In the expansion of $(1+bx)^n$, the coefficient of the x term is -6 and the coefficient of the x^2 term is 27. Work out the values of b and n

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Using the formula:

$$(1+bx)^n = 1 + n(bx) + \frac{n(n-1)}{2!}(bx)^2 + \cdots$$

$$\begin{aligned} nb &= -6 \\ \frac{n(n-1)}{2}b^2 &= 27 \end{aligned}$$

A good way to solve these equations is to substitute $nb = -6$ from the first equation into the second one.

Re-writing this second equation helps to see this…

$$\begin{aligned} \frac{(n^2-n)b^2}{2} &= 27 \\ \frac{(nb)^2 - nb(b)}{2} &= 27 \\ \frac{(-6)^2 - (-6)(b)}{2} &= 27 \\ \frac{36+6b}{2} &= 27 \\ 36+6b &= 54 \\ 6b &= 18 \\ b &= 3 \\ n &= -2 \end{aligned}$$