Mathematics: applications and interpretation

Higher level

Paper 1

2 hours

Instructions to candidates

- Write your session number in the boxes above
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working.You are therefore advised to show all working.

1. [Maximum mark: 7]

A marine biologist is studying the population of a rare species of fish in a protected lagoon. On 1 January 2025, the population is estimated to be 100 fish. Due to conservation efforts, the population is expected to increase each month, following a geometric sequence where r = 1.05.

(a) Use this model to estimate the fish population in December 2025. [2]

(b) Use this model to estimate the total fish population increase during the year

(i) 2025.

(ii) 2026. [5]

2. [Maximum mark: 6]

Cecilia wants to construct a pool in her house and deposits $80,000 into a savings account with a nominal annual interest rate of r%, **compounded monthly**. After one year, the amount in the account has increased to $84,100.

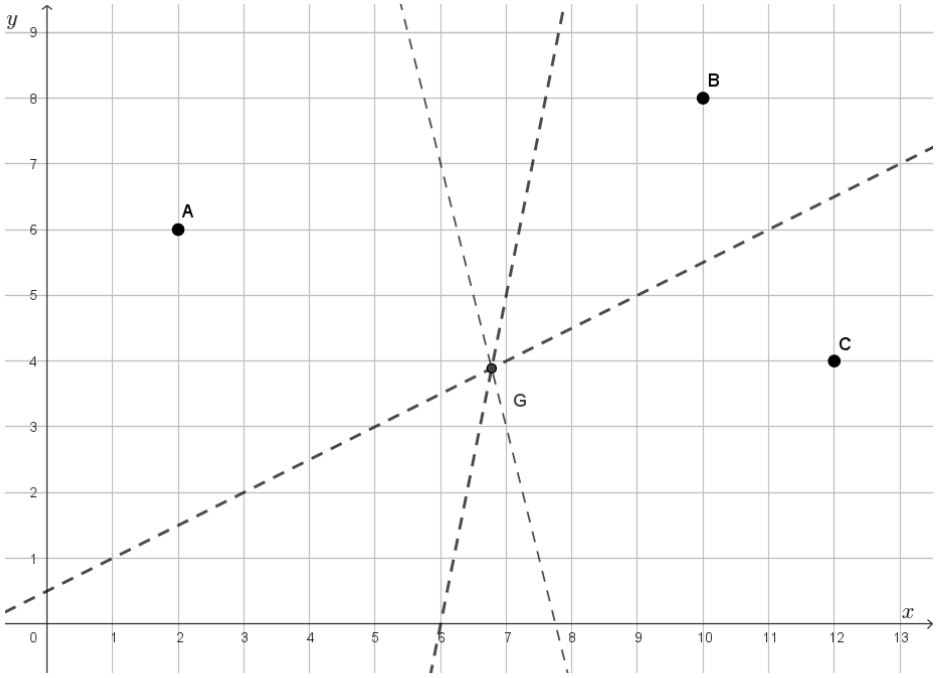
(a) Find the value of **r**. [3]

Cecilia withdraws the $84,100 and places it in an annuity that earns a nominal annual interest rate of 5.8%, **compounded monthly**. She will withdraw $7,000 at the end of each month to cover the construction costs.

(b) Find the amount of money remaining in the annuity at the end of **six months**. Express your answer to the nearest dollar. [3]

3. [Maximum mark: 6]

Three cellphone towers are installed in a rural area at locations $A(2,\ 6)$, $B(10,\ 8)$, and $C(12,\ 4)$. The signal coverage is analyzed using perpendicular bisectors of the segments joining the towers.



The perpendicular bisector of $[BC]$ intercepts the axes at coordinates $(0,\ 0.5)$ and $(-\ 1,\ 0)$.

(a) Write down the equation of the perpendicular bisector of $[BC]$ [2]

The equation of the perpendicular bisector of [AB] is $y\ =\ -\ 4x\ +\ 31$.

(b) Find the coordinates of point G, where the perpendicular bisectors meet. Give your answer to four significant figures.
[2]

A Voronoi diagram is used to determine the area best covered by each cellphone tower.

(c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

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(Question 3 continued)

4. [Maximum mark: 6]

The speed of a car, S, in miles per hour (mph), can be converted to kilometers per hour (km/h) using the formula:

$$K = 1.609S$$

where K is the speed in km/h and S is the speed in mph.

(a) (i) Find a formula for converting a speed in kilometers per hour to miles per hour.

(ii) Find the speed in miles per hour that corresponds to 100 km/h. [3]

Over a long journey, the mean speed of a car was recorded as 65 mph with a standard deviation of 8 mph.

(b) (i) Find the mean speed of the car in kilometers per hour.

(ii) Find the variance of the car's speed in kilometers per hour. [3]

5. [Maximum mark: 8]

When studying trees, researchers use a model in which the volume ($V\ cubic\ meters$) of a tree trunk is directly proportional to the cube of its height ($h\ meters$).

(a) (i) Use the model to find an expression for V in terms of h.

(ii) Hence find the volume of a tree with a height of **6.5 meters**, given that a tree of height **7 meters** has a volume of **8.75 cubic meters**. [4]

'Kleiber's law' states that the water uptake (W) of a tree is inversely proportional to the square of its height.

The water uptake of a $10m$ tree is k times the water uptake of a $7\ m$ tree.

(b) Find the value of k. [4]

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(Question 5 continued)

6. [Maximum mark: 5]

The following diagram shows a semicircular window with center O and diameter XY. A rectangular glass panel ODEF is also shown, such that DE = 10 cm and OD = 6 cm.

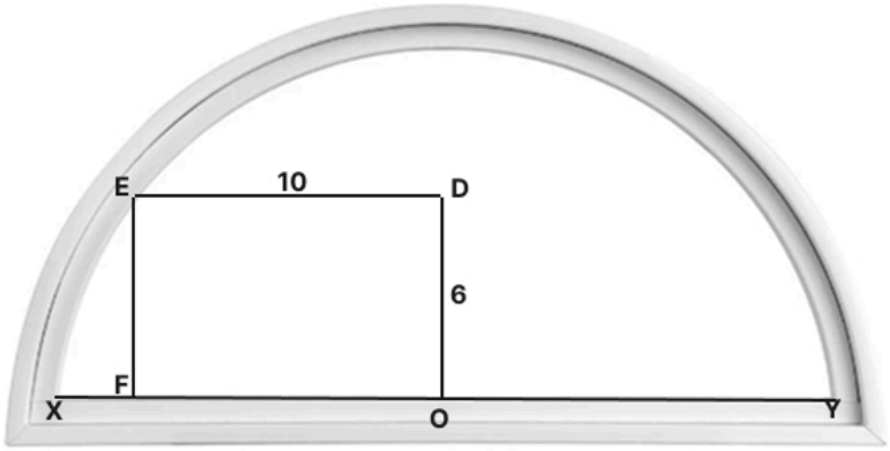


Diagram not to scale

Find the length of the arc EY.

7. [Maximum marks: 7]

Sonia, a mathematics teacher, wants to determine if there is a correlation between the time her students spend studying and their test results. To investigate this, she randomly selects five students from her class who took the same mathematics test and records the number of hours (t) they spent studying and the marks (m) they achieved.

The data is shown in the table below:

Hours	0	1.4	1.8	2.5	4.5
Marks	40	52	60	75	85

Sonia believes that there might be a linear relationship between the time spent studying and the marks obtained.

(a) (i) Find the Pearson's product moment correlation coefficient, r, for this data.
(ii) Find the least squares regression line of mmm on t for this data. [4]

(b) Using the regression model, estimate how many more marks a student would achieve if they studied an extra 1.5 hours. [2]

(c) State one reason why the value obtained in part (b) might not be valid. [1]

8. [Maximum mark: 4]

Consider the differential equation

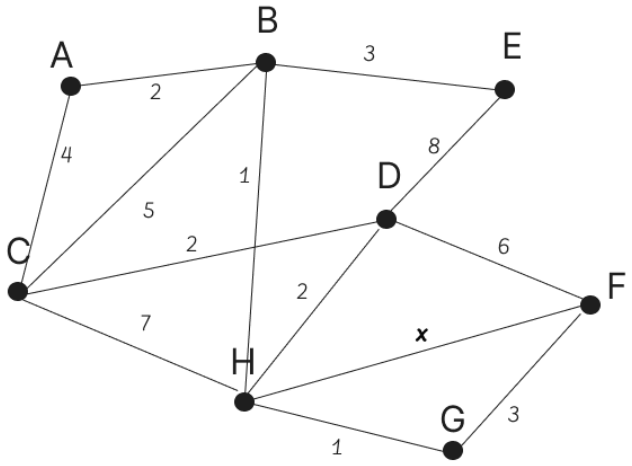
$$\frac{dy}{dx} = e^{-(x+y)}$$

where $x \geq 0$ and $y > 0$

Given that $y = 2$ when $x = 0$, use Euler's method with a step length of 0.1 to find an approximate value for y when $x = 2$

9, [Maximum mark: 5]

The weights on the following graph represent the time, in minutes, required to complete different tasks in a factory process.



(Diagram not to scale)

(a) Write down the vertices with odd degree. [1]

The total time required to complete all tasks is $41 + x$ minutes.

(b) Find two expressions, in terms of x, for the shortest time needed to complete all tasks at least once, starting and ending at the same vertex. Include in your answer the interval of values of x for which each expression is valid. [4]

10. [Maximum mark: 7]

(a) Find $\int \frac{16x}{2x^2+1} dx$ [3]

(b) Hence find the exact area between the curve $y = \frac{16x}{2x^2+1}$, the x-axis and the lines $x = 0$ and $x = 4$. Give your answer in the form $a \ln b$, where $a\, b, \in R$. [4]

11. [Maximum mark: 8]

Let $T(\theta)$ be the matrix representing a reflection in the line $y = tan(\theta)x$

(a) (i) Show that $T(\theta) \times T(\theta)$ is the identity matrix. [2]

(ii) Explain why this means that T(θ) is **self-inverse**, [2]

(iii) Use your result to explain why applying the reflection transformation twice returns every point to its original position. [2]

(b) Show that the determinant of $T(\theta)$ is -1, and explain what this means geometrically. [2]

12. [Maximum mark: 8]

A biologist is studying the spread of a certain bacteria colony in a controlled environment. They propose that the population P of the bacteria (in thousands) can be modeled by:

$$P = 700 - 600\times3^{-0.5t}$$

where t is the number of hours since the bacteria were introduced to the environment.

(a) Find the time taken for the population to grow from 0 to 300 thousand, according to this model. [2]

The biologist collects actual data for the population at different times and records the following values:

$t\ (hours)$	1	2	3	4
$P\ (thousands)$	260	420	560	620

(b) Find the sum of the square residuals $(SSres)$ when using $P = 700 - 600\times3^{-0.5t}$ to fit this data. [4]

The biologist now considers using a logistic model instead. They find that, when using this model, the sum of the square residuals for these four points is 201.

(c) State one reason why the biologist might prefer to use:

(i) the logistic model.

(ii) $P = 700 - 600\times3^{-0.5t}$ [2]

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(Question 12 continued)

13. [Maximum mark: 6]

Consider the complex numbers: $z_1 = 2 + 2i$ and $z_2 = \sqrt{3} + i$

Let $w = z_1 z_2$

By expressing z_1 and z_2 in modulus-argument form ,

a) write down the modulus of w [3]

b) write down the argument of w [1]

c) Find the smallest positive integer value of n such that w^n is a real number. [2]

14. [Maximum mark: 4]

Two boats are moving along a straight river. Boat A departs from the dock at $t = 0$ and its displacement x_A (in meters) from the dock at t hours is given by the equation:

$$x_A = \frac{1}{3}t^2, 0 \leq t \leq 10.$$

A faster boat, Boat B, leaves the dock at $t = 2$
After starting its motion, the time taken by Boat B to reach any point on the river is **half** the time that was taken by Boat A.

(a) Write down the equation for the displacement of Boat B, x_B in terms of t. [2]

(b) Find the value of t at which Boat B catches up with Boat A. [2]

15. [Maximum mark: 6]

A system of differential equations of the form

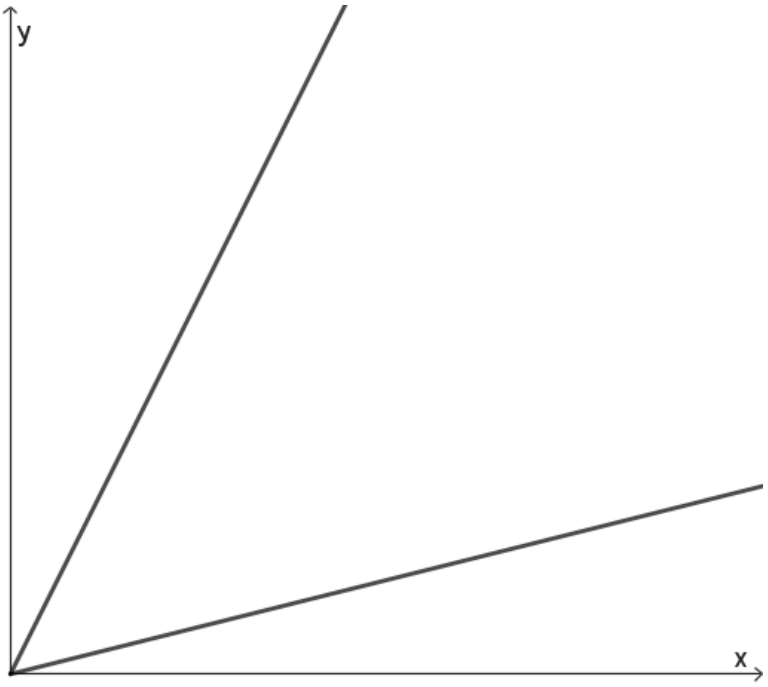
$$\frac{dx}{dt} = mx + ny$$

$$\frac{dy}{dt} = px + qy$$

has eigenvalues $\lambda = -3$ and $\lambda = 1$ with corresponding eigenvectors

$(4\ 1)$ and $(1\ 2)$

The following incomplete phase portrait for this system, with $x,\ y \geq 0$, shows lines through (0,0) parallel to the eigenvectors.



(a) On the phase portrait:

(i) Indicate the direction of motion along the eigenvectors.

(ii) Sketch one trajectory in each of the three regions. [3]

In this system, x and y represent the populations of two species, X and Y.
The population of Y is vulnerable, so additional members of species Y will be introduced to help maintain its population.
Currently, $x = 300$ and $y = 50$

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(Question 15 continued)

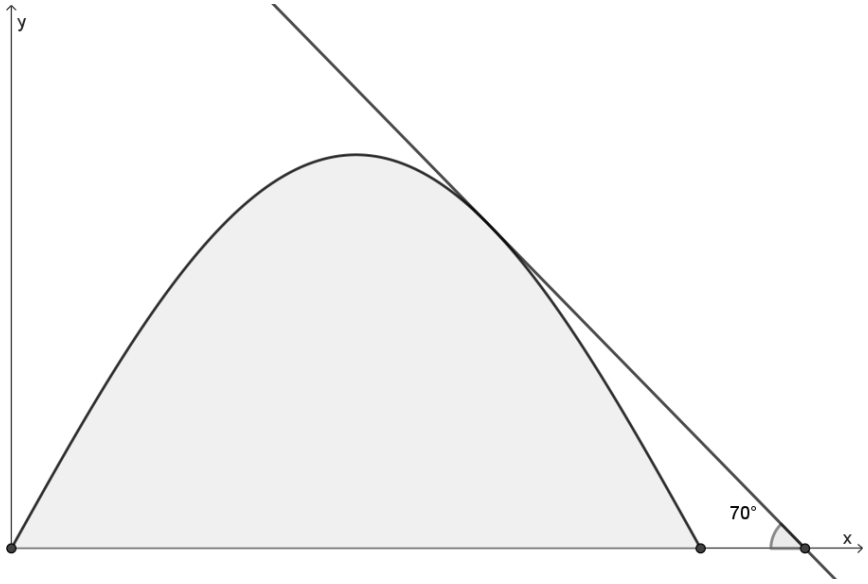
(b) Find the minimum number of new animals from species Y that need to be added for the population not to reduce to 0 over time. [3]

16. [Maximum mark: 8]

The cross-section of a bridge arch can be modeled by the curve with equation

$$y = 3 \sin sin (0.6x), \quad 0 \leq x \leq 5.24$$

where distances are measured in metres.



A worker places a straight support beam against the bridge arch to carry out maintenance. For stability, the angle between the beam and the horizontal ground must be **70°**.

Find the height above the ground at which the beam touches the bridge arch.

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(Question 16 continued)

17. [Maximum mark: 9]

A town has two mobile network providers, Alpha Mobile and Beta Telecom. After analyzing customer preferences, the town's telecommunications authority finds that:

- 12% of customers who used Alpha Mobile in one-month switch to Beta Telecom the following month.
- 18% of customers who used Beta Telecom in one-month switch to Alpha Mobile the following month.

These changes can be represented by the following transition matrix:

$$\begin{pmatrix} 0.88 & 0.18 \\ 0.12 & 0.82 \end{pmatrix}$$

The two eigenvalues for this matrix are 1 and 0.7. An eigenvector corresponding to the eigenvalue of 1 is:

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

(a) Find an eigenvector corresponding to the eigenvalue of 0.7. [2]

A diagonal matrix of eigenvalues is:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}$$

(b) Write down an expression for D^n, giving your answer as a 2×2 matrix in terms of n. [1]

When the study began, Alpha Mobile had 5000 customers while Beta Telecom had 3000 customers.

(c) Assuming all customers remain with one of these two providers, find an expression for the number of customers Alpha Mobile will have after n months. [6]

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(Question 17 continued)