Mathematics: analysis and approaches

Standard level

Paper 1

Markscheme

1. (a) $a = 5$ *A1*

[1 mark]

(b) (i) period $= \pi$ *A1*

(ii) $b = \frac{2\pi}{\pi} = 2$ *A1A1*

[3 marks]

c) substituting $\frac{\pi}{12}$ into their $f(x)$ *M1*

$cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ *(A1)*

$= 5\frac{\sqrt{3}}{2}$ *A1*

[3 marks]

Total [7 marks]

2. (a) Attempt to form $(g \circ f)(x)$ *M1*

$(g \circ f)(x) = (x + 3)^2 + h^2 = x^2 + 6x + 9 + h^2$ *A1*

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression $= 34$ *(M1)*

$h^2 = 9$ *A1*

$h = \pm 3$ *A1*

[3 marks]

Total [5 marks]

3. (a) P(A∪B) = P(A) + P(B) - P(A)xP(B) *A1*

P(A∪B) = 0.73 *A1*

[2 marks]

(b) P(B'∩ A) using a Venn diagram *M1*

P(B'∩ A) = 0.18 *A1*

[2 marks]

Total [4 marks]

4. (a) $u_9 = S_5 - S_4$ *M1*

$u_9 = 35 - 24 = 11$ *A1*

[2 marks]

(b) $24 = \frac{4}{2}(2u_1 + (4-1)d$

$47 = \frac{5}{2}(2u_1 + (5-1)d$ *M2*

attempt to solve the simultaneous equations

$d = 2$ *A1*

$u_1 = 3$ *A1*

[4 marks]

Total [6 marks]

5. attempt to use Pythagorean identity $cos^2\alpha + sin^2\alpha = 1$ *M1*

$sin^2\alpha = 1 - \frac{1}{25} = \frac{\sqrt{24}}{5}$ *(A1) A1*

attempt to use the Area formula, $A = \frac{1}{2}a\,b\,Sin\,C$ *M1*

$= \frac{1}{2}\ 15\sqrt{24}\ \frac{\sqrt{24}}{5}$ *(A1)*

$=\ 36$ (cm^2) *A1*

[6 marks]

6. attempt to apply binomial expansion (*M1)*

$mC1k = \frac{8}{3}$ $mk = \frac{8}{3}$ *(A1)*

$mC2\ k^2 = \frac{8}{3}$ *A1*

attempt to simplify

$\frac{m!!}{(m-2)!\ 2!}k^2 = \frac{8}{3}$

$(m^2 - m)\ k^2 = \frac{16}{3}$ *A1*

attempt to solve the system

$(m^2 - m)\ k^2 = \frac{16}{3}$

$mk = \frac{8}{3}$ *M1*

$m = 4$ *A1*

$k = \frac{2}{3}$ *A1*

[7 marks]

7. (a) attempt to integrate (*M1)*

$s(t) = -\frac{1}{12}t^4 + \frac{1}{2}t^3 + 4t^2 + t\ (+\ C)$ *A2*

substitution of t by 1 *M1*

displacement = $\frac{41}{12}\ (m)$ *A1*

[5 marks]

(b) attempt to differentiate v (*M1)*

$a(t) = -\ t^2 + 3t + 4$ *A1*

[2 marks]

c) $v(t) = 0$ *M1*

$0 = -t^2 + 3t + 4$ *M1*

$t = -1 \quad t = 4$ *A1*

Do not use $t = -1$

substitute the positive value of t into $v(t)$ *M1*

greatest speed is $\frac{35}{3}\ ms^{-1}$ *A1*

[5 marks]

(d) identify the correct intervals where speed increases

$t = 0\ to\ t = 4 \quad and \quad t = p\ \ to\ \ t = 8$ *(A1)(A1)*

$\int_0^2 v(t)dt + \int_p^8 v(t)\ dt$ *A1*

[3 marks]

8. (a) (i) valid approach to find the required logarithm

$3^x = \frac{1}{27}$ *M1*

$3^x = 3^{-3}$

$x = -3$ *A1*

(ii) valid approach to find the required logarithm

$8^x = 2$ *M1*

$2^{3x} = 2$

$x = \frac{1}{3}$ *A1*

(iii) valid approach to find the required logarithm

$\sqrt{5}^{\,x} = \frac{1}{125}$ *M1*

$5^{\frac{1}{2}x} = 5^{-3}$ *A1*

$x = -6$ *A1*

[7 marks]

(b) (i) attempt to use change of base *M1*

$log_b a = \frac{log_a a}{log_a b}$

substitute $log_a b = 5$ into their expression *A1*

$log_b a = \frac{1}{5}$ *A1*

(ii) attempt to use properties of exponents *M1*

$log_b \frac{\left(a^{\frac{1}{2}}\right)^5}{\left(b^4\right)^5}$ *A1*

applying quotient rule of logarithms

$log_b a^{\frac{5}{2}} - log_b b^{20}$ *A1*

correct working

$\frac{5}{2}\ \frac{1}{5} - 20 = -\frac{39}{2} = -19.5$ *A1*

[7 marks]

Total [14 marks]

9. (a) attempt to use the table to calculate *M2*

$p = 50$ and $q = 170$ *A2*

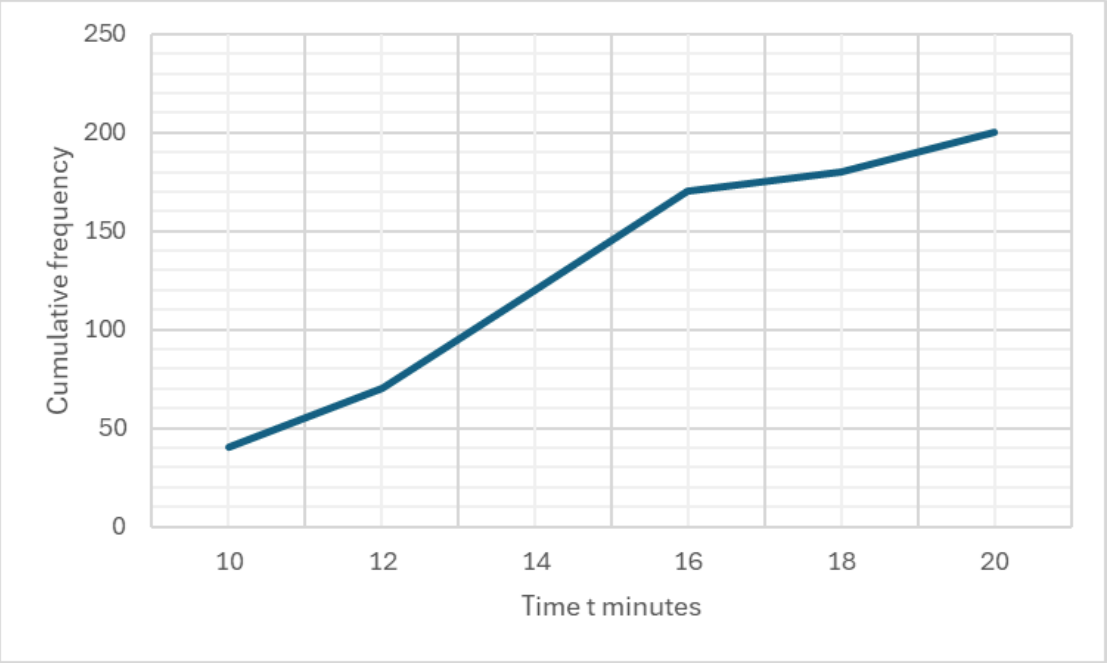
[4 marks]

(b) $P(t < 12) = \frac{70}{200} = 0.35$ *M1A1*

$P(t > 16) = \frac{30}{200} = 0.15$ *M1A1*

[4 marks]

(c)



[3 marks]

(d) (i) The number of boys not selected is 120 *M1*

boys selected = 80 *A1*

(ii) Enter the their cumulative frequency curve at 120 *M1*

$x = 14$ *A1*

[4 marks]

Total [15 marks]