IB Mathematics AI HL - Prediction Exams May 2025 - Paper 2

Paper 2 | 7 questions | 120 mins | 110 marks

Question 1

CALCULATOR | Easy

[Maximum mark: 17]

Professor Smith teaches a calculus course to a group of students in a school. He has noticed that the length of the lectures he gives follows a normal distribution with mean m minutes.

There is a 20% chance that any lecture lasts at most 48.6 minutes.

(a) Sketch a diagram to represent this probability. [2]

There is also a 20% chance that any lecture lasts longer than 55.4 minutes.

(b) Show that $m = 52$. [2]

The standard deviation of the number of minutes a lecture lasts is 4 minutes.

(c) Find the probability that a lecture

(i) lasts between 45 and 51 minutes;

(ii) lasts more than 55 minutes. [4]

There is a 70% chance that a lecture lasts less than x minutes.

(d) Find the value of x. [2]

Professor Smith delivers one lecture per day to this group of students, Monday to Friday. It can be assumed that the length of each lecture is independent.

(e) Find the probability that the lecture on Monday lasts between 45 and 51 minutes but the lecture on Tuesday lasts more than 55 minutes. [3]

(f) Calculate the probability that during a given week, at least 3 lectures last more than 55 minutes. [4]

Question 2

CALCULATOR

Medium

[Maximum mark: 16]

At the start 2001, a team of zoologists introduced a new species of rabbit onto a large island. 1000 rabbits were initially introduced and there were 1728 rabbits at the start of 2004.

The size of the population of the species, N, t years after the start of 2001, can be modelled by the following function

$$N(t) = p \times q^t, \qquad t \geq 0$$

(a) (i) Show that the value of p is 1000.

(ii) Verify that $q = 1.2$. [2]

(b) State the annual growth rate of the population as a percentage. [1]

(c) Use the model to predict the population size at the start of 2011. [2]

In 2012, the team observed that the growth rate of the rabbit species was changing and that the overall population was now in decline. Upon investigation, they found that the rabbits had became the prey of a new type of snake on the island. Zoologists adjusted their model to the following new function, where t is still the number of years after the start of 2001.

$$N(t) = 1380 \times (0.87)^{(t-12)} \qquad t \geq 12$$

(d) Use this model to find the size of the population at the start of 2019. [2]

(e) Find the year in which the population of the species will first drop below 200. [3]

(f) In the period from 2013 onwards, find the number of complete years in which the size of the population of the species was greater than or equal to 800. [3]

At the start of 2020, the remaining population of the species was transferred to another island along with 300 more rabbits. The team of zoologists now expect the population growth will recover and will increase at a rate of 10% per year.

(g) Estimate the size of the population at the start of 2030. [3]

Question 3

CALCULATOR

Medium

[Maximum mark: 16]

The cost to manufacture an electronic micro-component at a company can be modelled by the cost function

$$C(x) = x^3 - 3x^2 + 4x$$

where x is in hundreds of micro-components, and $C(x)$ is in hundreds of dollars.

(a) Find $C'(x)$. [2]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find the marginal cost when 200 micro-components are produced and interpret its meaning in this context. [3]

The revenue from selling the micro-components is given by the function

$$R(x) = 0.6x^3 + x^2 + 10x - 2$$

where x is in hundreds of micro-components and $R(x)$ is in hundreds of dollars.

(c) Given that Profit $=$ Revenue $-$ Cost, determine a function for the profit, $P(x)$, in hundreds of dollars from selling x hundreds of micro-components. [2]

(d) Find $P'(x)$. [2]

(e) Determine the intervals where $P(x)$ is increasing and decreasing. [4]

The derivative $P'(x)$ gives the marginal profit. The production will reach its optimal level when the marginal profit is zero and $P(x)$ is positive.

(f) Find the optimal production level and the expected profit at this level. [3]

Question 4

CALCULATOR

Medium

[Maximum mark: 14]

A submarine is moving in a straight line such that $x = 5 + 12t$ and $y = -10 - 18t$, where t is the travel time, in minutes, after 3:40 pm. x represents displacement due east, and y represents displacement due north, relative to an origin O. Distances are measured in metres.

(a) Find the coordinates of the submarine's position at 4:25 pm. [2]

(b) (i) Write down the velocity vector of the submarine.

(ii) Hence find the speed of the submarine. [3]

An iceberg is located at $\mathrm{P}(1500, -1400)$.

(c) Find the distance from the submarine to the iceberg at 4:55 pm. [3]

(d) Find the time, to the nearest minute, when the submarine is closest to the iceberg. [3]

(e) If the submarine changed course at the time found in part (d), to go directly underneath the iceberg, determine the time at which it would be directly underneath. [3]

Question 5

CALCULATOR | Hard

[Maximum mark: 19]

A proposed differential equation modelling the displacement of a person descending a water slide is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 10 - 0.8\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

where x is the displacement of the person in metres at time t seconds.

It can be assumed that initially, $x = \frac{\mathrm{d}x}{\mathrm{d}t} = 0$.

(a) By using the substitution $v = \frac{\mathrm{d}x}{\mathrm{d}t}$, find an expression for $\frac{\mathrm{d}v}{\mathrm{d}t}$ in terms of v. [2]

(b) Hence use Euler's method for coupled systems, with a step size of 0.5, to approximate a person's velocity after 5 seconds. [3]

(c) Solve the differential equation formed in part (a) to find an expression for v in terms of t. [6]

The $\lim_{t\to\infty} v(t)$ is known as the terminal velocity.

(d) State the value of the terminal velocity. [1]

(e) Determine the time at which $v(t)$ reaches 96% of the terminal velocity. Give your answer to the nearest second. [3]

(f) Calculate the percentage error in the approximation of the velocity found by Euler's method at this time. [2]

(g) Explain why Euler's method has overestimated the velocity at this time. [2]

Question 6

CALCULATOR Hard

[Maximum mark: 15]

A water park lends life jackets to its visitors. The park has two stations, A and B, where clients can take a life jacket and they can return the life jacket to either of the two stations. It is found that at the end of a day, 81% of the life jackets are returned to the same station they were taken from, while the other 19% are returned to the other station.

(a) Write down a transition matrix $\boldsymbol{T}$ representing the proportion of the life jackets moving between the two stations. [2]

On February 1st, before opening the park, the staff counted the number of life jackets and found there were 400 life jackets at station A and 500 at station B.

(b) Determine the number of life jackets expected to be at each station when the park opens on February 5th. Give your answer to the nearest whole number. [2]

(c) (i) Show that the eigenvalues of $\boldsymbol{T}$ are $\lambda_1 = 1$ and $\lambda_2 = 0.62$.

(ii) Find a corresponding eigenvector for each eigenvalue from **part (c) (i)**.

(iii) Hence express $\boldsymbol{T}$ in the form $\boldsymbol{T} = \boldsymbol{PDP}^{-1}$. [6]

(d) Show that $\boldsymbol{T}^n = \frac{1}{2}\begin{bmatrix} 1+(0.62)^n & 1-(0.62)^n \\ 1-(0.62)^n & 1+(0.62)^n \end{bmatrix}$, where $n \in \mathbb{Z}^+$. [2]

(e) Hence, find an expression for the number of life jackets in station A after n days, where $n \in \mathbb{Z}^+$ [3]

Question 7

CALCULATOR

Hard

[Maximum mark: 13]

The sunset time in Paris can be modelled by the function

$$S(t) = 1.85 \sin(0.52t - 1.57) + 19.5$$

where t is time in months, from the beginning of January.

The sunrise time can similarly be modelled by

$$R(t) = 1.35 \sin(0.52t - 4.7) + 7.5$$

Both $S(t)$ and $R(t)$ are measured in hours since midnight.

$S(t)$ can be expressed as $\mathrm{im}(z_1) + 19.5$ and $R(t)$ can be expressed as $\mathrm{im}(z_2) + 7.5$, where $z_1, z_2 \in \mathbb{C}$.

(a) Write down expressions for z_1 and z_2. Give your answers in the form $z_{1,2} = r\,e^{\theta i}$. [2]

The number of daylight hours in Paris, $D(t)$, is given by $D(t) = S(t) - R(t)$.

(b) Hence show that the number of hours of daylight in Paris can be modelled by the function

$$D(t) = 3.20 \sin(0.52t - 1.57) + 12$$

[6]

(c) According to the model, find:

(i) the highest number of hours of daylight in a year;

(ii) The approximate number of months per year in which there is at least 14 hours of daylight. [5]