IB Mathematics AA HL - Prediction Exams May 2025 - Paper 1

Paper 1 | 12 questions | 120 mins | 110 marks

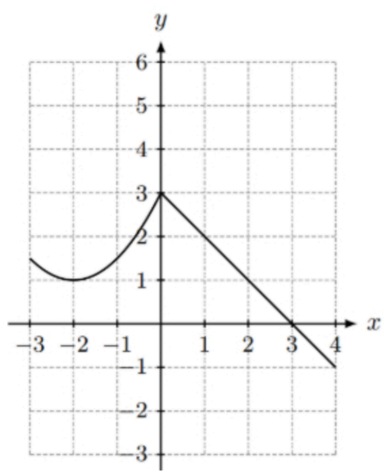
Section A

Question 1

NO CALCULATOR | Easy

[Maximum mark: 5]

The graph of $y = f(x)$ for $-3 \leq x \leq 4$ is shown in the following diagram.



(a) Write down the value of $f(2)$. [1]

Let $g(x) = 2f(x) - 1$ for $-3 \leq x \leq 4$.

(b) On the axes above, sketch the graph of g. [2]

(c) Hence determine the value of $(g \circ f)(2)$. [1]

(d) Hence solve the equation $(f \circ g)(x) = 0$ when $x > 0$. [1]

Question 2

NO CALCULATOR   Easy

[Maximum mark: 5]

(a) Show that $12\log_x 2 = \dfrac{12}{\log_2 x}$. [1]

(b) Hence solve the equation $\log_2 x = 8 - 12\log_x 2$. [4]

Question 3

NO CALCULATOR   Easy

[Maximum mark: 4]

When the resulting product of $3x^2 + 7x - 6$ multiplied by $ax + 1$ is divided by $x - 1$ the remainder is -4.

Find the integer a.

Question 4

NO CALCULATOR   Medium

[Maximum mark: 7]

(a) Show that $4 - 3\cos 2x = 6\sin^2 x + 1$. [1]

(b) Hence or otherwise solve $4 - 3\cos(4\theta + \frac{2\pi}{3}) - 9\sin(2\theta + \frac{\pi}{3}) = -2$ for $0 \leq \theta < \pi$. [6]

Question 5

NO CALCULATOR   Medium

[Maximum mark: 5]

Consider $f(x) = 2\cos\left(x - \frac{\pi}{2}\right) + 3$ and $g(x) = 4\cos\left(x + \frac{\pi}{2}\right) + 2$.

The function f is mapped onto g by three transformations.

(a) Fully describe each of the transformations and the order in which they must be applied. [3]

A new function is such that $h(x) = g(x) + k$ where $k \in \mathbb{R}$.

(b) Find the minimum value of k such that $h(x) \geq 0$ for all x. [2]

Question 6

NO CALCULATOR   Medium

[Maximum mark: 7]

(a) (i) Consider the following equation $2\binom{n}{r} = \binom{n}{r+1}$.

Show that it can be written as $3r + 2 = n$.

(ii) Now consider the following equation $7\binom{n}{r-1} = 2\binom{n}{r}$.

Show that it can be written as $9r - 2 = 2n$. [4]

Consider the expansion

$$(1 + x)^n = 1 + a_1x + \ldots + a_{k-1}x^{k-1} + a_kx^k + a_{k+1}x^{k+1} + \ldots + x^n$$

Where $a_i \in \mathbb{Q}$ and $k \in \mathbb{Z}$.

The coefficients of three consecutive terms of the expansion are such that

$$7 \times a_{k-1} = 2 \times a_k \quad \text{and} \quad 14 \times a_k = 7 \times a_{k+1}$$

(b) Find n. [3]

Question 7

NO CALCULATOR

Medium

[Maximum mark: 8]

Consider the function $f(x) = \dfrac{\cos(mx) - \cos(nx)}{x^2}$ where $m, n \in \mathbb{R}$.

The function has a maximum value of $f_{\max}$ and it is known that $f_{\max} = \lim_{x \to 0} f(x)$.

(a) Show that $f_{\max} = \dfrac{n^2 - m^2}{2}$. [6]

It is now known that $m > 0$ and $n = 4\sqrt{m}$.

(b) Hence, using these conditions find the largest possible value of $f_{\max}$. [2]

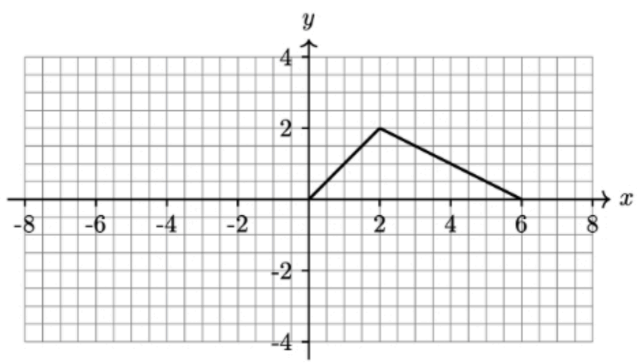
Question 8

NO CALCULATOR

Medium

[Maximum mark: 8]

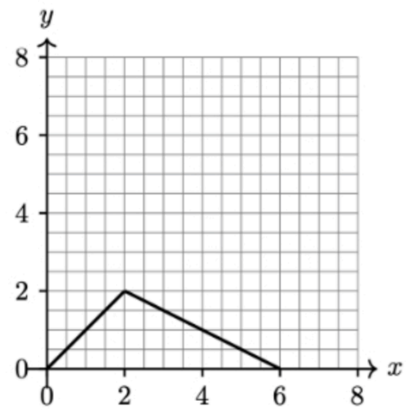
The graph of $y = f(x)$ for $0 \le x \le 6$ is shown below



The odd function $h(x)$ has the domain $-6 \le x \le 6$ and $h(x) = 2f(x)$ for $0 \le x \le 6$.

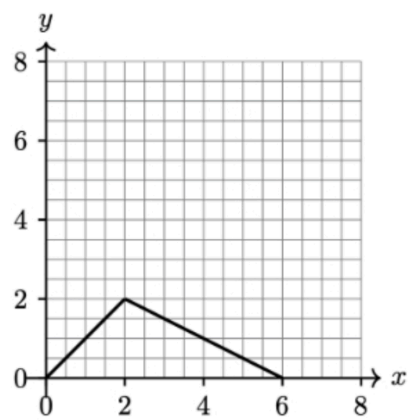
(a) Sketch $h(x)$ on the axes above. [2]

$f(x)$ is shown again below.



(b) Sketch the graph of $y = [f(x)]^2$ on the axes above. [3]

$f(x)$ is shown one more time below.



(c) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes above. [3]

Question 9

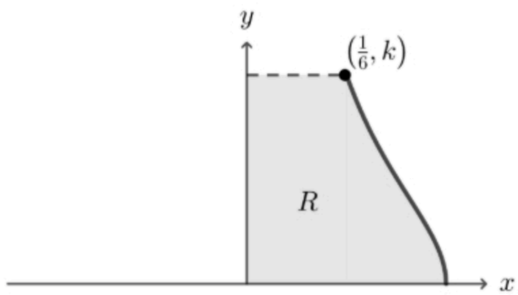
NO CALCULATOR

Hard

[Maximum mark: 8]

The function f is defined by $f(x) = \frac{\sqrt{1-9x^2}}{2x}$ for $x \geq 0$.

The region R is bounded by the curves $y = f(x)$ and the lines $x = 0$ and $y = 0$ as shown in the following diagram.



The shape of a solid clay sculpture can be modeled by rotating the region R though 2π radians about the $y-$axis.

The top edge of the sculpture has coordinates of $\left(\frac{1}{6}, k\right)$.

The volume of clay used to make the sculpture is $a\pi^2$ units^2. Where $a \in \mathbb{Q}$.

Find a.

Section B

Question 10

NO CALCULATOR Medium

[Maximum mark: 23]

Consider the function $f(x) = \frac{\cos x}{2 + \sin x}$ for $-\pi \leq x \leq \pi$.

(a) Evaluate $f(0)$. [1]

(b) Find all possible values of a if $f(a) = 0$. [2]

(c) (i) Show that $f'(x) = -\frac{2\sin x + 1}{(2 + \sin x)^2}$.

(ii) Hence find the $x-$coordinates of any stationary points of f. [7]

(d) Given that $f''(x) = -\frac{2\cos x(1 - \sin x)}{(2 + \sin x)^3}$ find the nature of any stationary points of f. [5]

(e) Hence sketch the graph of f, clearly showing the values of the axes intercepts and the $x-$coordinates of any stationary points. [3]

The function f is positive and decreasing in the region $s < x < t$.

The area enclosed by f and the $x-$axis from $x = s$ to $x = t$ is $\ln c$ where $c \in \mathbb{Z}$.

(f) Find c. [5]

Question 11

NO CALCULATOR Hard

[Maximum mark: 16]

(a) Use mathematical induction to prove that

$$2^n \times \cos x \times \cos 2x \times \cos 4x \times \ldots \times \cos 2^{n-1}x = \frac{\sin 2^n x}{\sin x}$$

where $n \in \mathbb{Z}^+$. [4]

(b) (i) Find the first two non-zero terms of the Maclaurin series for $\sin 8x$.

(ii) Hence find the first two non-zero terms of the Maclaurin series for $\frac{\sin 8x}{\sin x}$.

(iii) Hence find an estimate for $\int_0^{0.1} \cos x \cos 2x \cos 4x \, dx$. [12]

Question 12

NO CALCULATOR

Hard

[Maximum mark: 14]

The complex number z is a root of the equation $|z + 4i| = |z - 10i|$.

(a) Show that the imaginary part of z is 3. [2]

(b) Let ω_1 and ω_2 be two possible values of z such that $|z| = 6$.

(i) If ω_1 is in the first quadrant sketch both solutions on an Argand diagram.

(ii) Hence find the arguments of ω_1 and ω_2. [4]

A different complex number, v, is defined such that

$$v = \frac{\omega_1^k \, \omega_2}{-i}$$

Where k is a real number that can take any value in the interval $-10 \leq k \leq 10$.

(c) (i) Find $\arg(v)$ in terms of k and π.

(ii) Hence find all possible values of k such that v is a real number. [8]