Mathematics: analysis and approaches

Higher level

Paper 2

Markscheme

1.

(a) (i) Recognizing that f' is needed (M1)

$(f'(2) =)\ -2$ A1

(ii) $\frac{1}{2}$ A1

[3 marks]

(b) $y + 8 = \frac{1}{2}(x - 2)\ \ (y = \frac{1}{2}x - 9)$ A1

[1 mark]

(c) *Attempt to find intersection of curve and their normal either graphically or analytically, sketch showing intersection OR* $x^2 - 6x = \frac{1}{2}x - 9$ (M1)

(4.5, -6.75) A1A1

[3 marks]

Total [7 marks]

2. (a) *Attempt to use the formula of distance* *(A1)*

AV = 12.7 *A1*

[2 marks]

(b) AB = 6 seen anywhere (A1)

Attempt to use cosine rule in triangle $A\hat{V}B$ (M1)

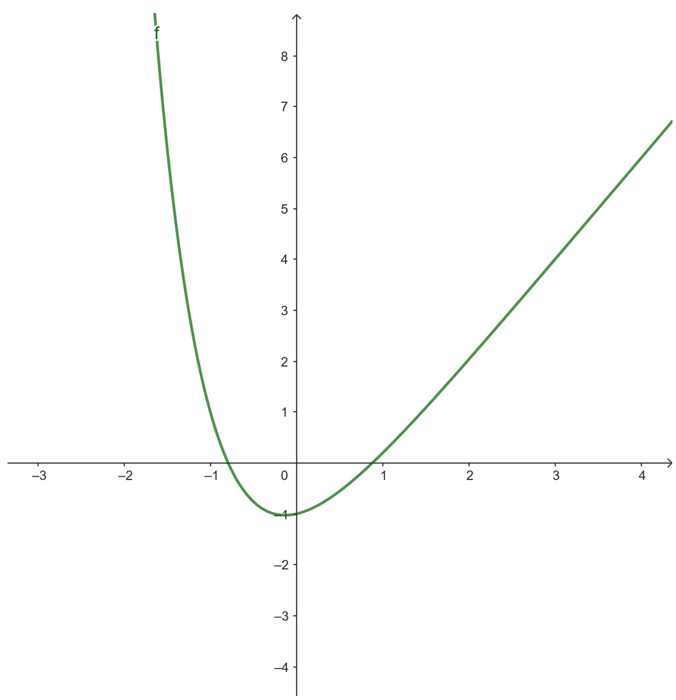
Use of the formula A1

$V\hat{A}B$ = 76.3° A1

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

[3 marks]

(b) Attempt to multiply by 2 the x A1

Add 3 to the function A1

$g(x) = 5^{-2x} + 4x+1$ *[2 marks]*

Total [5 marks]

4. (a) a = -0.04098 b = 5.36044

a = -0.041 b = 5.36 A1A1

[2 marks]

(b) r = -0.8949247

r = -0.894 A1

[1 mark]

(c) Attempt to substitute m into their equation (M1)

Time = 1.875 A1

[2 marks]

(d) Size of the sample and outlier A2

[2 marks]

Total [7 marks]

5. (a) *Recognized at rest when* $\frac{dy}{dx} = 0$ OR *x is a minimum* *(M1)*

$k = 1.57392078 \quad k = 1.57$ *A1*

[2 marks]

(b) Recognition that the total distance travelled is the difference between the initial displacement and the displacement at the minimum.

(M1)

initial displacement 3.54 and displacement at minimum 1.5 *(A1)*

total distance travelled: 3.54 - (1.5)

2.04 (m) *A1*

[3 marks]

Total [5 marks]

6. $1p^2 + 2p^2 + 3q + 4p^3 = 2.1$ (A1)

$p^2 + p^2 + q + p^3 = 1$ (A1)

Attempt to make q the subject *(M1)*

Use a graph to find the intersection *(M1)*

p = 0.61 and q = 0.0398 *A1*

Total [5 marks]

7. (a) (i) $8! = 40320$ *A1*

(ii) attempt to consider boys as a single object *(M1)*

$4!\ 5! = 2880$ *A1*

[3 marks]

(b) Recognition of case with 2 girls, and case with 3 girls *(M1)*

$5C4\ \ 3C2$= 5 x 4=20

$5C3\ \ 3C3$= 10 x 1=10 *A1*

20+10= 30 *A1*

8. (a) $AB = 1i - 1j - 4k$ *A1*

$AC = 2i - 2j - 2k$ *A1*

attempt to evaluate their AB and AC using the formula *(M1)*

$AB\ x\ AC = \ - 6i - 6j(+\ 0k)$ *A1*

[4 marks]

(b) Attempt to use the formula to calculate the cosine

$AB\ .\ AC = 12$ *A1*

$|AB| = \sqrt{18} = 4.24 \quad |AC| = \sqrt{12} = 3.46$ *A1*

$Sin\,\theta = \sqrt{1 - 0.816^2} = 0.578$

$area = \frac{1}{2} 4.24\, x\, 3.46x\, 0.578 = 4.24$ *A1*

[3 marks]

c) $-6(x-4) - 6(y-2) + 0(z-1) =$ *A1*

$-6x + 24 - 6y + 12$ *A1*

$-6x - 6y + 36$ AG

[2 marks]

Total [9 marks]

(a) (i) $g(x) = sin\, x$ $g(0) = 0$

$g'(x) = cosx$ $g'(0) = 1$

$g''(x) =- sinx$ $g''(x) = 0$

$g'''(x) =- cosx$ $g'''(x) =- 1$

$g''''(x) = sinx$ $g''''(x) = 0$

$g'''''(x) = cosx$ $g'''''(x) = 1$ *A1A1*

attempt to use Maclaurin's Theorem *M1*

$sin\, x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!}$ *A1*

(ii) $h(x) = cos\, x$ $h(0) = 1$

$h'(x) =- \sin x \qquad h'(0) = 0$

$h''(x) =- \cos x \qquad h''(0) =- 1$

$h'''(x) = \sin x \qquad h'''(0) = 0$

$h''''(x) = \cos x \qquad h''''(0) = 1$ *A1A1*

attempt to use Maclaurin's Theorem *M1*

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ *A1*

[8 marks]

(b) $f(x) = \sin(2\theta)$

attempt to use double angle identity *M1*

$f(0.15) = 2\sin(0.15)\cos(0.15)$

attempt to substitute in their (a) and (b) *M1*

$\sin 0.15 = \frac{0.15}{1!} - \frac{0.15^3}{3!} + \frac{0.15^5}{5!} = 0.1494381328$ *A1*

$\cos 0.15 = 1 - \frac{0.15^2}{2!} + \frac{0.15^4}{4!} = 0.9887710938$ *A1*

attempt to substitute

$f(0.15) = 2\ x\ 0.1494381328\ x\ 0.9887710938$= 0.2955202 *A1*

[5 marks]

Total [13 marks]

10. (a) Recognizing probabilities sum 1 M1

0.0388+P$(31 < W < 33)$+0.2781 = 1

P$(31 < W < 33)$ = 0.6831 A1

[2 marks]

(b) Use of inverse normal to find at least one z-score for $P(Z < z) = 0.0388$, or

$P(Z > z) = 0.2781$

$z_1 = -1.76478$ OR $z_2 = 0.58849$ (A1)

$\frac{31-\mu}{\sigma} = -1.76478$, $\frac{33-\mu}{\sigma} = 0.58849$ (A1) (A1)

Attempt to solve their equations (M1)

$\mu = 32.5$

$\sigma = 0.85$ A1

[5 marks]

c) (i) Recognized binomial distribution

$X \sim B(100, 0.6831)$ (M1)

$P(X = 70) = 0.0812$ A1

(ii) $P(31 < W < 33)$ seen anywhere

$P(31 < W < 33) = 0.44362563$ A1

Recognition of conditional probability M1

$P(W > 32/ 31 < W < 33) = \frac{P(32<W<33)}{P(31<W<33)} = \frac{0.44362563}{0.6831}$ A1

0.65 OR 65 % A1

[6 marks]

(d) $P(w < a) = 0.25 \;\; P(w < b) = 0.75$ A1A1

$a = 31.9$

b =33.1

IQR= 1.2 A1

[3 marks]

Total [16 marks]

11. (a) Vertical asymptote equation $x =- 3$ A1

[1 mark]

(b) roots (3,0) (10,0) A1A1

[2 marks]

(c) $a = \frac{1}{3}$ A1

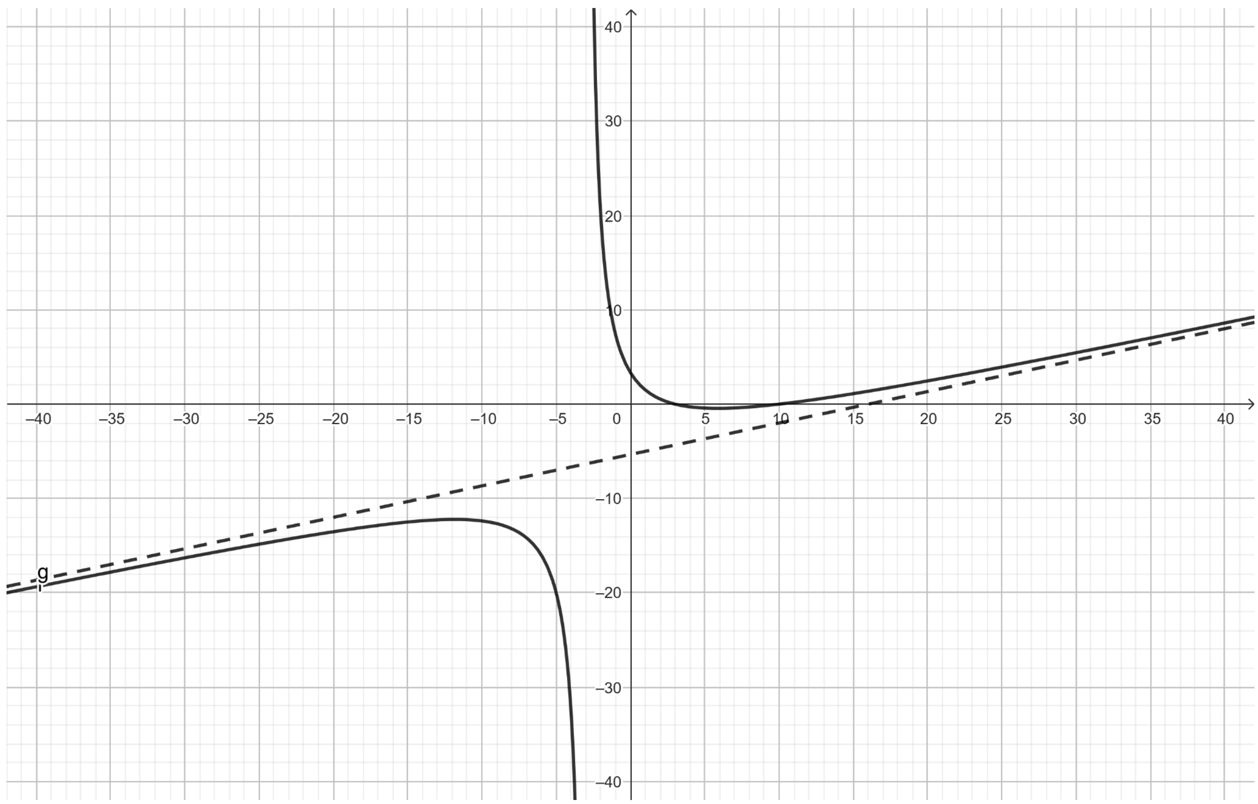
$\frac{x^2-13x+30}{3x+9} - \frac{1}{3}x = \frac{-16x+30}{3x+9}$ *A1* attempt to calculate the limit *M1*

$\lim_{x \to \infty} \frac{-16x+30}{3x+9} = -\frac{16}{3} = b$ *A1*

$y = \frac{1}{3}x - \frac{16}{3}$

[4 marks]

(d)



two branches with approximately correct shape (for $-40 \le x \le 40$) *A1*

their vertical and oblique asymptotes in approximately correct positions with both branches showing correct asymptotic behaviour to these asymptotes. *A1A1*

their axes intercept in approximately the correct positions. *A1*

[4 marks]

(e) Range

-12.22 and -0.45 see anywhere *A1*

attempt to write the range using at least one value in an interval or an inequality in y *M1*

$y >- 0.45$ and $y <- 12.2$ *A1A1*

[4 marks]

(f) $(-12.2, -3) \cup (1.22, +\infty)$ *A1A1A1A1*

[4 marks]

Total [19 marks]

12.(a) attempt to use $y_1 = y_0 + h\, x\, f(x_0, y_0)$ *M1*

$y_1 = 3 + 0.1\ x \frac{2x3^2+1^2}{1x3}$

$y_1 = 3.63$ *A1*

$y_1 = 3.63$ and $x_1 = 1.1$ $f(1.1, 3.63)$ *A1*

$y_2 = 4.32$ *A1*

[4 marks]

(b) let $y = vx$ *M1*

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ *A1*

$v + x\frac{dv}{dx} = \frac{x^2(2v^2+1)}{vx^2}$ *M1*

$x\frac{dv}{dx} = \frac{v^2+1}{v}$ *A1*

attempt to separate x and v

$\frac{v}{v^2+1}dv = \frac{dx}{x}$ *M1*

$\frac{1}{2}ln(v^2 + 1) = lnx + C$ *A1*

$\frac{1}{2}ln10 = C$ *M1*

$\frac{y^2}{x^2} + 1 = 10x^2$ *A1*

$y = x\sqrt{10x^2 - 1}$ *AG*

[8 marks]

c) $y = 1.2\sqrt{10 x 1.2^2 - 1}$

$y = 4.39$ *A1*