Mathematics: analysis and approaches

Higher level

Paper 1

Markscheme

1. (a) $a = 5$ *A1*

[1 mark]

(b) (i) period $=\pi$ *A1*

(ii) $b = \frac{2\pi}{\pi} = 2$ *A1A1*

[3 marks]

c) substituting $\frac{\pi}{12}$ into their $f(x)$ *M1*

$cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ *(A1)*

$=5\frac{\sqrt{3}}{2}$ *A1*

[3 marks]

Total [7 marks]

2. (a) Attempt to form $(g \circ f)(x)$ *M1*

$(g \circ f)(x) = (x + 3)^2 + h^2 = x^2 + 6x + 9 + h^2$ *A1*

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression =34 *(M1)*

$h^2 = 9$ *A1*

$h = \pm 3$ *A1*

[3 marks]

Total [5 marks]

3. (a) P(A∪B) = P(A) + P(B) - P(A)xP(B) *A1*

P(A∪B) = 0.73 *A1*

[2 marks]

(b) P(B'∩ A) using a Venn diagram *M1*

P(B'∩ A) = 0.18 *A1*

[2 marks]

Total [4 marks]

4. (a) $u_9 = S_5 - S_4$ *M1*

$u_9 = 35 - 24 = 11$ *A1*

[2 marks]

(b) $24 = \frac{4}{2}(2u_1 + (4 - 1)d$

$47 = \frac{5}{2}(2u_1 + (5 - 1)d$ *M2*

attempt to solve the simultaneous equations

$d = 2$ *A1*

$u_1 = 3$ *A1*

[4 marks]

Total [6 marks]

5. attempt to use Pythagorean identity $cos^2\alpha + sin^2\alpha = 1$ *M1*

$sin^2\alpha = 1 - \frac{1}{25} = \frac{\sqrt{24}}{5}$ *(A1) A1*

attempt to use the Area formula, $A = \frac{1}{2} a\, b\, Sin\, C$ *M1*

$= \frac{1}{2}\, 15\sqrt{24}\ \frac{\sqrt{24}}{5}$ *(A1)*

$= 36$ (cm^2) *A1*

Total [6 marks]

6. attempt to apply binomial expansion (M1)

$mC1k = \frac{8}{3}$ $\quad$ $mk = \frac{8}{3}$ (A1)

$mC2\,k^2 = \frac{8}{3}$ A1

attempt to simplify

$\frac{m!!}{(m-2)!\,2!}k^2 = \frac{8}{3}$

$(m^2 - m)\,k^2 = \frac{16}{3}$ A1

attempt to solve the system

$(m^2 - m)\,k^2 = \frac{16}{3}$

$mk = \frac{8}{3}$ M1

$m = 4$ A1

$k = \frac{2}{3}$ A1

[7 marks]

7. attempt to substitute solution into given equation (M1)

$(4 + hi)^2 - 2i(4 + hi) = k + 32i$

$16 + 8hi - h^2 - 8i + 2h = k + 32i$ A1

$16 - h^2 + 2h + (8h - 8)i = k + 32i$

attempt to equate real or imaginary parts (M1)

$16 - h^2 + 2h = k$ $\quad$ $8h - 8i = 32i$

$h = 5,\ k = 1$ A1A1

[5 marks]

8.(a) attempt to integrate by parts (M1)

$u = (ln\,x)^2,\ dv = x^2dx$ (M1)

$\int x^2(lnx)^2dx = (ln\,x)^2\frac{x^3}{3} - \int\frac{x^3}{3}\,2\,(ln\,x)\,\frac{1}{x}dx$ A1

$$\int x^2(lnx)^2dx = (ln\,x)^2\frac{x^3}{3} - \frac{2}{3}\left[ln\,x\,\frac{x^3}{3} - \int\frac{x^3}{3}\,\frac{1}{x}dx\right]$$

$=(ln\,x)^2\frac{x^3}{3} - \frac{2}{9}x^3ln\,x + \frac{2}{27}x^3(+\,C)$ *A1A1*

[7 marks]

(b) attempt to substitute limits in their integrated expression

attempt to replace $ln(\frac{1}{2})$ with $-\ ln2$ *(M1)*

correct working *A1*

$$\int_{0.5}^{1} x^2(lnx)^2dx = \frac{7}{108} - \frac{1}{24}(ln2)^2 - \frac{1}{36}ln2$$

[2 marks]

Total [9 marks]

9. (a) $f(x) = f(-\,x)$ *M1*

$\frac{sin^2(\frac{x}{k})}{3x^2} = \frac{sin^2(\frac{-x}{k})}{3(-x)^2} = \frac{(-sin\,(\frac{x}{k}))^2}{3x^2} = \frac{sin^2(\frac{x}{k})}{3x^2}$ *A1*

[2 marks]

(b) $\lim_{x\to 0}\frac{sin^2(\frac{x}{k})}{3x^2} = \frac{0}{0}$ *M1*

attempt to differentiate numerator and denominator *M1*

$\lim_{x\to o}\frac{2sin(\frac{x}{k})\,cos(\frac{x}{k})\,\frac{1}{k}}{6x} = \lim_{x\to o}\frac{sin(2\frac{x}{k})\,\,\frac{1}{k}}{6x}$ *A1*

$$\lim_{x \to o} \frac{\sin(2\frac{x}{k})\ \frac{1}{k}}{6x} = \frac{0}{0}$$

differentiate a second time *M1*

$$\lim_{x \to o} \frac{\frac{1}{k}\cos(2\frac{x}{k})\ \frac{2}{k}}{6} = 12$$ *A1*

$\frac{1}{3k^2}$ =12

$k^2 = \frac{1}{36}$

$k = \frac{1}{6}$ positive *A1*

[6 marks]

Total [8 marks]

10. (a) attempt to integrate *(M1)*

$s(t) = -\frac{1}{12}t^4 + \frac{1}{2}t^3 + 4t^2 + t\ (+\ C)$ *A2*

substitution of t by 1 *M1*

displacement = $\frac{41}{12}\ (m)$ *A1*

[5 marks]

(b) attempt to differentiate v *(M1)*

$a(t) = -t^2 + 3t + 4$ *A1*

[2 marks]

c) $v(t) = 0$ *M1*

$0 = -t^2 + 3t + 4$ *M1*

$t = -1 \quad t = 4$ *A1*

Do not use $t = -1$

substitute the positive value of t into $v(t)$ *M1*

greatest speed is $\frac{35}{3}\ ms^{-1}$ *A1*

[5 marks]

(d) identify the correct intervals where speed increases

$t = 0\ to\ t = 4\ \ and\ \ t = p\ \ to\ \ t = 8$ *(A1)(A1)*

$\int_0^2 v(t)dt + \int_p^8 v(t)\, dt$ *A1*

[3 marks]

Total [15 marks]

11. (a) y-intercept $\left(0, -\frac{1}{2}\right)$ *A1*

vertical asymptotes $x = -2 \quad x = 1$ *A1*

horizontal asymptote $y = 0$ *A1*

valid method to find the x coordinate of the local maximum point

local maximum point $\left(-\frac{1}{2}, \ -\frac{4}{9}\right)$ *A1*

three correct branches with correct asymptotic behavior and the key features in approximately correct relative positions to each other. *A1*

[5 marks]

(b) (i) $x = \frac{1}{y^2+y-2}$ *M1*

attempt to complete the square *M1*

$y^2 + y - 2 = (y + 0.5)^2 - 2.25$ *A1*

$x = \frac{1}{(y+0.5)^2-2.25}$

$(y + 0.5)^2 - 2.25 = \frac{1}{x}$

$y + 0.5 = \pm\sqrt{\frac{1}{x} + 2.25}$ *A1*

Award ***R1*** for concluding that the expression must have a + sign

R1

$$h^{-1}(x) = \frac{\sqrt{x+2.25x^2}}{x} - 0.5$$ *AG*

(ii) domain of h^{-1} is $x > 0$ *A1*

[6 marks]

c) attempts to find $(g \circ h)(a) = \frac{\pi}{4}$ *M1*

$(g \circ h) = \left[arctan\left(\frac{2}{a^2+a-2}\right) = \frac{\pi}{4}\right]$ *A1*

$arctan\ 2h(a) = \frac{\pi}{4}$ *A1*

$2(h(a)) = 1$

$\frac{1}{a^2+a-2} = \frac{1}{2}$ *A1*

attempt to solve their quadratic *M1*

$a^2 + a - 4 = 0$ *A1*

$-\frac{1}{2} \pm \frac{\sqrt{17}}{2}$ *A1*

[7 marks]

Total [18 marks]

12. (a) Recognized the position vector $15k$ *M1*

Recognized the vector $4i + 3j$ as velocity *M1*

attempt to find the unit vector of $4i + 3j$

$|(4i + 3j)| = \sqrt{4^2 + 3^2} = 5$ *A1*

unit vector $\frac{1}{5}(4i + 3j)$ *A1*

direction vector $500\frac{1}{5}(4i + 3j) = (400i + 300j + 0k)$ *A1*

$A = 15k + t_1(400i + 300j)$ *A1A1*

[7 marks]

(a) Recognized the point as position vector *M1*

$(1600i + 1200j + 15k) = (15k) + t_1(400i + 300j)$ *A1*

attempt to equate each component *M1*

$1600 = 400t_1$

$1200 = 300t_1$

$15 = 15$

$t_1 = 4$ *A1*

At 16 hs *A1*

[5 marks]

(b) Correct vector equation of the line for plane B

$B = (100i + 2000j + 15k) + t_2(200i - 400j)$ *A1A1*

attempt to equate both equations *M1*

$400t_1 = 100 + 200t_2$

$300t_1 = 2000 - 400t_2$

$15 = 15$ *A1*

Solve the simultaneous equations

$t_1 = 2$

$t_2 = 3.5$ *A1A1*

state that the planes will cross at the position vector

$A = 15k + t_1(400i + 300j)$ when $t_1 = 2$

$15k + 2(400i + 300j) = 800i + 600j + 15k$ *A1*

Verify that the planes passes a different time through this point R1

[8 marks]

Total [20 marks]