a) Write $f(x) = \frac{2+x}{1+x-2x^2}$ as the sum of two partial fractions

b) Find the binomial expansion of $f(x)$ in ascending powers of x up to an including the x^2 terms

c) State the values of x for which the series is valid

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a)

$$\frac{2+x}{1+x-2x^2} \equiv \frac{2+x}{(1-x)(1+2x)}$$

$$\equiv \frac{A}{1-x} + \frac{B}{1+2x}$$

$$\equiv \frac{A(1+2x)}{(1-x)(1+2x)} + \frac{B(1-x)}{(1-x)(1+2x)}$$

$$\frac{2+x}{(1-x)(1+2x)} \equiv \frac{A(1+2x)+B(1-x)}{(1-x)(1+2x)}$$

$$2+x \equiv A(1+2x)+B(1-x)$$

Let $x = 1$ $\quad 3 = A(1+2)$

$$A = 1$$

Let $x = -\frac{1}{2}$ $\quad \frac{3}{2} = B\left(1+\frac{1}{2}\right)$

$$B = 1$$

$$f(x) = \frac{1}{1-x} + \frac{1}{1+2x}$$

b)

$$\frac{1}{1-x} = (1-x)^{-1}$$

Using the formula:

$$(1+bx)^n = 1 + n(bx) + \frac{n(n-1)}{2!}(bx)^2 + \cdots$$

$$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \cdots$$

$$= 1 + x + x^2 + \cdots$$

$$\frac{1}{1+2x} \equiv (1+2x)^{-1}$$

$$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \cdots$$

$$= 1 - 2x + 4x^2 + \cdots$$

$$f(x) = (1 + x + x^2 + \cdots) + (1 - 2x + 4x^2 + \cdots)$$

$$= 2 - x + 5x^2 + \cdots$$

c) $(1-x)^{-1} = 1 + x + x^2 + \cdots$ is valid for $|-x| < 1$

is valid for $|x| < 1$

$(1+2x)^{-1} = 1 - 2x + 4x^2 + \cdots$ is valid for $|2x| < 1$

is valid for $|x| < \frac{1}{2}$

Therefore,

$f(x) = 2 - x + 5x^2 + \cdots$ is valid for $|x| < \frac{1}{2}$