IB Mathematics AI SL - Prediction Exams May 2025 - Paper 1

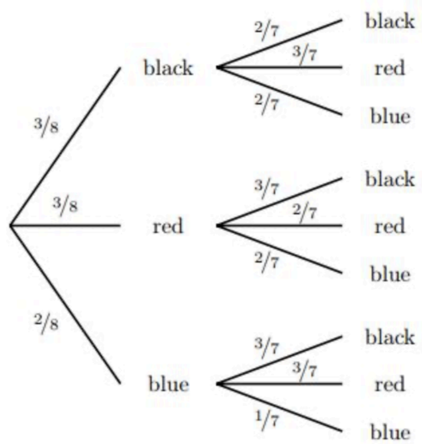
Paper 1 | 12 questions | 90 mins | 80 marks

Question 1

CALCULATOR | Easy

[Maximum mark: 4]

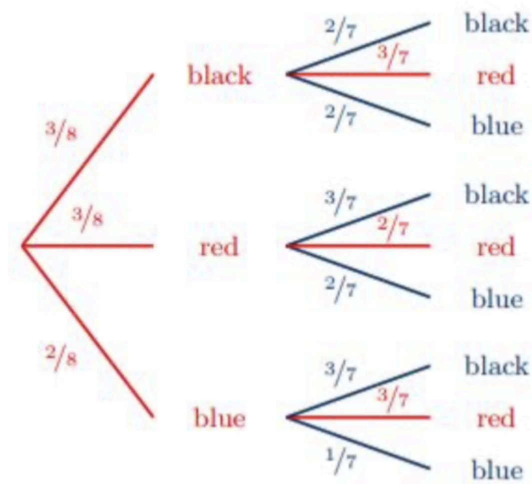
Oliver has 8 pens in his desk drawer, three black, three red and two blue. He randomly selects two pens, one after the other, from the drawer. The following tree diagram shows all the possible outcomes of taking the two pens.



(a) Find the probability that the second pen is red. [2]

(b) Given that the second pen is red, find the probability that the first pen is **not** red. [2]

(a) There are three ways in which the second pen could be red, as shown by the red branches in the following tree diagram.



Using the tree diagram above, we get

$$P(R_2) = \frac{3}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{3}{7} \qquad \textbf{(M1)}$$

$$= \frac{21}{56} \quad \left(= \frac{3}{8}\right) \qquad \textbf{A1}$$

(b) Using the conditional probability formula, we have

$$P(R_1'|R_2) = \frac{P(R_1' \cap R_2)}{P(R_2)}$$

In the denominator, we have $P(R_2)$, which we know to be $\frac{21}{56}$ from part (a).

In the numerator, we have $P(R_1' \cap R_2)$ which is the same as selecting a black pen followed by a red pen, or selecting a blue pen followed by a red pen.

We actually have both of these as the first and third terms of our working in part (a).

This is shown below, with the relevant parts in red:

$$P(R_2) = \frac{3}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{3}{7}$$

Hence the calculation we require is

$$P(R_1'|R_2) = \frac{3/8 \times 3/7 + 2/8 \times 3/7}{21/56} \qquad \textbf{(M1)}$$

Given that the second pen is red, the probability that the first pen is **not** red is

$$\boxed{P(R_1'|R_2) = \frac{15}{21} \quad (= \tfrac{5}{7})} \qquad \textbf{A1}$$

Question 2

CALCULATOR

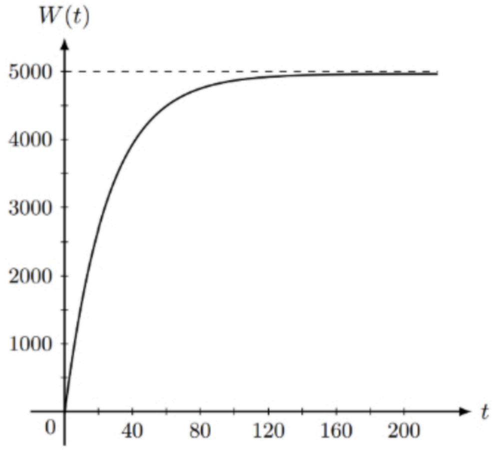
Easy

[Maximum mark: 6]

A water tank has a float valve that allows water to enter while the tank is not full. Over time, the float valve reduces the water flow until the tank is almost at its maximum capacity.

The amount of water in the tank, in litres, is given by the function $W(t) = 5\,000 - C\left(1.04^{-t}\right)$

where t is minutes after the tank starts to fill.



(a) Given that the tank was initially empty, find the value of C. [2]

(b) State what the horizontal asymptote represents in the context of the question. [1]

(c) Find the total amount of water in the tank after one and half hours. Give your answer in cubic metres. [3]

(a) Substituting $t = 0$, and letting $W(0) = 0$, to solve for C, we get

$$0 = 5\,000 - C(1.04^{-0}) \quad \textbf{(M1)}$$

$$= 5\,000 - C$$

$$\boxed{C = 5\,000} \quad \textbf{A1}$$

(b) The horizontal asymptote represents the maximum capacity of the tank, which is 5 000 litres.

R1

(c) First, we must convert the given time to minutes, because minutes is the unit used in the question.

$$1.5 \text{ hours} = 90 \text{ minutes}$$

Then the amount of water, in litres, after 90 minutes is

$$W(90) = 5\,000 - 5\,000\left(1.04^{-90}\right) \quad \textbf{(M1)}$$

$$= 4\,853.45\ldots \quad \textbf{(A1)}$$

Now we must convert from litres to cubic metres, using the fact that $1\,000$ litres $= 1\text{ m}^3$. We can divide the amount in litres by 1 000.

Expressed in cubic metres, we obtain

$$4\,853.45 \text{ litres} = \boxed{4.85\text{ m}^3\ (3\text{ s.f.})} \quad \textbf{A1}$$

Question 3

CALCULATOR

Easy

[Maximum mark: 7]

An e-commerce company has eight distribution centres and wants to know if there is a relationship between the number of employees at each centre and the average delivery time of sales. The data involved are summarized in the following table.

Distribution Centre	Number of employees x	Average delivery time, y (hours)
1	689	38.9
2	531	43.2
3	451	67.3
4	395	71.8
5	735	32.5
6	490	65.2
7	502	51.7
8	623	40.1

(a) From the data for these eight centres

(i) Calculate the Pearson's product-moment correlation coefficient, r.

(ii) Describe the correlation between the number of employees and the average delivery time. [3]

(b) Find the equation of the regression line, y on x, in the form $y = mx + c$. Give the values of m and c to 3 significant figures. [2]

The company opened a new distribution centre recently with 425 employees.

(c) Use your regression line, the solution to part (b), to estimate the average delivery time for this new distribution centre. [2]

(a) (i) $r = -0.922$ (3 s.f.) [by using G.D.C.]

A2

(ii) Since the correlation coefficient is negative and close to 1, there is a (very) strong negative linear correlation between the number of employees and the average delivery time.

R1

(b) $y = -0.116x + 115$ (3 s.f.) [by using G.D.C.]

A1A1

(c) Substituting $x = 425$ into the equation of the regression line, we get

$$y = -0.116(425) + 115$$ **(M1)**

$$= 65.7$$ **A1**

Question 4

CALCULATOR Medium

[Maximum mark: 7]

A'ja is a keen basketball player. Each time A'ja attempts a free throw, she has a 75% chance of scoring. Suppose she practises 120 free throws and counts the total number of free throws she makes. It can be assumed that the probability of scoring on any given free throw is independent of her other previous free throw attempts.

(a) (i) Write down the mean number of free throws she makes.

(ii) Calculate the variance of the number of free throws she makes. [4]

(b) Find the probability that the number of free throws she makes is less than one standard deviation away from the mean. [3]

(a) Let T be the number of free throws A'ja makes when practicing 120 free throws.

We have $T \sim \mathrm{B}(120, 0.75)$.

(i) Hence, using the formula for the expected value of T, we get

$$\begin{aligned} \mathrm{E}(T) &= np \\ &= (120)(0.75) && \textbf{(M1)} \\ &= 90 && \textbf{A1} \end{aligned}$$

(ii) Using the formula for the variance of T, we obtain

$$\begin{aligned} \mathrm{Var}(T) &= np(1-p) \\ &= (120)(0.75)(1-0.75) && \textbf{(M1)} \\ &= 22.5 && \textbf{A1} \end{aligned}$$

(b) Using the formula for the standard deviation of T, we have

$$\begin{aligned}\sigma &= \sqrt{\mathrm{Var}(T)} \\ &= \sqrt{22.5} \\ &= 4.743...\end{aligned} \qquad \textbf{(A1)}$$

Hence we find

$$\begin{aligned}\mathrm{P}(|T-90|<4.743...) &= \mathrm{P}(90-4.743...<T<90+4.743...) \\ &= \mathrm{P}(85.25...<T<94.74...) \\ &= \mathrm{P}(86 \le T \le 94) \\ &= \mathrm{binomCdf}(120, 0.75, 86, 94) \qquad \textbf{(M1)} \\ &= 0.657\ (3\,\mathrm{s.f.}) \qquad \textbf{A1}\end{aligned}$$

Question 5

CALCULATOR Easy

[Maximum mark: 6]

Bruno rides his bike to school each morning. During the first minute, he travels 160 metres. In each subsequent minute, he travels 80 % of the distance travelled during the previous minute.

The distance from his home to school is 750 metres. Bruno leaves his house at 8:30 am and must be at school by 8:40 am.

(a) Verify that Bruno will not arrive at school on time. [3]

Bruno realises that if he can increase the distance he travels each minute, from 80 % of the distance travelled during the previous minute to k %, he will make it to school on time.

(b) Determine the minimum value of k, given that Bruno still travels 160 m in the first minute. [3]

(a) We could consider each minute of Bruno's journey as a term of a geometric sequence, and we need the sum of the first 10 terms to determine how far Bruno travels between 8:30 and 8 :40 am.

We have a geometric sequence with the first term $u_1 = 160$ and common ratio $r = 0.8$.

If we use the sum of n terms formula $S_n = \frac{u_1(1-r^n)}{1-r}$ with $n = 10$, we get

$$S_{10} = \frac{u_1(1-r^{10})}{1-r}$$

$$= \frac{160(1-0.8^{10})}{1-0.8} \quad \textbf{(M1)}$$

$$= 714.10\ldots \quad \textbf{A1}$$

$$714.10\ldots < 750 \quad \textbf{R1}$$

Bruno will only travel 714 metres by 8:40 am, which is less than the 750 metres required.

Therefore, Bruno **will not** arrive at school on time.

(b) Again, we require the sum of the first 10 terms of a geometric sequence with first term $u_1 = 160$. This sum must be **at least** 750 to ensure that Bruno will make it to school on time. We do not know the common ratio, but if each minute Bruno travels $k\,\%$ of the distance travelled in the previous minute, then we will have a common ratio of $r = \frac{k}{100}$.

Using this information we can form an inequality.

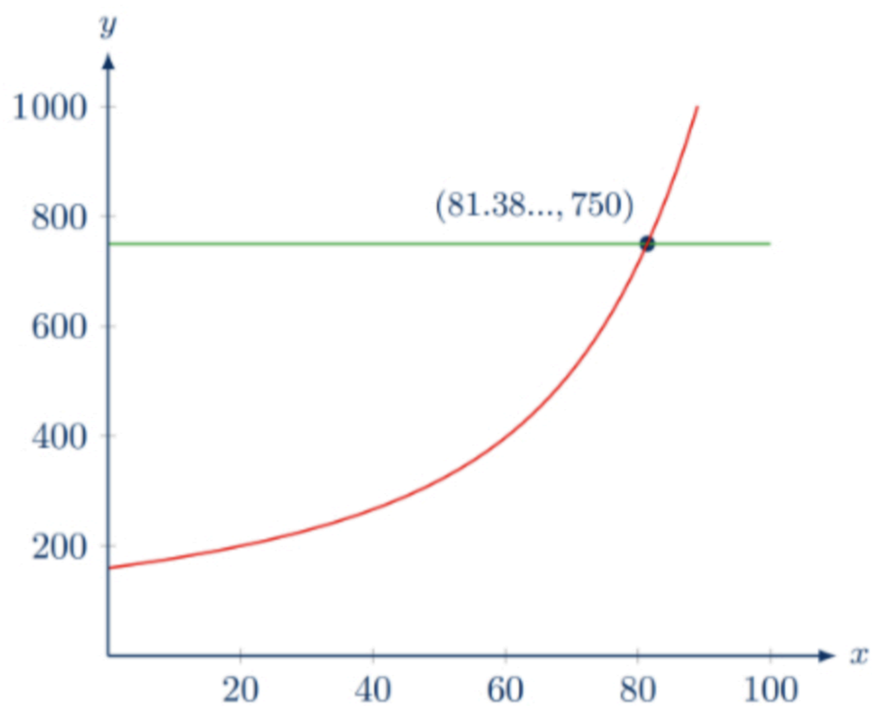
$$S_{10} \geq 750$$

$$\frac{160\left(1-\left(\frac{k}{100}\right)^{10}\right)}{1-\frac{k}{100}} \geq 750$$ **(M1)**

To solve this, we can sketch both sides of the inequality and find the point of intersection.

(M1)

The diagram below shows the graph of $y = S_{10}$ in **red** and the graph of $y = 750$ in **green**.



Hence the smallest value of k such that the sum of the first ten terms is at least 750 metres is 81.4% (3 s.f.)

A1

Question 6

CALCULATOR

Medium

[Maximum mark: 7]

Jack, a volleyball player, serves the ball with a trajectory modelled by the function

$$h_1(x) = -0.12x^2 + x + 0.725$$

where h is the height of the ball above the ground, in metres, and x is the horizontal distance from the serving point, in metres.

(a) Find the height of the ball when Jack makes his serve (i.e. the height above the ground at the point where he connects with the ball). [2]

Unfortunately, the serve is short and misses. For his next attempt, Jack will serve the ball c metres higher at the serving point, so that the ball has the same trajectory (only the vertical height changes, increasing by c metres).

(b) Write down a second function, $h_2(x)$, modelling the new path of the ball in terms of x and c. [1]

(c) (i) Calculate the value of c, if the horizontal distance is 10 metres from the serving point to the point where the ball would hit the ground.

(ii) Determine the vertical height of the ball from which Jack makes his successful serve. [4]

(a) Letting $x = 0$ and solving for $h_1(0)$, we get

$$h_1(x) = -0.12x^2 + x + 0.725$$

$$h_1(0) = -0.12(0)^2 + (0) + 0.725 \quad \textbf{(M1)}$$

$$= 0.725$$

Therefore the ball was served at a height of $\boxed{0.725 \text{ metres}}$

A1

(b) As c is a vertical height, we get

$$\boxed{h_2(x) = -0.12x^2 + x + 0.725 + c} \quad \textbf{A1}$$

(c) (i) Given that the distance is 10 metres, we have $h_2(10) = 0$, thus

$$h_2(10) = 0 \quad \textbf{(M1)}$$

$$-0.12(10)^2 + 10 + 0.725 + c = 0 \quad \textbf{(M1)}$$

$$\boxed{c = 1.275} \quad \textbf{A1}$$

(ii) Height of ball at service point $= 0.725 + 1.275$

$$\boxed{\text{height} = 2 \text{ metres}} \quad \textbf{A1}$$

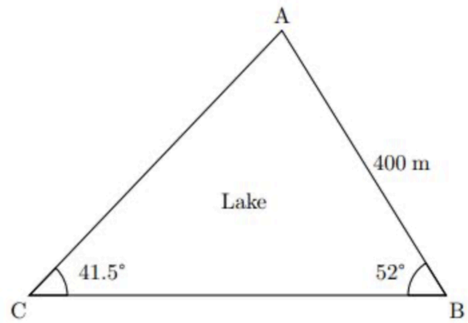
Question 7

CALCULATOR

Medium

[Maximum mark: 8]

A walking trail surrounding a man-made lake in a park is in the shape of a triangle, ABC, as shown in the following diagram. The side length AB is 400 m. Angle $A\hat{B}C$ is $52°$ and angle $A\hat{C}B$ is $41.5°$.



(a) Calculate the side length BC. [5]

(b) Hence, calculate the area of the man-made lake surrounded by the walking trail, rounding your answer **to the nearest square metre**. [3]

(a) The angle sum of a triangle is $180°$. Hence we can work out $\mathrm{B\hat{A}C}$.

$$\mathrm{B\hat{A}C} = 180 - \mathrm{A\hat{C}B} - \mathrm{A\hat{B}C}$$

$$= 180 - 41.5 - 52 \qquad \textbf{(M1)}$$

$$\mathrm{B\hat{A}C} = 86.5° \qquad \textbf{(A1)}$$

Therefore using the sine rule, we get

$$\frac{\mathrm{BC}}{\sin(86.5°)} = \frac{400}{\sin(41.5°)} \qquad \textbf{(M1)}$$

$$\mathrm{BC} = \frac{400 \times \sin(86.5°)}{\sin(41.5°)} \qquad \textbf{(M1)}$$

$$= 602.5...$$

$$\boxed{\mathrm{BC} = 603\,\mathrm{m}\,(3\,\mathrm{s.f.})} \qquad \textbf{A1}$$

(b) From part (a), we have the length BC, and we were given length AB and angle $\mathrm{A\hat{B}C}$ initially. We have an angle and the two adjacent sides, hence we can use the area of a triangle formula. We have

$$A = \frac{1}{2}(\mathrm{AB})(\mathrm{BC})\sin(\mathrm{A\hat{B}C})$$

$$= \frac{1}{2}(400)(602.5...)\sin(52°) \qquad \textbf{(M1)}$$

$$= 94\,961.32... \qquad \textbf{(A1)}$$

$$\boxed{= 94\,961\ \mathrm{m}^2\ \text{(nearest square metre)}} \qquad \textbf{A1}$$

Question 8

CALCULATOR | Medium

[Maximum mark: 7]

Give all answers for this question to 2 decimal places, unless otherwise stated.

On 1 January 2024, Emily invests $600 000 in a savings account which pays a nominal annual interest rate of 4.5%, compounded annually.

(a) Determine the amount of money that will be in the account after 12 years. [3]

After these 12 years, Emily is planning to retire and place the money she has saved into an annuity fund which pays a nominal annual interest rate of 3.5%, compounded monthly.

Emily wants to withdraw money from this account at the end of each month.

(b) (i) Calculate the amount Emily can withdraw at the end of each month if she wants the money to last for 18 years after her retirement.

(ii) Find how many **complete months**, counted from 1 January 2036, it will take for the balance of the fund to fall below $500 000. [4]

(a) In 12 years we have

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
12	4.5	−600 000	0	1 017 528.859...	1	1	END

(M1)(A1)

Hence, using the finance solver on the GDC (or the compound interest formula), we obtain

$$\text{FV} = 600\,000\left(1+\frac{4.5}{100(1)}\right)^{12(1)}$$

$= \$1\,017\,528.86\ (2\,\text{d.p.})$ [by using G.D.C.] **A1**

(b) (i) In 18 years we have 216 months. Hence, using the finance solver on the G.D.C., we have

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
216	3.5	1 017 528.86	−6 356.11	0	12	12	END

(A1)

Therefore, Emily can withdraw $6 356.11 each month.

A1

(ii) This time we need to calculate N, given that FV $= -500\,000$, thus, we get

N	I%	PV	PMT	FV	P/Y	C/Y	PMT
126.5	3.5	1 017 528.86	−6 356.11	−500 000	12	12	END

(A1)

Therefore, it takes more than 126.5 months to fall below $500 000, so in total 127 months.

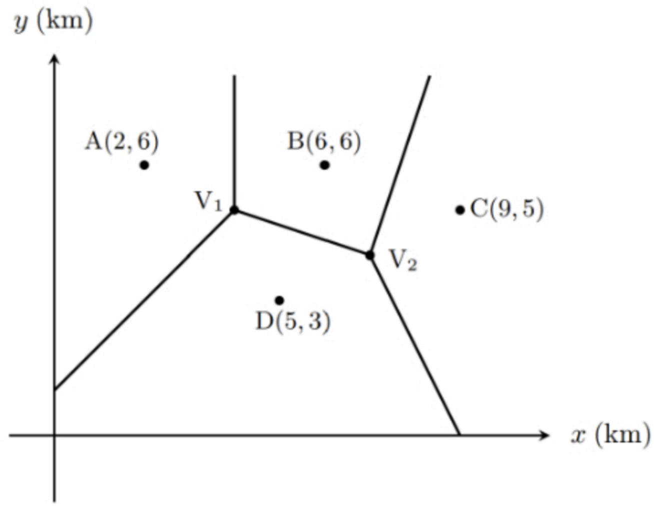
A1

Question 9

CALCULATOR Medium

[Maximum mark: 9]

Consider the Voronoi diagram below for a town centre that contains four coffee shops A, B, C, and D.



The equation of the perpendicular bisector between sites B and C is $y = 3x - 17$. The coordinates of the midpoint between C and D is (7,4).

(a) Determine the equation of the perpendicular bisector between C and D. [3]

(b) Hence, determine the coordinates of the intersection point V_2. [3]

The perpendicular bisectors of AB and AD intersect at the point $V_1(4,5)$, which is 2.236 km from location D, correct to 4 significant figures. A new coffee shop will be built at either V_1 or V_2.

(c) Given that the new shop is to be as far away as possible from an existing coffee shop, determine which of the locations the new coffee shop should be built at. [3]

(a) First we find the gradient of CD. Using the gradient formula, we get

$$\begin{aligned} m_{\text{CD}} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3-5}{5-9} \qquad \textbf{(M1)} \\ &= \frac{1}{2} \end{aligned}$$

Hence the gradient of the perpendicular bisector to CD is -2.

Substituting the coordinates of the given midpoint (7,4) and the slope -2 into the equation of a line, we get

$$4 = -2(7) + c \qquad \textbf{(M1)}$$

$$c = 18$$

Hence, the equation of the perpendicular bisector CD is

$$\boxed{y = -2x + 18} \qquad \textbf{A1}$$

(b) The coordinates of V_2 can be found by finding the coordinates where perpendicular bisector of BC and the perpendicular bisector of CD intersect. Hence, letting these two perpendicular bisectors be equal to each other, we get

$$3x - 17 = -2x + 18 \qquad \textbf{(M1)}$$

$$x = 7 \qquad \textbf{(A1)}$$

Hence $y = 4$ and therefore the coordinates of V_2 are $\boxed{(7, 4)}$.

A1

(c) The new coffee shop must be at V_1 or V_2, as these locations are the furthest distance away from each of the sites. We need to determine which of these points is the furthest from its closest site.

Given V_1 and its distance to D

$$V_1D = 2.236 \text{ km (4 s.f.)}$$

Consider V_2 and its distance to D

$$V_2D = \sqrt{(7-5)^2 + (4-3)^2} \qquad \textbf{(M1)}$$

$$= 2.2360... \text{ km} \qquad \textbf{(A1)}$$

The intersections V_1 and V_2 are at the same distance from any of its nearby coffee shops, and hence the optimal location for the new coffee shop is at either

R1

$$\boxed{V_1(4,5) \text{ or } V_2(7,4)}$$

Question 10

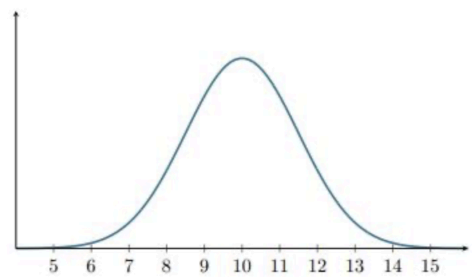
CALCULATOR

Medium

[Maximum mark: 6]

The number of hours that a fully charged laptop battery lasts is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours.

(a) On the following diagram, shade the region representing the probability that after a full charge, the battery lasts less than 8 hours. [1]



The value a hours is exactly two standard deviations below the mean.

(b) (i) Write down the value of a.

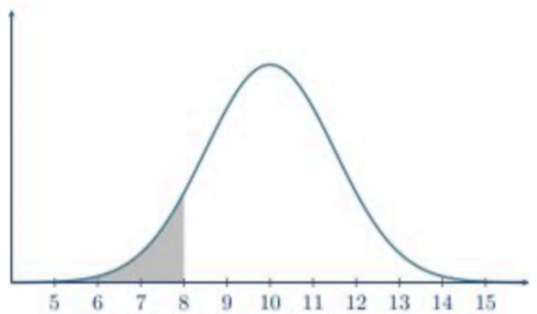
(ii) Find the probability that a full charge lasts between a hours and 8 hours. [3]

The manufacturer states that 25% of batteries will last at least x hours when fully charged.

(c) Find the value of x. [2]

revisionvillage

(a) We have



A1

(b) (i) As the mean is 10 hours and the standard deviation is 1.5 hours, the number of hours corresponding to two standard deviations below the mean is 7 hours.

A1

(ii) Let H be the number of hours the battery lasts. We have $H \sim \mathrm{N}(10, 1.5^2)$. Hence we find

$$P(7 < H < 8) = \text{normalCdf}(7, 8, 10, 1.5) \quad \textbf{(M1)}$$

$$= 0.06846...$$

$$P(7 < H < 8) = 0.0685 \text{ (3 s.f.)} \quad \text{[using G.D.C.]} \quad \textbf{A1}$$

(c) This is the minimum number of hours the longest lasting 25% of batteries last. Therefore 75% of batteries last **less** than this. Using G.D.C., we get

$$P(H < x) = 0.75$$

$$x = \text{InvNorm}(0.75, 10, 1.5) \quad \textbf{(M1)}$$

$$= 11.01...$$

$$x = 11.0 \text{ hours (3 s.f.)} \quad \textbf{A1}$$

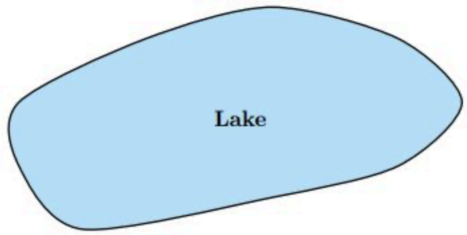
Question 11

CALCULATOR

Medium

[Maximum mark: 5]

A town's water distribution system supplies water from a lake. The operating cost is $3.25 per cubic metre of water. The following table shows an estimate of the areas of horizontal cross-sections at 50-metre intervals, of the water in the lake, where the depth is measured in metres below the ground level.



Depth (m)	0	50	100	150	200	250	300
Area (m^2)	623	592	560	568	537	519	494

(a) Use the trapezoidal rule to find an estimate of the volume of water in the lake. [3]

(b) Hence or otherwise, estimate the operating cost of extracting all the water. [2]

(a) Let x be the depth below the ground level. Thus, the volume of water is the integral from 0 to 300 of the area of the horizontal cross-sections multiplied by the differential $\mathrm{d}x$.

According to the trapezoidal rule, $h = \frac{b-a}{n}$. In this case we have a minimum depth of 0 metres, a maximum depth of 300 metres, and 6 intervals. Therefore

$$\begin{aligned} h &= \frac{300-0}{6} \\ &= 50 \end{aligned}$$

Using the trapezoidal rule we obtain

$$\begin{aligned} V &= \int_0^{300} A(x)\,\mathrm{d}x \\ &\approx \frac{h}{2}\big(623 + 494 + 2(592 + 560 + 568 + 537 + 519)\big) \\ &\approx \frac{50}{2}\big(623 + 494 + 2(592 + 560 + 568 + 537 + 519)\big) && \textbf{(A1)(A1)} \\ &\approx 166\,725\ \mathrm{m}^3 \\ &= 167\,000\ \mathrm{m}^3\ (3\text{ s.f.}) && \textbf{A1} \end{aligned}$$

(b) The operating cost of extracting all the water is

$$(166\,725)(3.25) = \$\,541\,856.25 \qquad \textbf{(M1)}$$

$$= \$\,542\,000\ (3\text{ s.f.}) \qquad \textbf{A1}$$

Question 12

CALCULATOR

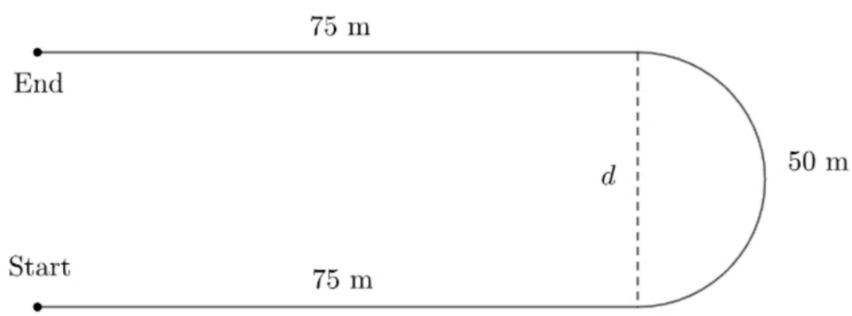
Hard ●●●●

[Maximum mark: 8]

Samira is designing a running track for the 200 metre race of her school sports day.

The 200 metre track is formed by two straight lines of 75 m each and a semi-circular part of 50 m.

Here is her initial design.



d is the diameter of the semi-circular part of the track.

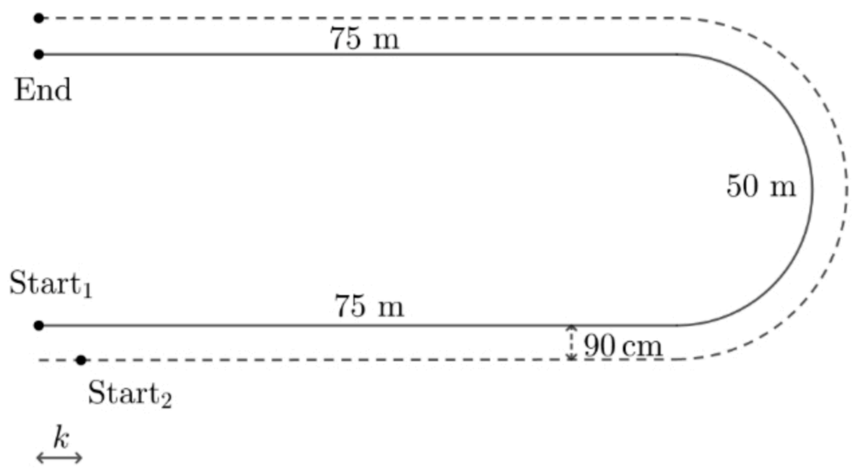
(a) Find the value of d. [2]

Samira wants to add a second lane to the track.

The width of each lane is 90 centimetres.

The runner in lane 1 starts at Start_1 and follows the solid line around the track, while the runner in lane 2 starts at Start_2 and follows the dashed line around the track.

This is shown below.



In order to make sure both lanes are exactly 200 metres in length, Samira must move Start_2 forward k metres.

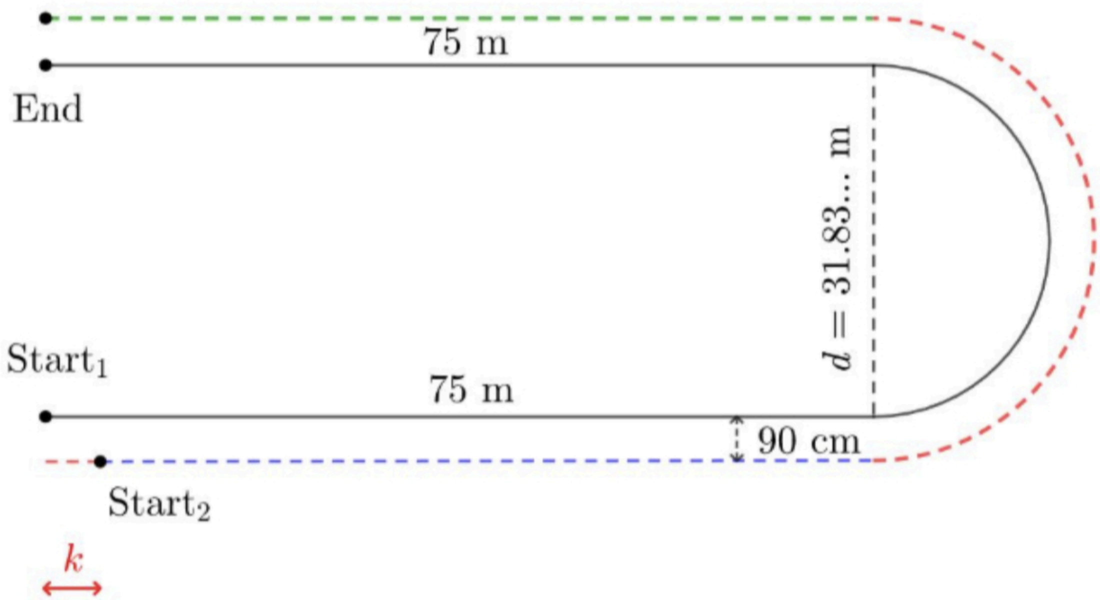
(b) Find the value of k. [6]

(a) We can use the formula for the circumference of a circle.

We know the perimeter of the semi-circle is 50 metres, therefore the circle has a circumference of 100 metres. The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius. We have to find d, the diameter, which is twice the radius. We have

$$C = 2\pi r$$
$$= \pi \times d$$
$$100 = \pi \times d$$
$$d = \frac{100}{\pi} \quad \textbf{(M1)}$$
$$= 31.83...$$
$$d = \boxed{31.8 \text{ metres}} \quad \textbf{A1}$$

(b) The path followed by the runner in lane 2 is shown below, split into 3 different parts.



The green dotted line is the finishing straight, and this is 75 m.

The blue dotted line is the starting straight, and this is $(75 - k)$ m.

(A1)

The red dotted line is half the circumference of a circle.

The sum of these three parts must be exactly 200 m, so that both runners run the same distance.

We know from part (a) that $d = 31.83...$ m, and we have been told that each lane is 90 cm wide, which is 0.9 m.

So we are looking for half the circumference of a circle that has a diameter of $d = 0.9 + 0.9 + 31.83...$ m.

(A1)

Using the formula for the circumference of a circle, we have

$$\begin{aligned} C &= 2\pi r \\ &= \pi \times d \\ &= \pi \times (0.9 + 0.9 + 31.83...) && \textbf{(M1)} \\ \frac{C}{2} &= \frac{\pi \times (0.9 + 0.9 + 31.83...)}{2} \\ &= 52.82... && \textbf{(A1)} \end{aligned}$$

We can now form an equation using the three separate parts from the diagram.

$$\text{green dotted line} + \text{blue dotted line} + \text{red dotted line} = 200$$

Substituting in our values from above, we have

$$\begin{aligned} 75 + (75 - k) + 52.82... &= 200 && \textbf{(M1)} \\ k &= 2.827... && \text{[by using G.D.C.]} \\ \boxed{k = 2.83\,\text{m}\,(3\text{ s.f.})} && && \textbf{A1} \end{aligned}$$