IB Mathematics AI SL - Prediction Exams May 2025 - Paper 2

Paper 2 | 5 questions | 90 mins | 80 marks

Question 1

CALCULATOR

Medium

[Maximum mark: 17]

Professor Smith teaches a calculus course to a group of students in a school. He has noticed that the length of the lectures he gives follows a normal distribution with mean m minutes.

There is a 20% chance that any lecture lasts at most 48.6 minutes.

(a) Sketch a diagram to represent this probability. [2]

There is also a 20% chance that any lecture lasts longer than 55.4 minutes.

(b) Show that $m = 52$. [2]

The standard deviation of the number of minutes a lecture lasts is 4 minutes.

(c) Find the probability that a lecture

(i) lasts between 45 and 51 minutes;

(ii) lasts more than 55 minutes. [4]

There is a 70% chance that a lecture lasts less than x minutes.

(d) Find the value of x. [2]

Professor Smith delivers one lecture per day to this group of students, Monday to Friday. It can be assumed that the length of each lecture is independent.

(e) Find the probability that the lecture on Monday lasts between 45 and 51 minutes but the lecture on Tuesday lasts more than 55 minutes. [3]

(f) Calculate the probability that during a given week, at least 3 lectures last more than 55 minutes. [4]

Question 2

CALCULATOR

Medium

[Maximum mark: 16]

A hotel is built so that the highest rooms have an outstanding view of a local tourist attraction. The hotel has 26 floors, and the number of rooms per floor decreases in a regular pattern by a fixed amount of d rooms. There are 96 rooms on the third floor and 84 on the sixth floor.

(a) Write down an equation, in terms of u_1 and d, for the number of rooms on

(i) the third floor.

(ii) the sixth floor. [2]

(b) Hence find the value of u_1 and d. [3]

(c) Calculate the total number of rooms in the hotel. Give your answer to the nearest whole number. [2]

The cost of booking a room increases the higher the floor on which the room is located. The price of booking a room on the first floor is \$98 dollars and the price increases by 5% per floor. Thus the price for booking a room on the second floor is \$102.90 and \$108.05 on the third floor, and so on.

(d) Find the price of booking a room on the eighth floor. Give your answer to two decimal places. [2]

(e) Determine the floor at which the price of booking a room first increases above \$200. [3]

(f) Calculate the total revenue for the hotel if three rooms on each of the 26 floors are booked. Give your answer to the nearest dollar. [4]

Question 3

CALCULATOR

Medium

[Maximum mark: 16]

At the start 2001, a team of zoologists introduced a new species of rabbit onto a large island. 1000 rabbits were initially introduced and there were 1728 rabbits at the start of 2004.

The size of the population of the species, N, t years after the start of 2001, can be modelled by the following function

$$N(t) = p \times q^t, \qquad t \geq 0$$

(a) (i) Show that the value of p is 1000.

(ii) Verify that $q = 1.2$. [2]

(b) State the annual growth rate of the population as a percentage. [1]

(c) Use the model to predict the population size at the start of 2011. [2]

In 2012, the team observed that the growth rate of the rabbit species was changing and that the overall population was now in decline. Upon investigation, they found that the rabbits had became the prey of a new type of snake on the island. Zoologists adjusted their model to the following new function, where t is still the number of years after the start of 2001.

$$N(t) = 1380 \times (0.87)^{(t-12)} \qquad t \geq 12$$

(d) Use this model to find the size of the population at the start of 2019. [2]

(e) Find the year in which the population of the species will first drop below 200. [3]

(f) In the period from 2013 onwards, find the number of complete years in which the size of the population of the species was greater than or equal to 800. [3]

At the start of 2020, the remaining population of the species was transferred to another island along with 300 more rabbits. The team of zoologists now expect the population growth will recover and will increase at a rate of 10% per year.

(g) Estimate the size of the population at the start of 2030. [3]

Question 4

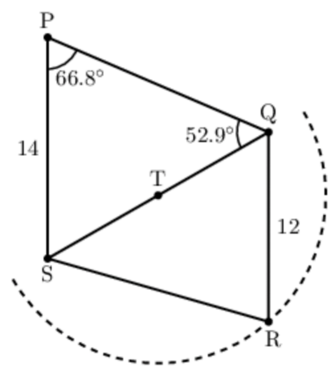
CALCULATOR

Medium

[Maximum mark: 15]

Melissa is playing with a geometric wood puzzle. She can join two triangles, shown in the following figure as $\triangle PQS$ and $\triangle QRS$, along a common side, QS. It is given that $PS = 14\,\text{cm}$, $QR = 12\,\text{cm}$, $Q\hat{P}S = 66.8°$, and $P\hat{Q}S = 52.9°$.

Note that $\triangle PQS$ and $\triangle QRS$ are **not** congruent.



(a) Calculate the length of QS. Give your answer to 2 decimal places. [3]

It is given that T is the midpoint of QS and $RT = 11\,\text{cm}$.

(b) Hence calculate $T\hat{Q}R$. Give your answer to 2 decimal places. [4]

(c) Calculate the area of triangle QRT. [3]

Melissa cuts a cardboard circle of a radius of 11 cm and puts the two triangles so the point T lies on the centre and point R is on the circumference of the circle.

(d) Show that the point P lies outside the circle. [5]

Question 5

CALCULATOR   Medium

[Maximum mark: 16]

The cost to manufacture an electronic micro-component at a company can be modelled by the cost function

$$C(x) = x^3 - 3x^2 + 4x$$

where x is in hundreds of micro-components, and $C(x)$ is in hundreds of dollars.

(a) Find $C'(x)$. [2]

The marginal cost of production is the cost of producing one additional unit. This can be approximated by the gradient of the cost function.

(b) Find the marginal cost when 200 micro-components are produced and interpret its meaning in this context. [3]

The revenue from selling the micro-components is given by the function

$$R(x) = 0.6x^3 + x^2 + 10x - 2$$

where x is in hundreds of micro-components and $R(x)$ is in hundreds of dollars.

(c) Given that Profit $=$ Revenue $-$ Cost, determine a function for the profit, $P(x)$, in hundreds of dollars from selling x hundreds of micro-components. [2]

(d) Find $P'(x)$. [2]

(e) Determine the intervals where $P(x)$ is increasing and decreasing. [4]

The derivative $P'(x)$ gives the marginal profit. The production will reach its optimal level when the marginal profit is zero and $P(x)$ is positive.

(f) Find the optimal production level and the expected profit at this level. [3]