

Markscheme

November 2020

Mathematics

On-screen examination



21 pages

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The markscheme may make use of the following abbreviations:

RM Assessor has the following annotations that should be used to award marks:

A0 only use to award a zero mark for an answer that has no merit e.g. awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with

Marks awarded by stamping the tick

SEEN

Seen; must be stamped on all blank response areas and on concatenated responses

? unclear

• Bullet notation means award 1 mark – see example 1 below

ECF Marks that can be awarded as error carried forward from previous results in the question

BOD Benefit of the doubt

MR misread

NWS no working shown

SC special case

OE or equivalent

WTTE or words to that effect or accept incomplete calculator display

AG Answer given

			Example 1 • 1 mark awarded and corr	responding notes are aligned		
b	•	Show clear line of reasoning in the m		45 & 49 seen OE		
				eg, 49 = 45 + x		2
						-
	•	4		ACCEPT 45 + X/10 = 4.9 and Ans	1	

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, Error Carried Forward (ECF) marks are awarded after an error.

- a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
- b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or sinx > 1) then subsequent marks should not be awarded.
- c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
- d) To award ECF marks for a question part, there must be working present for that part.
- e) ECF is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
- f) A misread (MR) is an error. ECF is normally awarded.

General points

- a) As this is an international examination, accept all alternative forms of **notation**, for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x² for x² unless noted otherwise in the MS.
- b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer**, then the last mark cannot be awarded.
- c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
- d) In the markscheme, equivalent examples of numerical and algebraic forms or simplified answers will generally be written in the notes preceded by OE or

equivalent e.g.
$$\frac{1}{2}$$
 or 1/2 or 1÷2 and $\frac{x}{2}$ x/2 or x÷2

e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.

- g) Special case marks SC can be allocated instead of but not in addition to the marks prescribed in the markscheme. Accept seeing equation not in-line,
- h) Accept notation errors in intermediate steps,
- i) When a calculator screenshot is taken, accept not seeing the whole operation

Question	Answers	Notes	Total
1 a	24 : 36		1
b	108 x 100		1
C	a ⁶		1

– 5 –



- 6 -

mathmmoeengtz0xxm

$$\begin{bmatrix} 9 \\ 10x^2 + x - 2 \end{bmatrix}$$

mathmmoeengtz0xxm

uestion	Answers	Notes	Total
а	 1 correctly identify the <i>x</i>-coordinate of vertex 2 correctly substitute their <i>x</i> into <i>f</i>(<i>x</i>) 	•1 2.5 OE seen as a result of substituting in $-\frac{b}{2a}$ or when substituting x in $f(x)$	
	•3 correctly calculate their value of $f(2.5)$ after substitution of 2 < their $x < 3$	 •2 – their2.5(their2.5 – 5) Accept only if 2 < their2.5 < 3 •3 their 6.25 OE 	3
b	2		1
С	AM1	AM1	
	•1 correct one factor	•1 $(x-2)$ or $(x-7)$ seen	
	•2 correct second factor AND negative sign	•2 $(g(x) =) - (x - 2)(x - 7)$ OE	
		(g(x) =) - (x - 2)(x - 7) OE without working award 2 marks	
	AM2	AM2	2
	•1 correctly write $g(x)$ in terms of $f(x)$	•1 $f(x-2)$	
	•2 correctly write $g(x)$	•2 - (x - 2)(x - 7) OR - (x - 4.5) ² + 6.25 OE	
d	correctly reflect their quadratic $g(x)$ in the x-axis	(h(x) =) their $(x - 2)(x - 7)$	1
е	opposite value of their positive maximum from (a)	their – 6.25 ACCEPT only if negative	1

Standard (9 marks)

Qı	uestion	Answers	Notes	Total
3	а	 1 seeing 40(%) 2 multiply their 40 % by 25 % AG 0.1 	 •1 0.4, ACCEPT 40(%) on the diagram •2 0.4 x 0.25 OE 	
	b	Correctly write 3 as the answer		1
	c	 1 correctly place their 3 and their 27 in 'First donor' 2 correct probabilities for their O- second branches 3 correct probabilities for their Not O- second branches 	First donor Second donor 2 0 $ 2$ 29 0 $ 27$ Not 0 $-$	
			27 30 Not O -	3
			.1 ACCEPT their 3 only if positive integer 0 <their3<30 .2 or .3 ACCEPT only 0<their probability<1<="" td=""><td></td></their></their3<30 	
	d	•1 correctly calculate the probability first O – and second not O – from their tree diagram	•1 $\frac{\text{their3}}{30} \times \frac{\text{their27}}{\text{their29}}$ or $\frac{27}{290}$ or 0.093(103) seen	
		•2 correctly calculate the probability first not O – and second O – from their tree diagram	•2 $\frac{their27}{30} \times \frac{their3}{their29}$ or $their \frac{27}{290}$ or 0.093(103) seen	3
			•3 <i>their</i> $\frac{27}{145}$ or 0.186(2) OE	

•3 correctly add their two products of probabilities above	.3 ACCEPT 0.19 or 19% .3 DO NOT ACCEPT 0.18 or decimal number that rounds to 0.18 .3 DO NOT ACCEPT 1dp rounding (ex: 0.2 or 20%)	
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Pool uestion	Answers	Notes	Total
a	 1 correctly substitute 1.3 and 4 into the correct volume formula 2 multiply their capacity by 0.9 OR subtract 10% of their capacity from their capacity 3 correctly calculate their volume before rounding 58.8 AG 	 •1 V = π x 16 x 1.3 or V = π x 4² x 1.3 or 20.8 π or 65.3(45) •2 their 65.3(45) x 0.9 OR their 65.3(45)-6.53(45) •3 their 58.81() ACCEPT only if their answer rounds to 58.8 	3
b	 •1 divide their 58800 by 11 200 or their 58.8 by 11.2 •2 5.25 (hours) •3 correctly write their time as hours and minutes 	 1 their 58800 OE 11200 2 5.25 (hours) or 315 (minutes) seen 3 their 5 hours and 15 min 5 hours and 15 min without working award 2 marks SC seeing 5.25 x 11.2 = 58.8 AWARD .1 and .2 	3
C	 1 correctly substitute 4 into the correct area of circle formula 2 correctly substitute 4 into the correct circumference of a circle formula 3 multiply their circumference of circle by 0.3 4 correctly add their •1 and •3 5 correctly round their .4 to the nearest square metre 	•1 $\pi \times 16$ or $\pi \times 4^2$ or 50.27 or 50.24 (using 3.14) •2 $\pi \times 8$ or $2 \times \pi \times 4$ or $25.1(327)$ •3 their $\pi \times 8 \times 0.3$ or their $2 \times \pi \times 4 \times 0.3$ or $7.5(398)$ •4 their 57.8(05) •5 their 58 ACCEPT earlier all correct rounding	5
d	correctly multiply their 57.8(05) or their 58 by 3.4	their (\$)196.52 or (\$)197.2 ACCEPT their 196.52 without working DO NOT ACCEPT their 57.8(05) x 3.4 incorrectly rounded in d)	1

Ques	stion	Answers	Notes	Total
5	a	 1 correctly determine 1.95 2 correct trig ratio used 3 correctly write their answer before rounding or correct inverse trig ratio AG 70(°) 	•1 12.55 - 10.6 •2 $\cos y = \frac{1.95}{5.7}$ accept not seeing this step •3 $y = \cos^{-1} \frac{1.95}{5.7}$ or 69.99(480991) Accept only if their answer rounds to 70	3
	b	AM1 •1 double the 70 •2 correctly subtract 140 from 180	AM1 •1 2×70 or 140 seen •2 40	2
		 AM2 1 subtract 70 from 90 or 160 from 180 2 correctly double 20 	 AM2 1 90 - 70 or 20 seen 2 40 40 without working AWARD 2 marks Accept correct answer on the canvas in part (a) 	2
	C	 AM1 •1 correct trig ratio used with 10.6 OR correctly substitute into sine rule 	AM1 •1 $\cos 70 = \frac{10.6}{v}$ or $\sin 20 = \frac{10.6}{v}$ OE OR $\frac{v}{\sin 70} = \frac{21.2}{\sin 40}$ OE •1 ACCEPT 10.6 and $\cos 70$ or $\sin 20$ seen	
		 2 correctly rearrange their trig ratio for v 3 correctly calculate their v after dividing by their trig ratio 	•2 their $(v =) \frac{10.6}{\cos 70}$ or their $(v =) \frac{10.6}{\sin 20}$ or their $(v =) \frac{21.2 \times \sin 70}{\sin 40}$ •3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded	3

	AM2	AM2	
	 1 correct trig ratio used with 12.55 OR correctly substitute into sine rule 2 correctly calculate their hypotenuse 3 correctly calculate their v after subtracting 5.7 from their calculated hypotenuse 	•1 $\cos 70 = \frac{12.55}{hyp}$ OR $\frac{a}{sin70} = \frac{25.1}{\sin 40}$ OE •2 their 36.69(37) ACCEPT $(hyp =)\frac{12.55}{\cos 70}$ •3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded	
	 1 seeing evidence of using similar triangles correctly 2 correctly rearrange for v OR correctly apply their scale factor 3 correctly calculate their v after dividing their product 	AM3 •1 seeing $\frac{12.55}{1.95}$ or $\frac{10.6}{12.55}$ OE or $\frac{v}{10.6} = \frac{v+5.7}{12.55}$ OE •2 $10.6 \times \frac{5.7}{1.95}$ OR $1.95v = 60.42$ •3 their 30.99(23) or 31 or 31.2(11) ACCEPT only if .1 or .2 is awarded	
d	 1 correctly substitute their v or theirv+5.7 into correct area of circle formula 2 subtract their areas of circles 3 multiply at least one area by their40/360 4 correctly calculate their area after multiplying their40/360 by their difference of areas 	•1 $\pi \times their 36.7^2$ or $\pi \times their 31^2$ seen •2 $\pi \times their 36.7^2 - \pi \times their 31^2$ OE •3 their area $\times their \frac{40}{360}$ •4 their 134.6() or 134.7() or 135	4
e	 AM1 1 correctly calculate the material wasted 2 correctly write their fraction of material wasted 3 correctly calculate percentage of their fraction of area AM2 1 correctly calculate the fraction area of material used 2 Subtract their fraction from 1 3 correctly write their difference above as percentage 	AM1 •1 (25.1 x 7.6 – their 134.6 =) their 56.16 ACCEPT [55.7, 56.2] •2 $\frac{their 56.16}{their 25.1 \times 7.6}$ •3 their 29() (%) AM2 •1 $\frac{their 134.6}{their 25.1 \times 7.6}$ •2 1 – their 0.705() •3 their 29() (%)	3

Ques	stion	Answers	Notes	Total
6	a	 1 correctly drag three heights 2 correctly drag the other three heights 	26 24 22 39 10 10 10 10 10 20 30 40 50 60 Annual exposure / (E)	2
	b	A valid statement including both 1990 and 2016	SC: All six bars correct and thinner award 1 mark DO NOT ACCEPT a comment on only one interval Ex: in 1990, 3 countries had E between 0 and 10 while in 2016 there are 8 Ex: Highest in 1990 is 17 while highest in 2016 is 24 ACCEPT Overall correct comment ex: Less particulates or less exposure in the air in 2016 than 1990 OE Comments including more than one interval. Ex: In 2016 no countries had E more than 40 but in 1990 there were	1
	C	 1 seeing two correct midpoints and two correct frequencies 2 Add their four products of midpoints and frequencies 3 Divide the sum of their products by 39 4 Correctly write their mean before rounding AG 15.3 (µg/m³) 	 Any two of 5,15,25,35 AND any two of 8, 24, 5, 2 seen 2 5 x 8 + 15 x 24 + 25 x 5 + 35 x 2 OR 595 <i>their</i> ⁵⁹⁵/₃₉ 4 15.25(6). ACCEPT only if their answer rounds to 15.3 	4
	d	•1 (a =) $-\frac{1}{5}$ OE •2 (b =) 18	 1 ACCEPT - 0.21< a < -0.14 2 ACCEPT correctly calculated <i>b</i> from their correct <i>a</i> 	2

6	e	Mark	1	2	3	
		F Identification of Factors	One factor mentioned from: The model/equation used to predict. or Whether the data will follow same trend or not or Acknowledge the E is decreasing or The fact that these are models based on scatter plots. ACCEPT factors affecting the trend like more data in the last 5 years collected or using electric cars or new technology that affects the environment or using solar power instead of petrol or air pollution or control of particular matter DO NOT ACCEPT only saying many factors			10
		PL (Prediction from line)	Attempt to calculate the E in 2030 using their line equation Ex: Substitute 40 in the equation and make incorrect calculation OR Substitute 2030 into the equation and make correct calculation OR Attempt to use the drop every 5 years to predict the drop in 2030 from the graph OR Describe the trend in words and predict it will reach below 13	Correctly calculate the E in 2030 using their line equation Ex: their (-0.15(40)+18 =)12 OR correctly use the drop every 5 years to predict the drop in 2030 from the graph		

PE	Attempt to calculate the E in 2030	Correctly calculate the E in 2030 using the	
(Prediction	using the exponential equation	exponential equation	
from		(6×0.91 ⁴⁰ +13.8 =)13.9(379), accept 14	
Exponential)	Ex: Substitute 40 in the equation		
	$(6 \times 0.91^{40} + 13.8)$ and make		
	incorrect calculation		
	OR	OR	
	Substitute 2030 into the equation	use the drop every 5 years to predict the	
	and make correct calculation	drop in 2030 from the graph	
	OR	ACCEPT if they mention that the	
	Attempt to use the drop every 5 years to predict the drop in 2030	ACCEPT if they mention that the exponential will never reach 13 and refer to	
	from the graph	horizontal assymptote being E = 13.8	
	OR		
	Describe the trend in words and		
	predict it will not reach 13		
D	Inaccurate with weak reason	Inaccurate with a valid reason related to	
degree of	(Interpolating)	variables affecting the future	
accuracy	Example:	(extrapolating)	
	inaccurate because I used line of	Ex:	
	best fit	The prediction I made not very accurate	
	Inaccurate because of sudden	because many factors may vary in the future	
	increase of pollution happening OR	or	
	Accurate with valid reason	predictions using line of best fit for the	
	Example:	future not guaranteed	
	accurate because I used my line of	or	
	best fit to estimate and made correct	The prediction using the line equation not	
	calculations	very accurate because it is taking only a	
	OR	window or isolated time	
	Rounding to a whole number used	or	
	for their estimated year	The prediction using the line equation not	
	Ex: 14 for the E from exponential	very accurate because it assumes the	
	DO NOT ACCEPT if they just write	future follows same pattern OR	
	down a year without any reference	My predictions not very accurate because	
	or calculations	a disaster may happen and affect pollution	
	DO NOT ACCEPT accurate or	DO NOT ACCEPT if they did not make a	
	inaccurate without reason	prediction	

Justifying the model(even implicitly ex: substituting only in exponential model) without justificationOR Select the line with acceptable justification Ex: it passes through	Selecting the Exponential model (even implicitly) with general justification (no explicit data) Ex: I advise using graph 2 because it takes most of the points into account or Line will decrease in a short time while exponential will take longer or exponential more fitting	Selecting the Exponential model (even implicitly) With good justification involving data in the last years Ex: graph 2 is better because in the last years, the E did not vary much or it is nearly constant in the last years and it takes this into account while graph 1 doesn't or the decrease in the line is constant while E is not is not and the exponential does not have constant decrease rate	
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	Answers	Notes	Total
а	adding the four sides 12 AG	4 x 3 or 3 + 3 + 3 + 3	1
b	1 correctly place 442 correctly place 52		2
C	 1 correctly describe one pattern in words 2 correctly describe a second pattern in words 	ACCEPT (P) goes up by 8WTTE, ACCEPT expands They are all even numbers or divisible by 2 or multiples of 2 The difference is constant It is linear or arithmetic DO NOT ACCEPT The rule in words ex: 8 times n then add 4 P is four times L P = 4L n goes up by 1 It is increasing general rules in terms of n, example: $P = 8n + 4$	2

d	 1 The correct general rule 2 The correct simplified general rule with correct notation 	 1 8n + 4 or 12+8(n - 1) or Un = 8n + 4 OE 2 P = 8n + 4, ACCEPT P = 4(2n + 1) DO NOT ACCEPT description in words 	2
e	 1 correctly substitute n ≥ 5 into their rule 2 correctly calculate their value of P after substitution n ≥ 5 3 recognize that their result is the same as their predicted value 	 1 Ex: 8 x 5 + 4 2 Ex: 44 (for the <i>n</i> = 5) 3 Same as value I predicted in table (and we find the candidate has 44 in the table for <i>n</i> = 5) OR same as when we continue the pattern and explains how 44 is obtained from pattern of adding 8 to 36 3 ACCEPT seeing the 44 in the table and seeing their calculated P = 44 when <i>n</i> = 5 SC for 1 mark Correctly test by applying the steps of verification mentioned in the left column with a value of <i>n</i> ≤ 4 SC for 1 mark Correctly verify their described pattern or rule (e.g. recursive rule) 	3

7 f	Mark	1	2	3	4	
	Predictions (P)	Correctly predict one value for A	Correctly predict two values for A			
	Description (D)	Attempt to describe a pattern in words or to write a rule Ex: Comment on difference, all are odd numbers OR Correctly describe one pattern in words for L OR Correctly describe in words their pattern for A OR Attempt to write down a general rule for L, example: 2n	Correctly describe one pattern in words for A Ex: A has second difference constant A is quadratic A is square numbers DO NOT ACCEPT the general rule in words OR Correctly write down general rule for L in terms of n. Rule: (L=)2n + 1 OR	Correctly describe one pattern in words for A AND Acceptable attempt to write down a general rule for A OR Correctly write down general rule for A in terms of n. $(2n + 1)^2$ OE OR Correctly describe one pattern in words for A AND correctly write down general rule for L For notation see N	Correctly describe one pattern in words for A AND Correctly write down the general rule for A in terms of <i>n</i> For notation see N	20

Testing (T)	ORWeak attempt to write down a general rule for A, example: n^2 DO NOT ACCEPT L or A is increasing 	Acceptable attempt to write down a general rule for A, example: $2n+1^2$ or recursive rule For notation see N Correctly test their general rule for A using $n \le 4$		
	Ex: correctly substitute in their general rule value of $n \le 4$ Ex: substitute in their general rule value of $n \le 4$ OR Correctly test their described pattern or their rule (e.g. recursive rule)	Ex: Correctly calculate their value for A in their general rule using $n \le 4$ AND Recognise that their correctly calculated value for A is the same as the given value. ACCEPT seeing their correctly calculated value for A and the given value in the table being equal		
(V)	Attempt to verify their general rule for A using $n \ge 5$ Ex: correctly substitute in their general rule value of $n \ge 5$ OR Correctly verify their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value of A using their $n \ge 5$ in their general rule	Correctly calculate their value for A in their general rule using $n \ge 5$ AND Recognise that their correctly calculated value for A is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for A and their predicted value in the table being equal	

· · · · · ·					1.1
J	Justify/proof (J)	Weak attempt to justify their described pattern or their general rule Examples: trying at least two more values and arguing as justification that they are the same or rule works OR Just seeing their rule as $(2n + 1)^2$ without any explanation OR Assuming Quadratic model and valid attempt to find coefficients DO NOT ACCEPT if D2 not	Good attempt to justify their general rule for A Examples: Assuming Quadratic model and get correct values of coefficients using any method OR squaring incorrect rule for L with a comment on squaring sides	correctly justify the correct general rule for A in relation to geometry Examples: Writing in words OE that squaring the side we get the area if their L rule is L= $2n + 1$ and their general rule is $(2n + 1)^2$ J3 automatically gains T2 and V3	
		achieved			
	Notation and terminology (N)	Correct notation of <u>their</u> rule OR Correct terminology describing a pattern DO NOT ACCEPT if they don't have any rules and they don't describe any patterns correct	Correct notation of <u>the general</u> rule for A OR The notation of <u>the general</u> rule includes errors AND Correct terminology describing pattern in words for A DO NOT ACCEPT if they don't have a general rule	Correct notation of <u>the general</u> rule for A AND Correct terminology describing pattern in words for A ACCEPT using Un instead of A only if they mention that A = U_n For notation of the general rule, DO NOT ACCEPT $(2n+1)^2$ or $(2xn+1)^2$ or $4xn^2+4n+1$ or The rule for A is $(2n + 1)^2$ Or non simplified rules ex: $A = (n + n + 1)^2$	

Communicati	Very weak communication	Weak communication	Good communication	
on (L)	More than two lines of communication that lacks organisation and coherence OR	At least three lines of communication with an attempt of organisation but lacks coherence	More than three lines of communication with acceptable organisation and coherence	
	Only calculations or algebraic steps		Can be awarded only if J2 is achieved	

Stage number (<i>n</i>)	Side length (L)	Area of square (A)
1	3	9
2	5	25
3	7	49
4	9	81
5	11	121
6	13	169
7	15	225
8	17	289

General rules:

L = 2n + 1 $A = (2n + 1)^2$ or $4n^2 + 4n + 1$