Mathematics: analysis and approaches

Higher level

Paper 3

Markscheme

1. (a) $Sin\ (x)\ =\ Cos\ (y)$

$x\ +\ y\ =\ 90°$ or the angles are complementary **_A1_**

domain $x\ +\ y\ \leq\ 90°$ **_A1_**

[2 marks]

(b) $Sin\ (17°)\ =\ Cos\ (z°)$

$z\ =\ 90\ -\ 17$

$z\ =\ 73°$ **_A1_**

[1 mark]

(c) If $\alpha\ =\ 90°$, show that $sin\ (90°\ -\ \beta)\ =\ -\ cos\beta$

Attempt to use the formula

$sin(\alpha\ -\ \beta)\ =\ sin\alpha cos\beta\ -\ cos\alpha sin\beta$ **_M1_**

$sin(90°\ -\ \beta)\ =\ sin\ 90\ cos\beta\ -\ cos\ 90°\ sin\ \beta$

$sin(90°\ -\ \beta)\ =\ 1\ cos\beta\ -\ 0\ sin\ \beta$

$sin(90°\ -\ \beta)\ =\ cos\beta$ **_A1_**

[2 marks]

(d) State $cos\ (90°\ -\ \beta)$ in terms of $Sin\ \beta$

Attempt to use the formula **_M1_**

$cos(90°\ -\ \beta)\ =\ cos\ 90°cos\beta\ +\ sin\ 90°\ sin\beta$

$cos(90°\ -\ \beta)\ =\ sin\beta$ **_A1_**

[2 marks]

(e) $\sum_{n=0}^{90} cos^2(n°)$

attempt to write the sum

$cos^2(0°) + cos^2(1°) + cos^2(2°) + cos^2(3°) + ... + cos^2(89°) + cos^2(90°)$ ***M1***

Attempt to use $sin(90° - \beta) = cos\beta$ ***M1***

$cos^2(90° - \beta) = sin^2\beta$ seen anywhere ***A1***

use of the Pitagoric identity ***M1***

$sin^2x + cos^2x = 1$

Replace

$cos^2x + cos^2(90° - x) = 1$ ***A1***

So

$cos^2 1° + cos^2(90° - 1°) = 1$ ***A1***

$cos^2 1° + cos^2 89° = 1$ ***A1***

Realized there are 44 pairs ***A1***

Realized their is only one $cos^2 45°$ ***R1***

$cos\ 45° = \frac{\sqrt{2}}{2}$

$cos^2 45° = \frac{2}{4} = \frac{1}{2}$ ***A1***

$cos^2 0° = 1$ ***A1***

Total addition : $44\ x\ 1 + \frac{1}{2} + 1$ ***M1***

Total addition : 45.5 ***A1***

[13 marks]

[Total 26 marks]

(f) $sin^2(0°) + sin^2(1°) + sin^2(2°) + sin^2(3°) + ... + sin^2(89°) + sin^2(90°)$ ***M1***

$sin^2(1°) + sin^2(89°) = sin^2(x°) + sin^2(90° - x°)$

Attempt to use $sin(90° - \beta) = cos\beta$ ***M1***

Replacing

$sin^2(x°) + sin^2(90° - x°)$

$sin^2(x°) + cos^2(x°) = 1$

$sin\ 45° = \frac{\sqrt{2}}{2}$

$sin^2 45° = \frac{2}{4} = \frac{1}{2}$ ***A1***

$sin^2 0° = 0$ *A1*

Total addition : 44.5-1=43.5 *M1*

Total addition:21.75 *AG*

[6 marks]

2. (a)

Proof for $n = 2$ *M1 A1*

State $n = k$ *A1*

State $n = k + 1$ *M1*

Split the exponent into a product *M1*

Use the product property *A1*

Conclusion *R1*

[6 marks]

(b)

i) Horizontal translation 1 unit to the right *A1*

Horizontal stretch scale factor -1 *A1A1*

ii) attempt to $1 - x > 0$ *M1*

$x < 0$ *A1*

iii)Sketch asymptote, root *A1A1A1*

[8 marks]

(c)

Prove $f(x) = f(- x)$ *M1*

Correct calculus *A1*

Prove $f(x) =- f(- x)$ *M1*

Correct calculus *A1*

[4 marks]

(d)

$x = \frac{1}{2}$ seen anywhere *A1*

$ln(1 - \frac{1}{2}) = ln \frac{1}{2} = ln\, 2^{-1} =- 1ln2 =- ln2$ *A1A1*

[3 marks]

(e) Use the Maclaurin series

$f(0) = ln(1 - 0) = 0$ **A1**

$f'(x) = \frac{-1}{1-x}$ $f'(0) =- 1$ **A1**

$f''(x) = \frac{-1}{(1-x)^2}$ $f''(0) =- 1$ **A1**

$f'''(x) = \frac{-2}{(1-x)^3}$ $f'''(0) =- 2$ **A1**

$f''''(x) = \frac{-6}{(1-x)^4}$ $f''''(0) =- 6$ **A1**

$f'''''(x) = \frac{-24}{(1-x)^5}$ $f'''''(0) = - 24$ **A1**

attempt to replace in the Maclaurin series **M1**

correct math **A1**

$ln(1 - x) = x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}$ **AG**

[8 marks]

[Total 29 marks]