Mathematics: applications and interpretation

Standard level

Paper 2

Markscheme

1. (a) $rs = -1$ ***A1***

(b) i) $r = -0.988(-0.98767.....)$ ***A2***

ii) strong **AND** negative ***A1A1***

> **Note:** Award at most ***A1A0*** if additional answers are seen.
> Due to the demand of the question, do not accept "negative (from the graph)" if their *r* value is positive.

(c) (i) $a = -5.57\ (-5.5701...)$ ***A1***

(ii) $b = 239(239.12....)$ ***A1***

(iii) *b* represents the electricity bill when the temperature is $0°C$ ***A1***

(d) i) attempt to substitute 15 for *x* ***(M1)***

$y = -5.57\times15 + 239$ ***A1***

$= 155(155.45.....)$ ***A1***

ii) Interpolation ***R1***

strong correlation ***R1***

e) H_1: $\mu_1 \neq \mu_2$ ***A1***

(f) p = 0.112 (0.11164855) … ***A2***
Note: Award ***A1*** for 0.11 (2sf)
Award ***A1*** for an answer of p = 0.111675… , from use of unpooled GDC settings.

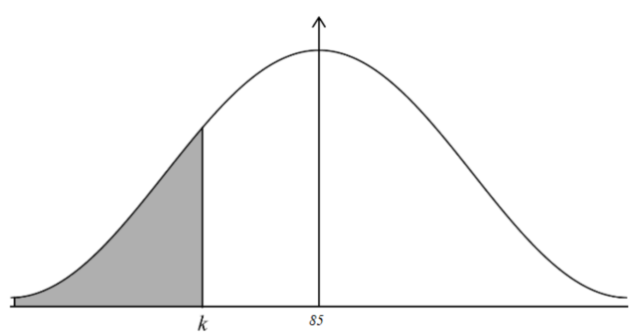
(g) $0.112 > 0.05$ ***R1***
there is insufficient evidence to reject the null hypothesis therefore the null hypothesis is accepted ***A1***
Note: Do not award ***R0A1***.

(h) (the two populations are) normally distributed ***A1***
Note: Do not accept "independent" as that applies to the samples, not the populations.

[Total 19 marks]

2. (a) 0.5 ***A1***
(b) 0.394 (0.39435022…) ***A2***

(c) (i)



A1A1

Note: Award ***A1*** for a normal curve (with symmetry and some evidence of change of curvature towards the extreme values).
Award ***A1*** for a shaded region $w < k$, where $k < mean$.

(ii) $P(W < k) = 0.25$
solving a cumulative distribution function **OR** use of inverse function on GDC ***(M1)***
$k = 71.5$ (71.5102…) ***A1***

(d) recognizing binomial distribution ***(M1)***
B(5, 0.25)
$P(w = 3)$ ***(A1)***
0.0879 (0.087890….) ***A1***

(e) $20(w - 3) + 50$ ***or*** $20w + 30$ ***A1A1***

Note: Award ***A1*** for a linear expression with a gradient of 2,
A1 for a completely correct expression in w.

(f) ($)154 ***A1***

(g) attempt to solve $20(w - 3) + 50 = 110$ ***or*** $20w + 30 = 110$ ***(M1)***

$w = 4$ ***A1***

[Total 15 marks]

3. (a) $1200 - 30x^2 = 600$ ***(M1)***

$x = 4.47$ (pesos) (since x is positive) ***A1***

(b) (i) $1200 - 30\times5^2 = 450$ ***A1***

(ii) $450\times5 = 2250$ (pesos) ***A1***

(c) (i) $profit = revenue - costs = V\times x - V\times 4$ ***(M1)***

$P = \left(1200 - 30x^2\right)x - \left(1200 - 30x^2\right)4$ ***A1***

$P = -30x^3 + 120x^2 + 1200x - 4800$ ***AG***

(ii) $\frac{dP}{dx} = -90x^2 + 240x + 1200$ ***(M1)A1A1***

(iii) attempt to find *x*-value ***(M1)***

e.g. sketch of $\frac{dP}{dx}$ with *x*-intercept indicated **OR** recognition that it occurs at the maximum of P **OR** algebraic approach

$-90x^2 + 240x + 1200 = 0$

$x = 5.22$ ***A1***

(iv) attempt to substitute their *x*-value into equation for V ***(M1)***

$1200 - 30 \times 5.22^2 = 382$ or 383 ***A1***

[Total 13 marks]

4. (a) (i) 0.92 (ii) 0.25 (iii) 0.75 ***A2***

Note: Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

(b) $0.08 \times 0.25 = 0.02$ ***A1***

(c) $P(not\ fail) = 0.69$ ***A1***
multiplying by 200 ***M1***
=138 ***AG***

Note: Award ***A0M0*** for a flawed approach to find $P(not\ fail)$ for example $\frac{172.5}{250}$, which is reverse engineering.

(d) Attempt to find the probability of one sensor failing. ***(M1)***

Then

No sensor fails	one sensor fails	both sensors fail
138	58	4

(A1)

degrees of freedom = 2 ***(A1)***

Note*:* Award ***A1*** for df = 2 seen anywhere and may be awarded independent of the ***M1*** mark.
The df cannot be implied from chi square statistic = 40.9

$p - value = 1.31\times10^{-9}$ $(1.309.... \times10^{-9})$ ***(A1)***

$0.05 > 1.31\times10^{-9}$ ***R1***

hence there is sufficient evidence to reject *H*0; the manufacturers claims are not both correct ***A1A1***

Note: The ***R1A1*** can be awarded as follow through within part (d) from their (explicitly labelled) incorrect p-value.
An unrealistic p-value ($p \geq 1$) should preclude awarding the final ***R1A1***.
Accept either a conclusion to reject the null hypothesis or the manufacturer's claims are not both correct.
Do not award ***R0A1***.

[Total 12 marks]

5. (a) attempt to substitute 4 into the $h'(x)$ ***(M1)***

$\frac{1}{16} \times 4^2 - \frac{3}{8}\times4 =- \frac{1}{2}$ ***A1***

(b) recognition of need to integrate ***(M1)***

$\frac{1}{48}x^3 - \frac{3}{16}x^2 + c$ ***(A1)***

attempt to substitute given condition to find *c* ***(M1)***

$\frac{1}{48}8^3 - \frac{3}{16}8^2 + c = \frac{5}{3}$ $\qquad c = 3$

$h(x) = \frac{1}{48}x^3 - \frac{3}{16}x^2 + 3$ ***A1***

(c) (i) $\int_0^8 \left(\frac{1}{48}x^3 - \frac{3}{16}x^2 + 3\right)dx$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location.
Award at most ***A0A1*** if d*x* is omitted.
Award at most ***A0A1*** for "integral *y dx*" this is not the correct integrand.

(ii) $13.3\ (m^2)$ ***A2***

Note: A negative area for their integrand is unrealistic. Award at most ***A1A0*** for their ***FT*** answer expressed as negative area or a negative area converted to a positive answer."

(d) $13.3 \times 1.5 = 20.0\ m^3\ (19.95 m^3)$ ***A1***

Note: Correct unit must be seen for the ***A1*** to be awarded.

(e) attempt to substitute one of the given conditions, both *x* and *h*, into either the function or the derivative ***(M1)***

$h(4) = 16a + 4b + c = \frac{4}{3}$ ***A1***

attempt to differentiate *h*(*x*) ***(M1)***

$h'(x) = 2ax + b$ ***(A1)***

$8a + b = -\frac{1}{2}$ ***A1***

$16a + b = 0$ ***A1***

Note: The equations can be found in any order, and hence the associated mark(s) should be awarded independently.

Award at most ***M1A1M1A1A1A0*** if the equations are not simplified to integer values.
E.g . $h(4) = a4^2 + b4 + c$........

(f) $a = \frac{1}{16}$ (0.0625), $b = -1$ and $c = \frac{13}{3}(4.33)$ ***A2***

Note: Award ***A1*** if only two are correct, ***A0*** otherwise.
Only follow through from three explicit equations given in part (e).

(g) gradients are the same **OR** $h'(4) = -\frac{1}{2}$ for both curves ***A1***
heights are the same **OR** $h(4) = \frac{4}{3}$ for both curves ***A1***

[Total 21 marks]