Markscheme

November 2020

Mathematics

On-screen examination

21 pages

The markscheme may make use of the following abbreviations:

ECF Marks that can be awarded as **error carried forward** from previous results in the question
BOD Benefit of the doubt
MR misread
NWS no working shown
SC special case
OE or equivalent
WTTE or words to that effect
AG Answer given

- Bullet notation means award 1 mark – see example 1 below

Example
•[1] mark awarded and corresponding notes are aligned

b	•[1] Show clear line of reasoning in the method •[2] their correct result	•[1] 45 & 49 seen **OE** eg, $49 = 45 + x$ •[2] their 4	**2**

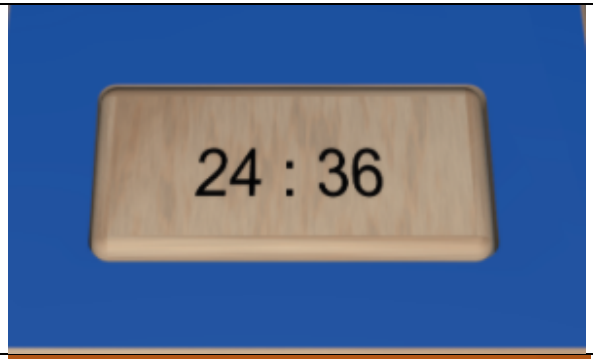
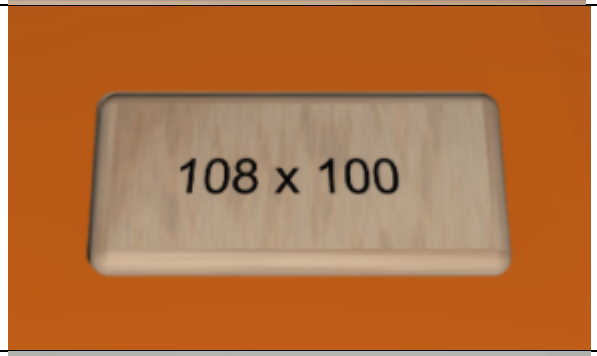
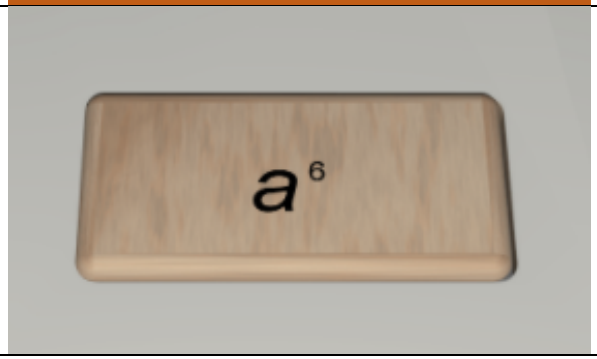
Error Carried Forward (ECF) marks

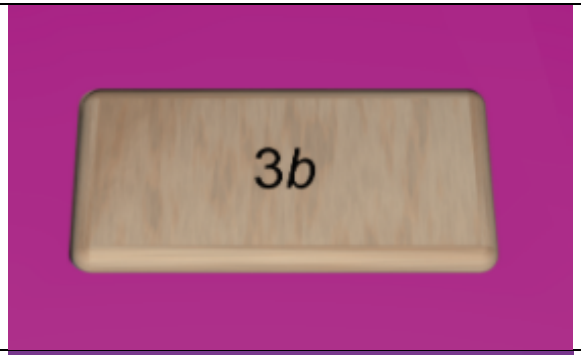
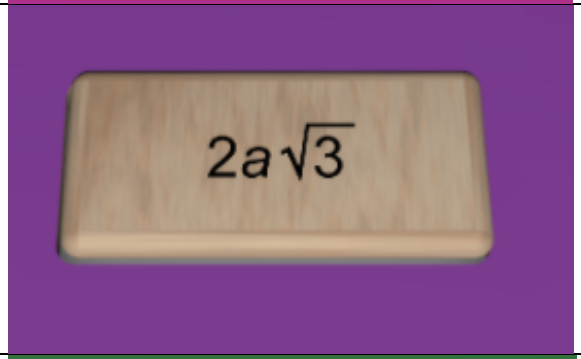
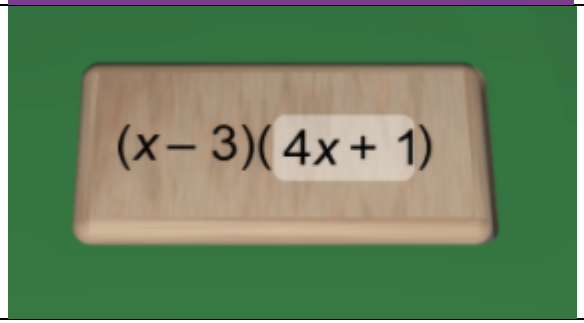
Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

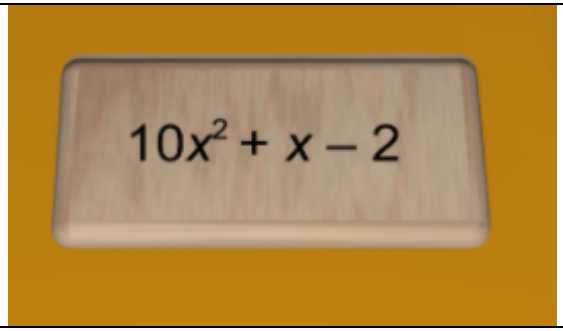
a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
b) If an answer resulting from **ECF** is inappropriate (eg, negative distances or sinx > 1) then subsequent marks should not be awarded.
c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
d) To award **ECF** marks for a question part, **there must be working present for that part**.
e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
f) A misread (**MR**) is an error**. ECF** is normally awarded.

General points

a) As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 or 1 000 or 1.000. However, **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 in final answers unless noted otherwise in the MS.
b) Accept notation errors in intermediate steps.
c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,** then the last mark cannot be awarded.
d) In the case when a correct result is obtained by coincidence based on incorrect seen method, do not award the mark for the result.
e) Where candidates have written two solutions to a question, mark the response that deserves more marks.
f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g. $\frac{1}{2}$ or 1/2 or $1 \div 2$ and $\frac{x}{2}$ or $x / 2$ or $x \div 2$
g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme..
i) When a calculator screenshot is taken, accept not seeing the whole operation.
j) Accept seeing an equation not in-line

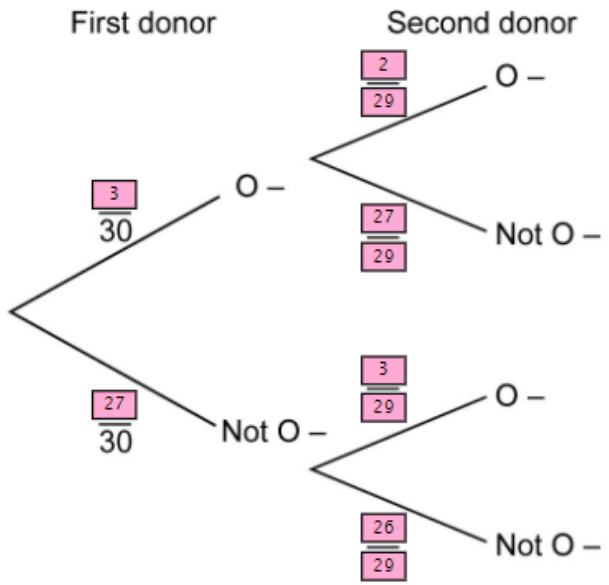
Question		Answers	Notes	Total
1	a			1
	b			1
	c			1

	d			**1**
	e			**1**
	f			**1**

	g				**1**

Question		Answers	Notes	Total
2	a	•1 correctly identify the x-coordinate of vertex •2 correctly substitute their x into $f(x)$ •3 correctly calculate their value of $f(2.5)$ after substitution of $2 <$ their $x < 3$	•1 2.5 **OE** seen as a result of substituting in $-\frac{b}{2a}$ or when substituting x in $f(x)$ •2 – their2.5(their2.5 – 5) Accept only if $2 <$ their2.5 < 3 •3 their 6.25 **OE**	**3**
	b	2		**1**
	c	**AM1** •1 correct one factor •2 correct second factor AND negative sign **AM2** •1 correctly write $g(x)$ in terms of $f(x)$ •2 correctly write $g(x)$	**AM1** •1 $(x – 2)$ or $(x – 7)$ seen •2 $(g(x) =) – (x – 2)(x – 7)$ **OE** $(g(x) =) – (x – 2)(x – 7)$ **OE** without working award 2 marks **AM2** •1 $f(x – 2)$ •2 $– (x – 2)(x – 7)$ OR $– (x – 4.5)^2 + 6.25$ **OE**	**2**
	d	correctly reflect their quadratic $g(x)$ in the x-axis	$(h(x) =)$ their $(x – 2)(x – 7)$	**1**
	e	opposite value of their positive maximum from (a)	their – 6.25 ACCEPT only if negative	**1**

Standard (9 marks)

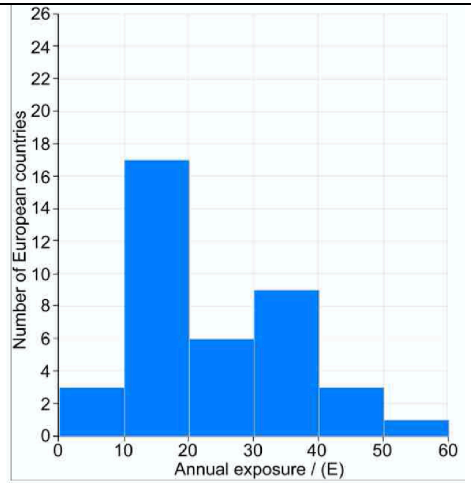
Question		Answers	Notes	Total
3	a	•1 seeing 40(%) •2 multiply their 40 % by 25 % **AG** 0.1	•1 0.4, ACCEPT 40(%) on the diagram •2 0.4 x 0.25 **OE**	**2**
	b	Correctly write 3 as the answer		**1**
	c	•1 correctly place their 3 and their 27 in 'First donor' •2 correct probabilities for their O- second branches •3 correct probabilities for their Not O- second branches	 .1 ACCEPT their 3 only if positive integer 0<their3<30 .2 or .3 ACCEPT only 0<their probability<1	**3**
	d	•1 correctly calculate the probability first **O–** and second not **O–** from their tree diagram •2 correctly calculate the probability first not **O–** and second **O–** from their tree diagram	•1 $\frac{their3}{30}\times\frac{their27}{their29}$ or $their\frac{27}{290}$ or 0.093(103…) seen •2 $\frac{their27}{30}\times\frac{their3}{their29}$ or $their\frac{27}{290}$ or 0.093(103…) seen •3 $their\frac{27}{145}$ or 0.186(2…) **OE**	**3**

		•3 correctly add their two products of probabilities above	.3 ACCEPT 0.19 or 19% .3 DO NOT ACCEPT 0.18 or decimal number that rounds to 0.18 .3 DO NOT ACCEPT 1dp rounding (ex: 0.2 or 20%)	

Pool Question		Answers	Notes	Total
4	a	•1 correctly substitute 1.3 and 4 into the correct volume formula •2 multiply their capacity by 0.9 OR subtract 10% of their capacity from their capacity •3 correctly calculate their volume before rounding 58.8 **AG**	•1 V = π x 16 x 1.3 or V = π x 4^2 x 1.3 or 20.8π or 65.3(45….) •2 their 65.3(45..) x 0.9 OR their65.3(45..)–6.53(45..) •3 their 58.81(…) ACCEPT only if their answer rounds to 58.8	**3**
	b	•1 divide their 58800 by 11 200 or their 58.8 by 11.2 •2 5.25 (hours) •3 correctly write their time as hours and minutes	•1 $\frac{\textit{their}58800}{11200}$ **OE** •2 5.25 (hours) or 315 (minutes) seen •3 their 5 hours and 15 min 5 hours and 15 min without working award 2 marks **SC** seeing 5.25 x 11.2 = 58.8 AWARD .1 and .2	**3**
	c	•1 correctly substitute 4 into the correct area of circle formula •2 correctly substitute 4 into the correct circumference of a circle formula •3 multiply their circumference of circle by 0.3 •4 correctly add their •1 and •3 •5 correctly round their .4 to the nearest square metre	•1 $\pi\times16$ **or** $\pi\times4^2$ or 50.27 **or** 50.24 (using 3.14) •2 $\pi\times8$ or $2\times\pi\times4$ or 25.1(327…) •3 their $\pi\times8\times0.3$ or their $2\times\pi\times4\times0.3$ or 7.5(398…) •4 their 57.8(05…) •5 their 58 ACCEPT earlier all correct rounding	**5**
	d	correctly multiply their 57.8(05…) or their 58 by 3.4	their ($)196.52 or ($)197.2 ACCEPT their 196.52 without working DO NOT ACCEPT their 57.8(05…) x 3.4 incorrectly rounded in d)	**1**

Question		Answers	Notes	Total
5	a	•1 correctly determine 1.95 •2 correct trig ratio used •3 correctly write their answer before rounding or correct inverse trig ratio AG 70(°)	•1 12.55 – 10.6 •2 $\cos y = \frac{1.95}{5.7}$ accept not seeing this step •3 $y = \cos^{-1}\frac{1.95}{5.7}$ or 69.99(480991) Accept only if their answer rounds to 70	3
	b	**AM1** •1 double the 70 •2 correctly subtract 140 from 180 **AM2** •1 subtract 70 from 90 **or** 160 from 180 •2 correctly double 20	**AM1** •1 2×70 or 140 seen •2 40 **AM2** •1 90 – 70 or 20 seen •2 40 40 without working AWARD 2 marks Accept correct answer on the canvas in part (a)	2
	c	**AM1** •1 correct trig ratio used with 10.6 OR correctly substitute into sine rule •2 correctly rearrange their trig ratio for v •3 correctly calculate their v after dividing by their trig ratio	**AM1** •1 $\cos70 = \frac{10.6}{v}$ or $\sin20 = \frac{10.6}{v}$ **OE** OR $\frac{v}{\sin70} = \frac{21.2}{\sin40}$ **OE** •1 ACCEPT 10.6 and cos70 or sin 20 seen •2 their $(v =)\frac{10.6}{\cos70}$ or their $(v =)\frac{10.6}{\sin20}$ or *their* $(v =)\frac{21.2\times\sin70}{\sin40}$ •3 their 30.99(23.…) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded	3

		AM2 •1 correct trig ratio used with 12.55 OR correctly substitute into sine rule •2 correctly calculate their hypotenuse •3 correctly calculate their v after subtracting 5.7 from their calculated hypotenuse **AM3** •1 seeing evidence of using similar triangles correctly •2 correctly rearrange for v OR correctly apply their scale factor •3 correctly calculate their v after dividing their product	**AM2** •1 $\cos 70 = \frac{12.55}{hyp}$ OR $\frac{a}{sin70} = \frac{25.1}{\sin 40}$ **OE** •2 their 36.69(37…) ACCEPT $(hyp =)\frac{12.55}{\cos 70}$ •3 their 30.99(23.…) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded **AM3** •1 seeing $\frac{12.55}{1.95}$ or $\frac{10.6}{12.55}$ OE or $\frac{v}{10.6} = \frac{v+5.7}{12.55}$ **OE** •2 $10.6 \times \frac{5.7}{1.95}$ OR 1.95v = 60.42 •3 their 30.99(23.…) or 31 or 31.2(11..) ACCEPT only if .1 or .2 is awarded	
	d	•1 correctly substitute their v or theirv+5.7 into correct area of circle formula •2 subtract their areas of circles •3 multiply at least one area by their40/ 360 •4 correctly calculate their area after multiplying their40/360 by their difference of areas	•1 $\pi \times their 36.7^2$ or $\pi \times their 31^2$ seen •2 $\pi \times their 36.7^2 - \pi \times their 31^2$ **OE** •3 $their\ area \times their \frac{40}{360}$ •4 their 134.6(…) or 134.7(…) or 135	**4**
	e	**AM1** •1 correctly calculate the material wasted •2 correctly write their fraction of material wasted •3 correctly calculate percentage of their fraction of area **AM2** •1 correctly calculate the fraction area of material used •2 Subtract their fraction from 1 •3 correctly write their difference above as percentage	**AM1** •1 (25.1 x 7.6 – their 134.6 =) their 56.16 ACCEPT [55.7 , 56.2] •2 $\frac{their\ 56.16}{their\ 25.1 \times 7.6}$ •3 their 29(…..) (%) **AM2** •1 $\frac{their\ 134.6}{their\ 25.1 \times 7.6}$ •2 1 – their 0.705(…) •3 their 29(….) (%)	**3**

Question		Answers	Notes	Total
6	a	•1 correctly drag three heights •2 correctly drag the other three heights	 **SC:** All six bars correct and thinner award 1 mark	**2**
	b	A valid statement including both 1990 and 2016	DO NOT ACCEPT a comment on only one interval Ex: in 1990, 3 countries had E between 0 and 10 while in 2016 there are 8 Ex: Highest in 1990 is 17 while highest in 2016 is 24 ACCEPT Overall correct comment ex: Less particulates or less exposure in the air in 2016 than 1990 **OE** Comments including more than one interval. Ex: In 2016 no countries had E more than 40 but in 1990 there were	**1**
	c	•1 seeing two correct midpoints and two correct frequencies •2 Add their four products of midpoints and frequencies •3 Divide the sum of their products by 39 •4 Correctly write their mean before rounding AG 15.3 (μg/m^3)	•1 Any two of 5,15,25,35 AND any two of 8, 24, 5, 2 seen •2 5 x 8 + 15 x 24 + 25 x 5 + 35 x 2 OR 595 •3 *their* $\frac{595}{39}$ •4 15.25(6….). ACCEPT only if their answer rounds to 15.3	**4**
	d	•1 (a =) $-\frac{1}{5}$ **OE** •2 (b =) 18	•1 ACCEPT $-0.21 < a < -0.14$ •2 ACCEPT correctly calculated b from their correct a	**2**

6	**e**		**Mark**	**1**	**2**	**3**	**10**
			F Identification of Factors	One factor mentioned from: The model/equation used to predict. or Whether the data will follow same trend or not or Acknowledge the E is decreasing or The fact that these are models based on scatter plots. ACCEPT factors affecting the trend like more data in the last 5 years collected or using electric cars or new technology that affects the environment or using solar power instead of petrol or air pollution or control of particular matter DO NOT ACCEPT only saying many factors		
			PL (Prediction from line)	Attempt to calculate the E in 2030 using their line equation Ex: Substitute 40 in the equation and make incorrect calculation **OR** Substitute 2030 into the equation and make correct calculation **OR** Attempt to use the drop every 5 years to predict the drop in 2030 from the graph **OR** Describe the trend in words and predict it will reach below 13	Correctly calculate the E in 2030 using their line equation Ex: their $(-0.15(40)+18=)12$ **OR** correctly use the drop every 5 years to predict the drop in 2030 from the graph	

			PE (Prediction from Exponential)	Attempt to calculate the E in 2030 using the exponential equation Ex: Substitute 40 in the equation $(6\times 0.91^{40}+13.8)$and make incorrect calculation **OR** Substitute 2030 into the equation and make correct calculation **OR** Attempt to use the drop every 5 years to predict the drop in 2030 from the graph OR Describe the trend in words and predict it will <u>not</u> reach 13	Correctly calculate the E in 2030 using the exponential equation $(6\times 0.91^{40}+13.8=)13.9(379)$, accept 14 **OR** use the drop every 5 years to predict the drop in 2030 from the graph ACCEPT if they mention that the exponential will never reach 13 and refer to horizontal assymptote being E = 13.8		
			D degree of accuracy	Inaccurate with weak reason (Interpolating) Example: inaccurate because I used line of best fit Inaccurate because of sudden increase of pollution happening OR Accurate with valid reason Example: accurate because I used my line of best fit to estimate and made correct calculations OR Rounding to a whole number used for their estimated year Ex: 14 for the E from exponential DO NOT ACCEPT if they just write down a year without any reference or calculations DO NOT ACCEPT accurate or inaccurate without reason	Inaccurate with a valid reason related to variables affecting the future (extrapolating) Ex: The prediction I made not very accurate because many factors may vary in the future or predictions using line of best fit for the future not guaranteed or The prediction using the line equation not very accurate because it is taking only a window or isolated time or The prediction using the line equation not very accurate because it assumes the future follows same pattern OR My predictions not very accurate because a disaster may happen and affect pollution DO NOT ACCEPT if they did not make a prediction		

		J Justifying the model	Selecting the Exponential model (even implicitly ex: substituting only in exponential model) without justification **OR** Select the line with acceptable justification Ex: it passes through more points **OR** comparison comment without selection Ex: Line will decrease in a short time while exponential will take longer **OE**	Selecting the Exponential model (even implicitly) with general justification (no explicit data) Ex: I advise using graph 2 because it takes most of the points into account or Line will decrease in a short time while exponential will take longer or exponential more fitting	Selecting the Exponential model (even implicitly) With good justification involving data in the last years Ex: graph 2 is better because in the last years, the E did not vary much or it is nearly constant in the last years and it takes this into account while graph 1 doesn't or the decrease in the line is constant while E is not is not and the exponential does not have constant decrease rate	

		Answers	**Notes**	**Total**
7	**a**	adding the four sides 12 **AG**	4 x 3 or 3 + 3 + 3 + 3	**1**
	b	•1 correctly place 44 •2 correctly place 52		**2**
	c	•1 correctly describe one pattern in words •2 correctly describe a second pattern in words	ACCEPT (P) goes up by 8 …WTTE , ACCEPT expands They are all even numbers or divisible by 2 or multiples of 2 The difference is constant It is linear or arithmetic DO NOT ACCEPT The rule in words ex: 8 times n then add 4 P is four times L P = 4L n goes up by 1 It is increasing general rules in terms of n, example: P = 8n + 4	**2**

	d	•1 The correct general rule •2 The correct simplified general rule with correct notation	•1 $8n + 4$ or $12+8(n – 1)$ or $U_n = 8n + 4$ OE •2 $P = 8n + 4$, ACCEPT $P = 4(2n + 1)$ DO NOT ACCEPT description in words	**2**
	e	•1 correctly substitute $n \geq 5$ into their rule •2 correctly calculate their value of P after substitution $n \geq 5$ •3 recognize that their result is the same as their predicted value	•1 Ex: 8 x 5 + 4 •2 Ex: 44 (for the $n = 5$) •3 Same as value I predicted in table (and we find the candidate has 44 in the table for $n = 5$) **OR** same as when we continue the pattern and explains how 44 is obtained from pattern of adding 8 to 36 •3 ACCEPT seeing the 44 in the table and seeing their calculated P = 44 when $n = 5$ **SC for 1 mark** Correctly test by applying the steps of verification mentioned in the left column with a value of $n \leq 4$ **SC for 1 mark** Correctly verify their described pattern or rule (e.g. recursive rule)	**3**

7	**f**	**Mark**	**1**	**2**	**3**	**4**	**20**
		Predictions (P)	Correctly predict one value for A	Correctly predict two values for A			
		Description (D)	Attempt to describe a pattern in words or to write a rule Ex: Comment on difference, all are odd numbers **OR** Correctly describe one pattern in words for L **OR** Correctly describe in words their pattern for A **OR** Attempt to write down a general rule for L, example: 2n	Correctly describe one pattern in words for A Ex: A has second difference constant A is quadratic A is square numbers DO NOT ACCEPT the general rule in words **OR** Correctly write down general rule for L in terms of n. Rule: $(L=)2n + 1$ **OR**	Correctly describe one pattern in words for A **AND** Acceptable attempt to write down a general rule for A **OR** Correctly write down general rule for A in terms of n. $(2n+1)^2$ **OE** **OR** Correctly describe one pattern in words for A **AND** correctly write down general rule for L For notation see N	Correctly describe one pattern in words for A **AND** Correctly write down the general rule for A in terms of n For notation see N	

			OR Weak attempt to write down a general rule for A, example: n^2 **DO NOT ACCEPT** L or A is increasing *n* goes up by 1	Acceptable attempt to write down a general rule for A, example: 2*n*+1^2 or recursive rule For notation see N			
		Testing (T)	Attempt to test their general rule for A using $n \leq 4$ Ex: correctly substitute in their general rule value of $n \leq 4$ Ex: substitute in their general rule value of $n \leq 4$ **OR** Correctly test their described pattern or their rule (e.g. recursive rule)	Correctly test their general rule for A using $n \leq 4$ Ex: Correctly calculate their value for A in their general rule using $n \leq 4$ **AND** Recognise that their correctly calculated value for A is the same as the given value. ACCEPT seeing their correctly calculated value for A and the given value in the table being equal			
		Verifying (V)	Attempt to verify their general rule for A using $n \geq 5$ Ex: correctly substitute in their general rule value of $n \geq 5$ **OR** Correctly verify their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value of A using their $n \geq 5$ in their general rule	Correctly calculate their value for A in their general rule using $n \geq 5$ **AND** Recognise that their correctly calculated value for A is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for A and their predicted value in the table being equal		

			Justify/proof (J)	Weak attempt to justify their described pattern or their general rule Examples: trying at least two more values and arguing as justification that they are the same or rule works OR Just seeing their rule as $(2n + 1)^2$ without any explanation OR Assuming Quadratic model and valid attempt to find coefficients DO NOT ACCEPT if D2 not achieved	Good attempt to justify their general rule for A Examples: Assuming Quadratic model and get correct values of coefficients using any method OR squaring incorrect rule for L with a comment on squaring sides	correctly justify the correct general rule for A in relation to geometry Examples: Writing in words OE that squaring the side we get the area if their L rule is L= $2n + 1$ and their general rule is $(2n + 1)^2$ J3 automatically gains T2 and V3		
			Notation and terminology (N)	Correct notation of <u>their</u> rule **OR** Correct terminology describing a pattern DO NOT ACCEPT if they don't have any rules and they don't describe any patterns correct	Correct notation of <u>the general</u> rule for A **OR** The notation of <u>the general</u> rule includes errors **AND** Correct terminology describing pattern in words for A DO NOT ACCEPT if they don't have a general rule	Correct notation of <u>the general</u> rule for A **AND** Correct terminology describing pattern in words for A ACCEPT using Un instead of A only if they mention that A = U_n For notation of the general rule, DO NOT ACCEPT (2*n*+1)^2 or (2x*n*+1)^2 or 4x*n*^2+4*n*+1 **or** The rule for A is $(2n+1)^2$ Or non simplified rules ex: $A = (n + n + 1)^2$		

		Communication (L)	Very weak communication More than two lines of communication that lacks organisation and coherence **OR** Only calculations or algebraic steps	Weak communication At least three lines of communication with an attempt of organisation but lacks coherence	Good communication More than three lines of communication with acceptable organisation and coherence Can be awarded only if J2 is achieved			

Stage number (*n*)	**Side length (L)**	**Area of square (A)**
1	3	9
2	5	25
3	7	49
4	9	81
5	11	121
6	13	169
7	15	225
8	17	289

General rules:

$L = 2n + 1$

$A = (2n + 1)^2$ *or* $4n^2 + 4n + 1$