## Mathematics: applications and interpretation

## Higher level

## Paper 3

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or
  correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is
  required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 29]

**The following question explores a method for analyzing the movement of objects under geometric transformations, taking the design of a robotic arm as a practical application.**

A company is designing a robotic arm that moves objects on a coordinate plane. The arm applies a sequence of transformations to the position of an object.

(a) A point $P(3.\ 2)$ is transformed by the matrix

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

i) Determine the coordinates of the transformed point. [2]
ii) Describe the geometric transformation represented by. [2]

(b) The point is then further transformed by the matrix

$$B = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

i) Find the coordinates of the final image of $P$ after applying $A$ followed by $B$. [2]
ii) Determine the single matrix that represents the combined transformation. [2]

**(This question continues on the following page)**

**(Question 1 continued)**

(c) The transformation matrix $C = (a\ b\ c\ d)$ satisfies $C^2 = I$, where is $I$ the identity matrix.

i) Show that this condition implies $a^2 + bc = 1$ and $d^2 + bc = 1$. [3]

ii) If $C$ represents a reflection and $det(C) = -1$, determine possible values of $a, b, c$ and $d$. [3]

(d) The company needs to analyze the effect of applying a transformation matrix to an eigenvector. Given that

$$D = (3\ 4\ 2\ 1)$$

i) Find the eigenvalues of . [4]

ii) Find an eigenvector corresponding to the larger eigenvalue. [4]

(e) The company tests a new transformation matrix $E$ that rotates points by $\theta$ counterclockwise.

i) Write the general form of $E$. [2]

ii) Show that for $\theta = 90°$, applying E twice results in a reflection. [3]

2. [Maximum mark: 26]

**The purpose of this question is to help a company determine whether switching to a new manufacturing process improves the durability of their product.**

A company produces smartphone screens and has developed a new manufacturing process. They conduct a durability test on 130 screens made using the new process and 260 screens made using the current process. After 500 hours of simulated use, they categorize each screen as having no damage, minor scratches, or major cracks.

The data from the test are recorded in the table below.

| Damage Level | New Process | Current Process |
|---|---|---|
| No damage | a | 95 |
| Minor scratches | 60 | 110 |
| Major cracks | b | 55 |

In total, 155 screens showed no damage.

(a) i) Show that the value of a is 60. [1]

ii) Find the value of b [1]

One of the tested screens is selected at random.

(b) Given that the screen has minor scratches, find the probability that it was produced using the new process. [2]

**(This question continues on the following page)**

**(Question 2 continued)**

(c) A x2 test for independence is performed at the 5% significance level to determine whether screen durability (damage level) is independent of the manufacturing process.

i) State the null and alternative hypotheses. [1]
ii) Find the $p - value$. [2]
iii) State the conclusion of the test in context, justifying your answer. [2]

(d) For the screens made using the current process, show that the proportion which developed scratches or cracks is $\frac{33}{52}$. [1]

To further analyze the data, the researchers define $p$ as the probability that a screen manufactured with the new process develops scratches or cracks. They then test the hypotheses:

$H_0: p = \frac{33}{52}$ ,

$H_1: p < \frac{33}{52}$

For a random sample of 120 screens made with the new process, let $X$ be the number of screens that develop scratches or cracks. The researchers assume that under the null hypothesis,

$X \sim B(120, \frac{33}{52})$

(e) State one additional assumption that the researchers are making in choosing this distribution. [1]

(f) Use the trial data to perform the test at the 5% significance level. State the conclusion of the test, justifying your answer. [5]

(g) In comparison with the test in part (c), state one mathematical reason why

i) the test in part (f) might be preferred. [1]

ii) the test in part (f) might not be preferred. [1]

(h) The researchers also analyze the mean time before damage occurs. A second test is conducted, and the time (in hours) before damage appears is recorded for both processes.

The mean time $\bar{t}$ and standard deviation $s_{n-1}$ for each process are given in the table below.

| Process | Number of screens tested | $\bar{t}$ (hours) | $s_{n-1}$ (hours) |
|---|---|---|---|
| New process | 100 | 500.3 | 6.1 |
| Current process | 200 | 498.7 | 6.1 |

Perform an appropriate hypothesis test at the 5% significance level to determine whether the new process increases the mean time before damage occurs. [7]

(i) The company decides to implement the new manufacturing process and publishes the following statement:

"Statistical tests prove that the new process significantly increases the durability of our screens."

Comment on the validity of this statement. [1]