Mathematics: applications and interpretation

Higher level

Paper 3

Markscheme

1. (a) i) $(0 \ -1 \ 1 \ 0)(3 \ 2) = (-2 \ 3)$ ***M1A1***

ii) $90°$ Counterclockwise rotation about the origin ***A1A1***

(b) i) $(2 \ 0 \ 0 \ -2)(-2 \ 3) = (-4 \ -6)$ ***M1A1***

ii) $(2 \ 0 \ 0 \ -2)(0 \ -1 \ 1 \ 0) = (0 \ -2 \ -2 \ 0)$ ***M1A1***

c) i) $(a \ b \ c \ d)(a \ b \ c \ d) = (1 \ 0 \ 0 \ 1)$ ***M1***

$\left(a^2 + bc \ ab + bd \ ac + dc \ cb + d^2\right) = (1 \ 0 \ 0 \ 1)$ ***M1A1***

Then

$a^2 + bc = 1$ and $cb + d^2 = 1$ ***AG***

(ii) $det(C) = ad - bc = -1$ ***M1***

$a^2 + bc = 1$ and $cb + d^2 = 1$

$ab + bd = 0$ and $ac + dc = 0$

$bc = 1 - a^2$

$bc = 1 - d^2$

Then $a^2 = d^2$ so either $a = d$ *or* $a = -d$ ***A1***

When $a = d$

$ad - bc = -1$

$ad - (1 - a^2) = -1$

$ad - 1 + a^2 = -1$

$a(d + a) = 0$

Either $a = 0$ which is not possible or $d + a = 0$ which means that $a = -d$

As $ab + bd = 0$ and $ac + dc = 0$

$b(a + d) = 0$ and $c(a + d) = 0$ and $d + a = 0$

This holds for any value of b *and* c

So $a = -d$ *and* $bc = 1 - a^2 = (1 + a)(1 - a)$

So $a = -d$ $b = (1 + a)$ *and* $c = (1 - a)$ ***A1***

(d)i) $|3 - \lambda\ 4\ 2\ 1 - \lambda| = (3 - \lambda)(1 - \lambda) - 8 = 0$ ***M1A1***

$\lambda = 5, \lambda = -1$ ***A1A1***

ii) $(3\ 4\ 2\ 1)(x\ y) = 5(x\ y)$ ***M1A1***

$x = 2y$

$(2\ 1)$ ***A1***

e) i) $(cos\theta\ -\ sin\theta\ sin\theta\ cos\theta)$ ***A1A1***

ii) $(cos\theta\ -\ sin\theta\ sin\theta\ cos\theta)(cos\theta\ -\ sin\theta\ sin\theta\ cos\theta) =$ ***M1***

$= \left(cos^2\theta - sin^2\theta\ -\ 2sin\theta cos\theta\ 2sin\theta cos\theta\ cos^2\theta\ -\ sin^2\theta\right)$

$\left(cos^2\theta - sin^2\theta \ - 2sin\theta cos\theta \ 2sin\theta cos\theta \ cos^2\theta - sin^2\theta\right) = (cos2\theta \ - sin2\theta \ sin2\theta \ cos2\theta)$

A1

$\left(cos2\frac{\pi}{2} \ - sin2\frac{\pi}{2} \ sin2\frac{\pi}{2} \ cos2\frac{\pi}{2}\right) = (cos\pi \ - sin\pi \ sin\pi \ cos\pi)$ which represents a reflection on the line $y = -x$ ***A1***

[Total 29 marks]

2. (a) i) $a + 95 = 155$ ***M1***

$a = 60$ ***AG***

ii) $60 + 60 + b = 130$ ***M1***

$b = 10$

(b) Restricting the size of the sample space to 170 ***M1***

$P(Minor\ scratches) = 0.353(0.3529411..., \frac{60}{170})$ **A1**

(c) i) H_0: $Damage\ level\ and\ manufacturing\ process\ are\ independent.$

A1

H_1: $Damage\ level\ and\ manufacturing\ process\ are\ not\ independent.$

Note: Condone equivalent statements such as 'not dependent' but do not accept "uncorrelated" or "not related" in place of "independent".

(ii) $p - value = 0.199\ (0.198547.....)$ ***M1 A1***

(iii) $0.199 > 0.05$ ***R1***

Hence there is insufficient evidence to reject the null hypothesis then we can

conclude that the damage level and the manufacturing processes used are independent. ***A1***

Note: Do not award R0A1.

(d) $\frac{110+55}{260}\left(=\frac{165}{260}\right)$ ***A1***

$\frac{33}{52}$ ***AG***

(e) **Either**

Each screen fails independently. ***R1***

or

Probability of failure remains constant. ***R1***

or

Sample size is sufficiently large. ***R1***

(f) 80 seen ***(A1)***

EITHER

attempt to find a probability ≤ 70 (condone strict inequality for ***(M1)***) ***(M1)***

$(P(X \leq 70) =)\ 0.142\ (0.14212\ ...)$ ***A1***

Note: Award ***(A1)(M1)A0*** for an unsupported $p = 0.104$, from use of strict inequality.

$0.142 > 0.05$ ***R1***

OR

attempt to find the critical region ***(M1)***

critical region is $X \leq 69$ ***A1***

69 < 70 or '69 is not in the critical region' ***R1***

THEN

EITHER

do not reject the null hypothesis (as there is insufficient evidence that the new manufacturing process reduces damage). ***A1***

OR

do not accept the alternative hypothesis (as there is insufficient evidence that the new manufacturing process reduces damage). ***A1***

(g) (i) the test for a proportion is directional and so considers whether the new manufacturing reduces the number of components damaged..

(ii) there could be variation in the value of *p* chosen for the null hypothesis / the value of p from the sample might not be a representative of the current manufacturing process ***R1***

(h) let

μ_1 be the mean length of time before damage appears with the new manufacturing process

and

μ_2 be the mean length with the current manufacturing process

$H_0: \mu_1 = \mu_2$

$H_0: \mu_1 > \mu_2$

Note: Award ***A1A0*** for correct hypotheses in which the two population means are not clearly defined (e.g. unsupported $\mu_1 and\ \mu_2$).

THEN

recognition of the need to use of a two-sample test ***(M1)***

$p - value = 0.0165\ (0.016516...)$ ***A2***

Note: If not pooled, answer is 0.016722... award ***(M1)A2***

$0.0165 < 0.05$ ***R1***

reject the null hypothesis (**OR** accept the alternative hypothesis) ***A1***

(there is sufficient evidence to conclude that the new manufacturing process increases the mean length of time before any damage appears)

Note: Do not award ***R0A1***. Follow through within part (h) for the last ***R1A1***, provided their *p*-value is between 0 and 1 inclusive.

(i) *The test shows statistical significance, but practical significance is not guaranteed. Other factors may influence screen durability.* ***A1***

[Total 26 marks]