IB Mathematics AI HL - Prediction Exams May 2025 - Paper 1

Paper 1 | 15 questions | 120 mins | 110 marks

Question 1

CALCULATOR

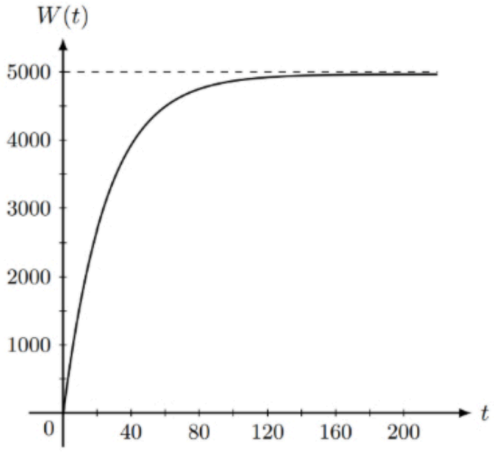
Easy

[Maximum mark: 6]

A water tank has a float valve that allows water to enter while the tank is not full. Over time, the float valve reduces the water flow until the tank is almost at its maximum capacity.

The amount of water in the tank, in litres, is given by the function $W(t) = 5\,000 - C\,(1.04^{-t})$

where t is minutes after the tank starts to fill.



(a) Given that the tank was initially empty, find the value of C. [2]

(b) State what the horizontal asymptote represents in the context of the question. [1]

(c) Find the total amount of water in the tank after one and half hours. Give your answer in cubic metres. [3]

Question 2

CALCULATOR

Easy

[Maximum mark: 7]

A'ja is a keen basketball player. Each time A'ja attempts a free throw, she has a 75% chance of scoring. Suppose she practises 120 free throws and counts the total number of free throws she makes. It can be assumed that the probability of scoring on any given free throw is independent of her other previous free throw attempts.

(a) (i) Write down the mean number of free throws she makes.

(ii) Calculate the variance of the number of free throws she makes. [4]

(b) Find the probability that the number of free throws she makes is less than one standard deviation away from the mean. [3]

Question 3

CALCULATOR

Easy

[Maximum mark: 6]

Bruno rides his bike to school each morning. During the first minute, he travels 160 metres. In each subsequent minute, he travels 80 % of the distance travelled during the previous minute.

The distance from his home to school is 750 metres. Bruno leaves his house at 8:30 am and must be at school by 8:40 am.

(a) Verify that Bruno will not arrive at school on time. [3]

Bruno realises that if he can increase the distance he travels each minute, from 80 % of the distance travelled during the previous minute to k %, he will make it to school on time.

(b) Determine the minimum value of k, given that Bruno still travels 160 m in the first minute. [3]

Question 4

CALCULATOR Easy

[Maximum mark: 7]

Give all answers for this question to 2 decimal places, unless otherwise stated.

On 1 January 2024, Emily invests \$600 000 in a savings account which pays a nominal annual interest rate of 4.5%, compounded annually.

(a) Determine the amount of money that will be in the account after 12 years. [3]

After these 12 years, Emily is planning to retire and place the money she has saved into an annuity fund which pays a nominal annual interest rate of 3.5%, compounded monthly.

Emily wants to withdraw money from this account at the end of each month.

(b) (i) Calculate the amount Emily can withdraw at the end of each month if she wants the money to last for 18 years after her retirement.

(ii) Find how many **complete months**, counted from 1 January 2036, it will take for the balance of the fund to fall below \$500 000. [4]

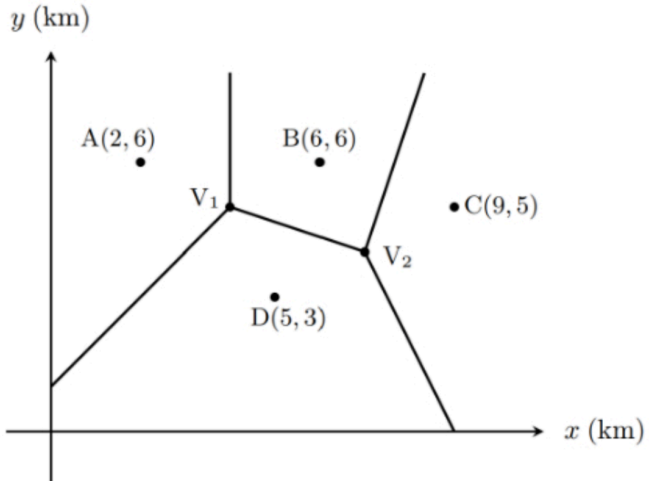
Question 5

CALCULATOR

Easy

[Maximum mark: 9]

Consider the Voronoi diagram below for a town centre that contains four coffee shops A, B, C, and D.



The equation of the perpendicular bisector between sites B and C is $y = 3x - 17$. The coordinates of the midpoint between C and D is (7,4).

(a) Determine the equation of the perpendicular bisector between C and D. [3]

(b) Hence, determine the coordinates of the intersection point V_2. [3]

The perpendicular bisectors of AB and AD intersect at the point $V_1(4,5)$, which is 2.236 km from location D, correct to 4 significant figures. A new coffee shop will be built at either V_1 or V_2.

(c) Given that the new shop is to be as far away as possible from an existing coffee shop, determine which of the locations the new coffee shop should be built at. [3]

Question 6

CALCULATOR

Easy

[Maximum mark: 5]

On an island, there is a population of cats that like to hunt a certain species of birds.

The populations of birds and cats can be modelled by the coupled system

$$\begin{cases} x'(t) = 2x - xy \\ y'(t) = 2xy - y \end{cases}$$

Where x is the number of birds and y is the number of cats, both measured in hundreds, at time t years. Initially there were 70 birds and 40 cats.

(a) Find the non-zero equilibrium point for the system. [2]

(b) Using Euler's method with a step size of 0.1, estimate the population of birds and cats after 1 year. [3]

Question 7

CALCULATOR Medium

[Maximum mark: 8]

A group of 12 long jumpers received a 2 month intensive training programme.

To assess the effectiveness of the programme, each athlete's average jump distance (in metres) was recorded immediately before starting the programme (X_1), and then again immediately after completing the programme (X_2).

The difference, $D = X_2 - X_1$, was recorded for each of the 12 athletes.

It was found that the mean difference for the 12 athletes was $\bar{x}_D = 0.233$ and the standard deviation was $\sigma_D = 0.490$.

A t-test was then conducted at the 5% significance level to determine whether there is any evidence that the training programme improves athlete performance.

(a) State one assumption that needs to be made in order to conduct a valid t-test. [1]

(b) State suitable hypotheses to test the claim that the training program improves athlete performance. [1]

(c) Calculate the p-value for the test. [4]

(d) State the conclusion to the test and justify your answer. [2]

Question 8

CALCULATOR Medium

[Maximum mark: 8]

The number of fish in a lake, f, is given by the function

$$f(x) = 450\,e^{-0.1x} + 50$$

Where x is the approximate concentration of algae in the lake, measured in parts per billion (ppb) and $x \geq 0$.

(a) Determine the range of f. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

The lake is considered "healthy" if there are at least 430 fish at any given time.

(c) Calculate the range of algae concentrations for which the lake can be considered healthy. [3]

Question 9

CALCULATOR Medium

[Maximum mark: 5]

Consider the following transformations T_1 and T_2:

- Transformation T_1 is a reflection in the line $y = \left(\tan \frac{\pi}{4}\right) x$ followed by a translation of 3 units in the direction of the positive x axis and 4 units in the direction of the positive y axis.
- Transformation T_2 is an enlargement by a scale factor of 3, centred at the origin, followed by a translation of 1 unit in the direction of the negative x axis and 2 units in the direction of the negative y axis.

Given that the image of point (a, b) is the same under transformation T_1 as it is under transformation T_2, find the value of a and the value of b.

Question 10

CALCULATOR Medium

[Maximum mark: 7]

The number of fatal crashes on a highway in a month is assumed to follow a Poisson distribution with mean μ. It is known from the Department of Transportation that there are 1.6 fatal crashes on this highway per month on average.

In an effort to reduce the number of fatal crashes, multiple speed cameras are installed along the highway.

To determine whether this solution effectively reduced the value of μ, the total number of fatal crashes, X, happening in the 12 months after deploying the platform is recorded, and an appropriate hypothesis test is performed.

The null hypothesis for the test is $H_0 : \mu = 1.6$.

(a) State the alternate hypothesis. [1]

(b) The critical region for the hypothesis test is decided to be $X \leq 12$.

(i) Calculate the significance level for the test.

(ii) Given that the number of fatal crashes was in fact reduced to 0.8 per month, determine the probability that a Type II error was made. [6]

Question 11

CALCULATOR Medium

[Maximum mark: 8]

A pharmaceutical company is testing a new treatment that is designed to lower triglyceride levels in the blood. Normal triglyceride levels in the blood are less than 150 mg per decilitre (mg/dL). A person is diagnosed with high triglyceride levels if their triglyceride level is higher than 200 mg/dL.

The new treatment has been tested with a randomly selected group of people diagnosed with high triglycerides. The pharmaceutical company claims that the treatment lowers the triglyceride levels to an average of 100 mg/dL. The data obtained from a random sample of 10 patients from the test group are as follows:

103.2, 101.7, 97.9, 93.4, 103.2, 98.5, 96.2, 100.5, 115.3, 99.3

It can be assumed that the treatment data follows a normal distribution $\mathrm{N}(\mu, \sigma^2)$.

(a) Determine unbiased estimates for μ and σ. [2]

(b) Calculate a 95% confidence interval for μ. [3]

To assess reliability, a re-test is performed. A second random sample of 10 patients from the test group is selected. For this group it is found that $\bar{x} = 102.48$ mg/dL, and $s_{n-1} = 4.181$.

(c) Calculate a second 95% confidence interval based on the data from the second group. [2]

(d) Hence, comment on the claim made by the pharmaceutical company. [1]

Question 12

CALCULATOR

Medium

[Maximum mark: 6]

An amusement park is recording a promotional video. For this, they will release a drone from the highest point in the park. The drone moves horizontally in a straight line for the first 12.5 seconds, then descends vertically.

The velocity vector of the drone, v m s^{-1}, at time t seconds, is given by

$$v(t) = \begin{pmatrix} -0.08t^2 + t \\ 0 \end{pmatrix} \text{ for } 0 \leq t \leq 12.5$$

And

$$v(t) = \begin{pmatrix} 0 \\ e^{25-2t} - \frac{12.5}{t} \end{pmatrix} \text{ for } t > 12.5$$

(a) Calculate the distance travelled by the drone in the first 12.5 seconds. [2]

The velocity of the drone when it reaches the ground is exactly -0.25 m s^{-1}.

(b) Given that the drone is airborne for more than 20 seconds, determine the height from which the drone was released. [4]

Question 13

CALCULATOR

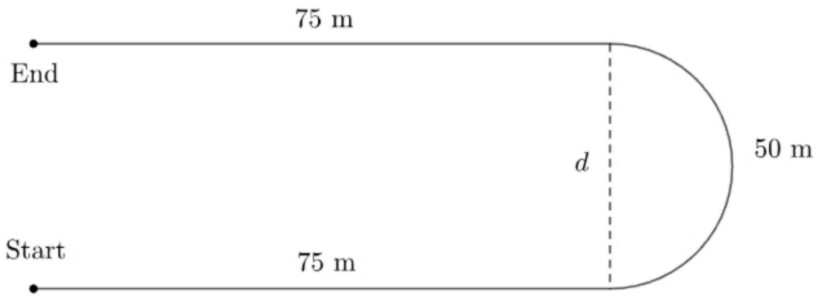
Medium

[Maximum mark: 8]

Samira is designing a running track for the 200 metre race of her school sports day.

The 200 metre track is formed by two straight lines of 75 m each and a semi-circular part of 50 m.

Here is her initial design.



d is the diameter of the semi-circular part of the track.

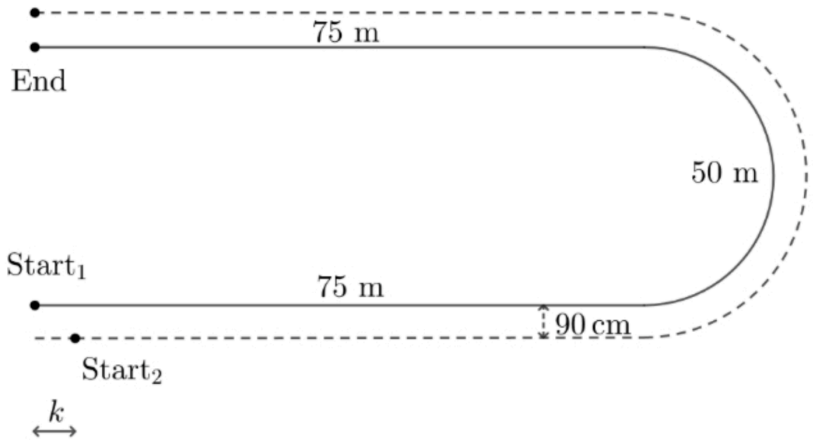
(a) Find the value of d. [2]

Samira wants to add a second lane to the track.

The width of each lane is 90 centimetres.

The runner in lane 1 starts at Start_1 and follows the solid line around the track, while the runner in lane 2 starts at Start_2 and follows the dashed line around the track.

This is shown below.



In order to make sure both lanes are exactly 200 metres in length, Samira must move Start_2 forward k metres.

(b) Find the value of k. [6]

Question 14

CALCULATOR Medium

[Maximum mark: 10]

An insurance salesman has four meetings scheduled with some prospective clients (A, B, C and D), and is planning the order in which to visit them, starting from his home at E and returning there afterwards.

The distances between each home, in km, are shown in the following weighted adjacency table. Each distance is a positive integer.

	A	B	C	D	E
A	–	8	3	7	6
B	8	–	10	x	9
C	3	10	–	4	7
D	7	x	4	–	8
E	6	9	7	8	–

(a) Starting from E, use the nearest neighbour algorithm to determine an upper bound for the salesman's journey, in terms of x. [3]

(b) (i) By removing client D, determine a minimum spanning tree for the remaining subgraph. Use Prim's algorithm, starting from vertex E.

(ii) Hence use the deleted vertex algorithm to find two possible expressions for the lower bound. [4]

(c) Given that the lower bound is exactly 1 km less than the upper bound, find the range of possible values for x. [3]

Question 15

CALCULATOR

Hard ●●●●○

[Maximum mark: 10]

The number of subscribers to a new video streaming service is recorded in the following table. The first row shows the number of weeks (t) since the service launched, and the second row shows the number of subscribers, S, in thousands.

Weeks since launching (t)	5	14	26	32	40	46	51	53	57	61
Subscribers $\times 1000\,(S)$	12	47	143	277	480	831	1160	1305	1535	1640

An analyst for the streaming service believes an exponential model may help future planning around expected subscriber numbers. Her first step is to linearize the data.

(a) Complete the following table, giving each value to 3 significant figures. [2]

t	5	14	26	32	40	46	51	53	57	61
$\ln S$	2.485	3.850								

(b) Using the values from part (a), determine the equation of the regression line $\ln S$ on t. [2]

(c) Hence find an expression for S in terms of t. Give your answer in the form $S = a(b)^t$. [4]

The streaming company has offered all employees a bonus when they reach 10 million subscribers.

(d) Use your model from part (c) to determine the number of whole weeks after launch that the company expects to reach 10 million subscribers. [2]