Mathematics: applications and interpretation

Higher level

Paper 1

Markscheme

1. (a) attempt to substitute into geometric sequence formula for twelfth term ***(M1)***

$u_{12} = 100 \times 1.05^{12-1}$ or 100,105, 110.25………

171 (171.0339…) ***A1***

(b) (i) attempt to substitute into the geometric series formula **OR** a sum of at least the first three terms ***(M1)***

$S_{12} = \frac{100(1.05^{12}-1)}{1.05-1}$ or $100 + 105 + 110.25 + \ldots$..

Note: Award ***M1*** for $u_1 = 100$ and $r = 1.05$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their u_n formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 1590$ (1591.712652….) ***A1***

(ii) finding $S_{24} = 4450.199887$ … or attempt to find the sum between u_{13} and u_{24} ***(M1)***

Note: Award ***M1*** for $S_{24} = 4450.199887$ **or** sigma notation that includes correct limits and their u_n formula

A1

$4450 - 1590 = 2860.$ ***A1***

[Total: 7 marks]

2. (a) **EITHER**

$N = 12$
$PV = \pm 80000$
$FV = \pm 84100$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

OR

$N = 1$
$PV = \pm 80000$
$FV = \pm 84100$
$P/Y = 1$
$C/Y = 12$ ***(M1)(A1)***

THEN

$I = 5.00$ (5.0008416…) ***A1***

$N = 6$
$I\% = 5,8$
$PV = \pm 84100$
$PMT = \pm 7000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award ***A1*** for all entries correct in the financial app (condone missing -/+ sign if the correct final answer is seen).

$FV =$ (\$) 44058 ***A1***

Note: Answer must be correct to the nearest dollar to award the final ***A1***.
Award ***(M1)(A1)A0*** for an unsupported final answer to a greater degree of accuracy eg. ($) 44057.778...
Award ***M1A1A0*** for a truncated answer of 44057 if no working is shown.

[Total: 6 marks]

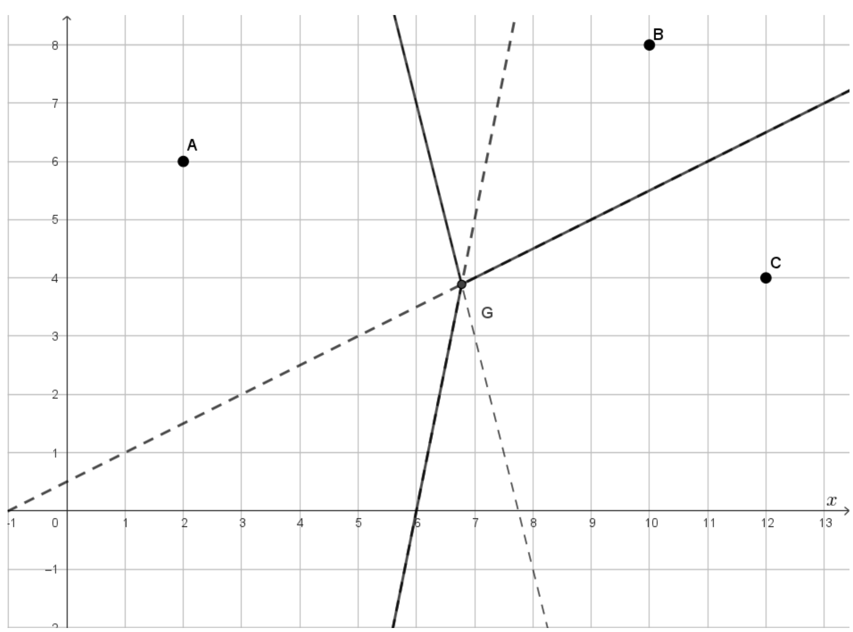
3. (a) $y = \frac{1}{2}x + \frac{1}{2}$ ***A1A1***

Note: Award ***A1*** for $\frac{1}{2}x$ and ***A1*** for $\frac{1}{2}$ (or equivalent equation). Award at most ***A1A0*** if the answer is not presented as an equation.

(b) G (6.778, 3.889) ***A1A1***

Note: If both answers are not correct to 4 sig figs, award at most ***A1A0***.

(c)



A2

Note: Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the *x*-axis.

Correct edges	Incorrect edges	Marks
3	0	***A2***
3	1	***A1A0***
3	2 or more	***A0A0***
2	0	***A1A0***
2	1	***A1A0***
2	2 or more	***A0A0***
1	0	***A1A0***
1	1 or more	***A0A0***

[Total: 6 marks]

4. (a) (i) attempt to rearrange to isolate S ***(M1)***

$S=\frac{1000}{1609}K$ or $S=\frac{1}{1.609}K$ ***A1***

Note: If the answer is not written as an equation, award at most ***M1A0.***

(ii) $S=\frac{1000}{1609}\times 100 = 62.2$ (62.1504039……) ***A1***

(b) i) $K = 1.609\times 65\approx 105$ (104.585) ***A1***

ii) recognizing that the variance is the square of the standard deviation ***M1***

$\sigma^2 = (1.609 \times 8)^2$

$\sigma^2 \approx 166$ or (165.688…………) ***A1***

[Total: 6 marks]

5. (a) (i) $V = c \times h^3$ ***(M1)***

(ii) replacing $V = 8.75$ and $h = 7$ in their equation ***A1***

$8.75 = c \times 7^3$

$c \approx 0.0255$ (0.02551020…..) ***A1***

$V = 0.0255 \times (6.5)^3$ ***A1***

(b) Attempt to express the equation for the water uptake ***(M1)***

$W = \frac{m}{h^2}$

Writing the ratio between the two trees ***(M1)***

$k = \frac{\frac{m}{10^2}}{\frac{m}{7^2}}$ ***A1***

$k = \frac{49}{100}$ ***A1***

[Total: 8 marks]

6. (recognition that OE is a radius and the hypothenuse of triangle ODE) ***A1***

$$\sqrt{6^2 + 10^2} = \sqrt{136}$$

(finding angle $Y\hat{O}E$)

correct calculation for finding $D\hat{O}E$ ***(A1)***

$D\hat{O}E = arctan(\frac{10}{6})$ **OR** tan $D\hat{O}E = \frac{10}{6}$

expressing $Y\hat{O}E$ as $90° + D\hat{O}E$ ***(M1)***

$Y\hat{O}E = 59.0°$

substituting *their* radius and angle $Y\hat{O}E$ correctly into arc length formula ***(M1)***

$Arc\ EY = \frac{59°}{360°} \times 2\pi \times \sqrt{136}$

$Arc\ EY = 12.0$ ***A1***

[Total: 5 marks]

7. (a) (i) $r = 0.966\ (0.96623561....\ )$ ... ***A2***

Note: Award ***A1*** for 0.97.

(ii) $m = 10.5\,t + 41.0\ \ (m = 10.4957767...\,t + 40.9886154...)$ ***A1A1***

Note: Second ***A1*** is for the correct variables.

(b) $10.5 \times 1.5\ \ (or 10.4957767... \times 1.5)$. ***(M1)***
15.75 (marks) (15.7436...) ***A1***

Note: Accept 16.

(c) *Accept any valid reason, e.g.:*
The students in the sample might not be of equal ability / she has not controlled for ability.
A student with full marks cannot be awarded an extra 15.75 would not be possible. ***R1***

[Total: 7 marks]

8. attempt to use Euler

$y_{n+1} = y_n + 0.1e^{-(x_n+y_n)}$ ***A1***

$y_1 = 2 + 0.1e^{-(0+2)}$ ***A1***

$y_1 = 2.10711$

$y_2 = 2.147862$ ***A1***

Then

$y(2) = 2.653838$ ***A1***

[Total: 4 marks]

9. (a) F and H ***A1***

(b) correct intervals seen ($x \leq 4$ (or $x < 4$) **AND** $x \geq 4$ (or $x > 4$)) ***A1***

Note: The case of $x = 4$ must be included for this ***A1*** to be awarded.

attempt to add edges to 41+ x ***(M1)***
(If $x < 4$ (or $x \leq 4$) then repeat FH and) length is $41 + 2x$ ***A1***
(If $x > 4$ (or $x \geq 4$) then repeat FG and GH and) length is $(41 + x + 4) = 45 + x$ ***A1***

Note: If the intervals are not explicit, award at most ***A0(M1)A1A1***.

[Total 5 marks]

10. (a) attempt to integrate by substitution or inspection **M1**

$4\ ln\left|2x^2 + 1\right| + c$

Note: Award ***M1*** for $ln(2x^2 + 1)$, ***A1*** for the 4 and ***A1*** for *c*. The ***A*** marks can only be awarded if the ***M*** mark is awarded. Condone absence of modulus signs.

(b) recognizing that area is $[4\ ln\left|2x^2 + 1\right|]_0^4$ ***(M1)***

$4\ ln\left(2\times4^2 + 1\right) - 4\ ln\left(2\times0^2 + 1\right) =$ ***(A1)***

$4\ ln(33) - 4\ ln(1) =$ ***(M1)***

$4\ ln(33) - 0 =$

$4\ ln(33)$ ***A1***

Note: Award ***(M1)A0M0A0*** for an unsupported final answer of 13.98603…   Award at most ***(M1)A1FTM0A0*** if their answer from part (a) does not include ln.

[Total 7 marks]

11.

a i) $i)$ $(\cos cos\ 2\theta\ \ \sin sin\ 2\theta\ \ \sin sin\ 2\theta\ \ 2\theta\ ) \times (\cos cos\ 2\theta\ \ \sin sin\ 2\theta\ \ \sin sin\ 2\theta\ \ 2\theta\ ) =$ ***A1***

$(2\theta\ + 2\theta\ \ \cos cos\ 2\theta \sin sin\ 2\theta - \cos cos\ 2\theta \sin sin\ 2\theta\ \ \ \cos cos\ 2\theta \sin sin\ 2\theta\ - \cos cos\ 2\theta \sin sin\ 2\theta\ \ 2\theta\ + 2\theta\ )$

= (1 0 0 1)

A1

ii) Correctly stating that if $A\times A = I$ then $A = A^{-1}$ satisfies the definition of an **inverse matrix** (i.e., a matrix whose product with itself gives the identity).

As $T(\theta)\times T(\theta)$ then $T(\theta) = T^{-1}(\theta)$ and is a self inverse matrix ***A1 A1***

iii) Explaining that matrix multiplication represents successive transformations.

Concluding that since $T(\theta)$ is self-inverse, applying it twice results in the identity transformation.

applying the transformation twice is equivalent to applying the **identity transformation**, which leaves all points unchanged.

A1 A1

b) $|\cos cos\, 2\theta \quad \sin sin\, 2\theta \quad \sin sin\, 2\theta \quad 2\theta \quad| =$

$= \cos cos\, 2\theta \times (-\cos cos\, 2\theta) - \sin sin\, 2\theta \times \sin sin\, 2\theta = -(2\theta + 2\theta) = -1$

A1 A1

12. (a) $300 = 700 - 600 \times 3^{-0.5t}$ ***(A1)***

0.738 (hours) (0.7381404...) ***A1***

Note: Accept 44.3 minutes.

(b) values of *P* are 354, 500, 584, 633 ***(A1)***

$(354 - 260)^2 + (500 - 420)^2 + (584 - 560)^2 + (633 - 620)^2 =$ ***(M1)(A1)***

= 16000 or 15981 ***A1***

(c) (i) The sum of the square residuals is smaller so it is a better fit ***R1***

(ii) Accept a valid argument in favour of model P or against the logistic model. ***R1***

The biologist might prefer the **logistic model** because it accounts for a **carrying capacity**, meaning that population growth slows down as it approaches an upper limit, which is often more realistic for biological populations than exponential models. Model P might be preferred as it is **simpler to interpret or easier to work with mathematically**

13. a) $\left|z_1\right| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$ ***A1***

$\left|z_2\right| = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$ ***A1***

Since $|w| = \left|z_1\right| \times \left|z_2\right| = 4\sqrt{2}$ ***A1***

b) $\arg arg\left(z_1\right) = tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$

$\arg arg\left(z_3\right) = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\arg arg\,(w) = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$ ***A1***

c) w^n is real when its argument is a multiple of π ***M1***

$n \times \frac{5\pi}{12} = k\pi$

$n = \frac{12}{5}k$

Then $k = 5$ ***A1***

14. (a) $x_B = \frac{1}{3}(2(t - 2))^2$ ***A1A1***

Note: Award ***A1*** for multiplying by 2 and ***A1*** for $t - 2$. Award ***A1A0*** $\frac{1}{3}(2t - 2)^2$

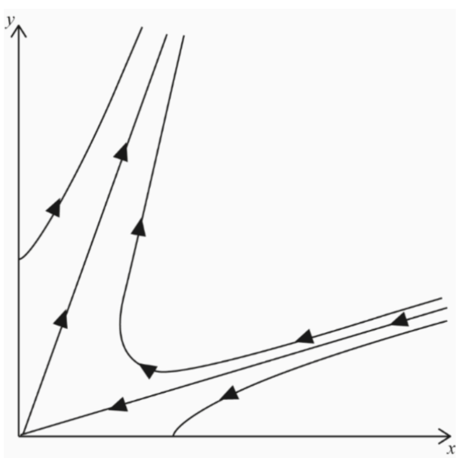
(b) equating their x_B to x_A ***(M1)***

$t = 4$ (hours) ***A1***

Note: Award ***A0 if*** $t = \frac{4}{3}$ *is also seen*

[Total 4 marks]

15. a)



A1A1A1

b) for Y not to die out $Y > \frac{1}{4}x$ ***(R1)***

as $x = 300$, $y > 75$ ***(M1)***

(minimum number of new animals is) 26 ***A1***

[Total 6 marks]

16. attempt to find gradient *(M1)*

EITHER

$gradient\ of\ tangent\ =\ -\ tan\ 46° =-\ 0.910...$ *(A1)(A1)*

Note: Award ***A1*** for negative and ***A1*** for $tan\ 70°$ (or equivalent).

OR

$gradient\ of\ tangent\ =\ tan\ 134° ==-\ 0.910$ *(A2)*

THEN

$\frac{dy}{dx} = 1.8\cos cos\ (0.6x) =-\ 0.910$ *(A1)*

Note: Award ***(A1)*** for a labelled sketch of the derivative function.

equating derivative to their gradient *(M1)*

$1.8\cos cos\ (0.6x)\ =-\ 0.910$

x=3.5 *(A1)*

$height\ =\ 3 \times sin(0.6\times3.5)$ *(M1)*

= 2.46 (m) ***A1***

[Total 8 marks]

17. a) $(0.18\ 0.18\ 0.12\ 0.12\ ) \times (x\ y\ ) = (0\ 0\ )$ *(M1)*

Note: Accept equivalent methods including only using one line of the matrix

$(1\ 1\ )$ (or any multiple) ***A1***

b) $D^n = \left(1\ 0\ 0\ 0.7^n\right)$ ***A1***

c) $(1\ 3\ \ -1\ 2)\left(1\ 0\ 0\ 0.7^{n}\right)(1\ 3\ \ -\ 1\ 2)^{-1}$ ***(M1)***

$(1\ 3\ \ -\ 1\ 2)\left(1\ 0\ 0\ 0.7^{n}\right)(1\ 3\ \ -\ 1\ 2)^{-1}(5000\ 3000)$ ***(A1)***

$(1\ 3\ \ -\ 1\ 2)\left(1\ 0\ 0\ 0.7^{n}\right)(1600\ 200)$ ***(A1)***

$(1\ 3\ \ -\ 1\ 2)\left(1600\ 200\ \times\ 0.7^{n}\right)$ ***(A1)***

$\left(1600\ +\ 600\times0.7^{n}\ -\ 1600\ +\ 400\times0.7^{n}\right)$ ***(A1)***

Note: Award ***A0*** if either term in the matrix is incorrect.