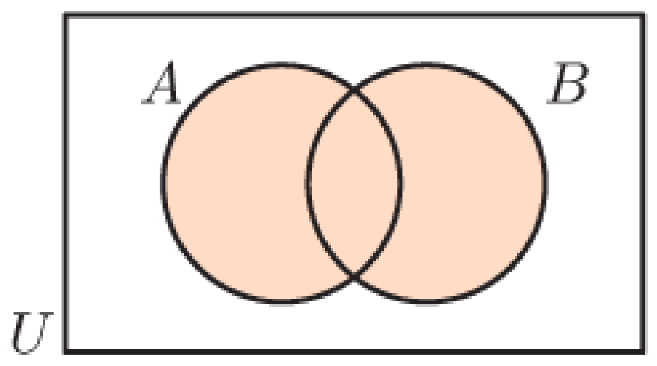
e



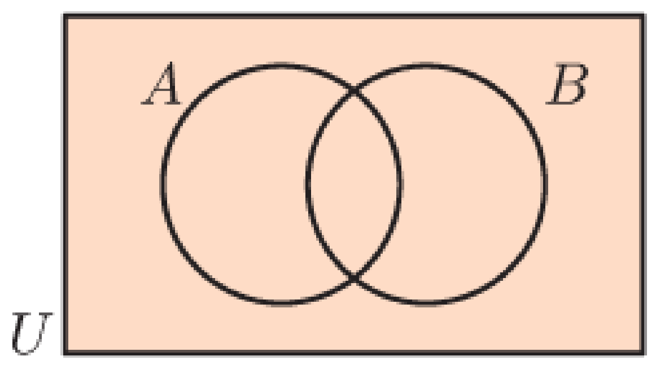
f



g



h



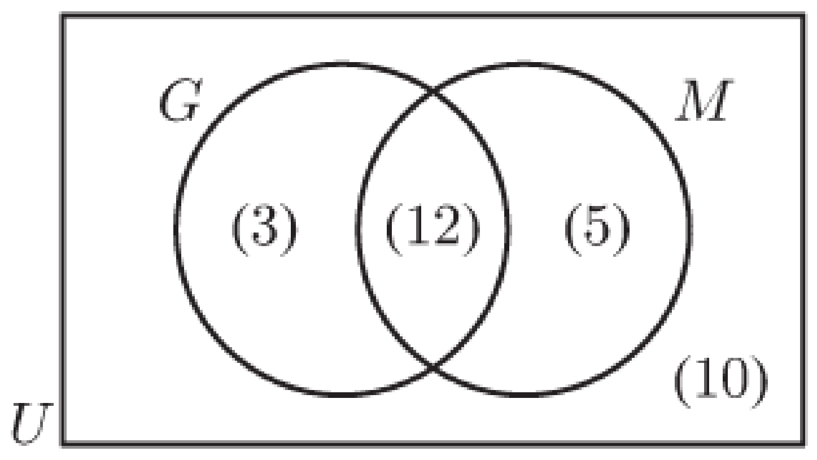
EXERCISE 17E.2

1 **a** 11 elements **b** 12 elements **c** 7 elements
d 19 elements **e** 5 elements **f** 12 elements

2 **a** 70 people
b **i** 40 people **ii** 31 people **iii** 11 people
iv 29 people **v** 10 people **vi** 60 people

3 **a** 32 members
b **i** 17 members **ii** 25 members **iii** 5 members
iv 12 members **v** 2 members

4 **a**



b **i** 15 movies
ii 17 movies
iii 12 movies
iv 3 movies

c Yes, as they either both liked or both disliked most of the movies they saw.

REVIEW SET 17A

1 **a** **i** pink $\notin P$ **ii** green $\in P$ **b** $n(P) = 6$

2 **a** $\{8, 16, 24, 32, 40, 48\}$ **b** $\{29, 31, 37\}$

3 **a** $A = \{26, 28, 30, 32, 34\}$ **b** $n(A) = 5$

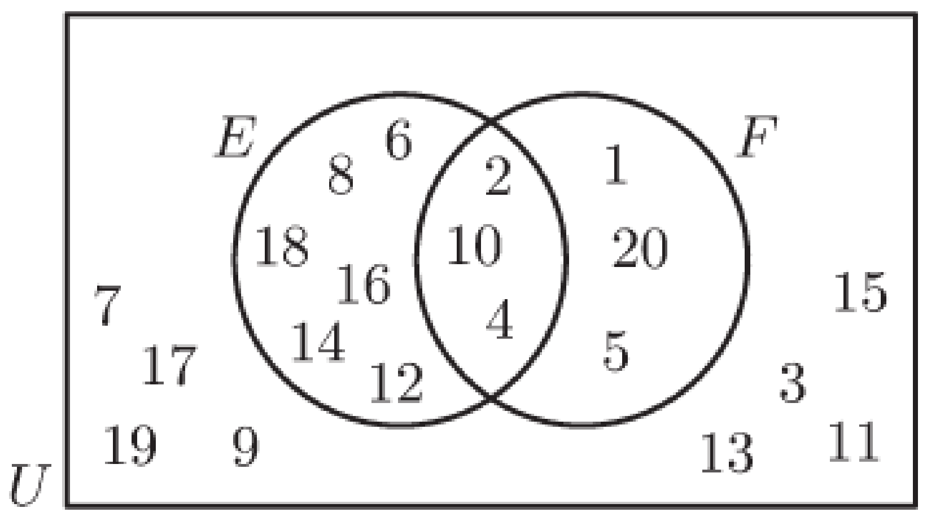
4 **a** **i** $A \cup B = \{c, d, f, j, m, p, s, v, w, z\}$
ii $A \cap C = \{f\}$
b Yes, as every element of C is also an element of B.

5 $\square = 4$

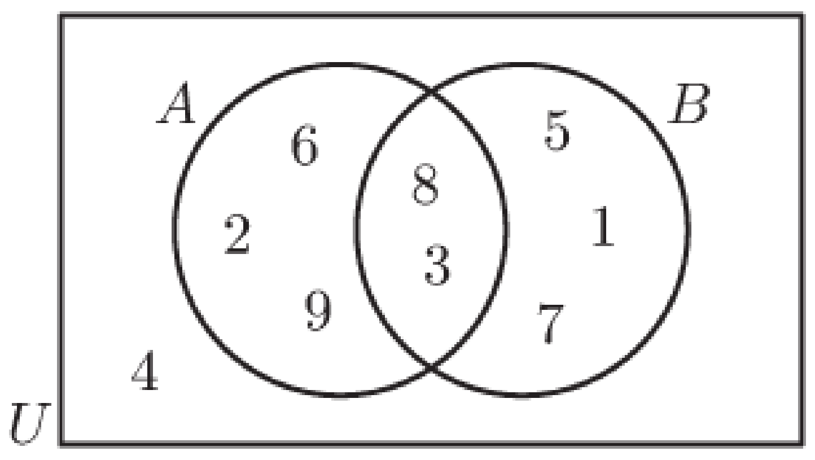
6 **a** $A \cup B = \{\text{dog, bird, sheep, giraffe, tiger, otter, bear}\}$
b $A \cup B = \{!, @, \div, ▶, \bullet, \#, +, *, \%\}$

7 **a** **i** $E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$
ii $F = \{1, 2, 4, 5, 10, 20\}$ **iii** $E \cap F = \{2, 4, 10\}$
iv $E \cup F = \{1, 2, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20\}$

b

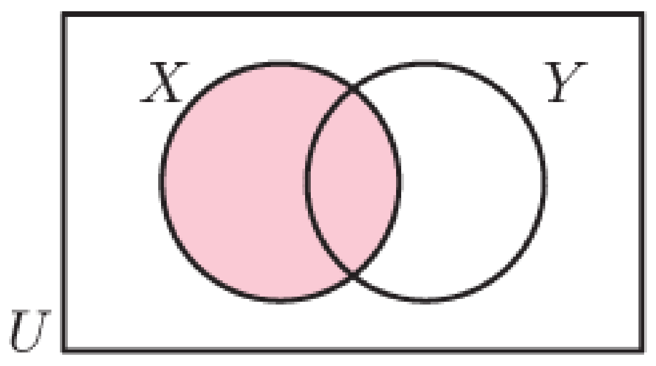


8 **a**

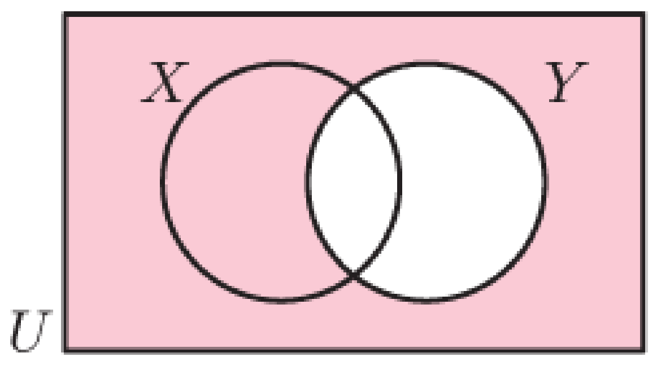


b **i** $n(A \cap B) = 2$
ii $n(A \cup B) = 8$

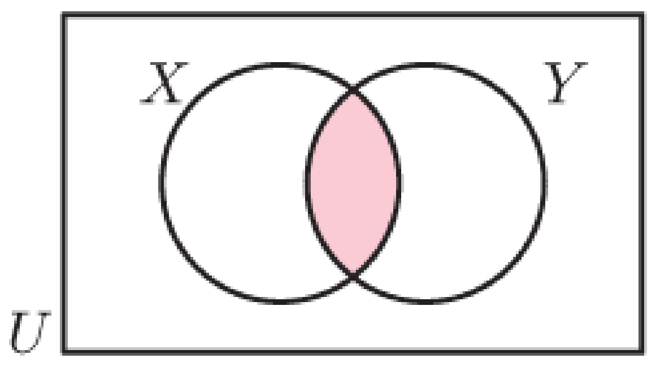
9 **a**



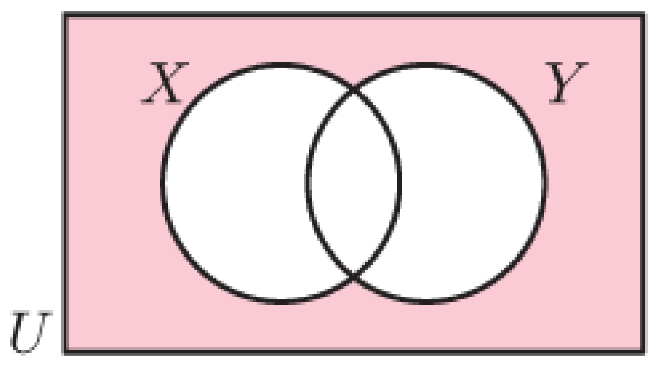
b



c



d



10 **a** 44 people
b **i** 25 people **ii** 18 people **iii** 7 people
iv 8 people **v** 11 people

REVIEW SET 17B

1 **a** $\{16, 25, 36, 49\}$ **b** $\{1, 2, 3, 6, 9, 18\}$ **2** **B** and **D**

3 **a** $M = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39\}$
$N = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$
b $M \cap N = \{12, 24, 36\}$

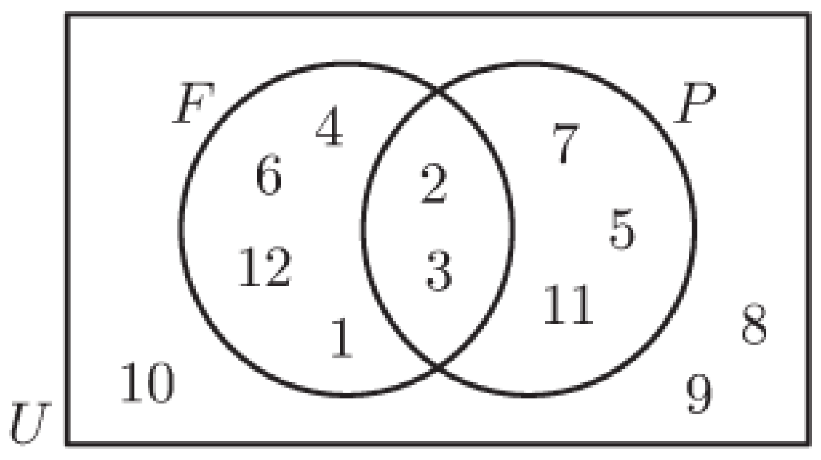
4 **a** $\{1, 5\}$, $\{1, 7\}$, $\{1, 9\}$, $\{5, 7\}$, $\{5, 9\}$, $\{7, 9\}$
b $\{1, 5, 7\}$, $\{1, 5, 9\}$, $\{1, 7, 9\}$, $\{5, 7, 9\}$

5 **a** **i** $X = \{\text{I, S, O, C, E L}\}$ **ii** $Y = \{\text{S, C, A, L, E, N}\}$
b $X \cup Y = \{\text{I, S, O, C, E, L, A, N}\}$ **c** $n(X \cup Y) = 8$

6 **a** $S \cap T = \{12, 24\}$ **b** $S \cap T = \{\text{blue, green}\}$

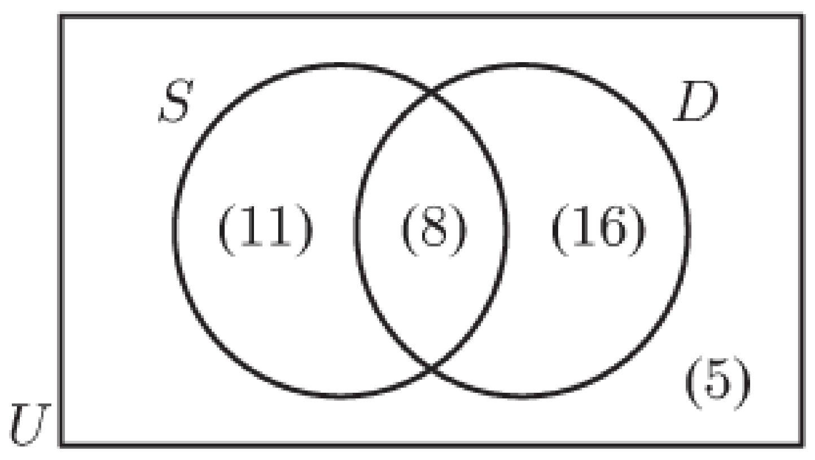
7 **a** 8 elements **b** 9 elements **c** 3 elements
d 14 elements **e** 21 elements

8



9 **a** The elements in either set M or set N.
b The elements in either set M or set N but not in both.
c The elements that are not in both set M and set N.

10 **a**



b **i** 19 members
ii 24 members
iii 8 members
iv 11 members
v 35 members
vi 27 members

EXERCISE 18A

1 **a** **A** **b** **C** **c** **B**

2 **a** 7 pm **b** 5 pm **c** 25 people
d There was an increase in customers (from 20 to 40).
e 10 people

3 **a** 20°C **b** 3 min **c** 1 min **d** 1 min and 7 min
e from 4 min to 10 min

4 **a** 18 min **b** 15 m **c** 25 m **d** 8 min and 12 min

5 **a** **i** January **ii** July **b** the temperature increases
c March, November, then April, November

6 **a** **i** 25 points **ii** 45 points **iii** 70 points **iv** 100 points
b fourth quarter
c from 9 min to 12 min, and from 27 min to 30 min
d from 39 min to 42 min

7 **a** 26°C **b** 60°C **c** after 4 minutes **d** 100°C

8 **a** 55 gigalitres **b** the start of March
c 60 gigalitres **d** 10 gigalitres
e June, July, August, September, October

EXERCISE 18B

1 **a** $5\frac{1}{2}$ hours **b** 300 km **c** 150 km **d** 2 hours
e **i** ≈ 70 km **ii** ≈ 30 km **iii** 50 km **iv** ≈ 120 km
f The family stopped for half an hour then continued to travel at a faster speed.

2 **a** the Williams family **b** 2 hours **c** 500 km

3 **a** 7:48 am **b** 7 km **c** 48 minutes **d** 6 minutes
e **i** 1 km **ii** 3 km **f** **i** 2 km **ii** 2 km

EXERCISE 18C

1 **a** 160 euros **b** 350 US dollars

2 **a** 900 Canadian dollars **b** 540 Canadian dollars
c 250 British pounds **d** 400 British pounds

3 **a** ≈ 28 miles **b** ≈ 17 miles **c** ≈ 77 km **d** ≈ 48 km

4 **a** **i** ≈ 4°C **ii** 140°F **b** 212°F **c** ≈ 29°C

REVIEW SET 18A

1 increasing

2 **a** **i** 7:45 pm **ii** 6 pm **b** $\approx$ 6:55 pm and 8:30 pm
c ≈ 43 diners

3 **a** 28°C **b** 45 min **c** 18°C **d** 14°C

4 **a** 250 euros **b** 450 euros **c** 400 British pounds
d 120 British pounds

5 **C** **6** **a** 11 km **b** Celia **c** 3 times **d** 3 km

REVIEW SET 18B

1 **a** 40 km/h **b** 2 seconds

2 **a** 20°C **b** after 3 minutes

3 **a** after 3 years **b** 200 salmon, after 9 years
c **i** between 0 and 3 years and between 9 and 10 years
ii between 3 and 9 years
d after 1 year and 5 years

4 **a** ≈ 6 m **b** ≈ 1.5 m **c** ≈ 26 feet **d** ≈ 7 feet

5 **a** 40% **b** 10 am to 11 am **c** 2 hours
d 6 pm **e** 8 am and 9 pm

6 **a** 24 km **b** Hissam **c** 80 minutes
d 16 km **e** Hissam

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