Mathematics: applications and interpretation

Standard level

Paper 2

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

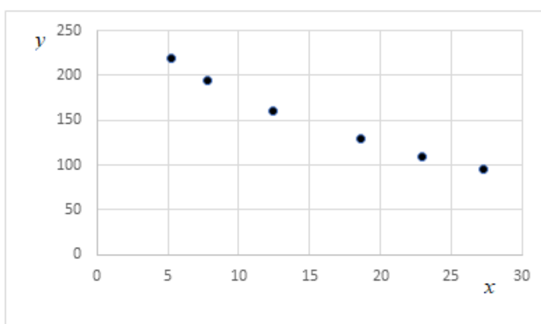
1. [Maximum mark: 19]

Sophia conducted a study to investigate whether there is a relationship between the monthly electricity bill, y, of a household and the average daily temperature, x, in a city.

She collected data from six randomly selected households during different months of the year and obtained the following results:

Average daily temperature(°C)	5.2	7.8	12.4	18.6	22.9	27.3
Electricity bill ($)	220	195	160	130	110	95

A scatter plot of these data is shown below.



(a) Write down the value of the Spearman's rank correlation coefficient, r_s. **[1]**

(b) (i) Find the Pearson's product-moment correlation coefficient, r.

(ii) Use your value of r to state which two of the following would best describe the correlation between the variables: Positive, Negative, Strong, Weak, No correlation. **[4]**

(This question continues on the following page)

(Question 1 continued)

The relationship between the variables can be modeled by the regression equation $y = ax + b$

(c) (i) Write down the value of a.

(ii) Write down the value of b.

(iii) According to this model, state in context what the value of b represents. **[3]**

(d) Sophia uses the regression equation to estimate the electricity bill for a household when the average daily temperature is 15.0°C.

(i) Find this estimated bill.

(ii) State two reasons Sophia might use to justify the validity of this estimate. **[5]**

To verify whether the relationship between electricity bill and temperature holds in different regions of the city, Sophia considers two neighborhoods, **Northville** and **Southport**, both experiencing similar temperature variations. She collects data from seven randomly selected households in each neighborhood and records their monthly electricity bills (in dollars) in the following tables:

Electricity bill Northville($)	Electricity bill Southport($)
120	140
125	145
130	155
135	160
150	175
170	190
185	210

Sophia conducts a **t-test** at the 5% level of significance to determine whether the mean electricity bill in Northville is different from the mean electricity bill in Southport. She assumes that the population variances are equal.

(This question continues on the following page)

(Question 1 continued)

For this test, Sophia sets the **null hypothesis** as:

$H_0\colon\ \mu_1 = \mu_2$

(e) Write down the alternative hypothesis. **[1]**

(f) Find the p-value for this test. **[2]**

(g) State the conclusion of the test. Justify your answer. **[2]**

(h) State one additional assumption Sophia has made about the distributions to conduct this test. **[1]**

2. [Maximum mark: 15]

A **hospital emergency department** tracks the waiting time, W, for patients before seeing a doctor. Based on past data, the waiting time follows a normal distribution with a mean of $85\ minutes$ and a standard deviation of $20\ minutes$.

(a) State $P(W < 85)$ **[1]**

(b) Find $P(60 < W < 85)$. **[2]**

25% of patients are seen by a doctor in less than k minutes.

(c) (i) Sketch a diagram of this normal distribution, shading the region that represents $P(W < k)$
(ii) Find the value of k. **[4]**

For a study, hospital staff randomly select five patients arriving at the emergency department. These selections are independent.

(d) Find the probability that exactly 3 of these patients are seen in less than k minutes. **[3]**

The hospital parking lot charges a flat rate of $50 for any car parked for up to 3 hours. For parking durations exceeding 3 hours, an additional $20 is charged for each extra hour or portion thereof.

For example, a car parked for 3.5 hours will be charged $50 plus an extra $10.

(e) Write an expression for the total parking charge, *C*, for a car parked for *x* hours, $x > 3$. **[2]**

(f) Calculate the total charge for a car parked for 6.2 hours. **[1]**

(g) Determine the total parking duration for a car that was charged $110. **[2]**

3. [Maximum mark: 13]

A water park tracks the number of visitors, V, per day in terms of x, the price of a single ticket (in dollars). The park uses the model:

$$V = 2400 - 30x^2$$

The maximum capacity of the water park is **600 visitors** per day.

(a) Find the maximum price they could charge per ticket for the park to have **600 visitors** in one day. **[2]**

(b) On a certain day the park sells tickets for **$5** each, use the model to find:

(i) the number of visitors.

(ii) the total income from ticket sales. **[2]**

The cost of running the park per visitor is **$4**. The profit per day, P, is the total income from ticket sales minus the cost of accommodating visitors.

(c) (i) Show that, according to the model,

$$P = -30x^3 + 120x^2 + 1200x - 4800$$

(ii) Find $\frac{dP}{dx}$

(iii) Find the value of x for which $\frac{dP}{dx} = 0$.

(iv) Find the number of visitors when the profit is maximized. **[9]**

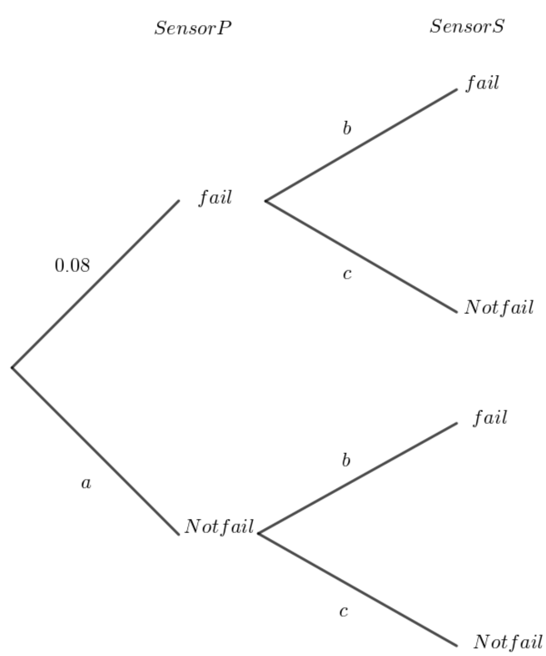
4. [Maximum mark: 12]

A new security system in a building requires at least one of two sensors to be functioning for the system to work. The system has a **primary sensor (P)** and a **secondary sensor (S)** for backup.

The manufacturer claims that the probability that **sensor P** fails within a month is **0.08**.The probability that **sensor S** fails within a month is **0.25**.The failures of the two sensors are **independent**.

If **both sensors fail**, the security system will go offline and need repairs. All sensors are replaced at the end of the month or when the system goes offline.

The following tree diagram shows the probabilities of sensor failures within one month, assuming the manufacturer's claims are correct.



(a) Write down the values of
(i) a (ii) b (iii) c **[2]**

(b) Hence find the probability that the security system goes offline within one month.**[1]**

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(Question 4 continued)

The building manager is skeptical of the manufacturer's claims, so they analyze records from the last 200 months. They classify each month based on whether **no sensors failed, one sensor failed, or both sensors failed**.

The data collected are shown in the table:

No sensor failed	one sensor failed	both sensors failed
110	75	15

(c) Show that the expected number of months in which no sensors failed, assuming the manufacturer's claims are correct, is **138**. **[2]**

(d) Perform a χ2 goodness-of-fit test at the **5% significance level** to test whether the manufacturer's claims are accurate, using the following hypotheses:

H_0: *The manufacturer's claims are correct.*

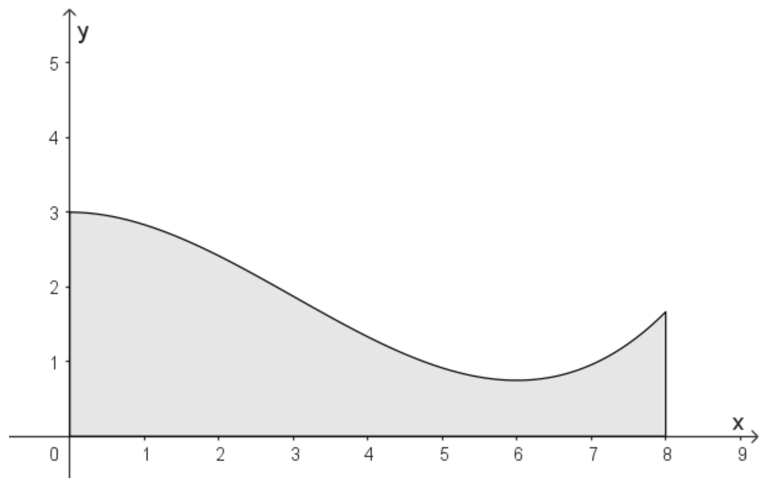
H_1: *The manufacturer's claims are not both correct.* **[7]**

5. [Maximum mark: 21]

Emma is designing a **skateboarding ramp** for a sports complex.

Let x be the horizontal distance, in metres, from the start of the ramp.
Let h be the height, in metres, of the ramp above the ground.

The following diagram shows a cross-section of the ramp. The ramp is supported by a **wooden platform**, represented by the shaded region in the diagram.



It is known that:

$$h'(x) = \frac{1}{16}x^2 - \frac{3}{8}x$$

(a) Find the gradient of the ramp when $x = 4$. **[2]**

At the end of the ramp, $x = 8$ and $h = \frac{5}{3}$

(b) Find an expression for $h(x)$. **[4]**

(c) (i) Write down an integral that can be used to find the area of the wooden platform.

(ii) Find the cross-sectional area of the wooden platform. **[4]**

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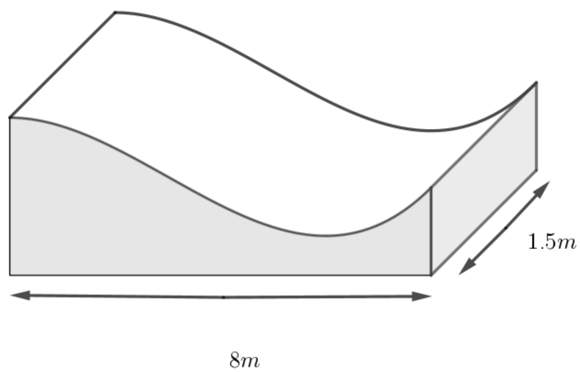
(Question 5 continued)

The ramp and wooden platform have a uniform cross-section and a width of **1.5 metres**, as shown in the following diagram.



(d) Find the volume of the wooden platform. **[1]**

To meet safety standards, Emma modifies the design. The starting section remains unchanged. Then, over the domain $4 \le x \le 8$, the updated design must satisfy the following conditions:

$h(x) = ax^2 + bx + c$ $\quad h(4) = \frac{4}{3}$ $\quad h'(4) = -\frac{1}{2}$ $\quad h'(8) = 0$

(e) Write down three equations in terms of a, b and c using the given conditions. **[6]**

(f) Hence, find the values of a, b and c. **[2]**

Emma claims that the updated design will ensure a smooth transition for skateboarders at $x = 4$

(g) Explain why Emma's claim is correct at this point. **[2]**