Mathematics: applications and interpretation

Higher level

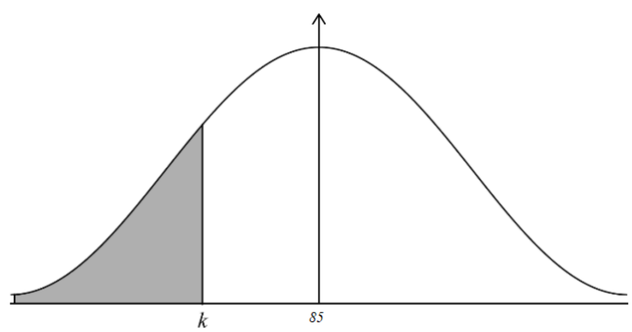
Paper 2

Markscheme

1. (a) 0.5 ***A1***

(b) 0.394 (0.39435022...) ***A2***

(c) (i)



A1A1

> **Note:** Award ***A1*** for a normal curve (with symmetry and some evidence of change of curvature towards the extreme values).
> Award ***A1*** for a shaded region $w < k$, where $k < mean$.

(ii) $P(W < k) = 0.25$

solving a cumulative distribution function **OR** use of inverse function on GDC ***(M1)***

k = 71.5 (71.5102...) ***A1***

(d) recognizing binomial distribution ***(M1)***

B(5, 0.25)

$P(w = 3)$ ***(A1)***

0.0879 (0.087890....) ***A1***

(e) $20(w - 3) + 50$ ***or*** $20w + 30$ ***A1A1***

Note: Award ***A1*** for a linear expression with a gradient of 2,
A1 for a completely correct expression in w.

(f) ($)154 ***A1***

(g) attempt to solve $20(w - 3) + 50 = 110$ ***or*** $20w + 30 = 110$ ***(M1)***

$w = 4$ ***A1***

[Total 15 marks]

2. (a) (i) $(6.8\ \ 3.4\ 4.1) - (2.5\ \ -1.3\ 5.2) = (4.3\ \ 4.7\ \ -1.1)$ ***A1***

Note: Accept alternate vector notation, e.g.$(4.3,\ 4.7, -1.1)$ $or \langle 4.3,\ 4.7, -1.1\rangle$

(ii) use of correct formula to find $\left|\overrightarrow{PQ}\right|$ ***(M1)***

$\sqrt{4.3^2 + 4.7^2 + (-1.1)^2} = 6.46\ (km)$ ***A1***

(b) magnitude of $(2.2\ \ -1.63\ 0)$ is

$\sqrt{2.2^2 + (-1.63)^2 + 0^2} = 2.74$ ***(A1)***

either $(4.3\ 4.7\ \ -1.1) \cdot (2.2\ \ -1.63\ 0)$ or

$4.3 \times 2.2 + 4.7 \times (-1.63) + (-1.1) \times 0$ ***(M1)***

$1.799\ or\ 1.80$ ***(A1)***

Note: The ***M*** mark can be implied by a partially correct ***A1*** line

$angle = arccos(\frac{1.799}{6.46\times2.74})$ ***(M1)***

Then $84.2°$ ***A1***

(c) using sum of angles in a triangle equals 180 ***(M1)***

$180° - 84.2° - 47.5° = 48.3°$ ***(A1)***

$\frac{6.46}{sin48.3°} = \frac{PR}{sin47.5°}$ ***(A1)***

$PR = 6.38\ (km)$ ***A1***

[Total 12 marks]

3.(a) $1200 - 30x^2 = 600$ ***(M1)***
x =4.47 (pesos) (since x is positive) ***A1***

(b) (i) $1200 - 30\times5^2 = 450$ ***A1***
(ii) $450\times5 = 2250$ (pesos) ***A1***

(c) (i) $profit = revenue - costs = V \times x - V\times4$ ***(M1)***
$P = (1200 - 30x^2)x - (1200 - 30x^2)4$ ***A1***
$P =- 30x^3 + 120x^2 + 1200x - 4800$ ***AG***

(ii) $\frac{dP}{dx} =- 90x^2 + 240x + 1200$ ***(M1)A1A1***

(iii) attempt to find x-value ***(M1)***
e.g. sketch of $\frac{dP}{dx}$ with x-intercept indicated **OR** recognition that it occurs at the maximum of P **OR** algebraic approach

$-90x^2 + 240x + 1200 = 0$

$x = 5.22$ ***A1***

(iv) attempt to substitue their x-value into equation for V ***(M1)***

$1200 - 30\times5.22^2 = 382$ or 383 ***A1***

[Total 13 marks]

4. (a) (i) 0.92 (ii) 0.25 (iii) 0.75 ***A2***

Note: Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

(b) $0.08\times0.25 = 0.02$ ***A1***

(c) $P(not\ fail) = 0.69$ ***A1***

multiplying by 200 ***M1***

=138 ***AG***

Note: Award ***A0M0*** for a flawed approach to find $P(not\ fail)$ for example $\frac{172.5}{250}$, which is reverse engineering.

(d) Attempt to find the probability of one sensor failing. ***(M1)***

Then

No sensor fails	one sensor fails	both sensors fail
138	58	4

(A1)

degrees of freedom = 2 ***(A1)***

Note*:* Award ***A1*** for df = 2 seen anywhere and may be awarded independent of the ***M1*** mark.
The df cannot be implied from chi square statistic = 40.9

$p - value = 1.31 \times 10^{-9} (1.309.... \times 10^{-9})$ ***(A1)***

$0.05 > 1.31 \times 10^{-9}$ ***R1***

hence there is sufficient evidence to reject *H*0; the manufacturers claims are not both correct ***A1A1***

Note: The ***R1A1*** can be awarded as follow through within part (d) from their (explicitly labelled) incorrect p-value.
An unrealistic p-value ($p \geq 1$) should preclude awarding the final ***R1A1***.
Accept either a conclusion to reject the null hypothesis or the manufacturer's claims are not both correct.
Do not award ***R0A1****.*

[Total 12 marks]

5. (a) A. ***A1***

Any valid reason for accepting A. or rejecting B. and C. ***R1***

for example:

- when $y = 0$ slopes have (or appear to have) zero gradient
- slopes along the vertical lines $\pm \frac{\pi}{2}$ $\pm \frac{3\pi}{2}$ should be horizontal.

Note: Allow ***A1R0***.

(b) $\int \frac{1}{y} dy = \int \cos cos\, x \ dx$ ***(M1)***

$ln|y| = sin(x) + C$ ***M1***

$y = C\, e^{sin(x)}$ ***(A1)(A1)***

Note: ***A1*** for left hand side, ***A1*** for right hand side.

substituting in $x = 0$, $y = 1$ ***(M1)***

$1 = C\,e^{sin(0)}$ ***(A1)***

$C = 1$

$y = e^{sin(x)}$ ***A1***

(c) $\frac{dy}{dx} = e^{\sin sin\,x} \times \cos cos\,x$ ***M1A1***

Note: Award ***M1*** for use of chain rule,

(d) substitution of $e^{\sin sin\,x} = y$ from part (b) into part (c) (i) or original differential equation ***M1***

$\frac{dy}{dx} = y\,cos(x)$ ***A1***

and hence $y = e^{sin(x)}$ is a solution for the differential equation ***AG***

[Total 13 marks]

6. (a)i) let C be the number of champion enemies eliminated and *M* the number of Jungle Monster.

$mean = 3.5 \times 3 = 10.5$ ***(A1)***

$P(M \leq 12) = 0.742$ ***A1***

(ii) attempt to add two means ***(M1)***

$3.5 + 2.1 = 5.6$

$P(M + C > 8) = P(M + C \geq 9)$ ***(M1)***

$0.114\ (0.11432247.....)$ ***A1***

(b) i) $E(G) = 4 \times 2.1 + 2 \times 3.5 = 15.4$ ***A1***

$Var(G) = Var(4C + 2M) = 16\times2.1 + 4\times3.5 = 47.6$ ***(M1)A1***

(c) any valid reason for example: ***R1***

mean is not equal to variance **OR** *G* cannot take all integer values

(d) distribution of mean score is $\overline{G}\sim N(15.4, \frac{47.6}{60})$ ***(A1)(A1)***

> **Note:** Award ***A1*** for normal distribution with mean 15.4 and ***A1*** for variance $\frac{47.6}{60}$

$P(\overline{G} > 15) = 0.673$ ***A2***

[Total 13 marks]

7. (a) attempt to use $V = \pi\int x^2 dy$ ***(M1)***

$x^2 = 4(y - 1)$ or any reasonable attempt to find *x* in terms of *y* ***(M1)***

$V = \pi\int_0^h (4y - 4)dy$ ***(A1)***

> **Note:** Correct limits must be seen for the ***A1*** to be awarded however the d*y* may be omitted (as not a final answer).If this is given as the final answer to this part the remaining marks can be awarded if seen in part (b).

$\int(4y - 4)dy = 2y^2 - 4y$ ***(A1)***

Note: Accept equivalent with alternate variable

$V = \pi\left[2y^2 - 4y\right]_0^h$

$V = \pi(2h^2 - 4h)$ ***A1***

Note: The final two ***A1*** marks can be awarded independently of the first ***A1***.
If $2h^2 - 4h$ or $2y^2 - 4y$ is the final (unsupported) answer award at most ***(M1)(M1)(A0)(A1)A0***.

(b) $volume\ of\ glass = \pi\left(2\times13^2 - 4\times13\right) = 898\ (898.49549....)$ ***(A1)***

time to fill the glass $\frac{898}{30} = 29.9\ seconds$ ***A1***

Note: Accept exact answers in terms of π $e.g$: $286\pi, \frac{143}{15}\pi$

(c) differentiating $V = \pi(2h^2 - 4h)$ implicitly ***(M1)***

$\frac{dV}{dt} = \pi(4h - 4)\frac{dh}{dt}$

(A1)

$\frac{dh}{dt} = 30\times\frac{1}{\pi(4h-4)}$ ***(M1) (A1)***

Note: Award ***M1*** for attempting to solve for $\frac{dh}{dt}$, ***A1*** for a correct expression.

substituting *h* =12 seen anywhere ***(M1)***

0.217 (0.21702946...) ***A1A1***

Note: Award ***A1*** for the correct value. Award ***A1*** for the correct units, independent of other marks.

[Total 14 marks]

8. (a) $(1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ )$ ***(M1)A1***

Note: Award ***M1*** is for a 3x3 matrix with at least one column correct.
Column order is not explicit in question and may not be labelled in candidate response; accept their correct adjacency matrix.

(b) $(1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ )^6$ ***(M1)***

$(22\ 21\ 21\ 21\ 22\ 1\ 21\ 21\ 22\ )$ ***A1***

$21\ different\ routes$ ***A1***

(c) i) $0.5^6\ (\frac{1}{64}, 0.015625)$ ***(A1)***

(ii) $21\ \times 0.5^6$ ***(M1)***

Then

$0.328\ (0.328125, \frac{21}{64})$ ***(A1)***

Note: Solutions to this part must be using the value (21) obtained from part (b) to be awarded any marks.

d) (i) $1\times 0.3 = 0.3$ ***A1***

(ii) $(0.5 \times 0.5) = 0.25$ ***A1***

(iii) $(0.5 \times 0.5 + 0.5 \times 0.5) = 0.5$ ***A1***

e) transition matrix is $(0\ 0.25\ 0.3\ 0.7\ 0.5\ 0.7\ 0.3\ 0.25\ 0)$

(with order PQ, PR and QR) ***(M1)(A1)***

Note: Column order is not explicit in question and may not be labelled in candidate response; accept their correct transition matrix.
Accept the transposed matrix.

$(0\ 0.25\ 0.3\ 0.7\ 0.5\ 0.7\ 0.3\ 0.25\ 0)^6$ ***(M1)***

$(0.2087165\ 0.20832\ 0.2079875\ 0.583296\ 0.58363\ 0.583296\ 0.2079875\ 0.20832\ 0.2087165)$

$0.208\ (0.2079875)$ ***A1***

(f) (Taking a high power of a matrix)

long term probabilities are 0.2087165, 0.583296 and 0.2079875 ***(M1)***

Q and 0.583 ***A1A1***

Note: Award ***(M0)A0A0*** for an unsupported answer of " Q" (with either no probability or an incorrect probability)

[Total 18 marks]