Mathematics: analysis and approaches

Standard level

Paper 2

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: Answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is
required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Consider the function defined by $f(x) = x^2 - 6x$. The graph of f passes through the point A(2,-8).

(a) (i) Find the gradient of the tangent to the graph of f at the point A.

(ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

(b) Write down the equation of the normal to the graph of f at point A. [1]

The normal to the graph of f at point A intersects the graph of f again at a second point B.

(c) Find the coordinates of B. [3]

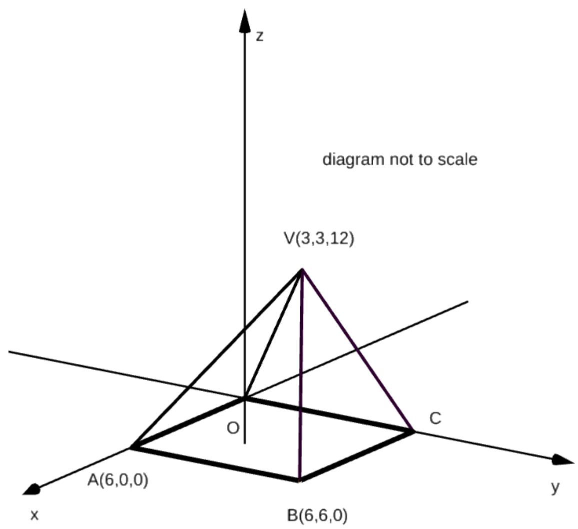
(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The following diagram shows a square pyramid with vertex V and a squared base OABC.

Point A has coordinates (6,0,0), point B has coordinates (6,6,0) and the vertex V has coordinates V(3,3,12).



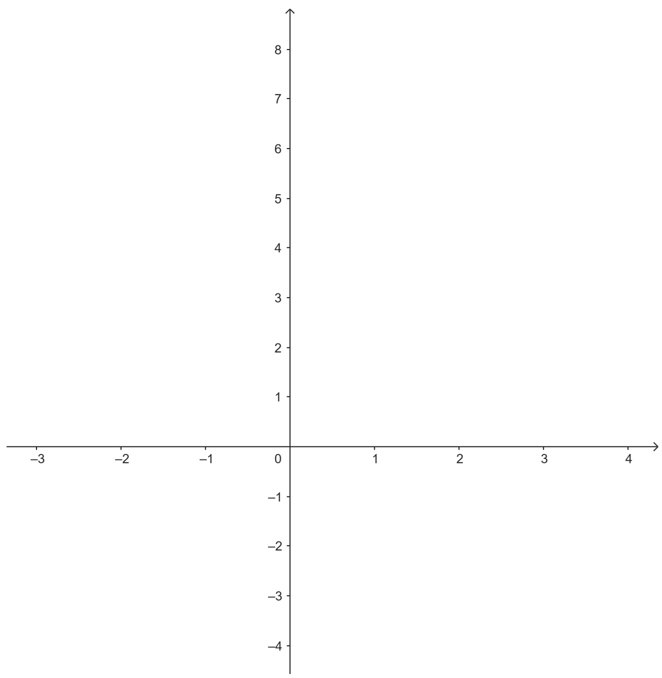
(a) Find AV. [2]

(b) Find the size of $V\hat{A}B$. [4]

3. [Maximum mark: 5]

Consider the function $f(x) = 5^{-x} + 2x - 2$

(a) On the following axes, sketch the graph of f for $-3 \leq x \leq 4$ [3]



The function $g(x)$ is obtained as the result of 2 transformations of the function f, first an horizontal stretch with scale factor $\frac{1}{2}$, followed by a vertical translation of 3 units.

(b) Find the function $g(x)$ [2]

4. [Maximum mark: 7]

A teacher wants to investigate if there is a relation between the results of their Math test and the hours spent by the students in social media. She decided to survey 8 students. The following table shows the results,

Mark (m)	65	77	23	88	27	90	56	47
Time (t)	2	3	5	2	4	1.5	2.5	3.5

(a) The regression line of t on m for this data can be written in the form $t = am + b$.

Find the value of a and the value of b. [2]

(b) Write down the Pearson's product moment coefficient r. [1]

(c) Use your regression line to estimate the time spent by a student that gets 85 marks. [2]

(d) Explain why this regression line is not useful for a student that gets 2 marks. [2]

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds in given by $s(t) = 3.5\, cos(\sqrt{5t + 2}) + 5$, where $0 \leq t \leq 8$.

The particle first comes at rest after k seconds.

(a) Find the value of k. [2]

(b) Find the total distance that the particle travels in the first k seconds. [3]

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $p, q \in R^{+}$.

x	1	2	3	4
$P(X = x)$	p^2	p^2	q	p^3

Given that $E(X) = 2.1$, find the value of p and q.

Do **not** write solutions on this page.

Section B

Answer all questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Maggie receives a bonus at the end of the year of $24000. On the 1st of January 2025 she wants to invest her money.

Bank A offers her to invest the money in an account that pays nominal annual interest rate of 6% per annum, compounded monthly. The interest is added on the last day of each month.

To invest the money in an account:

(a) Write down an expression for the value of Maggie´s investment after n years. [1]

(b) (i) Find the total value Maggie receives after a year and a half.

Maggie wants to buy a car for 28000.

(ii) Find the minimum number of complete months before the value of Maggie´s investment is at least 28000, investing in **Bank A.** [4]

Bank B offers Maggie another option: to receive 28000 after 3 years. The first year and a half compounding monthly at an annual interest rate of 6%, the rest of the time at a r nominal annual interest rate, compounded quarterly.

c) Find the value of the rate r correct to 1 decimal place. [3]

(This question continues on the following page)

(Question 7 continued)

Maggie bought the car the 1st of January 2021 for $22000. The value of the car depreciates at a rate of 15% per year.

(d) Calculate the actual value of the car to the nearest hundred. [4]

(e) Calculate the month and the year when the price of the car is halved. [3]

8. [Maximum mark: 16]

A factory is producing bags of sand. An automatic machine is filling the bags. The weight of each bag, W kgs, can be modelled by a normal distribution with a mean μ and a standard deviation σ.

It is known that P$(W < 31)$ = 0.0388 and P$(W > 33)$ = 0.2781

(a) Find the probability that the weight of a randomly selected bag is between 31 and 33 kilograms. [2]

(b) Find the value of μ and the value of σ. [5]

The company measures 100 bags selected at random. Any bag between 31 and 33 kilograms passes the control. Weights of bags are independent of each other.

(c) (i) Find the probability that exactly 70 bags pass the control.

(ii) Given that 70 bags pass the control, find the probability that exactly 60 bags weight more than 32kg. [6]

(d) Find the interquartile range. [3]

9. [Maximum mark: 14]

Paul needs to store seeds. The volume of seeds is 35 m^3. The shape of the storage should be an open cylinder, where the seed will be stored. To close the storage there is a top part with the shape of a hemisphere that fits exactly at the top of the cylinder. The cylinder has a height of h and a radius of x cm.

The total surface of the storage has a local maximum or minimum at $x = p$

(a) Show that the total surface area, A cm^2, of the solid is given by $A = 3\pi x^2 + \frac{70}{x}$ [3]

(b) (i) Find an expression for $\frac{dA}{dx}$

(ii) Find the exact value of p [5]

(c) (i) Find an expression for $\frac{d^2A}{dx^2}$

(ii) Use the second derivative of A to justify that A is a minimum when $x = p$.

(iii) Find the minimum surface area of the solid. [6]