

Mathematics: analysis and approaches

Higher level

Paper 1

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

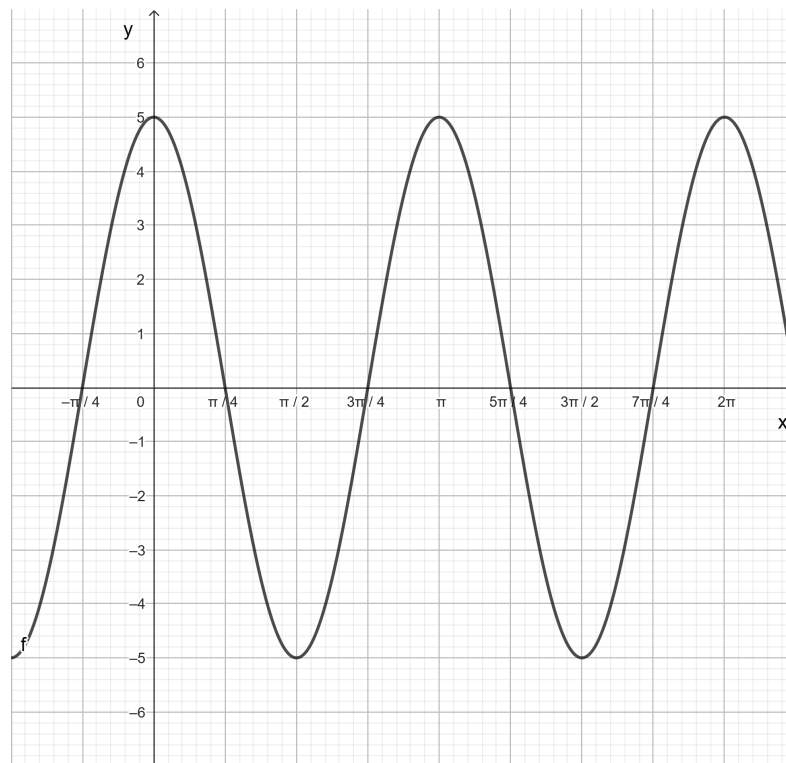
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a \cos(bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b) (i) Write down the period of f
- (ii) Hence find the value of b . [3]
- (c) Find the value of $f(\frac{\pi}{12})$. [3]

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(Question 1 continued)

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2. [Maximum mark: 5]

Consider the functions $f(x) = x + 3$ and $g(x) = x^2 + h^2$, where h is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 34$, find the possible values of h . [3]

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3. [Maximum mark: 4]

Events A and B are independent. $P(A) = 0.4$ and $P(B) = 0.55$

(a) Find $P(A \cup B)$. [2]

(b) Find $P(B' \cap A)$. [2]

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4. [Maximum mark 6]

In an arithmetic sequence it is given that $S_4 = 24$ and $S_5 = 35$.

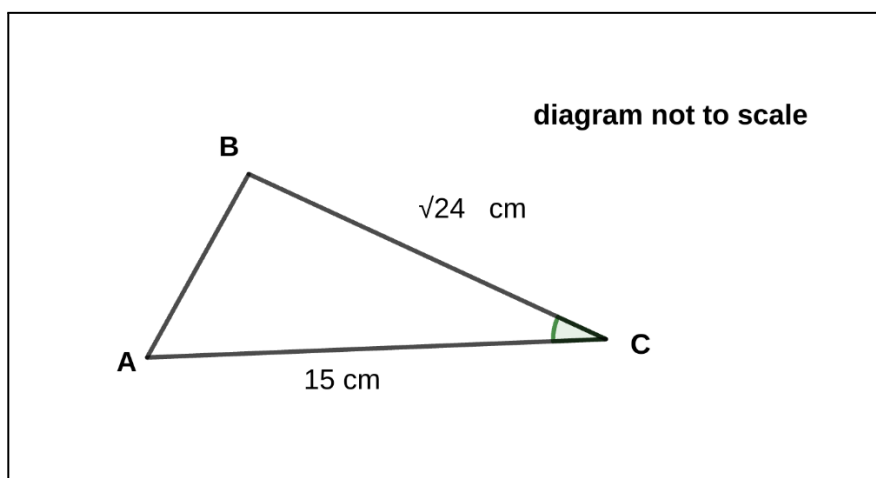
(a) Find u_5 . [2]

(b) Find u_1 and the common difference d [4]

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5. [Maximum mark 6]

In the following triangle ABC, $BC = \sqrt{24}$, $AC = 15$ cm, $\cos \hat{ACB} = \frac{1}{5}$



Find the exact area of triangle ABC.

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6.[Maximum mark: 7]

The binomial expansion of $(1 + kx)^m$ is given by $1 + \frac{8}{3}x + \frac{8}{3}x^2 + \dots + k^m x^m$

where $m \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of m and the value of k .

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7.[Maximum mark: 5]

It is given that $z = 4 + hi$ satisfies the equation $z^2 - 2iz = k + 32i$, where h and $k \in \mathbb{R}$.

Find the value of h and the value of k .

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8.[Maximum mark: 9]

(a) Find $\int x^2 (\ln x)^2 dx$ [7]

(b) Hence, show that $\int_{0.5}^1 x^2 (\ln x)^2 dx = \frac{7}{108} - \frac{1}{24}(\ln 2)^2 - \frac{1}{36}\ln 2$ [2]

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9. [Maximum mark: 8]

Consider the function $f(x) = \frac{\sin^2(\frac{x}{k})}{3x^2}$ where $k \neq 0$

(a) Show that f is an even function.

[2]

(b) Given that $\lim_{x \rightarrow 0} f(x) = 12$, find the value of k

[6]

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Section B

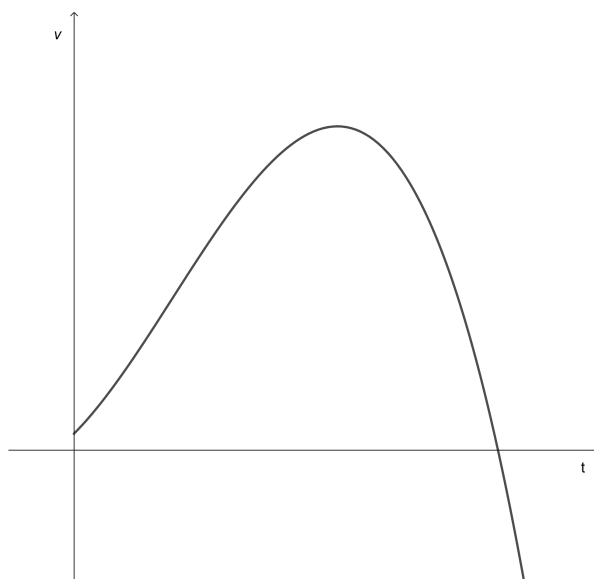
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

An object moves along a straight line. Its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by

$$v(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 4t + 1, \text{ for } 0 \leq t \leq 8$$

The object first comes to rest at $t = p$. The graph of v is shown in the following diagram:



At $t = 0$ the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$ [5]

(b) Find an expression for the acceleration of the object. [2]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

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(Question 10 continued)

- (d) Write down an expression that represents the distance travelled by the object in the first 8 seconds. Do not evaluate the expression. [3]

11. [Maximum mark: 18]

A function f is defined by $f(x) = \frac{1}{x^2+x-2}$ where $x \in \mathbb{R}$, $x \neq -2$, $x \neq 1$

- (a) Sketch the curve $y = f(x)$, clearly, indicating asymptotes, the equation of the asymptotes if any. State the coordinates of the point of intersection with the axes.
State the coordinates of any local maximum or minimum. [5]

A function h is defined by $h(x) = \frac{1}{x^2+x-2}$, where $x \in \mathbb{R}$ and $x > 1$.

- (b) Let h^{-1} be the inverse of h

(i) Show that $h^{-1}(x) = \frac{\sqrt{x+2.25x^2}}{x} - 0.5$

- (ii) State the domain of h^{-1} [6]

A function g is defined by $g(x) = \arctan(2x)$

- (c) Given that $(g \circ h)(k) = \frac{\pi}{4}$, find the exact value of k [7]

12. [Maximum mark: 20]

At midday a Plane A is passing in the vicinity of an airport at a height of 15 km and a speed of 600 km/h.

The plane is flying with a direction of $4i + 3j$. Consider that $v = i$ is a displacement due East of 1km, $v = j$ is a displacement due North of 1km, $v = k$ is an altitude of 1km.

(a) The airport is at the origin. Find the vector equation of the line r of the plane t hours after midday. [7]

(b) At what time the plane will pass through the point (1600, 1200, 15) [5]

Another plane, Plane B is flying toward the airport, from a location (100, 2000, 15). The velocity vector is $w = 200i - 300j$.

(c) Is there a danger of collision? Justify your answer. [8]

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