Mathematics: analysis and approaches

Higher level

Paper 2

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: Answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Consider the function defined by $f(x) = x^2 - 6x$. The graph of f passes through the point A(2,-8).

(a) (i) Find the gradient of the tangent to the graph of f at the point A.

(ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

(b) Write down the equation of the normal to the graph of f at point A. [1]

The normal to the graph of f at point A intersects the graph of f again at a second point B.

(c) Find the coordinates of B. [3]

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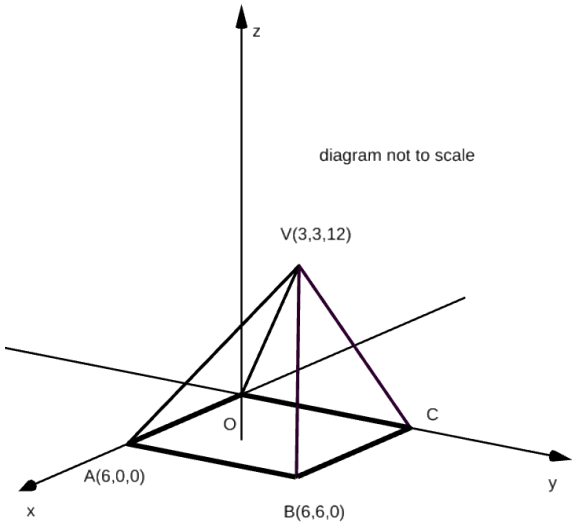
(Question 1 continued)

2. [Maximum mark: 6]

The following diagram shows a square pyramid with vertex V and a squared base OABC.

Point A has coordinates (6,0,0), point B has coordinates (6,6,0) and the vertex V has coordinates V(3,3,12).



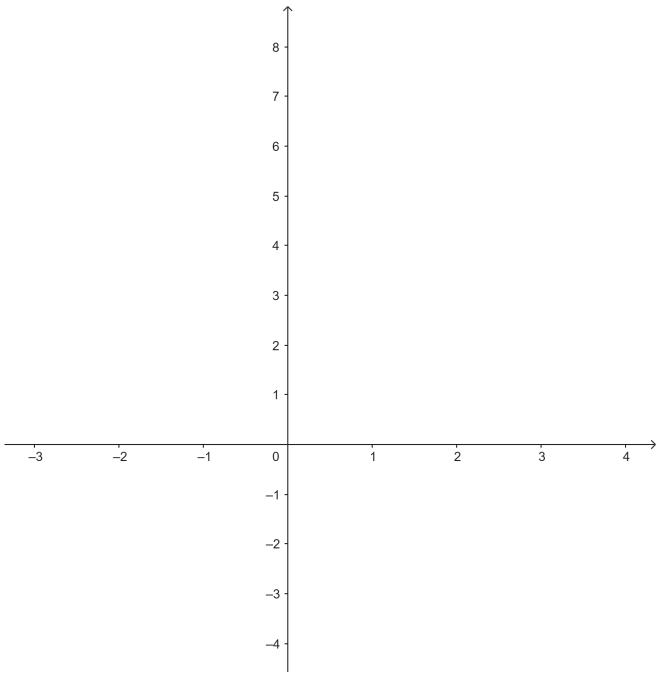
(a) Find AV. [2]

(b) Find the size of $V\hat{A}B$. [4]

3. [Maximum mark: 5]

Consider the function $f(x) = 5^{-x} + 2x - 2$

(a) On the following axes, sketch the graph of f for $-3 \leq x \leq 4$ [3]



The function $g(x)$ is obtained as the result of 2 transformations of the function f, first an horizontal stretch with scale factor $\frac{1}{2}$, followed by a vertical translation of 3 units.

(b) Find the function $g(x)$ [2]

4. [Maximum mark: 7]

A teacher wants to investigate if there is a relation between the results of their Math test and the hours spent by the students in social media. She decided to survey 8 students. The following table shows the results,

Mark (m)	65	77	23	88	27	90	56	47
Time (t)	2	3	5	2	4	1.5	2.5	3.5

(a) The regression line of t on m for this data can be written in the form $t = am + b$. Find the value of a and the value of b. [2]

(b) Write down the Pearson's product moment coefficient r. [1]

(c) Use your regression line to estimate the time spent by a student that gets 85 marks. [2]

(d) Explain why this regression line is not useful for a student that gets 2 marks. [2]

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds in given by $s(t) = 3.5\,cos(\sqrt{5t + 2}) + 5$, where $0 \leq t \leq 8$.

The particle first comes at rest after k seconds.

(a) Find the value of k. [2]

(b) Find the total distance that the particle travels in the first k seconds. [3]

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $p, q \in R^+$.

x	1	2	3	4
$P(X = x)$	p^2	p^2	q	p^3

Given that $E(X) = 2.1$, find the value of p and q.

7. [Maximum mark: 6]

The senior swim team of a school consist of 5 boys and three girls

The Team members are to be placed in a line to receive a prize.

(a) In how many ways can the team members placed if

(i) there are no restrictions;

(ii) the boys must be placed next to each other [3]

(b) Six members of the team are selected to attend a conference. Find the number of possible selections that contain at least girls. [3]

8. [Maximum mark: 9]

Given three points $A(0,2,6)$, $B(1,1,2)$, $C(2,0,4)$

(a) Find the cross product between vector AB and vector AC [4]

(b) Find the area of the triangle ABC [3]

(c) Show that the equation of a plane parallel to the plane that contains the triangle ABC and contains the point P (4,2,1) is $\Pi: \; -6x - 6y + 36$ [2]

9. [Maximum mark: 13]

Given $f(x) = sin(2\theta)$ where θ is measured in radians.

(a) Use the Maclaurin's theorem to find

(i) the first 3 non zero terms of the expansion of $sin\ \theta$ [4]

(ii) the series expansion for $cos\ \theta$ given all terms up to x^4 [4]

(b) Calculate using the results of (a) and (b) $f(0.15)$ correct to 7 decimal places. [5]

Do **not** write solutions on this page

Section B

Answer all questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

A factory is producing bags of sand. An automatic machine is filling the bags. The weight of each bag, W kgs, can be modelled by a normal distribution with a mean μ and a standard deviation σ.

It is known that $P(W < 31) = 0.0388$ and $P(W > 33) = 0.2781$

(a) Find the probability that the weight of a randomly selected bag is between 31 and 33 kilograms. [2]

(b) Find the value of μ and the value of σ. [5]

The company measures 100 bags selected at random. Any bag between 31 and 33 kilograms passes the control. Weights of bags are independent of each other.

(c) (i) Find the probability that exactly 70 bags pass the control.

(ii) Given that 70 bags pass the control, find the probability that exactly 60 bags weigh more than 32kg. [6]

(d) Find the interquartile range. [3]

11. [Maximum mark: 19]

Consider the function defined by $f(x) = \frac{x^2-13x+30}{3x+9}$, where $x \in R,\ x \neq 3$

(a) State the equation of the vertical asymptote. [1]

(b) Find the coordinates where the graph of f crosses the x-axis [2]

The graph of f also has an oblique asymptote of the form $y = ax + b$

(c) Find the equation of the oblique asymptote [4]

(d) Sketch the graph of f for $-40 < x < 40$, showing the asymptotes and the intersection with the axis. [4]

(e) Find the range of f. [4]

(f) Solve the inequality $f(x) < x$ [4]

12. [Maximum mark: 12]

Consider the equation $\frac{dy}{dx} = \frac{2y^2+x^2}{xy}$, where $x > 0, y > 0$

It is given that when $x = 1,\ y = 3$

(a) Use Eulerś method with a step length 0.1 to find an approximate value of y when $x = 1.2$ [4]

(b) By solving the differential equation, show that $y = x\sqrt{10x^2 - 1}$ [8]