Markscheme

November 2019

Extended mathematics

On-screen examination

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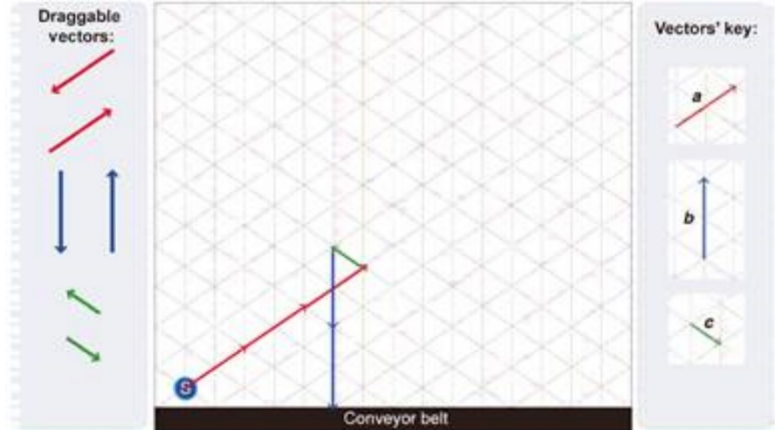
Error Carried Forward (ECF) marks
Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.
- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award **ECF** marks for a question part, **there must be working present for that part**.
- e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
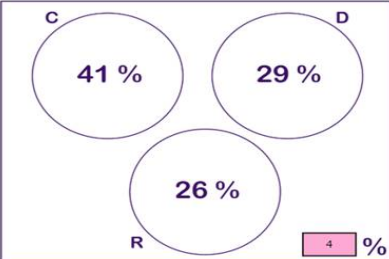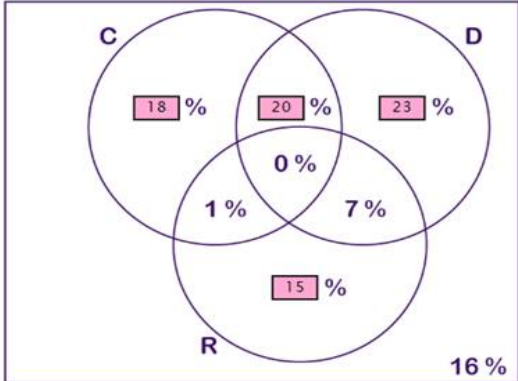- f) A misread (**MR**) is an error**. ECF** is normally awarded.
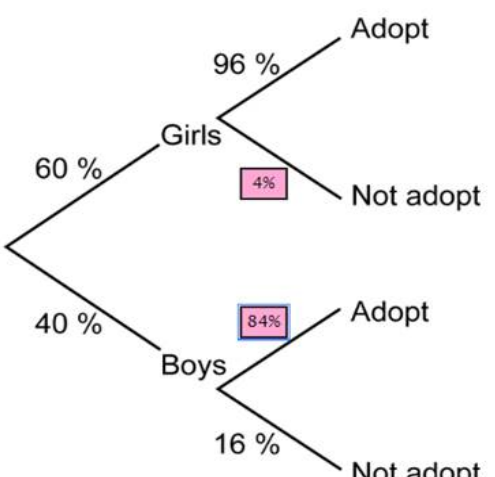
General points

- a) As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
- b) Accept notation errors in intermediate steps.
- c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,** then the last mark cannot be awarded.
- d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
- e) Where candidates have written two solutions to a question, mark the first solution.
- f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or equivalent) e.g. $\dfrac{1}{2}$ **OR** 1/2 **OR** $1 \div 2$ and $\dfrac{x}{2}$ $x / 2$ **OR** $x \div 2$
- g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
- h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
- i) Accept seeing equation not in-line.
- j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
- k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
- l) ACCEPT using the correct values regardless their previous result

Question		Answers	Notes	Total
1	a	**AM1 (adding volumes)** .1 correct Volume for one part .2 correct Volume for second part .3 correctly add their volumes **AM2 (Subtracting volumes)** .1 correct volume including empty part .2 correct volume of empty part .3 correctly subtract their volumes **AM3 (Area x depth)** .1 correct Area for one part .2 correct Area for the face .3 correctly multiply the total of their areas by 30	**AM1 (adding volumes)** .1 50x20x30 (=30000) OR 35x20x30(=21000) .2 20x15x30 (=9000) OR 40x15x30(=18000) .3 (their30000+their9000) OR (their21000+their18000) =39000 (ft³) **AM2 (Subtracting volumes)** .1 50x40x30 (=60000) .2 35x20x30 (=21000) .3 (their60000-their21000) =39000 (ft³) **AM3 (Area x depth)** .1 50x20 (=1000) OR 20x15 (=300) .2 (50x20 + 20x15) = 1300 .3 (their1300 x30) =39000 (ft³)	3
	b	.1 correct working step .2 correct sum of employees in working days .3 correct solution of their equation	.1 Ex: setting equation 5 x w + 2 x 3 = 7 x 58 OR 6 and 406 seen .2 (5w) = 400 .3 (w =) 80	3
	c	.1 correctly substitute their39000 and their80 .2 correctly calculate their P after their substitution involving at least one multiplication in numerator	.1 $(P =)\dfrac{(6 \times their39000 + 500 \times their80)}{9000}$ .2 their 30(.444…)	2

Question		Answers	Notes	Total
2	a	Correct vectors have been dragged		1
	b	.1 correct path in un-simplified form .1 correct path in simplest form	.1 −2***b*** + 3.5***a*** − 4***b*** or 4***a*** − 2***b*** − 0.5***a*** − 4***b*** .1 ACCEPT any path starting from M, passing by P and ending at conveyor belt .2 1.5***a*** – 4***b*** + 4***c*** ACCEPT 3.5***a*** – 6***b***	2
	c	.1 correct working step .2 another correct working step .3 correct answer Examples can be seen below: **AM1** .1 correctly calculate the number of minutes for 4 robots to prepare 300 orders .2 correctly calculate the number of minutes for 1 robot to prepare 300 orders .3 correct answer	.3 24 (min) **AM1** .1 60 (min) .2 240 (min) .3 24 (min)	3

		AM2 .1 correctly calculate the number of orders prepared by 1 robot in 3 minutes **and** 10 robots in 3 minutes .2 correctly calculate the number of orders prepared by 10 robots in 3 minutes .3 correct answer	**AM2** .1 1 robot 3.75 (orders) **and** 10 robots 37.5 (orders) .2 300/37.5 = 8 .3 24 (min)	
		AM3 .1 correctly calculate the number of robots or minutes to prepare 300 orders .2 correctly multiply the number of robots by the correct constant of proportionality .3 correct answer	**AM3** .1 80 (robots) or 60 (min) .2 0.125 or 2.5 .3 24 (min)	
		AM4 .1 correctly calculate the number of orders or minutes prepared by 10 robots .2 correctly multiply the number of orders by the correct constant of proportionality .3 correct answer	**AM4** .1 37.5 (orders) or 1.2 (min) .2 20 or 8 .3 (1.2x20=)24 (min)	
		AM5 .1 correctly calculate the number of minutes to prepare 1 order OR finding the number of orders prepared in 1 minute .2 correctly calculate the number of robots or minutes to prepare 300 orders .3 correct answer	**AM5** .1 0.2 (min) or 5 (orders) .2 60 (min) or 240 (robots) .3 (60/2.5 =) 24 (min) OR (1 x 24 =) 24 (min)	
		AM6 .1 correctly calculate the number of orders prepared by 1 robot .2 correctly calculate the number of orders prepared by 10 robots .3 correct answer	**AM6** .1 3.75 (order) .2 37.5 (order) .3 (3 x 8 =) 24 (min)	

Question		Answers	Notes	Total
3	a	correctly place 4	ACCEPT 4% 	1
	b	correctly state a reason for mutually exclusive related to sets representation	No intersection ACCEPT they are separate or apart WTTE $A \cap B \cap C = 0 \ or \ \varnothing$ DO NOT ACCEPT No girl selected both	1
	c	.1 correct working step .2 correct value of cat only .3 correct value of intersection .4 correct value of dog only .5 correct value of rabbit only	.1 16+39+50+23(-8-100) OE or 128 seen .2 18 .3 20 .4 23 .5 15 	5

| d | .1 0.84 seen
.2 probability of adopt from both branches seen
.3 their result correct after adding their multiplied values | .1 ACCEPT 84 seen on tree diagram
.2 0.6 x 0.96(=0.576) OE, ACCEPT 0.58 and 0.4 x 0.84(=0.336)OE, ACCEPT 0.34
.3 their 0.912 OE, ACCEPT 0.91

Adopt
96 %
Girls
60 %   4%   Not adopt
40 %   84% Adopt
Boys
16 %   Not adopt | **3** |

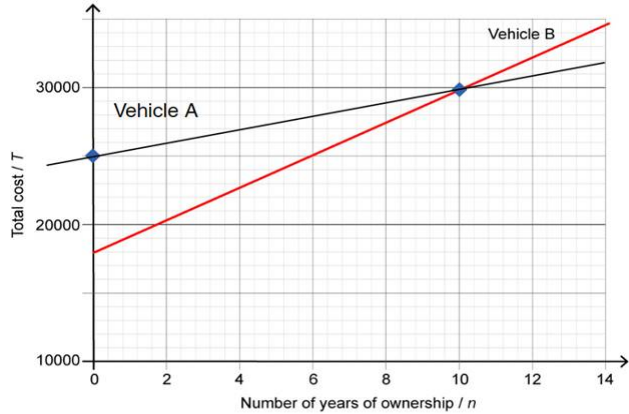| Question		Answers	Notes	Total
4	a	.1 (0, –2)	ACCEPT 0;-2 X=0, y=-2 DO NOT ACCEPT -2 C=-2	**1**
	b	.1 correctly set equation to solve .2 correctly factorize their equation or substitute into quadratic formula .3 correct coordinates of their point A .4 correct coordinates of their point B	.1 $3x^2 - 5x - 2 = 0$ .2 their $(3x+1)(x-2)$ .3 their (A =) (–1/3,0) OE, ACCEPT -0.3 .3 DO NOT ACCEPT if positive .4 Their (B =) (2,0) .4 DO NOT ACCEPT if negative	**4**
	c	correctly write coordinates of their point B after reflection on the y-axis	ACCEPT their-2,0 x=their-2 , y=0 DO NOT ACCEPT -2 Their(-2,0) if positive	**1**

Question		Answers	Notes	Total
5	a	.1 correctly write as a single log	.1 $(\log 2x =)\log\dfrac{12y}{3}$ or $\log 4y$ OR $\log 2 \times 3x = \log 12y$	
		.2 remove logs	.2 $2x = \dfrac{12y}{3}$ or $2x = 4y$ or $6x = 12y$	**2**
		AG $x = 2y$		
	b	.1 correctly remove log from first equation	.1 $3x - 2y = 16$	
		.2 correct equation in one unknown	.2 $6y - 2y = 16$ or $3x - x = 16$ OE or $\log_4(6y - 2y) = 2$ OE	
		.3 correct value of their x after substituting x=2y	.3 their $y = 4$	**4**
		.4 correct value of their y after substituting x=2y	.4 their $x = 8$	

Question		Answers	Notes	Total
6	a	400 + 6372 AG 6772	ACCEPT 6772-400=6372 or 6772-6372=400	**1**
	b	.1 recognising that angle XON is the same as angle XOS .2 recognising that angle ONX is equal to angle OXS AG similar triangles	.1 Shared angle .2 Both 90°, ACCEPT perpendicular	**2**
	c	**AM1** .1 equate ratios .2 rearrange correctly their equated ratios .3 correct answer .4 correctly round their answer to the nearest km **AM2** .1 correctly find value of angle XOS .2 correctly substitute into trig ratio or cosine rule their angle and 6372 .3 correct answer .4 correctly round their answer to the nearest km	**AM1** .1 $\dfrac{r}{OS}=\dfrac{ON}{r}$ **or** $\dfrac{6372}{6772}=\dfrac{ON}{6372}$ .2 $(ON=)\dfrac{r^2}{OS}$ **or** $(ON=)\dfrac{6372^2}{6772}$ .3 5995.626... .4 their 5996 **AM2** .1 $(\cos^{-1}\dfrac{6372}{6772}=)19.791...$ .2 $\cos(their\,19.791..)=\dfrac{ON}{6372}$, DO NOT ACCEPT their19.7911.. unless .1 seen .3 5995.626... .4 their 5996	**4**
	d	.1 correctly calculate their MN .2 correctly substitute theirMN and 6372 into the surface area of spherical cap formula .3 correctly calculate their result after substitution into the surface area of spherical cap formula .4 correctly write their answer rounded to 2 sf and in standard form	.1 (6372 – their ON =) their 376, ACCEPT answers in range [376,376.4] .2 $2\pi\times6372\times$their 376 or their 4791744π .3 their 15 068 653(.43…) .3 ACCEPT answers in the range [15 053 707.75,15 069 722.33] .4 their 1.5×10^7	**4**

	e	.1 correctly substitute 6372 into the Surface Area of sphere formula .2 divide their 6d by their surface area of sphere .3 correctly calculate their ratio as percentage	•1 $4\pi \times 6372^2$ or 510 224 605(.2..) •2 $\dfrac{\text{their}1.5 \times 10^7}{\text{their } 510\ 224\ 605}$ OE e.g. $\dfrac{\text{their}1.5}{\text{their}51}$ **or** 0.029 •3 their 2.9(…) ACCEPT 3(%)	**3**

Question		Answers	Notes	Total
7	a	**AM1** .1 recognise 80 .2 multiply 80 % by 31250 .3 correctly calculate their result after multiplying 31250 by their percentage **AM2** .1 multiply 20 % by 31250 .2 subtract 31250×0.2 from 31250 .3 correctly calculate their result after subtraction of their reduction from 31250	**AM1** .1 80 or 0.8 seen .2 0.8×31250 OE .3 their25000 **AM2** .1 31250×0.2 or 6250 seen .2 $31250 - 31250 \times 0.2$ or $31250 - 6250$ .3 their25000	**3**
	b	.1 multiply 14000 by 0.035 .2 correct answer	.1 14000×0.035 seen .2 490	**2**
	c	**AM1** .1 correctly calculate the difference of the annual fuel costs .2 divide their result by 1190 .3 correctly write their ratio as percentage	**AM1** .1 $1190 - \text{their } 490$ or $\lvert\text{their } 490 - 1190\rvert$ or 700 .2 $\dfrac{\text{their } 700}{1190}$ or 0.588235... OE .3 their58.8235... (%) ACCEPT 58.8 or 59 or 60	**3**

		AM2	**AM2**	
		.1 divide their (7b) by 1190	.1 $\dfrac{\text{their}490}{1190}$	
		.2 correctly write their ratio as percentage	.2 their41.17…	
		.3 correctly subtract their result from 100	.3 (100-their41.17…=)their58.8235... (%) ACCEPT 58.8 or 59 or 60	
	d	$T = 490n + 25000$ .1 their25 000 as the y intercept .2 their490 as the gradient	ACCEPT $y = 490x + 25000$, $y = (14000 \times 0.035)x + 25000$	**2**

7	e		**(1 mark)**	**(2 marks)**		10
		Factors (F)	**Two** factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment	**Three** factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment		
		Graph (G)	Attempt to plot their V_A line from earlier results Ex: correct slope or T-intercept from their earlier results or At least two points plotted satisfy their earlier results with acceptable accuracy	Correctly plot their V_A line from their earlier results ACCEPT T-intercept $\in their]24000,26000[$ and n at intersection $\in their]9.5,10.5[$		
		Number of years (N)	Attempt to determine after how many years their V_A and V_B will have the same total cost Ex: Attempt to solve simultaneously or trial and improvement OR Correctly calculate the total cost after a specific number of years for both vehicles	Correctly determine after how many years their V_A and V_B will have the same total cost Ex: Correctly calculate n=their10 after solving simultaneously or trial and improvement OR n=their10 and their graph intersects at n=their10		
		Justify (J)	Weak justification Ex: V_A is better for the environment even if its initial cost is more OR V_A is better for the environment AND it will cost less on the long run or overall cost will be less OR	Good justification Ex: V_A is better for the environment AND V_A is better if years of ownership are more than their n at point of intersection or V_B is better if years of ownership are less than their n point of intersection on the graph		

			V_A is better for the environment AND a justification matching their graph OR V_B is better as their calculations show V_B costs less even if not good for environment			
		Comment on Accuracy (A)	Not very accurate with weak reason. Ex: I rounded the number of years to get the total cost Didn't include other factors of owning a vehicle e.g. running costs OR Accurate with good reason. Ex: The values did not require rounding so the total cost is accurate. The values were whole numbers OR Sensible rounding used	Not very accurate with good reason. Ex: Average mileage 14 000 may vary and will affect the total cost		

Question		Answers	Notes	Total
8	a	.1 identify the base of the triangle .2 correctly substitute into Pythagoras OR trig ratio AG $\sqrt{2}$	.1 2-1 or 1 seen .1 ACCEPT triangle indicated on diagram .2 $\sqrt{1^2+1^2}$ OR $\sin 45 = \dfrac{1}{\sqrt{2}}$ or $\cos 45 = \dfrac{1}{\sqrt{2}}$ .2 ACCEPT $\sin 45$ and $1,1,\sqrt{2}$ seen .2 ACCEPT explanation in words	2
	b	correctly place $16\sqrt{2}$ and $32\sqrt{2}$	<table><tr><td>Stage (n)</td><td>Diagonal (D)</td></tr><tr><td>1</td><td>√2</td></tr><tr><td>2</td><td>2√2</td></tr><tr><td>3</td><td>4√2</td></tr><tr><td>4</td><td>8√2</td></tr><tr><td>5</td><td>16√2</td></tr><tr><td>6</td><td>32√2</td></tr></table>	1
	c	.1 correctly describe one pattern for D in words with correct terminology .2 correctly describe another pattern for D in words with correct terminology	ACCEPT Multiplying by 2, or doubles WTTF $\sqrt{2}$ is always there or number under root always the same WTTF Increasing by 1, 2, 4… Powers of 2	2

			DO NOT ACCEPT All even numbers or multiples of 2 Exponential or geometric General rule in words $2^{n-1}\sqrt{2}$ OE D is increasing **Note for more than two different patterns** All correct award 2 marks Ex: D is increasing, even numbers and $\sqrt{2}$ is always there At least one correct award 1 mark Ex: even numbers and $\sqrt{2}$ is always there and add 2	
	d	.1 the correct general rule .2 the correct simplified general rule with correct notation	.1 $2^{n-1}\sqrt{2}$ OE .2 $D = 2^{n-1}\sqrt{2}$ OE ACCEPT using D instead of D	**2**
	e	.1 correctly substitute $n \geq 5$ into their general rule .2 correctly calculate their value of S after substituting $n \geq 5$ .3 recognise that their correctly calculated value of S is the same as their predicted value	.1 Ex: $2^{5-1}\sqrt{2}$ .2 Ex: $16\sqrt{2}$ (for n=5) •3 Same as value I predicted in table (and we find the candidate has $16\sqrt{2}$ in the table for $n = 5$) **OR** same as when we continue the pattern and explains how $16\sqrt{2}$ is obtained from pattern of multiplying the 8 by 2 •3 ACCEPT seeing the $16\sqrt{2}$ in the table and seeing their calculated $D = 16\sqrt{2}$ when $n = 5$	**3**

8	g	**Mark**	1	2	3	4		
		Predictions (P)	Correctly predict two terms for P OR Correctly predict 4 terms without square root provided that $\sqrt{2}$ is in their response ACCEPT whether in the table or in the response box	Correctly predict four terms for P ACCEPT whether in the table or in the response box				20
		Description (D)	Attempt to describe a pattern in words for P OR a rule in words Ex: Multiplying by 2 every time Related to powers of 2 **OR** Attempt to describe pattern for P as general rule Ex for rule attempt: $2^{n-1}\sqrt{2}$ or $2^{n+1}+D$ $4s+s\sqrt{2}$ **OR** Correctly describe in words their pattern	Correctly describe one pattern in words for P Ex: The coefficient of $\sqrt{2}$ doubles each time adds to number multiplied by 2 **OR** Correctly describe the pattern for P as a general rule Ex: $(P=)2^{n+1}+2^{n-1}\sqrt{2}$ OE ACCEPT the general rule completely not simplified	Correctly describe one pattern in words for P Ex: The coefficient of $\sqrt{2}$ doubles each time **AND** Correctly describe the pattern for P as a general rule Ex: $(P=)2^{n+1}+2^{n-1}\sqrt{2}$ OE ACCEPT the general rule completely not simplified			
		Testing (T)	Attempt to test their general rule for P using $n \leq 4$ Ex: correctly substitute in their general rule value of $n \leq 4$	Correctly test their general rule for P using $n \leq 4$ Ex:				

				Correctly calculate their value for P in their general rule using $n \leq 4$ **AND** Recognise that their correctly calculated value for P is the same as the given value. ACCEPT seeing their correctly calculated value for P and the given value in the table being equal		
			OR Correctly test their described pattern or their rule (e.g. recursive rule)			
		Verifying (V)	Attempt to verify their general rule for P using $n \geq 5$ Ex: correctly substitute in their general rule value of $n \geq 5$ **OR** Correctly verify their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value for P in their general rule using $n \geq 5$	Correctly calculate their value for P in their general rule using $n \geq 5$ **AND** Recognise that their correctly calculated value for P is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for P and their predicted value in the table being equal	
		Justify/proof (J)	Attempt to justify their described pattern or their general rule Examples: trying at least two more values and arguing as justification that they are the same or rule works OR It is a geometric sequence or exponential used as justification OR	Justify their general rule arithmetically Examples: It is a geometric sequence with first term $4+\sqrt{2}$ and ratio 2 Setting equation for a given term and solve it correctly for n **OR** Attempt to justify their general rule geometrically Ex:	Good attempt to justify the general rule for P geometrically by using incorrect four lengths in terms of n Ex: adding incorrect sides in terms of n $2^{n+1}+2^{n+1}+2^n+2^{n-1}\sqrt{2}$ OE	Correctly justify the general rule for P geometrically by using correct lengths in terms of n Ex: $2^{n-1}+2^{n-1}+2^n+2^{n-1}\sqrt{2}$ or $4(2^{n-1})+2^{n-1}\sqrt{2}$ OE J4 automatically gains T2 and V3

		Recognise that P is the sum of 4S+D used as justification	weak attempt to get perimeter using three lengths in terms of n	
Notation and terminology (N)	Correct notation of <u>their</u> rule **OR** Correct terminology describing at least one pattern	Correct notation of <u>the general</u> rule for P **OR** The notation of <u>the general</u> rule includes errors **AND** Correct terminology describing at least one pattern	Correct notation of <u>the general</u> rule for P **AND** Correct terminology describing at least one pattern	
			The general rule: $P = 2^{n+1} + 2^{n-1}\sqrt{2}$ OE	
		DO NOT ACCEPT if they don't have a general rule	ACCEPT the general rule partially simplified but DO NOT ACCEPT the general rule completely not simplified	
	DO NOT ACCEPT if they don't have any rules and they don't describe any patterns		For notation of general rule,	
			DO NOT ACCEPT $P = 2^{\wedge n+1} + 2^{\wedge n-1}\sqrt{2}$ OE $P = 2^n * 2 + 2^n / \sqrt{2}$ OE	
	ACCEPT $(P=)2^{n+1} + 2^{n-1} \times \sqrt{2}$ OE		$(P=)2^{n+1} + 2^{n-1} \times \sqrt{2}$ OE The rule for P is $\mathbf{2^{n+1} + 2^{n-1}\sqrt{2}}$ OE	
Communication (L)	Very weak communication Two or three lines of communication **OR** Only calculations or algebraic steps	Weak communication More than three lines of communication but lack coherence	Good communication More than three lines of coherent communication Can be awarded only if J2 is achieved	

Predictions

Stage (n)	Side (S)	Diagonal (D)	Perimeter (P)
1	1	$\sqrt{2}$	$4+\sqrt{2}$
2	2	$2\sqrt{2}$	$8+2\sqrt{2}$
3	4	$4\sqrt{2}$	$16+4\sqrt{2}$
4	8	$8\sqrt{2}$	$32+8\sqrt{2}$
5	16	$16\sqrt{2}$	$64+16\sqrt{2}$
6	32	$32\sqrt{2}$	$128+32\sqrt{2}$
7	64	$64\sqrt{2}$	$256+64\sqrt{2}$
8	128	$128\sqrt{2}$	$512+128\sqrt{2}$