a) Use the binomial theorem to expand $\sqrt{4-9x}$ in ascending powers of x up to and including x^3

b) State the value of x for which the expansion is valid.

$$\sqrt{4-9x} = (4-9x)^{\frac{1}{2}}$$

We need to write the expansion in the correct form:

$$(a+b)^n = \left(a\left(1+\frac{b}{a}\right)\right)^n = a^n\left(1+\frac{b}{a}\right)^n$$

$$(4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$$

Using the formula:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$4^{\frac{1}{2}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} \quad = 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$$

$$= 2\left(1 + \frac{1}{2}\left(-\frac{9x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(-\frac{9x}{4}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}\left(-\frac{9x}{4}\right)^3 + \cdots\right)$$

$$= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^2 + \cdots\right)$$

$$= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^2 + \cdots$$

b) Converges when $\left|-\frac{9x}{4}\right| < 1$

$$|x| < \frac{4}{9}$$