IB Mathematics AI HL - Prediction Exams May 2025 - Paper 1

Paper 1 | 15 questions | 120 mins | 110 marks

Question 1

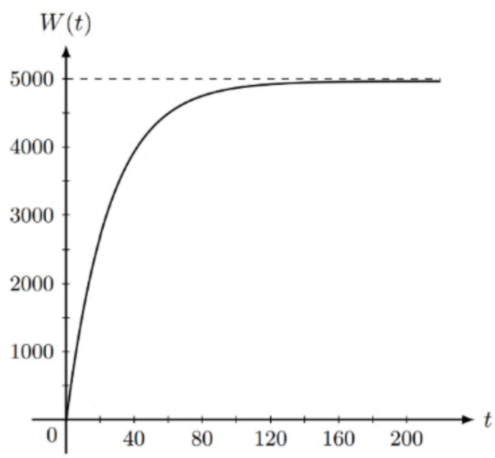
CALCULATOR | Easy

[Maximum mark: 6]

A water tank has a float valve that allows water to enter while the tank is not full. Over time, the float valve reduces the water flow until the tank is almost at its maximum capacity.

The amount of water in the tank, in litres, is given by the function $W(t) = 5\,000 - C\left(1.04^{-t}\right)$

where t is minutes after the tank starts to fill.



(a) Given that the tank was initially empty, find the value of C. [2]

(b) State what the horizontal asymptote represents in the context of the question. [1]

(c) Find the total amount of water in the tank after one and half hours. Give your answer in cubic metres. [3]

revisionvillage

(a) Substituting $t = 0$, and letting $W(0) = 0$, to solve for C, we get

$$0 = 5\,000 - C(1.04^{-0}) \qquad \textbf{(M1)}$$

$$= 5\,000 - C$$

$$\boxed{C = 5\,000} \qquad \textbf{A1}$$

(b)

The horizontal asymptote represents the maximum capacity of the tank, which is 5 000 litres.

R1

(c) First, we must convert the given time to minutes, because minutes is the unit used in the question.

$$1.5 \text{ hours} = 90 \text{ minutes}$$

Then the amount of water, in litres, after 90 minutes is

$$W(90) = 5\,000 - 5\,000\left(1.04^{-90}\right) \qquad \textbf{(M1)}$$

$$= 4\,853.45\ldots \qquad \textbf{(A1)}$$

Now we must convert from litres to cubic metres, using the fact that $1\,000$ litres $= 1\text{ m}^3$. We can divide the amount in litres by $1\,000$.

Expressed in cubic metres, we obtain

$$4\,853.45 \text{ litres} = \boxed{4.85 \text{ m}^3 \text{ (3 s.f.)}} \qquad \textbf{A1}$$

Question 2

CALCULATOR   Easy

[Maximum mark: 7]

A'ja is a keen basketball player. Each time A'ja attempts a free throw, she has a 75% chance of scoring. Suppose she practises 120 free throws and counts the total number of free throws she makes. It can be assumed that the probability of scoring on any given free throw is independent of her other previous free throw attempts.

(a) (i) Write down the mean number of free throws she makes.

(ii) Calculate the variance of the number of free throws she makes. [4]

(b) Find the probability that the number of free throws she makes is less than one standard deviation away from the mean. [3]

(a) Let T be the number of free throws A'ja makes when practicing 120 free throws.

We have $T \sim \mathrm{B}(120, 0.75)$.

(i) Hence, using the formula for the expected value of T, we get

$$\mathrm{E}(T) = np$$
$$= (120)(0.75) \qquad \textbf{(M1)}$$
$$\boxed{= 90} \qquad \textbf{A1}$$

(ii) Using the formula for the variance of T, we obtain

$$\mathrm{Var}(T) = np(1-p)$$
$$= (120)(0.75)(1-0.75) \qquad \textbf{(M1)}$$
$$\boxed{= 22.5} \qquad \textbf{A1}$$

(b) Using the formula for the standard deviation of T, we have

$$\begin{aligned}\sigma &= \sqrt{\mathrm{Var}(T)}\\ &= \sqrt{22.5}\\ &= 4.743... \qquad \textbf{(A1)}\end{aligned}$$

Hence we find

$$\begin{aligned}\mathrm{P}(|T-90| < 4.743...) &= \mathrm{P}(90 - 4.743... < T < 90 + 4.743...)\\ &= \mathrm{P}(85.25... < T < 94.74...)\\ &= \mathrm{P}(86 \le T \le 94)\\ &= \mathrm{binomCdf}(120, 0.75, 86, 94) \qquad \textbf{(M1)}\\ &= 0.657\,(3\,\text{s.f.}) \qquad \textbf{A1}\end{aligned}$$

Question 3

CALCULATOR | Easy

[Maximum mark: 6]

Bruno rides his bike to school each morning. During the first minute, he travels 160 metres. In each subsequent minute, he travels 80 % of the distance travelled during the previous minute.

The distance from his home to school is 750 metres. Bruno leaves his house at 8:30 am and must be at school by 8:40 am.

(a) Verify that Bruno will not arrive at school on time. [3]

Bruno realises that if he can increase the distance he travels each minute, from 80 % of the distance travelled during the previous minute to k %, he will make it to school on time.

(b) Determine the minimum value of k, given that Bruno still travels 160 m in the first minute. [3]

(a) We could consider each minute of Bruno's journey as a term of a geometric sequence, and we need the sum of the first 10 terms to determine how far Bruno travels between 8:30 and 8:40 am.

We have a geometric sequence with the first term $u_1 = 160$ and common ratio $r = 0.8$.

If we use the sum of n terms formula $S_n = \dfrac{u_1(1-r^n)}{1-r}$ with $n = 10$, we get

$$\begin{aligned} S_{10} &= \frac{u_1(1-r^{10})}{1-r} \\ &= \frac{160(1-0.8^{10})}{1-0.8} && \textbf{(M1)} \\ &= 714.10\ldots && \textbf{A1} \end{aligned}$$

$$714.10\ldots < 750 \qquad \textbf{R1}$$

Bruno will only travel 714 metres by 8:40 am, which is less than the 750 metres required.

Therefore, Bruno **will not** arrive at school on time.

(b) Again, we require the sum of the first 10 terms of a geometric sequence with first term $u_1 = 160$. This sum must be **at least** 750 to ensure that Bruno will make it to school on time. We do not know the common ratio, but if each minute Bruno travels $k\,\%$ of the distance travelled in the previous minute, then we will have a common ratio of $r = \frac{k}{100}$.

Using this information we can form an inequality.

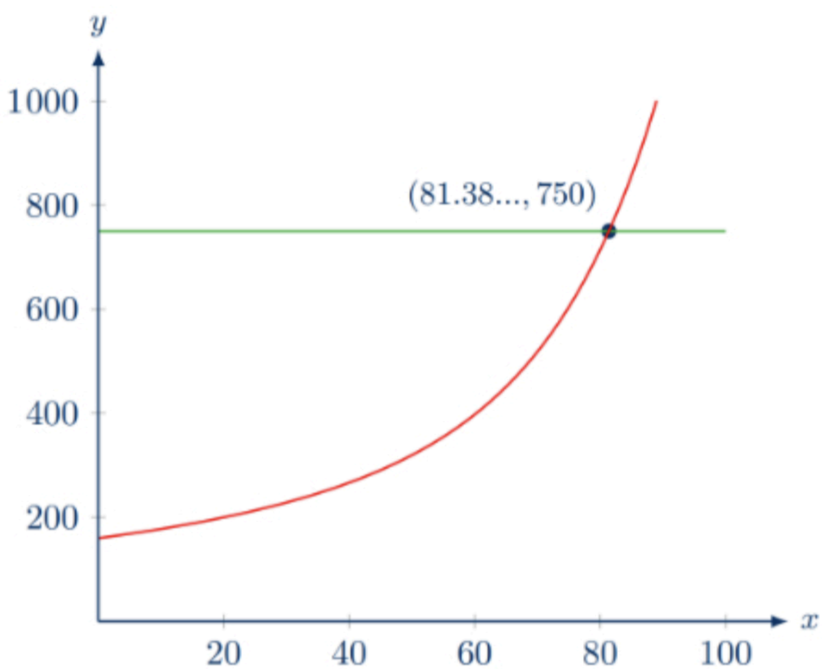
$$S_{10} \geq 750$$

$$\frac{160\left(1-\left(\frac{k}{100}\right)^{10}\right)}{1-\frac{k}{100}} \geq 750 \qquad \textbf{(M1)}$$

To solve this, we can sketch both sides of the inequality and find the point of intersection.

(M1)

The diagram below shows the graph of $y = S_{10}$ in **red** and the graph of $y = 750$ in **green**.



Hence the smallest value of k such that the sum of the first ten terms is at least 750 metres is 81.4% (3 s.f.)

A1

Question 4

CALCULATOR Easy

[Maximum mark: 7]

Give all answers for this question to 2 decimal places, unless otherwise stated.

On 1 January 2024, Emily invests \$600 000 in a savings account which pays a nominal annual interest rate of 4.5%, compounded annually.

(a) Determine the amount of money that will be in the account after 12 years. [3]

After these 12 years, Emily is planning to retire and place the money she has saved into an annuity fund which pays a nominal annual interest rate of 3.5%, compounded monthly.

Emily wants to withdraw money from this account at the end of each month.

(b) (i) Calculate the amount Emily can withdraw at the end of each month if she wants the money to last for 18 years after her retirement.

(ii) Find how many **complete months**, counted from 1 January 2036, it will take for the balance of the fund to fall below \$500 000. [4]

(a) In 12 years we have

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
12	4.5	−600 000	0	1 017 528.859...	1	1	END

(M1)(A1)

Hence, using the finance solver on the GDC (or the compound interest formula), we obtain

$$\text{FV} = 600\,000\left(1 + \frac{4.5}{100(1)}\right)^{12(1)}$$

$$= \$1\,017\,528.86 \text{ (2 d.p.)} \qquad \text{[by using G.D.C.]} \qquad \textbf{A1}$$

(b) (i) In 18 years we have 216 months. Hence, using the finance solver on the G.D.C., we have

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
216	3.5	1 017 528.86	−6 356.11	0	12	12	END

(A1)

Therefore, Emily can withdraw $6 356.11 each month.

A1

(ii) This time we need to calculate N, given that FV $= -500\,000$, thus, we get

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
126.5	3.5	1 017 528.86	−6 356.11	−500 000	12	12	END

(A1)

Therefore, it takes more than 126.5 months to fall below $500000, so in total 127 months.

A1

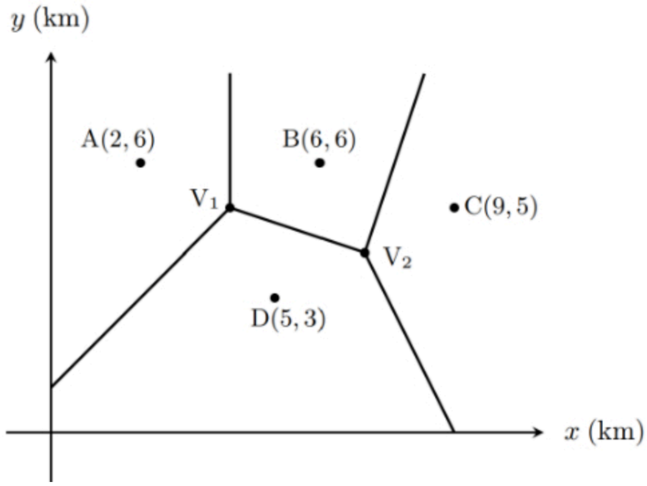
Question 5

CALCULATOR

Easy

[Maximum mark: 9]

Consider the Voronoi diagram below for a town centre that contains four coffee shops A, B, C, and D.



The equation of the perpendicular bisector between sites B and C is $y = 3x - 17$. The coordinates of the midpoint between C and D is (7,4).

(a) Determine the equation of the perpendicular bisector between C and D. [3]

(b) Hence, determine the coordinates of the intersection point V_2. [3]

The perpendicular bisectors of AB and AD intersect at the point $V_1(4,5)$, which is 2.236 km from location D, correct to 4 significant figures. A new coffee shop will be built at either V_1 or V_2.

(c) Given that the new shop is to be as far away as possible from an existing coffee shop, determine which of the locations the new coffee shop should be built at. [3]

(a) First we find the gradient of CD. Using the gradient formula, we get

$$\begin{aligned} m_{\mathrm{CD}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-5}{5-9} && \textbf{(M1)} \\ &= \frac{1}{2} \end{aligned}$$

Hence the gradient of the perpendicular bisector to CD is -2.

Substituting the coordinates of the given midpoint (7,4) and the slope -2 into the equation of a line, we get

$$4 = -2(7) + c \qquad \textbf{(M1)}$$

$$c = 18$$

Hence, the equation of the perpendicular bisector CD is

$$\boxed{y = -2x + 18} \qquad \textbf{A1}$$

(b) The coordinates of V_2 can be found by finding the coordinates where perpendicular bisector of BC and the perpendicular bisector of CD intersect. Hence, letting these two perpendicular bisectors be equal to each other, we get

$$3x - 17 = -2x + 18 \qquad \textbf{(M1)}$$

$$x = 7 \qquad \textbf{(A1)}$$

Hence $y = 4$ and therefore the coordinates of V_2 are $\boxed{(7, 4)}$.

A1

(c) The new coffee shop must be at V_1 or V_2, as these locations are the furthest distance away from each of the sites. We need to determine which of these points is the furthest from its closest site.

Given V_1 and its distance to D

$$V_1D = 2.236 \text{ km} \, (4 \text{ s.f.})$$

Consider V_2 and its distance to D

$$V_2D = \sqrt{(7-5)^2 + (4-3)^2} \quad \textbf{(M1)}$$

$$= 2.2360... \text{ km} \quad \textbf{(A1)}$$

The intersections V_1 and V_2 are at the same distance from any of its nearby coffee shops, and hence the optimal location for the new coffee shop is at either

R1

$$\boxed{V_1(4,5) \text{ or } V_2(7,4)}$$

Question 6

CALCULATOR   Easy

[Maximum mark: 5]

On an island, there is a population of cats that like to hunt a certain species of birds.

The populations of birds and cats can be modelled by the coupled system

$$\begin{cases} x'(t) = 2x - xy \\ y'(t) = 2xy - y \end{cases}$$

Where x is the number of birds and y is the number of cats, both measured in hundreds, at time t years. Initially there were 70 birds and 40 cats.

(a) Find the non-zero equilibrium point for the system. [2]

(b) Using Euler's method with a step size of 0.1, estimate the population of birds and cats after 1 year. [3]

(a) At an equilibrium point, both $x'(t)$ and $y'(t)$ must equal zero.

If we factorise both expressions and set them equal to zero, we have

$$x(2-y) = 0 \qquad y(2x-1) = 0$$ **(M1)**

Given that $x,\ y \neq 0$ for a non-zero equilibrium point, the solution to the first equation is $y = 2$ and the solution to the second equation is $x = \frac{1}{2}$.

Therefore, the non-zero equilibrium point is $\left(\frac{1}{2}, 2\right)$.

A1

(b) Using the formulae for Euler's method for coupled systems, with $h = 0.1$, we can generate expressions for x_{n+1}, y_{n+1} and t_{n+1}. We have

$$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$$
$$= x_n + 0.1(2x_n - x_n y_n)$$
$$y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$$
$$= y_n + 0.1(2x_n y_n - y_n)$$

(M1)

$$t_{n+1} = t_n + h$$
$$= t_n + 0.1$$

We also have initial values of $x_0 = 0.7$ and $y_0 = 0.4$ because x and y are measured in hundreds.

The following table shows the table of values generated using Euler's method for coupled systems using the G.D.C.

(M1)

n	0	1	2	3	4	5	6	7	8	9	10
t	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x	0.7	0.812	0.941	1.09	1.25	1.44	1.63	1.84	2.04	2.22	2.33
y	0.4	0.416	0.442	0.481	0.537	0.618	0.734	0.900	1.14	1.49	2.01

After one year, there are approximately 233 birds and 201 cats.

A1

Question 7

CALCULATOR Medium

[Maximum mark: 8]

A group of 12 long jumpers received a 2 month intensive training programme.

To assess the effectiveness of the programme, each athlete's average jump distance (in metres) was recorded immediately before starting the programme (X_1), and then again immediately after completing the programme (X_2)
.

The difference, $D = X_2 - X_1$, was recorded for each of the 12 athletes.

It was found that the mean difference for the 12 athletes was $\bar{x}_D = 0.233$ and the standard deviation was $\sigma_D = 0.490$
.

A t-test was then conducted at the 5% significance level to determine whether there is any evidence that the training programme improves athlete performance.

(a) State one assumption that needs to be made in order to conduct a valid t-test. [1]

(b) State suitable hypotheses to test the claim that the training program improves athlete performance. [1]

(c) Calculate the p-value for the test. [4]

(d) State the conclusion to the test and justify your answer. [2]

(a) For a t-test to be applied, the underlying distribution must be normal. Therefore,

We must assume that the differences, D, follow a normal distribution. **A1**

(b) The appropriate test is a one-tailed t-test on the differences, with the null hypothesis being that there is no difference in jump distance ($\mu_D = 0$), and the alternate hypothesis being that there is a positive difference.

The null and alternate hypotheses are

$$\mathrm{H}_0 : \mu_D = 0$$

$$\mathrm{H}_1 : \mu_D > 0$$ **A1**

(c) We do not have the data on each of the 12 athletes, therefore we are conducting the t-test using statistics. To do this, we require three values:

- The sample size;
- An unbiased estimate of the population mean;
- An unbiased estimate of the population standard deviation.

We have the sample size ($n = 12$), and we can use the sample mean ($\bar{x}_D = 0.233$) as an unbiased estimate of the population mean.

What we don't have is an unbiased estimate of the population standard deviation, but we do have the sample standard deviation ($\sigma_D = 0.490$).

Using the formula for an unbiased estimate of population variance, we can find the value we require.

$$s_{n-1}^2 = \frac{n}{n-1} s_n^2$$
$$\Rightarrow s_{n-1} = \sqrt{\frac{n}{n-1}} \times s_n$$

Where s_{n-1} is an unbiased estimate of the population standard deviation and s_n is the sample standard deviation. We have

$$s_{n-1} = \sqrt{\frac{12}{12-1}} \times 0.490 \qquad \textbf{(M1)}$$
$$= 0.5117\ldots \qquad \textbf{(A1)}$$

We can now run the t-test, using the three required values and the hypotheses stated in part (b).

(M1)

We obtain a p-value of $p = 0.0715$ (3 s.f.).

(A1)

(d) To conclude a t-test, we compare the p-value with the significance level.

In this case, we find that the p-value is greater than the significance level ($0.0715 > 0.05$).

R1

The p-value is greater than the significance level, therefore we do not reject the null hypothesis. There is insufficient evidence at the 5% significance level that the training programme improves athlete performance.

A1

Question 8

CALCULATOR

Medium

[Maximum mark: 8]

The number of fish in a lake, f, is given by the function

$$f(x) = 450\,e^{-0.1x} + 50$$

Where x is the approximate concentration of algae in the lake, measured in parts per billion (ppb) and $x \geq 0$.

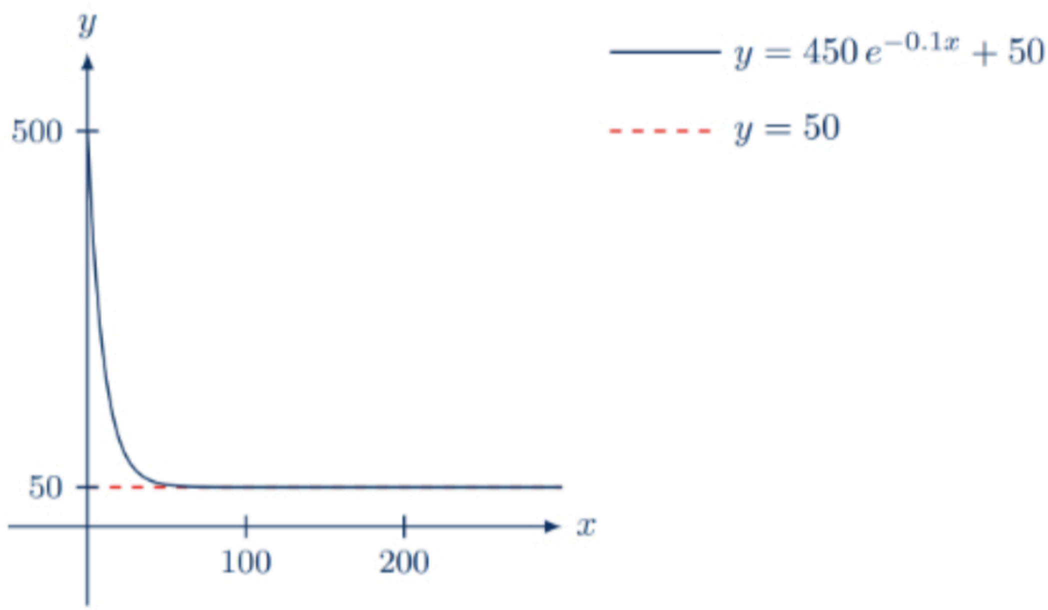
(a) Determine the range of f. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

The lake is considered "healthy" if there are at least 430 fish at any given time.

(c) Calculate the range of algae concentrations for which the lake can be considered healthy. [3]

(a) A sketch of $f(x)$ on the G.D.C. shows that the function has a y-intercept at 500, and is a decreasing function.



We can also see that as $x \to \infty$, $f(x) \to 50$ (but it never reaches 50). Therefore the range of f is

$$50 < f \leq 500$$

A2

(b) To find the inverse function, we will first re-write our function using y to represent $f(x)$, this will make manipulation of the function easier. We have

$$y = 450\,e^{-0.1x} + 50 \qquad \text{[Where } y = f(x)\text{]}$$

We now switch x and y, thereby creating the inverse function.

$$x = 450\,e^{-0.1y} + 50 \qquad \text{[Where } y = f^{-1}(x)\text{]}$$ **(M1)**

Now we can rearrange to make y the subject. This will be our inverse function.

$$x - 50 = 450\,e^{-0.1y}$$

$$\frac{x-50}{450} = e^{-0.1y}$$

$$\ln\left(\frac{x-50}{450}\right) = -0.1y$$ **(M1)**

$$-10\ln\left(\frac{x-50}{450}\right) = y$$

We can now substitute $y = f^{-1}(x)$ for our final answer.

$$f^{-1}(x) = -10\ln\left(\tfrac{x-50}{450}\right)$$ **A1**

(c) We found in part (a) that the maximum number of fish in the lake is 500, and this occurs when the concentration of algae is 0 ppb.

(A1)

For the lake to be considered healthy, the minimum number of fish is 430, and this will coincide with the maximum allowable algae concentration.

That is, the maximum algae concentration is a, such that $f(a) = 430$.

Using the relationship between a function and its inverse, we could therefore say $f^{-1}(430) = a$.

Substituting the value 430 into our inverse function from part (b), we have

$$\begin{aligned} f^{-1}(430) &= -10 \ln\left(\frac{430 - 50}{450}\right) \\ &= 1.6907... \\ &= 1.69 \text{ (3 s.f.)} \end{aligned}$$ **(M1)**

Hence the range of algae concentrations for which the lake can be considered healthy, in parts per billion, is

$$\boxed{0 \le x \le 1.69}$$ **A1**

Question 9

CALCULATOR

Medium

[Maximum mark: 5]

Consider the following transformations T_1 and T_2:

- Transformation T_1 is a reflection in the line $y = \left(\tan \frac{\pi}{4}\right) x$ followed by a translation of 3 units in the direction of the positive x axis and 4 units in the direction of the positive y axis.
- Transformation T_2 is an enlargement by a scale factor of 3, centred at the origin, followed by a translation of 1 unit in the direction of the negative x axis and 2 units in the direction of the negative y axis.

Given that the image of point (a, b) is the same under transformation T_1 as it is under transformation T_2, find the value of a and the value of b.

Using the formula sheet, we can create a matrix expression for T_1:

$$T_1 : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & -\cos\frac{\pi}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad \textbf{(M1)}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$T_1 : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} y+3 \\ x+4 \end{pmatrix} \qquad \textbf{(A1)}$$

Hence under transformation T_1, the image of point (a, b) would be $(b+3, a+4)$.

We can also create a matrix expression for T_2:

$$T_2 : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$T_2 : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3x-1 \\ 3y-2 \end{pmatrix} \qquad \textbf{(A1)}$$

Hence under transformation T_2, the image of point (a, b) would be $(3a-1, 3b-2)$.

By equating the x and y coordinates of both images, we can form a system of 2 linear equations:

$$b + 3 = 3a - 1$$

$$a + 4 = 3b - 2 \qquad \textbf{(M1)}$$

Using the G.D.C., we find that $a = 2.25$ and $b = 2.75$

A1

Question 10

CALCULATOR

Medium ●●●●●

[Maximum mark: 7]

The number of fatal crashes on a highway in a month is assumed to follow a Poisson distribution with mean μ. It is known from the Department of Transportation that there are 1.6 fatal crashes on this highway per month on average.

In an effort to reduce the number of fatal crashes, multiple speed cameras are installed along the highway.

To determine whether this solution effectively reduced the value of μ, the total number of fatal crashes, X, happening in the 12 months after deploying the platform is recorded, and an appropriate hypothesis test is performed.

The null hypothesis for the test is $H_0 : \mu = 1.6$.

(a) State the alternate hypothesis. [1]

(b) The critical region for the hypothesis test is decided to be $X \leq 12$.

(i) Calculate the significance level for the test.

(ii) Given that the number of fatal crashes was in fact reduced to 0.8 per month, determine the probability that a Type II error was made. [6]

(a) We are looking for evidence of a reduction in fatal crashes, therefore the alternate hypothesis is

$$\boxed{H_1 : \mu < 1.6}$$ **A1**

(b) (i) Assuming the null hypothesis to be true, that there are 1.6 fatal crashes per month, then in a 12 month period, the number of fatal crashes would be $1.6 \times 12 = 19.2$.

Again, assuming that the number of crashes per month follows a Poisson distribution, the number of crashes in a 12 month period will also follow a Poisson distribution. Therefore we have $X \sim \text{Po}(19.2)$.

(M1)

The significance level will be

$$\begin{aligned} \text{Significance Level} &= \text{P}(X \leq 12) \\ &= \text{poissCdf}(19.2, 0, 12) \quad \textbf{(M1)} \\ &= 0.05557... \end{aligned}$$

Converting this to a percentage and rounding to three significant figures, we have

$$\boxed{\text{Significance Level} = 5.56\%}$$ **A1**

(ii) A type II error occurs when a null hypothesis is not rejected, when in fact it should have been.

In this case, we will not reject the null hypothesis if $X > 12$.

We also have that the number of fatal crashes per month has reduced to 0.8, and so in a 12 month period, it has reduced to $0.8 \times 12 = 9.6$.

Hence $X \sim \mathrm{Po}(9.6)$. We have

$$\begin{aligned} \mathrm{P}(\text{Type II error}) &= \mathrm{P}(X > 12 \,|\, X \sim \mathrm{Po}(9.6)) && \textbf{(A1)} \\ &= 1 - \mathrm{P}(X \leq 12 \,|\, X \sim \mathrm{Po}(9.6)) \\ &= 1 - \mathrm{poissCdf}(9.6, 0, 12) && \textbf{(M1)} \\ &= 0.1721... \end{aligned}$$

$$\boxed{\mathrm{P}(\text{Type II error}) = 0.172 \text{ (3 s.f.)}} \qquad \textbf{A1}$$

Question 11

CALCULATOR Medium

[Maximum mark: 8]

A pharmaceutical company is testing a new treatment that is designed to lower triglyceride levels in the blood. Normal triglyceride levels in the blood are less than 150 mg per decilitre (mg/dL). A person is diagnosed with high triglyceride levels if their triglyceride level is higher than 200 mg/dL.

The new treatment has been tested with a randomly selected group of people diagnosed with high triglycerides. The pharmaceutical company claims that the treatment lowers the triglyceride levels to an average of 100 mg/dL. The data obtained from a random sample of 10 patients from the test group are as follows:

103.2, 101.7, 97.9, 93.4, 103.2, 98.5, 96.2, 100.5, 115.3, 99.3

It can be assumed that the treatment data follows a normal distribution $N(\mu, \sigma^2)$.

(a) Determine unbiased estimates for μ and σ. [2]

(b) Calculate a 95% confidence interval for μ. [3]

To assess reliability, a re-test is performed. A second random sample of 10 patients from the test group is selected. For this group it is found that $\bar{x} = 102.48$ mg/dL, and $s_{n-1} = 4.181$.

(c) Calculate a second 95% confidence interval based on the data from the second group. [2]

(d) Hence, comment on the claim made by the pharmaceutical company. [1]

(a) The unbiased estimates for μ and σ are

$\bar{x} = 101 \text{ (3 s.f.)}$ **A1**

$s_{n-1} = 5.91 \text{ (3 s.f.)}$ [by using G.D.C.] **A1**

(b) Since the population variance is unknown, we calculate a t-interval, as opposed to a z-interval.

(M1)

Using our list of data on the G.D.C.,

(M1)

We find the 95% confidence interval for μ, to 3 s.f. is

$96.7 < \mu < 105$ **A1**

(c) This time we do not have a set of data, but we have been given the relevant statistics.

Using these statistics on the G.D.C.,

(M1)

We find the second 95% confidence interval for μ, to 3 s.f. is

$$99.5 < \mu < 105$$ **A1**

(d) Both confidence intervals contain the value 100 mg/dL. This suggests that the testing methodology is reliable.

There is insufficient evidence to reject the claim. **A1**

Question 12

CALCULATOR   Medium

[Maximum mark: 6]

An amusement park is recording a promotional video. For this, they will release a drone from the highest point in the park. The drone moves horizontally in a straight line for the first 12.5 seconds, then descends vertically.

The velocity vector of the drone, v m s^{-1}, at time t seconds, is given by

$$v(t) = \begin{pmatrix} -0.08t^2 + t \\ 0 \end{pmatrix} \text{ for } 0 \leq t \leq 12.5$$

And

$$v(t) = \begin{pmatrix} 0 \\ e^{25-2t} - \frac{12.5}{t} \end{pmatrix} \text{ for } t > 12.5$$

(a) Calculate the distance travelled by the drone in the first 12.5 seconds. [2]

The velocity of the drone when it reaches the ground is exactly -0.25 m s^{-1}.

(b) Given that the drone is airborne for more than 20 seconds, determine the height from which the drone was released. [4]

(a) We only need to consider the horizontal component of the drone's velocity vector. Using the formula for distance travelled from time t_1 to time t_2, we have

$$\begin{aligned} d &= \int_{t_1}^{t_2} |v(t)| \, \mathrm{d}t \\ &= \int_{0}^{12.5} |-0.08t^2 + t| \, \mathrm{d}t \qquad \textbf{(M1)} \\ &= 26.04\ldots \qquad \text{[by using G.D.C.]} \end{aligned}$$

$d = 26.0$ m (3 s.f.)   **A1**

(b) We can use the same formula we used in part (a) with the vertical component of the velocity vector for $t > 12.5$. t_1 will be 12.5, because this is the time that the drone starts descending. However, we need a value for t_2, the time at which the drone reaches the ground.

Our first step therefore is to determine the time at which the drone reaches the ground. Solving the equation $v(t) = -0.25$ for t (and noting that t must be greater than 20), we get

$$e^{25-2t} - \frac{12.5}{t} = -0.25 \qquad \textbf{(M1)}$$

$$t = 50 \text{ s} \qquad \text{[by using G.D.C.]} \qquad \textbf{(A1)}$$

We can now find the total distance travelled by the drone vertically between $t_1 = 12.5$ and $t_2 = 50$ seconds.

We have

$$\begin{aligned} d &= \int_{t_1}^{t_2} |v(t)| \, \mathrm{d}t \\ &= \int_{12.5}^{50} \left|e^{25-2t} - \frac{12.5}{t}\right| \mathrm{d}t \qquad \textbf{(M1)} \\ &= 16.82... \qquad \text{[by using G.D.C.]} \end{aligned}$$

The drone was released from a height of 16.8 m (3 s.f.).

A1

Question 13

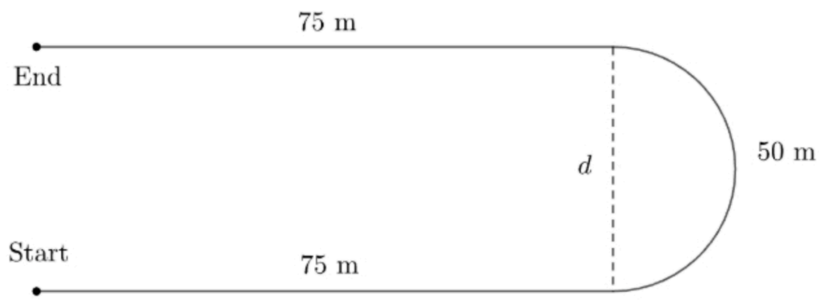
CALCULATOR Medium

[Maximum mark: 8]

Samira is designing a running track for the 200 metre race of her school sports day.

The 200 metre track is formed by two straight lines of 75 m each and a semi-circular part of 50 m.

Here is her initial design.



d is the diameter of the semi-circular part of the track.

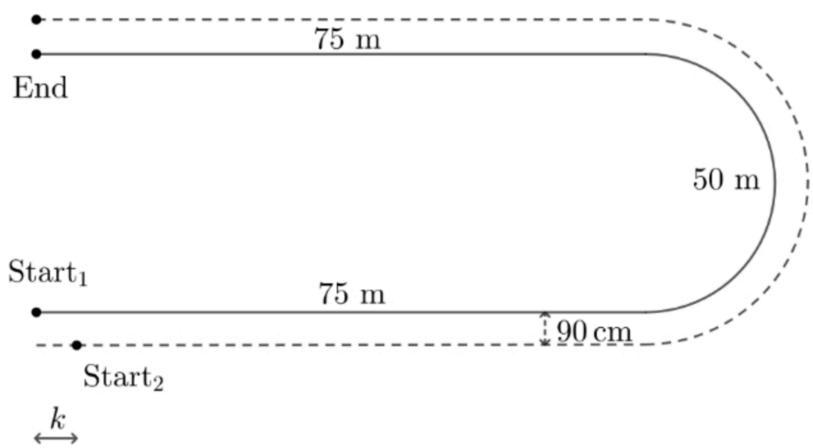
(a) Find the value of d. [2]

Samira wants to add a second lane to the track.

The width of each lane is 90 centimetres.

The runner in lane 1 starts at Start_1 and follows the solid line around the track, while the runner in lane 2 starts at Start_2 and follows the dashed line around the track.

This is shown below.



In order to make sure both lanes are exactly 200 metres in length, Samira must move Start_2 forward k metres.

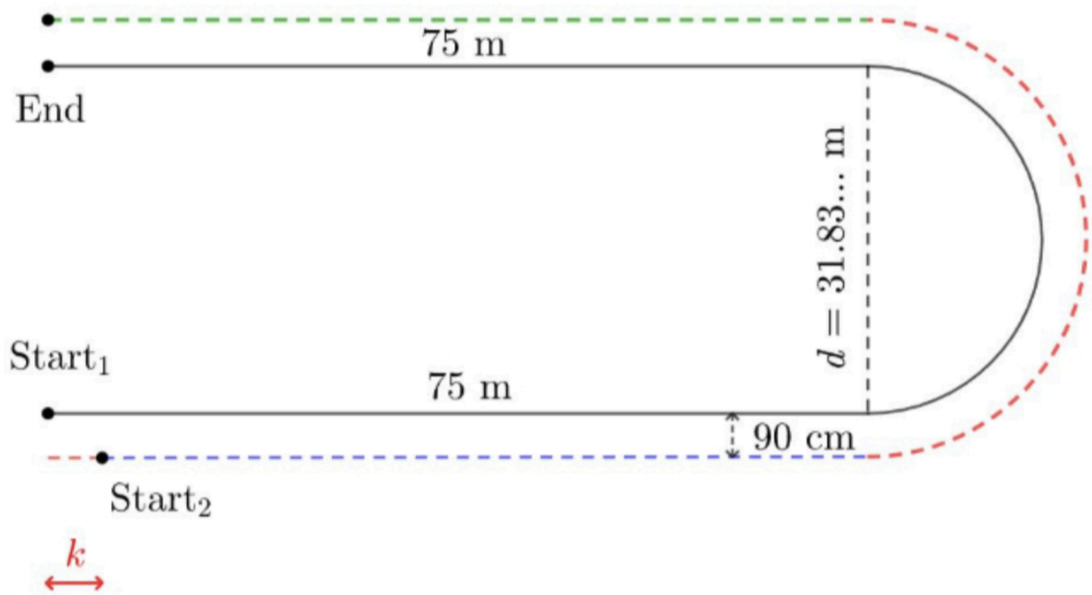
(b) Find the value of k. [6]

(a) We can use the formula for the circumference of a circle.

We know the perimeter of the semi-circle is 50 metres, therefore the circle has a circumference of 100 metres. The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius. We have to find d, the diameter, which is twice the radius. We have

$$\begin{aligned} C &= 2\pi r \\ &= \pi \times d \\ 100 &= \pi \times d \\ d &= \frac{100}{\pi} \qquad \textbf{(M1)} \\ &= 31.83... \\ d &= \boxed{31.8 \text{ metres}} \qquad \textbf{A1} \end{aligned}$$

(b) The path followed by the runner in lane 2 is shown below, split into 3 different parts.



The green dotted line is the finishing straight, and this is 75 m.

The blue dotted line is the starting straight, and this is $(75 - k)$ m.

(A1)

The red dotted line is half the circumference of a circle.

The sum of these three parts must be exactly 200 m, so that both runners run the same distance.

We know from part (a) that $d = 31.83...$ m, and we have been told that each lane is 90 cm wide, which is 0.9 m.

So we are looking for half the circumference of a circle that has a diameter of $d = 0.9 + 0.9 + 31.83...$ m.

(A1)

Using the formula for the circumference of a circle, we have

$$\begin{aligned} C &= 2\pi r \\ &= \pi \times d \\ &= \pi \times (0.9 + 0.9 + 31.83...) \qquad \textbf{(M1)} \\ \frac{C}{2} &= \frac{\pi \times (0.9 + 0.9 + 31.83...)}{2} \\ &= 52.82... \qquad \textbf{(A1)} \end{aligned}$$

We can now form an equation using the three separate parts from the diagram.

$$\text{green dotted line} + \text{blue dotted line} + \text{red dotted line} = 200$$

Substituting in our values from above, we have

$$75 + (75 - k) + 52.82... = 200 \qquad \textbf{(M1)}$$

$$k = 2.827... \qquad \text{[by using G.D.C.]}$$

$$\boxed{k = 2.83\,\text{m}\,(3\text{ s.f.})} \qquad \textbf{A1}$$

Question 14

CALCULATOR

Medium

[Maximum mark: 10]

An insurance salesman has four meetings scheduled with some prospective clients (A, B, C and D), and is planning the order in which to visit them, starting from his home at E and returning there afterwards.

The distances between each home, in km, are shown in the following weighted adjacency table. Each distance is a positive integer.

	A	B	C	D	E
A	–	8	3	7	6
B	8	–	10	x	9
C	3	10	–	4	7
D	7	x	4	–	8
E	6	9	7	8	–

(a) Starting from E, use the nearest neighbour algorithm to determine an upper bound for the salesman's journey, in terms of x. [3]

(b) (i) By removing client D, determine a minimum spanning tree for the remaining subgraph. Use Prim's algorithm, starting from vertex E.

(ii) Hence use the deleted vertex algorithm to find two possible expressions for the lower bound. [4]

(c) Given that the lower bound is exactly 1 km less than the upper bound, find the range of possible values for x. [3]

(a) The order in which we must choose the edges is as follows.

- We must first choose the edge of least weight connected to vertex E. This is edge AE, which has a weight of 6.
- From A, we must choose edge AC, which has a weight of 3.
- From C, we must choose edge CD, which has a weight of 4.
- From D, we must choose edge BD, which has a weight of x.
- From B, we must choose edge BE, which has a weight of 9.

(M2)

Therefore

$$\begin{aligned}\text{Upper Bound} &= 6 + 3 + 4 + x + 9 \\ &= \boxed{(22 + x)\,\text{km}}\end{aligned}$$

A1

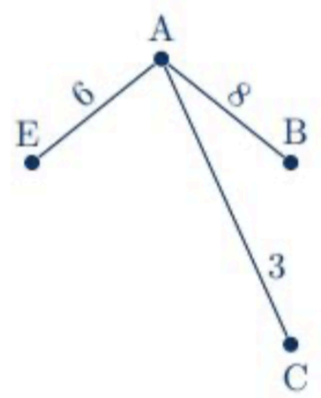
(b) (i) According to Prim's algorithm, we first select the edge of least weight. Starting from vertex E, we select edge AE, which has a weight of 6.

We now select the next edge of least weight that is incident to the tree (i.e. it must connect to vertex A or vertex E). We therefore select edge AC, which has a weight of 3.

We now only need to connect vertex B in order to complete our MST. The edge of least weight is AB, which has a weight of 8.

(M1)

Our minimum spanning tree is therefore



A1

(ii) We must now reconnect vertex D to the MST using the two edges of least weight. We have 4 options:

- AD - weight 7
- BD - weight x
- CD - weight 4
- DE - weight 8

Edge CD will definitely be included and edge DE will definitely be excluded.

If $x < 7$, then edge BD will be included.

If $x > 7$, then edge AD will be included.

If $x = 7$, then either AD or BD can be included.

Hence our first lower bound is $6 + 8 + 3 + 4 + 7 = \boxed{28\,\text{km}}$ (if $x \geq 7$).

A1

Our second lower bound is $6 + 8 + 3 + 4 + x = \boxed{(21 + x)\,\text{km}}$ (if $x < 7$).

A1

(c) We have our upper bound as $(22 + x)$ km. We now consider separately our two possible lower bounds, given that they must be exactly 1 km less than the upper bound.

Firstly, we have a lower bound of 28 km if $x \geq 7$. In this case the lower bound will only be 1 less than the upper bound if the upper bound is 29 km. In this case we must have $x = 7$.

(A1)

Secondly, we have a lower bound of $(21 + x)$ km if $x < 7$.

$21 + x$ will always be exactly one less than $22 + x$ hence this holds for all valid values of x.

(A1)

We were given initially that x is a positive integer, hence our range of possible values for x is

$$1 \leq x \leq 7, \; x \in \mathbb{Z}$$ **A1**

Question 15

CALCULATOR

Hard ●●●●

[Maximum mark: 10]

The number of subscribers to a new video streaming service is recorded in the following table. The first row shows the number of weeks (t) since the service launched, and the second row shows the number of subscribers, S, in thousands.

Weeks since launching (t)	5	14	26	32	40	46	51	53	57	61
Subscribers $\times 1\,000$ (S)	12	47	143	277	480	831	1160	1305	1535	1640

An analyst for the streaming service believes an exponential model may help future planning around expected subscriber numbers. Her first step is to linearize the data.

(a) Complete the following table, giving each value to 3 significant figures. [2]

t	5	14	26	32	40	46	51	53	57	61
$\ln S$	2.485	3.850								

(b) Using the values from part (a), determine the equation of the regression line $\ln S$ on t. [2]

(c) Hence find an expression for S in terms of t. Give your answer in the form $S = a(b)^t$. [4]

The streaming company has offered all employees a bonus when they reach 10 million subscribers.

(d) Use your model from part (c) to determine the number of whole weeks after launch that the company expects to reach 10 million subscribers. [2]

(a) Using the G.D.C., we have

(M1)

t	5	14	26	32	40	46	51	53	57	61
$\ln S$	2.48	3.85	4.96	5.62	6.17	6.72	7.06	7.17	7.34	7.40

A1

(b) Using the G.D.C., we have

(M1)

$$\ln S = 0.0870t + 2.53 \text{ (3 s.f.)}$$ **A1**

(c) We have

$$\ln S = 0.08702...t + 2.528...$$

$$S = e^{0.08702...t+2.528...}$$ **(M1)**

$$= e^{0.08702...t} \times e^{2.528...} \quad [\text{because } a^{m+n} = a^m \cdot a^n]$$ **(M1)**

$$= \left(e^{0.08702...}\right)^t \times e^{2.528...} \quad [\text{because } a^{m\times n} = (a^m)^n]$$

$$S = 12.5\,(1.09)^t \text{ (3 s.f.)}$$ **A2**

(d) Using the numerical solver on the G.D.C., we can form an equation and solve for t. Substituting $S = 10\,000$, because S is measured in thousands, we have

$$10,000 = 12.529...\,(1.0909...)^t$$

$$t = 76.78...$$ **(M1)**

It takes 77 whole weeks to reach 10 million subscribers.

A1