IB Mathematics AI SL - Prediction Exams May 2025 - Paper 2

Paper 2 | 5 questions | 90 mins | 80 marks

Question 1

CALCULATOR | Medium

[Maximum mark: 17]

Professor Smith teaches a calculus course to a group of students in a school. He has noticed that the length of the lectures he gives follows a normal distribution with mean m minutes.

There is a 20% chance that any lecture lasts at most 48.6 minutes.

(a) Sketch a diagram to represent this probability. [2]

There is also a 20% chance that any lecture lasts longer than 55.4 minutes.

(b) Show that $m = 52$. [2]

The standard deviation of the number of minutes a lecture lasts is 4 minutes.

(c) Find the probability that a lecture

(i) lasts between 45 and 51 minutes;

(ii) lasts more than 55 minutes. [4]

There is a 70% chance that a lecture lasts less than x minutes.

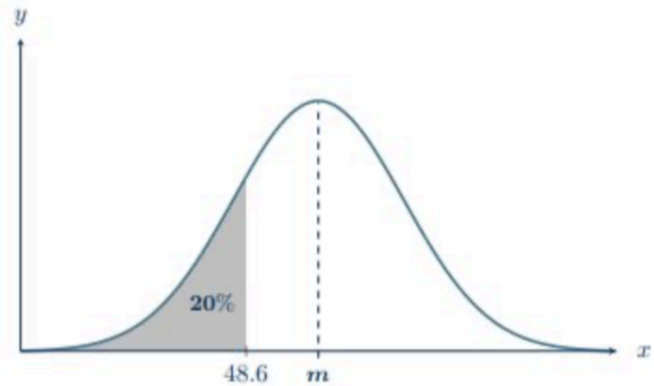
(d) Find the value of x. [2]

Professor Smith delivers one lecture per day to this group of students, Monday to Friday. It can be assumed that the length of each lecture is independent.

(e) Find the probability that the lecture on Monday lasts between 45 and 51 minutes but the lecture on Tuesday lasts more than 55 minutes. [3]

(f) Calculate the probability that during a given week, at least 3 lectures last more than 55 minutes. [4]

(a) We have



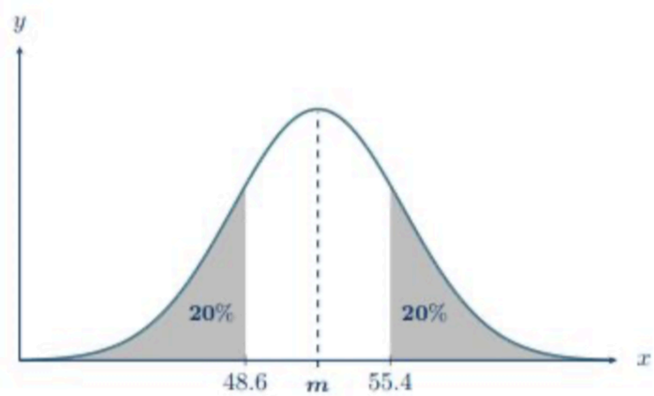
Normal distribution curve, 48.6 labelled to left of mean

A1

Shading to left of mean

A1

(b) If we consider the symmetry of the normal distribution curve, we can see that the values 48.6 and 55.4 must be equidistant from the mean:



Hence, the mean of the distribution, m, will be equal to the mean of 48.6 and 55.4.

(R1)

We have

$$m = \frac{48.6 + 55.4}{2} \quad \textbf{M1}$$

$$m = 52 \quad \textbf{AG}$$

(c) (i) Let the continuous random variable X represent the length of a randomly selected lecture.

We have $X \sim \mathrm{N}\left(52, 4^2\right)$. Hence we find

$$\begin{aligned} \mathrm{P}(45 < X < 51) &= \text{normalCdf}(45, 51, 52, 4) && \textbf{(M1)} \\ &= 0.3612... \\ &= 0.361\ (3\text{ s.f.}) && \text{[using G.D.C.]} \quad \textbf{A1} \end{aligned}$$

(ii) Again using the G.D.C., we have

$$\begin{aligned} \mathrm{P}(X > 55) &= \text{normalCdf}(55, \infty, 52, 4) && \textbf{(M1)} \\ &= 0.2266... \\ &= 0.227\ (3\text{ s.f.}) && \textbf{A1} \end{aligned}$$

(d) Using the G.D.C., we get

$$\begin{aligned} \mathrm{P}(X < x) &= 0.7 \\ x &= \text{InvNorm}(0.7, 52, 4) && \textbf{(M1)} \\ &= 54.09... \\ &= 54.1\ (3\text{ s.f.}) && \textbf{A1} \end{aligned}$$

(e) Taking the results in part (c) (i) and (ii), we can use the formula for independent events. We have

$$\begin{aligned} \mathrm{P}(45 < \mathrm{M} < 51 \cap \mathrm{T} > 55) &= \mathrm{P}(45 < \mathrm{M} < 51) \times \mathrm{P}(\mathrm{T} > 55) && \textbf{(M1)} \\ &= 0.3612... \times 0.2266... && \textbf{(M1)} \\ &= 0.08186... \\ &= 0.0819\ (3\text{ s.f.}) && \textbf{A1} \end{aligned}$$

(f) Let L represent the event that a randomly selected lecture during a given week lasts more than 55 minutes. We can say that L follows a binomial distribution, because

- We have a fixed number of trials ($n = 5$ lectures per week)
- Each trial is independent (given information)
- Each trial has two outcomes (a lecture must last either more than 55 minutes, or less than or equal to 55 minutes)
- We have a constant probability (from (c) (ii), $p = 0.2266...$)

The criteria for a binomial distribution are met, therefore $L \sim \mathrm{B}(5, 0.2266...)$

(R1A1)

We require $\mathrm{P}(L \geq 3)$. We have

$$\begin{aligned} \mathrm{P}(L \geq 3) &= \mathrm{binomCdf}(5, 0.2266..., 3, 5) \quad \textbf{(M1)} \\ &= 0.08041... \\ &= 0.0804 \text{ (3 s.f.)} \quad \textbf{A1} \end{aligned}$$

Question 2

CALCULATOR

Medium

[Maximum mark: 16]

A hotel is built so that the highest rooms have an outstanding view of a local tourist attraction. The hotel has 26 floors, and the number of rooms per floor decreases in a regular pattern by a fixed amount of d rooms. There are 96 rooms on the third floor and 84 on the sixth floor.

(a) Write down an equation, in terms of u_1 and d, for the number of rooms on

(i) the third floor.

(ii) the sixth floor. [2]

(b) Hence find the value of u_1 and d. [3]

(c) Calculate the total number of rooms in the hotel. Give your answer to the nearest whole number. [2]

The cost of booking a room increases the higher the floor on which the room is located. The price of booking a room on the first floor is $98 dollars and the price increases by 5% per floor. Thus the price for booking a room on the second floor is $102.90 and $108.05 on the third floor, and so on.

(d) Find the price of booking a room on the eighth floor. Give your answer to two decimal places. [2]

(e) Determine the floor at which the price of booking a room first increases above $200. [3]

(f) Calculate the total revenue for the hotel if three rooms on each of the 26 floors are booked. Give your answer to the nearest dollar. [4]

(a) The nth term of an arithmetic sequence can be found by $u_n = u_1 + (n-1)d$. Hence, we get

(i) $u_1 + 2d = 96$

A1

(ii) $u_1 + 5d = 84$

A1

(b) Using the equations we found in part (a), we get

$$\begin{cases} u_1 + 2d = 96 \\ u_1 + 5d = 84 \end{cases}$$

(M1)

Using G.D.C. to solve, we get $u_1 = 104$ and $d = -4$

A2

(c) Using the formula for the sum of terms of an arithmetic sequence, we get

$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

$$S_{26} = \frac{26}{2}(2(104) + (25)(-4))$$ **(M1)**

$$= 1\,404$$ **A1**

(d) Using the formula for the nth term in a geometric sequence, we get

$$u_n = u_1 r^{n-1}$$

$$u_8 = 98(1.05)^7 \quad \textbf{(M1)}$$

$$= 137.895...$$

$$\boxed{= \$137.90} \quad \textbf{A1}$$

(e) We can form an inequality using the formula for the nth term in a geometric sequence.

$$98(1.05)^{n-1} > 200$$

Using the numerical solver, we can determine the value of n for which both sides are equal, and then interpret our answer, given that n must be a positive integer.

$$98(1.05)^{n-1} = 200 \quad \textbf{(M1)}$$

$$n = 15.62... \quad \textbf{(A1)}$$

When $n = 15$, $98(1.05)^{14} = 194.03$.

When $n = 16$, $98(1.05)^{15} = 203.73$.

Therefore, booking first increases above \$200 on the 16th floor.

A1

(f) The sum of terms in a geometric sequence can be found by $s_n = \dfrac{u_1(1-r^n)}{1-r}$. Hence, the revenue for the hotel of having 1 room booked on each floor is given by

$$s_{26} = \frac{98(1-1.05^{26})}{1-1.05} \quad \textbf{(M1)}$$

$$= 5\,009.118... \quad \textbf{(A1)}$$

Thus, the income for 3 rooms booked on each floor can therefore be calculated by

$$3 \times s_{26} = 3 \times 5\,009.118... \quad \textbf{(M1)}$$

$$= 15\,027.35...$$

$$\boxed{= \$15\,027} \quad \textbf{A1}$$

Question 3

CALCULATOR

Medium

[Maximum mark: 16]

At the start 2001, a team of zoologists introduced a new species of rabbit onto a large island. 1000 rabbits were initially introduced and there were 1728 rabbits at the start of 2004.

The size of the population of the species, N, t years after the start of 2001, can be modelled by the following function

$$N(t) = p \times q^t, \qquad t \geq 0$$

(a) (i) Show that the value of p is 1000.

(ii) Verify that $q = 1.2$. [2]

(b) State the annual growth rate of the population as a percentage. [1]

(c) Use the model to predict the population size at the start of 2011. [2]

In 2012, the team observed that the growth rate of the rabbit species was changing and that the overall population was now in decline. Upon investigation, they found that the rabbits had became the prey of a new type of snake on the island. Zoologists adjusted their model to the following new function, where t is still the number of years after the start of 2001.

$$N(t) = 1380 \times (0.87)^{(t-12)} \qquad t \geq 12$$

(d) Use this model to find the size of the population at the start of 2019. [2]

(e) Find the year in which the population of the species will first drop below 200. [3]

(f) In the period from 2013 onwards, find the number of complete years in which the size of the population of the species was greater than or equal to 800. [3]

At the start of 2020, the remaining population of the species was transferred to another island along with 300 more rabbits. The team of zoologists now expect the population growth will recover and will increase at a rate of 10% per year.

(g) Estimate the size of the population at the start of 2030. [3]

(a) (i) We have been given that $N(0) = 1000$. Therefore

$$N(0) = p \times q^0 \quad \textbf{A1}$$

$$1000 = p$$

We have shown that $p = 1000$.

AG

(ii) Now we know that $N(t) = 1000 \times q^t$ and we have been given that $N(3) = 1728$. We can use the value $q = 1.2$ to verify that $N(3)$ gives the correct result.

$$N(3) = 1000 \times (1.2)^3 \quad \textbf{A1}$$

$$= 1728$$

Using the value $q = 1.2$, we have shown that the model predicts 1728 rabbits at the start of 2004. This verifies that $q = 1.2$.

AG

(b) We have shown that the model is $N(t) = 1000(1.2)^t$.

This tells us that each year, we multiply the existing population by 1.2 to generate the next population value.

Therefore the annual growth rate is 20%

A1

(c) After ten years the population size is

$$N(10) = 1000 \times 1.2^{10} \quad \textbf{(M1)}$$

$$= 6191.7...$$

$$= 6190\ (3\text{ s.f.}) \quad \textbf{A1}$$

(d) We have

$$N(t) = 1380 \times (0.87)^{(t-12)}$$

The start of 2019 will be when $t = 18$. We have

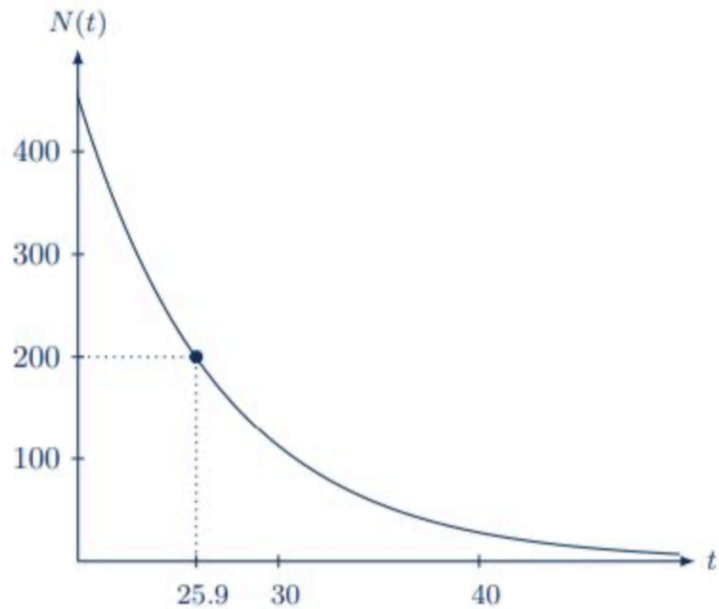
$$N(18) = 1380 \times (0.87)^{(18-12)} \quad \textbf{(M1)}$$

$$= 598.40...$$

$$= 598\ (3\text{ s.f.}) \quad \textbf{A1}$$

(e) The graph of $N(t) = 1380 \times (0.87)^{(t-12)}$ is shown below. We can sketch this on the G.D.C. and use it to determine the exact value of t when $N = 200$, and interpret this value in context.

(M1)



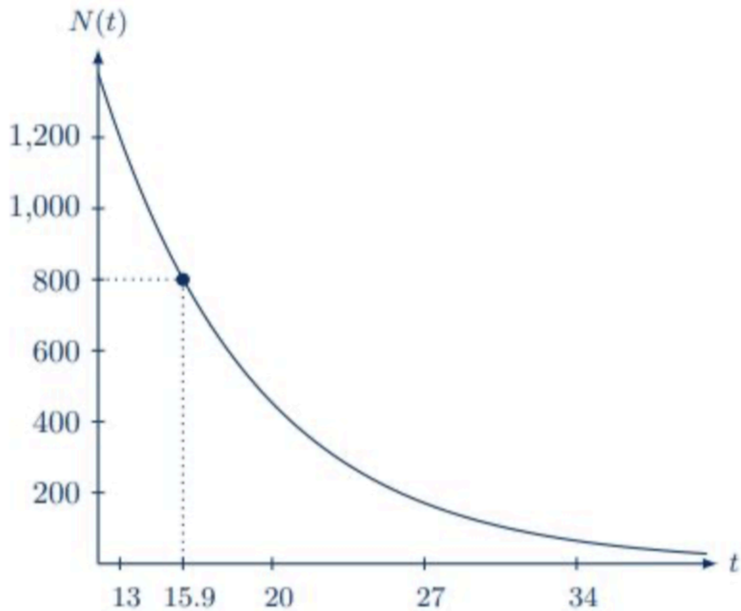
From the graph we have $N(t) < 200$ when $t > 25.9$.

(M1)

The population will first drop below 200 during the 25th year after the start of 2001, which is towards the end of the year 2026.

A1

(f) The graph of $N(t) = 1380 \times (0.87)^{(t-12)}$ is shown below



From the graph we have $N(t) \geq 800$ when $t \leq 15.9$

(M1)

This means we have a population of at least 800 for the full duration of 2013, 2014 and 2015. The value $t = 15.9$ occurs during the year 2016.

(M1)

The population size is greater than or equal to 800 for three complete years (2013 to 2015).

A1

(g) At the start of 2020, $t = 19$. We have

$$N(19) = 1380 \times (0.87)^{(19-12)}$$
$$= 520.61... \quad \textbf{(A1)}$$

We can round this to the nearest whole number, as we require a whole number of rabbits to be transferred to the new island.

The starting population on the new island will be $521 + 300 = 821$

With the predicted growth rate of 10%, the model for population growth on the new island will be $N(t) = 821 \times (1.1)^t$, where t will be time since the start of 2020, therefore we will use $t = 10$.

$$\text{Population at the start of 2030} = 821 \times (1.1)^{10} \quad \textbf{(M1)}$$
$$= 2129.4...$$
$$= 2130\ (3\text{ s.f.}) \quad \textbf{A1}$$

Question 4

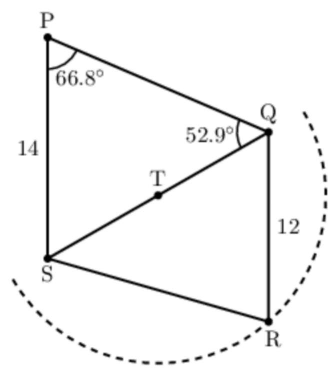
CALCULATOR

Medium

[Maximum mark: 15]

Melissa is playing with a geometric wood puzzle. She can join two triangles, shown in the following figure as $\triangle PQS$ and $\triangle QRS$, along a common side, QS. It is given that $PS = 14\,\text{cm}$, $QR = 12\,\text{cm}$, $Q\hat{P}S = 66.8°$, and $P\hat{Q}S = 52.9°$.

Note that $\triangle PQS$ and $\triangle QRS$ are **not** congruent.



(a) Calculate the length of QS. Give your answer to 2 decimal places. [3]

It is given that T is the midpoint of QS and $RT = 11$ cm.

(b) Hence calculate $T\hat{Q}R$. Give your answer to 2 decimal places. [4]

(c) Calculate the area of triangle QRT. [3]

Melissa cuts a cardboard circle of a radius of 11 cm and puts the two triangles so the point T lies on the centre and point R is on the circumference of the circle.

(d) Show that the point P lies outside the circle. [5]

(a) We can select the sine rule, because the length we are trying to find, QS, is opposite a known angle, and we have another known side, PS, opposite a known angle.

(M1)

Using the sine rule, we have

$$\frac{\mathrm{QS}}{\sin\left(\mathrm{Q\hat{P}S}\right)} = \frac{\mathrm{PS}}{\sin\left(\mathrm{P\hat{Q}S}\right)}$$

$$\mathrm{QS} = \frac{14\sin(66.8^\circ)}{\sin(52.9^\circ)} \quad \textbf{(M1)}$$

$$= 16.133...$$

$$= 16.13\ (2\text{ d.p.}) \quad \textbf{A1}$$

(b) We can select the cosine rule, because we know the lengths of all three sides in triangle QRT.

(M1)

$\mathrm{QR} = 12$, $\mathrm{RT} = 11$, and from part (a) we can determine that $\mathrm{QT} = \frac{16.13}{2}$.

(A1)

Using the cosine rule, we have

$$\cos\left(\mathrm{T\hat{Q}R}\right) = \frac{\mathrm{QT}^2 + \mathrm{QR}^2 - \mathrm{RT}^2}{2(\mathrm{QT})(\mathrm{QR})}$$

$$\mathrm{T\hat{Q}R} = \cos^{-1}\left(\frac{\left(\frac{16.13}{2}\right)^2 + 12^2 - 11^2}{2\left(\frac{16.13}{2}\right)(12)}\right) \quad \textbf{(M1)}$$

$$= 62.943...$$

$$= 62.94^\circ\ (2\text{ d.p.}) \quad \textbf{A1}$$

(c) We can use the area rule for triangles, because we have one known angle, and we know the length of the two adjacent sides.

We have been given $\mathrm{QR} = 12$. From part (b), $\mathrm{T\hat{Q}R} = 62.94^\circ$.

From part (a) we can determine that $\mathrm{QT} = \frac{16.13}{2}$.

(A1)

Using the area formula for a triangle given two sides and the angle between them, we get

$$\begin{aligned} A_{\triangle \mathrm{QRT}} &= \frac{1}{2}(\mathrm{QT})(\mathrm{QR})\sin(\mathrm{T\hat{Q}R}) \\ &= \frac{1}{2}\left(\frac{16.13}{2}\right)(12)\sin(62.94^\circ) \qquad \textbf{(M1)} \\ &= 43.09... \end{aligned}$$

$$\boxed{= 43.1\,\mathrm{cm}^2\ (3\text{ s.f.})}$$

A1

(d) To show that point P is outside the circle, we must show that length $\mathrm{PT} > 11$.

If we consider triangle PST, we have 2 known side lengths. $\mathrm{PS} = 14$ and $\mathrm{ST} = \frac{16.13}{2}$. We have no known angles, however we can use the fact that the sum of the angles in a triangle is 180° to determine the value of $\mathrm{P\hat{S}Q}$. We have

$$\begin{aligned} \mathrm{P\hat{S}Q} &= 180 - 66.8 - 52.9 \\ &= 60.3^\circ \qquad \textbf{(A1)} \end{aligned}$$

We now have enough information to use the cosine rule to calculate length PT. We have

$$\begin{aligned} \mathrm{PT}^2 &= \mathrm{PS}^2 + \mathrm{ST}^2 - 2(\mathrm{PS})(\mathrm{ST})\cos\left(\mathrm{P\hat{S}Q}\right) \\ &= 14^2 + \left(\frac{16.13}{2}\right)^2 - 2(14)\left(\frac{16.13}{2}\right)\cos(60.3^\circ) \qquad \textbf{(M1)} \\ &= 149.15... \qquad \textbf{(A1)} \\ \mathrm{PT} &= \sqrt{149.15...} \\ &= 12.21... \qquad \textbf{A1} \end{aligned}$$

Therefore, the point does not lie inside the circle because the length PT is greater than 11 cm.

A1

Question 5

CALCULATOR Medium

[Maximum mark: 16]

The cost to manufacture an electronic micro-component at a company can be modelled by the cost function

$$C(x) = x^3 - 3x^2 + 4x$$

where x is in hundreds of micro-components, and $C(x)$ is in hundreds of dollars.

(a) Find $C'(x)$. [2]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find the marginal cost when 200 micro-components are produced and interpret its meaning in this context. [3]

The revenue from selling the micro-components is given by the function

$$R(x) = 0.6x^3 + x^2 + 10x - 2$$

where x is in hundreds of micro-components and $R(x)$ is in hundreds of dollars.

(c) Given that Profit $=$ Revenue $-$ Cost, determine a function for the profit, $P(x)$, in hundreds of dollars from selling x hundreds of micro-components. [2]

(d) Find $P'(x)$. [2]

(e) Determine the intervals where $P(x)$ is increasing and decreasing. [4]

The derivative $P'(x)$ gives the marginal profit. The production will reach its optimal level when the marginal profit is zero and $P(x)$ is positive.

(f) Find the optimal production level and the expected profit at this level. [3]

(a) Differentiating the cost function, we get

$$C(x) = x^3 - 3x^2 + 4x$$

$$C'(x) = 3x^2 - 3(2)x + 4$$ **(M1)**

$$= 3x^2 - 6x + 4$$ **A1**

(b) Since x represents hundreds of micro-components, we substitute $x = 2$ into $C'(x)$ and evaluate:

$$C'(2) = 3(2)^2 - 6(2) + 4$$ **(M1)**

$$= \$4 \text{ [hundreds of dollars]}$$

$$= \$400$$ **A1**

Hence the marginal cost when 200 micro-components are produced is \$400.

For every additional 100 micro-components produced, the cost increases by \$400.

A1

(c) Given that Profit = Revenue − Cost, we get

$$P(x) = R(x) - C(x)$$

$$= 0.6x^3 + x^2 + 10x - 2 - (x^3 - 3x^2 + 4x)$$ **(M1)**

$$= -0.4x^3 + 4x^2 + 6x - 2$$ **A1**

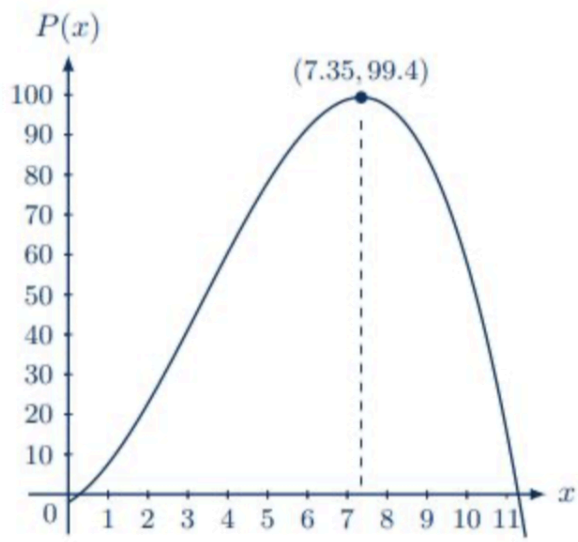
(d) Differentiating $P(x)$, we get

$$P'(x) = -0.4(3)x^2 + 4(2)x + 6(1)$$ **(M1)**

$$= -1.2x^2 + 8x + 6$$ **A1**

(e) The graph below shows the curve of $P(x)$ with the coordinates of the turning point shown. Note that x (number of micro-components) is restricted to be non-negative.

(M2)



We can see that $P(x)$ is increasing to the left of the turning point (the gradient is positive) and decreasing to the right of the turning point (the gradient is negative).

Hence, $P(x)$ is increasing for $0 \leq x < 7.35$ and decreasing for $x > 7.35$.

A2

(f) From part (e) we see that the marginal profit is zero at $x = 7.35$ (i.e. production of 735 micro-components). We can therefore calculate the expected profit at this production level

$$\begin{aligned} P(7.35) &= -0.4(7.35)^3 + 4(7.35)^2 + 6(7.35) - 2 \\ &\approx 99.36 \text{ hundreds of dollars} \\ &\approx \$9\,936 \end{aligned}$$ **(M1)**

Thus, the optimal production level is 735 micro-components and the expected profit at this level is $9 936.

A2