Mathematics: analysis and approaches

Standard level

Paper 2

Markscheme

1.

(a) (i) Recognizing that f' is needed (M1)

$(f'(2) =) \ -2$ A1

(ii) $\frac{1}{2}$ A1

[3 marks]

(b) $y + 8 = \frac{1}{2}(x - 2) \ \ (y = \frac{1}{2}x - 9)$ A1

[1 mark]

(c) *Attempt to find intersection of curve and their normal either graphically or analytically, sketch showing intersection OR* $x^2 - 6x = \frac{1}{2}x - 9$ (M1)

(4.5, -6.75) A1A1

[3 marks]

Total [7 marks]

2. (a) *Attempt to use the formula of distance* *(A1)*

AV = 12.7 *A1*

[2 marks]

(b) AB = 6 seen anywhere (A1)

Attempt to use cosine rule in triangle $A\hat{V}B$ (M1)

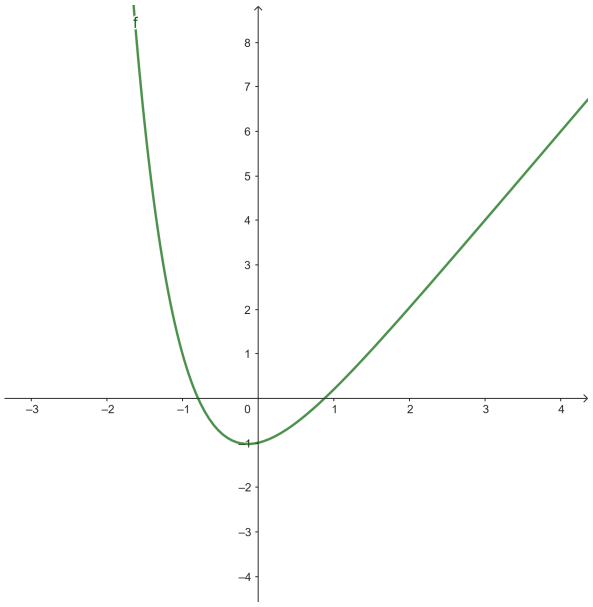
Use of the formula A1

$V\hat{A}B$ = 76.3° A1

[4 marks]

Total [6 marks]

3. (a)



A1A1A1

[3 marks]

(b) Attempt to multiply by 2 the x A1

Add 3 to the function A1

$g(x) = 5^{-2x} + 4x+1$ [*2 marks]*

Total [5 marks]

4. (a) a = -0.04098 b = 5.36044

a = -0.041 b = 5.36 A1A1

[2 marks]

(b) r = -0.8949247

r = -0.894 A1

[1 mark]

(c) Attempt to substitute m into their equation (M1)

Time = 1.875 A1

[2 marks]

(d) Size of the sample and outlier A2

[2 marks]

Total [7 marks]

5. (a) *Recognized at rest when* $\frac{dy}{dx} = 0$ OR *x is a minimum* *(M1)*

$k = 1.57392078$ $k = 1.57$ *A1*

[2 marks]

(b) Recognition that the total distance travelled is the difference between the initial displacement and the displacement at the minimum.

(M1)

initial displacement 3.54 and displacement at minimum 1.5 *(A1)*

total distance travelled: 3.54 - (1.5)

2.04 (m) *A1*

[3 marks]

Total [5 marks]

6. $1p^2 + 2p^2 + 3q + 4p^3 = 2.1$ (A1)

$p^2 + p^2 + q + p^3 = 1$ (A1)

Attempt to make q the subject *(M1)*

Use a graph to find the intersection *(M1)*

p = 0.61 and q = 0.0398 *A1*

Total [5 marks]

7. (a) Maggieś future value after n years = $24000\left(1 + \frac{6}{100x12}\right)^{12n}$ A1

[1 mark]

(b) (i) Total after a year and a half $24000\left(1 + \frac{6}{100x12}\right)^{12x1.5} = 26254.3$ A1

[1 mark]

(ii) $24000\left(1 + \frac{6}{100x12}\right)^{m} \geq 28000$ (A1)

$m \geq 30.9$ (A1)

$m = 31\ months$ A1

[3 marks]

(c) $26254.3\left(1 + \frac{r}{100x4}\right)^{4x1.5}$ (M1) (A1)

$(r =) 4.3147$

$(r =) 4.3\%$ A1

[3 marks]

(d) $22000\left(1 - \frac{15}{100}\right)^{4}$ (M1) (A1)

Actual value of the car 11484.1375 A1

Actual value of the car 11500 A1

[4 marks]

(e) $11000 = 22000\left(1 - \frac{15}{100}\right)^{t}$ (A1)

$t = 4.26$ A1

t = 4 years 3 months

t = 1st March 2025 A1

[3 marks]

Total [15 marks]

8. (a) Recognizing probabilities sum 1 M1

$0.0388 + P(31 < W < 33) + 0.2781 = 1$

$P(31 < W < 33) = 0.6831$ A1

[2 marks]

(b) Use of inverse normal to find at least one z-score for $P(Z < z) = 0.0388$, or $P(Z > z) = 0.2781$

$z_1 = -1.76478$ OR $z_2 = 0.58849$ (A1)

$\frac{31-\mu}{\sigma} = -1.76478$, $\frac{33-\mu}{\sigma} = 0.58849$ (A1) (A1)

Attempt to solve their equations (M1)

$\mu = 32.5$

$\sigma = 0.85$ A1

[5 marks]

c) (i) Recognized binomial distribution

$X \sim B(100, 0.6831)$ (M1)

$P(X = 70) = 0.0812$ A1

(ii) $P(31 < W < 33)$ seen anywhere

$P(31 < W < 33) = 0.44362563$ A1

Recognition of conditional probability M1

$P(W > 32 / 31 < W < 33) = \frac{P(32<W<33)}{P(31<W<33)} = \frac{0.44362563}{0.6831}$ A1

0.65 OR 65 % A1

[6 marks]

(d) $P(w < a) = 0.25 \;\; P(w < b) = 0.75$ A1A1

$a = 31.9$

b =33.1

IQR= 1.2 A1

[3 marks]

Total [16 marks]

9. (a) $35 = \pi r^2 h$ (A1)

Attempt to rearrange and substitute h into the expression of the area. M1

$A = 3\pi x^2 + \left(\frac{35}{\pi x^2}\right)2\pi x$

$A = 3\pi r^2 - \frac{70}{x}$ A1

[3 marks]

(b) (i) $\frac{dA}{dx} = 6\pi x - \frac{70}{x^2}$ A1A1

(ii) $\frac{dA}{dx} = 0$ (M1)

$6\pi p - \frac{70}{p^2} = 0$ (A1)

$(p =) \left(\frac{70}{6\pi}\right)^{\frac{1}{3}}$ *A1*

[3 marks]

c) (i) $\frac{d^2A}{dx^2} = 6\pi + \frac{140}{x^3}$ A1A1

(ii) Substituting their value of $x = p$ into $\frac{d^2A}{dx^2} = 6\pi + \frac{140}{x^3}$

The result is positive at $x = p$ A1

A is minimum at p

(iii) Attempt to substitute the value of p into A M1

minimum area = 67.8 A1

[6 marks]

Total [14 marks]