Mathematics: analysis and approaches

Higher level

Paper 3

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Answer all questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as a part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

In this question you will explore some properties of series and trigonometric identities.

(a) Given that $sin\ (x) = cos\ (y)$ State the relation between x and y, indicating the domain. [2]

(b) Hence $sin\ (17°) = cos\ (z)$ Find the value of z [1]

(c) If $\alpha = 90°$, show that $sin\ (90° - \beta) = cos\beta$ [2]

(d) State $cos\ (90° - \beta)$ in terms of $sin\ \alpha$ [2]

(e) Hence, calculate $\sum_{x=0}^{90} cos^2(x°)$ [13]

(f) Hence, show that $\sum_{x=0}^{45} sin^2(x°) = 21.75$ [6]

2. [Maximum mark: 29]

In this question you will explore logarithms.

(a) Prove by induction on n, $log\ (a^n) = n\ log\ a$, $n \in N$ and $n \geq 2$. [6]

(b) $f(x) = log\ x$, the function $g(x) = log(1 - x)$ is obtained by performing 2 transformation of $f(x)$.

i) Describe fully both transformations [3]

ii) Indicate the domain of $g(x)$ [1]

iii) Draw a sketch of $g(x)$ showing asymptotes and intersection with the axis if any. [3]

(c) Given that $h(x) = ln\ (1 - x)$, prove that $h(x)$ is not an even nor an odd function. [3]

(d) Solve $h(x) = -\ ln2\ \ x \in R$ and $-1 < x < 1$ [4]

(e) Prove using the Maclaurin series that

$$ln(1 - x) = x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5},\ \ x \in R \text{ and } -1 < x < 1$$

[8]